ELECTROSTATICS

Contents

JEE (ADVANCED) SYLLABUS

Electrostatics : Coulomb’s law; Electric field and potential; Electrical Potential energy of a system of point charges and of electrical dipoles in a uniform electrostatic field, Electric field lines; Flux of electric field; Gauss’s law and its application in simple cases, such as, to find field due to infinitely long straight wire, uniformly charged infinite plane sheet and uniformly charged thin spherical shell.

JEE (MAIN) SYLLABUS

Electrostatics : Electric charges : Conservation of charge, Coulomb's law-faces between two point charges, forces between multiple charges ; superposition principle and continuous charge distribution. Electric field : Electric field due to a point charge, Electric field lines, Electric dipole, Electric field due to a dipole, Torque on a dipole in a uniform electric field.

Electric flux, Gauss's law and its applications to find field due to infinitely long uniformly charged straight wire, uniformly charged infinite plane sheet and uniformly charged thin spherical shell. Electric potential and its calculation for a point charge, electric dipole and system of charges ; Equi-potential surfaces, Electrical potential energy of a system of two point charges in an electrostatics field. Conductors and insulators, Dielectrics and electric polarization, capacitor, combination of capacitors in series and in parallel, capacitance of a parallel plate capacitor with and without dielectric medium between the plates, Energy stored in a capacitance.

ELECTROSTATICS

1. INTRODUCTION

The branch of physics which deals with electric effect of static charge is called electrostatics.

2. ELECTRIC CHARGE

Charge of a material body or particle is the property (acquired or natural) due to which it produces and experiences electrical and magnetic effects. Some of naturally occurring charged particles are electrons, protons, α-particles etc.

Charge is a derived physical quantity & is measured in Coulomb in S.I. unit. In practice we use mC(10^{-3}C), μC (10^{-6}C), nC(10^{-9}C) etc.
C.G.S. unit of charge = electrostatic unit = esu.
1 coulomb = 3×10^9 esu of charge
Dimensional formula of charge = $[M^oL^oT^1I^1]$

2.1 Properties of Charge

(i) Charge is a scalar quantity : It adds algebraically and represents excess or deficiency of electrons.

(ii) Charge is of two types : (i) Positive charge and (ii) Negative charge Charging a body implies transfer of charge (electrons) from one body to another. Positively charged body means loss of electrons i.e. deficiency of electrons. Negatively charged body means excess of electrons. This also shows that **mass of a negatively charged body > mass of a positively charged identical body.**

(iii) Charge is conserved : In an isolated system, total charge (sum of positive and negative) remains constant whatever change takes place in that system.

(iv) Charge is quantized : Charge on any body always exists in integral multiples of a fundamental unit of electric charge. This unit is equal to the magnitude of charge on electron ($1e = 1.6 \times 10^{-19}$ coulomb). So charge on anybody is $Q = \pm ne$, where n is an integer and e is the charge of the electron. **Millikan's oil drop** experiment proved the quantization of charge or atomicity of charge

Note : Recently, the existence of particles of charge $\pm\frac{1}{3}e$ and $\pm\frac{2}{3}e$ has been postulated. These particles are called quarks but still this is not considered as the quantum of charge because these are unstable (They have very short span of life).

(v) Like point charges repel each other while unlike point charges attract each other.

(vi) Charge is always associated with mass, i.e., charge can not exist without mass though mass can exist without charge. The particle such as photon or neutrino which have no (rest) mass can never have a charge.

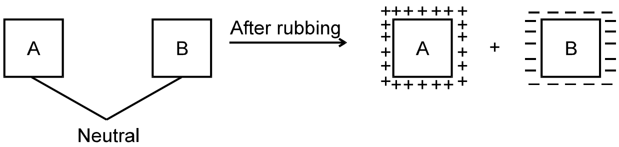
(vii) Charge is relativistically invariant: This means that charge is independent of frame of reference i.e. charge on a body does not change whatever be its speed. This property is worth mentioning as in contrast to charge, the mass of a body depends on its speed and increases with increase in speed.

(viii) A charge at rest produces only electric field around itself, a charge having uniform motion produces electric as well as magnetic field around itself while a charge having accelerated motion emits electromagnetic radiations.

2.2 Charging of a body

A body can be charged by means of (a) friction, (b) conduction, (c) induction, (d) thermionic ionization or thermionic emission (e) photoelectric effect and (f) field emission.

(a) Charging by Friction : When a neutral body is rubbed against other neutral body then some electrons are transferred from one body to other. The body which can hold electrons tightly, draws some electrons and the body which can not hold electrons tightly, loses some electrons. The body which draws electrons becomes negatively charged and the body which loses electrons becomes positively charged.



For example : Suppose a glass rod is rubbed with a silk cloth. As the silk can hold electrons more tightly and a glass rod can hold electrons less tightly (due to their chemical properties), some electrons will leave the glass rod and get transferred to the silk. So, in the glass rod their will be deficiency of electrons, therefore it will become positively charged. And in the silk, there will be some extra electrons, so it will become negatively charged

(b) Charging by conduction (flow): There are three types of materials in nature

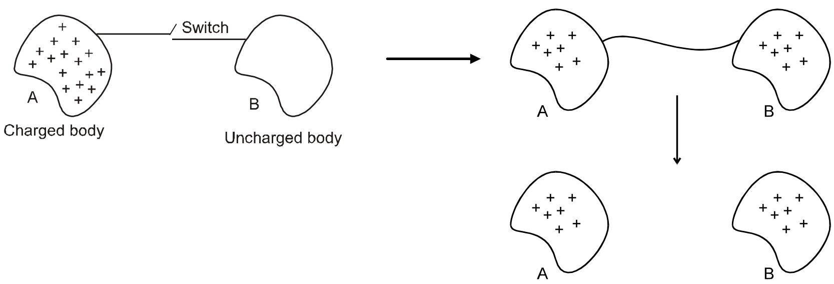
(i) Conductor : Conductors are the material in which the outer most electrons are very loosely bound, so they are free to move (flow). So in a conductor, there are large number of free electrons.

Ex. Metals like Cu, Ag, Fe, Al............

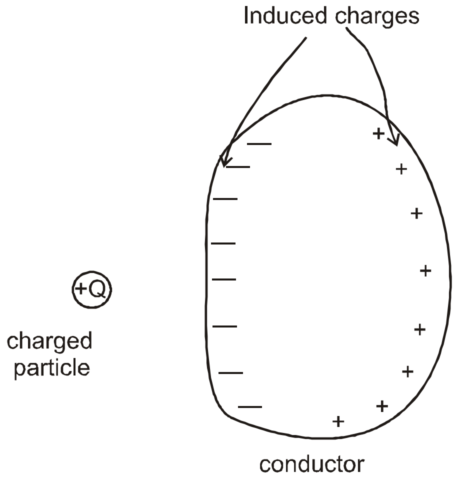
(ii) Insulator or Dielectric or Nonconductor : Non-conductors are the materials in which outer most electrons are very tightly bound, so that they cannot move (flow). Hence in a non-conductor there are no free electrons. Ex. plastic, rubber, wood etc.

(iii) Semi conductor : Semiconductors are the materials which have free electrons but very less in number.

Now lets see how the charging is done by conduction. In this method, we take a charged conductor 'A' and an uncharged conductor 'B'. When both are connected, some charge will flow from the charged body to the uncharged body. If both the conductors are identical & kept at large distance and connected to each other, then charge will be divided equally in both the conductors otherwise they will flow till their electric potential becomes same. Its detailed study will be done in last section of this chapter.



(c) Charging by Induction : To understand this, let's have introduction to induction.



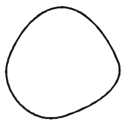
We have studied that there are lot of free electrons in the conductors. When a charged particle +Q is brought near a neutral conductor, due to attraction of +Q charge, many electrons (–ve charges) come closer and accumulate on the closer surface.

On the other hand, a positive charge (deficiency of electrons) appears on the other surface. The flow of charge continues till the resultant force on free electrons of the conductor becomes zero. This phenomena is called induction and charges produced are called induced charges.

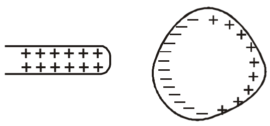
A body can be charged by induction in the following two ways :

Method I :

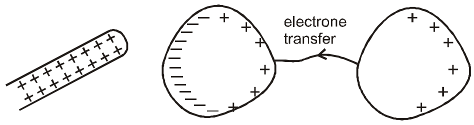
Step 1 : Take an isolated neutral conductor..



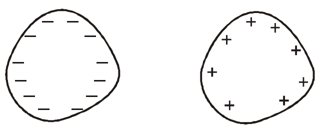
Step 2: Bring a charged rod near it. Due to the charged rod, charges will induce on the conductor.



Step 3 : Connect another neutral conductor with it. Due to attraction of the rod, some free electrons will move from the right conductor to the left conductor and due to deficiency of electrons positive charges will appear on right conductor and on the left conductor, there will be excess of electrons due to transfer from right conductor.



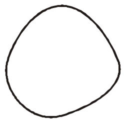
Step 4 : Now disconnect the connecting wire and remove the rod.



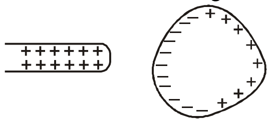
The first conductor will be negatively charged and the second conductor will be positively charged.

Method II

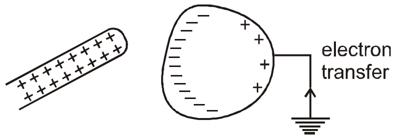
Step 1: Take an isolated neutral conductor.



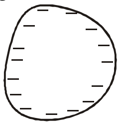
Step 2 : Bring a charged rod near it. Due to the charged rod, charges will induce on the conductor.



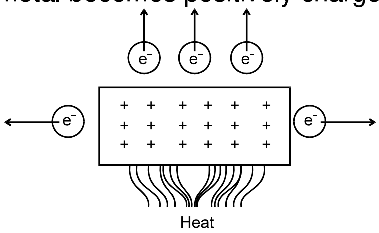
Step 3 : Connect the conductor to the earth (this process is called grounding or earthing). Due to attraction of the rod, some free electrons will move from earth to the conductor, so in the conductor there will be excess of electrons due to transfer from the earth, so net charge on conductor will be negative.



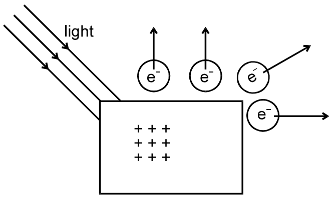
Step 4. Now disconnect the connecting wire. Conductor becomes negatively charged.



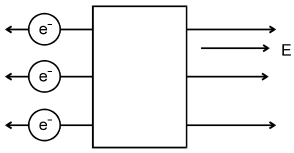
(d) Thermionic emission : When the metal is heated at a high temperature then some electrons of metals are ejected and the metal becomes positively charged.



(e) Photoelectric effect : When light of sufficiently high frequency is incident on metal surface then some electrons gain energy from light and come out of the metal surface and remaining metal becomes positively charged.



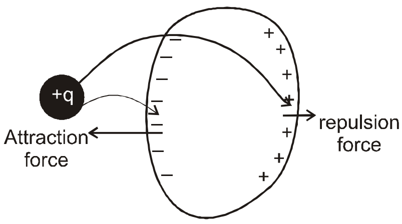
(f) Field emission : When electric field of large magnitude is applied near the metal surface then some electrons come out from the metal surface and hence the metal gets positively charged.



Solved Examples

Example 1. If a charged body is placed near a neutral conductor, will it attract the conductor or repel it ?

Solution :



If a charged body (+ve) is placed left side near a neutral conductor, (–ve) charge will induce at left surface and (+ve) charge will induce at right surface. Due to positively charged body –ve induced charge will feel attraction and the +ve induced charge will feel repulsion. But as the –ve induced charge is nearer, so the attractive force will be greater than the repulsive force. So the net force on the conductor due to positively charged body will be attractive. Similarly, we can prove for negatively charged body also.

From the above example we can conclude that. "A charged body can attract a neutral body."

If there is attraction between two bodies then one of them may be neutral. But if there is repulsion between two bodies, both must be charged (similarly charged). So **"repulsion is the sure test of electrification".**

Example 2. A positively charged body 'A' attracts a body 'B' then charge on body 'B' may be:
(A) positive (B) negative (C) zero (D) can't say

Answer : B, C

Example 3. Five styrofoam balls A, B, C, D and E are used in an experiment. Several experiments are performed on the balls and the following observations are made :
(i) Ball A repels C and attracts B.
(ii) Ball D attracts B and has no effect on E.
(iii) A negatively charged rod attracts both A and E.
For your information, an electrically neutral styrofoam ball is very sensitive to charge induction and gets attracted considerably, if placed nearby a charged body. What are the charges, if any, on each ball ?

	A	B	C	D	E
(A)	+	–	+	0	+
(B)	+	–	+	+	0
(C)	+	–	+	0	0
(D)	–	+	–	–	0

Answer : C

Solution : From (i), as A repels C, so both A and C must be charged similarly. Either both are +ve or both are –ve. As A also attract B, so charge on B should be opposite of A or B may be uncharged conductor.
From (ii) as D has no effect on E, so both D and E should be uncharged and as B attracts uncharged D, so B must be charged and D must be an uncharged conductor.
From (iii), a –vely charged rod attracts the charged ball A, so A must be +ve and from exp. (i) C must also be +ve and B must be –ve.

Example 4. Charge conservation is always valid. Is it also true for mass?

Solution : No, mass conservation is not always. In some nuclear reactions, some mass is lost and it is converted into energy.

Example 5. What are the differences between charging by induction and charging by conduction ?

Solution : Major differences between two methods of charging are as follows :
(i) In induction, two bodies are close to each other but do not touch each other while in conduction they touch each other. (Or they are connected by a metallic wire)
(ii) In induction, total charge of a body remains unchanged while in conduction it changes.
(iii) In induction, induced charge is always opposite in nature to that of source charge while in conduction charge on two bodies finally is of same nature.

Example 6. If a glass rod is rubbed with silk, it acquires a positive charge because :
(A) protons are added to it (B) protons are removed from it
(C) electrons are added to it (D) electrons are removed from it.

Answer : D

3. COULOMB'S LAW (INVERSE SQUARE LAW)

On the basis of experiments Coulomb established the following law known as Coulomb's law :
The magnitude of electrostatic force between two point charges is directly proportional to the product of charges and inversely proportional to the square of the distance between them.

i.e. $F \propto q_1q_2$ and $F \propto \frac{1}{r^2}$ $\Rightarrow F \propto \frac{q_1q_2}{r^2}$ $\Rightarrow$ $F = \frac{Kq_1q_2}{r^2}$

Important points regarding Coulomb's law :

(i) It is applicable only for point charges.

(ii) The constant of proportionality K in SI units in vacuum is expressed as $\frac{1}{4\pi\varepsilon_0}$ and in any other medium expressed as $\frac{1}{4\pi\varepsilon}$. If charges are dipped in a medium then electrostatic force on one charge is $\frac{1}{4\pi\varepsilon_0\varepsilon_r}\frac{q_1q_2}{r^2}$ where ε_0 and ε are called permittivity of vacuum and absolute permittivity of the medium respectively. The ratio $\varepsilon/\varepsilon_0 = \varepsilon_r$ is called relative permittivity of the medium, which is a dimensionless quantity.

(iii) The value of relative permittivity ε_r is constant for a medium and can have values between 1 to ∞. For vacuum, by definition it is equal to 1. For air it is nearly equal to 1 and may be taken to be equal to 1 for calculations. For metals, the value of ε_r is ∞ and for water is 81. The material in which more charge can induce ε_r will be higher.

(iv) The value of $\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9$ Nm^2 C^{-2} & $\varepsilon_0 = 8.855 \times 10^{-12}$ C^2/Nm^2.

Dimensional formula of ε is $[M^{-1} L^{-3} T^4 A^2]$

(v) The force acting on one point charge due to the other point charge is always along the line joining these two charges. It is equal in magnitude and opposite in direction on two charges, irrespective of the medium in which they lie.

(vi) The force is conservative in nature i.e., work done by electrostatic force in moving a point charge along a closed loop of any shape is zero.

(vii) Since the force is a central force, in the absence of any other external force, angular momentum of one particle w.r.t. the other particle (in two particle system) is conserved.

(viii) In vector form formula can be given as below.

$$\vec{F} = \frac{1}{4\pi\varepsilon_0\varepsilon_r}\frac{q_1q_2}{|\vec{r}|^3}\vec{r} = \frac{1}{4\pi\varepsilon_0\varepsilon_r}\frac{q_1q_2}{|\vec{r}|^2}\hat{r}$$; (q_1 & q_2 are to be substituted with sign.)

Here, $\vec{r}$ is position vector of the test charge (on which force is to be calculated) with respect to the source charge (due to which force is to be calculated).

Solved Examples

Example 7. Find out the electrostatic force between two point charges placed in air (each of +1 C) if they are separated by 1m.

Solution : $F_e = \frac{kq_1q_2}{r^2} = \frac{9\times10^9\times1\times1}{1^2} = 9\times10^9$ N

From the above result, we can say that 1 C charge is too large to realize. In nature, charge is usually of the order of μC

Example 8. A particle of mass m carrying charge q_1 is revolving around a fixed charge $-q_2$ in a circular path of radius r. Calculate the period of revolution and its speed also.

Solution : $\frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r^2} = mr\omega^2 = \frac{4\pi^2 mr}{T^2}$,

$T^2 = \frac{(4\pi\varepsilon_0)r^2(4\pi^2 mr)}{q_1q_2}$ or $T = 4\pi r\sqrt{\frac{\pi\varepsilon_0 mr}{q_1q_2}}$

and also we can say that

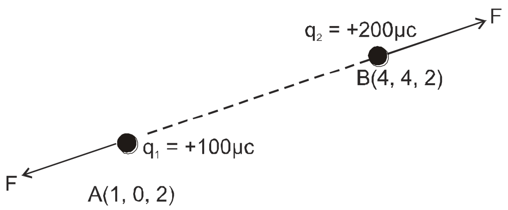
$\frac{q_1q_2}{4\pi\varepsilon_0 r^2} = \frac{mv^2}{r}$ $\Rightarrow$ $V = \sqrt{\frac{q_1q_2}{4\pi\varepsilon_0 mr}}$

Example 9. A point charge q_A = + 100 μc is placed at point A (1, 0, 2) m and another point charge q_B= +200μc is placed at point B (4, 4, 2) m. Find :

(i) Magnitude of electrostatic interaction force acting between them

(ii) Find $\vec{F}_A$ (force on A due to B) and $\vec{F}_B$ (force on B due to A) in vector form

Solution : (i)



Value of F : $\left|\vec{F}\right| = \frac{kq_Aq_B}{r^2} = \frac{(9\times10^9)\ (100\times10^{-6})\ (200\ \times10^{-6})}{\left(\sqrt{(4-1)^2\ +\ (4-0)^2\ +\ (2-2)^2}\right)^2} = 7.2$ N

(ii) Force on B, $\vec{F}_B = \frac{kq_Aq_B}{|\vec{r}|^3}\vec{r} = \frac{(9\times10^9)(100\times10^{-6})(200\times10^{-6})}{\left(\sqrt{(4-1)^2+(4-0)^2+(2-2)^2}\right)^3}\left[(4-1)\hat{i}+(4-0)\hat{j}+(2-2)\hat{k}\right]$

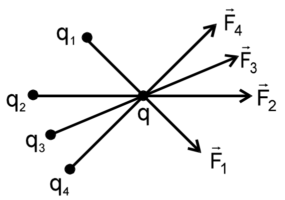
$= 7.2\left(\frac{3}{5}\hat{i}+\frac{4}{5}\hat{j}\right)$ N

Similarly $\vec{F}_A = 7.2\text{N}\left(-\frac{3}{5}\hat{i}-\frac{4}{5}\hat{j}\right)$ N

Action($\vec{F}_A$)and Reaction ($\vec{F}_B$) are equal but in opposite direction.

4. PRINCIPLE OF SUPERPOSITION

The electrostatic force is a two body interaction i.e. electrical force

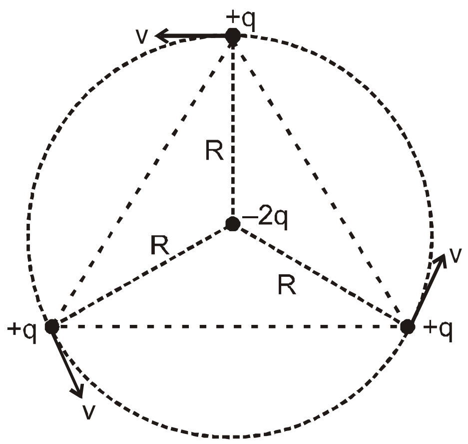


The electrostatic force is a two body interaction i.e., electrical force between two point charges is independent of presence or absence of other charges and so the principle of superposition is valid i.e., force on charged particle due to number of point charges is the resultant of forces due to individual point charges. Therefore, force on a point test charge due to many charges is given by.

$\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + ...$

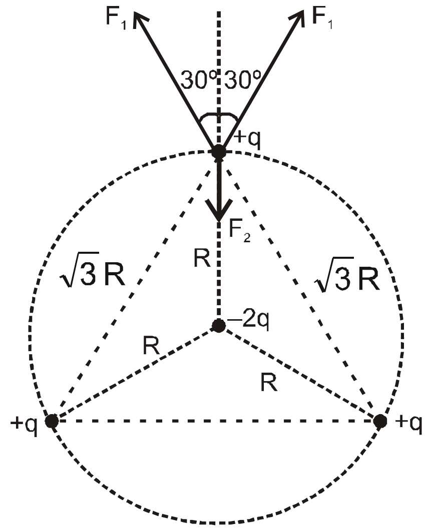
Solved Examples

Example 10. Three equal point charges of charge +q each are moving along a circle of radius R and a point charge –2q is also placed at the centre of circle (as shown in figure). If charges are revolving with constant and same speed in the circle then calculate speed of charges



Solution : $F_2 - 2F_1 \cos 30º = \dfrac{mv^2}{R}$

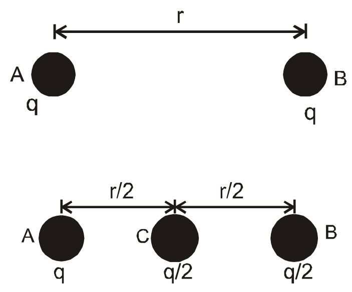
$\Rightarrow \dfrac{K(q)\ \ (2q)}{R^2} - \dfrac{2(Kq^2)}{(\sqrt{3}R)^2} \cos 30 = \dfrac{mv^2}{R}$



$\Rightarrow v = \sqrt{\dfrac{kq^2}{Rm}\left[2 - \dfrac{1}{\sqrt{3}}\right]}$

Example 11. Two equally charged identical small metallic spheres A and B repel each other with a force 2×10^{-5} N when placed in air (neglect gravitational attraction). Another identical uncharged sphere C is touched to B and then placed at the mid point of line joining A and B. What is the net electrostatic force on C ?

Solution : Let, initially the charge on each sphere be q and separation between their centres be r. Then according to given problem : F



$= \dfrac{1}{4\pi\varepsilon_0}\dfrac{q \times q}{r^2} = 2 \times 10^{-5}$ N

When sphere C touches B, the charge of B i.e. q will distribute equally on B and C as sphere are identical how charges on spheres;

$q_B = q_C = (q/2)$

So sphere C will experience a force

$F_{CA} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q(q/2)}{(r/2)^2} = 2F$ along $\overrightarrow{AB}$ due to charge on A.

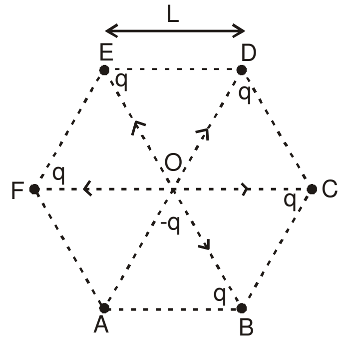
and, $F_{CB} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{(q/2)(q/2)}{(r/2)^2} = F$, along $\overrightarrow{BA}$ due to charge on B :

So the net force F_C on C due to charges on A and B,

$F_C = F_{CA} - F_{CB} = 2F - F = 2 \times 10^{-5}$ N along $\overrightarrow{AB}$.

Example 12. Five point charges, each of value q are placed on five vertices of a regular hexagon of side L. What is the magnitude of the force on a point charge of value – q coulomb placed at the centre of the hexagon?

Solution : **Method-I :** If there had been a sixth charge +q at the remaining vertex of hexagon, force due to all the six charges on –q at O would have been zero (as the forces due to individual charges will balance each other), i.e., $\vec{F_R} = 0$



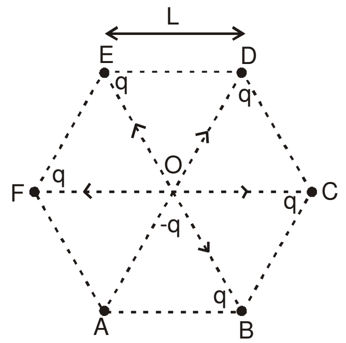
Now if $\vec{f}$ is the force due to sixth charge and $\vec{F}$ due to remaining five charges.

From $\vec{F} + \vec{f} = 0$ i.e. $\vec{F} = -\vec{f}$

or, $|F| = |f| = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q \times q}{L^2} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q^2}{L^2}$

$\vec{F}_{Net} = \vec{F}_{OD} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q^2}{L^2}$ along OD

Method-II : In the diagram, we can see that force due to charge A and D are opposite to each other



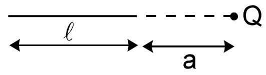
$\vec{F}_{oF} + \vec{F}_{OC} = \vec{0}$ ....(i)

Similarly $\vec{F}_{OB} + \vec{F}_{OE} = \vec{0}$ ....(ii)

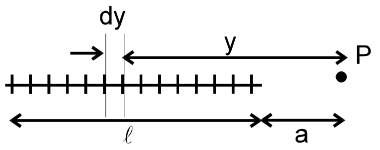
So $\vec{F}_{oF} + \vec{F}_{OB} + \vec{F}_{OC} + \vec{F}_{OD} + \vec{F}_{OE} = \vec{F}_{Net}$

Using (i) and (ii) $\vec{F}_{Net} = \vec{F}_{OD} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q^2}{L^2}$ along OD.

Example 13 A thin straight rod of length ℓ carrying a uniformly distributed charge q is located in vacuum. Find the magnitude of the electric force on a point charge 'Q' kept as shown in the figure.



Solution : As the charge on the rod is not point charge, therefore, first we have to find force on charge Q due to charge over a very small part on the length of the rod. This part, called element of length dy can be considered as point charge.



Charge on element, $dq = \lambda dy = \frac{q}{\ell} dy$

Electric force on 'Q' due to element

$= \frac{K.dq.Q}{y^2} = \frac{K.Q.q.dy}{y^2.\ell}$

All forces are along the same direction,

$\therefore \quad F = \sum dF$. This sum can be calculated using integration,

therefore, $F = \int_{y=a}^{a+\ell} \frac{KQqdy}{y^2\ell} = \frac{KqQ}{\ell} = \left[-\frac{1}{y}\right]_a^{a+\ell} = \frac{KQ.q}{\ell}\left[\frac{1}{a} - \frac{1}{a+\ell}\right] = \frac{KQq}{a(a+\ell)}$

Note : (1) The total charge of the rod cannot be considered to be placed at the centre of the rod as we do in mechanics for mass in many problems.

Note : (2) If a >> ℓ then, $F = \frac{KQq}{a^2}$

i.e., Behavior of the rod is just like a point charge.

5. ELECTROSTATIC EQUILIBRIUM

The point where the resultant force on a charged particle becomes zero is called equilibrium position.

5.1 Stable Equilibrium: A charge is initially in equilibrium position and is displaced by a small distance. If the charge tries to return back to the same equilibrium position then this equilibrium is called position of stable equilibrium.

5.2 Unstable Equilibrium: If charge is displaced by a small distance from its equilibrium position and the charge has no tendency to return to the same equilibrium position. Instead it goes away from the equilibrium position.

5.3 Neutral Equilibrium: If charge is displaced by a small distance and it is still in equilibrium condition then it is called neutral equilibrium.

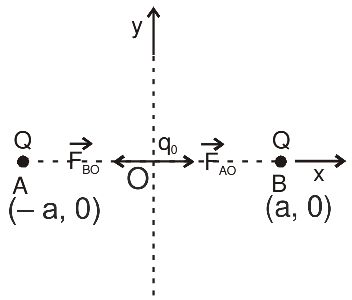
Solved Examples

Example 14. Two equal positive point charges 'Q' are fixed at points B(a, 0) and A(–a, 0). Another test charge q_0 is also placed at O(0, 0). Show that the equilibrium at 'O' is

(i) Stable for displacement along X-axis.

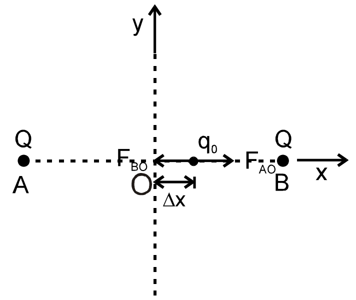
(ii) Unstable for displacement along Y-axis.

Solution : (i)



Initially $\vec{F}_{AO} + \vec{F}_{BO} = 0 \Rightarrow |\vec{F}_{AO}| = |\vec{F}_{BO}| = \dfrac{KQq_0}{a^2}$

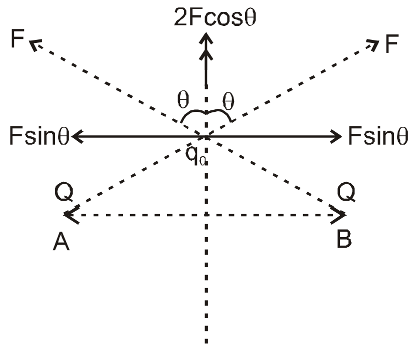
When charge is slightly shifted towards + x axis by a small distance Δx, then.



$|\vec{F}_{AO}| < |\vec{F}_{BO}|$

Therefore, the particle will move towards origin (its original position). Hence, the equilibrium is stable.

(ii) When charge is shifted along y axis:



After resolving components, net force will be along y axis So, the particle will not return to its original position & it is unstable equilibrium. Finally, the charge will move to infinity.

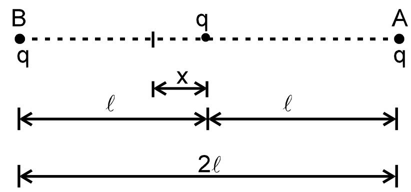
Example 15. Two point charges of charge q_1 and q_2 (both of same sign) and each of mass m are placed such that gravitational attraction between them balances the electrostatic repulsion. Are they in stable equilibrium? If not then what is the nature of equilibrium?

Solution : In given example : $\dfrac{Kq_1q_2}{r^2} = \dfrac{Gm^2}{r^2}$

We can see that irrespective of distance between them charges will main in equilibrium. If now distance is increased or decreased then there is no effect in their equilibrium. Therefore it is a neutral equilibrium.

Example 16. A particle of mass m and charge q is located midway between two fixed charged particles each having a charge q and a distance 2ℓ apart. Prove that the motion of the particle will be SHM if it is displaced slightly along the line connecting them and released. Also find its time period.

Solution : Let the charge q at the mid-point is displaced slightly to the left. The force on the displaced charge q due to charge q at A,



$$F_1 = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{(\ell + x)^2}$$

The force on the displaced charge q due to charge at B,

$$F_2 = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{(\ell - x)^2}$$

Net restoring force on the displaced charge q.

$$F = F_2 - F_1 \text{ or } F = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{(\ell - x)^2} - \frac{1}{4\pi\varepsilon_0}\frac{q^2}{(\ell + x)^2}$$

$$\text{or} \quad F = \frac{q^2}{4\pi\varepsilon_0}\left[\frac{1}{(\ell - x)^2} - \frac{1}{(\ell + x)^2}\right] = \frac{q^2}{4\pi\varepsilon_0}\frac{4\ell x}{(\ell^2 - x^2)^2}$$

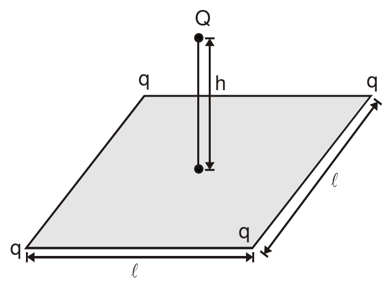
$$\text{Since} \quad \ell >> x, \quad \therefore F = \frac{q^2 \ell x}{\pi\varepsilon_0 \ell^4} \quad \text{or } F = \frac{q^2 x}{\pi\varepsilon_0 \ell^3}$$

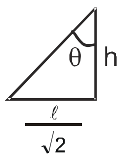
Hence we see that $F \propto x$ and it is opposite to the direction of displacement. Therefore, the motion is SHM.

$$T = 2\pi\sqrt{\frac{m}{k}}, \text{ (here } k = \frac{q^2}{\pi \in_0 \ell^3}) \ T = 2\pi\sqrt{\frac{m\pi \in_0 \ell^3}{q^2}}$$

Example 17. Find out mass of the charge Q, so that it remains in equilibrium for the given configuration.

Solution :





$\Rightarrow$ 4 Fcosθ = mg

$$\Rightarrow 4 \times \frac{KQq}{\left(\frac{\ell^2}{2} + h^2\right)^{3/2}} h = mg \qquad \therefore \qquad m = \frac{4KQqh}{g\left(\frac{\ell^2}{2} + h^2\right)^{3/2}}$$

Example 18. Two identical charged spheres are suspended by strings of equal length. Each string makes an angle θ with the vertical. When suspended in a liquid of density σ = 0.8 gm/cc, the angle remains the same. What is the dielectric constant of the liquid? (Density of the material of sphere is ρ = 1.6 gm/cc.)

Solution : Initially as the forces acting on each ball are tension T, weight mg and electric force F, for its equilibrium along vertical

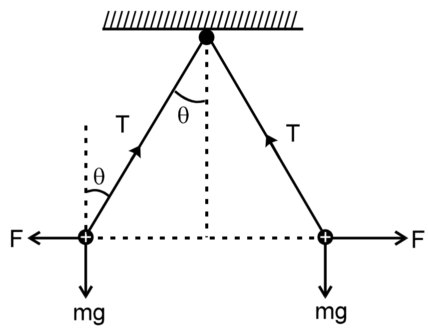
$$T \cos\theta = mg \qquad .....(1)$$

and along horizontal

$$T \sin\theta = F \qquad .....(2)$$

Dividing Eqn. (2) by (1), we have

$$\tan\theta = \frac{F}{mg} \qquad .....(3)$$



When the balls are suspended in a liquid of density σ and dielectric constant K, the electric force will become (1/K) times, i.e., F' = (F/K) while weight

$mg' = mg - F_B = mg - V\sigma g$ [as $F_B = V\sigma g$, where σ is density of material of sphere]

i.e., $mg' = mg\left[1-\frac{\sigma}{\rho}\right]$ $\left[\text{as } V = \frac{m}{\rho}\right]$

So, for equilibrium of ball,

$$\tan\theta' = \frac{F'}{mg'} = \frac{F}{Kmg[1-(\sigma/\rho)]} \quad ...(4)$$

According to given information $\theta' = \theta$; so from equations (4) and (3), we have :

$$K = \frac{\rho}{(\rho-\sigma)} = \frac{(1.6)}{(1.6-0.8)} = 2$$ **Ans.**

6. ELECTRIC FIELD

Electric field is the region around charged particle or charged body in which if another charge is placed, it experiences electrostatic force.

6.1 Electric field intensity $\vec{E}$:

Electric field intensity at a point is equal to the electrostatic force experienced by a unit positive point charge both in magnitude and direction.

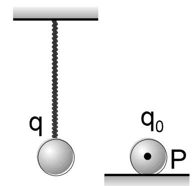
If a test charge q_0 is placed at a point in an electric field and experiences a force $\vec{F}$ due to some charges (called source charges), the electric field intensity at that point due to source charges is given by $\vec{E} = \frac{\vec{F}}{q_0}$

If the $\vec{E}$ is to be determined practically then the test charge q_0 should be small otherwise it will affect the charge distribution on the source which is producing the electric field and hence modify the quantity which is measured.

Solved Examples

Example 19. A positively charged ball hangs from a long silk thread. We wish to measure E at a point P in the same horizontal plane as that of the hanging charge. To do so, we put a positive test charge q_0 at the point and measure F/q_0. Will F/q_0 be less than, equal to, or greater than E at the point in question?

Solution : When we try to measure the electric field at point P then after placing the test charge at P, it repels the source charge (suspended charge) and the measured value of electric field $E_{measured} = \frac{F}{q_0}$ will be less than the actual value E_{act} ,that we wanted to measure.



6.2 Properties of electric field intensity $\vec{E}$:

(i) It is a vector quantity. Its direction is the same as the force experienced by positive charge.

(ii) Direction of electric field due to positive charge is always away from it while due to negative charge, always towards it.

(iii) Its S.I. unit is Newton/Coulomb.

(iv) Its dimensional formula is $[MLT^{-3}A^{-1}]$

(v) Electric force on a charge q placed in a region of electric field at a point where the electric field intensity is $\vec{E}$ is given by $\vec{F} = q\vec{E}$.

Electric force on point charge is in the same direction of electric field on positive charge and in opposite direction on a negative charge.

(vi) It obeys the superposition principle, that is, the field intensity at a point due to a system of charges is vector sum of the field intensities due to individual point charges.

i.e. $\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + ...$

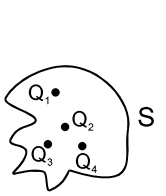
(vii) It is produced by source charges. The electric field will be a fixed value at a point unless we change the distribution of source charges.

Solved Examples

Example 20. Electrostatic force experienced by –3μC charge placed at point 'P' due to a system 'S' of fixed point charges as shown in figure is $\vec{F} = (21\hat{i} + 9\hat{j})$ μN.

(i) Find out electric field intensity at point P due to S.

(ii) If now, 2μC charge is placed and –3 μC is removed at point P then force experienced by it will be.



Solution : **(i)** $\vec{F} = q\vec{E} \Rightarrow (21\hat{i} + 9\hat{j})\mu N = -3\mu C\,(\vec{E})$

$\Rightarrow \vec{E} = -7\hat{i} - 3\hat{j}\,\frac{N}{C}$

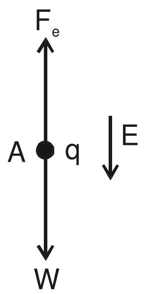
(ii) Since the source charges are not disturbed the electric field intensity at 'P' will remain same.

$\vec{F}_{2\mu C} = +2(\vec{E}) = 2(-7\hat{i} - 3\hat{j}) = (-14\hat{i} - 6\hat{j})\ \mu N$

Example 21. Calculate the electric field intensity which would be just sufficient to balance the weight of a particle of charge –10 μc and mass 10 mg. (Take g = 10 ms^2)

Solution : As force on a charge q in an electric field $\vec{E}$ is $\vec{F}_q = q\vec{E}$

So, according to given problem:

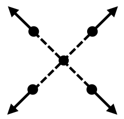
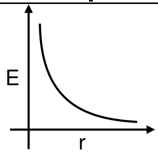
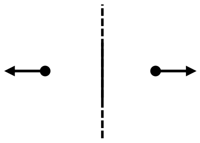
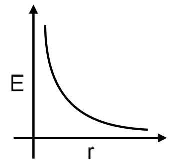
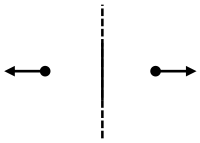
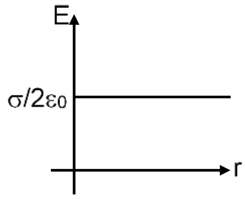
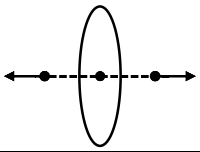
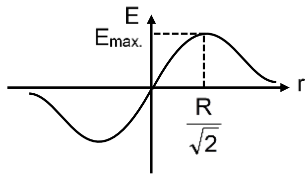
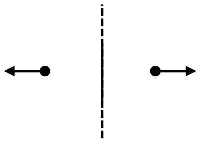
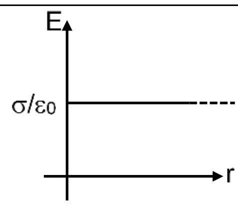
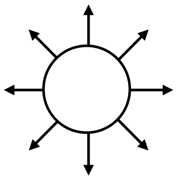
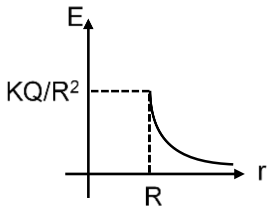
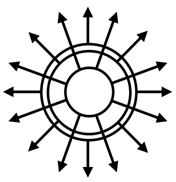
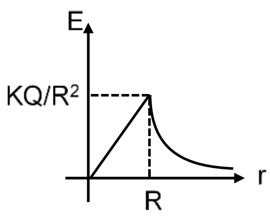


[W = weight of particle]

$|\vec{F}_q| = |\vec{W}|$ i.e., $|q|E = mg$

i.e., $E = \frac{mg}{|q|}$ = 10 N/C., in downward direction.

List of formula for Electric Field Intensity due to various types of charge distribution :

Name/Type	Formula	Note	Graph
Point charge 	$\vec{E} = \frac{Kq}{\lvert \vec{r} \rvert^2} \cdot \hat{r}$	* q is source charge. * $\vec{r}$ is vector drawn from source charge to the test point. Outwards due to +charges & inwards due to –charges.	
Infinitely long line charge 	$\frac{\lambda}{2\pi\varepsilon_0 r}\hat{r} = \frac{2K\lambda\hat{r}}{r}$	* q is linear charge density (assumed uniform) * r is perpendicular distance of point from line charge. * r is radial unit vector drawn from the charge to test point.	
Infinite non-conducting thin sheet 	$\frac{\sigma}{2\varepsilon_0}\hat{n}$	* σ is surface charge density. (assumed uniform) * n is unit normal vector x = distance of point on the axis from centre of the ring. * electric field is always along the axis.	
Uniformly charged ring 	$E = \frac{KQx}{(R^2 + x^2)^{3/2}}$ $E_{centre} = 0$	* Q is total charge of the ring * x = distance of point on the axis from centre of the ring. * electric field is always along the axis.	
Infinitely large charged conducting sheet 	$\frac{\sigma}{\varepsilon_0}\hat{n}$	*σ is the surface charge. density (assumed uniform) * n is the unit vector perpendicular to the surface.	
Uniformly charged hollow conducting / non-conducting / solid conducting sphere 	(i) for $r \geq R$ $\vec{E} = \frac{kQ}{\lvert \vec{r} \rvert^2}\hat{r}$ (ii) for $r < R$ $E = 0$	*R is radius of the sphere. * $\vec{r}$ is vector drawn from centre of sphere to the point. *Sphere acts like a point charge placed at centre for points outside the sphere. * $\vec{E}$ is always along radial direction. *Q is total charge ($= \sigma 4\pi R^2$.) (σ = surface charge density)	
Uniformly charged solid non-conducting sphere (insulating material) 	(i) for $r \geq R$ $\vec{E} = \frac{kQ}{\lvert \vec{r} \rvert^2}\hat{r}$ (ii) for $r \leq R$ $\vec{E} = \frac{kQ}{R^3}\vec{r}$	* $\vec{r}$ is vector drawn from centre of sphere to the point * Sphere acts like a point charge placed at the centre of points outside the sphere * $\vec{E}$ is always along radial dir^n *Q is total charge $\left(\rho.\frac{4}{3}\pi R^3\right)$. ($\rho$ = volume charge density) * Inside the sphere $E \propto r$. * Outside the sphere $E \propto 1/r^2$.	

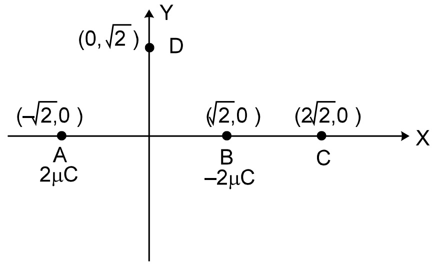
Example 22. Find out electric field intensity at point A (0, 1m, 2m) due to a point charge $-20\mu C$ situated at point B($\sqrt{2}$ m, 0, 1m).

Solution : $E = \frac{KQ}{|\vec{r}|^3}\vec{r} = \frac{KQ}{|\vec{r}|^2}\hat{r} \Rightarrow \vec{r}$ = P.V. of A – P.V. of B (P.V. = Position vector)

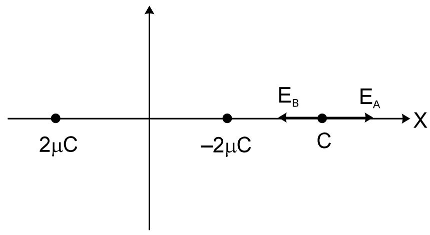
$= \left(-\sqrt{2}\hat{i} + \hat{j} + \hat{k}\right)$ $|\vec{r}| = \sqrt{(\sqrt{2})^2 + (1)^2 + (1)^2} = 2$

$E = \frac{9 \times 10^9 \times (-20 \times 10^{-6})}{8}\left(-\sqrt{2}\hat{i} + \hat{j} + \hat{k}\right) = -22.5 \times 10^3 \left(-\sqrt{2}\hat{i} + \hat{j} + \hat{k}\right)$ N/C.

Example 23. Two point charges $2\mu c$ and $-2\mu c$ are placed at points A and B as shown in figure. Find out electric field intensity at points C and D. [All the distances are measured in meter].



Solution : Electric field at point C (E_A, E_B are magnitudes only and arrows represent directions).



Electric field due to positive charge is away from it while due to negative charge, it is towards the charge.

It is clear that $E_B > E_A$.

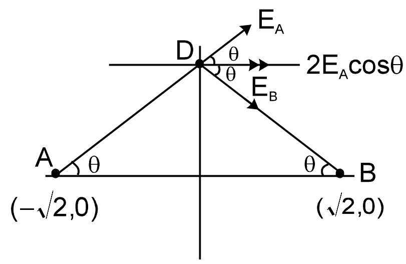
$\therefore$ $E_{Net} = (E_B - E_A)$ towards negative X-axis

$= \frac{K(2\mu c)}{(\sqrt{2})^2} - \frac{K(2\mu c)}{(3\sqrt{2})^2}$ towards negative X-axis = 8000 $(-\hat{i})$ N/C

Electric field at point D :

Since magnitude of charges are same and also AD = BD

So, $E_A = E_B$

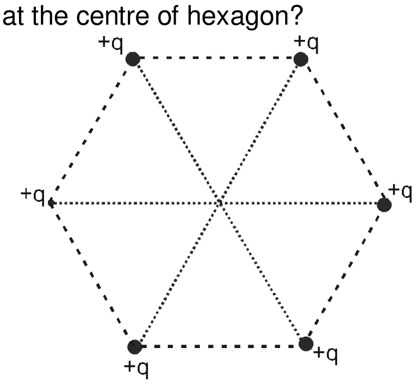


Vertical components of $\vec{E}_A$ and $\vec{E}_B$ cancel each other while horizontal components are in the same direction.

So, $E_{net} = 2E_A\cos\theta = \frac{2.K(2\mu c)}{2^2}\cos 45°$

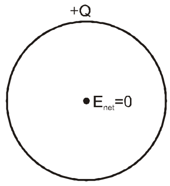
$= \frac{K \times 10^{-6}}{\sqrt{2}} = \frac{9000}{\sqrt{2}}\hat{i}$ N/C.

Example 24. Six equal point charges are placed at the corners of a regular hexagon of side 'a'. Calculate electric field intensity at the centre of hexagon?



Answer : Zero (By symmetry)

Similarly electric field due to a uniformly charged ring at the centre of ring :



Note : (i) Net charge on a conductor remains only on the outer surface of a conductor. This property will be discussed in the article of the conductor. (Article no.17)
(ii) On the surface of isolated spherical conductor charge is uniformly distributed.

6.3 Electric field due to a uniformly charged ring and arc.

Solved Examples

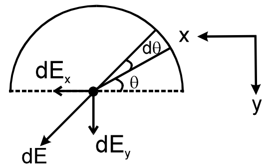
Example 25. Find out electric field intensity at the centre of a uniformly charged semicircular ring of radius R and linear charge density λ.

Solution : λ = linear charge density.

The arc is the collection of large no. of point charges. Consider a part of ring as an element of length $Rd\theta$ which subtends an angle $d\theta$ at centre of ring and it lies between θ and $\theta + d\theta$

$\overrightarrow{dE} = dE_x\hat{i} + dE_y\hat{j}$; $E_x = \int dE_x = 0$ (due to symmetry)

& $E_y = \int dE_y = \int_0^{\pi} dE\sin\theta = \frac{K\lambda}{R}\int_0^{\pi}\sin\theta.d\theta = \frac{2K\lambda}{R}$

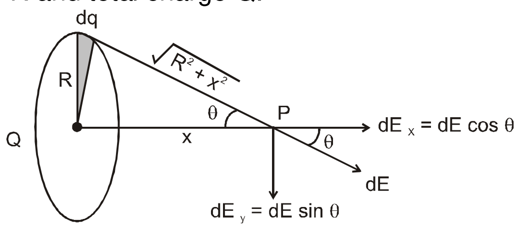


Example 26. Find out electric field intensity at the centre of uniformly charged quarter ring of radius R and linear charge density λ.

Solution : Refer to the previous question $\overrightarrow{dE} = dE_x\hat{i} + dE_y\hat{j}$ $\therefore$ on solving $E_{net} = \frac{K\lambda}{R}(\hat{i} + \hat{j})$

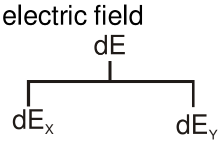
By use of symmetry and from the formula of electric field due to half ring.
Above answer can be justified.

(ii) Derivation of electric field intensity at a point on the axis at a distance x from centre of uniformly charged ring of radius R and total charge Q.



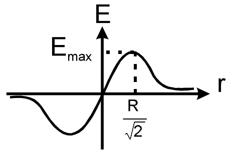
Consider an element of charge dq. Due to this element, the electric field at the point on axis, which is at a distance x from the centre of the ring is dE.

There are two components of this electric field



The y-component of electric field due to all the elements will be cancelled out to each other. So net electric field intensity at the point will be only due to X-component of each element.

$$E_{net} = \int dE_x = \int dE\cos\theta = \int_O^Q \frac{K(dq)}{R^2+x^2} \times \frac{x}{\sqrt{R^2+X^2}} = \frac{k \quad x}{(R^2+x^2)^{3/2}}\int_O^Q dq$$



$$E_{net} = \frac{KQx}{[R^2+x^2]^{3/2}}$$

Graph for variation of E with r.

E will be max when $\frac{dE}{dx} = 0$, that is at $x = \frac{R}{\sqrt{2}}$ and $E_{max} = \frac{2KQ}{3\sqrt{3}\ R^2}$

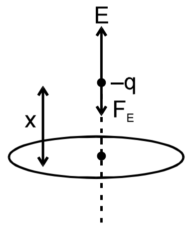
Case (i) : if x>>R, $E = \frac{KQ}{x^2}$ Hence the ring will act like a point charge

Case (ii) : if x<<R, $E = \frac{KQ\ x}{R^3}$

Solved Example

Example 27. Positive charge Q is distributed uniformly over a circular ring of radius a. A point particle having a mass m and a negative charge –q, is placed on its axis at a distance y from the centre. Find the force on the particle. Assuming y << a, find the time period of oscillation of the particle if it is released from there. (Neglect gravity)

Solution : When the negative charge is shifted at a distance x from the centre of the ring along its axis then force acting on the point charge due to the ring :



$$F_E = qE \text{ (towards centre)} = q\left[\frac{KQy}{(a^2+y^2)^{3/2}}\right]$$

If a >> y then $a^2 + y^2 \simeq a^2$

$$\therefore \quad F_E = \frac{1}{4\pi\varepsilon_0}\frac{Qqy}{a^3} \text{ (Towards centre)}$$

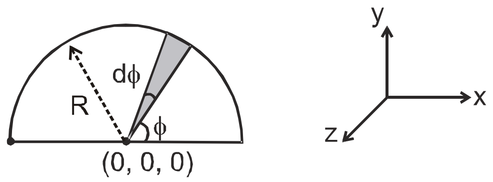
Since, restoring force $F_E \propto y$, therefore motion of charge the particle will be S.H.M.

Time period of SHM

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{m}{\left(\frac{Qq}{4\pi\varepsilon_0 a^3}\right)}} = \left[\frac{16\pi^3\varepsilon_0 ma^3}{Qq}\right]^{1/2}$$

Example 28. Calculate electric field intensity at a point on the axis which is at distance x from the centre of half ring, having total charge Q distributed uniformly on it. The radius of half ring is R.

Solution : Consider an element of small angle $d\phi$ at an angle ϕ as shown.



Coordinates of element : $(R\cos\phi, R\sin\phi, 0)$

Coordinates of point : (0, 0, x)

Now electric field due to element :

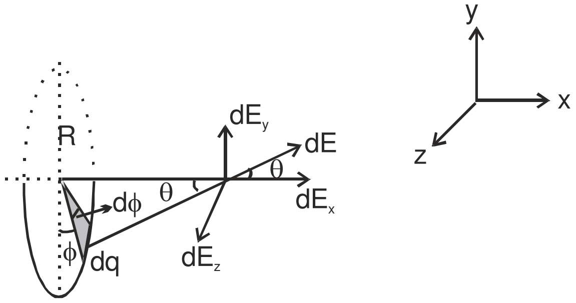
$$\overline{dE} = \frac{K(\lambda R d\phi)\cdot[-R\cos\phi\hat{i} - R\sin\phi\hat{j} + x\hat{k}]}{(R^2\cos^2\phi + R^2\sin^2\phi + x^2)^{3/2}} \Rightarrow E_x = \Sigma dE_x = -\int_0^\pi \frac{K\lambda R^2\cos\phi d\phi}{(R^2+x^2)^{3/2}} = 0$$

$$E_y = \Sigma dE_y = -\int_0^\pi \frac{K\lambda R^2\sin\phi d\phi}{(R^2+x^2)^{3/2}} = \frac{2K\lambda R^2}{(R^2+x^2)^{3/2}} = \frac{2KQR}{\pi(R^2+x^2)^{3/2}}$$

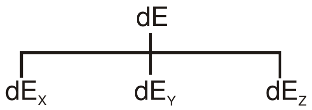
$$E_z = \Sigma dE_z = \int_0^{\pi} \frac{K\lambda Rxd\phi}{(R^2+x^2)^{3/2}} = \frac{KQx}{(R^2+x^2)^{3/2}}$$

$$E_{net} = \sqrt{E_x^2 + E_y^2 + E_z^2} = \frac{KQ}{(R^2+x^2)^{3/2}}\sqrt{\frac{4R^2}{\pi^2} + x^2}$$

Alternate solution :



Consider an element of charge dq at an angle ϕ on circumference of half ring. Due to this element electric field at the point on axis, which is at a distance x from the centre of half ring is dE. This electric field can be resolved into three component.



$$E_Z = \int_{-\pi/2}^{\pi/2} dE\sin\theta\sin\phi = 0$$

$$E_X = \int_{-\pi/2}^{\pi/2} dE\cos\theta = \frac{KQx}{[R^2+x^2]^{3/2}} \qquad .....(1)$$

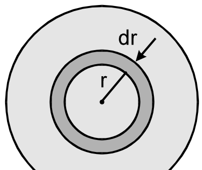
$$E_Y = \int_{-\pi/2}^{\pi/2} dE\sin\theta\cos\phi = \int \frac{Kdq}{R^2+x^2}\ \sin\theta.\cos\phi = \frac{2K\lambda R\sin\theta}{R^2+x^2} \qquad .....(2)$$

$\therefore \quad dq = \lambda R d\phi,\ \sin\theta = \dfrac{R}{\sqrt{R^2+x^2}} \qquad \Rightarrow \quad E_{net} = \sqrt{E_X^2 + E_Y^2}$

Example 29. Derive the expression of electric field intensity at a point 'P' which is situated at a distance x on the axis of uniformly charged disc of radius R and surface charge density σ. Also, derive results for

(i) x >> R (ii) x << R

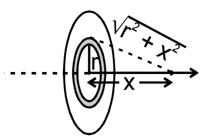
Solution : The disc can be considered to be a collection of large number of concentric rings. Consider an element of the shape of rings of radius r and of width dr. Electric field due to this ring at P is



$$dE = \frac{K.\sigma 2\pi r.dr.x}{(r^2+x^2)^{3/2}}$$

Put, $r^2 + x^2 = y^2$

$2rdr = 2ydy$



$$\therefore \quad dE = \frac{K.\sigma 2\pi y.dy.x}{y^3} = 2K\sigma\pi.x\ \frac{ydy}{y^3}$$

Electric field at P due to all rings is along the axis :

$$\therefore \quad E = \int dE \;\Rightarrow\; E = 2K\sigma\pi x \int_x^{\sqrt{R^2+x^2}} \frac{1}{y^2}dy = 2K\rho\pi x.\left[-\frac{1}{y}\right]_x^{\sqrt{R^2+x^2}}$$

$$= 2K\sigma\pi x\left[+\frac{1}{x}-\frac{1}{\sqrt{R^2+x^2}}\right] = 2K\sigma\pi\left[1-\frac{x}{\sqrt{R^2+x^2}}\right]$$

$$= \frac{\sigma}{2\varepsilon_0}\left[1-\frac{x}{\sqrt{R^2+x^2}}\right] \text{ along the axis}$$

Cases :(i) If x >> R

$$E = \frac{\sigma}{2\varepsilon_0}\left[1-\frac{x}{x\sqrt{\frac{R^2}{x^2}+1}}\right] = \frac{\sigma}{2\varepsilon_0}\left[1-\left(1+\frac{R^2}{x^2}\right)^{-1/2}\right]$$

$$\frac{\sigma}{2\varepsilon_0} = \left[1-1+\frac{1}{2}\frac{R^2}{x^2} + \text{higher order terms}\right] = \frac{\sigma}{4\varepsilon_0}\frac{R^2}{x^2} = \frac{\sigma\pi R^2}{4\pi\varepsilon_0 x^2} = \frac{Q}{4\pi\varepsilon_0 x^2}$$

i.e., behaviour of the disc is like a point charge.

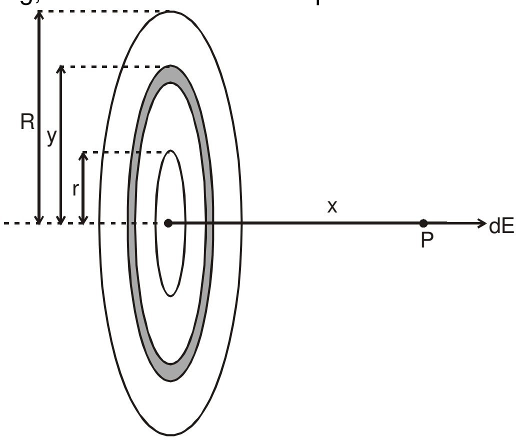
(ii) If x << R

$$E = \frac{\sigma}{2\varepsilon_0}[1-0] = \frac{\sigma}{2\varepsilon_0}$$

i.e., behaviour of the disc is like infinite sheet.

Example 30. Calculate electric field at a point on axis, which at a distance x from centre of uniformly charged disc having surface charge density σ and R which also contains a concentric hole of radius r.

Solution : Consider a ring of radius y (r < y < R) and width dy concentric with disc and in the plane of the disc. Due to this ring, the electric field at the point P :



$$dE = \frac{K(dq)x}{[X^2+Y^2]^{3/2}}$$

$$E_{net} = \int_r^R \frac{Kx.\sigma(2\pi y)dy}{[x^2+y^2]^{3/2}} \qquad [\because dq = \sigma 2\pi y dy]$$

$$E_{net} = \frac{2\pi\sigma kx}{2}\int_{x^2+r^2}^{x^2+R^2}\frac{dt}{t^{3/2}}, \text{put } x^2+y^2 = t,\ 2y.\,dy = dt$$

$$= \frac{\sigma x}{2\varepsilon_0}\left[\frac{1}{\sqrt{x^2+r^2}}-\frac{1}{\sqrt{x^2+R^2}}\right] \quad \text{away from centre}$$

Alternate method

We can also use superposition principle to solve this problem.

(i) Assume a disc without hole of radius R having surface charge density + σ.

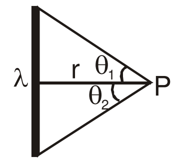
(ii) Also assume a concentric disc of radius r in the same plane of first disc having charge density – σ.

Now using derived formula in last example the net electric field at the centre is :

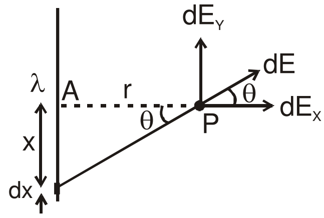
$$\vec{E}_{net} = \vec{E}_R + \vec{E}_r = \frac{\sigma x}{2\varepsilon_0}\left[\frac{1}{\sqrt{r^2+x^2}} - \frac{1}{\sqrt{R^2+x^2}}\right] \text{ away from centre.}$$

6.4 Electric field due to uniformly charged wire:

(i) Line charge of finite length : Derivation of expression for intensity of electric field at a point due to line charge of finite size of uniform linear charge density λ. The perpendicular distance of the point from the line charge is r and lines joining ends of line charge distribution make angle θ_1 and θ_2 with the perpendicular line.



Consider a small element dx on line charge distribution at distance x from point A (see fig.). The charge of this element will be dq = λdx. Due to this charge (dq), the intensity of electric field at the point P is dE.



Then $dE = \frac{K(dq)}{r^2+x^2} = \frac{K(\lambda dx)}{r^2+x^2}$

There will be two components of this field : $E_x = \int dE_x = \int dE\cos\theta = \int \frac{K\lambda dx}{r^2+x^2}.\cos\theta$

dE

dE_X $\qquad$ dE_Y

Assuming, $x = r\tan\theta \Rightarrow dx = r\sec^2\theta.\, d\theta$

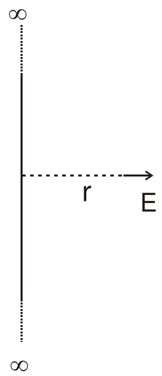
$$\text{so } E_x = \int_{-\theta_2}^{+\theta_1} \frac{K\ \lambda r\sec^2\theta\cdot\cos\theta\cdot d\theta}{r^2+r^2\tan^2\theta} = \frac{K\lambda}{r}\int_{-\theta_2}^{+\theta_1}\cos\theta.d\theta = \frac{K\lambda}{r}\ [\sin\theta_1 + \sin\theta_2] \quad .....(1)$$

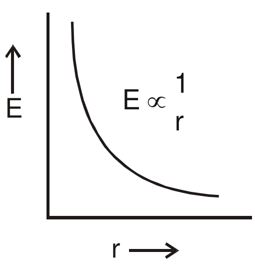
Similarly y-component. $E_y = \frac{K\lambda}{r}\int_{-\theta_2}^{+\theta_1}\sin\theta.d\theta = \frac{K\lambda}{r}\ [\cos\theta_2 - \cos\theta_1]$

Net electric field at the point: $E_{net} = \sqrt{E_x^2 + E_y^2}$

(ii) We can derive a result for infinitely long line charge: In above eq. (1) & (2), if we put $\theta_1 = \theta_2 = 90^{\circ}$, we can get required result.

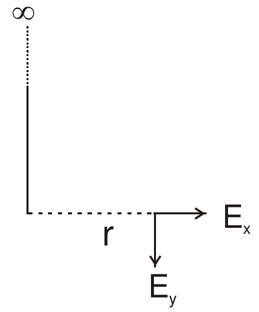
$E_{net} = E_x = \frac{2K\lambda}{r}$





(iii) For Semi- infinite wire : $\theta_1 = 90^\circ$ and $\theta_2 = 0^\circ$, so,

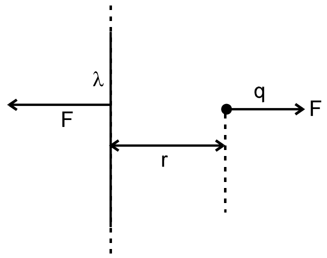
$E_x = \dfrac{K\lambda}{r}$, $E_y = \dfrac{K\lambda}{r}$



Solved Examples

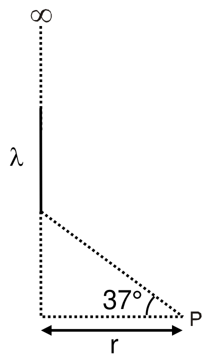
Example 31. A point charge q is placed at a distance r from a very long charged thread of uniform linear charge density λ. Find out total electric force experienced by the line charge due to the point charge. (Neglect gravity).

Solution : Force on charge q due to the thread, $F = \left(\dfrac{2K\lambda}{r}\right).q$

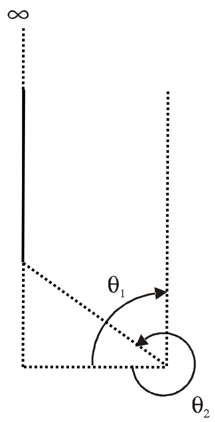


By Newton's III law, every action has equal and opposite reaction So, force on the thread $= \dfrac{2K\lambda}{r}.q$ (away from point charge)

Example 32. Figure shows a long wire having uniform charge density λ as shown in figure. Calculate electric field intensity at point P.

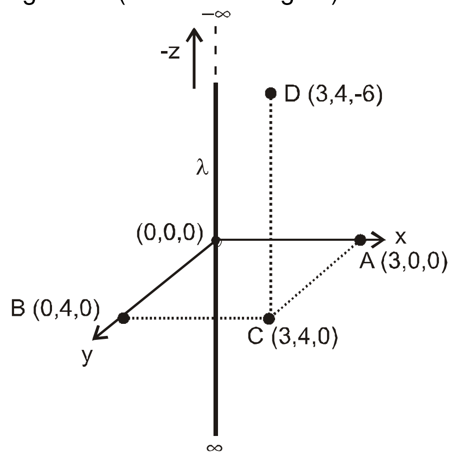


Solution : $\theta_1 = 90^\circ$ and $\theta_2 = 360^\circ - 37^\circ$ So



$E_x = \dfrac{K\lambda}{r}$ $[\sin\theta_1 + \sin\theta_2]$; $E_y = \dfrac{K\lambda}{r}$ $[\cos\theta_2 - \cos\theta_1]$

Example 33. Find electric field at point A, B, C, D due to infinitely long uniformly charged wire with linear charge density λ and kept along z-axis (as shown in figure). Assume that all the parameters are in S.I. units.



Solution : $E_A = \frac{2K\lambda}{3}(\hat{i}) \Rightarrow E_B = \frac{2K\lambda}{4}(\hat{j})$

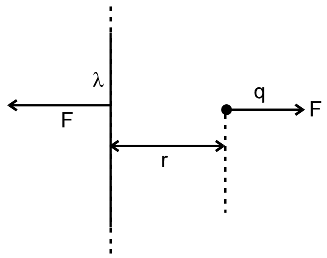
$E_C = \frac{2K\lambda}{5}\hat{OC} = \frac{2K\lambda}{5}\left(\frac{3\hat{i}+4\hat{j}}{5}\right) \qquad E_D = \frac{2K\lambda}{5}\left(\frac{3\hat{i}+4\hat{j}}{5}\right) \qquad \Rightarrow E_D = E_C$

Example 34. A point charge q is placed at a distance r from a very long charged thread of uniform linear charge density λ. Find out total electric force experienced by the line charge due to the point charge. (Neglect gravity).

Solution : Force on charge q due to the thread, $F = \left(\frac{2K\lambda}{r}\right).q$

By Newton's III law, every action has equal and opposite reaction, so force on the thread = $\frac{2K\lambda}{r}.q$

(away from point charge)

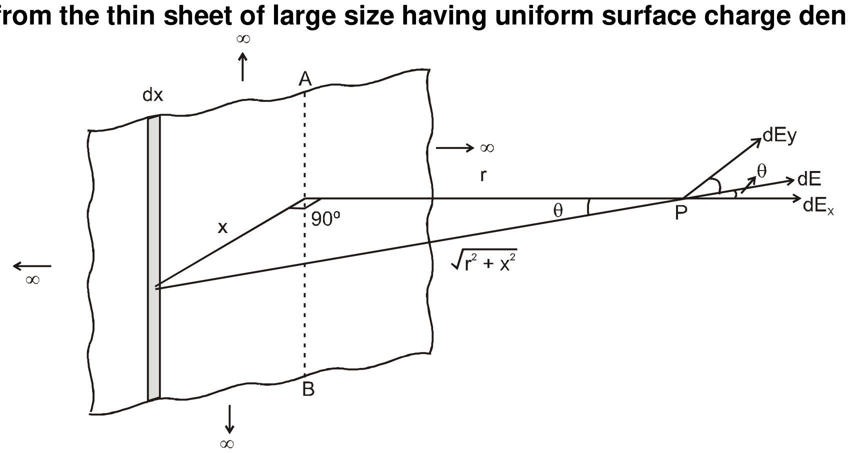


6.5 Electric field due to uniformly charged infinite sheet

$E_{net} = \frac{\sigma}{2\varepsilon_o}$ towards normal direction

ELECTRIC FIELD DUE TO AN INFINITELY LARGE, UNIFORMLY CHARGED SHEET

Derivation of expression for intensity of electric field at a point which is at a perpendicular distance r from the thin sheet of large size having uniform surface charge density σ.



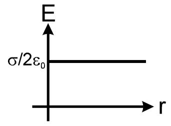
Assume a thin strip of width dx at distance x from line AB (see figure), which can be considered as a infinite line charge of charge density $\lambda = \sigma dx$

Due to this line charge the electric field intensity at point P will be $dE = \dfrac{\sigma K(dx)}{\sqrt{r^2 + x^2}}$

Take another element similar to the first element on the other side of AB. Due to symmetry, Y-component of all such elements will be cancelled out.

So net electric field will be given by : $E_{net} = \int dE_x = \int dE\cos\theta = \int \dfrac{2K(\sigma dx)}{\sqrt{r^2 + x^2}} \times \cos\theta$

Assume, $x = r\tan\theta \Rightarrow dx = r\sec^2\theta.\, d\theta$

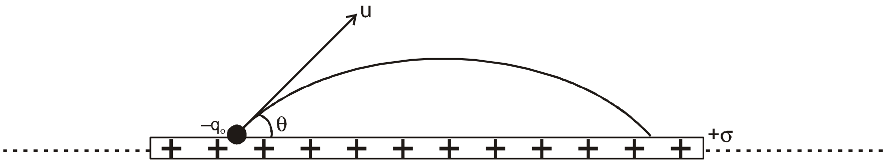


$\therefore \quad E_{net} = 2K\sigma \displaystyle\int_{-\pi/2}^{+\pi/2} \dfrac{r\sec^2\theta.d\theta.\cos\theta}{\sqrt{r^2 + r^2\tan^2\theta}} = \dfrac{\sigma}{2\varepsilon_o}$ away from sheet

Note : (1) The direction of electric field is always perpendicular to the sheet.
(2) The magnitude of electric field is independent of distance from sheet.

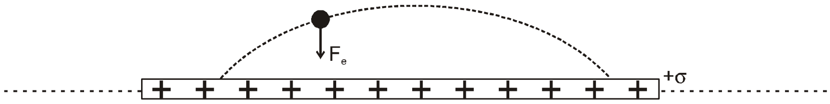
Solved Examples

Example 35. An infinitely large plate of surface charge density $+\sigma$ is lying in horizontal xy-plane. A particle having charge $-q_o$ and mass m is projected from the plate with velocity u making an angle θ with sheet. Find :



(i) The time taken by the particle to return on the plate..
(ii) Maximum height achieved by the particle.
(iii) At what distance will it strike the plate (Neglect gravitational force on the particle)

Solution :



Electric force acting on the particle $F_e = q_oE$: $F_e = (q_o)\left(\dfrac{\sigma}{2\varepsilon_o}\right)$ downward

So, acceleration of the particle : $a = \dfrac{F_e}{m} = \dfrac{q_o\sigma}{2\varepsilon_o m}$ = uniform

This acceleration will act like 'g' (acceleration due to gravity)

So, the particle will perform projectile motion.

(i) $T = \dfrac{2u\sin\theta}{g} = \dfrac{2u\sin\theta}{\left(\dfrac{q_o\sigma}{2\varepsilon_o m}\right)}$ (ii) $H = \dfrac{u^2\sin^2\theta}{2g} = \dfrac{u^2\sin^2\theta}{2\left(\dfrac{q_o\sigma}{2\varepsilon_o m}\right)}$

(iii) $R = \dfrac{u^2\sin 2\theta}{g} = \dfrac{u^2\sin 2\theta}{\left(\dfrac{q_o\sigma}{2\varepsilon_o m}\right)}$

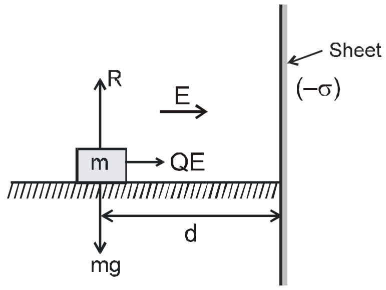
Example 36. A block having mass m and charge Q is resting on a frictionless plane at a distance d from fixed large non-conducting infinite sheet of uniform charge density $-\sigma$ as shown in Figure. Assuming that collision of the block with the sheet is perfectly elastic, find the time period of oscillatory motion of the block. Is it SHM?

Solution : The situation is shown in Figure. Electric force produced by sheet will accelerate the block towards the sheet producing an acceleration. Acceleration will be uniform because electric field E due to the sheet is uniform.

$a = \frac{F}{m} = \frac{QE}{m}$, where $E = \sigma/2\varepsilon_0$

As initially the block is at rest and acceleration is constant, from second equation of motion, time taken by the block to reach the wall

$d = \frac{1}{2}\,at^2$ i.e., $t = \sqrt{\frac{2L}{a}} = \sqrt{\frac{2md}{QE}} = \sqrt{\frac{4md\varepsilon_0}{Q\sigma}}$



As collision with the wall is perfectly elastic, the block will rebound with same speed and as now its motion is opposite to the acceleration, it will come to rest after traveling same distance d in same time t.

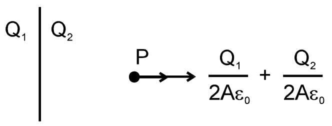
After stopping, it will again be accelerated towards the wall and so the block will execute oscillatory motion with 'span' d and time period.

$T = 2t = 2\sqrt{\frac{2md}{QE}} = 2\sqrt{\frac{4md\varepsilon_0}{Q\sigma}}$

However, as the restoring force F = QE is constant and not proportional to displacement x, the motion is not simple harmonic.

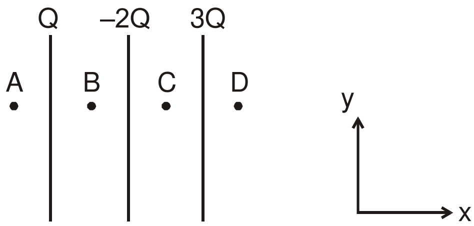
Example 37. If an isolated infinite sheet contains charge Q_1 on its one surface and charge Q_2 on its other surface then prove that electric field intensity at a point in front of sheet will be $\frac{Q}{2A\varepsilon_O}$, where $Q = Q_1 + Q_2$

Solution : Electric field at point P : $\vec{E} = \vec{E}_{Q_1} + \vec{E}_{Q_2} = \frac{Q_1}{2A\varepsilon_0}\hat{n} + \frac{Q_2}{2A\varepsilon_0}\hat{n} = \frac{Q_1 + Q_2}{2A\varepsilon_0}\hat{n} = \frac{Q}{2A\varepsilon_0}\hat{n}$

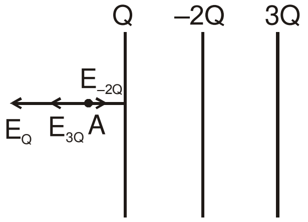


[This shows that the resultant field due to a sheet depends only on the total charge of the sheet and not on the distribution of charge on individual surfaces].

Example 38. Three large conducting parallel sheets are placed at a finite distance from each other as shown in figure. Find out electric field intensity at points A, B, C & D.

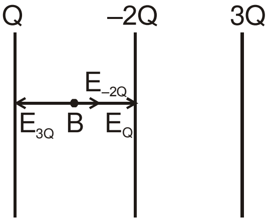


Solution : **For point A :**



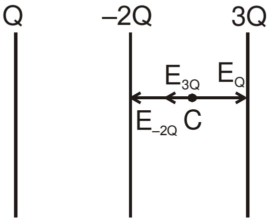
$$\vec{E}_{net} = \vec{E}_Q + \vec{E}_{3Q} + \vec{E}_{-2Q} = -\frac{Q}{2A\varepsilon_0}\hat{i} - \frac{3Q}{2A\varepsilon_0}\hat{i} + \frac{2Q}{2A\varepsilon_0}\hat{i} = -\frac{Q}{A\varepsilon_0}\hat{i}$$

For point B:



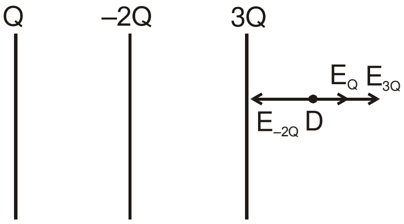
$$\vec{E}_{net} = \vec{E}_{3Q} + \vec{E}_{-2Q} + \vec{E}_Q = -\frac{3Q}{2A\varepsilon_0}\hat{i} + \frac{2Q}{2A\varepsilon_0}\hat{i} + \frac{Q}{2A\varepsilon_0}\hat{i} = \vec{0}$$

For point C :



$$\vec{E}_{net} = \vec{E}_Q + \vec{E}_{3Q} + \vec{E}_{-2Q} = +\frac{Q}{2A\varepsilon_0}\hat{i} - \frac{3Q}{2A\varepsilon_0}\hat{i} - \frac{Q}{2A\varepsilon_0}\hat{i} = -\frac{2Q}{A\varepsilon_0}\hat{i}$$

For point D :

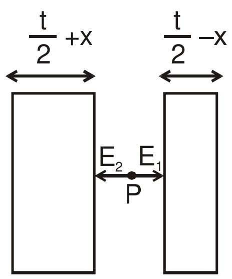


$$\vec{E}_{net} = \vec{E}_Q + \vec{E}_{3Q} + \vec{E}_{-2Q} = +\frac{Q}{2A\varepsilon_0}\hat{i} + \frac{3Q}{2A\varepsilon_0}\hat{i} - \frac{2Q}{2A\varepsilon_0}\hat{i} = \frac{2Q}{A\varepsilon_0}\hat{i}$$

Example 39. Determine and draw the graph of electric field due to infinitely large non-conducting sheet of thickness t and uniform volume charge density ρ as a function of distance x from its symmetry plane.

(a) $x \le \frac{t}{2}$ (b) $x \ge \frac{t}{2}$

Solution : We can consider two sheets of thickness $\left(\frac{t}{2} - x\right)$



and $\left(\frac{t}{2} + x\right)$

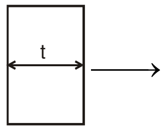
Where the point P lies inside the sheet.

Now, net electric field at point P :

$E = E_1 - E_2 = \dfrac{Q_1}{2A\varepsilon_0} - \dfrac{Q_2}{2A\varepsilon_0}$ [Q_1 : charge of left sheet; Q_2 : charge of right sheet.]

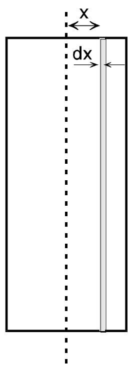
$$= \frac{A\rho\left(\frac{t}{2}+x\right) - \rho A\left(\frac{t}{2}-x\right)}{2A\varepsilon_0} = \frac{\rho x}{\varepsilon_0}$$

For point which lies outside the sheet we can consider a complete sheet of thickness t



$$\frac{Q}{2A\varepsilon_0}$$

$$E = \frac{\sigma t A}{2A\varepsilon_0} = \frac{\sigma t}{2\varepsilon_0}$$

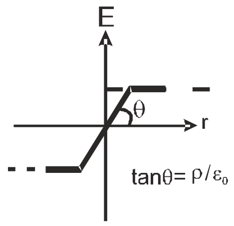


Alternate : We can assume thick sheet to be made of large number of uniformly charged thin sheets. Consider an elementary thin sheet of width dx at a distance x from symmetry plane.

Charge in sheet = $\rho A dx$ (A : assumed area of sheet)

Surface charge density, $\sigma = \dfrac{\rho A dx}{A}$

so, electric field intensity due to elementary sheet : $dE = \dfrac{\rho dx}{2\varepsilon_0}$



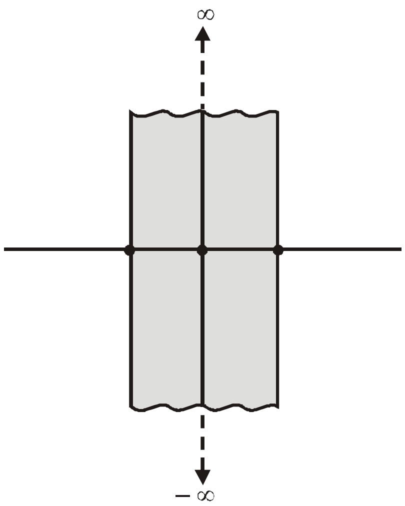
(a) When $x < \dfrac{t}{2} \Rightarrow$ $E_{Net} = \displaystyle\int_{-t/2}^{x} \frac{\rho dx}{2\varepsilon_0} - \int_{x}^{t/2} \frac{\rho dx}{2\varepsilon_0} = \frac{\rho x}{\varepsilon_0}$

(b) When $x > \dfrac{t}{2} \Rightarrow$ $E_{Net} = \displaystyle\int_{-t/2}^{t/2} \frac{\rho dx}{2\varepsilon_0} = \frac{\rho t}{2\varepsilon_0}$

Example 40. Thin infinite sheet of width w contains uniform charge distribution σ. Find out electric field intensity at following points :

(a) A point which lies in the same plane at a distance d from one of its edge.

(b) A point which is on the symmetry plane of sheet at a perpendicular distance d from it.



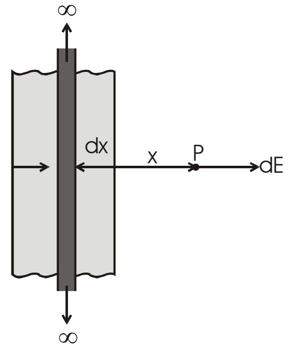
Solution : (a) Consider a thin strip of width dx. Linear charge density of strip : $\lambda = \sigma dx$

So, electric field due to this strip at point P

$$dE = \frac{2k\sigma dx}{x}$$

$$E_{net} = \int_d^{d+w} \frac{\sigma}{2\pi\varepsilon_0}\frac{dx}{x}$$

$$= \frac{\sigma}{2\pi\varepsilon_0}\ell n\left(\frac{d+w}{d}\right)$$



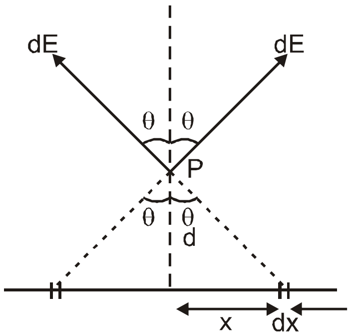
(b) Consider a thin strip of width dx. Linear charge density of strip :

$\lambda = \sigma dx$

$\therefore \quad E_p = \int 2dE\cos\theta$

or $\quad E_p = 2.\int_0^{w/2} \frac{\sigma dx}{2\pi\varepsilon_0\sqrt{d^2+x^2}}.\frac{d}{\sqrt{d^2+x^2}}$

$$= \frac{\sigma d}{\pi\varepsilon_0}\int_0^{w/2}\frac{dx}{d^2+x^2} = \frac{\sigma}{\pi\varepsilon_0}\tan^{-1}\frac{w}{2d}$$

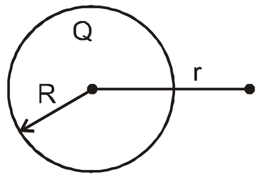


6.6 Electric field due to uniformly charged spherical shell

$E = \frac{KQ}{r^2}$ $r \geq R$ $\Rightarrow$ For the outside points & point on the surface the uniformly charged spherical shell behaves as a point charge placed at the centre

$E = 0 \qquad r < R$

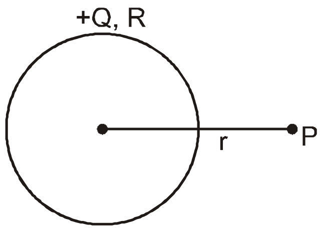
Electric field due to spherical shell outside it is always along the radial direction.



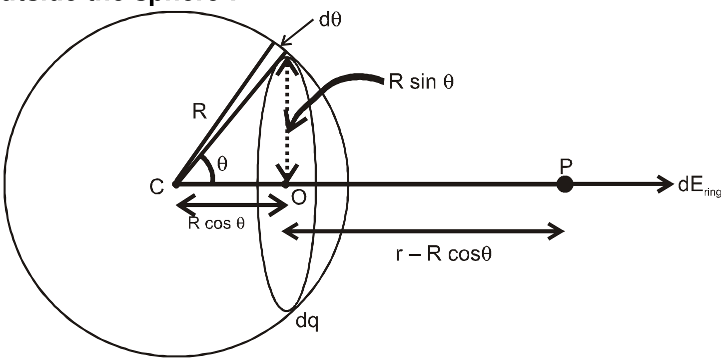
Finding electric field due to a uniformly charged spherical shell :

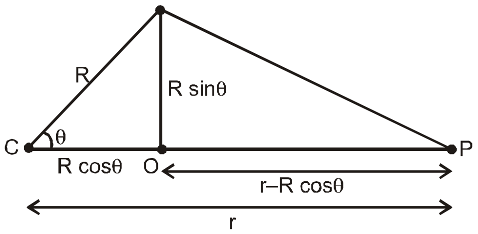
Suppose we have a spherical shell of radius R and charge +Q uniformly distributed on its surface. We have to find electric field at a point P, which is at a distance 'r' from the centre of the sphere.

For this, we can divide the shell into thin rings. Let's consider a ring making an angle θ with the axis and subtending a small angle dθ. Its width will be Rdθ. (arc = radius × angle = Rdθ).



For the points outside the sphere :





Electric field due to this small ring element :

$$dE = \frac{Kdqx}{[(\text{ring radius})^2 + x^2]^{3/2}} \qquad .....(1)$$

So, total electric field $E_{net} = \int \frac{Kdq\,x}{[(\text{ring radius})^2 + x^2]^{3/2}}$

Here, radius of the ring element = R sinθ & x = axial distance of point P from the ring = r – R cos θ

Area of the ring element = (length) (width) = (2π (radius of the ring)) Rdθ = (2πR sinθ) Rdθ

dq = charge of the small ring element. We can find dq by unitary method.

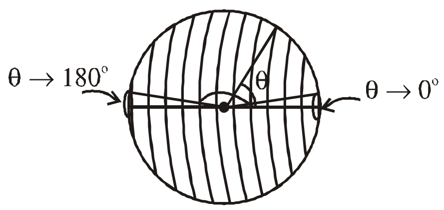
In $4\pi R^2$ Area, charge is Q

In unit Area, charge is $\frac{Q}{4\pi R^2}$.

In (2πR sinθ) Rdθ Area, charge = $\frac{Q}{4\pi R^2} \times (2\pi R \sin\theta)\, R d\theta = dq$

Putting values of r and dq in equation ..(1)

We get $E_{out} = \int\limits_{\theta=0}^{\theta=\pi} \frac{K\left(\frac{Q}{4\pi R^2} \times 2\pi(R\sin\theta)Rd\theta\right)(r - R\cos\theta)}{\left[(R\sin\theta)^2 + (r - R\cos\theta)^2\right]^{3/2}}$



(The first ring will make angle θ = 0 and the last ring will make θ = 180º. So, limit will be from θ = 0 to θ = 180º)

Steps of integration : From above integral :

$$E_{out} = \frac{KQ}{2} \int_0^{\pi} \frac{(r - R\cos\theta)\sin\theta d\theta}{(R^2 + r^2 - 2Rr\cos\theta)^{3/2}}$$

Now, let $z^2 = R^2 + r^2 - 2Rr\cos\theta \Rightarrow \quad 2\,zdz = 0 + 0 - 2Rr\,(-\sin\theta)\,d\theta$

$\therefore \quad zdz = Rr\sin\theta\, d\theta \qquad \& \qquad \cos\theta = \frac{R^2 + r^2 - z^2}{2Rr}$

Now, when θ = 0 $\quad \rightarrow \quad z = (r - R)$

$\theta = \pi \quad \rightarrow \quad z = r + R$

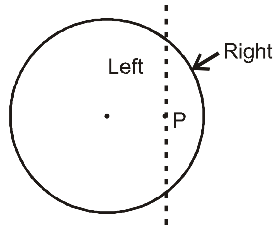
$$\therefore \quad E_{out} = \frac{KQ}{2} \int_{r-R}^{r+R} \frac{\left[r - R\frac{(R^2 + r^2 - z^2)}{2Rr}\right]\frac{zdz}{Rr}}{z^3} = \frac{KQ}{2} \int_{r-R}^{r+R} \frac{(2Rr^2 - R^3 - Rr^2 + Rz^2)dz}{2R^2r^2z^2}$$

$$= \frac{KQ}{4R^2r^2}\left[\int_{r-R}^{r+R} \frac{Rr^2dz}{z^2} - \int_{r-R}^{r+R} \frac{R^3dz}{z^2} + \int_{r-R}^{r+R} Rdz\right] = \frac{KQ}{4R^2r^2}\left[-\frac{Rr^2}{z} + \frac{R^3}{z} + Rz\right]_{r-R}^{r+R}$$

On solving above, we will get : $E_{out} = \frac{KQ}{r^2}$ if r > R

For the points inside the sphere:

Now lets derive the electric field due to uniformly charged solid sphere at a point 'P' inside it. The sphere is divided into two parts, the rings on the left part of point 'P' will produce electric field towards right and the rings on right part will produce electric field towards left and $E_{net} = E_{right} - E_{left}$. For this, limit of integration is divided into two parts.



$$E_{net} = \int_{\theta=0}^{\theta=\cos^{-1}\left(\frac{r}{R}\right)} \begin{pmatrix}\text{Electric} \\ \text{field due} \\ \text{to rings} \\ \text{of right} \\ \text{part}\end{pmatrix} - \int_{\theta=\cos^{-1}\left(\frac{r}{R}\right)}^{\theta=\pi} \begin{pmatrix}\text{Electric} \\ \text{field due} \\ \text{to rings} \\ \text{of left} \\ \text{part}\end{pmatrix}$$

As $\quad z^2 = R^2 + r^2 - 2rR\cos\theta$

When $\quad \theta = \cos^{-1}\left(\frac{r}{R}\right) \quad \Rightarrow \quad z = \sqrt{R^2 - r^2}$

When $\quad \theta = 0 \quad \Rightarrow \quad z = R - r$

When $\quad \theta = \pi \quad \Rightarrow \quad z = R + r$

From the result of previous case and just by changing limits we can write

$$E_{in} = \left[-\frac{Rr^2}{z} + \frac{R^3}{z} + Rz\right]_{R-r}^{\sqrt{R^2-r^2}} - \left[-\frac{Rr^2}{z} + \frac{R^3}{z} + Rz\right]_{\sqrt{R^2-r^2}}^{R+r}$$

On solving this expression, we will get E and E = 0 if r < R.

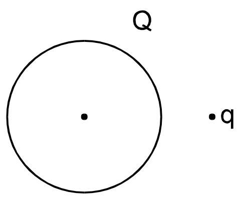
Finding electric field due to shell by integration is very lengthy, so we will not use this method. The given hand-out was just for knowledge. The best method to find E due to shell is by Gauss theorem which we will study later.

Solved Examples

Example 41. Figure shows a uniformly charged sphere of radius R and total charge Q. A point charge q is situated outside the sphere at a distance r from centre of sphere. Find out the following :

(i) Force acting on the point charge q due to the sphere.

(ii) Force acting on the sphere due to the point charge.



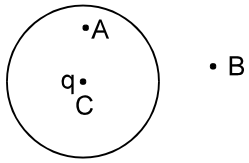
Solution : (i) Electric field at the position of point charge $\vec{E} = \frac{KQ}{r^2}\hat{r}$

So, $\vec{F} = \frac{KqQ}{r^2}\hat{r} \qquad |\vec{F}| = \frac{KqQ}{r^2}$

(ii) Since we know that every action has equal and opposite reaction so $\vec{F}_{sphere} = \frac{KqQ}{r^2}\hat{r}$

$\vec{F}_{sphere} = \frac{KqQ}{r^2}$

Example 42. Figure shows a uniformly charged thin sphere of total charge Q and radius R. A point charge q is also situated at the centre of the sphere. Find out the following :



(i) Force on charge q
(ii) Electric field intensity at A.
(iii) Electric field intensity at B.

Solution : (i) Electric field at the centre of the uniformly charged hollow sphere = 0
So force on charge q = 0

(ii) Electric field at A

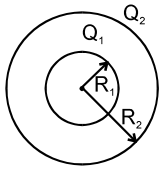
$$\vec{E}_A = \vec{E}_{Sphere} + \vec{E}_q = 0 + \frac{Kq}{r^2} \text{ ; } r = CA$$

E due to sphere = 0 , because point lies inside the charged hollow sphere.

(iii) Electric field $\vec{E}_B$ at point B $= \vec{E}_{Sphere} + \vec{E}_q = \frac{KQ}{r^2}.\hat{r} + \frac{Kq}{r^2}.\hat{r} = \frac{K(Q+q)}{r^2}.\hat{r}$; r = CB

Note : Here, we can also assume that the total charge of sphere is concentrated at the centre, for calculation of electric field at B.

Example 43. Two concentric uniformly charged spherical shells of radius R_1 and R_2 ($R_2 > R_1$) have total charges Q_1 and Q_2 respectively. Derive an expression of electric field as a function of r for following positions.



(i) $r < R_1$ (ii) $R_1 \le r < R_2$
(iii) $r \ge R_2$

Solution : (i) For $r < R_1$, therefore, point lies inside both the spheres

$E_{net} = E_{Inner} + E_{outer} = 0 + 0$

(ii) For $R_1 \le r < R_2$, point lies outside inner sphere but inside outer sphere :

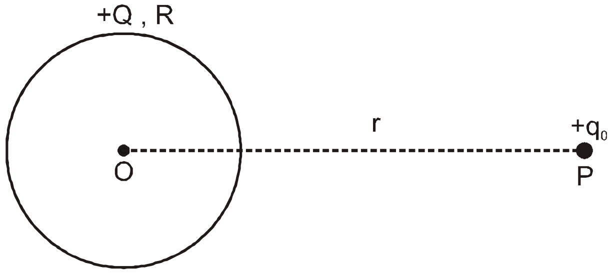
$\therefore \quad E_{net} = E_{inner} + E_{outer}$

$$\frac{KQ_1}{r^2}\,\hat{r} + 0 = \frac{KQ_1}{r^2}\hat{r}$$

(iii) For $r \ge R_2$
point lies outside inner as well as outer sphere.

Therefore, $E_{Net} = E_{inner} + E_{outer} = \frac{KQ_1}{r^2}\hat{r} + \frac{KQ_2}{r^2}\hat{r} = \frac{K(Q_1+Q_2)}{r^2}\hat{r}$

Example 44. A spherical shell having charge +Q (uniformly distributed) and a point charge + q_0 are placed as shown. Find the force between shell and the point charge(r >> R).

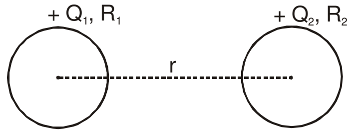


(i) Force on the point charge + q_0 due to the shell = $q\vec{E}_{0shell} = (q_0)\left(\frac{KQ}{r^2}\right)\hat{r} = \frac{KQq_0}{r^2}\hat{r}$ where $\hat{r}$, is unit vector along OP.

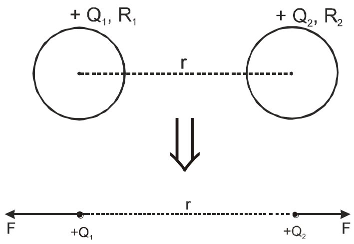
From action - reaction principle, force on the shell due to the point charge will also be : $F_{shell} = \frac{KQq_0}{r^2}(-\hat{r})$

Conclusion : To find the force on a hollow sphere due to outside charges, we can replace the sphere by a point charge kept at centre.

Example 45. Find force acting between two shells of radius R_1 and R_2 which have uniformly distributed charges Q_1 and Q_2 respectively and distance between their centers is r.



Solution : The shells can be replaced by point charges kept at centre, so force between them

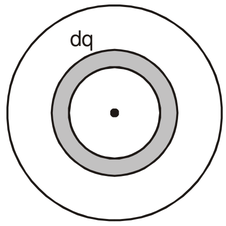


$$F = \frac{KQ_1Q_2}{r^2}$$

6.7 Electric field due to uniformly charged solid sphere:

Derive an expression for electric field due to solid sphere of radius R and total charge Q which is uniformly distributed in the volume, at a point which is at a distance r from centre for given two cases.

(i) $r \geq R$ (ii) $r \leq R$

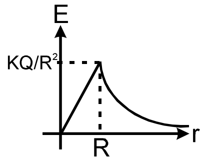


Assume an elementary concentric shell of charge dq. Due to this shell, the electric field at the point (r > R) will be:

$dE = \frac{Kdq}{r^2}$ [from above result of hollow sphere]

$$E_{net} = \int dE = \frac{KQ}{r^2}$$

For r < R, there will be no electric field due to shell of radius greater than r, so electric field at the point will be present only due to shells having radius less than r.



$$E'_{net} = \frac{KQ'}{r^2}$$

Here, $Q' = \dfrac{Q}{\frac{4}{3}\pi R^3} \times \dfrac{4}{3}\pi r^3 = \dfrac{Qr^3}{R^3}$

$\therefore$ $E'_{net} = \dfrac{KQ'}{r^2} = \dfrac{KQr}{R^3}$; away from the centre.

Note : The electric field inside and outside the sphere is always in radial direction.

Solved Examples

Example 46. A solid non conducting sphere of radius R and uniform volume charge density ρ has its centre at origin. Find out electric field intensity in vector form at following positions :

(i) (R/2, 0, 0) (ii) $\left(\frac{R}{\sqrt{2}}, \frac{R}{\sqrt{2}}, 0\right)$ (iii) (R, R, 0)

Solution : (i) At (R/2, 0, 0) : Distance of point from centre $= \sqrt{(R/2)^2 + 0^2 + 0^2}$ = R/2 < R, so point lies inside the sphere, so $\vec{E} = \frac{\rho\vec{r}}{3\varepsilon_0} = \frac{\rho}{3\varepsilon_0}\left[\frac{R}{2}\hat{i}\right]$

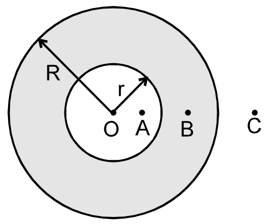
(ii) At $\left(\frac{R}{\sqrt{2}}, \frac{R}{\sqrt{2}},\ 0\right)$ distance of point from centre $= \sqrt{(R/\sqrt{2})^2 + (R/\sqrt{2})^2 + 0^2}$ = R = R, so point lies at the surface of sphere, therefore

$$\vec{E} = \frac{KQ}{R^3}\vec{r} = \frac{K\frac{4}{3}\pi R^3\rho}{R^3} = \left[\frac{R}{\sqrt{2}}\hat{i} + \frac{R}{\sqrt{2}}\hat{j}\right] = \frac{\rho}{3\varepsilon_0}\left[\frac{R}{\sqrt{2}}\hat{i} + \frac{R}{\sqrt{2}}\hat{j}\right]$$

(iii) The point is outside the sphere

$$\text{So, } \vec{E} = \frac{KQ}{r^3}\vec{r} = \frac{K\frac{4}{3}\pi R^3\rho}{(\sqrt{2}R)^3}\left[R\hat{i} + R\hat{j}\right] = \frac{\rho}{6\sqrt{2}\varepsilon_0}\left[R\hat{i} + R\hat{j}\right]$$

Example 47. A Uniformly charged solid non-conducting sphere of uniform volume charge density ρ and radius R is having a concentric spherical cavity of radius r. Find out electric field intensity at following points, as shown in the figure :



(i) Point A (ii) Point B

(iii) Point C (iv) Centre of the sphere

Solution : **Method-I :**

(i) For point A : We can consider the solid part of sphere to be made of large number of spherical shells which have uniformly distributed charge on its surface. Now, since point A lies inside all spherical shells so electric field intensity due to all shells will be zero.

$\overrightarrow{E_A} = 0$

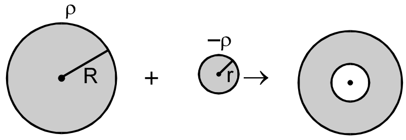
(ii) For point B : All the spherical shells for which point B lies inside will make electric field zero at point B. So electric field will be due to charge present from radius r to OB.

$$\text{So, } \overrightarrow{E_B} = \frac{K\frac{4}{3}\pi(OB^3 - r^3)\rho}{OB^3}\overrightarrow{OB} = \frac{\rho}{3\varepsilon_0}\frac{[OB^3 - r^3]}{OB^3}\overrightarrow{OB}$$

(iii) For point C, similarly we can say that for all the shell points C lies outside the shell

$$\text{So, } \overrightarrow{E_C} = \frac{K[\frac{4}{3}\pi(R^3 - r^3)]}{[OC]^3}\overrightarrow{OC} = \frac{\rho}{3\varepsilon_0}\frac{R^3 - r^3}{[OC]^3}\overrightarrow{OC}$$

Method-II : We can consider that the spherical cavity is filled with charge density ρ and also –ρ, thereby making net charge density zero after combining. We can consider two concentric solid spheres: One of radius R and charge density ρ and other of radius r and charge density –ρ. Applying superposition principle :



(i) $\vec{E_A} = \vec{E_\rho} + \vec{E_{-\rho}} = \dfrac{\rho(\vec{OA})}{3\varepsilon_0} + \dfrac{[-\rho(\vec{OA})]}{3\varepsilon_0} = 0$

(ii) $\vec{E_B} = \vec{E_\rho} + \vec{E_{-\rho}} = \dfrac{\rho(\vec{OB})}{3\varepsilon_0} + \dfrac{K\left[\frac{4}{3}\pi r^3(-\rho)\right]}{(OB)^3}\vec{OB}$

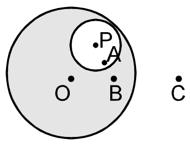
$= \left[\dfrac{\rho}{3\varepsilon_0} - \dfrac{r^3\rho}{3\varepsilon_0(OB)^3}\right]\vec{OB} = \dfrac{\rho}{3\varepsilon_0}\left[1 - \dfrac{r^3}{OB^3}\right]\vec{OB}$

(iii) $\vec{E_C} = \vec{E_\rho} + \vec{E_{-\rho}} = \dfrac{K\left(\frac{4}{3}\pi R^3\rho\right)}{OC^3}\vec{OC} + \dfrac{K\left(\frac{4}{3}\pi r^3(-\rho)\right)}{OC^3}\vec{OC} = \dfrac{\rho}{3\varepsilon_0(OC)^3}[R^3 - r^3]\vec{OC}$

(iv) $\vec{E_O} = \vec{E_\rho} + \vec{E_{-\rho}} = 0 + 0 = 0$

Example 48. In above question, if cavity is not concentric and centered at point P then repeat all the steps.

Solution : Again assume ρ and –ρ in the cavity, (similar to the previous example) :



(i) $\vec{E_A} = \vec{E_\rho} + \vec{E_{-\rho}} = \dfrac{\rho[\vec{OA}]}{3\varepsilon_0} + \dfrac{(-\rho)\vec{PA}}{3\varepsilon_0}$

$\dfrac{\rho}{3\varepsilon_0}[\vec{OA} - \vec{PA}] = \dfrac{\rho}{3\varepsilon_0}[\vec{OP}]$

Note : Here, we can see that the electric field intensity at point A is independent of position of point A inside the cavity. Also the electric field is along the line joining the centres of the sphere and the spherical cavity.

(ii) $\vec{E_B} = \vec{E_\rho} + \vec{E_{-\rho}} = \dfrac{\rho(\vec{OB})}{3\varepsilon_0} + \dfrac{K[\frac{4}{3}\pi r^3(-\rho)]}{[PB]^3}\vec{PB}$

(iii) $\vec{E_C} = \vec{E_\rho} + \vec{E_{-\rho}} = \dfrac{K[\frac{4}{3}\pi R^3\rho]}{[OC]^3}\vec{OC} + \dfrac{K[\frac{4}{3}\pi r^3(-\rho)]}{[PC]^3}\vec{PC}$

(iv) $\vec{E_O} = \vec{E_\rho} + \vec{E_{-\rho}} = 0 + \dfrac{K[\frac{4}{3}\pi r^3(-\rho)]}{[PO]^3}\vec{PO}$

Example 49. A non-conducting solid sphere has volume charge density that varies as $\rho = \rho_0 r$, where ρ_0 is a constant and r is distance from centre. Find out electric field intensities at following positions.

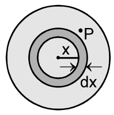
(i) r < R (ii) r ≥ R

Solution : **Method I :**

(i) For r < R :

The sphere can be considered to be made of large number of spherical shells. Each shell has uniform charge density on its surface. So the previous results of the spherical shell can be used. Consider a shell of radius x and thickness dx as an element. Charge on shell $dq = (4\pi x^2 dx)\rho_0 x$.

$\therefore$ Electric field intensity at point P due to shell, $dE = \dfrac{Kdq}{x^2}$



Since all the shell will have electric field in same direction

$$\therefore \quad E = \int_0^R dE = \int_0^r dE + \int_r^R dE$$

Due to shells which lie between region $r < x \le R$, electric field at point P will be zero.

$$\therefore \quad |\vec{E}| = \int_0^r \frac{Kdq}{r^2} + 0 = \int_0^r \frac{K.4\pi x^2 dx \rho_0 x}{r^2} = \frac{4\pi K\rho_0}{r^2}\left[\frac{x^4}{4}\right]_0^r = \frac{\rho_0 r^2}{4\varepsilon_0}$$

(ii) For $r \ge R$, $\quad \vec{E} = \int_0^R dE = \int_0^R \dfrac{K.4\pi x^2 dx \rho_0 x}{r^2} = \dfrac{\rho_0 R^4}{4\varepsilon_0 r^2}\hat{r}$

Method II :

(i) The sphere can be considered to be made of large number of spherical shells. Each shell has uniform charge density on its surface. So the previous results of the spherical shell can be used. We can say that all the shells for which point lies inside will make electric field zero at that point,

$$\text{So} \quad \vec{E}_{(r<R)} = \frac{K\int_0^r (4\pi x^2 dx)\rho_0 x}{r^2} = \frac{\rho_0 r^2}{4\varepsilon_0}\hat{r}$$

(ii) Similarly, for $r \ge R$, all the shells will contribute in electric field. Therefore :

$$\vec{E}_{(r<R)} = \frac{K\int_0^R (4\pi x^2 dx)\rho_0 x}{r^2} = \frac{\rho_0 R^4}{4\varepsilon_0 r^2}\hat{r}$$

7. ELECTRIC POTENTIAL :

In electrostatic field, the electric potential (due to some source charges) at a point P is defined as the work done by external agent in taking a unit point positive charge from a reference point (generally taken at infinity) to that point P without changing its kinetic energy.

7.1 Mathematical representation :

If $(W_{\infty \to P})_{ext}$ is the work required in moving a point charge q from infinity to a point P, the electric potential of the point P is

$$V_p = \left.\frac{W_{\infty\to p})_{ext}}{q}\right]_{\Delta K=0} = \frac{(-W_{elc})_{\infty\to p}}{q}$$

Note : (i) $(W_{\infty \to P})_{ext}$ can also be called as the work done by external agent against the electric force on a unit positive charge due to the source charge.

(ii) Write both W and q with proper sign.

7.2 Properties :

(i) Potential is a scalar quantity, its value may be positive, negative or zero.

(ii) S.I. Unit of potential is volt = $\frac{\text{joule}}{\text{coulmb}}$ and its dimensional formula is $[M^1L^2T^{-3}I^{-1}]$.

(iii) Electric potential at a point is also equal to the negative of the work done by the electric field in taking the point charge from reference point (i.e. infinity) to that point.

(iv) Electric potential due to a positive charge is always positive and due to negative charge it is always negative except at infinity. (Taking $V_\infty = 0$).

(v) Potential decreases in the direction of electric field.

(vi) $V = V_1 + V_2 + V_3 + ...$

7.3 Use of potential :

If we know the potential at some point (in terms of numerical value or in terms of formula) then we can find out the work done by electric force when charge moves from point 'P' to ∞ by the formula

$W_{ep})_{p\to\infty} = qV_p$

Solved Example

Example 50 A charge 2μC is taken from infinity to a point in an electric field, without changing its velocity. If work done against electrostatic forces is – 40μJ, then find the potential at that point.

Solution : $V = \frac{W_{ext}}{q} = \frac{-40\mu J}{2\mu C} = -20\ V$

Example 51 When charge 10 μC is shifted from infinity to a point in an electric field, it is found that work done by electrostatic forces is –10 μJ. If the charge is doubled and taken again from infinity to the same point without accelerating it, then find the amount of work done by electric field and against electric field.

Solution : $W_{ext})_{\infty\to p} = -W_{el})_{\infty\to p} = W_{el})_{p\to\infty} = 10\ \mu J$

because $\Delta KE = 0$

$\therefore\ V_p = \frac{(W_{ext})_{\infty\to p}}{20\mu C} = \frac{10\mu J}{10\mu C} = 1V$

So, if now the charge is doubled and taken from infinity then

$1 = \frac{W_{ext})_{\infty\to p}}{20\mu C} \Rightarrow$ or $W_{ext})_{\infty\to P} = 20\ \mu J$

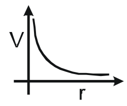
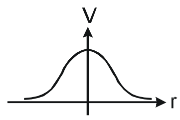
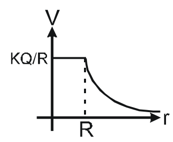
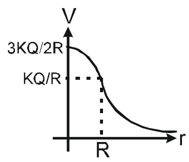
$\Rightarrow\ W_{el})_{\infty\to P} = -20\ \mu J$

Example 52 A charge 3μC is released from rest from a point P where electric potential is 20 V then its kinetic energy when it reaches infinity is :

Solution : $W_{el} = \Delta K = K_f - 0$

$\therefore\ W_{el})_{P\to\infty} = qV_P = 60\ \mu J$ So, $K_f = 60\ \mu J$

Electric Potential due to various charge distributions are given in table.

Name / Type	Formula	Note	Graph
Point charge	$\dfrac{Kq}{r}$	* q is source charge. * r is the distance of the point from the point charge.	
Ring (uniform/nonuniform charge distribution)	at centre: $\dfrac{KQ}{R}$ at the axis: $\dfrac{KQ}{\sqrt{R^2+x^2}}$	* Q is source chage. * x is the distance of the point on the axis from centre f ring	
Uniformly charged hollow conducting/nonconducting /solid conducting sphere	for $r \geq R$, $V = \dfrac{kQ}{r}$ for $r \leq R$, $V = \dfrac{kQ}{R}$	* R is radius of sphere * r is the distance from centre of sphere to the point * Q is total charge = $\sigma 4\pi R^2$.	
Uniformly charged solid nonconducting sphere .	For $r \geq R$, $V = \dfrac{kQ}{r}$ For $r \leq R$ $V = \dfrac{KQ(3R^2 - r^2)}{2R^3}$ $= \dfrac{\rho}{6\varepsilon_0}(3R^2 - r^2)$	* R is radius of sphere * r is distance from centre to the point * $V_{centre} = \dfrac{3}{2} V_{surface}$. * Q is total charge = $\rho \dfrac{4}{3}\pi R^3$. * Inside the sphere potential varies parabolically * Outside the sphere potential varies hyperbolically.	
Infinite line charge	Not defined	* Absolute potential is not defined. * Potential difference between two points is given by formula : $V_B - V_A = -2K\lambda\ \ln (r_B / r_A)$	
Infinite nonconducting thin sheet	Not defined	* Absolute potential is not defined. * Potential difference between two points is given by formula $V_B - V_A = -\dfrac{\sigma}{\varepsilon_0}(r_B - r_A)$	
Infinite charged conducting thin sheet	Not defined	* Absolute potential is not defined. * Potential difference between two points is given by formula $V_B - V_A = -\dfrac{\sigma}{\varepsilon_0}(r_B - r_A)$	

7.4 Potential due to a point charge :

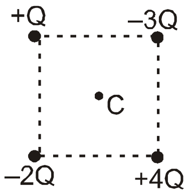
Derivation of expression for potential due to point charge Q, at a point which is at a distance r from the point charge.

From definition of potential

$$V = \frac{W_{ext(\infty \to p)}}{q_o} = \frac{-\int_{\infty}^{r} (q_0\vec{E})\cdot d\vec{r}}{q_o} = -\int_{\infty}^{r} \vec{E}\cdot\vec{dr} \quad \Rightarrow \quad V = -\int_{\infty}^{r} \frac{KQ}{r^2}(-dr)\cos 180^{\circ} = \frac{KQ}{r}$$

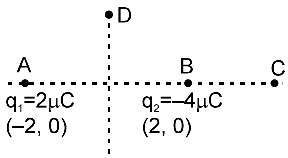
Solved Examples

Example 53. Four point charges are placed at the corners of a square of side ℓ. Calculate potential at the centre of square.



Solution : V = 0 at 'C'. [Use $V = \frac{Kq}{r}$]

Example 54. Two point charges 2μC and – 4μC are situated at points (–2m, 0m) and (2 m, 0 m) respectively. Find out potential at point C(4 m, 0 m) and D(0 m, $\sqrt{5}$ m).



Solution : Potential at point C

$$V_C = V_{q_1} + V_{q_2} = \frac{K(2\mu C)}{6} + \frac{K(-4\mu C)}{2} = \frac{9\times10^9\times2\times10^{-6}}{6} - \frac{9\times10^9\times4\times10^{-6}}{2} = -15000 \text{ V.}$$

$$\text{Similarly, } V_D = V_{q_1} + V_{q_2} = \frac{K(2\mu C)}{\sqrt{(\sqrt{5})^2+2^2}} + \frac{K(-4\mu C)}{\sqrt{(\sqrt{5})^2+2^2}} = \frac{K(2\mu C)}{3} + \frac{K(-4\mu C)}{3} = -6000 \text{ V.}$$

Finding potential due to continuous charges

If formula of E is tough, then we take a small element and integrate

V = $\int dv$

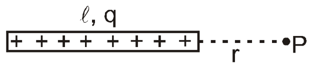
If formula of E is easy then, we use

V = $-\int_{r\to\infty}^{r=r} \vec{E}.d\vec{r}$

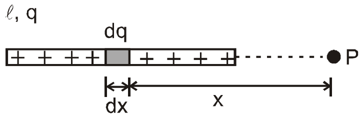
(i.e. for sphere, plate, infinite wire etc.)

Solved Examples

Example 55. A rod of length ℓ is uniformly charged with charge q. Calculate potential at point P.



Solution : Take a small element of length dx, at a distance x from left end. Potential due to this small element



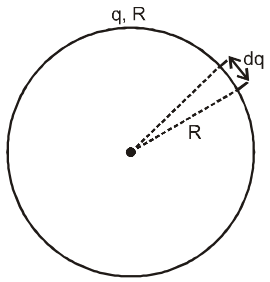
$$dV = \frac{K(dq)}{x} \qquad \therefore \qquad \text{Total potential} \Rightarrow \qquad V = \int_{x=0}^{x=\ell} \frac{kdq}{x}$$

$$\text{Where } dq = \frac{q}{\ell}dx \qquad \Rightarrow \qquad V = \int_{x=r}^{x=r+\ell} \frac{K\left(\frac{q}{\ell}dx\right)}{x} = \frac{Kq}{\ell}\log_e\left(\frac{\ell+r}{r}\right)$$

7.5 Potential due to a ring :

(i) Potential at the centre of uniformly charged ring :

Potential due to the small element dq

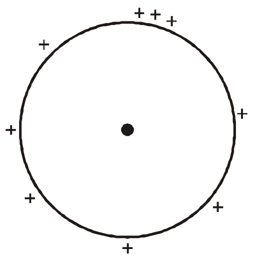


$$dV = \frac{Kdq}{R}$$

$$\therefore \quad \text{Net potential : } \quad V = \int \frac{Kdq}{R}$$

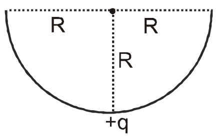
$$\therefore \quad V = \frac{K}{R}\int dq = \frac{Kq}{R}$$

(ii) For non-uniformly charged ring potential at the center is



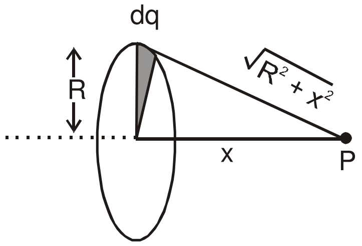
$$V = \frac{Kq_{total}}{R}$$

(iii) Potential due to half ring at center is :



$$V = \frac{Kq}{R}$$

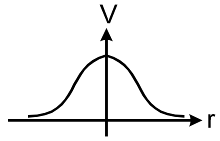
(iv) Potential at the axis of a ring : Calculation of potential at a point on the axis which is a distance x from centre of uniformly charged (total charge Q) ring of radius R.



Consider an element of charge dq on the ring. Potential at point P due to charge dq will be

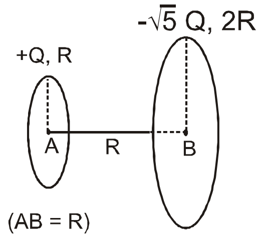
$$dV = \frac{K(dq)}{\sqrt{R^2+x^2}}$$

$\therefore$ Net potential at point P due to all such element will be : $V = \int dv = \frac{KQ}{\sqrt{R^2+x^2}}$



Solved Examples

Example 56. Figure shows two rings having charges Q and $-\sqrt{5}$ Q. Find Potential difference between A and B i.e. $(V_A - V_B)$.



Solution : $V_A = \frac{KQ}{R} + \frac{K\left(-\sqrt{5}\,Q\right)}{\sqrt{(2R)^2+(R)^2}}$; $V_B = \frac{K\left(-\sqrt{5}\,Q\right)}{2\,R} + \frac{K(\,Q)}{\sqrt{(R)^2+(R)^2}}$

From above, we can easily find $V_A - V_B$.

Example 57. A point charge q_0 is placed at the centre of uniformly charged ring of total charge Q and radius R. If the point charge is slightly displaced with negligible force along axis of the ring then find out its speed when it reaches a large distance.

Solution : Only electric force is acting on q_0

$\therefore$ $W_{el} = \Delta K = \frac{1}{2}mv^2 - 0$ $\Rightarrow$ Now $W_{el})_{c\to\infty} = q_0\,V_c = q_0\,\frac{KQ}{R}$.

$\therefore$ $\frac{Kq_0Q}{R} = \frac{1}{2}mv^2$ $\Rightarrow$ $v = \sqrt{\frac{2Kq_0Q}{mR}}$

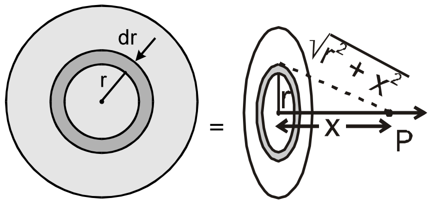
7.6 Potential due to uniformly charged disc :

$V = \frac{\sigma}{2\varepsilon_0}\left(\sqrt{R^2+x^2} - x\right)$, where σ is the charge density and x is the distance of the point on the axis from the center of the disc & R is the radius of disc.

Finding potential due to a uniformly charged disc:

A disc of radius 'R' has surface charge density (charge/area) = σ. We have to find potential at its axis, at point 'P' which is at a distance x from the centre.

For this, we can divide the disc into thin rings and let's consider a thin ring of radius r and thickness dr. Suppose charge on the small ring element = dq. Potential due to this ring at point 'P' is:



$$dV = \frac{Kdq}{\sqrt{r^2 + x^2}}$$

So, net potential : $V_{net} = \int \frac{Kdq}{\sqrt{r^2 + x^2}}$

Here, σ = charge/area = $\frac{dq}{d(\text{area})}$

So, $dq = \sigma \times d\,(\text{area}) = \sigma\,(2\pi r\,dr)$

(Here, d (area) = area of the small ring element = (length of ring) × (width of the ring) = $(2\pi r)\,.\,(dr)$)

So, $V_{net} = \int_{r=0}^{r=R} \frac{K\sigma(2\pi r\,dr)}{\sqrt{r^2 + x^2}}$

To integrate it, let $r^2 + x^2 = y^2$

$2r\,dr = 2y\,dy$. Substituting we will get :

$$V_{net} = \int_{r=0}^{r=R} \frac{1}{4\pi\varepsilon_0} \frac{\sigma(2\pi)y\,dy}{y} \quad \Rightarrow \quad V_{net} = \frac{\sigma}{2\varepsilon_0}[y]_{r=0}^{r=R}$$

$$V_{net} = \frac{\sigma}{2\varepsilon_0}\left(\sqrt{r^2 + x^2}\right)_{r=0}^{r=R} \quad \Rightarrow \quad V_{net} = \frac{\sigma}{2\varepsilon_0}\left(\sqrt{R^2 + x^2} - x\right)$$

If a test charge q_0 is placed at point P, then potential energy of this charge q_0 due to the disc = $U = q_0V$

$$\Rightarrow \quad U = q_0\left[\frac{\sigma}{2\varepsilon_0}\left(\sqrt{R^2 + x^2} - x\right)\right]$$

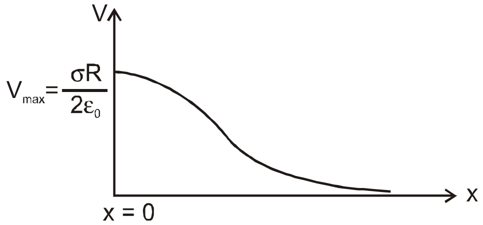
Graph of V v/s x : $V = \frac{\sigma}{2\varepsilon_0}\left(\sqrt{R^2 + x^2} - x\right)$

at x = 0, $V = \frac{\sigma R}{2\varepsilon_0}$ to check whether V will increase with x or decrease, lets multiply and divide by conjugate.

$$V = \frac{\sigma}{2\varepsilon_0}\left(\sqrt{R^2 + x^2} - x\right) \times \frac{\left(\sqrt{R^2 + x^2} + x\right)}{\left(\sqrt{R^2 + x^2} + x\right)}$$

$$\Rightarrow \quad V = \frac{\sigma R^2}{2\varepsilon_0}\left(\frac{1}{\left(\sqrt{R^2 + x^2} + x\right)}\right)$$

Now, we can say that as $x \uparrow \Rightarrow V\downarrow$ so curve will be



7.7 Potential Due To Uniformly Charged Spherical shell :

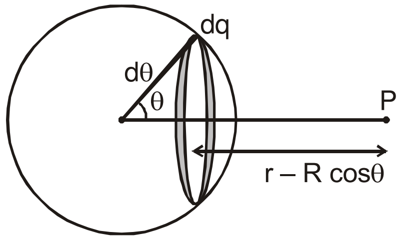
Derivation of expression for potential due to uniformly charged hollow sphere of radius R and total charge Q, at a point which is at a distance r from centre for the following situation

(i) $r > R$ (ii) $r < R$

Assume a ring of width $Rd\theta$ at angle θ from X axis (as shown in figure). Potential due to the ring at the point P will be

$$dV = \frac{K(dq)}{\sqrt{(r - R\cos\theta)^2 + (R\sin\theta)^2}}$$

Where $dq = 2\pi R \sin\theta \, (Rd\theta)\sigma$ where $Q = 4\pi R^2\sigma$



then, net potential $V = \int dV = \frac{KQ}{2}\int_0^{\pi} \frac{\sin\theta . d\theta}{\sqrt{(r - R\cos\theta)^2 + (R\sin\theta)^2}}$

Solving this eq. we find $V = \frac{KQ}{r}$ (for $r > R$)

& $V = \frac{KQ}{R}$ for ($r < R$)

Alternate Method : As the formula of E is easy, we use $V = -\int_{r \to \infty}^{r = r} \vec{E} \cdot d\vec{r}$

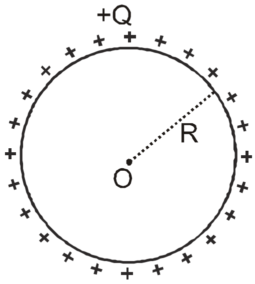
(i) At outside point ($r \geq R$) : $V_{out} = -\int_{r \to \infty}^{r = r} \left(\frac{KQ}{r^2}\right) dr \Rightarrow V_{out} = \frac{KQ}{r} = \frac{KQ}{(\text{Distance from centre})}$

For outside point, the hollow sphere acts like a point charge.

(ii) Potential at the centre of the sphere (r = 0) : As all the charges are at a distance R from the centre,

So, $V_{centre} = \frac{KQ}{R} = \frac{KQ}{(\text{Radius of the sphere})}$

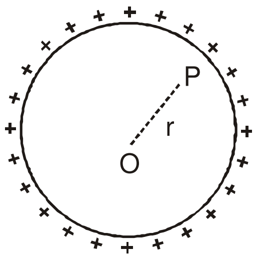
+Q



(iii) Potential at inside point (r < R) : Suppose we want to find potential at point P, inside the sphere.

∴ Potential difference between Point P and O :

+Q, R



$V_P - V_O = -\int_O^P \vec{E}_{in} \cdot d\vec{r}$ Where, $E_{in} = 0$

So $V_P - V_O = 0 \Rightarrow V_P = V_O = \frac{KQ}{R} \Rightarrow V_{in} = \frac{KQ}{R} = \frac{KQ}{(\text{Radius of the sphere})}$

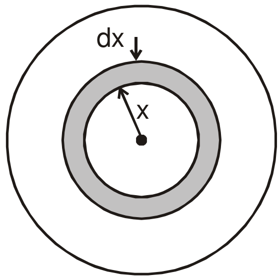
7.8 Potential Due To Uniformly Charged Solid Sphere :

Derivation of expression for potential due to uniformly charged solid sphere of radius R and total charge Q (distributed in volume), at a point which is at a distance r from centre for the following situations.

(i) $r \geq R$ (ii) $r \leq R$

Consider an elementary shell of radius x and width dx

(i) For $r \geq R$: $V = \int_0^R \frac{K \cdot 4\pi x^2 dx \rho}{r} = \frac{KQ}{r}$



(ii) For $r \leq R$: $V = \int_0^r \frac{K \cdot 4\pi x^2 dx \rho}{r} + \int_r^R \frac{K 4\pi x^2 dx \rho}{x}$

$= \frac{KQ}{2R^3}(3R^2 - r^2) \Rightarrow \left(\rho = \frac{Q}{\frac{4}{3}\pi R^3}\right)$

From definition of potential

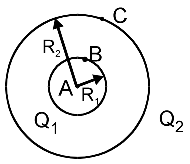
(i) For $r \geq R$: $V = -\int_\infty^r \frac{KQ}{r^2}\hat{r} \cdot dr = \frac{KQ}{r}$

(ii) For $r \leq R$: $V = \int_\infty^R \frac{KQ}{r^2}.dr - \int_R^r \frac{KQr}{R^3} dr$

$V = \frac{KQ}{R} - \frac{KQ}{2R^3}[r^2 - R^2] = \frac{KQ}{2R^3}[2R^2 - r^2 + R^2] = \frac{KQ}{2R^3}(3R^2 - r^2)$

Solved Examples

Example 58. Two concentric spherical shells of radius R_1 and R_2 ($R_2 > R_1$) are having uniformly distributed charges Q_1 and Q_2 respectively. Find out potential



(i) at point A
(ii) at surface of smaller shell (i.e. at point B)
(iii) at surface of larger shell (i.e. at point C)
(iv) at $r \leq R_1$
(v) at $R_1 \leq r \leq R_2$
(vi) at $r \geq R_2$

Solution : Using the results of hollow sphere as given in the table 7.4.

(i) $V_A = \frac{KQ_1}{R_1} + \frac{KQ_2}{R_2}$

(ii) $V_B = \frac{KQ_1}{R_1} + \frac{KQ_2}{R_2}$

(iii) $V_C = \frac{KQ_1}{R_2} + \frac{KQ_2}{R_2}$

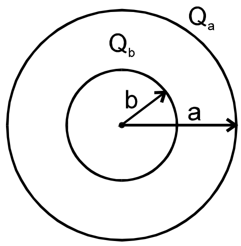
(iv) for $r \leq R_1$, $V = \frac{KQ_1}{R_1} + \frac{KQ_2}{R_2}$

(v) for $R_1 \leq r \leq R_2$, $V = \frac{KQ_1}{r} + \frac{KQ_2}{R_2}$

(vi) for $r \geq R_2$, $V = \frac{KQ_1}{r} + \frac{KQ_2}{r}$

Example 59. Two hollow concentric non-conducting spheres of radius the a and b (a > b) contain charges Q_a and Q_b respectively. Prove that potential difference between the two spheres is independent of charge on outer sphere. If outer sphere is given an extra charge, is there any change in potential difference?

Solution : $V_{\text{inner sphere}} = \dfrac{KQ_b}{b} + \dfrac{KQ_a}{a}$



$V_{\text{outer sphere}} = \dfrac{KQ_b}{a} + \dfrac{KQ_a}{a}$

$V_{\text{inner sphere}} - V_{\text{outer sphere}} = \dfrac{KQ_b}{b} - \dfrac{KQ_b}{a}$

$\therefore \quad \Delta V = KQ_b\left[\dfrac{1}{b} - \dfrac{1}{a}\right]$

Which is independent of charge on outer sphere. If outer sphere in given any extra charge, then there will be no change in potential difference.

8. POTENTIAL DIFFERENCE

The potential difference between two points A and B is work done by external agent against electric field in taking a unit positive charge from A to B with no change in kinetic energy between initial and final points ie. $\Delta K = 0$ or $K_i = K_f$

(a) Mathematical representation :

If $(W_{A\to B})_{ext}$ = Work done by external agent against electric field in taking the unit charge from A to B

Then, $V_B - V_A = \left.\dfrac{(W_{A\to B})_{ext}}{q}\right)_{\Delta K=0} = \dfrac{-(W_{A\to B})_{electric}}{q} = \dfrac{U_B - U_A}{q} = \dfrac{-\int_A^B \vec{F}_e.\vec{dr}}{q} = -\int_A^B \vec{E}.\vec{dr}$

Note : Take W and q both with sign

(b) Properties :

(i) The difference of potential between two points is called potential difference. It is also called voltage.

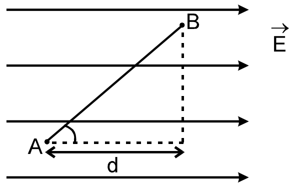
(ii) Potential difference is a scalar quantity. Its S.I. unit is also volt.

(iii) If V_A and V_B be the potential of two points A and B, then work done by an external agent in taking the charge q from A to B is $(W_{ext})_{AB} = q\,(V_B - V_A)$ or $(W_{el})_{AB} = q\,(V_A - V_B)$.

(iv) Potential difference between two points is independent of reference point.

8.1 Potential difference in a uniform electric field :

$\mathbf{V_B - V_A = -\ \vec{E}\cdot\overrightarrow{AB}}$



$\Rightarrow \quad V_B - V_A = -\,|E|\,|AB|\cos\theta$

$= -\,|E|\,d$

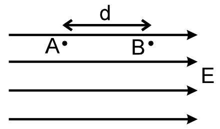
$= -\,Ed$

where d = effective distance between A and B along electric field.

or we can also say that $E = \dfrac{\Delta V}{\Delta d}$

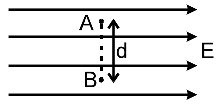
Special Cases :

Case-1. Line AB is parallel to electric field.



$\therefore$ $V_A - V_B = Ed$

Case-2. Line AB is perpendicular to electric field.



$\therefore$ $V_A - V_B = 0 \quad \Rightarrow \quad V_A = V_B$

Note : In the direction of electric field potential always decreases.

Solved Examples

Example 60. $1\mu C$ charge is shifted from A to B and it is found that work done by an external force is $40\mu J$ in doing so against electrostatic forces, then find potential difference $V_A - V_B$

Solution : $(W_{AB})_{ext} = q(V_B - V_A) \quad \Rightarrow \quad 40\ \mu J = 1\mu C\ (V_B - V_A) \quad \Rightarrow \quad V_A - V_B = -40\ V$

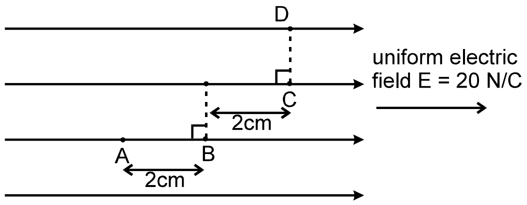
Example 61. A uniform electric field is present in the positive x-direction. If the intensity of the field is 5N/C then find the potential difference $(V_B - V_A)$ between two points A (0m, 2 m) and B (5 m ,3 m)

Solution : $V_B - V_A = -\vec{E}\,.\,\vec{AB} = -(5\hat{i}).(5\hat{i}+\hat{j}) = -25V.$

The electric field intensity in uniform electric field, $E = \dfrac{\Delta V}{\Delta d}$

Where ΔV = potential difference between two points.
Δd = effective distance between the two points.
(projection of the displacement along the direction of electric field.)

Example 62. Find out following



(i) $V_A - V_B$ (ii) $V_B - V_C$ (iii) $V_C - V_A$ (iv) $V_D - V_C$
(v) $V_A - V_D$ (vi) Arrange the order of potential for points A, B, C and D.

Solution : (i) $|\Delta V_{AB}| = Ed = 20 \times 2 \times 10^{-2} = 0.4$ so, $V_A - V_B = 0.4\ V$

because **In the direction of electric field potential always decreases.**

(ii) $|\Delta V_{BC}| = Ed = 20 \times 2 \times 10^{-2} = 0.4$ so, $V_B - V_C = 0.4\ V$

(iii) $|\Delta V_{CA}| = Ed = 20 \times 4 \times 10^{-2} = 0.8$ so, $V_C - V_A = -0.8\ V$

because **In the direction of electric field potential always decreases.**

(iv) $|\Delta V_{DC}| = Ed = 20 \times 0 = 0$ so, $V_D - V_C = 0$

because the effective distance between D and C is zero.

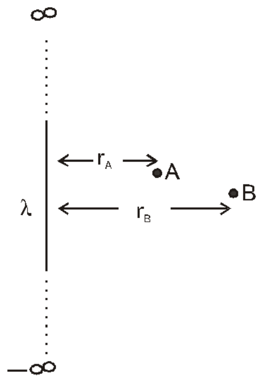
(v) $|\Delta V_{AD}| = Ed = 20 \times 4 \times 10^{-2} = 0.8$ so, $V_A - V_D = 0.8\ V$

because **In the direction of electric field potential always decreases.**

(vi) The order of potential is : $V_A > V_B > V_C = V_D$.

8.2 Potential difference due to infinitely long wire :

Derivation of expression for potential difference between two points, which have perpendicular distance r_A and r_B from infinitely long line charge of uniform linear charge density λ. :

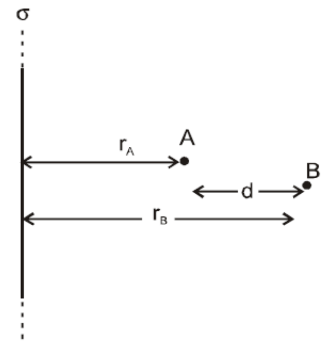


From definition of potential difference : $V_{AB} = V_B - V_A = -\int_{r_A}^{r_B} \vec{E}\cdot\overrightarrow{dr} = -\int_{r_A}^{r_B} \frac{2K\lambda}{r}\hat{r}\cdot\overrightarrow{dr}$

$\therefore \quad V_{AB} = -2K\lambda\, \ell n\left(\frac{r_B}{r_A}\right)$

8.3 Potential difference due to infinitely long thin sheet:

Derivation of expression for potential difference between two points, having separation d in the direction perpendicularly to a very large uniformly charged thin sheet of uniform surface charge density σ.



Let the points A and B have perpendicular distance r_A and r_B respectively then from definition of potential difference.

$V_{AB} = V_B - V_A = -\int_{r_A}^{r_B} \vec{E}\cdot\overrightarrow{dr} = -\int_{r_A}^{r_B} \frac{\sigma}{2\varepsilon_0}\hat{r}\cdot\overrightarrow{dr} \Rightarrow V_{AB} = -\frac{\sigma}{2\varepsilon_o}(r_B - r_A) = -\frac{\sigma d}{2\varepsilon_o}$

9. EQUIPOTENTIAL SURFACE :

9.1 Definition : If potential of a surface (imaginary or physically existing) is same throughout, then such surface is known as an equipotential surface.

9.2 Properties of equipotential surfaces :

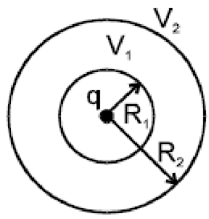
(i) When a charge is shifted from one point to another point on an equipotential surface, then work done against electrostatic forces is zero.

(ii) Electric field is always perpendicular to equipotential surfaces.

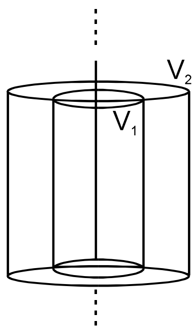
(iii) Two equipotential surfaces do not cross each other.

9.3 Examples of equipotential surfaces :

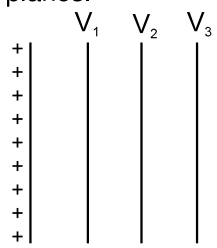
(i) Point charge : Equipotential surfaces are concentric and spherical as shown in figure. In figure, we can see that sphere of radius R_1 has potential V_1 throughout its surface and similarly for other concentric sphere potential is same.



(ii) Line charge : Equipotential surfaces have curved surfaces as that of coaxial cylinders of different radii.



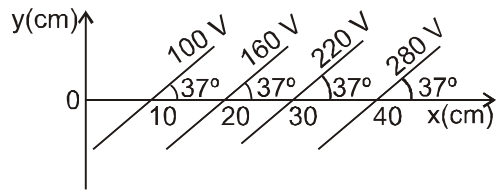
(iii) Uniformly charged large conducting / non conducting sheets :
Equipotential surfaces are parallel planes.



Note : In uniform electric field equi-potential surfaces are always parallel planes.

Solved Examples

Example 63. Some equipotential surfaces are shown in figure. What can you say about the magnitude and the direction of the electric field ?

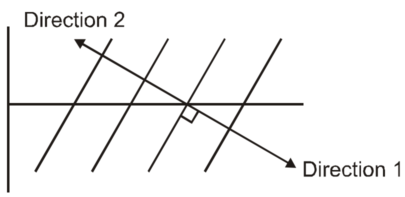


Solution : Here, we can say that the electric will be perpendicular to equipotential surfaces.

Also, $|\vec{E}| = \dfrac{\Delta V}{\Delta d}$

Where, ΔV = potential difference between two equipotential surfaces.

Δd = perpendicular distance between two equipotential surfaces.

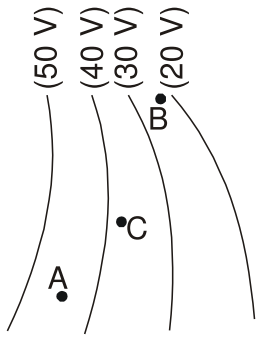


So $|\vec{E}| = \dfrac{60}{(10\sin 37^{\circ}) \times 10^{-2}}$ = 1000 V/m

Now there are two perpendicular directions: either direction 1 or direction 2 as shown in figure, but since we know that in the direction of electric field, electric potential decreases, so the correct direction is direction 2.

Hence E = 1000 V/m, making an angle 127° with the x-axis

Example 64. Figure shows some equipotential surfaces produce by some charges. At which point, the value of electric field is greatest?

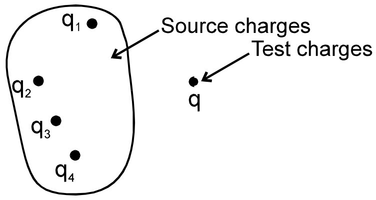


Solution : E is larger where equipotential surfaces are closer. ELOF are $\perp$ to equipotential surfaces. In the figure, we can see that for point B, they are closer so E at point B is maximum

10. ELECTROSTATIC POTENTIAL ENERGY

10.1 Electrostatic potential energy of a point charge due to many charges :

The electrostatic potential energy of a point charge at a point in electric field is the work done in taking the charge from reference point (generally at infinity) to that point without change in kinetic energy.



Its Mathematical formula is $U = W_{\infty \to P)ext}|_{\Delta K = 0} = qV = -W_{P\to\infty)ele}$

Here, q is the charge whose potential energy is being calculated and V is the potential at its position due to the source charges.

Note : Always put q and V with sign.

10.2 Properties :

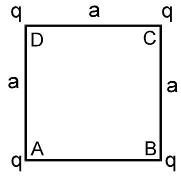
(i) Electric potential energy is a scalar quantity but may be positive, negative or zero.

(ii) Its unit is same as unit of work or energy i.e., joule (in S.I. system). Some times energy is also given in electron-volts. Where, 1eV = 1.6×10^{-19} J

(iii) Electric potential energy depends on reference point. (Generally Potential Energy at $r = \infty$ is taken zero)

Solved Examples

Example 65. The four identical charges (q each) are placed at the corners of a square of side a. Find the potential energy of one of the charges due to the remaining charges.



Solution : The electric potential of point A due to the charges placed at B, C and D is

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{a} + \frac{1}{4\pi\varepsilon_0}\frac{q}{\sqrt{2}a} + \frac{1}{4\pi\varepsilon_0}\frac{q}{a} = \frac{1}{4\pi\varepsilon_0}\left(2+\frac{1}{\sqrt{2}}\right)\frac{q}{a}$$

$\therefore$ Potential energy of the charge at A is = qV = $\frac{1}{4\pi\varepsilon_0}\left(2+\frac{1}{\sqrt{2}}\right)\frac{q^2}{a}$.

Example 66. A particle of mass 40 mg and carrying a charge 5×10^{-9} C is moving directly towards a fixed positive point charge of magnitude 10^{-8} C. When it is at a distance of 10 cm from the fixed point charge it has speed of 50 cm/s. At what distance from the fixed point charge will the particle come momentarily to rest? Is the acceleration constant during the motion?

Solution : If the particle comes to rest momentarily at a distance r from the fixed charge, then from conservation of energy, we have ?

$$\frac{1}{2}mu^2 + \frac{1}{4\pi\varepsilon_0}\frac{Qq}{a} = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{r}$$

Substituting the given data, we get: $\frac{1}{2} \times 40 \times 10^{-6} \times \frac{1}{2} \times \frac{1}{2} = 9 \times 10^9 \times 5 \times 10^{-8} \times 10^{-9}\left[\frac{1}{r} - 10\right]$

or, $\frac{1}{r} - 10 = \frac{5\times10^{-6}}{9\times5\times10^{-8}} = \frac{100}{9}$ $\Rightarrow$ $\frac{1}{r} = \frac{190}{9}$ $\Rightarrow$ $r = \frac{9}{190}$ m

or, i.e., $r = 4.7 \times 10^{-2}$ m. As here, $F = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{r^2}$ So, acc. $= \frac{F}{m} \propto \frac{1}{r^2}$

i.e., Acceleration is not constant during the motion.

Example 67. A proton moves from a large distance with a speed u m/s directly towards a free proton originally at rest. Find the distance of closest approach for the two protons in terms of mass of proton m and its charge e.

Solution : As here the particle at rest is free to move, when one particle approaches the other, due to electrostatic repulsion other will also start moving and so the velocity of first particle will decrease while of other will increase and at closest approach, both will move with same velocity. So, if v is the common velocity of each particle at closest approach, then by 'conservation of momentum' of the two proton system.

$$mu = mv + mv \text{ i.e., } v = \frac{1}{2}u$$

And by conservation of energy, $\frac{1}{2}mu^2 = \frac{1}{2}mv^2 + \frac{1}{2}mv^2 + \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r}$

$$\Rightarrow \frac{1}{2}mu^2 - m\left(\frac{u}{2}\right)^2 = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r} \qquad [\text{as } v = \frac{u}{2}]$$

$$\Rightarrow \frac{1}{4}mu^2 = \frac{e^2}{4\pi\varepsilon_0 r} \qquad \Rightarrow \qquad r = \frac{e^2}{\pi m\varepsilon_0 u^2}$$

11. ELECTROSTATIC POTENTIAL ENERGY OF A SYSTEM OF CHARGES

(This concept is useful when more than one charges move.)

It is the work done by an external agent against the internal electric field required to make a system of charges in a particular configuration from infinite separation without change in their kinetic energies.

11.1 Types of system of charges :

(i) Point charge system (ii) Continuous charge system.

11.2 Derivation for a system of point charges:

(i) Keep all the charges at infinity. Now bring the charges one by one to its corresponding position and find work required. PE of the system is algebraic sum of all the works.

Let W_1 = Work done in bringing first charge.

W_2 = Work done in bringing second charge against force due to 1st charge.

W_3 = Work done in bringing third charge against force due to 1st and 2nd charge.

PE = $W_1 + W_2 + W_3 + ...$. (This will contain $\frac{n(n-1)}{2} = {}^nC_2$ terms)

(ii) Method of calculation (to be used in problems) :

U = sum of the interaction energies of the charges.

$= (U_{12} + U_{13} + ... + U_{1n}) + (U_{23} + U_{24} + ... + U_{2n}) + (U_{34} + U_{35} + ... + U_{3n})$ .....

(iii) Method of calculation useful for symmetrical point charge systems.

Find PE of each charge due to rest of the charges.

If U_1 = PE of first charge due to all other charges.

$= (U_{12} + U_{13} + ... + U_{1n})$

U_2 = PE of second charge due to all other charges.

$= (U_{21} + U_{23} + ... + U_{2n})$ then U = PE of the system = $\frac{U_1 + U_2 + ...U_n}{2}$

Solved Examples

Example 68. Find out potential energy of the two point charge system having charges q_1 and q_2 separated by distance r.

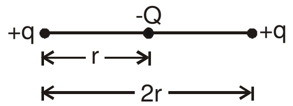
Solution : Let both the charges be placed at a very large separation initially.

Let, W_1 = work done in bringing charge q_1 in absence of $q_2 = q_1(V_f - V_i) = 0$

W_2 = work done in bringing charge q_2 in presence of $q_1 = q_2(V_f - V_i) = q_2(Kq_1/r - 0)$

$\therefore$ PE = $W_1 + W_2 = 0 + Kq_1q_2/r = Kq_1q_2/r$

Example 69. Figure shows an arrangement of three point charges. The total potential energy of this arrangement is zero. Calculate the ratio $\frac{q}{Q}$.

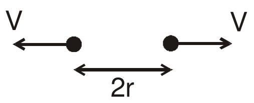


Solution : $U_{sys} = \frac{1}{4\pi\varepsilon_0}\left[\frac{-qQ}{r} + \frac{(+q)(+q)}{2r} + \frac{Q(-q)}{r}\right] = 0$

$-Q + \frac{q}{2} - Q = 0$ or $2Q = \frac{q}{2}$ or $\frac{q}{Q} = \frac{4}{1}$.

Example 70. Two point charges, each of mass m and charge q are released when they are at a distance r from each other. What is the speed of each charged particle when they are at a distance 2r ?

Solution : According to momentum conservation, both the charge particles will move with same speed. Now applying energy conservation:



$$0 + 0 + \frac{Kq^2}{r} = 2\frac{1}{2}mv^2 + \frac{Kq^2}{2r} \Rightarrow v = \sqrt{\frac{Kq^2}{2rm}}$$

Example 71. Two charged particles each having equal charges 2×10^{-5} C are brought from infinity to within a separation of 10 cm. Calculate the increase in potential energy during the process and the work required for this purpose.

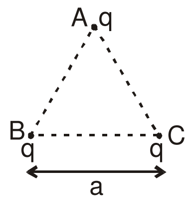
Solution : $\Delta U = U_f - U_i = U_f - 0 = U_f$

We have to simply calculate the electrostatic potential energy of the given system of charges

$$\therefore \quad \Delta U = U_f = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r} = \frac{9\times10^9\times2\times10^{-5}\times2\times10^{-5}\times100}{10} \text{ J} = 36 \text{ J}$$

$\therefore$ Work required = 36 J = equal to change in potential energy of system

Example 72. Three equal charges q each are placed at the corners of an equilateral triangle of side a.

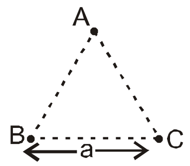


(i) Find out potential energy of charge system.

(ii) Calculate work required to decrease the side of triangle to a/2.

(iii) If the charges are released from the shown position and each of them has same mass m then find the speed of each particle when they lie on triangle of side 2a.

Solution : **(i)** **Method I** (Derivation)



Assume all the charges are at infinity initially. Work done in putting charge q at corner A

$\Rightarrow$ $W_1 = q(v_f - v_i) = q(0 - 0)$

Since potential at A is zero in absence of charges, work done in putting q at corner B in presence of charge at A :

$$\Rightarrow W_2 = \left(\frac{Kq}{a} - 0\right)q = \frac{Kq^2}{a}$$

Similarly work done in putting charge q at corner C in presence of charge at A and B.

$$\Rightarrow W_3 = q(v_f - v_i) = q\left[\left(\frac{Kq}{a} + \frac{Kq}{a}\right) - 0\right] = \frac{2Kq^2}{a}$$

So, net potential energy PE = $W_1 + W_2 + W_3 = 0 + \frac{Kq^2}{a} + \frac{2Kq^2}{a} = \frac{3Kq^2}{a}$

Method II (using direct formula) :

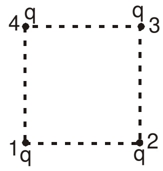
$$U = U_{12} + U_{13} + U_{23} = \frac{Kq^2}{a} + \frac{Kq^2}{a} + \frac{Kq^2}{a} = \frac{3Kq^2}{a}$$

(ii) Work required to decrease the sides $W = U_f - U_i = \frac{3Kq^2}{a/2} - \frac{3Kq^2}{a} = \frac{3Kq^2}{a}$ Joules

(iii) Work done by electrostatic forces = Change is kinetic energy of particles.

$$U_i - U_f = K_f - K_i \qquad \Rightarrow \frac{3Kq^2}{a} - \frac{3Kq^2}{2a} = 3\left(\frac{1}{2}mv^2\right) - 0 \Rightarrow v = \sqrt{\frac{Kq^2}{am}}$$

Example 73. Four identical point charges q each are placed at four corners of a square of side a. Find out potential energy of the charge system



Solution : **Method 1 (using direct formula) :** $U = U_{12} + U_{13} + U_{14} + U_{23} + U_{24} + U_{34}$

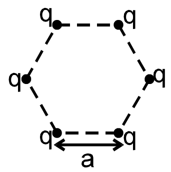
$$= \frac{Kq^2}{a} + \frac{Kq^2}{a\sqrt{2}} + \frac{Kq^2}{a} + \frac{Kq^2}{a} + \frac{Kq^2}{a\sqrt{2}} + \frac{Kq^2}{a} = \left[\frac{4Kq^2}{a} + \frac{2Kq^2}{a\sqrt{2}}\right] = \frac{2Kq^2}{a}\left[2 + \frac{1}{\sqrt{2}}\right]$$

Method 2 [Using, $U = \frac{1}{2}(U_1 + U_2 + ...)$] **:**

U_1 = total P.E. of charge at corner 1 due to all other charges.
U_2 = total P.E. of charge at corner 2 due to all other charges.
U_3 = total P.E. of charge at corner 3 due to all other charges.
U_4 = total P.E. of charge at corner 4 due to all other charges.
Since, due to symmetry, $U_1 = U_2 = U_3 = U_4$

$$U_{Net} = \frac{U_1 + U_2 + U_3 + U_4}{2} = 2U_1 = 2\left[\frac{Kq^2}{a} + \frac{Kq^2}{a} + \frac{Kq^2}{\sqrt{2}a}\right] = \frac{2Kq^2}{a}\left[2 + \frac{1}{\sqrt{2}}\right]$$

Example 74. Six equal point charges q each are placed at six corners of a hexagon of side a. Find out potential energy of charge system



Solution : $U_{Net} = \frac{U_1 + U_2 + U_3 + U_4 + U_5 + U_6}{2}$

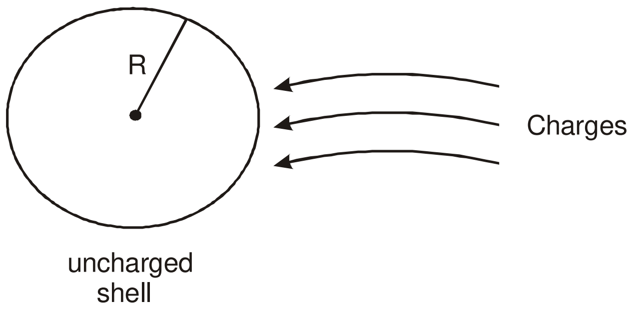
Due to symmetry $U_1 = U_2 = U_3 = U_4 = U_5 = U_6$ So $U_{net} = 3U_1 = \frac{3Kq^2}{a}\left[2 + \frac{2}{\sqrt{3}} + \frac{1}{2}\right]$

11.3 Derivation of electric potential energy for continuous charge system :

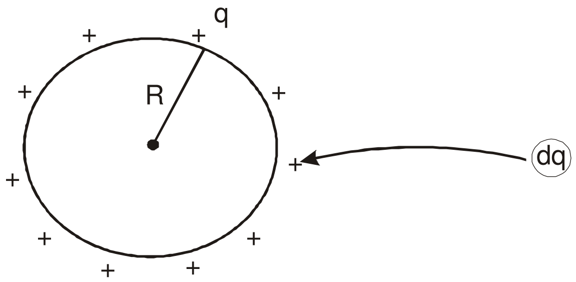
This energy is also known as self energy.

(i) Finding P.E., (Self Energy) of a uniformly Charged spherical shell :

For this, lets use method 1 : Take an uncharged shell. Now bring charges one by one from infinity to the surface fo the shell. The work required in this process will be stored as potential Energy.



Suppose, we have given charge q to the sphere and now we are giving extra charge dq to it. Work required to bring dq charge from infinity to the shell is



dW = (dq) $(V_f - V_i)$ $\Rightarrow$ dW = (dq) $\left(\frac{Kq}{R} - 0\right) = \frac{Kq}{R}dq$

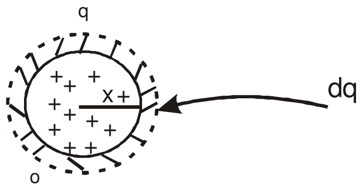
$\Rightarrow$ Total work required to give charge Q is W = $\int_{q=o}^{q=Q} \frac{Kq}{R}dq = \frac{KQ^2}{2R}$

This work will be stored in the form of P.E. (self energy)

So, P.E. of a charged spherical shell : U = $\frac{KQ^2}{2R}$

(ii) Self energy of uniformly charged solid sphere :

In this case we have to assemble a solid charged sphere. So we bring the charges one-by-one from infinity to the sphere so that the size of the sphere increases.



Suppose we have given charge q to the sphere, and its radius becomes 'x' .Now we are giving extra charge dq to it, which will increase its radius by 'dx'

$\therefore$ Work required to bring charge dq from infinity to the sphere

$= dq\,(V_f - V_i) = (dq)\left(\frac{Kq}{x} - o\right) = \frac{Kqdq}{x}$

$\therefore$ Total work required to give charge Q to the sphere

W = $\int \frac{Kqdq}{x}$, where q = $\rho\left(\frac{4}{3}\pi x^3\right)$

& dq = $\rho\,(4\,\pi x^2\,dx)$ $\Rightarrow$ W = $\int_{x=o}^{x=R} K\frac{\rho\left(\frac{4}{3}\pi x^3\right)\rho(4\pi x^2 dx)}{x}$

Solving, well get : W = $\frac{3}{5}\frac{KQ^2}{R}$ = U_{self} (for a solid sphere)

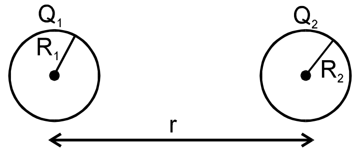
Solved Examples

Example 75. A spherical shell of radius R with uniform charge q is expanded to a radius 2R. Find the work performed by the electric forces and external agent against electric forces in this process.

Solution : $W_{ext} = U_f - U_i = \frac{q^2}{16\pi\varepsilon_0 R} - \frac{q^2}{8\pi\varepsilon_0 R} = -\frac{q^2}{16\pi\varepsilon_0 R}$

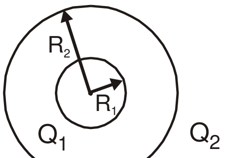
$W_{elec} = U_i - U_f = \frac{q^2}{8\pi\varepsilon_0 R} - \frac{q^2}{16\pi\varepsilon_0 R} = \frac{q^2}{16\pi\varepsilon_0 R}$

Example 76. Two non-conducting hollow uniformly charged spheres of radii R_1 and R_2 with charge Q_1 and Q_2 respectively are placed at a distance r. Find out total energy of the system.



Solution : $U_{total} = U_{self} + U_{Interaction} = \dfrac{Q_1^2}{8\pi\varepsilon_0 R_1} + \dfrac{Q_2^2}{8\pi\varepsilon_0 R_2} + \dfrac{Q_1 Q_2}{4\pi\varepsilon_0 r}$

Example 77. Two concentric spherical shells of radius R_1 and R_2 ($R_2 > R_1$) are having uniformly distributed charges Q_1 and Q_2 respectively. Find out total energy of the system.



Solution : $U_{total} = U_{self\ 1} + U_{self\ 2} + U_{Interaction} = \dfrac{Q_1^2}{8\pi\varepsilon_0 R_1} + \dfrac{Q_2^2}{8\pi\varepsilon_0 R_2} + \dfrac{Q_1 Q_2}{4\pi\varepsilon_0 R_2}$

11.4 Energy density :

Def : Energy density is defined as energy stored in unit volume in any electric field. Its mathematical formula is given as following :

Energy density = $\dfrac{1}{2}\varepsilon E^2$

where E = electric field intensity at that point

. $\varepsilon = \varepsilon_0\varepsilon_r$ electric permittivity of medium

Solved Examples

Example 78. Find out energy stored in an imaginary cubical volume of side a in front of a infinitely large non-conducting sheet of uniform charge density σ.

Solution : Energy stored : $U = \int \dfrac{1}{2}\varepsilon_0 E^2 dV$; where dV is small volume

$\therefore\ U = \dfrac{1}{2}\varepsilon_0 E^2 \int dV$ $\qquad\because$ E is constant.

$\therefore\ U = \dfrac{1}{2}\varepsilon_0 \dfrac{\sigma^2}{4\varepsilon_0^2} . a^3 . = \dfrac{\sigma^2 a^3}{8\varepsilon_0}$

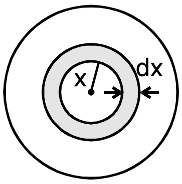
Example 79. Find out energy stored in the electric field of uniformly charged thin spherical shell of total charge Q and radius R.

Solution : We know that electric field inside the shell is zero, so the energy is stored only outside the shell, which can be calculated by using energy density formula.

$U_{self} = \int_{x=R}^{x\to\infty} \dfrac{1}{2}\varepsilon_0 E^2 dV$

Consider an elementary shell of thickness dx and radius x (x > R).

Volume of the shell = $(4\pi x^2 dx) = dV$



$$U = \int_R^\infty \frac{1}{2}\varepsilon_0\left[\frac{KQ}{x^2}\right]^2 .\ 4\pi x^2\, dx = \frac{1}{2}\varepsilon_0\, K^2Q^2 4\pi \int_R^\infty \frac{1}{x^2}\, dx$$

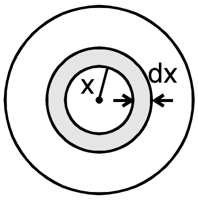
$$= \frac{4\pi\varepsilon_0}{2}\,\frac{Q^2}{(4\pi\varepsilon_0)^2}\cdot\left(\frac{1}{R}\right) = \frac{Q^2}{8\pi\varepsilon_0 R} = \frac{KQ^2}{2R}.$$

Example 80. Find out energy stored inside a solid non-conducting sphere of total charge Q and radius R. [Assume charge is uniformly distributed in its volume.]

Solution : We can consider solid sphere to be made of large number of concentric spherical shells. Also electric field intensity at the location of any particular shell is constant.

$$\therefore\quad U_{inside} = \int_0^R \frac{1}{2}\varepsilon_0 E^2 dV$$

Consider an elementary shell of thickness dx and radius x.



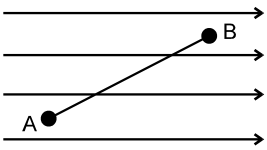
Volume of the shell = $(4\pi x^2 dx)$

$$U_{inside} = \int_0^R \frac{1}{2}\varepsilon_0\left[\frac{KQx}{R^3}\right]^2 .4\pi x^2\, dx = \frac{1}{2}\varepsilon_0\,\frac{K^2Q^2 4\pi}{R^6}\int_0^R x^4 dx$$

$$= \frac{4\pi\varepsilon_0}{2R^6}\,\frac{Q^2}{(4\pi\varepsilon_0)^2}\cdot\frac{R^5}{5} = \frac{Q^2}{40\pi\varepsilon_0 R} = \frac{KQ^2}{10R}.$$

12. RELATION BETWEEN ELECTRIC FIELD INTENSITY AND ELECTRIC POTENTIAL :

12.1 For uniform electric field :



(i) Potential difference between two points A and B

$$V_B - V_A = -\vec{E}\cdot\vec{AB}$$

12.2 Non uniform electric field

(i) $E_x = -\dfrac{\partial V}{\partial x}$, $E_y = -\dfrac{\partial V}{\partial y}$, $E_z = -\dfrac{\partial V}{\partial z}$

$$\Rightarrow\quad \vec{E} = E_x\hat{i} + E_y\hat{j} + E_z\hat{k} = -\left[\hat{i}\frac{\partial}{\partial x}V + \hat{j}\frac{\partial}{\partial y}V + \hat{k}\frac{\partial}{\partial z}V\right] = -\left[\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right]V = -\nabla V = -\text{grad } V$$

Where, $\dfrac{\partial V}{\partial x}$ = derivative of V with respect to x (keeping y and z constant)

$\dfrac{\partial V}{\partial y}$ = derivative of V with respect to y (keeping z and x constant)

$\dfrac{\partial V}{\partial z}$ = derivative of V with respect to z (keeping x and y constant)

12.3 If electric potential and electric field depends only on one coordinate, say r :

(i) $\vec{E} = -\frac{\partial V}{\partial r}\hat{r}$ where, $\hat{r}$ is a unit vector along increasing r.

(ii) $\int dV = -\int \vec{E}\cdot\vec{dr} \Rightarrow V_B - V_A = -\int_{r_A}^{r_B} \vec{E}.\vec{dr}$

$\vec{dr}$ is along the increasing direction of r.

(iii) The potential of a point $V = -\int_{\infty}^{r} \vec{E}.\vec{dr}$

Solved Examples

Example 81. A uniform electric field is along x-axis. The potential difference $V_A - V_B = 10$ V is between two points A (2m, 3m) and B (4m, 8m). Find the electric field intensity.

Solution : $E = \frac{\Delta V}{\Delta d} = \frac{10}{2} = 5$ V / m. (It is along + ve x-axis)

Example 82. $V = x^2 + y$. Find $\vec{E}$.

Solution : $\frac{\partial V}{\partial x}$ c = 2x, $\frac{\partial V}{\partial y} = 1$ and $\frac{\partial V}{\partial z} = 0$

$$\vec{E} = -\left(\hat{i}\frac{\partial V}{\partial x} + \hat{j}\frac{\partial V}{\partial y} + \hat{k}\frac{\partial V}{\partial z}\right) = -(2x\,\hat{i} + \hat{j})$$

$\therefore$ Electric field is non-uniform.

Example 83. For given $\vec{E} = 2x\hat{i} + 3y\hat{j}$, find the potential at (x, y) if V at origin is 5 volts.

Solution : $\int_5^v dV = -\int \vec{E}\cdot\overline{dr} = -\int_0^x E_x dx - \int_0^y E_y dy \Rightarrow V - 5 = -\frac{2x^2}{2} - \frac{3y^2}{2} \Rightarrow V = -\frac{2x^2}{2} - \frac{3y^2}{2} + 5.$

13. ELECTRIC DIPOLE

13.1 Electric Dipole

If two point charges, equal in magnitude 'q' and opposite in sign separated by a distance 'a' such that the distance of field point r>>a, the system is called a dipole. The electric dipole moment is defined as a vector quantity having magnitude p = (q × a) and direction from negative charge to positive charge.

Note: [In chemistry, the direction of dipole moment is assumed to be from positive to negative charge.]

The C.G.S unit of electric dipole moment is **debye** which is defined as the dipole moment of two equal and opposite point charges each having charge 10^{-10} Franklin and separation of 1 Å, i.e.,

1 debye (D) = $10^{-10} \times 10^{-8} = 10^{-18}$ Fr × cm

or 1 D = $10^{-18} \times \frac{C}{3\times10^9} \times 10^{-2}$ m = 3.3×10^{-30} C × m.

S.I. Unit is coulomb × metre = C . m

Solved Examples

Example 84. A system has two charges $q_A = 2.5 \times 10^{-7}$ C and $q_B = -2.5 \times 10^{-7}$ C located at points A : (0, 0, – 0.15 m) and B ; (0, 0, + 0.15 m) respectively. What is the net charge and electric dipole moment of the system ?

Solution : Net charge = $2.5 \times 10^{-7} - 2.5 \times 10^{-7} = 0$

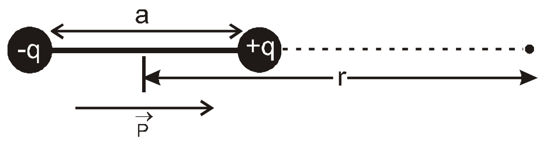
Electric dipole moment, P = (Magnitude of charge) × (Separation between charges)

$= 2.5 \times 10^{-7}\,[0.15 + 0.15]$ C m $= 7.5 \times 10^{-8}$ C m

The direction of dipole moment is from B to A.

13.2 Electric Field Intensity Due to Dipole :

(i) At the axial point :



$$\vec{E} = \frac{Kq}{\left(r-\frac{a}{2}\right)^2} - \frac{Kq}{\left(r+\frac{a}{2}\right)^2} \text{ (along the } \vec{P}) = \frac{Kq(2ra)}{\left(r^2-\frac{a^2}{4}\right)^2}\hat{P}$$

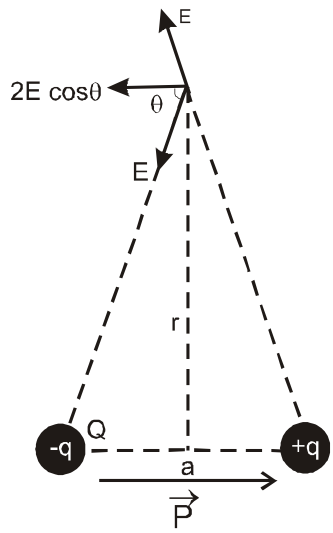
If r >> a then, $\vec{E} = \dfrac{Kq2ra}{r^4}\hat{P} = \dfrac{2K\vec{P}}{r^3}$,

As the direction of electric field at axial position is along the dipole moment ($\vec{P}$)

So, $\vec{E}_{axial} = \dfrac{2K\vec{P}}{r^3}$

(ii) Electric field at perpendicular Bisector (Equatorial Position)

$E_{net} = 2\,E\cos\theta$ (along $-\hat{P}$)



$$\vec{E}_{net} = 2\left(\frac{Kq}{\left(\sqrt{r^2+\left(\frac{a}{2}\right)^2}\right)^2}\right)\frac{\frac{a}{2}}{\sqrt{r^2+\left(\frac{a}{2}\right)^2}}(-\hat{P})$$

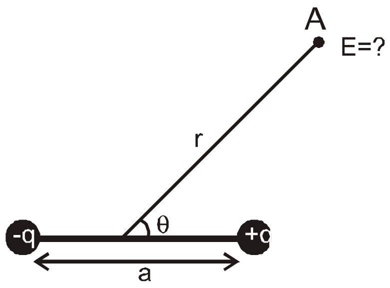
$$= \frac{Kqa}{\left(r^2+\left(\frac{a}{2}\right)^2\right)^{3/2}}(-\hat{P})$$

If r >> a then $\vec{E}_{net} = \dfrac{KP}{r^3}(-\hat{P})$

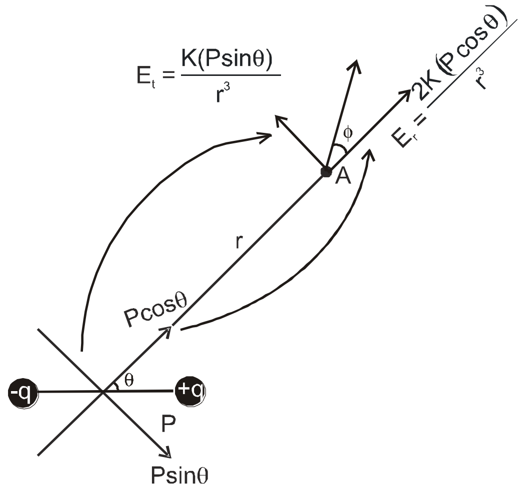
As the direction of $\vec{E}$ at equatorial position is opposite of $\vec{P}$ so we can write in vector form:

$$\vec{E}_{eqt} = -\frac{K\vec{P}}{r^3}$$

(iii) Electric field at general point (r, θ) :



For this, let’s resolve the dipole moment $\vec{P}$ into components.



One component is along radial line (=P cosθ) and other component is $\perp_r$ to the radial line (= Psinθ)

From the given figure $E_{net} = \sqrt{E_r^{\,2} + E_t^{\,2}} = \sqrt{\left(\frac{2KP\cos\theta}{r^3}\right)^2 + \left(\frac{KP\sin\theta}{r^3}\right)^2} = \frac{KP}{r^3}\sqrt{1+3\cos^2\theta}$

$$\tan\phi = \frac{E_t}{E_r} = \frac{\frac{KP\sin\theta}{r^3}}{\frac{2KP\cos\theta}{r^3}} = \frac{\tan\theta}{2}$$

$$E_{net} = \frac{KP}{r^3}\sqrt{1+3\cos^2\theta} \;\; ; \;\; \tan\phi = \frac{\tan\theta}{2}$$

Solved Examples

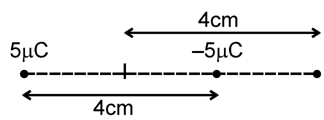
Example 85. The electric field due to a short dipole at a distance r, on the axial line, from its mid point is the same as that of electric field at a distance r', on the equatorial line, from its mid-point. Determine the ratio $\frac{r}{r'}$.

Solution : $\frac{1}{4\pi\varepsilon_0}\frac{2p}{r^3} = \frac{1}{4\pi\varepsilon_0}\frac{p}{r'^3}$ or $\frac{2}{r^3} = \frac{1}{r'^3}$ or $\frac{r^3}{r'^3} = 2$ or, $\frac{r}{r'} = 2^{1/3}$

Example 86. Two charges, each of 5 μC but opposite in sign, are placed 4 cm apart. Calculate the electric field intensity of a point that is at a distance 4 cm from the mid point on the axial line of the dipole.

Solution : We cannot use formula of short dipole here because distance of the point is comparable to the distance between the two point charges.

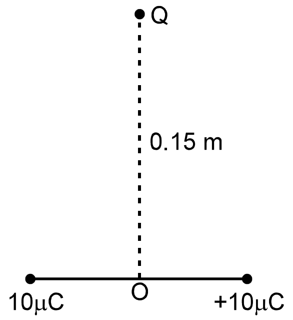
$q = 5 \times 10^{-6}$ C, $a = 4 \times 10^{-2}$ m, $r = 4 \times 10^{-2}$ m



$E_{res} = E_+ + E_- = \frac{K(5\mu C)}{(2cm)^2} - \frac{K(5\mu C)}{(6\ \ cm)^2} = \frac{144}{144\times10^{-8}}\ NC^{-1} = 10^8\ NC^{-1}$

Example 87. Two charges $\pm 10\mu C$ are placed 5×10^{-3} m apart as shown in figure. Determine the electric field at a point Q which is 0.15 m away from O, on the equatorial line.

Solution : In the given problem, r >> a

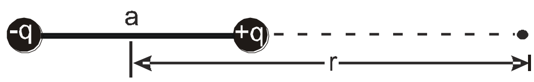


$$\therefore \quad E = \frac{1}{4\pi\varepsilon_0}\frac{p}{r^3} = \frac{1}{4\pi\varepsilon_0} \quad \text{or} \quad E = 9 \times 10^9 \times \frac{10 \times 10^{-6} \times 5 \times 10^{-3}}{0.15 \times 0.15 \times 0.15} NC^{-1} = 1.33 \times 10^5 \ NC^{-1}$$

13.3 Electric Potential due to a small dipole :

(i) Potential at axial position :

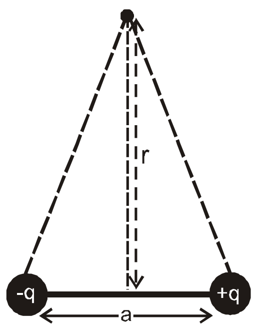
$$V = \frac{Kq}{\left(r - \frac{a}{2}\right)} + \frac{K(-q)}{\left(r + \frac{a}{2}\right)} \Rightarrow V = \frac{Kqa}{\left(r^2 - \left(\frac{a}{2}\right)^2\right)}$$



If r >> a then $V = \frac{Kqa}{r^2}$; where, qa = p

$$\boxed{\therefore \quad V_{axial} = \frac{KP}{r^2}}$$

(ii) Potential at equatorial position :



$$V = \frac{Kq}{\sqrt{r^2 + \left(\frac{a}{2}\right)^2}} + \frac{K(-q)}{\sqrt{r^2 + \left(\frac{a}{2}\right)^2}} = 0 \qquad \boxed{V_{eqt} = 0}$$

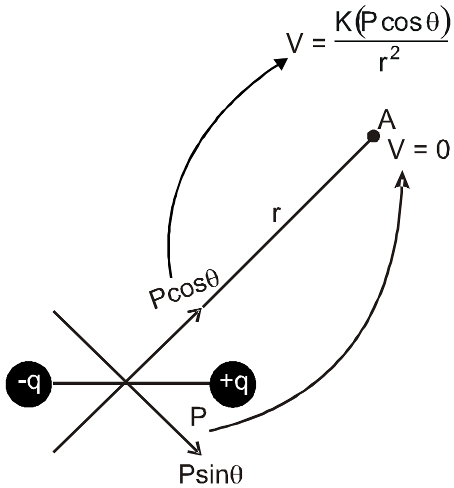
(iii) Potential at general point (r, θ) : Lets resolve the dipole moment $\vec{P}$ into components : $P\cos\theta$ along radial line and $P\sin\theta$ $\perp_r$ to the radial line

For the $P\cos\theta$ component, the point A is an axial point,

So, potential at A due to $P\cos\theta = \frac{K(P\cos\theta)}{r^2}$

And for $P\sin\theta$ component, the point A is an equatorial point,

So potential at A due to $P\sin\theta = 0$



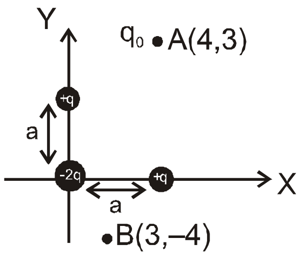
$$V_{net} = \frac{K(P\cos\theta)}{r^2}$$

$$\therefore\ V = \frac{K\left(\vec{P}\cdot\vec{r}\right)}{r^3}$$

Solved Examples

Example 88. (i) Find potential at point A and B due to the small charge - system fixed near origin. (Distance between the charges is negligible).

(ii) Find work done to bring a test charge q_0 from point A to point B, slowly. All parameters are in S.I. units.



Solution : (i) Dipole moment of the system is $\vec{P} = (qa)\hat{i} + (qa)\hat{j}$

Potential at point A due to the dipole

$$V_A = K\,\frac{(\vec{P}\cdot\vec{r})}{r^3} = \frac{K[(qa)\hat{i} + (qa)\hat{j}]\cdot(4\hat{i} + 3\hat{j})}{5^3} = \frac{k(qa)}{125}\quad(7)$$

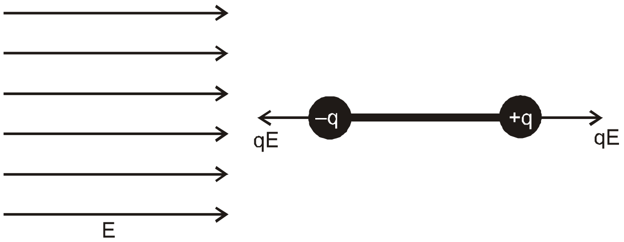
$$\Rightarrow\ V_B = \frac{K\left[(qa)\hat{i} + (qa)\hat{j}\right]\left(3\hat{i} - 4\hat{j}\right)}{(5)^3} = \frac{-K(qa)}{125}$$

(ii) $W_{A\to B} = U_B - U_A = q_0\,(V_B - V_A) = q_0\left[-\frac{K(qa)}{125} - \left(\frac{K(qa)(7)}{125}\right)\right]$

$$\Rightarrow\ W_{A\to B} = \frac{-Kqq_0a}{125}(8)$$

13.4 Dipole in uniform electric field

(i) Dipole is placed along electric field :



In this case, $F_{net} = 0$, $\tau_{net} = 0$, so it is an equilibrium state. And it is a stable equilibrium position.

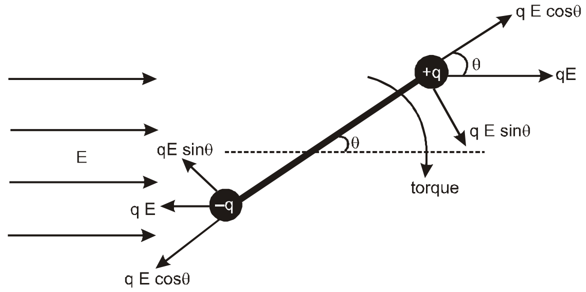
(ii) If the dipole is placed at angle θ from $\vec{E}$:

In this case $F_{net} = 0$ but

Net torque $\tau = (qE\sin\theta)$ (a)

Here $qa = P \Rightarrow \tau = PE\sin\theta$

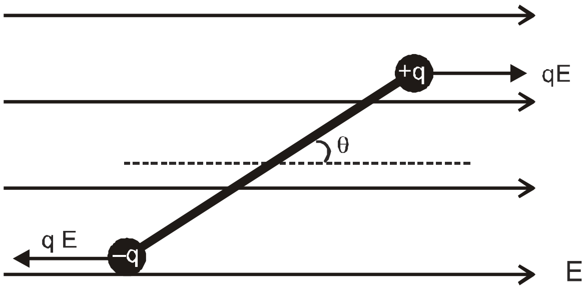
in vector form : $\vec{\tau} = \vec{P} \times \vec{E}$



Solved Examples

Example 89. A dipole is formed by two point charge –q and +q, each of mass m, and both the point charges are connected by a rod of length ℓ and mass m. This dipole is placed in uniform electric field $\vec{E}$.
If the dipole is disturbed by a small angle θ from stable equilibrium position, prove that its motion will be almost SHM. Also find its time period.

Solution : If the dipole is disturbed by θ angle, $\tau_{net} = -PE\sin\theta$ (Here -ve sign indicates that direction of torque is opposite to θ)



If θ is very small, $\sin\theta \approx \theta$

$\therefore \quad \tau_{net} = -(PE)\theta$

$\tau_{net} \propto (-\theta)$ so motion will be almost SHM & C = PE (where, $P = q\ell$)

$$\because \quad T = 2\pi\sqrt{\frac{I}{C}}$$

$$\therefore \quad T = 2m^{-}\sqrt{\frac{\frac{m\ell}{12} + 2m\left(\frac{\ell}{2}\right)^2}{P.E}} = 2\pi\sqrt{\frac{\frac{m\ell}{12} + \frac{m\ell^2}{2}}{q\ell E}} = 2\pi\sqrt{\frac{7m\ell^2}{12q\ell E}} = 2\pi\sqrt{\frac{7m\ell}{12qE}}$$

$$T = \pi\sqrt{\frac{7m\ell}{3qE}}$$

(iii) Potential energy of a dipole placed in uniform electric field :

$$U_B - U_A = -\int_A^B \vec{F} \cdot \overrightarrow{dr} \qquad \text{(for translational motion)}$$

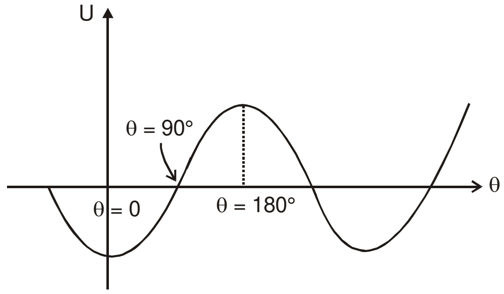
Here, $$U_B - U_A = -\int_A^B \vec{\tau}.\overrightarrow{d\theta} \qquad \text{(for rotational motion)}$$

In the case of dipole, at $\theta = 90°$, P.E. is assumed to be zero.

$$U_\theta - U_{90°} = -\int_{\theta=90°}^{\theta=\theta} (-PE\sin\theta)\ (d\theta) \qquad \text{(As the direction of torque is opposite of } \theta)$$

$U_\theta - 0 = -$ PE cos θ

$\theta = 90°$ is chosen as reference, so that the lower limit comes out to be zero.



$$\boxed{U_\theta = -\vec{P}\cdot\vec{E}}$$

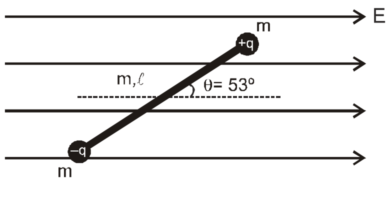
From the potential energy curve, we can conclude :

(i) At $\theta = 0$, there is minimum of P.E. so it is a stable equilibrium position.

(ii) At $\theta = 180°$, there is maxima of P.E. so it is a position of unstable equilibrium.

Solved Examples

Example 90. Two point masses of mass m and equal and opposite charge of magnitude q are attached on the corners of a non-conducting uniform rod of mass m and the system is released from rest in uniform electric field E as shown in figure from $\theta = 53°$



(i) Find angular acceleration of the rod just after releasing

(ii) What will be angular velocity of the rod when it passes through stable equilibrium.

(iii) Find work required to rotate the system by 180°.

Solution : (i) τ_{net} = PE sin53° = I α

$$\therefore \quad \alpha = \frac{(q\ell)\ E\ \left(\frac{4}{5}\right)}{\frac{m\ell^2}{12} + m\left(\frac{\ell}{2}\right)^2 + m\left(\frac{\ell}{2}\right)^2} = \frac{48qE}{35\ m\ell}$$

(ii) From energy conservation : $K_i + U_i = K_f + U_f$

$\therefore \quad 0 + (-\text{PE}\cos 53°) = \frac{1}{2}I\omega^2 + (-\text{PE}\cos 0°)$

where $I = \frac{m\ell^2}{12} + m\left(\frac{\ell}{2}\right)^2 + m\left(\frac{\ell}{2}\right)^2 = \frac{7m\ell^2}{12}$ $\quad\therefore\quad \frac{1}{2}I\omega^2 = \text{PE}(1-3/5) = \frac{2}{5}\text{PE}$

$\therefore \quad \frac{1}{2} \times \frac{7m\ell^2}{12} \times \omega^2 = \frac{2}{5}\,q\ell E$ $\quad$ or $\quad \omega = \sqrt{\frac{48qE}{35\,m\ell}}$

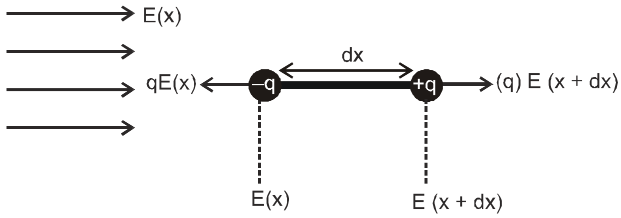
(iii) $\because \quad W_{ext} = U_f - U_i$

$\therefore \quad W_{ext} = (-\text{PE}\cos(180° + 53°)) - (-\text{PE}\cos 53°)$

or $\quad W_{ext} = (q\ell)E\left(\frac{3}{5}\right) + (q\ell)E\left(\frac{3}{5}\right) \quad \Rightarrow \quad W_{ext} = \left(\frac{6}{5}\right)q\ell E$

13.5 Dipole in non-uniform electric field :

If the dipole is placed along $\vec{E}$, (shown in figure)

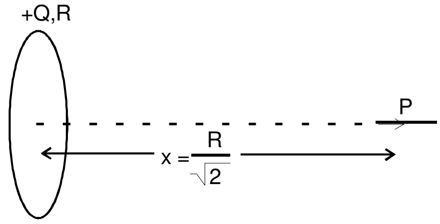


Then, Net force on the dipole : $F_{net} = qE(x + dx) - qE(x)$

$F_{net} = q\,\frac{E(x+dx)-E(x)}{dx}\,(dx)$; here $(q(dx) = P)$ $\quad \boxed{\therefore \quad F_{net} = P\left(\frac{dE}{dx}\right)}$

Solved Examples

Example 91. A short dipole is placed on the axis of a uniformly charged ring (total charge –Q, radius R) at a distance $\frac{R}{\sqrt{2}}$ from centre of ring as shown in figure. Find the Force on the dipole due to the ring



Solution : $\because \quad F = P\left(\frac{dE}{dx}\right)$

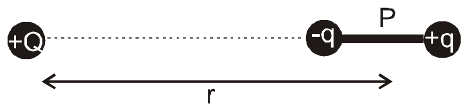
$\therefore \quad F = P\frac{d}{dx}\left(\frac{KQx}{(R^2+x^2)^{3/2}}\right)$; (at $x = \frac{R}{\sqrt{2}}$)

Solving we get, $F = 0$

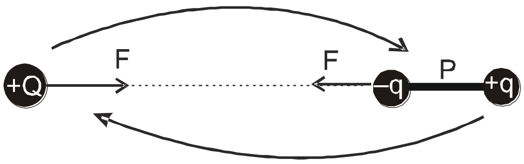
13.6 Force between a dipole and a point charge :

Solved Examples

Example 92. A short dipole of dipole moment P is placed near a point charge Q as shown in figure. Find force on the dipole due to the point charge



Solution :



Force on the point charge due to the dipole F = (Q) E_{dipole}

$$F = (Q)\left(\frac{2KP}{r^3}\right) \quad \text{(towards right)}$$

From action reaction concept, force on the dipole due to point charge will be equal to the force on charge due to dipole

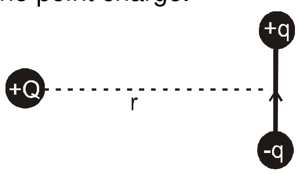
$$F = \frac{2KPQ}{r^3} \quad \text{(towards left)}$$

is force on dipole due to point Charge

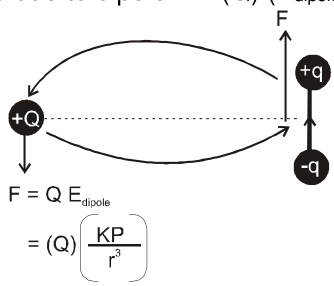
13.7 Force between two dipoles :

Solved Examples

Example 93. A short dipole of dipole moment P is placed near a point charge Q as shown in figure. Find force on the dipole due to the point charge.



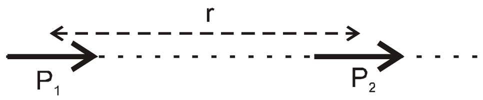
Solution : Force on the point charge due to dipole F = (Q) (E_{dipole})



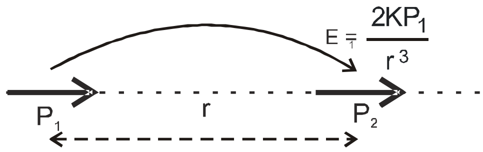
$$F = (Q)\left(\frac{KP}{r^3}\right)(\downarrow)$$

So force on the dipole due to the point charge will also be $F = \left(\frac{KPQ}{r^3}\right)(\uparrow)$ (but in opposite direction) as shown

Example 94. Find force on short dipole P_2 due to short dipole P_1 if they are placed at a distance r a part as shown in figure.



Solution : Force on P_2 due to P_1

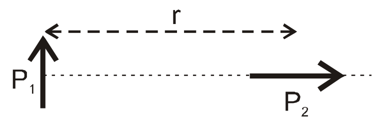


$F_2 = (P_2)\left(\frac{dE_1}{dr}\right)$

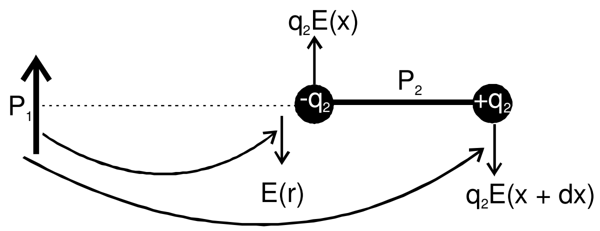
$\therefore \quad F_2 = (P_2)\left(\frac{d}{dr}\left(\frac{2\ \ kP_1}{r^3}\right)\right)$ or $F_2 = -\frac{6KP_1P_2}{r^4}$

Here – sign indicates that this force will be attractive (opposite to r)

Example 95. Find force on short dipole P_2 due to short dipole P_1 if they are placed a distance r apart as shown in figure.



Solution :



$F_{net} = q_2\ E\ (x + dx) - q_2\ E\ (x)$

$F_{net} = q_2\ \left(\frac{E(x+dx)-E(x)}{dx}\right)dx$

or $F_{net} = (P_2)\left(\frac{dE}{dx}\right)$

(Usually this formula is valid when the dipole is placed along $\vec{E}$. However, in this case also, we are getting the same formula)

$\therefore \quad F_{net} = (P_2)\left(\frac{d}{dr}\left(\frac{KP_1}{r^3}\right)\right)$

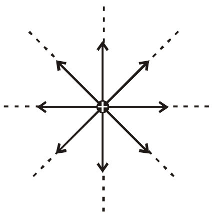
$\Rightarrow \quad F_{net} = \frac{3KP_1P_2}{r^4}$ (in magnitude) & (direction upwards)

14. ELECTRIC LINES OF FORCE (ELOF)

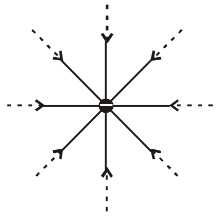
The line of force in an electric field is an imaginary line, the tangent to which at any point on it represents the direction of electric field at the given point.

14.1 Properties :

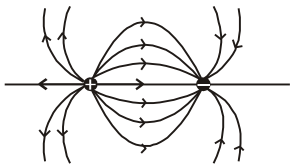
(i) Line of force originates out from a positive charge and terminates on a negative charge. If there is only one positive charge, then lines starts from positive charge and terminates at ∞. If there is only one negative charge, then lines starts from ∞ and terminates at negative charge.



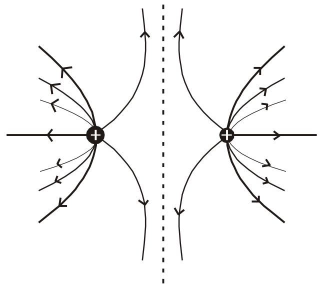
ELOF of Isolated positive charge



ELOF of Isolated negative charge

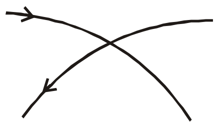


ELOF due to positive and negative charges



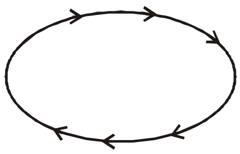
ELOF due to two positive charges

(ii) Two lines of force never intersect each other because there cannot be two directions of $\vec{E}$ at a single Point

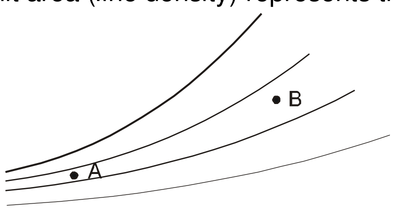


impossilble

(iii) Electric lines of force produced by static charges do not form closed loop.
If lines of force make a closed loop, then work done to move a +q charge along the loop will be non-zero. So it will not be conservative field. So these type of lines of force are not possible in electrostatics.



(iv) The Number of lines per unit area (line density) represents the magnitude of electric field.



$E_A > E_B$

If lines are dense $\Rightarrow$ E will be more
If Lines are rare $\Rightarrow$ E will be less and if E = O, no line of force will be found there

(v) Number of lines originating (terminating) at a charge is proportional to the magnitude of charge

Solved Examples

Example 96. If number of electric lines of force from charge q are 10, then find out number of electric lines of force from 2q charge.

Solution : No. of ELOF $\propto$ charge

$10 \propto q \quad \Rightarrow \quad 20 \propto 2q$

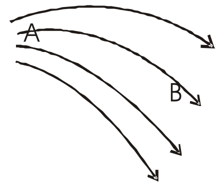
So, number of ELOF will be 20.

(vi) Electric lines of force end or start perpendicularly on the surface of a conductor.

(vii) Electric lines of force never enter into conductors.

Solved Examples

Example 97. Some electric lines of force are shown in figure. For points A and B

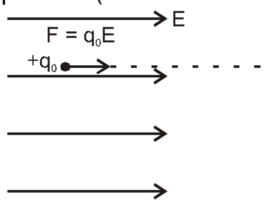


(A) $E_A > E_B$ (B) $E_B > E_A$ (C) $V_A > V_B$ (D) $V_B > V_A$

Solution : Lines are more dense at A, so $E_A > E_B$ In the direction of Electric field, potential decreases so $V_A > V_B$

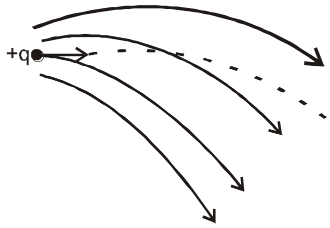
Example 98. If a charge is released in electric field, will it follow lines of force?

Solution : **Case I :** If lines of force are parallel (in uniform electric field) :



In this type of field, if a charge is released, force on it will be q_0E and its direction will be along. So the charge will move in a straight line, along the lines of force.

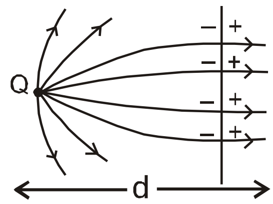
Case II : If lines of force are curved (in non-uniform electric field) :



The charge will not follow lines of force

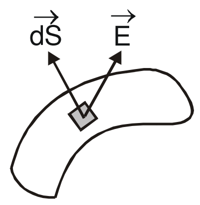
Example 99. A charge + Q is fixed at a distance d in front of an infinite metal plate. Draw the lines of force indicating the directions clearly.

Solution : There will be induced charge on two surfaces of conducting plate, so ELOF will start from +Q charge and terminate at conductor and then will again start from other surface of conductor.



15. ELECTRIC FLUX

Consider some surface in an electric field $\vec{E}$. Let us select a small area element $\vec{dS}$ on this surface. The electric flux of the field over the area element is given by $d\phi_E = \vec{E}.\vec{dS}$



Direction of $\vec{dS}$ is normal to the surface. It is along $\hat{n}$

or $d\phi_E = EdS \cos\theta$

or $d\phi_E = (E\cos\theta)\, dS$

or $d\phi_E = E_n\, dS$

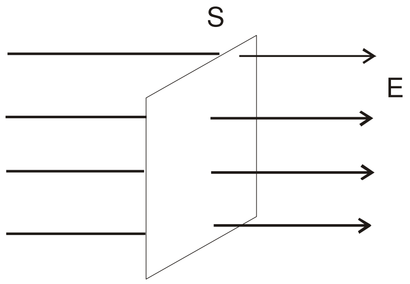
where E_n is the component of electric field in the direction of $\vec{dS}$.

The electric flux over the whole area is given by $\phi_E = \int_S \vec{E}.\vec{dS} = \int_S E_n dS$

If the electric field is uniform over that area then $\phi_E = \vec{E}\cdot\vec{S}$

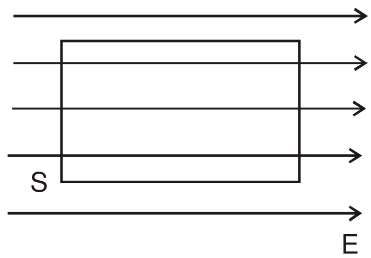
Special Cases :

Case I : If the electric field is normal to the surface, then angle of electric field $\vec{E}$ with normal will be zero



So $\phi = ES\cos 0$ or $\phi = ES$

Case II : If electric field is parallel of the surface (grazing), then angle made by $\vec{E}$ with normal = 90º



So $\phi = ES\cos 90º = 0$

15.1 Physical Meaning :

The electric flux through a surface inside an electric field represents the total number of electric lines of force crossing the surface. It is a property of electric field

15.2 Unit

(i) The SI unit of electric flux is $Nm^2\, C^{-1}$ (Gauss) or $J\, m\, C^{-1}$.

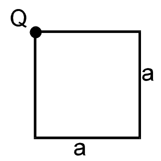
(ii) Electric flux is a scalar quantity. (It can be positive, negative or zero)

Solved Examples

Example 100. The electric field in a region is given by $\vec{E} = \frac{3}{5}E_0\hat{i} + \frac{4}{5}E_0\hat{j}$ with $E_0 = 2.0 \times 10^3$ N/C. Find the flux of this field through a rectangular surface of area $0.2m^2$ parallel to the Y–Z plane.

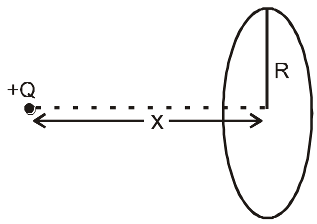
Solution : $\phi_E = \vec{E}\cdot\vec{S} = \left(\frac{3}{5}E_0\hat{i} + \frac{4}{5}E_0\hat{j}\right).\left(0.2\hat{i}\right) = 240\frac{N-m^2}{C}$

Example 101. A point charge Q is placed at the corner of a square of side a, then find the flux through the square.

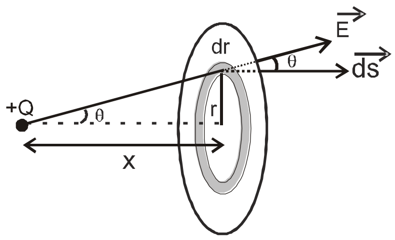


Solution : The electric field due to Q at any point of the square will be along the plane of square and the electric field lines are perpendicular to square ; so $\phi = 0$. In other words we can say that no line is crossing the square so, flux = 0.

Example 102. Find the electric flux due to a point charge 'Q' through the circular region of radius R if the charge is placed on the axis of ring at a distance x.



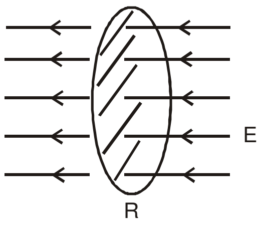
Solution : We can divide the circular region into small rings.



Lets take a ring of radius r and width dr. Flux through this small element $d\phi = E\, ds \cos\theta$

$$\therefore \quad \phi_{net} = \int E\, ds \cos\theta = \int_{r=o}^{r=R} \frac{KQ}{(x^2+r^2)}\,(2\pi\, r\, dr)\left(\frac{x}{\sqrt{x^2+r^2}}\right) = \frac{Q}{2\in_0}\left[1-\frac{x}{\sqrt{x^2+R^2}}\right]$$

Case III : Curved surface in uniform electric field. Suppose a circular surface of radius R is placed in a uniform electric field as shown.



Flux passing through the surface $\phi = E\,(\pi R^2)$

(ii) Now suppose, a hemispherical surface, is placed in the electric field. Flux through hemispherical surface:

$\phi = \int E ds \cos\theta$

$\phi = E \int ds \cos\theta$

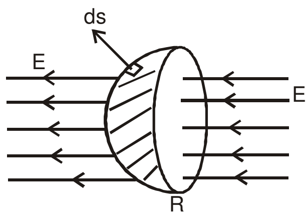
where, $\int ds \cos\theta$ is

projection of the spherical surface Area on base.

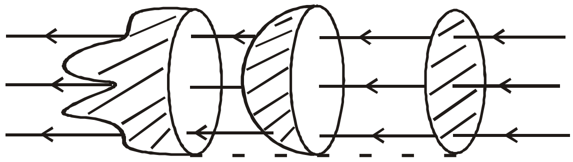
& $\int ds \cos\theta = \pi R^2$

So, $\phi = E(\pi R^2)$ = same Ans. as in previous case

So, we can conclude that



<u>If the number of electric field lines passing through two surfaces are same, then flux passing through these surfaces will also be same, irrespective of the shape of surface</u>



$\phi_1 = \phi_2 = \phi_3 = E(\pi R^2)$

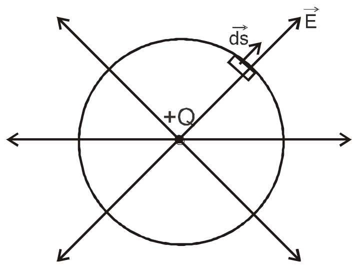
Case IV : Flux through a closed surface : Suppose there is a spherical surface and a charge 'q' is placed at centre.

$\therefore$ Flux through the spherical surface

$\phi = \int \vec{E}.\vec{ds} = \int E\,ds$ (as $\vec{E}$ is along $\vec{ds}$ (normal))

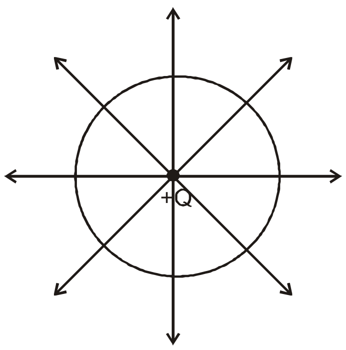
$\therefore \quad \phi = \frac{1}{4\pi\varepsilon_o}\frac{Q}{R^2}\int ds$ where, $\int ds = 4\pi R^2$

$\phi = \left(\frac{1}{4\pi\varepsilon_0}\frac{Q}{R^2}\right)(4\pi R^2) \quad \Rightarrow \quad \phi = \frac{Q}{\varepsilon_o}$

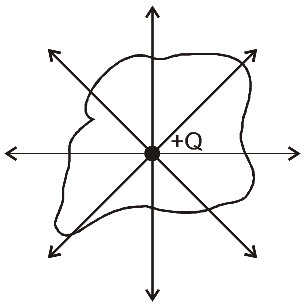


Now if the charge Q is enclosed by any other closed surface, still same no of lines of force will pass through the surface.

So, here also flux will be $\phi = \frac{Q}{\varepsilon_o}$. That's what Gauss Theorem is.



$\phi = \frac{Q}{\varepsilon_o}$



$\phi = \frac{Q}{\varepsilon_o}$

16. GAUSS'S LAW IN ELECTROSTATICS OR GAUSS'S THEOREM

This law was stated by a mathematician Karl F Gauss. This law gives the relation between the electric field at a point on a closed surface and the net charge enclosed by that surface. This surface is called Gaussian surface. It is a closed hypothetical surface. Its validity is shown by experiments. It is used to determine the electric field due to some symmetric charge distributions.

16.1 Statement and Details :

Gauss's law is stated as given below :

The surface integral of the electric field intensity over any closed hypothetical surface (called Gaussian surface) in free space is equal to $\frac{1}{\varepsilon_0}$ times the total charge enclosed within the surface. Here, ε_0 is the permittivity of free space.

If S is the Gaussian surface and $\sum_{i=1}^{n} q_i$ is the total charge enclosed by the Gaussian surface, then according to Gauss's law,

$$\phi_E = \oint \vec{E} \cdot \overrightarrow{dS} = \frac{1}{\varepsilon_0} \sum_{i=1}^{n} q_i .$$

The circle on the sign of integration indicates that the integration is to be carried out over the closed surface.

Note :

(i) Flux through Gaussian surface is independent of its shape.

(ii) Flux through Gaussian surface depends only on total charge present inside Gaussian surface.

(iii) Flux through Gaussian surface is independent of position of charges inside Gaussian surface.

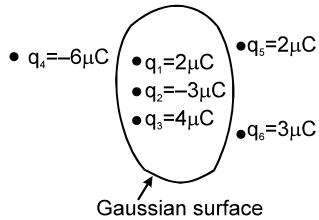
(iv) Electric field intensity at the Gaussian surface is due to all the charges present inside as well as outside the Gaussian surface.

(v) In a closed surface incoming flux is taken negative, while outgoing flux is taken positive, because $\hat{n}$ is taken positive in outward direction.

(vi) In a Gaussian surface, $\phi = 0$ does not imply E = 0 at every point of the surface but E = 0 at every point implies $\phi = 0$.

Solved Examples

Example 103. Find out flux through the given Gaussian surface.



Solution : $\phi = \frac{Q_{in}}{\varepsilon_0} = \frac{2\mu C - 3\mu C + 4\mu C}{\varepsilon_0} = \frac{3 \times 10^{-6}}{\varepsilon_0}$ Nm²/C

Example 104. If a point charge q is placed at the centre of a cube, then find out flux through any one face of cube.

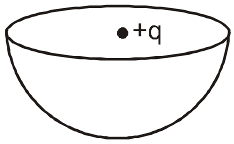
Solution : Flux through all 6 faces = $\frac{q}{\varepsilon_0}$. Since, all the surfaces are symmetrical

So, flux through one face = $\frac{1}{6}\frac{q}{\varepsilon_0}$

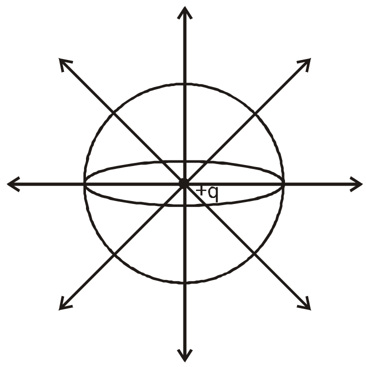
16.2 Flux through open surfaces using Gauss's Theorem :

Solved Examples

Example 105. A point charge +q is placed at the centre of curvature of a hemisphere. Find flux through the hemispherical surface.



Solution : Lets put an upper half hemisphere. Now, flux passing through the entire sphere = $\frac{q}{\varepsilon_0}$

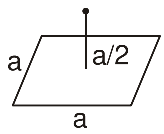


As the charge q is symmetrical to the upper half and lower half hemispheres, so half-half flux will emit from both the surfaces.

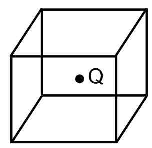
Flux emitting from lower half surface = $\frac{q}{2\varepsilon_0}$

Flux emitting from upper half surface = $\frac{q}{2\varepsilon_0}$

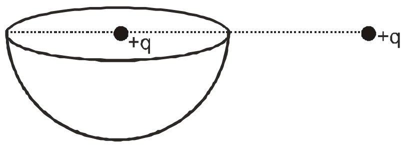
Example 106. A charge Q is placed at a distance a/2 above the centre of a horizontal, square surface of edge a as shown in figure. Find the flux of the electric field through the square surface.



Solution : We can consider imaginary faces of cube such that the charge lies at the centre of the cube. Due to symmetry, we can say that flux through the given area (which is one face of cube), $\phi = \frac{Q}{6\varepsilon_0}$

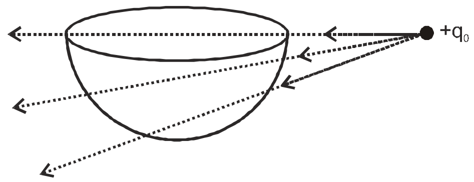


Example 107. Find flux through the hemispherical surface



Solution : (i) Flux through the hemispherical surface due to +q = $\frac{q}{2\varepsilon_0}$ (we have seen in previous examples)

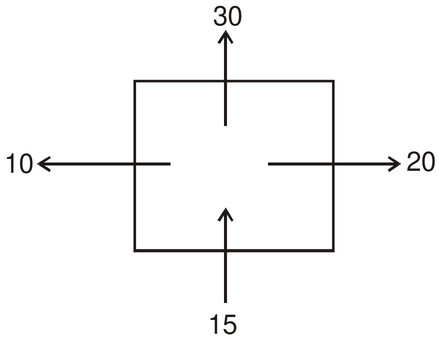
(ii) Flux through the hemispherical surface due to $+q_0$ is 0, because due to $+q_0$, field lines entering the surface = field lines coming out of the surface.



16.3 Finding q_{in} from flux :

Solved Examples

Example 108.



Flux (in S.I. units) coming out and entering a closed surface is shown in the figure. Find charge enclosed by the closed surface.

Solution : Net flux through the closed surface = + 20 + 30 + 10 -15 = 45 N.m^2/c

From Gauss's theorem : $\phi_{net} = \frac{q_{in}}{\varepsilon_0}$

or $45 = \frac{q_{in}}{\varepsilon_0}$ $\therefore$ $q_{in} = (45)\varepsilon_0$

16.4 Finding electric field from Gauss`s Theorem :

From Gauss`s theorem, we can say $\int \vec{E}.\vec{ds} = \phi_{net} = \frac{q_{in}}{\varepsilon_0}$

16.4.1 Finding E due to a spherical shell :

Electric field outside the Sphere :

Since, electric field due to a shell will be radially outwards. So lets choose a spherical Gaussian surface Applying Gauss's theorem for this spherical Gaussian surface,

$\int \vec{E}\,\vec{ds} = \phi_{net} = \frac{q_{in}}{\varepsilon_o} = \frac{q}{\varepsilon_o}$

↓

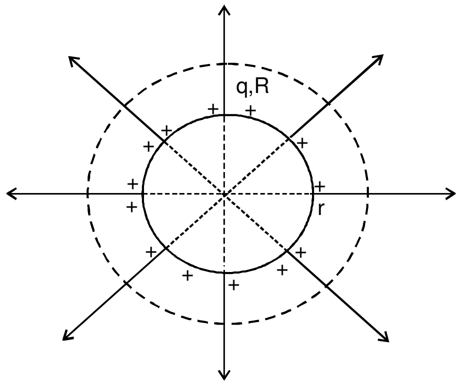
$\int |\vec{E}||\vec{ds}|\cos 0$ (because $\vec{E}$ is normal to the surface)

↓

$E\int ds$ (because value of E is constant at the surface)

$E(4\pi r^2)$ ($\therefore \int ds$ total area of the spherical surface = $4\pi r^2$)

$\Rightarrow E(4\pi r^2) = \frac{q_{in}}{\varepsilon_0} \quad \therefore E_{out} = \frac{q}{4\pi\varepsilon_o r^2}$



Electric field inside a spherical shell : Lets choose a spherical Gaussian surface inside the shell. Applying Gauss`s theorem for this surface

$\int \vec{E}\,\vec{ds} = \phi_{net} = \frac{q_{in}}{\varepsilon_0} = 0$

↓

$\int |\vec{E}||\vec{ds}|\cos 0$

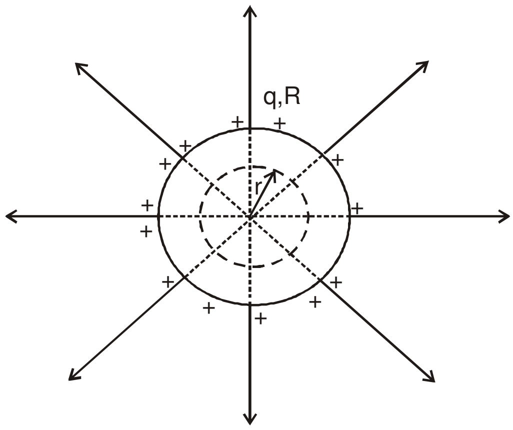
↓

$E\int ds$

↓

$E(4\pi r^2) \Rightarrow E(4\pi r^2) = 0$

$\therefore E_{in} = 0$



16.4.2 Electric field due to solid sphere (having uniformly distributed charge Q and radius R) :

Electric field outside the sphere :

Direction of electric field is radially outwards, so we will choose a spherical Gaussian surface Applying Gauss`s theorem

$\int \vec{E}\,\vec{ds} = \phi_{net} = \frac{q_{in}}{\varepsilon_o} = \frac{Q}{\varepsilon_o}$

↓

$\int |\vec{E}||\vec{ds}|\cos 0$

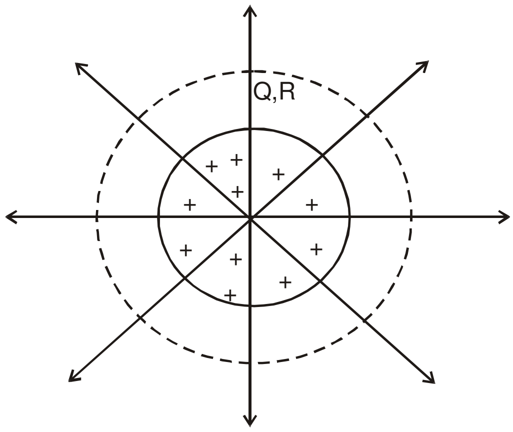
↓

$E\int ds$

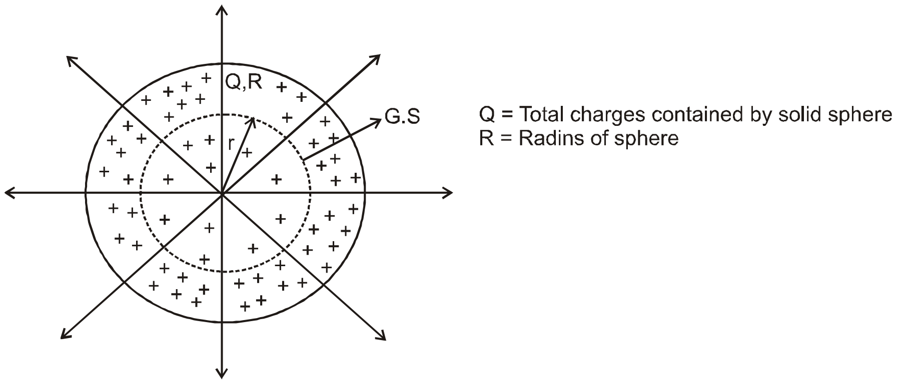
↓

$E(4\pi r^2) \Rightarrow E(4\pi r^2) = \frac{Q}{\varepsilon_o}$

or $E_{out} = \frac{Q}{4\pi\varepsilon_o r^2}$



Electric field inside a solid sphere :



For this choose a spherical Gaussian surface inside the solid sphere Applying Gauss`s theorem for this surface

$$\int \vec{E}\,d\vec{s} = \phi_{net} = \frac{q_{in}}{\varepsilon_o} = \frac{\frac{Q}{\frac{4}{3}\pi R^3} \times \frac{4}{3}\pi r^3}{\varepsilon_o} = \frac{Qr^3}{\varepsilon_o R^3}$$

$\downarrow$

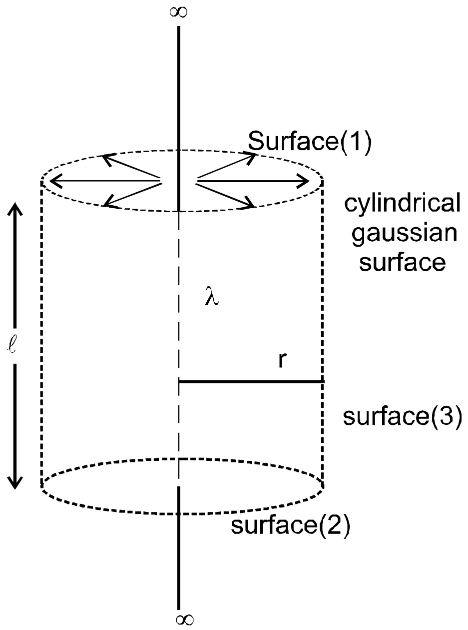
$\int E ds$

$\downarrow$

$$E\,(4\pi r^2) \qquad \Rightarrow \quad E(4\pi r^2) = \frac{Qr^3}{\varepsilon_o R^3}$$

$$E = \frac{Qr}{4\pi\varepsilon_o R^3} \qquad \therefore \quad E_{in} = \frac{kQ}{R^3}\,r$$

16.4.3 Electric field due to infinite line charge (having uniformly distributed charged of charge density λ) :



Electric field due to infinitely wire is radial so we will choose cylindrical Gaussian surface as shown is figure:

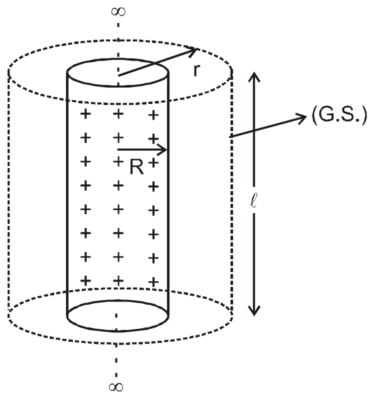
ϕ_{net}

$\phi_1 = 0 \quad \phi_2 = 0 \quad \phi_3 \neq 0 = \dfrac{q_{in}}{\varepsilon_o} = \dfrac{\lambda\ell}{\varepsilon_o}$

$\phi_3 = \int \vec{E}.\vec{ds} = \int E\,ds = E\int ds = E\,(2\pi r\ell)$

$\because \quad E\,(2\pi r\ell) = \dfrac{\lambda\ell}{\varepsilon_o} \qquad \therefore \qquad E = \dfrac{\lambda}{2\pi\varepsilon_o r} = \dfrac{2k\lambda}{r}$

16.4.4 Electric field due to infinitely long charged tube (having uniform surface charge density σ and radius R)) :



(i) E outside the tube : Lets choose a cylindrical Gaussian surface of length ℓ :

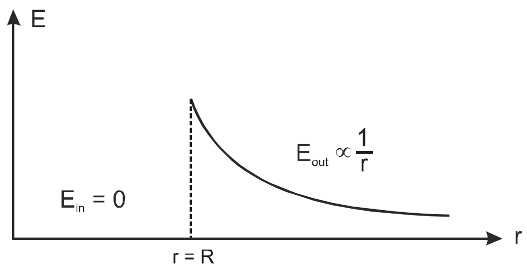
$\therefore\ \phi_{net} = \dfrac{q_{in}}{\varepsilon_o} = \dfrac{\sigma 2\pi R\ell}{\varepsilon_o} \quad \Rightarrow \quad E_{out} \times 2\pi r\ell = \dfrac{\sigma 2\pi R\ell}{\varepsilon_o} \quad \therefore \quad E = \dfrac{\sigma R}{r\,\varepsilon_0}$

(ii) E inside the tube :

Lets choose a cylindrical Gaussian surface inside the tube.

$\phi_{net} = \dfrac{q_{in}}{\varepsilon_o} = 0$

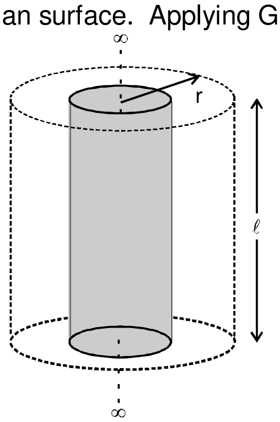
So $E_{in} = 0$



16.4.5 E due to infinitely long solid cylinder of radius R (having uniformly distributed charge in volume (volume charge density ρ)) :

(i) E at outside point :

Lets choose a cylindrical Gaussian surface. Applying Gauss`s theorem :

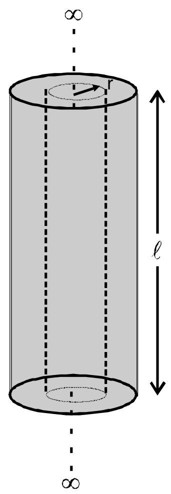


$$E \times 2\pi r\ell = \frac{q_{in}}{\varepsilon_o} = \frac{\rho \times \pi R^2 \ell}{\varepsilon_o}$$

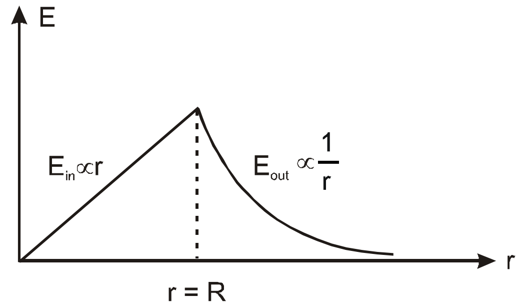
$$E_{out} = \frac{\rho R^2}{2r\,\varepsilon_0}$$

(ii) E at inside point :

Lets choose a cylindrical Gaussian surface inside the solid cylinder. Applying Gauss`s theorem



$$E \times 2\pi r\ell = \frac{q_{in}}{\varepsilon_o} = \frac{\rho \times \pi r^2 \ell}{\varepsilon_o} \Rightarrow E_{in} = \frac{\rho r}{2\varepsilon_0}$$



17. CONDUCTOR AND IT'S PROPERTIES [FOR ELECTROSTATIC CONDITION]

(i) Conductors are materials which contain large number of free electrons which can move freely inside the conductor.

(ii) In electrostatics conductors are always equipotential surfaces.

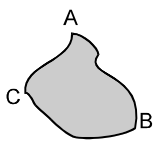
(iii) Charge always resides on outer surface of conductor.

(iv) If there is a cavity inside the conductor having no charge then charge will always reside only on outer surface of conductor.

(v) Electric field is always perpendicular to conducting surface.

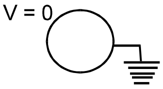
(vi) Electric lines of force never enter into conductors.

(vii) Electric field intensity near the conducting surface is given by formula $\vec{E} = \frac{\sigma}{\varepsilon_0}\hat{n}$



$$\overrightarrow{E_A} = \frac{\sigma_A}{\varepsilon_0}\hat{n}\ ;\ \overrightarrow{E_B} = \frac{\sigma_B}{\varepsilon_0}\hat{n} \text{ and } \overrightarrow{E_C} = \frac{\sigma_C}{\varepsilon_0}\hat{n}$$

(viii) When a conductor is grounded its potential becomes zero.



(ix) When an isolated conductor is grounded then its charge becomes zero.

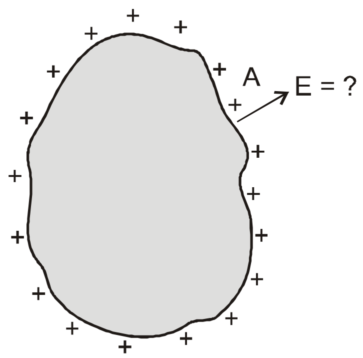
(x) When two conductors are connected there will be charge flow till their potentials become equal.

(xi) Electric pressure : Electric pressure at the surface of a conductor is given by formula

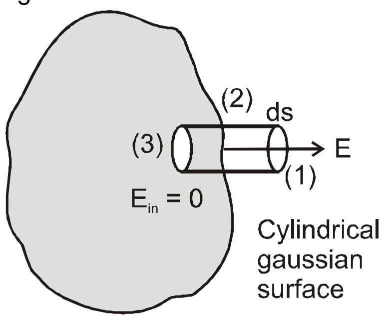
$P = \dfrac{\sigma^2}{2\varepsilon_0}$, where σ is the local surface charge density.

FINDING FIELD DUE TO A CONDUCTOR

Suppose we have a conductor and at any 'A', local surface charge density = σ. We have to find electric field just outside the conductor surface.



For this, let's consider a small cylindrical Gaussian surface, which is partly inside and partly outside the conductor surface, as shown in figure. It has a small cross section area ds and negligible height.



Applying Gauss's theorem for this surface :

$$\phi_{net} = \frac{q_{in}}{\varepsilon_0} = \frac{\sigma\, ds}{\varepsilon_0}$$

flux through surface (1)
$\phi_1 = Eds$
(because $\vec{E}$ is normal to the surface of conductor)

flux through surface (2)
$\phi_2 = 0$
($\vec{E}$ is normal to curved Gaussian surface)

flux through surface (3)
$\phi_3 = 0$
(as E inside the conductor = 0)

So, $Eds = \dfrac{\sigma ds}{\varepsilon_0} \quad \Rightarrow \quad E = \dfrac{\sigma}{\varepsilon_0}$

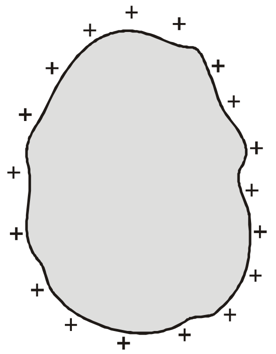
Electric field just outside the surface of conductor :

$E = \dfrac{\sigma}{\varepsilon_0}$ (direction will be normal to the surface)

in vector form: $\vec{E} = \dfrac{\sigma}{\varepsilon_0}\hat{n}$ (Here, $\hat{n}$ = unit vector normal to the conductor surface)

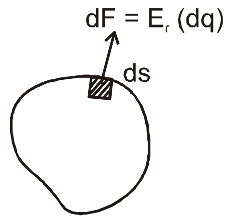
ELECTROSTATIC PRESSURE AT THE SURFACE OF THE CONDUCTOR :

Suppose a conductor is given some charge. Due to repulsion, all the charges will reach the surface of the conductor. But the charges will still repel each other. So an outward force will be felt by each charge due to others. Due to this force, there will be some pressure at the surface, which is called electrostatic pressure.



To find the electrostatic pressure, lets take a small surface element having Area 'ds'.

Force on this element due to the remaining charges :



$$dF = \begin{pmatrix} \text{electric field at} \\ \text{that place due to} \\ \text{remainig charges} \end{pmatrix} \begin{pmatrix} \text{charge of} \\ \text{the small} \\ \text{element} \end{pmatrix}$$

Let electric field at that point due to the remaining charges = E_r

and charge of the small element = dq = σds

$\Rightarrow$ dF = (E_r) (dq) = (E_r) (σds)

So, pressure on this small element

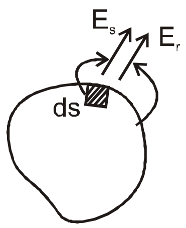
$$P = \frac{dF}{ds} = \frac{(E_r)(\sigma ds)}{ds} \Rightarrow P = (E_r)(\sigma) \qquad .....(1)$$

Now to find pressure, we have to find E_r (electric field at that position due to the remaining charges)

Suppose,

Electric field due to the small element near the surface = E_s

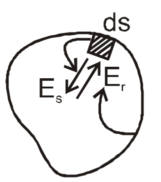
Electric field due to the remaining part near the surface = E_r



At a point just outside the surface, electric field due to the small element (E_s) will be normally out wards, and electric field due to the remaining part (E_r) will also be normally out wards.

So Net electric field just outside the surface = E_s + E_r and we have proved that electric field just outside the conductor surface = $\frac{\sigma}{\varepsilon_o}$

$$\Rightarrow \quad E_s + E_r = \frac{\sigma}{\varepsilon_o} \qquad .....(2)$$



Now, lets see the electric field just inside the metal surface. Here, electric field due to the remaining charges (E_r) will be in the same direction (normally outward), but the electric field due to the small element will be in opposite direction (normally inward)

So net electric field just inside the metal surface = E_r – E_s and we know that electric field inside a conductor = 0

So, $E_r - E_s = 0 \Rightarrow E_r = E_s$ .....(3)

from eqn. (2) and eqn. (3), we can say that :

$2E_r = \frac{\sigma}{\varepsilon_o} \quad \Rightarrow \quad E_r = \frac{\sigma}{2\varepsilon_o}$

Now, we can easily find the pressure from eqn.(1)

$$P = (E_r)(\sigma) = \frac{\sigma}{2\varepsilon_o}(\sigma) = \frac{\sigma^2}{2\varepsilon_o}$$

So, electrostatic pressure at the surface of the conductor $P = \frac{\sigma^2}{2\varepsilon_o}$

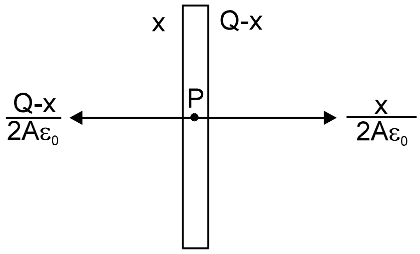
where, σ = local surface charge density.

Solved Examples

Example 109. Prove that if an isolated (isolated means no charges are near the sheet) large conducting sheet is given a charge then the charge distributes equally on its two surfaces.

Solution : Let there is x charge on left side of sheet and Q–x charge on right side of sheet.

Since, point P lies inside the conductor, so $E_P = O$



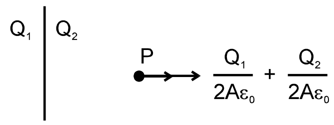
or $\frac{x}{2A\varepsilon_O} - \frac{Q-x}{2A\varepsilon_O} = 0 \quad \Rightarrow \quad \frac{2x}{2A\varepsilon_O} = \frac{Q}{2A\varepsilon_O}$

$\Rightarrow \; x = \frac{Q}{2} \;\; \& \;\; Q - x = \frac{Q}{2}$

So, charge is equally distributed on both sides.

Example 110. If an isolated infinite sheet contains charge Q_1 on its one surface and charge Q_2 on its other surface, then prove that electric field intensity at a point in front of sheet will be $\frac{Q}{2A\varepsilon_O}$, where $Q = Q_1 + Q_2$

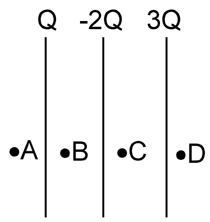
Solution : Electric field at point P :



$$\vec{E} = \vec{E}_{Q_1} + \vec{E}_{Q_2} = \frac{Q_1}{2A\varepsilon_0}\hat{n} + \frac{Q_2}{2A\varepsilon_0}\hat{n} = \frac{Q_1 + Q_2}{2A\varepsilon_0}\hat{n} = \frac{Q}{2A\varepsilon_0}\hat{n}$$

[This shows that the resultant field due to a sheet depends only on the total charge of the sheet and not on the distribution of charge on individual surfaces].

Example 111. Three large conducting sheets placed parallel to each other at finite distance contain charges Q, –2Q and 3Q respectively. Find electric field at points A, B, C and D



Solution : (i) $E_A = E_Q + E_{-2Q} + E_{3Q}$. Here E_Q means electric field due to 'Q'.

$$E_A = \frac{(Q-2Q+3Q)}{2A\varepsilon_0} = \frac{2Q}{2A\varepsilon_0} = \frac{Q}{A\varepsilon_0}, \text{ towards left}$$

(ii) $E_B = \dfrac{Q-(-2Q+3Q)}{2A\varepsilon_0} = 0$

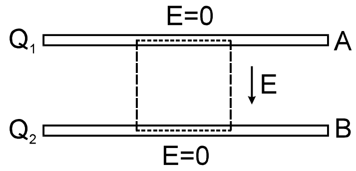
(iii) $E_C = \dfrac{(Q-2Q)-(3Q)}{2A\varepsilon_0} = \dfrac{-4Q}{2A\varepsilon_0} = \dfrac{-2Q}{A\varepsilon_0}$, towards right

$\Rightarrow \dfrac{2Q}{A\varepsilon_0}$ towards left

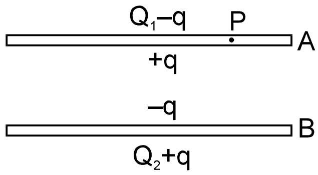
(iv) $E_D = \dfrac{(Q-2Q+3Q)}{2A\varepsilon_0} = \dfrac{2Q}{2A\varepsilon_0} = \dfrac{Q}{A\varepsilon_0}$, towards right

Example 112. Two conducting plates A and B are placed parallel to each other. A is given a charge Q_1 and B a charge Q_2. Prove that the charges on the inner facing surfaces are of equal magnitude and opposite sign.

Solution : Consider a Gaussian surface as shown in figure. Two faces of this closed surface lie completely inside the conductor where the electric field is zero. The flux through these faces is, therefore, zero. The other parts of the closed surface which are outside the conductor are parallel to the electric field and hence the flux on these parts is also zero. The total flux of the electric field through the closed surface is, therefore zero. From Gauss's law, the total charge inside this closed surface should be zero. The charge on the inner surface of A should be equal and opposite to that on the inner surface of B.



The distribution should be like the one shown in figure. To find the value of q, consider the field at a point P inside the plate A. Suppose, the surface area of the plate (one side) is A. Using the equation, $E = \sigma / (2\varepsilon_0)$, the electric field at P



Due to the charge $Q_1 - q = \dfrac{Q_1 - q}{2A\varepsilon_0}$ (downward)

Due to the charge $+ q = \dfrac{q}{2A\varepsilon_0}$ (upward),

Due to the charge $- q = \dfrac{q}{2A\varepsilon_0}$ (downward),

and due to the charge $Q_2 + q = \dfrac{Q_2 + q}{2A\varepsilon_0}$ (upward).

The net electric field at P due to all the four charged surfaces is (in the downward direction)

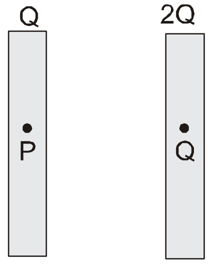
$$E_p = \frac{Q_1 - q}{2A\varepsilon_0} - \frac{q}{2A\varepsilon_0} + \frac{q}{2A\varepsilon_0} - \frac{Q_2 + q}{2A\varepsilon_0}$$

As the point P is inside the conductor, this field should be zero. Hence,

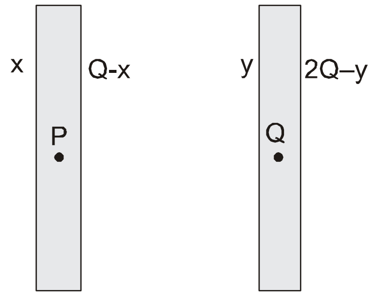
$Q_1 - q - q + q - Q_2 - q = 0$ or, $q = \dfrac{Q_1 - Q_2}{2}$

This result is a special case of the following result. When charged conducting plates are placed parallel to each other, the two outermost surfaces get equal charges and the facing surfaces get equal and opposite charges.

Example 113. Two large parallel conducting sheets (placed at finite distance) are given charges Q and 2Q respectively. Find out charges appearing on all the surfaces.



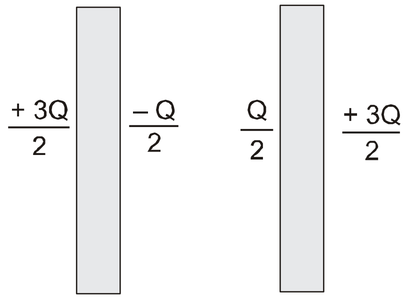
Solution : Let there is x amount of charge on left side of first plate. So, on its right side charge will be Q–x. Similarly, for second plate there is y charge on left side and 2Q – y charge is on right side,



$E_p = 0$ (By property of conductor)

$$\Rightarrow \quad \frac{x}{2A\varepsilon_o} - \left\{\frac{Q-x}{2A\varepsilon_o} + \frac{y}{2A\varepsilon_o} + \frac{2Q-y}{2A\varepsilon_o}\right\} = 0$$

We can also say that charge on left side of P = charge on right side of P



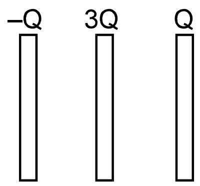
$$x = Q - x + y + 2Q - y \quad \Rightarrow \quad x = \frac{3Q}{2},\ Q - x = \frac{-Q}{2}$$

Similarly, for point Q : $x + Q - x + y = 2Q - y$

$\Rightarrow \quad y = Q/2,\ 2Q - y = 3Q/2$

So, final charge distribution of plates is

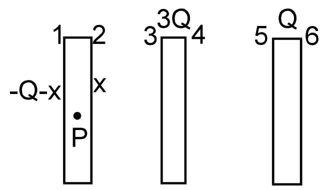
Example 114. Figure shows three large metallic plates with charges – Q, 3Q and Q respectively. Determine the final charges on all the surfaces.



Solution : We assume that charge on surface 2 is x. Following conservation of charge, we see that surfaces 1 has charge (– Q – x). The electric field inside the metal plate is zero. So, field at P is zero.

Resultant field at P

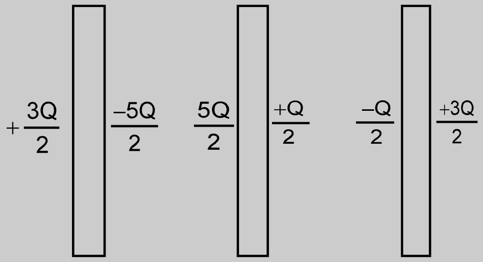
$E_P = 0$



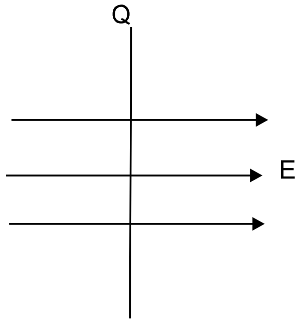
$$\Rightarrow \quad \frac{-Q-x}{2A\varepsilon_0} = \frac{x+3Q+Q}{2A\varepsilon_0} \quad \text{or} \quad -Q-x = x+4Q$$

$$\therefore \quad x = \frac{-5Q}{2}$$

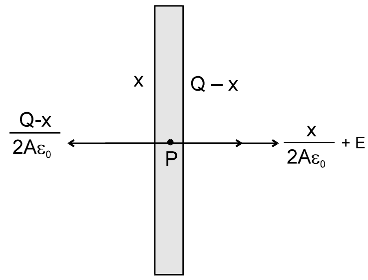
Note : We see that charges on the facing surfaces of the plates are of equal magnitude and opposite sign. This can be in general proved by Gauss theorem also. Remember this, it is important result. Thus the final charge distribution on all the surfaces is as shown in figure :



Example 115. An isolated conducting sheet of area A and carrying a charge Q is placed in a uniform electric field E, such that electric field is perpendicular to sheet and covers all the sheet. Find out charges appearing on its two surfaces.



Solution : Let there is x charge on left side of plate and Q – x charge on right side of plate



$\because$ $E_P = 0$

$\therefore$ $\frac{x}{2A\varepsilon_0} + E = \frac{Q-x}{2A\varepsilon_0}$ or $\frac{x}{A\varepsilon_0} = \frac{Q}{2A\varepsilon_0} - E$

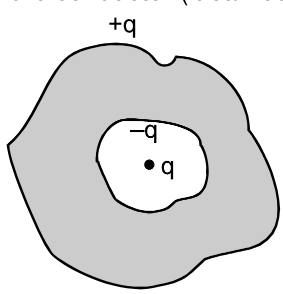
$\therefore$ $x = \frac{Q}{2} - EA\varepsilon_0$ and $Q - x = \frac{Q}{2} + EA\varepsilon_0$

So, charge on one side is $\frac{Q}{2}$ – $EA\varepsilon_0$ and other side $\frac{Q}{2}$ + $EA\varepsilon_0$

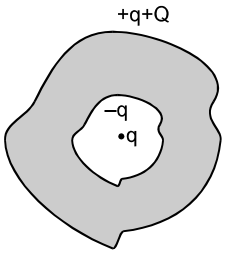
Note : Solve this question for Q = 0 without using the above answer and match that answer with the answer that you will get by putting Q = 0 in the above answer.

17.1 Some other important results for a closed conductor:

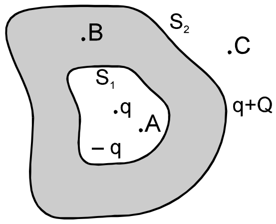
(i) If a charge q is kept in the cavity then –q will be induced on the inner surface and +q will be induced on the outer surface of the conductor (it can be proved using Gauss theorem)



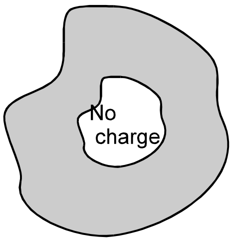
(ii) If a charge q is kept inside the cavity of a conductor and conductor is given a charge Q then –q charge will be induced on inner surface and total charge on the outer surface will be q + Q. (it can be proved using Gauss theorem)



(iii) Resultant field, due to q (which is inside the cavity) and induced charge on S_1, at any point outside S_1 (like B, C) is zero. Resultant field due to **q + Q on S_2 and any other charge outside S_2**, at any point inside of surface S_2 (like A, B) is zero

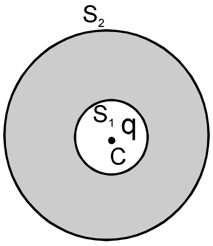


(iv) Resultant field in a charge free cavity in a closed conductor is zero. There can be charges outside the conductor and on the surface also. Then also, this result is true. No charge will be induced on the inner most surface of the conductor.



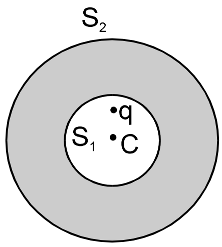
(v) Charge distribution for different types of cavities in conductors

(A)



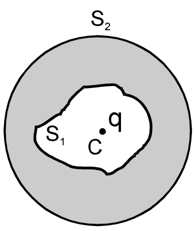
charge is at the common centre
(S_1, S_2 $\rightarrow$ spherical)

(B)



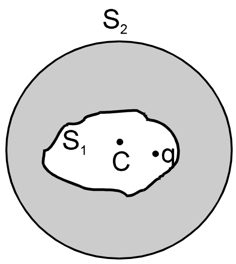
charge is not at the common centre
(S_1, S_2 $\rightarrow$ spherical)

(C)



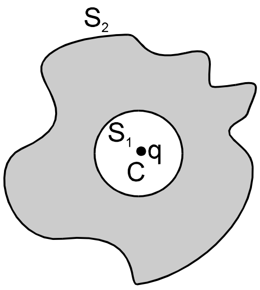
charge is at the centre of S_2
(S_2 $\rightarrow$ spherical)

(D)



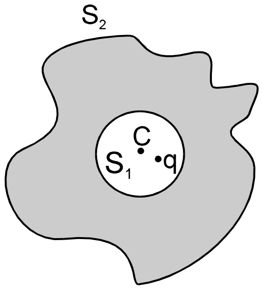
charge is not at the centre of S_2
(S_2 $\rightarrow$ spherical)

(E)



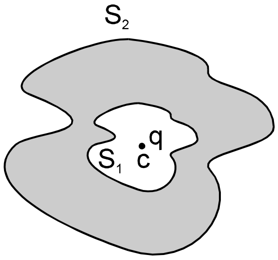
charge is at the centre of S_1 (Spherical)

(F)



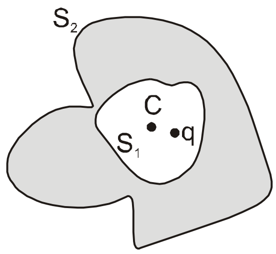
charge not at the centre of S_1(Spherical)

(G)



charge is at the geometrical centre

(H)



charge is not at the geometrical centre

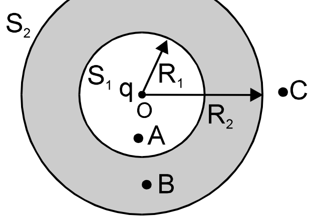
Using the result that $\vec{E}_{res}$ in the conducting material should be zero and using result (iii) we can show that

Case	A	B	C	D	E	F	G	H
S_1	Uniform	Nonuniform	Nonuniform	Nonuniform	Uniform	Nonuniform	Nonuniform	Nonuniform
S_2	Uniform	Uniform	Uniform	Uniform	Nonuniform	Nonuniform	Nonuniform	NonUniform

Note : In all cases, charge on inner surface S_1 = –q and on outer surface S_2 = q. The distribution of charge on 'S_1' will not change even if some charges are kept outside the conductor (i.e. outside the surface S_2). But the charge distribution on 'S_2' may change if some charges(s) is/are kept outside the conductor.

Solved Examples

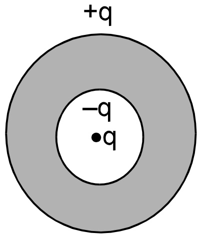
Example 116. An uncharged conductor of inner radius R_1 and outer radius R_2 contains a point charge q at the centre as shown in figure



(i) Find $\vec{E}$ and V at points A, B and C

(ii) If a point charge Q is kept outside the sphere at a distance 'r' (>>R_2) from centre, then find out resultant force on charge Q and charge q.

Solution : At point A :



$$V_A = \frac{Kq}{OA} + \frac{Kq}{R_2} + \frac{K(-q)}{R_1}\ ,\ \vec{E}_A = \frac{Kq}{OA^3}\overrightarrow{OA}$$

Note : Electric field at 'A' due to –q of S_1 and +q of S_2 is zero individually because they are uniformly distributed

At point B : $V_B = \frac{Kq}{OB} + \frac{K(-q)}{OB} + \frac{Kq}{R_2} = \frac{Kq}{R_2}$, $E_B = 0$

At point C : $V_C = \frac{Kq}{OC}$, $\vec{E}_C = \frac{Kq}{OC^3}\overrightarrow{OC}$

(ii) Force on point charge Q :

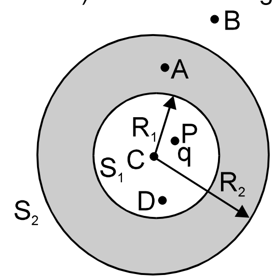
(**Note :** Here, force on 'Q' will be only due to 'q' of S_2 (see result (iii))

$\vec{F}_Q = \frac{KqQ}{r^2}\hat{r}$ (r = distance of 'Q' from centre 'O')

Force on point charge q:

$\vec{F}_q = 0$ (using result (iii) & charge on S_1 uniform)

Example 117. An uncharged conductor of inner radius R_1 and outer radius R_2 contains a point charge q placed at point P (not at the centre) as shown in figure. Find out the following :



(i) V_C (ii) V_A (iii) V_B (iv) E_A

(v) E_B (vi) Force on charge Q, if it is placed at B.

Solution : (i) $V_C = \frac{Kq}{CP} + \frac{K(-q)}{R_1} + \frac{Kq}{R_2}$

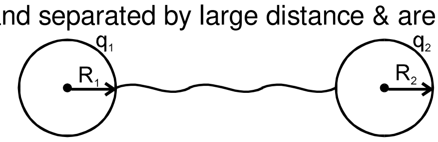
Note : –q on S_1 is non-uniformly distributed. Still it produces potential $\frac{K(-q)}{R_1}$ at 'C' because 'C' is at distance 'R_1' from each point of 'S_1'.

(ii) $V_A = \frac{Kq}{R_2}$ (iii) $V_B = \frac{Kq}{CB}$

(iv) E_A = O (point is inside metallic conductor)

(v) $E_B = \frac{Kq}{CB^2}\hat{CB}$ (vi) $F_Q = \frac{KQq}{CB^2}\hat{CB}$

(vi) Sharing of charges : Two conducting hollow spherical shells of radii R_1 and R_2 having charges Q_1 and Q_2 respectively and separated by large distance & are joined by a conducting wire



Let final charges on spheres are q_1 and q_2 respectively. Potential on both spherical shell becomes equal after joining. Therefore,

$$\frac{Kq_1}{R_1} = \frac{Kq_2}{R_2} \Rightarrow \frac{q_1}{q_2} = \frac{R_1}{R_2} \quad .....(i)$$

and, $q_1 + q_2 = Q_1 + Q_2$ .....(ii)

from (i) and (ii) : $q_1 = \frac{(Q_1+Q_2)R_1}{R_1+R_2}$ $q_2 = \frac{(Q_1+Q_2)R_2}{R_1+R_2}$

ratio of charges : $\frac{q_1}{q_2} = \frac{R_1}{R_2} \Rightarrow \frac{\sigma_1 4\pi R_1^2}{\sigma_2 4\pi R_2^2} = \frac{R_1}{R_2}$

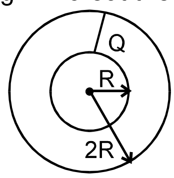
$\therefore$ ratio of surface charge densities : $\frac{\sigma_1}{\sigma_2} = \frac{R_2}{R_1}$

Ratio of final charges : $\frac{q_1}{q_2} = \frac{R_1}{R_2}$

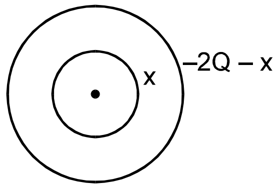
Ratio of final surface charge densities : $\frac{\sigma_1}{\sigma_2} = \frac{R_2}{R_1}$

Solved Example

Example 118. The two conducting spherical shells are joined by a conducting wire which is cut after some time when charge stops flowing. Find out the charge on each sphere after that.



Solution : After cutting the wire, the potential of both the shells is equal



Thus, potential of inner shell, $V_{in} = \frac{Kx}{R} + \frac{K(-2Q-x)}{2R} = \frac{k(x-2Q)}{2R}$

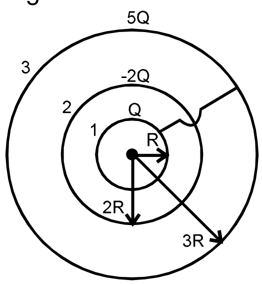
and potential of outer shell, $V_{out} = \frac{Kx}{2R} + \frac{K(-2Q-x)}{2R} = \frac{-KQ}{R}$

As, $V_{out} = V_{in}$

$\Rightarrow \quad \frac{-KQ}{R} = \frac{K(x-2Q)}{2R} \quad \Rightarrow \quad -2Q = x - 2Q \quad \Rightarrow \quad x = 0$

So, charge on inner spherical shell = 0 and outer spherical shell = – 2Q.

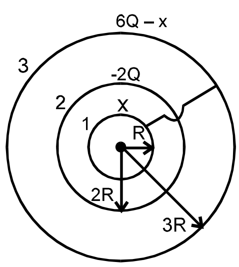
Example 119. Find charge on each spherical shell after joining the inner most shell and outer most shell by a conducting wire. Also find charges on each surface.



Solution : Let the charge on the innermost sphere be x.

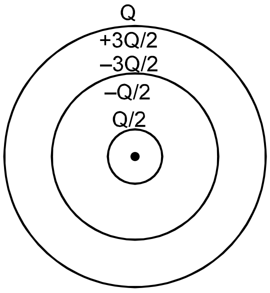
Finally potential of shell 1 = Potential of shell 3

$$\therefore \quad \frac{Kx}{R} + \frac{K(-2Q)}{2R} + \frac{K(6Q-x)}{3R} = \frac{Kx}{3R} + \frac{K(-2Q)}{3R} + \frac{K(6Q-x)}{3R}$$



$3x - 3Q + 6Q - x = 4Q$; $2x = Q$; $x = \frac{Q}{2}$

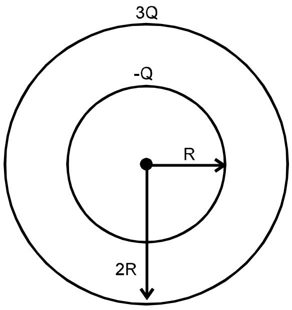
$\therefore$ Charge on innermost shell = $\frac{Q}{2}$ & Charge on outermost shell = $\frac{5Q}{2}$



Charge on middle shell = –2Q

$\therefore$ Final charge distribution is as shown in figure.

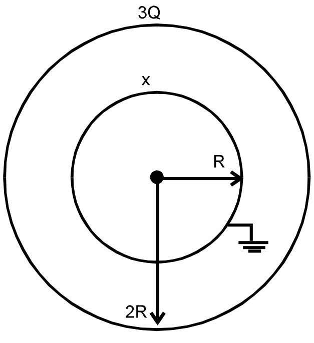
Example 120. Two conducting hollow spherical shells of radii R and 2R carry charges – Q and 3Q respectively. How much charge will flow into the earth if inner shell is grounded ?



Solution : When inner shell is grounded to the Earth then the potential of inner shell will become zero because potential of the Earth is taken to be zero.

$$\frac{Kx}{R} + \frac{K3Q}{2R} = 0$$

or $x = \frac{-3Q}{2}$, (the charge that has appeared on inner shells after grounding)



$\Rightarrow$ $\frac{-3Q}{2} - (-Q) = \frac{-Q}{2}$ [hence, charge flown into the Earth = $\frac{Q}{2}$]

Example 121. An isolated conducting sphere of charge Q and radius R is connected to a similar uncharged sphere (kept at a large distance) by using a high resistance wire. After a long time, what is the amount of heat loss ?

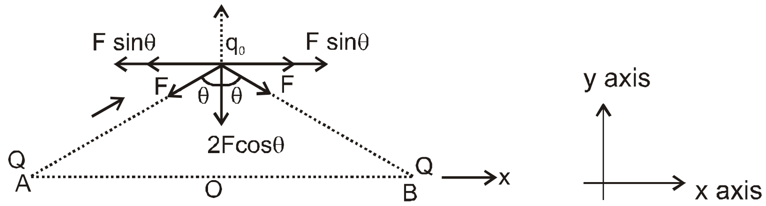
Solution : When two conducting spheres of equal radii are connected, charge is equally distributed on them (Result VI). So, we can say that heat loss of system :

$$\Delta H = U_i - U_f = \left(\frac{Q^2}{8\pi\varepsilon_0 R} - 0\right) - \left(\frac{Q^2/4}{8\pi\varepsilon_0 R} + \frac{Q^2/4}{8\pi\varepsilon_0 R}\right) = \frac{Q^2}{16\pi\varepsilon_0 R}$$

Solved Miscellaneous Problems

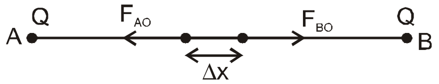
Problem 1. Two equal positive point charges 'Q' each are fixed at points B(a, 0) and A(–a, 0). Another negative point charge q_0 is also placed at O(0, 0) then prove that the equilibrium at 'O' is
(i) Stable for displacement in Y-direction.
(ii) Unstable for displacement in X-direction.

Solution : (i) When charge is shifted along y-axis:
Let x-y direction as :



After resolving into components, net force will be along negative y-axis so the particle will return to its original position. So, it is stable equilibrium

(ii) When negative charge q_0 is shifted along x-axis.



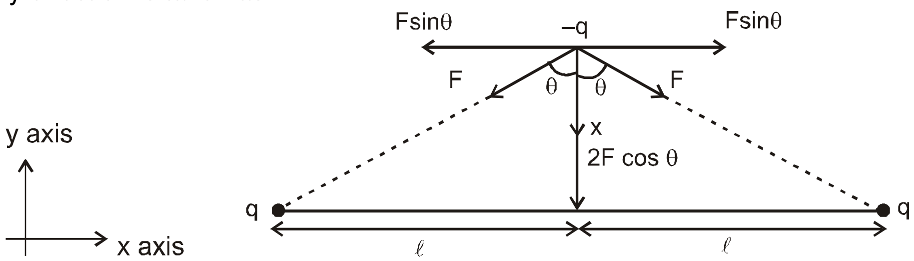
Initially, $\vec{F}_{AO} + \vec{F}_{BO} = \vec{0}$ $\Rightarrow$ $\left|\vec{F}_{AO}\right| = \left|\vec{F}_{BO}\right| = \dfrac{KQq_0}{d^2}$

When charge q_0 is slightly shifted towards + x axis by small distance Δx then $\left|\vec{F}_{BO}\right| > \left|\vec{F}_{AO}\right|$

Also, these forces are attractive forces (due to negative charge)
Therefore, the particle will move towards positive x-axis and will not return to its original position So, it is unstable equilibrium for negative charge.

Problem 2. A particle of mass m and charge –q is located midway between two fixed charged particles each having a charge q and a distance 2ℓ apart. Prove that the motion of the particle will be SHM if it is displaced slightly along perpendicular bisector and released. Also find its time period.

Solution : Let x-y direction is taken as :



Particle is shifted along y-axis by a small displacement x.
After resolving component of forces between q and –q charges :
By figure. F_{net} in x-axis = 0 [F_{net} = net force on –q charge]

Net force on – q charge in y direction = –2F cosθ = $-2.\ \dfrac{kqq}{(x^2+\ell^2)} \cdot \dfrac{x}{(x^2+\ell^2)^{1/2}}$

$|\vec{F}| = \dfrac{2Kq^2x}{(x^2+\ell^2)^{3/2}}$ $\Rightarrow$ $ma = \dfrac{2Kq^2x}{\ell^3}$ (for $x << \ell$) (a = acceleration of – q charge)

$\Rightarrow$ $a = \dfrac{2Kq^2}{m\ell^3}.\ x$ (downwards)

This is equation of S.H.M. ($a = -\omega^2 x$)
So, time period of this charge (–q) :

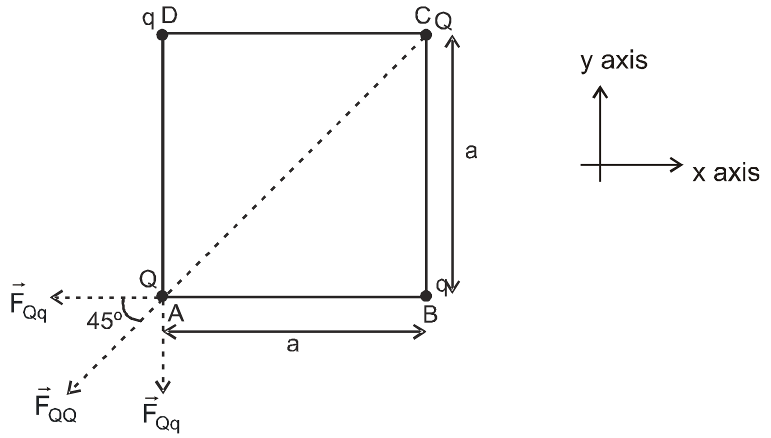
$T = 2\pi\sqrt{\dfrac{m\ell^3}{2Kq^2}}$ **Ans.**

Problem 3. Two charges Q each, are placed at two opposite corners of a square. A charge q is placed at each of the other two corners.

(a) If the resultant force on Q is zero, how are Q and q related ?

(b) Could q be chosen to make the resultant force on each charge zero ?

Solution : (a) Let on a square ABCD, charges are placed as shown



Now, forces on charge Q (at point A) due to other charge are $\vec{F}_{QQ}$, $\vec{F}_{Qq}$ and $\vec{F}_{Qq}$ respectively as shown in figure.

F_{net} on Q $= \vec{F}_{Q.Q} + \vec{F}_{Qq} + \vec{F}_{Qq}$ (at point A)

But $F_{net} = 0$

So, $\Sigma F_x = 0$

$\Sigma F_x = -F_{QQ} \cos 45° - F_{Qq}$

$$\Rightarrow \frac{KQ^2}{(\sqrt{2}a)^2} \cdot \frac{1}{\sqrt{2}} + \frac{KQq}{a^2} = 0 \quad \Rightarrow \quad q = -\frac{Q}{2\sqrt{2}} \quad \textbf{Ans.}$$

(b) For resultant force on each charge to be zero :

From previous data, force on charge Q is zero when $q = -\frac{Q}{2\sqrt{2}}$ If for this value of charge q, force on q is zero, then and only then the value of q exists for which the resultant force on each charge is zero.

Force on q :

Forces on charge q (at point D) due to other three charges are $\vec{F}_{qQ}$, $\vec{F}_{qq}$ and $\vec{F}_{qQ}$ respectively as shown in figure.

Net force on charge q :

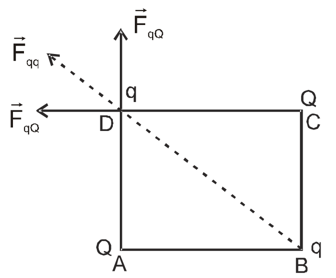
$\vec{F}_{net} = \vec{F}_{qq} + \vec{F}_{qQ} + \vec{F}_{qQ}$ But $\vec{F}_{net} = 0$

So, $\Sigma F_x = 0$

$$\Sigma F_x = -\frac{Kq^2}{(\sqrt{2}a)^2} \cdot \frac{1}{\sqrt{2}} - \frac{KQq}{(a)^2} \quad \Rightarrow \quad q = 2\sqrt{2} - Q$$

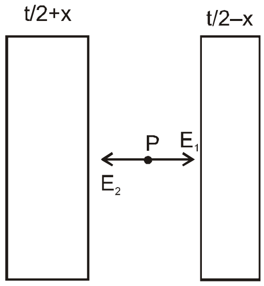
But from previous condition, $q = -\frac{Q}{2\sqrt{2}}$

So, no value of q makes the resultant force on each charge zero.



Problem 4. An infinitely large non-conducting sheet of thickness t and uniform volume charge density ρ is given in which left half of the sheet contains charge density ρ and right half contains charge density. Find the electric field at the symmetry plane of this sheet

Solution : We can consider two sheets of thickness $\left(\frac{t}{2}-x\right)$ and $\left(\frac{t}{2}+x\right)$



When a point lies inside the sheet.

Net electric field at point P : $E = E_1 - E_2 = \frac{Q_1}{2A\varepsilon_0} - \frac{Q_2}{2A\varepsilon_0}$

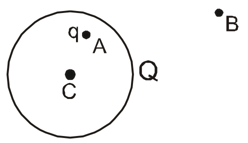
[Q_1 : charge on left sheet; Q_2 = charge of right sheet]

$$= \frac{A\rho\left(\frac{t}{2}+x\right) - 2\rho A\left(\frac{t}{2}-x\right)}{2A\varepsilon_0} = \frac{\rho\left[3x - \frac{t}{2}\right]}{2\varepsilon_0}$$

At the symmetry plane, x = 0 So, $E = -\frac{\rho t}{4\varepsilon_0}$ **Ans.**

Problem 5. Figure shows a uniformly charged thin non-conducting sphere of total charge Q and radius R. If point charge q is situated at point 'A' which is at a distance r < R from the centre of the sphere, then find out following:

(i) Force acting on charge q.
(ii) Electric field at centre of sphere.
(iii) Electric field at point B.

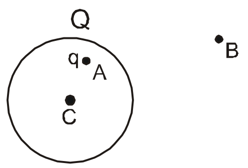


Solution : (i) Electric field inside a hollow sphere = 0

$\therefore$ Force on charge q.

F = qE = q × 0 = 0

(ii) Net electric field at centre of sphere



$\vec{E}_{net} = \vec{E}_1 + \vec{E}_2$

E_1 = field due to sphere = 0

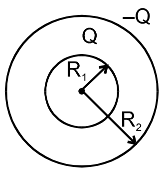
E_2 = field due to this charge = $\frac{Kq}{r^2}$

$\therefore$ $E_{net} = \frac{Kq}{r^2}$

(iii) Electric field at B due to charge on sphere, $\vec{E}_1 = \frac{KQ}{r_1^2}\hat{r}_1$ and due to charge q at A, $\vec{E}_2 = \frac{Kq}{r_2^2}\hat{r}_2$

So, $\vec{E}_{net} = \vec{E}_1 + \vec{E}_2 = \frac{KQ}{r_1^2}\hat{r}_1 + \frac{Kq}{r_2^2}\hat{r}_2$ where r_1 = CB and r_2 = AB

Problem 6. Figure shows two concentric spheres of radii R_1 and R_2 ($R_2 > R_1$) which contain uniformly distributed charges Q and –Q respectively. Find out electric field intensities at the following positions :

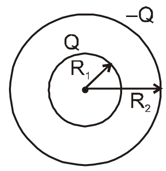


(i) $r < R_1$ (ii) $R_1 \le r < R_2$ (iii) $r \ge R_2$

Solution : Net electric field = $E_1 + E_2$

E_1 = field due to sphere of radius R_1

E_2 = field due of sphere of radius R_2



(i) $E_1 = 0, E_2 = 0$ $\therefore\ E_{net} = 0$

(ii) $E_1 = \frac{KQ}{r^2}, E_2 = 0$ $\Rightarrow\ \vec{E} = \frac{Kq}{r^2}\hat{r}$

(iii) $\vec{E}_1 = \frac{Kq}{r^2}\hat{r}\ \vec{E}_2 = \frac{Kq}{r^2}(-\hat{r})$ $\Rightarrow\ \vec{E}_{net} = \vec{E}_1 + \vec{E}_2 = 0$

Problem 7. A solid non conducting sphere of radius R and uniform volume charge density ρ has centre at origin. Find out electric field intensity in vector form at following positions :

(i) (R, 0, 0) (ii) (0, 0, R/2) (iii) (R, R, R)

Solution : For uniformly charged non-conducting sphere, electric field inside the sphere :

$$\vec{E} = k\frac{Q\vec{r}}{R^3} = \frac{\rho\vec{r}}{3\varepsilon_o} \text{ (for } r < R)$$

and electric field outside the sphere

$$\vec{E}_o = \frac{KQ}{r^2}.\hat{r} = \frac{1}{4\pi\varepsilon_o}.\frac{\rho\frac{4}{3}\pi R^3}{r^2}\hat{r} = \frac{\rho R^3}{3\varepsilon_o r^2}.\hat{r} \quad (\text{for } r \geq R)$$

(i) (R, 0, 0) means it is at the surface $\vec{r} = R\hat{r}$ and $\hat{r} = \hat{i}$

$$\therefore\ \vec{E}_o = \frac{\rho R^3}{3\varepsilon_o R^2}(\hat{i}) = \frac{\rho R}{3\varepsilon_o}.\hat{i}$$

(ii) $(0, 0, \frac{R}{2})$

means point is inside the sphere

$$\vec{r} = \frac{R}{2}\hat{k} \quad \Rightarrow \quad \vec{E} = \frac{\rho R}{6\varepsilon_o}\hat{k}$$

(iii) For position (R, R, R)

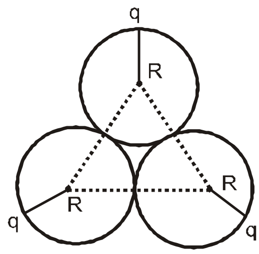
$$\vec{r} = R(\hat{i} + \hat{j} + \hat{k}) \Rightarrow \quad \hat{r} = \frac{\hat{i}+\hat{j}+\hat{k}}{\sqrt{3}},\ r = R\sqrt{3}$$

means point (R,R,R) is outside the sphere

$$\therefore\ \vec{E} = \frac{\rho R^3}{3\varepsilon_o(3R^2)}.\frac{(\hat{i}+\hat{j}+\hat{k})}{\sqrt{3}} = \frac{\rho R}{9\sqrt{3}\varepsilon_o}(\hat{i}+\hat{j}+\hat{k})$$ **Ans.**

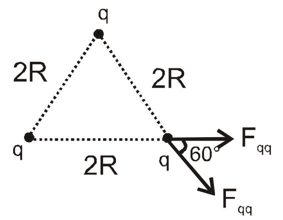
Problem 8. Three identical spheres each having a charge q (uniformly distributed) and radius R, are kept in such a way that each touches the other two. Find the magnitude of the electric force on any one sphere due to other two.

Solution : Given three identical spheres each having a charge q and radius R are kept as shown.



For any external point, sphere behaves like a point charge. So it becomes a triangle having point charges at its corners.

$$|\vec{F}_{qq}| = \frac{kq^2}{4R^2}$$



So, net force (F) = $2.\ \frac{kq^2}{4R^2}.\cos\frac{60}{2} = 2.\ \frac{kq^2}{4R^2}\ \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{4}.\ \frac{kq^2}{R^2}$ **Ans.**

Problem 9. A uniform electric field of 20 N/C exists in the vertically downward direction. Find the increase in the electric potential as one goes up through a height of 40cm.

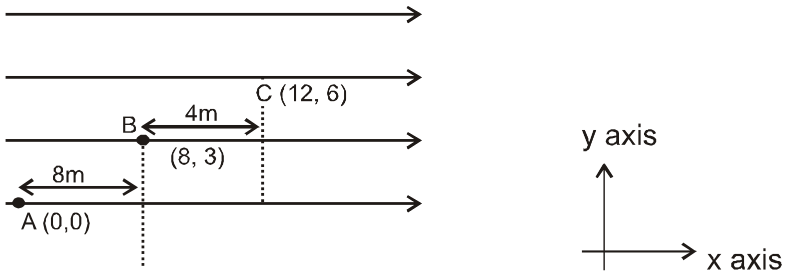
Solution : $E = -\frac{dv}{dr} \Rightarrow dv = -\vec{E}.d\vec{r}$

for $\vec{E}$ = constant $\Rightarrow$ $\Delta V = -\vec{E}.\vec{\Delta r}$

$\Delta V = -20(-\hat{j}).(40 \times 10^{-2})\hat{j} = 8$ volts.

Problem 10. An electric field of 10 N/C exists along the x-axis in space. Calculate the potential difference $V_B - V_A$, where the points A and B are given by –
(a) A = (0,0) ; B = (8m , 3m) (b) A = (8m, 3m) ; B = (12m , 6m)
(c) A = (0,0) ; B = (12m , 6m)

Solution : Electric field in x - axis means $\vec{E} = 10\hat{i}$

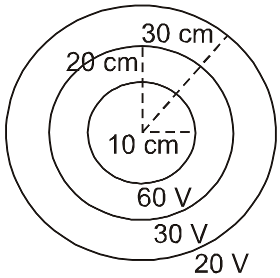


(a) $|\Delta V_{AB}| = \vec{E}.\vec{d} = 10\hat{i}\ .\ 8\ \hat{i} = 80$ V $\Rightarrow$ $V_B - V_A = -80V$

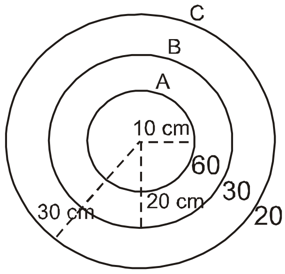
(b) $|\Delta V_{BC}| = \vec{E}.\vec{d} = 10\hat{i}\ .\ 4\hat{i} = 40$ volt $\Rightarrow$ $V_C - V_B = -40V$

(c) $|\Delta V_{AC}| = \vec{E}.\vec{d} = 10\hat{i}\ .\ 12\hat{i} = 120$ volt $\Rightarrow$ $V_C - V_A = -120$ V

Problem 11. Some equi-potential surfaces are shown in figure. What can you say about the magnitude and the direction of the electric field ?



Solution : We know, that the electric field is always perpendicular to equipotential surface. So, making electric field lines perpendicular to the surface, we find that these lines are originating from the centre. So, the field is similar to that due to a point charge placed at the centre. So, comparing the given potentials with that due to point charge, we have,



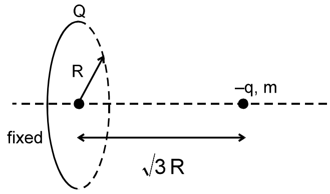
$V = \dfrac{KQ}{r} \quad \Rightarrow \quad KQ = V_A r_A = V_B r_B = V_C r_C = 6$ V-m

Hence, electric field at distance r can be given by

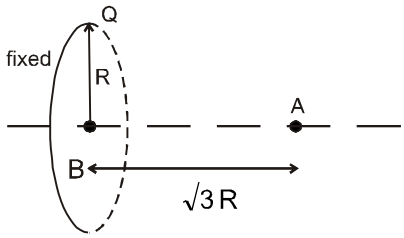
$E = \dfrac{KQ}{r^2} = \dfrac{6}{r^2}$ V/m

As the electric field lines are directed towards the decreasing potential. So, electric field is along radially outward direction.

Problem 12. A point charge of charge –q and mass m is released with negligible speed from a distance $\sqrt{3}R$ on the axis of a fixed uniformly charged ring of charge Q and radius R. Find out its velocity when it reaches at the centre of the ring.



Solution : As potential due to uniformly charged ring at its axis (at x distance) is :



$V = \dfrac{kQ}{\sqrt{R^2 + x^2}}$;

So, potential at point A due to ring

$V_1 = \dfrac{kQ}{\sqrt{R^2 + 3R^2}} = \dfrac{kQ}{2R}$

So potential energy of charge –q at point A

$P.E._1 = \dfrac{-kQq}{2R}$ and potential at point B, $V_2 = \dfrac{kQ}{R}$

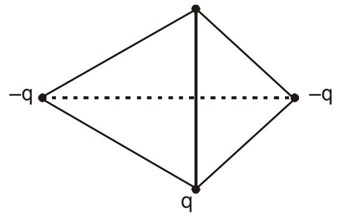
So, potential energy of charge –q at point B : $P.E._2 = \dfrac{-kQq}{R}$

Now by energy conservation : $P.E._1 + K.E._1 = P.E_2 + K.E_2$

$\dfrac{-kQq}{2R} + 0 = \dfrac{-kQq}{R} + \dfrac{1}{2}mv^2 \quad \Rightarrow \quad v^2 = \dfrac{kQq}{mR}$

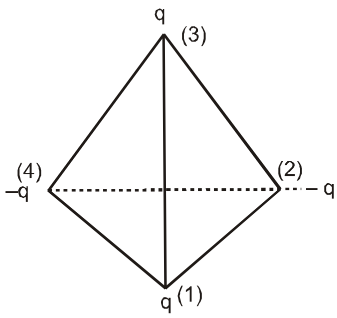
So velocity of charge – q at point B $\quad v = \sqrt{\dfrac{kQq}{mR}}$ **Ans.**

Problem 13. Four small point charges (each of equal magnitude q) are placed at four corners of a regular tetrahedron of side a. Find out potential energy of charge system



Solution : Potential energy of system : $U = U_{12} + U_{13} + U_{14} + U_{23} + U_{24} + U_{34}$

$$\therefore \quad U = \frac{-kq^2}{a} + \frac{kq^2}{a} + \frac{-kq^2}{a} + \frac{-kq^2}{a} + \frac{kq^2}{a} + \frac{-kq^2}{a}$$



Total potential energy of this charge system $U = \frac{-2kq^2}{a}$

Problem 14. If $V = x^2y + y^2z$ then find $\vec{E}$ at (x, y, z)

Solution : Given $V = x^2y + y^2z$ and $\vec{E} = -\frac{\partial v}{\partial r}$

$$\vec{E} = -\left[\frac{\partial V}{\partial x}\hat{i} + \frac{\partial V}{\partial y}\hat{j} + \frac{\partial V}{\partial z}\hat{k}\right] \quad \Rightarrow \quad \vec{E} = -[2xy\,\hat{i} + (x^2 + 2yz)\,\hat{j} + y^2\hat{k}]$$

Problem 15. Magnitude of electric field depends only on the x – coordinate as $\vec{E} = \frac{20}{x^2}\hat{i}$ V/m. Find

(i) The potential difference between two points A (5m, 0) and B(10m, 0).

(ii) Potential at x = 5 if V at ∞ is 10 volt.

(iii) In part (i), does the potential difference between A and B depend on whether the potential at ∞ is 10 volt or something else.

Solution : Given, $\vec{E} = \frac{20}{x^2}\hat{i}$ V/m

We know that : $\int dV = -\int \vec{E}.\vec{dr} \quad \Rightarrow \quad \int_{V_1}^{V_2} dV = -\int_{x_1}^{x_2} E_x dx = -\int_{x_1}^{x_2} \frac{20}{x^2} dx$

$\therefore$ Potential difference, $\Delta V = \frac{20}{x}\Big|_{x_1}^{x_2} \quad \Rightarrow \quad V_2 - V_1 = \frac{20}{x_2} - \frac{20}{x_1}$

(i) Potential difference between point A and B (ΔV for A to B)

$V_B - V_A = \frac{20}{10} - \frac{20}{5} = -2$ volt

(ii) ΔV for $x = \infty$ to $x = 5$

$V_5 - V_\infty = \frac{20}{5} - \frac{20}{\infty} \quad \therefore \quad V_5 = 10 + 4 = 14$ volt

(iii) Potential difference between two points does not depend on reference value of potential. So, the potential difference between A and B does not depend on whether the potential at ∞ is 10 volt or something else.

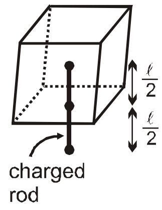
Problem 16. If $E = 2r^2$ then find V(r)

Solution : Given : $E = 2r^2$

we know that : $\int dv = -\int \vec{E}.\vec{dr} = -\int 2r^2\, dr \quad \Rightarrow \quad V(r) = \frac{-2r^3}{3} + c$ **Ans.**

Problem 17. A charge Q is uniformly distributed over a rod of length ℓ. Consider a hypothetical cube of edge ℓ with the centre of the cube at one end of the rod. Find the minimum possible flux of the electric field through the entire surface of the cube.

Solution : According to Gauss law : Flux depends upon charge inside the closed hypothetical surface. So, for minimum possible flux through the entire surface of the cube, charge inside it should be minimum.



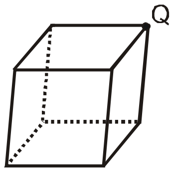
Linear charge density of rod = $\frac{Q}{\ell}$

and minimum length of rod inside the cube = $\frac{\ell}{2}$

So, charge inside the cube = $\frac{\ell}{2} \cdot \frac{Q}{\ell} = \frac{Q}{2}$

So, flux through the entire surface of the cube = $\frac{\Sigma q}{\epsilon_0} = \frac{Q}{2\,\epsilon_0}$

Problem 18. A charge Q is placed at a corner of a cube. Find the flux of the electric field through the six surfaces of the cube.

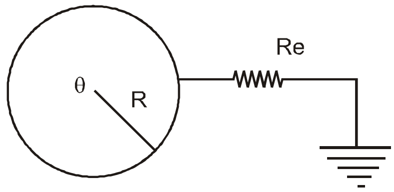


Solution : By Gauss's law, $\phi = \frac{q_{in}}{\varepsilon_0}$. Here, since Q is kept at the corner, so only $\frac{q}{8}$ charge is inside the cube. (Since, complete charge can be enclosed by 8 such cubes)

$\therefore \quad q_{in} = \frac{Q}{8}$ So, $\phi = \frac{q_{in}}{\varepsilon_0} = \frac{Q}{8\varepsilon_0}$ **Ans.**

Problem 19. An isolated conducting sphere of charge Q and radius R is grounded by using a high resistance wire. What is the amount of heat loss ?

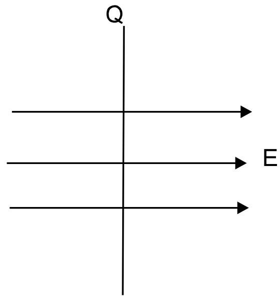
Solution : When sphere is grounded, it's potential become zero which means all charge goes to earth (since sphere is conducting and isolated)



So, all energy in sphere is converted into heat

So, total heat loss = $\frac{kQ^2}{2R}$

Problem 20. An isolated conducting sheet of area A and carrying a charge Q is placed in a uniform electric field E, such that electric field is perpendicular to sheet and covers all the sheet. Find charges appearing on left and right surfaces of the conducting sheet. Also find the resultant electric field on the left and right side of the plate.

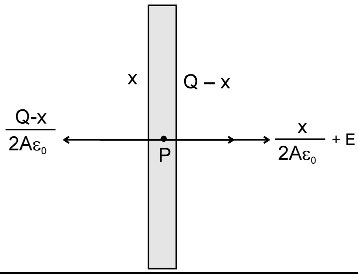


Solution : Let there is x charge on left side of plate and Q – x charge on right side of plate

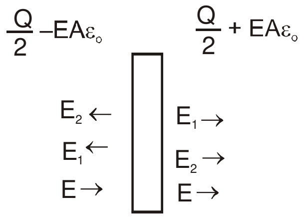
$\because \quad E_P = 0$

$\therefore \quad \frac{x}{2A\varepsilon_0} + E = \frac{Q-x}{2A\varepsilon_0}$ or $\frac{x}{A\varepsilon_0} = \frac{Q}{2A\varepsilon_0} - E$

$\therefore \quad x = \frac{Q}{2} - EA\varepsilon_0$ and $Q - x = \frac{Q}{2} + EA\varepsilon_0$



So, charge on one side is $\frac{Q}{2} - EA\varepsilon_0$ and other side $\frac{Q}{2} + EA\varepsilon_0$



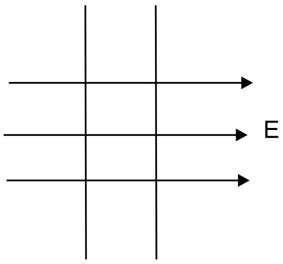
Electric field due to conducting sheet at a point outside the sheet $= \frac{Q_{net}}{2A\varepsilon_o} = \frac{Q}{2A\varepsilon_o}$ and having direction away from plate E_1 and E_2 are electric field due to charges on left and right side of the plate respectively.

Now,

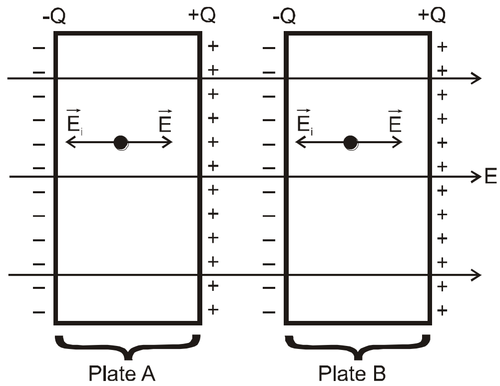
$$E_{left} = E - E_1 - E_2 = E - \frac{\left(\frac{Q}{2} - EA\varepsilon_o\right) + \left(\frac{Q}{2} + EA\varepsilon_o\right)}{2A\varepsilon_o} = E - \frac{Q}{2A\varepsilon_o} \text{ (towards right)}$$

$$E_{right} = E + E_1 + E_2 = E + \frac{Q}{2A\varepsilon_o} \text{ (towards right)}$$

Problem 21. Two uncharged and parallel conducting sheets, each of area A are placed in a uniform electric field E at a finite distance from each other, such that electric field is perpendicular to sheets and covers all the sheets. Find out charges appearing on its two surfaces.



Solution : Plates are conducting so net electric field inside these plates should be zero. So, electric field due to induced charges (on the surface of the plate) balance the outside electric field.



Here $\vec{E}_i$ = induced electric field

For both plates, $\vec{E}_i + \vec{E} = 0$

$\Rightarrow \quad \vec{E}_i = -\vec{E}$ .....(1)

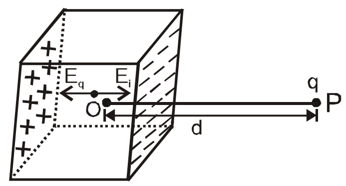
Let charge induced on surfaces are +Q and – Q, then $|\vec{E}_i| = \frac{Q}{A\varepsilon_0}$

By equation (1)

$\frac{Q}{A\varepsilon_0} = E \quad \Rightarrow \quad Q = AE\varepsilon_0$ **Ans.**

Problem 22. A positive charge q is placed in front of a conducting solid cube at a distance d from its centre. Find the electric field at the centre of the cube due to the charges appearing on its surface.

Solution : Here $\vec{E}_i = \vec{E}$ electric field due to induced charges and



E_q = electric field due to charge q

We know that net electric field in a conducting cavity is equal to zero.

i.e. $\vec{E} = \vec{0}$ at the centre of the cube.

$\Rightarrow \quad \vec{E}_i + \vec{E}_q = \vec{0}$

$\Rightarrow \quad \vec{E}_i = -\vec{E}_q \quad \Rightarrow \quad \vec{E}_i = -\frac{kq}{d^2}\overrightarrow{PO}$ **Ans.**

Exercise-1

✍ Marked Questions may have for Revision Questions.

PART - I : SUBJECTIVE QUESTIONS

SECTION (A) : PROPERTIES OF CHARGE AND COULOMB'S LAW

A-1. Two point charges $q_1 = 2 \times 10^{-3}$ C and $q_2 = -3 \times 10^{-6}$C are separated by a distance x = 10 cm. Find the magnitude and nature of the force between the two charges.

A-2. Two point charges $q_1 = 20\mu C$ and $q_2 = 25\mu C$ are placed at (–1, 1, 1) m and (3, 1, –2)m, with respect to a coordinate system. Find the magnitude and unit vector along electrostatic force on q_2?

A-3. 20 positively charged particles are kept fixed on the X-axis at points x = 1 m, 2 m, 3 m, ..., 20 m. The first particle has a charge 1.0×10^{-6} C, the second 8×10^{-6} C, the third 27×10^{-6} C and so on. Find the magnitude of the electric force acting on a 1 C charge placed at the origin.

A.4. (i) Two charged particles having charge 4.0×10^{-6} C and mass 24×10^{-3} Kg each are joined by an insulating string of length 1 m and the system is kept on a smooth horizontal table. Find the tension in the string.
(ii) If suddenly string is cut then what is the acceleration of each particle?
(iii) Are they having equal acceleration?

A-5.✍ Two identical conducting spheres (of negligible radius), having charges of opposite sign, attract each other with a force of 0.108 N when separated by 0.5 meter. The spheres are connected by a conducting wire, which is then removed (when charge stops flowing), and thereafter repel each other with a force of 0.036 N keeping the distance same. What were the initial charges on the spheres?

A-6. Two small spheres, each of mass 0.1 gm and carrying same charge 10^{-9} C are suspended by threads of equal length from the same point. If the distance between the centres of the sphere is 3 cm, then find out the angle made by the thread with the vertical. (g = 10 m/s^2) & $\tan^{-1}\left(\frac{1}{100}\right) = 0.6^{\circ}$

A-7. The distance between two fixed positive charges 4e and e is ℓ. How should a third charge 'q' be arranged for it to be in equilibrium? Under what condition will equilibrium of the charge 'q' be stable (for displacement on the line joining 4e and e) or will it be unstable?

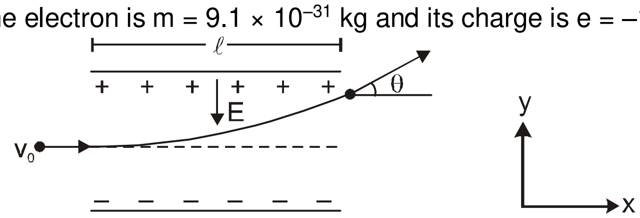
A-8.✍ Three charges, each of value q, are placed at the corners of an equilateral triangle. A fourth charge Q is placed at the centre O of the triangle.
(a) If Q = –q, will the charges at corners start to move towards centre or away from it.
(b) For what value of Q at O will the charges remain stationary?

A-9. Two charged particles A and B, each having a charge Q are placed a distance d apart. Where should a third particle of charge q be placed on the perpendicular bisector of AB so that it experiences maximum force? Also find the magnitude of the maximum force.

SECTION (B) : ELECTRIC FIELD

B-1. The electric force experienced by a charge of 5×10^{-6} C is 25×10^{-3} N. Find the magnitude of the electric field at that position of the charge due to the source charges.

B-2.✍ A uniform electric field E = 91×10^{-6} V/m is created between two parallel, charged plates as shown in figure. An electron enters the field symmetrically between the plates with a speed $v_0 = 4 \times 10^3$ m/s. The length of each plate is ℓ = 1m. Find the angle of deviation of the path of the electron as it comes out of the field. (Mass of the electron is m = 9.1×10^{-31} kg and its charge is e = -1.6×10^{-19} C).



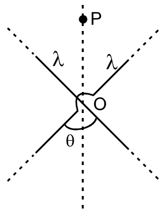
B-3. Two point particles A and B having charges of 4×10^{-6} C and -64×10^{-6} C respectively are held at a separation of 90 cm. Locate the point(s) on the line AB or on its extension where the electric field is zero.

B-4. Three point charges q_0 are placed at three corners of square of side a. Find out electric field intensity at the fourth corner.

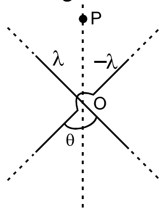
B-5. Two point charges $3\mu C$ and $2.5\ \mu C$ are placed at point A (1, 1, 2)m and B (0, 3, –1)m respectively. Find out electric field intensity at point C(3, 3, 3)m.

B-6. A hollow sphere of radius a carries a total charge Q distributed uniformly over its surface. A small area dA of the sphere is cut off. Find the electric field at the centre due to the remaining sphere.

B-7. (i) Two infinitely long line charges each of linear charge density λ are placed at an angle θ as shown in figure. Find out electric field intensity at a point P, which is at a distance x from point O along angle bisector of line charges.

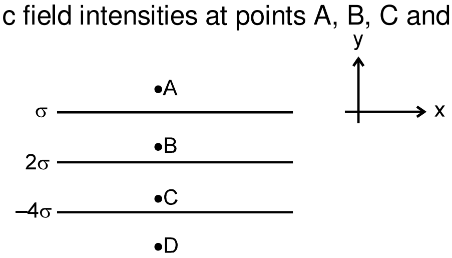


(ii) Repeat the above question if the line charge densities are λ and $-\lambda$. as shown in figure.



B-8. The bob of a simple pendulum has a mass of 60 g and a positive charge of 6×10^{-6} C. It makes 30 oscillations in 50 s above earth's surface. A vertical electric field pointing upward and of magnitude 5×10^4 N/C is switched on. How much time will it now take to complete 60 oscillations? ($g = 10\ m/s^2$)

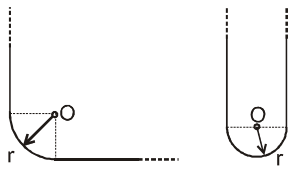
B-9. If three infinite charged sheets of uniform surface charge densities σ, 2σ and -4σ are placed as shown in figure, then find out electric field intensities at points A, B, C and D.



B-10. Find out electric field intensity due to uniformly charged solid non-conducting sphere of volume charge density ρ and radius R at following points :
(i) At a distance r from surface of sphere (inside)
(ii) At a distance r from the surface of sphere (outside)

B-11. Repeat the question if sphere is a hollow non-conducting sphere of radius R and has uniform surface charge density σ.

B-12. A thread carrying a uniform charge λ per unit length has the configuration shown in figure a and b. Assuming a curvature radius r to be considerably less than the length of the thread, find the magnitude of the electric field strength at the point O.



(a) (b)

SECTION (C) : ELECTRIC POTENTIAL AND POTENTIAL DIFFERENCE

C-1. A point charge 20 μC is shifted from infinity to a point P in an electric field with zero acceleration. If the potential of that point is 1000 volt, then
(i) Find out work done by external agent against electric field?
(ii) What is the work done by electric field?
(iii) If the kinetic energy of charge particle is found to increase by 10 mJ when it is brought from infinity to point P, then what is the total work done by external agent?
(iv) What is the work done by electric field in the part (iii)
(v) If a point charge 30 μC is released at rest at point P, then find out its kinetic energy at a large distance?

C-2. Two particles A and B having charges of 4×10^{-6} C and -8×10^{-6} C respectively, are held fixed at a separation of 60 cm. Locate the point(s) on the line AB where the electric potential is zero.

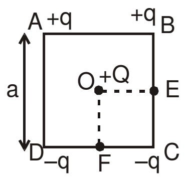
C-3. Six equal point charges q_0' each are placed at six corners of a regular hexagon of side 'a'. Find out work required to take a point charge 'q' slowly :
(i) From infinity to the centre of hexagon.
(ii) From infinity to a point on the axis which is at a distance '$\sqrt{3}$ a' from the centre of hexagon.
(iii) Does your answer to part (i) and (ii) depends on the path followed by the charge.

C-4. 20 J of work has to be done against an existing electric field to take a charge of 0.05 C from A to B. How much is the potential difference $V_B - V_A$?

C-5. A charge of 8 mC is located at the origin. Calculate the work done by external agent in taking a small charge of -2×10^{-9} C from a point A(0, 0, 0.03 m) to a point B(0, 0.04 m, 0) via a point C(0, 0.06 m, 0.09 m).

C-6. A positive charge Q = 50 μ C is located in the xy plane at a point having position vector $\vec{r}_0 = \left(2\hat{i} + 3\hat{j}\right)$ m where $\hat{i}$ and $\hat{j}$ are unit vectors in the positive directions of X and Y axis respectively. Find:
(a) The electric intensity vector and its magnitude at a point having co-ordinates (8 m, – 5 m).
(b) Work done by external agent in transporting a charge q = 10 μC from (8 m, 6 m) to the point (4 m, 3 m).

C-7. Four charges + q, + q, – q, – q are fixed respectively at the corners of A, B, C and D of a square of side 'a' arranged in the given order. Calculate the electric potential and intensity at O (Center of square). If E and F are the midpoints of sides BC, CD respectively, what will be the work done by external agent in carrying a charge Q slowly from O to E and from O to F?



C-8. A charge Q is distributed over two concentric hollow spheres of radius r and R (R > r), such that the surface densities of charge are equal. Find the potential at the common centre.

C-9. Two uniformly charged concentric hollow spheres of radii R and 2 R are charged. The inner sphere has a charge of 1 μC and the outer sphere has a charge of 2 μC of the same sign. The potential is 9000 V at a point P at a distance 3R from the common centre O. What is the value of R?

C-10. In front of a uniformly charged infinite non-conducting sheet of surface charge density σ, a point charge q_0 is shifted slowly from a distance a to b (b > a). If work done by external agent is W, then find out relation between the given parameters.

C-11. An electric field of 20 N/C exists along the negative x-axis in space. Calculate the potential difference $V_B - V_A$, where the points A and B are given by :
(a) A = (0, 0) ; B = (0, 4m) (b) A = (2m, 1m) ; B = (4m, 3m)

C-12. A uniform field of 8 N/C exists in space in positive x-direction.
(a) Taking the potential at the origin to be zero, write an expression for the potential at a general point (x, y, z). (b) At which points, the potential is 160 V? (c) If the potential at the origin is taken to be 80V, what will be the expression for the potential at a general point? (d) What will be the potential at the origin if the potential at x = infinity is taken to be zero ?

C-13. A particle of charge $+3 \times 10^{-9}$ C is in a uniform field directed to the left. It is released from rest and moves a distance of 5 cm, after which its kinetic energy is found to be 4.5×10^{-5} J.
(a) What work was done by the electrical force?
(b) What is the magnitude of the electrical field?
(c) What is the potential of the starting point with respect to the end point?

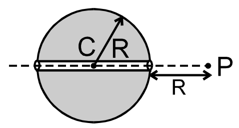
C-14. In the previous problem, suppose that another force in addition to the electrical force acts on the particle so that when it is released from rest, it moves to the right. After it has moved 5 cm, the additional force has done 9×10^{-5} J of work and the particle has 4.5×10^{-5} J of kinetic energy.
(a) What work was done by the electrical force?
(b) What is the magnitude of the electric field?
(c) What is the potential of the starting point with respect to the end point?

SECTION (D) : ELECTRIC POTENTIAL ENERGY OF A POINT CHARGE

D-1. An α particle is placed in an electric field at a point having electric potential 5V. Find its potential energy ?

D-2. Find the potential energy of a charge q_0 placed at the centre of regular hexagon of side a, if charge q is placed at each vertex of regular hexagon?

D-3. A solid uniformly charged fixed non-conducting sphere of total charge Q and radius R contains a tunnel of negligible diameter. If a point charge '–q' of mass 'm' is released at rest from point P as shown in figure then find out its velocity at following points



(i) At the surface of sphere (ii) At the centre of the sphere

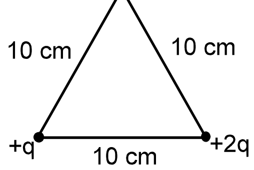
D-4. Two identical charges, 5 μC each are fixed at a distance 8 cm and a charged particle of mass 9×10^{-6} kg and charge – 10 μC is placed at a distance 5 cm from each of them and is released. Find the speed of the particle when it is nearest to the two charges.

D-5. A particle of mass m, charge q > 0 and initial kinetic energy K is projected from infinity towards a heavy nucleus of charge Q assumed to have a fixed position.
(a) If the aim is perfect, how close to the centre of the nucleus is the particle when it comes instantaneously to rest?
(b) With a particular imperfect aim, the particle's closest approach to nucleus is twice the distance determined in (a). Determine speed of particle at the closest distance of approach.

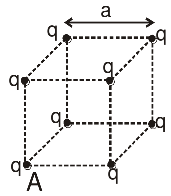
SECTION (E) : POTENTIAL ENERGY OF A SYSTEM OF POINT CHARGES

E-1. Two positive point charges 15 μC and 10 μC are 30 cm apart. Calculate the work done in bringing them closer to each other by 15 cm.

E-2. Three point charges are arranged at the three vertices of a triangle as shown in Figure. Given : $q = 10^{-7}$ C, calculate the electrostatic potential energy of the system.

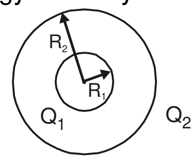


E-3. Eight equal point charges each of charge 'q' and mass 'm' are placed at eight corners of a cube of side 'a'.
(i) Find out potential energy of charge system
(ii) Find out work done by external agent against electrostatic forces and by electrostatic forces to increase all sides of cube from a to 2a.
(iii) If all the charges are released at rest, then find out their speed when they are at the corners of cube of side 2a.
(iv) If keeping all other charges fixed, charge of corner 'A' is released then find out its speed when it is at infinite distance?
(v) If all charges are released simultaneously from rest then find out their speed when they are at a very large distance from each other.



SECTION (F) : SELF ENERGY AND ENERGY DENSITY

F-1. Two concentric spherical shells of radius R_1 and R_2 ($R_2 > R_1$) are having uniformly distributed charges Q_1 and Q_2 respectively. Find out total energy of the system.



F-2 A spherical shell of radius R with a uniform charge q has point charge q_0 at its centre. Find the work performed by the electric forces during the shell expansion slowly from radius R to 2R. Also find out work done by external agent against electric forces.

F-3. Two identical non-conducting spherical shells having equal charge Q, which is uniformly distributed on it, are placed at a distance d apart. From where they are released. Find out kinetic energy of each sphere when they are at a large distance.

F-4. In a solid uniformly charged sphere of total charge Q and radius R, if energy stored out side the sphere is U_0 joules then find out self energy of sphere in term of U_0?

SECTION (G) : QUESTIONS BASED ON RELATION BETWEEN $\vec{E}$ AND V :

G-1. If $\vec{E} = 2y\hat{i} + 2x\hat{j}$, then find V(x, y, z).

G-2. If $V = x^2y + y^2z$ then find $\vec{E}$(x, y, z).

G-3. If $V = 2r^2$ then find out (i) $\vec{E}(1, 0, -2)$ (ii) $\vec{E}(r = 2)$

G-4. An electric field $\vec{E} = (10\hat{i} + 20\hat{j})$ N/C exists in the space. If the potential at the origin is taken to be zero, find the potential at (3m, 3m).

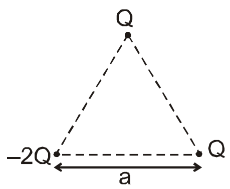
G-5. An electric field $\vec{E} = Bx\hat{i}$ exists in space, where B = 20 V/m^2. Taking the potential at (2 m, 4 m) to be zero, find the potential at the origin.

G-6. If $E = 2r^2$, then find V(r)

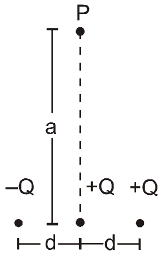
G-7. If $\vec{E} = 2x^2\hat{i} - 3y^2\hat{j}$, then find V(x, y, z)

SECTION (H) : DIPOLE

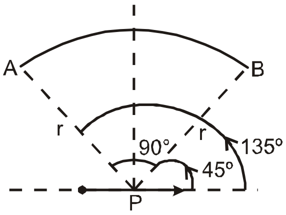
H-1. Three charges are arranged on the vertices of an equilateral triangle as shown in figure. Find the dipole moment of the combination.



H-2. Three point charges –Q, Q and Q are placed on a straight line with distance d between chargesas shown. Find the magnitude of the electric field at the point P in the configuration shown which is at a distance a from middle charge Q in the system provided that a >> d. Take 2Qd = p.



H-3. A charge ' q ' is carried slowly from a point A (r, 135º) to a point B (r, 45º) following a path which is a quadrant of circle of radius ' r '. If the dipole moment is $\vec{P}$, then find out the work done by external agent.



H-4. Find out the magnitude of electric field intensity and electric potential due to a dipole of dipole moment $\vec{P} = \hat{i} + \sqrt{3}\hat{j}$ kept at origin at following points.

(i) (2, 0, 0) (ii) (–1, $\sqrt{3}$, 0)

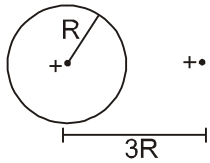
H-5. A molecule of a substance has permanent electric dipole moment equal to 10^{-29} C-m. A mole of this substance is polarised (at low temperature) by applying a strong electrostatic field of magnitude (10^6 Vm^{-1}). The direction of the field is suddenly changed by an angle of 60º. Estimate the heat released by the substance in aligning its dipoles along the new direction of the field. For simplicity, assume 100% polarisation to the sample.

SECTION (I) : ELECTRIC LINES OF FORCE, FLUX CALCULATION AND GAUSS'S LAW

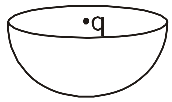
I-1. Find out the electric flux through an area 10 m^2 lying in XY plane due to an electric field $\vec{E} = 2\hat{i} - 10\hat{j} + 5\hat{k}$

I-2. In a uniform electric field E if we consider an imaginary cubical closed suface of side a, then find the net flux through the cube ?

I-3. Find the flux of the electric field through a spherical surface of radius R due to a charge of 8.85 × 10^{-8}C at the centre and another equal charge at a point 3R away from the centre
(Given : ε_0 = 8.85 × 10^{-12} units)



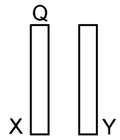
I-4. A charge q is placed at the centre of an imaginary hemispherical surface. Using symmetry arguments and the Gauss's law, find the electric flux due to this charge through the given surface.



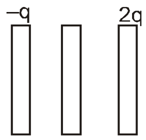
I-5. What do you predict by the given statement about the nature of charge (positive or negative) enclosed by the closed surface. "In a closed surface, lines which are leaving the surface are double than the lines which are entering it".

SECTION (J) : CONDUCTOR, IT'S PROPERTIES & ELECTRIC PRESSURE

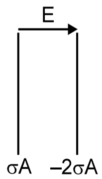
J-1. Two conducting plates X and Y, each having large surface area A (on one side), are placed parallel to each other as shown in figure. The plate X is given a charge Q whereas the other is neutral. Find:
(a) The surface charge density at the inner surface of the plate X,
(b) The electric field at a point to the left of the plates,
(c) The electric field at a point in between the plates and
(d) The electric field at a point to the right of the plates.



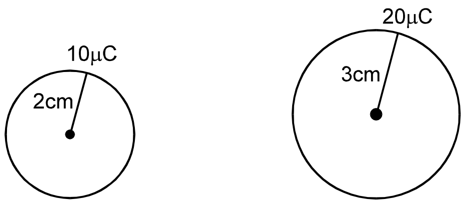
J-2. Three identical metal plates with large equal surface areas are kept parallel to each other as shown in figure. The leftmost plate is given a charge –q, the rightmost a charge 2q and the middle one remains neutral. Find the charge appearing on the outer surface of the leftmost plate.



J-3. Two thin conducting plates (very large) parallel to each other carrying total charges σA and $-2\sigma A$ respectively (where A is the area of each plate), are placed in a uniform external electric field E as shown. Find the surface charge on each surface.



J-4. Figure shows two conducting spheres separated by large distance and of radius 2cm and 3cm containing charges 10μC and 20μC respectively. When the spheres are connected by a conducting wire then find out following :

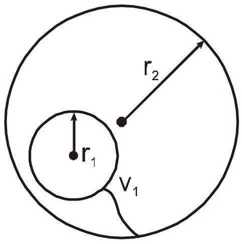


(i) Ratio of the final charge.
(ii) Final charge on each sphere.
(iii) Ratio of final charge densities.
(iv) Heat produced during the process.

J-5. Two concentric hollow conducting spheres of radius a and b (b > a) contains charges Q_a and Q_b respectively. If they are connected by a conducting wire then find out following
(i) Final charges on inner and outer spheres.
(ii) Heat produced during the process.

J-6. There are two concentric metal shells of radii r_1 and r_2 (> r_1). If initially, the outer shell has a charge q and the inner shell is having zero charge and then inner shell is grounded. Find :
(i) Charge on the inner surface of outer shell.
(ii) Final charges on each sphere.
(iii) Charge flown through wire in the ground.

J-7. A metal sphere of radius r_1 charged to a potential V_1 is then palced in a thin-walled uncharged conducting spherical shell of radius r_2. Determine the potential acquired by the spherical shell after it has been connected for a short time to the sphere by a conductor.



PART - II : ONLY ONE OPTION CORRECT TYPE

SECTION (A) : PROPERTIES OF CHARGE AND COULOMB'S LAW

A-1. A charged particle q_1 is at position (2, –1, 3). The electrostatic force on another charged particle q_2 at (0, 0, 0) is :

(A) $\frac{q_1q_2}{56\pi\epsilon_0}(2\hat{i}-\hat{j}+3\hat{k})$ (B) $\frac{q_1q_2}{56\sqrt{14}\pi\epsilon_0}(2\hat{i}-\hat{j}+3\hat{k})$

(C) $\frac{q_1q_2}{56\pi\epsilon_0}(\hat{j}-2\hat{i}-3\hat{k})$ (D) $\frac{q_1q_2}{56\sqrt{14}\pi\epsilon_0}(\hat{j}-2\hat{i}-3\hat{k})$

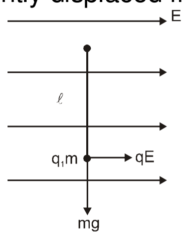
A-2. Three charges +4q, Q and q are placed in a straight line of length ℓ at points at distance 0, $\ell/2$ and ℓ respectively from one end of line. What should be the value of Q in order to make the net force on q to be zero?
(A) –q (B) –2q (C) –q/2 (D) 4q

A-3. Two similar very small conducting spheres having charges 40 μC and –20 μC are some distance apart. Now they are touched and kept at the same distance. The ratio of the initial to the final force between them is :
(A) 8 : 1 (B) 4 : 1 (C) 1 : 8 (D) 1 : 1

A-4. Two point charges placed at a distance r in air exert a force F on each other. The value of distance R at which they experience force 4F when placed in a medium of dielectric constant K = 16 is :
(A) r (B) r/4 (C) r/8 (D) 2r

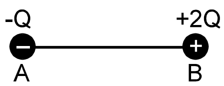
SECTION (B) : ELECTRIC FIELD

B-1. A simple pendulum has a length ℓ & mass of bob m. The bob is given a charge q coulomb. The pendulum is suspended in a uniform horizontal electric field of strength E as shown in figure, then calculate the time period of oscillation when the bob is slightly displaced from its mean position.



(A) $2\pi\sqrt{\frac{\ell}{g}}$ (B) $2\pi\sqrt{\left\{\frac{\ell}{g+\frac{qE}{m}}\right\}}$ (C) $2\pi\sqrt{\left\{\frac{\ell}{g-\frac{qE}{m}}\right\}}$ (D) $2\pi\sqrt{\frac{\ell}{\sqrt{g^2+\left(\frac{qE}{m}\right)^2}}}$

B-2. Charges 2Q and –Q are placed as shown in figure. The point at which electric field intensity is zero will be:



(A) Somewhere between –Q and 2Q (B) Somewhere on the left of –Q
(C) Somewhere on the right of 2Q
(D) Somewhere on the perpendicular bisector of line joining –Q and 2Q

B-3. The maximum electric field intensity on the axis of a uniformly charged ring of charge q and radius R will be :

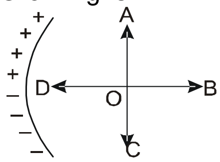
(A) $\frac{1}{4\pi\varepsilon_0}\frac{q}{3\sqrt{3}R^2}$ (B) $\frac{1}{4\pi\varepsilon_0}\frac{2q}{3R^2}$ (C) $\frac{1}{4\pi\varepsilon_0}\frac{2q}{3\sqrt{3}R^2}$ (D) $\frac{1}{4\pi\varepsilon_0}\frac{3q}{2\sqrt{3}R^2}$

B-4. A charged particle of charge q and mass m is released from rest in a uniform electric field E. Neglecting the effect of gravity, the kinetic energy of the charged particle after time 't' seconds is

(A) $\frac{Eqm}{t}$ (B) $\frac{E^2q^2t^2}{2m}$ (C) $\frac{2E^2t^2}{mq}$ (D) $\frac{Eq^2m}{2t^2}$

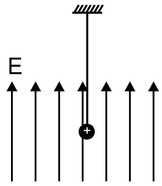
B-5. A flat circular fixed disc has a charge +Q uniformly distributed on the disc. A charge +q is thrown with kinetic energy K, towards the disc along its axis. The charge q :
(A) may hit the disc at the centre
(B) may return back along its path after touching the disc
(C) may return back along its path without touching the disc
(D) any of the above three situations is possible depending on the magnitude of K

B-6. The linear charge density on upper half of semi-circular section of ring is λ and that at lower half is $-\lambda$. The direction of electric field at centre O of ring is :



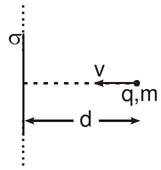
(A) along OA (B) along OB (C) along OC (D) along OD

B-7. A positively charged pendulum is oscillating in a uniform electric field as shown in Figure. Its time period of SHM as compared to that when it was uncharged. (mg > qE)



(A) Will increase (B) Will decrease
(C) Will not change (D) Will first increase then decrease

B-8. The particle of mass m and charge q will touch the infinitely large plate of uniform charge density σ if its velocity v is more than: {Given that $\sigma q > 0$}



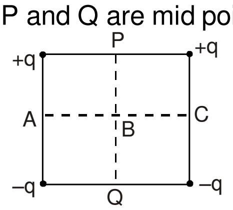
(A) 0 (B) $\sqrt{\dfrac{2\sigma q d}{m \epsilon_0}}$ (C) $\sqrt{\dfrac{\sigma q d}{m \epsilon_0}}$ (D) none of these

B-9. There is a uniform electric field in X-direction. If the work done by external agent in moving a charge of 0.2 C through a distance of 2 metre slowly along the line making an angle of 60º with X-direction is 4 joule, then the magnitude of E is :
(A) $\sqrt{3}$N/C (B) 4 N/C (C) 5 N/C (D) 20 N/C

SECTION (C) : ELECTRIC POTENTIAL AND POTENTIAL DIFFERENCE

C-1. At a certain distance from a point charge, the electric field is 500 V/m and the potential is 3000 V. What is the distance ?
(A) 6 m (B) 12 m (C) 36 m (D) 144 m

C-2. Figure represents a square carrying charges +q, +q, –q, –q at its four corners as shown. Then the potential will be zero at points : (A, C, P and Q are mid points of sides)



(A) A, B, C, P and Q (B) A, B and C (C) A, P, C and Q (D) P, B and Q

C-3. Two equal positive charges are kept at points A and B. The electric potential, while moving from A to B along straight line :
(A) continuously increases (B) remains constant
(C) decreases then increases (D) increases then decreases

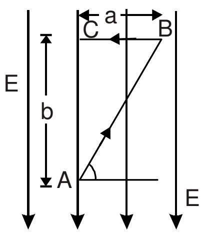
C-4. A semicircular ring of radius 0.5 m is uniformly charged with a total charge of 1.5×10^{-9} coul. The electric potential at the centre of this ring is :
(A) 27 V (B) 13.5 V (C) 54 V (D) 45.5 V

C-5. When a charge of 3 coul is placed in a uniform electric field, it experiences a force of 3000 newton. The potential difference between two points separated by a distance of 1 cm along field within this field is:
(A) 10 volt (B) 90 volt (C) 1000 volt (D) 3000 volt

C-6. A 5 coulomb charge experiences a constant force of 2000 N when moved between two points separated by a distance of 2 cm in a uniform electric field. The potential difference between these two points is:
(A) 8 V (B) 200 V (C) 800 V (D) 20,000 V

C-7. The kinetic energy which an electron acquires when accelerated (from rest) through a potential difference of 1 volt is called :
(A) 1 joule (B) 1 electron volt (C) 1 erg (D) 1 watt

C-8. The potential difference between points A and B in the given uniform electric field is :



(A) Ea (B) $E\sqrt{(a^2+b^2)}$ (C) Eb (D) $(Eb/\sqrt{2})$

C-9. An equipotential surface and an electric line of force :
(A) never intersect each other (B) intersect at 45º
(C) intersect at 60º (D) intersect at 90º

C-10. A particle of charge Q and mass m travels through a potential difference V from rest. The final momentum of the particle is :
(A) $\frac{mV}{Q}$ (B) $2Q\sqrt{mV}$ (C) $\sqrt{2mQV}$ (D) $\sqrt{\frac{2QV}{m}}$

C-11. If a uniformly charged spherical shell of radius 10 cm has a potential V at a point distant 5 cm from its centre, then the potential at a point distant 15 cm from the centre will be :
(A) $\frac{V}{3}$ (B) $\frac{2V}{3}$ (C) $\frac{3}{2}V$ (D) 3V

C-12. A hollow uniformly charged sphere has radius r. If the potential difference between its surface and a point at distance 3r from the centre is V, then the electric field intensity at a distance 3r from the centre is:
(A) V/6r (B) V/4r (C) V/3r (D) V/2r

C-13. A hollow sphere of radius 5 cm is uniformly charged such that the potential on its surface is 10 volts then potential at centre of sphere will be :
(A) Zero
(B) 10 volt
(C) Same as at a point 5 cm away from the surface
(D) Same as at a point 25 cm away from the centre

C-14. A charge +q is fixed at each of the points $x = x_0$, $x = 3x_0$, $x = 5x_0$, ... upto infinity on the x-axis and a charge –q is fixed at each of the points $x = 2x_0$, $x = 4x_0$, $x = 6x_0$, ... upto infinity. Here x_0 is a positive constant. Take the electric potential at a point due to a charge Q at a distance r from it to be. $\frac{Q}{4\pi \in_0 r}$
Then the potential at the origin due to the above system of charges is: **[JEE 1998 Screening, 2/200]**
(A) 0 (B) $\frac{q}{8\pi \in_0 x_0 \ell n2}$ (C) ∞ (D) $\frac{q\,\ell n2}{4\pi \in_0 x_0}$

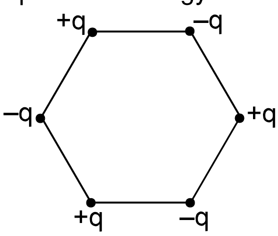
SECTION (D) : ELECTRIC POTENTIAL ENERGY OF A PARTICLE

D-1. If a charge is shifted from a high potential region to low potential region, the electrical potential energy:
(A) Increases (B) Decreases
(C) May increase or decrease. (D) Remains constant

D-2. A particle of mass 2 g and charge 1μC is held at rest on a frictionless horizontal surface at a distance of 1 m from a fixed charge of 1 mC. If the particle is released it will be repelled. The speed of the particle when it is at distance of 10 m from the fixed charge is:
(A) 100 m/s (B) 90 m/s (C) 60 m/s (D) 45 m/s

SECTION (E) : POTENTIAL ENERGY OF A SYSTEM OF POINT CHARGES

E-1. Six charges of magnitude + q and –q are fixed at the corners of a regular hexagon of edge length a as shown in the figure. The electrostatic potential energy of the system of charged particles is :



(A) $\frac{q^2}{\pi \in_0 a}\left[\frac{\sqrt{3}}{8}-\frac{15}{4}\right]$ (B) $\frac{q^2}{\pi \in_0 a}\left[\frac{\sqrt{3}}{2}-\frac{9}{4}\right]$ (C) $\frac{q^2}{\pi \in_0 a}\left[\frac{\sqrt{3}}{4}-\frac{15}{2}\right]$ (D) $\frac{q^2}{\pi \in_0 a}\left[\frac{\sqrt{3}}{2}-\frac{15}{8}\right]$

E-2. You are given an arrangement of three point charges q, 2q and xq separated by equal finite distances so that electric potential energy of the system is zero. Then the value of x is :
(A) $-\frac{2}{3}$ (B) $-\frac{1}{3}$ (C) $\frac{2}{3}$ (D) $\frac{3}{2}$

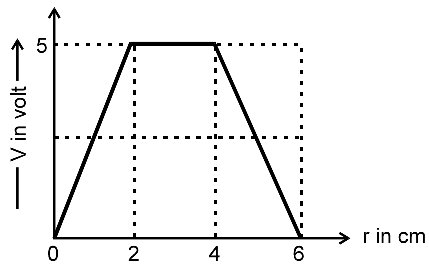
SECTION (F) : SELF ENERGY AND ENERGY DENSITY

F-1. A uniformly charged sphere of radius 1 cm has potential of 8000 V at surface. The energy density near the surface of sphere will be:
(A) 64×10^5 J/m^3 (B) 8×10^3 J/m^3 (C) 32 J/m^3 (D) 2.83 J/m^3

F-2. If ' n ' identical water drops (assumed spherical each) each charged to a potential energy U coalesce to form a single drop, the potential energy of the single drop is(Assume that drops are uniformly charged):
(A) $n^{1/3}$ U (B) $n^{2/3}$ U (C) $n^{4/3}$ U (D) $n^{5/3}$ U

SECTION (G) : QUESTIONS BASED ON RELATION BETWEEN $\vec{E}$ AND V :

G-1. The variation of potential with distance r from a fixed point is shown in Figure. The electric field at r = 5 cm, is :



(A) (2.5) V/cm (B) (–2.5) V/cm (C) (–2/5) cm (D) (2/5) V/cm

G-2. In the above question, the electric force acting on a point charge of 2 C placed at the origin will be :
(A) 2 N (B) 500 N (C) –5 N (D) –500 N

G-3. The electric potential V as a function of distance x (in metre) is given by $V = (5x^2 + 10x - 9)$ volt. The value of electric field at x = 1 m would be :
(A) – 20 volt/m (B) 6 volt/m (C) 11 volt/m (D) –23 volt/m

G-4. A uniform electric field having a magnitude E_0 and direction along positive x-axis exists. If the electric potential V is zero at x = 0, then its value at x = +x will be :
(A) $V_x = xE_0$ (B) $V_x = -xE_0$ (C) $V_x = x^2E_0$ (D) $V_x = -x^2 E_0$

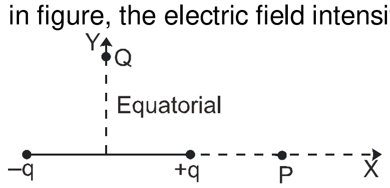
G-5. Let E be the electric field and V, the electric potential at a point.
(A) If $E \neq 0$, V cannot be zero (B) If E = 0, V must be zero
(C) If V = 0, E must be zero (D) None of these

G-6. The electric field in a region is directed outward and is proportional to the distance r from the origin. Taking the electric potential at the origin to be zero, the electric potential at a distance r :
(A) increases as one goes away from the origin. (B) is proportional to r^2
(C) is proportional to r (D) is uniform in the region

G-7. A non-conducting ring of radius 0.5 m carries a total charge of 1.11×10^{-10} C distributed non-uniformly on its circumference producing an electric field $\vec{E}$ every where in space. The value of the line integral $\int_{\ell=\infty}^{\ell=0} -\vec{E}.d\vec{\ell}$ ($\ell = 0$ being centre of the ring) in volts is : (Approximately) **[JEE 1997, 1]**
(A) +2 (B) –1 (C) –2 (D) zero

SECTION (H) : DIPOLE

H-1. Due to an electric dipole shown in figure, the electric field intensity is parallel to dipole axis :



(A) at P only (B) at Q only (C) both at P and at Q (D) neither at P nor at Q

H-2. An electric dipole of dipole moment $\vec{p}$ is placed at the origin along the x-axis. The angle made by electric field with x-axis at a point P, whose position vector makes an angle θ with x-axis, is :
(where, $\tan\alpha = 1/2 \tan\theta$)
(A) α (B) θ (C) $\theta + \alpha$ (D) $\theta + 2\alpha$

H-3. An electric dipole consists of two opposite charges each of magnitude 1.0 μC, separated by a distance of 2.0 cm. The dipole is placed in an external electric field of 1.0×10^5 N/C. The maximum torque on the dipole is :
(A) 0.2×10^{-3} N-m (B) 1.0×10^{-3} N-m (C) 2.0×10^{-3} N-m (D) 4.0×10^{-3} N-m

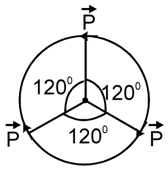
H-4. A dipole of electric dipole moment P is placed in a uniform electric field of strength E. If θ is the angle between positive directions of P and E, then the potential energy of the electric dipole is largest when θ is :
(A) zero (B) $\pi/2$ (C) π (D) $\pi/4$

H-5. Two opposite and equal charges of magnitude 4×10^{-8} coulomb each when placed 2×10^{-2} cm apart form a dipole. If this dipole is placed in an external electric field of 4×10^{8} N/C, the value of maximum torque and the work required in rotating it through 180º from its initial orientation which is along electric field will be : (Assume rotation of dipole about an axis passing through centre of the dipole):
(A) 64×10^{-4} N-m and 44×10^{-4} J (B) 32×10^{-4} N-m and 32×10^{-4} J
(C) 64×10^{-4} N-m and 32×10^{-4} J (D) 32×10^{-4} N-m and 64×10^{-4} J

H-6. At a point on the axis (but not inside the dipole and not at infinity) of an electric dipole
(A) The electric field is zero
(B) The electric potential is zero
(C) Neither the electric field nor the electric potential is zero
(D) The electric field is directed perpendicular to the axis of the dipole

H-7. The force between two short electric dipoles separated by a distance r is directly proportional to :
(A) r^2 (B) r^4 (C) r^{-2} (D) r^{-4}

H-8. Three dipoles each of dipole moment of magnitude p are placed tangentially on a circle of radius R in its plane positioned at equal angle from each other as shown in the figure. Then the magnitude of electric field intensity at the centre of the circle will be :



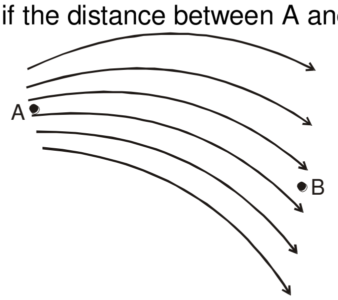
(A) $\frac{4kp}{R^3}$ (B) $\frac{2kp}{R^3}$ (C) $\frac{kp}{R^3}$ (D) 0

SECTION (I) : ELECTRIC LINES OF FORCE, FLUX CALCULATION AND GAUSS'S LAW

I-1. A square of side 'a' is lying in xy plane such that two of its sides are lying on the axis. If an electric field $\vec{E} = E_0 x\hat{k}$ is applied on the square. The flux passing through the square is :
(A) E_0a^3 (B) $\frac{E_0a^3}{2}$ (C) $\frac{E_0a^3}{3}$ (D) $\frac{E_0a^2}{2}$

I-2. If electric field is uniform, then the electric lines of forces are:
(A) Divergent (B) Convergent (C) Circular (D) Parallel

I-3. The figure shows the electric lines of force emerging from a charged body. If the electric fields at A and B are E_A and E_B respectively and if the distance between A and B is r, then



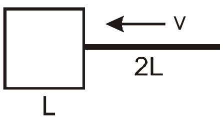
(A) $E_A < E_B$ (B) $E_A > E_B$ (C) $E_A = \frac{E_B}{r}$ (D) $E_A = \frac{E_B}{r^2}$

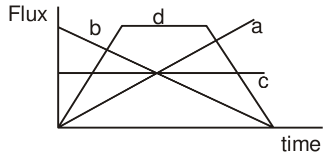
I-4. Select the correct statement :
(A) The electric lines of force are always closed curves
(B) Electric lines of force are parallel to equipotential surface
(C) Electric lines of force are perpendicular to equipotential surface
(D) Electric line of force is always the path of a positively charged particle.

I-5. If the electric flux entering and leaving a closed surface are respectively of magnitude ϕ_1 and ϕ_2, then the electric charge inside the surface will be :
(A) $\frac{\phi_2 - \phi_1}{\varepsilon_0}$ (B) $(\phi_1 - \phi_2)\varepsilon_0$ (C) $\varepsilon_0(\phi_2 - \phi_1)$ (D) $\varepsilon_0(\phi_2 + \phi_1)$

I-6. An electric dipole is placed at the centre of a sphere. Mark the correct options.
(A) The electric field is zero at every point of the sphere.
(B) The flux of the electric field through the sphere is non-zero.
(C) The electric field is zero on a circle on the sphere.
(D) The electic field is not zero anywhere on the sphere.

I-7. Figure (i) shows an imaginary cube of edge length L. A uniformly charged rod of length 2L moves towards left at a small but constant speed v. At t = 0, the left end of the rod just touches the centre of the face of the cube opposite to it. Which of the graphs shown in figure (ii) represents the flux of the electric field through the cube as the rod goes through it ?

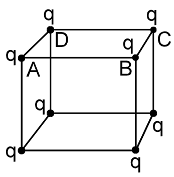
(i) 

(ii) 

(A) a (B) b (C) c (D) d

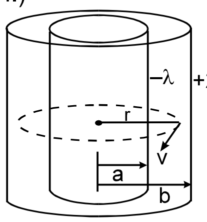
I-8. Electric charges are distributed in a small volume. The flux of the electric field through a spherical surface of radius 20 cm surrounding the total charge is 50 V-m. The flux over a concentric sphere of radius 40 cm will be:
(A) 50 V-m (B) 75 V-m (C) 100 V-m (D) 200 V-m

I-9. Eight point charges (can be assumed as uniformly charged small spheres and their centres at the corner of the cube) having value q each are fixed at vertices of a cube. The electric flux through square surface ABCD of the cube is



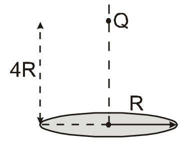
(A) $\frac{q}{24 \epsilon_0}$ (B) $\frac{q}{12 \epsilon_0}$ (C) $\frac{q}{6 \epsilon_0}$ (D) $\frac{q}{8 \epsilon_0}$

I-10. Figure shows two large cylindrical shells having uniform linear charge densities + λ and – λ. Radius of inner cylinder is 'a' and that of outer cylinder is 'b'. A charged particle of mass m, charge q revolves in a circle of radius r. Then, its speed 'v' is : (Neglect gravity and assume the radii of both the cylinders to be very small in comparison to their length.)



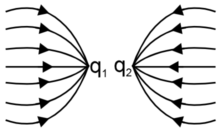
(A) $\sqrt{\frac{\lambda q}{2\pi \epsilon_0 m}}$ (B) $\sqrt{\frac{2 \lambda q}{\pi \epsilon_0 m}}$ (C) $\sqrt{\frac{\lambda q}{\pi \epsilon_0 m}}$ (D) $\sqrt{\frac{\lambda q}{4\pi\varepsilon_0 m}}$

I-11. A positive charge Q is placed at a distance of 4R above the centre of a disc of radius R. The magnitude of flux through the disc is ϕ. Now a hemispherical shell of radius R is placed over the disc such that it forms a closed surface. The flux through the curved surface (taking direction of area vector along outward normal as positive), is :



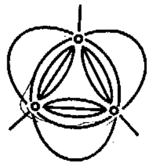
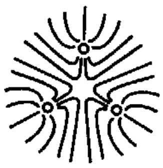
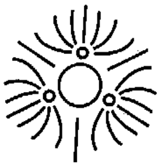
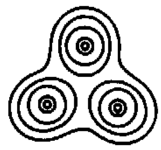
(A) zero (B) ϕ (C) $-\phi$ (D) 2ϕ

I-12. The given figure gives electric lines of force due to two charges q_1 and q_2. What are the signs of the two charges?

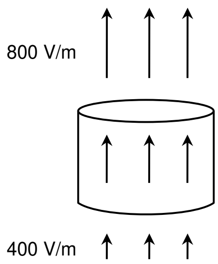


(A) Both are negative (B) Both are positive
(C) q_1 is positive but q_2 is negative (D) q_1 is negative but q_2 is positive

I-13. Three positive charges of equal value q are placed at the vertices of an equilateral triangle. The resulting lines of force should be sketched as in : **[JEE 2001(Scr.), 3/105]**

(A)  (B)  (C)  (D) 

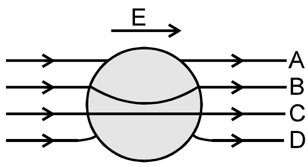
I-14. A cylinder on whose surfaces there is a vertical electric field of varying magnitude as shown. The electric field is uniform on the top surface as well as on the bottom surface therefore, this cylinder encloses **[Olympiad (stage-1) 2017]**



(A) no net charge
(B) net positive charge
(C) net negative charge
(D) There is not enough information to determine whether or not there is net charge inside the cylinder.

SECTION (J) : CONDUCTOR, IT'S PROPERTIES & ELECTRIC PRESSURE

J-1. A metallic solid sphere is placed in a uniform electric field. The lines of force follow the path(s) shown in figure as : **[JEE 1996, 2/100]**



(A) A (B) B (C) C (D) D

J-2. A neutral spherical metallic object A is placed near a finite metal plate B carrying a positive charge. The electric force on the object will be :
(A) away from the plate B (B) towards the plate B
(C) parallel to the plate B (D) zero

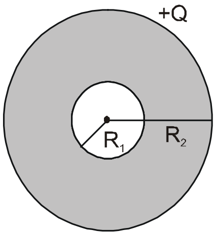
J-3. A positive point charge q is brought near a neutral metal sphere.
(A) The sphere becomes negatively charged.
(B) The sphere becomes positively charged.
(C) The interior remains neutral and the surface gets non-uniform charge distribution.
(D) The interior becomes positively charged and the surface becomes negatively charged.

J-4. Three concentric conducting spherical shells carry charges as follows : $+4Q$ on the inner shell, $-2Q$ on the middle shell and $-5Q$ on the outer shell. The charge on the inner surface of the outer shell is:
(A) 0 (B) 4 Q (C) – Q (D) – 2Q

J-5. A charge q is uniformly distributed over a large plastic plate. The electric field at a point P close to the centre and just above the surface of the plate is 50 V/m. If the plastic plate is replaced by a copper plate of the same geometrical dimensions and carrying the same uniform charge q, the electric field at the point P will become:

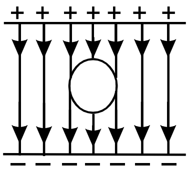
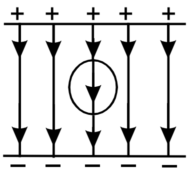
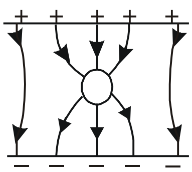
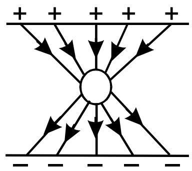
(A) zero (B) 25 V/m (C) 50 V/m (D) 100 V/m

J-6. Figure shows a thick metallic sphere. If it is given a charge +Q, then electric field will be present in the region

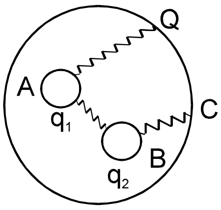


(A) $r < R_1$ only (B) $r > R_1$ and $R_1<r<R_2$ (C) $r \geq R_2$ only (D) $r \leq R_2$ only

J-7. An uncharged sphere of metal is placed in a uniform electric field produced by two large conducting parallel plates having equal and opposite charges, then lines of force look like:

(A) 
(B) 
(C) 
(D) 

J-8. Two small conductors A and B are given charges q_1 and q_2 respectively. Now they are placed inside a hollow metallic conductor (C) carrying a charge Q. If all the three conductors A, B and C are connected by conducting wires as shown, the charges on A, B and C will be respectively:



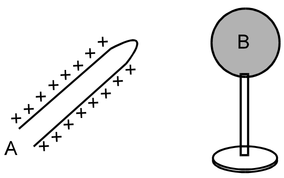
(A) $\frac{q_1+q_2}{2}, \frac{q_1+q_2}{2}, Q$ (B) $\frac{Q+q_1+q_3}{3}, \frac{Q+q_1+q_2}{3}, \frac{Q+q_1+q_2}{3}$

(C) $\frac{q_1+q_2+Q}{2}, \frac{q_1+q_2+Q}{2}, \ 0$ (D) 0, 0, $Q + q_1 + q_2$

J-9. You are travelling in a car during a thunder storm. In order to protect yourself from lightening, would you prefer to :

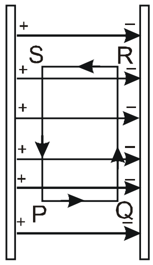
(A) Remain in the car (B) Take shelter under a tree
(C) Get out and be flat on the ground (D) Touch the nearest electrical pole

J-10. A positively charged body 'A' has been brought near a neutral brass sphere B mounted on a glass stand as shown in the figure. The potential of B will be:



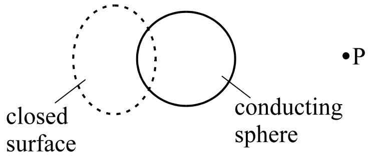
(A) Zero (B) Negative (C) Positive (D) Infinite

J-11. The amount of work done by electric field in joules in carrying a charge +q along the closed path PQRSP between the oppositely charged metal plates is: (where, E is electric field between the plates)



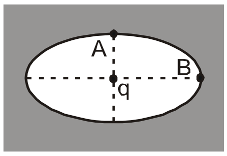
(A) zero (B) q
(C) qE (PQ + QR + SR + SP) (D) q/ε_0

J-12. Figure shows a closed surface which intersects a conducting sphere. If a positive charge is placed at the point P, the flux of the electric field through the closed surface:



(A) will become positive (B) will remain zero
(C) will become undefined (D) will become negative

J-13. An ellipsoidal cavity is carved within a perfect conductor. A positive charge q is placed at the center of the cavity. The points A and B are on the cavity surface as shown in the figure. Then : **[JEE 1999 (Scr.), 3/100]**

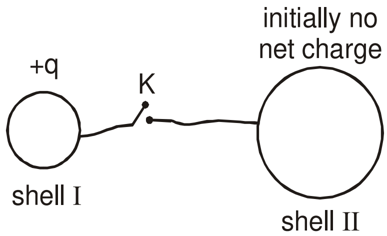


(A) Electric field near A in the cavity = electric field near B in the cavity
(B) Charge density at A = Charge density at B
(C) Potential at A = Potential at B
(D) Total electric field flux through the surface of the cavity is q/ε_0.

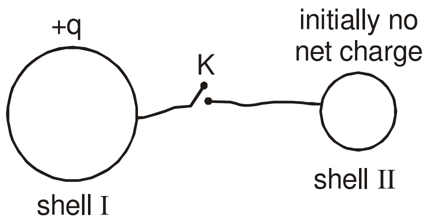
PART - III : MATCH THE COLUMN

1. Column I gives certain situations involving two thin conducting shells connected by a conducting wire via a key K. In all situations, one sphere has net charge +q and other sphere has no net charge. After the key K is pressed, column II gives some resulting effects. Match the figures in Column I with the statements in Column II.

Column-I **Column-II**

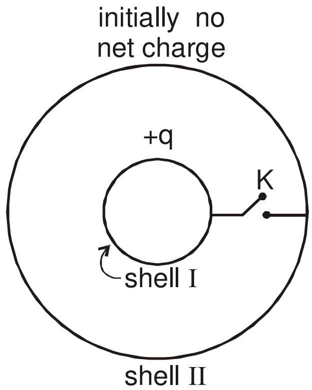
(A) 

(p) Charge flows through connecting wire

(B) 

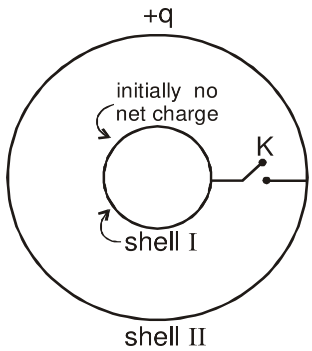
(q) Potential energy of system of spheres decreases.

(C)



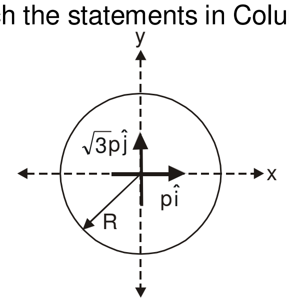
(r) No heat is produced.

(D)



(s) The shell I has no charge after equilibrium is reached.

2. Column I gives a situation in which two dipoles of dipole moment $p\hat{i}$ and $\sqrt{3}p\hat{j}$ are placed at origin. A circle of radius R with centre at origin is drawn as shown in figure. Column II gives coordinates of certain positions on the circle. Match the statements in Column I with the statements in Column II.



Column-I	Column-II
(A) The coordinate(s) of point on circle where potential is maximum	(p) $\left(\frac{R}{2}, \frac{\sqrt{3}\,R}{2}\right)$
(B) The coordinate(s) of point on circle where potential is zero	(q) $\left(-\frac{R}{2}, -\frac{\sqrt{3}\,R}{2}\right)$
(C) The coordinate(s) of point on circle where magnitude of electric field intensity is $\frac{1}{4\pi\varepsilon_0}\frac{4p}{R^3}$	(r) $\left(-\frac{\sqrt{3}\,R}{2}, \frac{R}{2}\right)$
(D) The coordinate(s) of point on circle where magnitude of electric field intensity is $\frac{1}{4\pi\varepsilon_0}\frac{2p}{R^3}$	(s) $\left(\frac{\sqrt{3}\,R}{2}, -\frac{R}{2}\right)$

Exercise-2

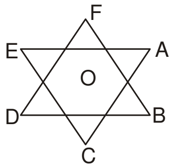
✍ **Marked Questions may have for Revision Questions.**

PART - I : ONLY ONE OPTION CORRECT TYPE

1. A total charge of 20 μC is divided into two parts and placed at some distance apart. If the charges experience maximum coulombian repulsion, the charges should be :

(A) 5μC, 15 μC (B) 10 μC, 10 μC (C) 12 μC, 8 μC (D) $\frac{40}{3}$μC, $\frac{20}{3}$μC

2. The magnitude of electric force on 2 μ c charge placed at the centre O of two equilateral triangles each of side 10 cm, as shown in figure is P. If charge A, B, C, D, E & F are 2 μc, 2 μc, 2 μc, –2 μc, – 2 μc, –2 μc respectively, then P is:

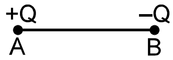


(A) 21.6 N (B) 64.8 N (C) 0 (D) 43.2 N

3. Five balls, numbered 1 to 5, are suspended using separate threads. Pairs (1, 2), (2, 4), (4, 1) show electrostatic attraction, while pairs (2, 3) and (4, 5) show repulsion. Therefore ball 1 :

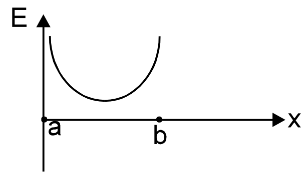
(A) Must be positively charged (B) Must be negatively charged
(C) May be neutral (D) Must be made of metal

4. Two point charges of same magnitude and opposite sign are fixed at points A and B. A third small point charge is to be balanced at point P by the electrostatic force due to these two charges. The point P :



(A) lies on the perpendicular bisector of line AB (B) is at the mid point of line AB
(C) lies to the left of A (D) none of these.

5. Two point charges a & b, whose magnitudes are same are positioned at a certain distance from each other with a at origin. Graph is drawn between electric field strength at points between a & b and distance x from a. E is taken positive if it is along the line joining from a to b. From the graph, it can be decided that

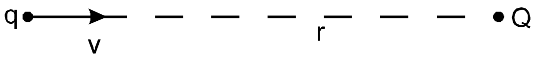


(A) a is positive, b is negative (B) a and b both are positive
(C) a and b both are negative (D) a is negative, b is positive

6. A solid sphere of radius R has a volume charge density $\rho = \rho_0 r^2$ (Where ρ_0 is a constant and r is the distance from centre). At a distance x from its centre (for x < R), the electric field is directly proportional to :

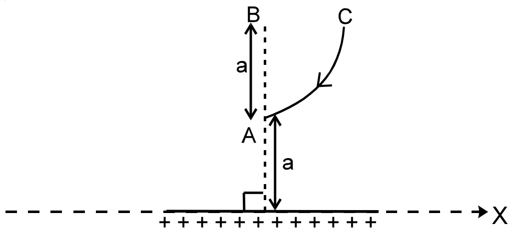
(A) $1/x^2$ (B) $1/x$ (C) x^3 (D) x^2

7. A charged particle 'q' is shot from a large distance with speed v towards a fixed charged particle Q. It approaches Q upto a closest distance r and then returns. If q were given a speed '2v', the closest distance of approach would be :



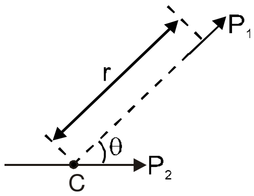
(A) r (B) 2r (C) r/2 (D) r/4

8. For an infinite line of charge having charge density λ lying along x-axis, the work required in moving charge q from C to A along arc CA is :



(A) $\frac{q\lambda}{\pi\varepsilon_0}\log_e\sqrt{2}$ (B) $\frac{q\lambda}{4\pi\varepsilon_0}\log_e\sqrt{2}$ (C) $\frac{q\lambda}{4\pi\varepsilon_0}\log_e 2$ (D) $\frac{q\lambda}{2\pi\varepsilon_0}\log_e\frac{1}{2}$

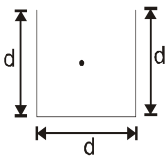
9. Two short electric dipoles are placed as shown (r is the distance between their centres). The energy of electric interaction between these dipoles will be:



(C is centre of dipole of moment P_2)

(A) $\frac{2k\,P_1P_2\cos\theta}{r^3}$ (B) $\frac{-2k\,P_1P_2\cos\theta}{r^3}$ (C) $\frac{-2k\,P_1P_2\sin\theta}{r^3}$ (D) $\frac{-4k\,P_1P_2\cos\theta}{r^3}$

10. A charge q is placed at the centre of the cubical vessel (with one face open) as shown in figure. The flux of the electric field through the surface of the vessel is :

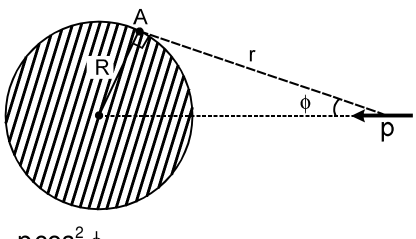


(A) zero (B) q/ε_0 (C) $\frac{q}{4\varepsilon_0}$ (D) $5q/6\varepsilon_0$

11. The electric field above a uniformly charged nonconducting sheet is E. If the nonconducting sheet is now replaced by a conducting sheet, with the charge same as before, the new electric field at the same point is :

(A) 2E (B) E (C) E/2 (D) None of these

12. A dipole having dipole moment p is placed in front of a solid uncharged conducting sphere as shown in the diagram. The net potential at point A lying on the surface of the sphere is :



(A) $\frac{kp\cos\phi}{r^2}$ (B) $\frac{k\ p\cos^2\phi}{r^2}$ (C) zero (D) $\frac{2kp\cos^2\phi}{r^2}$

13. The net charge given to an isolated conducting solid sphere:

(A) must be distributed uniformly on the surface (B) may be distributed uniformly on the surface
(C) must be distributed uniformly in the volume (D) may be distributed uniformly in the volume.

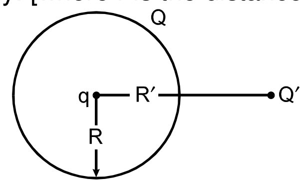
14. The net charge given to a solid insulating sphere:
(A) must be distributed uniformly in its volume
(B) may be distributed uniformly in its volume
(C) must be distributed uniformly on its surface
(D) the distribution will depend upon whether other charges are present or not.

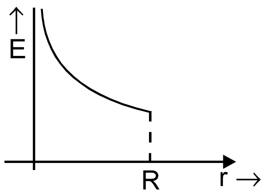
15. A charge Q is kept at the centre of a conducting sphere of inner radius R_1 and outer radius R_2. A point charge q is kept at a distance r (> R_2) from the centre. If q experiences an electrostatic force 10 N then assuming that no other charges are present, electrostatic force experienced by Q will be:
(A) – 10 N (B) 0 (C) 20 N (D) none of these

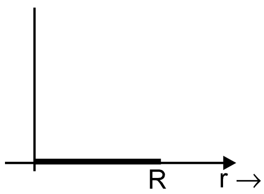
16. Two uniformly charged non-conducting hemispherical shells each having uniform charge density σ and radius R form a complete sphere (not stuck together) and surround a concentric spherical conducting shell of radius R/2. If hemispherical parts are in equilibrium then minimum surface charge density of inner conducting shell is:
(A) -2σ (B) $-\sigma/2$ (C) $-\sigma$ (D) 2σ

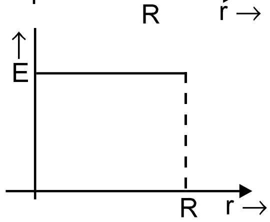
17. A solid metallic sphere has a charge +3Q. Concentric with this sphere is a conducting spherical shell having charge –Q. The radius of the sphere is a and that of the spherical shell is b(>a). What is the electric field at a distance r(a < r < b) from the centre?
(A) $\frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$ (B) $\frac{1}{4\pi\varepsilon_0}\frac{3Q}{r}$ (C) $\frac{1}{4\pi\varepsilon_0}\frac{3Q}{r^2}$ (D) $\frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$

18. A charge 'q' is placed at the centre of a conducting spherical shell of radius R, which is given a charge Q. An external charge Q′ is also present at distance R′ (R′ > R) from 'q'. Then the resultant field will be best represented for region r < R by: [where r is the distance of the point from q]



(A)


(B)


(C)


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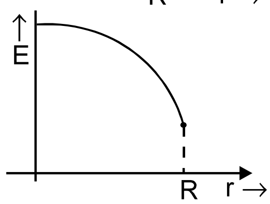
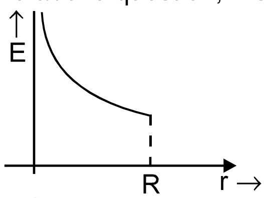
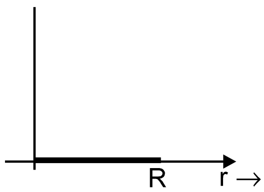
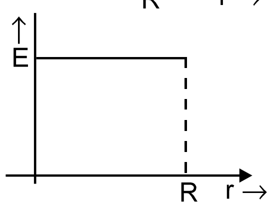
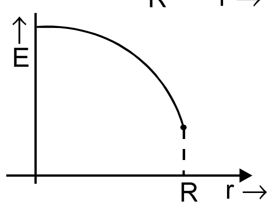


19. In the above question, if Q' is removed then which option is correct:

(A)


(B)


(C)


(D)


20. Two identical charged spheres suspended from a common point by two light strings of length l, are initially at a distance d(<< l) apart due to their mutual repulsion. The charges begin to leak from both the spheres at a constant rate. As a result, the spheres approach each other with a velocity ν. If x denotes the distance between the spheres, the ν varies as **[Olympiad (Stage-1) 2017]**

(A) x^{-1} (B) $x^{1/2}$ (C) $x^{-1/2}$ (D) x

21. Acidified water from certain reservoir kept at a potential V falls in the form of small droplets each of radius r through a hole into a hollow conducting sphere of radius a. The sphere is insulated and is initially at zero potential. If the drops continue to fall until the sphere is half full, the potential acquired by the sphere is **[Olympiad (Stage-1) 2017]**

(A) $\frac{a^2V}{2r^2}$ (B) $\sqrt{\frac{a}{r}}\frac{V}{2}$ (C) $\frac{a^3V}{2r^3}$ (D) $\frac{aV}{r}$

PART - II : SINGLE AND DOUBLE VALUE INTEGER TYPE

1. Two small equally charged identical conducting balls are suspended from long threads from the same point. The charges and masses of the balls are such that they are in equilibrium. The distance between them is a = $(108)^{1/3}$ cm (the length of the threads L >> a) .One of the ball is discharged. After sometime both balls comes to rest in equilibrium. What will be the distance b (in cm) between the balls when equilibrium is restored?

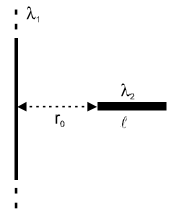
2. Two identical spheres of same mass and specific gravity (which is the ratio of density of a substance and density of water) 2.4 have different charges of Q and – 3Q. They are suspended from two strings of same length ℓ fixed to points at the same horizontal level, but distant ℓ from each other. When the entire set up is transferred inside a liquid of specific gravity 0.8, it is observed that the inclination of each string in equilibrium remains unchanged. The dielectric constant of the liquid is K. Find the value of 4K.

3. Two small balls of masses 3m and 2m and each having charges Q are connected by a string passing over a fixed pulley. Calculate the acceleration of the balls (in m/sec^2) if the whole assembly is located in a uniform electric field E = mg/2Q acting vertically downwards. Neglect any interaction between the balls. Take g = 10 m/s^2

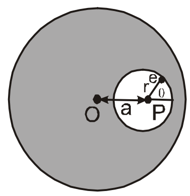
4. Two like charged, infinitely long parallel wires with the same linear charge density of 3×10^{-8} C/cm are 2 cm apart. The work done against electrostatic force per unit length to be done in bringing them closer by 1 cm is $\frac{x}{100}$ J/m: Find the integer closest to x.

5. The electric field at a point A on the perpendicular bisector of a uniformly charged wire of length ℓ = 3m and total charge q = 5 nC is x V/m. The distance of A from the centre of the wire is b = 2m. Find the value of x.

6. An infinitely long string uniformly charged with a linear charge density λ_1 and a segment of length ℓ uniformly charged with linear charge density λ_2 lie in a plane at right angles to each other and separated by a distance r_0 as shown in figure. The force with which these two interact is $\frac{\lambda_1 \lambda_2}{4\pi\varepsilon_0}\ell n(x)$. If $\ell = r_0$, then find the value of x.

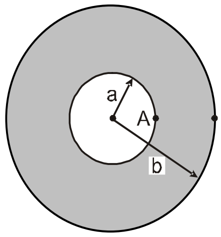


7. A cavity of radius r is present inside a solid dielectric sphere of radius R, having a volume charge density of ρ. The distance between the centres of the sphere and the cavity is a. An electron e is kept inside the cavity at an angle $\theta = 45^0$ as shown. The electron (mass m and charge –e) touches the sphere again after time $\left(\frac{P\sqrt{2}\, mr\varepsilon_0}{ea\rho}\right)^{1/2}$? Find the value of P. Neglect gravity.



8. A solid conductor sphere having a charge Q is surrounded by an uncharged concentric conducting hollow spherical shell. Let the potential difference between the surface of the solid sphere and that of the outer surface of the hollow shell be 30V. If the shell is now given a charge –3Q, the new potential difference between the same two surfaces is x V Find the value of x :

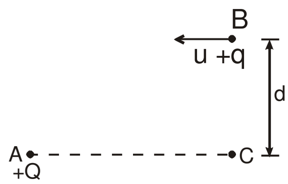
9. A hollow sphere having uniform charge density ρ (charge per unit volume) is shown in figure. If b = 2a and potential difference between A and B is $\frac{\rho a^2}{n\varepsilon_0}$. Then find the value of n :



10. Two identical particles of mass m carry a charge Q each. Initially, one is at rest on a smooth horizontal plane and the other is projected along the plane directly towards the first particle from a large distance, with a speed V. The closest distance of approach is $\frac{xQ^2}{4\pi\varepsilon_0 mv^2}$. Find the value of x :

11. A particle having charge + q is fixed at a point O and a second particle of mass m and having charge – q_0 moves with constant speed in a circle of radius r about the charge + q. The energy required to be supplied to the moving charge to increase radius of the path to 2 r is $\frac{qq_0}{n\pi\varepsilon_0 r}$. Find the value of n.

12. A positive charge +Q is fixed at a point A. Another positively charged particle of mass m and charge +q is projected from a point B with velocity u as shown in the figure. The point B is at large distance from A and at distance 'd' from the line AC. The initial velocity is parallel to the line AC. The point C is at very large distance from A. Find the minimum distance (in meter) of +q from +Q during the motion. Take Qq = $4\pi\epsilon_0$ mu^{2}d and $d = (\sqrt{2} - 1)$ meter.

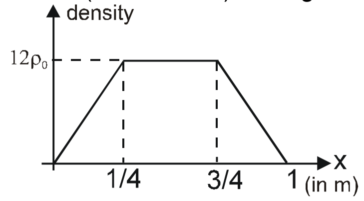


13. Small identical balls with equal charges of magnitude 'q' each are fixed at the vertices of a regular 2019-gon (a polygon of 2019 sides) with side 'a' = 4μm. At a certain instant, one of the balls is released and a sufficiently long time interval later, the ball adjacent to the first released ball is freed. The kinetic energies of the released balls are found to differ by K = 9 $\times 10^9$ J at a sufficiently large distance from the polygon. Determine the charge q in mC.

14. The electric potential varies in space according to the relation: V = 3x + 4y. A particle of mass 10 Kg starts from rest from point (2, 3.2) under the influence of this field. Find the speed in m/s of the particle when it crosses the x-axis. The charge on the particle is +1C. Assume V and (x, y) are in S.I. units.

15. The electric field in a region is given by $\vec{E} = E_0 x\hat{i}$. The charge contained inside a cubical volume bounded by the surface x = 0, x = 2m, y = 0, y = 2m, z = 0 and z = 2m is $n\varepsilon_0 E_0$. Find the value of n.

16. The volume charge density as a function of distance X from one face inside a unit cube is varying as shown in the figure. Find the total flux (in S.I. units) through the cube (If $\rho_0 = 8.85 \times 10^{-12}$ C/m^3). :



17. A very long uniformly charged thread oriented along the axis of a circle of radius R = 1m rests on its centre with one of the ends. The charge per unit length on the thread is $\lambda = 16\varepsilon_0$. Find the flux of the vector E through the circle area in (Vm).

18. The electric field in a region is radially outward with magnitude E = 2r. The charge contained in a sphere of radius a = 2m centred at the origin is $4x\pi\varepsilon_0$. Find the value of x.

19. Two isolated metallic solid spheres of radii R and 2R are charged such that both of these have same charge density 12 $\mu C/m^2$. The spheres are located far away from each other, and connected by a thin conducting wire. Find the new charge density on the bigger sphere in $\mu C/m^2$.

20. A metallic sphere of radius R is cut in two parts along a plane whose minimum distance from the sphere's centre is h = R/2 and the sphere is uniformly charged by a total electric charge Q. The minimum force necessary (to be applied on each of the two parts) to hold the two parts of the sphere together is $\frac{3kQ^2}{pR^2}$.
Then find the value of p ?

PART - III : ONE OR MORE THAN ONE OPTIONS CORRECT TYPE

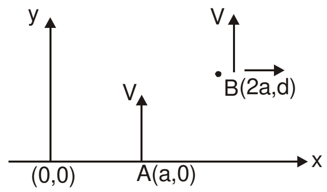
1. Select the correct alternative :
(A) The charge gained by the uncharged body from a charged body due to conduction is equal to half of the total charge initially present.
(B) The magnitude of charge increases with the increase in velocity of charge
(C) Charge cannot exist without matter although matter can exist without net charge
(D) Between two non–magnetic substances repulsion is the true test of electrification (electrification means body has net charge)

2. Two equal negative charges –q each are fixed at the points (0, a) and (0, –a) on the y-axis .A positive charge Q is released from rest at the point (2a, 0) on the x-axis. The charge Q will :
(A) Execute simple harmonic motion about the origin
(B) At origin velocity of particle is maximum.
(C) Move to infinity
(D) Execute oscillatory but not simple harmonic motion.

3. An oil drop has a charge – 9.6×10^{-19} C and mass 1.6×10^{-15} gm. When allowed to fall, due to air resistance force it attains a constant velocity. Then if a uniform electric field is to be applied vertically to make the oil drop ascend up with the same constant speed, which of the following are correct. ($g = 10\ ms^{-2}$) (Assume that the magnitude of resistance force is same in both the cases)
(A) The electric field is directed upward
(B) The electric field is directed downward
(C) The intensity of electric field is $1/3 \times 10^2\ NC^{-1}$
(D) The intensity of electric field is $1/6 \times 10^5\ NC^{-1}$

4. A non-conducting solid sphere of radius R is uniformly charged. The magnitude of the electric field due to the sphere at a distance r from its centre.
(A) increases as r increases, for $r \le R$
(B) decreases as r increases, for $0 < r < \infty$.
(C) decreases as r increases, for $R < r < \infty$.
(D) is discontinuous at r = R

5. A uniform electric field of strength E exists in a region. An electron (charge –e, mass m) enters a point A with velocity $V\hat{j}$. It moves through the electric field & exits at point B.Then



(A) $\vec{E} = -\frac{2amv^2}{ed^2}\hat{i}$.
(B) Rate of work done by the electric field at B is $\frac{4\,ma^2v^3}{d^3}$.
(C) Rate of work by the electric field at A is zero.
(D) Velocity at B is $\frac{2av}{d}\hat{i} + v\hat{j}$.

6. Which of the following quantites depends on the choice of zero potential or zero potential energy ?
(A) Potential at a particular point
(B) Change in potential energy of a two-charge system
(C) Potential energy of a two - charge system
(D) Potential difference between two points

7. The electric field intensity at a point in space is equal in magnitude to :
(A) Magnitude of the potential gradient there
(B) The electric charge there
(C) The magnitude of the electric force, a unit charge would experience there
(D) The force, an electron would experience there

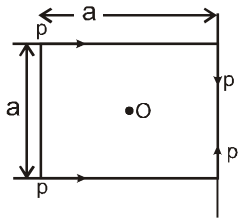
8. The electric field produced by a positively charged particle, placed in an xy-plane is 7.2 (4i + 3j) N/C at the point (3 cm, 3cm) and 100 $\hat{i}$ N/C at the point (2 cm, 0).
(A) The x-coordinate of the charged particle is –2cm.
(B) The charged particle is placed on the x-axis.
(C) The charge of the particle is 10 x 10^{-12} C.
(D) The electric potential at the origin due to the charge is 9V.

9. At distance of 5cm and 10cm outwards from the surface of a uniformly charged solid sphere, the potentials are 100V and 75V respectively. Then :
(A) Potential at its surface is 150V.
(B) The charge on the sphere is (5/3) × 10^{-9}C.
(C) The electric field on the surface is 1500 V/m
(D) The electric potential at its centre is 225V.

10. The electric potential decreases uniformly from 180 V to 20 V as one moves on the X-axis from x = – 2 cm to x = + 2 cm. The electric field at the origin :
(A) must be equal to 40V/cm.
(B) may be equal to 40V/cm.
(C) may be greater than 40V/cm.
(D) may be less than 40V/cm.

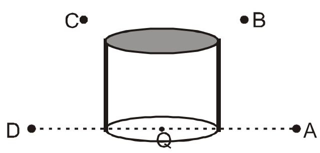
11. An electric dipole is kept in the electric field produced by a point charge.
(A) dipole will experience a force.
(B) dipole will experience a torque.
(C) it is possible to find a path (not closed) in the field on which work required to move the dipole is zero.
(D) dipole can be in stable equilibrium.

12. Four short dipoles each of dipole moment 'p' are placed at the vertices of a square of side a. The direction of the dipole moments are shown in the figure.



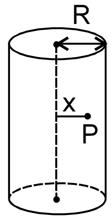
(A) Electric field at O is $\frac{\sqrt{2}\,p}{2\pi \epsilon_0 a^3}$
(B) Electric field at O is $\frac{\sqrt{2}\,p}{\pi \epsilon_0 a^3}$
(C) Electrostatic potential at O is zero
(D) Net dipole moment is 2p

13. Figure shows a charge Q placed at the centre of open face of a cylinder as shown in figure. A second charge q is placed at one of the positions A, B, C and D, out of which positions A and D are lying on a straight line parallel to open face of cylinder. In which position(s) of this second charge, the flux of the electric field through the cylinder remains unchanged ?



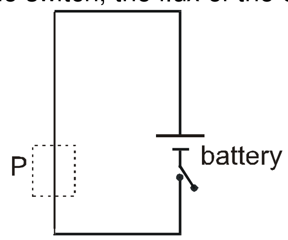
(A) A
(B) B
(C) C
(D) D

14. A long cylindrical volume (of radius R) contains a uniformly distributed charge of density ρ. Consider a point P inside the cylindrical volume at a distance x from its axis as shown in the figure. Here x can be more than or less than R.Electric field at point P is :



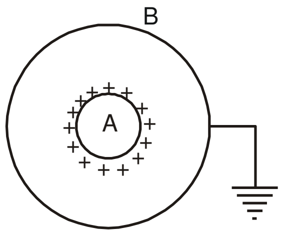
(A) $\frac{\rho x}{2\varepsilon_0}$ if $x < R$ (B) $\frac{\rho x}{\varepsilon_0}$ if $x < R$ (C) $\frac{\rho R^2}{4\varepsilon_0 x}$ if $x > R$ (D) $\frac{\rho R^2}{2\varepsilon_0 x}$ if $x > R$

15. An imaginary closed surface P is constructed around a neutral conducting wire connected to a battery and a switch as shown in figure. As the switch is closed, the free electrons in the wire start moving along the wire. In any time interval, the number of electrons entering the closed surface P is equal to the number of electrons leaving it. On closing the switch, the flux of the electric field through the closed surface:



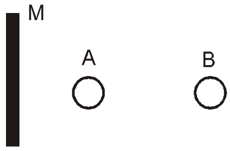
(A) remains unchanged (B) remains zero (C) is increased (D) is decreased

16. A and B are two conducting concentric spherical shells. A is given a charge Q while B is uncharged. If now B is earthed as shown in figure. Then :



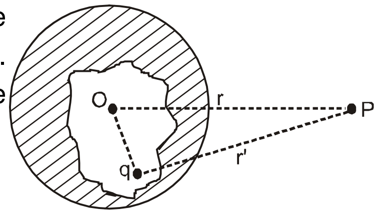
(A) The charge appearing on inner surface of B is -Q
(B) The field inside and outside A is zero.
(C) The field between A and B is not zero.
(D) The charge appearing on outer surface of B is zero.

17. A large nonconducting sheet M is given a uniform charge density. Two uncharged small metal spheres A and B are placed near the sheet as shown in figure.



(A) M attracts A (B) A attracts B (C) M attracts B (D) B attracts A

18. A point charge 'q' is within an electrically neutral conducting shell whose outer surface is a sphere of radius R. The centre of outer surface is at O. Consider a point P outside the conductor as shown in the figure. The magnitude of electric field at P

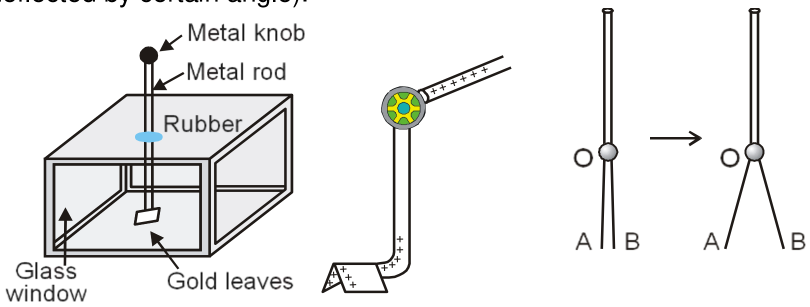


(A) due to charge induced on inner surface of the conductor is zero
(B) due to charge induced on inner surface of the conductor is $kq/(r')^2$
(C) due to charge induced on outer surface of the conductor is kq/r^2
(D) due to charge induced on surface of the conductor is kq/r^2

PART - IV : COMPREHENSION

COMPREHENSION # 1

A leaf electroscope is a simple apparatus to detect any charge on a body. It consists of two metal leaves OA and OB, free to rotate about O. Initially both are very slightly separated. When a charged object is touched to the metal knob at the top of the conducting rod, charge flows from knob to the leaves through the conducting rod. As the leaves are now charged similarly, they start repelling each other and get separated, (deflected by certain angle).



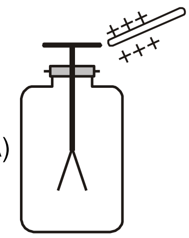
The angle of deflection in static equilibrium is an indicator of the amount of charge on the charged body.

1. When a + 20 C rod is touched to the knob, the deflection of leaves was 5°, and when an identical rod of – 40 C is touched, the deflection was found to be 9°. If an identical rod of +30 C is touched, then the deflection may be :

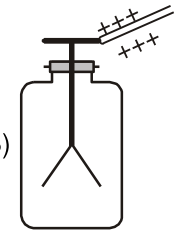
(A) 0 (B) 2° (C) 7° (D) 11°

2. If we perform these steps one by one.

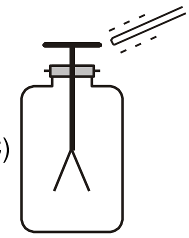
(i) A positively charged rod is brought closer to initially uncharged knob (A)



(ii) Then the positively charged rod is touched to the knob (B)



(iii) Now the +vely charged rod is removed, and a negatively charged. (C)



rod of same magnitude is brought closer at same distance

In which case, the leaves will converge (come closer), as compared to the previous state ?

(A) (i) (B) (i) and (iii)

(C) only (iii) (D) In all cases, the leaves will diverge

3. In an electroscope, both leaves are hinged at the top point O. Each leaf has mass m, length ℓ and gets charge q. Assuming the charge to be concentrated at ends A and B only, the small angle of deviation (θ) between the leaves in static equilibrium, is equal to :

(A) $\left(\frac{4kq^2}{\ell^2 mg}\right)^{1/3}$ (B) $\left(\frac{kq^2}{\ell^2 mg}\right)^{1/3}$ (C) $\left(\frac{2kq^2}{\ell^2 mg}\right)^{1/2}$ (D) $\left(\frac{64kq^2}{\ell^2 mg}\right)^{1/3}$

COMPREHENSION # 2

A charged particle is suspended at the centre of two thin concentric spherical charged shells, made of non conducting material. Figure A shows cross section of the arrangement. Figure B gives the net flux ϕ through a Gaussian sphere centered on the particle, as a function of the radius r of the sphere.

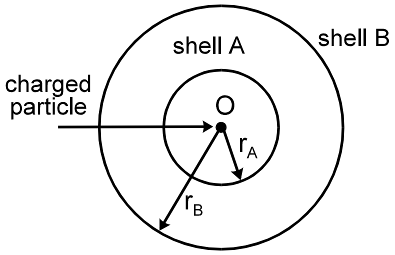


Figure A

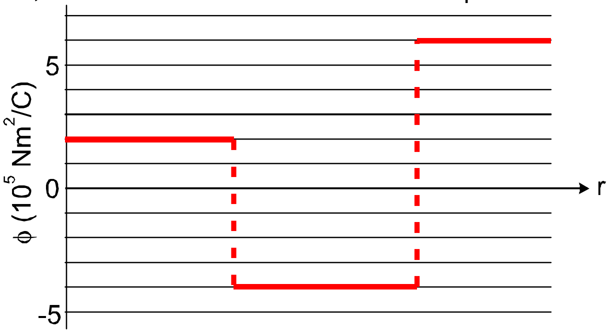


Figure B

4. What is the charge on the central particle ?

(A) 0.2 μC (B) 2 μC (C) 1.77 μC (D) 3.4 μC

5. What is the charge on shell A ?

(A) 5.31×10^{-6} C (B) -5.31×10^{-6} C (C) -3.54×10^{-6} C (D) -1.77×10^{-6} C

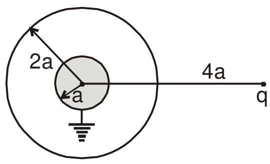
6. In which range of the values of r is the electric field zero ?

(A) 0 to r_A (B) r_A to r_B (C) For $r > r_B$

(D) For no range of r, electric field is zero.

COMPREHENSION # 3

A solid conducting sphere of radius 'a' is surrounded by a thin uncharged concentric conducting shell of radius 2a. A point charge q is placed at a distance 4a from common centre of conducting sphere and shell. The inner sphere is then grounded.



7. The charge on solid sphere is :

(A) $-\frac{q}{2}$ (B) $-\frac{q}{4}$ (C) $-\frac{q}{8}$ (D) $-\frac{q}{16}$

8. Pick up the correct statement.

(A) Charge on surface of inner sphere is non-uniformly distributed.
(B) Charge on inner surface of outer shell is non-uniformly distributed.
(C) Charge on outer surface of outer shell is non-uniformly distributed.
(D) All the above statements are false.

9. The potential of outer shell is :

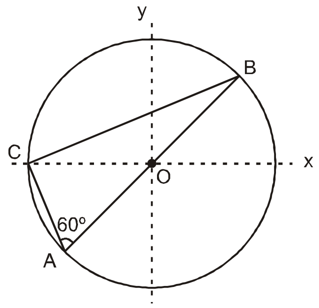
(A) $\frac{q}{32\pi\varepsilon_o a}$ (B) $\frac{q}{16\pi\varepsilon_o a}$ (C) $\frac{q}{8\pi\varepsilon_o a}$ (D) $\frac{q}{4\pi\varepsilon_o a}$

Exercise-3

✍ **Marked Questions can be used as Revision Questions.**
*** Marked Questions may have more than one correct option.**

PART - I : JEE (ADVANCED) / IIT-JEE PROBLEMS (PREVIOUS YEARS)

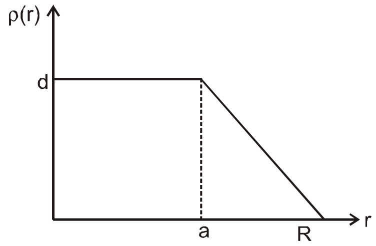
1.✍ Consider a system of three charges $\frac{q}{3}$, $\frac{q}{3}$ and $-\frac{2q}{3}$ placed at points A, B and C, respectively, as shown in the figure. Take O to be the centre of the circle of radius R and angle CAB = 60º. **[JEE 2008, 3/163]**



(A) The electric field at point O is $\frac{q}{8\pi\varepsilon_0 R^2}$ directed along the negative x-axis.
(B) The potential energy of the system is zero.
(C) The magnitude of the force between the charges at C and B is $\frac{q^2}{54\pi\varepsilon_0 R^2}$
(D) The potential at point O is $\frac{q}{12\pi\varepsilon_0 R}$.

Paragraph for Question Nos. 2 to 4 ✍

The nuclear charge (Ze) is non–uniformly distributed within a nucleus of radius R. The charge density $\rho(r)$ [charge per unit volume] is dependent only on the radial distance r from the centre of the nucleus as shown in figure. The electric field is only along the radial direction. **[JEE 2008 ; 4 × 3 = 12/163]**
Figure :



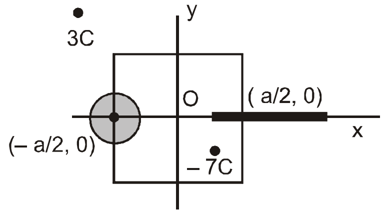
2.✍ The electric field at r = R is :
(A) independent of a
(B) directly proportional to a
(C) directly proportional to a^2
(D) inversely proportional to a

3.✍ For a = 0, the value d (maximum value of ρ as shown in the figure) is :
(A) $\frac{3Ze^2}{4\pi R^3}$ (B) $\frac{3Ze}{\pi R^3}$ (C) $\frac{4Ze}{3\pi R^3}$ (D) $\frac{Ze}{3\pi R^3}$

4.✍ The electric field within the nucleus is generally observed to be linearly dependent on r. This implies :
(A) a = 0 (B) a = R/2 (C) a = R (D) a = 2R/3

5. **STATEMENT-1 :** For practical purposes, the earth is used as a reference at zero potential in electrical circuits. **[JEE 2008, 3, –1/163]**
and
STATEMENT-2 : The electrical potential of a sphere of radius R with charge Q uniformly distributed on the surface is given by $\frac{Q}{4\pi\varepsilon_0 R}$.
(A) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 **is** a correct explanation for STATEMENT-1
(B) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 **is NOT** a correct explanation for STATEMENT-1
(C) STATEMENT-1 is True, STATEMENT-2 is False
(D) STATEMENT-1 is False, STATEMENT-2 is True.

6. A disk of radius a/4 having a uniformly distributed charge 6C is placed in the x-y plane with its centre at (–a/2, 0, 0). A rod of length a carrying a uniformly distributed charge 8C is placed on the x-axis from x = a/4 to x = 5a/4. Two point charges –7C and 3C are placed at (a/4, –a/4, 0) and (–3a/4, 3a/4, 0), respectively. Consider a cubical surface formed by six surfaces x = ± a/2, y = ± a/2, z = ±a/2. The electric flux through this cubical surface is : **[JEE 2009, 3/160, –1]**



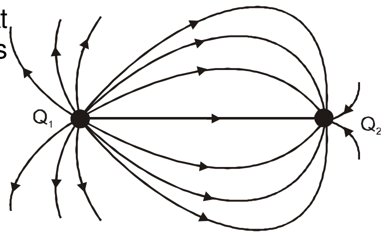
(A) $\frac{-2C}{\varepsilon_0}$ (B) $\frac{2C}{\varepsilon_0}$ (C) $\frac{10C}{\varepsilon_0}$ (D) $\frac{12C}{\varepsilon_0}$

7. Three concentric metallic spherical shells of radii R, 2R, 3R, are given charges Q_1, Q_2, Q_3, respectively. It is found that the surface charge densities on the outer surfaces of the shells are equal. Then, the ratio of the charges given to the shells, $Q_1 : Q_2 : Q_3$, is **[JEE 2009, 3/160, –1]**
(A) 1 : 2 : 3 (B) 1 : 3 : 5 (C) 1 : 4 : 9 (D) 1 : 8 : 18

8. Under the influence of the Coulomb field of charge +Q, a charge –q is moving around it in an elliptical orbit. Find out the correct statement(s). **[JEE 2009, 4/160, –1]**
(A) The angular momentum of the charge –q is constant.
(B) The linear momentum of the charge –q is constant.
(C) The angular velocity of the charge – q is constant.
(D) The linear speed of the charge –q is constant.

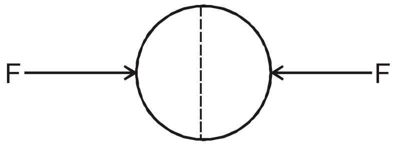
9. A solid sphere of radius R has a charge Q distributed in its volume with a charge density $\rho = kr^a$, where k and a are constants and r is the distance from its centre. If the electric field at r = R/2 is 1/8 times that at r = R, find the value of a. **[JEE 2009, 4/160, –1]**

10. A few electric field lines for a system of two charges Q_1 and Q_2 fixed at two different points on the x-axis are shown in the figure. These lines suggest that : **[JEE 2010, 3/163]**



(A) $|Q_1| > |Q_2|$
(B) $|Q_1| < |Q_2|$
(C) at a finite distance to the left of Q_1 the electric field is zero
(D) at a finite distance to the right of Q_2 the electric field is zero

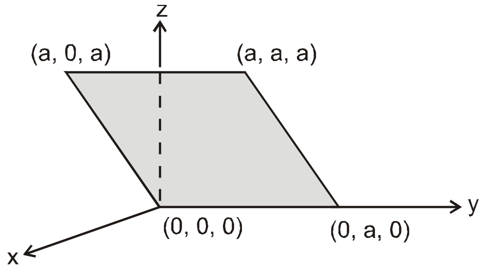
11. A uniformly charged thin spherical shell of radius R carries uniform surface charge density of σ per unit area. It is made of two hemispherical shells, held together by pressing them with force F (see figure). F is proportional to **[JEE 2010, 5/163, –2]**



(A) $\frac{1}{\varepsilon_0}\sigma^2R^2$ (B) $\frac{1}{\varepsilon_0}\sigma^2R$ (C) $\frac{1}{\varepsilon_0}\frac{\sigma^2}{R}$ (D) $\frac{1}{\varepsilon_0}\frac{\sigma^2}{R^2}$

12. A tiny spherical oil drop carrying a net charge q is balanced in still air with a vertical uniform electric field of strength $\frac{81\pi}{7}\times10^5$ Vm^{-1}. When the field is switched off, the drop is observed to fall with terminal velocity 2×10^{-3} m s^{-1}. Given g = 9.8 m s^{-2}, viscosity of the air = 1.8×10^{-5} Ns m^{-2} and the density of oil = 900 kg m^{-3}, the magnitude of q is : **[JEE 2010, 5/163, –2]**
(A) 1.6×10^{-19} C (B) 3.2×10^{-19} C (C) 4.8×10^{-19} C (D) 8.0×10^{-19} C

13. Consider an electric field $\vec{E} = E_0\hat{x}$, where E_0 is a constant. The flux through the shaded area (as shown in the figure) due to this field is : **[JEE-2011, 3/160, –1]**

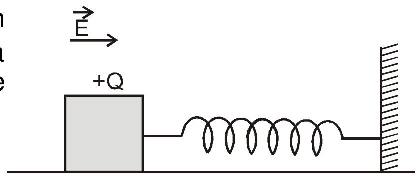


(A) $2E_0a^2$ (B) $\sqrt{2}\,E_0a^2$ (C) E_0a^2 (D) $\dfrac{E_0a^2}{\sqrt{2}}$

14*. A spherical metal shell A of radius R_A and a solid metal sphere B of radius R_B (< R_A) are kept far apart and each is given charge '+Q'. Now they are connected by a thin metal wire. Then **[JEE-2011, 4/160]**

(A) $E_A^{inside} = 0$ (B) $Q_A > Q_B$ (C) $\dfrac{\sigma_A}{\sigma_B} = \dfrac{R_B}{R_A}$ (D) $E_A^{on\,surface} < E_B^{on\,surface}$

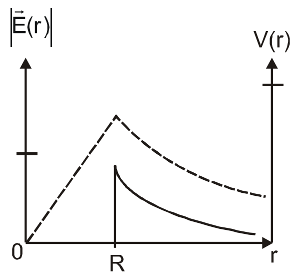
15. **A** wooden block performs SHM on a frictionless surface with frequency, ν_0. The block carries a charge +Q on its surface. If now a uniform electric field **E** is switched-on as shown, then the SHM of the block will be **[JEE 2011, 3/160, –1]**

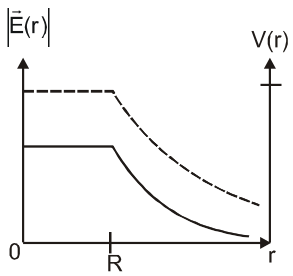


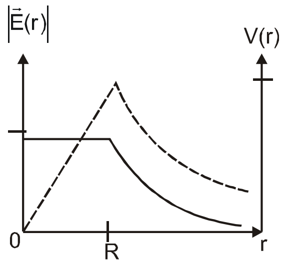
(A) of the same frequency and with shifted mean position.
(B) of the same frequency and with the same mean position.
(C) of changed frequency and with shifted mean position.
(D) of changed frequency and with the same mean position.

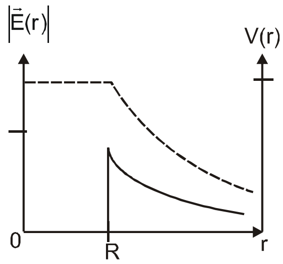
16. Which of the following statement(s) is/are correct? **[JEE 2011, 4/160]**
(A) If the electric field due to a point charge varies as $r^{-2.5}$ instead of r^{-2}, then the Gauss law will still be valid.
(B) The Gauss law can be used to calculate the field distribution around an electric dipole.
(C) If the electric field between two point charges is zero somewhere, then the sign of the two charges is the same.
(D) The work done by the external force in moving a unit positive charge from point A at potential V_A to point B at potential V_B is ($V_B - V_A$).

17. Consider a thin spherical shell of radius R with its centre at the origin, carrying uniform positive surface charge density. The variation of the magnitude of the electric field $|\vec{E}(r)|$ and the electric potential V(r) with the distance r from the centre, is best represented by which graph? **[JEE 2012, Paper-1 : 3/70, –1]**

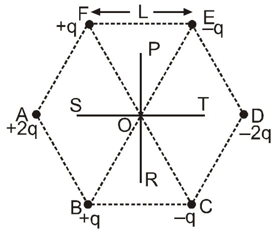
(A)


(B)


(C)


(D)


18*. Six point charges are kept at the vertices of a regular hexagon of side L and centre O, as shown in the figure. Given that $K = \frac{1}{4\pi\varepsilon_0}\frac{q}{L^2}$, which of the following statement (s) is (are) correct? **[JEE 2012, Paper-1 : 4/66]**

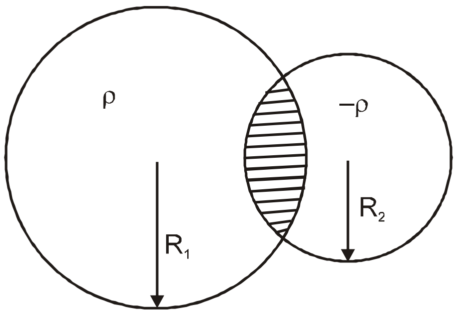


(A) the elecric field at O is 6K along OD
(B) The potential at O is zero
(C) The potential at all points on the line PR is same
(D) The potential at all points on the line ST is same.

19*. Two non-conducting solid spheres of radii R and 2R, having uniform volume charge densities ρ_1 and ρ_2 respectively, touch each other. The net electric field at a distance 2R from the centre of the smaller sphere, along the line joining the centres of the spheres, is zero. The ratio ρ_1/ρ_2 can be ; **[JEE (Advanced)-2013; 3/60, –1]**

(A) –4 (B) $-\frac{32}{25}$ (C) $\frac{32}{25}$ (D) 4

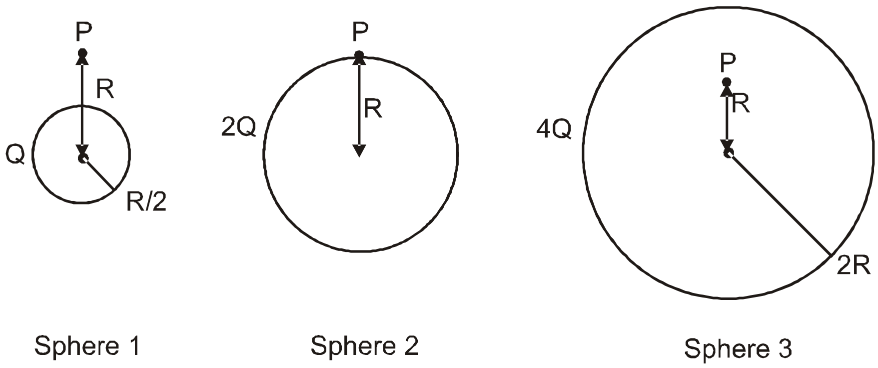
20*. Two non–conducting spheres of radii R_1 and R_2 and carrying uniform volume charge densities $+\rho$ and $-\rho$, respectively, are placed such that they partially overlap, as shown in the figure. At all points in the overlapping region : **[JEE (Advanced) 2013; 3/60, –1]**



(A) the electrostatic field is zero (B) the electrostatic potential is constant
(C) the electrostatic field is constant in magnitude (D) the electrostatic field has same direction

21. Charges Q, 2Q and 4Q are uniformly distributed in three dielectric solid spheres 1, 2 and 3 of radii R/2, R and 2R respectively, as shown in figure. If magnitudes of the electric fields at point P at a distance R from the centre of spheres 1, 2 and 3 are E_1 E_2 and E_3 respectively, then

[JEE (Advanced) 2014, 3/60, –1]



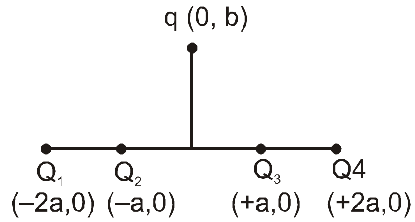
(A) $E_1 > E_2 > E_3$ (B) $E_3 > E_1 > E_2$ (C) $E_2 > E_1 > E_3$ (D) $E_3 > E_2 > E_1$

22. Let $E_1(r)$, $E_2(r)$ and $E_3(r)$ be the respective electric fields at a distance r from a point charge Q, an infinitely long wire with constant linear charge density λ, and an infinite plane with uniform surface charge density σ. if $E_1(r_0) = E_2(r_0) = E_3(r_0)$ at a given distance r_0, then **[JEE (Advanced) 2014, P-1, 3/60]**

(A) $Q = 4\sigma\pi r_0^2$ (B) $r_0 = \frac{\lambda}{2\pi\sigma}$ (C) $E_1(r_0/2) = 2E_2(r_0/2)$ (D) $E_2(r_0/2) = 4E_3(r_0/2)$

23. Four charge Q_1, Q_2, Q_3 and Q_4 of same magnitude are fixed along the x axis at x = –2a –a, +a and +2a, respectively. A positive charge q is placed on the positive y axis at a distance b > 0. Four options of the signs of these charges are given in List-I. The direction of the forces on the charge q is given in List-II Match List-1 with List-II and select the correct answer using the code given below the lists.

[JEE (Advanced)-2014, 3/60, –1]

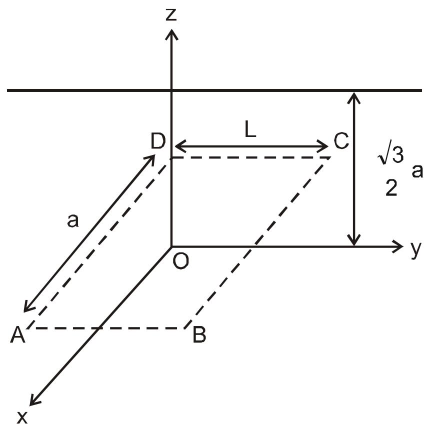


	List-I		List-II
P.	Q_1, Q_2, Q_3, Q_4 all positive	**1.**	+x
Q.	Q_1, Q_2 positive Q_3, Q_4 negative	**2.**	–x
R.	Q_1, Q_4 positive Q_2, Q_3 negative	**3.**	+y
S.	Q_1, Q_3 positive Q_2, Q_4 negative	**4.**	–y

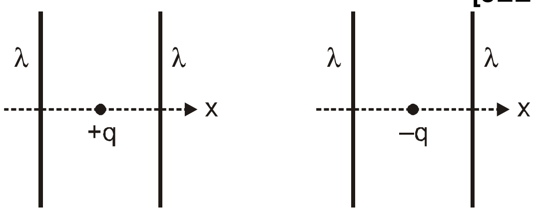
Code :

(A) P-3, Q-1, R-4, S-2 (B) P-4, Q-2, R-3, S-1 (C) P-3, Q-1, R-2, S-4 (D) P-4, Q-2, R-1, S-3

24. An infinitely long uniform line charge distribution of charge per unit length λ lies parallel to the y-axis in the y-z plane at $z = \frac{\sqrt{3}}{2}a$ (see figure). If the magnitude of the flux of the electric field through the rectangular surface ABCD lying in the x-y plane with its centre at the origin is $\frac{\lambda L}{n\varepsilon_0}$ (ε_0 = permittivity of free space), then the value of n is : **[JEE (Advanced) 2015 ; P-1, 4/88]**



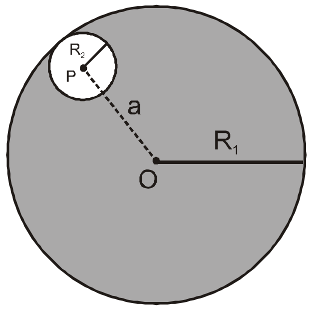
25. The figures below depict two situations in which two infinitely long static line charges of constant positive line charge density λ are kept parallel to each other. In their resulting electric field, point charges q and –q are kept in equilibrium between them. The point charges are confined to move in the x-direction only. If they are given a small dispacement about their equilibrium positions, then the correct ststement(s) is (are) : **[JEE (Advanced) 2015 ; P-1 4/88, –2]**



(A) Both charges execute simple harmonic motion.
(B) Both charges will continue moving in the direction of their displacement.
(C) Charge +q executes simple haramonic motion while charge –q continues moving in the direction of its displacement.
(D) Charge –q executes simple haramonic motion while charge +q continues moving in the direction of its displacement.

26. Consider a uniform speherical charge distribution of radius R_1 centred at the origin O. In this distribution, a spherical cavity of radius R_2, centred at P with distance OP = a = $R_1 - R_2$ (see figure) is made. If the electric field inside the cavity at position $\vec{r}$ is $\vec{E}(\vec{r})$, then the correct statement(s) is(are)

[JEE (Advanced) 2015 ; P-2,4/88, –2]



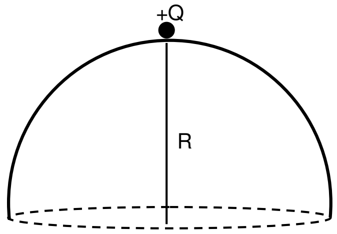
(A) $\vec{E}$ is uniform, its magnitude is independent of R_2 but its direction depends on $\vec{r}$

(B) $\vec{E}$ is uniform, its magnitude depends on R_2 and its direction depends on $\vec{r}$

(C) $\vec{E}$ is uniform, its magnitude is independent of a but its direction depends on $\vec{a}$

(D) $\vec{E}$ is uniform, and both its magnitude and direction depends on $\vec{a}$

27. A point charge +Q is placed just outside an imaginary hemispherical surface of radius R as shown in the figure. Which of the following statements is/are correct ? **[JEE (Advanced) 2017 ; 4/61, –2]**



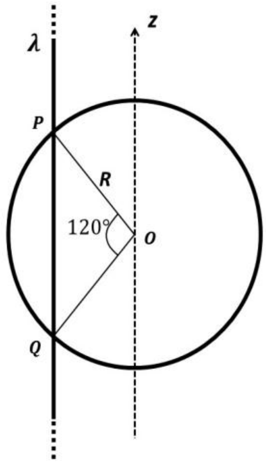
(A) Total flux through the curved and the flat surface is Q/ε_0

(B) The component of the electric field normal to the flat surface is constant over the surface

(C) The circumference of the flat surface is an equipotential

(D) The electric flux passing through the curved surface of the hemisphere is $-\frac{Q}{2\varepsilon_0}\left(1-\frac{1}{\sqrt{2}}\right)$

28*. An infinitely long thin non-conducting wire is parallel to the z-axis and carries a uniform line charge density λ. It pierces a thin non-conducting spherical shell of radius R in such a way that the arc PQ subtends an angle 120° at the centre O of the spherical shell, as shown in the figure. The permittivity of free space is ε_0. Which of the following statements is (are) true? **[JEE (Advanced) 2018 ; 4/60, –2]**



(A) The electric flux through the shell is $\sqrt{3}R\lambda / \varepsilon_0$

(B) The z-component of the electric field is zero at all the points on the surface of the shell

(C) The electric flux through the shell is $\sqrt{2}R\lambda / \varepsilon_0$

(D) The electric field is normal to the surface of the shell at all points

29. A particle, of mass 10^{-3}kg and charge 1.0 C, is initially at rest. At time t = 0, the particle comes under the influence of an electric field $\vec{E}(t) = E_0 \sin\omega t \hat{i}$ where E_0 = 1.0 NC^{-1} and $\omega = 10^3$ rad s^{-1}. Consider the effect of only the electrical force on the particle. Then the maximum speed, in ms^{-1}, attained by the particle at subsequent times is ____________. **[JEE (Advanced) 2018 ; 3/60]**

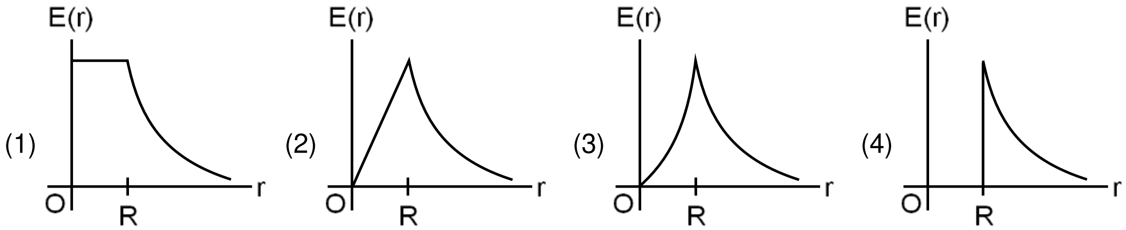
30. The electric field E is measured at a point P(0, 0, d) generated due to various charge distributions and the dependence of E on d is found to be different for different charge distributions. List-I contains different relations between E and d. List-II describes different electric charge distributions, along with their locations. Match the functions in List-I with the related charge distributions in List-II. **[JEE (Advanced) 2018 ; 3/60, –1]**

List-I		**List-II**	
P.	E is independent of d	1.	A point charge Q at the origin
Q.	$E \propto 1/d$	2.	A small dipole with point charges Q at (0, 0, ℓ) and –Q at (0, 0, $-\ell$) Take $2\ell << d$
R.	$E \propto 1/d^2$	3.	An infinite line charge coincident with the x-axis, with uniform linear charge density λ.
S.	$E \propto 1/d^3$	4.	Two infinite wires carrying uniform linear charge density parallel to the x-axis. The one along (y = 0, z = ℓ) has a charge density $+\lambda$ and the one along (y = 0, z = $-\ell$) has a charge density $-\lambda$. Take $2\ell << d$
		5.	Infinite plane charge coincident with the xy-plane with uniform surface charge density.

(A) P → 5 ; Q → 3, 4 ; R → 1 ; S → 2
(B) P → 5 ; Q → 3 ; R → 1, 4 ; S → 2
(C) P → 5 ; Q → 3 ; R → 1, 2; S → 4
(D) P → 4 ; Q → 2, 3 ; R → 1 ; S → 5

PART - II : JEE (MAIN) / AIEEE PROBLEMS (PREVIOUS YEARS)

1. A thin spherical shell of radius R has charge Q spread uniformly over its surface. Which of the following graphs most closely represents the electric field E (r) produced by the shell in the range $0 \le r < \infty$, where r is the distance from the centre of the shell? **[AIEEE-2008, 3/105]**



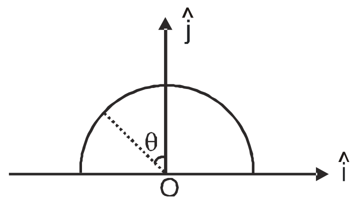
2. Two points P and Q are maintained at the potentials of 10 V and –4 V respectively. The work done in moving 100 electrons from P to Q is : **[AIEEE-2009, 4/144]**
(1) 9.60×10^{-17} J (2) -2.24×10^{-16} J (3) 2.24×10^{-16} J (4) -9.60×10^{-17} J

3. A charge Q is placed at each of the opposite corners of a square. A charge q is placed at each of the other two corners. If the net electrical force on Q is zero, then Q/q equals: **[AIEEE-2009, 4/144]**
(1) –1 (2) 1 (3) $-\frac{1}{\sqrt{2}}$ (4) $-2\sqrt{2}$

4. **STATEMENT 1 :** For a charged particle moving from point P to point Q, the net work done by an electrostatic field on the particle is independent of the path connecting point P to point Q. **[AIEEE 2009, 6/144]**
STATEMENT 2 : The net work done by a conservative force on an object moving along a closed loop is zero.
(1) Statement-1 is true, Statement-2 is true; Statement-2 is the correct explanation of Statement-1.
(2) Statement-1 is true, Statement-2 is true; Statement-2 is not the correct explanation of Statement-1.
(3) Statement-1 is false, Statement-2 is true.
(4) Statement-1 is true, Statement-2 is false.

5.✍ Let $\rho(r) = \frac{Q}{\pi R^4} r$ be the charge density distribution for a solid sphere of radius R and total charge Q. For a point 'P' inside the sphere at distance r_1 from the centre of sphere, the magnitude of electric field is : **[AIEEE 2009, 4/144]**
(1) $\frac{Q}{4\pi\varepsilon_0 r_1^2}$ (2) $\frac{Qr_1^2}{4\pi\varepsilon_0 R^4}$ (3) $\frac{Qr_1^2}{3\pi\varepsilon_0 R^4}$ (4) 0

6. A thin semi-circular ring of radius r has a positive charge q distributed uniformly over it. The net field $\vec{E}$ at the centre O is : **[AIEEE 2010, 4/144]**



(1) $\frac{q}{4\pi^2\varepsilon_0 r^2}\hat{j}$ (2) $-\frac{q}{4\pi^2\varepsilon_0 r^2}\hat{j}$ (3) $-\frac{q}{2\pi^2\varepsilon_0 r^2}\hat{j}$ (4) $\frac{q}{2\pi^2\varepsilon_0 r^2}\hat{j}$

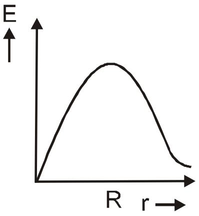
7. Let there be a spherically symmetric charge distribution with charge density varying as $\rho(r) = \rho_0\left(\frac{5}{4} - \frac{r}{R}\right)$ upto r = R, and $\rho(r) = 0$ for r > R, where r is the distance from the origin. The electric field at a distance r (r < R) from the origin is given by **[AIEEE 2010, 4/144]**
(1) $\frac{4\pi\rho_0 r}{3\varepsilon_0}\left(\frac{5}{3} - \frac{r}{R}\right)$ (2) $\frac{\rho_0 r}{4\varepsilon_0}\left(\frac{5}{3} - \frac{r}{R}\right)$ (3) $\frac{4\rho_0 r}{3\varepsilon_0}\left(\frac{5}{4} - \frac{r}{R}\right)$ (4) $\frac{\rho_0 r}{3\varepsilon_0}\left(\frac{5}{4} - \frac{r}{R}\right)$

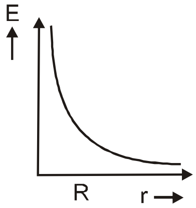
8.✍ Two identical charged spheres are suspended by strings of equal lengths. The strings make an angle of 30º with each other. When suspended in a liquid of density 0.8 g cm^{-3}, the angle remains the same. If density of the material of the sphere is 1.6 g cm^{-3}, the dielectric constant of the liquid is **[AIEEE 2010, 8/144]**
(1) 4 (2) 3 (3) 2 (4) 1

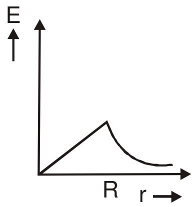
9. The electrostatic potential inside a charged spherical ball is given by $\phi = ar^2 + b$ where r is the distance from the centre; a,b are constants. Then the charge density inside the ball is : **[AIEEE 2011, 4/120, –1]**
(1) $-24\pi a\varepsilon_0 r$ (2) $-6\pi a\varepsilon_0 r$ (3) $-24\pi a\varepsilon_0$ (4) $-6 a\varepsilon_0$

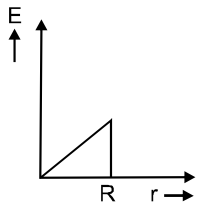
10. Two positive charges of magnitude 'q' are placed at the ends of a side (side 1) of a square of side '2a'. Two negative charges of the same magnitude are kept at the other corners. Starting from rest, if a charge Q moves from the middle of side 1 to the centre of square, its kinetic energy at the centre of square is : **[AIEEE 2011, 11 May; 4, –1]**
(1) zero (2) $\frac{1}{4\pi\varepsilon_0}\frac{2qQ}{a}\left(1+\frac{1}{\sqrt{5}}\right)$ (3) $\frac{1}{4\pi\varepsilon_0}\frac{2qQ}{a}\left(1-\frac{2}{\sqrt{5}}\right)$ (4) $\frac{1}{4\pi\varepsilon_0}\frac{2qQ}{a}\left(1-\frac{1}{\sqrt{5}}\right)$

11. In a uniformly charged sphere of total charge Q and radius R, the electric field E is plotted as function of distance from the centre. The graph which would correspond to the above will be :
[AIEEE 2012 ; 4/120, –1]

(1) 

(2) 

(3) 

(4) 

12. This question has statement-1 and statement-2. Of the four choices given after the statements, choose the one that best describes the two statements. **[AIEEE 2012 ; 4/120, –1]**
An insulating solid sphere of radius R has a unioformly positive charge density ρ. As a result of this uniform charge distribution there is a finite value of electric potential at the centre of the sphere, at the surface of the sphere and also at a point out side the sphere. The electric potential at infinite is zero.
STATEMENT-1 : When a charge 'q' is taken from the centre to the surface of the sphere its potential energy changes by $q\rho/3\varepsilon_0$.
STATEMENT-2 : The electric field at a distance r (r < R) from the centre of the sphere is $\rho r/3\varepsilon_0$.
(1) Statement-1 is true, Statement-2 is true; Statement-2 is not the correct explanation of statement-1.
(2) Statement-1 is true Statement-2 is false.
(3) Statement-1 is false Statement-2 is true.
(4) Statement-1 is true, Statement-2 is true, Statement-2 is the correct explanation of Statement-1.

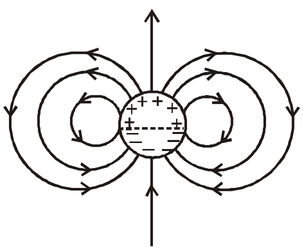
13. Two charges, each equal to q, are kept at x = – a and x = a on the x-axis. A particle of mass m and charge $q_0 = q/2$ is placed at the origin. If charge q_0 is given a small displacement (y <<a) along the y-axis, the net force acting on the particle is proportional to : **[JEE (Main) 2013, 4/120, –1]**
(1) y (2) –y (3) 1/y (4) – 1/y

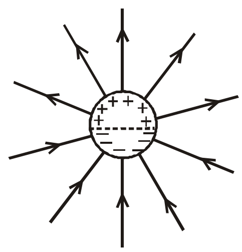
14. A charge Q is uniformly distributed over a long rod AB of length L as shown in the figure. The electric potential at the point O lying at distance L from the end A is : **[JEE (Main) 2013, 4/120, –1]**

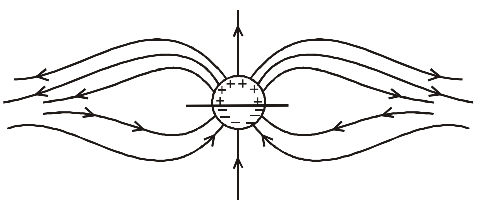
(1) $\dfrac{Q}{8\pi \in_0 L}$ (2) $\dfrac{3Q}{4\pi \in_0 L}$ (3) $\dfrac{Q}{4\pi \in_0 L\ln 2}$ (4) $\dfrac{Q\ln 2}{4\pi \in_0 L}$

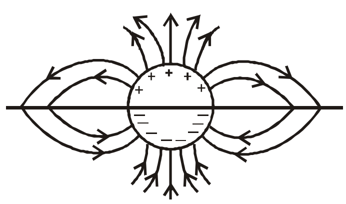
15. Assume that an electric field $\vec{E} = 30x^2\hat{i}$ exists in space. Then the potential difference $V_A - V_O$, where V_O is the potential at the origin and V_A the potential at x = 2 m is : **[JEE (Main) 2014, 4/120, –1]**
(1) 120 J (2) –120 J (3) – 80 J (4) 80 J

16. A long cylindrical shell carries positive surface charge σ in the upper half and negative surface charge $-\sigma$ in the lower half. The electric field lines around the cylinder will look like figure given in : (figures are schematic and not drawn to scale) **[JEE (Main) 2015; 4/120, –1]**

(1) 

(2) 

(3) 

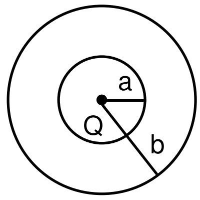
(4) 

17.* A uniformly charged solid sphere of radius R has potential V_0 (measured with respect to ∞) on its surface. For this sphere the equipotential surfaces with potentials $\frac{3V_0}{2}, \frac{5V_0}{4}, \frac{3V_0}{4}$ and $\frac{V_0}{4}$ have radius R_1, R_2, R_3 and R_4 respectively. Then **[JEE (Main) 2015; 4/120, –1]**

(1) $R_1 = 0$ and $R_2 > (R_4 - R_3)$ (2) $R_1 \neq 0$ and $(R_2 - R_1) > (R_4 - R_3)$

(3) $R_1 = 0$ and $R_2 < (R_4 - R_3)$ (4) $2R < R_4$

18. The region between two concentric spheres of radii 'a' and 'b', respectively (see figure), has volume charge density $\rho = A/r$, where A is a constant and r is the distance from the centre. At the centre of the spheres is a point charge Q. The value of A such that the electric field in the region between the spheres will be constant, is : **[JEE (Main) 2016 ; 4/120, –1]**



(1) $\frac{Q}{2\pi(b^2 - a^2)}$ (2) $\frac{2Q}{\pi(a^2 - b^2)}$ (3) $\frac{2Q}{\pi a^2}$ (4) $\frac{Q}{2\pi a^2}$

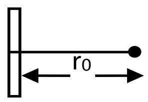
19. An electric dipole has a fixed dipole moment $\vec{p}$, which makes angle θ with respect to x-axis. When subjected to an electric field $\vec{E}_1 = E\hat{i}$, it experiences a torque $\vec{T}_1 = \tau\hat{k}$. When subjected to another electric field $\vec{E}_2 = \sqrt{3}E_1\hat{j}$ it experiences a torque $\vec{T}_2 = -\vec{T}_1$. The angle θ is : **[JEE (Main) 2017; 4/120, –1]**

(1) 90° (2) 30° (3) 45° (4) 60°

20. Three concentric metal shells A, B and C of respective radii a, b and c (a < b < c) have surface charge densities $+\sigma$, $-\sigma$ and $+\sigma$ respectively. The potential of shell B is : **[JEE (Main) 2018; 4/120, –1]**

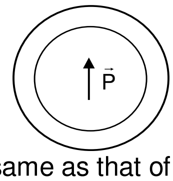
(1) $\frac{\sigma}{\varepsilon_0}\left[\frac{b^2 - c^2}{b} + a\right]$ (2) $\frac{\sigma}{\varepsilon_0}\left[\frac{b^2 - c^2}{c} + a\right]$ (3) $\frac{\sigma}{\varepsilon_0}\left[\frac{a^2 - b^2}{a} + c\right]$ (4) $\frac{\sigma}{\varepsilon_0}\left[\frac{a^2 - b^2}{b} + c\right]$

21. A positive point charge is released from rest at a distance r_0 from a positive line charge with uniform density. The speed (υ) of the point charge, as a function of instantaneous distance r from line charge, is proportional to : **[JEE (Main) 2019; 4/120, –1]**



(1) $\upsilon \propto \left(\frac{r}{r_0}\right)$ (2) $\upsilon \propto \ell n\left(\frac{r}{r_0}\right)$ (3) $\upsilon \propto \sqrt{\ell n\left(\frac{r}{r_0}\right)}$ (4) $\upsilon \propto e^{+r/r_0}$

22. Shown in the figure is a shell made of a conductor. It has inner radius a and outer radius b, and carries charge Q. At its centre is a dipole $\vec{P}$ as shown. In this case : **[JEE (Main) 2019; 4/120, –1]**



(1) electric field outside the shell is the same as that of a point charges at the centre of the shell

(2) surface charge density on the inner surface of the shell is zero everywhere

(3) surface charge density on the inner surface is uniform and equal to $\frac{(Q/2)}{4\pi a^2}$

(4) surface charge density on the outer surface depends on $|\vec{P}|$

23. Let a total charge 2Q be distributed in a sphere of radius R, with the charge density given by $\rho(r) = kr$, where r is the distance from the centre. Two charges A and B, of –Q each, are placed on diametrically opposite points, at equal distance, a, from the centre. If A and B do not experience any force, then : **[JEE (Main) 2019; 4/120, –1]**

(1) $a = 3R/2^{1/4}$ (2) $a = 2^{-1/4}R$ (3) $a = 8^{-1/4}R$ (4) $a = R/\sqrt{3}$

Answers

EXERCISE-1
PART - I

SECTION (A) :

A-1. 5400 N, attractive.

A-2. |F| = 0.18 N, $\hat{F} = \frac{(4\ \hat{i} - 3\ \hat{k})}{5}$.

A-3. $q_0 K\left[\frac{q_1}{r_1^2} + \frac{q_2}{r_2^2} .... \frac{q_{20}}{r_{20}^2}\right] = 1.89 \times 10^6$ N

A.4. (i) $\frac{Kq_1q_2}{r^2}$ = 0.144 N

(ii) $\frac{Kq_1q_2}{mr^2}$ = 6 m/s^2

(iii) No (Magnitude is same but direction is different)

A-5. $\pm 1.0 \times 10^{-6}$ C, $\mp 3 \times 10^{-6}$ C

A-6. $\tan^{-1}(1/100) = 0.6^\circ$

A-7. $\frac{2\ell}{3}$ from charge 4 e (If q is positive stable, If q is negative unstable)

A-8. (a) moves towards the centre

(b) $-\frac{q}{\sqrt{3}}$

A-9. $\frac{d}{2\sqrt{2}}$, $\frac{4Qq}{3\sqrt{3}\pi\varepsilon_0 d^2}$

SECTION (B) :

B-1. F/q = 5×10^3 N/C

B-2. The electron deviates by an angle $\theta = \tan^{-1}\frac{eE\ell}{mv_0^2}$ (from x axis) = 45º

B-3. 30 cm from A along BA

B-4. $\left(\sqrt{2} + \frac{1}{2}\right)\frac{Kq_0}{a^2}$

B-5. $2540\hat{i} + 2000\hat{j} + 1720\hat{k}$ N/C.

B-6. $\frac{QdA}{16\pi^2\varepsilon_0 a^4}$

B-7. (i) $\frac{4K\lambda}{x}$; along OP.

(ii) $\frac{4K\lambda}{x}\cot\frac{\theta}{2}$; Perpendicular to OP.

B-8. $100\sqrt{2} \approx 141$s

B-9. $\vec{E}_A = \frac{-\sigma}{2\varepsilon_0}\hat{j}$, $\vec{E}_B = \frac{-3\sigma}{2\varepsilon_0}\hat{j}$,

$\vec{E}_C = \frac{-7\sigma}{2\varepsilon_0}\hat{j}$, $\vec{E}_D = \frac{\sigma}{2\varepsilon_0}$, $\hat{j}$

B-10. (i) $\frac{\rho(R-r)}{3\varepsilon_0}$ (ii) $\frac{\rho R^3}{3\varepsilon_0(r+R)^2}$

B-11. (i) 0 (ii) $\frac{\sigma R^2}{\varepsilon_0(r+R)^2}\hat{r}$

B-12. (a) E = $\frac{\sqrt{2}\ \lambda}{4\pi\epsilon_0 r}$ (b) E = 0

SECTION (C) :

C-1. (i) q(ΔV) = 20 mJ. (ii) – 20 mJ
(iii) q(ΔV) +ΔK.E. = 30 mJ
(iv) – 20mJ (v) 30 mJ

C-2. 60 cm from A along BA and 20 cm from A along AB

C-3. (i) $\frac{6Kqq_0}{a}$ (ii) $\frac{3Kqq_0}{a}$ (iii) No

C-4. 400 volts

C-5. W = $Kqq_0\left(\frac{1}{r_B} - \frac{1}{r_A}\right)$ = 1.2 J

C-6. (a) $450\left(6\hat{i} - 8\hat{j}\right)$ V/m, 4.5 k V/m

(b) $4.5\left[\frac{1}{2} - \frac{1}{\sqrt{45}}\right]$ = 1.579 J

C-7. 0, $\frac{4\sqrt{2}qk}{a^2}$, 0, $\frac{4Qqk}{a}\left[\frac{1}{\sqrt{5}} - 1\right]$

where k = $\frac{1}{4\pi\epsilon_0}$

C-8. V = $\frac{Q}{4\pi\varepsilon_0}\cdot\frac{(R+r)}{R^2+r^2}$ **C-9.** 1m

C-10. $\frac{W}{q_0} = -\frac{\sigma}{2\varepsilon_0}(b-a)$

C-11. (a) 0 (b) E.d = 40 V

C-12. (a) –(8V/m) x
(b) points on the plane x = –20 m
(c) 80 V – (8 V/m) x (d) infinity

C-13. (a) $+4.5 \times 10^{-5}$ J (b) 3×10^5 N/C
(c) 1.5×10^4 V

C-14. (a) -4.5×10^{-5} J (b) 3×10^5 N/C
(c) -1.5×10^4 V

SECTION (D) :

D-1. 10 eV **D-2.** $\frac{6Kqq_0}{a}$

D-3. (i) $v_{surface} = \sqrt{\frac{qQ}{4\pi\epsilon_0 mR}}$

(ii) $v_{centre} = \sqrt{\frac{qQ}{2\pi\epsilon_0 mR}}$

D-4. 10^3 m/s **D-5.** (a) $\frac{Qq}{4\pi\varepsilon_0 K}$ (b) $\sqrt{\frac{K}{m}}$

SECTION (E) :

E-1. 4.5 J **E-2.** -9.0×10^{-3} J.

E-3. (i) $\frac{4Kq^2}{a}\left[3+\frac{3}{\sqrt{2}}+\frac{1}{\sqrt{3}}\right]$

(ii) $W_{ext} = -\frac{2Kq^2}{a}\left[3+\frac{3}{\sqrt{2}}+\frac{1}{\sqrt{3}}\right]$,

$W_{el} = \frac{2Kq^2}{a}\left[3+\frac{3}{\sqrt{2}}+\frac{1}{\sqrt{3}}\right]$

(iii) $\sqrt{\frac{Kq^2}{2ma}\left[3+\frac{3}{\sqrt{2}}+\frac{1}{\sqrt{3}}\right]}$

(iv) $\sqrt{\frac{2Kq^2}{ma}\left[3+\frac{3}{\sqrt{2}}+\frac{1}{\sqrt{3}}\right]}$

(v) $\sqrt{\frac{Kq^2}{ma}\left[3+\frac{3}{\sqrt{2}}+\frac{1}{\sqrt{3}}\right]}$

SECTION (F) :

F-1. $\frac{Q_1^2}{8\pi\varepsilon_0 R_1} + \frac{Q_2^2}{8\pi\varepsilon_0 R_2} + \frac{Q_1Q_2}{4\pi\varepsilon_0 R_2}$

F-2 $W_{el} = \frac{q(q_0+q/2)}{8\pi\varepsilon_0 R}$, $W_{ext} = -\frac{q(q_0+q/2)}{8\pi\varepsilon_0 R}$

F-3. K.E. = $\frac{1}{2}\frac{Q^2}{4\pi\varepsilon_0 d}$ **F-4.** $\frac{6U_0}{5}$ Joules

SECTION (G) :

G-1. $-2xy + C$

G-2. $-2xy\,\hat{i} - (x^2 + 2yz)\,\hat{j} - y^2\hat{k}$

G-3. (i) $-4(\hat{i} - 2\hat{k})$ (ii) $\vec{E} = -8\hat{r}$

G-4. $V_{(3,3)} - V_{(0,0)} = -90$ V

G-5. 40 V **G-6.** $-2r^3/3 + C$

G-7. $-2r^3/3 + y^3 + C$

SECTION (H) :

H-1. $Qa\sqrt{3}$, along the bisector of the angle at $-2Q$, towards the triangle

H-2. $\frac{1}{4\pi\varepsilon_0 a^3}\sqrt{Q^2a^2+p^2}$ **H-3.** $\frac{\sqrt{2}qP}{4\pi\varepsilon_0 r^2}$

H-4. **(i)** $|\vec{E}| = \frac{\sqrt{7}K}{8}$, $V = \frac{K}{4}$

(ii) $|\vec{E}| = \frac{K\sqrt{7}}{8}$, $V = \frac{K}{4}$ [where $K = 1/4\pi\epsilon_0$].

H-5. $\frac{6.023}{2}$ J = 3.016 J (Approx)

SECTION (I) :

I-1. 50 Nm^2/C. **I-2.** 0

I-3. $10^4\ \frac{N-m^2}{C}$, 0 **I-4.** $q/(2\varepsilon_0)$

I-5. There is net positive charge in the close surface.

SECTION (J) :

J-1. (a) $\frac{Q}{2A}$ (b) $\frac{Q}{2A\varepsilon_0}$ towards left

(c) $\frac{Q}{2A\varepsilon_0}$ towards right (d) $\frac{Q}{2A\varepsilon_0}$ towards right

J-2. q/2

J-3. $(\sigma - x)A,\ xA,\ -xA, (x-2\sigma)A$
where, $x = (2\varepsilon_0 E + 3\sigma)/2$

J-4. (i) $\frac{Q'_1}{Q'_2} = \frac{2}{3}$

(ii) 2/5 × 30 = 12μC, 3/5 × 30 = 18 μC

(iii) $\frac{\sigma'_1}{\sigma'_2} = \frac{3}{2}$

(iv) $2\pi\varepsilon_0\left(\frac{r_1r_2}{r_1+r_2}\right)(v_1 - v_2)^2$ = 3/2 Joules

J-5. (i) on inner shell = 0, on outer shell = $Q_a + Q_b$

(ii) $\frac{KQ_a^2}{2}\left[\frac{1}{a}-\frac{1}{b}\right]$

J-6. (i) $(r_1/r_2)q$

(ii) Charge on inner shell= – $(r_1/r_2)q$ and charge on the outer shell = q

(iii) Charge flown in to the earth = $(r_1/r_2)q$

J-7. $V_2 = V_1\frac{r_1}{r_2}$

PART - II

SECTION (A) :

A-1. (D) **A-2.** (A) **A-3.** (A)
A-4. (C)

SECTION (B) :

B-1. (D) **B-2.** (B) **B-3.** (C)
B-4. (B) **B-5.** (D) **B-6.** (C)
B-7. (A) **B-8.** (C) **B-9.** (D)

SECTION (C) :

C-1. (A) **C-2.** (B) **C-3.** (C)
C-4. (A) **C-5.** (A) **C-6.** (A)
C-7. (B) **C-8.** (C) **C-9.** (D)
C-10. (C) **C-11.** (B) **C-12.** (A)
C-13. (B) **C-14.** (D)

SECTION (D) :

D-1. (C) **D-2.** (B)

SECTION (E) :

E-1. (D) **E-2.** (A)

SECTION (F) :

F-1.	(D)			**F-2.**	(D)

SECTION (G) :

G-1.	(A)	**G-2.**	(D)	**G-3.**	(A)
G-4.	(B)	**G-5.**	(D)	**G-6.**	(B)
G-7.	(A)				

SECTION (H) :

H-1.	(C)	**H-2.**	(C)	**H-3.**	(C)
H-4.	(C)	**H-5.**	(D)	**H-6.**	(C)
H-7.	(D)	**H-8.**	(B)		

SECTION (I) :

I-1.	(B)	**I-2.**	(D)	**I-3.**	(B)
I-4.	(C)	**I-5.**	(C)	**I-6.**	(D)
I-7.	(D)	**I-8.**	(A)	**I-9.**	(C)
I-10.	(A)	**I-11.**	(C)	**I-12.**	(A)
I-13.	(B)	**I-14.**	(B)		

SECTION (J) :

J-1.	(D)	**J-2.**	(B)	**J-3.**	(C)
J-4.	(D)	**J-5.**	(C)	**J-6.**	(C)
J-7.	(C)	**J-8.**	(D)	**J-9.**	(A)
J-10.	(C)	**J-11.**	(A)	**J-12.**	(A)
J-13.	(C)				

PART - III

1. (A) – p,q ; (B) – p,q ; (C) – p,q,s ; (D) – r,s

2. (A) – p ; (B) – r,s ; (C) – p,q ; (D) – r,s

EXERCISE-2

PART - I

1.	(B)	**2.**	(D)	**3.**	(C)
4.	(D)	**5.**	(A)	**6.**	(C)
7.	(D)	**8.**	(A)	**9.**	(B)
10.	(D)	**11.**	(B)	**12.**	(B)
13.	(A)	**14.**	(B)	**15.**	(B)
16.	(A)	**17.**	(C)	**18.**	(A)
19.	(A)	**20.**	(C)	**21.**	(A)

PART - II

1.	3	**2.**	6	**3.**	2
4.	11	**5.**	9	**6.**	4
7.	6	**8.**	30	**9.**	3
10.	4	**11.**	16	**12.**	1
13.	2	**14.**	2	**15.**	8
16.	9	**17.**	8	**18.**	16
19.	10	**20.**	32		

PART - III

1.	(CD)	**2.**	(BD)	**3.**	(BC)
4.	(AC)	**5.**	(ABCD)	**6.**	(AC)
7.	(AC)	**8.**	(BCD)	**9.**	(ABCD)
10.	(BC)	**11.**	(AC)	**12.**	(BD)
13.	(AD)	**14.**	(AD)	**15.**	(AB)
16.	(ACD)	**17.**	(BD)	**18.**	(BC)

PART - IV

1.	(C)	**2.**	(C)	**3.**	(A)
4.	(C)	**5.**	(B)	**6.**	(D)
7.	(B)	**8.**	(C)	**9.**	(A)

EXERCISE-3

PART - I

1.	(C)	**2.**	(A)	**3.**	(B)
4.	(C)	**5.**	(B)	**6.**	(A)
7.	(B)	**8.**	(A)	**9.**	2
10.	(AD)	**11.**	(A)	**12.**	(D)
13.	(C)	**14.**	(ABCD)	**15.**	(A)
16.	(C)	**17.**	(D)	**18.**	(ABC)
19.	(BD)	**20.**	(CD)	**21.**	(C)
22.	(C)	**23.**	(A)	**24.**	6
25.	(C)	**26.**	(D)	**27.**	(CD)
28.	(AB)	**29.**	2.00	**30.**	(B)

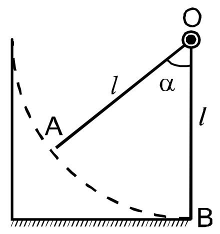
PART - II

1.	(4)	**2.**	(3)	**3.**	(4)
4.	(1)	**5.**	(2)	**6.**	(3)
7.	(2)	**8.**	(3)	**9.**	(4)
10.	(4)	**11.**	(3)	**12.**	(3)
13.	(1)	**14.**	(4)	**15.**	(3)
16.	(1)	**17.**	(3,4)	**18.**	(4)
19.	(4)	**20.**	(4)	**21.**	(3)
22.	(1)	**23.**	(3)		

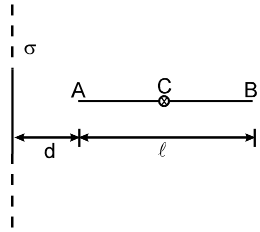
High Level Problems (HLP)

SUBJECTIVE QUESTIONS

1. An electrometer consists of a fixed vertical metal bar OB at the top of which is attached a thin rod OA which gets deflected from the bar under the action of an electric charge (fig.). The rod can rotate in vertical plane about fixed horizontal axis passing through O. The reading is taken on a quadrant graduated in degrees. The length of the rod is ℓ and its mass is m. What will be the charge when the rod of such an electrometer is deflected through an angle α in equilibrium? Find the answer using the following two assumptions: The charge on the electrometer is equally distributed between the bar & the rod and the charges are concentrated at point A on the rod & at point B on the bar.



2. A uniform rod AB of mass m and length ℓ is hinged at its mid point C. The left half (AC) of the rod has linear charge density $-\lambda$ and the right half (CB) has $+\lambda$, where λ is constant. A large non conducting sheet of uniform surface charge density σ is also present near the rod. Initially, the rod is kept perpendicular to the sheet. The end A of the rod is initially at a distance d. Now the rod is rotated by a small angle in the plane of the paper and released. Prove that the rod will perform SHM and find its time period.

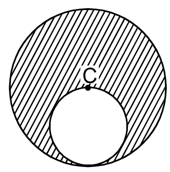


3. (i) Three equal charges q_0 each are placed at three corners of an equilateral triangle of side 'a'. Find out force acting on one of the charge due to other two charges?

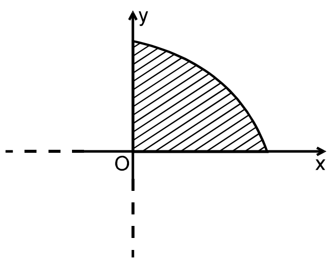
(ii) In the above question, if one of the charge is replaced by negative charge then find out force acting on it due to other charges ?

(iii) Repeat the part (i) if magnitude of each charge is doubled and side of triangle is reduced to half.

4. A solid sphere of radius 'R' is uniformly charged with charge density ρ in its volume. A spherical cavity of radius R/2 is made in the sphere as shown in the figure. Find the electric potential at the centre of the sphere.

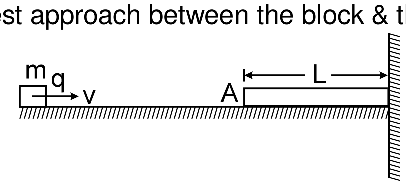


5. A uniform surface charge of density σ is given to a quarter of a disc extending upto infinity in the first quadrant of x – y plane. The centre of the disc is at the origin O. Find the z – component of the electric field at the point (0, 0, z) and the potential difference between the points (0, 0, d) & (0, 0, 2d).



6. Figure shows a rod of length L which is uniformly charged with linear charge density λ kept on a smooth horizontal surface. Right end of rod is in contact with a vertical fixed wall.

A block of mass m and charge q is projected with a velocity v from a point very far from rod in the line of rod. Find the distance of closest approach between the block & the left end A of the rod.



7. A charged ball of mass 5.88×10^{-4} kg is suspended from two silk strings of equal lengths so that the strings are inclined at 90° with each other. Another ball carrying a charge which is equal in magnitude but opposite in sign of the first one is placed vertically below the first one at a distance of 4.2×10^{-2} m. Due to this, the tension in the strings is doubled. Determine the charge on the ball and the tension in the strings after electrostatic interaction. ($g = 9.8$ m/s^2)

8. A charge of 16×10^{-9} C is fixed at the origin of coordinates. A second charge of unknown magnitude is at x = 3m, y = 0 and a third charge of 12×10^{-9} C is at x = 6m, y = 0. What is the value of the unknown charge if the resultant field at x = 8m, y = 0 is 20.25 N/C directed towards positive x-axis?

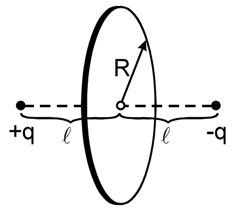
9. Two large conducting plates are placed parallel to each other with a separation of 2.00 cm between them. An electron starting from rest near one of the plates reaches the other plate in 2.00 microseconds. Find the surface charge density on the inner surfaces. Can you find out the charge density on outer surface?

10. A ball of mass 10^{-2} kg & having charge $+3 \times 10^{-6}$ C is tied at the end of a 1 m long thread. The other end of the string is fixed and a charge of -3×10^{-6} C is placed at this end. The ball can move in a circular orbit of radius 1 m in a vertical plane. Initially the ball is at the bottom. Find the minimum initial horizontal velocity of the ball so that it will be able to complete the full circle. [$g = 10$ m/s^2.] **[REE 1996, 5]**

11. A system consists of a thin charged wire ring of radius R and a very long uniformly charged thread oriented along the axis of the ring, with one of its ends coinciding with the centre of the ring. The total charge of the ring is equal to q. The charge of the thread (per unit length) is equal to. Find the interaction force between the ring and the thread.

12. A thin non-conducting ring of radius R has a linear charge density $\lambda = \lambda_0 \cos \varphi$, where λ_0 is a constant, φ is the azimuthal angle. Find the magnitude of the electric field strength
(a) At the centre of the ring.
(b) On the axis of the ring as a function of the distance x from its centre. Investigate the obtained function at x >> R.

13. Two point charges q and –q are separated by the distance 2ℓ (Figure). Find the flux of the electric field strength vector across a circle of radius R.



14. A system consists of a ball of radius R carrying a spherically symmetric charge and the surrounding space filled with a charge of volume density $\rho = \alpha/r$, where α is a constant, r is the distance from the centre of the ball. Find the ball's charge for which the magnitude of the electric field strength vector is independent of r outside the ball. How high is this strength ? The permittivities of the ball and the surrounding space are assumed to be equal to unity.

15. Find the electric field potential and strength at the centre of a hemisphere of radius R charged uniformly with the surface density σ.

16. A non-conducting disc of radius a and uniform positive surface charge density σ is placed on the ground, with its axis vertical. A particle of mass m and positive charge q is dropped, along the axis of the disc, from a height H with zero initial velocity. The particle has $\frac{q}{m} = \frac{4\varepsilon_0 g}{\sigma}$. **[JEE 1999 (Mains), 5+5 /100]**

(i) Find the value of H if the particle just reaches the disc.
(ii) Sketch the potential energy of the particle as a function of its height and find its equilibrium position.

17. The potential difference between two large parallel plates is varied as v = at ; a is a positive constant and t is time. An electron starts from rest at t = 0 from the plate which is at lower potential. If the distance between the plates is L, mass of electron m and charge on electron -e then find the velocity of the electron when it reaches the other plate.

18. Four point charges $+8\mu C$, $-1\mu C$, $-1\mu C$ and $+8\mu C$, are fixed at the points, $-\sqrt{\frac{27}{2}}$ m, $-\sqrt{\frac{3}{2}}$ m, $+\sqrt{\frac{3}{2}}$ m and $+\sqrt{\frac{27}{2}}$ m respectively on the y – axis. A particle of mass 6×10^{-4} kg and of charge + 0.1 μC moves along the -x direction. Its speed at x = + ∞ is v_0. Find the least value of v_0 for which the particle will cross the origin. Find also the kinetic energy of the particle at the origin. Assume that space is gravity free. Given : $1/(4\pi\varepsilon_0) = 9 \times 10^9$ Nm^2/C^2 **[JEE 2000 (Mains),10/ 100]**

19. A small ball of mass 2×10^{-3} kg having a charge of 1 μC is suspended by a string of length 0.8 m. Another identical ball having the same charge is kept at the point of suspension. Determine the minimum horizontal velocity which should be imparted to the lower ball so that it can make complete revolution. (g = 10 m/s^2) **[JEE 2001 (Mains), 5/100 ; REE 1996]**

20. Three point charges Q, 2Q and 8Q are to be placed on a 10 cm long straight line. Find the position where the charges should be placed such that the potential energy of this system is minimum. In this situation, what is the electric field at the position of the charge Q due to the other two charges? **[JEE 1987]**

21. A particle of charge - q and mass m moves in a circular orbit about a fixed charge + Q. Show that the "$r^3 \alpha T^2$ " law, is satisfied, where r is the radius of orbit and T is time period.

22. The field potential in a certain region of space depends only on the x coordinate as $\varphi = -ax^3 + b$, where a and b are constants. Find the distribution of the space charge $\rho(x)$.

23. A conducting sphere S_1 of radius r is attached to an insulating handle. Another conducting sphere S_2 of radius R is mounted on an insulating stand. S_2 is initially uncharged. S_1 is given a charge Q, brought into contact with S_2 and removed. S_1 is then recharged such that the charge on it is again Q & it is again brought into contact with S_2 & removed. This procedure is repeated n times **[JEE 1998 Mains, 7+1/200]**

(a) Find the electrostatic energy of S_2 after n such contacts with S_1.
(b) What is the limiting value of this energy as $n \to \infty$?

24. The electric field strength depends only on the x and y coordinates according to the law

$$\vec{E} = a(x\hat{i} + y\hat{j})/(x^2 + y^2),$$

where a is a constant, $\hat{i}$ and $\hat{j}$ are the unit vectors of the x and y axes. Find the flux of the vector $\vec{E}$ through a sphere of radius R with its centre at the origin of coordinates.Using the above result, also calculate total charge enclosed by the sphere.

25. A positive charge is distributed in a spherical region with charge density $\rho = \rho_0 r$ for $r \le R$ (where ρ_0 is a positive constant and r is the distance from centre). Find out electric potential and electric field at following locations.

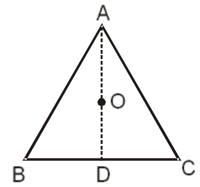
(a) At a distance r from centre inside the sphere. (b) At a distance r from centre outside the sphere.

26. Two point charges q and –2q are placed at a distance 6m apart on a horizontal plane (x–y plane). Find the locus of the zero potential points in the x–y plane.

27. Two metalic balls of radii R_1 and R_2 are kept in vacuum at a large distance compared to their radii. Find the ratio of the charges on the two balls for which electrostatic energy of the system is minimum. What is the potential difference between the two balls for this ratio ? Total charge of the balls is constant. Neglect the interaction energy. (Charge distribution on each ball is uniform)

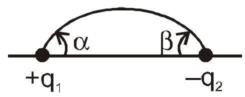
28. A ball of radius R carries a positive charge whose volume density depends only on the separation r from the ball's centre as $\rho = \rho_0 (1 - r/R)$, where ρ_0 is a constant. Assuming the perimittivities of the ball and the environment to be equal to unity, find :
(i) The magnitude of the electric field strength as a function of the distance r both inside and outside the ball;
(ii) The maximum intensity E_{max} and the corresponding distance r_m.

29. Consider an equilateral triangle ABC of side 2a in the plane of the paper as shown. The centroid of the triangle is O. Equal charges (Q) are fixed at the vertices A, B and C. In what follows consider all motion and situations to be confined the plane of the paper.

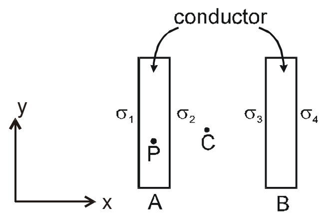


(a) A test charge (q), of same sign as Q, is placed on the median AD at a point at a distance δ below O. Obtain the force ($\vec{F}$) felt by the test charge.
(b) Assuming δ << a discuss the motion of the test charge when it is released.
(c) Obtain the force ($\vec{F}_D$) on this test charge if it is placed at the point D as shown in the figure.
(d) In the figure below mark the approximate locations of the equilibrium point(s) for this system. Justify your answer.
(e) Is the equilibrium at O stable or unstable if we displace the test charge in the direction of OP ? The line PQ is parallel to the base BC. Justify your answer.
(f) Consider a rectangle ABCD. Equal charges are fixed at the vertices A, B, C and D. O is the centroid. In the figure below mark the approximate locations of all the neutral points of the system for a test charge with same sign as the charges on the vertices. Dotted lines are drawn for the reference.
(g) How many neutral points are possible for a system in which N charges are placed at the N vertices of a regular N sided polygon ?

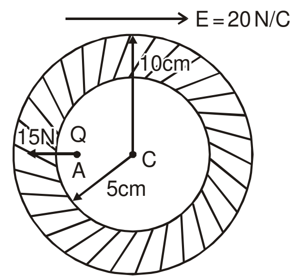
30. An electrostatic field line leaves at angle α from point charge q_1 and connects with point charge $-q_2$ at angle β (see figure).
Then find the relationship between α and β :



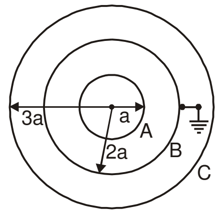
31. Figure shows two infinitely large conducting plates A and B. If electric field at C due to charge densities σ_1, σ_2, σ_3 and σ_4 is $E\hat{i}$, find σ_2 and σ_3 in terms of E. State whether this much information is sufficient to find σ_1 and σ_4 in terms of E. Derive a relation between σ_1 and σ_4.



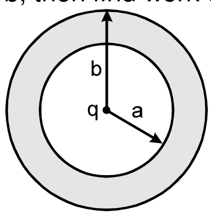
32. In a neutral conducting hollow sphere of inner and outer radii 5 cm and 10 cm respectively, a point charge Q = 1 C is placed at point A, that is 3 cm from the centre C of the hollow sphere. An external uniform electric field of magnitude 20 N/C is also applied. Net electric force on this charge is 15 N, away from the centre of the sphere as shown. Then find :
(a) Force due to external electric field on the outer surface of the shell.
(b) Net force on shell.
(c) Net force on point charge due to shell.



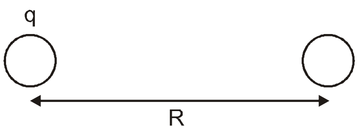
33. Figure shows a system of three concentric metal shells A, B and C with radii a, 2a and 3a respectively. Shell B is earthed and shell C is given a charge Q. Now if shell C is connected to shell A, then find the final charge on the shell B



34. A point charge q is brought slowly from infinity and is placed at the centre of a conducting neutral spherical shell of inner radius a and outer radius b, then find work done by external agent :

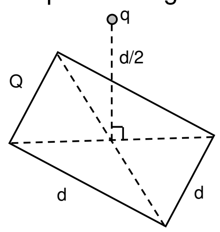


35. Consider two solid dielectric spheres of radius a, separated by a distance R(R >>a). One of the spheres has a charge q, and the other is neutral (see figure.) We double the distance between two sphere. How much charge should reside on the first sphere now so that the force between the spheres remains the same ?



36. A triangle is made from thin insulating rods of different lengths, and the rods are uniformly charged, i.e. the linear charge density on each rod is uniform and the same for all three rods. Find a particular point in the plane of the triangle at which the electric field strength is zero.

37. A square of side d, made from a thin insulating plate, is uniformly charged and carries a total charge of Q. A point charge q is placed on the symmetrical normal axis of the square at a distance d/2 from the plate. How large is the force acting on the point charge?



38. A point charge q is located between two mutually perpendicular conducting half-planes. Its distance from each half-plan is equal to ℓ. Find the modulus of the vector of the force acting on the charge.

39. A point charge q is located at a distance ℓ from an infinite conducting plane. Determine the surface density of charges induced on the plane as a function of separation r from the base of the perpendicular drawn to the plane from the charge.

40. A very long straight thread is oriented at right angles to an infinite conducting plane; its end is separated from the plane by a distance ℓ. The thread carries a uniform charge of linear density λ. Suppose the point O is the trace of the thread on the plane. Find the surface density of the induced charge on the plane

(a) At the point O (b) As a function of a distance r from the point O.

HLP Answers

1. $q = 4\ell\sqrt{4\pi \in_0 mg\sin\left(\frac{\alpha}{2}\right)}\ \sin\frac{\alpha}{2}$

2. $T = 2\pi\sqrt{\frac{2m \in_0}{3\lambda\sigma}}$

3. (i) $\frac{\sqrt{3}Kq_0^2}{a^2}$, away from the charges along perpendicular bisector of line joining remaining two charges.

(ii) $\frac{\sqrt{3}Kq_0^2}{a^2}$; towards the charges along perpendicular bisector of line joining remaining two charges.

(iii) $\frac{16\sqrt{3}Kq_0^2}{a^2}$; away from the charges along angle bisector.

4. $V = \frac{5\rho R^2}{12 \in_0}$

5. $\frac{\sigma}{8 \in_0}, \frac{\sigma}{8 \in_0}\ |d|$

6. $r = \frac{L}{\left(e^{\frac{2\pi\in_0 mv^2}{\lambda q}} - 1\right)}$

7. 3.36×10^{-8}C, 8.15×10^{-3} N

8. -25×10^{-9} C

9. 0.505×10^{-12} C/m^2, No

10. $u = \sqrt{58.1} = 7.6$ m/s

11. $F = \frac{q\lambda}{4\pi\varepsilon_0 R}$

12. (a) $\frac{\lambda}{4\varepsilon_0 R}$ (b) $E = \frac{\lambda_0 R^2}{4\varepsilon_0(x^2+R^2)^{3/2}}$.

For x >> R this strength $E \approx \frac{p}{4\pi\varepsilon_0 x^3}$, where $p = \pi R^2\lambda_0$.

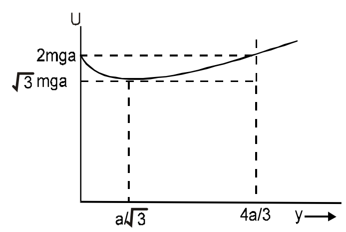
13. $|\phi| = \frac{q}{\varepsilon_0}\left(1 - \frac{1}{\sqrt{1+(R/\ell)^2}}\right)$. The sign of ϕ depends on how the direction of the normal to the circle is chosen.

14. $q = 2\pi\alpha R^2$, $E = \frac{1}{2}\frac{\alpha}{\varepsilon_0}$

15. $V = \frac{\sigma R}{2 \in_0}$, $E = \frac{\sigma}{4 \in_0}$

16. (i) H = 4 a/3

(ii) $U(y) = 2mg\left[\sqrt{y^2+a^2} - y\right] + mgy$; at equilibrium $\frac{dU}{dy} = 0 \Rightarrow y = \frac{a}{\sqrt{3}}$



17. $V = \left(\frac{9}{2}\frac{eaL}{m}\right)^{1/3}$

18. $v_0 = 3$ m/s; K.E. at origin $(27 - 10\sqrt{6}) \times 10^{-4}$ J $= 2.5 \times 10^{-4}$ J

19. $\sqrt{\frac{275}{8}} = 5.86$ m/s

20. Q at a distance of 10/3 cm from 2Q between 2Q and 8Q, E = 0.

22. $6a\varepsilon_0 x$

23. (a) $U_2 = \dfrac{a^2 Q^2}{8\pi \in_0 R}\left(\dfrac{1-a^n}{1-a}\right)^2$ where $a = \dfrac{R}{r+R}$ (b) $U_2\ (n \to \infty) = \dfrac{RQ^2}{8\pi \in_0 r^2}$

24. $\phi = 4\pi Ra$, $Q = 4\pi Ra\in_0$

25. (a) $\vec{E} = \dfrac{\rho_0 r^2}{4\varepsilon_0}\hat{r}$; $V = \dfrac{\rho_0[4R^3 - r^3]}{12\varepsilon_0}$ (b) $\vec{E} = \dfrac{\rho_0 R^4}{4\varepsilon_0 r^2}\hat{r}$; $V = \dfrac{\rho_0 R^4}{4\varepsilon_0 r}$

26. Locus is a circle (equation depends on choice of coordinate system)

27. $\dfrac{Q_1}{Q_2} = \dfrac{R_1}{R_2}$; 0

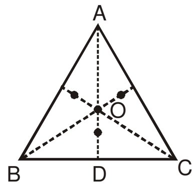
28. (i) $E = \dfrac{\rho_0 r}{3\ \varepsilon_0}\left(1-\dfrac{3r}{4R}\right)$ for r < R, $E = \dfrac{\rho_0 R^3}{12\varepsilon_0 r^2}$ for r > R (ii) $E_{max} = \dfrac{1}{9}\dfrac{\rho_0 R}{\varepsilon_0}$ for $r_m = \dfrac{2}{3}R$.

29. (a) $$\vec{F} = \frac{2KQq\left(\frac{a}{\sqrt{3}}-\delta\right)}{\left(a^2+\left(\frac{a}{\sqrt{3}}-\delta\right)^2\right)^{3/2}} - \frac{KQq}{\left(\frac{2a}{\sqrt{3}}+\delta\right)^2}$$ Here $K = 1/4\pi\varepsilon_0$ and direction is upward (towards A)

(b) Using binomial approximation, $\vec{F} = KQq\dfrac{9\sqrt{3}}{16}\dfrac{\delta}{a^3}$ (upward) which is linear in δ. Hence charge will oscillate simple harmonically about O when released.

(c) $\vec{F}_D = \dfrac{KQq}{3a^2}$ (downward)

(d) For small δ force on the test charge is upwards while for large δ (eg. at D) force is downwards. So there is a neutral point between O and D. By symmetry there will be neutral points on other medians also. In figure x. Below all possible (4) neutral points are shown by •.

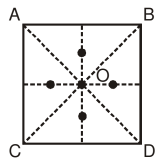


(e) Let the distance along P be x and O to be at (0, 0). Electric potential of a test charge along OP can be written as

$$V(x) = \frac{KQ}{\sqrt{x^2+(4/3)}} + \frac{KQ}{\sqrt{(x+1)^2+(1/3)}} + \frac{KQ}{\sqrt{(x-1)^2+(1/3)}} \approx KQ\sqrt{\frac{3}{4}}\left(3+\frac{9}{16}x^2\right)$$

We can see that $V(x) \propto x^2$, hence it is a stable equilibrium.

(f) Equilibrium points are indicated by •.



(g) N + 1

30. $q_1 \sin^2\dfrac{\alpha}{2} = q_2 \sin^2\dfrac{\beta}{2}$

31. $\sigma_1 = \sigma_4$, $\sigma_2 = \in_0 E$, $\sigma_3 = -\in_0 E$

32. (a) 20 N right hand side (b) 35 N right hand side (c) 35 N left hand side

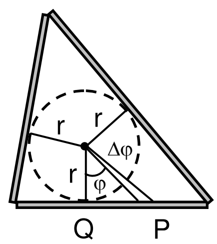
33. $-\dfrac{8Q}{11}$

34. $\dfrac{k\,q^2}{2\,b} - \dfrac{k\,q^2}{2\,a}$

35. $q' = 4\sqrt{2}q$

36. We are going to prove that the electric field strength is zero at the socalled incentre, the centre of the triangle's inscribed circle (which has radius r in the figure)

Let us consider a small length of rod at position P on one of the sides of the triangle; let it subtend an angle $\Delta\varphi$ at the incentre (see figure). Its distance from the incentre is $r/\cos\varphi$. Its small length Δx can be found by noting that P is a distance $x = r\tan\varphi$ along the rod from the fixed point Q and so $\Delta x = (r\,\Delta\varphi)/(\cos^2\varphi)$.



Consequently the charge it carries is$\Delta q = \frac{\lambda r \Delta\varphi}{\cos^2\varphi}$

where λ is the linear charge density on the rods. The magnitude of the elementary contribution of this small piece to the electric field at the incentre is

$$\Delta E = \frac{1}{4\pi\varepsilon_0}\frac{\Delta q \cos^2\varphi}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{\lambda r \Delta\varphi}{r^2}$$

It can be seen from this result that the same electric field (in both magnitude and direction) would be produced by an arc of the inscribed circle that subtends $\Delta\varphi$ at the circle's centre and carries the same linear charge density λ as the rod.

Summing up the contributions of the small arc pieces correspondingto all threesides of the triangle, we will, because of the circular symmetry, obtain zero net field. It follows that the electric field strength produced by the charged sides of the triangle is also zero at the incentre.

37. According to Newton's third law, the insulating plate acts on the point charge with a force of the same magnitude (but opposite direction) as the point charge does on the plate. We calculate the magnitude of this latter force.

Divide the plate (notionally) into small pieces, and denote the area of the i[th] piece by ΔA_i. Because of the uniform charge distribution, the charge on this small piece is

$$\Delta Q_i = \frac{Q}{d^2}\Delta A_i$$

and so the electric force acting on it is Fi = Ei_Qi, where Ei is the magnitude of the electric field produced by the point charge q at the position of the small piece.

The force acting on the insulating plate, as a whole, can be calculated as the vector sum of the forces acting on the individual pieces of the plate. Because of the axial symmetry, the net force is erpendicular to the plate, and so it is sufficient to sum the perpendicular components of the forces :

$$F = \sum_i F_i \cos\theta_i = \sum_i E_i \frac{Q}{d^2}\Delta A_i \cos\theta_i = \frac{Q}{d^2}\sum_i E_i \Delta A_i \cos\theta_i$$

where θ_i is the angle between the normal to the plate and the line that connects the point charge to the ith piece of it.

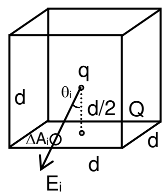
The sum in the given expression is nothing other than the electric flux through the square sheet produced by the point charge q :

$$\psi = \sum_i E_i \Delta A_i \cos\theta_i$$

and can be evaluated as follows.

Let us imagine that a cube of edge d is constructed symmetrically around the point charge (see figure). Then, the distance of the point charge from each side of the cube is just d/2. According to Gauss's law, the total electric flux passing through the six sides of the cube is q/ε_0 and so the flux through a single side is one-sixth of this :

$$\psi = \frac{q}{6\varepsilon_0}$$



Using this and our previous observations, we calculate the magnitude of the force acting on the point charge due to the presence of the charged insulating plate as

$$F = \frac{Qq}{6e_0 d^2}$$

38. $F = \frac{(2\sqrt{2}-1)q^2}{32\pi\varepsilon_0\ell^2}$

39. $\sigma = -\frac{q\ell}{2\pi(\ell^2+r^2)^{3/2}}$, $q_{ind} = -q$

40. (a) $\sigma = \frac{\lambda}{2\pi\ell}$; (b) $\sigma(r) = \frac{\lambda}{2\pi\sqrt{\ell^2+r^2}}$