NUCLEAR PHYSICS

Contents

JEE (ADVANCED) SYLLABUS

Atomic nucleus; α, β and γ radiations; Law of radioactive decay; Decay constant ; Half-life and mean life; Binding energy and its calculation; Fission and fusion processes; Energy calculation in these processes.

JEE (MAIN) SYLLABUS

Composition and size of nucleus, atomic masses, isotopes, isobars; isotones. Radioactivity-alpha, beta and gamma particles / rays and their properties ; radioactive decay law. Mass-energy relation, mass defect; binding energy per nucleon and its variation with mass number, nuclear fission and fusion.

NUCLEAR PHYSICS

It is the branch of physics which deals with the study of nucleus.

1. NUCLEUS :

(a) Discoverer : Rutherford

(b) Constituents : neutrons (n) and protons (p) [collectively known as nucleons]

1. **Neutron :** It is a neutral particle. It was discovered by J. Chadwick (in 1932).
   Mass of neutron, $m_n = 1.6749286 \times 10^{-27}$ kg.
2. **Proton :** It has a charge equal to +e. It was discovered by Goldstein.
   Mass of proton, $m_p = 1.6726231 \times 10^{-27}$ kg

$$m_p \tilde{\lesssim} m_n$$

(c) Representation :

$_Z X^A$ or $^A_Z X$

where X $\Rightarrow$ symbol of the atom

Z $\Rightarrow$ Atomic number = number of protons

A $\Rightarrow$ Atomic mass number = total number of nucleons.

= no. of protons + no. of neutrons.

Atomic mass number :

It is the nearest integer value of mass represented in a.m.u. (atomic mass unit).

1 a.m.u. = $\frac{1}{12}$ [mass of one atom of $_6C^{12}$ atom at rest and in ground state]

1.6603×10^{-27} kg ; 931.478 MeV/c^2

mass of proton (m_p) = mass of neutron (m_n) = 1 a.m.u.

Some definitions :

(1) Isotopes :

The nuclei having the same number of protons but different number of neutrons are called isotopes.

(2) Isotones :

Nuclei with the same neutron number N but different atomic number Z are called isotones.

(3) Isobars :

The nuclei with the same mass number but different atomic number are called isobars.

(d) Size of nucleus : Order of 10^{-15} m (fermi)

Radius of nucleus ; $R = R_0A^{1/3}$

where $R_0 = 1.1 \times 10^{-15}$ m (which is an empirical constant)

A = Atomic mass number of atom.

(e) Density : density = $\frac{\text{mass}}{\text{volume}} \cong \frac{Am_p}{\frac{4}{3}\pi R^3} = \frac{Am_p}{\frac{4}{3}\pi(R_0A^{1/3})^3} = \frac{3m_p}{4\pi\ \ R_0^3}$

$$= \frac{3 \times 1.67 \times 10^{-27}}{4 \times 3.14 \times (1.1 \times 10^{-15})^3} = 3 \times 10^{17} \text{ kg/m}^3$$

Nuclei of almost all atoms have almost same density as nuclear density is independent of the mass number (A) and atomic number (Z).

Solved Example

Example 1. Calculate the radius of ^{70}Ge.

Solution : We have, $R = R_0 A^{1/3} = (1.1 \text{ fm}) (70)^{1/3}$
$= (1.1 \text{ fm}) (4.12) = 4.53 \text{ fm}$.

Example 2. Calculate the electric potential energy of interaction due to the electric repulsion between two nuclei of ^{12}C when they 'touch' each other at the surface

Solution : The radius of a ^{12}C nucleus is
$R = R_0 A^{1/3}$
$= (1.1 \text{ fm}) (12)^{1/3} = 2.52 \text{ fm}$.
The separation between the centres of the nuclei is 2R = 5.04 fm. The potential energy of the pair is

$$U = \frac{q_1 q_2}{4\pi\varepsilon_0 r} = (9 \times 10^9 \text{ N–m}^2/\text{C}^2) \frac{(6 \times 1.6 \times 10^{-19} \text{C})^2}{5.04 \times 10^{-15} \text{m}} = 1.64 \times 10^{-12} \text{ J} = 10.2 \text{ MeV}.$$

2. MASS DEFECT

It has been observed that there is a difference between expected mass and actual mass of a nucleus.

$M_{expected} = Z m_p + (A – Z)m_n$

$M_{observed} = M_{atom} – Zm_e$

It is found that $M_{observed} < M_{expected}$

Hence, mass defect is defined as Mass defect = $M_{expected} – M_{observed}$

$\Delta m = [Zm_p + (A – Z)m_n] – [M_{atom} – Zm_e]$

3. BINDING ENERGY

It is the minimum energy required to break the nucleus into its constituent particles.

or

Amount of energy released during the formation of nucleus by its constituent particles and bringing them from infinite separation.

Binding Energy (B.E.) = Δmc^2

BE = Δm (in amu) × 931.5 MeV/amu

= Δm × 931.5 MeV

Note : If binding energy per nucleon is more for a nucleus then it is more stable.

For example

If $\left(\frac{B.E_1}{A_1}\right) > \left(\frac{B.E_2}{A_2}\right)$ then nucleus 1 would be more stable.

Solved Example

Example 3. Following data is available about 3 nuclei P, Q & R. Arrange them in decreasing order of stability

	P	Q	R
Atomic mass number (A)	10	5	6
Binding Energy (MeV)	100	60	66

Solution : $\left(\frac{B.E}{A}\right)_P = \frac{100}{10} = 10$ $\Rightarrow$ $\left(\frac{BE}{A}\right)_Q = \frac{60}{5} = 12$

$\left(\frac{B.E.}{A}\right)_R = \frac{66}{6} = 11$ $\therefore$ Stability order is Q > R > P.

Example 4. The three stable isotopes of neon: ${}^{20}_{10}Ne$, ${}^{21}_{10}Ne$ and ${}^{22}_{10}Ne$ have respective abundances of 90.51% 0.27% and 9.22%. The atomic masses of three isotopes are 19.99 u. 20.99 u, respectively. Obtain the average atomic mass of neon.

Solution : $m = \dfrac{90.51 \times 19.99 + 0.27 \times 20.99 + 9.22 \times 22}{100} = 20.18$ u

Example 5. A nuclear reaction is given as A + B → C + D. Binding energies of A, B, C and D are given as B_1, B_2, B_3 and B_4. Find the energy released in the reaction

Solution : $(B_3 + B_4) – (B_1 + B_2)$

Example 6 Calculate the binding energy of an alpha particle from the following data:

mass of ${}^{1}_{1}H$ atom = 1.007826 u

mass of ${}^{4}_{2}He$ neutron = 1.008665 u

mass of atom = 4.00260 u Take 1 u = 931 MeV/c^2.

Solution : The alpha particle contains 2 protons and 2 neutrons. The binding energy is

$B = (2 \times 1.007826 \text{ u} + 2 \times 1.008665 \text{ u} – 4.00260 \text{ u})c^2$

$= (0.03038 \text{ u})c^2 = 0.03038 \times 931 \text{ MeV} = 28.3 \text{ MeV}.$

Example 7. Find the binding energy of ${}^{56}_{26}Fe$. Atomic mass of ${}^{56}Fe$ is 55.9349 u and that of ${}^{1}H$ is 1.00783 u. Mass of neutron = 1.00867 u.

Solution : The number of protons in ${}^{56}_{26}Fe$ = 26 and the number of neutrons = 56 – 26 = 30.

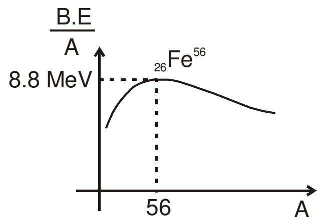
The binding energy of ${}^{56}_{26}Fe$ is

$= [26 \times 1.00783 \text{ u} + 30 \times 1.00867 \text{ u} – 55.9349 \text{ u}]\, c^2 = (0.52878 \text{ u})c^2$

$= (0.52878 \text{ u})\,(931 \text{ MeV/u}) = 492 \text{ MeV}.$

3.1 Variation of binding energy per nucleon with mass number :

The binding energy per nucleon first increases on an average and reaches a maximum of about 8.8 MeV for A 50 → 80. For still heavier nuclei, the binding energy per nucleon slowly decreases as A increases. Binding energy per nucleon is maximum for ${}_{26}Fe^{56}$, which is equal to 8.8 MeV. Binding energy per nucleon is more for medium nuclei than for heavy nuclei. Hence, medium nuclei are highly stable.



- ∗ The heavier nuclei being unstable have tendency to split into medium nuclei. This process is called **Fission**.
- ∗ The Lighter nuclei being unstable have tendency to fuse into a medium nucleus. This process is called **Fusion**.

4. RADIOACTIVITY :

It was discovered by Henry Becquerel.

Spontaneous emission of radiations (α, β, γ) from unstable nucleus is called **radioactivity**. Substances which shows radioactivity are known as **radioactive substance**.

Radioactivity was studied in detail by Rutherford.

In radioactive decay, an unstable nucleus emits α particle or β particle. After emission of α or β the remaining nucleus may emit γ-particle, and converts into more stable nucleus.

α-particle :

It is a doubly charged helium nucleus. It contains two protons and two neutrons.

Mass of α-particle = Mass of ${}_{2}He^{4}$ atom – $2m_e \approx 4\, m_p$

Charge of α-particle = + 2 e

β-particle :

(a) β^- (electron) :

Mass = m_e ; Charge = –e

(b) β^+ (positron) :

Mass = m_e ; Charge = +e

positron is an antiparticle of electron.

Antiparticle :

A particle is called antiparticle of other if on collision both can annihilate (destroy completely) and converts into energy. For example : (i) electron (– e, m_e) and positron (+ e, m_e) are anti particles. (ii) neutrino (ν) and antineutrino ($\bar{\nu}$) are antiparticles.

γ-particle : They are energetic photons of energy of the order of Mev and having rest mass zero.

5. RADIOACTIVE DECAY (DISPLACEMENT LAW) :

5.1 α-decay :

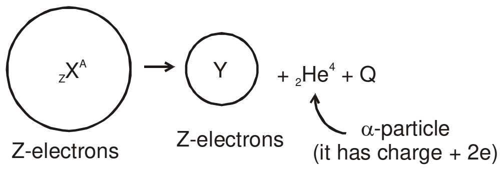
Nuclei with mass number greater than 210 undergo α-decay.

$_ZX^A \rightarrow \; _{Z-2}Y^{A-4} + {_2He^4} + Q$

Q value : It is defined as energy released during the decay process.

Q value = rest mass energy of reactants – rest mass energy of products.

This energy is available in the form of increase in K.E. of the products.



Let, M_x = mass of atom $_ZX^A$

M_y = mass of atom $_{Z-2}Y^{A-4}$

M_{He} = mass of atom $_2He^4$.

Q value = $[(M_x - Zm_e) - \{(M_y - (Z-2)m_e) + (M_{He} - 2m_e)\}]c^2$

$= [M_x - M_y - M_{He}]\, c^2$

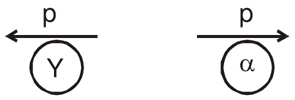
Considering actual number of electrons in α-decay

Q value = $[M_x - (M_y + 2m_e) - (M_{He} - 2m_e)]c^2$

$= [M_x - M_y - M_{He}]\, c^2$

Calculation of kinetic energy of final products :

As atom X was initially at rest and no external forces are acting, so final momentum also has to be zero. Hence both Y and α-particle will have same momentum in magnitude but in opposite direction.

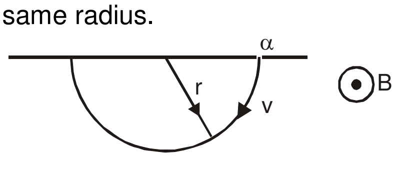


$p_\alpha^2 = p_Y^2$ $\qquad 2m_\alpha T_\alpha = 2m_Y T_Y$ $\qquad$ (Here we are representing T for kinetic energy)

$Q = T_y + T_\alpha$ $\qquad m_\alpha T_\alpha = m_Y T_Y$

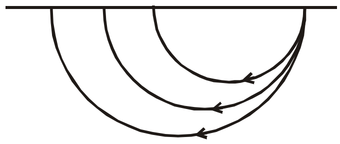
$$T_\alpha = \frac{m_Y}{m_\alpha + m_Y} Q \;;\quad T_Y = \frac{m_\alpha}{m_\alpha + m_Y} Q \;\Rightarrow\; T_\alpha = \frac{A-4}{A}\, Q \quad ; \quad T_Y = \frac{4}{A} Q$$

From the above calculation, one can see that all the α-particles emitted should have same kinetic energy. Hence, if they are passed through a region of uniform magnetic field having direction perpendicular to velocity, they should move in a circle of same radius.



$$r = \frac{mv}{qB} = \frac{mv}{2eB} = \frac{\sqrt{2Km}}{2eB}$$

Experimental Observation :



Experimentally it has been observed that all the α-particles do not move in the circle of same radius, but they move in `circles having different radii.

This shows that they have different kinetic energies. But it is also observed that they follow circular paths of some fixed values of radius i.e. yet the energy of emitted α-particles is not same but it is quantized. The reason behind this is that all the daughter nuclei produced are not in their ground state but some of the daughter nuclei may be produced in their excited states and they emits photon to acquire their ground state.

$$X \longrightarrow Y^*(\text{excited state}) + \alpha + Q$$
$$\downarrow$$
$$Y + \text{photon } (\gamma \text{ particle})$$

The only difference between Y and Y* is that Y* is in excited state and Y is in ground state.

Let, the energy of emitted γ-particles be E

$\therefore \quad Q = T_\alpha + T_Y + E$

where $Q = [M_x - M_y - M_{He}]\,c^2$

$T_\alpha + T_Y = Q - E$

$$T_\alpha = \frac{m_Y}{m_\alpha + m_Y}(Q-E) \; ; \qquad T_Y = \frac{m_\alpha}{m_\alpha + m_Y}(Q-E)$$

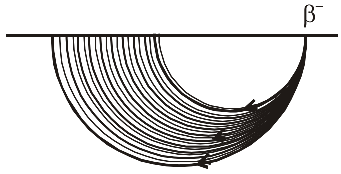
5.2 β^-- decay :

$${}_ZX^A \longrightarrow {}_{Z+1}Y^A + {}_{-1}e^0 + Q$$

${}_{-1}e^0$ can also be written as ${}_{-1}\beta^0$.

Here also one can see that by momentum and energy conservation, we will get

$$T_e = \frac{m_Y}{m_e + m_Y}Q \; ; \; T_Y = \frac{m_e}{m_e + m_Y}Q$$



as $m_e << m_Y$, we can consider that all the energy is taken away by the electron.

From the above results, we will find that all the β-particles emitted will have same energy and hence they have same radius if passed through a region of perpendicular magnetic field. But, experimental observations were completely different.

On passing through a region of uniform magnetic field perpendicular to the velocity, it was observed that β-particles take circular paths of different radius having a continuous spectrum.

To explain this, Paulling has introduced the extra particles called neutrino and antineutrino (antiparticle of neutrino).

$\bar{\nu} \rightarrow$ antineutrino, $\nu \rightarrow$ neutrino

Properties of antineutrino($\bar{\nu}$) & neutrino(ν) :

(1) They have rest mass equal to zero or, at most, the mass equivalent of a few electronvolts.
speed = c (or nearly equal to c)
Energy, $E = mc^2$

(2) They are chargeless (neutral)

(3) They have spin quantum number, $s = \pm \frac{1}{2}$

Considering the emission of antineutrino, the equation of β^- - decay can be written as

$${}_ZX^A \longrightarrow {}_{Z+1}Y^A + {}_{-1}e^0 + Q + \bar{\nu}$$

(4) They are not electromagnetic in nature as is the photon, the neutrino can pass unimpeded through vast amounts of matter.

Production of antineutrino along with the electron helps to explain the continuous spectrum because the energy is distributed randomly between electron and $\bar{\nu}$ and it also helps to explain the spin quantum number balance (p, n and ± e each has spin quantum number ± 1/2).

During β^- - decay, inside the nucleus a neutron is converted to a proton with emission of an electron and antineutrino.

$n \rightarrow p + {}_{-1}e^0 + \bar{\nu}$

Let, M_x = mass of atom ${}_ZX^A$

M_y = mass of atom ${}_{Z+1}Y^A$

m_e = mass of electron

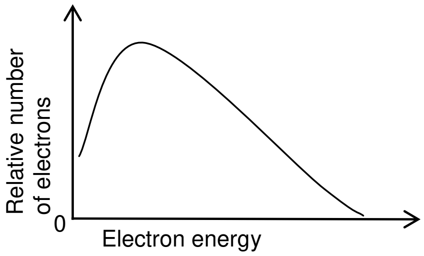
Q value = $[(M_X - Zm_e) - \{(M_Y - (z + 1)\, m_e) + m_e\}]\, c^2 = [M_X - M_Y]\, c^2$

Considering actual number of electrons.

Q value = $[M_X - \{(M_Y - m_e) + m_e\}]\, c^2 = [M_X - M_Y]\, c^2$

Energy spectrum of β-particles :

The figure below shows the energy spectrum of the electrons emitted in the beta decay.



Solved Examples

Example 8. Consider the beta decay $^{198}Au \rightarrow {}^{198}Hg^* + \beta^- + \bar{\nu}$ where $^{198}Hg^*$ represents a mercury nucleus in an excited state at energy 1.088 MeV above the ground state. What can be the maximum kinetic energy of the electron emitted? The atomic mass ^{198}Au is 197.968233 u and that of ^{198}Hg is 197.966760 u.

Solution : If the product nucleus ^{198}Hg is formed in its ground state, the kinetic energy available to the electron and the antineutrino is $Q = [m(^{198}Au) - m(^{198}Hg)]c^2$.

As $^{198}Hg^*$ has energy 1.088 MeV more than ^{198}Hg in ground state, the kinetic energy actually available is $Q = [m(^{198}Au) - m(^{198}Hg)]c^2 - 1.088$ MeV

$= (197.968233 \text{ u} - 197.966760 \text{ u}) \left(931 \dfrac{\text{MeV}}{\text{u}}\right) - 1.088 \text{ MeV}$

$= 1.3686 \text{ MeV} - 1.088 \text{ MeV} = 0.2806 \text{ MeV}.$

This is also the maximum possible kinetic energy of the electron emitted.

5.3 β^+ - decay :

${}_ZX^A \rightarrow {}_{Z-1}Y^A + {}_{+1}e^0 + \nu + Q$

In β^+ decay, inside a nucleus a proton is converted into a neutron, positron and neutrino.

$p \rightarrow n + {}_{+1}e^0 + \nu$

As mass increases during conversion of proton to a neutron, hence it requires energy for β^+ decay to take place,

∴ β^+ decay is rare process. It can take place in the nucleus where a proton can take energy from the nucleus itself.

Q value = $[(M_X - Zm_e) - \{(M_Y - (Z - 1)\, m_e) + m_e\}]\, c^2 = [M_X - M_Y - 2m_e]\, c^2$

Considering actual number of electrons.

Q value = $[M_X - \{(M_Y + m_e) + m_e\}]\, c^2 = [M_X - M_Y - 2m_e]\, c^2$

Solved Examples

Example 9. Calculate the Q-value in the following decays :

(a) $^{19}O \rightarrow {}^{19}F + e^- + \bar{\nu}$ (b) $^{25}Al \rightarrow {}^{25}Mg + e^+ + \nu$.

The atomic masses needed are as follows:

^{19}O	^{19}F	^{25}Al	^{25}Mg
19.003576 u	18.998403 u	24.990432 u	24.985839 u

Solution : (a) The Q-value of β^- -decay is

$Q = [m(^{19}O) - m(^{19}F)]c^2$

$= [19.003576\text{ u} - 18.998403\text{ u}]\ (931\text{ MeV/u}) = 4.816\text{ MeV}$

(b) The Q-value of β^+ -decay is

$Q = [m(^{25}Al) - m(^{25}Mg) - 2m_e]c^2$

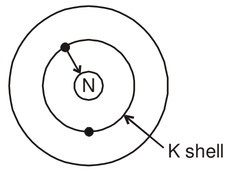
$$= \left[24.99032u - 24.985839u - 2 \times 0.511\frac{\text{MeV}}{c^2}\right]c^2$$

$= (0.004593\text{ u})\ (931\text{ MeV/u}) - 1.022\text{ MeV}$

$= 4.276\text{ MeV} - 1.022\text{ MeV} = 3.254\text{ MeV}.$

5.4 K capture :

It is a rare process which is found only in few nucleus. In this process the nucleus captures one of the atomic electrons from the K shell. A proton in the nucleus combines with this electron and converts itself into a neutron. A neutrino is also emitted in the process and is emitted from the nucleus.



Electron capture is competitive with positron emission. It occurs more often than positron emission in heavy nuclides because electrons are relatively closer to nucleus which allows more interaction.

$p + {}_{-1}e^0 \rightarrow n + \nu$

If X and Y are atoms then reaction is written as :

${}_ZX^A \rightarrow {}_{Z-1}Y^A + \nu + Q$ + characteristic x-rays of Y.

If X and Y are taken as nucleus, then reaction is written as :

${}_ZX^A + {}_{-1}e^0 \rightarrow {}_{Z-1}Y^A + \nu$

5.5 γ-decay :

Like an atom a nucleus can also exist in states whose energies are higher than that of its ground state. Excited nuclei return to their ground states by emitting photons whose energies correspond to the energy differences between the various initial and final states in the transitions involved. The photons emitted by nuclei have energy up to several Mev, and are traditionally called gamma rays.

${}_{12}Mg^{27} \rightarrow {}_{13}^{*}Al^{27} + {}_{-1}e^0$

${}_{13}^{*}Al^{27} \rightarrow {}_{13}Al^{27} + \gamma$

Al* represents aluminium nucleus in its excited state.

When γ-rays are passed through a slab their intensity decreases exponentially with slab thickness x. $I = I_0e^{-\mu x}$ where μ is absorption coefficient. It depends on the slab.

Note : (1) Nuclei having atomic numbers from Z = 84 to 112 shows radioactivity.

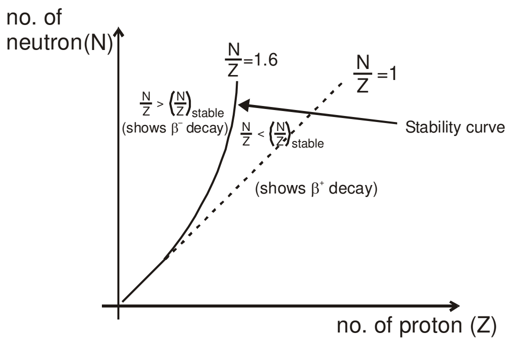
(2) Nuclei having Z = 1 to 83 are stable (only few exceptions are there)

(3) Whenever a neutron is produced, a neutrino is also produced.

(4) Whenever a neutron is converted into a proton, a antineutrino is produced.

6. NUCLEAR STABILITY :

Figure shows a plot of neutron number N versus proton number Z for the nuclides found in nature. The solid line in the figure represents the stable nuclides. For light stable nuclides, the neutron number is equal to the proton number so that ratio N/Z is equal to 1. The ratio N/Z increases for the heavier nuclides and becomes about 1.6 for the heaviest stable nuclides. The points (Z, N) for stable nuclides fall in a rather well-defined narrow region. There are nuclides to the left of the stability belt as well as to the right of it. The nuclides to the left of the stability region have excess neutrons, whereas, those to the right of the stability belt have excess protons.



These nuclides are unstable and decay with time according to the laws of radioactive disintegration. Nuclides with excess neutrons (lying above stability belt) show β^- decay while nuclides with excess protons (lying below stability belt) show β^+ decay and K - capture.

7. NUCLEAR FORCE :

(i) Nuclear forces are basically attractive and are responsible for keeping the nucleons bound in a nucleus in spite of repulsion between the positively charge protons.
(ii) It is strongest force with in nuclear dimensions (F_n ; 100 F_e)
(iii) It is short range force (acts only inside the nucleus)
(iv) It acts only between neutron-neutron, neutron-proton and proton-proton i.e. between nucleons.
(v) It does not depend on the nature of nucleons.
(vi) An important property of nuclear force is that it is not a central force. The force between a pair of nucleons is not solely determined by the distance between the nucleons. For example, the nuclear force depends on the directions of the spins of the nucleons. The force is stronger if the spins of the nucleons are parallel (i.e., both nucleons have m_s = + 1/2 or – 1/2) and is weaker if the spins are antiparallel (i.e., one nucleon has m_s = + 1/2 and the other has m_s = – 1/2). Here m_s is spin quantum number.

8. RADIOACTIVE DECAY : STATISTICAL LAW :

(Given by Rutherford and Soddy)

Rate of radioactive decay $\propto$ N

where N = number of active nuclei = λ N

where λ= decay constant of the radioactive substance.

Decay constant is different for different radioactive substances, but it does not depend on amount of substance and time.

SI unit of λ is s^{-1}

If $\lambda_1 > \lambda_2$ then first substance is more radioactive (less stable) than the second one.

For the case, if A decays to B with decay constant λ

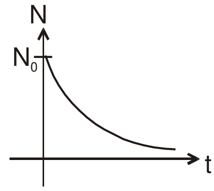
$$A \xrightarrow{\lambda} B$$

t = 0 N_0 0 where N_0 = number of active nuclei of A at t = 0

t = t N N' where N = number of active nuclei of A at t = t

Rate of radioactive decay of A $= -\frac{dN}{dt} = \lambda N$

$-\int_{N_0}^{N} \frac{dN}{N} = \int_0^t \lambda dt \Rightarrow \quad N = N_0\, e^{-\lambda t}$ (it is exponential decay)



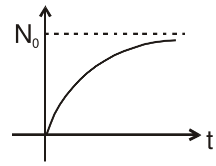
Number of nuclei decayed (i.e. the number of nuclei of B formed)

$N' = N_0 - N$

$= N_0 - N_0 e^{-\lambda t}$

$N' = N_0(1 - e^{-\lambda t})$

$N' = N_0 - N$



8.1 Half life ($T_{1/2}$) :

It is the time in which number of active nuclei becomes half.

$N = N_0\, e^{-\lambda t}$

After one half life, $N = \frac{N_0}{2}$

$\frac{N_0}{2} = N_0\, e^{-\lambda t} \Rightarrow t = \frac{\ln 2}{\lambda} \Rightarrow \frac{0.693}{\lambda} = t_{1/2}$

$t_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda}$ **(to be remembered)**

Number of nuclei present after n half lives i.e. after a time $t = n\, t_{1/2}$

$N = N_0 e^{-\lambda t} = N_0\, e^{-\lambda n t_{1/2}} = N_0\, e^{-\lambda n \frac{\ln 2}{\lambda}}$

$= N_0\, e^{\ln 2^{(-n)}} = N_0\, (2)^{-n} = N_0 (1/2)^n = \frac{N_0}{2^n}$

$\{n = \frac{t}{t_{1/2}}$. It may be a fraction, need not to be an integer}

or $N_0 \xrightarrow[\text{half life}]{\text{after 1st}} \frac{N_0}{2} \xrightarrow{2} N_0\left(\frac{1}{2}\right)^2 \xrightarrow{3} N_0\left(\frac{1}{2}\right)^3 \ldots\ldots\ldots\ldots\ldots \xrightarrow{n} N_0\left(\frac{1}{2}\right)^n$

Solved Example

Example 10. A radioactive sample has 6.0×10^{18} active nuclei at a certain instant. How many of these nuclei will still be in the same active state after two half-lives?

Solution : In one half-life the number of active nuclei reduces to half the original number. Thus, in two half lives the number is reduced to $\left(\frac{1}{2}\right)\times\left(\frac{1}{2}\right)$ of the original number. The number of remaining active nuclei is, therefore, $6.0 \times 10^{18} \times \left(\frac{1}{2}\right)\times\left(\frac{1}{2}\right) = 1.5 \times 10^{18}$.

Example 11. The number of ^{238}U atoms in an ancient rock equals the number of ^{206}Pb atoms. The half-life of decay of ^{238}U is 4.5×10^{9} y. Estimate the age of the rock assuming that all the ^{206}Pb atoms are formed from the decay of ^{238}U.

Solution : Since the number of ^{206}Pb atoms equals the number of ^{238}U atoms, half of the original ^{238}U atoms have decayed. It takes one half-life to decay half of the active nuclei. Thus, the sample is 4.5×10^{9} y old.

8.2 Activity :

Activity is defined as rate of radioactive decay of nuclei

It is denoted by A or R $A = \lambda N$

If a radioactive substance changes only due to decay then

$$A = -\frac{dN}{dt}$$

As in that case, $N = N_0 e^{-\lambda t}$

$A = \lambda N = \lambda N_0 e^{-\lambda t}$

$A = A_0 e^{-\lambda t}$

SI Unit of activity : becquerel (Bq) which is same as 1 dps (disintegration per second)

The popular unit of activity is curie which is defined as

1 curie = 3.7×10^{10} dps (which is activity of 1 gm Radium)

Solved Example

Example 12. The decay constant for the radioactive nuclide ^{64}Cu is 1.516×10^{-5} s^{-1}. Find the activity of a sample containing 1 μg of ^{64}Cu. Atomic weight of copper = 63.5 g/mole. Neglect the mass difference between the given radioisotope and normal copper.

Solution : 63.5 g of copper has 6×10^{23} atoms. Thus, the number of atoms in 1 μg of Cu is

$$N = \frac{6 \times 10^{23} \times 1\mu g}{63.5 g} = 9.45 \times 10^{15}$$

The activity = λN

$= (1.516 \times 10^{-5}\ s^{-1}) \times (9.45 \times 10^{15}) = 1.43 \times 10^{11}$ disintegrations/s

$= \frac{1.43 \times 10^{11}}{3.7 \times 10^{10}}$ Ci = 3.86 Ci.

Activity after n half lives : $\frac{A_0}{2^n}$

Example 13. The half-life of a radioactive nuclide is 20 hours. What fraction of original activity will remain after 40 hours?

Solution : 40 hours means 2 half lives.

Thus $A = \frac{A_0}{2^2} = \frac{A_0}{4}$ or $\frac{A}{A_0} = \frac{1}{4}$.

So one fourth of the original activity will remain after 40 hours.

Specific activity : The activity per unit mass is called specific activity.

8.3 Average Life :

$$T_{avg} = \frac{\text{sum of ages of all the nuclei}}{N_0} = \frac{\int_0^\infty \lambda N_0 e^{-\lambda t} dt.t}{N_0} = \frac{1}{\lambda}$$

Solved Examples

Example 14. The half-life of ^{198}Au is 2.7 days. Calculate (a) the decay constant, (b) the average-life and (c) the activity of 1.00 mg of ^{198}Au. Take atomic weight of ^{198}Au to be 198 g/mol.

Solution : (a) The half-life and the decay constant are related as

$$t_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda} \quad \text{or,} \qquad \lambda = \frac{0.693}{t_{1/2}} = \frac{0.693}{2.7\text{days}} = \frac{0.693}{2.7 \times 24 \times 3600\text{s}} = 2.9 \times 10^{-6}\ s^{-1}.$$

(b) The average-life is $t_{av} = \frac{1}{\lambda} = 3.9$ days.

(c) The activity is $A = \lambda N$. Now, 198 g of ^{198}Au has 6×10^{23} atoms.
The number of atoms in 1.00 mg of ^{198}Au is

$$N = 6 \times 10^{23} \times \frac{1.0\ \ mg}{198\ \ g} = 3.03 \times 10^{18}.$$

Thus, $A = \lambda N = (2.9 \times 10^{-6}\ s^{-1})\ (3.03 \times 10^{18})$

$$= 8.8 \times 10^{12} \text{ disintegrations/s} = \frac{8.8 \times 10^{12}}{3.7 \times 10^{10}} \text{Ci} = 240 \text{ Ci}.$$

Example 15. Suppose, the daughter nucleus in a nuclear decay is itself radioactive. Let λ_p and λ_d be the decay constants of the parent and the daughter nuclei. Also, let N_p and N_d be the number of parent and daughter nuclei at time t. Find the condition for which the number of daughter nuclei becomes constant.

Solution : The number of parent nuclei decaying in a short time interval t to t + dt is $\lambda_p N_p dt$. This is also the number of daughter nuclei decaying during the same time interval is $\lambda_d N_d dt$. The number of the daughter nuclei will be constant if

$$\lambda_p N_p dt = \lambda_d N_d dt \qquad \text{or} \qquad \lambda_p N_p = \lambda_d N_d.$$

Example 16. A radioactive sample decays with an average-life of 20 ms. A capacitor of capacitance 100 μF is charged to some potential and then the plates are connected through a resistance R. What should be the value of R so that the ratio of the charge on the capacitor to the activity of the radioactive sample remains constant in time?

Solution : The activity of the sample at time t is given by

$$A = A_0\, e^{-\lambda t}$$

where λ is the decay constant and A_0 is the activity at time t = 0 when the capacitor plates are connected. The charge on the capacitor at time t is given by

$$Q = Q_0\, e^{-t/CR}$$

where Q_0 is the charge at t = 0 and C = 100 μF is the capacitance. Thus $\frac{Q}{A} = \frac{Q_0}{A_0} \frac{e^{-t/CR}}{e^{-\lambda t}}$, .

It is independent of t if $\lambda = \frac{1}{CR}$ or $R = \frac{1}{\lambda C} = \frac{t_{av}}{C} = \frac{20 \times 10^{-3} s}{100 \times 10^{-6} F} = 200\ \Omega$.

Example 17. A radioactive nucleus can decay by two different processes. The half-life for the first process is t_1 and that for the second process is t_2. Show that the effective half-life t of the nucleus is given by $\frac{1}{t} = \frac{1}{t_1} + \frac{1}{t_2}$.

Solution : The decay constant for the first process is $\lambda_1 = \frac{\ln 2}{t_1}$ and for the second process it is $\lambda_2 = \frac{\ln 2}{t_1}$.

The probability that an active nucleus decays by the first process in a time interval dt is $\lambda_1 dt$. Similarly, the probability that it decays by the second process is $\lambda_2 dt$. The probability that it either decays by the first process or by the second process is $\lambda_1 dt + \lambda_2 dt$. If the effective decay constant is λ, this probability is also equal to λdt. Thus.

$$\lambda dt = \lambda_1 dt + \lambda_2 dt$$

or $\lambda = \lambda_1 + \lambda_2$ or $\frac{1}{t} = \frac{1}{t_1} + \frac{1}{t_2}$ (**To be remembered**)

Example 18. A factory produces a radioactive substance A at a constant rate R which decays with a decay constant λ to form a stable substance. Find (i) the no. of nuclei of A and (ii) Number of nuclei of B, at any time t assuming the production of A starts at t = 0. (iii) Also find out the maximum number of nuclei of 'A' present at any time during its formation.

Solution : Factory $\xrightarrow[\text{const. rate}]{R} A \xrightarrow[\text{decay}]{\lambda}$ B

Let N be the number of nuclei of A at any time t

$\therefore \quad \frac{dN}{dt} = R - \lambda N \qquad\qquad \int_0^N \frac{dN}{R-\lambda N} = \int_0^t dt$

On solving we will get $N = R/\lambda(1 - e^{-\lambda t})$

(ii) Number of nuclei of B at any time t, $N_B = R\,t - N_A = Rt - R/\lambda(1 - e^{-\lambda t}) = R/\lambda\,(\lambda\,t - 1 + e^{-\lambda t})$.

(iii) Maximum number of nuclei of 'A' present at any time during its formation = R/λ.

Example 19. A radioactive substance "A" having N_0 active nuclei at t = 0, decays to another radioactive substance "B" with decay constant λ_1. B further decays to a stable substance "C" with decay constant λ_2.

(a) Find the number of nuclei of A, B and C after time t.

(b) What would be the answer of part (a) if $\lambda_1 >> \lambda_2$ and $\lambda_1 << \lambda_2$.

Solution : (a) The decay scheme is as shown

$$A \xrightarrow{\lambda_1} B \xrightarrow{\lambda_2} C \text{ (stable)}$$

t = 0	N_0	0	0
t	N_1	N_2	N_3

Here N_1, N_2 and N_3 represent the nuclei of A, B and C at any time t.

For A, we can write

$N_1 = N_0\, e^{-\lambda 1 t}$ .... (1)

For B, we can write

$\frac{dN_2}{dt} = \lambda_1 N_1 - \lambda_2 N_2$ .....(2) or, $\frac{dN_2}{dt} + \lambda_2 N_2 = \lambda_1 N_1$

This is a linear differential equation with integrating factor I.F. = $e^{\lambda 2t}$

$e^{\lambda_2 t}\frac{dN_2}{dt} + e^{\lambda_2 t}\lambda_2 N_2 = \lambda_1 N_1\, e^{\lambda_2 t}$; $\int d(N_2 e^{\lambda_2 t}) = \int \lambda_1 N_1 e^{\lambda_2 t}\, dt$

$N_2\, e^{\lambda_2 t} = \lambda_1 N_0\, e^{\lambda_2 t}$ ...... using (1)

$N_2\, e^{\lambda_2 t} = \lambda_1 N_0 \frac{e^{(\lambda_2 - \lambda_1)\,t}}{\lambda_2 - \lambda_1} + C$ ......(3)

At t = 0, $N_2 = 0$ $\qquad 0 = \frac{\lambda_1 N_0}{\lambda_2 - \lambda_1} + C$

Hence $C = \frac{\lambda_1 N_0}{\lambda_1 - \lambda_2}$ Using C in eqn. (3), we get $N_2 = \frac{\lambda_1 N_0}{\lambda_2 - \lambda_1}\left(e^{-\lambda_1 t} - e^{-\lambda_2 t}\right)$

and $N_1 + N_2 + N_3 = N_0$ $\qquad \therefore\ N_3 = N_0 - (N_1 + N_2)$

(b) For $\lambda_1 >> \lambda_2$ $\qquad N_2 = \frac{\lambda_1 N_0}{-\lambda_1}\,(-e^{-\lambda_2 t}) = N_0\, e^{-\lambda 2t}$

For $\lambda_1 << \lambda_2$ $\qquad N_2 = \frac{\lambda_1 N_0}{\lambda_2}\,(e^{-\lambda_1 t}) = 0$

Alternate solution of (b) part without use of answer of part (a) :

If $\lambda_1 > \lambda_2$ that means A will decay very fast to 'B' and B will then decay slowly. We can say that practically N_1 vanishes in very short time & B has initial no. of atoms as N_0.

$\therefore$ Now $N_2 = N_0\, e^{-\lambda_2 t}$ & $N_1 = N_0\, e^{-\lambda_1 t}$

If $\lambda_1 << \lambda_2$ then B is highly unstable and it will soon decay into C.

So, it's rate of formation $\approx$ its rate of decay.

$\therefore\ \lambda_1 N_1 \approx \lambda_2 N_2 \Rightarrow \qquad N_2 = \frac{\lambda_1 N_1}{\lambda_2} = \frac{\lambda_1 N_0}{\lambda_2}\,(e^{-\lambda_1 t})$

9. NUCLEAR FISSION :

In nuclear fission heavy nuclei of A, above 200, break up into two or more fragments of comparable masses. The most attractive bid, from a practical point of view, to achieve energy from nuclear fission is to use $_{92}U^{236}$ as the fission material. The technique is to hit a uranium sample by slow-moving neutrons (kinetic energy $\approx$ 0.04 eV, also called thermal neutrons). A $_{92}U^{235}$ nucleus has large probability of absorbing a slow neutron and forming $_{92}U^{236}$ nucleus. This nucleus then fissions into two or more parts. A variety of combinations of the middle-weight nuclei may be formed due to the fission. For example, one may have

$_{92}U^{235} + {_0n^1} \rightarrow {_{92}U^{236}} \rightarrow X + Y + 2{_0n^1}$, or $_{92}U^{235} + {_0n^1} \rightarrow {_{92}U^{236}} \rightarrow X' + Y' + 3{_0n^1}$

and a number of other combinations.

* On an average 2.5 neutrons are emitted in each fission event.
* Mass lost per reaction $\approx$ 0.2 a.m.u.
* In nuclear fission the total B.E. increases and excess energy is released.
* In each fission event, about 200 MeV of energy is released a large part of which appears in the form of kinetic energies of the two fragments. Neutrons take away about 5MeV.

  eg. ${}^{235}_{92}U + {}_{o}n^1 \rightarrow {}^{236}_{92}U \rightarrow {}^{141}_{56}Ba + {}^{92}_{36}Kr + 3\,{}_{o}n^1$ + energy

  Q value = $[(M_U - 92m_e + m_n) - \{(M_{Ba} - 56\,m_e) + (M_{Kr} - 36m_e) + 3m_n\}]c^2$

  $= [(M_U + m_n) - (M_{Ba} + M_{Kr} + 3m_n)]c^2$
* A very important and interesting feature of neutron-induced fission is the chain reaction.

10. NUCLEAR REACTOR

Nuclear reactors utilize energy released in nuclear fission reaction to produce power. Some nuclear reactor are research reactors. Their primary aim is to provide a facility for research on different aspects of nuclear science and technology. Some reactors are used to produce power.

Important components of a nuclear reactor :

(i) Moderators : The average energy of neutrons liberated in fission of a $_{92}U^{235}$ is 2 Mev. These neutrons unless slowed down will escape from the reactor without interacting with uranium nuclei. Fast neutrons need to be slowed down for them to be able to get absorbed by Uranium.
When neutrons are made to strike a light nuclei like that of a hydrogen it looses almost all of it K. E. In reactor light nuclei called moderators are used. Commonly used moderators are water, heavy water (D_2O) and graphite 'Apsara' reactor in BARC user H_2O
RAPP uses D_2O as moderator

(ii) Multiplication factor (K) : The ratio of number of fissions produced by given generation of neutrons to the number of fissions of the preceding generation
If K = 1, the operation of reactor is said to be critical.
For steady generation of power K must be equal to 1
If K > 1, the reaction rate and reactor power increases exponentially if K is not brought down the reactor will become super critical and may explode.

(iii) Control Rods :- The reaction rate is controlled through control-rods made out of neutron absorbing material such as cadmium.

(iv) Safety Rods : - These rods are provided in reactors in addition to control rods These, when required, can be inserted into the reactor and K can be reduced.

11. NUCLEAR FUSION (THERMO NUCLEAR REACTION):

(a) Some unstable light nuclei of A below 20, fuse together, the B.E. per nucleon increases and hence the excess energy is released. The easiest thermonuclear reaction that can be handled on earth is the fusion of two deuterons (D–D reaction) or fusion of a deuteron with a triton (D–T reaction).

$_1H^2 + {_1H^2} \rightarrow {_2He^3} + {_0n^1}$ + 3.3 MeV (D–D)

Q value = $[2(M_D - m_e) - \{(M_{He3} - 2m_e) + m_n\}]c^2 = [2M_D - (M_{He3} + m_n)]c^2$

$_1H^2 + {_1H^2} \rightarrow {_1H^3} + {_1H^1}$ + 4.0 MeV (D–D)

Q value = $[2(M_D - m_e) - \{(M_T - m_e) + (M_H - m_e)\}]c^2 = [2M_D - (M_T + M_H)]c^2$

$_1H^2 + {_1H^3} \rightarrow {_2He^4} + n$ + 17.6 MeV (D – T)

Q value = $[\{(M_D - m_e) + (M_T - m_e)\} - \{(M_{He4} - 2m_e) + m_n\}]c^2 = [(M_D + M_T) - (M_{He4} + m_n)]c^2$

Note : In case of fission and fusion, $\Delta m = \Delta m_{atom} = \Delta m_{nucleus}$.

(b) These reactions take place at ultra high temperature ($\cong 10^7$ to 10^9). At high pressure it can take place at low temperature also. For these reactions to take place nuclei should be brought upto 1 fermi distance which requires very high kinetic energy.

(c) Energy released in fusion exceeds the energy liberated in the fission of heavy nuclei.

FUSION REACTIONS IN SUN

The fusion reaction in sun is multi-step process which involves conversion of hydrogen in helium.

The below set of reactions is called as p–p cycle of nuclear fusion in stars

${}_1H^1 + {}_1H^1 \rightarrow {}_1H^2 + {}_{+1}e^0 + \nu + 0.42$ Mev …(i)

${}_{+1}e^0 + {}_{-1}e^0 \rightarrow \gamma + \gamma + 1.02$ Mev …(ii)

${}_1H^2 + {}_1H^1 \rightarrow {}_2He^3 + \gamma + 5.49$ Mev …(iii)

${}_2He^3 + {}_2He^3 \rightarrow {}_2He^4 + 2\,{}_1H^1 + 12.86$ Mev …(iv)

2[(i) + (ii) + (iii)] + (iv)

$6\,{}_1H^1 + 2\,{}_{-1}e^0 \rightarrow {}_2He^4 + 2\,{}_1H^1 + 6\gamma + 2\nu$

$4\,{}_1H^1 + 2\,{}_{-1}e^0 \rightarrow {}_2He^4 + 2\nu + 6\gamma + 26.7$ Mev

Solved Example

Example 20. Calculate the energy released when three alpha particles combine to form a ^{12}C nucleus. The atomic mass of ${}^4_2He$ is 4.002603 u.

Solution : The mass of a ^{12}C atom is exactly 12 u.

The energy released in the reaction $3\left({}^4_2He\right) \rightarrow {}^{12}_6C$ is

$[3\,m({}^4_2He) - m({}^{12}_6C)]\,c^2 = [3 \times 4.002603\text{ u} - 12\text{ u}]\,(931\text{ MeV/u}) = 7.27$ MeV.

Example 21. Consider two deuterons moving towards each other with equal speeds in a deutron gas. What should be their kinetic energies (when they are widely separated) so that the closest separation between them becomes 2fm? Assume that the nuclear force is not effective for separations greater than 2 fm. At what temperature will the deuterons have this kinetic energy on an average?

Solution : As the deuterons move, the Coulomb repulsion will slow them down. The loss in kinetic energy will be equal to the gain in Coulomb potential energy. At the closest separation, the kinetic energy is zero and the potential energy is $\frac{e^2}{4\pi\varepsilon_0 r}$. If the initial kinetic energy of each deuteron is K and the closest separation is 2fm, we shall have

$$2K = \frac{e^2}{4\pi\varepsilon_0(2\ \text{fm})} = \frac{(1.6\times10^{-19}C)^2\times(9\times10^9 N-m^2/C^2)}{2\times10^{-15}m}$$

or, $K = 5.7 \times 10^{-14}$ J.

If the temperature of the gas is T, the average kinetic energy of random motion of each nucleus will be 1.5 kT. The temperature needed for the deuterons to have the average kinetic energy of 5.7×10^{-14} J will be given by

$1.5\,kT = 5.7 \times 10^{-14}$ J

where k = Botzmann constant

$$\text{or, } T = \frac{5.7\times10^{-14}J}{1.5\times1.38\times10^{-23}J/K} = 2.8 \times 10^9\text{ K.}$$

Exercise-1

☞ **If required, you can use the following data:**
Mass of proton m_p = 1.007276 u, Mass of $_1H^1$ atom = 1.007825 u, Mass of neutron m_n = 1.008665 u, Mass of electron = 0.0005486 u = 511 KeV/c^2, 1 u = 931 MeV/c^2. N_A = 6.023 × 10^{23}
Atomic mass of : H^2 = 2.01410 u, Be^8 = 8.00531u, B^{11} = 11.00930u, Li^7 = 7.01601u, He^4 = 4.002603u.

✍ **Marked Questions can be used as Revision Questions.**

PART - I : SUBJECTIVE QUESTIONS

Section (A) : Properties of Nucleus

A-1 A neutron star has a density equal to that of the nuclear matter($\approx 3 \times 10^{17}$ kg/m^3). Assuming the star to be spherical, find the radius of a neutron star whose mass is (i) 4.0 × 10^{30} kg (twice the mass of the sun) (ii) 6 × 10^{24} Kg (around mass of the earth).

A-2. Assuming the radius of a nucleus to be equal to R = 1.3 $A^{1/3} \times 10^{-15}$ m, where A is its mass number, evaluate the density of nuclei and the number of nucleons per unit volume of the nucleus. Take mass of one nucleon = 1.67 × 10^{-27} kg

Section (B) : Mass defect and binding energy

B-1. Find the binding energy of the nucleus of lithium isotope $_3Li^7$ and hence find the binding energy per nucleon in it. ($M_{_3Li^7}$ = 7.014353 amu, $M_{_1H^1}$ = 1.007826, mass of neutron= 1.00867 u)

B-2.✍ Find the energy required for separation of a $_{10}Ne^{20}$ nucleus into two α – particles and a $_6C^{12}$ nucleus if it is known that the binding energies per nucleon in $_{10}Ne^{20}$, $_2He^4$ and $_6C^{12}$ nuclei are equal to 8.03, 7.07 and 7.68 MeV respectively.

Section (C) : Radioactive decay & Displacement law

C-1. The kinetic energy of an α – particle which flies out of the nucleus of a Ra^{226} atom in radioactive disintegration is 4.78 MeV. Find the total energy evolved during the escape of the α – particle.

C-2. In the decay $^{64}Cu \rightarrow {}^{64}Ni + e^+ + \nu$, the maximum kinetic energy carried by the positron is found to be 0.680 MeV (a) Find the energy of the neutrino which was emitted together with a positron of energy 0.180 MeV (b) What is the momentum of this neutrino in kg–m /s ?Use the formula applicable to photon.

Section (D) : Statistical law of Radioactive decay

D-1.✍ Beta decay of a free neutron takes place with a half life of 14 minutes. Then find (a) decay constant (b) energy liberated in the process.

D-2.✍ How many β – particles are emitted during one hour by 1.0 μg of Na^{24} radionuclide whose half–life is 15 hours? [Take $e^{(-0.693/15)}$ = 0.955, and avagadro number = 6 × 10^{23}]

D-3. Calculate the specific activities of Na^{24} & U^{235} nuclides whose half lives are 15 hours and 7.1 × 10^8 years respectively.

Section (E) : Nuclear Fission and Fusion

E-1. Consider the case of bombardment of U^{235} nucleus with a thermal neutron. The fission products are Mo^{95} & La^{139} and two neutrons. Calculate the energy released by one U^{235} nucleus. (Rest masses of the nuclides are U^{235} = 235.0439 u, $_0^1n$ = 1.0087 u, Mo^{95} = 94.9058 u, La^{139} = 138.9061 u).

E-2. Energy evolved from the fusion reaction $2\,{}_1^2H = {}_2^4He + Q$ is to be used for the production of power. Assuming the efficiency of the process to be 30 %. Find the mass of deuterium that will be consumed in a second for an output of 50 MW. $_2He^4$ = 4.002603 amu and $_1H^2$ = 2.014102 amu.

E-3. For the D–T fusion reaction, find the rate at which deuterium & tritium are consumed to produce 1 MW. The Q–value of D–T reaction is 17.6 MeV & assume all the energy from the fusion reaction is available.

PART - II : ONLY ONE OPTION CORRECT TYPE

Section (A) : Properties of Nucleus

A-1. The mass number of a nucleus is
(A) always less than its atomic number
(B) always more than its atomic number
(C) equal to its atomic number
(D) sometimes more than and sometimes equal to its atomic number

A-2. The stable nucleus that has a radius 1/3 that of Os^{189} is -
(A) ${}_3Li^7$ (B) ${}_2He^4$ (C) ${}_5B^{10}$ (D) ${}_6C^{12}$

A-3. The graph of $\ell n\,(R/R_0)$ versus $\ell n\,A$ (R = radius of a nucleus and A = its mass number) is
(A) a straight line (B) a parabola (C) an ellipse (D) none of them

A-4. For uranium nucleus how does its mass vary with volume? **[JEE 2003 (Screening) 3,–1/84]**
(A) $m \propto V$ (B) $m \propto 1/V$ (C) $m \propto \sqrt{V}$ (D) $m \propto V^2$

A-5. Let F_{pp}, F_{pn} and F_{nn} denote the magnitudes of the nuclear force by a proton on a proton, by a proton on a neutron and by a neutron on a neutron respectively. When the separation is 1 fm,
(A) $F_{pp} > F_{pn} = F_{nn}$ (B) $F_{pp} = F_{pn} = F_{nn}$ (C) $F_{pp} > F_{pn} > F_{nn}$ (D) $F_{pp} < F_{pn} = F_{nn}$

Section (B) : Mass Defect and Binding Energy

B-1. As the mass number A increases, the binding energy per nucleon in a nucleus
(A) increases (B) decreases (C) remains the same
(D) varies in a way that depends on the actual value of A.

B-2. Which of the following is a wrong description of binding energy of a nucleus ?
(A) It is the energy required to break a nucleus into its constituent nucleons.
(B) It is the energy released when free nucleons combine to from a nucleus
(C) It is the sum of the rest mass energies of its nucleons minus the rest mass energy of the nucleus
(D) It is the sum of the kinetic energy of all the nucleons in the nucleus

B-3. The energy of the reaction $Li^7 + p \longrightarrow 2\,He^4$ is (the binding energy per nucleon in Li^7 and He^4 nuclei are 5.60 and 7.06 MeV respectively.)
(A) 17.3 MeV (B) 1.73 MeV (C) 1.46 MeV
(D) depends on binding energy of proton

B-4. The atomic weight of boron is 10.81 g/mole and it has two isotopes ${}^{10}{}_5B$ and ${}^{11}{}_5B$. The ratio (by number) of ${}^{10}{}_5B$: ${}^{11}{}_5B$ in nature would be :
(A) 19 : 81 (B) 10 : 11 (C) 15 : 16 (D) 81 : 19

Section (C) : Radioactive Decay & Displacement law

C-1 Which of the following processes represents a gamma decay?
(A) ${}^AX_Z + \gamma \longrightarrow {}^AX_{Z-1} + a + b$ (B) ${}^AX_Z + {}^1n_0 \longrightarrow {}^{A-3}X_{Z-2} + c$
(C) ${}^AX_Z \longrightarrow {}^AX_Z + f$ (D) ${}^AX_Z + e_{-1} \longrightarrow {}^AX_{Z-1} + g$

C-2. An α-particle is bombarded on ^{14}N. As a result, a ^{17}O nucleus is formed and a particle is emitted. This particle is a
(A) neutron (B) proton (C) electron (D) positron

C-3. A free neutron decays into a proton, an electron and :
(A) A neutrino (B) An antineutrino (C) An α-particle (D) A β-particle

C-4. Nuclei X decay into nuclei Y by emitting α particles. Energies of α particle are found to be only 1 MeV & 1.4 MeV. Disregarding the recoil of nuclei Y. The energy of γ photon emitted will be
(A) 0.8 MeV (B) 1.4 MeV (C) 1 MeV (D) 0.4 MeV

Section (D) : Statistical Law of Radioactive decay

D-1. In one average-life
(A) half the active nuclei decay (B) less than half the active nuclei decay
(C) more than half the active nuclei decay (D) all the nuclei decay

D-2. A freshly prepared radioactive source of half-life 2 h emits radiation of intensity which is 64 times the permissible safe level. The minimum time after which it would be possible to work safely with this source is -
(A) 6 h (B) 12 h (C) 24 h (D) 128 h

D-3. Two isotopes P and Q of atomic weight 10 and 20, respectively are mixed in equal amount by weight. After 20 days their weight ratio is found to be 1 : 4. Isotope P has a half-life of 10 days. The half-life of isotope Q is
(A) zero (B) 5 days (C) 20 days (D) inifinite

D-4. 10 grams of ^{57}Co kept in an open container beta-decays with a half-life of 270 days. The weight of the material inside the container after 540 days will be very nearly -
(A) 10 g (B) 7.5 g (C) 5 g (D) 2.5 g

D-5. $A \xrightarrow{\lambda} B \xrightarrow{2\lambda} C$

	A	B	C
t = 0	N_0	0	0
t	N_1	N_2	N_3

The ratio of N_1 to N_2 when N_2 is maximum is :
(A) at no time this is possible (B) 2
(C) 1/2 (D) $\frac{\ell n2}{2}$

D-6. The half-life of ^{131}I is 8 days. Given a sample of ^{131}I at time t = 0, we can assert that **[JEE-1999]**
(A) No nucleus will decay before t = 4 days
(B) No nucleus will decay before t = 8 days
(C) All nuclei will decay before t = 16 days
(D) A given nucleus may decay at any time after t = 0.

Section (E) : Nuclear Fission and Fusion

E-1. $_{92}U^{235}$ nucleus absorbs a slow neutron and undergoes fission into $_{54}X^{139}$ and $_{38}Sr^{94}$ nuclei. The other particles produced in this fission process are
(A) 1 β and 1 α (B) 2 β and 1 neutron (C) 2 neturons (D) 3 neutrons

E-2. Two lithium ^6Li nuclei in a lithium vapour at room temperature do not combine to form a carbon ^{12}C nucleus because
(A) a lithium nucleus is more tightly bound than a carbon nucleus
(B) carbon nucleus is an unstable particle
(C) it is not energetically favourable
(D) Coulomb repulsion does not allow the nuclei to come very close

E-3. In a uranium reactor whose thermal power is P = 100 MW, if the average number of neutrons liberated in each nuclear splitting is 2.5. Each splitting is assumed to release an energy E = 200 MeV. The number of neutrons generated per unit time is -
(A) 4×10^{18} s^{-1} (B) 8×10^{23} s^{-1} (C) 8×10^{19} s^{-1} (D) $\frac{125}{16} \times 10^{18}$ s^{-1}

E-4. Choose the statement which is true.
(A) The energy released per unit mass is more in fission than in fusion
(B) The energy released per atom is more in fusion than in fission.
(C) The energy released per unit mass is more in fusion and that per atom is more in fission.
(D) Both fission and fusion produce same amount of energy per atom as well as per unit mass.

E-5. Fusion reaction is possible at high temperature because -
(A) atoms are ionised at high temperature
(B) molecules break-up at high temperature
(C) nuclei break-up at high temperature
(D) kinetic energy is high enough to overcome repulsion between nuclei.

E-6. In a fission reaction ${}^{236}{}_{92}U \longrightarrow {}^{117}X + {}^{117}Y + n + n$ the average binding energy per nucleon of X and Y is 8.5 MeV whereas that of ${}^{236}U$ is 7.6 MeV. The total energy liberated will be about :
(A) 200 keV (B) 2 MeV (C) 200 MeV (D) 2000 MeV

E-7. A heavy nucleus having mass number 200 gets disintegrated into two small fragments of mass number 80 and 120. If binding energy per nucleon for parent atom is 6.5 M eV and for daughter nuclei is 7 M eV and 8 M eV respectively, then the energy released in each decay will be :
(A) 200 M eV (B) – 220 M eV (C) 220 M eV (D) 180 M eV

E-8. Assuming that about 20 MeV of energy is released per fusion reaction, ${}_1H^2 + {}_1H^3 \rightarrow {}_0n^1 + {}_2He^4$, the mass of ${}_1H^2$ consumed per day in a future fusion reactor of power 1 MW would be approximately
(A) 0.1 gm (B) 0.01 gm (C) 1 gm (D) 10 gm

PART - III : MATCH THE COLUMN

1. Match the column-I of properties with column-II of reactions

Column-I	**Column-II**
(A) Mass of product formed is less than the original mass of the system in	(P) α-decay
(B) Binding energy per nucleon increase in	(Q) β-decay
(C) Mass number is conserved in	(R) Nuclear fission
(D) Charge number is conserved in	(S) Nuclear fusion

2. In column-I, consider each process just before and just after it occurs. Initial system is isolated from all other bodies. Consider all product particles (even those having rest mass zero) in the system. Match the system in column-I with the result they produce in column-II.

Column-I	**Column-II**
(A) Spontaneous radioactive decay of an uranium nucleus initially at rest as given by reaction ${}^{238}_{92}U \rightarrow {}^{234}_{90}Th + {}^{4}_{2}He + ...$	(P) Number of protons is increased
(B) Fusion reaction of two hydrogen nuclei as given by reaction ${}^{1}_{1}H + {}^{1}_{1}H \rightarrow {}^{2}_{1}H + ...$	(Q) Momentum is conserved
(C) Fission of U^{235} nucleus initiated by a thermal neutron as given by reaction ${}^{1}_{0}n + {}^{235}_{92}U \rightarrow {}^{144}_{56}Ba + {}^{89}_{36}Kr + 3{}^{1}_{0}n + ...$	(R) Mass is converted to energy or vice versa
(D) β^- decay (negative beta decay)	(S) Charge is conserved

3. Four physical quantities are listed in column I. Their values are listed in Column II in a random order.

Column I	**Column II**
(a) Thermal energy of air molecules at room temperature	(e) 0.04 eV
(b) Binding energy of heavy nuclei per nucleon	(f) 2 eV
(c) X–ray photon energy	(g) 1 KeV
(d) Photon energy of visible light	(h) 7 MeV

The correct matching of columns I & II is given by :
(A) a – e, b – h, c – g, d – f (B) a – e, b – g, c – f, d – h
(C) a – f, b – e, c – g, d – h (D) a – f, b – h, c – e, d – g

Exercise-2

✍ **Marked Questions can be used as Revision Questions.**

PART - I : ONLY ONE OPTION CORRECT TYPE

1. Choose the wrong statement.
(A) The nuclear force becomes weak if the nucleus contains too many protons compared to the number of neutrons
(B) The nuclear force becomes weak if the nucleus contains too many neutrons compared to the number of protons.
(C) Nuclei with atomic number greater than 82 show a tendency to disintegrate.
(D) The nuclear force becomes very strong if the nucleus contains a large number of nucleons.

2. Binding Energy per nucleon of a fixed nucleus X^A is 6 MeV. It absorbs a neutron moving with KE = 2 MeV, and converts into Y at ground state, emitting a photon of energy 1 MeV. The Binding Energy per nucleon of Y (in MeV) is -
(A) $\frac{(6A+1)}{(A+1)}$ (B) $\frac{(6A-1)}{(A+1)}$ (C) 7 (D) $\frac{7}{6}$

3. The half life of ^{215}At is 100 μs. The time taken for the radioactivity of a sample of ^{215}At to decay to 1/16th of its initial value is : **[JEE 2002 (Screening) 2 × 3, –1 = 6/90]**
(A) 400 μs (B) 6.3 μs (C) 40 μs (D) 300 μs

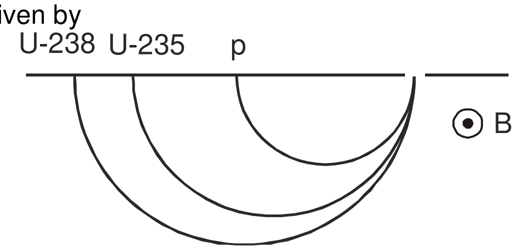
4. A free neutron decays to a proton but a free proton does not decay to a neutron. This is because
(A) neutron is a composite particle made of a proton and an electron whereas proton is fundamental particle
(B) neutron is an uncharged particle whereas proton is a charged particle
(C) neutron has larger rest mass than the proton
(D) weak forces can operate in a neutron but not in a proton.

5. **Match the following :**

Column I	**Column II**
(a) Photoelectric effect	I. Photon
(b) Wave	II. Frequency
(c) X rays	III. K capture
(d) Nucleus	IV. γ rays

(A) a – I, b – II, c – III, d – IV (B) a – II, b – I, c – IV, d – III
(C) a – II, b – I, c – III, d – IV (D) None of these

6.✍ Protons and singly ionized atoms of U^{235} & U^{238} are passed in turn (which means one after the other and not at the same time) through a velocity selector and then enter a uniform magnetic field. The protons describe semicircles of radius 10 mm. The separation between the ions of U^{235} and U^{238} after describing semicircle is given by



(A) 60 mm (B) 30 mm (C) 2350 mm (D) 2380 mm

7. When a β^--particle is emitted from a nucleus, the neutron-proton ratio :
(A) is decreased (B) is increased (C) remains the same (D) first (A) then (B)

8. Consider a sample of a pure beta-active material
(A) All the beta particles emitted have the same energy
(B) The beta particles originally exist inside the nucleus and are ejected at the time of beta decay
(C) The antineutrino emitted in a beta decay has zero rest mass and hence zero momentum.
(D) The active nucleus changes to one of its isobars after the beta decay

9. Masses of two isobars ${}^{64}{}_{29}Cu$ and ${}^{64}{}_{30}Zn$ are 63.9298 u and 63.9292 u respectively. It can be concluded from these data that : **[IIT - 1997]**
(A) Both the isobars are stable
(B) ${}^{64}Zn$ is radioactive, decaying to ${}^{64}Cu$ through β-decay
(C) ${}^{64}Cu$ is radioactive, decaying to ${}^{64}Zn$ through γ-decay
(D) ${}^{64}Cu$ is radioactive, decaying to ${}^{64}Zn$ through β-decay

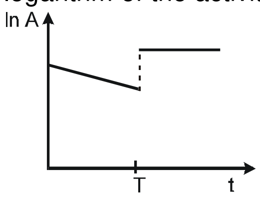
10. In an α-decay the Kinetic energy of α particle is 48 MeV and Q-value of the reaction is 50 MeV. The mass number of the mother nucleus is:- (Assume that daughter nucleus is in ground state)
(A) 96 (B) 100 (C) 104 (D) none of these

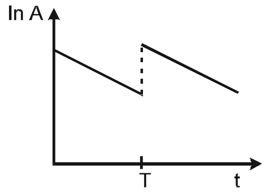
11. Free ${}^{238}U$ nuclei kept in a train emit alpha particles. When the train is stationary and a uranium nucleus decays, a passenger measures that the separation between the alpha particle and the recoiling nucleus becomes x in time t after the decay. If a decay takes place when the train is moving at a uniform speed υ, the distance between the alpha particle and the recoiling nucleus at a time t after the decay, as measured by the passenger will be –
(A) $x + \upsilon t$ (B) $x - \upsilon t$ (C) x
(D) depends on the direction of the train

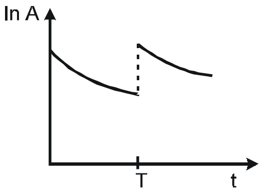
12. A nucleus with mass number 220 initially at rest emits an α-particle. If the Q value of the reaction is 5.5 MeV, calculate the kinetic energy of the α-particle **[JEE 2003 (Screening) 3,–1/84]**
(A) 4.4 MeV (B) 5.4 MeV (C) 5.6 MeV (D) 6.5 MeV

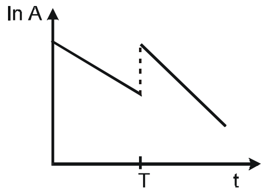
13. A charged capacitor of capacitance C is discharged through a resistance R. A radioactive sample decays with an average life τ. Find the value of R for which the ratio of the electrostatic field energy stored in the capacitor to the activity of the radioactive sample is independent of time.
(A) $\frac{\tau}{C}$ (B) $\frac{2\tau}{C}$ (C) $\frac{\tau}{2C}$ (D) $\frac{3\tau}{2C}$

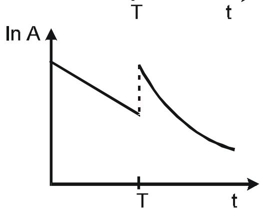
14. At time t = 0, some radioactive gas is injected into a sealed vessel. At time T, some more of the same gas is injected into the same vessel. Which one of the following graphs best represents the variation of the logarithm of the activity A of the gas with time t ?

(A) 

(B) 

(C) 

(D) 

(E) 

15. A sample of radioactive material has mass m, decay constant λ, and molecular weight M. Avogadro constant = N_A. The initial acitvity of the sample is :

(A) λm (B) $\frac{\lambda m}{M}$ (C) $\frac{\lambda m N_A}{M}$ (D) mN_Ae^{λ}

16. Two radioactive sources A and B initially contain equal number of radioactive atoms. Source A has a half-life of 1 hour and source B has a half-life of 2 hours. At the end of 2 hours, the ratio of the rate of disintegration of A to that of B is :

(A) 1 : 2 (B) 2 : 1 (C) 1 : 1 (D) 1 : 4

17. Two identical samples (same material and same amount initially) P and Q of a radioactive substance having mean life T are observed to have activities A_P & A_Q respectively at the time of observation. If P is older than Q, then the difference in their ages is:

(A) $T\,\ell n\left(\frac{A_P}{A_Q}\right)$ (B) $T\,\ell n\left(\frac{A_Q}{A_P}\right)$ (C) $\frac{1}{T}\ell n\left(\frac{A_P}{A_Q}\right)$ (D) $T\left(\frac{A_P}{A_Q}\right)$

18. N atoms of a radioactive element emit n alpha particles per second at an instant. Then the half - life of the element is

(A) $\frac{n}{N}$ sec. (B) $1.44\,\frac{n}{N}$ sec. (C) $0.69\,\frac{n}{N}$ sec. (D) $0.69\frac{N}{n}$ sec.

19. The radioactivity of an old sample of a liquid due to tritium (half life 12.5 years) was found to be only about 3% of that measured in a recently purchased bottle marked '7 year old'. The sample must have been prepared about :

(A) 70 year (B) 220 year (C) 420 year (D) 300 year

20. $A \xrightarrow{\lambda_1} B \xrightarrow{\lambda_2} C$

t = 0	N_0	0	0
t	N_1	N_2	N_3

In the above radioactive decay C is stable nucleus. Then:

(A) rate of decay of A will first increase and then decrease

(B) number of nuclei of B will first increase and then decrease

(C) if $\lambda_2 > \lambda_1$, then activity of B will always be higher than activity of A

(D) if $\lambda_1 >> \lambda_2$, then number of nucleus of C will always be less than number of nucleus of B.

21. Ninety percent of a radioactive sample is left over after a time interval t. The percentage of initial sample that will disintegrate in an interval 2t is **[OLYMPIAD 2011]**

(A) 38% (B) 19% (C) 9% (D) 62%

22. The intensity of gamma radiation from a given source is I. On passing through 36 mm of lead, it is reduced to 1/8. The thickness of lead, which will reduce the intensity to 1/2 will be : **[AIEEE 2005 4/300]**

(A) 6 mm (B) 9 mm (C) 18 mm (D) 12 mm

23. The fraction of the original number of nuclei of a radioactive atom having a mean life of 10 days, that decays during the 5th day is : **[Olympiad (State-1) 2017]**

(A) 0.15 (B) 0.30 (C) 0.045 (D) 0.064

PART - II : SINGLE AND DOUBLE VALUE INTEGER TYPE

1. Consider a point source emitting α-particles and receptor of area 1 cm^2 placed 1 m away from source. Receptor records any α-particle falling on it. If the source contains $N_0 = 3.0 \times 10^{16}$ active nuclei and the receptor records a rate of A = 50000 counts/second. Assume that the source emits alpha particles uniformly in all directions and the alpha particles fall nearly normally on the window. If decay constant is $3n \times 10^{-(n+1)}$, then find the value of n

2. In an ore containing uranium, the ratio (by number) of U-238 to Pb-206 is 3. Assuming that all the lead present in the ore is the final stable product of U-238. If age of the ore is 1.868×10^n years, then value of the n (Take the half life of U-238 to be 4.5×10^9 years. ($\ell n\ 4/3 = 0.2876$)) **[IIT - 1997]**

3. A Bi^{210} radionuclide decays via the chain (stable), $Bi^{210} \xrightarrow[\lambda_1]{\beta^- - \text{decay}} Po^{210} \xrightarrow[\lambda_2]{\alpha - \text{decay}} Pb^{206}$ where the decay constants are $\lambda_1 = 1.6 \times 10^{-6}\ s^{-1}$, $T_{1/2} \approx 5$ days, $\lambda_2 = 5.8 \times 10^{-8}\ s^{-1}$, $T_{1/2} \approx 4.6$ months. α activity of the Bi^{210} sample of mass 1.00 mg a month after its manufacture is $\frac{x}{5} \times 10^{11}$ Find x. $2^{-\frac{1}{4.6}} = 0.86$

4. A sample has two isotopes A^{150} and B having masses 50 g and 30 g respectively. A is radioactive and B is stable. A decays to A′ by emitting α particles. The half life of A is 2 hrs. The mass of total sample after 4 hours is nearly $4n \times 10^{-n}$ kg. Find n

5. A radionuclide with half life T = 693.1 days emits β-particles of average kinetic energy $E = 8.4 \times 10^{-14}$ joule. This radionuclide is used as source in a machine which generates electrical energy with efficiency $\eta = 12.6\%$. Number of moles of the nuclide required to generate electrical energy at an initial rate of 441 KW is $n \times 10^m$ then find out value of $\frac{n}{m}$ ($\log_e 2 = 0.6931$, $N_A = 6.023 \times 10^{23}$)

6. There is a stream of neutrons with a kinetic energy of 0.0327 eV. If the half-life of neutrons is 700 seconds, if the fraction of neutrons will decay before they travel a distance of 10 m is 3.90×10^{-n}. Find n **[1986; 6M]**

7. A sealed box was found which stated to have contained alloy composed of equal parts by weight of two metals A and B. These metals are radioactive, with half lives of 12 years and 18 years, respectively and when the container was opened it was found to contain 0.53 kg of A and 2.20 kg of B. The age of the alloy is M ×10 + n then find M – n.

8. The half-life of ^{40}K is $T = 1.30 \times 10^9$ y. A sample of m = 1.00 g of pure KCl gives c = 480 counts/s. If the relative percentage abundance of ^{40}K (fraction of ^{40}K present in term of number of atoms) in natural potassium is $n \times 10^{-2}$ % then value of n. Molecular weight of KCl is M = 74.5 , Avogadro number $N_A = 6.02 \times 10^{23}$, $1y = 3.15 \times 10^7$ s

9. Consider a fusion reaction $^4He + {}^4He = {}^8Be$. For the reaction Q-value is –(90 + n) KeV. Find n. Take 1 amu = $\frac{930}{c^2}$ MeV. Atomic mass of ^8Be is 8.0053 u and that of ^4He is 4.0026 u.

10. About 185 MeV of usable energy is released in the neutron induced fissioning of a $^{235}_{92}U$ nucleus. If the reactor using $^{235}_{92}U$ as fuel continuously generates 100 MW power. The time it will take for 1 Kg of the uranium $^{235}_{92}U$ to be used up is n days. Find [n]? [n] is grestest integer value of n.

11. Consider a nuclear reaction A + B → C. A nucleus ‘A’ moving with kinetic energy of 5 MeV collides with a nucleus ‘B’ moving with kinetic energy of 3 MeV and form a nucleus ‘C’ in excited state. If the kinetic energy of nucleus ‘C’ just after its formation is E MeV then find [E] . If it is formed in a state with excitation energy 10 MeV. Take masses of nuclei of A, B and C as 25.0, 10.0, 34.995 amu respectively. 1 amu = 930 MeV/c^2 [E] is greatest integer of E

12. The binding energy per nucleon of $^{16}_{8}O$ is 7.97 MeV and that of $^{17}_{8}O$ of 7.75 MeV. The energy required to remove a neutron from $^{17}_{8}O$ is 0.423×10^n MeV then find n

13. A π^o meson at rest decays into two photons of equal energy. If the wavelength (in m) of the photons is 1.8×10^{-n} then find n/2 (The mass of the π^o is 135 MeV/c^2)

PART - III : ONE OR MORE THAN ONE OPTIONS CORRECT TYPE

1. If a nucleus $^{A}_{Z}x$ emits one α particle and one β (negative β) particle in succession, then the daughter nucleus will have which of the following configurations?

(A) A – 4 nucleons (B) 4 nucleons (C) A – Z – 3 neutrons (D) Z – 2 protons

2. The heavier stable nuclei tend to have larger N/Z ratio because -

(A) a neutron is heavier than a proton

(B) a neutron is an unstable particle

(C) a neutron does not exert electric repulsion

(D) Coulomb forces have longer range compared to nuclear forces

3. A U^{238} sample of mass 1.0 g emits alpha particles at the rate 1.24×10^4 particles per second. ($N_A = 6.023 \times 10^{23}$)

(A) The half life of this nuclide is 4.5×10^9 years

(B) The half life of this nuclide is 9×10^9 years

(C) The activity of the prepared sample is 2.48×10^4 particles/sec

(D) The activity of the prepared sample is 1.24×10^4 particles/sec.

4. A nitrogen nucleus $_7N^{14}$ absorbs a neutron and can transform into lithium nucleus $_3Li^7$ under suitable conditions, after emitting

(A) 4 protons and 4 neutrons

(B) 5 protons and 1 negative beta particle

(C) 2 alpha particles and 2 gamma particles

(D) 1 alpha particle, 4 protons and 2 negative beta particles.

5. The decay constant of a radioactive substance is 0.173 $(\text{years})^{-1}$. Therefore:

(A) Nearly 63% of the radioactive substance will decay in (1/0.173) year.

(B) half life of the radioactive substance is (1/0.173) year.

(C) one -forth of the radioactive substance will be left after nearly 8 years.

(D) half of the substance will decay in one average life time.

Use approximation $\ell n2 = 0.692$

6. Let m_p be the mass of a proton, m_n the mass of a neutron, M_1 the mass of a $^{20}_{10}Ne$ nucleus & M_2 the mass of a $^{40}_{20}Ca$ nucleus. Then : **[JEE 1998, 2]**

(A) $M_2 = 2M_1$ (B) $M_2 > 2M_1$ (C) $M_2 < 2M_1$ (D) $M_1 < 10 (m_n + m_p)$

7. Nuclei of radioactive element A are being produced at a constant rate α. The element has a decay constant λ. At time t = 0, there are N_0 nuclei of the element. **[IIT - 1998]**

(A) Number of nuclei of A at time t is $\frac{1}{\lambda}[\alpha - (\alpha - \lambda N_0) e^{-\lambda t}]$

(B) Number of nuclei of A at time t is $\frac{1}{\lambda}[(\alpha - \lambda N_0) e^{-\lambda t}]$

(C) If $\alpha = 2N_0\lambda$, then the limiting value of number of nuclei of A $(t \to \infty)$ will be $2N_0$.

(D) If $\alpha = 2N_0\lambda$, then the number of nuclei of A after one half-life of A will be $N_0/2$.

PART - IV : COMPREHENSION

Comprehension-1

The radionuclide ^{56}Mn is being produced in a cyclotron at a constant rate P by bombarding a manganese target with deutrons. ^{56}Mn has a half life of 2.5 hours and the target contains large number of only the stable manganese isotope ^{55}Mn. The reaction that produces ^{56}Mn is :

$^{55}Mn + d \rightarrow {}^{56}Mn + p$

After being bombarded for a long time, the activity of ^{56}Mn becomes constant equal to $13.86 \times 10^{10}\ s^{-1}$.

(Use $\ell n2 = 0.693$; Avogadro No = 6×10^{23}; atomic weight ^{56}Mn = 56 gm/mole)

1. At what constant rate P, ^{56}Mn nuclei are being produced in the cyclotron during the bombardment ?
(A) 2×10^{11} nuclei/s
(B) 13.86×10^{10} nuclei/s
(C) 9.6×10^{10} nuclei/s
(D) 6.93×10^{10} nuclei/s

2. After the activity of ^{56}Mn becomes constant, number of ^{56}Mn nuclei present in the target, is equal to
(A) 5×10^{11}
(B) 20×10^{11}
(C) 1.2×10^{14}
(D) 1.8×10^{15}

3. After a long time bombardment, number of ^{56}Mn nuclei present in the target depends upon
(a) the number of ^{56}Mn nuclei present at the start of the process.
(b) half life of the ^{56}Mn
(c) the constant rate of production P.
(A) All (a), (b) and (c) are correct
(B) only (a) and (b) are correct
(C) only (b) and (c) are correct
(D) only (a) and (c) are correct

Comprehension-2

Consider the following nuclear decay : (initially $^{236}U_{92}$ is at rest)

$${}^{236}_{92}U \longrightarrow {}^{232}_{90}Th + X$$

4. Regarding this nuclear decay select the correct statement :
(A) The nucleus X may be at rest.
(B) The ${}^{232}_{90}Th$ nucleus may be in excited state.
(C) The X may have kinetic energy but ${}^{232}_{90}Th$ will be at rest
(D) The Q value is Δmc^2 where Δm is mass difference of (${}^{236}_{92}U$ and ${}^{232}_{90}Th$) and c is speed of light.

5. If the uranium nucleus is at rest before its decay, which one of the following statement is true concerning the final nuclei ?
(A) They have equal kinetic energies, but the thorium nucleus has much more momentum.
(B) They have equal kinetic energies and momenta of equal magnitudes.
(C) The have momenta of equal magnitudes, but the thorium nucleus has much more kinetic energy.
(D) They have momentum of equal magnitudes, but X has much more kinetic energy.

6. Following atomic masses and conversion factor are provided

${}^{236}_{92}U$ = 236.045 562 u ;

${}^{232}_{90}Th$ = 232.038054 u ;

${}^{1}_{0}n$ = 1.008665 u ; ${}^{1}_{1}p$ = 1.007277 u ;

${}^{4}_{2}He$ = 4.002603 u and 1 u = 1.5×10^{-10} J

The amount of energy released in this decay is equal to :
(A) 3.5×10^{-8} J
(B) 4.6×10^{-12} J
(C) 6.0×10^{-10} J
(D) 7.4×10^{-13} J

Exercise-3

✍ Marked Questions can be used as Revision Questions.

* Marked Questions may have more than one correct option.

PART - I : JEE (ADVANCED) / IIT-JEE PROBLEMS (PREVIOUS YEARS)

1. Half life of a radioactive substance 'A' is 4 days. The probability that a nucleus will decay in two half **[JEE 2006 3/184]**

(A) $\frac{1}{4}$ (B) $\frac{3}{4}$ (C) $\frac{1}{2}$ (D) 1

2. **Match the following** **[JEE 2006 5/184]**

Column 1	Column 2
(A) Nuclear fission	(p) Converts some matter into energy
(B) Nuclear fusion	(q) Possible for nuclei with low atomic number
(C) β - decay	(r) Possible for nuclei with high atomic number
(D) Exothermic nuclear reaction	(s) Essentially proceeds by weak nuclear forces.

3. In the options given below, let E denote the rest mass energy of a nucleus and n a neutron. The correct option is : **[IIT-JEE 2007' 3/81]**

(A) $E\left({}^{236}_{92}U\right) > E\left({}^{137}_{53}I\right) + E\left({}^{97}_{39}Y\right) + 2E(n)$

(B) $E\left({}^{236}_{92}U\right) < E\left({}^{137}_{53}I\right) + E\left({}^{97}_{39}Y\right) + 2E(n)$

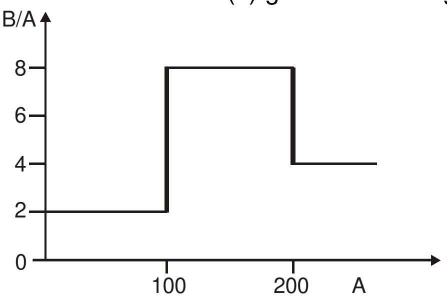
(C) $E\left({}^{236}_{92}U\right) < E\left({}^{140}_{56}Ba\right) + \left({}^{94}_{36}Kr\right) + 2E(n)$

(D) $E\left({}^{236}_{92}U\right) = E\left({}^{140}_{56}Ba\right) + E\left({}^{94}_{36}Kr\right) + 2E(n)$

4. Some laws / processes are given in **Column I**. Match these with the physical phenomena given in **Column II** and indicate your answer by darkening appropriate bubbles in the 4 × 4 matrix given in the ORS. **[IIT-JEE 2007' 6/81]**

Column I	Column II
(A) Transition between two atomic energy levels	(p) Characteristic X-rays
(B) Electron emission from a material	(q) Photoelectric effect
(C) Mosley's law	(r) Hydrogen spectrum
(D) Change of photon energy into kinetic energy of electrons	(s) β-decay

5*. Assume that the nuclear binding energy per nucleon (B/A) versus mass number (A) is as shown in the figure. Use this plot to choose the correct choice(s) given below. Figure : **[JEE 2008, 4/163]**



(A) Fusion of two nuclei with mass numbers lying in the range of 1 < A < 50 will release energy
(B) Fusion of two nuclei with mass numbers lying in the range of 51 < A < 100 will release energy
(C) Fission of a nucleus lying in the mass range of 100 < A < 200 will release energy when broken into two equal fragments
(D) Fission of a nucleus lying in the mass range of 200 < A < 260 will release energy when broken into two equal fragments

6. A radioactive sample S_1 having an activity of 5μCi has twice the number of nuclei as another sample S_2 which has an activity of 10μCi. The half lives of S_1 and S_2 can be **[JEE 2008, 3/163]**

(A) 20 years and 5 years, respectively (B) 20 years and 10 years, respectively
(C) 10 years each (D) 5 years each

Paragraph for Question Nos. 7 to 9

Scientists are working hard to develop nuclear fusion reactor. Nuclei of heavy hydrogen, ${}_{1}^{2}H$, known as deuteron and denoted by D, can be thought of as a candidate for fusion reactor. The D-D reaction is ${}_{1}^{2}H + {}_{1}^{2}H \rightarrow {}_{2}^{3}He + n + \text{energy}$. In the core of fusion reactor, a gas of heavy hydrogen is fully ionized into deuteron nuclei and electrons. This collection of ${}_{1}^{2}H$ nuclei and electrons is known as plasma. The nuclei move randomly in the reactor core and occasionally come close enough for nuclear fusion to take place. Usually, the temperatures in the reactor core are too high and no material wall can be used to confine the plasma. Special techniques are used which confine the plasma for a time t_0 before the particles fly away from the core. If n is the density (number/volume) of deuterons, the product nt_0 is called Lawson number. In one of the criteria, a reactor is termed successful if Lawson number is greater than 5×10^{14} s/cm³. It may be helpful to use the following: Boltzman constant $k = 8.6\times10^{-5}$ eV/K ; $\frac{e^2}{4\pi\varepsilon_0} = 1.44 \times 10^{-9}$ eVm. **[JEE 2009, 4/160, –1]**

7. In the core of nuclear fusion reactor, the gas becomes plasma because of
(A) strong nuclear force acting between the deuterons
(B) Coulomb force acting between the deuterons
(C) Coulomb force acting between deuterons-electrons pairs
(D) the high temperature maintained inside the reactor core

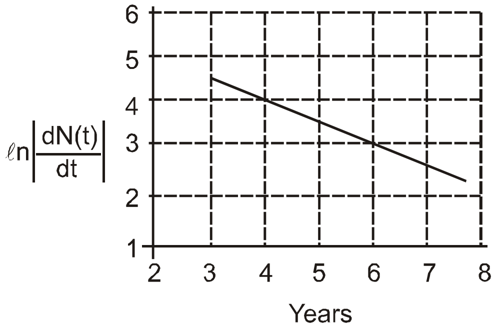
8. Assume that two deuteron nuclei in the core of fusion reactor at temperature T are moving towards each other, each with kinetic energy 1.5 kT, when the separation between them is large enough to neglect Coulomb potential energy. Also neglect any interaction from other particles in the core. The minimum temperature T required for them to reach a separation of 4×10^{-15} m in the range.
(A) 1.0×10^9 K < T < 2.0×10^9 K
(B) 2.0×10^9 K < T < 3.0×10^9 K
(C) 3.0×10^9 K < T < 4.0×10^9 K
(D) 4.0×10^9 K < T < 5.0×10^9 K

9. Results of calculations for four different designs of a fusion reactor using D-D reaction are given below. Which of these is most promising based on Lawson criterion ?
(A) deuteron density = 2.0×10^{12} cm^{-3}, confinement time = 5.0×10^{-3} s
(B) deuteron density = 8.0×10^{14} cm^{-3}, confinement time = 9.0×10^{-1} s
(C) deuteron density = 4.0×10^{23} cm^{-3}, confinement time = 1.0×10^{-11} s
(D) deuteron density = 1.0×10^{24} cm^{-3}, confinement time = 4.0×10^{-12} s

10. **Column II** gives certain systems undergoing a process. **Column I** suggests changes in some of the parameters related to the system. Match the statements in **Column-I** to the appropriate process(es) from **Column II**. **[JEE 2009,8/160]**

Column-I	Column-II
(A) The energy of the system is increased.	(p) System: A capacitor, initially uncharged Process: It is connected to a battery.
(B) Mechanical energy is provided to the system, which is converted into energy of random motion of its parts	(q) System: A gas in an adiabatic container fitted with an adiabatic piston. Process: The gas is compressed by pushing the piston
(C) Internal energy of the system is converted into its mechanical energy	(r) System: A gas in a rigid container Process: The gas gets cooled due to colder atmosphere surrounding it
(D) Mass of the system is decreased	(s) System: A heavy nucleus, initially at rest Process: The nucleus fissions into two fragments of nearly equal masses and some neutrons are emitted
	(t) System: A resistive wire loop Process: The loop is placed in a time varying magnetic field perpendicular to its plane

11. To determine the half life of a radioactive element, a student plots a graph of $\ell n\left|\frac{dN(t)}{dt}\right|$ versus t. Here $\frac{dN(t)}{dt}$ is the rate of radioactive decay at time t. If the number of radioactive nuclei of this element decreases by a factor of p after 4.16 years, the value of p is : **[JEE 2010, 3/163]**



12. The activity of a freshly prepared radioactive sample is 10^{10} disintegrations per second, whose mean life is 10^9 s. The mass of an atom of this radioisotope is 10^{-25} kg. The mass (in mg) of the radioactive sample is **[IIT-JEE 2011; 4/160]**

13. A proton is fired from very far away towards a nucleus with charge Q = 120 e, where e is the electronic charge. It makes a closest approach of 10 fm to the nucleus. The de Brogle wavelength (in units of fm) of the proton at its start is : (take the proton mass, $m_p = (5/3) \times 10^{-27}$ kg, $h/e = 4.2 \times 10^{-15}$ J.s/C ; $\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9$ m/F ; 1 fm = 10^{-15} m) **[IIT-JEE-2012, Paper-1; 4/70]**

Paragraph for Questions 14 and 15

The β^- decay process, discovered around 1900, is basically the decay of a neutron (n). In the laboratory, a proton (p) and an electron (e^-) are observed as the decay products of the neutron. Therefore, considering the decay of a neutron as a tri-body decay process, it was predicted theoretically that the kinetic energy of the electron should be a constant. But experimentally, it was observed that the electron kinetic energy has a continuous spectrum. Considering a three-body decay process, i.e. $n \to p + e^- + \bar{\nu}_e$, around 1930, Pauli explained the observed electron energy spectrum. Assuming the anti-neutrino ($\bar{\nu}_e$) to be massless and possessing negligible energy, and neutron to be at rest, momentum and energy conservation principles are applied. From this calculation, the maximum kinetic energy of the electron is 0.8×10^6 eV. The kinetic energy carried by the proton is only the recoil energy.

14. What is the maximum energy of the anti-neutrino ? **[IIT-JEE-2012, Paper-2; 4/66]**

(A) Zero
(B) Much less than 0.8×10^6 eV
(C) Nearly 0.8×10^6 eV
(D) Much larger than 0.8×10^6 eV

15. If the anti-neutrino had a mass of $3eV/c^2$ (where c is the speed of light) instead of zero mass, what should be the range of the kinetic energy, K, of the electron ? **[IIT-JEE-2012, Paper-2; 4/66]**

(A) $0 \le K \le 0.8 \times 10^6$ eV
(B) $3.0\text{ eV} \le K \le 0.8 \times 10^6$ eV
(C) $3.0\text{ eV} \le K < 0.8 \times 10^6$ eV
(D) $0 \le K < 0.8 \times 10^6$ eV

16. A freshly prepared sample of a radioisotope of half-life 1386 s has activity 10^3 disintegrations per second. Given that ln 2 = 0.693, the fraction of the initial number of nuclei (expressed in nearest integer percentage) that will decay in the first 80s after preparation of the sample is : **[JEE (Advanced) 2013; 3/60]**

17. Match List I of the nuclear processes with List II containing parent nucleus and one of the end products of each process and then select the correct answer using the codes given below the lists :

[JEE (Advanced) 2013 ; 3/60, –1]

List I	**List II**
P. Alpha decay	1. ${}^{15}_{8}O \rightarrow {}^{15}_{7}N + ...$
Q. β^+ decay	2. ${}^{238}_{92}U \rightarrow {}^{234}_{90}Th + ...$
R. Fission	3. ${}^{185}_{83}Bi \rightarrow {}^{184}_{82}Pb + ...$
S. Proton emission	4. ${}^{239}_{94}Pu \rightarrow {}^{140}_{57}La + ...$

Codes :

	P	Q	R	S
(A)	4	2	1	3
(B)	1	3	2	4
(C)	2	1	4	3
(D)	4	3	2	1

Paragraph for Questions 18 and 19

The mass of a ${}^{A}_{Z}X$ nucleus is less than the sum of the masses of (A – Z) number of neutrons and Z number of protons in the nucleus. The energy equivalent to the corresponding mass difference is known as the binding energy of the nucleus. A heavy nucleus of mass M can break into two light nuclei of masses m_1 and m_2 only if $(m_1 + m_2) < M$. Also two light nuclei of masses m_3 and m_4 can undergo complete fusion and form a heavy nucleus of mass M' only if $(m_3 + m_4) > M'$. The masses of some neutral atoms are given in the table below : **[JEE (Advanced) 2013 ; 3/60, –1]**

${}^{1}_{1}H$	1.007825u	${}^{2}_{1}H$	2.014102u	${}^{3}_{1}H$	3.016050u	${}^{4}_{2}He$	4.002603u
${}^{6}_{3}Li$	6.015123u	${}^{7}_{3}Li$	7.016004u	${}^{70}_{30}Zn$	69.925325u	${}^{82}_{34}Se$	81.916709u
${}^{152}_{64}Gd$	151.919803u	${}^{206}_{82}Pb$	205.974455u	${}^{209}_{83}Bi$	208.980388u	${}^{210}_{84}Po$	209.982876u

18. The correct statement is :
(A) The nucleus ${}^{6}_{3}Li$ can emit an alpha particle
(B) The nucleus ${}^{210}_{84}Po$ can emit a proton
(C) Deuteron and alpha particle can undergo complete fusion.
(D) The nuclei ${}^{70}_{30}Zn$ and ${}^{82}_{34}Se$ can undergo complete fusion.

19. The kinetic energy (in keV) of the alpha particle, when the nucleus ${}^{210}_{84}Po$ at rest undergoes alpha decay, is:
(A) 5319 (B) 5422 (C) 5707 (D) 5818

20. A nuclear power plant supplying electrical power to a village uses a radioactive material of half life T years as the fuel. The amount of fuel at the beginning is such that the total power requirement of the village is 12.5% of the electrical power available from the plant at that time. If the plant is able to meet the total power needs of the village for a maximum period of nT years, then the value of n is.

[JEE (Advanced) 2015 ; P-1, 4/88]

21. Match the nuclear processes given in **Column I** with the appropriate option(s) in **Column II**.

[JEE(Advanced) 2015 ; P-1, 8/88, –1]

Column-I	**Column-II**
(A) Nuclear fusion	(P) Absorption of thermal neutrons by ${}^{235}_{92}U$
(B) Fission in a nuclear reactor	(Q) ${}^{60}_{27}Co$ nucleus
(C) β–decay	(R) Energy production in stars via hydrogen conversion to helium
(D) γ–ray emission	(S) Heavy water
	(T) Neutrino emission

22. For a radioactive material, its activity A and rate of change of its activity R are defined as $A = \frac{-dN}{dt}$ and $R = \frac{-dA}{dt}$, where N(t) is the number of nuclei at time t. Two radioactive sources P (mean life τ) and Q (mean life 2τ) have the same activity at t = 0. Their rates of change of activities at $t = 2\tau$ are R_P and R_Q, respectively. If $\frac{R_P}{R_Q} = \frac{n}{e}$, then the value of n is : **[JEE(Advanced) 2015 ; P-2,4/88]**

23. A fission reaction is given by ${}^{236}_{92}U \rightarrow {}^{140}_{54}Xe + {}^{94}_{38}Sr + x + y$, where x and y are two particles. Considering ${}^{236}_{92}U$ to be at rest, the kinetic energies of the products are denoted by K_{Xe}, K_{Sr}, K_x (2MeV) and K_y (2MeV), respectively. Let the binding energies per nucleon of ${}^{236}_{92}U$, ${}^{140}_{54}Xe$ and ${}^{94}_{38}Sr$ be 7.5 MeV, 8.5 MeV and 8.5 MeV, respectively. Considering different conservation laws, the correct option(s) is(are) **[JEE (Advanced) 2015 ; P-2,4/88, –2]**

(A) x = n, y = n, K_{Sr} = 129 MeV, K_{Xe} = 86 MeV (B) x = p, y = e^-, K_{Sr} = 129 MeV, K_{Xe} = 86 MeV
(C) x = p, y = n, K_{Sr} = 129 MeV, K_{Xe} = 86 MeV (D) x = n, y = n, K_{Sr} = 86 MeV, K_{Xe} = 129 MeV

24. The isotope ${}^{12}_{5}B$ having a mass 12.014 u undergoes β-decay to ${}^{12}_{6}C$. ${}^{12}_{6}C$ has an excited state of the nucleus (${}^{12}_{6}C^*$) at 4.041 MeV above its ground state. If ${}^{12}_{5}B$ decays to ${}^{12}_{6}C^*$, the maximum kinetic energy of the β-particle in units of MeV is : (1u = 931.5 MeV/c^2, where c is the speed of light in vacuum)
[JEE (Advanced) 2016, 3/62]

25. An accident in a nuclear laboratory resulted in deposition of a certain amount of radioactive material of half-life 18 days inside the laboratory. Tests revealed that the radiation was 64 times more than the permissible level required for safe operation of the laboratory. What is the minimum number of days after which the laboratory can be considered safe for use ? **[JEE (Advanced) 2016 ; P-2, 3/62, –1]**

(A) 64 (B) 90 (C) 108 (D) 120

26. The electrostatic energy of Z protons uniformly distributed throughout a spherical nucleus of radius R is given by $E = \frac{3}{5}\frac{Z(Z-1)e^2}{4\pi\varepsilon_0 R}$. The measured masses of the neutron, ${}^{1}_{1}H$, ${}^{15}_{7}N$, and ${}^{15}_{8}O$ are 1.008665 u, 1.007825 u, 15.000109 u and 15.003065 u, respectively. Given that the radii of both the ${}^{15}_{7}N$ and ${}^{15}_{8}O$ nuclei are same, 1 u = 931.5 MeV/c^2 (c is the speed of light) and $e^2/(4\pi\varepsilon_0)$=1.44 MeV fm. Assuming that the difference between the binding energies of ${}^{15}_{7}N$ and ${}^{15}_{8}O$ is purely due to the electrostatic energy, the radius of either of the nuclei is (1 fm = 10^{-15} m) **[JEE (Advanced) 2016 ; P-2, 3/62, –1]**

(A) 2.85 fm (B) 3.03 fm (C) 3.42 fm (D) 3.80 fm

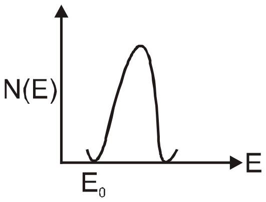
27. ${}^{131}I$ is an isotope of Iodine that β decays to an isotope of Xenon with a half-life of 8days. A small amount of a serum labelled with ${}^{131}I$ is injected into the blood of a person. The activity of the amount of ${}^{131}I$ injected was 2.4×10^5 Becquerel (Bq). It is known that the injected serum will get distributed uniformly in the blood stream in less than half an hour. After 11.5 hours, 2.5 ml of blood is drawn from the person's body, and gives an activity of 115 Bq. The total volume of blood in the person's body, in liters is approximately (you may use $e^x \approx 1 + x$ for |x| << 1 and ln 2 $\approx$ 0.7).**[JEE (Advanced) 2017 ; P-1, 3/61]**

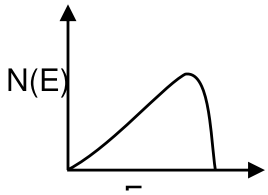
28*. In a radioactive decay chain, ${}^{232}_{90}Th$ nucleus decays to ${}^{212}_{90}Pb$ nucleus. Let N_α and N_β be the number of α and β^- particles, respectively, emitted in this decay process. Which of the following statements is (are) true? **[JEE (Advanced) 2018 ; P-2, 4/60, –2]**

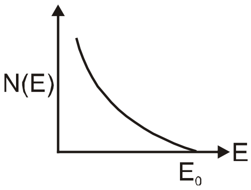
(A) $N_\alpha = 5$ (B) $N_\alpha = 6$ (C) $N_\beta = 2$ (D) $N_\beta = 4$

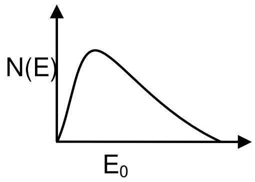
PART - II : JEE (MAIN) / AIEEE PROBLEMS (PREVIOUS YEARS)

1. The energy spectrum of β-particles (number N(E) as a function of β-energy E) emitted from a radioactive source is : **[AIEEE 2006 ; 3/180, –1]**

(1)


(2)


(3)


(4)


2. When $_3Li^7$ nuclei are bombarded by protons, and the resultant nuclei are $_4Be^8$, the emitted particles will be **[AIEEE 2006 ; 4.5/180]**
(1) neutrons (2) alpha particles (3) beta particles (4) gamma photons

3. The 'rad' is the correct unit used to report the measurement of **[AIEEE 2006 ; 4.5/180]**
(1) the rate of decay of radioactive source
(2) the ability of a beam of gamma ray photons to produce ions in a target
(3) the energy delivered by radiation to a target.
(4) the biological effect of radiation

4. If the binding energy per nucleon in ^7_3Li and ^4_2He nuclei are 5.60 MeV and 7.06 MeV respectively, then in the reaction $p + ^7_3Li \rightarrow 2^4_2He$ energy of proton must be : **[AIEEE 2006 ; 4.5/180]**
(1) 39.2 MeV (2) 28.24 MeV (3) 17.28 MeV (4) 1.46 MeV

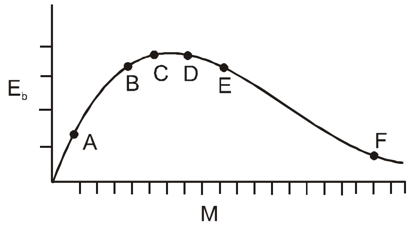
5. If M_o is the mass of an oxygen isotope $_8O^{17}$, M_p and M_N are the masses of a proton and a neutron respectively, the nuclear binding energy of the isotope is : **[AIEEE 2007 ; 3/120, –1]**
(1) $(M_o - 8M_P)C^2$ (2) $(M_o - 8M_P - 9M_N)C^2$ (3) M_oC^2 (4) $(M_o - 17M_N)C^2$

6. In gamma ray emission from a nucleus : **[AIEEE 2007 ; 3/120, –1]**
(1) both the neutron number and the proton number change
(2) there is no change in the proton number and the neutron number
(3) only the neutron number changes
(4) only the proton number changes

7. The half-life period of a radio-active element X is same as the mean life time of another radio-active element Y. Initially they have the same number of atoms. Then : **[AIEEE 2007 ; 3/120, –1]**
(1) X will decay faster than Y (2) Y will decay faster than X
(3) X and Y have same decay rate initially (4) X and Y decay at same rate always

8. This question contains Statement-1 and Statement-2. Of the four choices given after the statements, choose the one that best describes the two statements. **[AIEEE 2008 ; 3/105, –1]**
Statement-1 : Energy is released when heavy nuclei undergo fission or light nuclei undergo fusion.
and
Statement-2 : For heavy nuclei, binding energy per nucleon increases with increasing Z while for light nuclei it decreases with increasing Z.
(1) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1
(2) Statment-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1
(3) Statement-1 is true, Statement-2 is false
(4) Statement-1 is false, Statement-2 is true

9.



The above is a plot of binding energy per nucleon E_b, agains the nuclear mass M; A, B, C, D, E, correspond to different nuclei. Consider four reactions : **[AIEEE 2009 ; 4/144]**

(i) $A + B \rightarrow C + \varepsilon$ (ii) $C \rightarrow A + B + \varepsilon$ (iii) $D + E \rightarrow F + \varepsilon$ and (iv) $F \rightarrow D + E + \varepsilon$,

where ε is the energy released? In which reactions is ε positive?

(1) (i) and (iii) (2) (ii) and (iv) (3) (ii) and (iii) (4) (i) and (iv)

Directions : Question number 10 – 12 are based on the following paragraph.

The nucleus of mass $M + \Delta m$ is at rest and decays into two daughter nuclei of equal mass $\frac{M}{2}$ each. Speed of light is c. **[AIEEE 2010 3/144, –1]**

10. This binding energy per nucleon for the parent nucleus is E_1 and that for the daughter nuclei is E_2. Then

(1) $E_1 = 2E_2$ (2) $E_1 > E_2$ (3) $E_2 > E_1$ (4) $E_2 = 2E_1$

11. The speed of daughter nuclei is

(1) $c\,\frac{\Delta m}{M+\Delta m}$ (2) $c\sqrt{\frac{2\Delta m}{M}}$ (3) $c\sqrt{\frac{\Delta m}{M}}$ (4) $c\sqrt{\frac{\Delta m}{M+\Delta m}}$

12. A radioactive nucleus (initial mass number A and atomic number Z) emits 3 α-particles and 2 positrons. The ratio of number of neutrons to that of protons in the final nucleus will be

(1) $\frac{A-Z-8}{Z-4}$ (2) $\frac{A-Z-4}{Z-8}$ (3) $\frac{A-Z-12}{Z-4}$ (4) $\frac{A-Z-4}{Z-2}$

13. The half life of a radioactive substance is 20 minutes. The approximate time interval $(t_2 - t_1)$ between the time t_2 when $\frac{2}{3}$ of it has decayed and time t_1 when $\frac{1}{3}$ of it had decayed is : **[AIEEE - 2011, 4/120, –1]**

(1) 7 min (2) 14 min (3) 20 min (4) 28 min

14. **Statement - 1 :** A nucleus having energy E_1 decays by β^- emission to daughter nucleus having energy E_2, but the β^- rays are emitted with a continuous energy spectrum having end point energy $E_1 - E_2$.

Statement – 2 : To conserve energy and momentum in β-decay at least three particles must take part in the transformation. **[AIEEE 2011, 11 May; 4/120, –1]**

(1) Statement-1 is correct but statement-2 is not correct.
(2) Statement-1 and statement-2 both are correct and stateemnt-2 is the correct explanation of statement-1.
(3) Statement-1 is correct, statement-2 is correct and statement-2 is not the correct explanation of statement-1
(4) Statement-1 is incorrect, statement-2 is correct.

15. Assume that a neutron breaks into a proton and an electron. The energy released during this process is (mass of neutron = 1.6725×10^{-27} kg, Mass of proton = 1.6725×10^{-27} kg, mass of electron = 9×10^{-31} kg) **[AIEEE 2012 ; 4/120, –1]**

(1) 0.73 MeV (2) 7.10 MeV (3) 6.30 MeV (4) 5.4 MeV

16. Half-lives of two radioactive elements A and B are 20 minutes and 40 minutes, respectively, Initially, the samples have equal number of nuclei. After 80 minutes, the ratio of decayed numbers of A and B nuclei will be : **[JEE (Main) 2016 ; 4/120, –1]**

(1) 4 : 1 (2) 1 : 4 (3) 5 : 4 (4) 1 : 16

17. A radioactive nucleus A with a half life T, decays into a nucleus B. At t = 0, there is no nucleus B. At sometime t, the ratio of the number of B to that of A is 0.3. Then, t is given by : **[JEE (Main) 2017 ; 4/120, –1]**

(1) $t = \frac{T}{\log(1.3)}$ (2) $t = \frac{T}{2}\frac{\log 2}{\log 1.3}$ (3) $t = T\,\frac{\log 1.3}{\log 2}$ (4) $t = T \log (1.3)$

Answers

EXERCISE-1

PART – I

Section (A) :

A-1 (i) $r_1 = \left[\frac{4\times10^{30}}{3\times10^{17}}\times\frac{3}{4\pi}\right]^{1/3}$ = 14.71 km

(ii) $r_2 = \left[\frac{6\times10^{24}}{3\times10^{17}}\times\frac{3}{4\pi}\right]^{1/3}$ = 168.4 m

A-2. 2×10^{11} kg/cm^3, 1×10^{38} nucl. /cm^3

Section (B) :

B-1 $\text{B.E.} = \left[3M_{_1H^1} + 4m_{_0n^1} - M_{_3Li^7}\right]931\,\text{MeV}$

= 39.22 MeV, $\frac{\text{B.E.}}{A} = \frac{39.22}{7}$ = 5.6 MeV

B-2 E = 20 × (8.03) – 2×4 (7.07) – 12(7.68)

= 11.9 MeV

Section (C) :

C-1 $\frac{226}{222}\times4.78 = 4.87$ MeV.

C-2 (a) (0.680 – 0.180) Me V = 500 ke V

(b) $\frac{500\times10^3 e}{C}$ = 2.67×10^{-22} kg–m/s

Section (D) :

D-1. (a) $\frac{0.693}{14\times60}$ = $8.25\times10^{-4}\ s^{-1}$

(b) $(m_n - m_p - m_e)$ 931 = 782 keV

D-2. $\frac{6\times10^{23}\times10^{-6}}{24}\left[1-e^{-0.693/15}\right]$ = 1.128×10^{15}

D-3. $\frac{N_A}{24}\times\frac{0.693}{15\times60\times60}$ = 3.2×10^{17} dps

& $\frac{N_A}{235}\times\frac{0.693}{7.1\times10^8\times365\times86400}$

= 0.8×10^5 dps

Section (E) :

E-1. $[M_U + m_n - M_{Mo} - M_{La} - 2m_n]$ 931

= 207.9 MeV

E-2. $\frac{2}{Q}\times\frac{100}{30}\times\frac{50}{1.6\times10^{-19}}\times\frac{2}{N_A}\times10^{-3}\text{Kg}$

= 2.9×10^{-7} kg ;

where Q= $(2M_{1H^2} - M_{2He^4})\times931$ = 23.834531 MeV

E-3. $\frac{2}{N_A}\times\frac{1}{17.6\ e}\times10^{-3}$ Kg/s = 1.179×10^{-9} kg/s,

$\frac{3}{N_A}\times\frac{1}{17.6\ e}\times10^{-3}$ Kg/s = 1.769×10^{-9} kg/s

PART – II

Section (A) :

A-1.	(D)	**A-2.**	(A)	**A-3.**	(A)
A-4.	(A)	**A-5.**	(B)		

Section (B) :

B-1.	(D)	**B-2.**	(D)	**B-3.**	(A)
B-4.	(A)				

Section (C) :

C-1	(C)	**C-2.**	(B)	**C-3.**	(B)
C-4.	(D)				

Section (D) :

D-1.	(C)	**D-2.**	(B)	**D-3.**	(D)
D-4.	(A)	**D-5.**	(B)	**D-6.**	(D)

Section (E) :

E-1.	(D)	**E-2.**	(D)	**E-3.**	(D)
E-4.	(C)	**E-5.**	(D)	**E-6.**	(C)
E-7.	(C)	**E-8.**	(A)		

PART – III

1. (A) → P,Q,R,S ; (B) → P,Q,R,S ;
(C) → P,Q,R,S ; (D) → P,Q,R,S

2. (A) → Q,R,S ; (B) → Q,R,S ;
(C) → Q,R,S ; (D) → P,Q,R,S

3. (A)

EXERCISE-2

PART - I

1.	(D)	**2.**	(B)	**3.**	(A)
4.	(C)	**5.**	(A)	**6.**	(A)
7.	(A)	**8.**	(D)	**9.**	(D)
10.	(B)	**11.**	(C)	**12.**	(B)
13.	(B)	**14.**	(B)	**15.**	(C)
16.	(C)	**17.**	(B)	**18.**	(D)
19.	(A)	**20.**	(B)	**21.**	(B)
22.	(D)	**23.**	(D)		

PART - II

1.	7	**2.**	9	**3.**	7
4.	2	**5.**	2	**6.**	6
7.	3	**8.**	36	**9.**	3
10.	8	**11.**	2	**12.**	1
13.	7				

PART - III

1.	(AC)	**2.**	(CD)	**3.**	(AD)
4.	(ACD)	**5.**	(AC)	**6.**	(CD)
7.	(AC)				

PART - IV

1.	(B)	**2.**	(D)	**3.**	(C)
4.	(B)	**5.**	(D)	**6.**	(D)

EXERCISE-3

PART - I

1. (B)

2. (A) $\rightarrow$ (p) and (r), (B) $\rightarrow$ (p) and (q),
(C) $\rightarrow$ (p), (q), (r) and (s), (D) $\rightarrow$ (p), (q) and (r)

3. (A)

4. (A)$\rightarrow$ (p), (r); (B)$\rightarrow$(q), (s); (C)$\rightarrow$(p); (D)$\rightarrow$(q)

5.	(BD)	**6.**	(A)	**7.**	(D)
8.	(A)	**9.**	(B)		

10. (A) $\rightarrow$ p,q,t ; (B) $\rightarrow$ q, t ; (C) $\rightarrow$ s, (D) $\rightarrow$ s

11.	8	**12.**	1	**13.**	7
14.	(C)	**15.**	(D)	**16.**	4
17.	(C)	**18.**	(C)	**19.**	(A)
20.	3				

21. (A) $\rightarrow$ R (B) $\rightarrow$ P,S; (C) Q,T; (D) R,T

22.	2	**23.**	(A)	**24.**	9
25.	(C)	**26.**	(C)	**27.**	(5)
28.	(AC)				

PART – II

1.	(4)	**2.**	(4)	**3.**	(4)
4.	(3)	**5.**	(2)	**6.**	(2)
7.	(2)	**8.**	(3)	**9.**	(4)
10.	(3)	**11.**	(2)	**12.**	(2)
13.	(3)	**14.**	(2)	**15.**	(1)
16.	(3)	**17.**	(3)		

High Level Problems (HLP)

SUBJECTIVE QUESTIONS

1. A radioactive material decays by β-particle emission. During the first 2 seconds of a measurement, n β-particles are emitted and the next 2 seconds 0.75 n β-particles are emitted. Calculate the mean-life of this material in seconds to the nearest whole number. ($\ell n\ 3 = 1.0986$ and $\ell n\ 2 = 0.6931$). **[JEE 2003 Main) 2/60]**

2. What kinetic energy must an α-particle possess to split a deuteron H^2 whose binding energy is $E_b = 2.2$ MeV ?

3. A nucleus at rest undergoes α – decay according to the equation, ${}^{225}_{92}X \longrightarrow Y + \alpha$. At time t = 0, the emitted α-particle enters in a region of space where a uniform magnetic field $\vec{B} = B_0\hat{i}$ and electric field $\vec{E} = E_0\hat{i}$ exist. The α-particle enters in the region with velocity $\vec{v} = v_0\hat{j}$ from origin. At time $t = \sqrt{3} \times 10^7 \dfrac{m_\alpha}{q_\alpha E_0}$ sec., where m_α is the mass and q_α is the charge of α – particle. The particle was observed to have speed twice the initial speed v_0. Then find :

(a) the initial speed v_0 of the α – particle

(b) the velocity of α – particle at time t

(c) the binding energy per nucleon of X .

Given that : m (Y) = 221.03 u, m (He) = 4.003 u, m(n) = 1.009 u,

m (p) = 1.0084 u and 1 u = 1.67×10^{-27} kg = 931 MeV/c^2

4. A neutron collides elastically with an initially stationary deuteron. Find the fraction of the kinetic energy lost by the neutron (a) in a head-on collision; (b) in scattering at right angles.

5. Find the binding energy of a nucleus consisting of equal numbers of protons and neutrons and having the radius one and a half time smaller than that of Al^{27} nucleus. [Atomic mass of ${}_4Be^8$ = 8.0053 u, ${}_1H^1$ = 1.007826u, ${}_0n^1$= 1.008665 u]

6. A radio nuclide with half life T = 69.31 second emits β-particles of average kinetic energy E = 11.25 eV. At an instant concentration of β-particles at distance, r = 2 m from nuclide is n = 3×10^{13} per m^3.

(i) Calculate number of nuclei in the nuclide at that instant.

(ii) If a small circular plate is placed at distance r from nuclide such that β-particles strike the plate normally and come to rest, calculate pressure experienced by the plate due to collision of β-particle. (Mass of β-particle = 9×10^{-31} kg) ($\log_e 2 = 0.693$)

7. (a) Find the energy needed to remove a neutron from the nucleus of the calcium isotope ${}^{42}_{20}Ca$

(b) Find the energy needed to remove a proton from this nucleus

(c) Why are these energies different ?

Atomic masses of ${}^{41}_{20}Ca$ and ${}^{42}_{20}Ca$ are 40.962278 u and 41.958622 u respectively.

Atomic mass of ${}^{41}_{19}K$ is 40.961825 u, Mass of Proton is 1.007826 u

8. A nucleus X, initially at rest, undergoes alpha-decay according to the equation. **[JEE 1991; 2+4+2M]**

${}^{A}_{92}X \rightarrow + {}^{228}_{Z}Y + \alpha$

(a) Find the values of A and Z in the above process.

(b) The alpha particle produced in the above process is found to move in a circular track of radius 0.11 m in a uniform magnetic field of 3 tesla. Find the energy (in MeV) released during the process and the binding energy of the parent nucleus X.

Given that m (Y) = 228.03 u; m(${}^{1}_{0}n$) = 1.009 u.

m ${}^{4}_{2}He$ = 4.003 u ; m(${}^{1}_{1}H$) = 1.008 u.

9. 100 millicuries of radon which emits 5.5 MeV α - particles are contained in a glass capillary tube 5 cm long with internal and external diameters 2 and 6mm respectively Neglecting and effects and assuming that the inside of the tube is uniformly irradiated by the particles which are stopped at the surface calculate the temperature difference between the walls of a tube when steady thermal conditions have been reached.
Thermal conductivity of glass = 0.025 Cal $cm^{-2}\ s^{-1}\ C^{-1}$
Curie = 3.7×10^{10} disintergration per second
J = 4.18 joule Cal^{-1}

10. Radium being a member of the uranium series occurs in uranium ores. If the half lives of uranium and radium are respectively 4.5×10^9 and 1620 years calculate the $\frac{N_{radiun}}{N_{Uranium}}$ in Uranium ore at equlibrium.

11. ^{90}Sr decays to ^{90}Y by β decay with a half-life of 28 years. ^{90}Y decays by β decay to ^{90}Zr with a half-life of 64h. A pure sample of ^{90}Sr is allowed to decay. What is the value of $\frac{N_{Sr}}{N_y}$ after (a) 1h (b) 10 years?

12. The element Curium $^{248}{}_{96}Cm$ has a mean life of 10^{13} seconds. Its primary decay modes are spontaneous fission and α-decay, the former with a probability of 8% and the latter with a probability of 92%. Each fission releases 200 MeV of energy. The masses involved in α-decay are as follows : atomic masses of atoms are $^{248}{}_{96}Cm$ = 248.072220 u, He^4 = 4.002603 u & Pu^{244} = 244.064100 u. (1 u = 931 MeV/c^2). Calculate the power output from a sample of 10^{20} Cm atoms. **[IIT - 1997]**

13. Nucleus $_3A^7$ has binding energy per nucleon of 10 MeV. It absorbs a proton and its mass increases by $\frac{99}{100}$ times the mass of proton. Find the new binding energy of the nucleus so formed.
[Take energy equivalent of proton = 930 MeV]

14. The nucleus of $^{230}_{90}Th$ is unstable against α-decay with a half-life of 7.6×10^3 years. Write down the equation of the decay and estimate the kinetic energy of the emitted α-particle from the following data : $m\left(^{230}_{90}Th\right)$=230.0381 amu, $m\left(^{226}_{88}Ra\right)$ = 226.0254 amu and $m\left(^{4}_{2}He\right)$ = 4.0026 amu.

15. When ^{30}Si is bombarded with a deuteron. ^{31}Si is formed in its ground state with the emission of a proton. The energy released in this reaction from the following information is E then find [E]:
$^{31}Si \rightarrow {}^{31}P + \beta^- + 1.51$ MeV
$^{30}Si + d \rightarrow {}^{31}P + n + 5.10$ MeV
$n \rightarrow P + \beta^- + \bar{v} + 0.78$ MeV

HLP Answers

1. 6.954 sec **2.** 6.6 MeV

3. (a) $v_0 = 10^7$ m/s Ans. (b) $\vec{v}\,(t) = \frac{qE_0}{m} t\,\hat{i} + 10^7 \cos \omega t\,\hat{j} - 10^7 \sin \omega t\,\hat{k}$ where $\omega = qB/m$
(c) 8.11 MeV/nucleon

4. (a) $\eta = 4mM/(m+M)^2 = 0.89$; (b) $\eta = 2m/(m + M) = 2/3$. Here m and M are the masses of a neutron and deuteron.

5. Be^8, E_b = 56.5 MeV. **6.** (i) $9.6\ \pi \times 10^{22}$, (ii) 1.08×10^{-4} Nm^{-2}

7. (a) 11.48 MeV (b) 10.27 MeV (c) The neutron was acted upon only by attractive nuclear forces whereas the proton was also acted upon by repulsive electric forces that decrease its binding energy.

8. (a) 232, 90 (b) 5.3 MeV, 1823.2 MeV **9.** $(T_1 - T_2) = 1.09$ °C

10. $1/2.78 \times 10^6$

11. (a) For t = 1 h and using the values for the decay constans $N_{sr}/N_y = 3.56 \times 10^5$
(b) For t = 10 years, $N_{sr}/N_y = 3823$

12. 3.32×10^{-5} Js^{-1} **13.** 79.3 MeV **14.** $^{230}_{90}Th \rightarrow {}^{226}_{88}Ra + {}^{4}_{2}He + Q$; 9.25 MeV.

15. 4