KINEMATICS

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JEE (ADVANCED) SYLLABUS

Kinematics in one Dimensions, Projectile Motion, Relative Motion,

JEE (MAIN) SYLLABUS

Motion in a straight line : Position time graph, speed and velocity. Uniform and non-uniform motion, average speed and instantaneous velocity Uniformly acceleration motion, velocity-time, position-time graphs, relations for uniformly accelerated motion. Projectile Motion, Relative motion.

RECTILINEAR MOTION

MECHANICS

Mechanics is the branch of physics which deals with the cause and effects of motion of a particle, rigid objects and deformable bodies etc. Mechanics is classified under two streams namely **Statics** and **Dynamics**. Dynamics is further divided into **Kinematics** and **Kinetics**.

Mechanics

- **Statics**
  Study of forces and their effect on objects at rest
- **Dynamics**
  Study of forces and their effect on objects in motion
  - Kinematics
    The word kinematics means 'science of motion'. Branch of mechanics which deals with study of motion without going into the cause of motion, i.e. force, torque etc.
  - Kinetics
    It is branch of mechanics which is concerned about the causes (i.e. the force , torque) that cause motion of bodies.

1. MOTION AND REST

Motion is a combined property of the object and the observer. There is no meaning of rest or motion without the observer. Nothing is in absolute rest or in absolute motion.

An object is said to be in motion with respect to a observer, if its position changes with respect to that observer. It may happen by both ways either observer moves or object moves.

2. RECTILINEAR MOTION

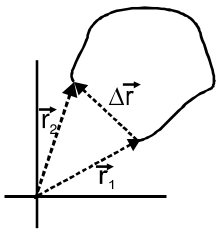
Rectilinear motion is motion, along a straight line or in one dimension. It deals with the kinematics of a particle in one dimension.

2.1 Position

The position of a particle refers to its location in the space at a certain moment of time. It is concerned with the question – "where is the particle at a particular moment of time?"

2.2 Displacement

The change in the position of a moving object is known as displacement. It is the vector joining the initial position ($\vec{r}_1$) of the particle to its final position ($\vec{r}_2$) during an interval of time.



$$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1$$

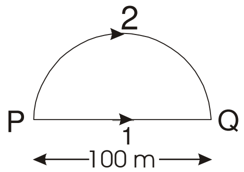
Displacement can be negative positive or zero.

2.3 Distance

The length of the actual path travelled by a particle during a given time interval is called as distance. The distance travelled is a scalar quantity which is quite different from displacement. In general, the distance travelled between two points may not be equal to the magnitude of the displacement between the same points.

Solved Example

Example 1. Ram takes path 1 (straight line) to go from P to Q and Shyam takes path 2 (semicircle).



(a) Find the distance travelled by Ram and Shyam?

(b) Find the displacement of Ram and Shyam?

Solution : (a) Distance travelled by Ram = 100 m

Distance travelled by Shyam = π(50 m) = 50π m

(b) Displacement of Ram = 100 m

Displacement of Shyam = 100 m

2.4 Average Velocity (in an interval) :

The average velocity of a moving particle over a certain time interval is defined as the displacement divided by the lapsed time.

Average Velocity = $\dfrac{\text{displacement}}{\text{time interval}}$

for straight line motion, along x-axis, we have

$$\mathbf{v_{av}} = \bar{v} = \langle v \rangle = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$$

The dimension of velocity is [LT^{-1}] and its SI unit is m/s.

The average velocity is a vector in the direction of displacement. For motion in a straight line, directional aspect of a vector can be taken care of by +ve and -ve sign of the quantity.

2.5 Instantaneous Velocity (at an instant) :

The velocity at a particular instant of time is known as instantaneous velocity. The term "velocity" usually means instantaneous velocity.

$$V_{inst.} = \lim_{\Delta t \to 0}\left(\frac{\Delta x}{\Delta t}\right) = \frac{dx}{dt}$$

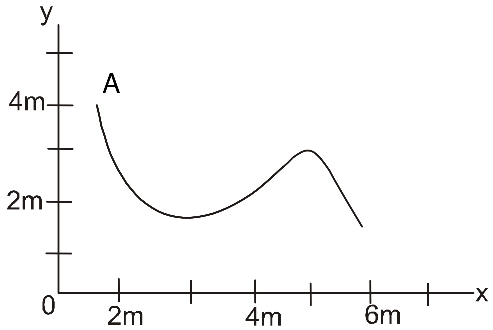
In other words, the instantaneous velocity at a given moment (say, t) is the limiting value of the average velocity as we let Δt approach zero. The limit as $\Delta t \to 0$ is written in calculus notation as dx/dt and is called the derivative of x with respect to t.

Note :

- The magnitude of instantaneous velocity and instantaneous speed are equal.
- The determination of instantaneous velocity by using the definition usually involves calculation of derivative. We can find v = $\dfrac{dx}{dt}$ by using the standard results from differential calculus.
- Instantaneous velocity is always tangential to the path.

Solved Example

Example 2. A particle starts from a point A and travels along the solid curve shown in figure. Find approximately the position B of the particle such that the average velocity between the positions A and B has the same direction as the instantaneous velocity at B.

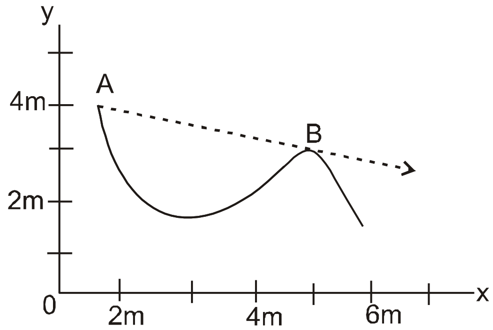


Answer : x = 5m, y = 3m

Solution : The given curve shows the path of the particle starting at y = 4 m.

Average velocity = $\frac{\text{displacement}}{\text{time taken}}$ **;** where displacement is straight line distance between points.

Instantaneous velocity at any point is the tangent drawn to the curve at that point.



Now, as shown in the graph, line AB shows displacement as well as the tangent to the given curve. Hence, point B is the point at which direction of AB shows average as well as instantaneous velocity.

2.6 Average Speed (in an interval)

Average speed is defined as the total path length travelled divided by the total time interval during which the motion has taken place. It helps in describing the motion along the actual path.

$$\textbf{Average Speed} = \frac{\text{distance travelled}}{\text{time interval}}$$

The dimension of velocity is $[LT^{-1}]$ and its SI unit is m/s.

Note :
- Average speed is always positive in contrast to average velocity which being a vector, can be positive or negative.
- If the motion of a particle is along a straight line and in same direction then, average velocity = average speed.
- Average speed is, in general, greater than the magnitude of average velocity.

Solved Example

Example 3. In the example 1, if Ram takes 4 seconds and Shyam takes 5 seconds to go from P to Q, find
(a) Average speed of Ram and Shyam?
(b) Average velocity of Ram and Shyam?

Solution : (a) Average speed of Ram = $\frac{100}{4}$ m/s = 25 m/s

Average speed of Shyam = $\frac{50\pi}{5}$ m/s = 10π m/s

(b) Average velocity of Ram = $\frac{100}{4}$ m/s = 25 m/s (From P to Q)

Average velocity of Shyam = $\frac{100}{5}$ m/s = 20 m/s (From P to Q)

Example 4. A particle travels half of total distance with speed v_1 and next half with speed v_2 along a straight line. Find out the average speed of the particle?

Solution : Let total distance travelled by the particle be 2s.

Time taken to travel first half = $\frac{s}{v_1}$

Time taken to travel next half = $\frac{s}{v_2}$

Average speed = $\frac{\text{Total distance covered}}{\text{Total time taken}} = \frac{2s}{\frac{s}{v_1}+\frac{s}{v_2}} = \frac{2v_1v_2}{v_1+v_2}$ (harmonic progression)

Example 5. A person travelling on a straight line moves with a uniform velocity v_1 for some time and with uniform velocity v_2 for the next equal time. The average velocity v is given by

Answer : $v = \frac{v_1+v_2}{2}$ (Arithmetic progression)

Solution :

As shown, the person travels from A to B through a distance S, where first part S_1 is travelled in time t/2 and next S_2 also in time t/2.

So, according to the condition : $v_1 = \frac{S_1}{t/2}$ and $v_2 = \frac{S_2}{t/2}$

Average velocity = $\frac{\text{Total displacement}}{\text{Total time taken}} = \frac{S_1+S_2}{t} = \frac{\frac{v_1t}{2}+\frac{v_2t}{2}}{t} = \frac{v_1+v_2}{2}$

2.7 Average acceleration (in an interval):

The average acceleration for a finite time interval is defined as :

Average acceleration = $\frac{\text{change in velocity}}{\text{time interval}}$

Average acceleration is a vector quantity whose direction is same as that of the change in velocity.

$$\vec{a}_{av} = \frac{\Delta\vec{v}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t}$$

Since for a straight line motion the velocities are along a line, therefore

$$a_{av} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t_f - t_i}$$

(where one has to substitute v_f and v_i with proper signs in one dimensional motion)

2.8 Instantaneous Acceleration (at an instant):

The instantaneous acceleration of a particle is its acceleration at a particular instant of time. It is defined as the derivative (rate of change) of velocity with respect to time. We usually mean instantaneous acceleration when we say "acceleration". For straight motion we define instantaneous acceleration as :

$$a = \frac{dv}{dt} = \lim_{\Delta t \to 0}\left(\frac{\Delta v}{\Delta t}\right) \quad \text{and in general} \quad \vec{a} = \frac{d\vec{v}}{dt} = \lim_{\Delta t \to 0}\left(\frac{\Delta \vec{v}}{\Delta t}\right)$$

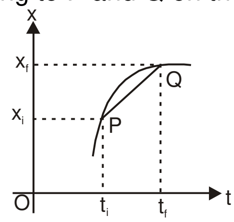
The dimension of acceleration is $[LT^{-2}]$ and its SI unit is m/s^2.

3. GRAPHICAL INTERPRETATION OF SOME QUANTITIES

3.1 Average Velocity

If a particle passes a point P (x_i) at time $t = t_i$ and reaches Q (x_f) at a later time instant $t = t_f$, its average velocity in the interval PQ is $V_{av} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$

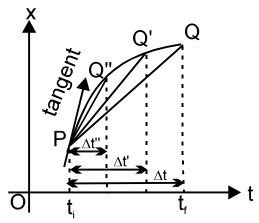
This expression suggests that the average velocity is equal to the slope of the line (chord) joining the points corresponding to P and Q on the x-t graph.



3.2 Instantaneous Velocity

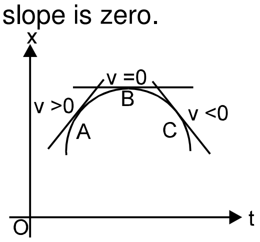
Consider the motion of the particle between the two points P and Q on the x-t graph shown. As the point Q is brought closer and closer to the point P, the time interval between PQ (Δt, $\Delta t'$, $\Delta t''$,......) get progressively smaller. The average velocity for each time interval is the slope of the appropriate dotted line (PQ, PQ′, PQ″......).

As the point Q approaches P, the time interval approaches zero, but at the same time the slope of the dotted line approaches that of the tangent to the curve at the point P. As $\Delta t \to 0$, V_{av} $(=\Delta x/\Delta t) \to V_{inst.}$



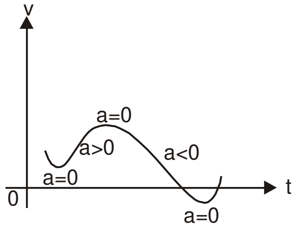
Geometrically, as $\Delta t \to 0$, chord PQ $\to$ tangent at P.

Hence the instantaneous velocity at P is the slope of the tangent at P in the x – t graph. When the slope of the x – t graph is positive, v is positive (as at the point A in figure). At C, v is negative because the tangent has negative slope. The instantaneous velocity at point B (turning point) is zero as the slope is zero.



3.3 Instantaneous Acceleration :

The derivative of velocity with respect to time is the slope of the tangent in velocity time (v–t) graph.



Solved Example

Example 6. Position of a particle as a function of time is given as $x = 5t^2 + 4t + 3$. Find the velocity and acceleration of the particle at t = 2 s?

Solution : Velocity; $v = \dfrac{dx}{dt} = 10t + 4$

At $t = 2$ s

$v = 10(2) + 4$

$v = 24$ m/s

Acceleration; $a = \dfrac{d^2x}{dt^2} = 10$

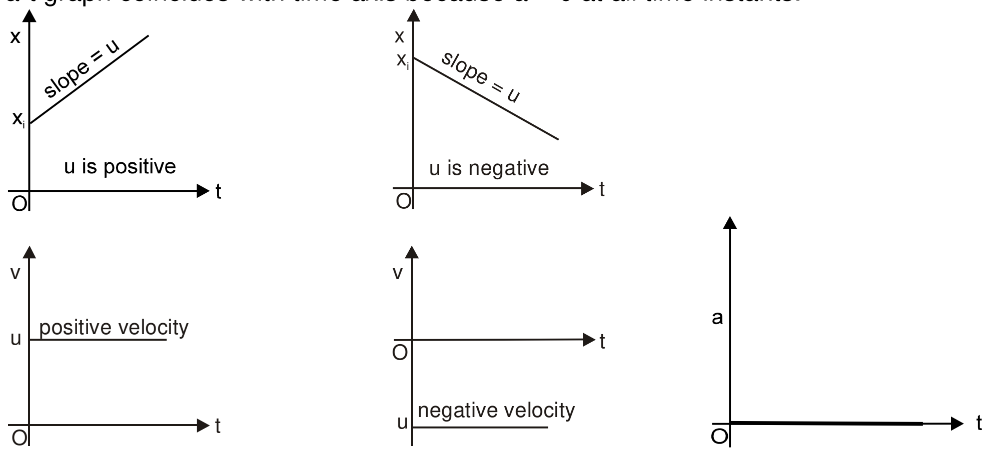
Acceleration is constant, so at t = 2 s $a = 10$ m/s^2

4. MOTION WITH UNIFORM VELOCITY

Consider a particle moving along x-axis with uniform velocity u starting from the point $x = x_i$ at t = 0.

Equations of x, v, a are : $x(t) = x_i + ut$; $v(t) = u$; $a(t) = 0$

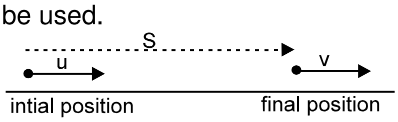
- x-t graph is a straight line of slope u through x_i.
- as velocity is constant, v – t graph is a horizontal line.
- a-t graph coincides with time axis because a = 0 at all time instants.



5. UNIFORMLY ACCELERATED MOTION :

If a particle is accelerated with constant acceleration in an interval of time, then the motion is termed as uniformly accelerated motion in that interval of time.

For uniformly accelerated motion along a straight line (x-axis) during a time interval of t seconds, the following important results can be used.



(a) $a = \frac{v-u}{t}$ (b) $V_{av} = \frac{v+u}{2}$ (c) $S = (v_{av})t$

(d) $S = \left(\frac{v+u}{2}\right)t$ (e) $v = u + at$

(f) $s = ut + 1/2\ at^2$; $s = vt - 1/2\ at^2$
$x_f = x_i + ut + 1/2\ at^2$

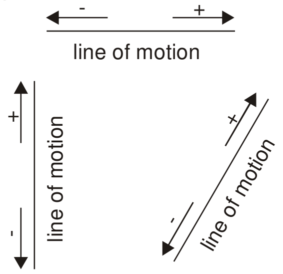
(g) $v^2 = u^2 + 2as$

(h) $s_n = u + a/2\ (2n - 1)$
u = initial velocity (at the beginning of interval)
a = acceleration
v = final velocity (at the end of interval)
s = displacement ($x_f - x_i$)
x_f = final coordinate (position)
x_i = initial coordinate (position)
s_n = displacement during the n^{th} sec

6. DIRECTIONS OF VECTORS IN STRAIGHT LINE MOTION

In straight line motion, all the vectors (position, displacement, velocity & acceleration) will have only one component (along the line of motion) and there will be only two possible directions for each vector.

- For example, if a particle is moving in a horizontal line (x-axis), the two directions are right and left. Any vector directed towards right can be represented by a positive number and towards left can be represented by a negative number.
- For vertical or inclined motion, upward direction can be taken +ve and downward as –ve



- For objects moving vertically near the surface of the earth, the only force acting on the particle is its weight (mg) i.e. the gravitational pull of the earth. Hence acceleration for this type of motion will always be $a = -g$ i.e., $a = -9.8\ m/s^2$ (–ve sign, because the force and acceleration are directed downwards, If we select upward direction as positive).

Note :
- If acceleration is in same direction as velocity, then speed of the particle increases.
- If acceleration is in opposite direction to the velocity then speed decreases i.e., the particle slows down. This situation is known as retardation.

Solved Examples

Example 7. A particle moving rectilinearly with constant acceleration is having initial velocity of 10 m/s. After some time, its velocity becomes 30 m/s. Find out velocity of the particle at the mid point of its path?

Solution : Let the total distance be 2x.
$\therefore$ distance upto midpoint = x
Let the velocity at the mid point be v and acceleration be a.
From equations of motion

$v^2 = 10^2 + 2ax$ .....(1)

$30^2 = v^2 + 2ax$ .....(2)

(2) – (1) gives

$v^2 - 30^2 = 10^2 - v^2$

$\Rightarrow$ $v^2 = 500$ $\Rightarrow$ $v = 10\sqrt{5}$ m/s

Example 8. Mr. Sharma brakes his car with constant acceleration from a velocity of 25 m/s to 15 m/s over a distance of 200 m.

(a) How much time elapses during this interval?
(b) What is the acceleration?
(c) If he has to continue braking with the same constant acceleration, how much longer would it take for him to stop and how much additional distance would he cover?

Solution : (a) We select positive direction for our coordinate system to be the direction of the velocity and choose the origin so that $x_i = 0$ when the braking begins. Then the initial velocity is $u_x = +25$ m/s at $t = 0$, and the final velocity and position are $v_x = +15$ m/s and $x = 200$ m at time t. Since the acceleration is constant, the average velocity in the interval can be found from the average of the initial and final velocities.

$$\therefore \qquad v_{av,\,x} = \frac{1}{2}(u_x + v_x) = \frac{1}{2}(15 + 25) = 20 \text{ m/s.}$$

The average velocity can also be expressed as $v_{av,\,x} = \dfrac{\Delta x}{\Delta t}$. With $\Delta x = 200$ m

and $\Delta t = t - 0$, we can solve for t : $t = \dfrac{\Delta x}{v_{av,x}} = \dfrac{200}{20} = 10$ s.

(b) We can now find the acceleration using $v_x = u_x + a_x t$

$$a_x = \frac{v_x - u_x}{t} = \frac{15 - 25}{10} = -1 \text{ m/s}^2.$$

The acceleration is negative, which means that the positive velocity is becoming smaller as brakes are applied (as expected).

(c) Now with known acceleration, we can find the total time for the car to go from velocity $u_x = 25$ m/s to $v_x = 0$. Solving for t, we find

$$t = \frac{v_x - u_x}{a_x} = \frac{0 - 25}{-1} = 25 \text{ s.}$$

The total distance covered is $x = x_i + u_x t + \dfrac{1}{2}a_x t^2$

$$= 0 + (25)(25) + \frac{1}{2}(-1)(25)^2 = 625 - 312.5 = 312.5 \text{ m.}$$

Additional distance covered = 312.5 – 200 = 112.5 m.

Example 9. A police inspector in a jeep is chasing a pickpocket an a straight road. The jeep is going at its maximum speed v (assumed uniform). The pickpocket rides on the motorcycle of a waiting friend when the jeep is at a distance d away, and the motorcycle starts with a constant acceleration a. Show that the pick pocket will be caught if $v \ge \sqrt{2ad}$.

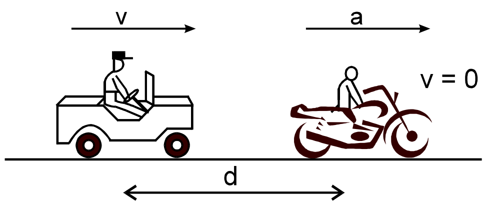
Solution : Suppose the pickpocket is caught at a time t after motorcycle starts. The distance travelled by the motorcycle during this interval is

$$s = \frac{1}{2}at^2 \qquad \text{\_\_\_\_ (1)}$$

During this interval the jeep travels a distance

$$s + d = vt \qquad \text{\_\_\_\_ (2)}$$

By (1) and (2),

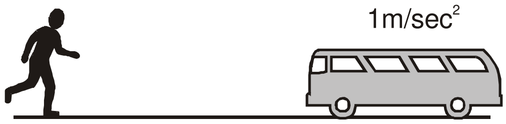


$$\frac{1}{2}at^2 + d = vt \qquad \text{or,} \qquad t = \frac{v \pm \sqrt{v^2 - 2ad}}{a}$$

The pickpocket will be caught if t is real and positive. This will be possible if

$$v^2 \ge 2ad \qquad \text{or,} \qquad v \ge \sqrt{2ad}$$

Example 10. A man is standing 40 m behind the bus. Bus starts with 1 m/sec^2 constant acceleration and also at the same instant the man starts moving with constant speed 9 m/s. Find the time taken by man to catch the bus.



Solution : Let after time 't' man will catch the bus

For bus

$x = x_0 + ut + \frac{1}{2}at^2$, $\quad x = 40 + 0(t) + \frac{1}{2}(1)t^2$

$x = 40 + \frac{t^2}{2}$ ....(i)

For man, $x = 9t$ ....(ii)

From (i) & (ii)

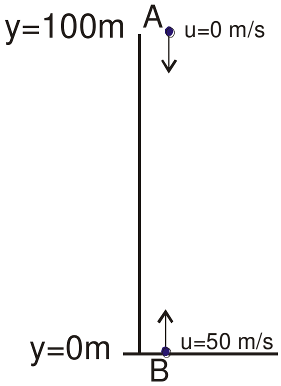
$40 + \frac{t^2}{2} = 9t$ or $t = 8$ s or $t = 10$s.

Example 11. A particle is dropped from height 100 m and another particle is projected vertically up with velocity 50 m/s from the ground along the same line. Find out the position where two particle will meet ? (take g = 10 m/s^2)

Solution : Let the upward direction is positive.

Let the particles meet at a distance y from the ground.

For particle A,



$y_0 = +100$ m

$u = 0$ m/s

$a = -10$ m/s^2

$y = 100 + 0(t) - \frac{1}{2}10 \times t^2 \left[y = y_0 + ut + \frac{1}{2}at^2\right]$

$= 100 - 5t^2$ .....(1)

For particle B,

$y_0 = 0$ m

$u = +50$ m/s

$a = -10$ m/s^2

$y = 50(t) - 10t^2$

$= 50t - 5t^2$ ......(2)

According to the problem;

$50t - 5t^2 = 100 - 5t^2$

$t = 2$ sec

Putting t = 2 sec in eqn. (1),

$y = 100 - 20 = 80$ m

Hence, the particles will meet at a height 80 m above the ground.

Example 12. A particle is dropped from a tower. It is found that it travels 45 m in the last second of its journey. Find out the height of the tower ? (take g = 10 m/s^2)

Solution : Let the total time of journey be n seconds.

Using; $s_n = u + \frac{a}{2}(2n-1)$ $\Rightarrow$ $45 = 0 + \frac{10}{2}(2n-1)$

n = 5 sec

Height of tower ; $\frac{1}{2}h = gt^2 = \frac{1}{2} \times 10 \times 5^2 = 125$ m

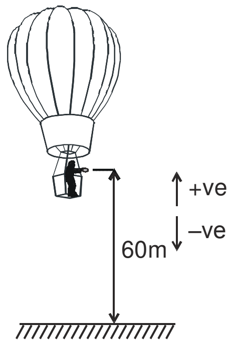
7. REACTION TIME

When a situation demands our immediate action. It takes some time before we really respond. Reaction time is the time a person takes to observe, think and act.

Solved Examples

Example 13. A stone is dropped from a balloon going up with a uniform velocity of 5 m/s. If the balloon was 60 m high when the stone was dropped, find its height when the stone hits the ground. Take g = 10 m/s^2.

Solution : $S = ut + \frac{1}{2}at^2$



$-60 = 5(t) + \frac{1}{2}(-10)\,t^2$

$-60 = 5t - 5t^2$

$5t^2 - 5t - 60 = 0$

$t^2 - t - 12 = 0$

$t^2 - 4t + 3t - 12 = 0$

$(t-4)(t+3) = 0$

$\therefore$ t = 4

Height of balloon from ground at this instant

= 60 + 4 × 5 = 80 m

Example 14. A balloon is rising with constant acceleration 2 m/sec^2. Two stones are released from the balloon at the interval of 2 sec. Find out the distance between the two stones 1 sec. after the release of second stone.

Solution : Acceleration of balloon = 2 m/sec^2

Let at t = 0, y = 0 when the first stone is released.

By the question, $y_1 = 0\,t_1 + \frac{1}{2}gt_1^2$ (taking vertical upward as – ve and downward as + ve)

$\therefore$ Position of Ist stone = $\frac{9}{2}g$

(1 second after release of second stone will be the 3rd second for the 1st stone)

For second stone $y_2 = ut_2 + \frac{1}{2}gt_2^2$

u = 0 + at = – 2 × 2 = – 4m/s (taking vertical upward as – ve and downward as + ve)

$\therefore \quad y_2 = -4 \times 1 + \frac{1}{2} g \times (1)^2 \qquad (t_2 = 1 \text{ second})$

The 2nd stone is released after 2 second

$\therefore \quad y = -\frac{1}{2}at^2 = -\frac{1}{2} \times 2 \times 4 = -4$

So, Position of second stone from the origin $= -4 + \frac{1}{2}g - 4$

Distance between two stones $= \frac{1}{2} g \times 9 - \frac{1}{2} g \times 1 + 8 = 48$ m.

Note :

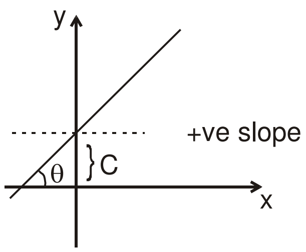
- As the particle is detached from the balloon it is having the same velocity as that of balloon, but its acceleration is only due to gravity and is equal to g.

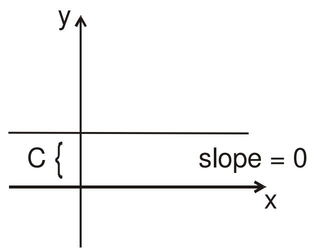
8. STRAIGHT LINE-EQUATION, GRAPH, SLOPE (+ve, –ve, zero slope).

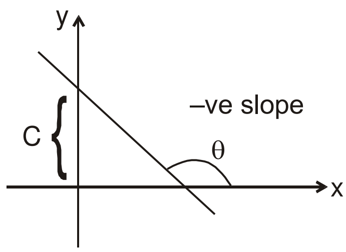
If θ is the angle at which a straight line is inclined to the positive direction of x-axis, & $0° \le \theta < 180°$, $\theta \ne 90°$, then the slope of the line, denoted by m, is defined by $m = \tan\theta$. If θ is 90°, m does not exist, but the line is parallel to the y-axis. If $\theta = 0$, then $m = 0$ & the line is parallel to the x-axis.

Slope – intercept form : $y = mx + c$ is the equation of a straight line whose slope is m & which makes an intercept c on the y-axis.

$$m = \text{slope} = \tan\theta = \frac{dy}{dx}$$

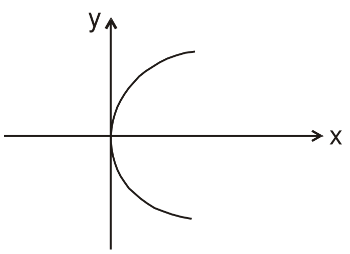




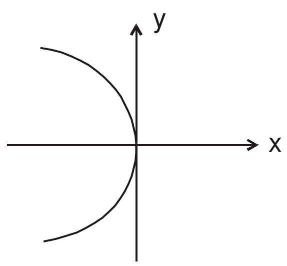


9. PARABOLIC CURVE-EQUATION, GRAPH (VARIOUS SITUATIONS UP, DOWN, LEFT, RIGHT WITH CONDITIONS)

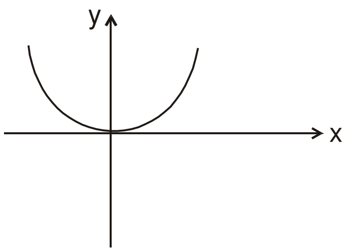
$y^2 = kx$



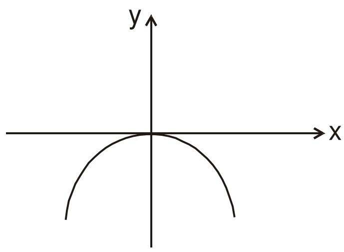
$y^2 = -kx$



$x^2 = ky$



$x^2 = -ky$



Where k is a positive constant.

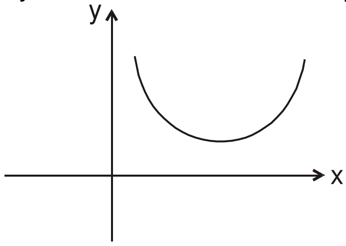
Equation of parabola :

Case (i) : $y = ax^2 + bx + c$

For $a > 0$

The nature of the parabola will be like that of the of nature $x^2 = ky$

Minimum value of y exists at the vertex of the parabola.



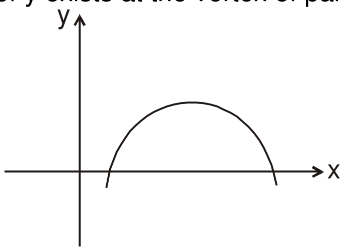
$y_{min} = \frac{-D}{4a}$ where $D = b^2 - 4ac$

Coordinates of vertex $= \left(\frac{-b}{2a}, \frac{D}{4a}\right)$

Case (ii) : $a < 0$

The nature of the parabola will be like that of the nature of $x^2 = -ky$

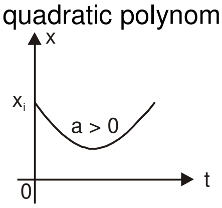
Maximum value of y exists at the vertex of parabola.

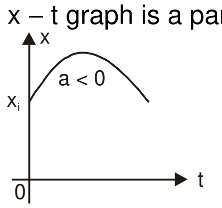


$y_{max} = \frac{D}{4a}$ where $D = b^2 - 4ac$

10. GRAPHS IN UNIFORMLY ACCELERATED MOTION ($a \neq 0$)

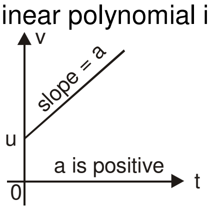
• x is a quadratic polynomial in terms of t. Hence x – t graph is a parabola.

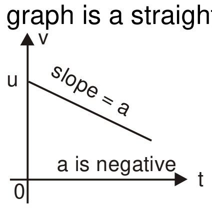




x-t graph

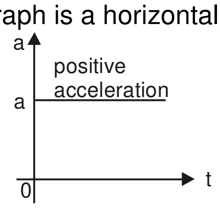
• v is a linear polynomial in terms of t. Hence v–t graph is a straight line of slope a.

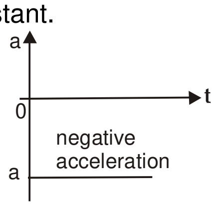




v-t graph

• a–t graph is a horizontal line because a is constant.





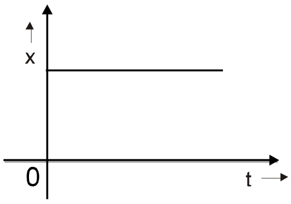
a-t graph

11. INTERPRETATION OF SOME MORE GRAPHS

11.1 Position vs Time graph

(i) Zero Velocity

As position of particle is fixed at all the time, so the body is at rest.



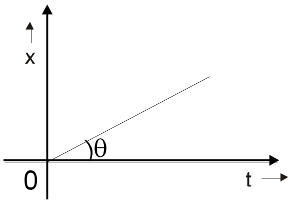
Slope; $\frac{dx}{dt} = \tan\theta = \tan 0^{\circ} = 0$

Velocity of particle is zero

(ii) Uniform Velocity

Here tan θ is constant $\tan\theta = \frac{dx}{dt}$

$\therefore$ $\frac{dx}{dt}$ is constant.



$\therefore$ velocity of particle is constant.

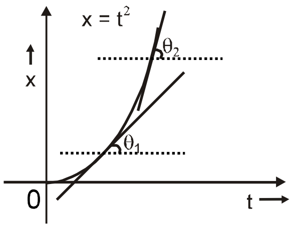
(iii) Non uniform velocity (increasing with time)

In this case;

As time is increasing, θ is also increasing.

$\therefore$ $\frac{dx}{dt} = \tan\theta$ is also increasing

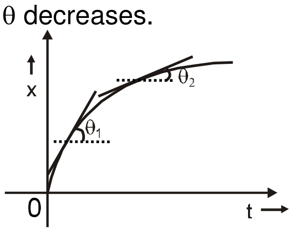
Hence, velocity of particle is increasing.



(iv) Non uniform velocity (decreasing with time)

In this case;

As time increases, θ decreases.



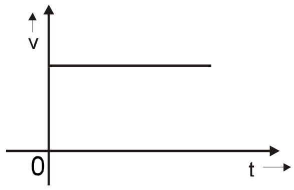
$\therefore$ $\frac{dx}{dt} = \tan\theta$ also decreases.

Hence, velocity of particle is decreasing.

11.2 Velocity vs time graph

(i) Zero acceleration

Velocity is constant.



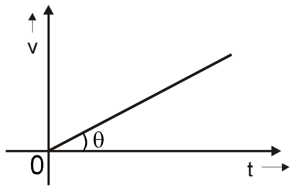
$\tan\theta = 0$

$\therefore \quad \frac{dv}{dt} = 0$

Hence, acceleration is zero.

(ii) Uniform acceleration

$\tan\theta$ is constant.



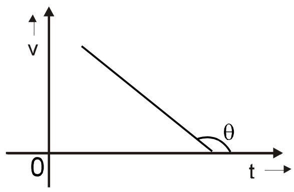
$\therefore \quad \frac{dv}{dt}$ = constant

Hence, it shows constant acceleration.

(iii) Uniform retardation

Since $\theta > 90^\circ$

$\therefore$ $\tan\theta$ is constant and negative.

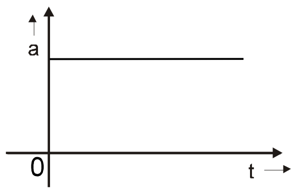


$\therefore \quad \frac{dv}{dt}$ = negative constant

Hence, it shows constant retardation.

11.3 Acceleration vs time graph

(i) Constant acceleration



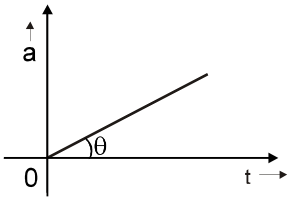
$\tan\theta = 0$

$\therefore \quad \frac{da}{dt} = 0$

Hence, acceleration is constant.

(ii) Uniformly increasing acceleration

θ is constant.



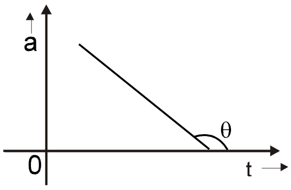
$0^\circ < \theta < 90^\circ \Rightarrow \tan\theta > 0$

$\therefore \dfrac{da}{dt} = \tan\theta = \text{constant} > 0$

Hence, acceleration is uniformly increasing with time.

(iii) Uniformly decreasing acceleration

Since $\theta > 90^\circ$



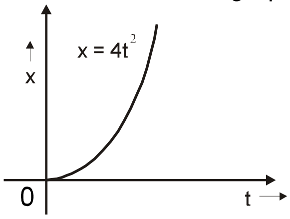
$\therefore$ $\tan\theta$ is constant and negative.

$\therefore \dfrac{da}{dt} = \text{negative constant}$

Hence, acceleration is uniformly decreasing with time

Solved Examples

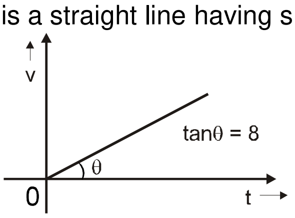
Example 15. The displacement vs time graph of a particle moving along a straight line is shown in the figure. Draw velocity vs time and acceleration vs time graph.



Solution : $x = 4t^2$

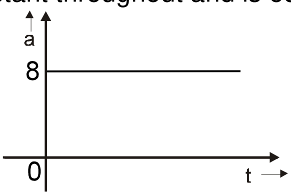
$v = \dfrac{dx}{dt} = 8t$

Hence, velocity-time graph is a straight line having slope i.e. $\tan\theta = 8$.

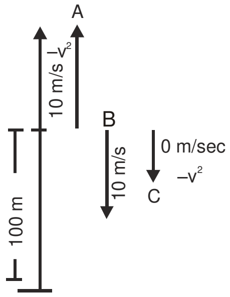


$a = \dfrac{dv}{dt} = 8$

Hence, acceleration is constant throughout and is equal to 8.



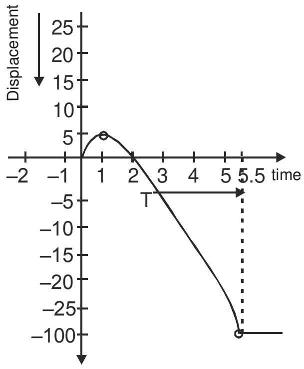
Example 16. At the height of 100 m, a particle A is thrown up with V = 10 m/s, B particle is thrown down with V = 10m/s and C particle released with V = 0 m/s. Draw graphs of each particle.



(i) Displacement–time (ii) Speed–time (iii) Velocity–time (iv) Acceleration–time

Solution : **For particle A :**

(i) Displacement vs time graph is



$y = ut + \frac{1}{2}at^2$

$u = +\ 10\ m/sec^2$

$y = 10t - \frac{1}{2} \times 10t^2 = 10\ t - 5t^2$

$v = \frac{dy}{dt} = 10 - 10\ t = 0$

t = 1 ; hence, velocity is zero at t = 1

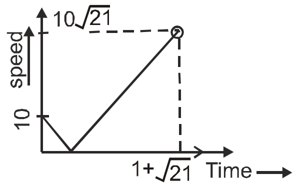
$10\ t - 5\ t^2 = -\ 100$

$t^2 - 2t - 20 = 0$

t = 5.5 sec.

i.e. particle travels up till 5.5 seconds.

(ii) Speed vs time graph : Particle has constant acceleration = g ↓ throughout the motion, so v-t curve will be straight line.



when moving up, v = u + at

0 = 10 – 10 t or t = 1 is the time at which speed is zero.

there after speed increases at constant rate of 10 m/s^2 .

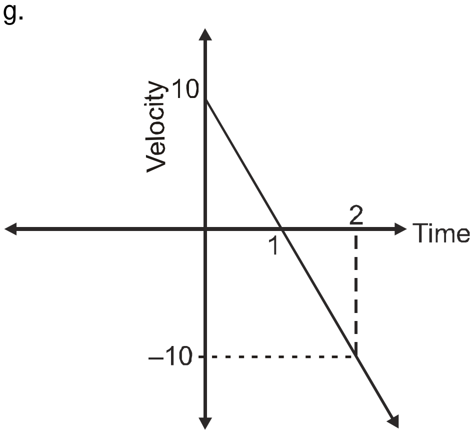
Resulting Graph is : (speed is always positive).

This shows that particle travels till a time of $1 + \sqrt{21}$ seconds

(iii) Velocity vs time graph :

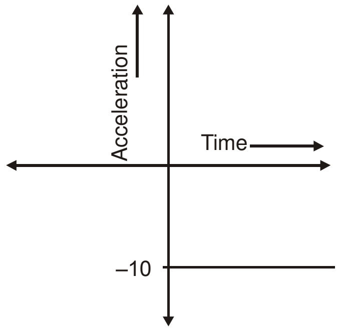
V = u + at

V = 10 – 10t ; this shows that velocity becomes zero at t = 1 sec and thereafter the velocity is negative with slope g.



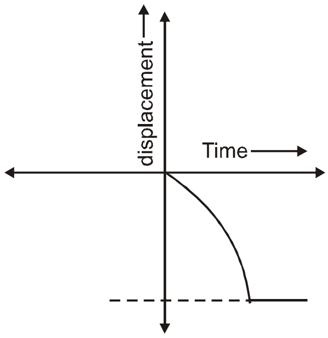
(iv) Acceleration vs time graph :

throughout the motion, particle has constant acceleration = –10 m/s^2.



For particle B : u = – 10 m/s. y = – 10t – $\frac{1}{2}$ (10) t^2 = – 10t – 5t^2

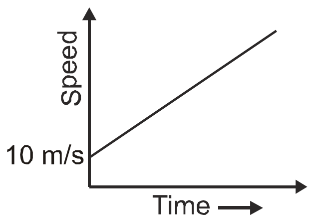
(i) Displacement time graph :



y = 10t – 5t^2 ; $\frac{dy}{dt}$ = – 10t – 5t^2 = – 10 – 10t

this shows that slope becomes more negative with time.

(ii) Speed time graph :

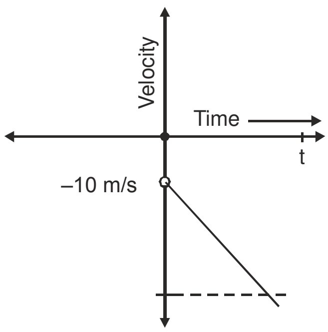


$\frac{dy}{dt}$ = – 10t – 5t^2 = – 10 – 10t

hence, speed is directly proportional to time with slope of 10 initial speed = 10 m/s

(iii) Velocity time graph :

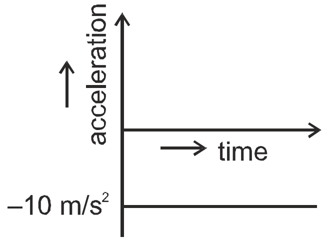
$\frac{dy}{dt}$ = – 10t – 5t^2 = – 10 – 10t



hence, velocity is directly proportional to time with slope of –10. Initial velocity = –10 m/s

(iv) Acceleration vs time graph :

throughout the motion, particle has constant acceleration = –10 m/s^2.

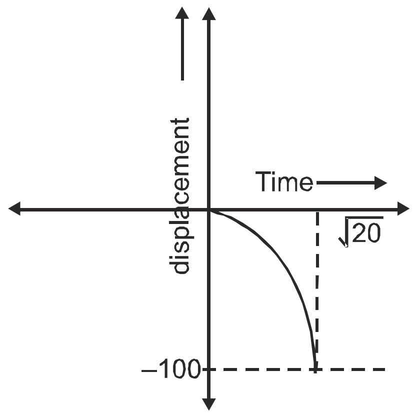


$a = \frac{dv}{dt} = -10$

For Particle C :

(i) Displacement time graph :

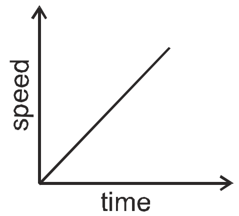
$u = 0,\ y = -\frac{1}{2} \times 10t^2 = -5t^2$



this shows that slope becomes more negative with time.

(ii) Speed vs time graph :

$v = \frac{dy}{dt} = -10\ t$

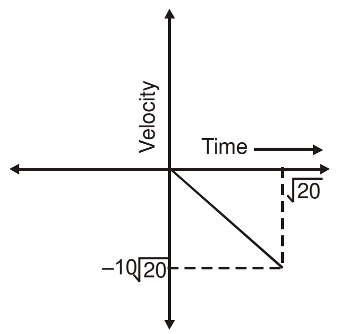


hence, speed is directly proportional to time with slope of 10.

(iii) Velocity time graph :

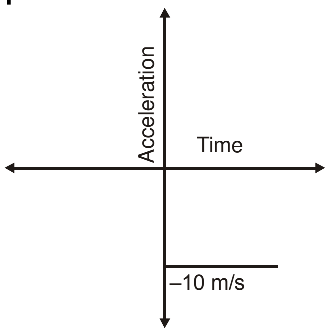
V = u + at

V = – 10t ;



hence, velocity is directly proportional to time with slope of –10.

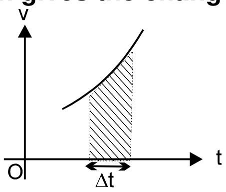
(iv) Acceleration vs time graph :



throughout the motion, particle has constant acceleration = –10 m/s^2.

12. DISPLACEMENT FROM v - t GRAPH & CHANGE IN VELOCITY FROM a -t GRAPH

Displacement = Δx = area under v-t graph. Since a negative velocity causes a negative displacement, areas below the time axis are taken negative. In similar way, can see that $\Delta v = a\ \Delta t$ leads to the conclusion that **area under a – t graph gives the change in velocity Δv during that interval.**



Solved Example

Example 17. Describe the motion shown by the following velocity-time graphs.

Solution : (a) **During interval AB:** velocity is +ve so the particle is moving in +ve direction, but it is slowing down as acceleration (slope of v-t curve) is negative. **During interval BC:** particle remains at rest as velocity is zero. Acceleration is also zero. **During interval CD:** velocity is -ve so the particle is moving in -ve direction and is speeding up as acceleration is also negative.

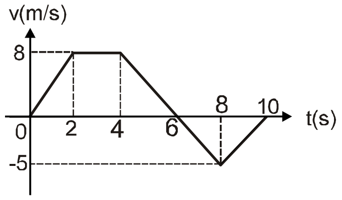
(b) **During interval AB:** particle is moving in +ve direction with constant velocity and acceleration is zero. **During interval BC:** particle is moving in +ve direction as velocity is +ve, but it slows down until it comes to rest as acceleration is negative. **During interval CD:** velocity is -ve so the particle is moving in -ve direction and is speeding up as acceleration is also negative.

Important Points to Remember

- For uniformly accelerated motion ($a \neq 0$), x–t graph is a parabola (opening upwards if $a > 0$ and opening downwards if $a < 0$). The slope of tangent at any point of the parabola gives the velocity at that instant.
- For uniformly accelerated motion ($a \neq 0$), v–t graph is a straight line whose slope gives the acceleration of the particle.
- In general, the slope of tangent in x–t graph is velocity and the slope of tangent in v–t graph is the acceleration.
- The area under a–t graph gives the change in velocity.
- The area between the v–t graph gives the distance travelled by the particle, if we take all areas as positive.
- Area under v–t graph gives displacement, if areas below the t–axis are taken negative.

Solved Example

Example 18. For a particle moving along x-axis, velocity-time graph is as shown in figure. Find the distance travelled and displacement of the particle?



Solution : Distance travelled = Area under v-t graph (taking all areas as +ve.)

Distance travelled = Area of trapezium + Area of triangle

$= \frac{1}{2}(2+6)\times 8 + \frac{1}{2}\times 4\times 5 = 32 + 10 = 42$ m

Displacement = Area under v-t graph (taking areas below time axis as –ve.)

Displacement = Area of trapezium − Area of triangle

$= \frac{1}{2}(2+6)\times 8 - \frac{1}{2}\times 4\times 5 = 32 - 10 = 22$ m

Hence, distance travelled = 42 m and displacement = 22 m.

13. MOTION WITH NON-UNIFORM ACCELERATION (USE OF DEFINITE INTEGRALS)

$\Delta x = \int_{t_i}^{t_f} v(t)dt$ (displacement in time interval $t = t_i$ to t_f)

The expression on the right hand side is called the definite integral of v(t) between $t = t_i$ and $t = t_f$. Similarly change in velocity

$\Delta v = v_f - v_i = \int_{t_i}^{t_f} a(t)dt$

13.1 Solving Problems which Involves Non uniform Acceleration

(i) Acceleration depending on velocity v or time t

By definition of acceleration, we have $a = \frac{dv}{dt}$. If a is in terms of t,

$\int_{v_0}^{v} dv = \int_0^t a(t)dt$. If a is in terms of v, $\int_{v_0}^{v} \frac{dv}{a(v)} = \int_0^t dt$.

On integrating, we get a relation between v and t, and then

using $\int_{x_0}^{x} dx = \int_0^t v(t)dt$, x and t can also be related.

(ii) Acceleration depending on velocity v or position x

$a = \frac{dv}{dt} \Rightarrow a = \frac{dv}{dx}\frac{dx}{dt} \Rightarrow a = \frac{dx}{dt}\frac{dv}{dx} \Rightarrow a = v\frac{dv}{dx}$

This is another important expression for acceleration. If a is in terms of x,

$\int_{v_0}^{v} v dv = \int_{x_0}^{x} a(x)dx$.

If a is in terms of v,

On integrating, we get a relation between x and v.

Using $\int_{x_0}^{x} \frac{dx}{v(x)} = \int_0^t dt$, we can relate x and t.

Solved Examples

Example 19. An object starts from rest at t = 0 and accelerates at a rate given by a = 6t. What is
(a) its velocity and (b) its displacement at any time t?

Solution : As acceleration is given as a function of time,

$$\therefore \quad \int_{v(t_0)}^{v(t)} dv = \int_{t_0}^{t} a(t)dt$$

Here $t_0 = 0$ and $v(t_0) = 0$ $\quad\therefore\quad v(t) = \int_0^t 6t\,dt = 6\left(\frac{t^2}{2}\right)\Big|_0^t = 6\left(\frac{t^2}{2} - 0\right) = 3t^2$

So, $v(t) = 3t^2$

As $\Delta x = \int_{t_0}^{t} v(t)dt \quad\therefore\quad \Delta x = \int_0^t 3t^2 dt = 3\left(\frac{t^3}{3}\right)\Big|_0^t = 3\left(\frac{t^3}{3} - 0\right) = t^3$

Hence, velocity $v(t) = 3t^2$ and displacement $\Delta x = t^3$.

Example 20. For a particle moving along v + x-axis, acceleration is given as a = x. Find the position as a function of time? Given that at t = 0, x = 1 v = 1.

Solution : $a = x \quad\Rightarrow\quad \frac{vdv}{dx} = x \quad\Rightarrow\quad \frac{v^2}{2} = \frac{x^2}{2} + C$

t = 0, x = 1 and v = 1

$\therefore C = 0 \quad\Rightarrow\quad v^2 = x^2$

$v = \pm x$ but given that x = 1 when v = 1

$\therefore \quad v = x \quad\Rightarrow\quad \frac{dx}{dt} = x \quad\Rightarrow\quad \frac{dx}{x} = dt$

$\ell nx = t + C \quad\Rightarrow\quad 0 = 0 + C \quad\Rightarrow\quad \ell nx = t$

$x = e^t$

Example 21. For a particle moving along x-axis, acceleration is given as a = v. Find the position as a function of time ?
Given that at t = 0 , x = 0 v = 1.

Solution : $a = v \quad\Rightarrow\quad \frac{dv}{dt} = v \quad\Rightarrow\quad \int\frac{dv}{v} = \int dt$

$\ell nv = t + c \quad\Rightarrow\quad 0 = 0 + c$

$v = e^t \quad\Rightarrow\quad \frac{dx}{dt} = e^t \quad\Rightarrow\quad \int dx = \int e^t dt$

$\Rightarrow\quad x = e^t + c \quad\Rightarrow\quad 0 = 1 + c$

$x = e^t - 1$

Miscellaneous Solved Problems

Problem 1. A particle covers $\frac{3}{4}$ of total distance with speed v_1 and next $\frac{1}{4}$ with v_2. Find the average speed of the particle?

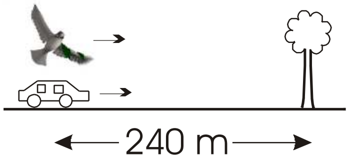
Answer : $\frac{4v_1v_2}{v_1 + 3v_2}$

Solution : Let the total distance be s

average speed $(< v >) = \frac{\text{Total distance}}{\text{Total time taken}}$

$$< v > = \frac{s}{\frac{3s}{4v_1} + \frac{s}{4v_2}} = \frac{1}{\frac{3}{4v_1} + \frac{1}{4v_2}} = \frac{4v_1v_2}{v_1 + 3v_2}$$

Problem 2. A car is moving with speed 60 Km/h and a bird is moving with speed 90 km/h along the same direction as shown in figure. Find the distance travelled by the bird till the time car reaches the tree?



Answer : 360 m

Solution : Time taken by a car to reaches the tree (t) = $\frac{240 \text{ m}}{60 \text{ km/hr}} = \frac{0.24}{60}$hr

Now, the distance travelled by the bird during this time interval (s)

$= 90 \times \frac{0.24}{60} = 0.12 \times 3$ km = 360 m.

Problem 3 The position of a particle moving on X-axis is given by $x = At^3 + Bt^2 + Ct + D$. The numerical values of A, B, C, D are 1, 4, –2 and 5 respectively and SI units are used. Find (a) the dimensions of A, B, C and D, (b) the velocity of the particle at t = 4 s, (c) the acceleration of the particle at t = 4s, (d) the average velocity during the interval t =0 to t = 4s, (e) the average acceleration during the interval t = 0 to t = 4 s.

Answer : **(a)** $[A] = [LT^{-3}]$, $[B] = [LT^{-2}]$, $[C] = [LT^{-1}]$ and $[D] = [L]$; **(b)** 78 m/s ; **(c)** 32 m/s^2 ; **(d)** 30 m/s ; **(e)** 20 m/s^2

Solution : As $x = At^3 + Dt^2 + Ct + D$

(a) Dimensions of A, B, C and D,

$[At^3] = [x]$ (by principle of homogeneity)

$[A] = [LT^{-3}]$

similarly, $[B] = [LT^{-2}]$, $[C] = [LT^{-1}]$ and $[D] = [L]$;

(b) As $v = \frac{dx}{dt} = 3At^2 + 2Bt + C$

velocity at t = 4 sec.

$v = 3(1)(4)^2 + 2(4)(4) - 2 = 78$ m/s.

(c) Acceleration $(a) = \frac{dv}{dt} = 6At + 2B$; $a = 32$ m/s^2

(d) average velocity as $x = At^3 + Bt^2 + Ct = D$

position at t = 0, is x = D = 5m.

Position at t = 4 sec is (1)(64) + (4)(16) – (2) (4) + 5 = 125 m

Thus the displacement during 0 to 4 sec. is 125 – 5 = 120 m

$\therefore \langle v \rangle = 120 / 4 = 30$ m/s

(e) $v = 3At^2 + 20\,t + C$, velocity at t = 0 is c = – 2 m/s

velocity at t = 4 sec is 78 m/s $\therefore \langle a \rangle = \frac{v_2 - v_1}{t_2 - t_1} = 20$ m/s^2

Problem 4. For a particle moving along x-axis, velocity is given as a function of time as $v = 2t^2 + \sin t$. At t = 0, particle is at origin. Find the position as a function of time?

Solution : $v = 2t^2 + \sin t \quad \Rightarrow \quad \frac{dx}{dt} = 2t^2 + \sin t$

$\int_0^x dx = \int_0^t (2t^2 + \sin t)\,dt = \; x = \frac{2}{3}t^3 - \cos t + 1$ **Ans.**

Problem 5. A car decelerates from a speed of 20 m/s to rest in a distance of 100 m. What was its acceleration, assumed constant?

Solution : v = 0 u = 20 m/s s = 100 m $\Rightarrow$ as $v^2 = u^2 + 2as$

$0 = 400 + 2a \times 100 \quad \Rightarrow \quad a = -2$ m/s

$\therefore$ Acceleration = 2 m/s^2 **Ans.**

Problem 6. A 150 m long train accelerates uniformly from rest. If the front of the train passes a railway worker 50 m away from the station at a speed of 25 m/s, what will be the speed of the back part of the train as it passes the worker?

Solution : $v^2 = u^2 + 2as$

$25 \times 25 = 0 + 100\,a$

$a = \frac{25}{4}$ m/s^2

Now, for time taken by the back end of the train to pass the worker

we have $v'^2 = v^2 + 2al = (25)^2 + 2 \times \frac{25}{48} \times 150$

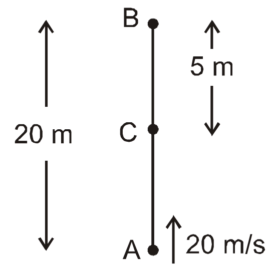
$v'^2 = 25 \times 25 \times 4$

$v' =$ **50 m/s.** **Ans.**

Problem 7. A particle is thrown vertically with velocity 20 m/s. Find (a) the distance travelled by the particle in first 3 seconds, (b) displacement of the particle in 3 seconds.

Answer : 25m, 15m

Solution : Highest point say B



$V_B = 0$

$v = u + gt$

$0 = 20 - 10\,t$

$t = 2$ sec.

$\therefore$ distance travel in first 2 seconds.

$s = s(t = 0 \text{ to } 2\text{sec}) + s\,(2\text{sec. to } 3\text{sec.})$

$s = [ut + 1/2\,at^2]_{t=0 \text{ to } t=2s} + [ut + 1/2at^2]_{t=2 \text{ to } t=3s}$

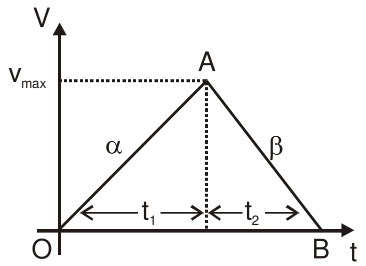
$s = 20 \times 2 - 1/2 \times 10 \times 4 + 1/2 \times 10 \times 1^2$

$= (40 - 20) + 5 =$ **25 m.**

and displacement $= 20 - 5 =$ **15 m.**

Problem 8. A car accelerates from rest at a constant rate α for some time after which it decelerates at a constant rate β to come to rest. If the total time elapsed is t. Find the maximum velocity acquired by the car.

Solution : $t = t_1 + t_2$



slope of OA curve $= \tan\theta = \alpha = \frac{v_{max}}{t_1}$

slope of AB curve $= \beta = \frac{v_{max}}{t_2}$

$t = t_1 + t_2$

$\Rightarrow\ t = \frac{v_{max}}{\alpha} + \frac{v_{max}}{\beta} \quad \Rightarrow \quad v_{max} = \left(\frac{\alpha\ \beta}{\alpha + \beta}\right)t$

Problem 9. In the above question find total distance travelled by the car in time 't'.

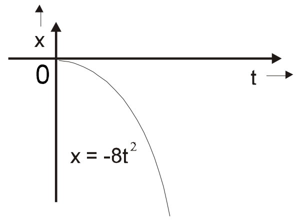
Solution : $v_{max} = \frac{\alpha\beta}{(\alpha+\beta)}t \quad \Rightarrow \quad t_1 = \frac{v_{max}}{\alpha} = \frac{\beta t}{(\alpha+\beta)} \quad \Rightarrow \quad t_2 = \frac{v_{max}}{\beta} = \frac{\alpha t}{(\alpha+\beta)}$

$\therefore$ Total distance travelled by the car in time 't'

$= \frac{1}{2}\alpha t_1^2 + v_{max}\,t_2 - \frac{1}{2}\beta t_2^2 = \frac{1}{2}\frac{\alpha\beta^2 t^2}{(\alpha+\beta)^2} + \frac{\alpha^2\beta t^2}{(\alpha+\beta)^2} - \frac{1}{2}\frac{\beta\alpha^2 t^2}{(\alpha+\beta)^2}$

Area under graph (directly) $= \frac{1}{2}\frac{\alpha\beta t^2}{(\alpha+\beta)} = \frac{\alpha\beta t^2}{2(\alpha+\beta)}$ **Ans.**

Problem 10. The displacement vs time graph of a particle moving along a straight line is shown in the figure. Draw velocity vs time and acceleration vs time graph.



Upwards direction is taken as positive, downwards direction is taken as negative.

Solution : (a) The equation of motion is : $x = -8t^2$

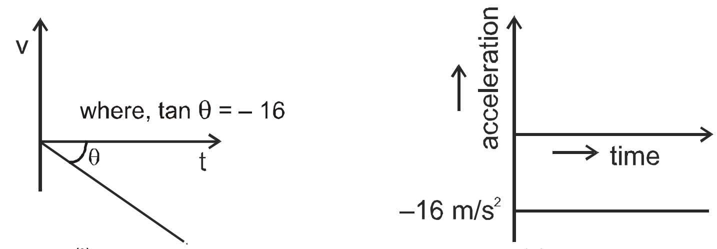
$\therefore$ $v = \frac{dx}{dt} = -16t$; this shows that velocity is directly proportional to time and slope of velocity-time curve is negative i.e. – 16.

Hence, resulting graph is (i)

(b) Acceleration of particle is : $a = \frac{dv}{dt} = -16$.

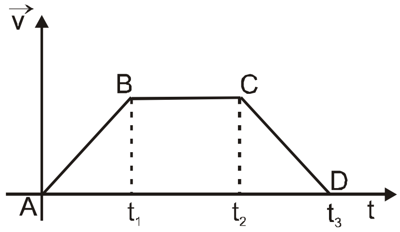
This shows that acceleration is constant but negative.

Resulting graph is (ii)



(i) (ii)

Problem 11. Draw displacement–time and acceleration–time graph for the given velocity–time graph.



Solution : **Part AB :** v-t curve shows constant slope

i.e., constant acceleration or Velocity increases at constant rate with time.

Hence, s-t curve will show constant increase in slope

and a-t curve will be a straight line.

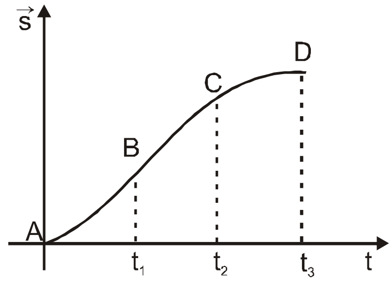
Part BC : v-t curve shows zero slope i.e. constant velocity. So, s-t curve will show constant slope and acceleration will be zero.

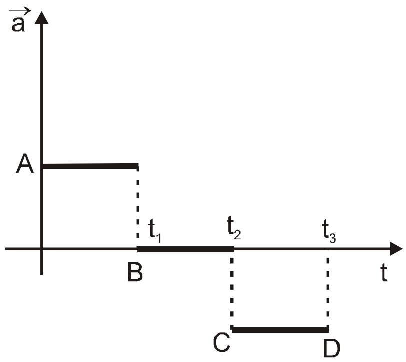
Part CD : v-t curve shows negative slope i.e. velocity is decreasing with time or acceleration is negative.

Hence, s-t curve will show decrease in slope becoming zero in the end.

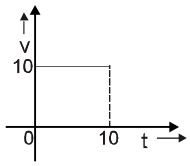
and a-t curve will be a straight line with negative intercept on y-axis.

RESULTING GRAPHS ARE :

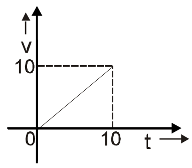




Problem 12. For a particle moving along x-axis, following graphs are given. Find the distance travelled by the particle in 10 s in each case?



(a)



(b)

Solution : **(a)** Distance area under the v - t curve

$\therefore$ Distance = 10 × 10 = 100 m **Ans.**

(b) Area under v – t curve

$\therefore$ Distance = $\frac{1}{2}$ × 10 × 10 = 50 m **Ans.**

Problem 13. For a particle moving along x-axis, acceleration is given as $a = 2v^2$. If the speed of the particle is v_0 at x = 0, find speed as a function of x.

Solution : $a = 2v^2 \Rightarrow$ or $\frac{dv}{dt} = 2v^2$ or $\frac{dv}{dx} \times \frac{dx}{dt} = 2v^2$

$v\frac{dv}{dx} = 2v^2 \Rightarrow \frac{dv}{dx} = 2v$

$\int_{v_0}^{v} \frac{dv}{v} = \int_{0}^{x} 2\ dx \Rightarrow \left[\ell nv\right]_{v_0}^{v} = \left[2x\right]_0^x$

$\ell n\frac{v}{v_0} = 2x \Rightarrow v = v_0e^{2x}$ **Ans.**

PROJECTILE MOTION

1. BASIC CONCEPT :

1.1 Projectile

Any object that is given an initial velocity obliquely, and that subsequently follows a path determined by the net constant force, (In this chapter constant force is gravitational force) acting on it is called a projectile.

Examples of projectile motion :

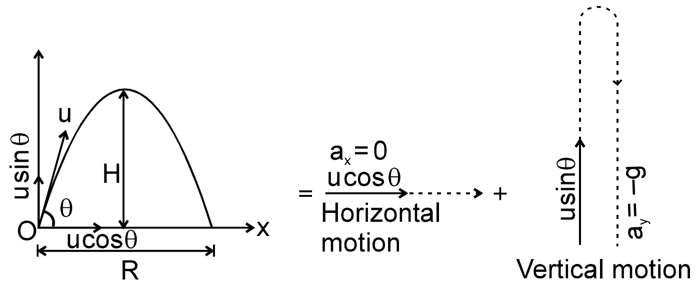
- A cricket ball hit by the batsman for a six
- A bullet fired from a gun.
- A packet dropped from a plane; but the motion of the aeroplane itself is not projectile motion because there are forces other than gravity acting on it due to the thrust of its engine.

1.2 Assumptions of Projectile Motion :

- We shall consider only trajectories that are of sufficiently short range so that the gravitational force can be considered constant in both magnitude and direction.
- All effects of air resistance will be ignored.
- Earth is assumed to be flat.

1.3 Projectile Motion :

- The motion of projectile is known as projectile motion.
- It is an example of two dimensional motion with constant acceleration.
- Projectile motion is considered as combination of two simultaneous motions in mutually perpendicular directions which are completely independent from each other i.e. horizontal motion and vertical motion.

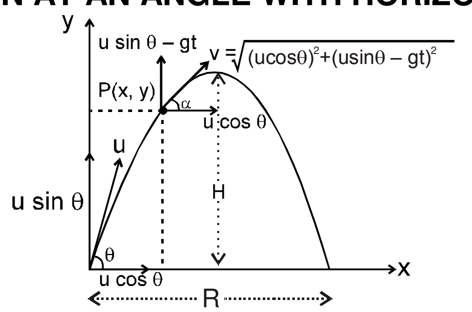


Parabolic path = vertical motion + horizontal motion.

Galileo's Statement :

Two perpendicular directions of motion are independent from each other. In other words any vector quantity directed along a direction remains unaffected by a vector perpendicular to it.

2. PROJECTILE THROWN AT AN ANGLE WITH HORIZONTAL



- Consider a projectile thrown with a velocity u making an angle θ with the horizontal.
- Initial velocity u is resolved in components in a coordinate system in which horizontal direction is taken as x-axis, vertical direction as y-axis and point of projection as origin.

$u_x = u\cos\theta \qquad\qquad u_y = u\sin\theta$

- Again this projectile motion can be considered as the combination of horizontal and vertical motion. Therefore,

	Horizontal direction	Vertical direction
(a)	Initial velocity $u_x = u\cos\theta$	Initial velocity $u_y = u\sin\theta$
(b)	Acceleration $a_x = 0$	Acceleration $a_y = g$
(c)	Velocity after time t, $v_x = u\cos\theta$	Velocity after time t, $v_y = u\sin\theta - gt$

2.1 Time of flight :

The displacement along vertical direction is zero for the complete flight.
Hence, along vertical direction net displacement = 0

$\Rightarrow (u\sin\theta)\,T - \frac{1}{2}gT^2 = 0 \quad \Rightarrow \quad T = \frac{2u\sin\theta}{g}$

2.2 Horizontal range :

$R = u_x.T \quad \Rightarrow \quad R = u\cos\theta.\frac{2u\sin\theta}{g}$

$R = \frac{u^2\sin 2\theta}{g}$

2.3 Maximum height :

At the highest point of its trajectory, particle moves horizontally, and hence vertical component of velocity is zero.
Using 3rd equation of motion i.e. $v^2 = u^2 + 2as$
we have for vertical direction

$0 = u^2\sin^2\theta - 2gH \quad \Rightarrow \quad H = \frac{u^2\sin^2\theta}{2g}$

2.4 Resultant velocity :

$\vec{v} = v_x\hat{i} + v_y\hat{j} = u\cos\theta\,\hat{i} + (u\sin\theta - gt)\,\hat{j}$

Where, $|\vec{v}| = \sqrt{u^2\cos^2\theta + (u\sin\theta - gt)^2}$ and $\tan\alpha = v_y / v_x$.

Also, $v\cos\alpha = u\cos\theta \quad \Rightarrow \quad v = \frac{u\cos\theta}{\cos\alpha}$

Note :
- Results of article 2.1, 2.2, and 2.3 are valid only if projectile lands at same horizontal level from which it was projected.
- Vertical component of velocity is positive when particle is moving up and vertical component of velocity is negative when particle is coming down if vertical upwards direction is taken as positive.

2.5 General result :

- For maximum range $\theta = 45º$

  $R_{max} = \frac{u^2}{g} \quad \Rightarrow \quad H_{max} = \frac{R_{max}}{2}$

- We get the same range for two angle of projections α and $(90 - \alpha)$ but in both cases, maximum heights attained by the particles are different.

  This is because, $R = \frac{u^2\sin 2\theta}{g}$, and $\sin 2(90 - \alpha) = \sin 180 - 2\alpha = \sin 2\alpha$

- If R = H

  i.e. $\frac{u^2\sin 2\theta}{g} = \frac{u^2\sin^2\theta}{2g} \quad \Rightarrow \quad \tan\theta = 4$

- Range can also be expressed as $R = \frac{u^2\sin 2\theta}{g} = \frac{2u\sin\theta . u\cos\theta}{g} = \frac{2u_xu_y}{g}$

Solved Examples

Example 1. A body is projected with a speed of 30 ms^{-1} at an angle of 30º with the vertical. Find the maximum height, time of flight and the horizontal range of the motion. [Take g = 10 m/s^2]

Solution : Here u = 30 ms^{-1}, Angle of projection, $\theta = 90 - 30 = 60º$

Maximum height, $H = \frac{u^2 \sin^2\theta}{2g} = \frac{30^2 \sin^2 60º}{2\times 10} = \frac{900}{20}\times\frac{3}{4} = \frac{135}{4}$ m

Time of flight, $T = \frac{2u\sin\theta}{g} = \frac{2\times 30\times \sin 60^0}{10} = 3\sqrt{3}$ sec.

Horizontal range = $R = \frac{u^2\sin 2\theta}{g} = \frac{30\times 30\times 2\sin 60º\cos 60º}{10} = 45\sqrt{3}$ m

Example 2. A projectile is thrown with a speed of 100 m/s making an angle of 60° with the horizontal. Find the minimum time after which its inclination with the horizontal is 45° ?

Solution : $u_x = 100 \times \cos 60° = 50$

$u_y = 100 \times \sin 60° = 50\sqrt{3}$

$v_y = u_y + a_y t = 50\sqrt{3} - gt$ and $v_x = u_x = 50$

When angle is 45°,

$\tan 45^0 = \frac{v_y}{v_x} \quad \Rightarrow \quad v_y = v_x$

$\Rightarrow \quad 50 - gt\sqrt{3} = 50 \quad \Rightarrow \quad 50(\sqrt{3}-1) = gt \quad \Rightarrow \quad t = 5(\sqrt{3}-1)$ s

Example 3. A large number of bullets are fired in all directions with the same speed v. What is the maximum area on the ground on which these bullets will spread ?

Solution : Maximum distance up to which a bullet can be fired is its maximum range, therefore

$R_{max} = \frac{v^2}{g}$

Maximum area = $\pi(R_{max})^2 = \frac{\pi v^4}{g^2}$.

Example 4. The velocity of projection of a projectile is given by : $\vec{u} = 5\hat{i} + 10\hat{j}$. Find

(a) Time of flight, (b) Maximum height, (c) Range

Solution : We have $u_x = 5$ $u_y = 10$

(a) Time of flight = $\frac{2u\sin\theta}{g} = \frac{2u_y}{g} = \frac{2\times 10}{10} = 2$ s

(b) Maximum height = $\frac{u^2\sin^2\theta}{2g} = \frac{u_y^{\,2}}{2g} = \frac{10\times 10}{2\times 10} = 5$ m

(c) Range = $\frac{2u\sin\theta . u\cos\theta}{g} = \frac{2u_x u_y}{g} = \frac{2\times 10\times 5}{10} = 10$ m

Example 5. A particle is projected at an angle of 30° w.r.t. horizontal with speed 20 m/s :

(i) Find the position vector of the particle after 1s.

(ii) Find the angle between velocity vector and position vector at t = 1s.

Solution : (i) $x = u\cos\theta\, t = 20 \times \frac{\sqrt{3}}{2} \times t = 10\sqrt{3}$ m

$y = u\sin\theta t - \frac{1}{2} \times 10 \times t^2 = 20 \times \frac{1}{2} \times (1) - 5(1)^2 = 5$m

Position vector, $\vec{r} = 10\sqrt{3}\,\hat{i} + 5\hat{j}$, $|\vec{r}| = \sqrt{(10\sqrt{3})^2 + 5^2}$

(ii) $v_x = 10\sqrt{3}\ \hat{i}$

$v_y = u_y + a_y t = 10 - g\,t = 0$

$\therefore\quad \vec{v} = 10\sqrt{3}\ \hat{i}\ ,\ |\vec{v}| = 10\sqrt{3}$

$\vec{v}\cdot\vec{r} = (10\sqrt{3}\hat{i})\cdot(10\sqrt{3}\hat{i}+5\hat{j}) = 300$

$\vec{v}\ .\ \vec{r} = |\vec{v}|\,|\vec{r}|\cos\theta$

$\Rightarrow\ \cos\theta = \dfrac{\vec{v}\cdot\vec{r}}{|\vec{v}|\,|\vec{r}|} = \dfrac{300}{10\sqrt{3}\sqrt{325}}\quad\Rightarrow\quad \theta = \cos^{-1}\left(2\sqrt{\dfrac{3}{13}}\right)$

3. EQUATION OF TRAJECTORY

The path followed by a particle (here projectile) during its motion is called its **Trajectory**. Equation of trajectory is the relation between instantaneous coordinates (Here x & y coordinate) of the particle.

If we consider the horizontal direction,

$x = u_x.t$

$x = u\cos\theta.\ t$ .....(1)

For vertical direction :

$y = u_y\ .\ t - 1/2\ gt^2$

$= u\sin\theta\ .\ t - 1/2\ gt^2$ .....(2)

Eliminating ‘t’ from equation (1) & (2)

$$y = u\sin\theta\ .\ \frac{x}{u\cos\theta} - \frac{1}{2}\,g\left(\frac{x}{u\cos\theta}\right)^2 \Rightarrow \quad y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$$

This is an equation of parabola called as trajectory equation of projectile motion.

Other forms of trajectory equation :

- $y = x\tan\theta - \dfrac{gx^2(1+\tan^2\theta)}{2u^2}$
- $y = x\tan\theta - \dfrac{gx^2}{2u^2\cos^2\theta}$

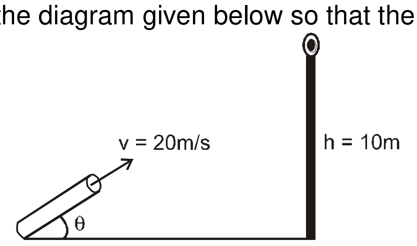
$\Rightarrow\ y = x\tan\theta\left[1 - \dfrac{gx}{2u^2\cos^2\theta\tan\theta}\right]$

$\Rightarrow\ y = x\tan\theta\left[1 - \dfrac{gx}{2u^2\sin\theta\cos\theta}\right]$

$\Rightarrow\ y = x\tan\theta\left[1 - \dfrac{x}{R}\right]$

Solved Examples

Example 6. Find the value of θ in the diagram given below so that the projectile can hit the target.



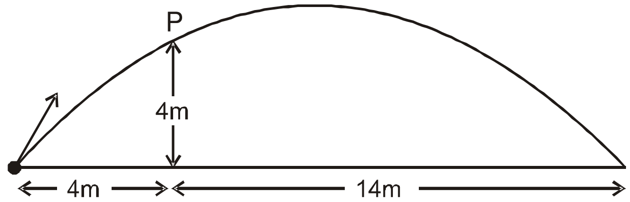
Solution. $y = x\tan\theta - \dfrac{gx^2(1+\tan^2\theta)}{2u^2}\qquad \Rightarrow 10 = 20\tan\theta - \dfrac{5\times(20)^2}{(20)^2}\ (1+\tan^2\theta)$

$\Rightarrow\ 2 = 4\tan\theta - (1+\tan^2\theta)\qquad \Rightarrow \tan^2\theta - 4\tan\theta + 3 = 0$

$\Rightarrow (\tan\theta - 3)(\tan\theta - 1) = 0\qquad \Rightarrow \tan\theta = 3, 1\quad \Rightarrow\ \theta = 45°, \tan^{-1}(3)$

Example 7. A ball is thrown from ground level so as to just clear a wall 4 m high at a distance of 4 m and falls at a distance of 14 m from the wall. Find the magnitude and direction of initial velocity of the ball figure is given below.

Solution.



The ball passes through the point P(4, 4). Also range = 4 + 14 = 18 m.

The trajectory of the the ball is, $y = x \tan\theta \left(1 - \frac{x}{R}\right)$

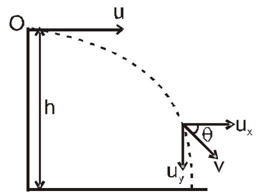
Now x = 4m, y = 4m and R = 18 m

$\therefore \quad 4 = 4\tan\theta\left[1 - \frac{4}{18}\right] = 4\tan\theta.\ \frac{7}{9}$ or $\tan\theta = \frac{9}{7}$ $\Rightarrow$ $\theta = \tan^{-1}\frac{9}{7}$

And $R = \frac{2u^2\sin\theta\cos\theta}{g}$ or $18 = \frac{2}{9.8} \times u^2 \times \frac{9}{\sqrt{130}} \times \frac{7}{\sqrt{130}} \Rightarrow u = \sqrt{182}$

4. PROJECTILE THROWN PARALLEL TO THE HORIZONTAL FROM SOME HEIGHT

Consider a projectile thrown from point O at some height h from the ground with a velocity u. Now we shall study the characteristics of projectile motion by resolving the motion along horizontal and vertical directions.



Horizontal direction	Vertical direction
(i) Initial velocity $u_x = u$	Initial velocity $u_y = 0$
(ii) Acceleration $a_x = 0$	Acceleration $a_y = g$ (downward)

4.1 Time of flight :

This is equal to the time taken by the projectile to return to ground. From equation of motion

$S = ut + \frac{1}{2}at^2$, along vertical direction, we get

$-h = u_y t + \frac{1}{2}(-g)t^2 \quad \Rightarrow \quad h = \frac{1}{2}gt^2 \quad \Rightarrow \quad t = \sqrt{\frac{\mathbf{2h}}{\mathbf{g}}}$

4.2 Horizontal range :

Distance covered by the projectile along the horizontal direction between the point of projection to the point on the ground.

$R = u_x \cdot t \quad \Rightarrow \quad R = u\sqrt{\frac{2h}{g}}$

4.3 Velocity at a general point P(x, y) :

$v = \sqrt{u_x^2 + u_y^2}$

Here horizontal velocity of the projectile after time t

$v_x = u$

velocity of projectile in vertical direction after time t

$v_y = 0 + (-g)t = -gt = gt$ (downward)

$\therefore \quad v = \sqrt{u^2 + g^2t^2}$ and $\tan\theta = v_y/v_x$

4.4 Velocity with which the projectile hits the ground :

$V_x = u$

$V_y^2 = 0^2 - 2g(-h)$

$V_y = \sqrt{2gh}$

$V = \sqrt{V_x^2 + V_y^2} \qquad \Rightarrow V = \sqrt{\mathbf{u^2 + 2gh}}$

4.5 Trajectory equation :

The path traced by projectile is called the trajectory.

After time t,

$x = ut$ .....(1)

$y = \frac{-1}{2} gt^2$ .....(2)

From equation (1)

$t = x/u$

Put the value of t in equation (2)

$y = \frac{-1}{2} g. \frac{\mathbf{x^2}}{\mathbf{u^2}}$

This is trajectory equation of the particle projected horizontally from some height.

Solved Examples

Examples based on horizontal projection from some height :

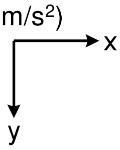
Example 8. A projectile is fired horizontally with a speed of 98 ms^{-1} from the top of a hill 490 m high. Find

(i) the time taken to reach the ground

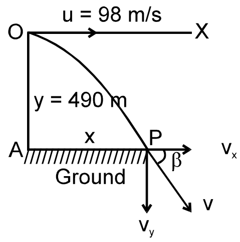
(ii) the distance of the target from the hill and

(iii) the velocity with which the projectile hits the ground. (take g = 9.8 m/s^2)

Solution : (i) The projectile is fired from the top O of a hill with speed u = 98 ms^{-1} along the horizontal as shown as OX. It reaches the target P at vertical depth OA, in the coordinate system as shown,



OA = y = 490 m



As, $y = \frac{1}{2} gt^2$

$\therefore \quad 490 = \frac{1}{2} \times 9.8\, t^2$

or $\quad t = \sqrt{100} = 10$ s.

(ii) Distance of the target from the hill is given by, AP = x = Horizontal velocity × time = 98 × 10 = 980 m.

(iii) The horizontal and vertical components of velocity v of the projectile at point P are

$v_x = u = 98\ ms^{-1}$

$v_y = u_y + gt = 0 + 9.8 \times 10 = 98\ ms^{-1}$

$V = \sqrt{v_x^2 + v_y^2} = \sqrt{98^2 + 98^2} = 98\sqrt{2}\ ms^{-1}$

Now if the resultant velocity v makes an angle β with the horizontal, then

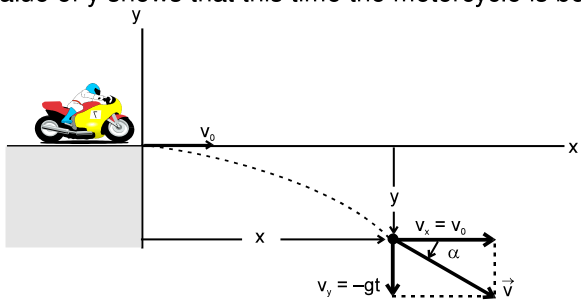
$\tan\beta = \frac{v_y}{v_x} = \frac{98}{98} = 1 \qquad \therefore \qquad \beta = 45º$

Example 9. A motorcycle stunt rider rides off the edge of a cliff. Just at the edge his velocity is horizontal, with magnitude 9.0 m/s. Find the motorcycle's position, distance from the edge of the cliff and velocity after 0.5 s.

Solution : At t = 0.50 s, the x and y-coordinates are $x = v_0 t$ = (9.0 m/s) (0.50 s) = 4.5 m

$y = -\frac{1}{2} gt^2 = -\frac{1}{2}\ (10\ m/s^2)\ (0.50\ s)^2 = -\frac{5}{4}$ m

The negative value of y shows that this time the motorcycle is below its starting point.



The motorcycle's distance from the origin at this time $r = \sqrt{x^2 + y^2} = \sqrt{\left(\frac{9}{2}\right)^2 + \left(\frac{5}{4}\right)^2} = \frac{\sqrt{349}}{4}$ m.

The components of velocity at this time are $v_x = v_0 = 9.0$ m/s

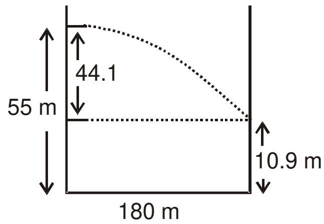
$v_y = -gt = (-10\ \text{m/s}^2)\ (0.50\ \text{s}) = -5$ m/s.

The speed (magnitude of the velocity) at this time is

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(9.0\,\text{m/s})^2 + (-5\,\text{m/s})^2} = \sqrt{106}\ \text{m/s}$$

Example 10. An object is thrown between two tall buildings 180 m from each other. The object is thrown horizontally from a window 55 m above ground from one building through a window 10.9 m above ground in the other building. Find out the speed of projection. (Use g = 9.8 m/s²)

Solution : $t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 44.1}{9.8}}$



t = 3 sec.

R = uT

$\frac{180}{3} = u$; u = 60 m/s

5. PROJECTION FROM A TOWER

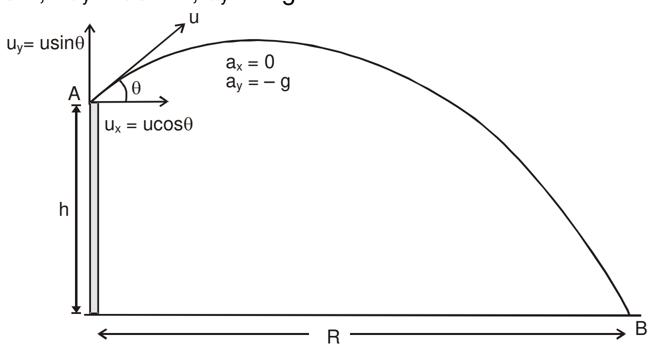
Case (i) : Horizontal projection

$u_x = u$; $u_y = 0$; $a_y = -g$

This is same as previous section (section 4)

Case (ii) : Projection at an angle θ above horizontal

$u_x = u\cos\theta$; $u_y = u\sin\theta$; $a_y = -g$



Equation of motion between A & B (in Y direction)

$S_y = -h$, $u_y = u\sin\theta$, $a_y = -g$, $t = T$

$S_y = u_y t + \frac{1}{2} a_y t^2 \quad \Rightarrow \quad -h = u\sin\theta\, t - \frac{1}{2} g t^2$

Solving this equation we will get time of flight, T.

And range, $R = u_x T = u\cos\theta\, T$; Also, $v_y^2 = u_y^2 + 2a_yS_y = u^2 \sin^2\theta + 2gh$; $v_x = u\cos\theta$

$v_B = \sqrt{v_y^2 + v_x^2} \quad \Rightarrow \quad v_B = \sqrt{u^2 + 2gh}$

Case (iii) : Projection at an angle θ below horizontal

$u_x = u\cos\theta$; $u_y = -u\sin\theta$; $a_y = -g$

$S_y = u_y t + \frac{1}{2} a_y t^2$

$S_y = -h$, $u_y = -u\sin\theta$, $t = T$, $a_y = -g$

$\Rightarrow -h = -u\sin\theta\, T - \frac{1}{2} gT^2 \Rightarrow h = u\sin\theta\, T + \frac{1}{2} gT^2$

Solving this equation we will get time of flight, T.

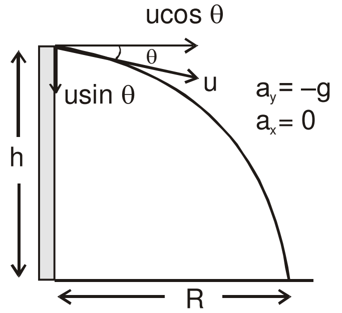
And range, $R = u_x T = u\cos\theta\, T$

$v_x = u\cos\theta$

$v_y^2 = u_y^2 + 2a_yS_y = u^2\sin^2\theta + 2(-g)(-h)$

$v_y^2 = u^2\sin^2\theta + 2gh$

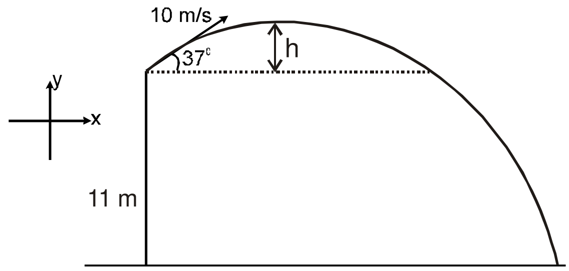
$v_B = \sqrt{v_x^2 + v_y^2} = \sqrt{u^2 + 2gh}$



Note : *objects thrown from same height in different directions with same initial speed will strike the ground with the same final speed. But the time of flight will be different.*

Solved Examples

Example 11. From the top of a 11 m high tower a stone is projected with speed 10 m/s, at an angle of 37º as shown in figure. Find
(a) Speed after 2s
(b) Time of flight.
(c) Horizontal range.
(d) The maximum height attained by the particle.
(e) Speed just before striking the ground.



Solution : (a) Initial velocity in horizontal direction = 10 cos 37 = 8 m/s
Initial velocity in vertical direction = 10 sin 37º = 6 m/s
Speed after 2 seconds

$v = v_x\hat{i} + v_y\hat{j} = 8\hat{i} + (u_y + a_yt)\hat{j} = 8\hat{i} + (6 - 10 \times 2)\hat{j} = 8\hat{i} - 14\hat{j}$

(b) $S_y = u_y t + \frac{1}{2} a_y t^2 \quad \Rightarrow -11 = 6 \times t + \frac{1}{2} \times (-10) t^2$

$5t^2 - 6t - 11 = 0 \quad \Rightarrow (t + 1)(5t - 11) = 0 \quad \Rightarrow t = \frac{11}{5}$ sec.

(c) Range $= 8 \times \frac{11}{5} = \frac{88}{5}$ m

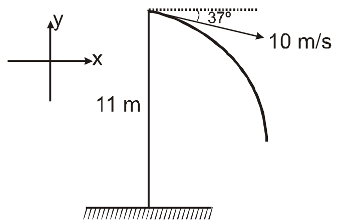
(d) Maximum height above the level of projection, $h = \frac{u_y^2}{2g} = \frac{6^2}{2 \times 10} = 1.8$ m

$\therefore$ maximum height above ground = 11 + 1.8 = 12.8 m

(e) $v = \sqrt{u^2 + 2gh} = \sqrt{100 + 2 \times 10 \times 11}$

$\Rightarrow v = 8\sqrt{5}$ m/s

Example 12. From the top of a 11 m high tower a stone is projected with speed 10 m/s, at an angle of 37º as shown in figure. Find
(a) Time of flight.
(b) Horizontal range.
(c) Speed just before striking the ground.



Solution : $u_x = 10 \cos 37^\circ = 8$ m/s , $u_y = -10 \sin 37^\circ = -6$ m/s

(a) $S_y = u_y t + \frac{1}{2} a_y t^2 \Rightarrow -11 = -6 \times t + \frac{1}{2} \times (-10)\, t^2 \Rightarrow 5t^2 + 6t - 11 = 0$

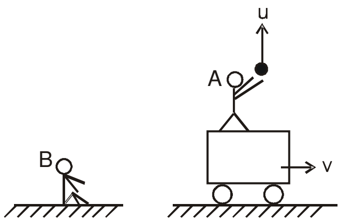
$\Rightarrow (t-1)(5t+11) = 0 \Rightarrow t = 1$ sec

(b) Range = 8 × 1 = 8 m

(c) $v = \sqrt{u^2 + 2gh} = \sqrt{100 + 2 \times 10 \times 11} \Rightarrow v = \sqrt{320}$ m/s $= 8\sqrt{5}$ m/s

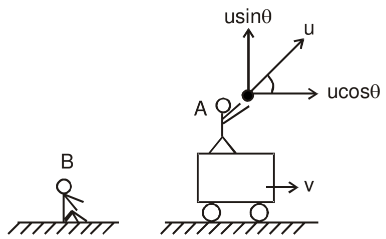
Note : that in Ex.11 and Ex.12, objects thrown from same height in different directions with same initial speed strike the ground with the same final speed, but after different time intervals.

6. PROJECTION FROM A MOVING PLATFORM

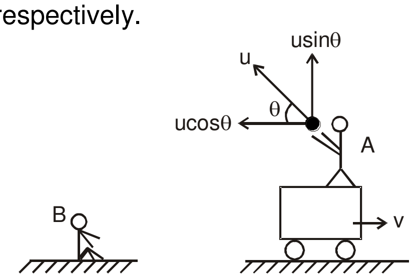


CASE (1) : When a ball is thrown upward from a truck moving with uniform speed, then observer A standing in the truck, will see the ball moving in straight vertical line (upward & downward).
The observer B sitting on road, will see the ball moving in a parabolic path. The horizontal speed of the ball is equal to the speed of the truck.

CASE (2) : When a ball is thrown at some angle 'θ' in the direction of motion of the truck, horizontal & vertical component of ball's velocity w.r.t. observer A standing on the truck, is $u\cos\theta$, and $u\sin\theta$ respectively.
Horizontal & vertical component of ball's velocity w.r.t. observer B sitting on the ground, is
$u_x = u\cos\theta + v$ and $u_y = u\sin\theta$ respectively.

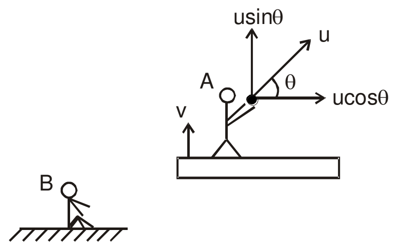


CASE (3) : When a ball is thrown at some angle 'θ' in the opposite direction of motion of the truck, horizontal & vertical component of ball's velocity w.r.t. observer A standing on the truck, is $u\cos\theta$, and $u\sin\theta$ respectively.
Horizontal & vertical component of ball's velocity w.r.t. observer B sitting on the ground, is
$u_x = u\cos\theta - v$ and $u_y = u\sin\theta$ respectively.

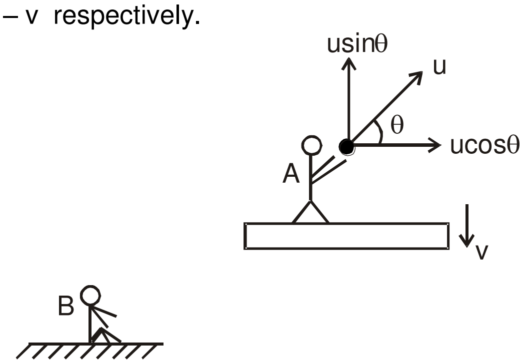


CASE (4) : When a ball is thrown at some angle 'θ' from a platform moving with speed v upwards, horizontal & vertical component of ball's velocity w.r.t. observer A standing on the moving platform, is $u\cos\theta$ and $u\sin\theta$ respectively.

Horizontal & vertical component of ball's velocity w.r.t. observer B sitting on the ground, is $u_x = u\cos\theta$ and $u_y = u\sin\theta + v$ respectively.



CASE (5) : When a ball is thrown at some angle 'θ' from a platform moving with speed v downwards, horizontal & vertical component of ball's velocity w.r.t. observer A standing on the moving platform, is $u\cos\theta$ and $u\sin\theta$ respectively.
Horizontal & vertical component of ball's velocity w.r.t. observer B sitting on the ground, is $u_x = u\cos\theta$ and $u_y = u\sin\theta - v$ respectively.



Solved Examples

Example 13. A boy standing on a long railroad car throws a ball straight upwards. The car is moving on the horizontal road with an acceleration of 1 m/s^2 and the projection speed in the vertical direction is 9.8 m/s. How far behind the boy will the ball fall on the car ?

Solution : Let the initial velocity of car be 'u'.

time of flight, $t = \frac{2u_y}{g} = 2$

where u_y = component of velocity in vertical direction

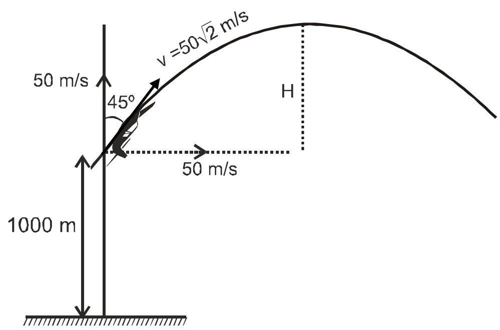
Distance travelled by car $x_c = u \times 2 + \frac{1}{2} \times 1 \times 2^2 = 2u + 2$

distance travelled by ball $x_b = u \times 2$

$x_c - x_b = 2u + 2 - 2u = 2m$ **Ans.**

Example 14. A fighter plane moving with a speed of $50\sqrt{2}$ m/s upward at an angle of 45º with the vertical, releases a bomb. Find
(a) Time of flight (b) Maximum height of the bomb above ground

Solution : (a) $y = u_y t + \frac{1}{2} a_y t^2$



$-1000 = 50t - \frac{1}{2} \times 10 \times t^2$; $t^2 - 10t - 200 = 0$

$(t - 20)(t + 10) = 0$; $t = 20$ sec

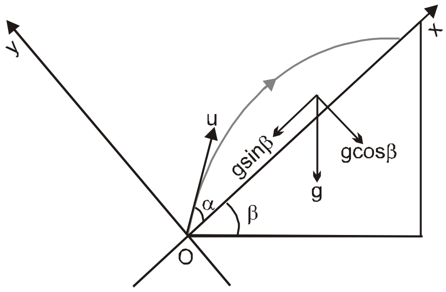
(b) $H = \frac{u_y^2}{2g} = \frac{50^2}{2g} = \frac{50 \times 50}{20} = 125$ m.

Hence maximum height above ground H = 1000 + 125 = 1125 m

7. PROJECTION ON AN INCLINED PLANE

Case (i) : Particle is projected up the incline

Here α is angle of projection w.r.t. the inclined plane. x and y axis are taken along and perpendicular to the incline as shown in the diagram.



In this case: $a_x = -\,g\sin\beta$

$u_x = u\cos\alpha$

$a_y = -\,g\cos\beta$

$u_y = u\sin\alpha$

Time of flight (T) : When the particle strikes the inclined plane y becomes zero

$$y = u_y t + \frac{1}{2} a_y t^2$$

$$\Rightarrow \quad 0 = u\sin\alpha T - \frac{1}{2} g\cos\beta T^2 \quad \Rightarrow \quad T = \frac{2u\sin\alpha}{g\cos\beta} = \frac{2u_\perp}{g_\perp}$$

Where $u_\perp$ and $g_\perp$ are component of u and g perpendicular to the incline.

Maximum height (H) : When half of the time is elapsed y coordinate is equal to maximum distance from the inclined plane of the projectile

$$H = u\sin\alpha\left(\frac{u\sin\alpha}{g\cos\beta}\right) - \frac{1}{2} g\cos\beta\left(\frac{u\sin\alpha}{g\cos\beta}\right)^2 \quad \Rightarrow \quad H = \frac{u^2\sin^2\alpha}{2g\cos\beta} = \frac{u^2{}_\perp}{2g_\perp}$$

Range along the inclined plane (R):

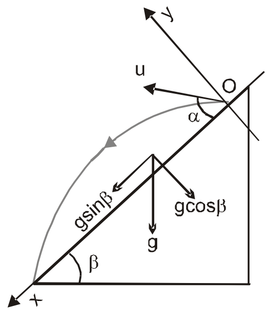
When the particle strikes the inclined plane x coordinate is equal to range of the particle

$$x = u_x t + \frac{1}{2} a_x t^2$$

$$\Rightarrow \quad R = u\cos\alpha\left(\frac{2u\sin\alpha}{g\cos\beta}\right) - \frac{1}{2} g\sin\beta\left(\frac{2u\sin\alpha}{g\cos\beta}\right)^2 \quad \Rightarrow \quad R = \frac{2u^2\sin\alpha\cos(\alpha+\beta)}{g\cos^2\beta}$$

Case (ii) : Particle is projected down the incline

In this case :



$a_x = g\sin\beta$; $u_x = u\cos\alpha$

$a_y = -g\cos\beta$

$u_y = u\sin\alpha$

Time of flight (T) : When the particle strikes the inclined plane y coordinate becomes zero

$$y = u_y t + \frac{1}{2} a_y t^2 \quad \Rightarrow \quad 0 = u\sin\alpha T - \frac{1}{2} g\cos\beta T^2 \quad \Rightarrow \quad T = \frac{2u\sin\alpha}{g\cos\beta} = \frac{2u_\perp}{g_\perp}$$

Maximum height (H) : When half of the time is elapsed y coordinate is equal to maximum height of the projectile

$$H = u\sin\alpha\left(\frac{u\sin\alpha}{g\cos\beta}\right) - \frac{1}{2} g\sin\beta\left(\frac{u\sin\alpha}{g\cos\beta}\right)^2 \quad \Rightarrow \quad H = \frac{u^2\sin^2\alpha}{2g\cos\beta} = \frac{u^2{}_\perp}{2g_\perp}$$

Range along the inclined plane (R): When the particle strikes the inclined plane x coordinate is equal to range of the particle

$x = u_x t + \frac{1}{2} a_x t^2$

$\Rightarrow$ R = $u\cos\alpha \left(\frac{2u\sin\alpha}{g\cos\beta}\right) + \frac{1}{2} g\sin\beta \left(\frac{2u\sin\alpha}{g\cos\beta}\right)^2 \Rightarrow \quad R = \frac{2u^2 \sin\alpha\cos(\alpha-\beta)}{g\cos^2\beta}$

Standard results for projectile motion on an inclined plane

	Up the Incline	Down the Incline
Range	$\frac{2u^2\sin\alpha\cos(\alpha+\beta)}{g\cos^2\beta}$	$\frac{2u^2\sin\alpha\cos(\alpha-\beta)}{g\cos^2\beta}$
Time of flight	$\frac{2u\sin\alpha}{g\cos\beta}$	$\frac{2u\sin\alpha}{g\cos\beta}$
Angle of projection for maximum range	$\frac{\pi}{4} - \frac{\beta}{2}$	$\frac{\pi}{4} + \frac{\beta}{2}$
Maximum Range	$\frac{u^2}{g(1+\sin\beta)}$	$\frac{u^2}{g(1-\sin\beta)}$

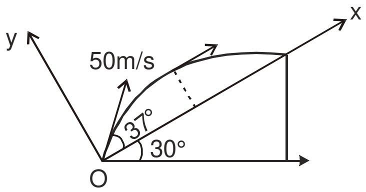
Here α is the angle of projection with the incline and β is the angle of incline.

Note : For a given speed, the direction which gives the maximum range of the projectile on an incline, bisects the angle between the incline and the vertical, for upward or downward projection.

Solved Examples

Example 15. A bullet is fired from the bottom of the inclined plane at angle θ = 37° with the inclined plane. The angle of incline is 30° with the horizontal. Find
(i) The position of the maximum height of the bullet from the inclined plane.
(ii) Time of light
(iii) Horizontal range along the incline.
(iv) For what value of θ will range be maximum.
(v) Maximum range.

Solution : (i) Taking axis system as shown in figure



At highest point $V_y = 0$

$V_y^2 = U_y^2 + 2a_y y$

$0 = (30)^2 - 2g\cos30^\circ y$

$y = 30\sqrt{3}$ (maximum height) ......(1)

(ii) Again for x coordinate $V_y = U_y + a_y t$

$0 = 30 - g\cos30^\circ \times t \Rightarrow t = 2\sqrt{3}$

$T = 2 \times 2\sqrt{3}$ sec Time of flight

(iii) $x = U_x t + \frac{1}{2} a_x t^2$

$x = 40 \times 4\sqrt{3} - \frac{1}{2} g\sin30^\circ \times (4\sqrt{3})^2$

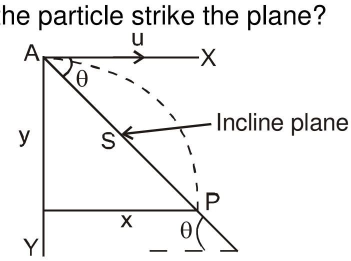
$x = 40\,(4\sqrt{3} - 3)$ m Range

(iv) $\frac{\pi}{4} - \frac{30^\circ}{2} = 45^\circ - 15^\circ = 30^\circ$

(v) $\frac{u^2}{g(1+\sin\beta)} = \frac{50\times50}{10\left(1+\frac{1}{2}\right)} = \frac{2500}{15} = \frac{500}{3}$ m

Example 16. A particle is projected horizontally with a speed u from the top of a plane inclined at an angle θ with the horizontal. How far from the point of projection will the particle strike the plane?

Solution : Take X, Y-axes as shown in figure. Suppose that the particle strikes the plane at a point P with coordinates (x, y). Consider the motion between A and P.



Suppose distance between A and P is S

Then position of P is,

$x = S\cos\theta$

$y = -S\sin\theta$

Using equation of trajectory (For ordinary projectile motion)

$y = x\tan\theta - \dfrac{gx^2}{2u^2\cos^2\theta}$

here $y = -S\sin\theta$

$x = S\cos\theta$

θ = angle of projection with horizontal = 0º

$-S\sin\theta = S\cos\theta\,(0) - \dfrac{g(S\cos\theta)^2}{2u^2}$ $\quad S = \dfrac{2u^2\sin\theta}{g\cos^2\theta}$

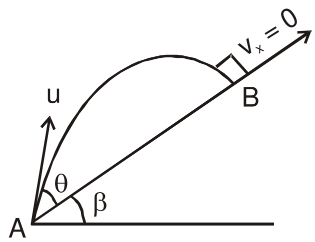
Aliter : $R = \dfrac{2u^2\sin\alpha\cos(\alpha-\beta)}{g\cos^2\beta}$ $\qquad$ Here $\alpha = \beta = \theta$ $\qquad \Rightarrow \qquad R = \dfrac{2u^2\sin\theta}{g\cos^2\theta}$

Example 17. A projectile is thrown at an angle θ with an inclined plane of inclination β as shown in figure. Find the relation between β and θ if :

(a) Projectile strikes the inclined plane perpendicularly, to the inclined plane

(b) Projectile strikes the inclined plane horizontally to the ground

Solution : (a) If projectile strikes perpendicularly.



$v_x = 0$ when projectile strikes

$v_x = u_x + a_x t$

$0 = u\cos\theta - g\sin\beta T \quad \Rightarrow \quad T = \dfrac{u\cos\theta}{g\sin\beta}$

we also know that $T = \dfrac{2u\sin\theta}{g\cos\beta}$

$\Rightarrow \quad \dfrac{u\cos\theta}{g\sin\beta} = \dfrac{2u\sin\theta}{g\cos\beta} \qquad \Rightarrow \qquad 2\tan\theta = \cot\beta$

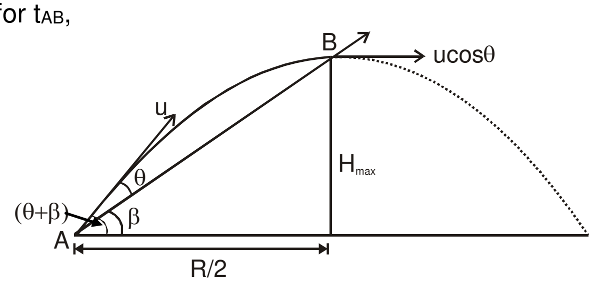
(b) If projectile strikes horizontally, then at the time of striking the projectile will be at the maximum height from the ground.

Therefore time taken to move from A to B, t_{AB} = 1/2 time of flight over horizontal plane

$= \dfrac{2u\sin(\theta+\beta)}{2\times g}$

Also, t_{AB} = time of flight over incline = $\dfrac{2u\sin\theta}{g\cos\beta}$

Equating for t_{AB},

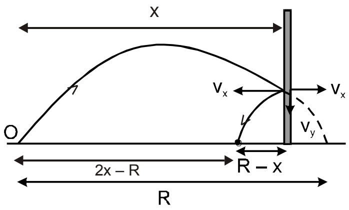


$\dfrac{2u\sin\theta}{g\cos\beta} = \dfrac{2u\sin(\theta+\beta)}{2g} \qquad \Rightarrow \qquad 2\sin\theta = \sin(\theta+\beta)\cos\beta.$

Elastic collision of a projectile with a wall :

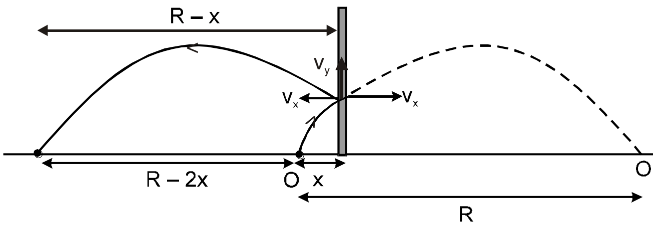
Suppose a projectile is projected with speed u at an angle θ from point O on the ground. Range of the projectile is R. A vertical, smooth wall is present in the path of the projectile at a distance x from the point O. The collision of the projectile with the wall is elastic. Due to collision, direction of x component of velocity is reversed but its magnitude remains the same and y component of velocity remains unchanged. Therefore the remaining distance (R – x) is covered in the backward direction and the projectile lands at a distance of R – x from the wall. Also time of flight and maximum height depends only on y component of velocity, hence they do not change despite of collision with the vertical, smooth and elastic wall.

Case I : If $x \geq \frac{R}{2}$



Here distance of landing place of projectile from its point of projection is 2x – R.

Case II : If $x < \frac{R}{2}$



Here distance of landing place of projectile from its point of projection is R – 2x.

Solved Examples

Example 18. A ball thrown from ground at an angle $\theta = 37^\circ$ with speed u = 20 m/s collides with A vertical wall 18.4 meter away from the point of projection. If the ball rebounds elastically to finally fall at some distance in front of the wall, find for this entire motion,

(i) Maximum height
(ii) Time of flight
(iii) Distance from the wall where the ball will fall
(iv) Distance from point of projection, where the ball will fall.

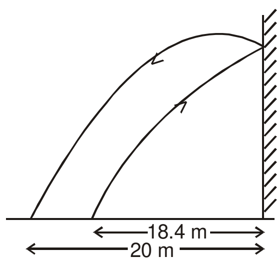
Solution : (i) $H = \frac{u^2 \sin^2\theta}{2g} = \frac{(20)^2 \sin^2 37^\circ}{2\times10} = \frac{20\times20}{2\times10}\times\frac{3}{5}\times\frac{3}{5} = 7.2$ m

(ii) $T = \frac{2u\sin\theta}{g} = \frac{2\times20\times\sin37^\circ}{10} = 2.4$ sec.

(iii) $R = \frac{u^2\sin2\theta}{g} = \frac{u^2}{g}\times2\sin\theta\cos\theta$

$\Rightarrow R = \frac{(20)^2}{10}\times2\sin37^\circ\cos37^\circ = 38.4$ m

Distance from the wall where the ball falls
= R – x = 38.4 – 18.4 = **20 m. Ans.**



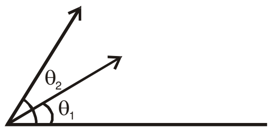
(iv) Distance from the point of projection = | R – 2x | = | 38.4 – 2 × 18.4|= 1.6 m

Solved Miscellaneous Problems

Problem 1. Two projectiles are thrown with different speeds and at different angles so as to cover the same maximum height. Find out the sum of the times taken by each to the reach to highest point, if time of flight is T.

Answer : Total time taken by either of the projectile.

Solution : $H_1 = H_2$ (given)



$$\frac{u_1^2 \sin^2\theta_1}{2g} = \frac{u_2^2 \sin^2\theta_2}{2g}$$

$u_1^2 \sin^2\theta_1 = u_2^2 \sin^2\theta_2$ .....(1)

at maximum height final velocity = 0

$v^2 = u_1^2 - 2gH_1$

$U_1^2 = 2gH_1$ similarly $U_2^2 = 2gH_2$

$U_1 = U_2$

on putting in equation (1)

$\because\ u_1^2 \sin^2\theta_1 = u_2^2 \sin^2\theta_2 \Rightarrow \theta_1 = \theta_2$

$T_1 = \dfrac{2u_1 \sin\theta_1}{g} \Rightarrow T_2 = \dfrac{2u_2 \sin\theta_2}{g} \Rightarrow \therefore T_1 = T_2$

Time taken to reach the maximum height by 1st projectile = $\dfrac{T_1}{2}$

Time taken to reach the maximum height by 2nd projectile = $\dfrac{T_2}{2}$

$\therefore$ sum of time taken by each to reach highest point = $\dfrac{T_1}{2} + \dfrac{T_2}{2} = 2\dfrac{T_1}{2}$ (or $2\dfrac{T_2}{2}$) = T_1 (or T_2)

Total time taken by either of the projectile

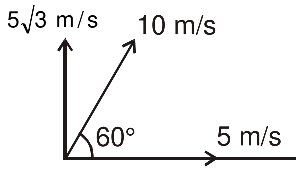
Problem 2. A particle is projected with speed 10 m/s at an angle 60^0 with horizontal. Find :

(a) Time of flight (b) Range
(c) Maximum height (d) Velocity of particle after one second.
(e) Velocity when height of the particle is 1 m

Answer : (a) $\sqrt{3}$ sec. (b) $5\sqrt{3}$ m (c) $\dfrac{15}{4}$ m (d) $10\sqrt{2-\sqrt{3}}$ m/s (e) $\vec{v} = 5\hat{i} \pm \sqrt{55}\hat{j}$

Solution : (a) $T = \dfrac{2u\sin\theta}{g} = \dfrac{2\times5\sqrt{3}}{10} = \sqrt{3}$ sec.

(b) Range $= \dfrac{u^2 \sin 2\theta}{g} = \dfrac{10\times10\times2\times\sin60^\circ\cos60^\circ}{10}$



$= 0.20 \times \dfrac{\sqrt{3}}{2} \times \dfrac{1}{2} = 5\sqrt{3}$ m.

(c) maximum height $H = \dfrac{u^2 \sin^2\theta}{2g} = \dfrac{10\times10\times.3}{2\times\ 10\times4} = \dfrac{15}{4}$ m

(d) velocity at any time 't'

$\vec{v} = v_x\hat{i} + v_y \Rightarrow \vec{v}_x = \vec{u}_x \Rightarrow v_x = 5$

$\vec{v}_y = \vec{u}_y + \vec{a}_y t \Rightarrow v_y = 5\sqrt{3} - 10 \times 1$

$\vec{v} = 5\hat{i} + (5\sqrt{3} - 10)\hat{j} \Rightarrow v = 10\left(\sqrt{2-\sqrt{3}}\right)$ m/s

(e) $v^2 = u^2 + 2gh$ velocity at any height 'h' is $\vec{v} = v_x\hat{i} + v_y\hat{j}$; $v_x = u_x = 5$

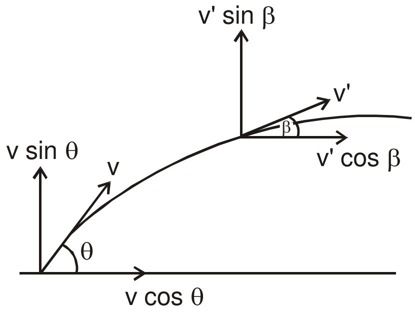
$v_y = u_y^2 - 2gh = \left(5\sqrt{3}\right)^2 - 2 \times 10 \times 1$

$v_y = \sqrt{55} \Rightarrow \vec{v} = 5\hat{i} \pm \sqrt{55}\hat{j}$

Problem 3. A stone is thrown with a velocity v at angle θ with horizontal. Find its speed when it makes an angle β with the horizontal.

Answer : $\frac{v\cos\beta}{\cos\theta}$

Solution : $v\cos\theta = v'\cos\beta$



$v' = \frac{v\cos\theta}{\cos\beta}$

Problem 4. Two paper screens A and B are separated by a distance of 100 m. A bullet pierces A and then B. The hole in B is 10 cm below the hole in A. If the bullet is travelling horizontally at the time of hitting the screen A, calculate the velocity of the bullet when it hits the screen A. Neglect the resistance of paper and air.

Answer : 700 m/s

Solution : Equation of motion in x direction $100 = v \times t$

$t = \frac{100}{v}$ ......(1)

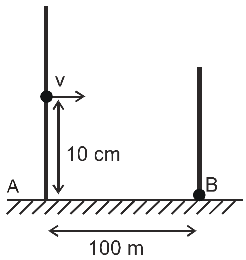
in y direction

$0.1 = 1/2 \times 9.8 \times t^2$ ......(2)

$0.1 = 1/2 \times 9.8 \times (100/v)^2$

From equation (1) & (2)

on solving we get $u = 700$ m/s



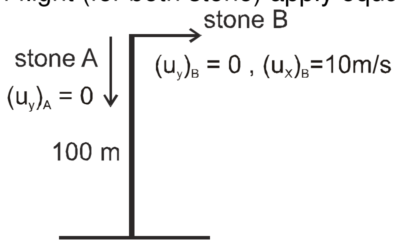
Problem 5. Two stones A and B are projected simultaneously from the top of a 100 m high tower. Stone B is projected horizontally with speed 10 m/s, and stone A is dropped from the tower. Find out the following (g = 10 m/s^2)

(a) Time of flight of the two stone (b) Distance between two stones after 3 sec.

(c) Angle of strike with ground (d) Horizontal range of particle B.

Answer : (a) $2\sqrt{5}$ sec. (b) $x_B = 30$ m, $y_B = 45$ m (c) $\tan^{-1} 2\sqrt{5}$ (d) $20\sqrt{5}$ m

Solution : (a) To calculate time of flight (for both stone) apply equation of motion in y direction



$100 = 1/2\ gt^2$

$t = 2$ sec.

(b) $X_B = 10 \times 3 = 30$ mt

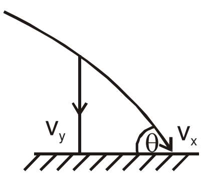
$Y_B = 1/2 \times g \times t^2 = 1/2 \times 10 \times 3 \times 3$

$Y_B = 45$ m

distance between two stones after 3 sec. $X_B = 30$,

$Y_B = 45$

So, distance = $\sqrt{(30)^2 + (45)^2}$



(c) angle of striking with ground $v_y^2 = u_y^2 + 2gh = 0 + 2 \times 10 \times 100$

$v_y = 20\sqrt{5}$ m/s $\Rightarrow$ $v_x = 10$ m/s

$\because \vec{v} = v_x\hat{i} + v_y\hat{j}$ $\Rightarrow$ $\tan\theta = \frac{v_y}{v_x}$ $\Rightarrow \theta = \tan^{-1}\left(\frac{20\sqrt{5}}{10}\right) = \tan^{-1}(2\sqrt{5})$

(d) Horizontal range of particle 'B' $X_B = 10 \times (2\sqrt{5}) = 20\sqrt{5}$ m

Problem 6. Two particles are projected simultaneously with the same speed V in the same vertical plane with angles of elevation θ and 2θ, where $\theta < 45^\circ$. At what time will their velocities be parallel.

Answer : $\frac{v}{g}\cos\left(\frac{\theta}{2}\right)\operatorname{cosec}\left(\frac{3\theta}{2}\right)$

Solution : Velocity of particle projected at angle 'θ' after time t

$\vec{V}_1 = (v\cos\theta\,\hat{i} + v\sin\theta\,\hat{j}) - (gt\,\hat{j})$

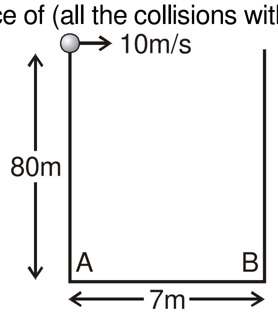
Velocity of particle projected at angle '2θ' after time t

$\vec{V}_2 = (v\cos 2\theta\,\hat{i} + v\sin 2\theta\,\hat{j}) - (gt\,\hat{j})$

Since velocities are parallel so $\frac{v_x}{v_x'} = \frac{v_y}{v_y'} \Rightarrow \frac{v\cos\theta}{v\cos 2\theta} = \frac{v\sin\theta - gt}{v\sin 2\theta - gt}$

Solving above equation we can get result. $\frac{v}{g}\cos\left(\frac{\theta}{2}\right)\operatorname{cosec}\left(\frac{3\theta}{2}\right)$

Problem 7. A ball is projected horizontally from top of a 80 m deep well with velocity 10 m/s. Then particle will fall on the bottom at a distance of (all the collisions with the wall are elastic and wall is smooth).



(A) 5 m from A (B) 5 m from B (C) 2 m from A (D) 2 m from B

Answer : (B) 5 m from B (C) 2 m from A

Solution : Total time taken by the ball to reach at bottom = $\sqrt{\frac{2H}{g}} = \sqrt{\frac{2x80}{10}} = 4$ sec.

Let time taken in one collision is t

Then t x 10 = 7

t = .7 sec.

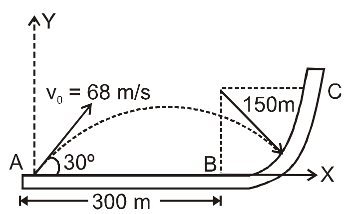
No. of collisions = $\frac{4}{.7} = 5\frac{5}{7}$ (5th collisions from wall B)

Horizontal distance travelled in between 2 successive collisions = 7 m

$\therefore$ Horizontal distance travelled in 5/7 part of collisions = $\frac{5}{7}x7 = 5$ m

Distance from A is 2 m. **Ans.**

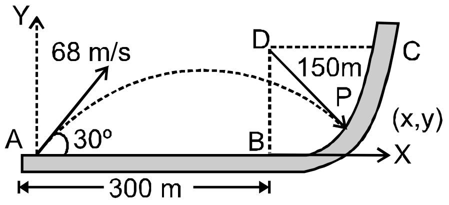
Problem 8. A projectile is launched from point 'A' with the initial conditions shown in the figure. BC part is circular with radius 150 m. Determine the 'x' and 'y' co-ordinates of the point of impact.



Solution : Let the projectile strikes the circular path at (x,y) and 'A' to be taken as origin. From the figure co-ordinates of the centre of the circular path is (300, 150). Then the equation of the circular path is

$(x - 300)^2 + (y - 150)^2 = (150)^2$ .....(1)

and the equation of the trajectory is



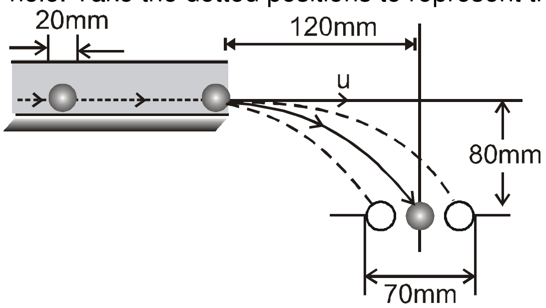
$$y = x \tan 30^\circ - \frac{1}{2}\frac{gx^2}{(68)^2 \cos^2 30^\circ}$$

$$y = \frac{x}{\sqrt{3}} - \frac{2x^2 g}{9248} \quad .....(2)$$

From Eqs. (1) and (2) we get

x = 373 m ; y = 18.75 m

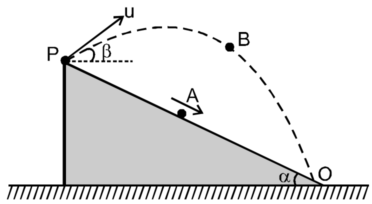
Problem 9. Ball bearings leave the horizontal through with a velocity of magnitude 'u' and fall through the 70 mm diameter hole as shown. Calculate the permissible range of 'u' which will enable the balls to enter the hole. Take the dotted positions to represent the limiting conditions.



Solution : $t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 0.08}{9.8}} = 0.13\text{s}$

$u_{min} = \frac{(120 - 25 + 10) \times 10^{-3}}{0.13} = 0.807$ m/s and $u_{max} = \frac{(120 + 25 - 10) \times 10^{-3}}{0.13} = 1.038$ m/s .

Problem 10. Particle A is released from a point P on a smooth inclined plane inclined at an angle α with the horizontal. At the same instant another particle B is projected with initial velocity u making an angle β with the horizontal. Both the particles meet again on the inclined plane. Find the relation between α and β.



Solution : Consider motion of B along the plane initial velocity = u cos $(\alpha + \beta)$

acceleration = g sin α

$\therefore \quad OP = u\cos(\alpha + \beta)\, t + \frac{1}{2} g \sin(\alpha) t^2$ .....(i)

For motion of particle A along the plane,

initial velocity = 0

acceleration = g sin α

$\therefore \quad OP = \frac{1}{2} g \sin \alpha\, t^2$ .....(ii)

From Equation. (i) and (ii) $\quad u\cos(\alpha + \beta)\, t = 0$

So, either $\quad t = 0$ or $\alpha + \beta = \frac{\pi}{2}$

Thus, the condition for the particles to collide again is $\alpha + \beta = \frac{\pi}{2}$.

RELATIVE MOTION

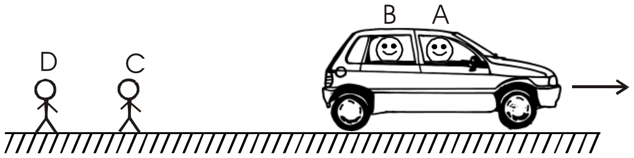
1 RELATIVE MOTION

Motion is a combined property of the object under study as well as the observer. It is always relative ; there is no such thing like absolute motion or absolute rest. Motion is always defined with respect to an observer or reference frame.

Reference frame :

Reference frame is an axis system from which motion is observed along with a clock attached to the axis, to measure time. Reference frame can be stationary or moving.

☞ Suppose there are two persons A and B sitting in a car moving at constant speed. Two stationary persons C and D observe them from the ground.



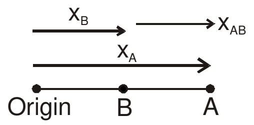
Here B appears to be moving for C and D, but at rest for A. Similarly C appears to be at rest for D but moving backward for A and B.

2 RELATIVE MOTION IN ONE DIMENSION :

2.1 Relative Position :

It is the position of a particle w.r.t. observer.

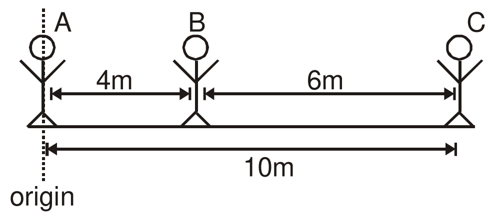
In general if position of A w.r.t. to origin is x_A and that of B w.r.t. origin is x_B then "Position of A w.r.t. B" x_{AB} is



$$\boxed{x_{AB} = x_A - x_B}$$

Solved Example

Example 1. See the figure (take +ve direction towards right and –ve towards left). Find x_{BA}, x_{CA}, x_{CB}, x_{AB} and x_{AC}.



Here, Position of B w.r.t. A is 4 m towards right. (x_{BA} = +4m)

Position of C w.r.t. A is 10 m towards right. (x_{CA} = +10m)

Position of C w.r.t. B is 6 m towards right (x_{CB} = +6m)

Position of A w.r.t. B is 4 m towards left. (x_{AB} = –4 m)

Position of A w.r.t. C is 10 m towards left. (x_{AC} = –10m)

2.2 Relative Velocity

Definition : Relative velocity of a particle A with respect to B is defined as the velocity with which A appears to move if B is considered to be at rest. In other words, it is the velocity with which A appears to move as seen by B considering itself to be at rest.

NOTE 1 : All velocities are relative & have no significance unless observer is specified. However, when we say "velocity of A", what we mean is , velocity of A w.r.t. ground which is assumed to be at rest.

Relative velocity in one dimension -

If x_A is the position of A w.r.t. ground, x_B is position of B w.r.t. ground and x_{AB} is position of A w.r.t. B then we can say v_A = velocity of A w.r.t. ground = $\frac{dx_A}{dt}$

v_B = velocity of B w.r.t. ground = $\frac{dx_B}{dt}$

and v_{AB} = velocity of A w.r.t. B = $\frac{dx_{AB}}{dt} = \frac{d}{dt}(x_A - x_B) = \frac{dx_A}{dt} - \frac{dx_B}{dt}$

Thus

$$\boxed{v_{AB} = v_A - v_B}$$

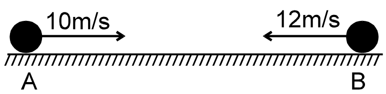
NOTE 2. : Velocity of an object w.r.t. itself is always zero.

Solved Examples

Example 2. An object A is moving with 5 m/s and B is moving with 20 m/s in the same direction. (Positive x-axis)
(i) Find velocity of B with respect to A.
(ii) Find velocity of A with respect to B

Solution : (i) $v_B = +20$ m/s, $v_A = +5$ m/s,
$v_{BA} = v_B - v_A = +15$ m/s
(ii) $v_B = +20$ m/s, $v_A = +15$ m/s
$v_{AB} = v_A - v_B = -15$ m/s
Note : $v_{BA} = -v_{AB}$

Example 3. Two objects A and B are moving towards each other with velocities 10 m/s and 12 m/s respectively as shown.
(i) Find the velocity of A with respect to B.
(ii) Find the velocity of B with respect to A



Solution : $v_A = +10$, $v_B = -12$
(i) $v_{AB} = v_A - v_B = (10) - (-12) = 22$ m/s.
(ii) $v_{BA} = v_B - v_A = (-12) - (10) = -22$ m/s.

2.3 Relative Acceleration

It is the rate at which relative velocity is changing.

$$a_{AB} = \frac{dv_{AB}}{dt} = \frac{dv_A}{dt} - \frac{dv_B}{dt} = a_A - a_B$$

Equations of motion when relative acceleration is constant.

$v_{rel} = u_{rel} + a_{rel}\,t$

$s_{rel} = u_{rel}\,t + \frac{1}{2} a_{rel}\,t^2$

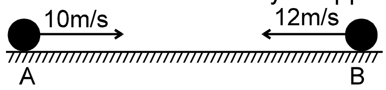
$v^2_{rel} = u^2_{rel} + 2a_{rel}\,s_{rel}$

2.4 Velocity of Approach / Separation

It is the component of relative velocity of one particle w.r.t. another, along the line joining them. If the separation is decreasing, we say it is velocity of approach and if separation is increasing, then we say it is velocity of separation. In one dimension, since relative velocity is along the line joining A and B, hence velocity of approach / separation is simply equal to magnitude of relative velocity of A w.r.t. B.

Solved Examples

Example 4. A particle A is moving with a speed of 10 m/s towards right and another particle B is moving at speed of 12 m/s towards left. Find their velocity of approach.

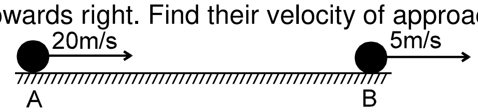


Solution : $V_A = +10$, $V_B = -12 \Rightarrow V_{AB} = V_A - V_B \Rightarrow 10 - (-12) = 22$ m/s

since separation is decreasing hence $V_{app} = |V_{AB}| = 22$ m/s

Ans. : 22 m/s

Example 5 A particle A is moving with a speed of 20 m/s towards right and another particle B is moving at a speed of 5 m/s towards right. Find their velocity of approach.



Solution : $V_A = +20$, $V_B = +5$

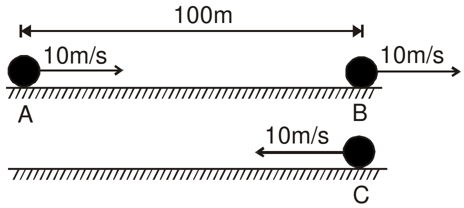
$V_{AB} = V_A - V_B$

$20 - (+5) = 15$ m/s

since separation is decreasing hence $V_{app} = |V_{AB}| = 15$ m/s

Ans. : 15 m/s

Example 6. A particle A is moving with a speed of 10 m/s towards right, particle B is moving at a speed of 10 m/s towards right and another particle C is moving at speed of 10 m/s towards left. The separation between A and B is 100 m. Find the time interval between C meeting B and C meeting A.



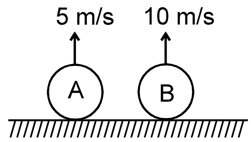
Solution : $t = \dfrac{\text{seperation between A and C}}{V_{app} \text{ of A and C}} = \dfrac{100}{10-(-10)} = 5$ sec.

Ans. : 5 sec.

Note : $a_{app} = \left(\dfrac{d}{dt}\right)v_{app}$, $a_{sep} = \dfrac{d}{dt}v_{sep}$

$v_{app} = \int a_{app}dt$, $v_{sep} = \int a_{sep}dt$

Example 7. A and B are thrown vertically upward with velocity, 5 m/s and 10 m/s respectively (g = 10 m/s^2. Find separation between them after one second



Solution : $S_A = ut - \dfrac{1}{2}gt^2$

$= 5t - \dfrac{1}{2} \times 10 \times t^2 = 5 \times 1 - 5 \times 1^2 = 5 - 5 = 0$

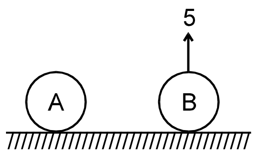
$S_B = ut - \dfrac{1}{2}gt^2 = 10 \times 1 - \dfrac{1}{2} \times 10 \times 1^2 = 10 - 5 = 5$

$\therefore S_B - S_A$ = separation = 5m.

Aliter : $\vec{a}_{BA} = \vec{a}_B - \vec{a}_A = (-10) - (-10) = 0$

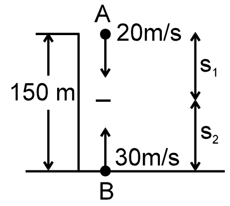
Also $\vec{v}_{BA} = \vec{v}_B - \vec{v}_A = 10 - 5 = 5$ m/s

$\therefore \vec{s}_{BA}$ (in 1 sec) $= \vec{v}_{BA} \times t = 5 \times 1 = 5$ m



$\therefore$ Distance between A and B after 1 sec = 5 m.

Example 8. A ball is thrown downwards with a speed of 20 m/s from the top of a building 150 m high and simultaneously another ball is thrown vertically upwards with a speed of 30 m/s from the foot of the building. Find the time after which both the balls will meet. (g = 10 m/s^2)



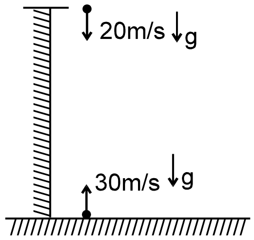
Solution : $S_1 = 20\,t + 5\,t^2$

$S_2 = 30\,t - 5\,t^2$

$S_1 + S_2 = 150$

$\Rightarrow\ 150 = 50\,t \ \Rightarrow\ t = 3\text{ s}$

Aliter : Relative acceleration of both is zero since both have same acceleration in downward direction



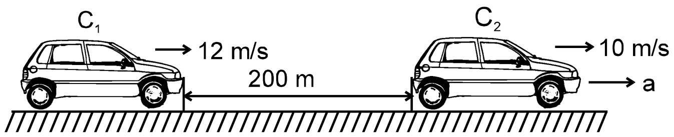
$\vec{a}_{AB} = \vec{a}_A - \vec{a}_B = g - g = 0$

$\vec{v}_{BA} = 30 - (-20) = 50$

$s_{BA} = v_{BA} \times t$

$t = \dfrac{s_{BA}}{v_{BA}} = \dfrac{150}{50} = 3\text{ s}$

Example 9. Two cars C_1 and C_2 moving in the same direction on a straight single lane road with velocities 12 m/s and 10 m/s respectively. When the separation between the two was 200 m C_2 started accelerating to avoid collision. What is the minimum acceleration of car C_2 so that they don't collide.



Solution : Acceleration of car 1 w.r.t. car 2

$\vec{a}_{12} = \vec{a}_1 - \vec{a}_2 = \vec{a}_{C_1} - \vec{a}_{C_2} = 0 - a = (-a)$

$\vec{u}_{12} = \vec{u}_1 - \vec{u}_2 = 12 - 10 = 2$ m/s.

The collision is just avoided if relative velocity becomes zero just at the moment the two cars meet each other.

i.e. $v_{12} = 0$ When $s_{12} = 200$

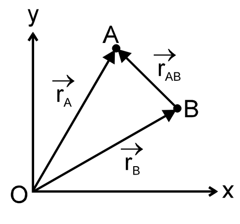
Now $v_{12} = 0$, $\vec{u}_{12} = 2$, $\vec{a}_{12} = -\,a$ and $s_{12} = 200$

$\therefore\ \ v_{12}^2 - u_{12}^2 = 2a_{12}s_{12}$

$\Rightarrow\ 0 - 2^2 = -\,2 \times a \times 200 \qquad \Rightarrow \qquad a = \dfrac{1}{100}$ m/s^2 = 0.1 m/s^2 = 1 cm/s^2.

$\therefore$ Minimum acceleration needed by car C_2 = 1 cm/s^2

3. RELATIVE MOTION IN TWO DIMENSION



$\vec{r}_A$ = position of A with respect to O

$\vec{r}_B$ = position of B with respect to O

$\vec{r}_{AB}$ = position of A with respect to B.

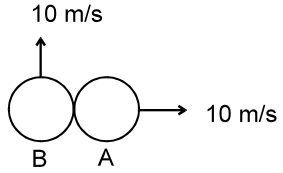
$\vec{r}_{AB} = \vec{r}_A - \vec{r}_B$ (The vector sum $\vec{r}_A - \vec{r}_B$ can be done by Δ law of addition or resolution method)

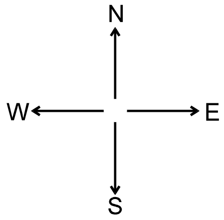
$\therefore\ \ \dfrac{d(\vec{r}_{AB})}{dt} = \dfrac{d(\vec{r}_A)}{dt} - \dfrac{d(\vec{r}_B)}{dt}$.

$\Rightarrow\ \vec{v}_{AB} = \vec{v}_A - \vec{v}_B\ ;\ \dfrac{d(\vec{v}_{AB})}{dt} = \dfrac{d(\vec{v}_A)}{dt} - \dfrac{d(\vec{v}_B)}{dt} \ \Rightarrow\ \vec{a}_{AB} = \vec{a}_A - \vec{a}_B$

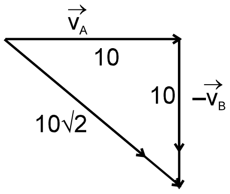
Solved Examples

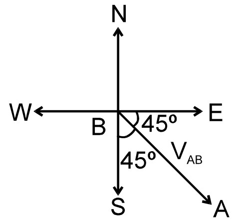
Example 10. Object A and B both have speed of 10 m/s. A is moving towards East while B is moving towards North starting from the same point as shown. Find velocity of A relative to B.





Solution : **Method 1 :** $\vec{v}_{AB} = \vec{v}_A - \vec{v}_B$ $\quad\therefore\ |\vec{v}_{AB}| = 10\sqrt{2}$



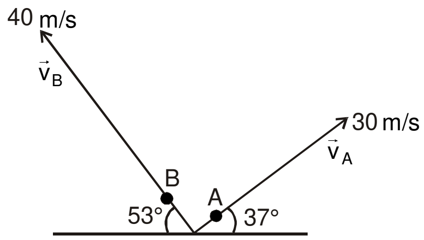


Method 2 : $\vec{v}_A = 10\,\hat{i}$, $\vec{v}_B = 10\,\hat{j}$

$\vec{v}_{AB} = \vec{v}_A - \vec{v}_B = 10\,\hat{i} - 10\,\hat{j}$ $\quad\therefore\ |\vec{v}_{AB}| = 10\sqrt{2}$

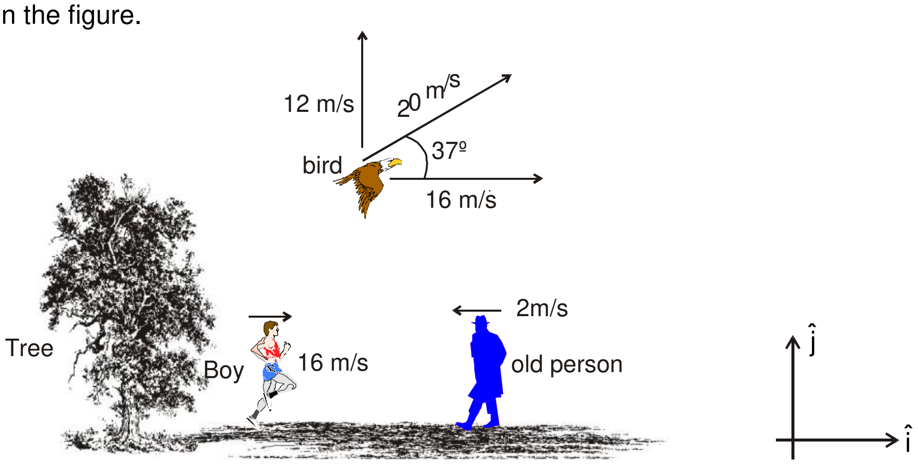
Note : $|\vec{v}_A - \vec{v}_B| = \sqrt{v_A^2 + v_B^2 - 2v_Av_B\cos\theta}$, where θ is angle between $\vec{v}_A$ and $\vec{v}_B$.

Example 11. Two particles A and B are projected in air. A is thrown with a speed of 30 m/sec and B with a speed of 40 m/sec as shown in the figure. What is the separation between them after 1 sec.



Solution : $\vec{a}_{AB} = \vec{a}_A - \vec{a}_B = \vec{g} - \vec{g} = 0$ $\quad\therefore\ \vec{v}_{AB} = \sqrt{30^2 + 40^2} = 50$ $\quad\therefore\ s_{AB} = v_{AB}t = 50\,t = 50$ m

Example 12. An old man and a boy are walking towards each other and a bird is flying over them as shown in the figure.



(1) Find the velocity of tree, bird and old man as seen by boy.
(2) Find the velocity of tree, bird and boy as seen by old man
(3) Find the velocity of tree, boy and old man as seen by bird.

Solution : (1) With respect to boy :

v_{tree} = 16 m/s (←) or $-16\hat{i}$

v_{bird} = 12 m/s (↑) or $12\hat{j}$

$v_{old\ man}$ = 18 m/s (←) or $-18\hat{i}$

(2) With respect to old man :

v_{Boy} = 18 m/s (→) or $18\hat{i}$

v_{Tree} = 2 m/s (→) or $2\hat{i}$

v_{Bird} = 18 m/s (→) and 12 m/s (↑) or $18\hat{i} + 12\hat{j}$

(3) With respect to Bird : v_{Tree} = 12 m/s (↓) and 16 m/s (←) or $-12\hat{j} - 16\hat{i}$

$v_{old\ man}$ = 18 m/s (←) and 12 m/s (↓) or $-18\hat{i} - 12\hat{j}$

v_{Boy} = 12 m/s (↓) or $-12\hat{j}$

3.1 Relative Motion in Lift

Projectile motion in a lift moving with acceleration a upwards

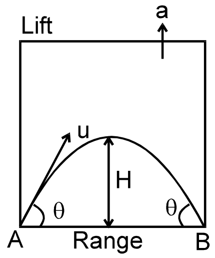
(1) In the reference frame of lift, acceleration of a freely falling object is (g + a)

(2) Velocity at maximum height = ucosθ

(3) $T = \dfrac{2u\sin\theta}{g+a}$

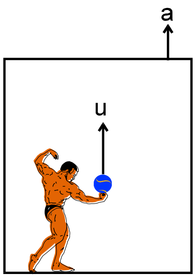
(4) Maximum height (H) = $\dfrac{u^2\sin^2\theta}{2(g+a)}$

(5) Range = $\dfrac{u^2\sin 2\theta}{g+a}$



Solved Example

Example 13. A lift is moving up with acceleration a. A person inside the lift throws the ball upwards with a velocity u relative to hand.



(a) What is the time of flight of the ball?

(b) What is the maximum height reached by the ball in the lift?

Solution : (a) $\vec{a}_{BL} = \vec{a}_B - \vec{a}_L = g + a$

$\vec{s} = \vec{u}\,t + \dfrac{1}{2}\vec{a}_{BL}\,t^2$

$0 = uT - \dfrac{1}{2}(g+a)T^2$

$\therefore\ T = \dfrac{2u}{(g+a)}$

(b) $v^2 - u^2 = 2as$

$0 - u^2 = -2(g+a)H$

$H = \dfrac{u^2}{2(g+a)}$

4. RELATIVE MOTION IN RIVER FLOW

If a man can swim relative to water with velocity $\vec{v}_{mR}$ and water is flowing relative to ground with velocity $\vec{v}_R$, velocity of man relative to ground $\vec{v}_m$ will be given by :

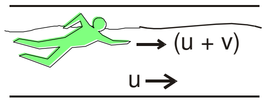
$\vec{v}_{mR} = \vec{v}_m - \vec{v}_R$ or $\vec{v}_m = \vec{v}_{mR} + \vec{v}_R$

If $\vec{v}_R = 0$, then $\vec{v}_m = \vec{v}_{mR}$ in words, velocity of man in still water = velocity of man w.r.t. river

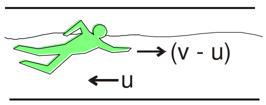
4.1 River Problem in One Dimension :

☞ Velocity of river is u & velocity of man in still water is v.

Case-1 : Man swimming downstream (along the direction of river flow).
In this case velocity of river $v_R = + u$
velocity of man w.r.t. river $v_{mR} = +v$
now $\vec{v}_m = \vec{v}_{mR} + \vec{v}_R = u + v$



Case-2 : Man swimming upstream (opposite to the direction of river flow). In this case velocity of river $\vec{v}_R = - u$
velocity of man w.r.t. river $\vec{v}_{mR} = +v$
now $\vec{v}_m = \vec{v}_{mR} + \vec{v}_R = (v - u)$



Solved Example

Example 14 A swimmer capable of swimming with velocity 'v' relative to water jumps in a flowing river having velocity 'u'. The man swims a distance d down stream and returns back to the original position. Find out the time taken in complete motion.

Solution : Total time = time of swimming downstream + time of swimming upstream

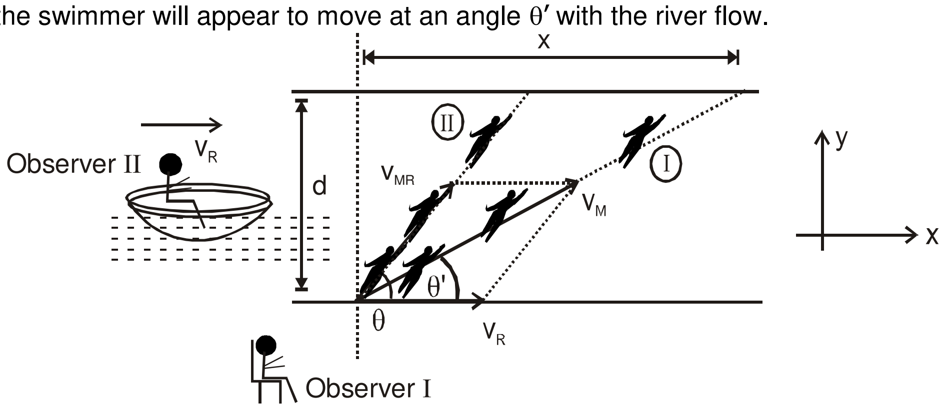
$$t = t_{down} + t_{up} = \frac{d}{v+u} + \frac{d}{v-u} = \frac{2dv}{v^2 - u^2}$$ **Ans.**

4.2 Motion of Man Swimming in a River

Consider a man swimming in a river with a velocity of $\vec{v}_{MR}$ relative to river at an angle of θ with the river flow. The velocity of river is $\vec{V}_R$. Let there be two observers I and II, observer I is on ground and observer II is on a raft floating along with the river and hence moving with the same velocity as that of river. Hence motion w.r.t. observer II is same as motion w.r.t. river. i.e., the man will appear to swim at an angle θ with the river flow for observer II .

For observer I the velocity of swimmer will be $\vec{v}_M = \vec{v}_{MR} + \vec{v}_R$,

Hence the swimmer will appear to move at an angle θ' with the river flow.

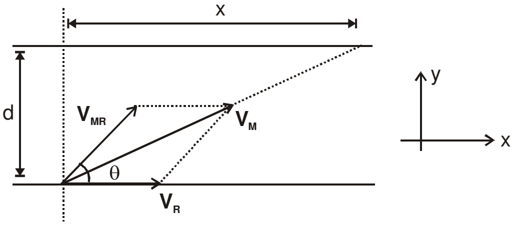


(I) : Motion of swimmer for observer I

(II) : Motion of swimmer for observer II

4.3 River problem in two dimension (crossing river) :

Consider a man swimming in a river with a velocity of $\vec{v}_{MR}$ relative to river at an angle of θ with the river flow. The velocity of river is V_R and the width of the river is d



$$\vec{v}_M = \vec{v}_{MR} + \vec{v}_R \Rightarrow \vec{v}_M = (v_{MR}\cos\theta\,\hat{i} + v_{MR}\sin\theta\,\hat{j}) + v_R\,\hat{i} \Rightarrow \vec{v}_M = (v_{MR}\cos\theta + v_R)\,\hat{i} + v_{MR}\sin\theta\,\hat{j}$$

Here $v_{MR}\sin\theta$ is the component of velocity of man in the direction perpendicular to the river flow. This component of velocity is responsible for the man crossing the river. Hence if the time to cross the river is t, then $t = \dfrac{d}{v_y} = \dfrac{d}{v_{MR}\sin\theta}$

DRIFT

It is defined as the displacement of man in the direction of river flow. (See the figure). It is simply the displacement along x-axis, during the period the man crosses the river. $(v_{MR}\cos\theta + v_R)$ is the component of velocity of man in the direction of river flow and this component of velocity is responsible for drift along the river flow. If drift is x then,

Drift = $v_x \times t$

$$x = (v_{MR}\cos\theta + v_R) \times \frac{d}{v_{MR}\sin\theta}$$

4.4 Crossing the river in shortest time

As we know that $t = \dfrac{d}{v_{MR}\sin\theta}$. Clearly t will be minimum when $\theta = 90°$ i.e. time to cross the river will be minimum if man swims perpendicular to the river flow. Which is equal to $\dfrac{d}{v_{MR}}$.

4.5 Crossing the river in shortest path, Minimum Drift

The minimum possible drift is zero. In this case the man swims in the direction perpendicular to the river flow as seen from the ground. This path is known as **shortest path**

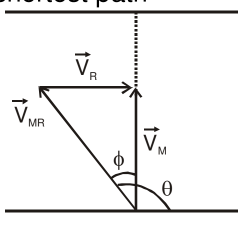
here $x_{min} = 0 \Rightarrow (v_{MR}\cos\theta + v_R) = 0$ or $\cos\theta = -\dfrac{v_R}{v_{MR}}$

☞ since $\cos\theta$ is –ve, $\therefore\ \theta > 90°$, i.e. for minimum drift the man must swim at some angle ϕ with the perpendicular in backward direction. Where $\sin\phi = \dfrac{v_R}{v_{MR}}$

☞ $\theta = \cos^{-1}\left(\dfrac{-v_R}{v_{MR}}\right) \quad \therefore \left|\dfrac{v_R}{v_{MR}}\right| \le 1$ i.e. $v_R \le v_{MR}$

i.e. minimum drift is zero if and only if velocity of man in still water is greater than or equal to the velocity of river.

☞ Time to cross the river along the shortest path



$$t = \frac{d}{v_{MR}\sin\theta} = \frac{d}{\sqrt{v_{MR}^2 - V_R^2}}$$

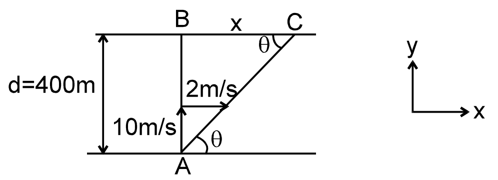
Note :

- **If v_R > v_{MR} then it is not possible to have zero drift. In this case the minimum drift (corresponding to shortest possible path is non zero and the condition for minimum drift can be proved to be $\cos\theta = -\left(\frac{v_{MR}}{v_R}\right)$ or** $\sin\phi = \left(\frac{v_{MR}}{v_R}\right)$ **for minimum but non zero drift.**

Solved Examples

Example 15. A 400 m wide river is flowing at a rate of 2.0 m/s.A boat is sailing with a velocity of 10 m/s with respect to the water, in a direction perpendicular to the river.
(a) Find the time taken by the boat to reach the opposite bank.
(b) How far from the point directly opposite to the starting point does the boat reach the opposite bank.
(c) In what direction does the boat actually move, with river flow (downstream).

Solution :



(a) time taken to cross the river $t = \frac{d}{v_y} = \frac{400m}{10m/s}$ = 40 s **Ans.**

(b) drift (x) = $(v_x)(t)$ = (2m/s) (40s) = 80 m **Ans.**

(c) Actual direction of boat, $\theta = \tan^{-1}\left(\frac{10}{2}\right) = \tan^{-1} 5$, (downstream) with the river flow.

Example 16. A man can swim at the rate of 5 km/h in still water. A 1 km wide river flows at the rate of 3 km/h. The man wishes to swim across the river directly opposite to the starting point.
(a) Along what direction must the man swim?
(b) What should be his resultant velocity?
(c) How much time will he take to cross the river?

Solution : The velocity of man with respect to river v_{mR}= 5 km/hr, this is greater than the river flow velocity, therefore, he can cross the river directly (along the shortest path). The angle of swim must be

$$\theta = \frac{\pi}{2} + \sin^{-1}\left(\frac{v_r}{v_{mR}}\right) = 90^{\circ} + \sin^{-1}\left(\frac{v_r}{v_{mR}}\right) = 90^{\circ} + \sin^{-1}\left(\frac{3}{5}\right) = 90^{\circ} + 37^{\circ}$$

= 127º w.r.t. the river flow or 37° w.r.t. perpendicular in backward direction **Ans.**

(b) Resultant velocity will be

$$v_m = \sqrt{v_{mR}^2 - v_R^2} = \sqrt{5^2 - 3^2} = 4 \text{ km/hr}$$

along the direction perpendicular to the river flow.

(c) time taken to cross the $t = \frac{d}{\sqrt{v_{mR}^2 - v_R^2}} = \frac{1km}{4km/hr} = \frac{1}{4}$ h = 15 min

Example 17. A man wishing to cross a river flowing with velocity u jumps at an angle θ with the river flow.
(i) Find the net velocity of the man with respect to ground if he can swim with speed v in still water.
(ii) In what direction does the man actually move.
(iii) Find how far from the point directly opposite to the starting point does the man reach the opposite bank, if the width of the river is d. (i.e. drift)

Solution : (i) $v_{MR} = v$, $v_R = u$; $\vec{v}_M = \vec{v}_{MR} + \vec{v}_R$

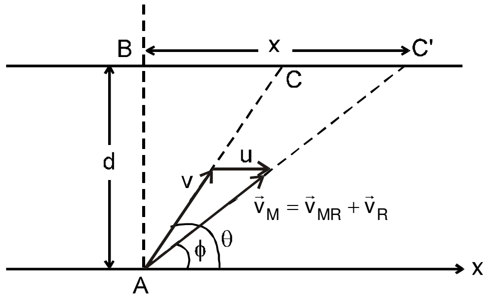
$\therefore$ Velocity of man,

$v_M = \sqrt{u^2 + v^2 + 2vu\cos\theta}$ **Ans.**

(ii) $\tan\phi = \dfrac{v\sin\theta}{u + v\cos\theta}$ **Ans.**

(iii) $(v \sin\theta)\, t = d \Rightarrow t = \dfrac{d}{v\sin\theta}$;

$x = (u + v\cos\theta)\, t = (u + v\cos\theta)\dfrac{d}{v\sin\theta}$



Example 18. A boat moves relative to water with a velocity v which is n times less than the river flow velocity u. At what angle to the stream direction must the boat move to minimize drifting ?

Solution : (In this problem, one thing should be carefully noted that the velocity of boat is less than the river flow velocity. Hence boat cannot reach the point directly opposite to its starting point. i.e. drift can never be zero)

Suppose boat starts at an angle θ from the normal direction up stream as shown.

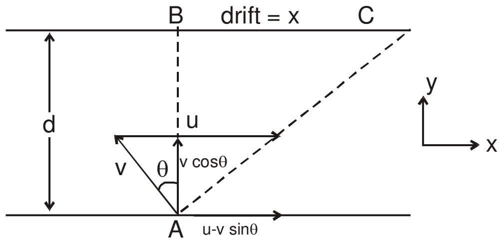
Component of velocity of boat along the river, $v_x = u - v\sin\theta$

and velocity perpendicular to the river,

$v_y = v\cos\theta$.

time taken to cross the river is

$t = \dfrac{d}{v_y} = \dfrac{d}{v\cos\theta}$.



Drift $x = (v_x)t = (u - v\sin\theta)\dfrac{d}{v\cos\theta}$

$= \dfrac{ud}{v}\sec\theta - d\tan\theta$

The drift x is minimum, when $\dfrac{dx}{d\theta} = 0$,

or $\left(\dfrac{ud}{v}\right)(\sec\theta \cdot \tan\theta) - d\sec^2\theta = 0$

or $\dfrac{u}{v}\sin\theta = 1$ or $\boxed{\sin\theta = \dfrac{v}{u}}$

This is the result we stated without proof as a note in section 4.5

so, for minimum drift, the boat must move at an angle $\theta = \sin^{-1}\left(\dfrac{v}{u}\right) = \sin^{-1}\dfrac{1}{n}$ from normal direction.

5. WIND AIRPLANE PROBLEMS

This is very similar to boat river flow problems. The only difference is that boat is replaced by aeroplane and river is replaced by wind.

Thus, velocity of aeroplane with respect to wind

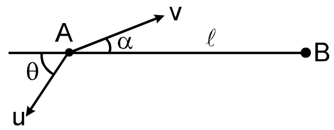
$\vec{v}_{aw} = \vec{v}_a - \vec{v}_w$ or $\vec{v}_a = \vec{v}_{aw} + \vec{v}_w$

where, $\vec{v}_a$ = velocity of aeroplane w.r.t. ground and, $\vec{v}_w$ = velocity of wind.

Solved Examples

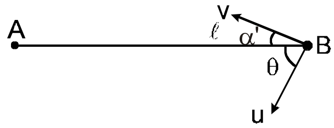
Example 19 An aeroplane flies along a straight path A to B and returns back again. The distance between A and B is ℓ and the aeroplane maintains the constant speed v w.r.t. wind. There is a steady wind with a speed u at an angle θ with line AB. Determine the expression for the total time of the trip.

Solution : Suppose plane is oriented at an angle α w.r.t. line AB while the plane is moving from A to B :



Velocity of plane along AB = v cosα – ucosθ,
and for no-drift from line AB ; v sinα = usinθ

$\Rightarrow \quad \sin\alpha = \dfrac{u\sin\theta}{v}$



time taken from A to B : $t_{AB} = \dfrac{\ell}{v\cos\alpha - u\cos\theta}$

Suppose plane is oriented at an angle α' w.r.t. line AB while the plane is moving from B to A :
velocity of plane along BA = vcosα + u cosθ and for no drift from line AB ; vsinα = usinθ

$\Rightarrow \quad \sin\alpha = \dfrac{u\sin\theta}{v} \qquad \Rightarrow \qquad \alpha = \alpha'$

time taken from B to A : $t_{BA} = \dfrac{\ell}{v\cos\alpha + u\cos\theta}$

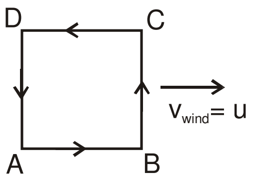
total time taken = $t_{AB} + t_{BA} = \dfrac{\ell}{v\cos\alpha - u\cos\theta} + \dfrac{\ell}{v\cos\alpha + u\cos\theta}$

$= \dfrac{2v\ell\cos\alpha}{v^2\cos^2\alpha - u^2\cos^2\theta} = \dfrac{2v\ell\sqrt{1-\dfrac{u^2\sin^2\theta}{v^2}}}{v^2 - u^2}$.

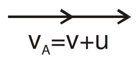
Example 20. Find the time an aeroplane having velocity v, takes to fly around a square with side a if the wind is blowing at a velocity u along one side of the square.

Answer : $\dfrac{2a}{v^2 - u^2}\left(v + \sqrt{v^2 - u^2}\right)$

Solution :

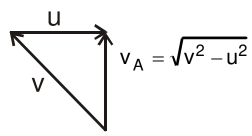


Velocity of aeroplane while flying through AB



$v_A = v + u \quad ; \qquad t_{AB} = \dfrac{a}{v+u}$

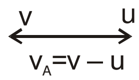
Velocity of aeroplane while flying through BC



$v_A = \sqrt{v^2 - u^2}$;

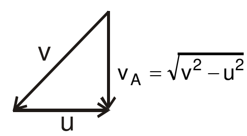
$t_{BC} = \dfrac{a}{\sqrt{v^2 - u^2}}$

Velocity of aeroplane while flying through CD



$v_A = v - u \quad ; \qquad t_{CD} = \dfrac{a}{v-u}$

Velocity of aeroplane while flying through DA



$v_A = \sqrt{v^2 - u^2}$; $t_{DA} = \dfrac{a}{\sqrt{v^2 - u^2}}$

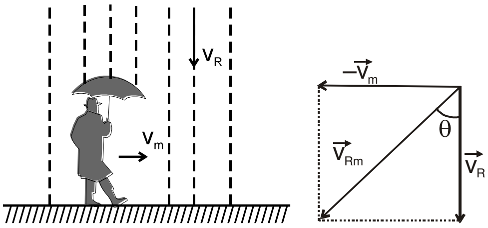
Total time = $t_{AB} + t_{BC} + t_{CD} + t_{DA} = \dfrac{a}{v+u} + \dfrac{a}{\sqrt{v^2 - u^2}} + \dfrac{a}{v-u} + \dfrac{a}{\sqrt{v^2 - u^2}} = \dfrac{2a}{v^2 - u^2}\left(v + \sqrt{v^2 - u^2}\right)$

6. RAIN PROBLEM

If rain is falling vertically with a velocity $\vec{v}_R$ and an observer is moving horizontally with velocity $\vec{v}_m$, the velocity of rain relative to observer will be :

$\vec{v}_{Rm} = \vec{v}_R - \vec{v}_m$ or $v_{Rm} = \sqrt{v_R^2 + v_m^2}$

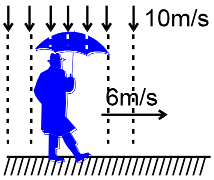
and direction $\theta = \tan^{-1}\left(\frac{v_m}{v_R}\right)$ with the vertical as shown in figure



Solved Examples

Example 21. Rain is falling vertically at speed of 10 m/s and a man is moving with velocity 6 m/s. Find the angle at which the man should hold his umbrella to avoid getting wet.

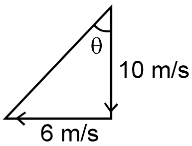
Solution :



$\vec{v}_{rain} = -10\,\hat{j} \Rightarrow \vec{v}_{man} = 6\,\hat{j}$

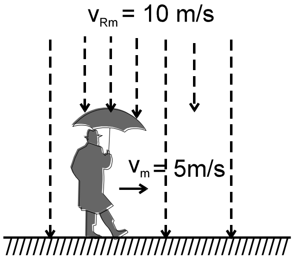
$\vec{v}_{r.w.r.t.\ man} = -10\,\hat{j} - 6\,\hat{i}$

$\tan\theta = \frac{6}{10} \Rightarrow \theta = \tan^{-1}\left(\frac{3}{5}\right)$



Where θ is angle with vertical

Example 22. A man moving with 5m/s observes rain falling vertically at the rate of 10 m/s. Find the speed and direction of the rain with respect to ground.



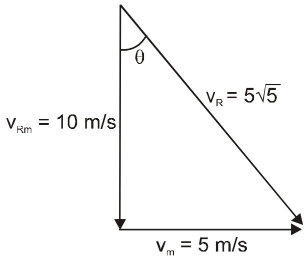
Solution : $v_{RM} = 10$ m/s, $v_M = 5$ m/s

$\vec{v}_{RM} = \vec{v}_R - \vec{v}_M$

$\Rightarrow \vec{v}_R = \vec{v}_{RM} + \vec{v}_M$

$\Rightarrow \vec{v}_R = 5\sqrt{5}$

$\tan\theta = \frac{1}{2}$, $\theta = \tan^{-1}\frac{1}{2}$.



Example 23. A standing man, observes rain falling with velocity of 20 m/s at an angle of 30º with the vertical.
(i) Find the velocity with which the man should move so that rain appears to fall vertically to him.
(ii) Now if he further increases his speed, rain again appears to fall at 30º with the vertical. Find his new velocity.

Solution : (i) $\vec{v}_m = -v\hat{i}$ (let)

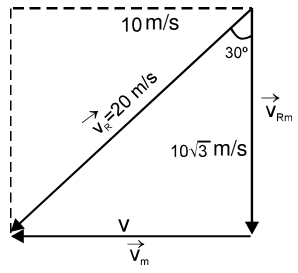
$\vec{v}_R = -10\hat{i} - 10\sqrt{3}\,\hat{j}$

$\vec{v}_{RM} = -(10\hat{i} - v) - 10\sqrt{3}\,\hat{j}$

$\Rightarrow\ -(10 - v) = 0$

(for vertical fall, horizontal component must be zero)

or $v = 10$ m/s **Ans.**



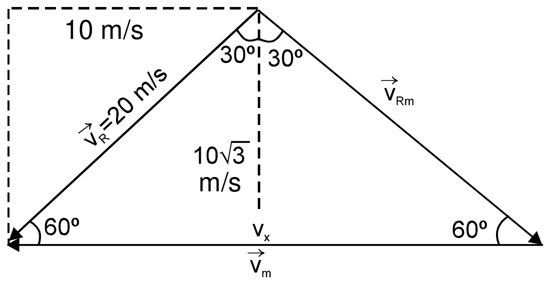
(ii) $\vec{v}_R = -10\hat{i} - 10\sqrt{3}\,\hat{j}$

$\vec{v}_m = -v_x\hat{i}$

$\vec{v}_{RM} = -(10 - v_x)\,\hat{i} - 10\sqrt{3}\,\hat{j}$

Angle with the vertical = 30º

$\Rightarrow\ \tan 30º = \dfrac{10 - v_x}{-10\sqrt{3}} \quad \Rightarrow\ v_x = 20$ m/s



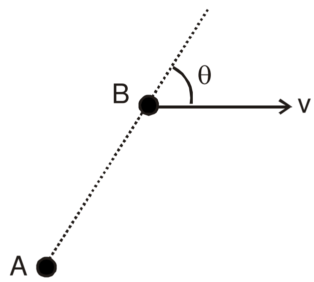
7. VELOCITY OF APPROACH / SEPARATION IN TWO DIMENSION

It is the component of relative velocity of one particle w.r.t. another, along the line joining them.

If the separation is decreasing, we say it is velocity of approach and if separation is increasing, then we say it is velocity of separation.

Solved Examples

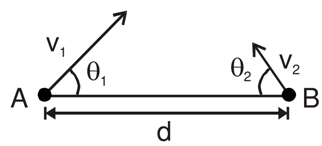
Example 24. Particle A is at rest and particle B is moving with constant velocity v as shown in the diagram at t = 0. Find their velocity of separation



Solution : $v_{BA} = v_B - v_A = v$

v_{sep} = component of v_{BA} along line AB = $v\cos\theta$

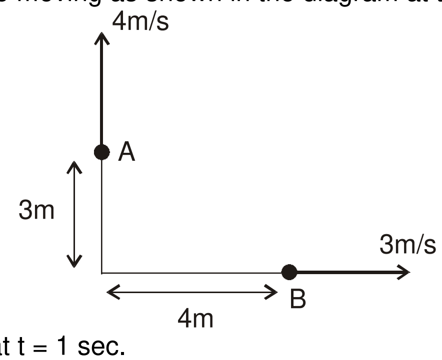
Example 25. Two particles A and B are moving with constant velocities v_1 and v_2. At t = 0, v_1 makes an angle θ_1 with the line joining A and B and v_2 makes an angle θ_2 with the line joining A and B. Find their velocity of approach.



Solution : Velocity of approach is relative velocity along line AB

$v_{APP} = v_1\cos\theta_1 + v_2\cos\theta_2$

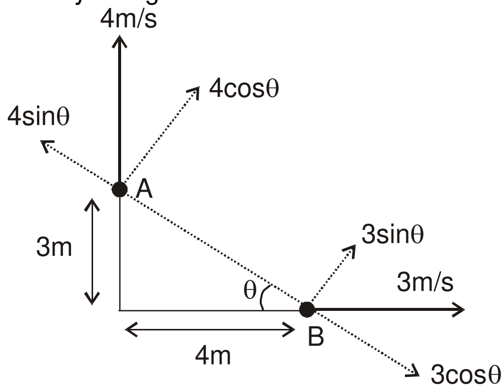
Example 26. Particles A and B are moving as shown in the diagram at t = 0. Find their velocity of separation



(i) at t = 0 (ii) at t = 1 sec.

Solution : (i) $\tan\theta = 3/4$

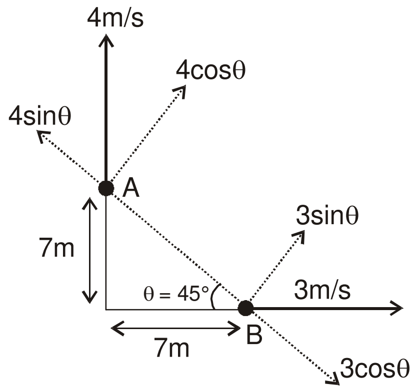
v_{sep} = relative velocity along line AB



$= 3\cos\theta + 4\sin\theta = 3.\frac{4}{5} + 4.\frac{3}{5} = \frac{24}{5} = 4.8$ m/s

(ii) $\theta = 45°$

v_{sep} = relative velocity along line AB



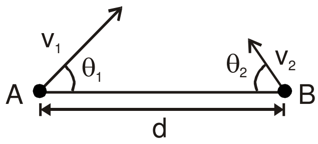
$= 3\cos\theta + 4\sin\theta = 3.\frac{1}{\sqrt{2}} + 4.\frac{1}{\sqrt{2}} = \frac{7}{\sqrt{2}}$ m/s

7.1 Condition for uniformly moving particles to collide

If two particles are moving with uniform velocities and the relative velocity of one particle w.r.t. other particle is directed towards each other then they will collide.

Solved Examples

Example 27. Two particles A and B are moving with constant velocities v_1 and v_2. At t = 0, v_1 makes an angle θ_1 with the line joining A and B and v_2 makes an angle θ_2 with the line joining A and B.



(i) Find the condition for A and B to collide.
(ii) Find the time after which A and B will collide if separation between them is d at t = 0

Solution : (i) For A and B to collide, their relative velocity must be directed along the line joining them. Therefore their relative velocity along the perpendicular to this line must be zero.
Thus $v_1 \sin\theta_1 = v_2 \sin\theta_2$.

(ii) $v_{APP} = v_1\cos\theta_1 + v_2\cos\theta_2$; $t = \dfrac{d}{v_{app}} = \dfrac{d}{v_1\cos\theta_1 + v_2\cos\theta_2}$

7.2 Minimum / Maximum distance between two particles

If the separation between two particles decreases and after some time it starts increasing then the separation between them will be minimum at the instant, velocity of approach changes to velocity of separation. (at this instant $v_{app} = 0$)

Mathematically S_{AB} is minimum when $\dfrac{dS_{AB}}{dt} = 0$

Similarly for maximum separation $v_{sep} = 0$.

Note :

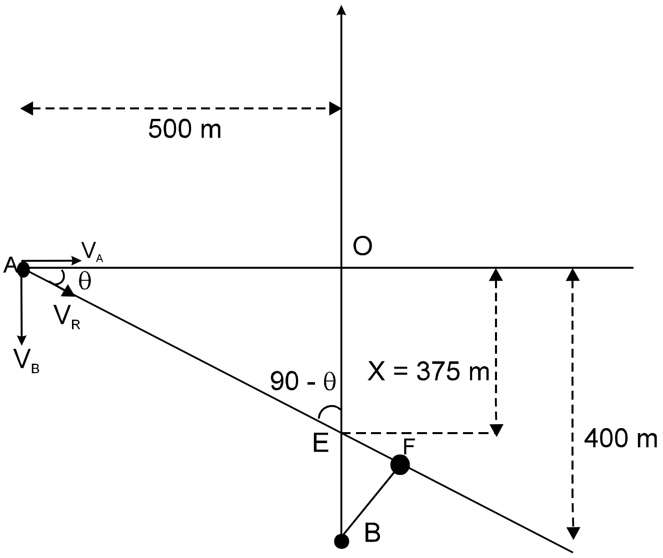
- If the initial position of two particles are $\vec{r}_1$ and $\vec{r}_2$ and their velocities are $\vec{v}_1$ and $\vec{v}_2$ then shortest distance between the particles, $d_{shortest} = \dfrac{|\vec{r}_{12} \times \vec{v}_{12}|}{|\vec{v}_{12}|}$ and time after which this situation will occur, $t = -\dfrac{\vec{r}_{12}\cdot\vec{v}_{12}}{|\vec{v}_{12}|^2}$

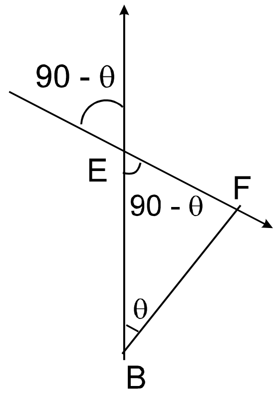
Solved Examples

Example 28. Two cars A and B are moving west to east and south to north respectively along crossroads. A moves with a speed of 72 kmh^{-1} and is 500 m away from point of intersection of cross roads and B moves with a speed of 54 kmh^{-1} and is 400 m away from point of intersection of cross roads. Find the shortest distance between them?

Solution : **Method – I (Using the concept of relative velocity)**
In this method we watch the velocity of A w.r.t. B. To do this we plot the resultant velocity V_r. Since the accelerations of both the bodies is zero, so the relative acceleration between them is also zero. Hence the relative velocity will remain constant. So the path of A with respect to B will be straight line and along the direction of relative velocity of A with respect to B. The shortest distance between A & B is when A is at point F (i.e., when we drop a perpendicular from B on the line of motion of A with respect to B).

From figure





$\tan\theta = \frac{V_B}{V_A} = \frac{15}{20} = \frac{3}{4}$ ....(i)

This θ is the angle made by the resultant velocity vector with the x-axis.
Also we know that from figure

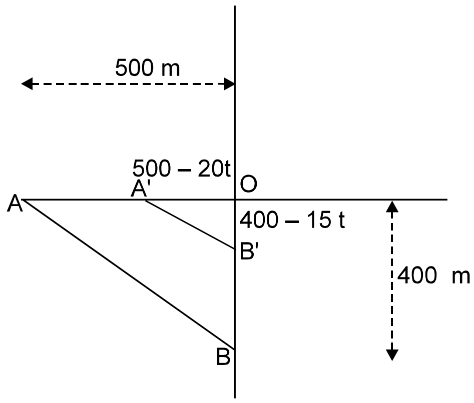
$OE = \frac{x}{500} = \frac{3}{4}$ ....(ii)

From equation (i) & (ii) we get
x = 375 m
∴ EB = OB – OE = 400 – 375 = 25 m
But the shortest distance is BF.

From magnified figure we see that BF = EB cosθ = $25 \times \frac{4}{5}$

∴ BF = 20 m

Method II (Using the concept of maxima – minima)
A & B be are the initial positions and A', B' be the final positions after time t.
B is moving with a speed of 15 m/sec so it will travel a distance of BB' = 15t during time t.
A is moving with a speed of 20 m/sec so it will travel a distance of AA' = 20t during time t.
So
OA' =500 – 20 t
OB' = 400 – 15 t
∴ $A'B'^2 = OA'^2 + OB'^2$
$= (500 - 20t)^2 + (400 - 15t)^2$ .....(i)
For A'B' to be minimum $A'B'^2$ should also be minimum



$\therefore \frac{d(A'B'^2)}{dt} = \frac{d(400-15t)^2 + (500-20t)^2}{dt} = 0$

= 2(400 – 15t) (–15) + 2(500 – 20t) (–20) = 0
= – 1200 + 45t = 2000 – 80 t
∴ 125 t = 3200

∴ $t = \frac{128}{5}$ s.

Hence A and B will be closest after $\frac{128}{5}$ s.

Now $\frac{d^2A'B'}{dt^2}$ comes out to be positive hence it is a minima.

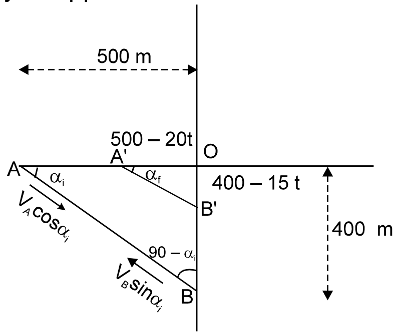
On substituting the value of t in equation (i) we get

$\therefore$ $A'B'^2 = \left(400 - 15\times\frac{128}{5}\right)^2 + \left(500 - 20.\frac{128}{5}\right)^2 = \sqrt{16^2 + (-12)^2} = 20$ m

$\therefore$ Minimum distance A'B' = 20 m.

Method III (Using the concept of relative velocity of approach)

After time t let us plot the components of velocity of A and B in the direction along AB. When the distance between the two is minimum, the relative velocity of approach is zero.



$\therefore$ $V_A \cos\alpha_f + V_B \sin\alpha_f = 0$

(where α_f is the angle made by the line A'B' with the x-axis)

$20 \cos\alpha_f = -15 \sin\alpha_f$

$\therefore$ $\tan\alpha_f = -\frac{20}{15} = -\frac{4}{3}$

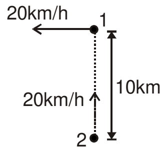
(Here do not confuse this angle with the angle θ in method (I) because that θ is the angle made by the net relative velocity with x-axis, but α_f is the angle made by the line joining the two particles with x-axis when velocity of approach in zero.)

$\therefore$ $\frac{400 - 15t}{500 - 20t} = -\frac{4}{3}$

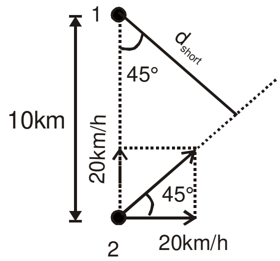
$\therefore$ $t = \frac{128}{5}$ So, OB' = 16 m and OA' = – 12m

$A'B' = \sqrt{16^2 + (-12)^2} = 20$ m

Example 29. Two ships are 10 km apart on a line joining south to north. The one farther north is steaming west at 20 km h^{-1}. The other is steaming north at 20 km h^{-1}. What is their distance of closest approach ? How long do they take to reach it ?



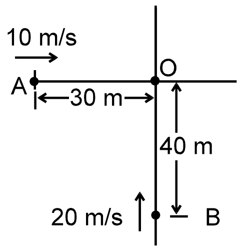
Solution : Solving from the frame of particle -1



we get $d_{short} = 10 \cos 45° = \frac{10}{\sqrt{2}} = 5\sqrt{2}$ km

$t = \frac{10\sin 45}{|\vec{V}_{21}|} = \frac{10\times 1/\sqrt{2}}{20\sqrt{2}} = \frac{1}{4}$ h = 15 min.

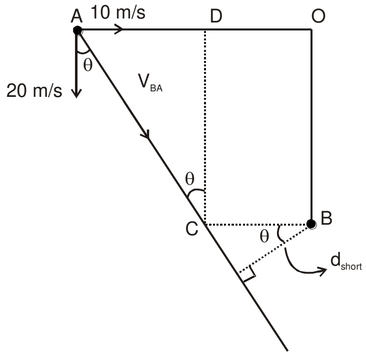
Example 30. Two particles A and B are moving with uniform velocity as shown in the figure given below at t = 0.



(i) Will the two particle collide

(ii) Find out shortest distance between two particles

Solution : Solving from the frame of B



we get $\tan\theta = \frac{10}{20} = \frac{1}{2}$

again $\tan\theta = \frac{AD}{CD} = \frac{AD}{40} = \frac{1}{2}$

$\Rightarrow$ AD = 20 $\Rightarrow$ DO = 10 $\Rightarrow$ BC = 10

d_{short} = BC cosθ = 10 cosθ = $\frac{10\times 2}{\sqrt{5}} = 4\sqrt{5}$ m

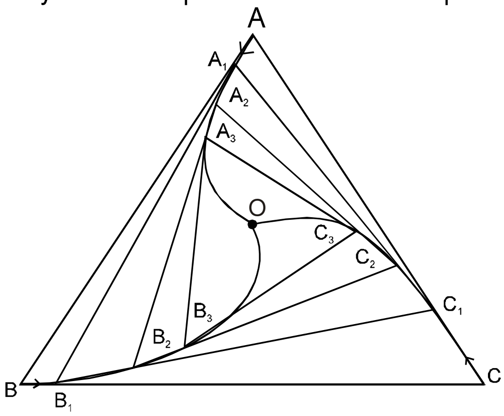
Since closest distance is non zero therefore they will not collide

7.3 Miscellaneous Problems on collision

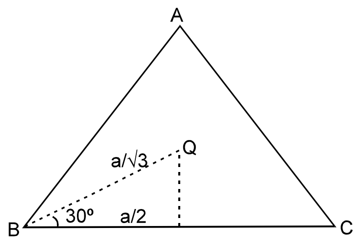
Solved Examples

Example 31. There are particles A, B and C are situated at the vertices of an equilateral triangle ABC of side a at t = 0. Each of the particles moves with constant speed v. A always has its velocity along AB, B along BC and C along CA. At what time will the particle meet each other?

Solution : The motion of the particles is roughly sketched in figure. By symmetry they will meet at the centroid O of the triangle. At any instant the particles will from an equilateral triangle ABC with the same



Centroid O. All the particles will meet at the centre. Concentrate on the motion of any one particle, say B. At any instant its velocity makes angle 30º with BO. The component of this velocity along BO is v cos 30º. This component is the rate of decrease of the distance BO. Initially

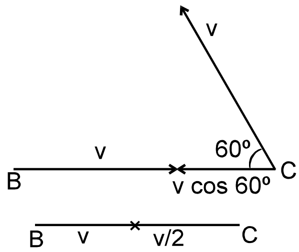


BO = $\frac{a/2}{\cos 30^\circ} = \frac{a}{\sqrt{3}}$ = displacement of each particle. Therefore,

the time taken for BO to become zero

$= \frac{a/\sqrt{3}}{v\cos 30^\circ} = \frac{2a}{\sqrt{3}v\times\sqrt{3}} = \frac{2a}{3v}$.

Aliter : Velocity of B is v along BC. The velocity of C is along CA. Its component along BC is v cos 60º = v/2. Thus, the separation BC decreases at the rate of approach velocity.



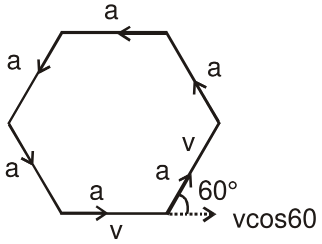
$\therefore$ approach velocity = v + $\frac{v}{2} = \frac{3v}{2}$

Since, the rate of approach is constant, the time taken in reducing

the separation BC from a to zero is t = $\frac{a}{\frac{3v}{2}} = \frac{2a}{3v}$

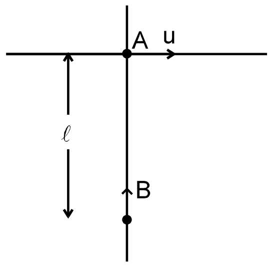
Example 32. Six particles situated at the corners of a regular hexagon of side a move at a constant speed v. Each particle maintains a direction towards the particle at the next corner. Calculate the time the particles will take to meet each other.

Solution : $V_{app} = V - V\cos 60° = V - V/2 = V/2$



$$t = \frac{a}{V_{app}} = \frac{a}{V/2} = \frac{2a}{V}$$

Example 33. 'A' moves with constant velocity u along the 'x' axis. B always has velocity towards A. After how much time will B meet A if B moves with constant speed v. What distance will be travelled by A and B.



Solution : Let at any instant the velocity of B makes an angle α with that of x axis and the time to collide is T.

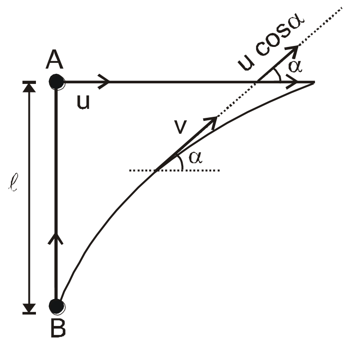
$v_{app} = v - u\cos\alpha$

$$\ell = \int_0^T v_{app}\,dt = \int_0^T (v - u\cos\alpha)dt \quad .... (1)$$

Now equating the displacement of A and B along x direction we get

$$uT = \int v\cos\alpha\,dt \quad .... (2)$$

Now from (1) and (2) we get



$$\ell = vT - \int_0^T u\cos\alpha\,dt = vT - \frac{u}{v}\int_0^T v\cos\alpha\,dt = vT - \frac{u}{v}.\,uT$$

$$\Rightarrow T = \frac{\ell v}{v^2 - u^2}$$

Now distance travelled by A and B

$$= u \times \frac{\ell v}{v^2 - u^2} \text{ and } v \times \frac{\ell v}{v^2 - u^2} = \frac{uv\ell}{v^2 - u^2} \text{ and } \frac{v^2\ell}{v^2 - u^2}$$

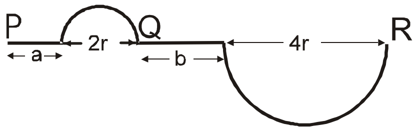
Exercise-1

✍ **Marked Questions can be used as Revision Questions.**

PART - I : SUBJECTIVE QUESTIONS

SECTION (A) : RECTILINEAR MOTION

A-1.✍ A car starts from P and follows the path as shown in figure. Finally car stops at R. Find the distance travelled and displacement of the car if a = 7 m, b = 8 m and r = $\frac{11}{\pi}$ m ? [Take $\pi = \frac{22}{7}$]



A-2. A man moves to go 50 m due south, 40 m due west and 20 m due north to reach a field.
(a) What distance does be have to walk to reach the field ?
(b) What is his displacement from his house to the field ?

A-3. A particle covers each 1/3 of the total distance with speed v_1, v_2 and v_3 respectively. Find the average speed of the particle ?

A-4. The position of a body is given by $x = At + 4Bt^3$, where A and B are constants, x is position and t is time. Find (a) acceleration as a function of time, (b) velocity and acceleration at t = 5 s.

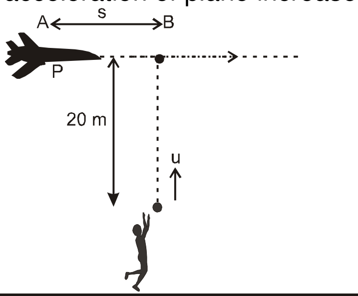
A-5. A boy start towards east with uniform speed 5m/s. After t = 2 second he turns right and travels 40 m with same speed. Again he turns right and travels for 8 second with same speed. Find out the displacement; average speed, average velocity and total distance travelled.

A-6.✍ A train starts from rest and moves with a constant acceleration of 2.0 m/s^2 for half a minute. The brakes are then applied and the train comes to rest in one minute after applying breaks. Find (a) the total distance moved by the train, (b) the maximum speed attained by the train and (c) the position(s) of the train at half the maximum speed. (Assume retardation to be constant)

A-7. A particle moving along a straight line with constant acceleration is having initial and final velocity as 5 m/s and 15 m/s respectively in a time interval of 5 s. Find the distance travelled by the particle and the acceleration of the particle. If the particle continues with same acceleration, find the distance covered by the particle in the 8th second of its motion. (Direction of motion remains same)

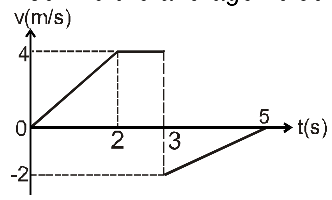
A-8. A ball is dropped from a tower. In the last second of its motion it travels a distance of 15 m. Find the height of the tower. [Take g = 10m/sec^2]

A-9.✍ A toy plane P starts flying from point A along a straight horizontal line 20 m above ground level starting with zero initial velocity and acceleration 2 m/s^2 as shown. At the same instant, a man P throws a ball vertically upwards with initial velocity 'u'. Ball touches (coming to rest) the base of the plane at point B of plane's journey when it is vertically above the man. 's' is the distance of point B from point A. Just after the contact of ball with the plane, acceleration of plane increases to 4 m/s^2. Find:

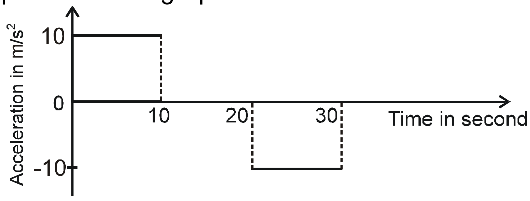


(i) Initial velocity 'u' of ball.
(ii) Distance 's'.
(iii) Distance between man and plane when the man catches the ball back. (g = 10 m/s^2) (Neglect the height of man)

A-10. For a particle moving along x-axis, velocity-time graph is as shown in figure. Find the distance travelled and displacement of the particle? Also find the average velocity of the particle in intervel 0 to 5 second.



A-11. A cart started at t = 0, its acceleration varies with time as shown in figure. Find the distance travelled in 30 seconds and draw the position-time graph.



A-12. Two particles A and B start from rest and move for equal time on a straight line. The particle A has an acceleration a for the first half of the total time and 2a for the second half. The particle B has an acceleration 2a for the first half and a for the second half. Which particle has covered larger distance?

SECTION (B) : PROJECTILE MOTION

B-1. Two bodies are projected at angles θ and $(90 - \theta)$ to the horizontal with the same speed. Find the ratio of their times of flight?

B-2. In above question find the ratio of the maximum vertical heights ?

B-3. A projectile can have the same range R for two angles of projections at a given speed. If T_1 & T_2 be the times of flight in two cases, then find out relation between T_1, T_2 and R ?

B-4. The direction of motion of a projectile at a certain instant is inclined at an angle α to the horizontal. After t seconds it is inclined an angle β. Find the horizontal component of velocity of projection in terms of g, t, α and β. (α and β are positive in anticlockwise direction)

B-6. A gun kept on a straight horizontal road is used to hit a car, travelling along the same road away from the gun with a uniform speed of $72 \times \sqrt{2}$ km/hour. The car is at a distance of 50 metre from the gun, when the gun is fired at an angle of 45° with the horizontal. Find (i) the distance of the car from the gun when the shell hits it, (ii) the speed of projection of the shell from the gun. [$g = 10$ m/s^2] **[IIT 1974]**

B-7. A projectile is fired horizontally with a velocity of 98 m/s from the top of a hill 490 m high. Find : (Take $g = 9.8$ m/s^2)
(i) The time taken to reach the ground
(ii) The distance of the target from the foot of hill
(iii) The velocity with which the particle hits the ground

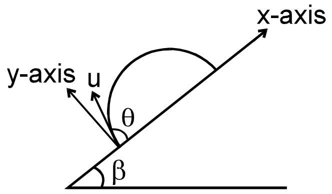
B-8. From the top of a tower of height 50m a ball is projected upwards with a speed of 30 m/s at an angle of 30° to the horizontal. Then calculate -
(i) Maximum height from the ground
(ii) At what distance from the foot of the tower does the projectile hit the ground.
(iii) Time of flight.

B-9. The equation of a projectile is $y = \sqrt{3}\,x - \frac{gx^2}{2}$, find the angle of projection. Also find the speed of projection. Where at t = 0, x = 0 and y = 0 also $\frac{d^2x}{dt^2} = 0$ & $\frac{d^2y}{dt^2} = -g$.

B-10. A bullet is fired from horizontal ground at some angle passes through the point $\left(\frac{3R}{4}, \frac{R}{4}\right)$, where 'R' is the range of the bullet. Assume point of the fire to be origin and the bullet moves in x-y plane with x-axis horizontal and y-axis vertically upwards. Angle of projection is $\frac{\alpha\pi}{180}$ radian. Find α :

B-11. The radius vector of a point A relative to the origin varies with time t as $\vec{r} = at\hat{i} - bt^2\hat{j}$, where a and b are positive constants and $\hat{i}$ and $\hat{j}$ are the unit vectors of the x and y axes. Find:
(i) The equation of the point's trajectory y (x) ; plot this function
(ii) The time dependence of the velocity **v** and acceleration **a** vectors as well as of the moduli of these quantities.

B-12. A particle is projected at an angle θ with an inclined plane making an angle β with the horizontal as shown in figure, speed of the particle is u, after time t find :



(a) x component of acceleration ?
(b) y component of acceleration ?
(c) x component of velocity ?
(d) y component of velocity ?
(e) x component of displacement ?
(f) y component of displacement ?
(g) y component of velocity when particle is at maximum distance from the incline plane ?

SECTION (C) : RELATIVE MOTION

C-1. Two parallel rail tracks run north-south. Train A moves due north with a speed of 54 km h^{-1} and train B moves due south with a speed of 90 km h^{-1}. A monkey runs on the roof of train A with a velocity of 18 km/h w.r.t. train A in a direction opposite to that of A. Calculate the (a) relative velocity of B with respect to A (b) relative velocity of ground with respect to B (c) velocity of a monkey as observed by a man standing on the ground. (d) velocity of monkey as observed by a passenger of train B.

C-2. A train is moving with a speed of 40 km/h. As soon as another train going in the opposite direction passes by the window, the passenger of the first train starts his stopwatch and notes that other train passes the window in 3 s. Find the speed of the train going in the opposite direction if its length is 75 m.

C-3. The driver of a train A running at 25 ms^{-1} sights a train B moving in the same direction on the same track with 15 ms^{-1}. The driver of train A applies brakes to produce a deceleration of 1.0 ms^{-2}. What should be the minimum distance between the trains to avoid the accident.

C-4. Two perpendicular rail tracks have two trains A & B respectively. Train A moves towards north with a speed of 54 km h^{-1} and train B moves towards west with a speed of 72 km h^{-1}. Assume that both trains start from same point. Calculate the
(a) Relative velocity of ground with respect to B
(b) Relative velocity of A with respect to B.
(c) Rate of separation of the two trains

C-5. A man is swimming in a lake in a direction of 30° East of North with a speed of 5 km/h and a cyclist is going on a road along the lake shore towards East at a speed of 10 km/h. In what direction and with what speed would the man appear to swim to the cyclist.

C-6. A motorboat is observed to travel 10 km h^{-1} relative to the earth in the direction 37º north of east. If the velocity of the boat due to the wind only is 2 km h^{-1} westward and that due to the current only is 4 km h^{-1} southward, what is the magnitude and direction of the velocity of the boat due to its own power?

C-7. A ship is sailing towards north at a speed of $\sqrt{2}$ m/s. The current is taking it towards East at the rate of 1 m/s and a sailor is climbing a vertical pole on the ship at the rate of 1 m/s. Find the velocity of the sailor with respect to ground.

C-8. A man can swim with a speed of 4 km h^{-1} in still water. How long does he take to cross a river 1 km wide if the river flows steadily at 3 km h^{-1} and he makes his strokes normal to the river current ? How far down the river does he go when he reaches the other bank ?

C-9. A river is flowing from west to east at a speed of 5 m/min. A man on the south bank of the river, capable of swimming at 10 m/min in still water, swims across the shortest path distance. In what direction should he swim ?

C-10. Rain seems to be falling vertically to a person sitting in a bus which is moving uniformly eastwards with 10 m/s. It appears to come from vertical at a velocity 20 m/s. Find the speed of rain drops with respect to ground.

C-11. To a man walking at the rate of 2 km/hour with respect to ground, the rain appears to fall vertically. When he increases his speed to 4 km/hour in same direction of his motion, rain appears to meet him at an angle of 45º with horizontal, find the real direction and speed of the rain.

C-12. A particle is kept at rest at origin. Another particle starts from (5m, 0) with a velocity of $-4\hat{i} + 3\hat{j}$ m/s. Find their closest distance of approach.

C-13. Four particles situated at the corners of a square of side 'a', move at a constant speed v. Each particle maintains a direction towards the next particle in succession. Calculate the time the particles will take to meet each other.

C-14. When two bodies move uniformly towards each other, the distance between them diminishes by 16 m every 10 s. If bodies move with velocities of the same magnitude and in the same direction as before the distance between then will decrease 3 m every 5 s. Calculate the velocity of each body.

PART - II : ONLY ONE OPTION CORRECT TYPE

SECTION (A) : RECTILINEAR MOTION

A-1. A hall has the dimensions 10 m × 10 m × 10 m. A fly starting at one corner ends up at a farthest corner. The magnitude of its displacement is:

(A) $5\sqrt{3}$ m (B) $10\sqrt{3}$ m (C) $20\sqrt{3}$ m (D) $30\sqrt{3}$ m

A-2. A person travelling on a straight line without changing direction moves with a uniform speed v_1 for half distance and next half distance he covers with uniform speed v_2. The average speed v is given by

(A) $v = \frac{2v_1v_2}{v_1+v_2}$ (B) $v = \sqrt{v_1v_2}$ (C) $\frac{v_1+v_2}{2}$ (D) $\frac{1}{v} = \frac{1}{v_1} + \frac{1}{v_2}$

A-3. A stone is thrown vertically upward with an initial speed u from the top of a tower, reaches the ground with a speed 3u. The height of the tower is:

(A) $\frac{3u^2}{g}$ (B) $\frac{4u^2}{g}$ (C) $\frac{6u^2}{g}$ (D) $\frac{9u^2}{g}$

A-4. A particle starts from rest with uniform acceleration a. Its velocity after n seconds is v. The displacement of the particle in the last two seconds is :

(A) $\frac{2v(n-1)}{n}$ (B) $\frac{v(n-1)}{n}$ (C) $\frac{v(n+1)}{n}$ (D) $\frac{2v(2n+1)}{n}$

A-5. A body starts from rest and is uniformly acclerated for 30 s. The distance travelled in the first 10 s is x_1, next 10 s is x_2 and the last 10 s is x_3. Then $x_1 : x_2 : x_3$ is the same as

(A) 1 : 2 : 4 (B) 1 : 2 : 5 (C) 1 : 3 : 5 (D) 1 : 3 : 9

A-6. A ball is dropped from the top of a building. The ball takes 0.5 s to fall past the 3 m height of a window some distance from the top of the building. If the speed of the ball at the top and at the bottom of the window are v_T and v_B respectively, then (g = 9.8 m/sec^2)

(A) $v_T + v_B = 12$ ms^{-1} (B) $v_T - v_B = 4.9$ m s^{-1} (C) $v_Bv_T = 1$ ms^{-1} (D) $\frac{v_B}{v_T} = 1$ ms^{-1}

A-7. A stone is released from an elevator going up with an acceleration a and speed u. The acceleration and speed of the stone just after the release is

(A) a upward, zero (B) (g-a) upward, u (C) (g-a) downward, zero (D) g downward, u

A-8. The initial velocity of a particle is given by u (at t = 0) and the acceleration by f, where f = at (here t is time and a is constant). Which of the following relation is valid ?

(A) $v = u + at^2$ (B) $v = u + \frac{at^2}{2}$ (C) $v = u + at$ (D) $v = u$

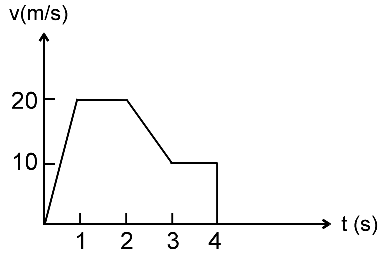
A-9. A student determined to test the law of gravity for himself walks off a sky scraper 320 m high with a stopwatch in hand and starts his free fall (zero initial velocity). 5 second later, superman arrives at the scene and dives off the roof to save the student. What must be superman's initial velocity in order that he catches the student just before reaching the ground ? [Assume that the superman's acceleration is that of any freely falling body.] ($g = 10\ m/s^2$)

(A) 98 m/s (B) $\frac{275}{3}$ m/s (C) $\frac{187}{2}$ m/s (D) It is not possible

A-10. In the above question, what must be the maximum height of the skyscraper so that even superman cannot save him.

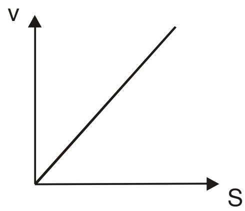
(A) 65 m (B) 85 m (C) 125 m (D) 145 m

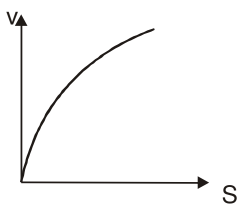
A-11. The variation of velocity of a particle moving along a straight line is shown in the figure. The distance travelled by the particle in 4 s is :

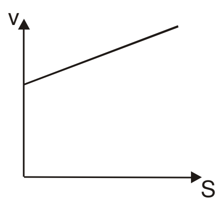


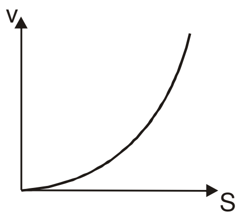
(A) 25 m (B) 30 m (C) 55 m (D) 60 m

A-12. A particle starts from rest and moves along a straight line with constant acceleration. The variation of velocity v with displacement S is :

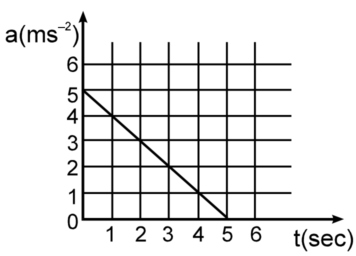
(A) 

(B) 

(C) 

(D) 

A-13. Starting from rest at t = 0, a car moves in a straight line with an acceleration given by the accompanying graph. The speed of the car at t = 3 s is :



(A) $1\ m\,s^{-1}$ (B) $2\ m\,s^{-1}$ (C) $6.0\ m\,s^{-1}$ (D) $10.5\ m\,s^{-1}$

SECTION (B) : PROJECTILE MOTION

B-1. It was calculated that a shell when fired from a gun with a certain velocity and at an angle of elevation $\frac{5\pi}{36}$ rad should strike a given target in the same horizontal plane. In actual practice, it was found that a hill just prevented the trajectory. At what angle of elevation should the gun be fired to hit the target :

(A) $\frac{5\pi}{36}$ rad (B) $\frac{11\pi}{36}$ rad (C) $\frac{7\pi}{36}$ rad (D) $\frac{13\pi}{36}$ rad.

B-2. A projectile is thrown with a speed v at an angle θ with the upward vertical. Its average velocity between the instants at which it crosses half the maximum height is

(A) v sin θ, horizontal and in the plane of projection
(B) v cos θ, horizontal and in the plane of projection
(C) 2v sin θ, horizontal and perpendicular to the plane of projection
(D) 2v cos θ, vertical and in the plane of projection.

B-3. A particle moves along the parabolic path $y = ax^2$ in such a way that the x component of the velocity remains constant, say c. The acceleration of the particle is

(A) $ac\,\hat{k}$ (B) $2ac^2\,\hat{j}$ (C) $ac^2\,\hat{j}$ (D) $a^2c\,\hat{j}$

B-4. The speed at the maximum height of a projectile is half of its initial speed u. Its range on the horizontal plane is:

(A) $\frac{2u^2}{3g}$ (B) $\frac{\sqrt{3}u^2}{2g}$ (C) $\frac{u^2}{3g}$ (D) $\frac{u^2}{2g}$

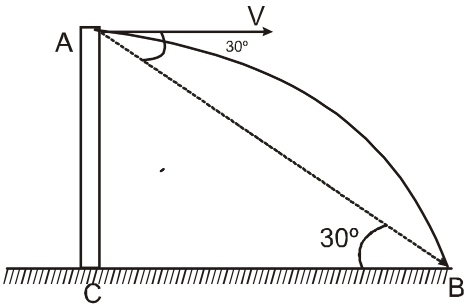
B-5. The velocity of projection of a projectile is $(6\hat{i} + 8\hat{j})$ ms^{-1}. The horizontal range of the projectile is (g = 10 m/sec^2)

(A) 4.9 m (B) 9.6 m (C) 19.6 m (D) 14 m

B-6. One stone is projected horizontally from a 20 m high cliff with an initial speed of 10 ms^{-1}. A second stone is simultaneously dropped from that cliff. Which of the following is true?

(A) Both strike the ground with the same speed.
(B) The stone with initial speed 10 ms^{-1} reaches the ground first.
(C) Both the stones hit the ground at the same time.
(D) The stone which is dropped from the cliff reaches the ground first.

B-7. An object is thrown horizontally from a point 'A' from a tower and hits the ground 3s later at B. The line from 'A' to 'B' makes an angle of 30º with the horizontal. The initial velocity of the object is : (take g = 10 m/s^2)

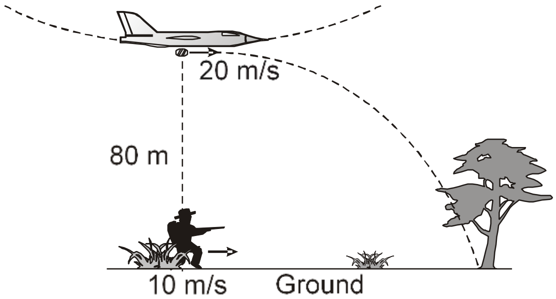


(A) $15\sqrt{3}$ m/s (B) 15 m/s (C) $10\sqrt{3}$ m/s (D) $25/\sqrt{3}$ m/s

B-8. A body is projected horizontally from the top of a tower with initial velocity 18 ms^{-1}. It hits the ground at angle 45º. What is the vertical component of velocity when it strikes the ground ?

(A) $18\sqrt{3}$ ms^{-1} (B) 18 ms^{-1} (C) $9\sqrt{2}$ ms^{-1} (D) 9 ms^{-1}

B-9. A bomber plane moving at a horizontal speed of 20 m/s releases a bomb at a height of 80 m above ground as shown. At the same instant a Hunter of negligible height starts running from a point below it, to catch the bomb with speed 10 m/s. After two seconds he realized that he cannot make it, he stops running and immediately holds his gun and fires in such direction so that just before bomb hits the ground, bullet will hit it. What should be the firing speed of bullet (Take g = 10 m/s^2)



(A) 10 m/s (B) $20\sqrt{10}$ m/s (C) $10\sqrt{10}$ m/s (D) None of these

B-10. A ball is projected from a certain point on the surface of a planet at a certain angle with the horizontal surface. The horizontal and vertical displacement x and y varies with time t in second as:

$x = 10\sqrt{3}t$ and $y = 10t - t^2$

The maximum height attained by the ball is

(A) 100 m (B) 75 m (C) 50 m (D) 25 m.

B-11. A plane surface is inclined making an angle θ with the horizontal. From the bottom of this inclined plane, a bullet is fired with velocity v. The maximum possible range of the bullet on the inclined plane is

(A) $\frac{v^2}{g}$ (B) $\frac{v^2}{g(1+\sin\theta)}$ (C) $\frac{v^2}{g(1-\sin\theta)}$ (D) $\frac{v^2}{g(1+\cos\theta)}$

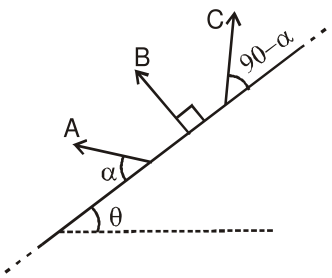
B-12. A ball is horizontally projected with a speed v from the top of a plane inclined at an angle 45º with the horizontal. How far from the point of projection will the ball strike the plane?

(A) $\frac{v^2}{g}$ (B) $\frac{\sqrt{2}v^2}{g}$ (C) $\frac{2v^2}{g}$ (D) $\left[\frac{2\sqrt{2}v^2}{g}\right]$

B-13. A particle is projected at angle 37° with the incline plane in upward direction with speed 10 m/s. The angle of incline plane is given 53°. Then the maximum distance from the incline plane attained by the particle will be –

(A) 3m (B) 4 m (C) 5 m (D) zero

B-14. Three stones A, B, C are projected from surface of very long inclined plane with equal speeds and different angles of projection as shown in figure. The incline makes an angle θ with horizontal. If H_A, H_B and H_C are maximum height attained by A, B and C respectively above inclined plane then : (Neglect air friction)



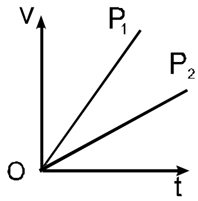
(A) $H_A + H_C = H_B$ (B) $H_A^2 + H_C^2 = H_B^2$ (C) $H_A + H_C = 2H_B$ (D) $H_A^2 + H_C^2 = 2H_B^2$

SECTION (C) : RELATIVE MOTION

C-1. An aeroplane is flying vertically upwards with a uniform speed of 500 m/s. When it is at a height of 1000 m above the ground a shot is fired at it with a speed of 700 m/s from a point directly below it. The minimum uniform acceleration of the aeroplane now so that it may escape from being hit ? ($g = 10$ m/s^2)
(A) 10 m/s^2 (B) 8 m/s^2 (C) 12 m/s^2 (D) None of these

C-2. A thief is running away on a straight road with a speed of 9 ms^{-1}. A police man chases him on a jeep moving at a speed of 10 ms^{-1}. If the instantaneous separation of the jeep from the motorcycle is 100m, how long will it take for the police man to catch the thief ?
(A) 1s (B) 19s (C) 90s (D) 100s

C-3. Shown in the figure are the velocity time graphs of the two particles P_1 and P_2. Which of the following statements about their relative motion is true? Magnitude of their relative velocity : (Consider 1-D motion)



(A) is zero (B) is non-zero but constant
(C) continuously decreases (D) continuously increases

C-4. Two trains A and B which are 100 km apart are travelling towards each other on different tracks with each having initial speed of 50 km/h. The train A accelerates at 20 km/h^2 and the train B retards at the rate 20 km/h^2. The distance covered by the train A when they cross each other is :
(A) 45 km (B) 55 km (C) 65 km (D) 60 km

C-5. A jet airplane travelling from east to west at a speed of 500 km h^{-1} eject out gases of combustion at a speed of 1500 km h^{-1} with respect to the jet plane. What is the velocity of the gases with respect to an observer on the ground ?
(A) 1000 km h^{-1} in the direction west to east (B) 1000 km h^{-1} in the direction east to west
(C) 2000 km h^{-1} in the direction west to east (D) 2000 km h^{-1} in the direction east to west

C-6. A ship is travelling due east at 10 km/h. A ship heading 30° east of north is always due north from the first ship. The speed of the second ship in km/h is -
(A) $20\sqrt{2}$ (B) 20 (C) $20\sqrt{3/2}$ (D) $20/\sqrt{2}$

C-7. Two billiard balls are rolling on a flat table. One has velocity components $v_x = 1$m/s, $v_y = \sqrt{3}$ m/s and the other has components $v_x = 2$m/s and $v_y = 2$ m/s. If both the balls start moving from the same point, the angle between their path is -
(A) 60° (B) 45° (C) 22.5° (D) 15°

C-8. A boat is rowed across a river (perpendicular to river flow) at the rate of 9 km/hr. The river flows at the rate of 12 km/hr. The velocity of boat in km/hr is:
(A) 14 (B) 15 (C) 16 (D) 17

C-9. A boat which can move with a speed of 5 m/s relative to water crosses a river of width 480 m flowing with a constant speed of 4 m/s. What is the time taken by the boat to cross the river along the shortest path :
(A) 80 s (B) 160 s (C) 240 s (D) 320 s

C-10. An airplane pilot sets a compass course due west and maintains an air speed of 240 km/h. After flying for $\frac{1}{2}$ h, he finds himself over a town that is 150 km west and 40 km south of his starting point. The wind velocity (with respect to ground) is :

(A) 100 km/h, 37° W of S (B) 100 km/h, 37° S of W
(C) 120 km/h, 37° W of S (D) 120 km/h, 37° S of W

C-11. It is raining vertically downwards with a velocity of 3 km h^{-1}. A man walks in the rain with a velocity of 4 kmh^{-1}. The rain drops will fall on the man with a relative velocity of :

(A) 1 kmh^{-1} (B) 3 kmh^{-1} (C) 4 kmh^{-1} (D) 5 kmh^{-1}

C-12. An aeroplane has to go along straight line from A to B, and back again. The relative speed with respect to wind is V. The wind blows perpendicular to line AB with speed υ. The distance between A and B is ℓ. The total time for the round trip is:

(A) $\frac{2\ell}{\sqrt{V^2-\upsilon^2}}$ (B) $\frac{2\upsilon\ell}{V^2-\upsilon^2}$ (C) $\frac{2V\ell}{V^2-\upsilon^2}$ (D) $\frac{2\ell}{\sqrt{V^2+\upsilon^2}}$

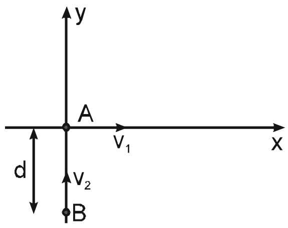
C-13. A person standing on the escalator takes time t_1 to reach the top of a tower when the escalator is moving. He takes time t_2 to reach the top of the tower when the escalator is standing. How long will he take if he walks up on a moving escalator ?

(A) $t_2 - t_1$ (B) $t_1 + t_2$ (C) $t_1 t_2/(t_1 - t_2)$ (D) $t_1 t_2/(t_1 + t_2)$

C-14. For two particles A and B, given that $\vec{r}_A = 2\hat{i} + 3\hat{j}$, $\vec{r}_B = 6\hat{i} + 7\hat{j}$, $\vec{v}_A = 3\hat{i} - \hat{j}$ and $\vec{v}_B = x\hat{i} - 5\hat{j}$. What is the value of x if they collide :

(A) 1 (B) – 1 (C) 2 (D) –2

C-15. Two particles A and B move with velocities v_1 and v_2 respectively along the x & y axis. The initial separation between them is ' d ' as shown in the figure. Find the least distance between them during their motion :



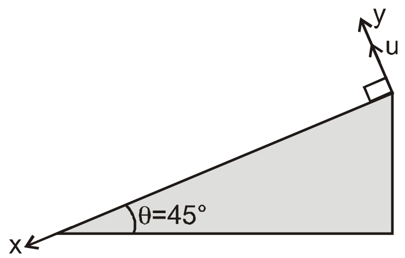
(A) $\frac{d.v_1^2}{v_1^2+v_2^2}$ (B) $\frac{d.v_2^2}{v_1^2+v_2^2}$ (C) $\frac{d.v_1}{\sqrt{v_1^2+v_2^2}}$ (D) $\frac{d.v_2}{\sqrt{v_1^2+v_2^2}}$

PART - III : MATCH THE COLUMN

1. **Match the following :**

(A) Rate of change of displacement	(p) Magnitude of average velocity
(B) Average speed is always greater than or equal to	(q) Initial to final position
(C) Displacement has the same direction as that of	(r) Velocity
(D) Motion under gravity is considered as the case of	(s) Uniform acceleration

2. An inclined plane makes an angle $\theta = 45°$ with horizontal. A stone is projected normally from the inclined plane, with speed u m/s at t = 0 sec. x and y axis are drawn from point of projection along and normal to inclined plane as shown. The length of incline is sufficient for stone to land on it and neglect air friction. Match the statements given in column I with the results in column II. (g in column II is acceleration due to gravity.)

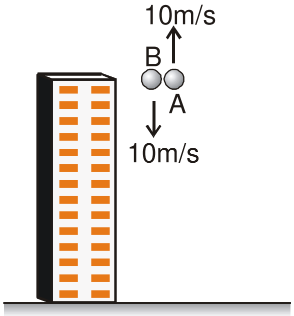
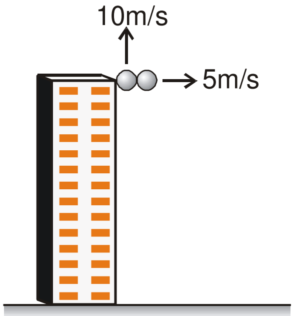
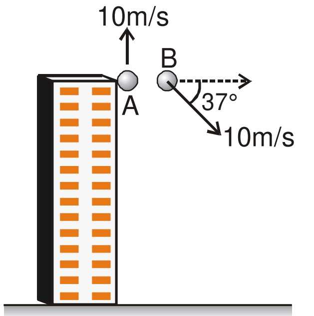
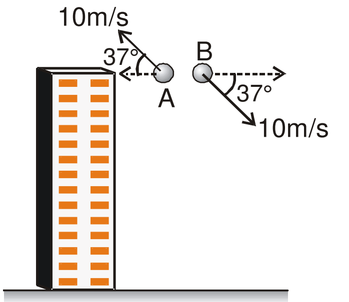


Column I	Column II
(A) The instant of time at which velocity of stone is parallel to x-axis	(p) $\frac{2\sqrt{2}u}{g}$
(B) The instant of time at which velocity of stone makes an angle $\theta = 45°$ with positive x-axis. in clockwise direction	(q) $\frac{2u}{g}$
(C) The instant of time till which (starting from t = 0) component of displacement along x-axis become half the range on inclined plane is	(r) $\frac{\sqrt{2}u}{g}$
(D) Time of flight on inclined plane is	(s) $\frac{u}{\sqrt{2}g}$

3. A particle is projected from level ground. Assuming projection point as origin, x-axis along horizontal and y-axis along vertically upwards. If particle moves in x-y plane and its path is given by $y = ax - bx^2$ where a, b are positive constants. Then match the physical quantities given in column-I with the values given in column-II. (g in column II is acceleration due to gravity.)

Column I	Column II
(A) Horizontal component of velocity	(p) $\frac{a}{b}$
(B) Time of flight	(q) $\frac{a^2}{4b}$
(C) Maximum height	(r) $\sqrt{\frac{g}{2b}}$
(D) Horizontal range	(s) $\sqrt{\frac{2a^2}{bg}}$

4. Both A & B are thrown simultaneously as shown from a very high tower.

	Column-I		Column-II
(A)		(p)	Distance between the two balls at two seconds is $16\sqrt{5}$ m.
(B)		(q)	distance between two balls at 2 seconds is 40 m.
(C)		(r)	distance between two balls at 2 sec is $10\sqrt{5}$ m.
(D)		(s)	Magnitude of relative velocity of B w.r.t A is $5\sqrt{2}$ m/s.
		(t)	magnitude of relative velocity of B with respect to A is $5\sqrt{5}$ m/s.

Exercise-2

✍ **Marked Questions can be used as Revision Questions.**

PART - I : ONLY ONE OPTION CORRECT TYPE

1. In the one-dimensional motion of a particle, the relation between position x and time t is given by $x^2 + 2x = t$ (here $x > 0$). Choose the correct statement :

(A) The retardation of the particle is $\frac{1}{4(x+1)^3}$ (B) The uniform acceleration of the particle is $\frac{1}{(x+1)^3}$

(C) The uniform velocity of the particle is $\frac{1}{(x+1)^3}$ (D) The particle has a variable acceleration of 4t + 6.

2.✍ A ball is thrown vertically upwards from the top of a tower of height h with velocity v. The ball strikes the ground after time.

(A) $\frac{v}{g}\left[1+\sqrt{1+\frac{2gh}{v^2}}\right]$ (B) $\frac{v}{g}\left[1-\sqrt{1+\frac{2gh}{v^2}}\right]$ (C) $\frac{v}{g}\left(1+\frac{2gh}{v^2}\right)^{1/2}$ (D) $\frac{v}{g}\left(1-\frac{2gh}{v^2}\right)^{1/2}$

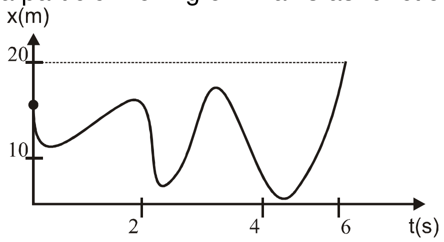
3.✍ A balloon is moving upwards with velocity 10 ms^{-1}. It releases a stone which comes down to the ground in 11 s. The height of the balloon from the ground at the moment when the stone was dropped is :

(A) 495 m (B) 592 m (C) 460 m (D) 500 m

4.✍ Water drops fall at regular intervals from a tap which is 5m above the ground. The third drop is leaving the tap at the instant the first drop touches the ground. How far above the ground is the second drop at that instant ? (Take g = 10 ms^{-2})

(A) $\frac{5}{4}$ m (B) 4 m (C) $\frac{5}{2}$ m (D) $\frac{15}{4}$ m

5. Figure shows the position of a particle moving on X-axis as function of time.



(A) The particle has come to rest 5 times
(B) Initial speed of particle was zero
(C) The velocity remains positive for t = 0 to t = 6 s
(D) The average velocity for the total period shown is negative.

6. A particle moves in the xy plane with only an x-component of acceleration of 2 m s^{-2}. The particle starts from the origin at t = 0 with an initial velocity having an x-component of 8 m s^{-1} and y-component of –15 ms^{-1}. Velocity of particle after time t is :

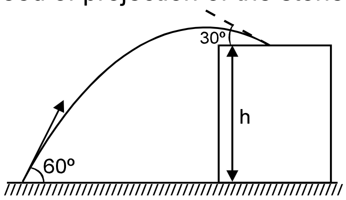
(A) $[(8 + 2t)\,\hat{i} - 15\,\hat{j}]$ m s^{-1} (B) zero

(C) $2t\,\hat{i} + 15\,\hat{j}$ (D) directed along z-axis.

7.✍ If R and h represent the horizontal range and maximum height respectively of an oblique projection whose start point (i.e. point of projecteion) & end point are in same horizontal level. Then $\frac{R^2}{8h}$ + 2h represents

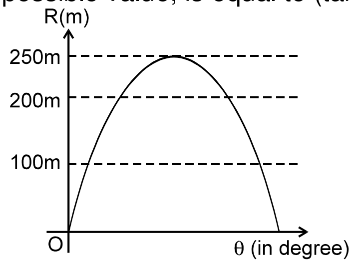
(A) maximum horizontal range (B) maximum vertical range
(C) time of flight (D) velocity of projectile at highest point

8. A stone projected at an angle of 60º from the ground level strikes at an angle of 30º on the roof of a building of height 'h'. Then the speed of projection of the stone is :



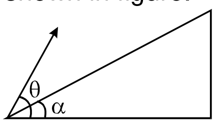
(A) $\sqrt{2gh}$ (B) $\sqrt{6gh}$ (C) $\sqrt{3gh}$ (D) $\sqrt{gh}$

9. From the ground level, a ball is to be shot with a certain speed. Graph shows the range R it will have versus the launch angle θ. The least speed the ball will have during its flight if θ is chosen such that the flight time is half of its maximum possible value, is equal to (take g = 10 m/s²)



(A) 250 m/s (B) 50 $\sqrt{3}$ m/s (C) 50 m/s (D) 25 $\sqrt{3}$ m/s

10. A projectile is fired at an angle θ with the horizontal. Find the condition under which it lands perpendicular on an inclined plane of inclination α as shown in figure.



(A) sin α = cos (θ – α) (B) cos α = sin (θ – α) (C) tan θ = cot (θ – α) (D) cot(θ – α) = 2tanα

11. A ball is thrown eastward across level ground. A wind blows horizontally to the east, and assume that the effect of wind is to provide a constant force towards the east, equal in magnitude to the weight of the ball. The angle θ (with horizontal east) at which the ball should be projected so that it travels maximum horizontal distance is

(A) 45° (B) 37° (C) 53° (D) 67.5°

12. Two cars are moving in the same direction with a speed of 30 km h^{-1}. They are separated from each other by 5 km. Third car moving in the opposite direction meets the two cars after an interval of 4 minutes. What is the speed of the third car?

(A) 35 km h^{-1} (B) 40 km h^{-1} (C) 45 km h^{-1} (D) 75 km h^{-1}

13. A bus is moving with a velocity 10 ms^{-1} on a straight road. A scooterist wishes to overtake the bus in 100s. If, the bus is at a distance of 1 km from the scooterist, with what velocity should the scooterist chase the bus? (Neglect size of the bus)

(A) 50 ms^{-1} (B) 40 ms^{-1} (C) 30 ms^{-1} (D) 20 ms^{-1}

14. A coin is released inside a lift at a height of 2 m from the floor of the lift. The height of the lift is 10 m. The lift is moving with an acceleration of 11 m/s² downwards. The time after which the coin will strike with the lift is :

(A) 4 s (B) 2 s (C) $\frac{4}{\sqrt{21}}$ s (D) $\frac{2}{\sqrt{11}}$ s

15. A police van moving on a highway with a speed of 30 km h^{-1} fires a bullet at a thief's car speeding away in the same direction with a speed of 192 km h^{-1}. If the muzzle speed of the bullet is 150 ms^{-1}, with what speed does the bullet hit the thief's car (as, seen by thief). According to thief in the car ?

(A) 105 m/s (B) 100 m/s (C) 110 m/s (D) 90 m/s

16. A bucket is placed in the open where the rain is falling vertically. If a wind begins to blow horizontally at double the velocity of the rain, how will be rate of filling of the bucket change?
(A) Remain unchanged (B) Doubled (C) Halved (D) Become four times

17. For four particles A, B, C & D the velocities of one with respect to other are given as $\vec{V}_{DC}$ is 20 m/s towards north, $\vec{V}_{BC}$ is 20 m/s towards east and $\vec{V}_{BA}$ is 20 m/s towards south. Then $\vec{V}_{DA}$ is
(A) 20 m/s towards north (B) 20 m/s towards south
(C) 20 m/s towards east (D) 20 m/s towards west

18. A battalion of soldiers is ordered to swim across a river 500 m wide. At what minimum rate should they swim perpendicular to river flow in order to avoid being washed away by the waterfall 300 m downstream. The speed of current being 3 m/sec :
(A) 6 m/sec. (B) 5 m/sec. (C) 4 m/sec (D) 2 m/sec

19. A man crosses the river perpendicular to river flow in time t seconds and travels an equal distance down the stream in T seconds. The ratio of man's speed in still water to the speed of river water will be :
(A) $\frac{t^2 - T^2}{t^2 + T^2}$ (B) $\frac{T^2 - t^2}{T^2 + t^2}$ (C) $\frac{t^2 + T^2}{t^2 - T^2}$ (D) $\frac{T^2 + t^2}{T^2 - t^2}$

20. A man is going up in a lift (open at the top) moving with a constant velocity 3 m/s. He throws a ball up at 5 m/sec relative to the lift when the lift is 50 m above the ground. Height of the lift when the ball meets it during its downward journey is (g = 10 m/s^2) **[Olympiad (Stage-1) 2017]**
(A) 53 m (B) 58 m (C) 63 m (D) 68 m

PART - II : SINGLE AND DOUBLE VALUE INTEGER TYPE

1. A particle moving in straight line, traversed half the distance with a velocity v_0. The remaining part of the distance was covered with velocity v_1 for half the time and with velocity v_2 for the other half of the time. Mean velocity of the particle averaged over the whole time of motion comes out to be $av_0\left(\frac{v_1 + v_2}{b\,v_0 + v_1 + v_2}\right)$, where a and b are positive integers. Find a + b.

2. A man walking with a speed v = 2 m/s constant in magnitude and direction passes under a lantern hanging at a height H above the ground (consider lantern as a point source). Find the velocity in m/s with which the edge of the shadow of the man's head moves over the ground, if his height is ' h ' given that H = 3h.

3. The displacement of a particle moving on a straight line is given by x = 16t – 2t^2. Distance travelled by the particle during the first 2 sec. is S_1 and during first 6 sec. is S_2. Find $\frac{3S_2}{S_1}$

4. A particle is thrown upwards from ground. It experiences a constant air resistance which can produce a retardation of 2 m/s^2 opposite to the direction of velocity of particle. The ratio of time of ascent to the time of descent is $\sqrt{\frac{\alpha}{\beta}}$. Where α and β are integers. Find minimum value of $\alpha + \beta$ [g = 10 m/s^2]

5. A lift is descending with uniform acceleration. To measure the acceleration, a person in the lift drops a coin at the moment when lift was descending with speed 6 ft/s. The coin is 5 ft above the floor of the lift at time it is dropped. The person observes that the coin strikes the floor in 1 second. Calculate from these data, the acceleration of the lift in ft/s^2. [Take g = 32 ft/s^2]

6. A lift starts from the top of a mine shaft and descends with a constant speed of 10 m/s. 4 s later a boy throws a stone vertically upwards from the top of the shaft with a speed of 30 m/s. If stone hits the lift at a distance x below the shaft write the value of x/3 (in m) [Take: g = 10 m/s^2] (Give value of $20\sqrt{6}$ = 49)

7. The maximum possible acceleration of a train starting from the rest and moving on straight track is 10 m/s^2 and maximum possible retardation is 5 m/s^2. The maximum speed that train can achieve is 85 m/s. Minimum time in which the train can complete a journey of 1000m ending at rest, is $n\sqrt{\frac{2}{3}}$ sec. Where n is integer. Find n.

8. A hunter in a valley is trying to shoot a deer on a hill. The distance of the deer along his line of sight is $10\sqrt{181}$ meters and the height of the hill is 90 meters. His gun has a muzzle velocity of 100 m/sec. Minimum how many meters above the deer should he aim his rifle in order to hit it? [g = 10 m/s^2]

9. A stone is thrown in such a manner that it would just hit a bird at the top of a tree and afterwards reach a maximum height double that of the tree. If at the moment of throwing the stone the bird flies away horizontally with constant velocity and the stone hits the bird after some time.The ratio of horizontal velocity of stone to that of the bird is $\frac{1}{n}+\frac{1}{\sqrt{n}}$. Find 2n.

10. If 4 seconds be the time in which a projectile reaches a point P of its path and 5 seconds the time from P till it reaches the horizontal plane passing through the point of projection. The height of P above the horizontal plane (in m) will be - [g = 9.8 m/sec^2]

11. From the top of a tower of height 40 m, a ball is projected upwards with a speed of 20 m/sec at an angle of 30º to the horizontal. Distance from the foot of the tower where the ball hit the ground is $40\sqrt{\beta}$ m. Here β is an integer. Find β

12. A ball starts falling with zero initial velocity on a smooth inclined plane forming an angle α with the horizontal. Having fallen the distance h, the ball rebounds elastically off the inclined plane. At distance $nh\sin\alpha$ from the impact point the ball rebounds for the second time. Here n is an integer. Find n.

13. A stone is projected horizontally with speed v from a height h above ground. A horizontal wind is blowing in direction opposite to velocity of projection and gives the stone a constant horizontal acceleration f (in direction opposite to initial velocity). As a result the stone falls on ground at a point vertically below the point of projection. Then find the value of $\frac{f^2h}{gv^2}$ (g is acceleration due to gravity)

14. A small ball rolls of the top of a stairway horizontally with a velocity of 4.5 m s^{-1}. Each step is 0.2 m high and 0.3 m wide. If g is 10 ms^{-2}, then the ball will strike the nth step where n is equal to (assume ball strike at the edge of the step).

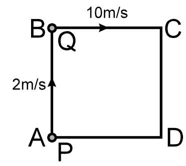
15. Men are running along a road at 15 km/h behind one another at equal intervals of 20 m. Cyclists are riding in the same direction at 25 km/h at equal intervals of 30 m. At what speed (in km/h) an observer travel along the road in opposite direction so that whenever he meets a runner he also meets a cyclist? (Neglect the size of cycle)

16. Two particles P and Q are moving with constant velocities of $(\hat{i}+\hat{j})$ m/s and $(-\hat{i}+2\hat{j})$ m/s respectively. At time t = 0, P is at origin and Q is at a point with position vector $(2\hat{i}+\hat{j})$ m. If the equation of the trajectory of Q as observed by P is x + 2y = n, then find n.

17. An aeroplane has to go from a point A to another point B, 1000 km away due 30º west of north. A wind is blowing due north at a speed of 20 m/s. The air-speed of the plane is 150 m/s. If the angle at which the pilot should head the plane to reach the point B is $\sin^{-1}(1/n)$ west of the line AB, Then find n.

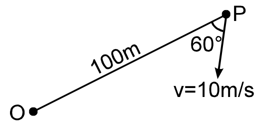
18. Rain appears to be falling at an angle of 37° with vertical to the driver of a car moving with a velocity of 7 m/sec. When he increases the velocity of the car to 25 m/sec, the rain again appears to fall at an angle 37° with vertical. If the actual velocity of rain relative to ground is 4n m/s then find n.

19. Two men P & Q are standing at corners A & B of square ABCD of side 8 m. They start moving along the track with constant speed 2 m/s and 10 m/s respectively. Find the time (in seconds) when they will meet for the first time.



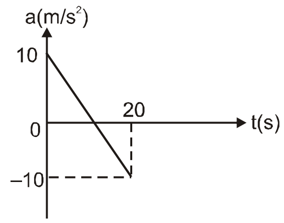
20. Two straight tracks AOB and COD meet each other at right angles at point O. A person walking at a speed of 5 km/h along AOB is at the crossing O at 12 o'clock noon. Another person walking at the same speed along COD reaches the crossing O at 1:30 PM. If the time at which the distance between them is least is 12:T PM, then find T.

21. P is a point moving with constant speed 10 m/s such that its velocity vector always maintains an angle 60° with line OP as shown in figure (O is a fixed point in space). The initial distance between O and P is 100 m. After what time (in sec) shall P reach O.



PART - III : ONE OR MORE THAN ONE OPTION CORRECT TYPE

1. The acceleration time plot for a particle (starting from rest) moving on a straight line is shown in figure. For given time interval,
(A) The particle has zero average acceleration
(B) The particle has never turned around.
(C) The particle has zero displacement
(D) The average speed in the interval 0 to 10s is the same as the average speed in the interval 10s to 20s.



2. The acceleration of a particle is zero at t = 0
(A) Its velocity must be constant.
(B) The speed at t = 0 may be zero.
(C) If the acceleration is zero from t = 0 to t = 5 s, the speed is constant in this interval.
(D) If the speed is zero from t = 0 to t = 5 s the acceleration is also zero in the interval.

3. A particle moves along the Y-axis and its y-coordinate(y) changes with time (t) as $y = u(t-2) + a(t-2)^2$
(A) the initial velocity (at t = 0) of the particle is u (B) the acceleration of the particle is a
(C) the acceleration of the particle is 2a (D) at t =2s particle is at the origin

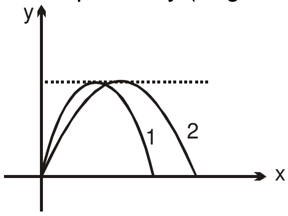
4. A projectile is projected at an angle α (> 45°) with an initial velocity u. The time t at which its horizontal component will equal the vertical component in magnitude:

(A) $t = \frac{u}{g}(\cos\alpha - \sin\alpha)$ (B) $t = \frac{u}{g}(\cos\alpha + \sin\alpha)$

(C) $t = \frac{u}{g}(\sin\alpha - \cos\alpha)$ (D) $t = \frac{u}{g}(\sin^2\alpha - \cos^2\alpha)$

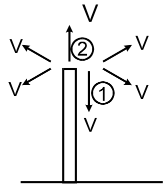
5. At what angle should a body be projected with a velocity 24 ms^{-1} just to pass over the obstacle 14 m high at a distance of 24 m. [Take g = 10 ms^{-2}]
(A) $\tan\theta = 19/5$ (B) $\tan\theta = 1$ (C) $\tan\theta = 3$ (D) $\tan\theta = 2$

6. Two stones are projected from level ground. Trajectories of two stones are shown in figure. Both stones have same maximum heights above level ground as shown. Let T_1 and T_2 be their time of flights and u_1 and u_2 be their speeds of projection respectively (neglect air resistance). Then



(A) $T_2 > T_1$ (B) $T_1 = T_2$ (C) $u_1 > u_2$ (D) $u_1 < u_2$

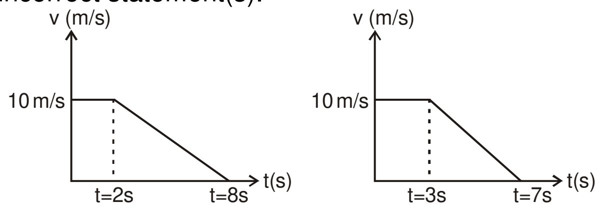
7. Particles are projected from the top of a tower with same speed at different angles as shown. Which of the following are True ?



(A) All the particles would strike the ground with (same) speed.
(B) All the particles would strike the ground with (same) speed simultaneously.
(C) Particle 1 will be the first to strike the ground.
(D) Particle 1 strikes the ground with maximum speed.

8. A man in a lift which is ascending with an upward acceleration 'a' throws a ball vertically upwards with a velocity ‘v’ with respect to himself and catches it after ‘t_1’ seconds. Afterwards when the lift is descending with the same acceleration 'a' acting downwards the man again throws the ball vertically upwards with the same velocity with respect to him and catches it after ‘t_2’ seconds
(A) the acceleration of the ball with respect to ground is g when it is in air
(B) the velocity v of the ball relative to the lift is $\frac{g\,(t_1+t_2)}{t_1\,t_2}$
(C) the acceleration 'a' of the lift is $\frac{g\,(t_2-t_1)}{t_1+t_2}$
(D) the velocity ‘v’ of the ball relative to the man is $\frac{g\,t_1\,t_2}{(t_1+t_2)}$

9. Car A and car B move on a straight road and their velocity versus time graphs are as shown in figure. Comparing the motion of car A in between t = 0 to t = 8 sec. and motion of car B in between t = 0 to t = 7 sec., pick up the incorrect statement(s).



Car A Car B

(A) Distance travelled by car A is less than distance travelled by car B.
(B) Distance travelled by car A is greater than distance travelled by car B.
(C) Average speed of both cars are equal.
(D) Average speed of car A is less than average speed of car B.

10. At an instant particle-A is at origin and moving with constant velocity $(3\hat{i}+4\hat{j})$ m/s and particle-B is at (4, 4)m and moving with constant velocity $(4\hat{i}-3\hat{j})$ m/s. Then :
(A) at this instant relative velocity of B w.r.t. A is $(\hat{i}-7\hat{j})$ m/s
(B) at this instant approach velocity of A and B is $3\sqrt{2}$ m/s
(C) relative velocity of B w.r.t. A remains constant
(D) approach velocity of A and B remains constant

PART - IV : COMPREHENSION

Comprehension-1

The position of a particle is given by x = 2 (t – t^2) where t is expressed in seconds and x is in meter. Positive direction is towards right.

1. The acceleration of the particle is
(A) 0 (B) 4 m/s^2 (C) –4 m/s^2 (D) None of these.

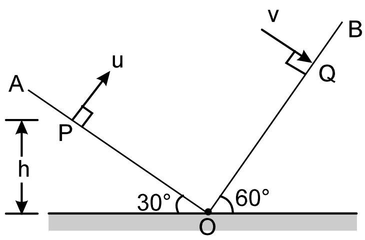
2. The maximum value of position co-ordinate of particle on positive x-axis is
(A) 1 m (B) 2 m (C) 1/2 m (D) 4 m

3. The particle
(A) never goes to negative x-axis
(B) never goes to positive x-axis
(C) starts motion from the origin then goes upto x = 1/2 in the positive x-axis then goes to negative x-axis
(D) final velocity of the particle is zero

4. The total distance travelled by the particle between t = 0 to t = 1 s is :
(A) 0 m (B) 1 m (C) 2 m (D) 1/2 m

Comprehension-2

Two inclined planes OA and OB having inclinations 30° and 60° with the horizontal respectively intersect each other at O, as shown in figure. A particle is projected from point P with velocity u = $10\sqrt{3}\ \frac{m}{s}$ along a direction perpendicular to plane OA. If the particle strikes plane OB perpendicular at Q (Take g = 10 m/s^2). Then



5. The time of flight from P to Q is :
(A) 5 Sec. (B) 2 sec (C) 1 sec (D) None of these

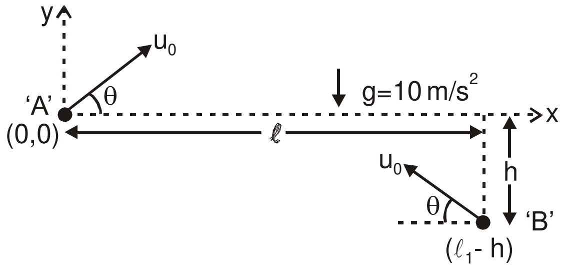
6. The speed with which the particle strikes the plane OB is :
(A) 10 m/s (B) 20 m/s (C) 30 m/s (D) 40 m/s

7. The height h of point P from the ground is :-
(A) $10\sqrt{3}$ m (B) 10 m (C) 5 m (D) 20 m

8. The distance PQ is :
(A) 20 m (B) $10\sqrt{3}$ m (C) 10 m (D) 5 m

Comprehension-3

Two particles 'A' and 'B' are projected in the vertical plane with same initial speed u_0 from position (0, 0) and (ℓ, – h) towards each other as shown in figure at t = 0.



9. The path of particle 'A' with respect to particle 'B' will be

(A) parabola (B) straight line parallel to x-axis.
(C) straight line parallel to y-axis (D) none of these.

10. Minimum distance between particle A and B during motion will be :

(A) ℓ (B) h (C) $\sqrt{\ell^2 + b^2}$ (D) ℓ + h

11. The time when separation between A and B is minimum is :

(A) $\frac{x}{u_0 \cos\theta}$ (B) $\sqrt{\frac{2h}{g}}$ (C) $\frac{\ell}{2u_0 \cos\theta}$ (D) $\frac{2\ell}{u_0 \cos\theta}$

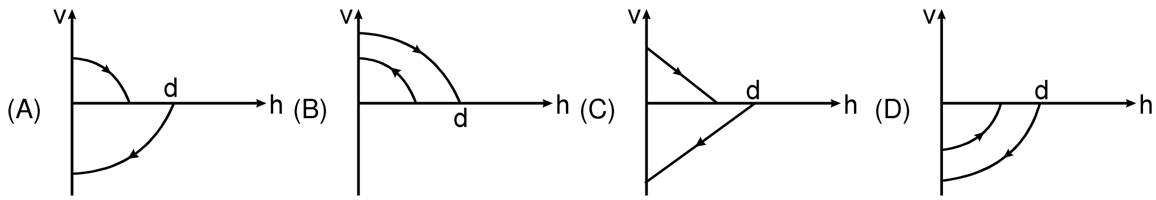
Exercise-3

PART - I : JEE (ADVANCED) / IIT-JEE PROBLEMS (PREVIOUS YEARS)

*** Marked Questions may have more than one correct option.**

RECTILINEAR MOTION

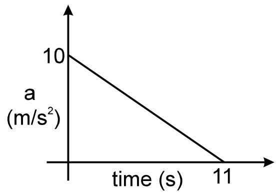
1. A ball is dropped vertically from a height d above the ground. It hits the ground and bounces up vertically to a height d/2. Neglecting subsequent motion and air resistance, its velocity v varies with the height h above the ground as **[JEE '2000, 1/35]**



2. A block is moving down a smooth inclined plane starting from rest at time t = 0. Let S_n be the distance travelled by the block in the interval t = n – 1 to t = n. The ratio $\frac{S_n}{S_{n+1}}$ is **[JEE (Scr.), 2004, 3/84, –1]**

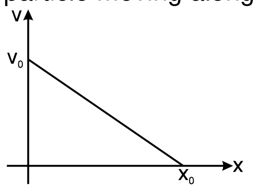
(A) $\frac{2n-1}{2n}$ (B) $\frac{2n-1}{2n+1}$ (C) $\frac{2n+1}{2n-1}$ (D) $\frac{2n}{2n-1}$

3. A particle is initially at rest, It is subjected to a linear acceleration a, as shown in the figure. The maximum speed attained by the particle is **[JEE (Scr.) 2004; 3/84, –1]**

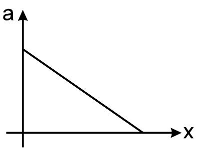
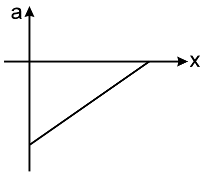
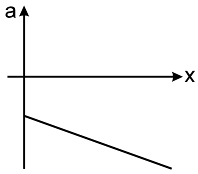
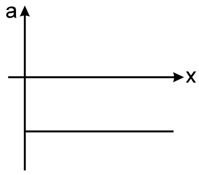


(A) 605 m/s (B) 110 m/s (C) 55 m/s (D) 550 m/s

4. The velocity displacement graph of a particle moving along a straight line is shown.



The most suitable acceleration-displacement graph will be **[JEE (Scr.) 2005 ; 3/84, –1]**

(A)  (B)  (C)  (D) 

PROJECTILE MOTION

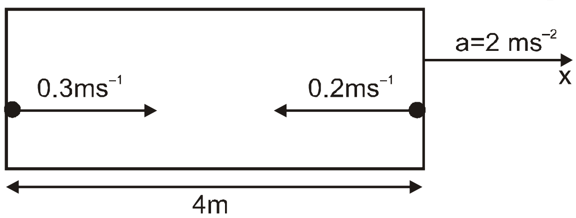
5. A ball is projected from the ground at an angle of 45° with the horizontal surface. It reaches a maximum height of 120m and returns to the ground. Upon hitting the ground for the first time, it loses half of its kinetic energy. Immediately after the bounce, the velocity of the ball makes an angle of 30° with the horizontal surface. The maximum height it reaches after the bounce, in metres, is ___________.
[JEE (Advanced) 2018, 3/60]

RELATIVE MOTION

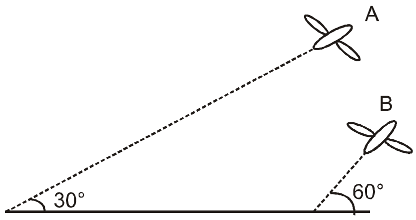
6. **STATEMENT-1 :** For an observer looking out through the window of a fast moving train, the nearby objects appear to move in the opposite direction to the train, while the distant objects appear to be stationary. **[JEE' 2008, 3/163]**
and
STATEMENT-2 : If the observer and the object are moving at velocities $\vec{v}_1$ and $\vec{v}_2$ respectively with reference to a laboratory frame, the velocity of the object with respect to the observer is $\vec{v}_2 - \vec{v}_1$

(A) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 **is** a correct explanation for STATEMENT-1
(B) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 **is NOT** a correct explanation for STATEMENT-1
(C) STATEMENT-1 is True, STATEMENT-2 is False
(D) STATEMENT-1 is False, STATEMENT-2 is True.

7. A rocket is moving in a gravity free space with a constant acceleration of 2ms^{-2} along +x direction (see figure). The length of a chamber inside the rocket is 4 m. A ball is thrown from the left end of the chamber in +x direction with a speed of 0.3 ms^{-1} relative to the rocket. At the same time, another ball is thrown in –x direction with a speed of 0.2 ms^{-1} from its right end relative to the rocket. The time in seconds when the two balls hit each other is **[JEE (Advanced) 2014, P-1, 3/60]**



8. Airplanes A and B are flying with constant velocity in the same vertical plane at angles 30° and 60° with respect to the horizontal respectively as shown in figure. The speed of A is $100\sqrt{3}$ ms^{-1}. At time t = 0s, an observer in A finds B at a distance of 500m. This observer sees B moving with a constant velocity perpendicular to the line of motion of A. If at t = t_0, A just escapes being hit by B, t_0 in seconds is: **[JEE (Advanced) 2014, P-1, 3/60]**



PART - II : JEE (MAIN) / AIEEE PROBLEMS (PREVIOUS YEARS)

RECTILINEAR MOTION

1. If a body loses half of its velocity on penetrating 3 cm in a wooden block, then how much will it penetrate more before coming to rest? **[AIEEE 2002, 4/300]**
(1) 1 cm (2) 2 cm (3) 3 cm (D) 4 cm

2. From a building two balls A and B are thrown such that A is thrown upwards and B downwards (both vertically with same speed). If V_A and V_B are their respective velocities on reaching the ground, then **[AIEEE 2002, 4/300]**
(1) $v_B > v_A$ (2) $v_A = v_B$
(3) $v_A > v_B$ (4) their velocities depends on their masses

3. Speeds of two identical cars are u and 4u at a specific instant. The ratio of the respective distances at which the two cars are stopped at the same instant is : **[AIEEE 2002, 4/300]**
(1) 1 : 1 (2) 1 : 4 (3) 1 : 8 (4) 1 : 16

4. The coordinates of a moving particle at any time t are given by x = αt^3 and y = βt^3. The speed of the particle at time t is given by : **[AIEEE 2003, 4/300]**
(1) $\sqrt{\alpha^2+\beta^2}$ (2) $3t^2\sqrt{\alpha^2+\beta^2}$ (3) $t^2\sqrt{\alpha^2+\beta^2}$ (4) $\sqrt{\alpha^2+\beta^2}$

5. A car moving with a speed of 50 km/hr, can be stopped by brakes after at least 6 m. if the same car is moving at a speed of 100 km/hr, the minimum stopping distance is : **[AIEEE 2003, 4/300]**
(1) 12 m (2) 18 m (3) 24 m (4) 6 m

6. A ball is released from the top of a tower of height h metres. It takes T seconds to reach the ground. What is the position of the ball in T/3 seconds? **[AIEEE 2004, 4/300]**
(1) h/9 metre from the ground (2) 7h/9 metre from the ground
(3) 8h/9 metre from the ground (4) 17h/9 metre from the ground

7. An automobile travelling with a speed of 60 km/h, can brake to stop within a distance of 20 m. If the car is going twice as fast, ie. 120 km/h, the stopping distance will be **[AIEEE 2004, 4/300]**
(1) 20 m (2) 40 m (3) 60 m (4) 80 m

8. The relation between time t and distance x is t = $ax^2 + bx$, where a and b are constants. The acceleration is : **[AIEEE 2005, 4.300]**
(1) $-2abv^2$ (2) $2bv^2$ (3) $-2av^3$ (4) $2av^3$

9. A car, starting from rest, accelerates at the rate f through a distance S, then continues at constant speed for time t and then decelerates at the rate f/2 to come to rest. If the total distance travelled is 15 S, then : **[AIEEE 2005, 4/300]**
(1) S = ft (2) S = $\frac{1}{6}ft^2$ (3) S = $\frac{1}{72}ft^2$ (4) S = $\frac{1}{4}ft^2$

10. A particle is moving eastwards with a velocity of 5 ms^{-1}. In 10 second the velocity changes to 5 ms^{-1} northwards. The average acceleration in this time is : **[AIEEE 2005, 4/300]**
(1) $\frac{1}{\sqrt{2}}$ ms^{-2} towards north-west (2) $\frac{1}{2}$ ms^{-2} towards north
(3) zero (4) $\frac{1}{2}$ ms^{-2} towards north-west.

11. A parachutist after bailing out falls 50 m without friction. When parachute opens, it decelerates at 2 m/s^2. He reaches the ground with a speed of 3 m/s. At what height approximately, did he bail out? **[AIEEE 2005, 4/300]**
(1) 91 m (2) 182 m (3) 293 m (4) 111 m

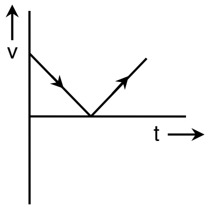
12. A particle located at x = 0 at time t = 0, starts moving along the positive x-direction with a velocity v that varies as v = $\alpha\sqrt{x}$. The displacement of the particle varies with time as **[AIEEE-2006, 3/180]**
(1) $t^{1/2}$ (2) t^3 (3) t^2 (4) t

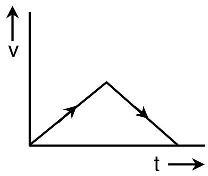
13. The velocity of a particle is $v = v_0 + gt + ft^2$. If its position is x = 0 at t = 0, then its displacement after unit time (t = 1) is **[AIEEE 2007, 3/120]**
(1) $v_0 + 2g + 3f$ (2) $v_0 + \frac{g}{2} + \frac{f}{3}$ (3) $v_0 + g + f$ (4) $v_0 + \frac{g}{2} + f$

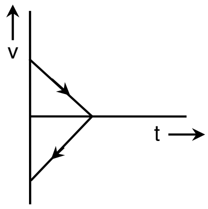
14. An object moving with a speed of 6.25 m/s, is decelerated at a rate given by $\frac{d\upsilon}{dt} = -2.5\sqrt{\upsilon}$ where υ is the instantaneous speed. The time taken by the object, to come to rest, would be : **[AIEEE 2011, 4/120, –1]**
(1) 1 s (2) 2 s (3) 4 s (4) 8 s

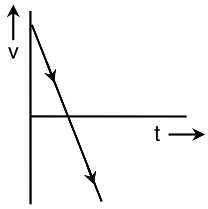
15. From a tower of height H, a particle is thrown vertically upwards with a speed u. The time taken by the particle, to hit the ground, is n times that taken by it to reach the highest point of its path. The relation between H, u and n is : **[JEE (Main) 2014 ; 4/120, –1]**
(1) $2gH = n^2u^2$ (2) $gH = (n-2)^2u^2$ (3) $2gH = nu^2(n-2)$ (4) $gH = (n-2)u^2$

16. A body is thrown vertically upwards. Which one of the following graphs correctly represent the velocity vs time ? **[JEE (Main) 2017 ; 4/120, –1]**

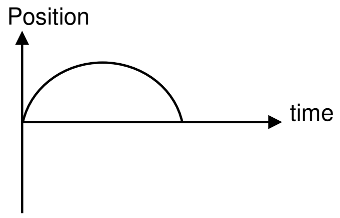
(1)


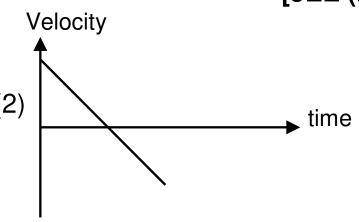
(2)


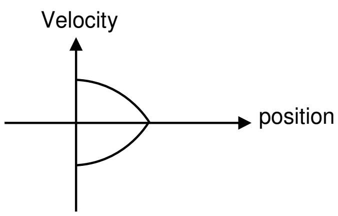
(3)


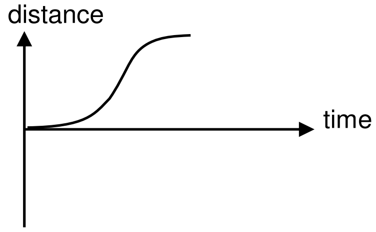
(4)


17. All the graphs below are intended to represent the same motion. One of them does it incorrectly. Pick it up. **[JEE (Main) 2018; 4/120, –1]**

(1)


(2)


(3)


(4)


PROJECTILE MOTION

18. A particle is projected at 60º to the horizontal with a kinetic energy K. The kinetic energy at the highest point is **[AIEEE 2007, 3/120]**
(1) K (2) Zero (3) K/4 (4) K/2

19. A particle has an initial velocity of $3\hat{i} + 4\hat{j}$ and an acceleration of $0.4\hat{i} + 0.3\hat{j}$. Its speed after 10 s is : **[AIEEE 2009, 4/144]**
(1) $7\sqrt{2}$ units (2) 7 units (3) 8.5 units (4) 10 units

20. A particle is moving with velocity $\vec{v} = K(y\hat{i} + x\hat{j})$, where K is a constant. The general equation for its path is: **[AIEEE 2010, 4/144]**

(1) $y = x^2$ + constant (2) $y^2 = x$ + constant
(3) xy = constant (4) $y^2 = x^2$ + constant

21. A water fountain on the ground sprinkles water all around it. If the speed of water coming out of the fountain is v, the total area around the fountain that gets wet is : **[AIEEE 2011, 4/120, –1]**

(1) $\pi\frac{v^2}{g}$ (2) $\pi\frac{v^4}{g^2}$ (3) $\frac{\pi}{2}\frac{v^4}{g^2}$ (4) $\pi\frac{v^2}{g^2}$

22. A boy can throw a stone up to a maximum height of 10m. The maximum horizontal distance that the boy can throw the same stone up to will be : **[AIEEE 2012 ; 4/120, –1]**

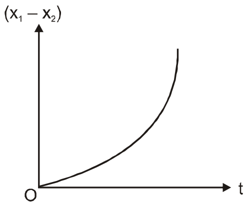
(1) $20\sqrt{2}$m (2) 10 m (3) $10\sqrt{2}$m (4) 20m

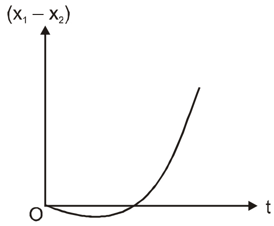
23. A projectile is given an initial velocity of $(\hat{i} + 2\hat{j})$ m/s, where $\hat{i}$ is along the ground and $\hat{j}$ is along the vertical. If g = 10 m/s^2, the equation of its trajectory is : **[JEE (Main) 2013, 4/120]**

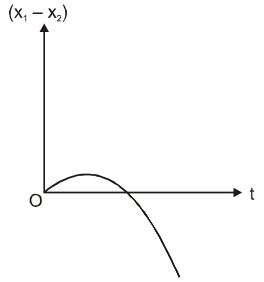
(1) $y = x - 5x^2$ (2) $y = 2x - 5x^2$ (3) $4y = 2x - 5x^2$ (4) $4y = 2x - 25x^2$

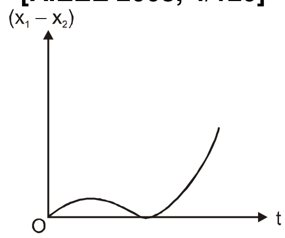
RELATIVE MOTION

24. A body is at rest at x = 0. At t = 0, it starts moving in the positive x-direction with a constant acceleration. At the same instant another body passes through x = 0 moving in the positive x-direction with a constant speed. The position of the first body is given by $x_1(t)$ after time 't' and that of second body by $x_2(t)$ after the same time interval. Which of the following graphs correctly describes $(x_1 - x_2)$ as a function of time 't' ? **[AIEEE 2008, 4/120]**

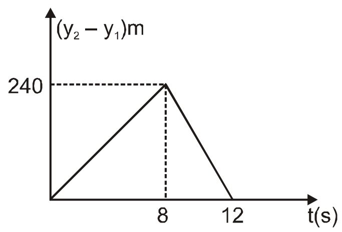
(1)


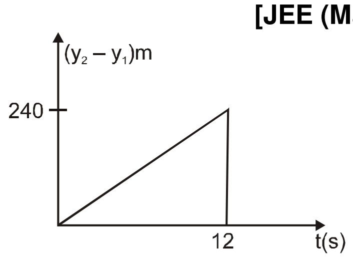
(2)


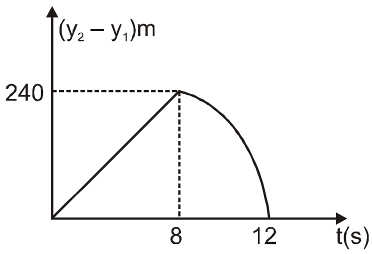
(3)


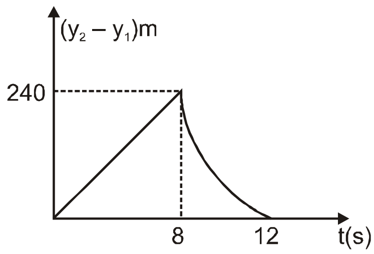
(4)


25. Two stones are thrown up simultaneously from the edge of a cliff 240 m high with initial speed of 10 m/s and 40 m/s respectively. Which of the following graph best represents the time variation of relative position of the second stone with respect to the first ? (Assume stones do not rebound after hitting the ground and neglect air resistance, take g = 10 m/s^2) (The figures are schematic and not drawn to scale.) **[JEE (Main) 2015; 4/120, –1]**

(1)


(2)


(3)


(4)


Answers

EXERCISE-1

PART - I

SECTION (A) :

A-1. Distance travelled by the car = 48 m,
Displacement of the car = 36 m

A-2. (a) 110 m (b) 50 m, $\tan^{-1}\frac{4}{3}$ west of south

A-3. $\frac{3v_1v_2v_3}{v_1v_2+v_2v_3+v_1v_3}$

A-4. (a) 24 Bt ; (b) A + 300 B, 120 B

A-5. 50m at 53° S of W, 5m/s, 25/9 m/s at 53° S of W, 90 m

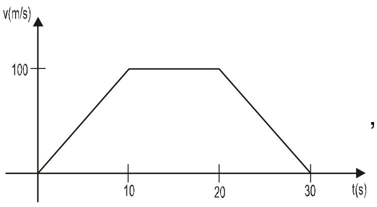
A-6. (a) 2700 m = 2.7 km,
(b) 60 m/s,
(c) 225 m and 2.25 km

A-7. 50m ; $2 m/s^2$; 20 m

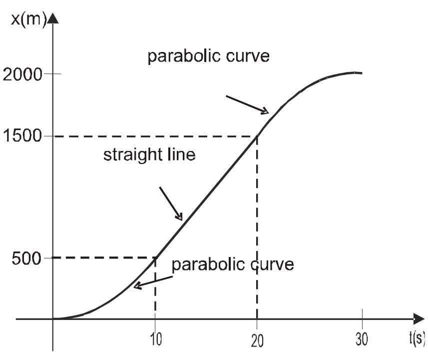
A-8. 20m

A-9. (i) 20 m/s (ii) 4 m
(iii) $\sqrt{656}$ m.

A-10. distance travelled = 10 m; displacement = 6 m; average velocity = $\frac{6}{5}$ = 1.2 m/s

A-11. 2000 m, 

,



A-12. Particle B

SECTION (B) :

B-1. $\tan\theta : 1$ **B-2.** $\tan^2\theta : 1$

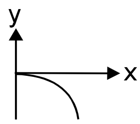
B-3. $T_1T_2 = \frac{2R}{g}$ **B-4.** $\frac{gt}{\tan\alpha - \tan\beta}$

B-6. (i) 250 m (ii) 50 m/sec.

B-7. (i) 10 sec. (ii) 980 m (iii) $98\sqrt{2}$ m/s

B-8. (i) 61.25 m (ii) $75\sqrt{3}$ m $\approx$ 130 m
(iii) 5 sec.

B-9. θ = 60°, 2 m/s **B-10.** 53

B-11. (i) $y = -\frac{bx^2}{a^2}$ 

Ans

(ii) $\vec{v} = a\hat{i} - 2bt\hat{j}$, acceleration $= -2b\hat{j}$,

$|\vec{v}| = \sqrt{a^2 + 4b^2t^2}$, |acceleration| = $2b$

B-12. (a) – g sinβ, (b) –g cosβ,
(c) u cosθ – g sinβ × t,
(d) u sinθ – gcosβ × t,
(e) u cosθ × t – $\frac{1}{2}$g sinβ × t^2,
(f) u sinθ × t – $\frac{1}{2}$g cosβ × t^2,
(g) zero.

SECTION (C) :

C-1. (a) 144 km/h due south,
(b) 90 km/h due north,
(c) 36 km/h due north,
(d) 126 km/h due north

C-2. 50 km/h **C-3.** 50 m

C-4. (a) 20 m/s or 72 km/h due east
(b) 25 m/s or 90 km/h at 37°N of E
(c) 25 m/s or 90 km/h

C-5. 30º N of W at $5\sqrt{3}$ km/h

C-6. $10\sqrt{2}$ km/h, 45º N of E

C-7. $\hat{i} + \sqrt{2}\hat{j} + \hat{k}$, $\hat{i} \rightarrow$ east, $\hat{j} \rightarrow$ north, $\hat{k} \rightarrow$ vertical upward

C-8. $\frac{1}{4}$h, $\frac{3}{4}$km

C-9. At an angle 30° west of north

C-10. $10\sqrt{5}$ m/s

C-11. $2\sqrt{2}$ m/s, 45° with vertical and away from the man.

C-12. 3 m **C-13.** a/v

C-14. $v_1 = \frac{11}{10}$ m/s and $v_2 = \frac{1}{2}$ m/s.

PART - II

SECTION (A) :

A-1.	(B)	**A-2.**	(A)	**A-3.**	(B)
A-4.	(A)	**A-5.**	(C)	**A-6.**	(A)
A-7.	(D)	**A-8.**	(B)	**A-9.**	(B)
A-10.	(C)	**A-11.**	(C)	**A-12.**	(B)
A-13.	(D)				

SECTION (B) :

B-1.	(D)	**B-2.**	(A)	**B-3.**	(B)
B-4.	(B)	**B-5.**	(B)	**B-6.**	(C)
B-7.	(A)	**B-8.**	(B)	**B-9.**	(C)
B-10.	(D)	**B-11.**	(B)	**B-12.**	(D)
B-13.	(A)	**B-14.**	(A)		

SECTION (C) :

C-1.	(A)	**C-2.**	(D)	**C-3.**	(D)
C-4.	(D)	**C-5.**	(A)	**C-6.**	(B)
C-7.	(D)	**C-8.**	(B)	**C-9.**	(B)
C-10.	(A)	**C-11.**	(D)	**C-12.**	(A)
C-13.	(D)	**C-14.**	(B)	**C-15.**	(C)

PART - III

1. (A) – r ; (B) – p ; (C) – q ; (D) – s

2. (A) – r ; (B) – s ; (C) – q ; (D) – p

3. (A) – r ; (B) – s ; (C) – q ; (D) – p

4. (A) – q ; (B) – r, t ; (C) – p ; (D) – q

EXERCISE-2

PART - I

1.	(A)	**2.**	(A)	**3.**	(A)
4.	(D)	**5.**	(A)	**6.**	(A)
7.	(A)	**8.**	(C)	**9.**	(D)
10.	(D)	**11.**	(D)	**12.**	(C)
13.	(D)	**14.**	(A)	**15.**	(A)
16.	(A)	**17.**	(D)	**18.**	(B)
19.	(C)	**20.**	(A)		

PART - II

1.	4	**2.**	3	**3.**	5
4.	5	**5.**	22	**6.**	43
7.	30	**8.**	10	**9.**	4
10.	98	**11.**	3	**12.**	8
13.	2	**14.**	9	**15.**	5
16.	4	**17.**	15	**18.**	5
19.	3	**20.**	45	**21.**	20

PART - III

1.	(ABD)	**2.**	(BCD)	**3.**	(CD)
4.	(BC)	**5.**	(AB)	**6.**	(BD)
7.	(AC)	**8.**	(ACD)	**9.**	(ABC)
10.	(ABC)				

PART - IV

1.	(C)	**2.**	(C)	**3.**	(C)
4.	(B)	**5.**	(B)	**6.**	(A)
7.	(C)	**8.**	(A)	**9.**	(B)
10.	(B)	**11.**	(C)		

EXERCISE-3

PART - I

1.	(A)	**2.**	(B)	**3.**	(C)
4.	(B)	**5.**	30	**6.**	(B)
7.	2	**8.**	5		

PART - II

1.	(1)	**2.**	(2)	**3.**	(2)
4.	(2)	**5.**	(3)	**6.**	(3)
7.	(4)	**8.**	(3)	**9.**	(3)
10.	(1)	**11.**	(3)	**12.**	(3)
13.	(2)	**14.**	(2)	**15.**	(3)
16.	(4)	**17.**	(4)	**18.**	(3)
19.	(1)	**20.**	(4)	**21.**	(2)
22.	(4)	**23.**	(2)	**24.**	(2)
25.	(3)				