HEAT TRANSFER

Content

JEE (ADVANCED) SYLLABUS

latent heat; Heat conduction in one dimension; Elementary concepts of convection and radiation; Newton's law of cooling

JEE (MAIN) SYLLABUS

Heat transfer-conduction, convection and radiation, Newton's law of cooling.

HEAT TRANSFER

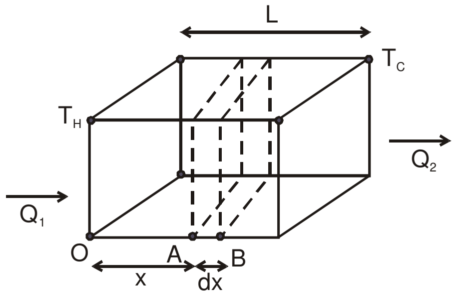
1. INTRODUCTION

Heat is energy in transit which flows due to temperature difference; from a body at higher temperature to a body at lower temperature. This transfer of heat from one body to the other takes place through three routes.

(i) Conduction (ii) Convection (iii) Radiation

2. CONDUCTION

The process of transmission of heat energy in which heat is transferred from one particle of the medium to the other, but each particle of the medium stays at its own position is called conduction, for example if you hold an iron rod with one of its end on a fire for some time, the handle will get hot. The heat is transferred from the fire to the handle by conduction along the length of iron rod. The vibrational amplitude of atoms and electrons of the iron rod at the hot end takes on relatively higher values due to the higher temperature of their environment. These increased vibrational amplitude are transferred along the rod, from atom to atom during collision between adjacent atoms. In this way a region of rising temperature extends itself along the rod to your hand.



Consider a slab of face area A, Lateral thickness L, whose faces have temperatures T_H and T_C ($T_H > T_C$). Now consider two cross sections in the slab at positions A and B separated by a lateral distance of dx. Let temperature of face A be T and that of face B be T + ΔT. Then experiments show that Q, the amount of heat crossing the area A of the slab at position x in time t is given by

$$\boxed{\frac{Q}{t} = -KA\frac{dT}{dx}} \qquad ...(2.1)$$

Here K is a constant depending on the material of the slab and is named thermal conductivity of the material, and the quantity $\left(\frac{dT}{dx}\right)$ is called temperature gradient. The (–) sign in equation (2.1) shows heat flows from high to low temperature (ΔT is a –ve quantity)

3. STEADY STATE

If the temperature of a cross-section at any position x in the above slab remains constant with time (remember, it does vary with position x), the slab is said to be in steady state.

Remember steady-state is distinct from thermal equilibrium for which temperature at any position (x) in the slab must be same.

For a conductor in steady state there is no absorption or emission of heat at any cross-section. (As temperature at each point remains constant with time). The left and right face are maintained at constant temperatures T_H and T_C respectively, and all other faces must be covered with adiabatic walls so that no heat escapes through them and same amount of heat flows through each cross-section in a given Interval of time. Hence $Q_1 = Q = Q_2$. Consequently the temperature gradient is constant throughout the slab.

Hence, $\frac{dT}{dx} = \frac{\Delta T}{L} = \frac{T_f - T_i}{L} = \frac{T_C - T_H}{L}$

and $\frac{Q}{t} = -KA\ \frac{\Delta T}{L}$

$\Rightarrow$ $\frac{Q}{t} = KA\left(\frac{T_H - T_C}{L}\right)$ .... (3.1)

Here Q is the amount of heat flowing through a cross-section of slab at any position in a time interval of t.

Solved Example

Example 1. One face of an aluminium cube of edge 2 metre is maintained at 100ºC and the other end is maintained at 0ºC. All other surfaces are covered by adiabatic walls. Find the amount of heat flowing through the cube in 5 seconds. (Thermal conductivity of aluminium is 209 W/m–ºC)

Solution : Heat will flow from the end at 100ºC to the end at 0ºC. Area of cross-section perpendicular to direction of heat flow,

$A = 4m^2$

then $\frac{Q}{t} = KA\frac{(T_H - T_C)}{L}$

$$Q = \frac{(209W/m^oC)(4m^2)(100^oC - 0^oC)(5\,sec)}{2m} = 209\ KJ$$ **Ans.**

4. THERMAL RESISTANCE TO CONDUCTION

If you are interested in insulating your house from cold weather or for that matter keeping the meal hot in your tiffin-box, you are more interested in poor heat conductors, rather than good conductors. For this reason, the concept of thermal resistance R has been introduced.

For a slab of cross-section A, Lateral thickness L and thermal conductivity K,

$$R = \frac{L}{KA} \quad ... (4.1)$$

In terms of R, the amount of heat flowing though a slab in steady-state (in time t)

$$\frac{Q}{t} = \frac{(T_H - T_C)}{R}$$

If we name $\frac{Q}{t}$ as thermal current i_T

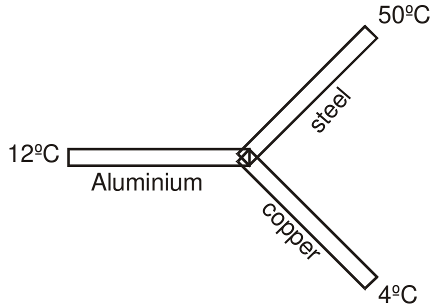
then, $i_T = \frac{T_H - T_C}{R}$ (4.2)

This is mathematically equivalent to OHM's law, with temperature playing the role of electric potential. Hence results derived from OHM's law are also valid for thermal conduction.

More over, for a slab in steady state we have seen earlier that the thermal current i_L remains same at each cross-section. This is analogous to kirchoff's current law in electricity, which can now be very conveniently applied to thermal conduction.

Solved Example

Example 2. Three identical rods of length 1m each, having cross-section area of 1cm^2 each and made of Aluminium, copper and steel respectively are maintained at temperatures of 12ºC, 4ºC and 50ºC respectively at their separate ends.



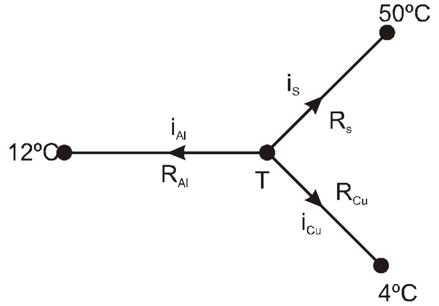
Find the temperature of their common junction. [K_{Cu} = 400 W/m-K, K_{Al} = 200 W/m-K, K_{steel} = 50 W/m-K]

Solution : $R_{Al} = \dfrac{L}{KA} = \dfrac{1}{10^{-4}\times 200} = \dfrac{10^4}{200}$

Similarly $R_{steel} = \dfrac{10^4}{50}$ and $R_{copper} = \dfrac{10^4}{400}$

Let temperature of common junction = T

then from Kirchoff;s current laws,



$i_{Al} + i_{steel} + i_{Cu} = 0$

$\Rightarrow \dfrac{T-12}{R_{Al}} + \dfrac{T-50}{R_{steel}} + \dfrac{T-4}{R_{Cu}} = 0$

$\Rightarrow$ (T – 12) 200 + (T – 50) 50 + (T – 4) 400

$\Rightarrow$ 4(T – 12) + (T – 50) + 8 (T – 4) = 0

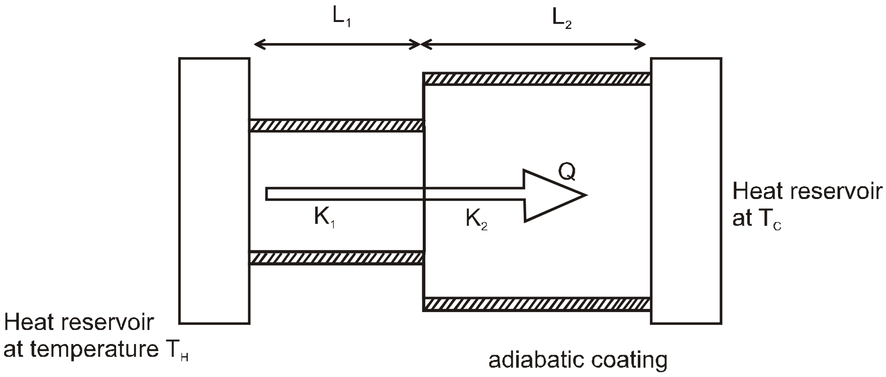
$\Rightarrow$ 13T = 48 + 50 + 32 = 130

$\Rightarrow$ T = 10ºC **Ans.**

5. SLABS IN PARALLEL AND SERIES

5.1 Slabs in series (in steady state)

Consider a composite slab consisting of two materials having different thicknesses L_1 and L_2 different cross-sectional areas A_1 and A_2 and different thermal conductivities K_1 and K_2. The temperature at the outer surface of the slabs are maintained at T_H and T_C, and all lateral surfaces are covered by an adiabatic coating.



Let temperature at the junction be T, since steady state has been achieved thermal current through each slab will be equal. Then thermal current through the first slab.

$i = \frac{Q}{t} = \frac{T_H - T}{R_1}$ or $T_H - T = iR_1$ ... (5.1)

and that through the second slab,

$i = \frac{Q}{t} = \frac{T - T_C}{R_2}$ or $T - T_C = iR_2$ ....(5.2)

adding eqn. 5.1 and eqn 5.2

$T_H - T_L = (R_1 + R_2)\, i$ or $i = \frac{T_H - T_C}{R_1 + R_2}$

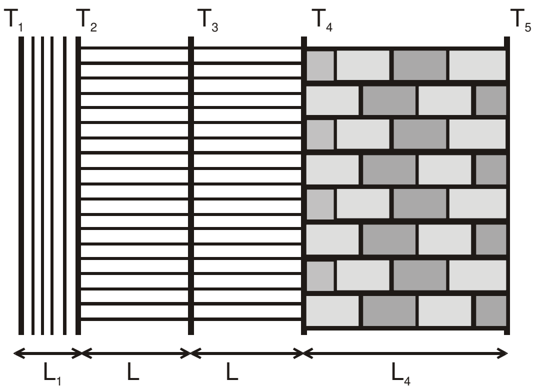
Thus these two slabs are equivalent to a single slab of thermal resistance $R_1 + R_2$.

If more than two slabs are joined in series and are allowed to attain steady state, then equivalent thermal resistance is given by

$R = R_1 + R_2 + R_3 + ...$ ...(5.3)

Solved Example

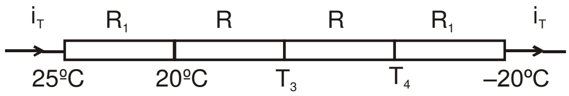
Example 3 The figure shows the cross-section of the outer wall of a house built in a hill-resort to keep the house insulated from the freezing temperature of outside. The wall consists of teak wood of thickness L_1 and brick of thickness ($L_2 = 5L_1$), sandwitching two layers of an unknown material with identical thermal conductivities and thickness. The thermal conductivity of teak wood is K_1 and that of brick is ($K_2 = 5K$). Heat conduction through the wall has reached a steady state with the temperature of three surfaces being known. (T_1 = 25ºC, T_2 = 20ºC and T_5 = –20ºC). Find the interface temperature T_4 and T_3.



Solution : Let interface area be A. then thermal resistance of wood, $R_1 = \frac{L_1}{K_1A}$

and that of brick wall $R_2 = \frac{L_2}{K_2A} = \frac{5L_1}{5K_1A} = R_1$

Let thermal resistance of the each sand witch layer = R. Then the above wall can be visualised as a circuit



thermal current through each wall is same. Hence $\frac{25-20}{R_1} = \frac{20-T_3}{R} = \frac{T_3-T_4}{R} = \frac{T_4+20}{R_1}$

$\Rightarrow$ $25 - 20 = T_4 + 20$ $\Rightarrow$ $T_4 = -15ºC$ **Ans.**

also, $20 - T_3 = T_3 - T_4$ $\Rightarrow$ $T_3 = \frac{20+T_4}{2} = 2.5ºC$ **Ans.**

Example 4 In example 3, $K_1 = 0.125$ W/m–ºC, $K_2 = 5K_1 = 0.625$ W/m–ºC and thermal conductivity of the unknown material is K = 0.25 W/mºC. L_1 = 4cm, $L_2 = 5L_1$ = 20cm. If the house consists of a single room of total wall area of 100 m^2, then find the power of the electric heater being used in the room.

Solution : **<u>Ist method</u>** $R_1 = R_2 = \frac{(4\times10^{-2}m)}{(0.125w/m\text{–}^{\circ}C)(100m^2)} = 32 \times 10^{-4}$ ºC/w

$\because$ $\frac{25-20}{R_1} = \frac{20-T_3}{R}$ $\Rightarrow$ $L = \frac{17.5}{5} \times \frac{K}{K_1} L_1 = 28$ cm

$R = \frac{L}{KA} = 112 \times 10^{-4}$ ºC/W

the equivalent thermal resistance of the entire wall = $R_1 + R_2 + 2R = 288 \times 10^{-4}$ ºC/W

$\therefore$ Net heat current, i.e. amount of heat flowing out of the house per second = $\frac{T_H - T_C}{R}$

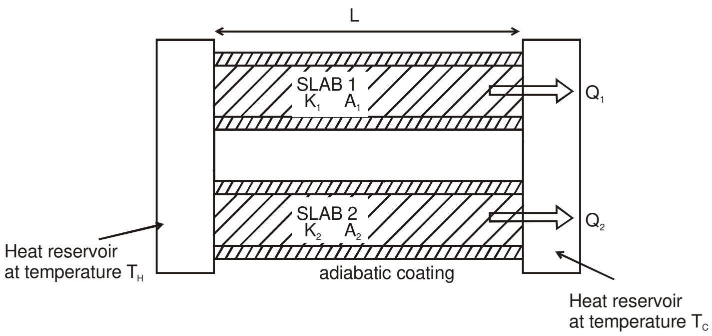
$= \frac{25^{\circ}C-(-20^{\circ}C)}{288\times10^{-4}\,^{\circ}C/w} = \frac{45\times10^4}{288}$ watt = 1.56 Kwatt

Hence the heater must supply 1.56 kW to compensate for the outflow of heat. **Ans.**

<u>IInd method</u>

$i = \frac{T_1-T_2}{R_1} = \frac{25-20}{32\times10^{-4}} = 1.56$ Kwatt

5.2 Slabs in parallel :



Consider two slabs held between the same heat reservoirs, their thermal conductivities K_1 and K_2 and cross-sectional areas A_1 and A_2

then $R_1 = \frac{L}{K_1A_1}$, $R_2 = \frac{L}{K_2A_2}$

thermal current through slab 1

$$i_1 = \frac{T_H - T_C}{R_1}$$

and that through slab 2

$$i_2 = \frac{T_H - T_C}{R_2}$$

Net heat current from the hot to cold reservoir

$$i = i_1 + i_2 = (T_H - T_C)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

Comparing with $i = \frac{T_H - T_C}{R_{eq}}$, we get,

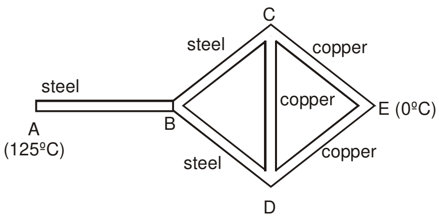
$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

If more than two rods are joined in parallel, the equivalent thermal resistance is given by

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + ... \qquad .... (5.4)$$

Solved Example

Example 5 Three copper rods and three steel rods each of length ℓ = 10 cm and area of cross-section 1 cm^2 are connected as shown



If ends A and E are maintained at temperatures 125ºC and 0ºC respectively, calculate the amount of heat flowing per second from the hot to cold function. [K_{Cu} = 400 W/m-K, K_{steel} = 50 W/m-K]

Solution : $R_{steel} = \frac{L}{KA} = \frac{10^{-1} m}{50(W/m\text{–}^oC) \times 10^{-4} m^2} = \frac{1000}{50}$ ºC/W.

Similarly $R_{Cu} = \frac{1000}{400}$ ºC/W

Junction C and D are identical in every respect and both will have same temperature. Consequently, the rod CD is in thermal equilibrium and no heat will flow through it. Hence it can be neglected in further analysis.

Now rod BC and CE are in series their equivalent resistance is $R_1 = R_S + R_{Cu}$ similarly rods BD and DE are in series with same equivalent resistance $R_1 = R_S + R_{Cu}$ these two are in parallel giving an equivalent resistance of

$$\frac{R_1}{2} = \frac{R_S + R_{Cu}}{2}$$

This resistance is connected in series with rod AB. Hence the net equivalent of the combination is

$$R = R_{steel} + \frac{R_1}{2} = \frac{3R_{steel} + R_{Cu}}{2} = 500\left(\frac{3}{50} + \frac{1}{400}\right) {}^\circ C/W$$

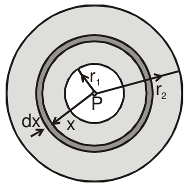
Now $\quad i = \frac{T_H - T_C}{R} = \frac{125\ {}^\circ C}{500\left(\frac{3}{50} + \frac{1}{400}\right) {}^\circ C/W}$ = 4 watt. **Ans.**

Example 6. Two thin concentric shells made of copper with radius r_1 and r_2 ($r_2 > r_1$) have a material of thermal conductivity K filled between them. The inner and outer spheres are maintained at temperatures T_H and T_C respectively by keeping a heater of power P at the centre of the two spheres. Find the value of P.

Solution : Heat flowing per second through each cross-section of the sphere = P = i.

Thermal resistance of the spherical shell of radius x and thickness dx,

$$dR = \frac{dx}{K.4\pi x^2}$$



$$\Rightarrow \quad R = \int_{r_1}^{r_2} \frac{dx}{4\pi x^2.K} = \frac{1}{4\pi K}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

thermal current $i = P = \dfrac{T_H - T_C}{R} = \dfrac{4\pi K(T_H - T_C)r_1r_2}{(r_2 - r_1)}$. **Ans.**

Example 7. A container of negligible heat capacity contains 1 kg of water. It is connected by a steel rod of length 10 m and area of cross-section 10cm^2 to a large steam chamber which is maintained at 100ºC. If initial temperature of water is 0ºC, find the time after which it becomes 50ºC. (Neglect heat capacity of steel rod and assume no loss of heat to surroundings) (Use table 3.1, take specific heat of water = 4180 J/kg ºC)

Solution : Let temperature of water at time t be T, then thermal current at time t,

$$i = \left(\frac{100 - T}{R}\right)$$

This increases the temperature of water from T to T + dT

$$\Rightarrow \quad i = \frac{dH}{dt} = ms\,\frac{dT}{dt}$$

$$\Rightarrow \quad \frac{100 - T}{R} = ms\,\frac{dT}{dt} \quad \Rightarrow \quad \int_0^{50} \frac{dT}{100 - T} = \int_0^t \frac{dT}{Rms}$$

$$\Rightarrow \quad -\ell n\left(\frac{1}{2}\right) = \frac{t}{Rms}$$

or $\quad t = Rms\ \ell n2 \text{ sec} = \dfrac{L}{KA}\,ms\ \ell n2 \text{ sec}$

$$= \frac{(10m)(1kg)(4180J/kg\text{–}^{\circ}C)}{46(w/m^{\circ}C)\times(10\times10^{-4}m^2)}\ell n2$$

$= \dfrac{418}{46}$ (0.69) × 10^5 = 6.27 × 10^5 sec = 174.16 hours **Ans.**

Can you now see how the following facts can be explained by thermal conduction ?

(a) In winter, iron chairs appear to be colder than the wooden chairs.
(b) Ice is covered in gunny bags to prevent melting.
(c) Woolen clothes are warmer.
(d) We feel warmer in a fur coat.
(e) Two thin blankets are warmer than a single blanket of double the thickness.
(f) Birds often swell their feathers in winter.
(g) A new quilt is warmer than old one.
(h) Kettles are provided with wooden handles.
(i) Eskimo's make double walled ice houses.
(j) Thermos flask is made double walled.

6. CONVECTION *(not in JEE Syllabus)

When heat is transferred from one point to the other through actual movement of heated particles, the process of heat transfer is called convection. In liquids and gases, some heat may be transported through conduction. But most of the transfer of heat in them occurs through the process of convection. Convection occurs through the aid of earth's gravity. Normally the portion of fluid at greater temperature is less dense, while that at lower temperature is denser. Hence hot fluid rises up while colder fluid sink down, accounting for convection. In the absence of gravity convection would not be possible.
Also, the anomalous behaviour of water (its density increases with temperature in the range 0-4ºC) give rise to interesting consequences vis-a-vis the process of convection. One of these interesting consequences is the presence of aquatic life in temperate and polar waters. The other is the rain cycle.

Can you now see how the following facts can be explained by thermal convection ?

(a) Oceans freeze top to down and not bottom to up. (this fact is singularly responsible for presence of aquatic life is temperate and polar waters.)
(b) The temperature in the bottom of deep oceans is invariably 4ºC, whether it is winter or summer.
(c) You cannot illuminate the interior of a lift in free fall or an artificial satellite of earth with a candle.
(d) You can Illuminate your room with a candle.

7. RADIATION :

The process of the transfer of heat from one place to another place without heating the intervening medium is called radiation. The term radiation used here is another word for electromagnetic waves. These waves are formed due to the superposition of electric and magnetic fields perpendicular to each other and carry energy.

Properties of Radiation:

(a) All objects emit radiations simply because their temperature is above absolute zero, and all objects absorb some of the radiation that falls on them from other objects.
(b) Maxwell on the basis of his electromagnetic theory proved that all radiations are electromagnetic waves and their sources are vibrations of charged particles in atoms and molecules.
(c) More radiations are emitted at higher temperature of a body and lesser at lower temperature.
(d) The wavelength corresponding to maximum emission of radiations shifts from longer wavelength to shorter wavelength as the temperature increases. Due to this the colour of a body appears to be changing. Radiations from a body at NTP has predominantly infrared waves.
(e) Thermal radiations travels with the speed of light and move in a straight line.
(f) Radiations are electromagnetic waves and can also travel through vacuum.
(g) Similar to light, thermal radiations can be reflected, refracted, diffracted and polarized.
(h) Radiation from a point source obeys inverse square law (intensity $\alpha \frac{1}{r^2}$).

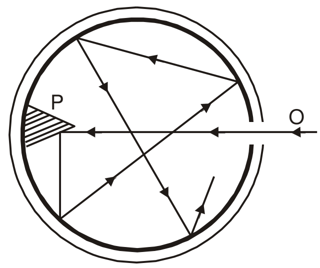
8. PREVOST THEORY OF EXCHANGE :

According to this theory, all bodies radiate thermal radiation at all temperatures. The amount of thermal radiation radiated per unit time depends on the nature of the emitting surface, its area and its temperature. The rate is faster at higher temperatures. Besides, a body also absorbs part of the thermal radiation emitted by the surrounding bodies when this radiation falls on it. If a body radiates more than what it absorbs, its temperature falls. If a body radiates less than what it absorbs, its temperature rises. And if the temperature of a body is equal to the temperature of its surroundings it radiates at the same rate as it absorbs.

9. PERFECTLY BLACK BODY AND BLACK BODY RADIATION

(FERY'S BLACK BODY)

A perfectly black body is one which absorbs all the heat radiations of whatever wavelength, incident on it. It neither reflects nor transmits any of the incident radiation and therefore appears black whatever be the colour of the incident radiation.



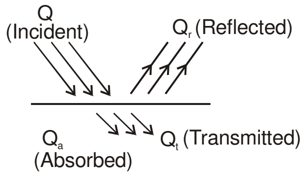
In actual practice, no natural object possesses strictly the properties of a perfectly black body. But the lamp-black and platinum black are good approximation of black body. They absorb about 99 % of the incident radiation. The most simple and commonly used black body was designed by Ferry.

It consists of an enclosure with a small opening which is painted black from inside. The opening acts as a perfect black body. Any radiation that falls on the opening goes inside and has very little chance of escaping the enclosure before getting absorbed through multiple reflections. The cone opposite to the opening ensures that no radiation is reflected back directly.

10. ABSORPTION, REFLECTION AND EMISSION OF RADIATIONS

$Q = Q_r + Q_t + Q_a$

$$1 = \frac{Q_r}{Q} + \frac{Q_t}{Q} + \frac{Q_a}{Q} \quad ; \quad 1 = r + t + a$$



where r = reflecting power, a = absorptive power
and t = transmission power.

(i) r = 0, t = 0, a = 1, perfect black body
(ii) r = 1, t = 0, a = 0, perfect reflector
(iii) r = 0, t = 1, a = 0, perfect transmitter

10.1 Absorptive power :

In particular absorptive power of a body can be defined as the fraction of incident radiation that is absorbed by the body. $a = \dfrac{\text{Energy absorbed}}{\text{Energy incident}}$

As all the radiations incident on a black body are absorbed, a = 1 for a black body.

10.2 Emissive power:

Energy radiated per unit time per unit area along the normal to the area is known as emissive power.

$E = \dfrac{Q}{\Delta A\, \Delta t}$ (Notice that unlike absorptive power, emissive power is not a dimensionless quantity).

10.3 Spectral Emissive power (E_λ) :

Emissive power per unit wavelength range at wavelength λ is known as spectral emissive power, E_λ. If E is the total emissive power and E_λ is spectral emissive power, they are related as follows,

$$E = \int_0^\infty E_\lambda\, d\lambda \quad \text{and} \quad \frac{dE}{d\lambda} = E_\lambda$$

10.4 Emissivity:

$$e = \frac{\text{Emissive power of a body at temperature T}}{\text{Emissive power of a black body at same temperature T}} = \frac{E}{E_0}.$$

11. KIRCHOFF'S LAW:

The ratio of the emissive power to the absorptive power for the radiation of a given wavelength is same for all substances at the same temperature and is equal to the emissive power of a perfectly black body for the same wavelength and temperature.

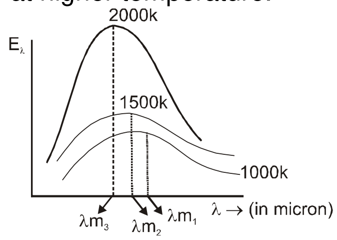
$$\frac{E(\text{body})}{a(\text{body})} = E(\text{black body})$$

Hence we can conclude that good emitters are also good absorbers.

12. NATURE OF THERMAL RADIATIONS : (WIEN'S DISPLACEMENT LAW)

From the energy distribution curve of black body radiation, the following conclusions can be drawn :

(a) The higher the temperature of a body, the higher is the area under the curve i.e. more amount of energy is emitted by the body at higher temperature.



(b) The energy emitted by the body at different temperatures is not uniform. For both long and short wavelengths, the energy emitted is very small.

(c) For a given temperature, there is a particular wavelength (λ_m) for which the energy emitted (E_λ) is maximum.

(d) With an increase in the temperature of the black body, the maxima of the curves shift towards shorter wavelengths.

From the study of energy distribution of black body radiation discussed as above, it was established experimentally that the wavelength (λ_m) corresponding to maximum intensity of emission decreases inversely with increase in the temperature of the black body.

i.e. $\lambda_m \propto \frac{1}{T}$ or $\lambda_m T = b$

This is called Wien's displacement law. Here b = 0.282 cm-K, is the Wien's constant.

Solved Example

Example 8. Solar radiation is found to have an intensity maximum near the wavelength range of 470 nm. Assuming the surface of sun to be perfectly absorbing (a = 1), calculate the temperature of solar surface.

Solution : Since a =1, sun can be assumed to be emitting as a black body from Wien's law for a black body

$$\lambda_m \,.\, T = b$$

$$\Rightarrow \quad T = \frac{b}{\lambda_m} = \frac{0.282(\text{cm}-\text{K})}{(470\times10^{-7}\text{cm})} \simeq 6000\text{ K.}$$ **Ans.**

13. STEFAN-BOLTZMANN'S LAW :

According to this law, the amount of radiation emitted per unit time from an area A of a black body at absolute temperature T is directly proportional to the fourth power of the temperature.

$$u = \sigma A T^4 \qquad \text{..... (13.1)}$$

where σ is Stefan's constant = 5.67×10^{-8} W/m^2 K^4

A body which in not a black body absorbs and hence emits less radiation then

For such a body, $u = e\sigma AT^4$ .....(13.2)

where e = emissivity (which is equal to absorptive power) which lies between 0 to 1

With the surroundings of temperature T_0, net energy radiated by an area A per unit time.

$$\Delta u = u - u_0 = e\sigma A(T^4 - T_0^4) \qquad \text{....(13.3)}$$

Solved Example

Example 9. A body of emissivity (e = 0.75), surface area of 300 cm^2 and temperature 227ºC is kept in a room at temperature 27ºC. Calculate the initial value of net power emitted by the body.

Solution: Using equation. (13.3) $P = e\,\sigma\,A\,(T^4 - T_0^4)$

$= (0.75)\,(5.67 \times 10^{-8}\ W/m^2\text{–}K^4)\,(300 \times 10^{-4}\ m^2) \times \{(500\ K)^4 - (300\ K)^4\} =$ 69.4 Watt. **Ans.**

Example 10. A hot black body emits the energy at the rate of 16 J m^{-2} s^{-1} and its most intense radiation corresponds to 20,000 Å. When the temperature of this body is further increased and its most intense radiation corresponds to 10,000 Å, then find the value of energy radiated in Jm^{-2} s^{-1}.

Solution : Wein's displacement law is : $\lambda_m.T = b$

i.e. $T \propto \dfrac{1}{\lambda_m}$; Here, λ_m becomes half.

$\therefore$ Temperature doubles. Also $e = \sigma T^4$

$\Rightarrow \dfrac{e_1}{e_2} = \left(\dfrac{T_1}{T_2}\right)^4 \Rightarrow e_2 = \left(\dfrac{T_2}{T_1}\right)^4 .e_1 = (2)^4 . 16 = 16.16 = 256$ J m^{-2} s^{-1} **Ans.**

14. NEWTON'S LAW OF COOLING :

For small temperature difference between a body and its surrounding, the rate of cooling of the body is directly proportional to the temperature difference and the surface area exposed.

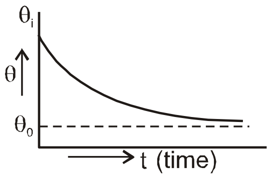
$\dfrac{d\theta}{dt} \propto (\theta - \theta_0)$, where θ and θ_0 are temperature corresponding to object and surroundings.

From above expression, $\dfrac{d\theta}{dt} = -k(\theta - \theta_0)$ .....(14.1)

This expression represents Newton's law of cooling. It can be derived directly from stefan's law, which gives,

$$k = \frac{4e\sigma\theta_0^3}{mc}\,A \qquad .....(14.2)$$

Now $\dfrac{d\theta}{dt} = -k[\theta - \theta_0] \Rightarrow \displaystyle\int_{\theta_i}^{\theta_f} \frac{d\theta}{(\theta - \theta_0)} = \int_0^t -k\,dt$



where θ_i = initial temperature of object and

θ_f = final temperature of object.

$\Rightarrow \ell n\dfrac{(\theta_f - \theta_0)}{(\theta_i - \theta_0)} = -kt \Rightarrow (\theta_f - \theta_0) = (\theta_i - \theta_0)\,e^{-kt}$

$\Rightarrow \theta_f = \theta_0 + (\theta_i - \theta_0)\,e^{-kt}$ .....(14.3)

14.1 Limitations of Newton's Law of Cooling:

(a) The difference in temperature between the body and surroundings must be small

(b) The loss of heat from the body should be by radiation only.

(c) The temperature of surroundings must remain constant during the cooling of the body.

14.2 Approximate method for applying Newton’s law of cooling

Sometime when we need only approximate values from Newton’s law, we can assume a constant rate of cooling, which is equal to the rate of cooling corresponding to the average temperature of the body during the interval.

$$\left\langle \frac{d\theta}{dt} \right\rangle = -\,k(\langle\theta\rangle - \theta_0) \qquad .....(14.4)$$

If θ_i & θ_f be initial and final temperature of the body then,

$$<\theta> = \frac{\theta_i + \theta_f}{2} \qquad .....(14.5)$$

Remember equation (14.5) is only an approximation and equation (14.1) must be used for exact values.

Solved Example

Example 11. A body at temperature 40ºC is kept in a surrounding of constant temperature 20ºC. It is observed that its temperature falls to 35ºC in 10 minutes. Find how much more time will it take for the body to attain a temperature of 30ºC.

Solution : from equation (14.3)

$$\Delta\theta_f = \Delta\theta_i \, e^{-kt}$$

for the interval in which temperature falls from 40 to 35ºC.

$$(35 - 20) = (40 - 20)\, e^{-k.\,10}$$

$$\Rightarrow \quad e^{-10\,k} = \frac{3}{4} \qquad \Rightarrow \qquad k = \frac{\ell n \frac{4}{3}}{10}$$

for the next interval

$$(30 - 20) = (35 - 20)e^{-kt}$$

$$\Rightarrow \quad e^{-kt} = \frac{2}{3} \qquad \Rightarrow \qquad kt = \ell n \frac{3}{2}$$

$$\Rightarrow \quad \frac{\left(\ell n \frac{4}{3}\right)t}{10} = \ell n \frac{3}{2} \qquad \Rightarrow \qquad t = 10 \frac{\left(\ell n \frac{3}{2}\right)}{\left(\ell n \frac{4}{3}\right)} \text{ minute} = 14.096 \text{ min} \qquad \textbf{Ans.}$$

Aliter : (by approximate method) for the interval in which temperature falls from 40 to 35ºC

$$<\theta> = \frac{40 + 35}{2} = 37.5ºC$$

from equation (14.4) $\left\langle \frac{d\theta}{dt} \right\rangle = -k(<\theta> - \theta_0)$

$$\Rightarrow \quad \frac{(35^\circ C - 40^\circ C)}{10(\text{min})} = -K(37.5ºC - 20ºC) \qquad \Rightarrow \qquad K = \frac{1}{35}(\text{min}^{-1})$$

for the interval in which temperature falls from 35ºC to 30ºC

$$<\theta> = \frac{35 + 30}{2} = 32.5ºC$$

from equation (14.4)

$$\frac{(30^\circ C - 35^\circ C)}{t} = -\,(32.5ºC - 20ºC)\,K$$

$$\Rightarrow \quad \text{required time,} \quad t = \frac{5}{12.5} \times 35 \text{min} = 14 \text{ min} \qquad \textbf{Ans.}$$

Exercise-1

Marked Questions can be used as Revision Questions.

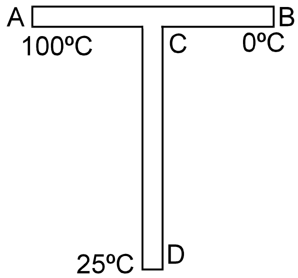
PART - I : SUBJECTIVE QUESTIONS

Section (A) : Thermal conduction in linear conductors at steady state

A-1. A uniform slab of dimension 10 cm × 10 cm × 1 cm is kept between two heat reservoirs at temperatures 10ºC and 90ºC. The larger surface areas touch the reservoirs. The thermal conductivity of the material is 0.80 W/m–ºC. Find the amount of heat flowing through the slab per second.

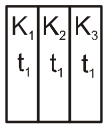
A-2. One end of a steel rod (K = 42 J/m–s–ºC) of length 1.0 m is kept in ice at 0ºC and the other end is kept in boiling water at 100ºC. The area of cross–section of the rod is 0.04cm^2. Assuming no heat loss to the atmosphere, find the mass of the ice melting per second. Latent heat of fusion of ice = 3.36 × 10^5 J/kg.

A-3. A rod CD of thermal resistance 5.0 K/W is joined at the middle of an identical rod AB as shown in figure. The ends A, B and D are maintained at 100ºC, 0ºC and 25ºC respectively. Find the heat current in CD.



A-4. A semicircular rod is joined at its ends to a straight rod of the same material and same cross-sectional area. The straight rod forms a diameter of the other rod. The junctions are maintained at different temperatures. Find the ratio of the heat transferred through a cross-section of the semicircular rod to the heat transferred through a cross-section of the straight rod in a given time.

A-5. Three slabs of same surface area but different conductivities k_1, k_2, k_3 and different thickness t_1, t_2, t_3 are placed in close contact. After steady state this combination behaves as a single slab. Find its effective thermal conductivity.



Section (B) : Thermal conduction in nonlinear conductors at steady state

B-1. A hollow metallic sphere of radius 20 cm surrounds a concentric metallic sphere of radius 5 cm. The space between the two spheres is filled with a nonmetallic material. The inner and outer spheres are maintained at 50°C and 10°C respectively and it is found that 160 π Joule of heat passes radially from the inner sphere to the outer sphere per second. Find the thermal conductivity of the material between the spheres.

B-2. A hollow tube has a length ℓ, inner radius R_1 and outer radius R_2. The material has thermal conductivity K. Find the heat flowing through the walls of the tube per second if the inside of the tube is maintained at temperature T_1 and the outside is maintained at T_2 [Assume $T_2 > T_1$]

Section (C) : Thermal conduction through conductors which have not achieved steady state

C-1. A metal rod of cross-sectional area 1.0 cm^2 is being heated at one end. At one time, the temperature gradient is 5.0°C/cm at cross-section A and is 2.6 °C/cm at cross-section B. Calculate the rate at which the temperature is increasing in the part AB of the rod. The heat capacity of the part AB = 0.40 J/°C, thermal conductivity of the material of the rod = 200 W/m–°C. Neglect any loss of heat to the atmosphere.

Section (D) : Radiation, stefen's law and wein's law

D-1. When q_1 joules of radiation is incident on a body it reflects and transmits total of q_2 joules. Find the emissivity of the body.

D-2. A blackbody of surface area 1 cm^2 is placed inside an enclosure. The enclosure has a constant temperature 27ºC and the blackbody is maintained at 327ºC by heating it electrically. What electric power is needed to maintain the temperature? $\sigma = 6.0 \times 10^{-8}$ $W/m^2 - K^4$.

D-3. Estimate the temperature at which a body may appear blue or red. The values of λ_{mean} for these are 5000 Å and 7500Å respectively. [Given Wein's constant b = 0.3 cm K]

D-4 The temperature of a hot liquid in a container of negligible heat capacity falls at the rate of 3 K/min due to heat emission to the surroundings, just before it begins to solidify. The temperature then remains constant for 30 min, by the time the liquid has all solidified. Find the ratio of specific heat capacity of liquid to specific latent heat of fusion.

D-5. The earth receives at its surface radiation from the sun at the rate of 1400 Wm^{-2}. The distance of the centre of the sun from the surface of the earth is 1.5×10^{11} m and the radius of the sun is 7×10^8 m. Treating the sun as a black body calculate temperature of sun. **[1989; 2M]**

D-6 A solid copper sphere (density ρ and specific heat c) of radius r at an initial temperature 200 K is suspended inside a chamber whose walls are at almost 0 K. Calculte the time required for the temperature of the sphere to drop to 100 K. (Assume sphere as a black body) **[1991; 2M]**

Section (E) : Newton's Law of cooling

E-1. A liquid cools from 70ºC to 60ºC in 5 minutes. Find the time in which it will further cool down to 50 ºC, if its surrounding is held at a constant temperature of 30ºC.

PART - II : ONLY ONE OPTION CORRECT TYPE

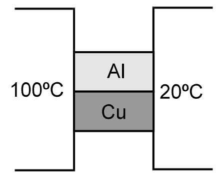
Section (A) : Thermal conduction in linear conductors at steady state

A-1. A wall has two layers A and B, each made of different material. Both the layers have the same thickness. The thermal conductivity for A is twice that of B. Under steady state, the temperature difference across the whole wall is 36°C. Then the temperature difference across the layer A is

(A) 6°C (B) 12°C (C) 18°C (D) 24°C

A-2. Two metal cubes with 3 cm-edges of copper and aluminium are arranged as shown in figure (assume no loss of heat from open surfaces) (K_{CU} = 385 W/m-K, K_{AL} = 209 W/m-K)

(a) The total thermal current from one reservoir to the other is :

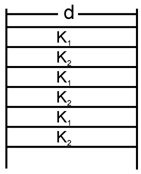


(A) 1.42×10^3 W (B) 2.53×10^3 W (C) 1.53×10^4 W (D) 2.53×10^4 W

(b) The ratio of the thermal current carried by the copper cube to that carried by the aluminium cube is

(A) 1.79 (B) 1.69 (C) 1.54 (D) 1.84

A-3. A wall consists of alternating blocks with length 'd' and coefficient of thermal conductivity k_1 and k_2. The cross sectional area of the blocks are the same. The equivalent coefficient of thermal conductivity of the wall between left and right is :



(A) $K_1 + K_2$ (B) $\frac{(K_1+K_2)}{2}$ (C) $\frac{K_1K_2}{K_1+K_2}$ (D) $\frac{2K_1K_2}{K_1+K_2}$

A-4. A boiler is made of a copper plate 2.4 mm thick with an inside coating of a 0.2 mm thick layer of tin. The surface area exposed to gases at 700°C is 400 cm^2. The amount of steam that could be generated per hour at atmospheric pressure is (K_{cu} = 0.9 and K_{tin} = 0.15 cal/cm/s/°C and L_{steam} = 540 cal/g)
(A) 5000 Kg (B) 1000 kg (C) 4000 kg (D) 200 kg

A-5. A lake surface is exposed to an atmosphere where the temperature is < 0°C. If the thickness of the ice layer formed on the surface grows from 2 cm to 4 cm in 1 hour, The atmospheric temperature, T_a will be (Thermal conductivity of ice K = 4 × 10^{-3} cal/cm/s/°C; density of ice = 0.9 gm/cc. Latent heat of fusion of ice = 80 cal/gm. Neglect the change of density during the state change. Assume that the water below the ice has 0º temperature every where)
(A) – 20 °C (B) 0 °C (C) – 30 °C (D) – 15 °C

Section (B) : Thermal conduction in nonlinear conductors at steady state

B-1. Heat flows radially outward through a spherical shell of outside radius R_2 and inner radius R_1. The temperature of inner surface of shell is θ_1 and that of outer is θ_2. The radial distance from centre of shell where the temperature is just half way between θ_1 and θ_2 is :

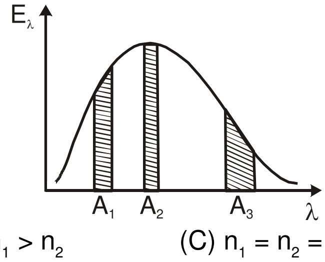
(A) $\frac{R_1+R_2}{2}$ (B) $\frac{R_1R_2}{R_1+R_2}$ (C) $\frac{2R_1R_2}{R_1+R_2}$ (D) $R_1 + \frac{R_2}{2}$

Section (C) : Radiation and stefen's law

C-1. A metallic sphere having radius 0.08 m and mass m = 10kg is heated to a temperature of 227°C and suspended inside a box whose walls are at a temperature of 27°C. The maximum rate at which its temperature will fall is : (Take e = 1, Stefan's constant σ = 5.8 × 10^{-8} Wm^{-2} K^{-4} and specific heat of the metal s = 90 cal/kg/deg J = 4.2 Joules/Calorie)
(A) .055 °C/sec (B) .066 °C/sec (C) .044 °C/sec (D) 0.03 °C/sec

C-2 A solid spherical black body of radius r and uniform mass distribution is in free space. It emits power 'P' and its rate of colling is R then
(A) R P α r^2 (B) R P α r (C) R P α 1/r^2 (D) R P α

C-3 Three separate segments of equal area A_1, A_2 and A_3 are shown in the energy distribution curve of a blackbody radiation. If n_1, n_2 and n_3 are number of photons emitted per unit time corresponding to each area segment respectively then :

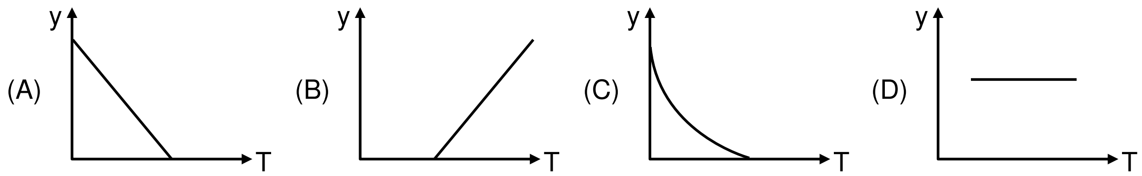


(A) $n_2 > n_1 > n_3$ (B) $n_3 > n_1 > n_2$ (C) $n_1 = n_2 = n_3$ (D) $n_3 > n_2 > n_1$

Section (D) : Newton's Law of cooling

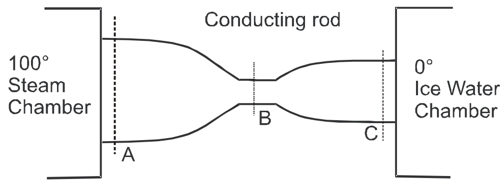
D-1. Which of the law can be understood in terms of Stefan's law
(A) Wien's displacement law (B) Kirchoff's law
(C) Newton's law of cooling (D) Planck's law

D-2. A hot liquid is kept in a big room. According to Newton's law of cooling rate of cooling of liquid (represented as y) is plotted against its temperature T. Which of the following curves may represent the plot ?



PART - III : MATCH THE COLUMN

1. A copper rod (initially at room temperature 20°C) of non-uniform cross section is placed between a steam chamber at 100°C and ice-water chamber at 0°C.



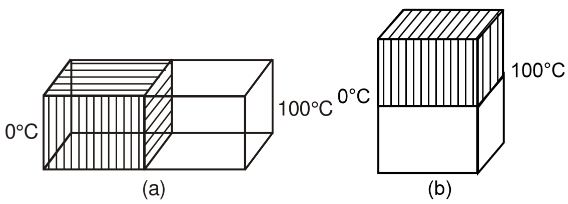
Column-I	Column-II
(A) Initially rate of heat flow $\left(\frac{dQ}{dt}\right)$ will be	(p) maximum at section A
(B) At steady state rate of heat flow $\left(\frac{dQ}{dt}\right)$ will be	(q) maximum at section B
(C) At steady state temperature gradient $\left\lvert\left(\frac{dT}{dx}\right)\right\rvert$ will be	(r) minimum at section A
(D) At steady state rate of change of temperature $\left(\frac{dT}{dt}\right)$ at a certain point will be	(s) minimum at section B
	(t) same for all section

Exercise-2

Marked Questions can be used as Revision Questions.

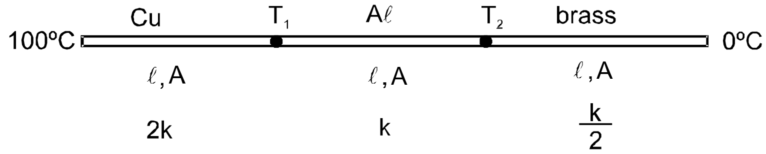
PART - I : ONLY ONE OPTION CORRECT TYPE

1. Two identical square rods of metal are welded end to end as shown in figure (a). Assume that 10 cal of heat flows through the rods in 2 min. Now the rods are welded as shown in figure, (b). The time it would take for 10 cal to flow through the rods now, is



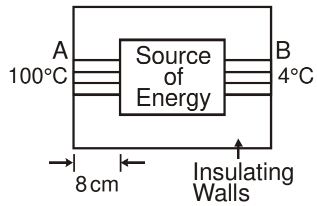
(A) 0.75 min (B) 0.5 min (C) 1.5 min (D) 1 min

2. Three metal rods made of copper, aluminium and brass, each 20 cm long and 4 cm in diameter, are placed end to end with aluminium between the other two. The free ends of copper and brass are maintained at 100 and 0°C respectively. Assume that the thermal conductivity of copper is twice that of aluminium and four times that of brass. The approximately equilibrium temperatures of the copper-aluminium and aluminium-brass junctions are respectively.



(A) 68 °C and 75 °C (B) 75 °C and 68 °C (C) 57 °C and 86 °C (D) 86 °C and 57 °C

3. A closed cubical box is made of a perfectly insulating material walls of thickness 8 cm and the only way for heat to enter or leave the box is through two solid metallic cylindrical plugs, each of cross-sectional area 12 cm^2 and length 8 cm, fixed in the opposite walls of the box. The outer surface A on one plug is maintained at 100°C while the outer surface B of the other plug is maintained at 4°C. The thermal conductivity of the material of each plug is 0.5 cal/°C/cm. A source of energy generating 36 cal/s is enclosed inside the box. Assuming the temperature to be the same at all points on the inner surface, the equilibrium temperature of the inner surface of the box is



(A) 62 °C (B) 46 °C (C) 76 °C (D) 52 °C

4. Two models of a windowpane are made. In one model, two identical glass panes of thickness 3 mm are separated with an air gap of 3 mm. This composite system is fixed in the window of a room. The other model consists of a single glass pane of thickness 6 mm, the temperature difference being the same as for the first model. The ratio of the heat flow for the double pane to that for the single pane is
($K_{glass} = 2.5 \times 10^{-4}$ cal/s.m. °C and $K_{air} = 6.2 \times 10^{-6}$ cal/s.m.°C)
(A) 1/20 (B) 1/70 (C) 31/1312 (D) 31/656

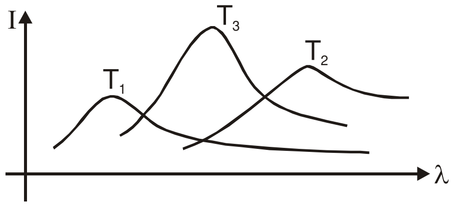
5. Heat is flowing through two cylindrical rods made of same materials whose ends are maintained at similar temperatures. If diameters of the rods are in ratio 1 : 2 and lengths in ratio 2 : 1, then the ratio of thermal current through them in steady state is :
(A) 1 : 8 (B) 1 : 4 (C) 1 : 6 (D) 4 : 1

6. The ends of a metre stick are maintained at 100ºC and 0ºC. One end of a rod is maintained at 25ºC. Where should its other end be touched on the metre stick so that there is no heat current in the rod in steady state?
(A) 25 cm from the hot end (B) 40 cm from the cold end
(C) 25 cm from the cold end (D) 60 cm from the cold end

7. A spherical solid black body of radius 'r' radiates power 'H' and its rate of cooling is 'C'. If density is constant then which of the following is/are true.
(A) $H \propto r$ and $c \propto r^2$ (B) $H \propto r^2$ and $c \propto \frac{1}{r}$ (C) $H \propto r$ and $c \propto \frac{1}{r^2}$ (D) $H \propto r^2$ and $c \propto r^2$

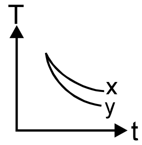
8. The earth is getting energy from the sun whose surface temperature is T_s and radius is R. Let the radius of the earth the r and the distance from the sun be d. Assume the earth and the sun both to behave as perfect black bodies and the earth is in thermal equilibrium at a constant temperature T_e. Therefore, the temperature T_s of the sun is xT_e where x is **[Olympiad 2015 stage-1]**
(A) $\sqrt{\frac{2d}{R}}$ (B) $\sqrt{\frac{2R}{r}}$ (C) $\sqrt{\frac{4d}{r}}$ (D) $\frac{d}{r}$

9. The plots of intensity vs. wavelength for three black bodies at temperatures T_1, T_2 and T_3 respectively are as shown. Their temperatures are such that- **[JEE (Scr) 2000, 3/35]**



(A) $T_1 > T_2 > T_3$ (B) $T_1 > T_3 > T_2$ (C) $T_2 > T_3 > T_1$ (D) $T_3 > T_2 > T_1$

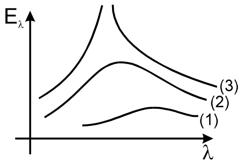
10. The temperature of bodies X and Y vary with time as shown in the figure. If emissivity of bodies X and Y are e_X & e_Y and absorptive powers are A_X and A_Y, (assume other conditions are identical for both) : then: **[JEE (Scr.) 2003, 3/84, –1]**



(A) $e_Y > e_X$, $A_Y > A_X$ (B) $e_Y < e_X$, $A_Y < A_X$ (C) $e_Y > e_X$, $A_Y < A_X$ (D) $e_Y < e_X$, $A_Y > A_X$

11. Three discs of same material A, B, C of radii 2 cm, 4 cm and 6 cm respectively are coated with carbon black. Their wavelengths corresponding to maximum spectral radiancy are 300, 400 and 500 nm respectively then maximum power will be emitted by **[JEE (Scr.) 2004, 3/84, –1]**
(A) A (B) B (C) C (D) same for all

12. Three graphs marked as 1, 2, 3 representing the variation of maximum emissive power and wavelength of radiation of the sun, a welding arc and a tungsten filament. Which of the following combination is correct **[JEE (Scr.) 2005, 3/84, –1]**

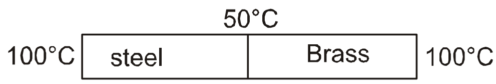


(A) 1- bulb, 2 $\rightarrow$ welding arc, 3 $\rightarrow$ sun (B) 2- bulb, 3 $\rightarrow$ welding arc, 1 $\rightarrow$ sun
(C) 3- bulb, 1 $\rightarrow$ welding arc, 2 $\rightarrow$ sun (D) 2- bulb, 1 $\rightarrow$ welding arc, 3 $\rightarrow$ sun

PART - II : SINGLE AND DOUBLE VALUE INTEGER TYPE

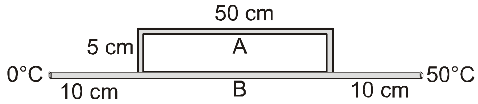
1. Two rods of same dimensions, but made of different materials are joined end to end with their free ends being maintained at 100ºC and 0ºC respectively. The temperature of the junction is 70ºC. Then the temperature of the junction if the rods are interchanged will be equal to T °C Find T :

2. Figure shows a steel rod joined to a brass rod. Each of the rods has length of 31 cm and area of cross-section 0.20 cm^2. The junction is maintained at a constant temperature 50°C and the two ends are maintained at 100°C.



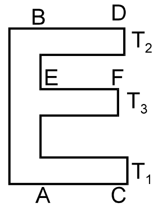
The amount of heat taken out from the cold junction in 10 minutes after the steady state is reached in $n \times 10^2$ J. Find 'n'. The thermal conductivities are K_{steel} = 46 W/m–°C and K_{brass} = 109 W/m–°C.

3. Consider the situation shown in figure. The frame is made of the same material and has a uniform cross-sectional area everywhere. If amount of heat flowing per second through a cross-section of part A is 60 J.



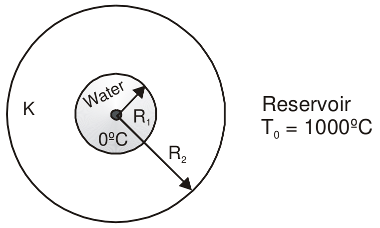
The amount of total heat taken out per second from the end at 50°C is 0.132×10^n J/s. Find 'n'.

4. Four thin identical rods AB, AC, BD and EF made of the same material are joined as shown. The free-ends C, D and F are maintained at temperatures T_1, T_2 and T_3 respectively. Assuming that there is no loss of heat to the surroundings, the temperature at joint E when the steady state is attained is $\frac{1}{K}(2T_1 + 2T_2 + 3T_3)$. Find K (E is mid point of AB)



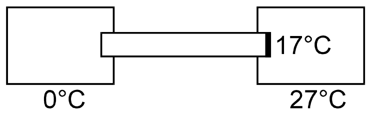
5. One end of copper rod of uniform cross-section and of length 1.45 m is in contact with ice at 0ºC and the other end with water at 100°C. The position of point along its length where a temperature of 200°C should be maintained so that in steady state the mass of ice melting is equal to that of steam produced in the same interval of time is x cm from hotter end of rod. Find x [Assume that the whole system is insulated from surroundings]. (Take L_v = 540 cal/g L_f = 80 cal/g)

6. A hollow spherical conducting shell of inner radius R_1 = 0.25 m and outer radius R_2 = 0.50 m is placed inside a heat reservoir of temperature T_0 = 1000 ºC. The shell is initially filled with water at 0ºC. The thermal conductivity of the material is $k = \frac{10^2}{4\pi}$ W/m-K and its heat capacity is negligible.



The time required to raise the temperature of water to 100ºC is 1100 K $\ell n \frac{10}{9}$ sec. Find K. Take specific heat of water s = 4.2 kJ/kg.°C, density of water d_w = 1000 kg/m^3, $\pi = \frac{22}{7}$

7. A cylindrical rod of length 1 m is fitted between a large ice chamber at 0°C and an evacuated chamber maintained at 27°C as shown in figure. Only small portions of the rod are inside the chambers and the rest is thermally insulated from the surrounding.



The cross-section going into the evacuated chamber is blackened so that it completely absorbs any radiation falling on it. The temperature of the blackened end is 17°C when steady state is reached. Stefan constant $\sigma = 6 \times 10^{-8}$W/m^2–K^4. The thermal conductivity of the material of the rod is 1.2 P (W/m – °C). Find P (29^4 = 707281)

8. A spherical tungsten piece of radius 1.0 cm is suspended in an evacuated chamber maintained at 300 K. The piece is maintained at 1000 K by heating it electrically.The rate at which the electrical energy must be supplied P Watt. Find P. The emissivity of tungsten is 0.30 and the stefan constant σ is 6.0×10^{-8} W/m^2–K^4.

PART - III : ONE OR MORE THAN ONE OPTIONS CORRECT TYPE

1. Assume transmitivity $t \to 0$ for all the cases :
(A) bad absorber is bad emitter
(B) bad absorber is good reflector
(C) bad reflector is good emitter
(D) bad emitter is good absorber

2. A hollow and a solid sphere of same material and having identical outer surface are heated under identical condition to the same temperature at the same time (both have same e, a) :
(A) in the beginning both will emit equal amount of radiation per unit time
(B) in the beginning both will absorb equal amount of radiation per unit time
(C) both spheres will have same rate of fall of temperature (dT/dt)
(D) both spheres will have equal temperatures at any moment

3. Two bodies A and B have thermal emissivities of 0.01 and 0.81 respectively. The surface areas of the two bodies are the same. The two bodies emit total radiant power at the same rate. The wavelength λ_B corresponding to maximum spectral radiancy in the radiation from B is shifted from the wavelength corresponding to maximum spectral radiancy in the radiation from A by 1.00 μm. If the temperature of A is 5802 K, **[JEE 1994, 2]**
(A) the temperature of B is 1934 K
(B) λ_B = 1.5 μm
(C) the temperature of B is 11604 K
(D) the temperature of B is 2901 K

4. The solar constant is the amount of heat energy received per second per unit area of a perfectly black surface placed at a mean distance of the Earth from the Sun, in the absence of Earth's atmosphere, the surface being held perpendicular to the direction of Sun's rays. Its value is 1388 W/m^2. If the solar constant for the earth is 's'. The surface temperature of the sun is TK, D is the diameter of the Sun, R is the mean distance of the Earth from the Sun. The sun subtends a small angle 'θ' at the earth. Then correct options is/are :

(A) $s = \sigma T^4\left(\frac{D}{R}\right)^2$ (B) $s = \frac{\sigma T^4}{4}\left(\frac{D}{R}\right)^2$ (C) $s = \frac{\sigma T^4}{4}\theta^2$ (D) $s = \frac{\sigma T^4}{4}\left(\frac{R}{D}\right)^2$

5. A heated body emits radiation which has maximum intensity at frequency ν_m. If the temperature of the body is doubled:

(A) the maximum intensity radiation will be at frequency $2\nu_m$

(B) the maximum intensity radiation will be at frequency ν_m.

(C) the total emitted power will increase by a factor 16

(D) the total emitted power will increase by a factor 2.

6. Two identical rods made of two different metals A and B with thermal conductivities K_A and K_B respectively are joined end to end. The free end of A is kept at a temperature T_1 while the free end of B is kept at a temperature T_2 (< T_1). Therefore, in the steady state **[Olympiad (Stage-1) 2017]**

(A) the temperature of the junction will be determined only by K_A and K_B

(B) if the lengths of the rods are doubled the rate of heat flow will be halved.

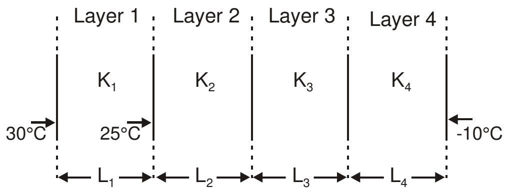
(C) if the temperature at the two free ends are interchanged the junction temperature will change

(D) the composite rod has an equivalent thermal conductivity of $\frac{2K_AK_B}{K_A + K_B}$

PART - IV : COMPREHENSION

Comprehension - 1

Figure shows in cross section a wall consisting of four layers with thermal conductivities K_1 = 0.06 W/mK; K_3 = 0.04 W/mK and K_4 = 0.10 W/mK. The layer thicknesses are L_1 = 1.5 cm ; L_3 = 2.8 cm and L_4 = 3.5 cm. The temperature of interfaces is as shown in figure. Energy transfer through the wall is steady.



1. The temperature of the interface between layers 3 and 4 is :

(A) – 1°C (B) – 3°C (C) 2°C (D) 0°C

2. The temperature of the interface between layers 2 and 3 is :

(A) 11°C (B) 8°C (C) 7.2°C (D) 5.4°C

3. If layer thickness L_2 is 1.4 cm, then its thermal conductivity K_2 will have value (in W/mK) :

(A) 2×10^{-2} (B) 2×10^{-3} (C) 4×10^{-2} (D) 4×10^{-3}

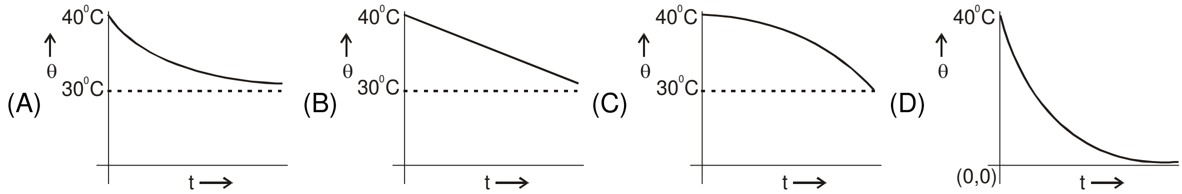
Comprehension-2

A body cools in a surrounding of constant temperature 30ºC. Its heat capacity is 2J/ºC. Initial temperature of the body is 40ºC. Assume Newton's law of cooling is valid. The body cools to 38ºC in 10 minutes.

4.✍ In further 10 minutes it will cool from 38ºC to :

(A) 36ºC (B) 36.4ºC (C) 37ºC (D) 37.5ºC

5.✍ The temperature of the body in ºC denoted by θ the variation of θ versus time t is best denoted as



6.✍ When the body temperature has reached 38ºC, it is heated again so that it reaches to 40ºC in 10 minutes.The total heat required from a heater by the body is:

(A) 3.6J (B) 7J (C) 8 J (D) 4 J

Comprehension-3

A metal ball of mass 2 kg is heated by means of a 40 W heater in a room at 25°C. The temperature of the ball becomes steady at 60°C.

7. Find the rate of loss of heat to the surrounding when the ball is at 60°C.

(A) 40 W (B) 16W (C) 96W (D) 100 W

8. Assuming Newton's law of cooling, calculate the rate of loss of heat to the surrounding when the ball is at 39°C.

(A) 40 W (B) 16W (C) 96W (D) 100 W

9. Assume that the temperature of the ball rises uniformly from 25°C to 39°C in 2 minutes. Find the total loss of heat to the surrounding during this period.

(A) 900 J (B) 940 J (C) 960 J (D) 1000 J

Exercise-3

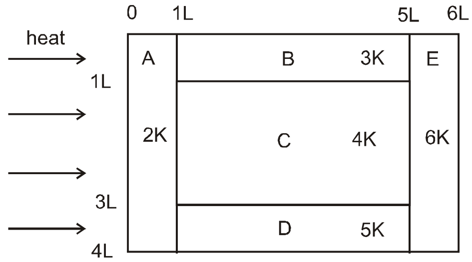
✍ Marked Questions can be used as Revision Questions.
*** Marked Questions may have more than one correct option.**

PART - I : JEE (ADVANCED) / IIT-JEE PROBLEMS (PREVIOUS YEARS)

1. A metal rod AB of length 10x has its one end A in ice at 0ºC and the other end B in water at 100ºC. If a point P on the rod is maintained at 400ºC, then it is found that equal amounts of water and ice evaporate and melt per unit time. The latent heat of evaporation of water is 540 cal/g and latent heat of melting of ice is 80 cal/g. If the point P is at a distance of λx from the ice end A, find the value of λ. [Neglect any heat loss to the surrounding] **[JEE, 2009, 4/160, –1]**

2.✍ Two spherical bodies A (radius 6 cm) and B (radius 18 cm) are at temperature T_1 and T_2 respectively. The maximum intensity in the emission spectrum of A is at 500 nm and in that of B is at 1500 nm. Considering them to be black bodies, what will be the ratio of the rate of total energy radiated by A to that of B ? **[JEE, 2010, 3/163]**

3.* A composite block is made of slabs A, B, C, D and E of different thermal conductivities (given in terms of a constant K) and sizes (given in terms of length, L) as shown in the figure. All slabs are of same width. Heat 'Q' flows only from left to right through the blocks. Then in steady state **[JEE, 2011, 4/160]**

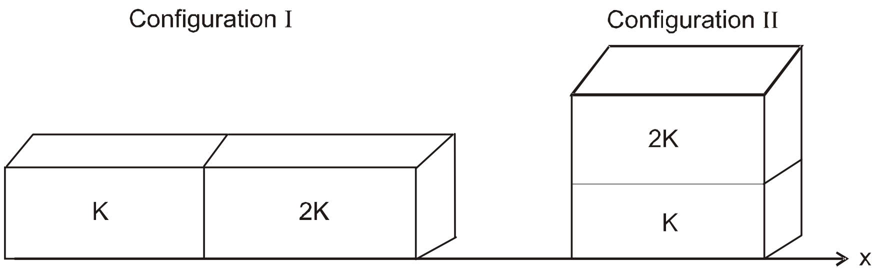


(A) heat flow through A and E slabs are same
(B) heat flow through slab E is maximum
(C) temperature difference across slab E is smallest
(D) heat flow through C = heat flow through B + heat flow through D.

4. Three very large plates of same area are kept parallel and close to each other. They are considered as ideal black surfaces and have very high thermal conductivity. The first and third plates are maintained at temperatures 2T and 3T respectively. The temperature of the middle (i.e. second) plate under steady state condition is **[IIT-JEE-2012, Paper-1; 3/70, –1]**

(A) $\left(\frac{65}{2}\right)^{\frac{1}{4}}T$ (B) $\left(\frac{97}{4}\right)^{\frac{1}{4}}T$ (C) $\left(\frac{97}{2}\right)^{\frac{1}{4}}T$ (D) $(97)^{\frac{1}{4}}T$

5. Two rectangular blocks, having indentical dimensions, can be arranged either in configuration I or in configuration II as shown in the figure, On of the blocks has thermal conductivity k and the other 2k. The temperature difference between the ends along the x-axis is the same in both the configurations. It takes 9s to transport a certain amount of heat from the hot end to the cold end in the configuration I. The time to transport the same amount of heat in the configuration II is : **[JEE (Advanced) 2013,3/60,–1]**



(A) 2.0 s (B) 3.0 s (C) 4.5 s (D) 6.0 s

6. Parallel rays of light of intensity $I = 912\ Wm^{-2}$ are incident on a spherical balck body kept in surroundings of temperature 300 K. Take Stefan-Biltzmann constant $\sigma = 5.7 \times 10^{-8}\ Wm^{-2}\ K^{-4}$ and assume that the energy exchange with the surroundings is only through radiation. Th final steady state temperature of the black body is close to: **[JEE (Advanced) 2014, 3/60, –1]**

(A) 330 K (B) 660 K (C) 990 K (D) 1550 K

7. Two spherical stars A and B emit blackbody radiation. The radius of A is 400 times that of B and A emits 10^4 times the power emitted from B. The ratio $\left(\frac{\lambda_A}{\lambda_B}\right)$ fo their wavelengths λ_A and λ_B at which the peaks occur in their respective radiation curves is : **[JEE (Advanced) 2015 ; P-1, 4/88]**

8.* An incandescent bulb has a thin filament of tungsten that is heated to high temperature by passing an electric current. The hot filament emits black-body radiation. The filament is observed to break up at random locations after a sufficiently long time of operation due to non-uniform evaporation of tungsten from the filament. If the bulb is powered at constant voltage, which of the following statement(s) is (are) true ? **[JEE (Advanced) 2016 ; P-1, 4/62, –2]**

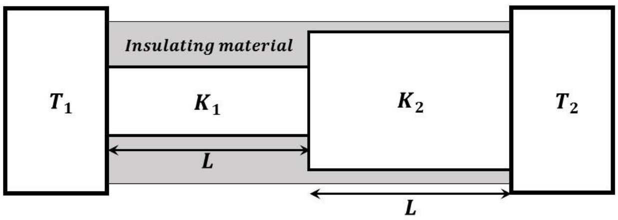
(A) The temperature distribution over the filament is uniform
(B) The resistance over small sections of the filament decreases with time
(C) The filament emits more light at higher band of frequencies before it breaks up
(D) The filament consumes less electrical power towards the end of the life of the bulb

9. A metal is heated in a furnace where a sensor is kept above the metal surface to read the power radiated (P) by the metal. The sensor has scale that displays $\log_2 (P / P_0)$, where P_0 is a constant. When the metal surface is at a temperature of 487 ºC, the sensor shows a value 1. Assume that the emissivity of the metallic surface remains constant. What is the value displayed by the sensor when the temperature of the metal surface is raised to 2767 ºC. **[JEE (Advanced) 2016 ; P-1, 3/62]**

10. A human body has surface area of approximately $1 m^2$. The normal body temperature is 10 K above the surrounding room temperature T_0. Take the room temperature to be T_0 = 300 K. For T_0 = 300K, the value of σT_0^4 = 460 Wm^{-2} (where σ is the Stefan-Boltzmann constant). Which of the following options is/are correct ? **[JEE (Advanced) 2017 ; P-1, 4/61, –2]**

(A) If the surrounding temperature reduces by a small amount $\Delta T_0 << T_0$, then to maintain the same body temperature the same (living) human being needs to radiate $\Delta W = 4\sigma T_0^3 \Delta T_0$ more energy per unit time.
(B) Reducing the exposed surface area of the body (e.g. by curling up) allows humans to maintain the same body temperature while reducing the energy lost by radiation
(C) If the body temperature rises significantly then the peak in the spectrum of electromagnetic radiation emitted by the body would shift to longer wavelengths
(D) The amount of energy radiated by the body in 1 second is close to 60 joules

11. Two conducting cylinders of equal length but different radii are connected in series between two heat baths kept at temperatures T_1 = 300K and T_2 = 100K, as shown in the figure. The radius of the bigger cylinder is twice that of the smaller one and the thermal conductivities of the materials of the smaller and the larger cylinders are K_1 and K_2 respectively. If the temperature at the junction of the two cylinders in the steady state is 200K, then K_1/K_2 =__________. **[JEE (Advanced) 2018 ; P-1, 3/60]**

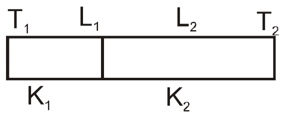


PART - II : JEE (MAIN) / AIEEE PROBLEMS (PREVIOUS YEARS)

1. Assuming the sun to be a spherical body of radius R at a temperature of T K, evaluate the total radiant power, incident on Earth, at a distance r from the Sun. (earth radius = r_0) **[AIEEE-2006; 3/180]**

(1) $\dfrac{R^2\sigma T^4}{r^2}$ (2) $\dfrac{4\pi r_0^2 R^2 \sigma T^4}{r^2}$ (3) $\dfrac{\pi r_0^2 R^2 \sigma T^4}{r^2}$ (4) $\dfrac{r_0^2 R^2 \sigma T^4}{4\pi r^2}$

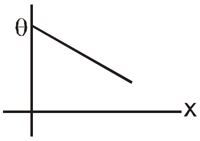
2. One end of a thermally insulated rod is kept at a temperature T_1 and the other at T_2. The rod is composed of two sections of lengths L_1 and L_2 and thermal conductivities k_1 and k_2 respectively. The temperature at the interface of the sections is

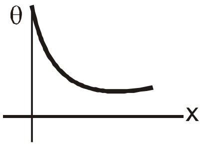


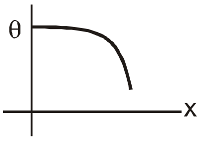
[AIEEE-2007; 3/120]

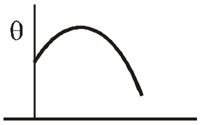
(1) $\dfrac{(K_2L_2T_1 + K_1L_1T_2)}{(K_1L_1 + K_2L_2)}$ (2) $\dfrac{(K_2L_1T_1 + K_1L_2T_2)}{(K_2L_1 + K_1L_2)}$ (3) $\dfrac{(K_1L_2T_1 + K_2L_1T_2)}{(K_1L_2 + K_2L_1)}$ (4) $\dfrac{(K_1L_1T_1 + K_2L_2T_2)}{(K_1L_1 + K_2L_2)}$

3. A long metallic bar is carrying heat from one of its ends to the other end under steady-state. The variation of temperature θ along the length x of the bar from its hot end is best described by which of the following figures **[AIEEE-2009, 4/144]**

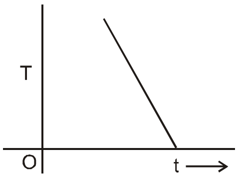
(1) 

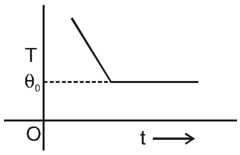
(2) 

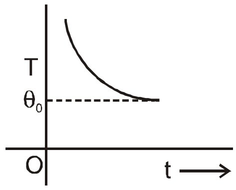
(3) 

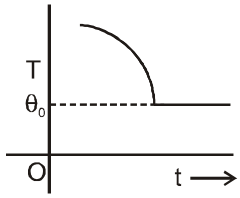
(4) 

4. If a piece of metal is heated to temperature θ and then allowed to cool ina room which is at temperature θ_0, the graph between the temperature T of the metal and time t will be closest to : **[JEE (Main) 2013, 4/120, –1]**

(1) 

(2) 

(3) 

(4) 

5. Three rods of Copper, brass and steel are welded together to form a Y-shaped structure. Area of cross-section of each rod = 4 cm^2. End of copper rod is maintained at 100°C where as ends of brass and steel are kept at 0°C. Lengths of the copper, brass and steel rods are 46, 13 and 12 cms respectively. The rods are thermally insulated from surroundings except at ends. Thermal conductivities of copper, brass and steel are 0.92, 0.26 and 0.12 CGS units respectively. Rate of heat flow through copper rod is: **[JEE (Main) 2014, 4/120, –1]**

(1) 1.2 cal/s (2) 2.4 cal/s (3) 4.8 cal/s (4) 6.0 cal/s

6. An ideal gas undergoes a quasi static, reversible process in which its molar heat capacity C remains constant. If during this process the relation of pressure P and volume V is given by PV^n = constant, then n is given by (Here C_p and C_V are molar specific heat at constant pressure and constant volume, respectively) : **[JEE (Main) 2016, 4/120, –1]**

(1) $n = \frac{C - C_p}{C - C_V}$ (2) $n = \frac{C_p - C}{C - C_V}$ (3) $n = \frac{C - C_V}{C - Cp}$ (4) $n = \frac{C_p}{C_V}$

Answers

EXERCISE-1

PART - I

Section (A) :

A-1.	64 J	**A-2.**	5×10^{-5} g/s
A-3.	4.0 W	**A-4.**	$2 : \pi$

A-5. $\dfrac{t_1 + t_2 + t_3}{\dfrac{t_1}{k_1} + \dfrac{t_2}{k_2} + \dfrac{t_3}{k_3}}$

Section (B) :

B-1.	15 W/m–°C	**B-2.**	$\dfrac{2\pi K\ell(T_2 - T_1)}{\ln(R_2 / R_1)}$

Section (C) :

C-1. 12 °C/s

Section (D) :

D-1.	$\dfrac{q_1 - q_2}{q_1}$	**D-2.**	0.73 W.
D-3.	6×10^3 K; 4×10^3 K		
D-4	$\dfrac{1}{90}$	**D-5.**	5803
D-6	1.71 ρrc		

Section (E) :

E-1. 7 minutes

PART - II

Section (A) :

A-1.	(B)	**A-2.**	(a) (A), (b) (D)		
A-3.	(B)	**A-4.**	(C)	**A-5.**	(C)

Section (B) :

B-1. (C)

Section (C) :

C-1.	(B)	**C-2.**	(B)	**C-3.**	(D)

Section (D) :

D-1.	(C)	**D-2.**	(B)

PART - III

1. (A) $\rightarrow$ p, s ; (B) $\rightarrow$ t ; (C) $\rightarrow$ q, r ; (D) $\rightarrow$ t

EXERCISE-2

PART - I

1.	(B)	**2.**	(D)	**3.**	(C)
4.	(D)	**5.**	(A)	**6.**	(C)
7.	(B)	**8.**	(A)	**9.**	(B)
10.	(A)	**11.**	(B)	**12.**	(A)

PART - II

1.	30	**2.**	3	**3.**	3
4.	7	**5.**	10	**6.**	5
7.	3	**8.**	22		

PART - III

1.	(ABC)	**2.**	(AB)	**3.**	(AB)
4.	(BC)	**5.**	(AC)	**6.**	(BCD)

PART - IV

1.	(B)	**2.**	(A)	**3.**	(A)
4.	(B)	**5.**	(A)	**6.**	(C)
7.	(A)	**8.**	(B)	**9.**	(C)

EXERCISE-3

PART - I

1.	9	**2.**	9	**3.**	(ACD)
4.	(C)	**5.**	(A)	**6.**	(A)
7.	2	**8.**	(CD)	**9.**	9
10.	(B)	**11.**	4.00		

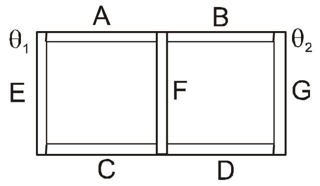
PART - II

1.	(3)	**2.**	(3)	**3.**	(1)
4.	(3)	**5.**	(3)	**6.**	(1)

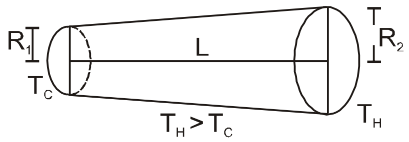
High Level Problems (HLP)

SUBJECTIVE QUESTIONS

1. Seven rods A, B, C, D, E, F and G are joined as shown in figure. All the rods have equal cross-sectional area A and length ℓ. The thermal conductivities of the rods are $K_A = 2K_C = 3K_B = 6K_D = K_0$. The rod E is kept at a constant temperature θ_1 and the rod G is kept at a constant temperature $\theta_2 (\theta_2 > \theta_1)$. (a) Show that the rod F has a uniform temperature $\theta = (3\theta_1 + \theta_2)/4$. (b) Find the rate of heat flow from the source which maintains the temperature θ_2.



2. Find the rate of heat flow through a cross-section of the rod shown in figure ($T_H > T_C$). Thermal conductivity of the material of the rod is K.



3. A solid aluminium sphere and a solid copper sphere of twice the radius of aluminium are heated to the same temperature and are allowed to cool under identical surrounding temperatures. Assume that the emissivity of both the sphere is the same. Find the initial ratio of
(a) the rate of heat loss from the aluminium sphere to the rate of heat loss from the copper sphere and
(b) the rate of fall of temperature of the aluminium sphere to the rate of fall of temperature of the copper sphere. The specific heat capacity of aluminium = 900 J/kg–°C and that of copper = 390 J/kg–°C. The density of copper = 3.4 times the density of aluminium.

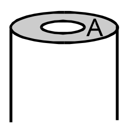
4. A hot body placed in a surrounding of temperature T_0. Its temperature at t = 0 is T_1. The specific heat capacity of the body is s and its mass is m. Assuming Newton's law of cooling to be valid, find
(a) the maximum heat that the body can lose and
(b) the time starting from t = 0 in which it will lose 50% of this maximum heat. (Answer in terms of k)

5. Find the total time elapsed for a hollow copper sphere of inner radius 3 cm outer radius 6 cm, density $\rho = 9 \times 10^3$ kg/m^3, specific heat $s = 4 \times 10^3$ J/kg K and emissivity e = 0.4 to cool from 727°C to 227°C when the surrounding temperature is 0 K. (for inner surface e = 1 Stefan's constant $\sigma = 5.6 \times 10^{-8}$ W/m^2 K^4)

6. A metal block of heat capacity 90 J/°C placed in a room at 25°C is heated electrically. The heater is switched off when the temperature reaches 35°C. The temperature of the block rises at the rate of 2°C/s just after the heater is switched on and falls at the rate of 0.2°C/s just after the heater is switched off. Assume Newton's law of cooling to hold.
(a) Find the power of the heater.
(b) Find the power radiated by the block just after the heater is switched off.
(c) Find the power radiated by the block when the temperature of the block is 30°C.
(d) Assuming that the power radiated at 30°C respresents the average value in the heating process, find the time for which the heater was kept on.

7. A hollow tube has a length l, inner radius R_1 and outer radius R_2. The material has thermal conductivity K. Find rate of heat flowing through the walls of the tube if the flat ends are maintained at temperatures T_1 and $T_2 (T_2 > T_1)$.

8. Calculate thermal conductance for radial flow of an annular cylinder of length ℓ and inner and outer radius r_1 and r_2. Assume that thermal conductivity of the material is K



9. Calculate thermal conductance for radial flow of a spherical shell of inner and outer radius r_1 and r_2. Assume that thermal conductivity of the material is K

10. A metallic cylindrical vessel whose inner and outer radii are r_1 and r_2 is filled with ice at 0ºC. The mass of the ice in the cylinder is m. Circular portions of the cylinder is sealed with completely adiabatic walls. The vessel is kept in air. Temperature of the air is 50ºC. Find time elapsed for the ice to melt completely. (Thermal conductivity of the cylinder is k, its length is ℓ. Latent heat of fusion is L).

11. A uniform cylinder of length L and thermal conductivity k is placed on a metal plate of the same area S of mass m and infinite conductivity. The specific heat of the plate is c. The top of the cylinder is maintained at T_0. Find the time required for the temperature of the plate to rise from T_1 to T_2 ($T_1 < T_2 < T_0$).

12. Assume that the total surface area of a human body is 1.6 m^2 and that it radiates like an ideal radiator. Calculate the amount of energy radiated per second by the body if the body temperature is 37°C. Stefan constant σ is 6.0×10^{-8} W/m^2–K^4. (31^4 = 923521)

13. The surface of a household radiator has an emissivity of 0.55 and an area of 1.5 m^2.
(a) At what rate is radiation emitted by the radiator when its temperature is 50°C?
(b) At what rate is the radiation absorbed by the radiator when the walls of the room are at 22°C?
(c) What is the net rate of radiation from the radiator? (stefan constant $\sigma = 6 \times 10^{-8}$ W/m^2 – K^4)

14. A man, the surface area of whose skin is 2m^2, is sitting in a room where the air temperature is 20°C. If the skin temperature is 28°C. Find the net rate at which his body loses heat. [Take the emissivity of skin 0.97 and stephen's constant = 5.67×10^{-8} W/m^2 – K^4

15. An electric heater is used in a room of total wall area 137 m^2 to maintain a temperature of 20°C inside it when outside temperature is –10°C. The walls have three different layers of materials. The innermost layer is of wood of thickness 2.5 cm, the middle layer is of cement of thickness 1.0 cm and the outermost layer is of brick of thickness 25 cm. Find the power of the electric heater : (Assume that there is no heat loss through the floor and the ceiling. The thermal conductivities of wood, cement and brick are 0.125, 1.5 and 1.0 W/m C° respectively). **[IIT 1986]**

16. A rod of length ℓ with thermally insulated lateral surface consists of material whose heat conductivity coefficient varies with temperature as $K = \alpha / T$, where α is a constant. The ends of the rod are kept at temperatures T_1 and T_2. Find the function T(x), where x is the distance from the end whose temperature is T_1 and the heat flow density,

17. Two chunks of metal with heat capacities C_1 and C_2, are interconnected by a rod of length ℓ and cross-sectional area S and fairly low heat conductivity K. The whole system is thermally insulated from the environment. At a moment t = 0 the temperature difference between the two chunks of metal equals $(\Delta T)_0$. Assuming the heat capacity of the rod to be negligible, find the temperature difference between the chunks as a function of time.

Answers

1. $\dfrac{3K_0A(\theta_2-\theta_1)}{8l}$

2. $\dfrac{K\pi R_1R_2(T_H-T_C)}{L}$

3. (a) 1 : 4 (b) 2.9 : 1

4. (a) $ms(T_1 - T_0)$ (b) $\dfrac{In\ \ 2}{k}$

5. 6.56×10^4 sec

6. (a) 180 W, (b) 18 W, (c) 9 W, (d) $\dfrac{100}{19}$ s

7. $\dfrac{K\pi(R_2^2-R_1^2)\ \ (T_2-T_1)}{l}$

8. $\dfrac{2\pi\ell k}{\ell n(r_2/r_1)}$

9. $\dfrac{4\pi k r_1 r_2}{(r_2-r_1)}$

10. $t = mL\ \ell n\ \dfrac{\left(\frac{r_2}{r_1}\right)}{100\pi k\ell}$

11. $\dfrac{mCL}{KS}\ell n\left(\dfrac{T_0-T_1}{T_0-T_2}\right)$

12. 887 J

13. 539 W, 375 W, 164 W

14. 92.2 W

15. 9 kW

16. $T(x) = T_1\left(\dfrac{T_2}{T_1}\right)^{x/\ell}$; $q = (\alpha/\ell)\ \ell n\left(\dfrac{T_1}{T_2}\right)$

17. $\Delta T = (\Delta T)_0\, e^{-\alpha t}$, where $\alpha = (1/C_1 + 1/C_2)\, SK/\ell$