MAGNETIC EFFECT OF CURRENT AND MAGNETIC FORCE ON CHARGE OR CURRENT

Contents

JEE (ADVANCED) SYLLABUS

Biot–Savart's law and Ampere's law; Magnetic field near a current-carrying straight wire, along the axis of a circular coil and inside a long straight solenoid; Force on a moving charge and on a current-carrying wire in a uniform magnetic field. Magnetic moment of a current loop; Effect of a uniform magnetic field on a current loop; Moving coil galvanometer, voltmeter, ammeter and their conversions.

JEE (MAIN) SYLLABUS

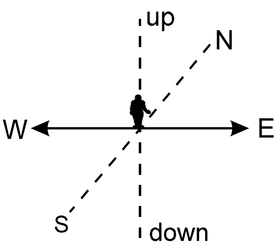
Biot-Savart law and its application to current carrying circular loop. Ampere's law and its applications to infinitely long current carrying straight wire and solenoid. Force on a moving charge in uniform magnetic and electric fields. Cyclotron. Force on a current-carrying conductor in a uniform magnetic field. Force between two parallel current-carrying conductors-definition of ampere. Torque experienced by a current loop in uniform magnetic field ; Moving coil galvanometer, its current sensitivity and conversion to ammeter and voltmeter. Current loop as a magnetic dipole and its magnetic dipole moment. Bar magnet as an equivalent solenoid, magnetic field lines ; Earth's magnetic field and magnetic elements. Para-, dia- and ferro- mangetic substances. Magnetic susceptibility and permeability, Hysteresis, Electromagnets and permanent magnets.

MAGNETIC EFFECT OF CURRENT AND MAGNETIC FORCE ON CHARGE OR CURRENT

1. MAGNET :

Two bodies even after being neutral (showing no electric interaction) may attract / repel strongly if they have a special property. This property is known as magnetism. The force with which they attract or repel is called magnetic force. Those bodies are called magnets. Later on we will see that it is due to circulating currents inside the atoms. Magnets are found in different shape but for many experimental purposes, a bar magnet is frequently used. When a bar magnet is suspended at its middle, as shown, and it is free to rotate in the horizontal plane it always comes to equilibrium in a fixed direction.

One end of the magnet (say A) is directed approximately towards north and the other end (say B) approximately towards south. This observation is made everywhere on the earth. Due to this reason the end A, which points towards north direction is called NORTH POLE and the other end which points towards south direction is called SOUTH POLE. They can be marked as 'N' and 'S' on the magnet. This property can be used to determine the north or south direction anywhere on the earth and indirectly east



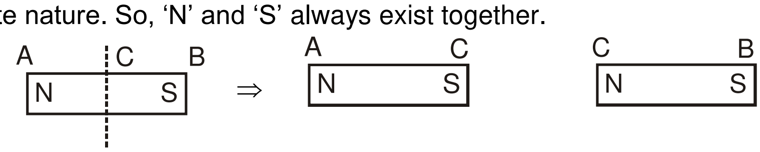
and west also if they are not known by other method (like rising of sun and setting of the sun). This method is used by navigators of ships and aeroplanes. The directions are as shown in the figure. All directions E, W, N, S are in the horizontal plane.
The magnet rotates due to the earths magnetic field about which we will discuss later in this chapter.

1.1 Pole strength magnetic dipole and magnetic dipole moment :

A magnet always has two poles 'N' and 'S' and like poles of two magnets repel each other and the unlike poles of two magnets attract each other they form action reaction pair.

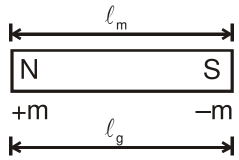
(i) (ii)

The poles of the same magnet do not come to meet each other due to attraction. They are maintained we cannot get two isolated poles by cutting the magnet from the middle. The other end becomes pole of opposite nature. So, 'N' and 'S' always exist together.



They are known as +ve and –ve poles. North pole is treated as positive pole (or positive magnetic charge) and the south pole is treated as –ve pole (or –ve magnetic charge). They are quantitatively represented by their "POLE STRENGTH" +m and –m respectively (just like we have charges +q and –q in electrostatics). Pole strength is a scalar quantity and represents the strength of the pole hence, of the magnet also).

A magnet can be treated as a dipole since it always has two opposite poles (just like in electric dipole we have two opposite charges –q and +q). It is called MAGNETIC DIPOLE and it has a MAGNETIC DIPOLE MOMENT. It is represented by $\vec{M}$. It is a vector quantity. It's direction is from –m to +m that means from 'S' to 'N')



$M = m.\ell_m$ here ℓ_m = magnetic length of the magnet. ℓ_m is slightly less than ℓ_g (it is geometrical length of the magnet = end to end distance). The 'N' and 'S' are not located exactly at the ends of the magnet. For calculation purposes we can assume $\ell_m = \ell_g$ [Actually $\ell_m/\ell_g \simeq 0.84$].

The units of m and M will be mentioned afterwards where you can remember and understand.

1.2 Magnetic field and strength of magnetic field.

The physical space around a magnetic pole has special influence due to which other pole experience a force. That special influence is called MAGNETIC FIELD and that force is called 'MAGNETIC FORCE'. This field is qualitatively represented by 'STRENGTH OF MAGNETIC FIELD' or "MAGNETIC INDUCTION" or "MAGNETIC FLUX DENSITY". It is represented by $\vec{B}$. It is a vector quantity.

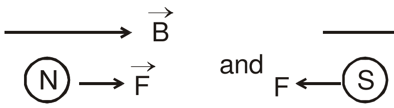
***Definition* of $\vec{B}$:** The magnetic force experienced by a north pole of unit pole strength at a point due to some other poles (called source) is called the strength of magnetic field at that point due to the source.

Mathematically, $\vec{B} = \dfrac{\vec{F}}{m}$

Here $\vec{F}$ = magnetic force on pole of pole strength m. m may be +ve or –ve and of any value.

S.I. unit of $\vec{B}$ is **Tesla** or **Weber/m²** (abbreviated as T and Wb/m²).

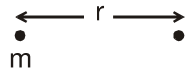
We can also write $\vec{F} = m\vec{B}$. According to this direction of on +ve pole (North pole) will be in the direction of field and on –ve pole (south pole) it will be opposite to the direction of $\vec{B}$.



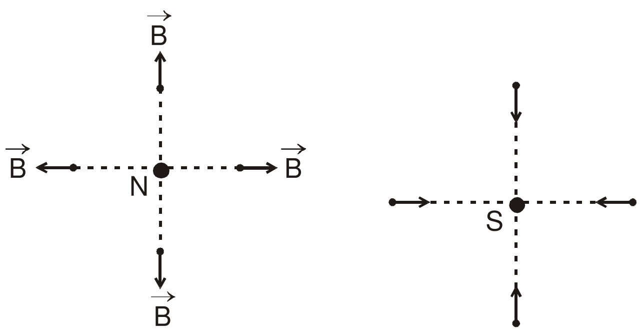
The field generated by sources does not depend on the test pole (for its any value and any sign).

(a) $\vec{B}$ due to various source

(i) Due to a single pole : (Similar to the case of a point charge in electrostatics)

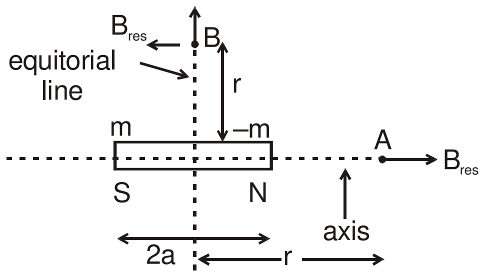


$B = \left(\dfrac{\mu_0}{4\pi}\right)\dfrac{m}{r^2}$. This is magnitude



Direction of B due to north pole and due to south poles are as shown in vector form $\vec{B} = \left(\frac{\mu_0}{4\pi}\right)\frac{m}{r^3}\vec{r}$ here m is with sign and $\vec{r}$ = position vector of the test point with respect to the pole.

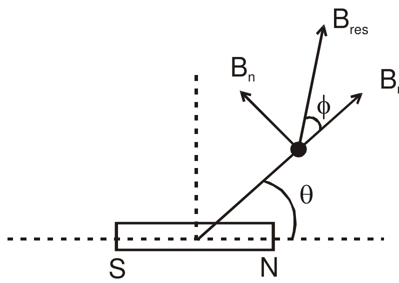
(ii) Due to a bar magnet : (Same as the case of electric dipole in electrostatics) Independent case never found. Always 'N' and 'S' exist together as magnet.



at A (on the axis) = $2\left(\frac{\mu_0}{4\pi}\right)\frac{\vec{M}}{r^3}$ for a << r

at B (on the equatorial) = $-\left(\frac{\mu_0}{4\pi}\right)\frac{\vec{M}}{r^3}$ for a << r

At General point :



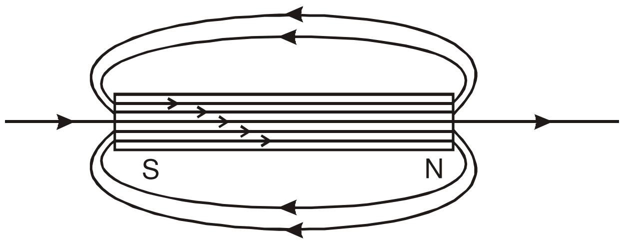
$$B_r = 2\left(\frac{\mu_0}{4\pi}\right)\frac{M\cos\theta}{r^3}$$

$$B_n = 2\left(\frac{\mu_0}{4\pi}\right)\frac{M\sin\theta}{r^3}$$

$$B_{res} = \frac{\mu_0 M}{4\pi r^3}\sqrt{1+3\cos^2\theta}$$

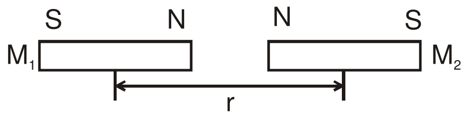
$$\tan\phi = \frac{B_n}{B_r} = \frac{\tan\theta}{2}$$

1.3 Magnetic lines of force of a bar magnet :



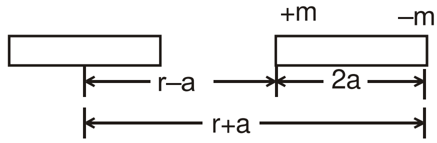
Solved Examples

Example 1. Find the magnetic force on a short magnet of magnetic dipole moment M_2 due to another short magnet of magnetic dipole moment M_1.



Solution : To find the magnetic force we will use the formula of 'B' due to a magnet. We will also assume m and –m as pole strengths of 'N' and 'S' of M_2. Also length of M_2 as 2a. B_1 and B_2 are the strengths of the magnetic field due to M_1 at +m and –m respectively. They experience magnetic forces F_1 and F_2 as shown.

$F_1 = 2\left(\frac{\mu_0}{4\pi}\right)\frac{M_1}{(r-a)^3}m$ and $F_2 = 2\left(\frac{\mu_0}{4\pi}\right)\frac{M_1}{(r+a)^3}m$



$\therefore \quad F_{res} = F_1 - F_2 = 2\left(\frac{\mu_0}{4\pi}\right)M_1 m\left[\left(\frac{1}{(r-a)^3}\right)-\left(\frac{1}{(r+a)^3}\right)\right]$

$= 2\left(\frac{\mu_0}{4\pi}\right)\frac{M_1 m}{r^3}\left[\left(1-\frac{a}{r}\right)^{-3}-\left(1+\frac{a}{r}\right)^{-3}\right]$

By using acceleration, Binomial expansion, and neglecting terms of high power we get

$F_{res} = 2\left(\frac{\mu_0}{4\pi}\right)\frac{M_1 m}{r^3}\left[1+\frac{3a}{r}-1+\frac{3a}{r}\right]$

$= 2\left(\frac{\mu_0}{4\pi}\right)\frac{M_1 m}{r^3}\frac{6a}{r} = 2\left(\frac{\mu_0}{4\pi}\right)\frac{M_1 3M_2}{r^4} = 6\left(\frac{\mu_0}{4\pi}\right)\frac{M_1 M_2}{r^4}$

Direction of F_{res} is towards right.

Alternative Method :

$B = \frac{\mu_0}{4\pi}.\frac{2M_1}{r^3} \quad \Rightarrow \quad \frac{dB}{dr} = -\frac{\mu_0}{4\pi}\times\frac{6M_1}{r^4}$

$F = -M_2 \times \frac{dB}{dr} \quad \Rightarrow \quad F = \left(\frac{\mu_0}{4\pi}\right)\frac{6M_1M_2}{r^4}$

Example 2. A magnet is 10 cm long and its pole strength is 120 CGS units (1 CGS unit of pole strength = 0.1 A-m). Find the magnitude of the magnetic field B at a point on its axis at a distance 20 cm from it.

Solution : The pole strength is m = 120 CGS units = 12A-m.

Magnetic length is 2ℓ = 10 cm or ℓ = 0.05 m.

Distance from the magnet is d = 20 cm = 0.2 m. The field B at a point in end-on position is

$B = \frac{\mu_0}{4\pi}\frac{2Md}{(d^2-\ell^2)^2} = \frac{\mu_0}{4\pi}\frac{4m\ell d}{(d^2-\ell^2)^2}$

$= \left(10^{-7}\frac{T-m}{A}\right)\frac{4\times(12A-m)\times(0.05m)\times(0.2m)}{[(0.2m)^2-(0.05m)^2]^2}$

$= 3.4 \times 10^{-5}$ T.

Example 3. Find the magnetic field due to a dipole of magnetic moment 1.2 A-m^2 at a point 1 m away from it in a direction making an angle of 60º with the dipole-axis.

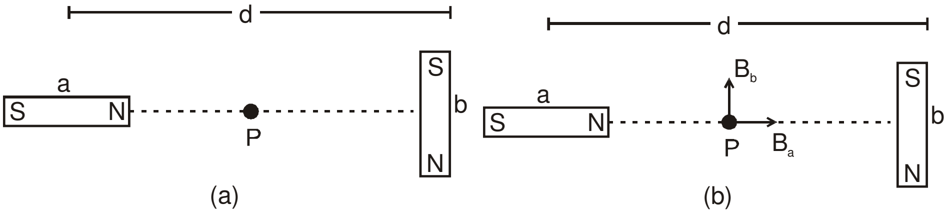
Solution : The magnitude of the field is $B = \frac{\mu_0}{4\pi}\frac{M}{r^3}\sqrt{1+3\cos^2\theta}$

$= \left(10^{-7}\frac{T-m}{A}\right)\frac{1.2A-m^2}{1m^3}\sqrt{1+3\cos^2 60º} = 1.6 \times 10^{-7}$ T.

The direction of the field makes an angle α with the radial line where

$\tan\alpha = \frac{\tan\theta}{2} = \frac{\sqrt{3}}{2}$

Example 4. Figure shows two identical magnetic dipoles a and b of magnetic moments M each, placed at a separation d, with their axes perpendicular to each other. Find the magnetic field at the point P midway between the dipoles.



(a) (b)

Solution : The point P is in end-on position for the dipole (a) and in broadside-on position for the dipole (b). The magnetic field at P due to a is $B_a = \frac{\mu_0}{4\pi}\frac{2M}{(d/2)^3}$ along the axis of a, and that due to b is

$B_b = \frac{\mu_0}{4\pi}\frac{M}{(d/2)^3}$ parallel to the axis of b as shown in figure. The resultant field at P is, therefore.

$$B = \sqrt{B_a^2 + B_b^2}$$

$$= \frac{\mu_0 M}{4\pi(d/2)^3}\sqrt{1^2+2^2} = \frac{2\sqrt{5}\mu_0 M}{\pi d^3}$$

The direction of this field makes an angle α with B_a such that $\tan\alpha = B_b/B_a = 1/2$.

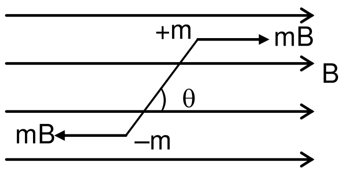
1.4 Magnet in an external uniform magnetic field :

(same as case of electric dipole)

$F_{res} = 0$ (for any angle)

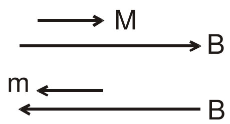
$\tau = MB \sin\theta$

* here θ is angle between $\vec{B}$ and $\vec{M}$



Note :

- $\vec{\tau}$ acts such that it tries to make $\vec{M} \times \vec{B}$.
- $\vec{\tau}$ is same about every point of the dipole it's potential energy is



$U = -MB\cos\theta = -\vec{M}\cdot\vec{B}$

$\theta = 0^\circ$ is stable equilibrium

$\theta = \pi$ is unstable equilibrium

for small 'θ' the dipole performs SHM about $\theta = 0^\circ$ position

$\tau = -MB\sin\theta$;

$I\alpha = -MB\sin\theta$

for small θ, $\sin\theta \simeq \theta$

$$\alpha = -\left(\frac{MB}{I}\right)\theta$$

Angular frequency of SHM

$$\omega = \sqrt{\frac{MB}{I}} = \frac{2\pi}{T} \qquad \Rightarrow \qquad T = 2\pi\sqrt{\frac{I}{MB}}$$

here $I = I_{cm}$ if the dipole is free to rotate $= I_{hinge}$ if the dipole is hinged

Solved Examples

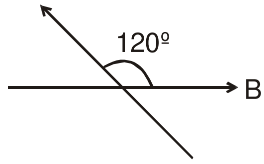
Example 5. A bar magnet having a magnetic moment of 1.0×10^{-4} J/T is free to rotate in a horizontal plane. A horizontal magnetic field $B = 4 \times 10^{-5}$ T exists in the space. Find the work done in rotating the magnet slowly from a direction parallel to the field to a direction 60º from the field.

Solution : The work done by the external agent = change in potential energy

$= (-MB \cos\theta_2) - (-MB \cos\theta_1) = -MB (\cos 60º - \cos 0º)$

$= \frac{1}{2} MB = \frac{1}{2} \times (1.0 \times 10^4 \text{ J/T}) (4 \times 10^{-5} \text{ T}) = 0.2 \text{ J}$

Example 6. A magnet of magnetic dipole moment M is released in a uniform magnetic field of induction B from the position shown in the figure. Find :



(i) Its kinetic energy at $\theta = 90º$

(ii) its maximum kinetic energy during the motion.

(iii) will it perform SHM? oscillation ? Periodic motion? What is its amplitude?

Solution : (i) Apply energy conservation at $\theta = 120º$ and $\theta = 90º$

$- MB \cos 120º + 0 = - MB \cos 90º + (K.E.) \Rightarrow \quad KE = \frac{MB}{2}$ **Ans.**

(ii) K.E. will be maximum where P.E. is minimum. P.E. is minimum at $\theta = 0º$. Now apply energy conservation between $\theta = 120º$ and $\theta = 0º$.

$- mB \cos 120º + 0 = -mB \cos 0º + (KE)_{max}$

$\frac{3}{2} (KE)_{max} = MB$ **Ans.**

The K.E. is max at $\theta = 0º$ can also be proved by torque method. From $\theta = 120º$ to $\theta = 0º$ the torque always acts on the dipole in the same direction (here it is clockwise) so its K.E. keeps on increases till $\theta = 0º$. Beyond that τ reverses its direction and then K.E. starts decreasing $\therefore$ $\theta = 0º$ is the orientation of M to here the maximum K.E.

(iii) Since 'θ' is not small.

$\therefore$ the motion is not S.H.M. but it is oscillatory and periodic amplitude is 120º.

Example 7. A bar magnet of mass 100 g, length 7.0 cm, width 1.0 cm and height 0.50 cm takes $\pi/2$ seconds to complete an oscillation in an oscillation magnetometer placed in a horizontal magnetic field of 25μT.

(a) Find the magnetic moment of the magnet.

(b) If the magnet is put in the magnetometer with its 0.50 cm edge horizontal, what would be the time period?

Solution : (a) The moment of inertia of the magnet about the axis of rotation is $I = \frac{m'}{12} (L^2 + b^2)$

$\frac{100 \times 10^{-3}}{12} = [(7 \times 10^{-2})^2 + (1 \times 10^{-2})^2] \text{ kg-m}^2. = \frac{25}{6} \times 10^{-5} \text{ kg-m}^2.$

We have, $T = 2\pi \sqrt{\frac{I}{MB}}$ or $M = \frac{4\pi^2 I}{BT^2} = \frac{4\pi^2 \times 25 \times 10^{-5} \text{kg/m}^2}{6 \times (25 \times 10^{-6} \text{T}) \times \frac{\pi^2}{4} \text{s}^2} = 27 \text{ A-m}^2.$

(b) In this case the moment of inertia becomes $I' = \frac{m'}{12}(L^2 + b^2)$ where b' = 0.5 cm.

The time period would be $T' = \sqrt{\frac{I'}{MB}}$ .... (ii)

Dividing by equation (i),

$$\frac{T'}{T} = \sqrt{\frac{I'}{I}} = \frac{\sqrt{\frac{m'}{12}(L^2 + b'^2)}}{\sqrt{\frac{m'}{12}(L^2 + b^2)}} = \frac{\sqrt{(7cm)^2 + (0.5cm)^2}}{\sqrt{(7cm)^2 + (1.0cm)^2}} = 0.992$$

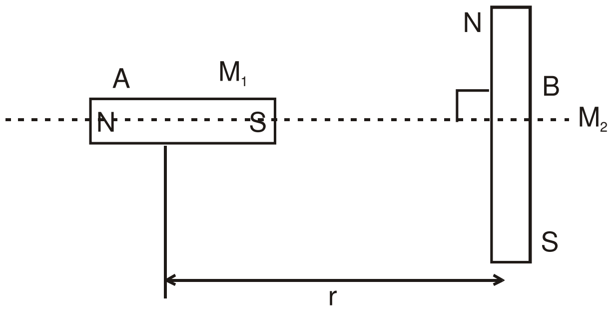
or $T' = \frac{0.992 \times \pi}{2}$ s $= 0.496\pi$ s.

1.5 Magnet in an External Non-uniform Magnetic Field :

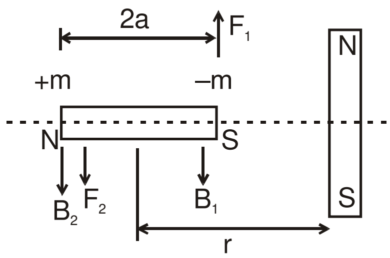
No special formulae are applied is such problems. Instead see the force on individual poles and calculate the resistant force torque on the dipole.

Solved Examples

Example 8. Find the torque on M_1 due to M_2 .



Solution : Due to M_2, magnetic fields at 'S' and 'N' of M_1 are B_1 and B_2 respectively. The forces on –m and +m are F_1 and F_2 as shown in the figure. The torque (about the centre of the dipole m_1) will be



$= F_1 a + F_2 a = (F_1 + F_2)a$

$$= \left[\left(\frac{\mu_0}{4\pi}\right)\frac{M_2}{(r-a)}m + \frac{\mu_0}{4\pi}\frac{M_2}{(r+a)}m\right]a$$

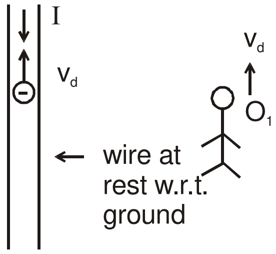
$$\cong \frac{\mu_0}{4\pi}M_2 m\left(\frac{1}{r^3} + \frac{1}{r^3}\right)a \qquad \because\ a << r$$

$$= \frac{\mu_0 M_2 m}{4\pi}\frac{2}{r^3}a = \frac{\mu_0 M_1 M_2}{4\pi r^3}$$ **Ans.**

2. MAGNETIC EFFECTS OF CURRENT (AND MOVING CHARGE)

It was observed by **OERSTED** that a current carrying wire produces magnetic field nearly it. It can be tested by placing a magnet in the near by space, it will show some movement (deflection or rotation of displacement). This observation shows that current or moving charge produces magnetic field.

2.1 Frame Dependence of $\vec{B}$.



(a) The motion of anything is a relative term. A charge may appear at rest by an observer (say O_1) and moving at same velocity $\vec{v}_1$ with respect to observer O_2 and at velocity $\vec{v}_2$ with respect to observers O_3 then $\vec{B}$ due to that charge w.r.t. O_1 will be zero and w.r. to O_2 and O_3 it will be $\vec{B}_1$ and $\vec{B}_2$ (that means different)

(b) In a current carrying wire electron move in the opposite direction to that of the current and +ve ions (of the metal) are static w.r.t. the wire. Now if some observer (O_1) moves with velocity V_d in the direction of motion of the electrons then electrons will have zero velocity and +ve ions will have velocity V_d in the downward direction w.r.t. O_1. The density (n) of +ve ions is same as the density of free electrons and their charges are of the same magnitudes

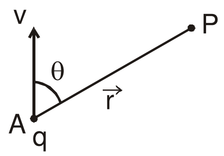
So, w.r.t. O_1 electrons will produce zero magnetic field but +ve ions will produce +ve same $\vec{B}$ due to the current carrying wire does not depend on the reference frame (this is true for any velocity of the observer).

(c) **$\vec{B}$ due to magnet :**

$\vec{B}$ produced by the magnet does not contain the term of velocity

So, we can say that the $\vec{B}$ due magnet does not depend on frame.

2.2 due to a point charge :



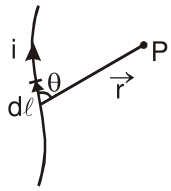
A charge particle 'q' has velocity v as shown in the figure. It is at position 'A' at some time. $\vec{r}$ is the position vector of point 'P' w.r. to position of the charge. Then $\vec{B}$ at P due to q is

$$B = \left(\frac{\mu_0}{4\pi}\right)\frac{qv\sin\theta}{r^2}$$; here θ = angle between $\vec{v}$ and $\vec{r}$

$$\vec{B} = \left(\frac{\mu_0}{4\pi}\right)\frac{q\vec{v}\times\vec{r}}{r^3}$$; with sign $\Rightarrow$ $\vec{B}\perp\vec{v}$ and also $\vec{B}\perp\vec{r}$.

Direction of $\vec{B}$ will be found by using the rules of vector product.

2.3 Biot-savart's law ($\vec{B}$ due to a wire)



It is an experimental law. A current 'i' flows in a wire (may be straight or curved). Due to '$d\ell$' length of the wire the magnetic field at 'P' is

$dB \propto i d\ell$

$\propto \frac{1}{r^2}$ $\qquad\Rightarrow\qquad$ $\propto \sin\theta$

$$\Rightarrow\ dB \propto \frac{i d\ell\sin\theta}{r^2} \qquad \Rightarrow\ dB = \left(\frac{\mu_0}{4\pi}\right)\frac{i d\ell\sin\theta}{r^2} \qquad \Rightarrow\ \overrightarrow{dB} = \left(\frac{\mu_0}{4\pi}\right)\frac{i\overrightarrow{d\ell}\times\vec{r}}{r^3}$$

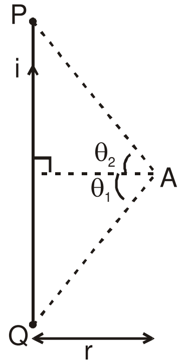
here $\vec{r}$ = position vector of the test point w.r.t. $\overrightarrow{d\ell}$

θ = angle between $\overrightarrow{d\ell}$ and $\vec{r}$. The resultant $\vec{B} = \int\overrightarrow{dB}$

Using this fundamental formula we can derive the expression of $\vec{B}$ due to a long wire.

2.3.1 $\vec{B}$ due to a straight wire :

Due to a straight wire 'PQ' carrying a current 'i' the $\vec{B}$ at A is given by the formula



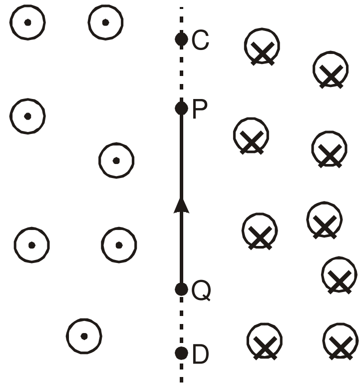
$B = \frac{\mu_0 I}{4\pi r}$ (sin θ_1 + sin θ_2)

(Derivation can be seen in a standard text book like your school book or concept of physics of HCV part-II)

Direction :

Due to every element of 'PQ' $\vec{B}$ at A is directed in wards. So its resultant is also directed inwards. It is represented by (x)

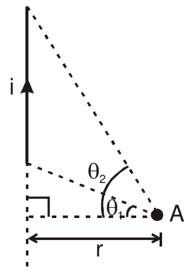
The direction of $\vec{B}$ at various points is shown in the figure shown



At points 'C' and 'D' $\vec{B}$ = 0 (think how).

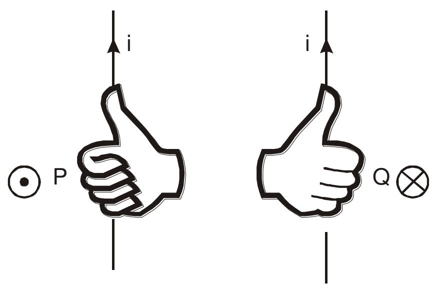
For the case shown in figure

B at A = $\frac{\mu_0 i}{4\pi r}$ (sin θ_2 – sinθ_1) $\otimes$

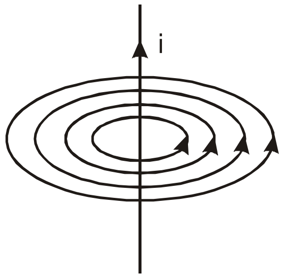


Shortcut for Direction :

The direction of the magnetic field at a point P due to a straight wire can be found by a slight variation in the right-hand thumb rule. If we stretch the thumb of the right hand along the current and curl our fingers to pass through the point P, the direction of the fingers at P gives the direction of the magnetic field there.

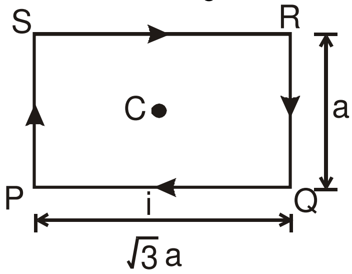


We can draw magnetic field lines on the pattern of electric field lines. A tangent to a magnetic field line given the direction of the magnetic field existing at that point. For a straight wire, the field lines are concentric circles with their centers on the wire and in the plane perpendicular to the wire. There will be infinite number of such lines in the planes parallel to the above mentioned plane.



Solved Examples

Example 9. Find resultant magnetic field at 'C' in the figure shown.



Solution : It is clear that 'B' at 'C' due all the wires is directed ⊗. Also B at 'C' due PQ and SR is same. Also due to QR and PS is same

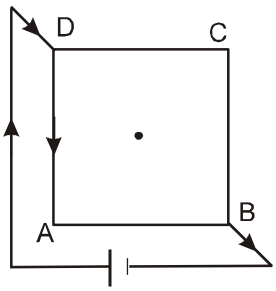
$\therefore\ B_{res} = 2(B_{PQ} + B_{SP})$

$$B_{PQ} = \frac{\mu_0 i}{4\pi \frac{a}{2}} (\sin 60^\circ + \sin 60^\circ),$$

$$B_{sp} = \frac{\mu_0 i}{4\pi \frac{\sqrt{3}a}{2}} (\sin 30^\circ + \sin 30^\circ)$$

$$\Rightarrow\ B_{res} = 2\left(\frac{\sqrt{3}\ \ \mu_0 i}{2\pi a} + \frac{\mu_0 i}{2\pi a\sqrt{3}}\right) = \frac{4\mu_0 i}{\sqrt{3}\pi a}$$

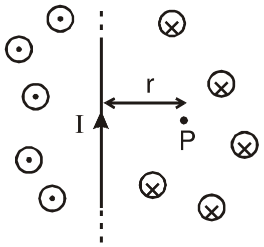
Example 10. Figure shows a square loop made from a uniform wire. Find the magnetic field at the centre of the square if a battery is connected between the points B and D as shown in the figure



Solution : The current will be equally divided at D. The fields at the centre due to the currents in the wires DA and DC will be equal in magnitude and opposite in direction. The resultant of these two fields will be zero. Similarly, the resultant of the fields due to the wires AB and CB will be zero. Hence, the net field at the centre will be zero.

Special case :

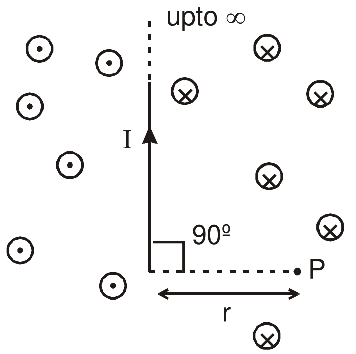
(i) If the wire is infinitely long then the magnetic field at 'P' (as shown in the figure) is given by (using $\theta_1 = \theta_2 = 90^o$ and the formula of 'B' due to straight wire)



$$B = \frac{\mu_0 I}{2\pi r} \Rightarrow B \propto \frac{I}{r}$$

The direction of $\vec{B}$ at various is as shown in the figure. The magnetic lines of force will be concentric circles around the wire (as shown earlier)

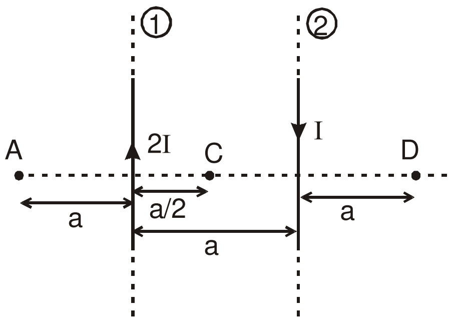
(ii) If the wire is infinitely long but 'P' is as shown in the figure. The direction of $\vec{B}$ at various points is as shown in the figure. At 'P'



$$B = \frac{\mu_0 I}{4\pi r}$$

Solved Example

Example 11. In the figure shown there are two parallel long wires (placed in the plane of paper) are carrying currents 2I and I consider points A, C, D on the line perpendicular to both the wires and also in the plane of the paper. The distances are mentioned. Find



(i) $\vec{B}$ at A, C, D

(ii) position of point on line A C D where $\vec{B}$ is O

Solution : (i) Let us call $\vec{B}$ due to (1) and (2) as $\vec{B}_1$ and $\vec{B}_2$ respectively. Then

at A : $\vec{B}_1$ is $\odot$ and $\vec{B}_2$ is $\otimes$

$B_1 = \frac{\mu_0 2I}{2\pi a}$ and $B_2 = \frac{\mu_0 I}{2\pi 2a}$

$\therefore \quad B_{res} = B_1 - B_2 = \frac{3}{4}\frac{\mu_0 I}{\pi a} \odot$ **Ans.**

at C : $\vec{B}_1$ is $\otimes$ and $\vec{B}_2$ also $\otimes$

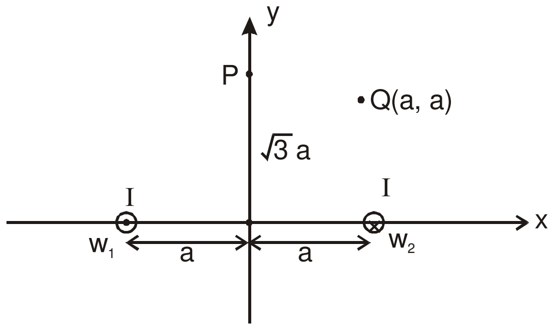
$\therefore \quad B_{res} = B_1 + B_2 = \frac{\mu_0 2I}{2\pi\frac{a}{2}} + \frac{\mu_0 I}{2\pi\frac{a}{2}} = \frac{6\mu_0 I}{2\pi a} = \frac{3\mu_0 I}{\pi a} \otimes$ **Ans.**

at D : $\vec{B}_1$ is $\otimes$ and $\vec{B}_2$ is $\odot$ and both are equal in magnitude.

$\therefore \quad B_{res} = 0$ **Ans.**

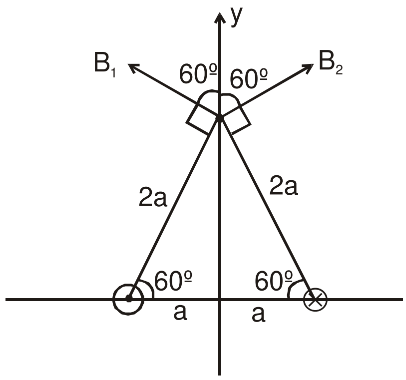
(ii) It is clear from the above solution that B = 0 at point 'D'.

Example 12. In the figure shown two long wires W_1 and W_2 each carrying current I are placed parallel to each other and parallel to z-axis. The direction of current in W_1 is outward and in W_2 it is inwards. Find the $\vec{B}$ at 'P' and 'Q'.



Solution : Let $\vec{B}$ due to W_1 be $\vec{B}_1$ and due to W_2 be $\vec{B}_2$.

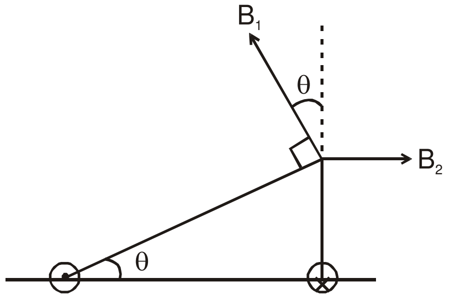
By symmetry $|\vec{B}_1| = |\vec{B}_2| = B$



$B_p = 2\,B\cos 60^\circ = B = \frac{\mu_0 I}{2\pi 2a} = \frac{\mu_0 I}{4\pi a}$

$\therefore \quad \vec{B_p} = \frac{\mu_0 I}{4\pi a}\hat{j}$ **Ans.**

For $\theta \quad B_1 = \frac{\mu_0 I}{2\pi\sqrt{5}a}, \quad \Rightarrow \quad B_2 = \frac{\mu_0 I}{2\pi a}$

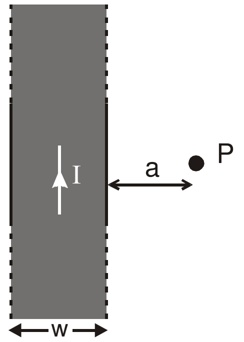


$\tan\theta = \dfrac{a}{2a} = \dfrac{1}{2}$ $\Rightarrow$ $\vec{B} = (B_1 \cos\theta\, \hat{j}) + (B_2 - B_1 \sin\theta)\, \hat{i}$

$\sin\theta = \dfrac{1}{\sqrt{5}}$ $\Rightarrow$ $\vec{B} = \dfrac{\mu_0 I}{5\pi a}\hat{j} + \left(\dfrac{\mu_0 I}{2\pi a} - \dfrac{\mu_0 I}{10\pi a}\right)\hat{i}$

$\cos\theta = \dfrac{2}{\sqrt{5}}$ $\Rightarrow$ $\vec{B} = \dfrac{2\mu_0 I}{5\pi a}\hat{i} + \dfrac{\mu_0 I}{5\pi a}\hat{j}$

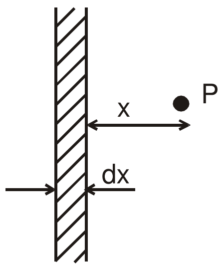
Example 13. In the figure shown a large metal sheet of width 'w' carries a current I (uniformly distributed in its width 'w'. Find the magnetic field at point 'P' which lies in the plane of the sheet.



Solution : To find 'B' at 'P' the sheet can be considered as collection of large number of infinitely long wires. Take a long wire distance 'x' from 'P' and of width 'dx'. Due to this the magnetic field at 'P' is 'dB'

$$dB = \frac{\mu_0 \left(\frac{I}{w}dx\right)}{2\pi x} \otimes$$

due to each such wire $\vec{B}$ will be directed inwards

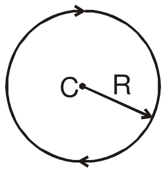


$$\therefore \quad B_{res} = \int dB = \frac{\mu_0 I}{2\pi w}\int_{x=a}^{a+w}\frac{dx}{x}\int_{x=a}^{a+w}\frac{dx}{x} = \frac{\mu_0 I}{2\pi w}.\ \ln\frac{a+w}{a}$$ **Ans.**

2.3.2 $\vec{B}$ due to circular loop

(a) At centre : Due to each $\vec{d\ell}$ element of the loop $\vec{B}$ at 'c' is inwards (in this case).

$\therefore$ $\overrightarrow{B_{res}}$ at 'c' is $\otimes$.



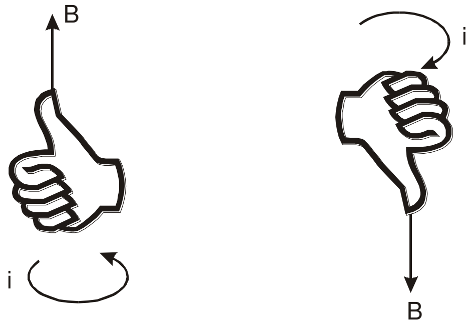
$B = \frac{\mu_0 NI}{2R}$,

N = No. of turns in the loop.

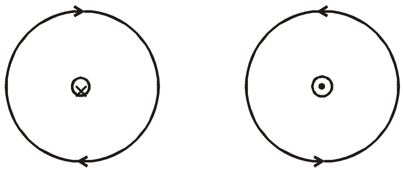
$= \frac{\ell}{2\pi R}$; ℓ = length of the loop.

N can be fraction $\left(\frac{1}{4}, \frac{1}{3}, \frac{11}{3} \text{etc.}\right)$ or integer

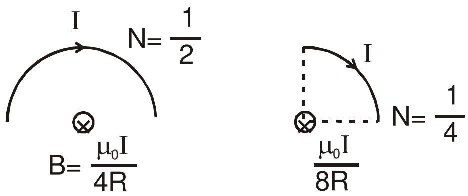
Direction of $\vec{B}$: The direction of the magnetic field at the centre of a circular wire can be obtained using the right-hand thumb rule. If the fingers are curled along the current, the stretched thumb will point towards the magnetic field (figure).



Another way to find the direction is to look into the loop along its axis. If the current is in anticlockwise direction, the magnetic field is towards the viewer. If the current is in clockwise direction, the field is away from the viewer.

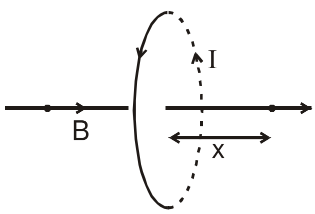


Semicircular and Quarter of a circle :



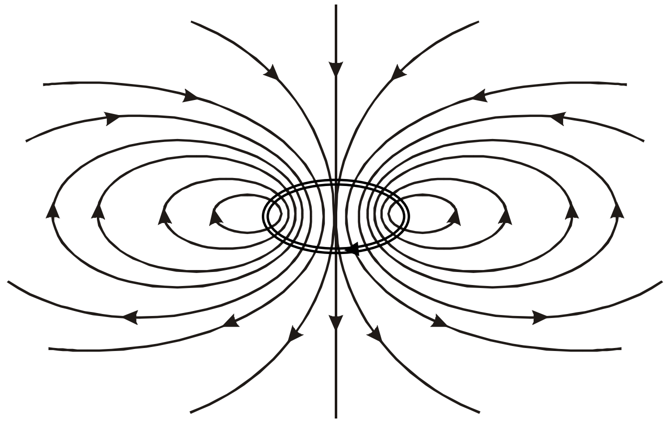
(b) On the axis of the loop : $B = \frac{\mu_0 NIR^2}{2(R^2 + x^2)^{3/2}}$

N = No. of turns (integer)



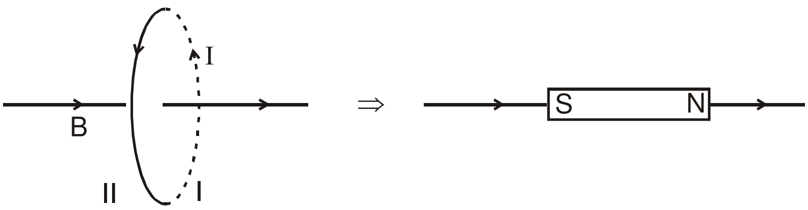
Direction can be obtained by right hand thumb rule. curl your fingers in the direction of the current then the direction of the thumb points in the direction of at the points on the axis.

The magnetic field at a point not on the axis is mathematically difficult to calculate. We show qualitatively in figure the magnetic field lines due to a circular current which will give some idea of the field.

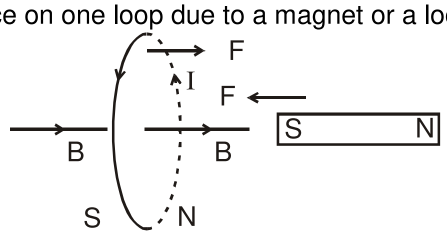


2.3.3 A loop as a magnet :

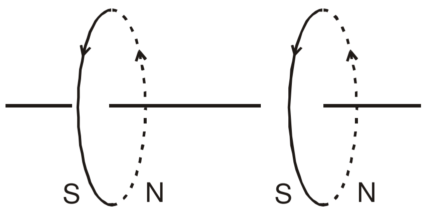
The pattern of the magnetic field is comparable with the magnetic field produced by a bar magnet.



The side 'I' (the side from which the $\vec{B}$ emerges out) of the loop acts as 'NORTH POLE' and side II (the side in which the $\vec{B}$ enters) acts as the 'SOUTH POLE'. It can be verified by studying force on one loop due to a magnet or a loop.



The loop and the magnet attract each other



The two loops attract each other.

Mathematically

$B_{axis} = \dfrac{\mu_0 NIR^2}{2(R^2 + x^2)^{3/2}} \cong \dfrac{\mu_0 NIR^2}{2x^3}$ for x >> R

$= 2\left(\dfrac{\mu_0}{4\pi}\right)\left(\dfrac{IN\pi R^2}{x^3}\right)$

it is similar to B_{axis} due to magnet $= 2\left(\dfrac{\mu_0}{4\pi}\right)\dfrac{m}{x^3}$

Magnetic dipole moment of the loop

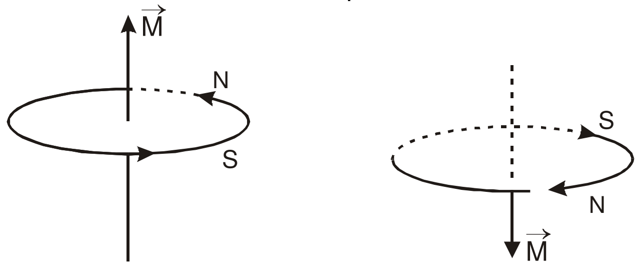
$M = IN\pi R^2$

$M = INA$ for any other shaped loop.
Unit of M is Amp. m^2.
Unit of m (pole strength) = Amp. m {$\because$ in magnet $M = m\ell$}

$\vec{M} = IN\vec{A}$,

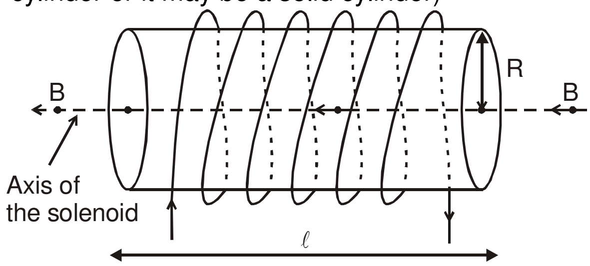
$\vec{A}$ = unit normal vector for the loop.

To be determined by right hand rule which is also used to determine direction of $\vec{B}$ on the axis. It is also from 'S' side to 'N' side of the loop.



2.3.4 Solenoid :

(i) Solenoid contains large number of circular loops wrapped around a non-conducting cylinder. (it may be a hollow cylinder or it may be a solid cylinder)



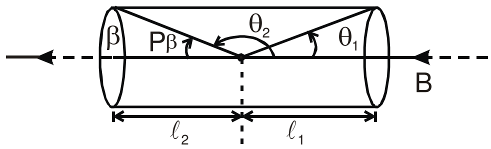
(ii) The winding of the wire is uniform direction of the magnetic field is same at all points of the axis.

(iii) $\vec{B}$ on axis (turns should be very close to each others).

$B = \frac{\mu_0 ni}{2}$ (cos θ_1 – cos θ_2)

where n : number of turns per unit length.

$\cos\theta_1 = \frac{\ell_1}{\sqrt{\ell_1^2 + R^2}}$; $\cos\beta = \frac{\ell_2}{\sqrt{\ell_2^2 + R^2}} = -\cos\theta_2$



$$B = \frac{\mu_0 ni}{2}\left[\frac{\ell_1}{\sqrt{\ell_1^2 + R^2}} + \frac{\ell_2}{\sqrt{\ell_2^2 + R^2}}\right] = \frac{\mu_0 ni}{2}(\cos\theta_1 + \cos\beta)$$

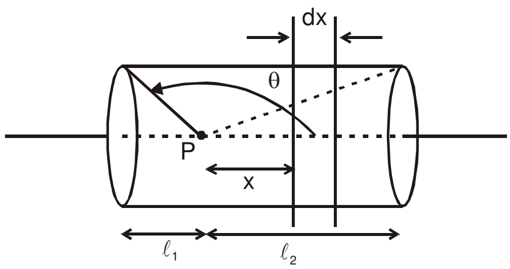
Note :

- Use right hand rule for direction (same as the direction due to loop).

Derivation :

Take an element of width dx at a distance x from point P. [point P is the point on axis at which we are going to calculate magnetic field. Total number of turns in the element dn = ndx where n : number of turns per unit length.

$$dB = \frac{\mu_0 i R^2}{2(R^2 + x^2)^{3/2}}(ndx)$$



$$B = \int dB = \int_{-\ell_1}^{\ell_2} \frac{\mu_0 i \ R^2 \ ndx}{2(R^2 + x^2)^{3/2}} = \frac{\mu_0 ni}{2}\left[\frac{\ell_1}{\sqrt{\ell_1^2 + R^2}} + \frac{\ell_2}{\sqrt{\ell_2^2 + R^2}}\right] = \frac{\mu_0 ni}{2}\left[\cos\theta_1 - \cos\theta_2\right]$$

(iv) For 'Ideal Solenoid' :

***Inside** (at the mid point)*

$\ell >> R$ or length is infinite

$\theta_1 \to 0$; $\theta_2 \to \pi$

$$B = \frac{\mu_0 ni}{2} [1 - (-1)]$$

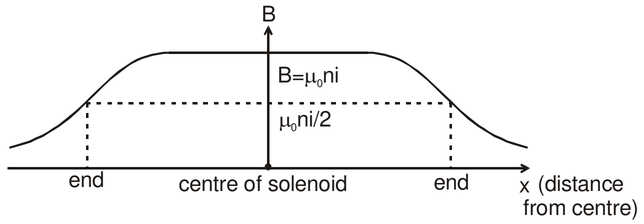
$$\mathbf{B = \mu_0 ni}$$

If material of the solid cylinder has relative permeability 'μ_r' then $B = \mu_0\mu_r ni$

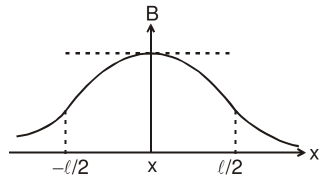
At the ends $B = \dfrac{\mu_0 ni}{2}$

(v) Comparison between ideal and real solenoid :

(a) Ideal Solenoid



Real Solenoid



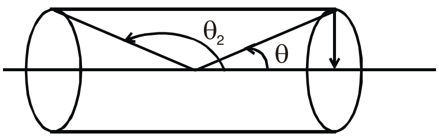
Solved Examples

Example 14. A solenoid of length 0.4 m and diameter 0.6 m consists of a single layer of 1000 turns of fine wire carrying a current of 5.0×10^{-3} ampere. Find the magnetic field on the axis at the middle and at the ends of the solenoid. (Given $\mu_0 = 4\pi \times 10^{-7} \dfrac{V-s}{A-m}$).

Solution : $B = \dfrac{1}{2}\mu_0 ni [\cos\theta_1 - \cos\theta_2]$ $\Rightarrow$ $n = \dfrac{1000}{0.4} = 2500$ per meter

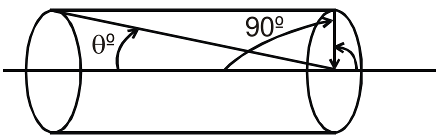
$i = 5 \times 10^{-3}$ A.

(i) $\cos\theta_1 = \dfrac{0.2}{\sqrt{(0.3)^2 + (0.2)^2}} = \dfrac{0.2}{\sqrt{0.13}}$



$\cos\theta_2 = \dfrac{-0.2}{\sqrt{0.13}}$ $\Rightarrow$ $B = \dfrac{1}{2} \times (4 \times \pi \times 10^{-7}) \times 2500 \times 5 \times 10^{-3} \dfrac{2 \times 0.2}{\sqrt{0.13}} = \dfrac{\pi \times 10^{-5}}{\sqrt{13}}$ T

(ii) At the end



$\cos\theta_1 = \dfrac{0.4}{\sqrt{(0.3)^2 + (0.4)^2}} = 0.8$

$\Rightarrow$ $\cos\theta_2 = \cos 90^\circ = 0$

$B = \dfrac{1}{2} \times (4 \times \pi \times 10^{-7}) \times 2500 \times 5 \times 10^{-3} \times 0.8$

$\Rightarrow$ $B = 2\pi \times 10^{-6}$ Wb/m^2

2.4 AMPERE's circuital law :

The line integral $\oint \vec{B}\cdot\vec{d\ell}$ on a closed curve of any shape is equal to μ_0 (permeability of free space) times the net current I through the area bounded by the curve.

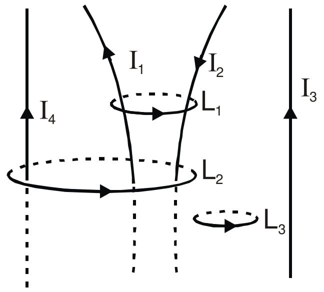
$$\oint \vec{B}\cdot\vec{d\ell} = \mu_0 I$$

Note :

- Line integral is independent of the shape of path and position of wire with in it.
- The statement $\oint \vec{B}\cdot\vec{d\ell} = 0$ does not necessarily mean that $\vec{B} = 0$ everywhere along the path but only that no net current is passing through the path.
- **Sign of current :** The current due to which $\vec{B}$ is produced in the same sense as $\vec{d\ell}$ (i.e. $\vec{B}\cdot\vec{d\ell}$ positive will be taken positive and the current which produces $\vec{B}$ in the sense opposite to $\vec{d\ell}$ will be negative.

Solved Examples

Example 15. Find the values of $\oint \vec{B}\cdot\vec{d\ell}$ for the loops L_1, L_2, L_3 in the figure shown. The sense of $\vec{d\ell}$ is mentioned in the figure.

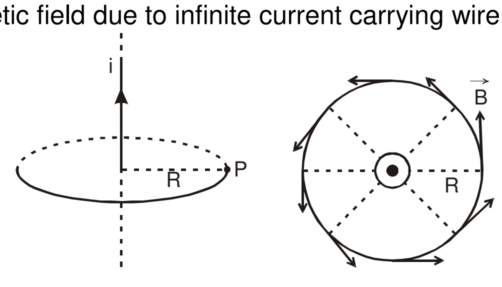


Solution : for L_1 = $\oint \vec{B}\cdot\vec{d\ell}$ $\mu_0(I_1 - I_2)$

here I_1 is taken positive because magnetic lines of force produced by I_1 is anti clockwise as seen from top. I_2 produces lines of $\vec{B}$ in clockwise sense as seen from top. The sense of $\vec{d\ell}$ is anticlockwise as seen from top.

for L_2 : $\oint \vec{B}\cdot\vec{d\ell} = \mu_0$ $(I_1 - I_2 + I_4)$ for L_3 : $\oint \vec{B}\cdot\vec{d\ell} = 0$

Uses : 2.4.1 To find out magnetic field due to infinite current carrying wire



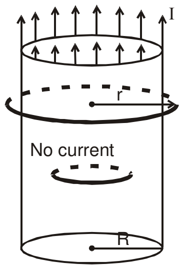
By B.S.L. $\vec{B}$ will have circular lines. $\vec{d\ell}$ is also taken tangent to the circle.

$\oint \vec{B}\cdot\vec{d\ell}$ = $\oint$ B.dℓ $\quad\because \theta = 0^\circ$ so B d $\oint d\ell$ = B $2\pi R$ ($\because$ B = const.)

Now by amperes law : B $2\pi R$ = μ_0 I $\quad\therefore\quad$ B = $\dfrac{\mu_0 i}{2\pi R}$

2.4.2. Hollow current carrying infinitely long cylinder :

(I is uniformly distributed on the whole circumference)



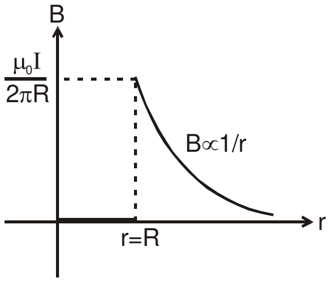
(i) for $r \geq R$

By symmetry the amperian loop is a circle.

$\oint \vec{B} \cdot \vec{d\ell} = \oint B d\ell \quad \because \theta = 0$

$= B \int_0^{2\pi r} d\ell \quad \because B = \text{const.} \quad \Rightarrow \quad B = \dfrac{\mu_0 I}{2\pi r}$

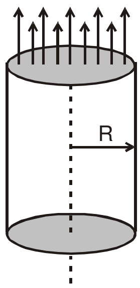
(ii) $r < R = \oint \vec{B} \cdot \vec{d\ell} = \oint B d\ell$



$= B(2\pi r) = 0 \quad \Rightarrow \quad B_{in} = 0$

2.4.3 Solid infinite current carrying cylinder :

Assume current is uniformly distributed on the whole cross section area



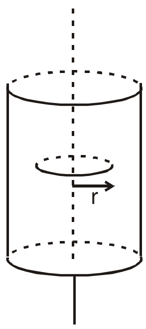
current density $J = \dfrac{I}{\pi R^2}$

Case (I) : $r \leq R$

take an amperian loop inside the cylinder. By symmetry it should be a circle whose centre is on the axis of cylinder and its axis also coincides with the cylinder axis on the loop.

$\oint \vec{B} \cdot \vec{d\ell} = \oint B \cdot d\ell = B\oint d\ell = B.\ 2\pi r = \mu_0 \dfrac{I}{\pi R^2} \pi r^2$

$B = \dfrac{\mu_0 I r}{2\pi R^2} = \dfrac{\mu_0 J r}{2} \Rightarrow \vec{B} = \dfrac{\mu_0 \vec{J} \times \vec{r}}{2}$



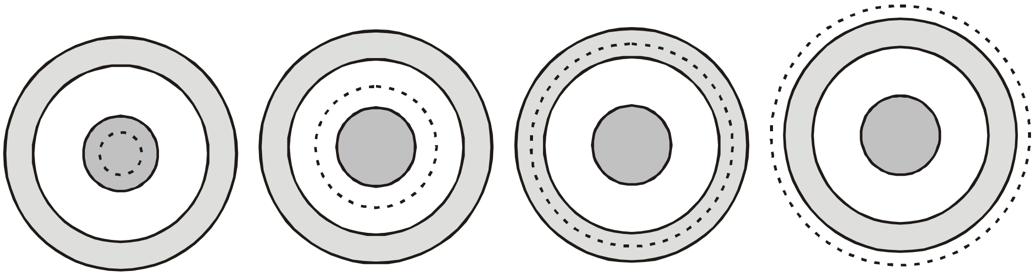
Case (II) : $r \geq R$ $\oint \vec{B}.\vec{d\ell} = \oint B d\ell = B\oint d\ell = B\ .\ (2\pi r) = \mu_0\ .\ I$

$\Rightarrow \quad B = \dfrac{\mu_0 I}{2\pi r}$ also $\vec{B}\ \dfrac{\mu_0 I}{2\pi r}\ (\hat{J} \times \hat{r}) = \dfrac{\mu_0 J \pi R^2}{2\pi r}$

Solved Examples

Example 16. Consider a coaxial cable which consists of an inner wire of radius a surrounded by an outer shell of inner and outer radii b and c respectively. The inner wire carries an electric current i_0 and the outer shell carries an equal current in same direction. Find the magnetic field at a distance x from the axis where (a) x < a, (b) a < x < b (c) b < x < c and (d) x > c. Assume that the current density is uniform in the inner wire and also uniform in the outer shell.

Solution :



(a) (b) (c) (d)

A cross-section of the cable is shown in figure. Draw a circle of radius x with the centre at the axis of the cable. The parts a, b, c and d of the figure correspond to the four parts of the problem. By symmetry, the magnetic field at each point of a circle will have the same magnitude and will be tangential to it. The circulation of B along this circle is, therefore,

$$\oint \vec{B}.d\vec{\ell} = B2\pi x$$

in each of the four parts of the figure.

(a) The current enclosed within the circle in part b is i_0 so that

$$\frac{i_0}{\pi a^2} \cdot \pi x^2 = \frac{i_0}{a^2}x^2 .$$

Ampere's law

$$\oint \vec{B}.d\vec{\ell} = \mu_0 i \text{ gives} \quad B.2\pi x = \frac{\mu_0 i_0 x^2}{a^2} \text{ or, } B = \frac{\mu_0 i_0 x}{2\pi a^2} .$$

The direction will be along the tangent to the circle.

(b) The current enclosed within the circle in part b is i_0 so that

$$B\ 2\pi x = \mu_0\ i_0 \qquad \text{or,} \qquad B = \frac{\mu_0 i_0}{2\pi x} .$$

(c) The area of cross-section of the outer shell is $\pi c^2 - \pi b^2$. The area of cross-section of the outer shell within the circle in part c of the figure is $\pi x^2 - \pi b^2$.

Thus, the current through this part is $\frac{i_0(x^2 - b^2)}{(c^2 - b^2)}$. This is in the same direction to the current i_0 in the inner wire. Thus, the net current enclosed by the circle is

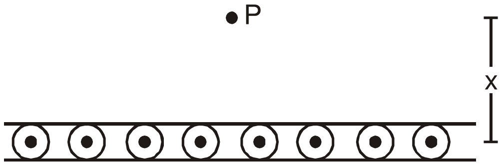
$$i_{net} = i_0 + \frac{i_0(x^2 - b^2)}{c^2 - b^2} = \frac{i_0(c^2 + x^2 - 2b^2)}{c^2 - b^2} .$$

From Ampere's law, $B\ 2\pi x = \frac{i_0(c^2 + x^2 - 2b^2)}{c^2 - b^2}$ or, $B = \frac{\mu_0 i_0(c^2 + x^2 - 2b^2)}{2\pi x(c^2 - b^2)}$

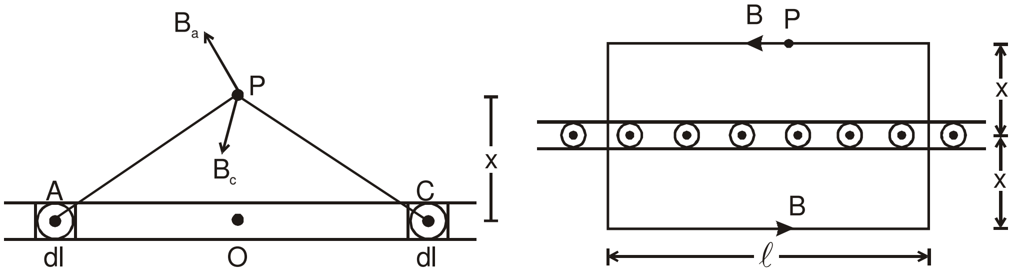
(d) The net current enclosed by the circle in part d of the figure is $2i_0$ and hence

$$B\ 2\pi x = \mu_0\ 2i_0 \qquad \text{or,} \qquad B = \frac{\mu_0 i_0}{\pi x} .$$

Example 17. Figure shows a cross-section of a large metal sheet carrying an electric current along its surface. The current in a strip of width dl is λdl where λ is a constant. Find the magnetic field at a point P at a distance y from the metal sheet.



Solution : Consider two strips A and C of the sheet situated symmetrically on the two sides of P (figure). The magnetic field at P due to the strip A is B_0 perpendicular to AP and that due to the strip C is B_C perpendicular to CP. The resultant of these two is parallel to the width AC of the sheet. The field due to the whole sheet will also be in this direction. Suppose this field has magnitude B.

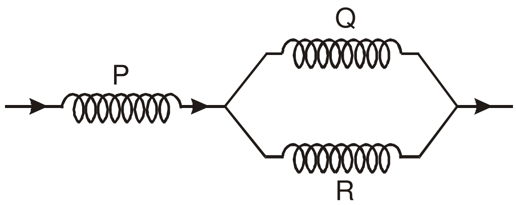


The field on the opposite side of the sheet at the same distance will also be B but in opposite direction. Applying Ampere's law to the rectangle shown in figure.

$2B\ell = \mu_0 \lambda\ell$ or $B = \frac{1}{2}\mu_0 \lambda.$

Note that it is independent of y.

Example 18. Three identical long solenoids P, Q and R are connected to each other as shown in figure. If the magnetic field at the centre of P is 4 T, what would be the field at the centre of Q? Assume that the field due to any solenoid is confined within the volume of that solenoid only.



Solution : As the solenoids are identical, the currents in Q and R will be the same and will be half the current in P. The magnetic field within a solenoid is given by B = $\mu_0$ni. Hence the field in Q will be equal to the field in R and will be half the field in P i.e., will be 2T.

3. MAGNETIC FORCE ON MOVING CHARGE

When a charge q moves with velocity $\vec{v}$, in a magnetic field $\vec{B}$, then the magnetic force experienced by moving charge is given by following formula :

$\vec{F} = q(\vec{v} \times \vec{B})$ Put q with sign.

$\vec{v}$: Instantaneous velocity

$\vec{B}$: Magnetic field at that point.

Note :

- $\vec{F} \perp \vec{v}$ and also $\vec{F} \perp \vec{B}$
- $\because \vec{F} \perp \vec{v}$ $\therefore$ power due to magnetic force on a charged particle is zero. (Use the formula of power $P = \vec{F}\cdot\vec{v}$ for its proof).
- Since work done by magnetic force is zero in every part of the motion. The magnetic force cannot increase or decrease the speed (or kinetic energy) of a charged particle. Its can only change the direction of velocity.
- On a stationary charged particle, magnetic force is zero.
- If $\vec{V} \,||\, \vec{B}$, then also magnetic force on charged particle is zero. It moves along a straight line if only magnetic field is acting.

Solved Examples

Example 19. A charged particle of mass 5 mg and charge q = +2μC has velocity $\vec{v} = 2\hat{i} - 3\hat{j} + 4\hat{k}$. Find out the magnetic force on the charged particle and its acceleration at this instant due to magnetic field $\vec{B} = 3\hat{j} - 2\hat{k}$. $\vec{v}$ and $\vec{B}$ are in m/s and Wb/m^2 respectively.

Solution : $\vec{F} = q\vec{v} \times \vec{B}$ = 2 × 10^{-6} ($2\hat{i} - 3\hat{j} \times 4\hat{k}$) × ($3\hat{j} - 2\hat{k}$)

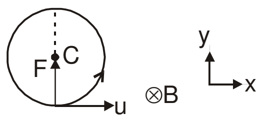
= 2 × 10^{-6} [$-6\hat{i} + 4\hat{j} + 6\hat{k}$] N

By Newton's Law $\vec{a} = \dfrac{\vec{F}}{m} = \dfrac{2\times10^{-6}}{5\times10^{-6}}(-6\hat{i} + 4\hat{j} + 6\hat{k})$

= 0.8 ($-3\hat{i} + 2\hat{j} + 3\hat{k}$) m/s^2

3.1 Motion of charged particles under the effect of magnetic force

- Particle released if v = 0 then $f_m = 0$
  $\therefore$ particle will remain at rest
- $\vec{V} \,||\, \vec{B}$ here θ = 0 or θ = 180º
  $\therefore$ $F_m = 0$ $\therefore$ $\vec{a} = 0$ $\therefore$ $\vec{V}$ = const.
  $\therefore$ particle will move in a straight line with constant velocity



- Initial velocity $\vec{u} \perp \vec{B}$ and $\vec{B}$ = uniform

In this case $\because$ B is in z direction so the magnetic force in z-direction will be zero $\left(\because \ \vec{F_m} \perp \vec{B}\right)$.

Now there is no initial velocity in z-direction.

$\therefore$ particle will always move in xy plane.

$\therefore$ velocity vector is always $\perp \vec{B}$ $\therefore$ F_m = quB = constant

now quB = $\dfrac{mu^2}{R}$ $\Rightarrow$ R = $\dfrac{mu}{qB}$ = constant.

The particle moves in a curved path whose radius of curvature is same every where, such curve in a plane is only a circle.

$\therefore$ path of the particle is circular.

$$R = \frac{mu}{qB} = \frac{p}{qB} = \frac{\sqrt{2mk}}{qB}$$

here p = linear momentum ; k = kinetic energy

now $v = \omega R \Rightarrow \omega = \frac{qB}{m} = \frac{2\pi}{T} = 2\pi f$

Time period $T = 2\pi m/qB$

frequency $f = qB/2\pi m$

Note :

- ω, f, T are independent of velocity.

Solved Examples

Example 20. A proton (p), α-particle and deuteron (D) are moving in circular paths with same kinetic energies in the same magnetic field. Find the ratio of their radii and time periods. (Neglect interaction between particles).

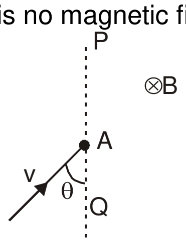
Solution : $R = \frac{\sqrt{2mK}}{qB}$

$$\therefore \quad R_p : R_\alpha : R_D = \frac{\sqrt{2mK}}{qB} : \frac{\sqrt{2.4mK}}{2qB} : \frac{\sqrt{2.2mK}}{qB} = 1 : 1 : \sqrt{2}$$

$T = 2\pi m/qB$

$$\therefore \quad T_p : T_\alpha : T_D = \frac{2\pi m}{qB} : \frac{2\pi 4m}{2qB} : \frac{2\pi 2m}{qB} = 1 : 2 : 2 \textbf{ Ans.}$$

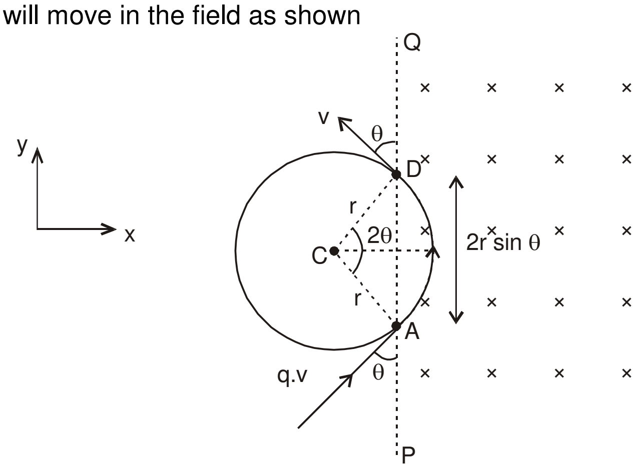
Example 21. A positive charge particle of charge q, mass m enters into a uniform magnetic field with velocity v as shown in the figure. There is no magnetic field to the left of PQ. Find



(i) time spent, (ii) distance travelled in the magnetic field

(iii) impulse of magnetic force.

Solution : The particle will move in the field as shown



Angle subtended by the arc at the centre = 2θ

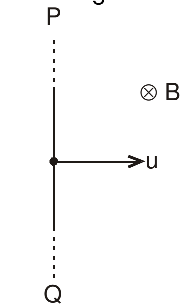
(i) Time spent by the charge in magnetic field $\omega t = 2\theta \Rightarrow \frac{qB}{m} t = 2\theta \Rightarrow t = \frac{2m\theta}{qB}$

(ii) Distance travelled by the charge in magnetic field = $r(2\theta) = \frac{mv}{qB} . 2\theta$

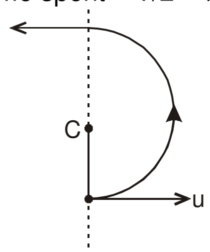
(iii) Impulse = change in momentum of the charge = $(-mv \sin\theta \hat{i} + mv \cos\theta \hat{j}) - (mv \sin\theta \hat{i} + mv \cos\theta \hat{j})$

$= -2mv \sin\theta \hat{i}$

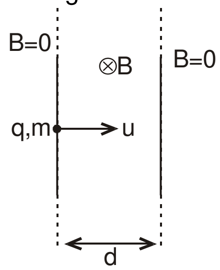
Example 22. In the figure shown the magnetic field on the left of 'PQ' is zero and on the right of 'PQ' it is uniform. Find the time spent in the magnetic field.



Solution : The path will be semicircular time spent = $T/2 = \pi m/qB$

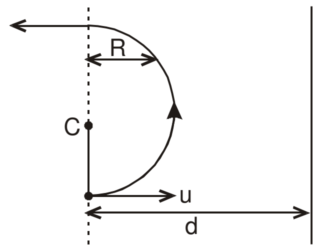


Example 23. A uniform magnetic field of strength 'B' exists in a region of width 'd'. A particle of charge 'q' and mass 'm' is shot perpendicularly (as shown in the figure) into the magnetic field. Find the time spend by the particle in the magnetic field if

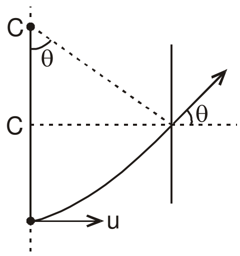


(i) $d > \frac{mu}{qB}$ (ii) $d < \frac{mu}{qB}$

Solution : (i) $d > \frac{mu}{qB}$ means $d > R$ $\therefore$ $t = \frac{T}{2} = \frac{\pi m}{qB}$

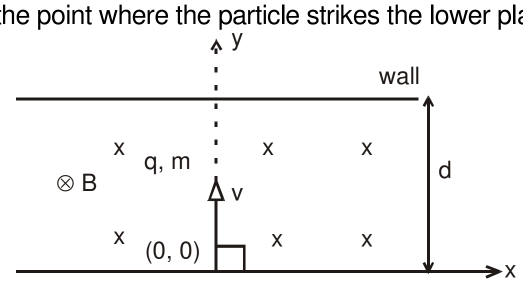


(ii) $\sin\theta = \dfrac{d}{R}$

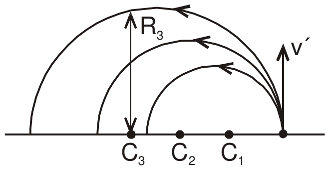


$\theta = \sin^{-1}\left(\dfrac{d}{R}\right)$; $\omega t = \theta \Rightarrow t = \dfrac{m}{qB}\sin^{-1}\left(\dfrac{d}{R}\right)$

Example 24. What should be the speed of charged particle so that it can't collide with the upper wall? Also find the coordinate of the point where the particle strikes the lower plate in the limiting case of velocity.



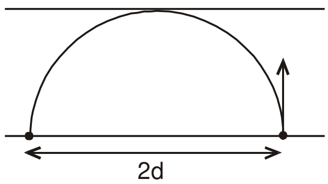
Solution : (i) The path of the particle will be circular larger the velocity, larger will be the radius. For particle not to s



strike R < d $\therefore$ $\dfrac{mv}{qB} < d$

$\Rightarrow$ $v < \dfrac{qBd}{m}$.

(ii) for limiting case $v = \dfrac{qBd}{m}$; $R = d$



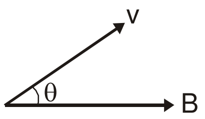
$\therefore$ coordinate = (–2d, 0, 0)

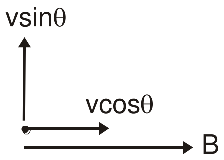
3.2 Helical path :

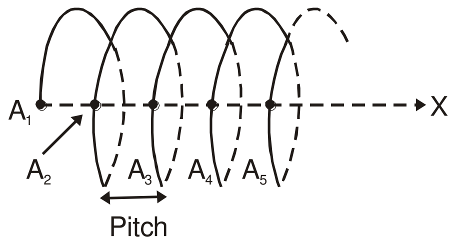
If the velocity of the charge is not perpendicular to the magnetic field, we can break the velocity in two components – $v_{||}$, parallel to the field and $v_{\perp}$, perpendicular to the field. The components $v_{||}$ remains unchanged as the force $q\vec{v}\times\vec{B}$ is perpendicular to it. In the plane perpendicular to the field, the particle traces a circle of radius $r = \dfrac{mv_{\perp}}{qB}$ as given by equation. The resultant path is helix.

Complete analysis :

Let a particle have initial velocity in the plane of the paper and a constant and uniform magnetic field also in the plane of the paper.







The particle starts from point A_1.

It completes its one revolution at A_2 and 2^{nd} revolution at A_3 and so on. X-axis is the tangent to the helix points

A_1, A_2, A_3, ...all are on the x-axis.

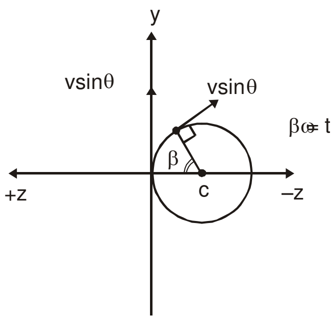
distance $A_1A_2 = A_3A_4 =$ .... $= v \cos\theta$. T = pitch

where T = Time period

Let the initial position of the particle be (0, 0, 0) and $v \sin\theta$ in +y direction. Then in x :

$F_x = 0$, $a_x = 0$, v_x = constant $= v \cos\theta$, $x = (v \cos\theta)t$

In y-z plane :



From figure it is clear that $y = R \sin\beta$, $v_y = v \sin\theta \cos\beta$

$z = -(R - R \cos\beta)$

$v_z = v \sin\theta \sin\beta$

acceleration towards centre $= (v\sin\theta)^2/R = \omega^2 R$

$\therefore$ $a_y = -\omega^2 R \sin\beta$, $a_z = -\omega^2 R \cos\beta$

At any time : the position vector of the particle (or its displacement w.r.t. initial position)

$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, x,y,z already found

velocity $\vec{v} = v_x\hat{i} + v_y\hat{j} + v_z\hat{k}$, v_x, v_y, v_z already found

$\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$, a_x, a_y, a_z already found

Radius $q(v \sin\theta)B = \dfrac{m(v\sin\theta)^2}{R}$ $\Rightarrow$ $R = \dfrac{mv\sin\theta}{qB}$ $\Rightarrow \omega = \dfrac{v\sin\theta}{R} = \dfrac{qB}{m} = \dfrac{2\pi}{T} = 2\pi f.$

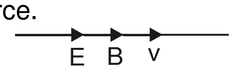
3.3 Charged Particle in $\vec{E}$ & $\vec{B}$

When a charged particle moves with velocity $\vec{V}$ in an electric field $\vec{E}$ and magnetic field $\vec{B}$, then. Net force experienced by it is given by following equation.

$$\vec{F} = q\vec{E} + q(\vec{V} \times \vec{B})$$

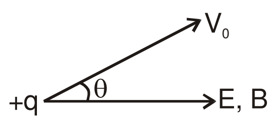
Combined force is known as Lorentz force.

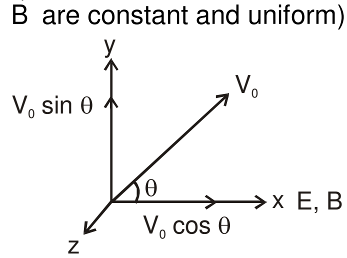
$\vec{E} \parallel \vec{B} \parallel \vec{v}$

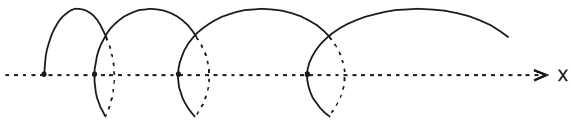


In above situation particle passes undeviated but its velocity will change due to electric field. Magnetic force on it = 0.

Case(i) : $\vec{E} \parallel \vec{B}$ and uniform $\theta \neq 0, 180°$ ($\vec{E}$ and $\vec{B}$ are constant and uniform)

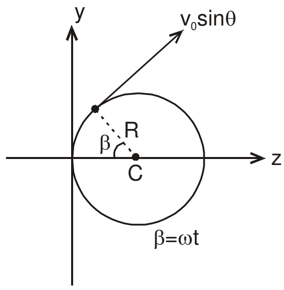






in x : $F_x = qE$, $a_x = \dfrac{qE}{m}$, $v_x = v_0 \cos\theta + a_x t$, $x = v_0 t + \dfrac{1}{2} a_x t^2$

in yz plane :



$qv_0 \sin\theta\, B = m(v_0 \sin\theta)^2/R \quad \Rightarrow \quad R = \dfrac{mv_0 \sin\theta}{qB}$

$\omega = \dfrac{v_0 \sin\theta}{R} = \dfrac{qB}{m} = \dfrac{2\pi}{T} = 2\pi f$

$\vec{r} = \{(V_0 \cos\theta)t + \dfrac{1}{2}\dfrac{qE}{m}t^2\}\,\hat{i} + R\sin\omega t\,\hat{j} + (R - R\cos\omega t)(-\hat{k})$

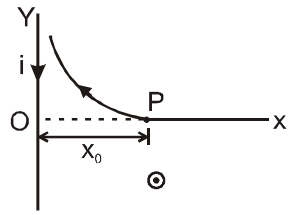
$\vec{V} = \left(V_0\cos\theta + \dfrac{qE}{m}t\right)\hat{i} + (V_0 \sin\theta)\cos\omega t\,\hat{j} + V_0\sin\theta\sin\omega t\,(-\hat{k})$

$\vec{a} = \dfrac{qE}{m}\,\hat{i} + \omega^2 R[-\sin\beta\,\hat{j} - \cos\beta\,\hat{k}]$

Solved Examples

Example 25. A long, straight wire carries a current i. A particle having a positive charge q and mass m kept at a distance x_0 from the wire is projected towards it with a speed v as shown in figure. Find the minimum separation between the wire and the particle

Solution : Let the particle be initially at P (figure). Take the wire as the Y-axis and the foot of perpendicular from P to the wire as the origin. Take the line OP as the X-axis. We have, OP = x_0. The magnetic field B at any point to the right of the wire is along the positive Z-axis. The magnetic force on the particle is, therefore, in the X-Y plane. As there is no initial velocity along the Z-axis, the motion will be in the X-Y plane. Also, its speed remains unchanged. As the magnetic field is not uniform, the particle does not go along a circle. The force at time t is



$\vec{F} = q\vec{v} \times \vec{B}$

$= q(v_x\vec{i} + v_y\vec{j}) \times \left(\dfrac{\mu_0 i}{2\pi x}\hat{k}\right)$

$= -\vec{j}qv_x\dfrac{\mu_0 i}{2\pi x} + \vec{i}\,qv_y\dfrac{\mu_0 i}{2\pi x}.$

Thus $a_x = \dfrac{F_x}{m} = \dfrac{\mu_0 qi}{2\pi m} = \lambda\dfrac{v_y}{x}$ .....(i)

where $\lambda = \dfrac{\mu_0 qi}{2\pi m}$.

Also, $a_x = \dfrac{dv_x}{dt} = \dfrac{dv_x}{dx}\dfrac{dx}{dt} = \dfrac{v_x dv_x}{dx}$ .....(ii)

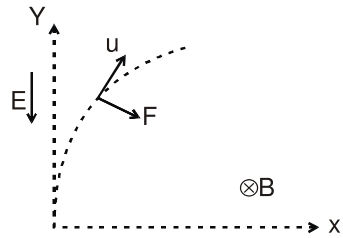
As, $v_x^2 + v_y^2 = v^2$,

giving $v_x\,dv_x = -v_y\,dv_y$. .....(iii)

From (i), (ii) and (iii), $\dfrac{v_y dv_y}{dx} = \dfrac{-\lambda v_y}{x}$ or, $\dfrac{dx}{x} = \dfrac{-dv_y}{\lambda}$

Initially $x = x_0$ and $v_y = 0$. At minimum separation from the wire, $v_x = 0$ so that $v_y = v$.

Thus $\displaystyle\int_{x_0}^{x}\frac{dx}{x} = \int_0^v \frac{dv_y}{-\lambda}$ or, $\ln\dfrac{x}{x_0} = -\dfrac{v}{\lambda}$ or, $x = x_0\,e^{v/\lambda} = x_0\,e^{-\frac{2\pi mv}{\mu_0 qi}}$



Example 26. An electron is released from the origin at a place where a uniform electric field E and a uniform magnetic field B exist along the negative Y-axis and the negative Z-axis respectively. Find the displacement of the electron along the Y-axis when its velocity becomes perpendicular to the electric field for the first time.

Solution : Let us take axes as shown in figure. According to the right-handed system, the Z-axis is upward in the figure and hence the magnetic field is shown downwards. At any time, the velocity of the electron may be written as

$\vec{u} = u_x\vec{i} + u_y\vec{j}$

The electric and magnetic fields may be written as

$\vec{E} = -E\vec{j}$ and $\vec{B} = -B\vec{k}$

respectively. The force on the electron is $\vec{F} = -e(\vec{E} + \vec{u}\times\vec{B}) = eE\vec{j} + eB(u_y\vec{i} - u_x\vec{j})$

Thus, $F_x = eu_y B$ and $F_y = e(E - u_xB)$.

The components of the acceleration are

$$a_x = \frac{du_x}{dt} = \frac{eB}{m}u_y \qquad .....(i)$$

and $$a_y = \frac{du_y}{dt} = \frac{e}{m}(E - u_xB). \qquad .....(ii)$$

We have,

$$\frac{d^2u_y}{dt^2} = -\frac{eB}{m}\frac{du_x}{dt} = -\frac{eB}{m}.\frac{eB}{m}\ u_y = -\omega^2 u_y$$

where $$\omega = \frac{eB}{m} \qquad .....(iii)$$

This equation is similar to that for a simple harmonic motion. Thus,

$$u_y = A\sin(\omega t + \delta) \qquad .....(iv)$$

and hence, $$\frac{du_y}{dt} = A\,\omega\cos(\omega t + \delta) \qquad .....(v)$$

At t = 0, $u_y = 0$ and $\frac{du_y}{dt} = \frac{F_y}{dt} = \frac{eE}{m}$.

Putting in (iv) and (v),

$$\delta = 0 \text{ and } A = \frac{eE}{m\omega}\ \frac{E}{B}.$$

Thus, $u_y = \frac{E}{B}\sin\omega t$.

The path of the electron will be perpendicular to the Y-axis when $u_y = 0$. This will be the case for the first time at t where

$\sin\omega t = 0$ or $\omega t = \pi$ or $t = \frac{\pi}{\omega} = \frac{\pi m}{eB}$

Also, $u_y = \frac{dy}{dt} = \frac{E}{B}\sin\omega t$

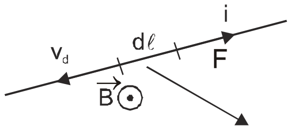
or $\int_0^y dy = \frac{E}{B}\sin\omega t\ dt$ or, $y = \frac{E}{B\omega}(1 - \cos\omega t)$.

At $t = \frac{\pi}{\omega}$, $y = \frac{E}{B\omega}(1 - \cos\pi) = \frac{2E}{B\omega}$

Thus, the displacement along the Y-axis is $\frac{2E}{B\omega} = \frac{2Em}{BeB} = \frac{2Em}{eB^2}$. **Ans.**

3.4 Magnetic force on A current carrying wire :

Suppose a conducting wire, carrying a current i, is placed in a magnetic field $\vec{B}$. Consider a small element $d\ell$ of the wire (figure). The free electrons drift with a speed v_d opposite to the direction of the current. The relation between the current i and the drift speed v_d is



$i = jA = nev_dA.$ ....(i)

Here A is the area of cross-section of the wire and n is the number of free electrons per unit volume. Each electron experiences an average (why average?) magnetic force

$\vec{f} = -e\vec{v}_d \times \vec{B}$

The number of free electrons in the small element considered in $nAd\ell$. Thus, the magnetic force on the wire of length $d\ell$ is

$d\vec{F} = (nAd\ell)(-e\vec{v}_d \times \vec{B})$

If we denote the length $d\ell$ along the direction of the current by $d\vec{\ell}$, the above equation becomes

$d\vec{F} = nAev_d\overrightarrow{d\ell} \times \vec{B}$.

Using (i), $d\vec{F} = id\vec{\ell} \times \vec{B}$.

The quantity $id\vec{\ell}$ is called a current element.

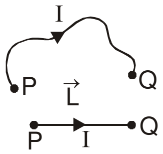
$\overrightarrow{F_{res}} = \int \overrightarrow{dF} = \int id\vec{\ell} \times \vec{B} = i \int \overrightarrow{d\ell} \times \vec{B}$ ($\because$ i is same at all points of the wire.)

If $\vec{B}$ is uniform then $\vec{F}_{res} = i(\int \overrightarrow{d\ell}) \times \vec{B}$

$\overrightarrow{F_{res}} = i \ \vec{L} \times \vec{B}$

Here $\vec{L}$ = $\int d\vec{\ell}$ = vector length of the wire = vector connecting the end points of the wire.

$\Rightarrow$

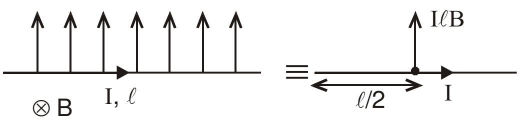


Note :

- If a current loop of any shape is placed in a uniform $\vec{B}$ then $\left.\overrightarrow{F_{res}}\right)_{magnetic}$ on it = 0 ($\because \vec{L}$ = 0).

3.5 Point of application of magnetic force :

On a straight current carrying wire the magnetic force in a uniform magnetic field can be assumed to be acting at its mid point.

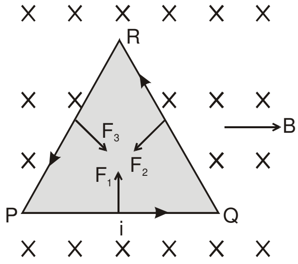


This can be used for calculation of torque.

Solved Examples

Example 27. A wire is bent in the form of an equilateral triangle PQR of side 20 cm and carries a current of 2.5 A. It is placed in a magnetic field B of magnitude 2.0 T directed perpendicularly to the plane of the loop. Find the forces on the three sides of the triangle.

Solution : Suppose the field and the current have directions as shown in figure. The force on PQ is



$$\vec{F}_1 = i\vec{\ell} \times \vec{B}$$

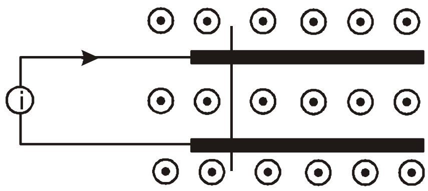
or $F_1 = 2.5\ A \times 20\ cm \times 2.0\ T = 1.0\ N$

The rule of vector product shows that the force F_1 is perpendicular to PQ and is directed towards the inside of the triangle.

The forces $\vec{F}_2$ and $\vec{F}_3$ on QR and RP can also be obtained similarly. Both the forces are 1.0 N directed perpendicularly to the respective sides and towards the inside of the triangle.

The three forces $\vec{F}_1$, $\vec{F}_2$ and $\vec{F}_3$ will have zero resultant, so that there is no net magnetic force on the triangle. This result can be generalised. Any closed current loop, placed in a homogeneous magnetic field, does not experience a net magnetic force.

Example 28. Figure shows two long metal rails placed horizontally and parallel to each other at a separation y. A uniform magnetic field B exists in the vertically upward direction. A wire of mass m can slide on the rails. The rails are connected to a constant current source which drives a current i in the circuit. The friction coefficient between the rails and the wire is μ.



(a) What soluble the minimum value of μ which can prevent the wire from sliding on the rails?

(b) Describe the motion of the wire if the value of μ is half the value found in the previous part.

Solution : (a) The force on the wire due to the magnetic field is

$$\vec{F} = i\vec{\ell} \times \vec{B} \qquad \text{or} \qquad F = iyB$$

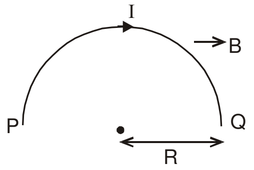
It acts towards right tin the given figure. If the wire does not slide on the rails, the force of friction by the rails should be equal to F. If μ_0 be the minimum coefficient of friction which can prevent sliding, this force is also equal to μ_0 mg. Thus,

$$\mu_0\, mg = iyB \qquad \text{or} \qquad \mu_0 = \frac{iyB}{mg}$$

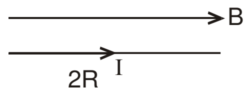
(b) If the friction coefficient is $\mu = \frac{\mu_0}{2} = \frac{iyB}{2\ mg}$, the wire will slide towards right. The frictional force by the rails is $f = \mu mg = \frac{iyB}{2}$ towards left.

The resultant force is $iyB - \frac{iyB}{2} = \frac{iyB}{2}$ towards right. The acceleration will be $a = \frac{iyB}{2m}$. The wire will slide towards right with this acceleration.

Example 29. In the figure shown a semicircular wire is placed in a uniform $\vec{B}$ directed toward right. Find the resultant magnetic force and torque on it.

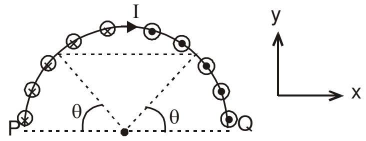


Solution : The wire is equivalent to



$\because \quad \theta = 0 \qquad \therefore \quad F_{res} = 0$ **Ans.**

forces on individual parts are marked in the figure by ⊗ and ⊙. By symmetry their will be pair of forces forming couples.

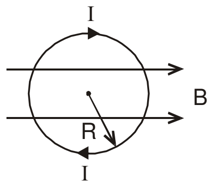


$$\tau = \int_0^{\pi/2} i(Rd\theta)B\sin(90-\theta).2R\cos\theta$$

$$\tau = \frac{i\pi R^2}{2}B \quad \Rightarrow \quad \vec{\tau} = \frac{i\pi R^2}{2}B(-\hat{j})$$ **Ans.**

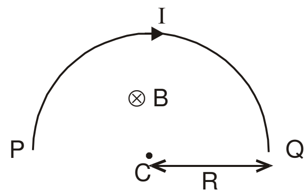
Example 30. Find the resultant magnetic force and torque on the loop.

Solution : $\overrightarrow{F_{res}} = 0$, ($\because$ loop)



and $\vec{\tau} = i\pi R^2 B(-\hat{j})$ using the above method

Example 31 In the figure shown find the resultant magnetic force and torque about 'C', and 'P'.

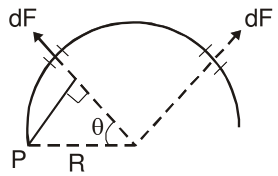


Solution : $\vec{F}_{nett} = I \cdot 2R \cdot B$

$\because$ wire is equivalent to

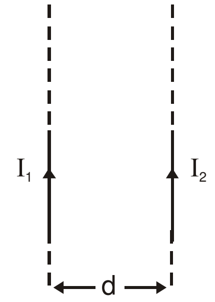
Force on each element is radially outward : $\tau_c = 0$

point about

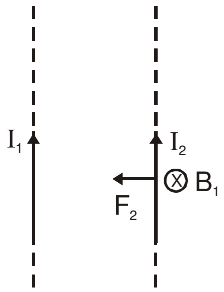


$P = \int_0^{\pi} [I(Rd\theta)B\sin 90^0]\ R\sin\theta = 2IBR^2$ **Ans.**

Example 32. Prove that magnetic force per unit length on each of the infinitely long wire due to each other is $\mu_0 I_1 I_2/2\pi d$. Here it is attractive also.



Solution :



On (2), B due to (i) is = $\frac{\mu_0 I_1}{2\pi d} \otimes$

$\therefore$ F on (2) on 1m length

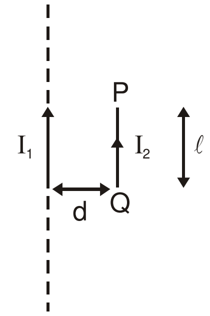
$= I_2 . \frac{\mu_0 I_1}{2\pi d} .1$ towards left it is attractive

$= \frac{\mu_0 I_1 I_2}{2\pi d}$ (hence proved)

Similarly on the other wire also.

Note :

- Definition of ampere (fundamental unit of current) using the above formula.If $I_1 = I_2 = 1A$, d = 1m then F = 2 × 10^{-7} N
  $\therefore$ "When two very long wires carrying equal currents and separated by 1m distance exert on each other a magnetic force of 2 × 10^{-7} N on 1m length then the current is 1 ampere."
- The above formula can also be applied if to one wire is infinitely long and the other is of finite length. In this case the force per unit length on each wire will not be same.

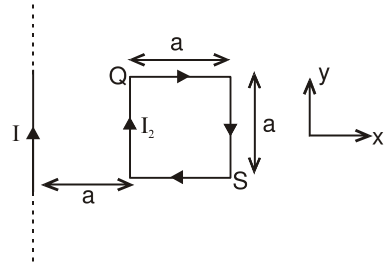


Force per unit length on PQ = $\frac{\mu_0 I_1 I_2}{2\pi d}$ (attractive)

- If the currents are in the opposite direction then the magnetic force on the wires will be repulsive.

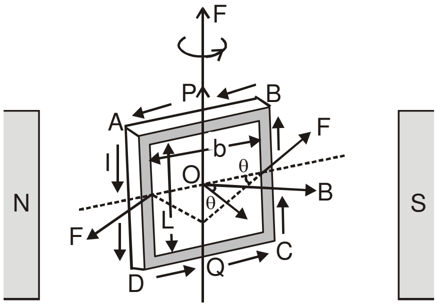
Example 33. Find the magnetic force on the loop 'PQRS' due to the loop wire.

Solution : $F_{res} = \frac{\mu_0 I_1 I_2}{2\pi a}\, a(-\hat{i}) + \frac{\mu_0 I_1 I_2}{2\pi(2a)} a(\hat{i}) = \frac{\mu_0 I_1 I_2}{4\pi}(-\hat{i})$



4. TORQUE ON A CURRENT LOOP :

When a current-carrying coil is placed in a uniform magnetic field the net force on it is always zero. However, as its different parts experience forces in different directions so the loop may experience a torque (or couple) depending on the orientation of the loop and the axis of rotation. For this, consider a rectangular coil in a uniform field B which is free to rotate about a vertical axis PQ and normal to the plane of the coil making an angle θ with the field direction as shown in figure (A).



Axis of rotation
(A)

The arms AB and CD will experience forces B(NI)b vertically up and down respectively. These two forces together will give zero net force and zero torque (as are collinear with axis of rotation), so will have no effect on the motion of the coil.

Now the forces on the arms AC and BD will be BINL in the direction out of the page and into the page respectively, resulting in zero net force, but an anticlockwise couple of value

τ = F × Arm = BINL × (b sinθ)

i.e. τ = BIA sinθ with A = NLb ......(i)

Now treating the current–carrying coil as a dipole of moment $\vec{M} = I\vec{A}$ Eqn. (i) can be written in vector form as

$\vec{\tau} = \vec{M} \times \vec{B}$ with $\vec{M} = I\vec{A} = NIA\vec{n}$ ......(ii)

This is the required result and from this it is clear that :

(1) Torque will be minimum (= 0) when sinθ = min = 0, i.e., θ = 0º, i.e. 180º i.e., the plane of the coil is perpendicular to magnetic field i.e. normal to the coil is collinear with the field [fig. (A) and (C)]

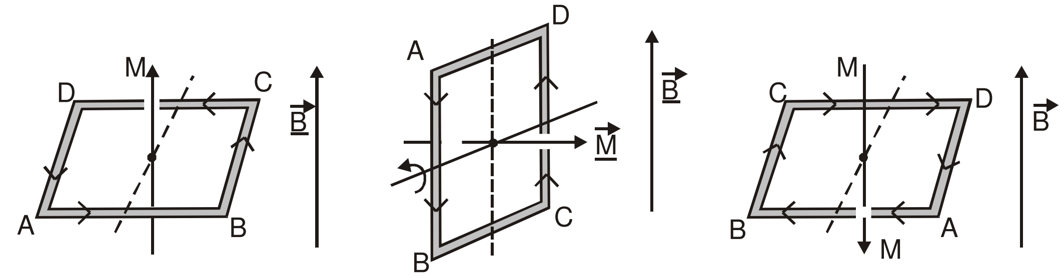
(2) Torque will be maximum (= BINA) when sinθ = max = 1, i.e., θ = 90º i.e. the plane of the coil is parallel to the field i.e. normal to the coil is perpendicular to the field. [fig.(B)].

(3) By analogy with dielectric or magnetic dipole in a field, in case of current–carrying in a field.

$U = -\vec{M} \bullet \vec{B}$ with $F = -\frac{dU}{dr}$

and W = MB(1 – cosθ)

The values of U and W for different orientations of the coil in the field are shown in fig.



$\theta = 0^\circ$ ($\vec{M}$ is parallel to $\vec{B}$)
$\tau = 0$ = min
W = 0 = min
U = –MB = min
Stable equilibrium
(A)

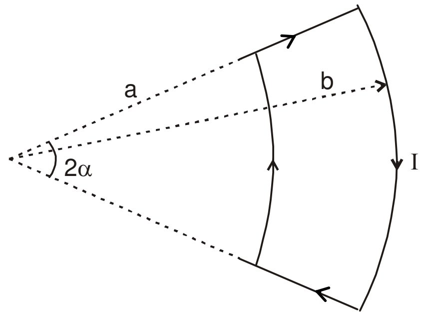
$\theta = 90^\circ$ ($\vec{M}$ is $\perp$ to $\vec{B}$)
τ = MB = max
W = MB
U = 0
No equilibrium
(B)

$\theta = 180^\circ$ ($\vec{M}$ is antiparallel to $\vec{B}$)
$\tau = 0$
W = 2MB = max
U = MB = max
Unstable equilibrium
(C)

(4) Instruments such as electric motor, moving coil galvanometer and tangent galvanometers etc. are based on the fact that a current–carrying coil in a uniform magnetic field experiences a torque (or couple).

Solved Examples

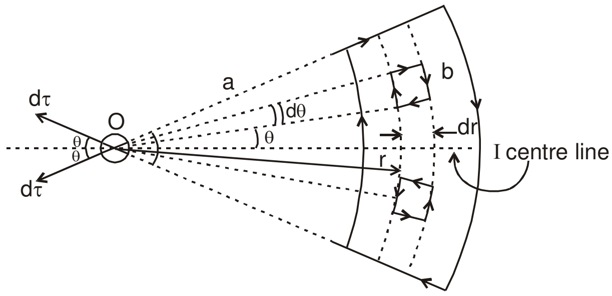
Example 34. A loop with current I is in the field of a long straight wire with current I_0. The plane of the loop is perpendicular to the straight wire. Find torque acting on the loop.



Solution : $d\vec{s} = (rd\theta\, dr)$ (inwards)

$d\vec{M} = (rI\, d\theta\, dr)$ (inwards)

$\vec{B} = \frac{\mu_0 I_0}{2\pi r}$ (tangential clockwise)



$d\tau = |d\vec{M} \times \vec{B}| = \frac{\mu_0 I I_0 d\theta dr}{2\pi}$ (towards centre)

$\therefore \quad \tau = \int_{-\alpha}^{\alpha} \int_a^b d\tau \cos\theta$

$= \frac{\mu_0 I I_0}{2\pi} \int_{-\alpha}^{\alpha} \int_a^b \cos\theta\, d\theta dr = \frac{\mu_0 I I_0 (b-a)\sin\alpha}{\pi}$ (to the left) **Ans.**

Example 35. A circular coil of radius R and a current I, which can rotate about a fixed axis passing through its diameter is initially placed such that its plane lies along magnetic field B. Kinetic energy of loop when it rotates through an angle 90° is : (Assume that I remains constant)

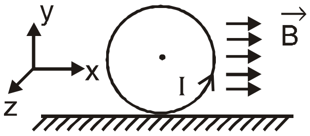
(A) $\pi R^2 BI$ (B) $\frac{\pi R^2 BI}{2}$ (C) $2\pi R^2 BI$ (D) $\frac{3}{2}\pi R^2 I$

Answer : **(A)**

Solution : Loss in potential energy = gain kinetic energy

(–MB cos90°) – (–MBcos0°) = KE

Example 36. In the shown figure a conducting ring of mass m = 2kg and radius R = 0.5 m. lies on a smooth horizontal plane with its plane vertical. The ring carries a current of $I = \frac{1}{\pi} A$. A horizontal uniform magnetic field of B = 12T is switched on at t = 0. The initial angular acceleration α in rad./sec^2 of the ring will be 4x if x is :



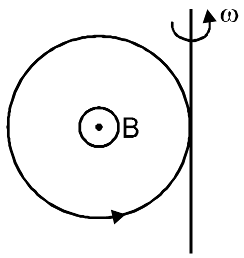
Answer : **3**

Solution : $\tau_{y\text{-axis}} = I_{y\text{-axis}}\, \alpha$

$(I.\pi r^2)\, B = 1/2\, mr^2\alpha$

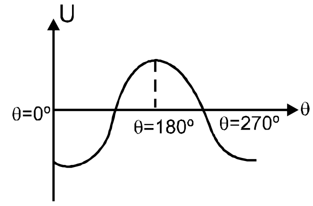
$\alpha = 12$ rad/sec^2 $\quad\therefore\quad X = 3$

Example 37. A current carrying ring, carrying a constant current $\frac{2}{\pi}$ Amp., radius 1m, mass $\frac{2}{3}$kg and having 10 windings is free to rotate about its tangential vertical axis. A uniform magnetic field of 1 tesla is applied perpendicular to its plane. How much minimum angular velocity (in rad/sec.) should be given to the ring in the direction shown, so that it can rotate 270º in that direction. Write your answer in nearest single digit in rad/sec.



Answer : **9**

Solution : to reach θ = 270º, it has to cross the potential energy barrier at θ = 180º and to cross θ = 180º angular velocity at θ = 180º should be 0^+



$k_i + U_i = k_f + U_y$

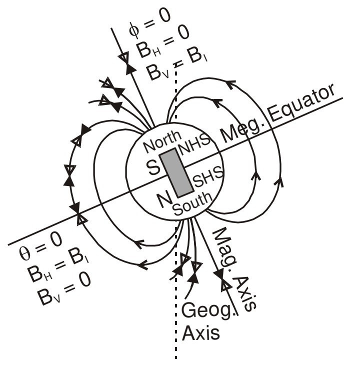
$$\frac{1}{2}\left(\frac{3}{2}MR^2\right)\omega^2 + (-Mi\ AB\cos 0º) = 0 + (-NiAB\cos 180º)$$

$\omega = \sqrt{80} \approx 9$ rad/sec.

5. TERRESTRIAL MAGNETISM (EARTH'S MAGNETISM) :

5.1 Introduction :

The idea that earth is magnetised was first suggested towards the end of the sixteen'th century by Dr. William Gilbert. The origin of earth's magnetism is still a mater of conjecture among scientists but it is agreed upon that the earth behaves as a magnetic dipole inclined at a small angle (11.5º) to the earth's axis of rotation with its south pole pointing north. The lines of force of earth's magnetic field are shown in figure which are parallel to the earth's surface near the equator and perpendicular to it near the poles. While discussing magnetism of the earth one should keep in mind that:



(a) The **magnetic meridian** at a place is not a line but a vertical plane passing through the axis of a freely suspended magnet, i.e., it is a plane which contains the place and the magnetic axis.

(b) The **geographical meridian** at a place is a vertical plane which passes through the line joining the geographical north and south, i.e., it is a plane which contains the place and earth's axis of rotation, i.e., geographical axis.

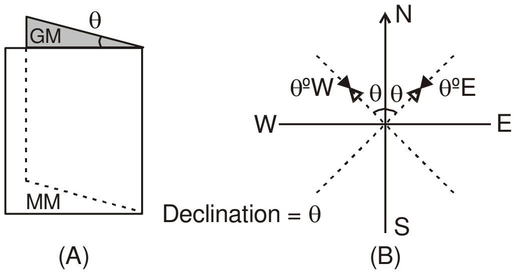
(c) The **magnetic Equator** is a great circle (a circle with the centre at earth's centre) on earth's surface which is perpendicular to the magnetic axis. The magnetic equator passing through Trivandrum in South India divides the earth into two hemispheres. The hemisphere containing south polarity of earth's magnetism is called the northern hemisphere (NHS) while the other, the southern hemisphere (SHS).

(d) The magnetic field of earth is not constant and changes irregularly from place to place on the surface of the earth and even at a given place it varies with time too.

5.2 Elements of the Earth's Magnetism :

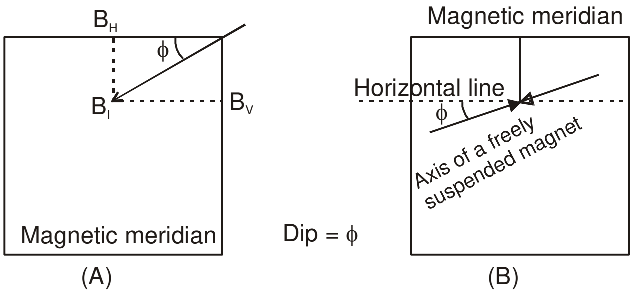
The magnetism of earth is completely specified by the following three parameters called elements of earth's magnetism :

(a) Variation or Declination θ : At a given place the angle between the geographical meridian and the magnetic meridian is called declination, i.e., at a given place it is the angle between the geographical north-south direction and the direction indicated by a magnetic compass needle, Declination at a place is expressed at θ° E or θ° W depending upon whether the north pole of the compass needle lies to the east (right) or to the west (left) of the geographical north-south direction. The declination at London is 10ºW means that at London the north pole of a compass needle points 10ºW, i.e., left of the geographical north.



(b) Inclination or Angle of Dip ϕ : It is the angle which the direction of resultant intensity of earth's magnetic field subtends with horizontal line in magnetic meridian at the given place. Actually it is the angle which the axis of a freely suspended magnet (up or down) subtends with the horizontal in magnetic meridian at a given place.

Here, it is worthy to note that as the northern hemisphere contains south polarity of earth's magnetism, in it the north pole of a freely suspended magnet (or pivoted compass needle) will dip downwards, i.e., towards the earth while the opposite will take place in the southern hemisphere.



Angle of dip at a place is measured by the instrument called Dip-Circle in which a magnetic needle is free to rotate in a vertical plane which can be set in any vertical direction. Angle of dip at Delhi is 42º.

(c) Horizontal Component of Earth's Magnetic Field B_H : At a given place it is defined as the component of earth's magnetic field along the horizontal in the magnetic meridian. It is represented by B_H and is measured with the help of a **vibration** or **deflection magnetometer**. At Delhi the horizontal component of the earth's magnetic field is 35 μT, i.e., 0.35 G.

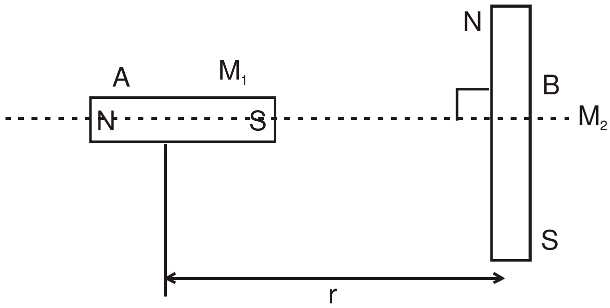
If at a place magnetic field of earth is B_I and angle of dip ϕ, then in accordance with figure (a).

$$B_H = B_I \cos\phi \quad \text{and} \quad B_v = B_I \sin\phi \qquad .....(1)$$

so that,
$$\tan\phi = \frac{B_v}{B_H} \quad \text{and} \quad I = \sqrt{B_H^2 + B_v^2} \qquad .....(2)$$

Solved Miscellaneous Problems

Problem 1. Two short magnet A and B of magnetic dipole moments M_1 and M_2 respectively are placed as shown. The axis of 'A' and the equatorial line of 'B' are the same. Find the magnetic force on one magnet due to the other.



Answer : $F = 3\left(\frac{\mu_0}{4\pi}\right)\frac{M_2M_1}{r^4}$ upwards on M_1 down wards on M_2

Solution : Magnetic field due to magnet B : $B = \frac{\mu_0}{4\pi}.\frac{M_2}{r^3}$

Magnetic force acting on magnet A : $F = M_1\frac{dB}{dr} = -3\left(\frac{\mu_0}{4\pi}\right)\frac{M_1M_2}{r^4}$

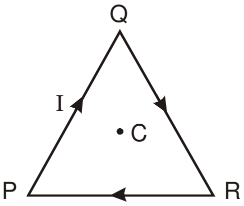
Problem 2. A bar magnet has a pole strength of 3.6 A-m and magnetic length 8 cm. Find the magnetic field at (a) a point on the axis at a distance of 6 cm from the centre towards the north pole and (b) a point on the perpendicular bisector at the same distance.

Answer : (a) 8.6×10^{-4} T ; (b) 7.7×10^{-5} T.

Solution : $M = 3.6 \times 8 \times 10^2$ A.m^2

(a) $B = \frac{\mu_0}{4\pi}.\frac{2Mr}{(r^2-a^2)^2} = 8.6 \times 10^{-4}$ T. (b) $B = \frac{\mu_0}{4\pi}.\frac{M}{(r^2+a^2)^{3/2}} = 7.7 \times 10^{-5}$ T

Problem 3. A loop in the shape of an equilateral triangle of side 'a' carries a current I as shown in the figure. Find out the magnetic field at the centre 'C' of the triangle.



Answer : $\frac{9\mu_0 i}{2\pi a}$

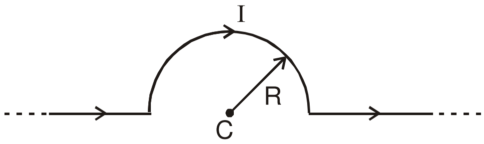
Solution : $B = B_1 + B_2 + B_3 = 3B_1 = 3\frac{\mu_0}{4\pi} \times \frac{i}{\left(\frac{a}{2\sqrt{3}}\right)} \times (\sin 60^\circ + \sin 60^\circ) = \frac{9\mu_0 i}{2\pi a}$

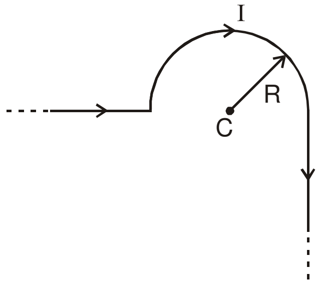
Problem 4. Two long wires are kept along x and y axes they carry currents I & I respectively in +ve x and +ve y directions respectively. Find $\vec{B}$ at a point (0, 0, d).

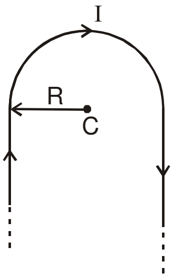
Answer : $\frac{\mu_0 I}{2\pi d}(\hat{i} - \hat{j})$

Solution : $\vec{B} = \vec{B}_1 + \vec{B}_2 = \frac{\mu_0}{2\pi}\frac{i}{d}(-\hat{j}) + \frac{\mu_0}{2\pi}\frac{i}{d}(\hat{i}) = \frac{\mu_0 I}{2\pi\ d}(\hat{i} - \hat{j})$

Problem 5. Find 'B' at centre 'C' in the following cases :

(i)


(ii)


(iii)


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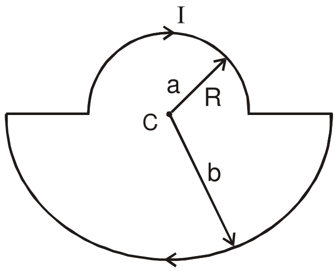
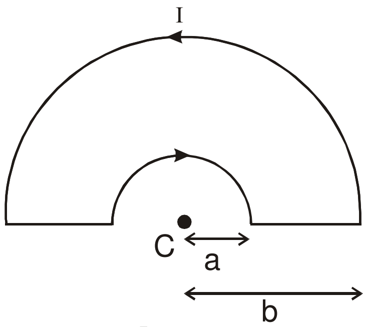
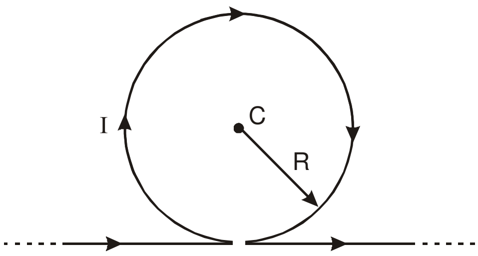
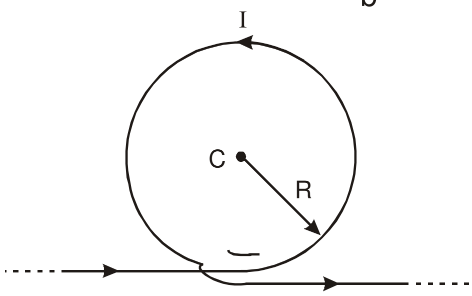
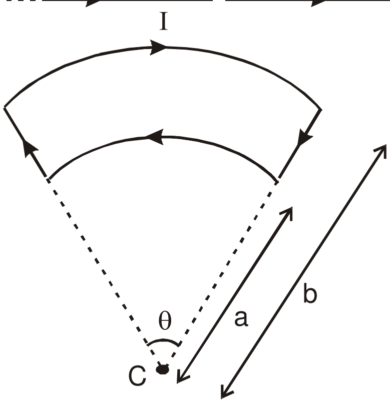


(v)


(vi)


(vii)


(viii)


Answer :

(i) $\frac{\mu_0 I}{4R} \otimes$ (ii) $\frac{\mu_0 I}{4R}\left(1+\frac{1}{\pi}\right) \otimes$ (iii) $\frac{\mu_0 I}{2R}\left(\frac{1}{2}+\frac{1}{\pi}\right) \otimes$ (iv) $\frac{\mu_0 I}{4}\left(\frac{1}{a}+\frac{1}{b}\right) \otimes$

(v) $\frac{\mu_0 I}{4}\left(\frac{1}{a}-\frac{1}{b}\right) \otimes$ (vi) $\frac{\mu_0 I}{2R}\left(1-\frac{1}{\pi}\right) \otimes$ (vii) $\frac{\mu_0 I}{2R}\left(1+\frac{1}{\pi}\right) \odot$ (viii) $\frac{\mu_0 I\theta}{4\pi}\left(\frac{1}{a}-\frac{1}{b}\right) \odot$

Solution :

(i) $B = \frac{\mu_0 I}{2R} \times \frac{1}{2} = \frac{\mu_0 I}{4R}$

(ii) $B = B_1 + B_2 = \left(\frac{\mu_0 I}{2R} \times \frac{1}{2}\right) + \left(\frac{\mu_0}{4\pi} \cdot \frac{I}{R}\right) = \frac{\mu_0 I}{4R}\left(1+\frac{1}{\pi}\right)$

(iii) $B = B_1 + B_2 + B_3 = 2B_1 + B_2 = \left(2 \times \frac{\mu_0}{4\pi}\frac{I}{R}\right) + \left(\frac{\mu_0 I}{2R} \times \frac{1}{2}\right) = \frac{\mu_0 I}{2R}\left(\frac{1}{2}+\frac{1}{\pi}\right)$

(iv) $B = B_1 + B_2 = \frac{\mu_0}{2a}\times\frac{1}{2}+\frac{\mu_0}{2b}\times\frac{1}{2} = \frac{\mu_0 I}{4}\left(\frac{1}{a}+\frac{1}{b}\right)$

(v) $B = B_1 - B_2 = \left(\frac{\mu_0 I}{2a}\times\frac{1}{2}-\frac{\mu_0 I}{2b}\times\frac{1}{2}\right) = \frac{\mu_0 I}{4}\left(\frac{1}{a}-\frac{1}{b}\right)$

(vi) $B = B_1 - B_2 = \frac{\mu_0 I}{2R}-\frac{\mu_0 I}{2\pi R} = \frac{\mu_0 I}{2\ R}\left(1-\frac{1}{\pi}\right)$

(vii) $B = B_1 + B_2 = \frac{\mu_0 I}{2R}+\frac{\mu_0 I}{2\pi R} = \frac{\mu_0 I}{2R}\left(1+\frac{1}{\pi}\right)$

(viii) $B = B_1 - B_2 = \frac{\mu_0 I}{2a}-\frac{\mu_0 I}{2b} \times \frac{\theta}{2\pi} = \frac{\mu_0 I\theta}{4\pi}\left(\frac{1}{a}-\frac{1}{b}\right)$

Problem 6. A thin solenoid of length 0.4 m and having 500 turns of wire carries a current 1A; then find the magnetic field on the axis inside the solenoid.

Answer : $5\pi \times 10^{-4}$ T.

Solution : $B = \mu_0 ni = \frac{\mu_0 Ni}{\ell} = 5\pi \times 10^{-4}$ T.

Problem 7. A charged particle of charge 2C thrown vertically upwards with velocity 10 m/s. Find the magnetic force on this charge due to earth's magnetic field. Given vertical component of the earth = 3μT and angle of dip = 37º.

Answer : $2 \times 10 \times 4 \times 10^{-6} = 8 \times 10^{-5}$ N towards west.

Solution : $\tan 37^\circ = \frac{B_V}{B_H} \Rightarrow B_H = \frac{4}{3} \times 3 \times 10^{-6}$ T

$F = q\,v\,B_H = 8 \times 10^{-5}$ N

Problem 8. A charged particle has acceleration $\vec{a} = 2\hat{i} + x\hat{j}$ in a magnetic field $\vec{B} = -3\hat{i} + 2\hat{j} - 4\hat{k}$. Find the value of x.

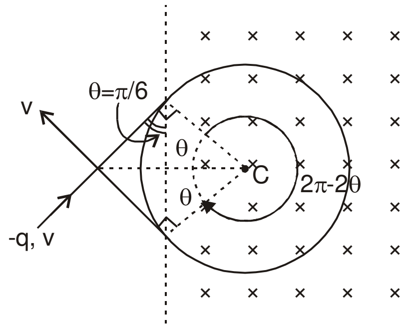
Solution : $\because \vec{F} \perp \vec{B} \quad \therefore \vec{a} \perp \vec{B} \quad \therefore \vec{a}\cdot\vec{B} = 0$

$\therefore (2\hat{i} + x\hat{j}) . (-3\hat{i} + 2\hat{j} - 4\hat{k}) = 0$

$\Rightarrow -6 + 2x = 0 \Rightarrow x = 3.$

Problem 9. Repeat above question if the charge is –ve and the angle made by the boundary with the velocity is $\frac{\pi}{6}$.

Solution : (i) $2\pi - 2\theta = 2\pi - 2\frac{\pi}{6} . = 2\pi - \frac{\pi}{3} = \frac{5\pi}{3} = \omega t = \frac{qBt}{m} \Rightarrow t = \frac{5\pi m}{3qB}$



(ii) Distance travelled $s = r(2\pi - 2\theta) = \frac{5\pi r}{3}$

(iii) Impulse = charge in linear momentum = $m(-v \sin\theta\,\hat{i} + v\cos\theta\,\hat{j}) - m(v\sin\theta\,\hat{i} + v\cos\theta\,\hat{j})$

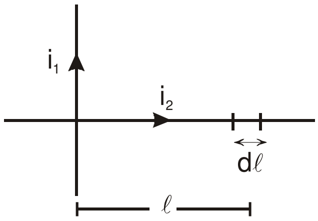
$= -2mv\sin\theta\,\hat{i} = -2mv\sin\frac{\pi}{6}\,\hat{i} = -mv\,\hat{i}$

Problem 10. A particle of charge q and mass m is projected in a uniform and constant magnetic field of strength B. The initial velocity vector $\vec{v}$ makes angle 'θ' with the $\vec{B}$. Find the distance travelled by the particle in time 't'.

Answer : vt

Solution : Speed of the particle does not change there fore distance covered by the particle is
s = vt

Problem 11. Two long wires, carrying currents i_1 and i_2, are placed perpendicular to each other in such a way that they just avoid a contact. Find the magnetic force on a small length $d\ell$ of the second wire situated at a distance ℓ from the first wire.



Solution : The situation is shown in figure. The magnetic field at the site of $d\ell$, due to the first wire is,

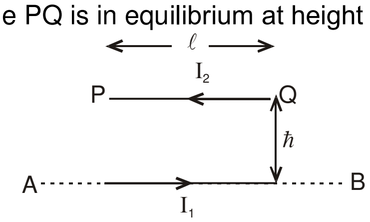
$$B = \frac{\mu_0 i_1}{2\pi\ell}$$

This field is perpendicular to the plane of the figure going into it. The magnetic force on the length $d\ell$ is,

$$dF = i_2\, d\ell\, B \sin 90^\circ = \frac{\mu_0 i_1 i_2 d\ell}{2\pi\ell}$$

This force is parallel to the current i_1.

Problem 12. In the figure shown the wires AB and PQ carry constant currents I_1 and I_2 respectively. PQ is of uniformly distributed mass 'm' and length 'ℓ'. AB and PQ are both horizontal and kept in the same vertical plane. The PQ is in equilibrium at height 'h'. Find



(i) 'h' in terms of I_1, I_2, ℓ, m, g and other standard constants.

(ii) If the wire PQ is displaced vertically by small distance prove that it performs SHM. Find its time period in terms of h and g.

Solution : (i) Magnetic repulsive force balances the weight.

$$\frac{\mu_0 I_1 I_2}{2\pi h}\,\ell\ mg \quad \Rightarrow \quad h = \frac{\mu_0 I_1 I_2 \ell}{2\pi mg}$$

(ii) Let the wire be displaced downward by distance x(<< h).
Magnetic force on it will increase, so it goes back towards its equilibrium position. Hence it performs oscillations.

$$F_{res} = \frac{\mu_0 I_1 I_2}{2\pi(h-x)}\ell - mg = \frac{mgh}{h-x} - mg = \frac{mg(h-h+x)}{h-x} = \frac{mg}{h-x}x \cong \frac{mg}{h}x \text{ for } x << h$$

$$\therefore \quad T = 2\pi\sqrt{\frac{m}{mg/h}} = 2\pi\sqrt{\frac{h}{g}}$$ **Ans.**

HANDOUT ELECTRO MAGNETIC FORCE & MOVE ON CHARGE OR CURRENT (EMF)

1. MAGNETIC CLASSIFICATION OF SUBSTANCES HYSTERESIS:

(A) Classification of substances according to their magnetic behaviour:

All substance show magnetic properties. An iron nail brought near a pole of a bar magnet is strongly attracted by it and sticks to it, Similar is the behaviour of steel, cobalt and nickel. Such substance are called 'ferromagnetic 'substance. Some substances are only weakly attracted by a magnet, while some are repelled by it. They are called 'paramagnetic' and 'diamagnetic 'substance respectively. All substance, solids, liquids and gases, fall into one or other of these classes.

(i) Diamagnetic substance : Some substance, when placed in a magnetic field, are feebly magnetised opposite to the direction of the magnetising field. These substances when brought close to a pole of a powerful magnet, are somewhat repelled away from the magnet. They are called 'diamagnetic' substances and their magnetism is called the 'diamagnetism'.

Examples of diamagnetic substances are bisuth, zinc, copper, silver, gold, lead, water, mercury, sodium chloride, nitrogen, hydrogen, etc.

(ii) Paramagnetic substances : Some substance when placed in a magnetic field, are feebly magnetised in the direction of the magnetising field. These substance, when brought close to a pole of a powerful magnet, are attracted towards the magnet. These are called 'paramagnetic' substance and their magnetism is called 'paramagnetism'.

(iii) Ferromagnetic substances : Some substance, when placed in a magnetic field, are strongly magnetised in the direction of the magnetising field. They are attracted fast towards a magnet when brought close to either of the poles of the magnet. These are called 'ferromagnetic' substances and their magnetism is called 'ferromagnetism'.

(B) Some important terms used in magnetism :

MAGNETISATION AND MAGNETIC INTENSITY

The earth abounds with a bewildering variety of elements and compounds.In addition, we have been synthesising new alloys, compounds and evenelements. One would like to classify the magnetic properties of these substances. In the present section, we define and explain certain terms which will help us to carry out this exercise.

We have seen that a circulating electron in an atom has a magnetic moment. In a bulk material, these moments add up vectorially and theycan give a net magnetic moment which is non-zero.

Magnetisation

Magnetisation M of a sample to be equal to its net magnetic moment per unit volume:

$$M = \frac{m_{net}}{V}$$

M is a vector with dimensions L^{-1} A and is measured in a units of A m^{-1}.Consider a long solenoid of n turns per unit length and carrying a current I. The magnetic field in the interior of the solenoid was shown to be given by $B_0 = \mu_0 nI$

If the interior of the solenoid is filled with a material with non-zero magnetisation, the field inside the solenoid will be greater than B_0. The net B field in the interior of the solenoid may be expressed as

$$B = B_0 + B_m$$

where B_m is the field contributed by the material core. It turns out thatthis additional field B_m is proportional to the magnetisation M of the material and is expressed as $B_m = \mu_0 M$

where μ_0 is the same constant (permeability of vacuum) that appears in Biot-Savart's law.

Magnetic Intensity

It is convenient to introduce another vector field H, called the magnetic intensity, which is defined by

$\Rightarrow \quad H = \dfrac{B}{\mu} \qquad \Rightarrow \quad H = \dfrac{B_0 + B_m}{\mu}$

where H has the same dimensions as M and is measured in units of A m^{-1}. Thus, the total magnetic field B is written as : $B = B_0 + B_m$

We repeat our defining procedure. We have partitioned the contributionto the total magnetic field inside the sample into two parts: one, due toexternal factors such as the current in the solenoid. This is represented by H. The other is due to the specific nature of the magnetic material,namely M.

$\Rightarrow \quad H = \dfrac{\mu_0 H + \mu_0 M}{\mu} \quad \Rightarrow \quad H = \dfrac{H}{\mu_r} + \dfrac{M}{\mu_r} \quad \Rightarrow \quad (\mu_r - 1)H = M \quad \Rightarrow \quad M = \chi H$

The latter quantity can be influenced by external factors. This influence is mathematically expressed as

$M = \chi H$

where χ , a dimensionless quantity, is appropriately called the **magnetic susceptibility**. It is a measure of how a magnetic material responds to an external field. Table lists χ for some elements. It is small and positive for materials, which are called paramagnetic. It is small and negative for materials, which are termed diamagnetic.

$\mu_r - 1 = \chi, \quad \mu_r = 1 + \chi$

is a dimensionless quantity called the relative magnetic permeability of the substance. It is the analog of the dielectric constant in electrostatics. The magnetic permeability of the substance is μ and it has the same dimensions and units as μ_0 ; $\mu = \mu_0\mu_r = \mu_0 (1 + \chi)$.

$\mu_r = (1 + \chi)$.

The three quantities χ, μ_r and μ are interrelated and only one of them is independent. Given one, the other two may be easily determined.

Table :1 Magnetic Susceptibility of Some Element At 300K

Diamagnetic substance	χ	Paramagnetic substance	χ
Bismuth	-1.66×10^{-5}	Aluminium	2.3×10^{-5}
Copper	-9.8×10^{-6}	Calcium	1.9×10^{-5}
Diamond	-2.2×10^{-5}	Chromium	2.7×10^{-4}
Gold	-3.6×10^{-5}	Lithium	2.1×10^{-5}
Lead	-1.7×10^{-5}	Magnesium	1.2×10^{-5}
Mercury	-2.9×10^{-5}	Niobium	2.6×10^{-5}
Nitrogen (STP)	-5.0×10^{-9}	Oxygen (STP)	2.1×10^{-6}
Silver	-2.6×10^{-5}	Platinum	2.9×10^{-4}
Silicon	-4.2×10^{-6}	Tungsten	6.8×10^{-5}

Example 1. A solenoid has a core of a material with relative permeability 400. The windings of the solenoid are insulated from the core and carry a current of 2A. If the number of turns is 1000 per metre, calculate

(a) H (b) M (c) B

(d) the magnetising current I_m.

Solution : (a) The field H is dependent of the material of the core, and is

$H = nI = 1000 \times 2.0 = 2 \times 10^3$ A/m.

(b) The magnetic field B is given by

$B = \mu_r \mu_0 H$

$= 400 \times 4\pi \times 10^{-7}$ (N/A^2) $\times 2 \times 10^3$ (A/m) = 1.0 T

(c) Magnetisation is given by
$M = (B - \mu_0 H)/\mu_0$
$= (\mu_r \mu_0 H - \mu_0 H) / \mu_0 = (\mu_r - 1)H = 399 \times H \approx 8 \times 10^5$ A/m

(d) The magnetising current I_M is the additional current that needs to be passed through the windings of the solenoid in the absence of the core which would give a B value as in the presence of the core. Thus
$B = \mu_r n_0 (I + I_M)$. Using $I = 2A$, $B = 1$ T, we get $I_M = 794$ A.

MAGNETIC PROPERTIES OF MATERIALS

The discussion in the previous section helps us to classify materials as diamagnetic, paramagnetic or ferromagnetic. In terms of the susceptibility χ , a material is diamagnetic if χ is negative, para- if χ is positive and small, and ferro- if χ is large and positive.

A glance at Table 5.3 gives one a better feeling for these materials. Here ε is a small positive number introduced to quantify paramagnetic materials. Next, we describe these materials in some detail.

Table :2

Diamagnetic	Paramagnetic	Ferromagnetic
$-1 \le \chi < 0$	$0 < \chi < \varepsilon$	$\chi >> 1$
$0 \le \mu_r < 1$	$1 < \mu_r < 1 + \varepsilon$	$\mu_r >> 1$
$\mu < \mu_0$	$\mu > \mu_0$	$\mu >> \mu_0$

Diamagnetism :

Diamagnetic substances are those which have tendency to move from stronger to the weaker part of the external magnetic field. In other words,unlike the way a magnet attracts metals like iron, it would repel adiamagnetic substance.

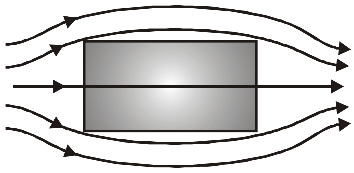
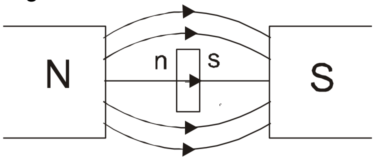


Figure shows a bar of diamagnetic material placed in an externalmagnetic field. The field lines are repelled or expelled and the field insidethe material is reduced. In most cases, as is evident from Table 1, this reduction is slight, being one part in 10^5. When placed in a non-uniform magnetic field, the bar will tend to move from high to low field.The simplest explanation for diamagnetism is as follows. Electrons in an atom orbiting around nucleus possess orbital angular momentum.These orbiting electrons are equivalent to current-carrying loop and thus possess orbital magnetic moment. Diamagnetic substances are the ones in which resultant magnetic moment in an atom is zero. When magnetic field is applied, those electrons having orbital magnetic moment in thesame direction slow down and those in the opposite direction speed up.This happens due to induced current in accordance with Lenz's law whichyou will study in Electro magnetic Induction. Thus, the substance develops a net magneticmoment in direction opposite to that of the applied field and hencerepulsion.Some diamagnetic materials are bismuth, copper, lead, silicon,nitrogen (at STP), water and sodium chloride. Diamagnetism is present in all the substances. However, the effect is so weak in most cases that it gets shifted by other effects like paramagnetism, ferromagnetism, etc.The most exotic diamagnetic materials are superconductors. These are metals, cooled to very low temperatures which exhibits both perfect conductivity and perfect diamagnetism. Here the field lines are completely expelled! $\chi = -1$ and $\mu_r = 0$. A superconductor repels a magnet and (byNewton's third law) is repelled by the magnet. The phenomenon of perfect diamagnetism in superconductors is called the Meissner effect, after the name of its discoverer. Superconducting magnets can be gainfully exploited in variety of situations, for example, for running magnetically levitated superfast trains.

Properties of diamagnetic substance :

Diamagnetic substance show following properties.

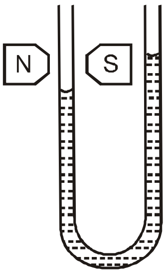
(i) When a rod of diamagnetic material is suspended freely between two magnetic poles, then its axis becomes perpendicular to the magnetic field.



(ii) In a non-uniform magnetic field a diamagnetic substance tends to move from the stronger to the weaker part of the field.

(a) **(b)**

(iii) If a diamagnetic solution is poured into a U-tube and one arm of this U-tube is placed between the poles of a strong magnet, the level of the solution in that arm is depressed.

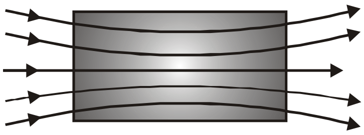


(iv) A diamagnetic gas when allowed to ascend in between the poles of a magnet spreads across the field.

(v) The susceptibility of a diamagnetic substance is independent of temperature.

Paramagnetism

Paramagnetic substances are those which get weakly magnetised when placed in an external magnetic field. They have tendency to move from a region of weak magnetic field to strong magnetic field, i.e., they get weakly attracted to a magnet.



The individual atoms (or ions or molecules) of a paramagnetic material possess a permanent magnetic dipole moment of their own. On account of the ceaseless random thermal motion of the atoms, no net magnetisation is seen. In the presence of an external field B_0, which is strong enough, and at low temperatures, the individual atomic dipole moment can be made to align and point in the same direction as B_0. Figure shows a bar of paramagnetic material placed in an external field. The field lines gets concentrated inside the material, and the field inside is enhanced. In most cases, as is evident from Table 1, this enhancement is slight, being one part in 10^5. When placed in a non-uniform magnetic field, the bar will tend to move from weak field to strong.Some paramagnetic materials are aluminium, sodium, calcium,oxygen (at STP) and copper chloride. Experimentally, one finds that the magnetisation of a paramagnetic material is inversely proportional to the absolute temperature T,

$$M = C\frac{B_0}{T}$$

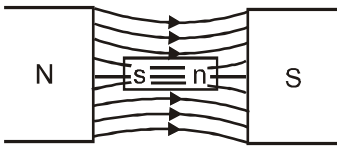
or equivalently, using Eqs $M = \chi H$ and $B_0 = \mu_0 H$

$$\chi = \frac{\mu_0}{T}$$

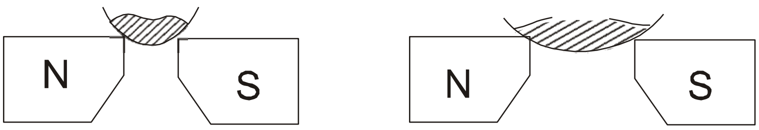
This is known as Curie's law, after its discoverer Pieree Curie (1859-1906). The constant C is called Curie's constant. Thus, for a paramagneticmaterial both χ and μ_r depend not only on the material, but also (in a simple fashion) on the sample temperature. As the field is increased or the temperature is lowered, the magnetisation increases until it reaches the saturation value M_s, at which point all the dipoles are perfectly aligned with the field. Beyond this, Curie's law is no longer valid.

Properties of Paramagnetic Substance

(i) When a rod of paramagnetic material is suspended freely between two magnetic poles, then its axis becomes parallel to the magnetic field The poles produced at the ends of the rod are opposite to the nearer magnetic poles.

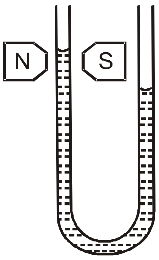


(ii) In a non-uniform magnetic field, the paramagnetic substances tend to move from weaker to stronger part of the magnetic field.



(a) **(b)**

(iii) If a paramagnetic solution is poured in a U-tube and one arm of the U-tube is placed between two strong poles, the level of the solution in that arm rises.



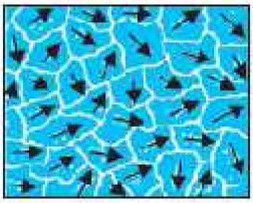
(iv) A paramagnetic gas when allowed to ascend between the pole-pieces of a magnet spreads along the field.

(v) The susceptibility of a paramagnetic substance varies inversely as the kelvin temperature of the substance, that is,

$\chi_m \propto \frac{1}{T}$. This known as curie's law.

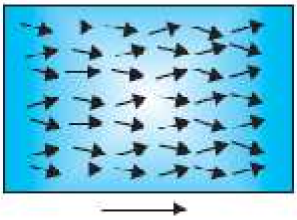
Ferromagnetism

Ferromagnetic substances are those which gets strongly magnetised when placed in an external magnetic field. They have strong tendency to move from a region of weak magnetic field to strong magnetic field, i.e., they get strongly attracted to a magnet.The individual atoms (or ions or molecules) in a ferromagnetic material possess a dipole moment as in a paramagnetic material. However, they interact with one another in such a way that they spontaneously align themselves in a common direction over a macroscopic volume called domain. The explanation of this cooperative effect requires quantum mechanics and is beyond the scope of this textbook. Each domain has a net magnetisation. Typical domain size is 1mm and the domain contains about 10^{11} atoms. In the first instant, the magnetisation varies randomly from domain to domain and there is no bulk magnetisation. This is shown in Fig.



Randomlyoriented domains,

When we apply an external magnetic field B_0, the domains orient themselves in the direction of B_0 and simultaneously the domain oriented in the direction of B_0 grow in size. This existence of domains and their motion in B_0 are not speculations. One may observe this under a microscope after sprinkling a liquid suspension of powdered ferromagnetic substance of samples. This motion of suspension can be observed. Figure (b) shows the situation when the domains have aligned and amalgamated to form a single 'giant' domain.



Aligned domains.

Thus, in a ferromagnetic material the field lines are highly concentrated. In non-uniform magnetic field, the sample tends to move towards the region of high field. We may wonder as to what happens when the external field is removed. In some ferromagnetic materials the magnetisation persists. Such materials are called hard magnetic materials or hard ferromagnets. Alnico, an alloy of iron, aluminium, nickel, cobalt and copper, is one such material. The naturally occurring lodestone is another. Such materials form permanent magnets to be used among other things as a compass needle. On the other hand, there is a class of ferromagnetic materials in which the magnetisation disappears on removal of the external field. Soft iron is one such material. Appropriately enough,such materials are called soft ferromagnetic materials. There are a number of elements, which are ferromagnetic: iron, cobalt, nickel, gadolinium, etc. The relative magnetic permeability is >1000!

The ferromagnetic property depends on temperature. At high enoughtemperature, a ferromagnet becomes a paramagnet. The domain structuredisintegrates with temperature. This disappearance of magnetisation with temperature is gradual. It is a phase transition reminding us of the melting of a solid crystal. The temperature of transition from ferromagnetic to paramagnetism is called the Curie temperature T_c. Table lists the Curie temperature of certain ferromagnets. The susceptibilityabove the Curie temperature, i.e., in the paramagnetic phase isdescribed by, $\chi = \dfrac{C}{T - T_c}$ $(T > T_c)$

Ferromagnetic substances : These substances which are strongly attracted by a magnet, show all the properties of a paramagnetic substance to a much higher degree. For example, they are strongly magnetised in relatively weak magnetising field in the same direction as the field. They have relative permeabilities of the order of hundreds and thousands. Similarly, the susceptibilities of ferromagnetic have large positive values.

Curie temperature : Ferromagnetism decreases with rise in temperature. If we heat a ferromagnetic substance, then at a definite temperature the ferromagnetic property of the substance "suddenly" disappears and the substance becomes paramagnetic. The temperature above which a ferromagnetic substance becomes paramagnetic is called the' Curie temperature' of the substance. The curie temperature of iron is 770ºC and that of nickel is 358ºC.

Example 2. A domain in ferromagnetic iron is in the form of a cubeof side length 1μm. Estimate the number of iron atoms in the domainand the maximum possible dipole moment and magnetisation of the domain. The molecular mass of iron is 55 g/mole and its density is 7.9 g/cm^3. Assume that each iron atom has a dipole momentof 9.27×10^{-24} A m^2.

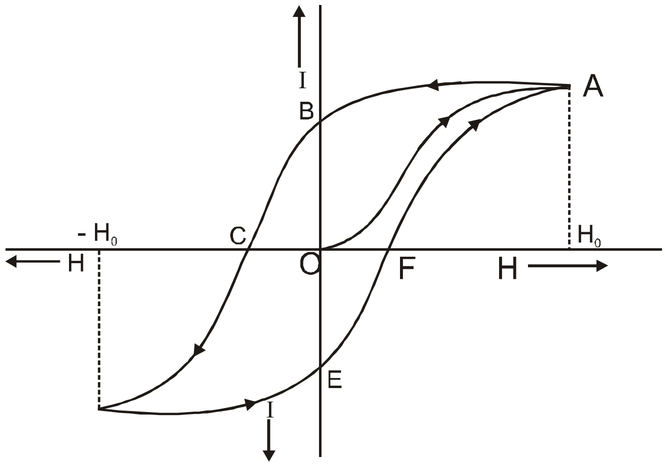
Solution : The volume of the cubic domain is$V = (10^{-6}\ m)^3 = 10^{-18}\ m^3 = 10^{-12}\ cm^3$. Its mass is volume × density = 7.9 g cm^{-3} × 10^{-12} cm^3= 7.9 × 10^{-12} gIt is given that Avagadro number (6.023 × 10^{23}) of iron atoms have a mass of 55 g. Hence, the number of atoms in the domain is

$$N = \frac{7.9 \times 10^{-12} \times 6.023 \times 10^{23}}{55} = 8.65 \times 10^{10} \text{ atoms}$$

The maximum possible dipole moment m_{max} is achieved for the(unrealistic) case when all the atomic moments are perfectly aligned.Thus,$m_{max} = (8.65 \times 10^{10}) \times (9.27 \times 10^{-24}) = 8.0 \times 10^{-13}$ A m^2

The consequent magnetisation is $M_{max} = m_{max}$/Domain volume = 8.0×10^{-13} Am^2/10^{-18} m^3 = 8.0×10^5 Am^{-1}

Hysterisis



Consider that a specimen of ferromagnetic material is placed in a magnetising field, whose strength and direction can be changed. Suppose that the specimen is unmagnetised initially. When the magnetising field (H) is increased, the intensity of magnetisation (I) of the material of the specimen also increases. It is found that when the magnetising field is made zero, the intensity of magnetisation does not become zero but still has some finite value. It becomes zero only, when magnetising field is increased inreversed direction. In other words, intensity of magnetisation does not become zero on making magnetising field zero but does so a little late and this effect is called hysteresis.

The lag of intensity of magnetisation behind the magnetising field during the process of magnetisation and demagnetisation of a ferromagnetic material is called hysteresis.

Fig shows the magnetisation curve of a ferromagnetic material, when it is taken over a complete cycle of magnetisation (I) is also zero. As magnetising field is increased, intensity of a mgnetisation also increases along.OA. Corresponding to point A, the intensity of magnetisation becomes maximum. The increase in value of the magnetising field beyond H_0 does not produce any increase in the intensity of magnetisation. In other words ,corresponding to point A, The specimen of the ferromagnetic material acquires a state of magnetic saturation. If magnetising field is now decreased slowly, intensity of magnetisation decreases but not a along the path AO.It decrease along the path AB. Corresponding to point B, magnetising field becomes zero but some magnetisation equal to OB is still left in the specimen. Here, OB gives the measure of retentivity of the material of the specimen.

The value of the intensity of magnetisation of a material, when the magnetising field is reduced to zero, is called retentivity of the material. It is also known residual magnetism or remanetism or remanence.

To reduce intensity of magnetisation to zero, the magnetising field has to be increased in reverse direction. As it is done so, the intensity of magnetisation decreases along BC, till it become zero corresponding to point C. Thus, to make intensity of magnetisation zero, magnetising field equal OC has to be applied in reverse direction. Here, OC gives the measure of coercivity of the material of the specimen .

The value of reverse magnetising field required so as to reduce residual magnetism to zero, is called coercivity of the material.

When the magnetising field is further increased in reverse direction, intensity of magnetisation increase along CD with the increase in magnetising field. Corresponding to point D (when the magnetising field becomes-H_0) it again acquires a saturation value, which is symmetrical to that corresponding to point A. If the magnetising field is decreased from–H_0 to zero, the intensity of magnetisation follows the path DE. Finally, when magnetising field is increased field in original direction, the point A is reached via EFA. If the magnetising field is repeatedly changed between H_0 and – H_0 , the curve ABCDEFA is retraced. The curve ABCDEFA is called the hysteresis loop. It is found that the area of the hysteresis loop (I – H curve)is proportional to the net energy absorbed per unit volume by the specimen, as it taken over a complete cycle of magnetisation and demagneti-sation. The energy so absorbed by the speciment appears as the heat energy.

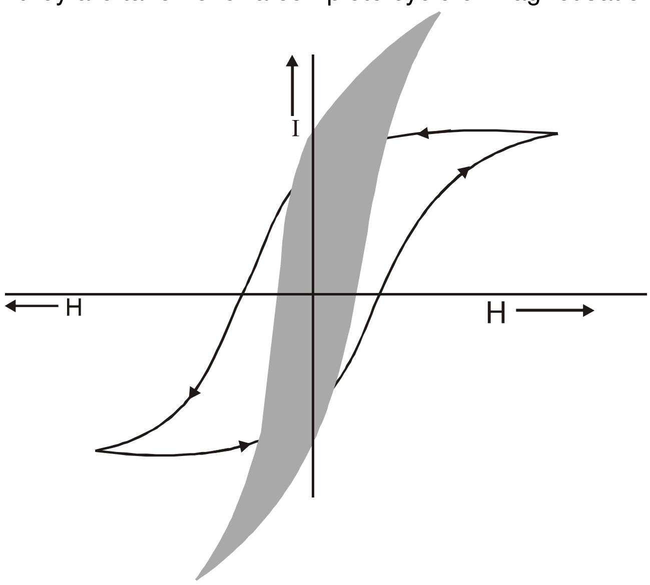
Note :- If hysteresis loop is drawn by plotting a graph between magnetic induction (B) and intensity of magnetisation, then area of the hysteresis loop is numerically equal to the work done per unit volume (or energy absorbed per unit volume) in taking the magnetic specimen over a complete cycle of magnetisation.

COMPARISON OF HYSTERESIS LOOPS FOR SOFT IRON AND STEEL :-

The shape of the hysteresis loop is a characteristics of a ferromagnetic substance. It gives the idea about many important magnetic properties of the substance.

Figure Shown hysteresis loops for soft iron and steel. Whereas the hysteresis loop for soft iron is narrow, the hysteresis loop for steel is quite wide. The following conclusion can be drawn from the study of the hysteresis loops of soft iron and steel :

1. The area of hysteresis loop for soft iron is much smaller then that for steel. Therefore, loss of energy per unit volume in case of soft iron will be very small as compared to that in case of steel, when they are taken over a complete cycle of magnetisation and demagnetisation.



2. Soft iron acquires maximum intensity of magnetisation for comparatively much lesser value of magnetising field than in case of steel. In other words, soft iron is much strongly magnetised (or more susceptible to magnetisation) than steel.
3. The retentivity of soft iron is greater then that of steel. On removing magnetising field, quite a large amount of magnetisation is retained by soft iron.
4. The coercivity of steel is much larger then that of soft iron. Therefore, the residula magnetism in steel can not be destroyed that easily as in case of soft iron.

Section of magnetic materials :

The choice of a magnetic material for making permanent magnet, electromagnet, core of transformer or diaphragm of telephone ear-piece can be decided from the hysteresis curve of the material.

(i) Permanent magnets :

The material for a permanent magnet should have high retentivity so that the magnet is strong, and high coercivity so that the magnetisation is not wiped out by stray external fields, mechanical ill treatement and temperature changes. The hysteresis loss is immaterial because the material in this case is never put to cyclic changes of magnetisation. From these considerations permanent magnets are made of steel. The fact that the retentivity of soft iron is a little greater than that of steel is outweighed by its much smaller coercivity, which makes it very easy to demagnetise.

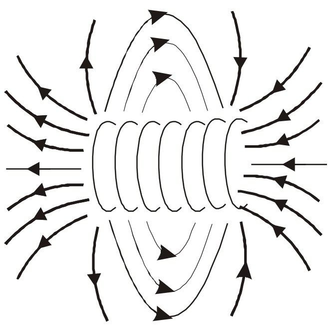
(ii) Electromagnets : The material for the cores of electromagnets should have high permeability (or high susceptibility), specially at low magnetising fields, and a low retentivity. Soft iron is suitable material for electromagnets).

(iii) Transformer cores and telephone diaphragms : In these cases the material goes through complete cycles of magnetisation continuously. The material must therefore have a low hysteresis loss to have less dissipation of energy and hence a small heating of the material (otherwise the insulation of windings may break), a high permeability (to obtain a large flux density at low field) and a high specific resistance (to reduce eddy current loses).

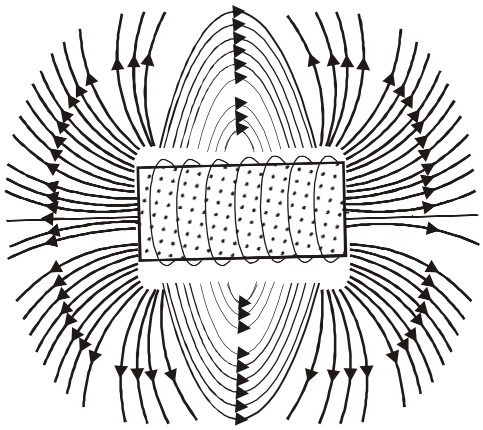
Soft -iron is used for making transformer cores and telephone diaphragms : More effective alloys have now been developed for transformer cores. They are permalloys, mumetals etc.

2. ELECTROMAGNET :

If we place a soft-iron rod in the solenoid, the magnetism of solenoid increases hundreds of time. Then the solenoid is called an ‘electromagnet’. It is a temporary magnet.

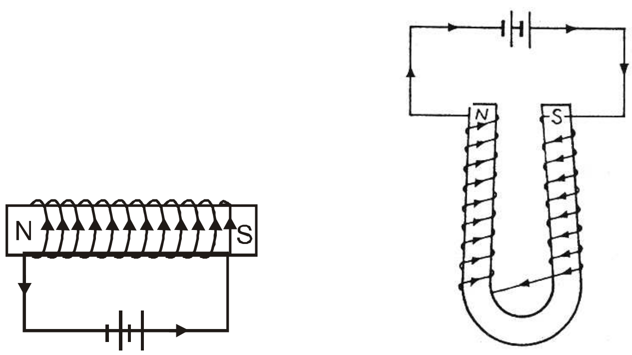


(a)



(b)

An electromagnet is made by winding closely a number of turns of insulated copper wire over a soft-iron straight rod or a horse-shoe rod. On passing current through this solenoid, a magnetic field is produced in the space within the solenoid.



(a) **(b)**

Example 3. A magnetising field of 1600 Am^{-1} produces a magnetic flux of 2.4×10^{-5} wb in an iron bar of cross-sectional area $0.2 cm^2$. Calculate permeability and susceptibility of the bar.

Solution : $B = \dfrac{\Phi}{A} = \dfrac{2.4 \times 10^{-5}\ \text{Wb}}{0.2 \times 10^{-4}\ \text{m}^2} = 1.2\ \text{Wb/m}^2 = 1.2\ \text{N A}^{-1}\ \text{m}^{-1}$.

The magnetising field (or magnetic intensity) H is 1600 Am^{-1}. Therefore, the magnetic permeability is given by -

$$\mu = \frac{B}{H} = \frac{1.2 NA^{-1} m^{-1}}{1600\, Am^{-1}} = 7.5 \times 10^{-4}\ N/A^2.$$

Now, from the relation $\mu = \mu_0 (1 + \chi_m)$, the susceptibility is given by

$$\chi_m = \frac{\mu}{\mu_0} - 1.$$

We known that $\mu_0 = 4\pi \times 10^{-7}\ N/A^2$. $\therefore$ $\chi_m = \dfrac{7.5 \times 10^{-4}}{4 \times 3.14 \times 10^{-7}} - 1 = 596.$

Example 4. The core of toroid of 3000 turns has inner and outer radii of 11 cm and 12 cm respectively. A current of 0.6 A produces a magnetic field of 2.5 T in the core. Compute relative permeability of the core.($\mu_0 = 4\pi \times 10^{-7}$ T m A^{-1}).

Solution : The magnetic field in the empty space enclosed by the windings of a toroid carrying a current i_0 is $\mu_0 n i_0$

where n is the number of turns per unit length of the toroid and μ_0 is permeability of free space.

If the space is filled by a core of some material of permeability μ, then the field is given by

$B = \mu n i_0$

But $\mu = \mu_0 u_r$, where μ_r is the relative permeability of the core material. Thus,

$B = \mu_0 u_r n i_0$ or $\mu_r = \dfrac{B}{\mu_0 n i_0}$

Here $B = 2.5$ T, $i_0 = 0.7$ A and $n = \dfrac{300}{2\pi r}\ m^{-1}$, where r is the mean radius of the toroid

$\left(r = \dfrac{11+12}{2} = 11.5 \text{ cm } 11.5 \times 10^{-2} m\right)$. Thus,

$$\mu_r = \frac{2.5}{(4\pi \times 10^{-7}) \times (3000/2\pi \times 11.5 \times 10^{-2}) \times 0.7} = \frac{2.5 \times 11.5 \times 10^{-2}}{2 \times 10^{-7} \times 3000 \times 0.7}$$

$\mu_r = 684.5$

Exercise-1

✍ **Marked Questions can be used as Revision Questions.**

PART - I : SUBJECTIVE QUESTIONS

Section (A) : Magnet and Magnetic field due to a moving charge

A-1. The magnetic moment of a short dipole is, 1 A m^2. What is the magnitude of the magnetic induction in air at 10 cm from centre of the dipole on a line making an angle of 30º from the axis of the dipole?

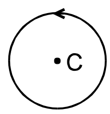
A-2. A point charge q = 2μC is at the origin. It has velocity $2\hat{i}$ m/s. Find the magnetic field at the following points in vector form (at the moment when the charged particle passes through the origin) :

(i) (2, 0, 0) (ii) (0, 2, 0) (iii) (0, 0, 2) (iv) (2, 1, 2)

(v) Is the magnitude of the magnetic field on the circumference of the circle (in yz plane) $y^2 + z^2 = c^2$ where 'c' is a constant is same every where. Is it same in direction also.

(vi) Answer the above (v) for the circle of same equation but in a plane x = a where 'a' is a constant.

A-3. A particle of negative charge of magnitude 'q' is revolving with constant speed 'V' in a circle of radius 'R' as shown in figure. Find the magnetic field (magnitude and direction) at the following points :

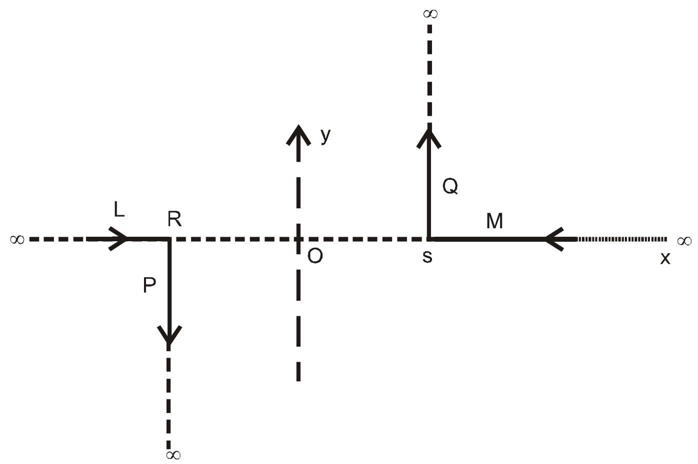


(i) centre of the circle (magnitude and direction)

(ii) a point on the axis and at a distance 'x' from the centre of the ring (magnitude only). Is its direction constant all the time?

Section (B) : Magnetic field due to a straight wire

B-1. A pair of stationary and infinitely long bent wires is placed in the X-Y plane as shown in figure. The wires carry currents of 10A each as shown. The segments L and M are along the x-axis. The segments P and Q are parallel to the Y-axis such that OS = OR = 0.02 m. Find the magnitude and direction of the magnetic induction at the origin O.

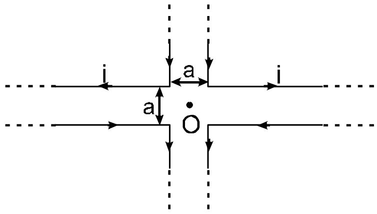


B-2. A current of 1 amp is flowing in the sides of an equilateral triangle of side 4.5 × 10^{-2} m. Find the magnetic field at the centroid of triangle. **[REE - 1991]**

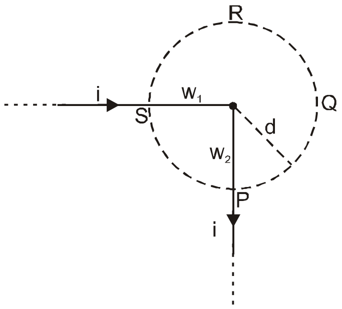
B-3. Two straight infinitely long and thin parallel wires are spaced 0.1 m apart and carry a current of 10 ampere each. Find the magnetic field at a point distant 0.1 m from both wires in the two cases when the currents are in the

(i) Same and (ii) Opposite direction.

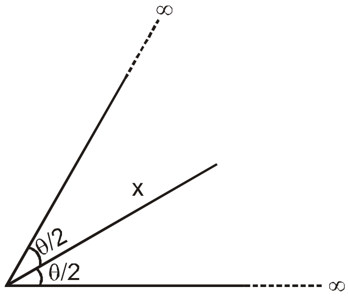
B-4. Four infinitely long 'L' shaped wires, each carrying a current i have been arranged as shown in the figure. Obtain the magnetic field strength at the point 'O' equidistant from all the four corners.



B-5. Figure shows a long wire bent at the middle to form a right angle. Show that the magnitudes of the magnetic fields at the points Q and R are unequal and find these magnitudes. The wire w_1 and the circumference of circle are coplaner and w_2 is perpendicular to plane of paper. Also find the ratio of field at Q and R.

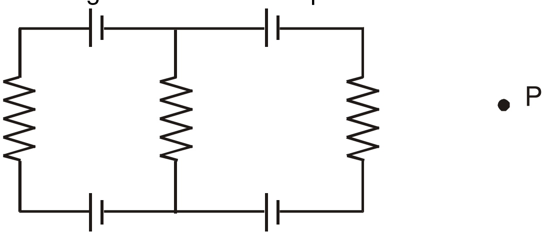


B-6. A long wire carrying a current i is bent to form a plane angle θ. Find the magnetic field B at a point on the bisector of this angle situated at a distance x from the vertex is written in the form of $K\cot\frac{\theta}{4}$ Tesla. Then, find the value of K.



B-7. Find the magnetic field B at the centre of a square loop of side 'a', carrying a current i.

B-8. Each of the batteries shown in figure has an emf equal to 10 V. Find the magnetic field B at the point p.



Section (C) : Magnetic field due to a circular loop

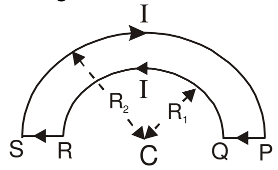
C-1. (i) Two circular coils of radii 5.0 cm and 10 cm carry equal currents of 1 A. The coils have 50 and 100 turns respectively and are placed in such a way that their planes as well as the centre coincide. Find the magnitude of the magnetic field B at the common centre when the currents in the coils are (a) in the same sense (b) in the opposite sense.

(ii) If the outer coil of the above problem is rotated through 90º about a diameter, what would be the magnitude of the magnetic field B at the centre?

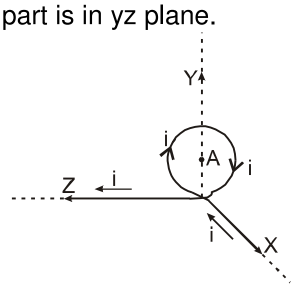
C-2. Two circular coils of wire each having a radius of 4 cm and 10 turns have a common axis and are 6 cm apart. If a current of 1 A passes through each coil in the opposite direction find the magnetic induction.
(a) At the centre of each coil ;
(b) At a point on the axis, midway between them.

Section (D) : Magnetic field due to a straight wire and circular arc

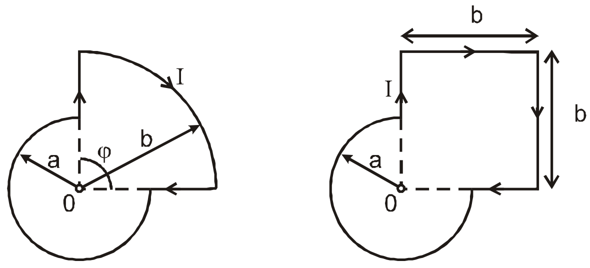
D-1. Two wire loops PQRSP formed by joining two semicircular wires of radii R_1 and R_2 carries a current I as shown in (fig.) The magnitude of the magnetic induction at the center C is



D-2. Find the magnitude of the magnetic induction B of a magnetic field generated by a system of thin conductors (along which a current i is flowing) at a point A (0, R, 0), that is the centre of a circular conductor of radius R. The circular part is in yz plane.

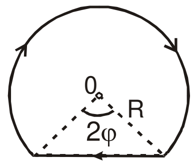


D-3. Find the magnetic induction of the field at the point O of a loop with current I, whose shape is illustrated in figure

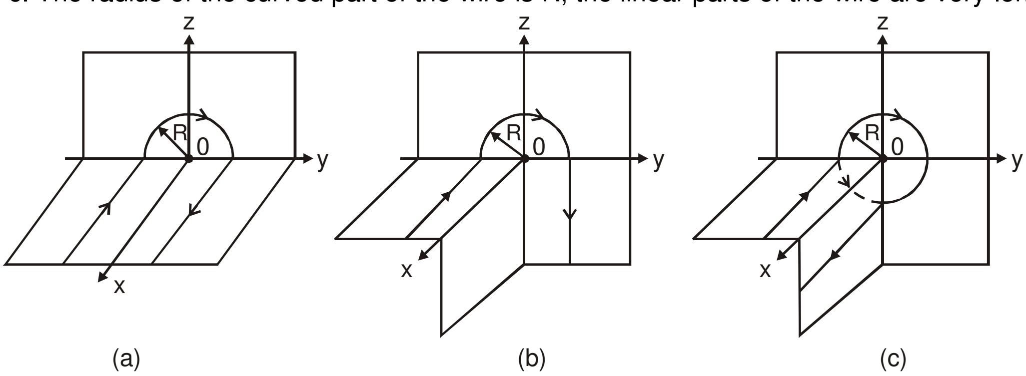


(a) (b)

(a) In figure 'a' the radii a and b, as well as the angle φ are known,
(b) In figure b, the radius a and the side b are known.
(c) A current I = 5.0 A flows along a thin wire shaped as shown in figure. The radius of a curved part of the wire is equal to R = 120 mm, the angle 2φ = 90°. Find the magnetic induction of the field at the point O.



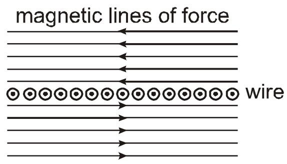
D-4. Find the magnetic induction at the point O if the wire carrying a current I has the shape shown in figure a, b, c. The radius of the curved part of the wire is R, the linear parts of the wire are very long.



(a) (b) (c)

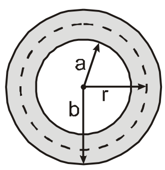
Section (E) : Magnetic field due to a cylinder, large sheet, solenoid, toroid and ampere's law

E-1. A conductor consists of an infinite number of adjacent wires, each infinitely long & carrying a current i. Show that the lines of B will be as represented in figure & that B for all points in front of the infinite current sheet will be given by, $B = (1/2)\ \mu_0\, ni$, where n is the number of conducting wires per unit length.



E-2. Figure shows a cylindrical conductor of inner radius a & outer radius b which carries a current *i* uniformly spread over its cross section. Show that the magnetic field B for points inside the body of the conductor (i.e. $a < r < b$) is given by $B = \dfrac{\mu_0 i}{2\pi(b^2 - a^2)}\ \dfrac{r^2 - a^2}{r}$.

Check this formula for the limiting case of a = 0.



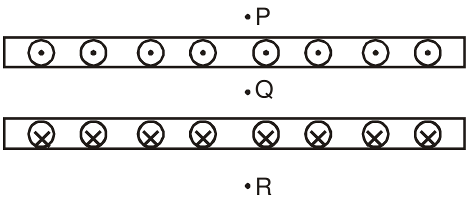
E-3. A thin but long, hollow, cylindrical tube of radius r carries a current i along its length. Find the magnitude of the magnetic field at a distance r/4 from the surface (a) inside the tube (b) outside the tube.

E-4. The magnetic field B inside a long solenoid, carrying a current of 10 A, is 3.14×10^{-2} T. Find the number of turns per unit length of the solenoid.

E-5. A copper wire having resistance 0.01 ohm in each metre is used to wind a 400 turn solenoid of radius 1.0 cm and length 20 cm. Find the emf of a battery which when connected across the solenoid will cause a magnetic field of 1.0×10^{-2} T near the centre of the solenoid.

E-6 A tightly-wound, long solenoid is kept with its axis parallel to a large metal sheet carrying a surface current. The surface current through a width dl of the sheet is Kdl and the number of turns per unit length of the solenoid is n. The magnetic field near the centre of the solenoid is found to be zero. (a) Find the current in the solenoid. (b) If the solenoid is rotated to make its axis 60º to the plane of metal sheet, what would be the magnitude of the magnetic field near its centre?

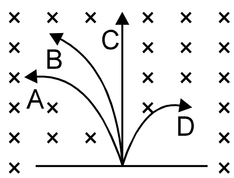
E-7. Two large metal sheets carry surface currents as shown in figure. The current through a strip of width dl is Kdl where K is a constant. Find the magnetic field at the point P, Q and R.



Section (F) : Magnetic force on a charge (Normal incidence)

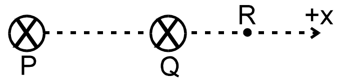
F-1. A charged particle is accelerated through a potential difference of 24 kV and acquires a speed of 2×10^6 m/s. It is then injected perpendicularly into a magnetic field of strength 0.2 T. Find the radius of the circle described by it.

F-2. A neutron, a proton, an electron and an α-particle enters a uniform magnetic field with equal velocities. The field is directed along the inward normal to the plane of the paper. Which of these tracks followed are by electron and α-particle. **[JEE- 1984]**



F-3. In the formula $X = 3\ YZ^2$, the quantities X and Z have the dimensions of capacitance and magnetic induction respectively. The dimensions of Y in the MKS system are **[JEE - 1988]**

F-4. Two long parallel wires carrying currents 2.5 amps and I amps in the same direction (directed into the plane of the paper) are held at P and Q respectively such that they are perpendicular to the plane of paper.



The points P and Q are located at a distance of 5m and 2m respectively from a collinear point R.

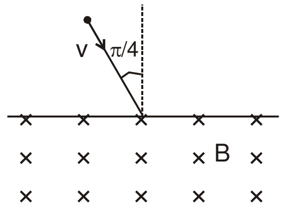
(a) An electron moving with a velocity of 4 x 10^5 m/s along the positive X-direction experiences a force of magnitude 3.2 x 10^{-20} N at the point R. Find the value of I.

(b) Find all the positions at which a third long-parallel wire carrying a current of magnitude 2.5 A may be placed so that the magnetic induction at R is zero. **[JEE - 1990]**

F-5. A magnetic field of $8\hat{k}$ mT exerts a force of $(4.0\,\hat{i} + 3.0\,\hat{j}) \times 10^{-10}$ N on a particle having a charge of 5×10^{-10} C and going in the X – Y plane. Find the velocity of the particle.

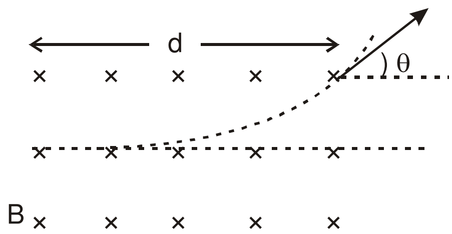
F-6. An experimenter's diary reads as follows; "a charged particle is projected in a magnetic field of $(7.0\,\hat{i}\ - 3.0\,\hat{j}) \times 10^{-3}$ T. The acceleration of the particle is found to be $(x\,\hat{i}\ + 7.0\,\hat{j}) \times 10^{-6}$ m/s^2. Find the value of x.

F 7. A particle of mass m and positive charge q, moving with a uniform velocity v, enters a magnetic field B as shown in figure. (a) Find the radius of the circular arc it describes in the magnetic field. (b) Find the angle subtended by the arc at the centre. (c) How long does the particle stay inside the magnetic field? (d) Solve the three parts of the above problem if the charge q on the particle is negative.

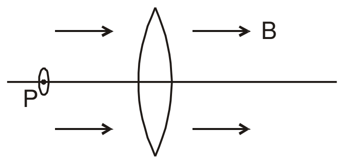


F 8. A particle of mass m and charge q is projected into a region having a perpendicular magnetic field B. Find the angle of deviation (figure) of the particle as it comes out of the magnetic field if the width d of the region is very slightly smaller than

(a) $\dfrac{mv}{qB}$ (b) $\dfrac{mv}{\sqrt{2}qB}$ (c) $\dfrac{3mv}{qB}$



F 9. Figure shows a convex lens of focal length 10 cm lying in a uniform magnetic field B of magnitude 1.2 T parallel to its principal axis. A particle having a charge 2.0×10^{-3} C and mass 2.0×10^{-5} kg is projected perpendicular to the plane of the diagram with a speed of 4.8 m/s. The particle moves along a circle with its centre on the principal axis at a distance of 15 cm from the lens. The axis of the lens and of the circle are same. Show that the image of the particle goes along a circle and find the radius of that circle.



F-10. A uniform magnetic field of magnitude 0.20 T exists in space from east to west. A particle having mass 10^{-5} kg and charge 10^{-5} C is projected from south to north so that it moves with a uniform velocity. Find velocity of projection of the particle? (g = 10 m/s^2)

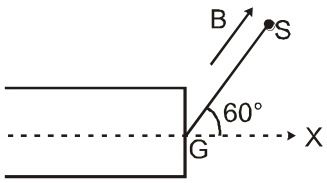
F-11. A tightly-wound, long solenoid has n turns per unit length, a radius r and carries a current i. A particle having charge q and mass m is projected from a point on the axis of the solenoid in a direction perpendicular to the axis. What can be the minimum speed for which the particle should be projected to strike the solenoid?

Section (G) : Magnetic force on a charge (oblique incidence)

G-1. A particle having a charge of 5.0 μC and a mass of 5.0×10^{-12} kg is projected with a speed of 1.0 km/s in a magnetic field of magnitude 5.0 mT. The angle between the magnetic field vector and the velocity vector is $\sin^{-1}$ (0.90). Show that the path of the particle will be a helix. Find the diameter of the helix and its pitch.

G-2. A proton projected in a magnetic field of 0.04 T travels along a helical path of radius 5.0 cm (of projected circle) and pitch 20 cm. Find the components of the velocity of the proton along and perpendicular to the magnetic field. Take the mass of the proton = 1.6×10^{-27} kg.

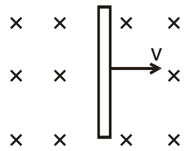
G-3. An electron gun G emits electron of energy 2keV traveling in the positive x-direction. The electrons are required to hit the spot S where GS = 0.1m and the line GS makes an angle of 60^0 with the x-axis, as shown in the fig. A uniform magnetic field $\vec{B}$ parallel to GS exists in the region outside of electron gun. Find the minimum value of B needed to make the electron hit S. [Take mass of electron = 9×10^{-31} kg]



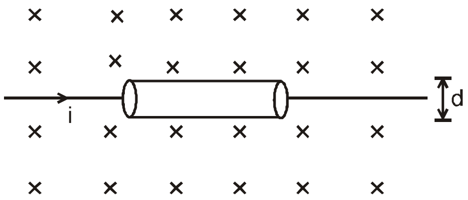
Section (H) : Electric and magnetic force on a charge

H-1. An electron beam passes through a magnetic field of 2×10^{-3} Wb/m^2 and an electric field of 3.2×10^4 V/m, both acting simultaneously. ($\vec{E} \perp \vec{B} \perp \vec{V}$) If the path of electrons remains undeflected calculate the speed of the electron. If the electric field is removed, what will be the radius of the electron path [mass of electron = 9.1×10^{-31} kg]?

H-2 A conducting wire of length ℓ, lying normal to a magnetic field B, moves with a velocity v as shown in figure. (a) Find the average magnetic force on a free electron of the wire. (b) Due to this magnetic force, electrons concentrate at one end resulting in an electric field inside the wire. The redistribution stops when the electric force on the free electrons balances the magnetic force. Find the electric field developed inside the wire when the redistribution stops. (c) What potential difference is developed between the ends of the wire?



H-3. A current i is passed through a cylindrical gold strip of radius r. The number of free electrons per unit volume is n. (a) Find the drift velocity v of the electrons. (b) If a magnetic field B exists in the region as shown in figure, what is the average magnetic force on the free electrons? (c) Due to the magnetic force, the free electrons get accumulated on one side of the conductor along its length. This produces a transverse electric field in the conductor which opposes the magnetic force on the electrons. Find the magnitude of the electric field which will stop further accumulation of electrons. (d) What will be the potential difference developed across the width of the conductor due to the electron accumulation? The appearance of a transverse emf, when a current-carrying wire is placed in a magnetic field, is called Hall effect.



H-4 A particle moves in a circle of radius 1.0 cm under the action of a magnetic field of 0.40 T. An electric field of 200 V/m makes the path straight. Find the charge/mass ratio of the particle.

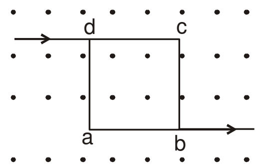
H-5 A proton goes undeflected in a crossed electric and magnetic field (the fields are perpendicular to each other) at a speed of 10^5 m/s. The velocity is perpendicular to both the fields. When the electric field is switched off, the proton moves along a circle of radius 2 cm. Find the magnitudes of the electric and the magnetic fields. Take the mass of the proton = 1.6×10^{-27} kg.

H-6. A particle having mass m and charge q is released from the origin in a region in which electric field and magnetic field are given by $\vec{B} = +B_0\hat{j}$ and $\vec{E} = +E_0\hat{i}$. Find the speed of the particle as a function of its X-coordinate.

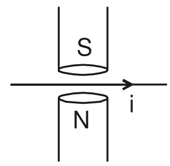
Section (I) : Magnetic force on a current carrying wire

I-1. Consider a 10 cm long portion of a straight wire carrying a current of 10 A placed in a magnetic field of 0.1 T making an angle of 37º with the wire. What magnetic force does the wire experience?

I-2. A current of 2 A enters at the corner d of a square frame abcd of side 10 cm and leaves at the opposite corner b. A magnetic field B = 0.1 T exists in the space in direction perpendicular to the plane of the frame as shown in figure. Find the magnitude and direction of the magnetic forces on the four sides of the frame.



I-3. A magnetic field of strength 1.0 T is produced by a strong electromagnet in a cylindrical region of diameter 4.0 cm as shown in figure. A wire, carrying a current of 2.0 A, is placed perpendicular to and intersecting the axis of the cylindrical region. Find the magnitude of the force acting on the wire.

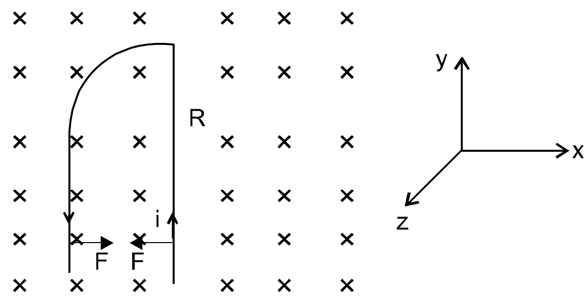


I-4. A wire of length ℓ carries a current i along the y-axis. A magnetic field exists which is given as $\vec{B} = B_0(\hat{i} + \hat{j} + \hat{k})T$. Find the magnitude of the magnetic force acting on the wire.

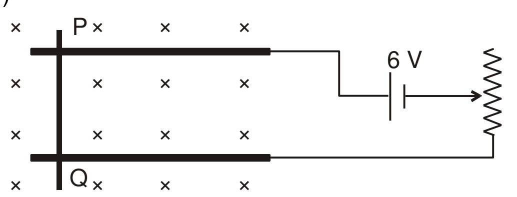
I-5. A thin straight horizontal wire of length 0.2 m whose mass is 10^{-4} kg floats in a magnetic induction field when a current of 10 ampere is passed through it. To make this possible, what should be the minimum magnetic strength ? (Take g = 10 m/s^2)

I-6. A wire, carrying a current, i, is kept in the X–Y plane along the curve $y = A \sin\left(\frac{2\pi}{\lambda}x\right)$. A uniform magnetic field B exists in the z-direction. Find the magnitude of the magnetic force on the portion of the wire between x = 0 and x = $\lambda/2$

I-7. A rigid wire consists of a quarter-circular portion of a circle of radius R and two straight sections (figure). The wire is partially immersed in a perpendicular magnetic field B as shown in the figure. Find the magnetic force on the wire if it carries a current i.



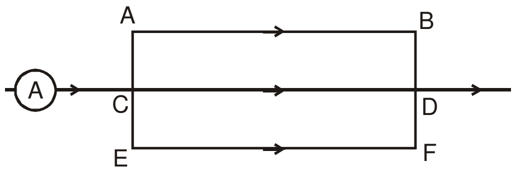
I-8. A metal wire PQ of mass 10 gm lies at rest on two horizontal metal rails separated by 4.90 cm (figure). A vertically downward magnetic field of magnitude 0.800 T exists in the space. The resistance of the circuit is slowly decreased and it is found that when the resistance goes below 20.0 Ω, the wire PQ starts sliding on the rails. Find the coefficient of friction. Neglect magnetic force acting on wire PQ due to metal rails (g = 9.8 m/s^2)



I-9. Two long wires carrying current in the same direction attract each other but two parallel beams of electrons moving in the same direction repel each other. Explain

I-10. The magnetic field existing in a region is given by $\vec{B} = B_0\left(1 - \frac{x}{\ell}\right)\hat{k}$, where B_0 and ℓ are constants, x is the x coordinate of a point and $\hat{k}$ is the unit vector along Z axis. A square loop of edge ℓ and carrying a current i, is placed with its edges parallel to the x, y axes. Find the magnitude of the net magnetic force experienced by the loop.

I-11. Figure shows a part of an electric circuit. The wires AB, CD and EF are very long and have identical resistances. The separation between the neighboring wires is 2 cm. The wires AE and BF have negligible resistance and the ammeter reads 60 A. Calculate the magnetic force per unit length on AB and CD.



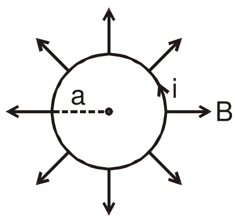
I-12. Two long straight parallel conductors are separated by a distance of r_1 = 5cm and carry constant currents i_1 = 10 A & i_2 = 20 A. What work per unit length of a conductor must be done to increase the separation between the conductors to r_2 = 10 cm if, currents flow in the same direction?

I-13. A straight rod of mass m and length ℓ can slide on two parallel plastic rails kept in a horizontal plane with a separation d. The coefficient of friction between the wire and the rails is μ. If the wire carries a current i, what minimum magnetic field should exist in the space in order to slide the wire on the rails.

Section (J) : Magnetic force and torque on a current carrying loop and magnetic dipole moment

J-1. A circular coil of 100 turns has an effective radius 0.05 m and carries a current of 0.1 amp (assume current remains constant). How much work is required to turn it in an external magnetic field of 1.5 wb/m^2 through 180° about an axis perpendicular to the magnetic field. The plane of the coil is initially perpendicular to the magnetic field. **[REE - 1986]**

J-2. (a) A circular loop of radius a, carrying a current i, is placed in a two-dimensional magnetic field. The centre of the loop coincides with the centre of the field (figure). The strength of the magnetic field at the periphery of the loop is B. Find the magnetic force on the wire.



(b) A hypothetical magnetic field existing in a region is given by $\vec{B} = B_0\vec{e}_r$, where $\vec{e}_r$ denotes the unit vector along the radial direction of a point relative to the origin and B_0 = constant. A circular loop of radius a, carrying a current i, is placed with its plane parallel to the X-Y plane and the centre at (0, 0, a). Find the magnitude of the magnetic force acting on the loop.

J-3 A rectangular coil of 100 turns has length 4 cm and width 5 cm. It is placed with its plane parallel to a uniform magnetic field and a current of 2A is sent through the coil. Find the magnitude of the magnetic field B, if the torque acting on the coil is 0.2 N-m.

J-4 A 50-turn circular coil of radius 4 cm carrying a current of 2.5 A is rotated in a magnetic field of strength 0.20 T. (a) What is the maximum torque that acts on the coil? (b) In a particular position of the coil, the torque acting on it is half of this maximum. What is the angle between the magnetic field and the plane of the coil?

J-5 A square loop of sides 10 cm carries a current of 10 A. A uniform magnetic field of magnitude 0.20 T exists parallel to one of the side of the loop. (a) What is the force acting on the loop? (b) What is the torque acting on the loop?

J-6. A circular coil of diameter 2.0 cm has 500 turns in it and carries a current of 1.0 A. Its axis makes an angle of 30º with the uniform magnetic field of magnitude 0.40 T that exists in the space. Find the torque acting on the coil.

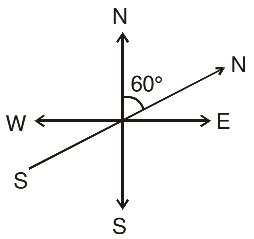
J-7. A circular loop carrying a current i has wire of total length L. A uniform magnetic field B exists parallel to the plane of the loop. (a) Find the torque on the loop. (b) If the same length of the wire is used to form a rectangular loop of side ratio 1 : 2, what would be the torque? Which is larger?

J-8. A charge Q is spread uniformly over an insulated ring of radius R. What is the magnetic moment of the ring if it is rotated with an angular velocity ω about its axis? **[JEE - 1990]**

Section (K) : Magnetic field due to a magnet and earth

K-1. Two circular coils each of 100 turns are held such that one lies in the vertical plane and the other in the horizontal plane with their centres coinciding. The radius of the vertical and the horizontal coils are respectively 20 cm and 30 cm. If the directions of the current in them are such that the earth's magnetic field at the centre of the coil is exactly neutralized, calculate the current in each coil. [horizontal component of the earth's field = $27.8\ \mu_0 A\ m^{-1}$; angle of dip = 30º] **[REE - 1988]**

K-2. A short magnet of magnetic moment 6 Amp.m^2 is lying in a horizontal plane with its North pole pointing 60º East of North. Find the net horizontal magnetic field at a point on the axis of the magnet 0.2 m away from it. [Horizontal component of earth's magnetic field = 0.3×10^{-4} tesla] **[REE-1991]**



K-3. A coil of 50 turns and 20 cm diameter is made with a wire of 0.2 mm diameter and resistivity $2 \times 10^{-6}\Omega$ cm. The coil is connected to a source of EMF. 20 V and negligible internal resistance

(a) Find the current through the coil.

(b) What must be the minimum potential difference across the coil so as to nullify the earth's horizontal magnetic induction of 3.14×10^{-5} tesla at the centre of the coil. How should the coil be placed to achieve the above result.

Section (L) : Magnetic material

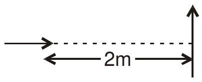
L-1. A magnetising field of 1600 Am^{-1} produces a magnetic flux of 2.4×10^{-5} wb in an iron bar of cross-sectional area 0.2cm^2. Calculate permeability and susceptibility of the bar.

L-2. The core of toroid of 3000 turns has inner and outer radii of 11 cm and 12 cm respectively. A current of 0.6 A produces a magnetic field of 2.5 T in the core. Compute relative permeability of the core. ($\mu_0 = 4\pi \times 10^{-7}$ T m A^{-1}).

PART - II : ONLY ONE OPTION CORRECT TYPE

Section (A) : Magnet and Magnetic field due to a moving charge

A-1. Two identical short magnetic dipoles of magnetic moments 1.0 A-m^2 each, placed at a separation of 2 m with their axes perpendicular to each other. The resultant magnetic field at a point midway between the dipole is: **[REE - 1995]**



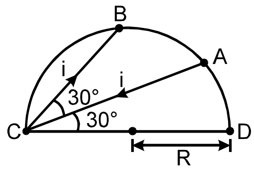
(A) 5×10^{-7} T B) $\sqrt{5} \times 10^{-7}$ T (C) 10^{-7} T (D) 2×10^{-7} T

A-2. A point charge is moving in a circle with constant speed. Consider the magnetic field produced by the charge at a fixed point P (not centre of the circle) on the axis of the circle.
(A) it is constant in magnitude only (B) it is constant in direction only
(C) it is constant in direction and magnitude both (D) it is not constant in magnitude and direction both.

Section (B) : Magnetic field due to a straight wire

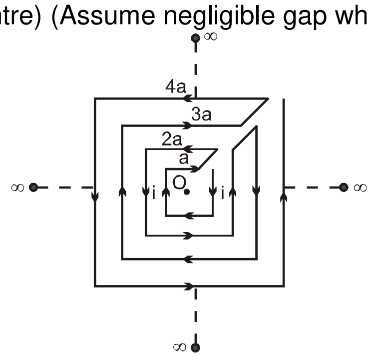
B-1. Two infinitely long, thin, insulated, straight wires lie in the x-y plane along the x and y-axis respectively. Each wire carries a current i, respectively in the positive x-direction and positive y-direction. The magnetic field will be zero at all points on the straight line: **[JEE - 1993]**
(A) $y = x$ (B) $y = -x$ (C) $y = x - 1$ (D) $y = -x + 1$

B-2. A current carrying wire is placed in the grooves of an insulating semi circular disc of radius 'R', as shown. The current enters at point A and leaves from point B. Determine the magnetic field at point D.



(A) $\frac{\mu_0 I}{8\pi R\sqrt{3}}$ (B) $\frac{\mu_0 I}{4\pi R\sqrt{3}}$
(C) $\frac{\sqrt{3}\mu_0 I}{4\pi R}$ (D) none of these

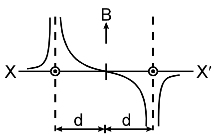
B-3. Determine the magnitude of magnetic field at the centre of the current carrying wire arrangement shown in the figure. The arrangement extends to infinity. (The wires joining the successive squares are along the line passing through the centre) (Assume negligible gap while wire is turned into squares)

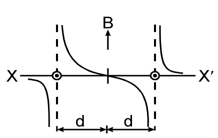


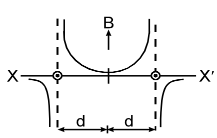
(A) $\frac{\mu_0 i}{\sqrt{2}\pi a}$ (B) 0 (C) $\frac{2\sqrt{2}\mu_0 i}{\pi a}\ell n2$ (D) none of these

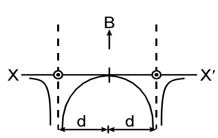
B-4. Two parallel, long wires carry currents i_1 and i_2 with $i_1 > i_2$. When the current are in the same direction, the magnetic field at a point midway between the wire is 20μT. If the direction of i_1 is reversed, the field becomes 30μT. The ratio i_1/i_2 is
(A) 4 (B) 3 (C) 5 (D) 1

B-5. Two long parallel wires are at a distance 2d apart. They carry steady equal currents flowing out of the plane of the paper, as shown. The variation of the magnetic field B along the XX' is given by

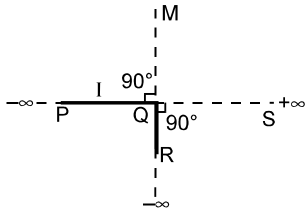
(A) 

(B) 

(C) 

(D) 

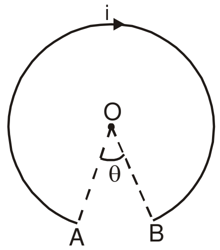
B-6. An infinitely long conductor PQR is bent to form a right angle as shown. A current I flows through PQR. The magnetic field due to this current at the point M is H_1. Now, another infinitely long straight conductor QS is connected at Q so that the current in PQ remaining unchanged. The magnetic field at M is now H_2. The ratio H_1/H_2 is given by



(A) 1/2 (B) 1 (C) 2/3 (D) 2

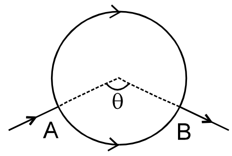
Section (C) : Magnetic field due to a circular loop, a straight wire and circular arc, cylinder, large sheet, solenoid, toroid and ampere's law

C-1. A current carrying wire AB of the length $2\pi R$ is turned along a circle, as shown in figure. The magnetic field at the centre O.



(A) $\frac{\mu_0 i}{2R}\left(\frac{2\pi-\theta}{2\pi}\right)^2$ (B) $\frac{\mu_0 i}{2R}\left(\frac{2\pi-\theta}{2\pi}\right)$ (C) $\frac{\mu_0 i}{2R}\ (2\pi-\theta)$ (D) $\frac{\mu_0 i}{2R}\ (2\pi+\theta)^2$

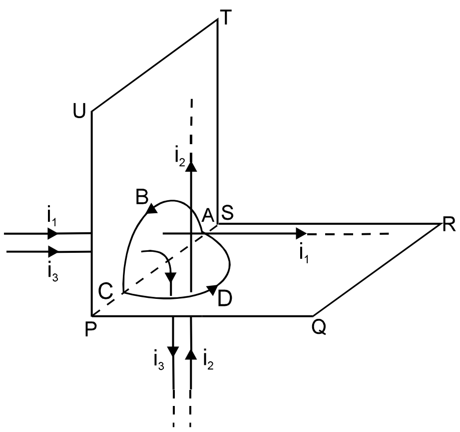
C-2. A battery is connected between two points A and B the circumference of a uniform conducting ring of radius r and resistance R. One of the arcs AB of the ring subtends an angle θ at the centre. The value of the magnetic induction at the centre due to the current in the ring is :



(A) zero, only if $\theta = 180º$ (B) zero for all values of θ
(C) proportional to 2 (180º - θ) (D) inversely proportional to r

C-3. A wire is wound on a long rod of material of relative permeability $\mu_r = 4000$ to make a solenoid. If the current through the wire is 5 A and number of turns per unit length is 1000 per metre, then the magnetic field inside the solenoid is :
(A) 25.12 mT (B) 12.56 m T (C) 12.56 T (D) 25.12 T

C-4. Figure shows an amperian path ABCDA. Part ABC is in vertical plane PSTU while part CDA is in horizontal plane PQRS. Direction of circumlation along the path is shown by an arrow near point B and at D. $\oint \vec{B}.d\vec{\ell}$ for this path according to Ampere's law will be :



(A) $(i_1 - i_2 + i_3)\ \mu_0$ (B) $(-i_1 + i_2)\ \mu_0$ (C) $i_3\ \mu_0$ (D) $(i_1 + i_2)\ \mu_0$

C-5. A circular loop of radius r carries a current i. How should a long, straight wire carrying a current 3i be placed in the plane of the circle so that the magnetic field at the centre of the loop becomes zero?

(A) At distance $\frac{3r}{\pi}$ from the centre of loop (B) At distance $\frac{2r}{\pi}$ from the centre of loop

(C) At distance $\frac{2r}{3\pi}$ from the centre of loop (D) Not possible

C-6. A long solenoid is fabricated by closely winding a wire of diameter 0.5 mm over a cylindrical nonmagnetic frame so that the successive turns nearly touch each other. What would be the magnetic field B at the centre of the solenoid if it carries a current of 2.5 A ?

(A) $\pi \times 10^{-3}$ T (B) $2\pi \times 10^{-3}$ T (C) $4\pi \times 10^{-3}$ T (D) None of these

C-7. A uniform current I flows along the length of an infinitely long, straight, thin walled pipe. Then **[JEE - 1993]**

(A) the magnetic field at all points inside the pipe is the same, but not zero

(B) the magnetic field at any point inside the pipe is zero

(C) the magnetic field is zero only on the axis of the pipe

(D) the magnetic field is different at different points inside the pipe.

C-8. A circular loop is kept in that vertical plane which contains the north-south direction. It carries a current that is towards south at the topmost point. Let A be a point on axis of the circle to the east of it and B a point on this axis to the west of it. The magnetic field due to the loop

(A) is towards east at A and towards west at B (B) is towards west at A and towards east at B

(C) is towards east at both A and B (D) is towards west at both A and B

C-9. A long, thick straight conductor of radius R carries current I uniformly distributed in its cross section area. The ratio of energy density of the magnetic field at distance R/2 from surface inside the conductor and outside the conductor is

(A) 1 : 16 (B) 1 : 1 (C) 1 : 4 (D) 9/16

Section (D) : Magnetic force on a charge (Normal incidence)

D-1. Which of the following particles will experience minimum magnetic force (magnitude) when projected with the same velocity perpendicular to a magnetic field?

(A) Be +++ (B) proton (C) α -particle (D) Li^{++}

D-2. Electric current i enters and leaves a square loop made of homogeneous wire of uniform cross-section through diagonally opposite corners. A charge particle q moving along the axis of the square loop. Passes through centre at speed υ. The magnetic force acting on the particle when it passes through the centre has a magnitude

(A) $qv\frac{\mu_0 i}{2a}$ (B) $qv\frac{\mu_0 i}{2\pi a}$ (C) $qv\frac{\mu_0 i}{a}$ (D) zero

D-3. Two particles X and Y having equal charges, after being accelerated through the same potential difference, enter a region of uniform magnetic field and describe circular paths of radii R_1 and R_2 respectively. The ratio of the masses of X to that of Y **[JEE -1988]**

(A) $\left(\frac{R_1}{R_2}\right)^{1/2}$ (B) $\frac{R_2}{R_1}$ (C) $\left(\frac{R_1}{R_2}\right)^2$ (D) $\frac{R_1}{R_2}$

D-4. A negative charged particle falling freely under gravity enters a region having uniform horizontal magnetic field pointing towards north. The particle will be deflected towards **[REE -1991]**

(A) East (B) West (C) North (D) South

D-5. A proton is moved along a magnetic field line. The magnetic force on the particle is
(A) along its velocity (B) opposite to its velocity
(C) perpendicular to its velocity (D) zero.

D-6. A proton beam is going from west to east and an electron beam is going from east to west. Neglecting the earth's magnetic field, the electron beam will be deflected
(A) towards the proton beam (B) away from the proton beam
(C) away from the electron beam (D) None of these

Section (E) : Magnetic force on a charge (oblique incidence)

E- 1. A proton of mass m and charge q enters a magnetic field B with a velocity v at an angle θ with the direction of B. The radius of curvature of the resulting path is

(A) $\frac{mv}{qB}$ (B) $\frac{mv\sin\theta}{qB}$ (C) $\frac{mv}{qB\sin\theta}$ (D) $\frac{mv\cos\theta}{qB}$

Section (F) : Electric and magnetic force on a charge

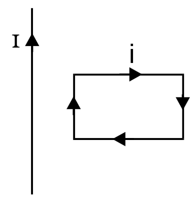
F-1. A positively charged particle moves in a region having a uniform magnetic field and uniform electric field in same direction. At some instant, the velocity of the particle is perpendicular to the field direction. The path of the particle will be
(A) a straight line (B) a circle
(C) a helix with uniform pitch (D) a helix with increasing pitch.

F-2. If a charged particle projected in a gravity-free room it does not deflect
(A) electric field and magnetic field must be zero
(B) both electric field and magnetic field may be present
(C) electric field will be zero and magnetic field may be zero
(D) electric field may be zero and magnetic field must be zero

Section (G) : Magnetic force on a current carrying wire

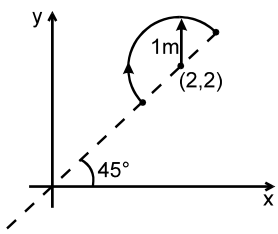
G-1. A conducting circular loop of radius r carries a constant current i. It is placed in a uniform magnetic field **B** such that **B** is perpendicular to the plane of the loop. The magnetic force acting on the loop is **[JEE - 1983]**
(A) i r B (B) 2π r i B (C) zero (D) π r i B

G-2. A rectangular loop carrying a current i is situated near a long straight wire such that the wire is parallel to one of the sides of the loop and the plane of the loop is same of the left wire. If a steady current I is established in the wire as shown in the (figure) the loop will



(A) Rotate about an axis parallel to the wire
(B) Move away from the wire
(C) Move towards the wire
(D) Remain stationary

G-3. A uniform magnetic field $\vec{B} = (3\hat{i}+4\hat{j}+\hat{k})$ exists in region of space. A semicircular wire of radius 1 m carrying current 1 A having its centre at (2, 2, 0) is placed in x-y plane as shown in fig. The force on semicircular wire will be

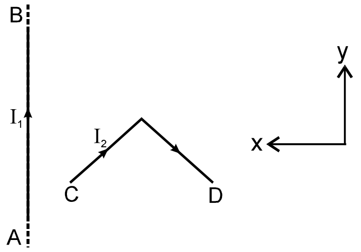


(A) $\sqrt{2}(\hat{i}+\hat{j}+\hat{k})$ (B) $\sqrt{2}(\hat{i}-\hat{j}+\hat{k})$
(C) $\sqrt{2}(\hat{i}+\hat{j}-\hat{k})$ (D) $\sqrt{2}(-\hat{i}+\hat{j}+\hat{k})$

G-4. Select the correct alternative(s): **[JEE-1986]**
Two thin long parallel wires separated by a distance 'b' are carrying a current 'i' ampere each. The magnitude of the force per unit length exerted by one wire on the other is

(A) $\frac{\mu_0 i^2}{b^2}$ (B) $\frac{\mu_0 i^2}{2\pi b}$ (C) $\frac{\mu_0 i}{2\pi b}$ (D) $\frac{\mu_0 i}{2\pi b^2}$

G-5. In the figure shown a current I_1 is established in the long straight wire AB. Another wire CD carrying current I_2 is placed in the plane of the paper. The line joining the ends of this wire is perpendicular to the wire AB. The resultant force on the wire CD is :



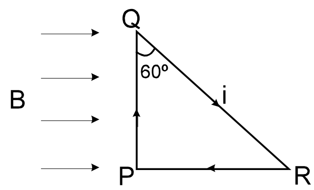
(A) zero (B) towards negative x-axis
(C) towards positive y-axis (D) none of these

G-6. A steady current 'I' flows in a small square loop of wire of side L in a horizontal plane. The loop is now folded about its middle such that half of it lies in a vertical plane. Let $\vec{\mu}_1$ and $\vec{\mu}_2$ respectively denote the magnetic moments of the current loop before and after folding. Then : **[JEE - 1993]**
(A) $\vec{\mu}_2 = 0$ (B) $\vec{\mu}_1$ and $\vec{\mu}_2$ are in the same direction
(C) $\frac{|\vec{\mu}_1|}{|\vec{\mu}_2|} = \sqrt{2}$ (D) $\frac{|\vec{\mu}_1|}{|\vec{\mu}_2|} = \frac{1}{\sqrt{2}}$

G-7. A current-carrying, straight wire is kept along the axis of a square loop carrying a current. The straight wire
(A) will exert an inward force on the square loop
(B) will exert an outward force on the squareloop
(C) will not exert any force on the square loop
(D) will exert a force on the square loop parallel to itself.

G-8. Two parallel wires carry currents of 10 A and 40 A in opposite directions. Another wire carrying a current antiparallel to 10 A is placed midway between the two wires. The magnetic force on it will be
(A) towards 20 A (B) towards 40 A
(C) zero (D) perpendicular to the plane of the currents

G-9. For the circuit shown in figure, the direction and magnitude of the force on PQR is :



(A) No resultant force act on the loop (B) ILB out of the page
(C) $\frac{1}{2}$ ILB into the page (D) ILB into the page

Section (H) : Magnetic force and torque on a current carrying loop and magnetic dipole moment

H-1. A bar magnet has a magnetic moment 2.5 JT^{-1} and is placed in a magnetic field of 0.2 T. Work done in turning the magnet from parallel to antiparallel position relative to the field direction is :
(A) 0.5 J (B) 1 J (C) 2.0 J (D) Zero

H-2. A circular loop of area 1 cm^2, carrying a current of 10 A, is placed in a magnetic field of 0.1 T perpendicular to the plane of the loop. The torque on the loop due to the magnetic field is
(A) zero (B) 10^{-4} N-m (C) 10^{-2} N-m (D) 1 N-m

H-3. A rod of length ℓ having uniformly distributed charge Q is rotated about one end with constant frequency ' f '. Its magnetic moment
(A) $\pi fQ\ell^2$ (B) $\frac{\pi fQ\ell^2}{3}$ (C) $\frac{2\pi fQ\ell^2}{3}$ (D) $2\pi fQ\ell^2$

Section (I) : Magnetic field due to earth

I-1. A power line lies along the east-west direction and carries a current of 10 ampere. The force per metre due to the earth's magnetic field of 10^{-4} T is

(A) 10^{-5} N (B) 10^{-4} N (C) 10^{-3} N (D) 10^{-2} N

I-2. A circular coil of radius 20 cm and 20 turns of wire is mounted vertically with its plane in magnetic meridian. A small magnetic needle (free to rotate about vertical axis) is placed at the center of the coil. It is deflected through 45° when a current is passed through the coil and in equilibrium (Horizontal component of earth's field is 0.34×10^{-4} T). The current in coil is :

(A) $\frac{17}{10\pi}$ A (B) 6A (C) 6×10^{-3} A (D) $\frac{3}{50}$ A

Section (J) : Magnetic material

J-1 The magnetic materials having negative magnetic susceptibility are:

(A) Non magnetic (B) Para magnetic (C) Diamagnetic (D) Ferromagnetic

J-2 When a small magnetising field H is applied to a magnetic material, the intensity of magnetisation (I) is proportional to :

(A) H^{-2} (B) $H^{1/2}$ (C) H (D) H^2

J-3✍ How does the magnetic susceptibility χ of a paramagnetic material change with absolute temperatue T ?

(A) $\chi \propto T$ (B) $\chi \propto T^{-1}$ (C) χ = constant (D) $\chi \propto e^T$

J-4 Consider the following statements for a paramagnetic substance kept in a magnetic field :

(a) If the magnetic field increases, the magnetisation increases.

(b) If temperature rises, the magnetisation increases.

(A) Both (a) and (b) are true (B) (a) is true but (b) is false

(C) (b) is true but (a) is false (D) Both (a) and (b) are false

J-5.✍ Which of the following relations is not correct ?

(A) $B = \mu_0 (H + I)$ (B) $B = \mu_0 H (1 + \chi_m)$ (C) $\mu_0 = \mu (1 + \chi_m)$ (D) $\mu_r = 1 + \chi_m$

J-6 The hysteresis loop for the material of a permanent magnet is :

(A) short and wide (B) tall and narrow (C) tall and wide (D) short and narrow

J-7 Select the incorrect alternative (s) :

When a ferromagnetic material goes through a complete cycle of magnetisation, the magnetic susceptibility :

(A) has a fixed value (B) may be zero (C) may be infinite (D) may be nagative

J-8 The material for making permanent magnets should have :

(A) high retentivity, high coercivity (B) high retentivity, low coercivity

(C) low retentivity, high coercivity (D) low retentivity, low coercivity

J-9 (a) Soft iron is a conductor of electricity. (b) It is a magnetic material.

(c) It is an alloy of iron.

(d) It is used for making permanent magents. State whether :

(A) a and c are true (B) a and b are true (C) c and d are true (D) b and d are true

J-10 Soft iron is used in many electrical machines for :

(A) low hysteresis loss and low permeability (B) low hysteresis loss and high permeability

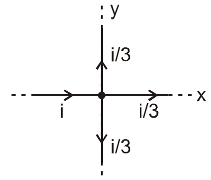
(C) high hysteresis loss and low permeability (D) high hysteresis loss and high permeability

PART - III : MATCH THE COLUMN

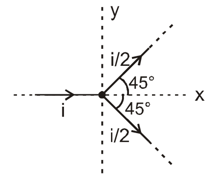
1. Column-II gives four situations in which three (in q, r, s, t) and four (in p) semi infinite current carrying wires are placed in xy-plane as shown. The magnitude and direction of current is shown in each figure. Column-I gives statements regarding the x, y and z components of magnetic field at a point P whose coordinates are P (0, 0, d). Match the statements in column-I with the corresponding figures in column-II.

Column I | **Column II**

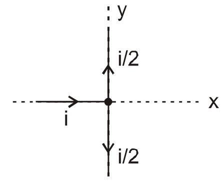
(A) The x component of magnetic field at point P is zero in

(p) 

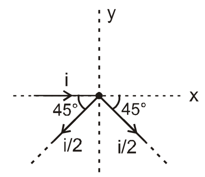
(B) The z component of magnetic field at point P is zero in

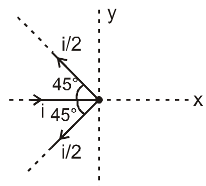
(q) 

(C) The magnitude of magnetic field at point P is $\frac{\mu_0 i}{4\pi d}$ in

(r) 

(D) The magnitude of magnetic field at point P is less than $\frac{\mu_0 i}{2\pi d}$ in

(s) 

(t) 

2. There are four situations given in column I involving a magnetic dipole of dipole moment $\vec{\mu}$ placed in uniform external magnetic field $\vec{B}$. Column II gives corresponding results. Match the situtations in column I with the corresponding results in column II

Column - I	**Column - II**
(A) Magnetic dipole moment $\vec{\mu}$ is parallel to uniform external magnetic field $\vec{B}$ (angle between both vectors is zero)	(p) force on dipole is zero
(B) Magnetic dipole moment $\vec{\mu}$, is perpendicular to uniform external magnetic field $\vec{B}$	(q) torque on dipole is zero
(C) Angle between magnetic dipole moment $\vec{\mu}$ and uniform external magnetic field $\vec{B}$ is acute	(r) magnitude of torque is (μB)
(D) Angle between magnetic dipole moment $\vec{\mu}$ and uniform external magnetic field $\vec{B}$ is 180°.	(s) potential energy of dipole due to external magnetic field is (μB)
	(t) Magnetic dipole is in stable equilibrium

3. A particle enters a space where exists uniform magnetic field $\vec{B} = B_x\vec{i} + B_y\vec{j} + B_z\vec{k}$ & uniform electric field $\vec{E} = E_x\vec{i} + E_y\vec{j} + E_z\vec{k}$ with initial velocity $\vec{u} = u_x\vec{i} + u_y\vec{j} + u_z\vec{k}$. Depending on the values of various components the particle selects a path. Match the entries of column A with the entries of column B. The components other than specified in column A in each entry are non-zero. Neglect gravity.

Column - I	**Column - II**
(A) $B_y = B_z = E_x = E_z = 0$ $u = 0$	(p) circle
	(q) helix with uniform pitch and constant radius
(B) $E = 0$; $u_xB_x + u_yB_y \neq -u_zB_z$	(r) cycloid
(C) $\vec{u} \times \vec{B} = 0, \vec{u} \times \vec{E} = 0$	(s) helix with uniform pitch and variable radius
(D) $\vec{u} \perp \vec{B}, \vec{B} \parallel \vec{E}$	(t) helix with variable pitch and constant radius
	(u) straight line

Exercise-2

Marked Questions can be used as Revision Questions.

PART - I : ONLY ONE OPTION CORRECT TYPE

1. An α particle is moving along a circle of radius R with a constant angular velocity ω. Point A lies in the same plane at a distance 2R from the centre. Point A records magnetic field produced by α particle. If the minimum time interval between two successive times at which A records zero magnetic field is 't', the angular speed ω, in terms of t is -

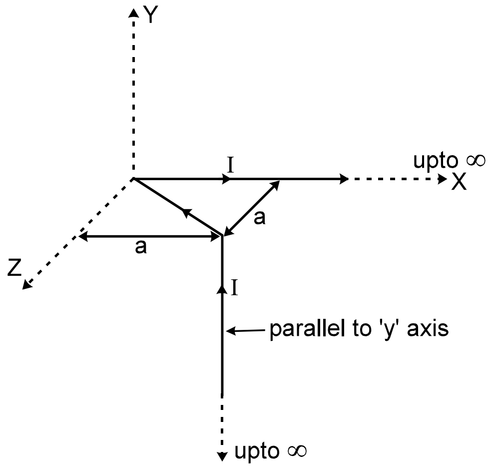
(A) $\frac{2\pi}{t}$ (B) $\frac{2\pi}{3t}$ (C) $\frac{\pi}{3t}$ (D) $\frac{\pi}{t}$

2. A particle is moving with velocity $\vec{v} = \hat{i} + 3\hat{j}$ and it produces an electric field at a point given by $\vec{E} = 2\hat{k}$. It will produce magnetic field at that point equal to (all quantities are in S.I. units) ($c = \frac{1}{\sqrt{\mu_0\varepsilon_0}}$)

(A) $\frac{6\hat{i} - 2\hat{j}}{c^2}$ (B) $\frac{6\hat{i} + 2\hat{j}}{c^2}$

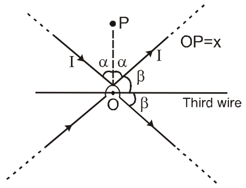
(C) zero (D) can not be determined from the given data

3. The magnetic field at the origin due to the current flowing in the wire is



(A) $-\frac{\mu_0 I}{8\pi a}(\hat{i} + \hat{k})$ (B) $\frac{\mu_0 I}{2\pi a}(\hat{i} + \hat{k})$ (C) $\frac{\mu_0 I}{8\pi a}(-\hat{i} + \hat{k})$ (D) $\frac{\mu_0 I}{4\pi a\sqrt{2}}(\hat{i} - \hat{k})$

4. Three infinite current carrying conductors are placed as shown in figure. Two wires carry same current while current in third wire is unknown. The three wires do not intersect with each other and all of them are in the plane of paper. Which of the following is correct about a point 'P' which is also in the same plane :



(A) Magnetic field intensity at P is zero for all values of x.

(B) If the current in the third wire is $\frac{2I}{\sin\alpha}$ (left to right) then magnetic field will be zero at P for all values of x.

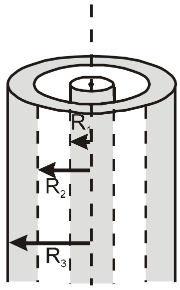
(C) If the current in the third wire is $\frac{2I}{\sin\alpha}$ (right to left) then magnetic field will be zero at P for all values of x.

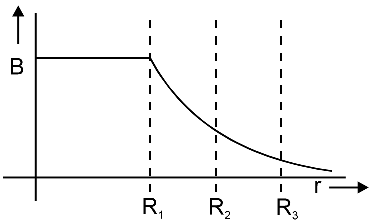
(D) None of these

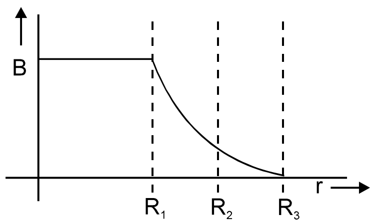
5. A long straight wire along the z-axis carries a current I in the negative Z direction. The magnetic vector field $\vec{B}$ at a point having coordinates (x, y) in the z = 0 plane is

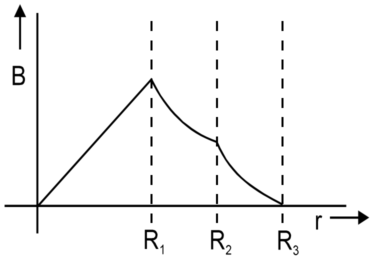
(A) $\frac{\mu_0 I}{2\pi}\frac{(y\hat{i} - x\hat{j})}{(x^2 + y^2)}$ (B) $\frac{\mu_0 I}{2\pi}\frac{(x\hat{i} + y\hat{j})}{(x^2 + y^2)}$ (C) $\frac{\mu_0 I}{2\pi}\frac{(x\hat{j} - y\hat{i})}{(x^2 + y^2)}$ (D) $\frac{\mu_0 I}{2\pi}\frac{(x\hat{i} - y\hat{j})}{(x^2 + y^2)}$

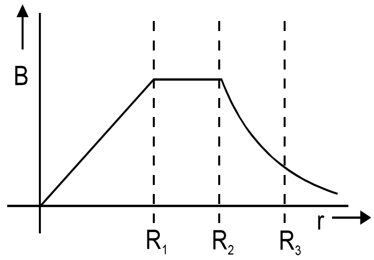
6.✍ A coaxial cable is made up of two conductors. The inner conductor is solid and is of radius R_1 & the outer conductor is hollow of inner radius R_2 and outer radius R_3. The space between the conductors is filled with air. The inner and outer conductors are carrying currents of equal magnitudes and in opposite directions. Then the variation of magnetic field with distance from the axis is best plotted as:



(A)


(B)


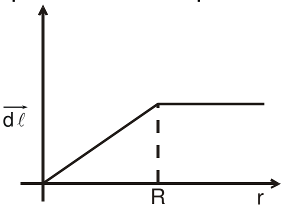
(C)


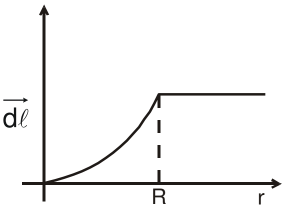
(D)


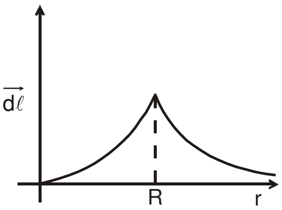
7. Axis of a solid cylinder of infinite length and radius R lies along y-axis it carries a uniformly distributed current ' i ' along +y direction. Magnetic field at a point $\left(\frac{R}{2}, y, \frac{R}{2}\right)$ is :

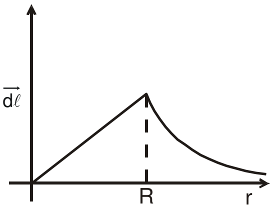
(A) $\frac{\mu_0 i}{4\pi R}(\hat{i} - \hat{k})$ (B) $\frac{\mu_0 i}{2\pi R}(\hat{j} - \hat{k})$ (C) $\frac{\mu_0 i}{4\pi R}\hat{j}$ (D) $\frac{\mu_0 i}{4\pi R}(\hat{i} + \hat{k})$

8. A cylindrical wire of radius R is carrying current i uniformly distributed over its cross-section. If a circular loop of radius ' r ' is taken as amperian loop, then the variation value of $\oint \vec{B}.d\vec{\ell}$ over this loop with radius ' r ' of loop will be best represented by

(A) $\oint \vec{B}\cdot\overrightarrow{d\ell}$ 

(B) $\oint \vec{B}\cdot\overrightarrow{d\ell}$ 

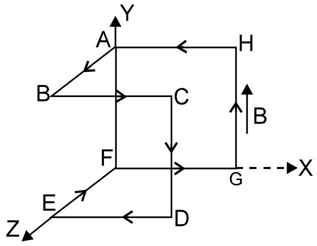
(C) $\oint \vec{B}\cdot\overrightarrow{d\ell}$ 

(D) $\oint \vec{B}\cdot\overrightarrow{d\ell}$ 

9. A constant direct current of uniform density $\vec{j}$ is flowing in an infinitely long cylindrical conductor. The conductor contains an infinitely long cylindrical cavity whose axis is parallel to that of the conductor and is at a distance $\vec{\ell}$ from it. What will be the magnetic induction $\vec{B}$ at a point inside the cavity at a distance $\vec{r}$ from the centre of cavity?

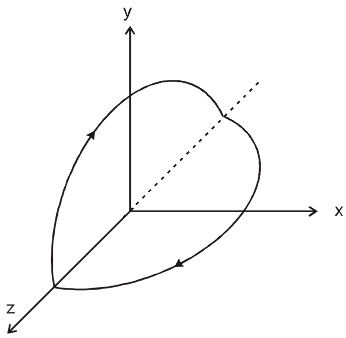
(A) $\frac{\mu_0(\vec{j}\times\vec{r})}{2}$ (B) $\frac{\mu_0(\vec{j}\times\vec{\ell})}{2}$ (C) $\frac{\mu_0(\vec{j}\times\vec{\ell})+\mu_0(\vec{j}\times\vec{r})}{2}$ (D) $\frac{\mu_0(\vec{j}\times\vec{\ell})-\mu_0(\vec{j}\times\vec{r})}{2}$

10. The given fig. shows a coil bent with AB = BC = CD = DE = EF = FG = GH = HA = 1 m and carrying current 1 A. There exists in space a vertical uniform magnetic field of 2 T in the y-direction. The torque on the loop is.



(A) $2\hat{k}$ (B) $-2\hat{k}$ (C) $2\hat{j}$ (D) $4\hat{k}$

11. A non-planar circular loop consists of two semi-circles one of which lies in yz-plane & the other is in xz-plane as shown. The magnetic force experienced by positive charge of value Q moving with velocity v along x direction when it is at the origin is:

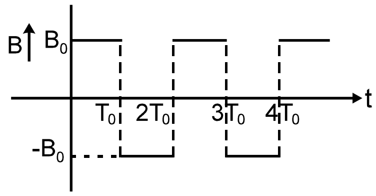


(A) $\frac{Qv\mu_0 I}{4R}$ (B) $\frac{Qv\mu_0 I}{2R}$ (C) $\frac{Qv\mu_0 I}{2\sqrt{2}R}$ (D) 0

12. A particle of mass 'm' and charge 'q' moves with a constant velocity 'v' along the positive x direction. It enters a region containing a uniform magnetic field $\vec{B}$ directed along the negative Z direction, extending from x = a to x = b. The minimum value of v required so that the particle can just enter the region x > b is

(A) $\frac{qbB}{m}$ (B) $\frac{q(b-a)B}{m}$ (C) $\frac{qaB}{m}$ (D) $\frac{q(b+a)B}{2m}$

13. In a region magnetic field along x axis changes with time according to the given graph.



If time period, pitch and radius of projected path are T_0, P_0 and R_0 respectively then which of the following is incorrect if the positively charged particle is projected at an angle θ_0 with the positive x-axis in x-y plane from origin:

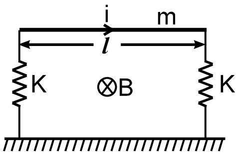
(A) At t = $\frac{T_0}{2}$, co-ordinates of charge are $\left(\frac{P_0}{2},\ 0,\ -2R_0\right)$.

(B) At t = $\frac{3T_0}{2}$, co-ordinates of charge are $\left(\frac{3P_0}{2},\ 0,\ 2R_0\right)$.

(C) The difference of maxismum and minimum z coordinate of the particle is $4R_0$

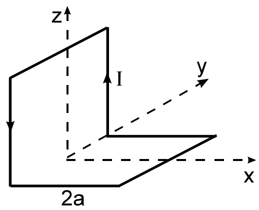
(D) The difference of maximum and minimum z coordinate of the particle is R_0

14. A horizontal metallic rod of mass 'm' and length 'ℓ' is supported by two vertical identical springs of spring constant ' K ' each and natural length ℓ_0. A current ' i ' is flowing in the rod in the direction shown. If the rod is in equilibrium then the length of each spring in this state is



(A) $\ell_0 + \frac{i\ell B - mg}{K}$ (B) $\ell_0 + \frac{i\ell B - mg}{2K}$ (C) $\ell_0 + \frac{mg - i\ell B}{2K}$ (D) $\ell_0 + \frac{mg - i\ell B}{K}$

15. A non-planer loop of conducting wire carrying a current I is placed as shown in the figure. Each of the straight sections of the loop is of length 2a. The magnetic field due to this loop at the point P(a, 0, a) points in the direction.



(A) $\frac{1}{\sqrt{2}}(-\hat{j}+\hat{k})$ (B) $\frac{1}{\sqrt{3}}(-\hat{j}+\hat{k}+\hat{i})$ (C) $\frac{1}{\sqrt{3}}(\hat{i}+\hat{j}+\hat{k})$ (D) $\frac{1}{\sqrt{2}}(\hat{i}+\hat{k})$

16. A circular coil of radius R and a current I, which can rotate about a fixed axis passing through its diameter is initially placed such that its plane lies along magnetic field B. Kinetic energy of loop when it rotates through an angle 90° is : (Assume that I remains constant)

(A) $\pi R^2 BI$ (B) $\frac{\pi R^2 BI}{2}$ (C) $2\pi R^2 BI$ (D) $\frac{3}{2}\pi R^2 I$

17._ A coil 2.0 cm in diameter has 300 turns. If the coil carries a current of 10 mA and lies in a magnetic field 5×10^{-2} T, the maximum torque experienced by the coil is : **[Olympiad (State-1) 2017]**

(A) 4.7×10^{-2} N–m (B) 4.7×10^{-4} N–m (C) 4.7×10^{-5} N–m (D) 4.7×10^{-8} N–m

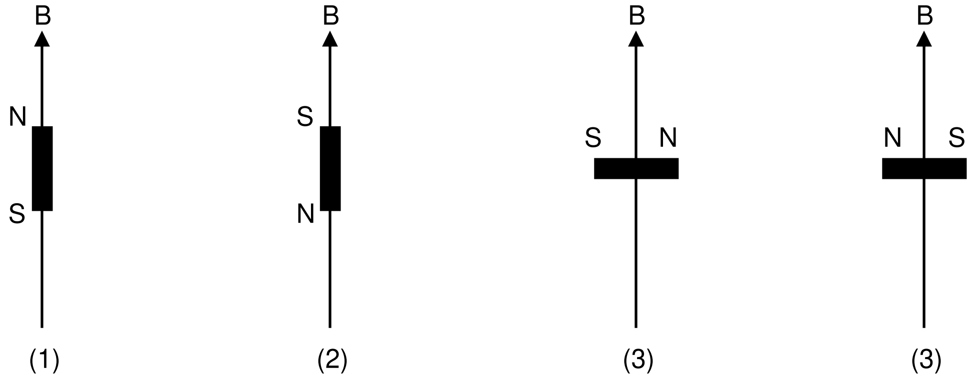
18._ An infinitely long straight non-magnetic conducting wire of radius a carries a dc current I. The magnetic field B, at a distance r (r < a) from axis of the wire is : **[Olympiad (State-1) 2017]**

(A) $\frac{\mu_0 I}{2\pi a}$ (B) $\frac{\mu_0 I r}{2\pi a^2}$ (C) $\frac{2\mu_0 I r}{\pi a^2}$ (D) $\frac{\mu_0 I r^2}{2\pi a^3}$

19._ The earth's magnetic field at a certain point is 7.0×10^{-5} T. This field is to be balanced by a magnetic field at the centre of a circular current carrying coil of radius 5.0 cm by suitably orienting it. If the coil has 100 turns then the required current is about **[Olympiad (State-1) 2017]**

(A) 28 mA (B) 56 mA (C) 100 mA (D) 560 mA

20._ Consider different orientations of a bar magnet lying in a uniform magnetic field as shown below. The potential energy is maximum in orientation **[Olympiad (State-1) 2017]**



(A) 1 (B) 2 (C) 3 (D) 4

PART - II : SINGLE AND DOUBLE VALUE INTEGER TYPE

1. A charge of 1 coulomb is placed at one end of a non-conducting rod of length 0.6m. The rod is rotated in a vertical plane about a horizontal axis passing through the other end of the rod with an angular velocity $10^4\pi$ rad/sec. Find the magnetic field at a point on the axis of rotation at a distance of 0.8m from the centre of the path is $N\pi \times 10^{-4}$T, then find value of N **[REE- 1990]**

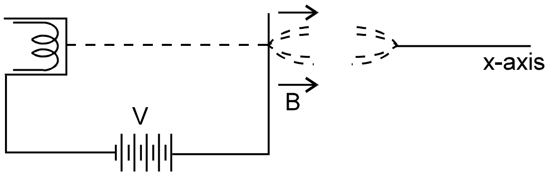
2. Magnetic field at the center of a rectangle of length 'ℓ' & width d, carrying a current i, is given by $\frac{N\mu_0 i}{\pi d}$, then find value of N (here $\ell >> d$)

3. A regular polygon of n sides is formed by bending a wire of length L which carries a current i. If the magnetic field B at the centre of the polygon $\frac{\mu_0 i n^2 \sin\frac{\pi}{n}\tan\frac{\pi}{n}}{N\pi L}$, then find the value of N

4. A solid cylindrical conductor of radius R carries a current along its length. The current density J, however, is not uniform over the cross section of the conductor but is a function of the radius according to J = br, where b is a constant. The magnetic field B. at $r_1 < R$ is $\frac{\mu_0 b r_1^2}{N}$. Then find value of N

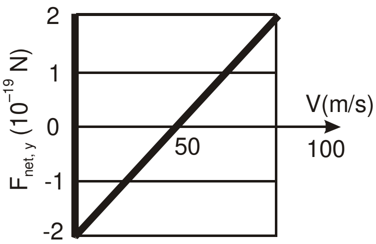
5. A capacitor of capacitance 50 μF is connected to a battery of 20 volts for a long time and then disconnected from it. It is now connected across a long solenoid having 8000 turns per metre. It is found that the potential difference across the capacitor drops to 90% of its maximum value in 2.0 seconds. The average magnetic field produced at the centre of the solenoid during this period is $N\pi \times 10^{-8}$ T. Then find value of N.

6. Electrons emitted with negligible speed from an electron gun are accelerated through a potential difference V along the X-axis. These electrons emerges from a narrow hole into a uniform magnetic field B directed along this axis. However, some of the electrons emerging from the hole make slightly divergent angles as shown in figure. These paraxial electrons meet for second time on the X-axis at a distance $\sqrt{\frac{N\pi^2 mV}{eB^2}}$. Then find value of N. (Neglect interaction between electrons)



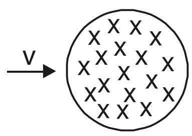
7. A particle of mass 1×10^{-26} kg and charge + 1.6×10^{-19} C traveling with a velocity of 1.28×10^{6} m/s in the +x direction enters a region in which a uniform electric field E and a uniform magnetic field of induction B are present such that $E_x = E_y = 0$, $E_z = -102.4$ kV/m and $B_x = B_z = 0$, $B_y = 8 \times 10^{-2}$ Wb m^{-2}. The particle enters this region at the origin at time t = 0. If X-coordinate of the particle is $\frac{64}{N}$ at $t = 5 \times 10^{-6}$ s. Then find value of N. **[JEE - 1982]**

8. At time t_1, an electron is sent along the positive direction of x-axis, through both an electric field $\vec{E}$ and a magnetic field $\vec{B}$, with $\vec{E}$ directed parallel to the y-axis. Graph gives the y-component $F_{net,\,y}$ of the net force on the electron due to the two fields, as a function of the electron's speed V at time t_1. Assuming $B_x = 0$, magnitude of electric field $\vec{E}$ in mN/C is 50X, then find X. (Use $e = 1.6 \times 10^{-19}$ C)



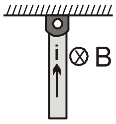
9. When a proton is released from rest in a gravity free room, it starts with an initial acceleration a_0 towards east. When it is projected towards south with a speed v_0, it moves with an initial acceleration $3a_0$ towards east. The minimum possible magnetic field in the room is $\frac{Nma_0}{ev_0}$. Find the value of N.

10. A uniform magnetic field exists in a circular region of radius r. The magnitude of magnetic field is B and points inward. An electron flies into the region radially as shown in the figure. After a certain time, the electron deflected by the magnetic field leaves the region. The time interval during which the electron moves in the region is $\frac{Nm\tan^{-1}\frac{eBr}{mV}}{eB}$. Then find value of N. (m is mass of electron and e is charge of electron)

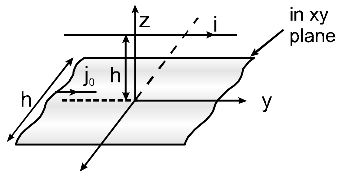


11. A long horizontal wire AB which is free to move in a vertical plane and carries a steady current of 20 A, is in equilibrium at a height of 0.01 m over another parallel infinitely long wire CD, which is fixed in a horizontal plane and carries a steady current of 30 A. Show that when AB is slightly depressed, it executes simple harmonic motion. The period of oscillations is $\frac{1}{N}$ seconds, then find the value of N. (Use $\pi^2 = 10$ & $g = 10 m/s^2$) **[JEE - 1994]**

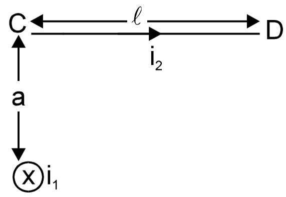
12. A uniform rod of length L and mass M is hinged at its upper point and is at rest at that moment in the vertical plane. A current i flows in it. A uniform magnetic field of strength B exists perpendicular to the rod and in horizontal direction (as shown). The force exerted by the hinge on the rod just after release in horizontal direction is $\frac{iLB}{N}$. Then find the value of N.



13. A conductor placed along the line z = h carrying a current i is placed parallel to a very long current strip of width ' h ' placed in the xy plane and carrying a current per unit width J_0 as shown in the figure. The force per unit length on the conductor is $\frac{\mu_0 i J_0}{\pi} \tan^{-1}\left(\frac{1}{N}\right)(-\hat{k})$. The sheet is placed symmetrically with respect to y-axis, Then find value of N.



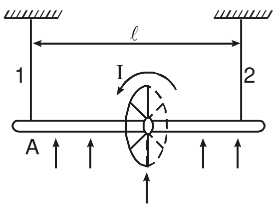
14. An infinitely long straight wire carries a current i_1, as shown in the figure. The total force on another wire CD of length ℓ which is placed so that c is at a distance a from the current carrying wire is $\frac{\mu_0 i_1 i_2}{N\pi} \ln\left(\frac{a^2 + \ell^2}{a^2}\right)$. Then find value of N



15. Consider a nonconducting ring of radius r and mass m which has a total charge q distributed uniformly on it. The ring is rotated about its axis with an angular speed ω. (a) The equivalent electric current in the ring is $\frac{q\omega}{N\pi}$. Then find value of N(b) The magnetic moment of the ring is $\frac{4q\omega r^2}{N}$. Then find value of N.

16. Consider a nonconducting disc of radius r and mass m which has a charge q distributed uniformly over it. The disk is rotated about its axis with an angular speed ω. Magnetic moment of the disc is $\frac{1}{N} q\omega r^2$. Then find value of N.

17. The circular current loop of radius b = 2cm shown in the figure is mounted rigidly on the axle, midway between the two supporting cords. Current in the loop is 100 amp. The length of axle is ℓ = 1m. Absence of an external magnetic field the tensions in the cords are equal to $T_0 = 10\pi$ N

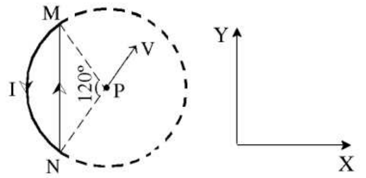


When the vertical magnetic field B = 100 T is present and the axle still in equilibrium, then the tensions (in newtons) in the cords are $T_1 = n\pi$; $T_2 = m\pi$. Find the magnitude of m – n

Note: Current in anticlockwise direction as seen from right side.

18. A wire loop of radius a carrying current I is placed in the X-Y plane as shown in the figure

(a) If a particle with charge +Q and mass m is placed at the centre p and given a velocity along NP (fig). Magnitude of its initial instantaneous acceleration acceleration is $\frac{QV}{m}\frac{\mu_0 I}{6a}\left(\frac{N\sqrt{3}}{\pi}-1\right)$, then find value of N

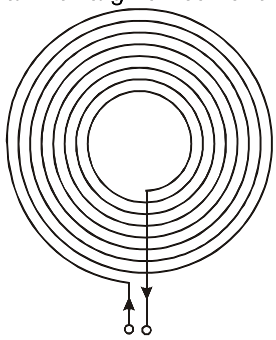


(b) If an external uniform magnetic induction field $\vec{B} = B\hat{i}$ is applied. The torque acting on the loop due to the field is $\vec{\tau} = BI\left(\frac{\pi}{3}-\frac{\sqrt{3}}{N}\right)a^2\ \hat{j}$.Then find value of N.

19. A thin insulated wire forms a plane spiral of N = 100 tight turns carrying a current I = 8 mA. The radii of inside and outside turns (figure) are equal to a = 50 mm and b = 100 mm. :

(a) The magnetic induction at the centre of the spiral is N μT; Then find value of N.

(b) The magnetic moment of the spiral with a given current is N mA.m^2. Then find value of N.

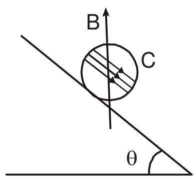


20. A square frame carrying a current I = 0.90A is located in the same plane as a long straight wire carrying a current I_0 = 5.0 A. The frame side has a length a = 8.0 cm. The axis of the frame passing through the midpoints of opposite sides is parallel to the wire and is separated from it by the distance which is η = 1.5 times greater than the side of the frame.

(a) Ampere force (magnetic force) acting on the frame is $\frac{9}{P}$ μN ; Then find value of P.

(b) The magntidute of mechanical work is (144ℓnP) × 10^{-9} J to be performed in order to turn the frame slowly through 180° about its axis, with the currents maintained constant. Then find value of P

21. Figure shows (only cross section) a wooden cylinder C with a mass m of 0.25 kg, a radius R and a length ℓ perpendicular to the plane of paper of 0.1 meter with N = 10 where N is number of turns of wire wrapped around it longitudinally, so that the plane of the wire loop contains the axis of the cylinder. The least current through the loop is $\frac{P}{2}$ A that will prevent the cylinder from moving down a plane whose surface is inclined at angle θ to the horizontal, in the presence of a vertical field of magnetic induction 0.5 weber/meter^2, if the plane of the windings is parallel to the inclined plane? (bottom most point of cylinder does not slip) then find value of P.

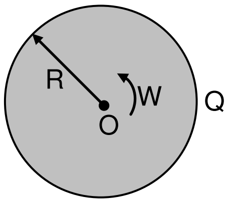


PART - III : ONE OR MORE THAN ONE OPTIONS CORRECT TYPE

1. A magnetic needle (small magnet) is kept in a nonuniform magnetic field. It **[JEE - 1982]**
(A) may experience a force and torque
(B) may experience a force but not a torque
(C) may experience a torque but not a force
(D) will experience neither a force nor a torque

2. The magnetic field at the origin due to a current element i $d\vec{\ell}$ placed at a position $\vec{r}$ is
(A) $\frac{\mu_0 i}{4\pi}\frac{d\vec{l}x\vec{r}}{r^3}$
(B) $-\frac{\mu_0 i}{4\pi}\frac{\vec{r}\times d\vec{\ell}}{r^3}$
(C) $\frac{\mu_0 i}{4\pi}\frac{\vec{r}\times d\vec{\ell}}{r^3}$
(D) $-\frac{\mu_0 i}{4\pi}\frac{d\vec{\ell}\times\vec{r}}{r^3}$

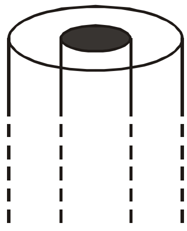
3. A non-conducting disc having uniform positive charge Q, is rotating about its axis with uniform angular velocity ω. The magnetic field at the center of the disc is.



(A) directed outward
(B) having magnitude $\frac{\mu_0 Q\omega}{4\pi R}$
(C) directed inwards
(D) having magnitude $\frac{\mu_0 Q\omega}{2\pi R}$

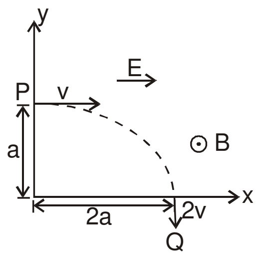
4. A hollow tube is carrying an electric current along its length distributed uniformly over its surface. The magnetic field
(A) increases linearly from the axis to the surface
(B) is constant inside the tube
(C) is zero at the axis
(D) is non-zero outside the tube at finite distance from surface

5. In a coaxial, straight cable, the central conductor and the outer conductor carry equal currents in opposite directions. The magnetic field is non-zero.



(A) outside the cable
(B) inside the inner conductor except axis of the conductor
(C) all the point inside the outer conductor and outside inner conductor
(D) only on axis of conductors.

6. A particle of charge +q and mass m moving under the influence of a uniform electric field E and a uniform magnetic field $B\hat{k}$ follows a trajectory from P and Q as shown in figure. The velocities at P and Q are $v\hat{i}$ and $-2v\hat{j}$. Which of the following statement(s) is/are correct?



(A) $E = \frac{3}{4}\left(\frac{mv^2}{qa}\right)$
(B) Rate of work done by the electric field at P is $\frac{3}{4}\left(\frac{mv^3}{a}\right)$
(C) Rate of work done by the electric field at P is zero.
(D) Rate of work done by both fields at Q is zero.

7. Let $\vec{E}$ and $\vec{B}$ denote the electric and magnetic fields in a certain region of space. A proton moving with a velocity along a straight line enters the region and is found to pass through it undeflected. Indicate which of the following statements are possible for the observation. **[JEE - 1993]**

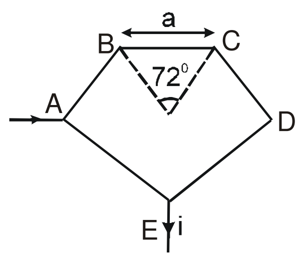
(A) $\vec{E} = 0$ and $\vec{B} = 0$

(B) $\vec{E} \neq 0$ and $\vec{B} = 0$

(C) $\vec{E} \neq 0$, $\vec{B} \neq 0$ and both $\vec{E}$ and $\vec{B}$ are parallel to $\vec{v}$

(D) $\vec{E}$ is parallel to $\vec{v}$ but $\vec{B}$ is perpendicular to $\vec{v}$

8. Magnetic field strength at the centre of regular pentagon made of a conducting wire of uniform cross section area as shown in figure is :



(A) $\frac{5\mu_0 i}{4\pi a}\left[2\sin\frac{72°}{2}\right]$

(B) 0

(C) Not zero if current 'i' leaves D point instead of E

(D) Zero even if the current 'i' leaves point D instead of point E

9. H^+, He^+ and O^{2+} all having the same kinetic energy pass through a thin region in which there is a uniform magnetic field perpendicular to their velocity. The masses of H^+, He^+ and O^{2+} are 1 amu, 4amu and 16 amu respectively, then

(A) H^+ will be deflected most
(B) O^{2+} will be deflected most
(C) He^+ and O^{2+} will be deflected equally
(D) All will be deflected equally

10. A beam of electrons moving with a momentum p enters a uniform magnetic field of flux density B perpendicular to its motion. Which of the following statement(s) is (are) true?

(A) Energy gained is $\frac{p^2}{2m}$

(B) Centripetal force on the electron is Be $\frac{m}{p}$

(C) Radius of the electron's path is $\frac{p}{Be}$

(D) Work done on the electrons by the magnetic field is zero

11. Two ions have equal masses but one is singly-ionized and other is tripply-ionized. They are projected from the same place in a uniform magnetic field with the same velocity perpendicular to the field.

(A) Both ions will go along circles of equal radii.
(B) The circle described by the single-ionized charge will have a radius tripply that of the other circle
(C) The two circles do not touch each other
(D) The two circles touch each other

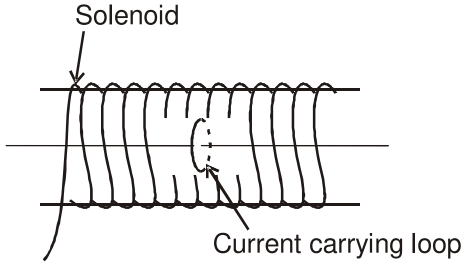
12. A charged particle goes undeflected in a region containing uniform electric and magnetic field. It is not possible at all ($\vec{\upsilon}$ = velocity of particle, $\vec{E}$ = uniform electric field, $\vec{B}$ = uniform magnetic field)

(A) $\vec{E} \parallel \vec{B}$, $\vec{\upsilon} \parallel \vec{E}$
(B) $\vec{E}$ is not collinear to $\vec{B}$
(C) $\vec{\upsilon} \parallel \vec{B}$ but $\vec{E}$ is not collinear to $\vec{B}$
(D) $\vec{E} \parallel \vec{B}$ but $\vec{\upsilon}$ is not collinear to $\vec{E}$

13. A charged particle moves along a circle under the action of possible uniform & constant electric and magnetic fields. Which of the following are not possible at all ?

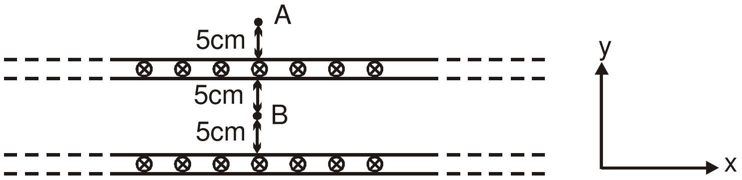
(A) E = 0, B = 0 (B) E = 0, B $\neq$ 0 (C) E $\neq$ 0, B = 0 (D) E $\neq$ 0, B $\neq$ 0

14. A single circular loop of wire with radius 0.02 cm carries a current of 8.0 A. It is placed at the centre of a solenoid that has length 0.65 m, radius 0.080 m and 1300 turns.



(A) The value of the current in the solenoid so that the magnetic field at the centre of the loop becomes zero, is equal to 4.4 A.
(B) The value of the current in the solenoid so that the magnetic field at the centre of the loop becomes zero, is equal to 10 A.
(C) The magnitude of the total magnetic field at the centre of the loop (due to both the loop and the solenoid) if the current in the loop is reversed in direction from that needed to make the total field equal to zero T esla, is $8\pi \times 10^{-3}$ T.
(D) The magnitude of the total magnetic field at the centre of the loop (due to both the loop and the solenoid) if the current in the loop is reversed in direction from that needed to make the total field equal to zero Tesla, is $16\pi \times 10^{-3}$ T.

15. Figure shows cross section of two large parallel metal sheets carrying electric currents along their surfaces. The current in each sheet is $\frac{10}{\pi}$ A/m along the width. Consider two points A and B, as shown in the figure with their positions.



(A) Magnetic field at A is 4μT along x-direction.
(B) Magnetic field at A is 4μT along negative x-direction.
(C) Magnetic field at B is zero.
(D) Magnetic field at B is 2μT along x-direction.

16. Let [$\in_0$] denote the dimensional formula of the permittivity of the vacuum and [μ_0] that of the permeability of the vacuum. If M = mass, L = length, T = time and I = electric current,
(A) $[\in_0] = M^{-1} L^{-3} T^2 I$ (B) $[\in_0] = M^{-1} L^{-3} T^4 I^2$ (C) $[\mu_0] = MLT^{-2} I^{-2}$ (D) $[\mu_0] = ML^2 T^{-1} I$

17. A positively charged particle is moving along the positive X-axis. You want to apply a magnetic field for a short time so that the particle may reverse its direction and move parallel to the negative X-axis. This can be done by applying the magnetic field along.
(A) Y-axis (B) Z-axis (C) Y-axis only (D) Z-axis only

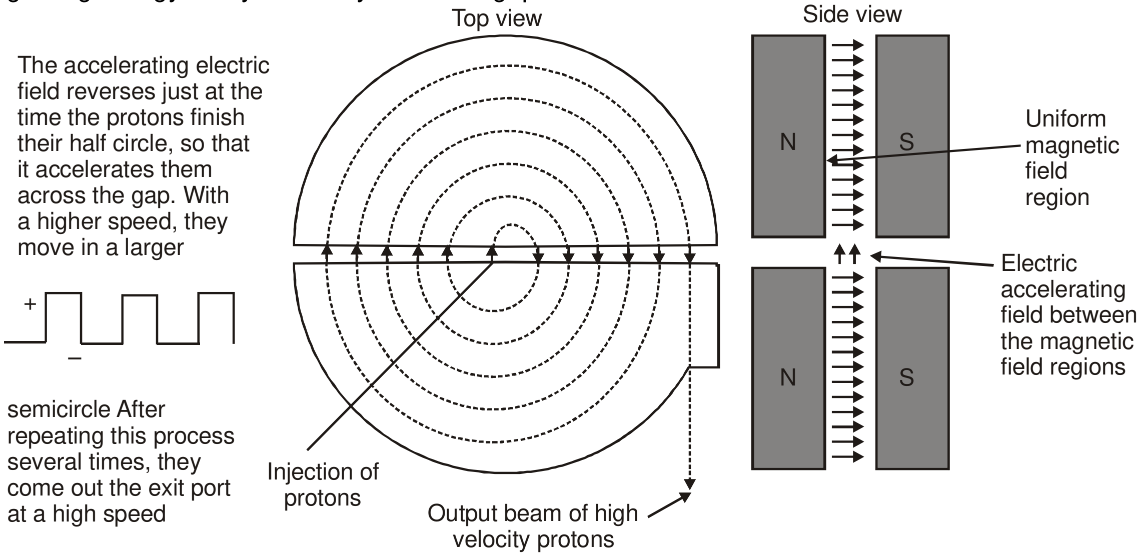
18._ A small bar magnet is suspended by a thread. A torque is applied and the magnet is found to execute angular oscillations. The time period of oscillations **[Olympiad (Stage-1) 2017]**
(A) decreases with the moment of the magnet
(B) increases with the increase of the horizontal component of the earth's magnetic filed
(C) will remain unchanged even if another magnet is kept at a distance
(D) depends on the mass of the magnet

PART - IV : COMPREHENSION

Comprehension-1

(Read the following passage and answer the questions. They have only one correct option)

In the given figure of a cyclotron, showing the particle source S and the dees. A uniform magnetic field is directed up from the plane of the page. Circulating protons spiral outward within the hollow dees, gaining energy every time they cross the gap between the dees.



Suppose that a proton, injected by source S at the centre of the cyclotron in figure initially moves toward a negatively charged dee. It will accelerate toward this dee and enter it. Once inside, it is shielded from electric field by the copper walls of the dee; that is the electric field does not enter the dee. The magnetic field, however, is not screened by the (nonmagnetic) copper dee, so the proton moves in circular path whose radius, which depends on its speed, is given by

$$r = \frac{mv}{qB} \qquad .....(1)$$

Let us assume that at the instant the proton emerges into the center gap from the first dee, the potential difference between the dees is reversed. Thus, the proton again faces a negatively charged dee and is again accelerated. Thus, the proton again faces a negatively charged dee and is again accelerated. This process continues, the circulating proton always being in step, with the oscillations of the dee potential, until the proton has spiraled out to the edge of the dee system. There a deflector plate sends it out through a portal.

The key to the operation of the cyclotron is that the frequency f at which the proton circulates in the field (and that does not depend on its speed) must be equal to the fixed frequency f_{osc} of the electrical oscillator, or

$$f = f_{osc} \text{ (resonance condition).} \qquad .....(2)$$

This resonance condition says that, if the energy of the circulating proton is to increase, energy must be fed to it at a frequency f_{osc} that is equal to the natural frequency f at which the proton circulates in the magnetic field.

Combining equation 1 and 2 allows us to write the resonance condition as

$$qB = 2\pi m f_{osc}. \qquad .....(3)$$

For the proton, q and m are fixed. The oscillator (we assume) is designed to work at a single fixed frequency f_{osc}. We then "tune" the cyclotron by varying B until eq. 3 is satisfied and then many protons circulate through the magnetic field, to emerge as a beam.

1. Ratio of radius of successive semi circular path

(A) $\sqrt{1} : \sqrt{2} : \sqrt{3} : \sqrt{4}$ .... (B) $\sqrt{1} : \sqrt{3} : \sqrt{5}$ ....

(C) $\sqrt{2} : \sqrt{4} : \sqrt{6}$: .... (D) 1 : 2 : 3 ....

2. Change in kinetic energy of charge particle after every time period is :

(A) 2qV (B) qV (C) 3qV (D) None of these

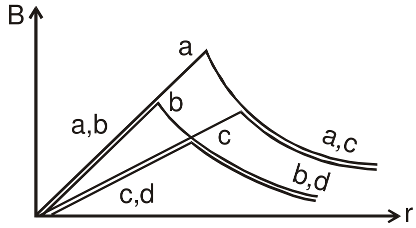
3. If q/m for a charge particle is 10^6, frequency of applied AC is 10^6 Hz. Then applied magnetic field is:
(A) 2π tesla (B) π tesla (C) 2 tesla (D) can not be defined

4. Distance travelled in each time period are in the ratio of:
(A) $\sqrt{1}+\sqrt{3}:\sqrt{5}+\sqrt{7}:\sqrt{9}+\sqrt{11}$ (B) $\sqrt{2}+\sqrt{3}:\sqrt{4}+\sqrt{5}:\sqrt{6}+\sqrt{7}$
(C) $\sqrt{1}:\sqrt{2}:\sqrt{3}$ (D) $\sqrt{2}:\sqrt{3}:\sqrt{4}$

5. For a given charge particle a cyclotron can be "tune" by :
(A) changing applied A.C. voltage only (B) changing applied A.C. voltage and magnetic field both
(C) changing applied magnetic field only (D) by changing frequency of applied A.C.

Comprehension-2

Curves in the graph shown give the magnitude B of the magnetic field (due to individual wire) inside and outside four long wires a, b, c and d, carrying currents that are uniformly distributed across the cross sections of the wires as functions of radial distance r (from the axis). Overlapping portions of the plots are indicated by double labels. All curves start from the origin.



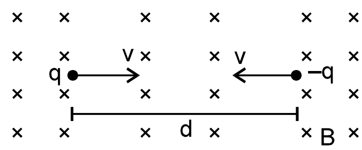
6. Which wire has the greatest radius ?
(A) a (B) b (C) c (D) d

7. Which wire has the greatest magnitude of the magnetic field on the surface ?
(A) a (B) b (C) c (D) d

8. The current density in wire a is
(A) greater than in wire c.
(B) less than in wire c.
(C) equl to that in wire c.
(D) not comparable to that of in wire c due to lack of information.

Comprehension-3

Two particles, each having a mass m are placed at a separation d in a uniform magnetic field B as shown in figure. They have opposite charges of equal magnitude q. At time t = 0, the particles are projected towards each other, each with a speed v. (Neglect interaction between two charges)



9. Find the maximum value v_m of the projection speed so that the two particles do not collide.
(A) $\frac{qBd}{4m}$ (B) $\frac{2qBd}{m}$ (C) $\frac{qBd}{m}$ (D) $\frac{qBd}{2m}$

10. What would be the minimum separation between the particles if $v = v_m/4$?
(A) $\frac{d}{4}$ (B) $\frac{3d}{5}$ (C) $\frac{3d}{4}$ (D) $\frac{5d}{4}$

11. What would be the maximum separation between the particles if $v = v_m/4$?
(A) $\frac{5d}{2}$ (B) $\frac{5d}{4}$ (C) $\frac{d}{4}$ (D) $\frac{3d}{4}$

12. At what instant will a collision occur between the particles if $v = 2v_m$?
(A) $\frac{\pi m}{6qB}$ (B) $\frac{\pi m}{qB}$ (C) $\frac{6\pi m}{qB}$ (D) $\frac{\pi m}{2qB}$

13. Suppose $v = 2v_m$ and they stick to each other after the collision. Describe the motion after the collision (neglect the magnetic, gravitational & electric forces between charges).
(A) circular motion (B) parabolic motion (C) elliptical motion (D) straight line

Exercise-3

✍ **Marked Questions can be used as Revision Questions.**

* **Marked Questions may have more than one correct option.**

PART - I : JEE (ADVANCED) / IIT-JEE PROBLEMS (PREVIOUS YEARS)

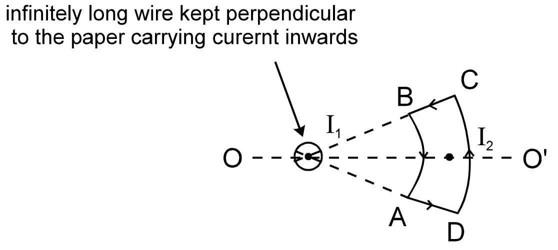
1*. Which of the following statement is correct in the given figure. **[JEE 2006 ; 5/35, –1]**

(A) net force on the loop is zero

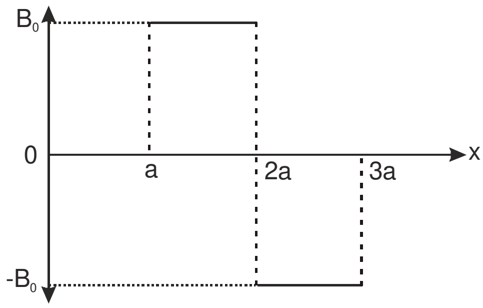
(B) net torque on the loop is zero

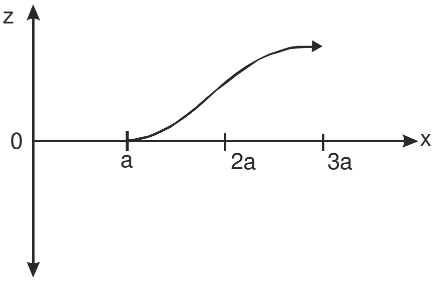
(C) loop will rotate clockwise about axis OO' when seen from O

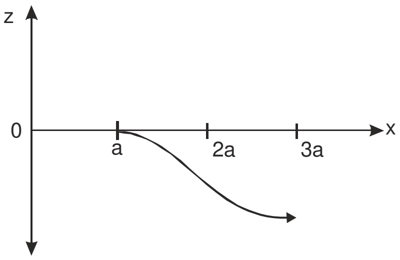
(D) loop will rotate anticlockwise about OO' when seen from O

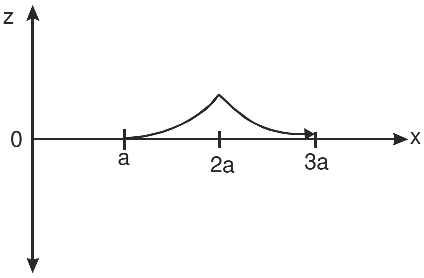


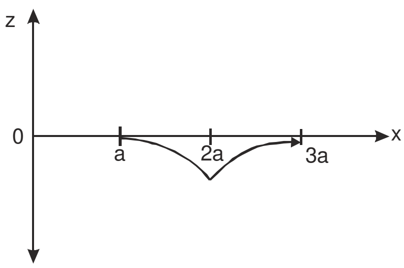
2.✍ A magnetic field $\vec{B} = B_0\hat{j}$ exists in the region a < x < 2a and $\vec{B} = -B_0\hat{j}$, in the region 2a < x < 3a, where B_0 is a positive constant. A positive point charge moving with a velocity $\vec{V} = v_0\hat{i}$, where v_0 is a positive constant, enters the magnetic field at x = a. The trajectory of the charge in this region can be like. **[JEE – 2007 ; 3/162, –1]**



(A) 

(B) 

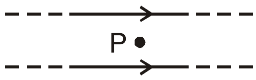
(C) 

(D) 

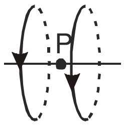
3. Two wires each carrying a steady current I are shown in four configurations in Column I. Some of the resulting effects are described in Column II. Match the statements in Column I with the statements in Column II and indicate your answer by darkening appropriate bubbles in the 4 × 4 matrix given in the ORS. **[JEE – 2007 ; 6/162]**

Column-I

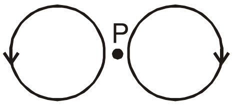
(A) Point P is situated midway between the wires.



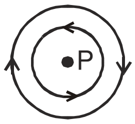
(B) Point P is situated at the mid–point of the line joining the centres of the circular wires, which have same radii.



(C) Point P is situated at the mid–point of the line joining the centers of the circular wires, which have same radii.



(D) Point P is situated at the common center of the wires.



Column-I

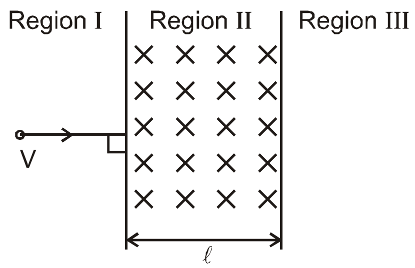
(p) The magnetic fields (B) at P due to the currents in the wires are in the same direction.

(q) The magnetic field B at P due to the currents in the wires are in opposite directions.

(r) There is no magnetic field at P.

(s) The wires repel each other.

4.* A particle of mass m and charge q, moving with velocity V enters region II normal to the boundary as shown in the figure. Region II has a uniform magnetic field B perpendicular to the plane of the paper. The length of the region II is ℓ. Choose the correct choice(s). **[JEE – 2008 ; 4/163]**



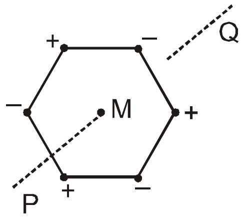
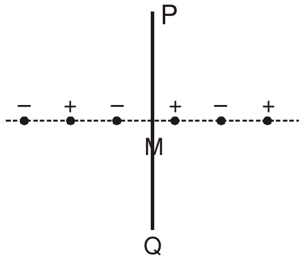
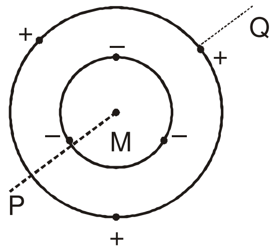
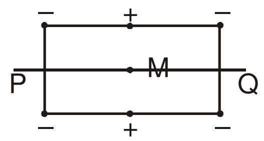
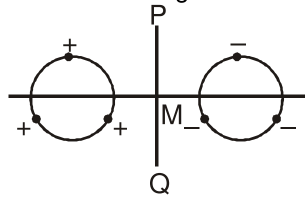
(A) The particle enters Region III only if its velocity $V > \frac{q\ell B}{m}$

(B) The particle enters Region III only if its velocity $V < \frac{q\ell B}{m}$

(C) Path length of the particle in Region II is maximum when velocity $V = q\ell B/m$

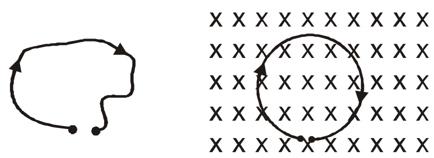
(D) Time spent in Region II is same for any velocity V as long as the particle returns to Region I

5. Six point charges, each of the same magnitude q, are arranged in different manners as shown in **Column–II**. In each case, a point M and a line PQ passing through M are shown. Let E be the electric field and V be the electric potential at M (potential at infinity is zero) due to the given charge distribution when it is at rest. Now, the whole system is set into rotation with a constant angular velocity about the line PQ. Let B be the magnetic field at M and μ be the magnetic moment of the system in this condition. Assume each rotating charge to be equivalent to a steady current. **[JEE - 2009, 8/160]**

Column-I		**Column-I**
(A) $E = 0$	(p)	 Charges are at the corners of a regular hexagon. M is at the centre of the hexagon. PQ is perpendicular to the plane of the hexagon.
(B) $V \neq 0$	(q)	 Charges are on a line perpendicular to PQ at equal intervals. M is the midpoint between the two innermost charges.
(C) $B = 0$	(r)	 Charges are placed on two coplanar insulating rings at equal intervals. M is the common centre of the rings. PQ is perpendicular to the plane of rings.
(D) $\mu \neq 0$	(s)	 Charges are placed at the corners of a rectangle of sides a and 2a and at the mid–points of the longer sides. M is at the centre of the rectangular. PQ is parallel to the longer sides.
	(t)	 Charges are placed on two coplanar, identical insulating rings at equal intervals. M is the mid–point between the centres of the rings. PQ is perpendicular to the line joining the centres and coplanar to the rings.

6. A steady current I goes through a wire loop PQR having shape of a right angle triangle with PQ = 3x, PR = 4x and QR = 5x. If the magnitude of the magnetic field at P due to this loop is $k\left(\frac{\mu_0 I}{48\pi x}\right)$, find the value of k. **[JEE 2009, 4/160, –1]**

7. A thin flexible wire of length L is connected to two adjacent fixed points carries a current I in the clockwise direction, as shown in the figure. When system is put in a uniform magnetic field of strength B going into the plane of paper, the wire takes the shape of a circle. The tension in the wire is : **[JEE 2010, 3/163, –1]**

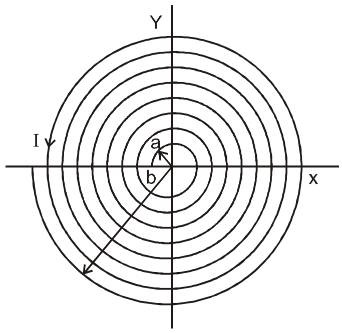


(A) IBL (B) $\frac{IBL}{\pi}$ (C) $\frac{IBL}{2\pi}$ (D) $\frac{IBL}{4\pi}$

8.* An electron and a proton are moving on straight parallel paths with same velocity. They enter a semi-infinite region of uniform magnetic field perpendicular to the velocity. Which of the following statement(s) is/are true? **[JEE 2011, 4/160]**

(A) They will never come out of the magnetic field region.
(B) They will come out travelling along parallel paths.
(C) They will come out at the same time.
(D) They will come out at different times.

9. A long insulated copper wire is closely wound as a spiral of *'N'* turns. The spiral has inner radius 'a' and outer radius *'b'.* The spiral lies in the X-Y plane and a steady current I flows through the wire. The Z-component of the magnetic field at the center of the spiral is **[JEE - 2011' 3/160, –1]**



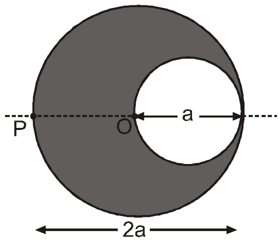
(A) $\frac{\mu_0 NI}{2(b-a)}\ell n\left(\frac{b}{a}\right)$ (B) $\frac{\mu_0 NI}{2(b-a)}\ell n\left(\frac{b+a}{b-a}\right)$ (C) $\frac{\mu_0 NI}{2b}\ell n\left(\frac{b}{a}\right)$ (D) $\frac{\mu_0 NI}{2b}\ell n\left(\frac{b+a}{b-a}\right)$

10.* Consider the motion of a positive point charge in a region where there are simultaneous uniform electric and magnetic fields $\vec{E} = E_0\hat{j}$ and $\vec{B} = B_0\hat{j}$. At time t = 0, this charge has velocity $\vec{v}$ in the x-y plane, making an angle θ with x-axis. Which of the following option(s) is(are) correct for time t > 0 ?

(A) If θ = 0°, the charge moves in a circular path in the x-z plane. **[IIT-JEE-2012, Paper-1; 4/70]**
(B) If θ = 0°, the charge undergoes helical motion with constant pitch along the y-axis.
(C) If θ = 10°, the charge undergoes helical motion with its pitch increasing with time, along the y-axis.
(D) If θ = 90°, the charge undergoes linear but accelerated motion along the y-axis.

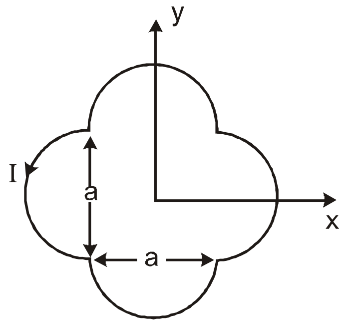
11. A cylindrical cavity of diameter a exists inside a cylinder of diameter 2a shown in the figure. Both the cylinder and the cavity are infinitely long. A uniform current density J flows along the length. If the magnitude of the magnetic field at the point P is given by $\frac{N}{12}\mu_0 aJ$, then the value of N is :

[IIT-JEE-2012, Paper-1; 4/70]



12. A loop carrying current I lies in the x-y plane as shown in the figure. The unit vector $\hat{k}$ is coming out of the plane of the paper. The magnetic moment of the current loop is :

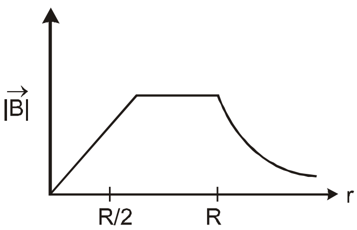
[IIT-JEE-2012, Paper-2; 3/66, –1]

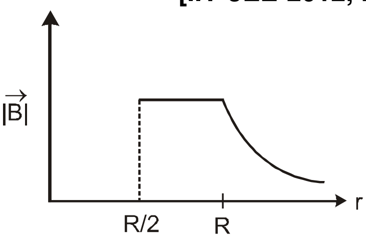


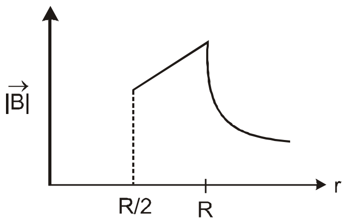
(A) $a^2 I\hat{k}$ (B) $\left(\frac{\pi}{2}+1\right)a^2 I\hat{k}$

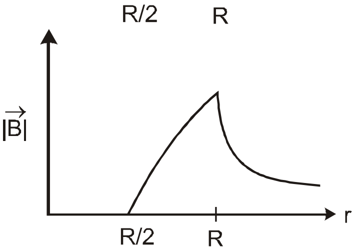
(C) $-\left(\frac{\pi}{2}+1\right)a^2 I\hat{k}$ (D) $(2\pi+1)a^2 I\hat{k}$

13. An infinitely long hollow conducting cylinder with inner radius R/2 and outer radius R carries a uniform current density along is length. The magnitude of the magnetic field, $|\vec{B}|$ as a function of the radial distance r from the axis is best represented by : **[IIT-JEE-2012, Paper-2; 3/66, –1]**

(A) 

(B) 

(C) 

(D) 

14.* A particle of mass M and positive charge Q, moving with a constant velocity $\vec{u}_1 = 4\hat{i}ms^{-1}$, enters a region of uniform static magnetic field normal to the x-y plane. The region of the magnetic field extends from x = 0 to x = L for all values of y. After passing through this region, the particle emerges on the other side after 10 milliseconds with a velocity $\vec{u}_2 = 2\left(\sqrt{3}\hat{i}+\hat{j}\right)ms^{-1}$. The correct statement(s) is (are) :

[JEE (Advanced)-2013, 4/60, –1]

(A) The direction of the magnetic field is –z direction.
(B) The direction of the magnetic field is +z direction
(C) The magnitude of the magnetic field $\frac{50\pi M}{3Q}$ units.
(D) The magnitude of the magnetic field is $\frac{100\pi M}{3Q}$ units.

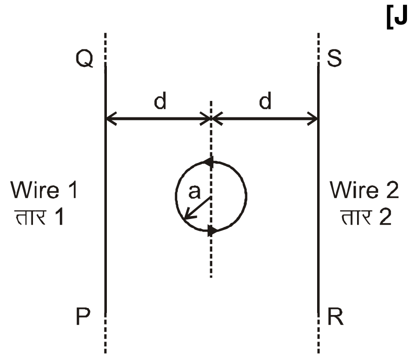
15.* A steady current I flows along an infinitely long hollow cylindrical conductor of radius R. This cylinder is placed coaxially inside an infinite solenoid of radius 2R. The solenoid has n turns per unit length and carries a steady current I. Consider a point P at a distance r from the common axis. The correct statement(s) is (are) : **[JEE (Advanced) 2013, 3/60, –1]**

(A) In the region $0 < r < R$, the magnetic field is non–zero.
(B) In the region $R < r < 2R$, the magnetic field is along the common axis.
(C) In the region $R < r < 2R$, the magnetic field is tangential to the circle of radius r, centered on the axis.
(D) In the region $r > 2R$, the magnetic field is non–zero.

16. Two parallel wires in the plane of the paper are distance X_0 apart. A point charge is moving with speed u between the wires in the same plane at a distance X_1 from one of the wires. When the wires carry current of magnitude I in the same direction, the radius of curvature of the path of the point charge is R_1. In contrast, if the currents I in the two wires have direction opposite to each other, the radius of curvature of the path is R_2. If $\frac{x_0}{x_1} = 3$, the value of $\frac{R_1}{R_2}$ is. **[JEE (Advanced) 2014, P-1, 3/60]**

Paragraph for Questions 17 to 18

The figure shows a circular loop of radius a with two long parallel wires (numbered 1 and 2) all in the plane of the paper. The distance of each wire from the centre of the loop is d. The loop and the wires are carrying the same current I. The current in the loop is in the counterclockwise direction if seen from above. **[JEE (Advanced) 2014, P-2, 3/60, –1]**



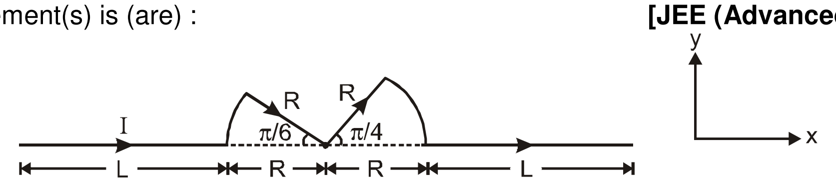
17. When $d \approx a$ but wires are not touching the loop, it is found that the net magnetic filed on the axis of the loop is zero at a height h above the loop. In that case **[JEE (Advanced) 2014, 3/60, –1]**

(A) current in wire 1 and wire 2 is the direction PQ and RS, respectively and $h \approx a$
(B) current in wire 1 and wire 2 is the direction PQ and SR, respectively and $h \approx a$
(C) current in wire 1 and wire 2 is the direction PQ and SR, respectively and $h \approx 1.2\ a$
(D) current in wire 1 and wire 2 is the direction PQ and RS, resepectively and $h \approx 1.2\ a$

18. Consider d >> a, and the loop is rotated about its diameter parallel to the wires by 30º from the position shown in the figure. If the currents in the wires are in the opposite directions, the torque on the loop at its new position will be (assume that the net field due to the wires is constant over the loop)
[JEE (Advanced) 2014, 3/60, –1]

(A) $\frac{\mu_0 I^2 a^2}{d}$ (B) $\frac{\mu_0 I^2 a^2}{2d}$ (C) $\frac{\sqrt{3}\mu_0 I^2 a^2}{d}$ (D) $\frac{\sqrt{3}\mu_0 I^2 a^2}{2d}$

19.* A conductor (shown in the figure) carrying contant current I is kept in the x-y plane in a uniform magnetic field $\vec{B}$. If F is the magnitude of the total magnetic force acting on the conductor, then the correct statement(s) is (are) : **[JEE (Advanced) 2015 ; 4/88, –2]**

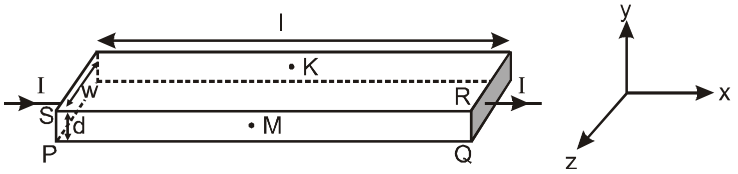


(A) If $\vec{B}$ is along $\hat{z}$, $F \propto (L + R)$ (B) If $\vec{B}$ is along $\hat{x}$, $F = 0$
(C) If $\vec{B}$ is along $\hat{y}$, $F \propto (L + R)$ (D) If $\vec{B}$ is along $\hat{z}$, $F = 0$

Paragraph for Questions 20 to 21

In a thin rectangular metallic strip a constant current I flows along the positive x-direction, as shown in the figure. The length, width and thicknes of the strip are l, w and d, respectively.

A uniform magnetic field $\vec{B}$ is applied on the strip along the positive y-direction. Due to this, the charge carriers experience a net deflection along the z-direction. This results in accumulation of charge carriers on the surface PQRS and is appearance of equal and opposite charges on the face opposite to PQRS. A potential difference along the z-direction is thus developed. Charge accumulation continues until the magnetic force is balanced by the electric force. The current is assumed to be uniformly distributed on the cross section of the strip and carried by electrons. **[JEE (Advanced) 2015 ; P-2, 4/88, –2]**



20.* Consider two different metallic strips (1 and 2) fo the same material. Their lengths are the same, width are w_1 and w_2 and thicknesses are d_1 and d_2, respectively. Two points K and M are symmetrically located on the opposite faces parallel to the x-y plane (see figure). V_1 and V_2 are the potential differences between K and M in strips 1 and 2, respectively. Then, for a given current I flowing through them in a given magnetic field strength B, the correct statement(s) is (are)

(A) If $w_1 = w_2$ and $d_1 = 2d_2$, then $V_2 = 2V_1$ (B) If $w_1 = w_2$ and $d_1 = 2d_2$, then $V_2 = V_1$

(C) If $w_1 = 2w_2$ and $d_1 = d_2$, then $V_2 = 2V_1$ (D) If $w_1 = 2w_2$ and $d_1 = d_2$, then $V_2 = V_1$

21.* Consider two different metallic strips (1 and 2) of same dimensions (length l, width w and thickness d) with carrier densities n_1 and n_2, respectively. Strip 1 is placed in magnetic field B_1 and strip 2 is placed in magnetic field B_2, both along positive y-directions. Then V_1 and V_2 are the potential differences developed between K and M in strips 1 and 2, respectively. Assuming that the current I is the same for both the strips, the correct option(s) is (are)

(A) If $B_1 = B_2$ and $n_1 = 2n_2$, then $V_2 = 2V_1$ (B) If $B_1 = B_2$ and $n_1 = 2n_2$, then $V_2 = V_1$

(C) If $B_1 = 2B_2$ and $n_1 = n_2$, then $V_2 = 0.5V_1$ (D) If $B_1 = 2B_2$ and $n_1 = n_2$, then $V_2 = V_1$

Answer Q.22. Q.23 and Q.24 by appropriately matching the information given in the three columns of the following table.

A charged particle (electron or proton) is introduced at the origin (x = 0, y = 0, z = 0) with a given initial velocity $\vec{v}$. A uniform electric field $\vec{E}$ and a uniform magnetic field $\vec{B}$ exist everywhere. The velocity $\vec{v}$, electric field $\vec{E}$ and magnetic field $\vec{B}$ are given in column 1, 2 and 3, respectively. The quantities E_0, B_0 are positive in magnitude.

	Column-1		**Column-2**		**Column-3**
(I)	Electron with $\vec{v} = 2\frac{E_0}{B_0}\hat{x}$	(i)	$\vec{E} = E_0\hat{z}$	(P)	$\vec{B} = -B_0\hat{x}$
(II)	Electron with $\vec{v} = \frac{E_0}{B_0}\hat{y}$	(ii)	$\vec{E} = -E_0\hat{y}$	(Q)	$\vec{B} = B_0\hat{x}$
(III)	Electron with $\vec{v} = 0$	(iii)	$\vec{E} = -E_0\hat{x}$	(R)	$\vec{B} = B_0\hat{y}$
(IV)	Electron with $\vec{v} = 2\frac{E_0}{B_0}\hat{x}$	(iv)	$\vec{E} = E_0\hat{x}$	(S)	$\vec{B} = B_0\hat{z}$

22. In which case will the particle move in a straight line with constant velocity ?

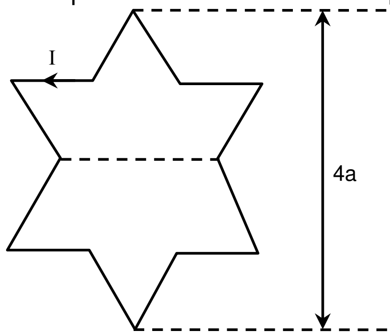
[JEE (Advanced) 2017 ; P-1, 3/61, –1]

(A) (IV) (i) (S) (B) (III) (ii) (R) (C) (II) (iii) (S) (D) (III) (iii) (P)

23. In which case will the particle describe a helical path with axis along the positive z direction ?
[JEE (Advanced) 2017 ; P-1, 3/61, –1]
(A) (IV) (i) (S) (B) (II) (ii) (R) (C) (III) (iii) (P) (D) (IV) (ii) (R)

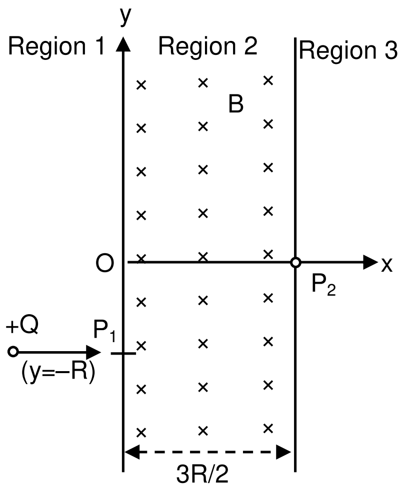
24. In which case would the particle move in a straight line along the negative direction of y-axis (i.e, move along $-\hat{y}$) ? **[JEE (Advanced) 2017 ; P-1, 3/61, –1]**
(A) (III) (ii) (R) (B) (IV) (ii) (S) (C) (III) (ii) (P) (D) (II) (iii) (Q)

25. A symmetric star shaped conducting wire loop is carrying a steady state current I as shown in the figure. The distance between the diametrically opposite vertices of the star is 4a. The magnitude of the magnetic field at the center of the loop is : **[JEE (Advanced) 2017 ; P-2, 3/61, –1]**



(A) $\frac{\mu_0 I}{4\pi a}6\left[\sqrt{3}-1\right]$ (B) $\frac{\mu_0 I}{4\pi a}6\left[\sqrt{3}+1\right]$ (C) $\frac{\mu_0 I}{4\pi a}3\left[2-\sqrt{3}\right]$ (D) $\frac{\mu_0 I}{4\pi a}3\left[\sqrt{3}-1\right]$

26*. A uniform magnetic field B exists in the region between x = 0 and x = $\frac{3R}{2}$ (region 2 in the figure) pointing normally into the plane of the paper. A particle with charge +Q and momentum p directed along x-axis enters region 2 from region 1 at point P_1 (y = –R). Which of the following option(s) is/are correct ?
[JEE (Advanced) 2017 ; P-2, 4/61, –2]



(A) When the particle re-enters region 1 through the longest possible path in region 2, the magnitude of the change in its linear momentum between P_1 and the farthest point from y-axis is $p/\sqrt{2}$.
(B) For a fixed B, particles of same charge Q and same velocity v, the distance between the point P_1 and the point of re-entry into region 1 is inversely proportional to the mass of the particle.
(C) For $B = \frac{8}{13}\frac{p}{QR}$, the particle will enter region 3 through the point P_2 on x-axis
(D) For $B > \frac{2}{3}\frac{p}{QR}$, the particle will re-enter region 1.

27*. Two infinitely long straight wires lie in the xy-plane along the lines $x = \pm R$. The wire located at $x = +R$ carries a constant current I_1 and the wire located at $x = -R$ carries a constant current I_2. A circular loop of radius R is suspended with its centre at $(0, 0, \sqrt{3}R)$ and in a plane parallel to the xy-plane. This loop carries a constant current I in the clockwise direction as seen from above the loop. The current in the wire is taken to be positive if it is in the $+\hat{j}$ direction. Which of the following statements regarding the magnetic field $\vec{B}$ is (are) true? **[JEE (Advanced) 2018 ; P-1, 4/60, –2]**

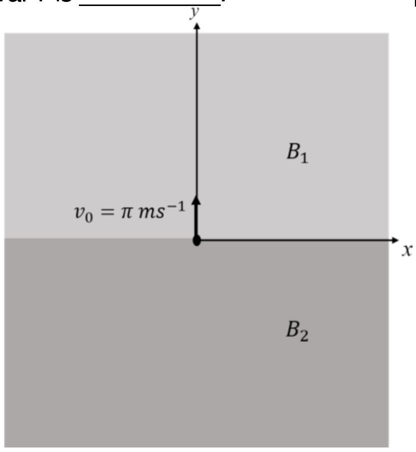
(A) If $I_1 = I_2$, then $\vec{B}$ **cannot** be equal to zero at the origin (0, 0, 0)

(B) If $I_1 > 0$ and $I_2 < 0$, then $\vec{B}$ can be equal to zero at the origin (0, 0, 0)

(C) If $I_1 < 0$ and $I_2 > 0$, then $\vec{B}$ can be equal to zero at the origin (0, 0, 0)

(D) If $I_1 = I_2$, then the z-component of the magnetic field at the centre of the loop is $\left(-\frac{\mu_0 I}{2R}\right)$

28. In the xy-plane, the region y > 0 has a uniform magnetic field $B_1\hat{k}$ and the region y < 0 has another uniform magnetic field $B_2\hat{k}$. A positively charged particle is projected from the origin along the positive y-axis with speed $v_0 = \pi ms^{-1}$ at t = 0, as shown in the figure. Neglect gravity in this problem. Let t = T be the time when the particle crosses the x-axis from below for the first time. If $B_2 = 4B_1$, the average speed of the particle, in ms^{-1}, along the x-axis in the time interval T is __________. **[JEE (Advanced) 2018 ; P-1, 3/60]**



PART - II : JEE (MAIN) / AIEEE PROBLEMS (PREVIOUS YEARS)

1. In a region, steady and uniform electric and magnetic fields are present. These two fields are parallel to each other. A charged particle is released from rest in this region. The path of the particle will be a : **[AIEEE 2006, 1.5/180]**

(1) circle (2) helix (3) straight line (4) ellipse

2. Needles N_1, N_2 and N_3 are made of a ferromagnetic, a paramagnetic and a diamagnetic substance respectively. A magnet when brought close to them will : **[AIEEE 2006, 1.5/180]**

(1) attract all three of them
(2) attract N_1 and N_2 strongly but repel N_3
(3) attract N_1 strongly, N_2 weakly and repel N_3 weakly
(4) attract N_1 strongly, but repel N_2 and N_3 weakly

3. A long solenoid has 200 turns per cm and carries a current i. The magnetic field at its centre is 6.28×10^{-2} Weber/m^2. Another long solenoid has 100 turns per cm and it carries a current i/3. The value of the magnetic field at its centre is : **[AIEEE 2006, 4.5/180]**

(1) 1.05×10^{-4} Weber/m^2 (2) 1.05×10^{-2} Weber/m^2
(3) 1.05×10^{-5} Weber/m^2 (4) 1.05×1010^{-3} Weber/m^2

4. A long straight wire of radius *a* carries a steady current *i*. The current is uniformly distributed across its crosssection. The ratio of the magnetic field at a/2 and 2a from axis is : **[AIEEE 2007, 3/120]**

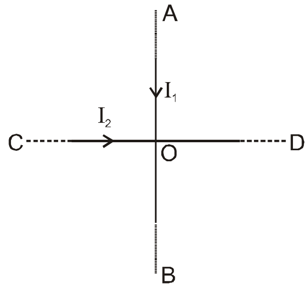
(1) 1/4 (2) 4 (3) 1 (4) 1/2

5. A current I flows along the length of an infinitely long, straight, thin walled pipe. Then : **[AIEEE 2007, 3/120]**
(1) the magnetic field is zero only on the axis of the pipe
(2) the magnetic field is different at different points inside the pipe
(3) the magnetic field at any point inside the pipe is zero
(4) the magnetic field at all points inside the pipe is the same, but not zero

6. A charged particle with charge q enters a region of constant, uniform and mutually orthogonal fields $\vec{E}$ and $\vec{B}$ with a velocity $\vec{v}$ perpendicular to both $\vec{E}$ and $\vec{B}$, and comes out without any change in magnitude or direction of $\vec{v}$. Then : **[AIEEE 2007, 3/120]**
(1) $\vec{v} = \vec{E} \times \vec{B}/B^2$ (2) $\vec{v} = \vec{E} \times \vec{E}/B^2$ (3) $\vec{v} = \vec{E} \times \vec{E}/E^2$ (4) $\vec{v} = \vec{B} \times \vec{E}/E^2$

7. A charged particle moves through a magnetic field perpendicular to its direction. Then :**[AIEEE 2007, 3/120]**
(1) the momentum changes but the kinetic energy is constant
(2) both momentum and kinetic energy of the particle are not constant
(3) both, momentum and kinetic energy of the particle are constant
(4) kinetic energy changes but the momentum is constant

8. Two identical conducting wires AOB and COD are placed at right angles to each other. The wire AOB carries an electric current I_1 and COD carries a current I_2. The magnetic field on a point lying at a distance d from O, in a direction perpendicular to the plane of the wires AOB and COD, will be given by **[AIEEE 2007, 3/120]**



(1) $\frac{\mu_0}{2\pi}\left(\frac{I_1 + I_2}{d}\right)^{1/2}$ (2) $\frac{\mu_0}{2\pi d}\left(I_1^2 + I_2^2\right)^{1/2}$ (3) $\frac{\mu_0}{2\pi d}$ $(I_1 + I_2)$ (4) $\frac{\mu_0}{2\pi d}\left(I_1^2 + I_2^2\right)$

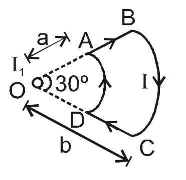
9. Relative permittivity and permeability of a material are ε_r and μ_r, respectively. Which of the following values of these quantities are allowed for a diamagnetic material ? **[AIEEE 2008, 3/105]**
(1) $\varepsilon_r = 1.5$, $\mu_r = 0.5$ (2) $\varepsilon_r = 0.5$, $\mu_r = 0.5$ (3) $\varepsilon_r = 1.5$, $\mu_r = 1.5$ (4) $\varepsilon_r = 0.5$, $\mu_r = 1.5$

10. A horizontal overhead powerline is at a height of 4 m from the ground and carries a current of 100 A from east to west. The magnetic field directly below it on the ground is ($\mu_0 = 4\pi \times 10^{-7}$ T mA^{-1}):
(1) 5×10^{-6} T northward (2) 5×10^{-6} T southward **[AIEEE 2008, 3/105]**
(3) 2.5×10^{-7} T northward (4) 2.5×10^{-7} T southward

Comprehension :

Direction : Question numbers 11 and 12 are based on the following paragraph :

A current loop ABCD is held fixed on the plane of the paper as shown in the figure. The arcs BC (radius = b) and DA (radius = a) of the loop are joined by two straight wires AB and. CD. A steady current I is flowing in the loop. Angle made by AB and CD at the origin O is 30º. Another straight thin wire with steady current I_1 flowing out of the plane of the paper is kept at the origin.



11. The magnitude of the magnetic field due to the loop ABCD at the origin (O) is: **[AIEEE 2009; 4/144, –1]**

(1) $\frac{\mu_0 I(b-a)}{24ab}$ (2) $\frac{\mu_0 I}{4\pi}\left[\frac{b-a}{ab}\right]$ (3) $\frac{\mu_0 I}{4\pi}\left[2(b-a)+\frac{\pi}{3}(a+b)\right]$ (4) zero

12. Due to the presence of the current I_1 at the origin : **[AIEEE 2009; 4/144, –1]**

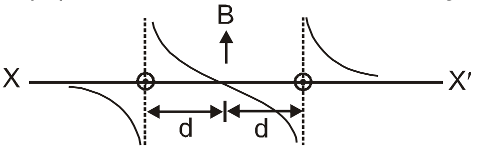
(1) The forces on AD and BC are zero.

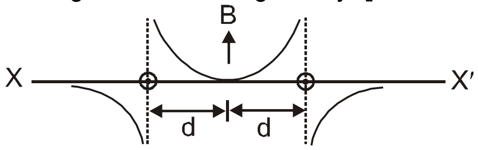
(2) The magnitude of the net force on the loop is given by $\frac{\mu_0 I_1 I}{4\pi}\left[2(b-a)+\frac{\pi}{3}(a+b)\right]$

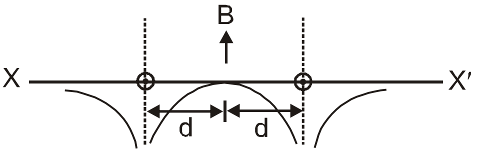
(3) the magnitude of the net force on the loop is given by $\frac{\mu_0 I I_1}{24ab}$ (b – a).

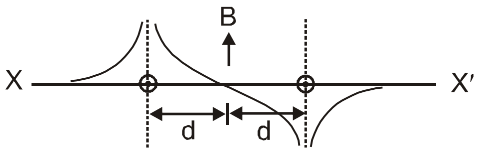
(4) the forces on AB and DC are zero.

13. Two long parallel wires are at a distance 2d apart. They carry steady equal currents flowing out of the plane of the paper as shown. The variation of magnetic field B along the line XX′ is given by **[AIEEE 4/144 2010]**

(1) 

(2) 

(3) 

(4) 

14. A current I flows in an infinitely long wire with cross-section in the form of a semicircular ring of radius R. The magnitude of the magnetic induction on its axis is : **[AIEEE - 2011, 1-May, 4/120, –1]**

(1) $\frac{\mu_0 I}{\pi^2 R}$ (2) $\frac{\mu_0 I}{2\pi^2 R}$ (3) $\frac{\mu_0 I}{2\pi R}$ (4) $\frac{\mu_0 I}{4\pi R}$

15. An electric charge +q moves with velocity $\vec{v}=3\hat{i}+4\hat{j}+\hat{k}$, in an electromagnetic field given by : $\vec{E}=3\hat{i}+\hat{j}+2\hat{k}$ and $\vec{B}=\hat{i}+\hat{j}+3\hat{k}$. The y-component of the force experienced by + q is :

[AIEEE 2011, 11 May; 4/120, –1]

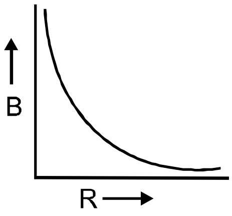
(1) 7 q (2) 5 q (3) 3 q (4) 2 q

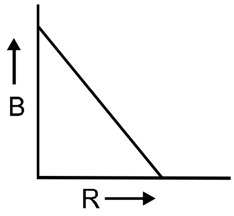
16. A thin circular disk of radius R is uniformly charged with density $\sigma > 0$ per unit area. The disk rotates about its axis with a uniform angular speed ω. The magnetic moment of the disk is :

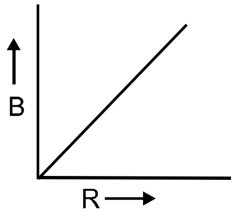
[AIEEE 2011, 11 May; 4/120, –1]

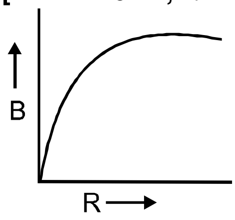
(1) $\pi R^4 \sigma\omega$ (2) $\frac{\pi R^4}{2}\sigma\omega$ (3) $\frac{\pi R^4}{4}\sigma\omega$ (4) $2\pi R^4 \sigma\omega$

17. A charge Q is uniformly distributed over the surface of non-condcting disc of radius R. The disc rotates about an axis perpendicular to its plane and passing through its centre with an angular velocity ω. As a result of this rotation a magnetic field of induction B is obtained at the centre of the disc. if we keep both the amount of charge placed on the disc and its angular velocity to be constant and vary the radius of the disc then the variation of the magnetic induction at the centre of the disc will be represented by the figure : **[AIEEE 2012 ; 4/120, –1]**

(1) 

(2) 

(3) 

(4) 

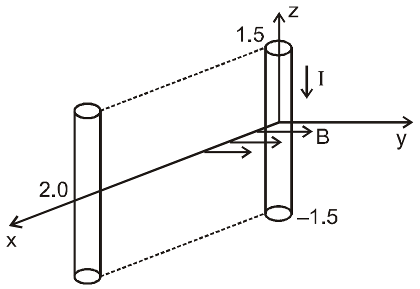
18. Two short bar magnets of length 1 cm each have magnetic moments 1.20 Am^2 and 1.00 Am^2 respectively. They are placed on a horizontal table parallel to each other with their N poles poining towards the South. They have a common magnetic equator and are separated by a distance of 20.0 cm. The value of the resultand horizontal magnetic induction at the mid - point O of the line joining their centres is close to (Horizontal component of earth's magnetic induction is 3.6×10^{-5} Wb/m^2)

[JEE (Main) 2013 ; 4/120, –1]

(1) 3.6×10^{-5} Wb/m^2 (2) 2.56×10^{-4} Wb/m^2 (3) 3.50×10^{-4} Wb/m^2 (4) 5.80×10^{-4} Wb/m^2

19. A conductor lies along the z-axis at $-1.5 \leq z < 1.5$ m and carries a fixed current of 10.0 A in $-\hat{a}_z$ direction (see figure). For a field $\vec{B} = 3.0 \times 10^{-4} e^{-0.2x}\, \hat{a}_y$ T, find the power required to move the conductor at constant speed to x = 2.0 m, y = 0 m in 5×10^{-3}s. Assume parallel motion along the x-axis

[JEE (Main) 2014, 4/120, –1]



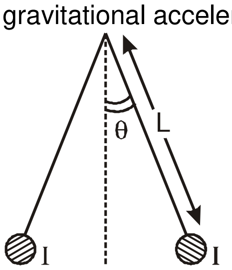
(1) 1.57 W (2) 2.97 W (3) 14.85 W (4) 29.7 W

20. Two coaxial solenoids of different radii carry current I in the same direction. Let $\vec{F_1}$ be the magnetic force on the inner solenoid due to the outer one and $\vec{F_2}$ be the magnetic force on the outer solenoid due to the inner one. Then : **[JEE (Main) 2015; 4/120, –1]**

(1) $\vec{F_1} = \vec{F_2} = 0$ (2) $\vec{F_1}$ is radially inwards and $\vec{F_2}$ is radially outwards

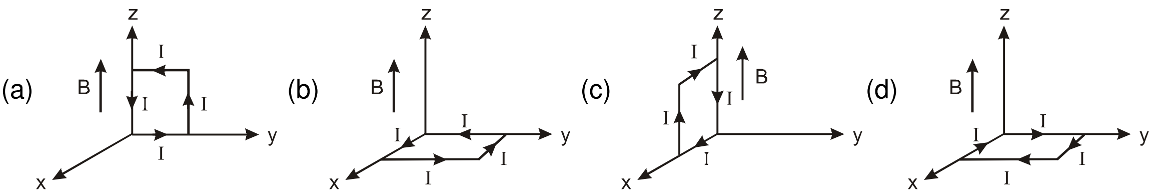
(3) $\vec{F_1}$ is radially inwards and $\vec{F_2} = 0$ (4) $\vec{F_1}$ is radially outwards and $\vec{F_2} = 0$

21. Two long current carrying thin wires, both with current I, are held by insulating threads of length L and are in equilibrium as shown in the figure, with threads making an angle 'θ' with the vertical. If wires have mass λ per unit length then the value of I is (g = gravitational acceleration) **[JEE (Main) 2015; 4/120, –1]**



(1) $\sin\theta\sqrt{\dfrac{\pi\lambda gL}{\mu_0 \cos\theta}}$ (2) $2\sin\theta\sqrt{\dfrac{\pi\lambda gL}{\mu_0 \cos\theta}}$ (3) $2\sqrt{\dfrac{\pi gL}{\mu_0}\tan\theta}$ (4) $\sqrt{\dfrac{\pi\lambda gL}{\mu_0}\tan\theta}$

22. A rectangular loop of sides 10 cm and 5 cm carrying a current I of 12 A is placed in different orientations as shown in the figures below : **[JEE (Main) 2015 ; 4/120, –1]**



If there is a uniform magnetic field of 0.3 T in the positive z direction, in which orientations the loop would be in (i) stable equilibrium and (ii) unstable equilibrium ?

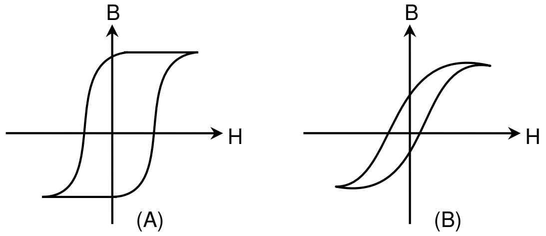
(1) (a) and (b), respectively (2) (a) and (c), respectively

(3) (b) and (d), respectively (4) (b) and (c), respectively

23. Two identical wires A and B, each of length l, carry the same current I. Wire A is bent into a circle of radius R and wire B is bent to form a square of side 'a'. If B_A and B_B are the values of magnetic field at the centres of the circle and square respectively, then the ratio $\frac{B_A}{B_B}$ is : **[JEE (Main) 2016 ; 4/120, –1]**

(1) $\frac{\pi^2}{16\sqrt{2}}$ (2) $\frac{\pi^2}{16}$ (3) $\frac{\pi^2}{8\sqrt{2}}$ (4) $\frac{\pi^2}{8}$

24. Hysteresis loops for two magnetic materials A and B are given below : **[JEE (Main) 2016 ; 4/120, –1]**



These materials are used to make magnets for electric generators, transformer core and electromagnet core. Then it is proper to use :
(1) A for electromgnets and B for electric generators.
(2) A for transformers and B for electric generators.
(3) B for electromgnets and transformers.
(4) A for electric generators and transformers.

25. A magnetic needle of magnetic moment 6.7×10^{-2} Am^2 and moment of inertia 7.5×10^{-6} kg m^2 is performing simple harmonic oscillations in a magnetic field of 0.01 T. Time taken for 10 complete oscillations is : **[JEE (Main) 2017 ; 4/120, –1]**
(1) 8.76 s (2) 6.65 s (3) 8.89 s (4) 6.98 s

26. An electron, a proton and an alpha particle having the same kinetic energy are moving in circular orbits of radii r_e, r_p, r_α respectively in uniform magnetic field B. The relation between r_e, r_p, r_α is : **[JEE (Main) 2018; 4/120, –1]**
(1) $r_e < r_p < r_\alpha$ (2) $r_e < r_\alpha < r_p$ (3) $r_e > r_p = r_\alpha$ (4) $r_e < r_p = r_\alpha$

27. The dipole moment of a circular loop carrying a current I, is m and the magnetic field at the centre of the loop is B_1. When the dipole moment is doubled by keeping the current constant, the magnetic field at the centre of loop is B_2. The ratio $\frac{B_1}{B_2}$ is : **[JEE (Main) 2018; 4/120, –1]**

(1) $\sqrt{2}$ (2) $\frac{1}{\sqrt{2}}$ (3) 2 (4) $\sqrt{3}$

Answers

EXERCISE-1

PART - I

Section (A) :

A-1. $\frac{\sqrt{13}}{2} \times 10^{-4}$ wb/m^2

A-2. (i) 0, (ii) $10^{-13}\hat{k}$ (iii) $-10^{-13}\hat{j}$
(iv) $\frac{4}{27}\times 10^{-13}(-2\hat{j}+\hat{k})$ (v) yes, no
(vi) yes, no.

A-3. (i) $\frac{\mu_0 qv}{4\pi R^2}$,inwards (ii) $\frac{\mu_0 qv}{4\pi(x^2+R^2)}$, No

Section (B) :

B-1. 1×10^{-4} wb/m^2, towards the reader

B-2. 4×10^{-5} wb/m^2

B-3. (i) $2\sqrt{3} \times 10^{-5}$T (ii) 2×10^{-5} T **B-4.** 0

B-5. $\frac{\mu_0 i}{4\pi d}$, $\sqrt{2}\ \frac{\mu_0 i}{4\pi d}$, $\frac{1}{\sqrt{2}}$ **B-6.** $\frac{\mu_0 i}{2\pi x}$

B-7. $\frac{2\sqrt{2}\mu_0 i}{\pi a}$ **B-8.** 0

Section (C) :

C-1. (i) (a) $4\pi \times 10^{-4}$ (b) zero (ii) $2\sqrt{2}\ \pi \times 10^{-4}$ T

C-2. (a) $5\pi\times 10^{-5}\left(1-\frac{8}{13\sqrt{13}}\right)$T (b) zero

Section (D) :

D-1. $\frac{\mu_0 I}{4}\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$

D-2. $B=\frac{\mu_0 i}{4\pi R}\sqrt{2\left(2\pi^2+2\pi+1\right)}$

D-3. (a) $B=\frac{\mu_0 I}{4\pi}\left(\frac{2\pi-\varphi}{a}+\frac{\varphi}{b}\right)$;

(b) $B=\frac{\mu_0 I}{4\pi}\left(\frac{3\pi}{2a}+\frac{\sqrt{2}}{b}\right)$ (c) $B = (\pi - \varphi + \tan\varphi)$

$\mu_0 I/2\pi R = 28\mu T$.

D-4. (a) $\vec{B}=\frac{\mu_0 I}{4\pi R}(-\pi\hat{i}-2\hat{k})$

(b) $\vec{B}=\frac{\mu_0 I}{4R}\left[-\hat{i}\left(1+\frac{1}{\pi}\right)-\frac{\hat{k}}{\pi}\right]$

(c) $\vec{B}=\frac{\mu_0 I}{4\pi R}\left(-\hat{j}-\hat{k}\right)$

Section (E) :

E-3. (a) zero (b) $\frac{2\mu_0 i}{5\pi r}$

E-4. 2500 turns/m

E-5. 1V

E-6 (a) $\frac{K}{2n}$ (b) $\frac{\mu_0 K}{2}\sqrt{3}$, $\frac{\mu_0 K}{2}$

E-7. 0, μ_0 K towards right in the figure, 0

Section (F) :

F-1. 12 cm

F-2. D – electron, B – α-particle

F-3. $M^{-3} L^{-2} T^8 A^4$

F-4. (a) 4 A,
(b) (i) current directed into the plane of paper, 1 m from R on RQ (away from Q)
(ii) current directed out of from paper, 1m from R on RQ (between R and Q)

F-5. $(-75\hat{i} + 100\hat{j})$ m/s **F-6.** 3.0

F 7. (a) $\frac{mv}{qB}$ (b) $\pi/2$ (c) $\frac{\pi m}{2qB}$

(d) $\frac{mv}{qB}, \frac{3\pi}{2}, \frac{3\pi m}{2qB}$

F 8. (a) $\pi/2$ (b) $\pi/4$ (c) π **F 9.** 8 cm

F-10. 50 m/s **F-11.** $\frac{\mu_0 qrni}{2m}$

Section (G) :

G-1. 36 cm, $4\pi\sqrt{19}$ cm

G-2. $\frac{4}{\pi}\times 10^5$ m/s, 2×10^5 m/s

G-3. $15\pi \times 10^{-4}$ T

Section (H) :

H-1. 16×10^6 m/s, $\frac{91}{20}$ cm

H-2. (a) evB (b) vB (c) $vB\ell$

H-3. (a) $\frac{i}{\pi r^2 ne}$ (b) $\frac{iB}{\pi r^2 n}$ upwards in the figure

(c) $\frac{iB}{\pi r^2 ne}$ (d) $\frac{2iB}{\pi rne}$

H-4. $\frac{5}{4}\times 10^5$ C/kg

H-5. 5×10^3 N/C. 5×10^{-2} T

H-6. $\sqrt{\frac{2qE_0x}{m}}$

Section (I) :

I-1. 6×10^{-2}N perpendicular to both the wire and the field

I-2. 1×10^{-2}N on each wire, on da and cb towards left and on dc and ab downward.

I-3. 8×10^{-2} N **I-4.** $\sqrt{2}B_0 i\ell$

I-5. 5×10^{-4} T, horizontal and $\perp$ to the wire.

I-6. $i\lambda B/2$ **I-7.** $-iRB\,\hat{j}$

I-8. 0.12 **I-10.** $iB_0\,\ell$

I-11. 6×10^{-3} N/m, downward zero 6×10^{-3} N/m,

I-12. $\frac{\mu_0 i_1 i_2}{2\pi}\ell n\frac{r_2}{r_1} = 40\,\ell n2\,\mu$ J/m

I-13. $\frac{\mu mg}{i\ell\sqrt{1+\mu^2}}$

Section (J) :

J-1. $\pm 75\,\pi \times 10^{-3}$ J

J-2. (a) $2\pi aiB$, perpendicular to the plane of the figure going into it.
(b) $\sqrt{2}\pi ai\cdot B_0$

J-3 $\frac{1}{2}$T

J-4 (a) $4\pi \times 10^{-2}$ N-m (b) 60º

J-5 (a) zero (b) 2×10^{-2} N-m parallel to the side.

J-6. $\pi \times 10^{-2}$ N - m

J-7. (a) $\frac{iL^2B}{4\pi}$ (b) $\frac{BiL^2}{18}$ **J-8.** $\frac{1}{2}Q\omega R^2$

Section (K) :

K-1. $I_V = 1112 \times 10^{-4}$ A, $I_H = 556\sqrt{3}\times 10^{-4}$ A.

K-2. $3\sqrt{31}\times 10^{-5}$ T, $\tan^{-1}\left(\frac{5\sqrt{3}}{7}\right)$, East of north

K-3. (a) 1.0A,
(b) 2.0V perpendicular to the magnetic meridian and in vertical plane

Section (L) :

L-1. 7.5×10^{-4} N/A^2, 596

L-2. $\mu_r = 798.6$

PART - II

Section (A) :

A-1.	(B)	**A-2.**	(A)

Section (B) :

B-1.	(A)	**B-2.**	(B)	**B-3.**	(C)
B-4.	(C)	**B-5.**	(B)	**B-6.**	(C)

Section (C)

C-1.	(A)	**C-2.**	(B)	**C-3.**	(D)
C-4.	(D)	**C-5.**	(A)	**C-6.**	(B)
C-7.	(B)	**C-8.**	(C)	**C-9.**	(D)

Section (D)

D-1.	(B)	**D-2.**	(D)	**D-3.**	(C)
D-4.	(B)	**D-5.**	(D)	**D-6.**	(A)

Section (E) :

E- 1. (C)

Section (F) :

F-1.	(D)	**F-2.**	(B)

Section (G) :

G-1.	(C)	**G-2.**	(C)	**G-3.**	(B)
G-4.	(B)	**G-5.**	(D)	**G-6.**	(C)
G-7.	(C)	**G-8.**	(B)	**G-9.**	(A)

Section (H) :

H-1.	(B)	**H-2.**	(A)	**H-3.**	(B)

Section (I) :

I-1.	(C)	**I-2.**	(A)

Section (J) :

J-1.	(C)	**J-2.**	(C)	**J-3.**	(B)
J-4.	(B)	**J-5.**	(C)	**J-6.**	(C)
J-7.	(A)	**J-8.**	(A)	**J-9.**	(B)
J-10.	(B)				

PART - III

1. (A)$\rightarrow$p,q,r,t ; (B)$\rightarrow$p, q,r,s,t ; (C)$\rightarrow$r ;(D)$\rightarrow$p,q,r, s,t

2. (A)$\rightarrow$p,q,t ; (B)$\rightarrow$p, r ; (C) $\rightarrow$ p ; (D) $\rightarrow$p, q, s

3. (A) $\rightarrow$ r, (B) $\rightarrow$ q,u (C) $\rightarrow$ u, (D) $\rightarrow$t

EXERCISE-2

PART - I

1.	(B)	**2.**	(A)	**3.**	(C)
4.	(C)	**5.**	(A)	**6.**	(C)
7.	(A)	**8.**	(B)	**9.**	(B)
10.	(A)	**11.**	(A)	**12.**	(B)
13.	(C)	**14.**	(B)	**15.**	(D)
16.	(A)	**17.**	(C)	**18.**	(B)
19.	(B)	**20.**	(B)		

PART - II

1.	6	**2.**	2	**3.**	1
4.	3	**5.**	16	**6.**	32
7.	10	**8.**	25	**9.**	2
10.	2	**11.**	5	**12.**	4
13.	2	**14.**	4	**15.**	(a) 2 (b) 8
16.	4	**17.**	(a) 8	**18.**	(a) 3 (b) 4
19.	(a) 7		(b) 15	**20.**	(a) 20 (b) 2
21.	5				

PART - III

1.	(ABC)	**2.**	(CD)	**3.**	(AD)
4.	(BCD)	**5.**	(BC)	**6.**	(ABD)
7.	(ABC)	**8.**	(BD)	**9.**	(AC)
10.	(CD)	**11.**	(BD)	**12.**	(CD)
13.	(ACD)	**14.**	(BD)	**15.**	(AC)
16.	(BC)	**17.**	(AB)	**18.**	(AD)

PART - IV

1.	(B)	**2.**	(A)	**3.**	(A)
4.	(A)	**5.**	(C)	**6.**	(C)
7.	(A)	**8.**	(A)	**9.**	(D)
10.	(C)	**11.**	(B)	**12.**	(A)
13.	(D)				

EXERCISE-3

PART - I

1. (AC) **2.** (A)

3. (A) $\rightarrow$ (q), (r) ; (B) $\rightarrow$ (p); (C) $\rightarrow$ (q), (r) ; (D) $\rightarrow$ (q,s)

4. (ACD)

5. (A) $\rightarrow$ (p), (r), (s) ; (B) $\rightarrow$ (r), (s);
(C) $\rightarrow$ (p), (q), (t) ; (D) $\rightarrow$ (r), (s)

6.	7	**7.**	(C)	**8.**	(BD)
9.	(A)	**10.**	(CD)	**11.**	5
12.	(B)	**13.**	(D)	**14.**	(AC)
15.	(AD)	**16.**	3	**17.**	(C)
18.	(B)	**19.**	(ABC)	**20.**	(AD)
21.	(AC)	**22.**	(C)	**23.**	(A)
24.	(A)	**25.**	(A)	**26.**	(CD)
27.	(ABD)	**28.**	2.00		

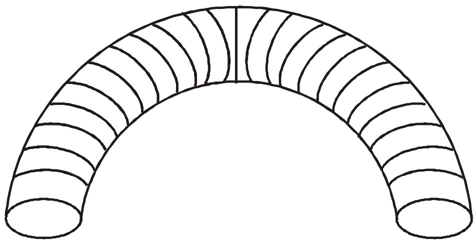
PART - II

1.	(3)	**2.**	(3)	**3.**	(2)
4.	(3)	**5.**	(3)	**6.**	(1)
7.	(1)	**8.**	(2)	**9.**	(1)
10.	(2)	**11.**	(1)	**12.**	(1)
13.	(1)	**14.**	(1)	**15.**	(1)
16.	(3)	**17.**	(1)	**18.**	(2)
19.	(2)	**20.**	(4)	**21.**	(2)
22.	(3)	**23.**	(3)	**24.**	(3)
25.	(2)	**26.**	(4)	**27.**	(1)

High Level Problems (HLP)

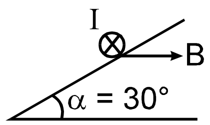
SUBJECTIVE QUESTIONS

1. Calculate the magnetic moment of a thin wire with a current $I = 0.8$ A, wound tightly on half a toroid (figure). The diameter of the cross-section of the tore is equal to d = 5.0 cm, the number of turns is N=500.



2. A small charged ball suspended on an inextensible thread of length L moves in a uniform time-independent vertically upward magnetic field of induction B. The mass of the ball is m and the charge is q, and angular speed is ω. Determine the radius of the horizontal circle in which the ball moves if the thread always remains taut. Take gravitational acceleration to be equal to g.

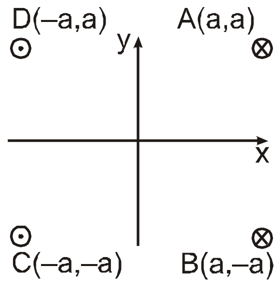
3. The figure shows a conductor of weight 1.0 N & length L = 0.5 m placed on a rough inclined plane making an angle 30^0 with the horizontal so that conductor is perpendicular to a uniform horizontal magnetic field of induction B = 0.10 T. The coefficient of static friction between the conductor and the plane is 0.1. A current of I = 10 A flows through the conductor inside the plane of this paper as shown. What is the force needed to be applied parallel to the inclined plane to keep the conductor at rest?



4. 4 long wires each carrying current I as shown in the figure are placed at the points A, B, C and D. Find the magnitude and direction of:

(a) magnetic field at the centre of the square

(b) force per metre acting on wire at point D.

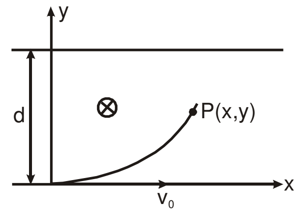


5. A uniformly charged ring of radius 0.1 m rotates at a frequency of 10^4 rps about its axis. Find the ratio of energy density of electric field to the energy density of the magnetic field at a point on the axis at distance 0.2 m from the centre. (Use speed of light $c = 3 \times 10^8$ m/s, $\pi^2 = 10$)

6. The current density $\vec{J}$ inside a long, solid, cylindrical wire of radius a = 12 mm is in the direction of the central axis, and its magnitude varies linearly with radial distance r from the axis according to $J = \frac{J_0 r}{a}$, where $J_0 = \frac{10^5}{4\pi}$ A/m^2. Find the magnitude of the magnetic field at $r = \frac{a}{2}$ in μT.

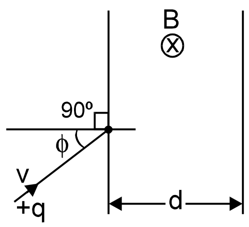
7. A neutral particle is at rest in a uniform magnetic field $\vec{B}$. At t = 0, particle decays into two particles each of mass ' m ' and one of them having charge ' q '. Both of these move off in separate paths lying in plane perpendicular to $\vec{B}$. At later time, the particles collide. Find this time of collision neglecting the interaction force.

8. A non-uniform magnetic field $\vec{B} = B_0\left(1+\frac{y}{d}\right)(-\hat{k})$ is present in region of space in between y = 0 & y = d. The lines are shown in the diagram. A particle of mass ' m ' and positive charge ' q ' is moving. Given an initial velocity $\vec{v} = v_0\hat{i}$. Find the components of velocity of the particle when it leaves the field.

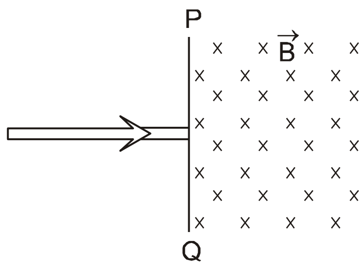


9. A neutral atom of atomic mass number 100 which is stationary at the origin in gravity free space emits an α-particle (A) in z-direction. The product ion is P. A uniform magnetic field exists in the x-direction. Disregard the electro magnetic interaction between A and P. If the angle of rotation of A after which A and P will meet for the first time is $\frac{n\pi}{25}$ radians, what is the value of n ?

10. In the figure shown a positively charged particle of charge 'q' and mass 'm' enters into a uniform magnetic field of strength 'B' as shown in the figure. The magnetic field points inwards and is present only within a region of width 'd'. The initial velocity of the particle is perpendicular to the magnetic field and ϕ = 30º. Find the time spent by the particle inside the magnetic field if d = 0.2m, B = 1T, q = 1C, m = 1kg and v = 1 m/s. Use sin 45° = 0.7 if required.



11. A thin beam of charged particles is incident normally on the boundary of a region containing a uniform magnetic field as shown. The beam comprise of mono energetic α and β⁻ particles, each possessing a kinetic energy T. Neglecting interaction between particles of the beam, find the separation between the points on line PQ where the α and β particles emerge out of the magnetic field. (express your answer in terms of T, B, α (2e, m_α) & β⁻ (–e, m_e), where all terms have their usual meanings.)

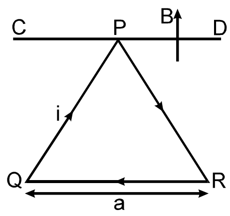


12. An electron moving with velocity $\vec{v}_1 = 1\hat{i}$ m/s at a point in a magnetic field experiences a force $\vec{F}_1 = -e\hat{j}N$, where e is the charge of electron. If the electron is moving with a velocity $\vec{v}_2 = \hat{i} - \hat{j}$ m/s at the same point, it experiences a force $\vec{F}_2 = -e(\hat{i} + \hat{j})$ N. Find the magnetic field and the force the electron would experience if it were moving with a velocity $\vec{v}_3 = \vec{v}_1 \times \vec{v}_2$ at the same point.

13. An electron is shot into one end of a solenoid. As it enters the uniform magnetic field within the solenoid, its speed is 800 m/s and its velocity vector makes an angle of 30° with the central axis of the solenoid. The solenoid carries 4.0 A current and has 8000 turn along its length. Find number of revolutions made by the electron within the solenoid by the time it emerges from the solenoid's opposite end. (Use charge to mass ratio $\frac{e}{m}$ for electron = $\sqrt{3} \times 10^{11}$ C/kg) Fill your answer in multiple of 10^3. (Neglect end effect)

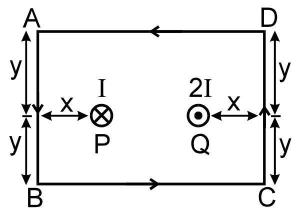
14. In the figure shown an infinitely long wire carries a current I_1 and another uniform rigid wire ACB (bent at 'C' at right angle) of mass 'm' carries a current I_2, it is hinged at corner 'C'. Find the angular acceleration of this wire ACB just after release. Is the direction of rotation clockwise or anticlockwise. (Assume gravity is absent) [Take $\ell n2 = 0.7$]

15. A loop PQR formed by three identical uniform conducting rods each of length ' a ' is suspended from one of its vertices (P) so that it can rotate about horizontal fixed smooth axis CD. Initially plane of loop is in vertical plane. A constant current ' i ' is flowing in the loop. Total mass of the loop is ' m '. At t = 0, a uniform magnetic field of strength B directed vertically upwards is switched on. Acceleration due to gravity is ' g '. Then find the minimum value of B so that the plane of the loop becomes horizontal (even for an instant) during its subsequent motion.



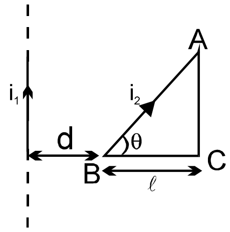
16. In order to impart an angular velocity to an earth satellite the geomagnetic field can be used. Find the maximum possible angular velocity about its own axis gained by the satellite if a storage battery with a capacity of Q = 5 Amp. hours is discharged suddenly through a coil of N = 20 turns wound around the satellite's surface along the circumference of the largest circle. The satellite has a mass of m = 10^3 kg and is a thin walled uniform sphere. The geomagnetic field is parallel to the winding plane and its flux density is B = 0.5 Gauss. (1 Gauss = 10^{-4} Tesla)

17. Let B_P and B_Q be the magnetic field produced by the wire P and Q which are placed symmetrically in a rectangular loop ABCD as shown in figure. Current in wire P is I directed inward and in Q is 2 I directed outwards.

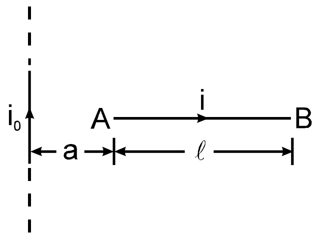


if $\int_A^B \vec{B}_Q.\vec{d\ell} = 2\mu_0$ tesla meter, $\int_D^A \vec{B}_P.\vec{d\ell} = -2\mu_0$ tesla meter & $\int_A^B \vec{B}_P.\vec{d\ell} = -\mu_0$ tesla meter. Then find the value of I

18. A triangular loop (ABC) having current i_2 and an infinite wire having current i_1 are placed in the same plane. Find the magnetic force of interaction between the infinite wire and the loop ABC.



19. A finite conductor AB carrying current i is placed near a fixed very long wire current carrying i_0 as shown in the figure. Find the point of application and magnitude of the net ampere force on the conductor AB. What happens to the conductor AB if it is free to move. (Neglect gravitational field)

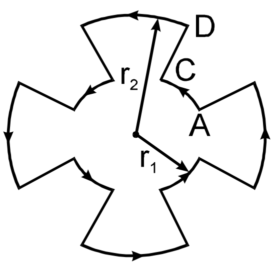


20. A particle of mass m and charge q is moving in a region where uniform, constant electric and magnetic $\vec{E}$ & $\vec{B}$ fields are present, $\vec{E}$ & $\vec{B}$ are parallel to each other. At time t = 0 the velocity of the particle is perpendicular to $\vec{E}$. (Assume that its speed is always <<c, the speed of light in vacuum). Find the velocity $\vec{v}$ of the particle at time t. You must express your answer in terms of t, q, m, the vectors $\vec{v}_0$, $\vec{E}$ & $\vec{B}$ and their magnitudes v_0, E and B. **[JEE-1998 Mains; 8 /200]**

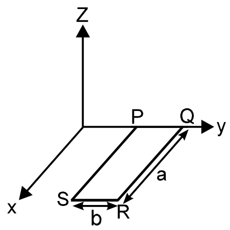
21. A current of 10 A flows around a closed path in a circuit which is in the horizontal plane. The circuit consists of eight alternating arcs of radii r_1 = 0.08 m and r_2 = 0.12 m. Each arc subtends the same angle at the centre. **[JEE 2001 (Mains) (10 marks)]**

(i) Find the magnetic field produced by this circuit at the centre.

(ii) An infinitely long straight wire carrying a current of 10 A is passing through the centre of the given circuit vertically with the direction of the current being into the plane of the circuit. What is the force acting on the wire at the centre due to the current in the circuit? What is the force acting on the arc AC and the straight segment CD due to the current in the central wire.



22. A rectangular loop PQRS made from a uniform wire has length a, width b and mass m. It is free to rotate about the arm PQ, which remains hinged along a horizontal line taken as the y-axis as shown in figure.Take the vertically upward direction as the z-axis. A uniform magnetic field $\vec{B} = \left(3\hat{i} + 4\hat{k}\right)B_0$ exists in the region. The loop is held in the x-y plane and a current I is passed through it. The loop is now released and is found to stay in the horizontal position in equilibrium. **[JEE 2002, 5/60 (Mains)]**

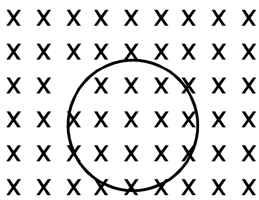


(a) What is the direction of the current I in PQ?

(b) Find the magnetic force on the arm RS.

(c) Find the expression for I in terms of B_0, a, b and m.

23. A ring of mass m and radius r is rotated in uniform magnetic field B which is perpendicular to the plane of the loop with constant angular velocity ω_0. Find the net ampere force on the ring and the tension developed in the ring if there is a current i in the ring. Current and rotation both are clockwise.



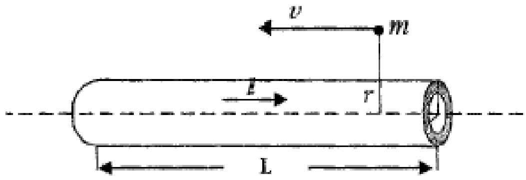
24. A positive charge particle of charge ' q ' & mass ' m ' is released at origin. There are uniform magnetic field and electric field in the space given by $\vec{E} = E_0\hat{j}$ & $\vec{B} = B_0\hat{k}$, where E_0 & B_0 are constants. Find the ' y ' co-ordinate of the particle at time ' t '.

25. A square loop of wire of edge a carries a current i. Magnetic induction B for a point on the axis of the loop at a distance $x = \frac{a}{\sqrt{2}}$ from its centre is $\frac{N\mu_0 i}{3\pi a}$, then find value of N.

26. A positive point charge q of mass m, kept at a distance x_0 (in the same plane) from a fixed very long straight current is projected normally away from it with speed v. Find the maximum separation between the wire and the particle.

27. Consider a solid sphere of radius r and mass m which has a charge q distributed uniformly over its volume. The sphere is rotated about a diameter with an angular speed ω. Find magnetic moment of the sphere.

28. A long non-magnetic cylindrical conductor with inner radius a and outer radius b carries a current I. The current density in the conductor is uniform. Assume end effects can be neglected. **[Olympiad 2013; Stage-2]**



(a) Prove by the Biot-Savart law that the magnetic field, if any, at a point is always tangential to the circle passing through the point, with the centre of the circles on the axis of the cylinder.

(b) Find the magnetic field due to the current as a function of radius.

(i) inside the hollow space ($r < a$)

(ii) within the conductor ($a < r < b$)

(iii) outside the conductor ($r > b$)

(c) A beam of particles, each with positive charge q and mass m travels with initial velocity v anti-parallel to the direction of the current. Assume that the length of cylinder is L. Find the deflection of a particle as a function of its initial distance r from the axis if $r > b$ when it goes from one end to other end. You can assume that the velocity is high enough such that the velocity is constant and the deflection is small.

HLP Answers

1. $\frac{1}{2}$ A.m^2

2. $r = \left[L^2 - \left\{\frac{mg}{m\omega^2 \pm q\omega B}\right\}^2\right]^{1/2}$

3. $\frac{3}{4}\left(1 - \frac{\sqrt{3}}{10}\right)$ N to $\frac{3}{4}\left(1 + \frac{\sqrt{3}}{10}\right)$ N

4. (a) $\frac{\mu_0}{4\pi}\left(\frac{4I}{a}\right)$ along Y-axis (b) $\frac{\mu_0 I^2(-\hat{j} - 3\hat{i})}{8\pi a}$

5. 9×10^9 J **6.** 10 **7.** $\frac{\pi m}{qB}$

8. $V_x = V_0 - \frac{3qB_0 d}{2m}$ and $V_y = \sqrt{V_0^2 - \left(V_0 - \frac{3q\ B_0}{2m} d\right)^2}$

9. 48 **10.** $\frac{\pi}{12}$ sec.

11. $\frac{2\sqrt{2T}}{eB}\left\{\frac{\sqrt{m_\alpha}}{2} + \sqrt{m_e}\right\}$

12. $\vec{B} = -\hat{k}, \vec{F} = 0$ **13.** 1600

14. $\frac{9\mu_0 I_1 I_2}{40\pi mL}$ a.c.w. **15.** $\frac{4mg}{3ia}$

16. $\omega = \frac{3}{2}\frac{BN\pi Q}{M} = 2.7\ \pi \times 10^{-2}$ rad/s.

17. 6A

18. $\frac{\mu_0 i_1 i_2 \tan\theta}{2\pi}\left[\ell n\frac{d+\ell}{d} - \frac{\ell}{d+\ell}\right]$ towards left.

19. $F = \frac{\mu_0}{2\pi}(i_0 i)\ell n\left(1 + \frac{\ell}{a}\right)$ in the direction of i_0.

$x = \frac{\ell}{\ell n\left(1 + \frac{\ell}{a}\right)}$, where x is the perpendicular distance from the wire i_0. It will move upward and will roted in the clockwise direction.

20. $\vec{v}_{(t)} = \cos\omega t\, \vec{v}_0 + \frac{(\sin\omega t)(\vec{V}_0 \times \vec{B}_0)}{B} + \frac{qt}{m}\vec{E}$

where $\omega = qB/m$

21. (i) $B_C = \frac{5\ \pi \times 10^{-4}}{24}$ T

(ii) F on central wire = 0 ; $F_{AC} = 0$;

$F_{CD} = 2 \times 10^{-5} \ln\left(\frac{3}{2}\right)$

22. (a) from P to Q (b) $IbB_0(3\hat{k} - 4\hat{i})$

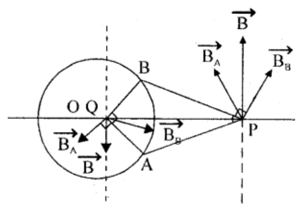
(c) $\frac{mg}{6bB_0}$

23. 0, $\frac{r}{2\pi}(m\omega_0^2 + 2\pi iB)$

24. $y = \frac{E_0 m}{qB_0^2}\left[1 - \cos\frac{q B_0}{m}t\right]$

25. N = 2 **27.** $\frac{1}{5}q\omega R^2$

28. (a) Consider a thin shell which is equivalent to a set of parallel long wires put around a circle. Consider two wires A and B which are equi-distant from OQP.



Let $\vec{B}_A$ be the magnetic field due to wire A and $\vec{B}_B$ be the magnetic field due to wire B. According to the Biot Savart law, both magnetic fields are equal in magnitude and their projections on the line OQP are also equal, but in opposite directions by geometry. The resultant magnetic field

$$\vec{B} = \vec{B}_A + \vec{B}_B$$

is always perpendicular to the line OQP and therefore always tangential to the circle through the point of observation.

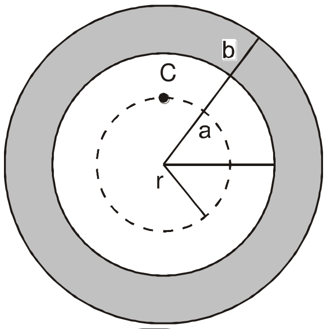
(b) (i) $r \le a$: Using Ampere's law

$\oint_c \vec{B}.d\vec{l} = \mu_0 i$

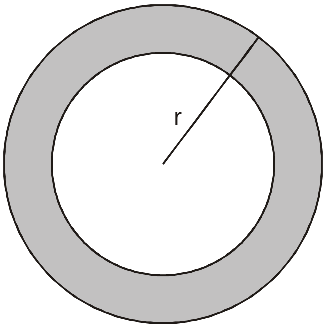
$B2\pi r = 0$

$B = 0$

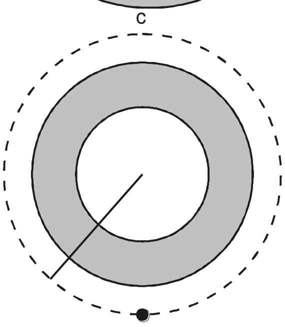
(i)



(ii)



(iii)



(ii) $a \le r \le b$

$$B2\pi r = \mu_0 I \frac{\pi\left(r^2 - a^2\right)}{\pi\left(b^2 - a^2\right)}$$

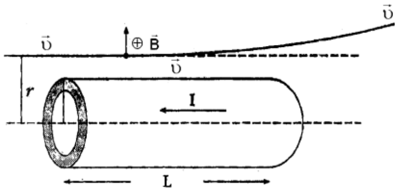
$$B = \frac{\mu_0 I\left(r^2 - a^2\right)}{2\pi r\left(b^2 - a^2\right)}$$

(iii) $r \ge b$:

$$B2\pi r = \mu_0 I$$

$$B = \frac{\mu_0 I}{2\pi r}$$

(c)



$$\vec{F} = q\left(\vec{\upsilon} \times \vec{B}\right);\ F_r = qvB = qv\frac{\mu_0 I}{2\pi r}$$

Impulse :

$$\int F_r dt = \frac{q\upsilon\mu_0 I}{2\pi}\int\frac{dt}{r} = \frac{q\upsilon\mu_0 I}{2\pi}\int\frac{dx}{r\upsilon} = \frac{q\mu_0 I}{2\pi}\int\frac{dx}{r} = \frac{q\mu_0 I}{2\pi}\frac{L}{r}$$

Change of momentum along

Radial direction :

$$P_r = \int F_r dt = \frac{q\mu_0 IL}{2\pi r}$$

Deflection :

$$\theta \approx \frac{P_r}{P} = \frac{q\mu_0 IL}{2\pi r m\upsilon} = \frac{\mu_0 IqL}{2\pi\, m\upsilon}\cdot\frac{1}{r}$$