WAVE ON A STRING

Contents

JEE (ADVANCED) SYLLABUS

Wave motion (plane waves only), longitudinal and transverse waves, superposition of waves; Progressive and stationary waves; Vibration of strings and air columns;

JEE (MAIN) SYLLABUS

Wave motion : Longitudinal and transverse waves, speed of a wave. Displacement relation for a progressive wave. Principle of superposition of waves, reflection of waves, Standing waves in strings..

WAVE ON A STRING

WAVES

Wave motion is the phenomenon that can be observed almost everywhere around us, as well it appears in almost every branch of physics. Surface waves on bodies of matter are commonly observed. Sound waves and light waves are essential to our perception of the environment. All waves have a similar mathematical description, which makes the study of one kind of wave useful for the study of other kinds of waves. In this chapter, we will concentrate on string waves, which are type of a mechanical waves. Mechanical waves require a medium to travel through. Sound waves, water waves are other examples of mechanical waves. Light waves are not mechanical waves, these are electromagnetic waves which do not require medium to propagate.

Mechanical waves originate from a disturbance in the medium (such as a stone dropping in a pond) and the disturbance propagates through the medium. The forces between the atoms in the medium are responsible for the propagation of mechanical waves. Each atom exerts a force on the atoms near it, and through this force the motion of the atom is transmitted to the others. The atoms in the medium do not, however, experience any net displacement. As the wave passes, the atoms simply move back and forth. Again for simplicity, we concentrate on the study of harmonic waves (that is those that can be represented by sine and cosine functions).

TYPES OF MECHANICAL WAVES

Mechanical waves can be classified according to the physical properties of the medium, as well as in other ways.

1. **Direction of particle motion :** Waves can be classified by considering the direction of motion of the particles in the medium as wave passes. If the disturbance travels in the x direction but the particles move in a direction, perpendicular to the x axis as the wave passes it is called a transverse wave. If the motion of the particles were parallel to the x axis then it is called a longitudinal wave. A wave pulse in a plucked guitar string is a transverse wave. A sound wave is a longitudinal wave.
2. **Number of dimensions :** Waves can propagate in one, two, or three dimensions. A wave moving along a taut string is a one dimensional wave. A water wave created by a stone thrown in a pond is a two dimensional wave. A sound wave created by a gunshot is a three-dimensional wave
3. **Periodicity :** A stone dropped into a pond creates a wave pulse, which travels outward in two dimensions. There may be more than one ripple created, but there is still only one wave pulse. If similar stones are dropped in the same place at even time intervals, then a periodic wave is created.
4. **Shape of wave fronts :** The ripples created by a stone dropped into a pond are circular in shape. A sound wave propagating outward from a point source has spherical wavefronts. A plane wave is a three dimensional wave with flat wave fronts.

   (Far away from a point source emitting spherical waves, the waves appear to be plane waves.)

   A solid can sustain transverse as well as longitudinal wave. A fluid has no well-defined form or structure to maintain and offer far more resistance to compression than to a shearing force. Consequently, only longitudinal wave can propagate through a gas or within the body of an ideal (non viscous) liquid.

   However, transverse waves can exist on the surface of a liquid. In the case of ripples on a pond, the force restoring the system to equilibrium is the surface tension of the water, whereas for ocean waves, it is the force of gravity.

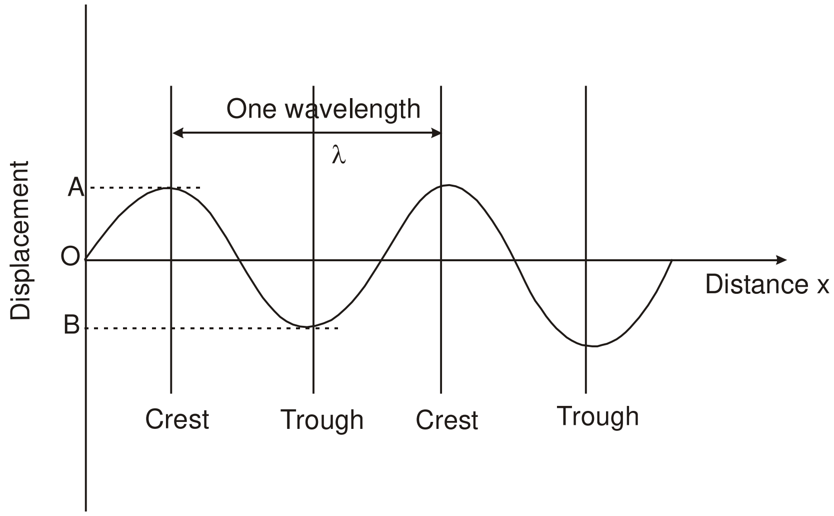
   Also, if disturbance is restricted to propagate only in one direction and there is no loss of energy during propagation, then shape of disturbance remains unchanged.

DESCRIBING WAVES :

Two kinds of graph may be drawn - displacement-distance and displacement-time.

A displacement - distance graph for a transverse mechanical wave shows the displacement y of the vibrating particles of the transmitting medium at different distance x from the source at a certain instant i.e. it is like a photograph showing shape of the wave at that particular instant.

The maximum displacement of each particle from its undisturbed position is the amplitude of the wave. In the figure 1, it is OA or OB.



The wavelength λ of a wave is generally taken as the distance between two successive crests or two successive trough. To be more specific, it is the distance between two consecutive points on the wave which have same phase.

A displacement-time graph may also be drawn for a wave motion, showing how the displacement of one particle at a particular distance from the source varies with time. If this is simple harmonic variation then the graph is a sine curve.

WAVE LENGTH, FREQUENCY, SPEED

If the source of a wave makes f vibrations per second, so too will the particles of the transmitting medium. That is, the frequency of the waves equals frequency of the source.

When the source makes one complete vibration , one wave is generated and the disturbance spreads out a distance λ from the source. If the source continues to vibrate with constant frequency f, then f waves will be produced per second and the wave advances a distance f λ in one second. If v is the wave speed then

$$v = f\lambda$$

This relationship holds for all wave motions.

Travelling wave : Imagine a horizontal string stretched in the x direction. Its equilibrium shape is flat and straight. Let y measure the displacement of any particle of the string from its equilibrium position, perpendicular to the string. If the string is plucked on the left end, a pulse will travel to the right. The vertical displacement y of the left end of the string (x = 0) is a function of time.

i.e., $y\,(x = 0, t) = f(t)$

If there are no frictional losses, the pulse will travel undiminished, retaining its original shape. If the pulse travels with a speed v, the 'position' of the wave pulse is x = vt. Therefore, the displacement of the particle at point x at time t was originated at the left end at time $t - \frac{x}{v}$. [y, (x, t) is function of both x and t]. But the displacement of the left end at time t is f(t) thus at time $t - \frac{x}{v}$, it is $f(t - \frac{x}{v})$.

Therefore :

$$y(x, t) = y\,(x = 0,\ t - \frac{x}{v}) = f\,(t - \frac{x}{v})$$

This can also be expressed as

$$\Rightarrow \quad \frac{f}{v}\,(vt - x) \quad \Rightarrow \quad -\frac{f}{v}\,(x - vt)$$

$$y(x, t) = g(x - vt)$$

Using any fixed value of t (i.e. at any instant), this shows shape of the string. If the wave is travelling in – x direction, then wave equation is written as

$$y(x, t) = f(t + \frac{x}{v})$$

The quantity x – vt is called phase of the wave function. As phase of the pulse has fixed value

$$x - vt = \text{const.}$$

Taking the derivative w.r.t. time $\frac{dx}{dt} = v$

where v is the phase velocity although often called wave velocity. It is the velocity at which a particular phase of the disturbance travels through space.

In order for the function to represent a wave travelling at speed v, the three quantities x, v and t must appear in the combination (x + vt) or (x – vt). Thus $(x - vt)^2$ is acceptable but $x^2 - v^2 t^2$ is not.

Solved Example

Example 1. A wave pulse is travelling on a string at 2 m/s. Displacement y of the particle at x = 0 at any time t is given by $y = \frac{2}{t^2 + 1}$. Find :

(i) Expression of the function y =(x, t) i.e. displacement of a particle at position x and time t.

(ii) Shape of the pulse at t = 0 and t = 1s.

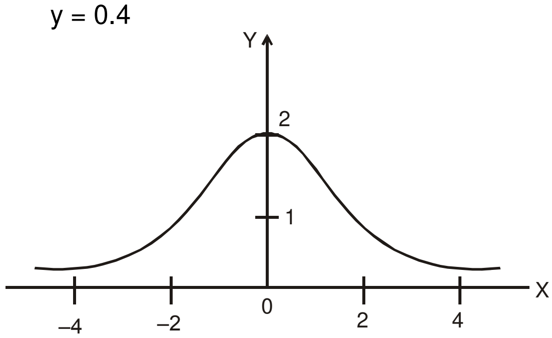
Solution : (i) By replacing t by $\left(t - \frac{x}{v}\right)$, we can get the desired wave function i.e.,

$$y = \frac{2}{\left(t - \frac{x}{2}\right)^2 + 1}$$

(ii) We can use wave function at a particular instant, say t = 0, to find shape of the wave pulse using different values of x.

at t = 0 $\quad y = \frac{2}{\frac{x^2}{4} + 1}$

at x = 0	y = 2
x = 2	y = 1
x = – 2	y = 1
x = 4	y = 0.4
x = –4	y = 0.4

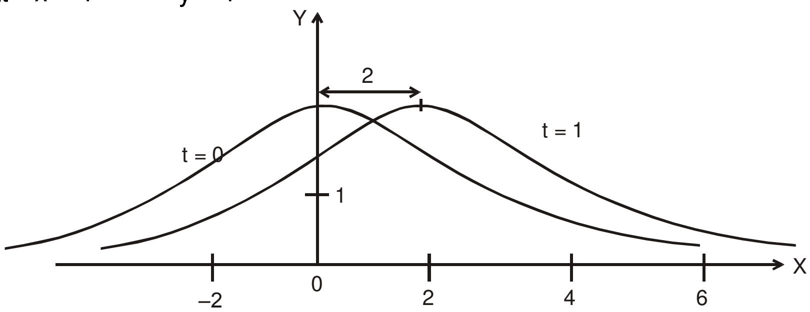


Using these value, shape is drawn.

Similarly for t = 1s, shape can be drawn. What do you conclude about direction of motion of the wave from the graphs? Also check how much the pulse has moved in 1s time interval. This is equal to wave speed. Here is the procedure :

$$y = \frac{2}{\left(1-\frac{x}{2}\right)^2 + 1}$$

at t = 1s
at x = 2 y = 2 (maximum value)
at x = 0 y = 1
at x = 4 y = 1



The pulse has moved to the right by 2 units in 1s interval.

Also as $t - \frac{x}{2}$ = constt.

Differentiating w.r.t. time

$$1 - \frac{1}{2} \cdot \frac{dx}{dt} = 0 \Rightarrow \frac{dx}{dt} = 2.$$

TRAVELLING SINE WAVE IN ONE DIMENSION (WAVE ON STRING) :

The wave equation $y = f\left(t - \frac{x}{v}\right)$ is quite general. If holds for arbitrary wave shapes, and for transverse as well as for longitudinal waves.

A complete description of the wave requires specification of f(x). The most important case, by far, in physics and engineering is when f(x) is sinusoidal, that is, when the wave has the shape of a sine or cosine function. This is possible when the source, that is moving the left end of the string, vibrates the left end x = 0 in a simple harmonic motion. For this, the source has to continuously do work on the string and energy is continuously supplied to the string.

The equation of motion of the left end may be written as

$$f(t) = A \sin \omega t$$

where A is amplitude of the wave, that is maximum displacement of a particle in the medium from its equilibrium position ω is angular frequency, that is $2\pi f$ where f is frequency of SHM of the source.

The displacement of the particle at x at time t will be

$$y = f\left(t - \frac{x}{v}\right) \quad \text{or} \quad y = A \sin \omega\left(t - \frac{x}{v}\right) \qquad y = A \sin(\omega t - kx)$$

where k = is called wave number. $T = \frac{2\pi}{\omega} = \frac{1}{f}$ is period of the wave, that is the time it takes to travel the distance between two adjacent crests or trough (it is wavelength λ).

The wave equation $y = A \sin(\omega t - kx)$ says that at x = 0 and t = 0, y = 0. This is not necessarily the case, of source. For the same condition, y may not equal to zero. Therefore, the most general expression would involve a phase constant ϕ, which allows for other possibilities,

$$y = A \sin(\omega t - kx + \phi)$$

A suitable choice of ϕ allows any initial condition to be met. The term $(kx - wt + \phi)$ is called the phase of the wave. Two waves with the same phase (on phase differing by a multiple of 2π) are said to be "in phase". They execute the same motion at the same time.

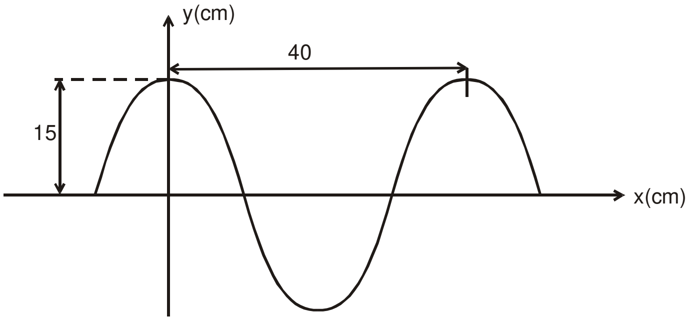
The velocity of the particle at position x and at time t is given by

$$\frac{\partial y}{\partial t} = A\omega \cos(\omega t - kx + \phi)$$

The wave equation has been partially differentiated keeping x as constant, to specify the particle. Note that wave velocity $\frac{dx}{dt}$ is different from particle velocity while waves velocity is constant for a medium and it along the direction of string, whereas particle velocity is perpendicular to wave velocity and is dependent upon x and t.

Solved Example

Example 2. A sinusoidal wave travelling in the positive x direction has an amplitude of 15 cm, wavelength 40 cm and frequency 8 Hz. The vertical displacement of the medium at t = 0 and x = 0 is also 15 cm, as shown.



(a) Find the angular wave number, period, angular frequency and speed of the wave.

(b) Determine the phase constant ϕ, and write a general expression for the wave function.

Solution : (a) $k = \frac{2\pi}{\lambda} = \frac{2\pi}{40}\frac{\text{rad}}{\text{cm}} = \frac{\pi}{20}$ rad/cm

$T = \frac{1}{f} = \frac{1}{8}$ s $\qquad \omega = 2\pi f = 16\ s^{-1}$

$v = f\lambda = 320$ cm/s

(b) It is given that A = 15 cm

and also y = 15 cm at x = 0 and t = 0

then using $y = A \sin(\omega t - kx + \phi)$

$15 = 15 \sin\phi \quad \Rightarrow \quad \sin\phi = 1$

or $\phi = \frac{\pi}{2}$ rad.

Therefore, the wave function is

$$y = A\sin\left(\omega t - kx + \frac{\pi}{2}\right) = (15\text{ cm})\sin\left[(16\pi s^{-1})t - \left(\frac{\pi}{20}\frac{\text{rad}}{\text{cm}}\right).x + \frac{\pi}{2}\right]$$

Example 3. A sinusoidal wave is travelling along a rope. The oscillator that generates the wave completes 60 vibrations in 30 s. Also, a given maximum travels 425 cm along the rope in 10.0 s. What is the wavelength?

Solution : $v = \frac{425}{10} = 42.5$ cm/s. $\qquad f = \frac{60}{30} = 2$ Hz

$\lambda = \frac{v}{f} = 21.25$ cm.

THE LINEAR WAVE EQUATION :

By using wave function y = A sin (ωt – kx + ϕ), we can describe the motion of any point on the string. Any point on the string moves only vertically, and so its x coordinate remains constant. The transverse velocity v_y of the point and its transverse acceleration a_y are therefore

$$v_y = \left[\frac{dy}{dt}\right]_{x=\text{constant}} \Rightarrow \frac{\partial y}{\partial t} = \omega A \cos(\omega t - kx + \phi) \quad .....(1)$$

$$a_y = \left[\frac{dv_y}{dt}\right]_{x=\text{constant}} \Rightarrow \frac{\partial v_y}{\partial t} = \frac{\partial^2 y}{\partial t^2} = -\omega^2 A \sin(\omega t - kx + \phi) \quad .....(2)$$

and hence $v_{y,\,max} = \omega A$

$a_{y,\,max} = \omega^2 A$

The transverse velocity and transverse acceleration of any point on the string do not reach their maximum value simultaneously. Infact, the transverse velocity reaches its maximum value (ωA) when the displacement y = 0, whereas the transverse acceleration reaches its maximum magnitude (ω^2A) when y = ± A

further $\left[\frac{dy}{dx}\right]_{t=\text{constant}}$

$$\Rightarrow \frac{\partial y}{\partial x} = -kA \cos(\omega t - kx + \phi) \quad .....(3)$$

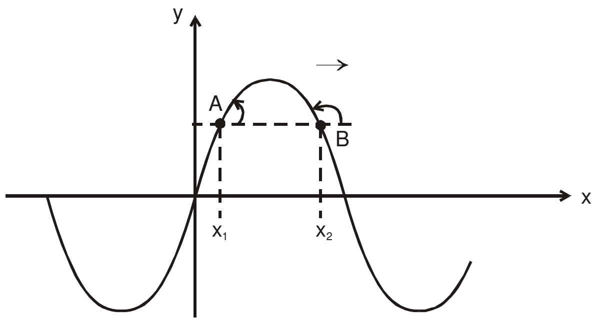
$$= \frac{\partial^2 y}{\partial x^2} = -k^2 A \sin(\omega t - kx + \phi) \quad .....(4)$$

From (1) and (3)

$$\frac{\partial y}{\partial t} = -\frac{\omega}{k}\frac{\partial y}{\partial x} \Rightarrow v_P = -v_w \times \text{slope}$$

i.e. if the slope at any point is negative, particle velocity is positive and vice-versa, for a wave moving along positive x axis i.e. v_w is positive.

For example, consider two points A and B on the y-x curve for a wave, as shown. The wave is moving along positive x-axis.



Slope at A is positive therefore at the given moment, its velocity is negative. That means it is coming downward. Reverse is the situation for particle at point B.

Now using equation (2) and (4)

$$\frac{\partial^2 y}{\partial x^2} = \frac{k^2}{\omega^2}\frac{\partial^2 y}{\partial t^2} \Rightarrow \frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$$

This is known as the linear wave equation or differential equation representation of the travelling wave model. We have developed the linear wave equation from a sinusoidal mechanical wave travelling through a medium, but it is much more general. The linear wave equation successfully describes waves on strings, sound waves and also electromagnetic waves.

Solved Example

Example 4. Verify that wave function $y = \dfrac{2}{(x-3t)^2+1}$ is a solution to the linear wave equation. x and y are in cm.

Solution : By taking partial derivatives of this function w.r.t. x and t

$$\frac{\partial^2 y}{\partial x^2} = \frac{12(x-3t)^2-4}{[(x-3t)^2+1]^3}, \text{ and } \frac{\partial^2 y}{\partial t^2} = \frac{108\ \ (x-3t)^2-36}{[(x-3t)^2+1]^3}$$

or $\dfrac{\partial^2 y}{\partial x^2} = \dfrac{1}{9}\dfrac{\partial^2 x}{\partial t^2}$

Comparing with linear wave equation, we see that the wave function is a solution to the linear wave equation if the speed at which the pulse moves is 3 cm/s. It is apparent from wave function therefore it is a solution to the linear wave equation.

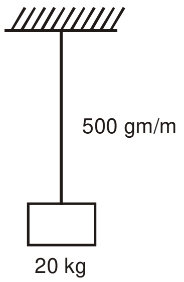
THE SPEED OF TRANSVERSE WAVES ON STRINGS

The speed of a wave on a string is given by $v = \sqrt{\dfrac{T}{\mu}}$

where T is tension in the string (in Newton's) and μ is mass per unit length of the string (kg/m).
It should be noted that v is speed of the wave w.r.t. the medium (string).
In case the tension is not uniform in the string or string has non-uniform linear mass density then v is speed at a given point and T and μ are corresponding values at that point.

Solved Example

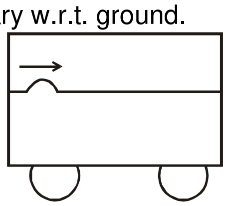
Example 5. Find speed of the wave generated in the string as in the situation shown. Assume that the tension is not affected by the mass of the cord.



Solution : T = 20 × 10 = 200 N

$$v = \sqrt{\frac{200}{0.5}} = 20 \ \ m/s$$

Example 6. A taut string having tension 100 N and linear mass density 0.25 kg/m is used inside a cart to generate a wave pulse starting at the left end, as shown. What should be the velocity of the cart so that pulse remains stationary w.r.t. ground.



Solution : Velocity of pulse = $\sqrt{\dfrac{T}{\mu}} = 20 \ \ m/s$

Now $\vec{v}_{PG} = \vec{v}_{PC} + \vec{v}_{CG}$

$0 = 20\,i + \vec{v}_{CG}$

$\vec{v}_{CG} = -\,20i$ m/s

POWER TRANSMITTED ALONG THE STRING BY A SINE WAVE

When a travelling wave is established on a string, energy is transmitted along the direction of propagation of the wave, in form of potential energy and kinetic energy

Average Power $\langle P \rangle = 2\pi^2 f^2 A^2 \mu v$

Energy Transferred = $\int_0^t P_{av} dt$

Energy transferred in one time period = P_{av} T

This is also equal to the energy stored in one wavelength.

Intensity : Energy transferred per second per unit cross sectional area is called intensity of the wave.

$$I = \frac{\text{Power}}{\text{Cross sectional area}} = \frac{P}{s} \Rightarrow I = \frac{1}{2}\rho\omega^2 A^2 v$$

This is average intensity of the wave.

Energy density : Energy per unit volume of the wave

$$= \frac{Pdt}{svdt} = \frac{I}{v}$$

Solved Example

Example 7. A string with linear mass density m = 5.00 × 10^{-2} kg/m is under a tension of 80.0 N. How much power must be supplied to the string to generate sinusoidal waves at a frequency of 60.0 Hz and an amplitude of 6.00 cm?

Solution : The wave speed on the string is $v = \sqrt{\frac{T}{\mu}} = \left(\frac{80.0\ \text{N}}{5.00 \times 10^{-2}\ \text{kg/m}}\right)^{1/2} = 40.0$ m/s

Because f = 60 Hz, the angular frequency ω of the sinusoidal waves on the string has the value

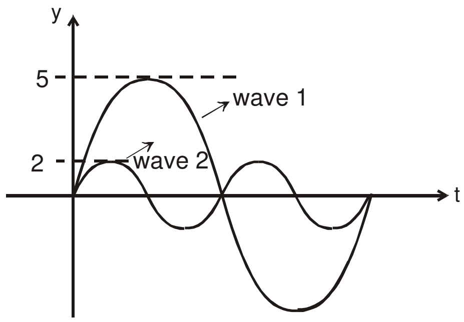
$\omega = 2\pi f = 2\pi(60.0\ \text{Hz}) = 377\ \text{s}^{-1}$

Using these values in following Equation for the power, with A = 6.00 × 10^{-2} m, gives

$$p = \frac{1}{2}\mu\omega^2 A^2 v$$

$$= \frac{1}{2}(5.00 \times 10^{-2}\ \text{kg/m})(377\text{s}^{-1})^2 \times (6.00 \times 10^{-2}\ \text{m})^2 (40.0\ \text{m/s}) = 512\ \text{W}.$$

Example 8. Two waves in the same medium are represented by y-t curves in the figure. Find ratio of their average intensities?



Solution : $\frac{I_1}{I_2} = \frac{\omega_1^2 A_1^2}{\omega_2^2 A_2^2} = \frac{f_1^2 \cdot A_1^2}{f_2^2 \cdot A_2^2} = \frac{1 \times 25}{4 \times 4} = \frac{25}{16}$

THE PRINCIPLE OF SUPERPOSITION

When two or more waves simultaneously pass through a point, the disturbance at the point is given by the sum of the disturbances each wave would produce in absence of the other wave(s).

In general, the principle of superposition is valid for small disturbances only. If the string is stretched too far, the individual displacements do not add to give the resultant displacement. Such waves are called nonlinear waves. In this course, we shall only be talking about linear waves which obey the superposition principle.

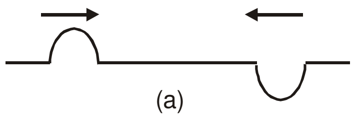
To put this rule in a mathematical form, let $y_1(x, t)$ and $y_2(x, t)$ be the displacements that any element of the string would experience if each wave travelled alone. The displacement $y(x, t)$ of an element of the string when the waves overlap is then given by

$$y(x, t) = y_1(x, t) + y_2(x, t)$$

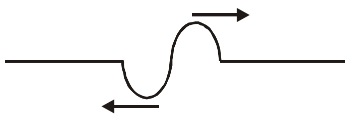
The principal of superposition can also be expressed by stating that overlapping waves algebraically add to produce a resultant wave. The principle implies that the overlapping waves do not in any way alter the travel of each other.

If we have two or more waves moving in the medium the resultant waveform is the sum of wave functions of individual waves.

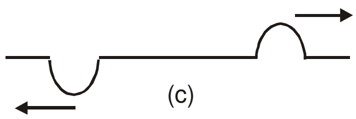
Fig: a sequence of pictures showing two pulses travelling in opposite directions along a stretched string. When the two disturbances overlap they give a complicated pattern as shown in (b). In (c), they have passed each other and proceed unchanged.



(a)



(b)



(c)

An Illustrative examples of this principle is phenomena of interference and reflection of waves.

Solved Example

Example 9. Two waves passing through a region are represented by

$$y_1 = 5 \text{ mm} \sin [(2\pi \text{ cm}^{-1})\, x - (50\,\pi \text{ s}^{-1})\, t]$$

and $$y_2 = 10 \text{ mm} \sin [(\pi \text{ cm}^{-1})x - (100\pi \text{ s}^{-1})\, t].$$

Find the displacement of the particle at x = 1 cm at time t = 5.0 ms.

Solution : According to the principle of superposition, each wave produces its disturbance independent of the other and the resultant disturbance is equal to the vector sum of the individual disturbance.

The displacements of the particle at x = 1 cm at time t = 5.0 ms due to the two waves are,

y_1 = 5 mm sin [(2π cm^{-1}) x – (50 π s^{-1}) t]

y_1 = 5 mm sin [(2π cm^{-1}) × 1cm – (50 π s^{-1}) 5 × 10^{-3} sec]

$$= 5 \text{ mm} \sin\left[2\pi - \frac{\pi}{4}\right] = -5 \text{ mm}$$

and y_2 = 10 mm sin [(π cm^{-1})x – (100π s^{-1}) t].

y_2 = 10 mm sin [(π cm^{-1}) ×1 cm – (100π s^{-1}) 5 × 10^{-3} sec]

$$= 10 \text{ mm} \sin\left[\pi - \frac{\pi}{2}\right] = 10 \text{ mm}$$

The net displacement is : y = y_1 + y_2 = 10 mm – 5 mm = 5 mm

INTERFERENCE OF WAVES GOING IN SAME DIRECTION

Suppose two identical sources send sinusoidal waves of same angular frequency ω in positive x-direction. Also, the wave velocity and hence, the wave number k is same for the two waves. One source may be situated at different points. The two waves arriving at a point then differ in phase. Let the amplitudes of the two waves be A_1 and A_2 and the two waves differ in phase by an angle ϕ. Their equations may be written as

$y_1 = A_1 \sin(kx - \omega t)$

and $y_2 = A_2 \sin(kx - \omega t + \phi)$.

According to the principle of superposition, the resultant wave is represented by

$y = y_1 + y_2 = A_1 \sin(kx - \omega t) + A_2 \sin(kx - \omega t + \phi)$.

we get $y = A \sin(kx - \omega t + \alpha)$

where, $A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\phi}$ (A is amplitude of the resultant wave)

Also, $\tan\alpha = \dfrac{A_2 \sin\phi}{A_1 + A_2\cos\phi}$ (α is phase difference of the resultant wave with the first wave)

Constructive and Destructive Interference

Constructive Interference :

When resultant amplitude A is maximum

$A = A_1 + A_2$

when $\cos\phi = +1$ or $\phi = 2n\pi$

where n is an integer.

Destructive interference :

When resultant amplitude A is minimum

or $A = |A_1 - A_2|$

When $\cos\phi = -1$ or $\phi = (2n + 1)\pi$ where n is an integer.

Solved Example

Example 10. Two sinusoidal waves of the same frequency travel in the same direction along a string. If A_1 = 3.0 cm, A_2 = 4.0 cm, ϕ_1 = 0, and $\phi_2 = \pi/2$ rad, what is the amplitude of the resultant wave?

Solution : Resultant amplitude = $\sqrt{3^2 + 4^2 + 2 \times 3 \times 4 \times \cos 90^\circ}$ = 5 cm.

REFLECTION AND TRANSMISSION OF WAVES

A travelling wave, at a rigid or denser boundary, is reflected with a phase reversal but the reflection at an open boundary (rarer medium) takes place without any phase change. The transmitted wave is never inverted, but propagation constant k is changed.

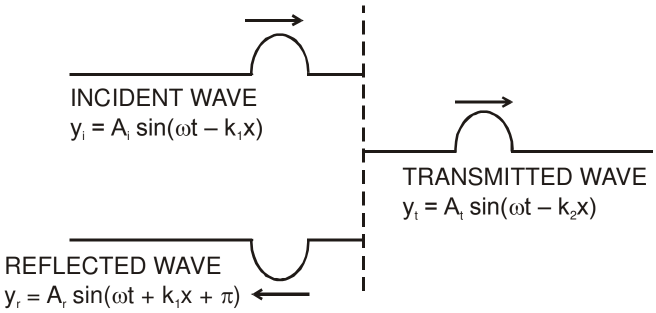


Fig. : Reflection at denser boundary

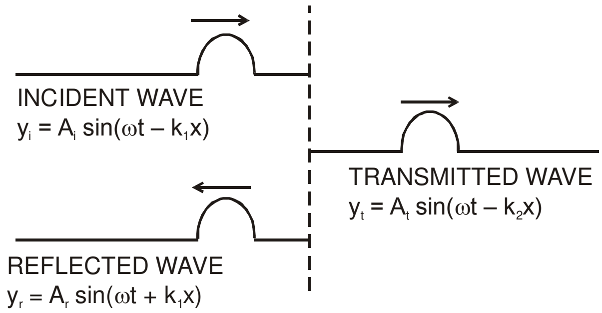


Fig. : Reflection at rarer boundary

Amplitude of reflected and transmitted waves :

v_1 and v_2 are speeds of the incident wave and transmitted wave in mediums respectively then

$$A_r = \frac{v_2 - v_1}{v_1 + v_2} A_i \qquad A_t = \frac{2v_2}{v_1 + v_2} . \ A_i$$

A_r is positive if $v_2 > v_1$, i.e., wave is reflected from a rarer medium.

Solved Examples

Example 11. A harmonic wave is travelling on string 1. At a junction with string 2 it is partly reflected and partly transmitted. The linear mass density of the second string is four times that of the first string, and that the boundary between the two strings is at x = 0. If the expression for the incident wave is, $y_i = A_i \cos(k_1 x - \omega_1 t)$. What are the expressions for the transmitted and the reflected waves in terms of A_i, k_1 and ω_1?

Solution : Since $v = \sqrt{T/\mu}$, $T_2 = T_1$ and $\mu_2 = 4\mu_1$

we have, $v_2 = \dfrac{v_1}{2}$ .....(i)

The frequency does not change, that is,

$\omega_1 = \omega_2$ .....(ii)

Also, because $k = \omega/v$, the wave numbers of the harmonic waves in the two strings are related by,

$k_2 = \dfrac{\omega_2}{v_2} = \dfrac{\omega_1}{v_1/2} = 2\dfrac{\omega_1}{v_1} = 2k_1$ .....(iii)

The amplitudes are,

$$A_t = \left(\frac{2v_2}{v_1+v_2}\right) A_i = \left[\frac{2(v_1/2)}{v_1+(v_1/2)}\right] A_i = \frac{2}{3}\, A_i \qquad .....(iv)$$

and $$A_r = \left(\frac{v_2-v_1}{v_1+v_2}\right) A_i = \left[\frac{(v_1/2)-v_1}{v_1+(v_1/2)}\right] A_i = \frac{A_i}{3} \qquad .....(v)$$

Now with equation (ii), (iii) and (iv), the transmitted wave can be written as,

$$y_t = \frac{2}{3}\, A_i \cos(2k_1 x - \omega_1 t)$$ **Ans.**

Similarly the reflected wave can be expressed as,

$$= \frac{A_i}{3} \cos(k_1 x + \omega_1 t + \pi)$$ **Ans.**

STANDING WAVES :

Suppose two sine waves of equal amplitude and frequency propagate on a long string in opposite directions. The equations of the two waves are given by

$y_1 = A \sin(\omega t - kx)$ and $y_2 = A \sin(\omega t + kx + \phi)$.

These waves interfere to produce what we call standing waves. To understand these waves, let us discuss the special case when $\phi = 0$.

The resultant displacements of the particles of the string are given by the principle of superposition as

$y = y_1 + y_2$

$= A[\sin(\omega t - kx) + \sin(\omega t + kx)] = 2A \sin \omega t \cos kx$

or, $y = (2A \cos kx) \sin \omega t$.

This is the required result and from this it is clear that :

1. As this equation satisfies the wave equation, $\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$

it represents a wave. However, as it is not of the form f(ax ± bt), the wave is not travelling and so is called standing or stationary wave.

2. The amplitude of the wave $A_s = 2A \cos kx$ is not constant but varies periodically with position (and not with time as in beats).

3. The points for which amplitude is minimum are called nodes and for these

$\cos kx = 0$, i.e., $kx = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}$

i.e., $x = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}, ...$ $\left[\text{as} \quad k = \frac{2\pi}{\lambda}\right]$

i.e., in a stationary wave, nodes are equally spaced.

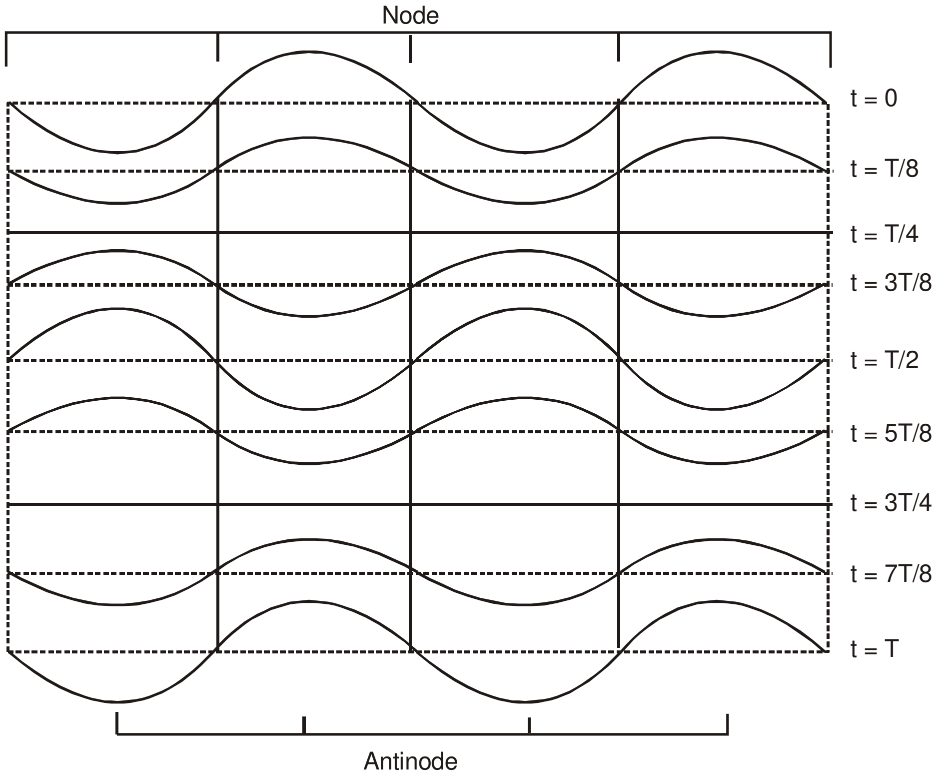
4. The points for which amplitude is maximum are called antinodes and for these,

$\cos kx = \pm 1$, i.e., $kx = 0, \pi, 2\pi, 3\pi, ...$

i.e., $x = 0, \frac{\lambda}{2}, \frac{2\lambda}{2}, \frac{3\lambda}{2}, ...$ $\left[\text{as} \quad k = \frac{2\pi}{\lambda}\right]$

i.e., like nodes, antinodes are also equally spaced with spacing ($\lambda/2$) and $A_{max} = \pm 2A$. Furthermore, nodes and antinodes are alternate with spacing ($\lambda/4$).

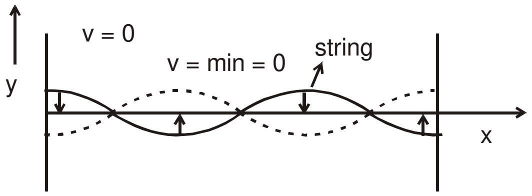
5. The nodes divide the medium into segments (or loops). All the particles in a segment vibrate in same phase, but in opposite phase with the particles in the adjacent segment. Twice in one period all the particles pass through their mean position simultaneously with maximum velocity ($A_s\omega$), the direction of motion being reversed after each half cycle.



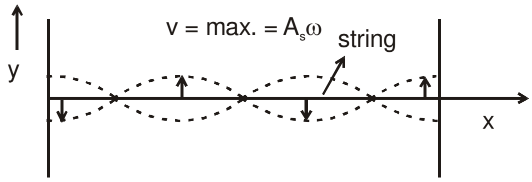
(a)

6. Standing waves can be transverse or longitudinal, e.g., in strings (under tension) if reflected wave exists, the waves are transverse-stationary, while in organ pipes waves are longitudinal-stationary.

7. As in stationary waves nodes are permanently at rest, so no energy can be transmitted across them, i.e., energy of one region (segment) is confined in that region. However, this energy oscillates between elastic potential energy and kinetic energy of the particles of the medium. When all the particles are at their extreme positions KE is minimum while elastic PE is maximum (as shown in figure A), and when all the particles (simultaneously) pass through their mean position KE will be maximum while elastic PE minimum (Figure B). The total energy confined in a segment (elastic PE + KE), always remains the same.



Elastic PE = max = E
Kinetic energy = min = 0
(A)



Elastic PE = min = 0
Kinetic energy = max = E
(B)

Solved Examples

Example 12. Two waves travelling in opposite directions produce a standing wave. The individual wave functions are

$y_1 = (4.0 \text{ cm}) \sin(3.0x - 2.0t)$

$y_2 = (4.0 \text{ cm}) \sin(3.0x + 2.0t)$

where x and y are in centimeter.

(a) Find the maximum displacement of a particle of the medium at x = 2.3 cm.

(b) Find the position of the nodes and antinodes.

Solution : (a) When the two waves are summed, the result is a standing wave whose mathematical representation is given by Equation, with A = 4.0 cm and k = 3.0 rad/cm;

$$y = (2A \sin kx) \cos \omega t = [(8.0 \text{ cm}) \sin 3.0\,x] \cos 2.0\,t$$

Thus, the maximum displacement of a particle at the position x = 2.3 cm is

$$y_{max} = [(8.0 \text{ cm}) \sin 3.0x]_{x = 2.3 \text{ cm}} = (8.0 \text{ m}) \sin (6.9 \text{ rad}) = 4.6 \text{ cm}$$

(b) Because $k = 2\pi/\lambda = 3.0$ rad/cm, we see that $\lambda = 2\pi/3$cm. Therefore, the antinodes are located at

$$x = n\left(\frac{\pi}{6.0}\right) \text{ cm} \qquad (n = 1, 3, 5, ...)$$

and the nodes are located at

$$x = n\,\frac{\lambda}{2}\left(\frac{\pi}{3.0}\right)\text{cm} \quad (n = 1, 2, 3, ...)$$

Example 13. Two travelling waves of equal amplitudes and equal frequencies move in opposite direction along a string. They interfere to produce a standing wave having the equation.

$y = A \cos kx \sin \omega t$

in which A = 1.0 mm, $k = 1.57 \text{ cm}^{-1}$ and $\omega = 78.5 \text{ s}^{-1}$. (a) Find the velocity and amplitude of the component travelling waves. (b) Find the node closest to the origin in the region x > 0. (c) Find the antinode closest to the origin in the region x > 0. (d) Find the amplitude of the particle at x = 2.33 cm.

Solution : (a) The standing wave is formed by the superposition of the waves

$$y_1 = \frac{A}{2} \sin (\omega t - kx) \text{ and } y_2 = \frac{A}{2} \sin (\omega t + kx).$$

The wave velocity (magnitude) of either of the waves is

$$v = \frac{\omega}{k} = \frac{78.5 \text{ s}^{-1}}{1.57 \text{ cm}^{-1}} = 50 \text{ cm/s; Amplitude} = 0.5 \text{ mm}.$$

(b) For a node, cos kx = 0.

The smallest positive x satisfying this relation is given by

$$kx = \pi/2 \qquad \text{or, } x = \frac{\pi}{2k} = \frac{3.14}{2 \times 1.57 \text{ cm}^{-1}} = 1 \text{ cm}$$

(c) For an antinode, $|\cos kx| = 1$.

The smallest positive x satisfying this relation is given by

$$kx = \pi \qquad \text{or,} \quad x = \frac{\pi}{k} = 2 \text{ cm}$$

(d) The amplitude of vibration of the particle at x is given by $|A \cos kx|$. For the given point,

$$kx = (1.57 \text{ cm}^{-1})(2.33 \text{ cm}) = \frac{7}{6}\pi = \pi + \frac{\pi}{6}.$$

Thus, the amplitude will be $(1.0 \text{ mm}) \,|\cos(\pi + \pi/6)| = \frac{\sqrt{3}}{3}$ mm = 0.86 mm.

VIBRATION OF STRING :

(a) Fixed at both ends : Suppose a string of length L is kept fixed at the ends x = 0 and x = L. In such a system suppose we send a continuous sinusoidal wave of a certain frequency, say, toward the right. When the wave reaches the right end. It gets reflected and begins to travel back. The left-going wave then overlaps the wave, which is still travelling to the right. When the left-going wave reaches the left end, it gets reflected again and the newly reflected wave begins to travel to the right. overlapping the left-going wave. This process will continue and, therefore, very soon we have many overlapping waves, which interfere with one another. In such a system, at any point x and at any time t, there are always two waves, one moving to the left and another to the right. We, therefore, have

$y_1(x, t) = y_m \sin(kx - \omega t)$ (wave travelling in the positive direction of x-axis)

and $y_2(x, t) = y_m \sin(kx + \omega t)$ (wave travelling in the negative direction of x-axis).

The principle of superposition gives, for the combined wave

$$y'(x, t) = y_1(x, t) + y_2(x, t)$$
$$= y_m \sin(kx - wt) + y_m \sin(kx + \omega t)$$
$$= (2y_m \sin kx) \cos \omega t$$

It is seen that the points of maximum or minimum amplitude stay at one position.

Nodes : The amplitude is zero for values of kx that give sin kx = 0 i.e. for,

$kx = n\pi$, for n = 0, 1, 2, 3,.....

Substituting $k = 2\pi/\lambda$ in this equation, we get

$x = n\frac{\lambda}{2}$, for n = 0, 1, 2, 3,.....

The positions of zero amplitude are called the **nodes.** Note that a distance of $\frac{\lambda}{2}$ or half a wavelength separates two consecutive nodes.

Antinodes : The amplitude has a maximum value of $2y_m$, which occurs for the values of kx that give $|\sin kx| = 1$. Those values are

$kx = (n + 1/2)\pi$ for n = 0, 1, 2, 3,....

Substituting $k = 2\pi/\lambda$ in this equation, we get.

$x = (n + 1/2)\frac{\lambda}{2}$ for n = 0, 1, 2, 3,....

as the positions of maximum amplitude. These are called the **antinodes**. The antinodes are separated by $\lambda/2$ and are located half way between pairs of nodes.

For a stretched string of length L, fixed at both ends, the two ends of the ends is chosen as position x = 0, then the other end is x = L. In order that this end is a node; the length L must satisfy the condition

$$L = n\frac{\lambda}{2}, \text{ for } n = 1, 2, 3,....$$

This condition shows that standing waves on a string of length L have restricted wavelength given by

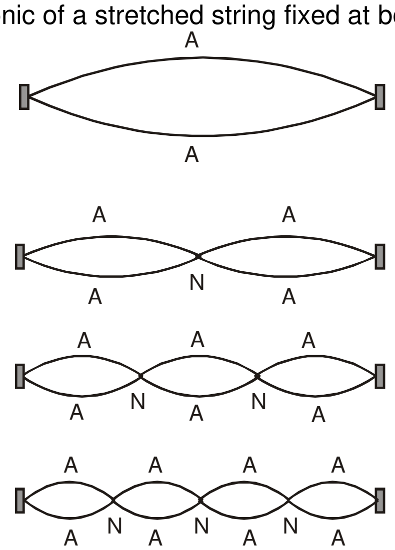
$$\lambda = \frac{2L}{n}, \text{ for } n = 1, 2, 3,.....$$

The frequencies corresponding to these wavelengths follow from Eq. as

$$f = n\,\frac{v}{2L}, \text{ for } n = 1, 2, 3,.....$$

where v is the speed of travelling waves on the string. The set of frequencies given by equation are called the natural frequencies or **modes** of oscillation of the system. This equation tells us that the natural frequencies of a string are integral multiples of the lowest frequency $f = \frac{v}{2L}$, which corresponds to n = 1. The oscillation mode with that lowest frequency is called the fundamental mode or the first harmonic. The second harmonic or first overtone is the oscillation mode with n = 2. The third harmonic and second overtone corresponds to n = 3 and so on. The frequencies associated with these modes are often labeled as ν_1, ν_2, ν_3 and so on. The collection of all possible modes is called the harmonic series and n is called the harmonic number.

Some of the harmonic of a stretched string fixed at both the ends are shown in figure.



(a) Fundamental or first harmonic $f_0 = v/2L$

(b) second harmonic or first overtone = $f_1 = 2f_0 = 2v/2L$

(c) third harmonic or 2nd overtone = $f_2 = 3f_0, = 3v/2L$

(d) fourth harmonic or 3rd overtone = $f_3 = 4f_0, = 4v/2L$

Solved Examples

Example 14. A middle C string on a piano has a fundamental frequency of 262 Hz, and the A note has fundamental frequency of 440 Hz. (a) Calculate the frequencies of the next two harmonics of the C string. (b) If the strings for the A and C notes are assumed to have the same mass per unit length and the same length, determine the ratio of tensions in the two strings.

Solution : (a) Because f_1 = 262 Hz for the C string, we can use Equation to find the frequencies f_2 and f_3;

$f_2 = 2f_1 = 524$ Hz

$f_3 = 3f_1 = 786$ Hz

Using Equation for the two strings vibrating at their fundamental frequencies gives

$$f_{1A} = \frac{1}{2L}\sqrt{\frac{T_A}{\mu}} \Rightarrow f_{1C} = \frac{1}{2L}\sqrt{\frac{T_C}{\mu}}$$

$$\therefore \frac{f_{1A}}{f_{1C}} = \sqrt{\frac{T_A}{T_C}} \Rightarrow \frac{T_A}{T_C} = \left(\frac{f_{1A}}{f_{1C}}\right)^2 = \left(\frac{440\ \text{Hz}}{262\ \text{Hz}}\right)^2 = 2.82. \quad \textbf{Ans.}$$

Example 15. A wire having a linear mass density 10^{-3} kg/m is stretched between two rigid supports with a tension of 90 N. The wire resonates at a frequency of 350 Hz. The next higher frequency at which the same wire resonates is 420 Hz. Find the length of the wire.

Solution : Suppose the wire vibrates at 350 Hz in its nth harmonic and at 420 Hz in its (n + 1)th harmonic.

$$350\ s^{-1} = \frac{n}{2\ L}\sqrt{\frac{F}{\mu}} \qquad .....(i)$$

and $$420\ s^{-1} = \frac{(n+1)}{2\ L}\sqrt{\frac{F}{\mu}} \qquad .....(ii)$$

This gives $\frac{420}{350} = \frac{n+1}{n}$ or, n = 5.

Putting the value in (i),

$$350 = \frac{5}{2\ell}\sqrt{\frac{90}{10^{-3}}} \Rightarrow 350 = \frac{5}{2\ell} \times 300 \qquad \Rightarrow \qquad \ell = \frac{1500}{700} = \frac{15}{7}\ m = 2.1\ m$$

(b) Fixed at one end : Standing waves can be produced on a string which is fixed at one end and whose other end is free to move in a transverse direction. Such a free end can be nearly achieved by connecting the string to a very light thread.

If the vibrations are produced by a source of “correct” frequency, standing waves are produced. If the end x = 0 is fixed and x = L is free, the equation is again given by

$$y = 2A \sin kx \cos \omega t$$

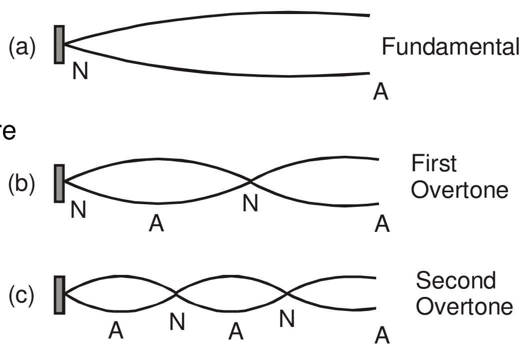
with the boundary condition that x = L is an antinode. The boundary condition that x = 0 is a node is automatically satisfied by the above equation. For x = L to be an antinode,

$$\sin kL = \pm 1$$

or, $kL = \left(n+\frac{1}{2}\right)\pi$ or, $\frac{2\pi L}{\lambda} = \left(n+\frac{1}{2}\right)\pi$

or, $\frac{2Lf}{v} = n + \frac{1}{2}$ or, $f = \left(n+\frac{1}{2}\right)\frac{v}{2\ L} = \frac{n+\frac{1}{2}}{2\ L}\sqrt{T/\mu}$ .....

These are the normal frequencies of vibration. The fundamental frequency is obtained when n = 0, i.e.,



$f_0 = v/4L$

The overtone frequencies are

$$f_1 = \frac{3v}{4L} = 3f_0$$

$$f_2 = \frac{5v}{4L} = 5f_0$$

We see that all the harmonic of the fundamental are not the allowed frequencies for the standing waves. Only the odd harmonics are the overtones. Figure shows shapes of the string for some of the normal modes.

LAWS OF TRANSVERSE VIBRATIONS OF A STRING - SONOMETER WIRE

(a) Law of length $f \,\alpha\, \frac{1}{L}$ so $\frac{f_1}{f_2} = \frac{L_2}{L_1}$; if T & μ are constant

(b) Law of tension $f \,\alpha\, \sqrt{T}$ so $\frac{f_1}{f_2} = \sqrt{\frac{T_1}{T_2}}$; L & μ are constant

(c) Law of mass $f \,\alpha\, \frac{1}{\sqrt{\mu}}$ so $\frac{f_1}{f_2} = \sqrt{\frac{\mu_2}{\mu_1}}$; T & L are constant

Solved Miscellaneous Problems

Problem 1. A wave pulse moving along the x axis is represented by the wave function $y(x, t) = \frac{2}{(x-3t)^2+1}$

where x and y are measured in cm and t is in seconds.

(i) In which direction is the wave moving?

(ii) Find speed of the wave.

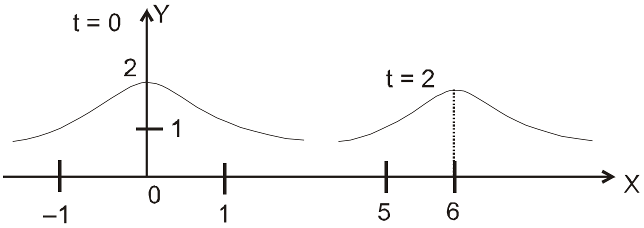
(iii) Plot the waveform at t = 0, t = 2s.

Solution : $y = \frac{2}{(x-3t)^2+1}$

(i) As wave is moving in +ve x direction because y = (x, t) = f(t – x/v) = f/v(vt – x)

(ii) Now x – v t is compared with x – 3t

$\therefore$ v = 3 cm/sec.



Ans. (i) Positive x axis (ii) 3 cm/s.

Problem 2. At t = 0, a transverse wave pulse in a wire is described by the function $y = \frac{6}{x^3+3}$ where x and y are in meters write the function y(x, t) that describes this wave if it is travelling in the positive x direction with a speed of 4.5 m/s.

Solution : $y = \frac{6}{x^2+3} = f(x)$

As $y(x, t) = f(x - vt) = \frac{6}{(x-4.5t)^2+3}$

Ans. $\frac{6}{(x-4.5t)^2+3}$

Problem 3. The wave function for a travelling wave on a string is given as

$$y(x, t) = (0.350 \text{ m}) \sin (10\pi t - 3\pi x + \frac{\pi}{4})$$

(a) What are the speed and direction of travel of the wave?
(b) What is the vertical displacement of the string at t = 0, x = 0.1 m?
(c) What are wavelength and frequency of the wave?

Solution : $Y(x, t) = (0.350\text{m}) \sin (10\pi t - 3\pi x + \frac{\pi}{4})$

comparing with equation ;

$Y = A \sin (\omega t - kx + \phi)$ $\omega = 10\pi$, $k = 3\pi$, $f = \frac{\pi}{4}$

(a) speed $= \frac{\omega}{k} = \frac{10}{3} = 3.33$ m/sec and along +ve x axis

(b) $y(0.1, 0) = 0.35 \sin (10\pi \times O - 3\pi (0.1) + \frac{\pi}{4}) = 0.35 \sin\left[\frac{\pi}{4} - \frac{3\pi}{10}\right] = -5.48$ cm

(c) $k = \frac{2\pi}{\lambda} = 3\pi \Rightarrow \lambda = \frac{2}{3}$ cm = 0.67 cm and $f = \frac{v}{\lambda} = \frac{10/3}{2/3} = 5$ Hz.

Problem 4. Show that the wave function $y = e^{b(x - vt)}$ is a solution of the linear wave equation.

Solution : $Y = e^{b(x-v t)}$ $\frac{\partial y}{\partial x} = be^{b(x-v t)}$ and $\frac{\partial y}{\partial t} = (bv)e^{b(xv- v t)}$

$\frac{\partial^2 y}{\partial x^2} = b^2 e^{b(x-v t)}$ and $\frac{\partial^2 y}{\partial t^2} = (bv)^2 e^{b(x - v t)}$

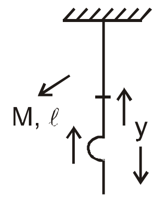
obviously ; $\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$ which is a Linear wave equation.

Problem 5. A uniform rope of mass m and length L hangs from a ceiling.

(a) Show that the speed of a transverse wave on the rope is a function of y, the distance from the lower end, and is given by $v = \sqrt{gy}$.

(b) Show that the time a transverse wave takes to travel the length of the rope is given by $t = 2\sqrt{L/g}$.

Solution : (a) As mass per unit length



$\mu = \frac{m}{\ell}$

$\therefore$ $V = \sqrt{\frac{T}{\mu}}$

$\therefore$ Tension at P = μyg

$\therefore$ $V = \sqrt{\frac{\mu yg}{\mu}} = \sqrt{yg}$

(b) Now $\frac{dy}{dt} = \sqrt{yg}$

$$\int_O^{\ell} \frac{dy}{\sqrt{y}} = \sqrt{g}\int_o^t dt$$

$t = 2\sqrt{\ell/g}$

Problem 6. Two sinusoidal waves of the same frequency are to be sent in the same direction along a taut string. One wave has an amplitude of 5.0 mm, the other 8.0 mm.

(a) What phase difference ϕ_1 between the two waves results in the smallest amplitude of the resultant wave?

(b) What is that smallest amplitude?

(c) What phase difference ϕ_2 results in the largest amplitude of the resultant wave?

(d) What is that largest amplitude?

(e) What is the resultant amplitude if the phase angle is $(\phi_1 - \phi_2)/2$?

Solution : (a) For smallest amplitude ;

$A_R = |A_1 - A_2|$ and that is possible when $\phi_1 = \pi$ between A_1 and A_2

(b) $A_R = |A_1 - A_2| = 3$ mm

(c) for largest amplitude ;

$A_R = |A_1 + A_2|$ and that is possible when $\phi_2 = 0$ between A_1 and A_2

(d) $A_R = |A_1 + A_2| = 13$ mm

(e) when $\phi = \dfrac{\phi_1 - \phi_2}{2} = \dfrac{\pi - o}{2} = \dfrac{\pi}{2}$

$\therefore \quad A_R = [A_1^2 + A_2^2 + 2A_1 A_2 \cos \dfrac{\pi}{2}]^{1/2} = 9.4$ mm

Ans. (a) π rad; (b) 3.0 mm; (c) 0 rad; (d) 13 mm; (e) 9.4 mm

Problem 7. A string fixed at both ends is 8.40 m long and has a mass of 0.120 kg. It is subjected to a tension of 96.0 N and set oscillating. (a) What is the speed of the waves on the string? (b) What is the longest possible wavelength for a standing wave? (c) Give the frequency of the wave.

Solution : (a) $V = \sqrt{\dfrac{T}{\mu}} = \sqrt{\dfrac{96}{\left(\dfrac{0.12}{8.4}\right)}} = 82$ m/sec.

(b) for longest possible wavelength ; $\dfrac{\lambda}{2} = \ell$

$\lambda = 2\ell = 2 \times 8.4 = 16.8$ m

(c) $V = f\lambda \Rightarrow f = \dfrac{v}{\lambda} = \dfrac{82}{16.8} = 4.88$ HZ.

Ans. (a) 82.0 m/s, (b) 16.8 m, (c) 4.88 Hz.

Exercise-1

✍ **Marked Questions can be used as Revision Questions.**

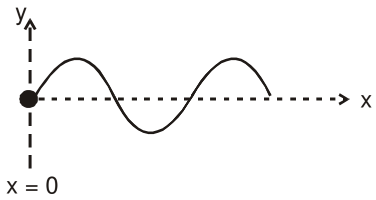
PART - I : SUBJECTIVE QUESTIONS

Section (A) : Equation of travelling wave (Including sine Wave)

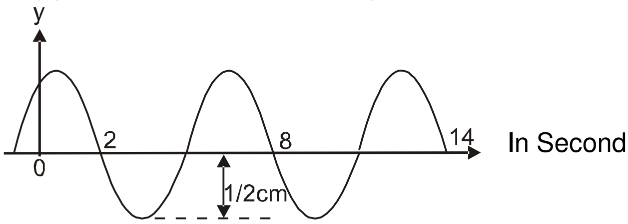
A-1. Consider the wave $y = (10 \text{ mm}) \sin [(5\pi \text{ cm}^{-1})\, x - (60\pi \text{ s}^{-1})\, t + \frac{\pi}{4}]$. Find (a) the amplitude (b) the wave number (c) the wavelength (d) the frequency (e) the time period and (f) the wave velocity (g) phase constant of SHM of praticle at x = 0.

A-2. Two identical traveling waves, moving in the same direction are out of phase by $\pi/2$ rad. What is the amplitude of the resultant wave in terms of the common amplitude y_m of the two combining waves?

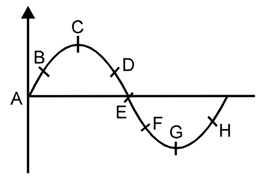
A-3. The string shown in figure is driven at a frequency of 5.00 Hz. The amplitude of the motion is 12.0 cm, and the wave speed is 20.0 m/s. Furthermore, the wave is such that y = 0 at x = 0 and t = 0. Determine (a) the angular frequency and (b) wave number for this wave (c) Write an expression for the wave function. Calculate (d) the maximum transverse speed and (e) the maximum transverse acceleration of a point on the string.



A-4.✍ The sketch in the figure shows displacement time curve of a sinusoidal wave at x = 8 m. Taking velocity of wave v = 6m/s along positive x-axis, write the equation of the wave.



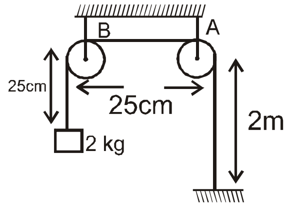
A-5. A transverse wave is travelling along a string from left to right. The fig. represents the shape of the string (snap-shot) at a given instant. At this instant
(a) which points have an upward velocity
(b) which points have downward velocity
(c) which points have zero velocity
(d) which points have maximum magnitude of velocity.



Section (B) : Speed of a wave on a string

B-1. A piano string having a mass per unit length equal to 5.00×10^{-3} kg/m is under a tension of 1350 N. Find the speed with which a wave travels on this string.

B-2.✍ In the arrangement shown in figure, the string has mass of 5g. How much time will it take for a transverse disturbance produced at the floor to reach the pulley A ? Take g = 10 m/s^2.



B-3. A uniform rope of length 20 m and mass 8 kg hangs vertically from a rigid support. A block of mass 2 kg is attached to the free end of the rope. A transverse pulse of wavelength 0.06 m is produced at the lower end of the rope. What is the wavelength of the pulse when it reaches the top of the rope?

B-4. A particle on a stretched string supporting a travelling wave, takes minimum time 5.0 ms to move from its mean position to the mean position. The distance between two consecutive particles, which are at their mean positions, is 2.0 cm. Find the frequency, the wavelength and the wave speed.

B-5. Two wires of different densities but same area of cross-section are soldered together at one end and are stretched to a tension T. The velocity of a transverse wave in the first wire is half of that in the second wire. Find the ratio of the density of the first wire to that of the second wire.

B-6. A 4.0 kg block is suspended from the ceiling of an elevator through a string having a linear mass density of 1.6×10^{-3} kg/m. Find (a) speed (with respect to the string) with which a wave pulse can proceed on the string if the elevator accelerates up at the rate of 6 m/s^2. (b) In part (a) at some instant speed of lift is 40 m/s in upward direction, then speed of the wave pulse with respect to ground at that instant is (Take g = 10 m/s^2)

Section (C) : Power transmitted along the string

C-1. A 6.00 m segment of a long string has a mass of 180 g. A high-speed photograph shows that the segment contains four complete cycles of a wave. The string is vibrating sinusoidally with a frequency of 50.0 Hz and a peak-to-valley displacement of 15.0 cm. (The "peak-to-valley" displacement is the vertical distance from the farthest positive displacement to the farthest negative displacement.) (a) Write the function that describes this wave traveling in the positive x direction. (b) Determine the average power being supplied to the string.

C-2. A transverse wave of amplitude 5 mm and frequency 10 Hz is produced on a wire stretched to a tension of 100 N. If the wave speed is 100 m/s, what average power is the source transmitting to the wire? ($\pi^2 = 10$)

Section (D) : Interference, Reflection, Transmission

D-1. The equation of a plane wave travelling along positive direction of x-axis is $y = a\sin\frac{2\pi}{\lambda}(\nu t - x)$ When this wave is reflected at a rigid surface and its amplitude becomes 80%, then find the equation of the reflected wave

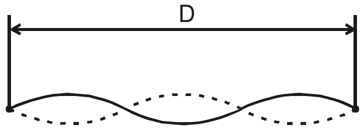
D-2. A series of pulses, each of amplitude 0.150 m, are sent on a string that is attached to a wall at one end. The pulses are reflected at the wall and travel back along the string without loss of amplitude. When two waves are present on the same string. The net displacement of a given point is the sum of the displacements of the individual waves at the point. What is the net displacement at point on the string where two pulses are crossing, (a) if the string is rigidly attached to the post ? (b) If the end at which reflection occurs is free to slide up and down?

D-3. Two waves, each having a frequency of 100 Hz and a wavelength of 2 cm, are travelling in the same direction on a string. What is the phase difference between the waves (a) if the second wave was produced 10 m sec later than the first one at the same place (b) if the two waves were produced at a distance 1 cm behind the second one? (c) If each of the waves has an amplitude of 2.0 mm, what would be the amplitudes of the resultant waves in part (a) and (b)?

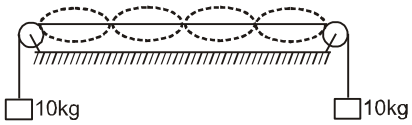
Section (E) : Standing waves and Resonance

E-1. What are (a) the lowest frequency (b) the second lowest frequency and (c) the third lowest frequency for standing waves on a wire that is 10.0 m long has a mass of 100 g and is stretched under a tension of 25 N which is fixed at both ends ?

E-2. A nylon guitar string has a linear density of 7.20 g/m and is under a tension of 150 N. The fixed supports are distance D = 90.0 cm apart. The string is oscillating in the standing wave pattern shown in figure. Calculate the (a) speed. (b) wavelength and (c) frequency of the traveling waves whose superposition gives this standing wave.



E-3. The length of the wire shown in figure between the pulleys is 1.5 m and its mass is 15 g. Find the frequency of vibration with which the wire vibrates in four loops leaving the middle point of the wire between the pulleys at rest. ($g = 10\ m/s^2$)

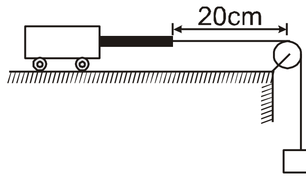


E-4. A string oscillates according to the equation $y' = (0.50\ cm)\sin\left[\left(\frac{\pi}{3}cm^{-1}\right)x\right]\cos[(40\,\pi\ s^{-1})t]$. What are the (a) amplitude and (b) speed of the two waves (identical except for direction of travel) whose superposition gives this oscillation? (c) What is the distance between nodes? (d) What is the transverse speed of a particle of the string at the position x = 1.5 cm when $t = \frac{9}{8}$ s ?

E-5. A string vibrates in 4 loops with a frequency of 400 Hz.
(a) What is its fundamental frequency ?
(b) What frequency will cause it to vibrate into 7 loops.

E-6. The vibration of a string of length 60 cm is represented by the equation, $y = 3\cos(\pi x/20)\cos(72\pi t)$ where x & y are in cm and t in sec.
(i) Write down the component waves whose superposition gives the above wave.
(ii) Where are the nodes and antinodes located along the string.
(iii) What is the velocity of the particle of the string at the position x = 5 cm & t = 0.25 sec.

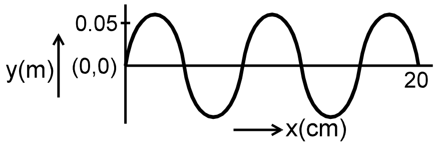
E-7. A heavy string is tied at one end to a movable support and to a light thread at the other end as shown in figure. The thread goes over a fixed pulley and supports a weight to produce a tension. The lowest frequency with which the heavy string resonates is 120 Hz. If the movable support is pushed to the right by 20 cm so that the joint is placed on the pulley, what will be the minimum frequency at which the heavy string can resonate?



PART - II : ONLY ONE OPTION CORRECT TYPE

Section (A) : Equation of travelling wave (Including sine Wave)

A-1. For the wave shown in figure, the equation for the wave, travelling along +x axis with velocity 350 ms^{-1} when its position at t = 0 is as shown



(A) $0.05\sin\left(\frac{314}{4}x - 27475\,t\right)$
(B) $0.05\sin\left(\frac{379}{5}x - 27475\,t\right)$
(C) $1\sin\left(\frac{314}{4}x - 27475\,t\right)$
(D) $0.05\sin\left(\frac{289}{5}x + 27475\,t\right)$

A-2. The displacement of a wave disturbance propagating in the positive x-direction is given by $y = 1/(1 + x^2)$ at time t = 0 and $y = 1/[1 + (x - 1)^2]$ at t = 2 seconds where x and y are in meters. The shape of the wave disturbance does not change during the propagation. The velocity of the wave is: **[JEE - 1990]**
(A) 2.5 m/s (B) 0.25 m/s (C) 0.5 m/s (D) 5 m/s

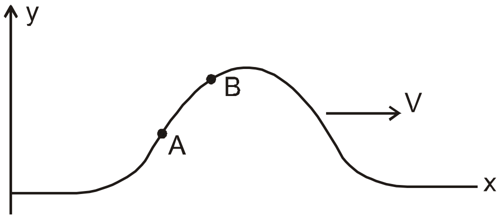
A-3. A transverse wave is described by the equation $Y = Y_0 \sin 2\pi\,(ft - x/\lambda)$. The maximum particle velocity is equal to four times the wave velocity if **[JEE - 1984]**
(A) $\lambda = \pi Y_0/4$ (B) $\lambda = \pi Y_0/2$ (C) $\lambda = \pi Y_0$ (D) $\lambda = 2\pi Y_0$

A-4. A travelling wave on a string is given by $y = A \sin [\alpha x + \beta t + \frac{\pi}{6}]$. If $\alpha = 0.56$ /cm, $\beta = 12$/sec, $A = 7.5$ cm, then position and velocity of particle at x = 1 cm and t = 1s is
(A) 4.6 cm, 46.5 cm s^{-1} (B) 3.75 cm, 77.94 cm s^{-1}
(C) 1.76 cm, 7.5 cm s^{-1} (D) 7.5 cm, 75 cm s^{-1}

A-5. A transverse wave of amplitude 0.50m, wavelength 1m and frequency 2 Hz is propagating on a string in the negative x-direction. The expression of the wave is **[REE - 1989]**
(A) $y(x, t) = 0.5 \sin (2\pi x - 4\pi t)$ (B) $y(x, t) = 0.5 \cos (2\pi x + 4\pi t)$
(C) $y(x, t) = 0.5 \sin (\pi x - 2\pi t)$ (D) $y(x, t) = 0.5 \cos (2\pi x - 2\pi t)$

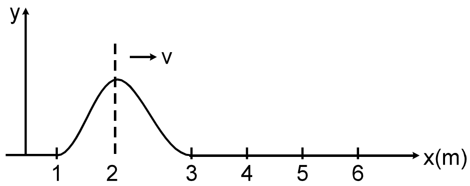
A-6. Two small boats are 10m apart on a lake. Each pops up and down with a period of 4.0 seconds due to wave motion on the surface of water. When one boat is at its highest point, the other boat is at its lowest point. Both boats are always within a single cycle of the waves. The speed of the waves is
(A) 2.5 m/s (B) 5.0 m/s (C) 14 m/s (D) 40 m/s

A-7. A wave pulse is generated in a string that lies along x-axis. At the points A and B, as shown in figure, if R_A and R_B are ratio of wave speed to the particle speed respectively then:



(A) $R_A > R_B$ (B) $R_B > R_A$
(C) $R_A = R_B$
(D) Information is not sufficient to decide.

A-8. Wave pulse on a string shown in figure is moving to the right without changing shape. Consider two particles at positions $x_1 = 1.5$ m and $x_2 = 2.5$ m. Their transverse velocities at the moment shown in figure are along directions:

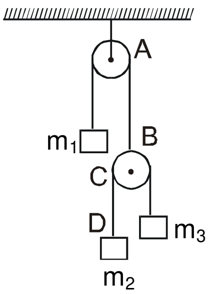


(A) positive y–axis and positive y–axis respectively
(B) negative y–axis and positive y–axis respectively
(C) positive y–axis and negative y–axis respectively
(D) negative y–axis and negative y–axis respectively

A-9. An observer standing at the sea-coast observes 54 waves reaching the coast per minute. If the wavelength of wave is 10m, The velocity of wave is **[REE - 1979]**
(A) 19 m/sec (B) 29 m/sec (C) 9 m/sec (D) 39 m/sec

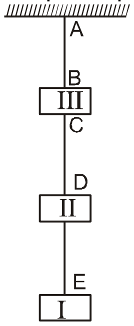
Section (B) : Speed of a wave on a string

B-1. Both the strings, shown in figure, are made of same material and have same cross-section. The pulleys are light. The wave speed of a transverse wave in the string AB is v_1 and in CD it is v_2. The v_1/v_2 is



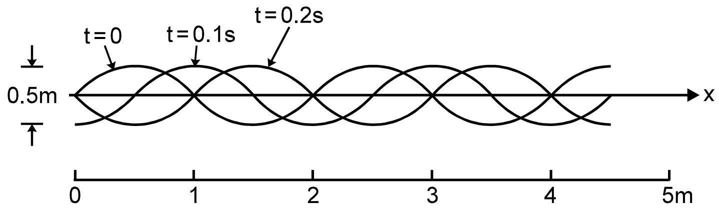
(A) 1 (B) 2 (C) $\sqrt{2}$ (D) $\frac{1}{\sqrt{2}}$

B-2. Three blocks I, II, & III having mass of 1.6 kg, 1.6 kg and 3.2 kg respectively are connected as shown in the figure. The linear mass density of the wire AB, CD and DE are 10 g/m, 8 g/m and 10 g/m respectively. The speed of a transverse wave pulse produced in AB, CD and DE are : ($g = 10$ m/sec^2)



(A) 80 m/s, $20\sqrt{10}$ m/s, 40 m/s (B) $20\sqrt{10}$ m/s, 40 m/s, 80 m/s
(C) $20\sqrt{10}$ m/s in all (D) 80 m/s in all

B-3. Three consecutive flash photographs of a travelling wave on a string are reproduced in the figure here. The following observations are made. Mark the one which is correct. (Mass per unit length of the string = 3 g/cm.)



(A) displacement amplitude of the wave is 0.25 m, wavelength is 1 m, wave speed is 2.5 m/s and the frequency of the driving force is 0.2/s.
(B) displacement amplitude of the wave is 2.0 m, wavelength is 2 m, wave speed is 0.4 m/s and the frequency of the driving force is 0.7/s.
(C) displacement amplitude of the wave is 0.25 m, wavelength is 2 m, wave speed is 5 m/s and the frequency of the driving force is 2.5 /s.
(D) displacement amplitude of the wave is 0.5 m, wavelength is 2 m, wave speed is 2.5 m/s and the frequency of the driving force is 0.2/s.

B-4. A heavy ball is suspended from the ceiling of a motor car through a light string. A transverse pulse travels at a speed of 50 cm/s on the string when the car is at rest and 52 cm/s when the car accelerates on a horizontal road. Then acceleration of the car is : (Take $g = 10$ m/s^2.)
(A) 2.7 m/s^2 (B) 3.7 m/s^2 (C) 2.4 m/s^2 (D) 4.1 m/s^2

Section (C) : Power transmitted along the string

C-1. A wave moving with constant speed on a uniform string passes the point $x = 0$ with amplitude A_0, angular frequency ω_0 and average rate of energy transfer P_0. As the wave travels down the string it gradually loses energy and at the point $x = \ell$, the average rate of energy transfer becomes $\frac{P_0}{2}$. At the point $x = \ell$, angular frequency and amplitude are respectively :
(A) ω_0 and $A_0/\sqrt{2}$ (B) $\omega_0/\sqrt{2}$ and A_0 (C) less than ω_0 and A_0 (D) $\omega_0/\sqrt{2}$ and $A_0/\sqrt{2}$

C-2. A sinusoidal wave with amplitude y_m is travelling with speed V on a string with linear density ρ. The angular frequency of the wave is ω. The following conclusions are drawn. Mark the one which is correct.
(A) doubling the frequency doubles the rate at which energy is carried along the string
(B) if the amplitude were doubled, the rate at which energy is carried would be halved
(C) if the amplitude were doubled, the rate at which energy is carried would be doubled
(D) the rate at which energy is carried is directly proportional to the velocity of the wave.

C-3. Sinusoidal waves 5.00 cm in amplitude are to be transmitted along a string having a linear mass density equal to 4.00×10^{-2} kg/m. If the source can deliver a average power of 90 W and the string is under a tension of 100 N, then the frequency at which the source can operate is (take $\pi^2 = 10$):
(A) 45 Hz (B) 50 Hz (C) 30 Hz (D) 62 Hz

C-4. The average power transmitted through a given point on a string supporting a sine wave is. 0.40 watt when the amplitude of wave is 2 mm. What average power will transmitted through this point its amplitude is increased to 4 mm.
(A) 0.40 watt (B) 0.80 watt (C) 1.2 watt (D) 1.6 watt

Section (D) : Interference, Reflection, Transmission

D-1. When two waves of the same amplitude and frequency but having a phase difference of ϕ, travelling with the same speed in the same direction (positive x), meets at a point then.
(A) their resultant amplitude will be twice that of a single wave but the frequency will be same
(B) their resultant amplitude and frequency will both be twice that of a single wave
(C) their resultant amplitude will depend on the phase angle while the frequency will be the same
(D) the frequency and amplitude of the resultant wave will depend upon the phase angle.

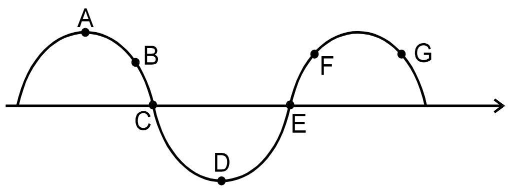
D-2. The rate of transfer of energy in a wave depends
(A) directly on the square of the wave amplitude and square of the wave frequency
(B) directly on the square of the wave amplitude and square root of the wave frequency
(C) directly on the wave frequency and square of the wave amplitude
(D) directly on the wave amplitude and square of the wave frequency

D-3. Two waves of amplitude A_1, and A_2 respectively and equal frequency travels towards same point. The amplitude of the resultant wave is
(A) $A_1 + A_2$ (B) $A_1 - A_2$
(C) between $A_1 + A_2$ and $A_1 - A_2$ (D) Can not say

D-4. A wave pulse, travelling on a two piece string, gets partially reflected and partially transmitted at the junction. The reflected wave is inverted in shape as compared to the incident one. If the incident wave has speed v and the transmitted wave v′,
(A) $v' > v$ (B) $v' = v$ (C) $v' < v$
(D) nothing can be said about the relation of v and v′.

D-5. The effects are produced at a given point in space by two waves described by the equations, $y_1 = y_m \sin \omega t$ and $y_2 = y_m \sin (\omega t + \phi)$ where y_m is the same for both the waves and ϕ is a phase angle. Tick the incorrect statement among the following.
(A) the maximum intensity that can be achieved at a point is twice the intensity of either wave and occurs if $\phi = 0$
(B) the maximum intensity that can be achieved at a point is four times the intensity of either wave and occurs if $\phi = 0$
(C) the maximum amplitude that can be achieved at the point is twice the amplitude of either wave and occurs at $\phi = 0$
(D) When the intensity is zero, the net amplitude is zero, and at this point $\phi = \pi$.

D-6. The following figure depicts a wave travelling in a medium. Which pair of particles are in phase.



(A) A and D (B) B and F
(C) C and E (D) B and G

D-7. Three waves of equal frequency having amplitudes 10 μm, 4 μm and 7 μm arrive at a given point with a successive phase difference of $\pi/2$. The amplitude of the resulting wave in μm is given by
(A) 7 (B) 6 (C) 5 (D) 4

Section (E) : Standing waves and Resonance

E-1. A wave represented by the equation $y = a \cos(kx - \omega t)$ is superposed with another wave to form a stationary wave such that the point x = 0 is a node. The equation for other wave is : **[JEE - 1988]**
(A) $a \sin(kx + \omega t)$ (B) $-a \cos(kx + \omega t)$ (C) $-a \cos(kx - \omega t)$ (D) $-a \sin(kx - \omega t)$

E-2. A stretched sonometer wire resonates at a frequency of 350 Hz and at the next higher frequency of 420 Hz. The fundamental frequency of this wire is
(A) 350 Hz (B) 5 Hz (C) 70 Hz (D) 170 Hz

E-3. Equations of a stationary wave and a travelling wave are $y_1 = a \sin kx \cos \omega t$ and $y_2 = a \sin(\omega t - kx)$. The phase difference between two points $x_1 = \frac{\pi}{3k}$ and $x_2 = \frac{3\pi}{2k}$ is ϕ_1 for the first wave and ϕ_2 for the second wave. The ratio $\frac{\phi_1}{\phi_2}$ is :
(A) 1 (B) 5/6 (C) 3/4 (D) 6/7

E-4. Two stretched wires A and B of the same lengths vibrate independently. If the radius, density and tension of wire A are respectively twice those of wire B, then the fundamental frequency of vibration of A relative to that of B is **[REE - 1990]**
(A) 1 : 1 (B) 1 : 2 (C) 1 : 4 (D) 1 : 8

E-5. A steel wire of mass 4.0 g and length 80 cm is fixed at the two ends. The tension in the wire is 50 N. The wavelength of the fourth harmonic of the fundamental will be :
(A) 80 cm (B) 60 cm (C) 40 cm (D) 20 cm

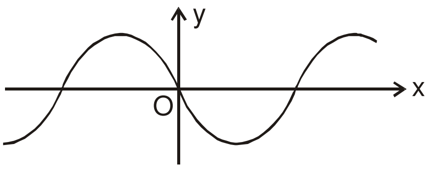
E-6. One end of two wires of the same metal and of same length (with radius, r and 2r) are joined together. The wire is used as sonometer wire and the junction is placed in between two bridges. The tension T is applied to the wire. If at a junction a node is formed then the ratio of number of loops formed in the wires will be: **[JEE - 1985]**
(A) 1 : 2 (B) 2 : 3 (C) 3 : 4 (D) 4 : 5

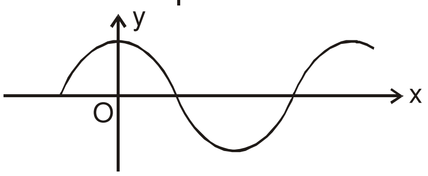
E-7. In a stationary wave represented by $y = a \sin \omega t \cos kx$, amplitude of the component progressive wave is
(A) $\frac{a}{2}$ (B) a (C) 2a (D) None of these

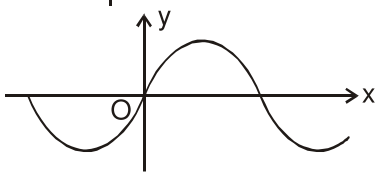
PART - III : MATCH THE COLUMN

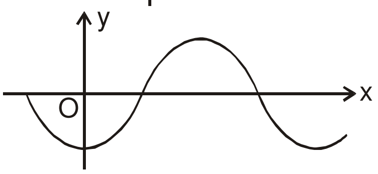
1. For four sinusoidal waves, moving on a string along positive x direction, displacement-distance curves (y-x curves) are shown at time t = 0. In the right column, expressions for y as function of distance x and time t for sinusoidal waves are given. All terms in the equations have general meaning. Correctly match y-x curves with corresponding equations.

Column-I

(A)


(B)


(C)


(D)


Column-II

(p) $y = A \cos(\omega t - kx)$

(q) $y = -A \cos(kx - \omega t)$

(r) $y = A \sin(\omega t - kx)$

(s) $y = A \sin(kx - \omega t)$

(t) $y = A \cos(kx - \omega t)$

2. In case of string waves, match the statements in column-I with the statements in column-II.

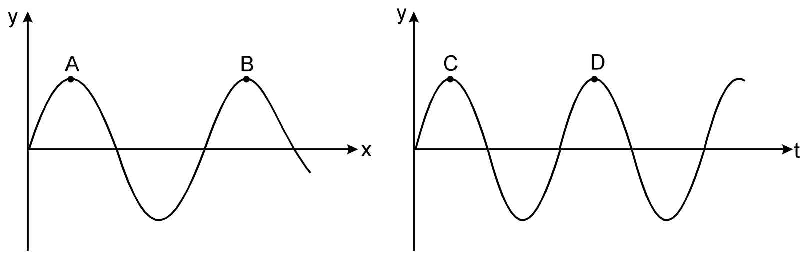
Column-I		Column-II	
(A)	A tight string is fixed at both ends and sustaining standing wave	(p)	At the middle, antinode is formed in odd harmonic
(B)	A tight string is fixed at one end and free at the other end	(q)	At the middle, node is formed in even harmonic
(C)	A tight string is fixed at both ends and vibrating in four loops	(r)	the frequency of vibration is 300% more than its fundamental frequency
(D)	A tight string is fixed at one end and free at the other end, vibrating in 2nd overtone	(s)	Phase difference between SHMs of any two particles will be either π or zero.
		(t)	The frequency of vibration is 400% more than fundamental frequency.

Exercise-2

Marked Questions can be used as Revision Questions.

PART - I : ONLY ONE OPTION CORRECT TYPE

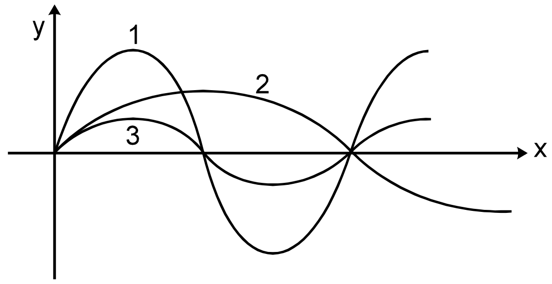
1. The same progressive wave is represented by two graphs I and II. Graph I shows how the displacement 'y' varies with the distance x along the wave at a given time. Graph II shows how y varies with time t at a given point on the wave. The ratio of measurements AB to CD, marked on the curves, represents :



(I) (II)

(A) wave number k
(B) wave speed V.
(C) frequency ν.
(D) angular frequency ω.

2. Graph shows three waves that are separately sent along a string that is stretched under a certain tension along x–axis. If ω_1, ω_2 and ω_3 are their angular frequencies respectively then :



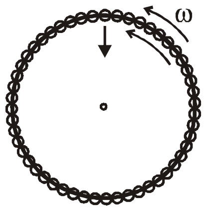
(A) $\omega_1 = \omega_3 > \omega_2$
(B) $\omega_1 > \omega_2 > \omega_3$
(C) $\omega_2 > \omega_1 = \omega_3$
(D) $\omega_1 = \omega_2 = \omega_3$

3. A transverse periodic wave on a string with a linear mass density of 0.200 kg/m is described by the following equation y = 0.05 sin(420t – 21.0 x), where x and y are in meters and t is in seconds. The tension in the string is equal to :
(A) 32 N (B) 42 N (C) 66 N (D) 80 N

4. A heavy but uniform rope of length L is suspended from a ceiling. A particle is dropped from the ceiling at the instant when the bottom end is given the jerk. Where will the particle meet the pulse :
(A) at a distance lf $\frac{2L}{3}$ from the bottom (B) at a distance $\frac{L}{3}$ from the bottom
(C) at a distance $\frac{3L}{4}$ from the bottom (D) None of these

5. An object of specific gravity ρ is hung from a thin steel wire. The fundamental frequency for transverse standing waves in the wire is 300 Hz. The object is immersed in water so that one half of its volume is submerged. The new fundamental frequency in Hz is **[JEE - 1995]**
(A) $300\left(\frac{2\rho-1}{2\rho}\right)^{1/2}$ (B) $300\left(\frac{2\rho}{2\rho-1}\right)^{1/2}$ (C) $300\left(\frac{2\rho}{2\rho-1}\right)$ (D) $300\left(\frac{2\rho-1}{2\rho}\right)$

6. (a) A circular loop of rope of length L rotates with uniform angular velocity ω about an axis through its centre on a horizontal smooth platform. Velocity of pulse (with respect to rope) produced due to slight radial displacement is given by



(A) ωL (B) $\frac{\omega L}{2\pi}$ (C) $\frac{\omega L}{\pi}$ (D) $\frac{\omega L}{4\pi^2}$

(b) In the above question if the motion of the pulse and rotation of the loop, both are in same direction then the velocity of the pulse w.r.t. to ground will be :
(A) ωL (B) $\frac{\omega L}{2\pi}$ (C) $\frac{\omega L}{\pi}$ (D) $\frac{\omega L}{4\pi^2}$

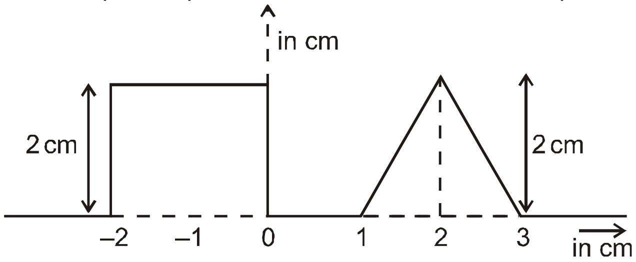
(c) In the above question if both are in opposite direction then the velocity of the pulse w.r.t. to ground will be:
(A) ωL (B) $\frac{\omega L}{2\pi}$ (C) $\frac{\omega L}{\pi}$ (D) 0

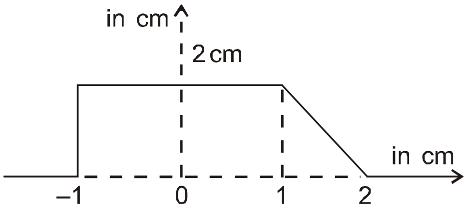
7. Which of the following function correctly represents the travelling wave equation for finite positive values of x and t :
(A) $y = x^2 - t^2$ (B) $y = \cos x^2 \sin t$
(C) $y = \log(x^2 - t^2) - \log(x - t)$ (D) $y = e^{2x} \sin t$

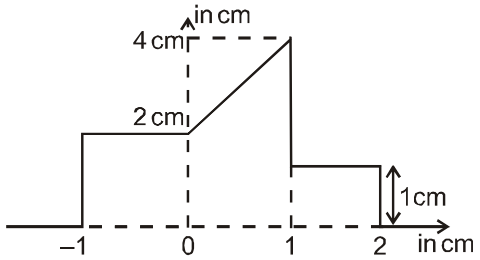
8. A 75 cm string fixed at both ends produces resonant frequencies 384 Hz and 288 Hz without there being any other resonant frequency between these two. Wave speed for the string is :
(A) 144 m/s (B) 216 m/s (C) 108 m/s (D) 72 m/s

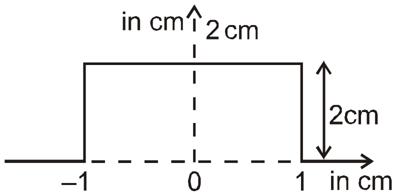
9. Two wave pulses travel in opposite directions on a string and approach each other. The shape of the one pulse is same with respect to the other.
(A) The pulses will collide with each other and vanish after collision
(B) The pulses will reflect from each other i.e., the pulse going towards right will finally move towards left and vice versa.
(C) The pulses will pass through each other but their shapes will be modified
(D) The pulses will pass through each other without any change in their shape

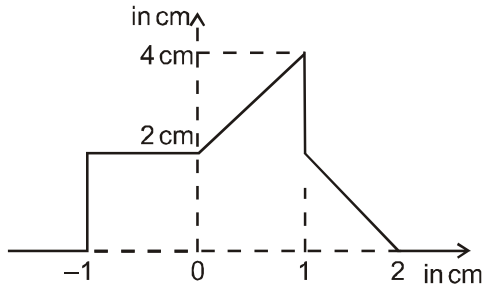
10. The figure shows at time t = 0 second, a rectangular and triangular pulse on a uniform wire are approaching each other. The pulse speed is 0.5 cm/s. The resultant pulse at t = 2 second is



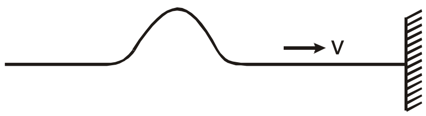
(A)


(B)


(C)


(D)


11. When a wave pulse travelling in a string is reflected from a rigid wall to which string is tied as shown in figure. For this situation two statements are given below.



(1) The reflected pulse will be in same orientation of incident pulse due to a phase change of π radians.
(2) During reflection the wall exerts a force on string in upward direction.
For the above given two statements choose the correct option given below.
(A) Only (1) is true (B) Only (2) is true (C) Both are true (D) Both are wrong

12. A wave travels on a light string.The equation of the wave is $Y = A \sin (kx - \omega t + 30°)$. It is reflected from a heavy string tied to an end of the light string at x = 0.If 64% of the incident energy is reflected the equation of the reflected wave is
(A) $Y = 0.8 A \sin (kx - \omega t + 30° + 180°)$
(B) $Y = 0.8 A \sin (kx + \omega t + 30° + 180°)$
(C) $Y = 0.8 A \sin (kx + \omega t - 30°)$
(D) $Y = 0.8 A \sin (kx + \omega t + 30°)$

13. The wave–function for a certain standing wave on a string fixed at both ends is $y(x, t) = 0.5 \sin (0.025\pi x) \cos 500t$ where x and y are in centimeters and t is in seconds. The shortest possible length of the string is :
(A) 126 cm (B) 160 cm (C) 40 cm (D) 80 cm

14. Equation of a standing wave is expressed as $y = 2A \sin\omega t \cos kx$. In the equation, quantity ω/k represents
(A) the transverse speed of the particles of the string.
(B) the speed of the component waves.
(C) the speed of the standing wave.
(D) a quantity that is independent of the properties of the string.

15. A sonometer wire is divided in many segments using bridges. If fundamental natural frequencies of the segments are n_1, n_2, n_3.... then the fundamental natural frequency of entire sonometer wire will be (If the divisions were not made) :

(A) $n = n_1 + n_2 + n_3 + ...$ (B) $n = \sqrt{n_1 \times n_2 \times n_3 \times ....}$ (C) $\frac{1}{n} = \frac{1}{n_1} + \frac{1}{n_2} + \frac{1}{n_3} + ...$ (D) none of the above

16. A string of length L, fixed at its both ends is vibrating in its first overtone mode. Consider two elements of the string of same small length at positions $\ell_1 = 0.2$ L and $\ell_2 = 0.45$ L from one end. If K_1 and K_2 are their respective maximum kinetic energies then

(A) $K_1 = K_2$ (B) $K_1 > K_2$ (C) $K_1 < K_2$

(D) it is not possible to decide the relation.

17. A stone is hung in air from a wire which is stretched over a sonometer. The bridges of the sonometer are 40 cm apart when the wire is in unison with a tuning fork of frequency 256. When the stone is completely immersed in water, the length between the bridges is 22 cm for re-establishing (same mode) unison with the same tuning fork. The specific gravity of the material of the stone is:

(A) $\frac{(40)^2}{(40)^2 + (22)^2}$ (B) $\frac{(40)^2}{(40)^2 - (22)^2}$ (C) $256 \times \frac{22}{40}$ (D) $256 \times \frac{40}{22}$

18. A wire of density 9 gm/cm^3 is stretched between two clamps 1.00 m apart while subjected to an extension of 0.05 cm. The lowest frequency of transverse vibrations in the wire is **[JEE - 1975]**

(Assume Young's modulus $Y = 9 \times 10^{10}$ N/m^2)

(A) 35 Hz (B) 45 Hz (C) 75 Hz (D) 90 Hz

19.✍ A 20 cm long rubber string fixed at both ends obeys Hook's law. Initially when it is stretched to make its total length of 24 cm, the lowest frequency of resonance is υ_0. It is further stretched to make its total length of 26 cm. The lowest frequency of resonance will now be :

(A) the same as υ_0 (B) greater than υ_0 (C) lower than υ_0 (D) None of these

20. Two wires made of the same material, one thick and the other thin, are connected to form a composite wire. The composite wire is subjected to some tension. A wave travelling along the wire crosses the junction point. The characteristic that undergoes a change at the junction point is

(A) Frequency only (B) Speed of propagation only

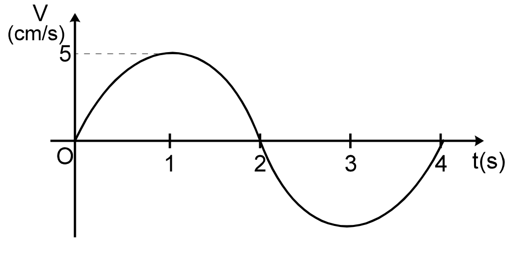
(C) Wavelength only (D) The speed of propagation as well as the wavelength

21. Standing waves are generated on string loaded with a cylindrical body. If the cylinder is immersed in water, the length of the loops changes by a factor of 2.2. The specific gravity of the material of the cylinder is **[Olympiad 2014, Stage–I]**

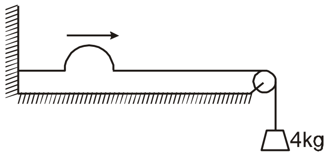
(A) 1.11 (B) 2.15 (C) 2.50 (D) 1.26

PART - II : SINGLE AND DOUBLE VALUE INTEGER TYPE

1. A certain transverse sinusoidal wave of wavelength 20 cm is moving in the positive x direction. The transverse velocity of the particle at x = 0 as a function of time is shown. The amplitude of the motion is equal to $\frac{x}{\pi}$ (in cm). Find x :

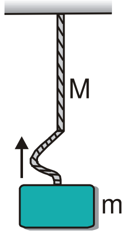


2. Figure shows a string of linear mass density 1.0 g/cm on which a wave pulse is travelling. Find the time taken (in milli second) by the pulse in travelling through a distance of 60 cm on the string. (Take $g = 10\ m/s^2$).



3. A wire of 9.8×10^{-3} kg per meter mass passes over a frictionless pulley fixed on the top of an inclined frictionless plane which makes an angle of 30° with the horizontal. Masses M_1 & M_2 are tied at the two ends of the wire. The mass M_1 rests on the plane and the mass M_2 hangs freely vertically downwards. The whole system is in equilibrium. Now a transverse wave propagates along the wire with a velocity of 100 m/sec. Find the ratio of masses M_1 to M_2. **[REE - 1993, 4]**

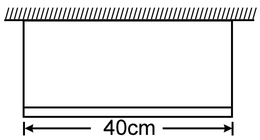
4. A uniform rope of length ℓ and mass M hangs vertically from a rigid support. A block of mass m (m = M) is attached to the free end of the rope. A transverse pulse of wavelength λ is produced at the lower end of the rope. The wavelength of the pulse, when it reaches the top of the rope, is $a\lambda$. Find a^2



5. A non–uniform rope of mass M and length L has a variable linear mass density given by $\mu = kx$, where x is the distance from one end of the wire and k is a constant. The time required for a pulse generated at one end of the wire to travel to the other end is given by $t = \sqrt{(pML/9F)}$ where F (constant) is the tension in the wire then find p

6. A man generates a symmetrical pulse in a string by moving his hand up and down. At t = 0 the point in his hand moves downward from mean position. The pulse travels with speed 3 m/s on the string & his hand passes 6 times in each second from the mean position. Then the point on the string at a distance 3m will reach its upper extreme first time at time t = n/12. Find n

7. A uniform horizontal rod of length 40 cm and mass 1.2 kg is supported by two identical wires as shown in figure. Where should a mass of 4.8 kg be placed on the rod from left end (in cm) so that the same tuning fork may excite the wire on left into its fundamental vibrations and that on right into its first overtone? Take $g = 10\ m/s^2$.



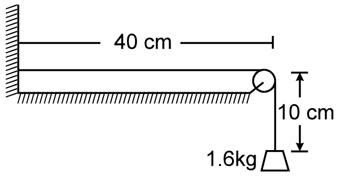
8. A string of length 'ℓ' is fixed at both ends. It is vibrating in its 3rd overtone with maximum amplitude 'a'. The amplitude at a distance $\frac{\ell}{3}$ from one end is $= \sqrt{p}\frac{a}{2}$. Find p

9. A string 120 cm in length and fixed at both ends sustains a standing wave, with the consecutive points of the string at which the displacement amplitude is equal to 3.5 mm (not maximum) being separated by 15.0 cm. The maximum displacement amplitude is $3.5\ x^{1/2}$ in mm. Then find x. To which overtone do these oscillations correspond?

10. A string of mass ' m ' and length ℓ, fixed at both ends is vibrating in its fundamental mode.The maximum amplitude is ' a ' and the tension in the string is ' T '. If the energy of vibrations of the string is $\frac{\pi^2 a^2 T}{\eta L}$. Find η **[JEE(Mains) 2003; 4/60]**

11. A travelling wave of amplitude 5A is partially reflected from a boundary with the amplitude 3 A. Due to superposition of two waves with different amplitudes in opposite directions a wave pattern is formed. Determine the ratio of amplitude at antinode to node.

12. A 50 cm long wire of mass 20 g supports a mass of 1.6 kg as shown in figure. Find the first overtone frequency of the portion of the string between the wall and the pulley. Take g = 10 m/s^2.



13. A 1 m long rope, having a mass of 40 g, is fixed at one end and is tied to a light string at the other end. The tension in the string is 400 N. Find the wavelength in second overtone (in cm).

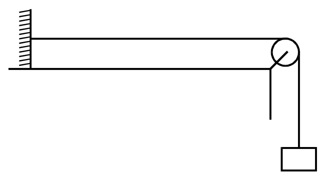
14. In an experiment of standing waves, a string 90 cm long is attached to the prong of an electrically driven tuning fork that oscillates perpendicular to the length of the string at a frequency of 60 Hz. The mass of the string is 0.044 kg. What tension (in newton) must the string be under (weights are attached to the other end) if it is to oscillate in four loops?

15. Three consecutive resonant frequencies of a string are 90, 150 and 210 Hz. If the length of the string is 80 cm, what would be the speed (in m/s) of a transverse wave on this string ?

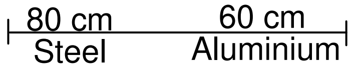
16. A steel wire of length 1 meter, mass 0.1 kg and uniform cross sectional area 10^{-6} m^2 is rigidly fixed at both ends without any stress. The temperature of the wire is lowered by 20ºC. If transverse waves are setup by plucking the string in the middle, calculate the frequency of the fundamental mode of vibration. Young's modulus of steel = 2×10^{11} N/m^2, coefficient of linear expansion of steel = 1.21×10^{-5}/ºC. **[JEE - 1984]**

17. A wire having some linear density is stretched between two rigid supports with tension. It is observed that the wire resonates at a frequency of 420 cycles/sec. The next higher frequency at which the same wire resonates is 490 cycles/sec. Find the possible mode of vibration of string initially: **[JEE - 1971]**

18. Figure shows a string stretched by a block going over a pulley. The string vibrates in its tenth harmonic in resonance with a particular tuning fork. When a beaker containing water is brought under the block so that the block is completely dipped into the beaker, the string vibrates in its eleventh harmonic. If the density of the material of the block is d gram/cm^3 then find greatest integer of d. (node is formed at the pulley)



19. Figure shows an aluminium wire of length 60 cm joined to a steel wire of length 80 cm and stretched between two fixed supports. The tension produced is 40 N. The cross-sectional area of the steel wire is 1.0 mm^2 and that of the aluminium wire is 3.0 mm^2. The minimum frequency of a tuning fork which can produce standing waves in the system with the joint as a node is 10P (in Hz) then find P. Given density of aluminium is 2.6 g/cm^3 and that of steel is 7.8 g/cm^3.



20. A metallic wire with tension T and at temperature 30°C vibrates with its fundamental frequency of 1 kHz. The same wire with the same tension but at 10°C temperature vibrates with a fundamental frequency of 1.001 kHz. The coefficient of linear expansion of the wire is equal to 10^{-K} ºC. Find 2K

PART - III : ONE OR MORE THAN ONE OPTIONS CORRECT TYPE

1. The plane wave represented by an equation of the form y = f(x – v t) implies the propagation along the positive x-axis without change of shape with constant velocity v :

(A) $\frac{\partial y}{\partial t} = -v\left(\frac{\partial y}{\partial x}\right)$ (B) $\frac{\partial y}{\partial t} = -v\left(\frac{\partial^2 y}{\partial x^2}\right)$ (C) $\frac{\partial^2 y}{\partial t^2} = -v^2\left(\frac{\partial^2 y}{\partial x^2}\right)$ (D) $\frac{\partial^2 y}{\partial t^2} = v^2\left(\frac{\partial^2 y}{\partial x^2}\right)$

2. A wave equation which gives the displacement along the Y direction is given by $y = 10^{-4}\sin(60t + 2x)$ where x and y are in meters and t is time in seconds. This represents a wave **[JEE - 1982]**

(A) travelling with a velocity of 30 m/s in the negative x direction

(B) of wavelength π meter

(C) of frequency $30/\pi$ hertz

(D) of amplitude 10^{-4} meter travelling along the negative x direction.

3. The displacement of a particle in a medium due to a wave travelling in the x-direction through the medium is given by $y = A\sin(\alpha t - \beta x)$, where t = time, and α and β are constants :

(A) the frequency of the wave is α (B) the frequency of the wave is $\alpha/2\pi$

(C) the wavelength is $2\pi/\beta$ (D) the velocity of the wave is α/β

4. The displacement of particles in a string stretched in x-direction is represented by y. Among the following expressions for y, those describing wave motion are : **[JEE - 1987, 2]**

(A) $\cos(kx)\sin(\omega t)$ (B) $k^2x^2 - \omega^2t^2$ (C) $\cos^2(kx + \omega t)$ (D) $\cos(k^2x^2 - \omega^2t^2)$

5. The particle displacement in a wave is given by $y = 0.2 \times 10^{-5}\cos(500\,t - 0.025\,x)$, where the distances are measured in meters and time in seconds. Now **[REE - 1994]**

(A) wave velocity is 2×10^4 ms^{-1} (B) particle velocity is 2×10^4 ms^{-1}

(C) initial phase difference is $\frac{\pi}{2}$ (D) wavelength of the wave is (80π) m

6. A transverse sinusoidal wave of amplitude a, wavelength λ and frequency f is travelling on a stretched string. The maximum speed of any point on the string is v /10, where v is the speed of propagation of the wave. if $a = 10^{-3}$ m and v = 10 m/s, then λ and f are given by

(A) $\lambda = 2\pi \times 10^{-2}$ m (B) $\lambda = 10^{-3}$ m (C) $f = 10^3/(2\pi)$ Hz (D) $f = 10^4$ Hz

7. If the tension in a string is increased by 21 percent, the fundamental frequency of the string changes by 15 Hz. Which of the following statements will also be correct?

(A) The original fundamental frequency is nearly 150 Hz

(B) The velocity of propagation changes nearly by 4.5%

(C) The velocity of propagation changes nearly by 10%

(D) The fundamental wavelength changes nearly by 10%

8. Two waves of equal frequency f and speed v travel in opposite directions along the same path. The waves have amplitudes A and 3A. Then :

(A) the amplitude of the resulting wave varies with position between maxima of amplitude 4A and minima of zero amplitude

(B) the distance between a maxima and adjacent minima of amplitude is $\frac{\nu}{2f}$

(C) at a point on the path the average displacement is zero $\Rightarrow$ Average displacement of medium particle at any point is zero.

(D) the position of a maxima or minima of amplitude does not change with time

9. Two particles A and B have a phase difference of π when a sine wave passes through the region
(A) A and B oscillates with same frequency
(B) A and B move in opposite directions
(C) A and B are separated by odd multiple of half of the wavelength
(D) the displacements at A and B have equal magnitudes

10. In a stationary wave,
(A) all the particles of the medium vibrate either in phase or in opposite phase
(B) all the antinodes vibrate in opposite phase
(C) the alternate antinodes vibrate in phase
(D) all the particles between consecutive nodes vibrate in phase

11. Following are equations of four waves :
(i) $y_1 = a \sin \omega \left(t - \frac{x}{\nu}\right)$ (ii) $y_2 = a \cos \omega \left(t + \frac{x}{\nu}\right)$
(iii) $z_1 = a \sin \omega \left(t - \frac{x}{\nu}\right)$ (iv) $z_2 = a \cos \omega \left(t + \frac{x}{\nu}\right)$
Which of the following statements is/are correct?
(A) On superposition of waves (i) and (iii), a travelling wave having amplitude $a\sqrt{2}$ will be formed
(B) Superposition of waves (ii) and (iii) is not possible
(C) On superposition of (i) and (ii), a stationary wave having amplitude $a\sqrt{2}$ will be formed
(D) On superposition of (iii) and (iv), a transverse stationary wave will be formed

12. A wave disturbance in a medium is described by $y(x, t) = 0.02 \cos (50\pi t + \pi/2) \cos 10(\pi x)$, where x and y are in meter and t in second. **[JEE - 1995]**
(A) A node occurs at x = 0.15 m (B) An antinode occurs at x = 0.3 m
(C) The speed of wave is 5 ms^{-1} (D) The wavelength is 0.2m.

13. The vibrations of a string of length 600 cm fixed at both ends are represented by the equation $y = 4 \sin\left(\pi \frac{x}{15}\right) \cos (96\pi t)$, where x and y are in cm and t in seconds. **[JEE - 1985]**
(A) The maximum displacement of a point x = 5 cm is $2\sqrt{3}$cm .
(B) The nodes located along the string are at a distance of 15n where integer n varies from 0 to 40.
(C) The velocity of the particle at x = 7.5 cm at t = 0.25 sec is 0
(D) The equations of the component waves whose superposition gives the above wave are $2 \sin 2\pi \left(\frac{x}{30} + 48t\right)$, $2 \sin 2\pi \left(\frac{x}{30} - 48t\right)$.

14. A wave given by $\xi = 10 \sin [80\pi t - 4\pi x]$ propagates in a wire of length 1m fixed at both ends. If another wave of similar amplitude is superimposed on this wave to produce a stationary wave then **[REE - 1998]**
(A) the superimposed wave is $\xi = -10 \sin [80\pi t + 4\pi x]$
(B) the maximum amplitude of the stationary wave is 20 m.
(C) the wave length of the wave is 0.5 m.
(D) the number of total nodes produced in the wire are 3.

15. Consider an element of a stretched string along which a wave travels. During its transverse oscillatory motion, the element passes through a point at y = 0 and reaches its maximum at $y = y_m$. Then, the string element has its maximum **[Olympiad 2014; Stage–I]**
(A) kinetic energy at $y = y_m$. (B) elastic potential energy at $y = y_m$.
(C) kinetic energy at y = 0. (D) elastic potential energy at y = 0.

PART - IV : COMPREHENSION

Comprehension-1

A pulse is started at a time t = 0 along the +x direction on a long, taut string. The shape of the pulse at t = 0 is given by function y with

$$y = \begin{cases} \frac{x}{4}+1 & \text{for} \quad -4 < x \le 0 \\ -x+1 & \text{for} \quad 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

here y and x are in centimeters. The linear mass density of the string is 50 g/m and it is under a tension of 5N,

1. The shape of the string is drawn at t = 0 and the area of the pulse enclosed by the string and the x-axis is measured. It will be equal to
(A) 2 cm^2 (B) 2.5 cm^2 (C) 4 cm^2 (D) 5 cm^2

2. The vertical displacement of the particle of the string at x = 7 cm and t = 0.01 s will be
(A) 0.75 cm (B) 0.5 cm (C) 0.25 cm (D) zero

3. The transverse velocity of the particle at x = 13 cm and t = 0.015 s will be
(A) –250 cm/s (B) –500 cm/s (C) 500 cm/s (D) –1000 cm/s

Comprehension-2

A transverse sinusoidal wave is generated at one end of a long, horizontal string by a bar that moves up and down through an amplitude of 1/2 cm. The motion is continuous and is repeated regularly 120 times per second. The string has linear density 90 gm/m and is kept under a tension of 900 N. Find:

4. The maximum value of the transverse component of the tension (in newton)
(A) 1.8 π (B) 10.8 π (C) 9 (D) 18 π

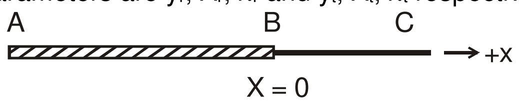
5. What is the maximum power (in watt) transferred along the string .
(A) 3.24 π^2 (B) 6.48 π^2 (C) 12.96 π^2 (D) 25.92 π^2

6. What is the transverse displacement y (in cm) when the minimum power transfer occurs
(A) 0 (B) $\frac{1}{2}$ (C) $\frac{1}{4}$ (D) 1

[Leave the answer in terms of π wherever it occurs]

Comprehension-3

In the figure shown a sinusoidal wave is generated at the end 'A'. The wave travels along positive x-axis and during its motion it encounter another string BC at the junction 'B' at x = 0. The density of strings AB and BC are ρ and 9ρ respectively and their radii of cross sections are 2r and r. The wave function, amplitude and wavelength of incident wave are respectively y_i, A_i and λ_i. Similarly for reflected and transmitted wave these parameters are y_r, A_r, λ_r and y_t, A_t, λ_t respectively.



7. Which of the following statement regarding phase difference, $\Delta\phi$ between waves at x = 0 is true ?
(A) $\Delta\phi$ = 0, between y_i and y_r (B) $\Delta\phi$ = 0, between y_r and y_t
(C) $\Delta\phi = \pi$, between y_i and y_t (D) $\Delta\phi = \pi$, between y_r and y_t

8. The ratio of wavelengths λ_r to λ_t (i.e. $\lambda_r : \lambda_t$) will be
(A) 1 : 1 (B) 3 : 2 (C) 2 : 3 (D) None of these

9. The ratio of amplitudes A_r to A_t is (i.e. $A_r : A_t$) will be
(A) 1 : 1 (B) 1 : 4 (C) 4 : 1 (D) None of these

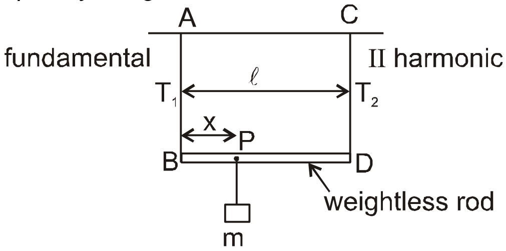
Exercise-3

*** Marked Questions may have more than one correct option.**
✍ Marked Questions can be used as Revision Questions.

PART - I : JEE (ADVANCED) / IIT-JEE PROBLEMS (PREVIOUS YEARS)

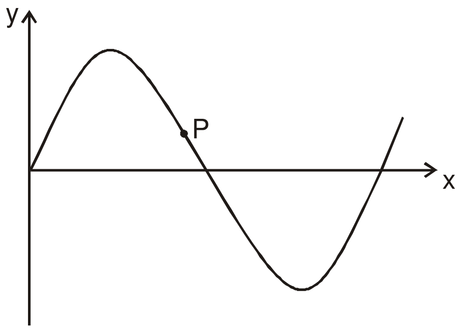
1. A transverse wave travelling in a string produces maximum transverse velocity of 3 m/s and maximum transverse acceleration 90 m/s^2 in a particle. If the velocity of wave in the string is 20 m/s. Determine the equation of the wave ? **[JEE(Mains) 2005 ; 4/60]**

2.✍ A massless rod BD is suspended by two identical massless strings AB and CD of equal lengths. A block of mass 'm' is suspended point P such that BP is equal to 'x', if the fundamental frequency of the left wire is twice the fundamental frequency of right wire, then the value of x is: **[JEE(Mains) 2006; 3/184]**



(A) $\ell/5$ (B) $\ell/4$ (C) $4\ell/5$ (D) $3\ell/4$

3. A transverse sinusoidal wave moves along a string in the positive x-direction at a speed of 10 cm/s. The wavelength of the wave is 0.5 m and its amplitude is 10 cm. At a particular time t, the snap-shot of the wave is shown in figure. The velocity of point P when its displacement is 5 cm is **[JEE 2008; 3/163]**
Figure :



(A) $\frac{\sqrt{3}\,\pi}{50}\hat{j}$ m/s (B) $-\frac{\sqrt{3}\,\pi}{50}\hat{j}$ m/s (C) $\frac{\sqrt{3}\,\pi}{50}\hat{i}$ m/s (D) $-\frac{\sqrt{3}\,\pi}{50}\hat{i}$ m/s

4. A 20cm long string, having a mass of 1.0 g, is fixed at both the ends. The tension in the string is 0.5 N. The string is set into vibrations using an external vibrator of frequency 100 Hz. Find the separation (**in cm**) between the successive nodes on the string. **[JEE 2009; 4/160, –1]**

5. When two progressive waves $y_1 = 4 \sin (2x - 6t)$ and $y_2 = 3 \sin \left(2x - 6t - \frac{\pi}{2}\right)$ are superimposed, the amplitude of the resultant wave is : **[JEE 2010; 3/163, –1]**

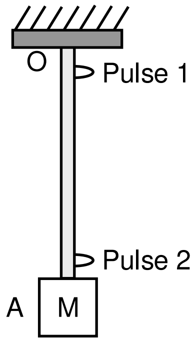
6*.✍ A horizontal stretched string, fixed at two ends, is vibrating in its fifth harmonic according to the equation, $y(x, t) = (0.01 \text{ m}) \sin [(62.8 \text{ m}^{-1})x] \cos [(628 \text{ s}^{-1})t]$. Assuming $\pi = 3.14$, the correct statement(s) is (are) : **[JEE-Advanced 2013, 3/60]**
(A) The number of nodes is 5.
(B) The length of the string is 0.25 m.
(C) The maximum displacement of the midpoint of the string its equilibrium position is 0.01 m.
(D) The fundamental frequency is 100 Hz.

7*. One end of a taut string of length 3m along the x-axis is fixed at x = 0. The speed of the waves in the string is 100m/s. The other end of the string is vibrating in the y-direction so that stationary waves are set up in the string. The possible waveform(s) of these stationary waves is (are) **[JEE (Advanced) 2014, P-1, 3/60]**

(A) $y(t) = A \sin \frac{\pi x}{6} \cos \frac{50\pi t}{3}$ (B) $y(t) = A \sin \frac{\pi x}{3} \cos \frac{100\pi t}{3}$

(C) $y(t) = A \sin \frac{5\pi x}{6} \cos \frac{250\pi t}{3}$ (D) $y(t) = A \sin \frac{5\pi x}{2} \cos 250\pi t$

8*. A block M hangs vertically at the bottom end of a uniform rope of constant mass per unit length. The top end of the rope is attached to a fixed rigid support at O. A transverse wave pulse (Pulse 1) of wavelength λ_0 is produced at point O on the rope. The pulse takes time T_{OA} to reach point A. If the wave pulse of wavelength λ_0 is produced at point A (Pulse 2) without disturbing the position of M it takes time T_{AO} to reach point O. Which of the following options is/are correct **[JEE (Advanced) 2017, P-1, 4/61, –2]**



(A) The velocities of the two pulses (Pulse 1 and Pulse 2) are the same at the midpoint of rope
(B) The velocity of any pulse along the rope is independent of its frequency and wavelength
(C) The wavelength of Pulse 1 becomes longer when it reaches point A
(D) The time $T_{AO} = T_{OA}$

PART - II : JEE (MAIN) / AIEEE PROBLEMS (PREVIOUS YEARS)

1. A string is stretched between fixed points separated by 75 cm. It is observed to have resonant frequencies of 420 Hz and 315 Hz. There are no other resonant frequencies between these two. Then, the lowest resonant frequency for this string is: **[AIEEE 2006 ; 3/180]**

(1) 10.5 Hz (2) 105 Hz (3) 1.05 Hz (4) 1050 Hz

2. A wave travelling along the x- axis is described by the equation y(x, t) = 0.005 cos (αx –βt). If the wavelength and the time period of the wave are 0.08 m and 2.0 s, respectively, then α and β in appropriate units are **[AIEEE 2008 ; 3/105, –1]**

(1) $\alpha = \frac{0.08}{\pi}, \beta = \frac{2.0}{\pi}$ (2) $\alpha = \frac{0.04}{\pi}, \beta = \frac{1.0}{\pi}$ (3) $\alpha = 12.50\pi, \beta = \frac{\pi}{2.0}$ (4) $\alpha = 25.00\,\pi,\ \beta = \pi$

3. The equation of a wave on a string of linear mass density 0.04 kg m^{-1} is given by y = 0.02 (m) sin $\left[2\pi\left(\frac{t}{0.04(s)} - \frac{x}{0.50(m)}\right)\right]$. The tension in the string is : **[AIEEE 2010; 144/4 –1]**

(1) 4.0 N (2) 12.5 N (3) 0.5 N (4) 6.25 N

4. The transverse displacement y(x, t) of a wave on a string is given by $y(x,t) = e^{-\left(ax^2 + bt^2 + 2\sqrt{ab}\,xt\right)}$. This represents a : **[AIEEE - 2011; 1 MAY; 4/120, –1]**

(1) wave moving in +x–direction with speed $\sqrt{\frac{a}{b}}$ (2) wave moving in –x-direction with speed $\sqrt{\frac{b}{a}}$

(3) standing wave of frequency $\sqrt{b}$ (4) standing wave of frequency $\frac{1}{\sqrt{b}}$

5. A travelling wave represented by y = A sin ((ωt – kx) is superimposed on another wave represented by y = A sin (ωt + kx). The resultant is : **[AIEEE 2011, 11 MAY; 4/120, –1]**
(1) A wave travelling along + x direction
(2) A wave travelling along – x direction
(3) A standing wave having nodes at x = $\frac{n\lambda}{2}$, n = 0, 1, 2 ....
(4) A standing wave having nodes at x = $\left(n + \frac{1}{2}\right)\frac{\lambda}{2}$; n = 0, 1, 2 .....

6. A sonometer wire of length 1.5 m is made of steel. The tension in it produces an elastic strain of 1%. What is the fundamental frequency of steel if density and elasticity of steel are 7.7×10^3 kg/m^3 and 2.2×10^{11} N/m^2 respectively ? **[JEE(Main) 2013; 4/120, –1]**
(1) 188.5 Hz (2) 178.2 Hz (3) 200.5 Hz (4) 770 Hz

7. A uniform string of length 20 m is suspended from a rigid support. A short wave pulse is introduced at its lowest end. It starts moving up the string. The time taken to reach the support is : (take g = 10 ms^{-2}) **[JEE(Main) 2016; 4/120, –1]**
(1) 2 s (2) $2\sqrt{2}$ s (3) $\sqrt{2}$ s (4) $2\pi\sqrt{2}$ s

Answers

EXERCISE - 1

PART - I

Section (A) :

A-1. (a) 10 mm (b) 5π cm^{-1} (c) $\frac{2}{5}$ cm
(d) 30 Hz (e) $\frac{1}{30}$ s (f) 12 cm/s.

A-2. 1.41 y_m

A-3. (a) 10π rad/s (b) $\pi/2$ rad/m
(c) $y = (12 \times 10^{-2}\ \text{m}) \sin\left(\frac{\pi}{2}x - 10\pi t\right)$
(d) $\frac{6}{5}\pi$ m/s (e) $12\pi^2$ m/s^2

A-4. $0.5 \sin\left(\frac{\pi}{3}t - \frac{\pi}{18}x + \frac{7\pi}{9}\right)$

A-5. (a) D, E, F; (b) A, B, H ; (c) C, G ; (d) A, E

Section (B) :

B-1. $300\sqrt{3}$ m/s **B-2.** $\frac{1}{50}$ sec

B-3. $\frac{3}{10\sqrt{5}}$ m.

B-4. 100 Hz, 4 cm, 4 m/s **B-5.** 4

B-6. (a) 200 m/s; (b) 240 m/s

Section (C) :

C-1. (a) $y = (7.50\ \text{cm}) \sin\left(\frac{4\pi}{3}x - 314t + \phi\right)$
(b) $\frac{2025\pi^2}{32} \simeq 625$ W

C-2. 50 mW

Section (D) :

D-1. $y' = 0.8\, a \sin. \frac{2\pi}{\lambda}\left(vt + x + \frac{\lambda}{2}\right)$

D-2. (a) Zero (b) 0.300 m.

D-3. (a) 2π (b) π (c) 4.0 mm, zero

Section (E) :

E-1. (a) 2.5 Hz; (b) 5 Hz; (c) 7.5 Hz.

E-2. (a) $\frac{250}{\sqrt{3}}$ m/s; (b) 60.0 cm; (c) $\frac{1250}{3\sqrt{3}}$ Hz

E-3. $\frac{400}{3}$ Hz

E-4. (a) 0.25 cm ; (b) 1.2×10^2 cm/s;
(c) 3.0 cm ; (d) 0

E-5. (a) 100 Hz (b) 700 Hz

E-6. **(i)** $y_1 = 1.5 \cos\{(\pi/20)x - 72\pi t\}$,
$y_2 = 1.5 \cos\{(\pi/20)x + 72\pi t\}$
(ii) 10 , 30 , 50 cm and 0 , 20 , 40 , 60 cm
(iii) 0

E-7. 240 Hz.

PART - II

Section (A) :

A-1. (A) **A-2.** (C) **A-3.** (B)
A-4. (B) **A-5.** (B) **A-6.** (B)
A-7. (B) **A-8.** (B) **A-9.** (C)

Section (B) :

B-1. (C) **B-2.** (A) **B-3.** (C)
B-4. (D)

Section (C) :

C-1. (A) **C-2.** (D) **C-3.** (C)
C-4. (D)

Section (D) :

D-1.	(C)	**D-2.**	(A)	**D-3.**	(C)
D-4.	(C)	**D-5.**	(A)	**D-6.**	(D)
D-7.	(C)				

Section (E) :

E-1.	(B)	**E-2.**	(C)	**E-3.**	(D)
E-4.	(B)	**E-5.**	(C)	**E-6.**	(A)
E-7.	(A)				

PART - III

1. (A) $\rightarrow$ r ; (B) $\rightarrow$ p, t ; (C) $\rightarrow$ s ; (D) $\rightarrow$ q

2. (A) $\rightarrow$ p,q,s; (B) $\rightarrow$ s; (C) $\rightarrow$ q,r,s; (D) $\rightarrow$ s,t

EXERCISE - 2

PART - I

1.	(B)	**2.**	(A)	**3.**	(D)
4.	(B)	**5.**	(A)		
6.	(a) (B); (b) (C); (c) (D)				
7.	(C)	**8.**	(A)	**9.**	(D)
10.	(D)	**11.**	(D)	**12.**	(C)
13.	(C)	**14.**	(B)	**15.**	(C)
16.	(B)	**17.**	(B)	**18.**	(A)
19.	(B)	**20.**	(D)	**21.**	(D)

PART - II

1.	10	**2.**	30	**3.**	2
4.	2	**5.**	8	**6.**	15
7.	5	**8.**	3	**9.**	2, 3
10.	4	**11.**	4	**12.**	50
13.	80	**14.**	36	**15.**	96
16.	11	**17.**	6	**18.**	5
19.	18	**20.**	8		

PART - III

1.	(AD)	**2.**	(ABCD)	**3.**	(BCD)
4.	(AC)	**5.**	(AD)	**6.**	(AC)
7.	(AC)	**8.**	(CD)	**9.**	(ABCD)
10.	(ACD)	**11.**	(AD)	**12.**	(ABCD)
13.	(ABCD)	**14.**	(ABC)	**15.**	(CD)

PART - IV

1.	(B)	**2.**	(C)	**3.**	(A)
4.	(B)	**5.**	(C)	**6.**	(B)
7.	(D)	**8.**	(B)	**9.**	(B)

EXERCISE - 3

PART - I

1. Equation of wave in string

$y = 0.1 \sin\left(30\ t\ \pm \frac{3}{2}x + \phi\right)$ [where ϕ is initial phase]

2.	(A)	**3.**	(A)	**4.**	5
6.	(BC)	**7.**	(ACD)		
8.	(BD)				

PART - II

1.	(2)	**2.**	(4)	**3.**	(4)
4.	(2)	**5.**	(4)	**6.**	(2)
7.	(2)				

High Level Problems (HLP)

SUBJECTIVE QUESTIONS

1. A string that is stretched between fixed supports separated by 75.0 cm has resonant frequencies of 420 and 315 Hz with no intermediate resonant frequencies. What are (a) the lowest resonant frequencies and (b) the wave speed?

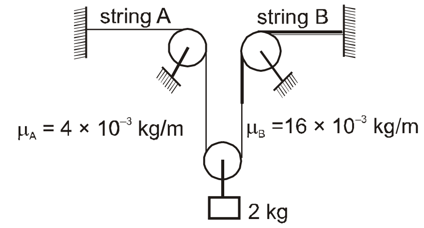
2. A steel wire of length $50\sqrt{3}$ cm is connected to an aluminium wire of length 60 cm and stretched between two fixed supports. The tension produced is 104 N, if the cross section area of each wire is $1mm^2$. If a transverse wave is set up in the wire, find the lowest frequency for which standing waves with node at the joint are produced. (density of aluminium = 2.6 gm/cm^3 and density of steel = 7.8 gm/cm^3)

3. An aluminium wire of cross-sectional area 1×10^{-6} m^2 is joined to a steel wire of same cross-sectional area. This compound wire is stretched on a sonometer, pulled by a weight of 10 kg. The total length of the compound wire between the bridges is 1.5 m, of which the aluminium is 0.6 m and the rest is steel wire. Transverse vibrations are set up in the wire by using an external force of variable frequency. Find the lowest frequency of excitation for which standing waves are formed such that the joint in the wire is a node. What is the total number of nodes observed at this frequency, excluding the two at the ends of the wire ? The density of aluminium is 2.6×10^3 kg/m^3 and that of steel is 1.04×10^4 kg/m^3. **[REE - 1983]**

4. The fundamental frequency of a sonometer wire increases by 6 Hz if its tension is increased by 44 % keeping the length constant. Find the change in the fundamental frequency of the sonometer when the length of the wire is increased by 20 % keeping the original tension in the wire. **[JEE - 1997; 5]**

5. A metal wire with volume density ρ and young's modulus Y is streatched between rigid supports. At temperature T, the speed of a transverse wave is found to be v_1. When temperature decreases $T - \Delta T$, the speed increases to v_2. Determine coefficient of linear expansion of wire.

6. Find velocity of wave in string A & B.



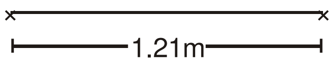
7. A string of length 50 cm is vibrating with a fundamental frequency of 400 Hz in horizontal position. In the fundamental mode the maximum displacement at the middle is 2 cm from equilibrium position and the tension in the string is 10 N. Determine the maximum value of the vertical component of force on the end supporting.

8. A string fixed at both ends is vibrating in the lowest mode of vibration for which a point at quarter of its length from one end is a point of maximum displacement. The frequency of vibration in this mode is 100 Hz. What will be the frequency emitted when it vibrates in the next higher mode such that this point is again a point of maximum displacement.

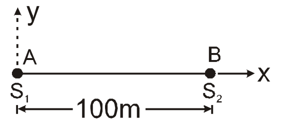
9. A guitar string is 180 cm long and has a fundamental frequency of 90 Hz. Where should it be pressed to produce a fundamental frequency of 135 Hz?

10. A piano wire weighing 4 g and having a length of 90.0 cm emits a fundamental frequency corresponding to the "Middle C" (v = 125 Hz). Find the tension in the wire.

11. Length of a sonometer wire is 1.21 m. Find the length of the three segments for fundamental frequencies to be in the ratio 1 : 2 : 3.



12. In the figure shown A and B are two ends of a string of length 100m. S_1 and S_2 are two sources due to which points 'A' and 'B' oscillate in 'y' and 'z' directions respectively according to the equation $y = 2 \sin (100 \pi t + 30°)$ and $z = 3 \sin (100 \pi t + 60°)$ where t is in sec and y is in mm. The speed of propagation of disturbance along the string is 50 m/s. Find the instantaneous position vector (in mm) and velocity vector in (m/s) of a particle 'P' of string which is at 25m from A. You have to find these parameters after both the disturbances from S_1 and S_2 have reached 'P'. Also find the phase difference between the waves at the point 'P' when they meet at 'P' first time.



13. A string of mass m is fixed at both ends. The fundamental tone oscillations are excited with angular frequency ω and maximum displacement amplitude a_{max}. Find :
(a) the maximum kinetic energy of the string;
(b) the mean kinetic energy of the string averaged over one oscillation period.

HLP Answers

1. (a) 105 Hz; (b) 157.5 m/s

2. 1000/3Hz

3. 162 vibrations/sec, 3

4. Decrease by 5Hz

5. $\alpha = \frac{\rho(v_2^2 - v_1^2)}{Y\Delta T}$

6. 50 m/sec; 25 m/sec

7. $\frac{2\pi}{5}$ N

8. 300 Hz

9. 120 cm from an end.

10. 225 N

11. 0.66 m, 0.33 m, 0.22 m

12. $\vec{r}_{(\text{in mm})} = 25000\hat{i} + 2\sin(100\pi t + 30°)\hat{j} + 3\sin(100\pi t + 60°)\hat{k}$

$\vec{v}(\text{in m/s}) = 0.2\pi\cos(100\pi t + 30°)\hat{j} + 0.3\pi\cos(100\pi t + 60°)\hat{k}$

Phase difference at time 't' is 30° constant always after they meet at 'P'.

13. (a) $T_{max} = 1/4\ m\ \omega^2 a^2_{max}$; (b) $T = 1/8\ m\omega^2\ a^2_{max}$.