WAVE OPTICS

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JEE (MAIN) SYLLABUS

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WAVE OPTICS

1. PRINCIPLE OF SUPERPOSITION

When two or more waves simultaneously pass through a point, the disturbance of the point is given by the sum of the disturbances each wave would produce in absence of the other wave(s). In case of wave on string disturbance means displacement, in case of sound wave it means pressure change, in case of Electromagnetic Waves. it is electric field or magnetic field. Superposition of two light travelling in almost same direction results in modification in the distribution of intensity of light in the region of superposition. This phenomenon is called interference.

1.1 Superposition of two sinusoidal waves :

Consider superposition of two sinusoidal waves (having same frequency), at a particular point.

Let, $x_1(t) = a_1 \sin \omega t$

and, $x_2(t) = a_2 \sin (\omega t + \phi)$

represent the displacement produced by each of the disturbances. Here we are assuming the displacements to be in the same direction. Now according to superposition principle, the resultant displacement will be given by,

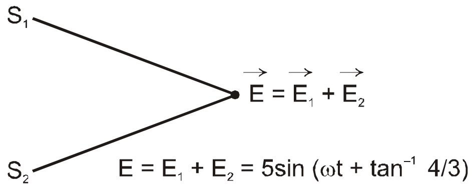
$$x(t) = x_1(t) + x_2(t)$$
$$= a_1 \sin \omega t + a_2 \sin (\omega t + \phi)$$
$$= A \sin (\omega t + \phi_0)$$

where $A^2 = a_1^2 + a_2^2 + 2a_1 . a_2 \cos \phi$ .... (1.1)

and $\tan \phi_0 = \dfrac{a_2 \sin\phi}{a_1 + a_2 \cos\phi}$ ....(1.2)

Solved Examples

Example 1. S_1 and S_2 are two sources of light which produce individually disturbance at point P given by $E_1 = 3\sin \omega t$, $E_2 = 4 \cos \omega t$. Assuming $\vec{E}_1$ & $\vec{E}_2$ to be along the same line, find the resultant after their superposition.



Solution : $E = 3 \sin\omega t + 4 \sin(\omega t + \frac{\pi}{2})$

$A^2 = 3^2 + 4^2 + 2(3)(4) \cos \frac{\pi}{2} = 5^2$

$\tan\phi_0 = \dfrac{4\sin\frac{\pi}{2}}{3 + 4\cos\frac{\pi}{2}} = \dfrac{4}{3} \Rightarrow \phi_0 = 53^\circ$

$E = 5\sin[\omega t + 53^\circ]$

1.2 SUPERPOSITION OF PROGRESSIVE WAVES; PATH DIFFERENCE :

Let S_1 and S_2 be two sources producing progressive waves (disturbance travelling in space given by y_1 and y_2)

At point P,

$$y_1 = a_1 \sin (\omega t - kx_1 + \theta_1)$$
$$y_2 = a_2 \sin (\omega t - kx_2 + \theta_2)$$
$$y = y_1 + y_2 = A \sin(\omega t + \Delta\phi)$$

Here, the phase difference,

$$\Delta\phi = (\omega t - kx_1 + \theta_1) - (\omega t - kx_2 + \theta_2)$$
$$= k(x_2 - x_1) + (\theta_1 - \theta_2) = k\Delta p - \Delta\theta$$

where $\Delta\theta = \theta_2 - \theta_1$

Here $\Delta p = \Delta x$ is the path difference

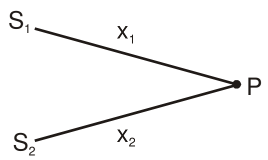


Figure: 1.3

Clearly, phase difference due to path difference = k (path difference)

where k = $\frac{2\pi}{\lambda}$

$\Rightarrow \quad \Delta\phi = k\Delta p = \frac{2\pi}{\lambda}\ \Delta x$ ..... (1.3)

For Constructive Interference :

$\Delta\phi = 2n\pi$, $n = 0, 1, 2$ .... or, $\Delta x = n\lambda$

$A_{max} = A_1 + A_2$

Intensity, $\sqrt{I_{max}} = \sqrt{I_1} + \sqrt{I_2} \Rightarrow \qquad I_{max} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2$ ... (1.4)

For Destructive interference :

$\Delta\phi = (2n + 1)\pi$, $n = 0, 1, 2$ ....

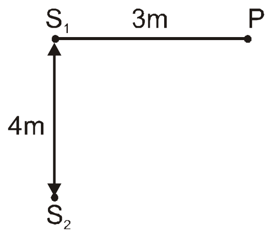
or, $\Delta x = (2n + 1)\lambda/2$

$A_{min} = |A_1 - A_2|$

Intensity, $\sqrt{I_{min}} = \left|\sqrt{I_1} - \sqrt{I_2}\right| \Rightarrow \qquad I_{min.} = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2$ ... (1.5)

Solved Examples

Example 2.



S_1 and S_2 are two coherent sources of frequency 'f' each. ($\theta_1 = \theta_2 = 0^{\circ}$) $V_{sound} = 330$m/s.

(i) so that constructive interference at 'p'

(ii) so that destructive interference at 'p'

Solution : For constructive interference $K\Delta x = 2n\pi$

$\frac{2\pi}{\lambda} \times 2 = 2n\pi$

$\lambda = \frac{2}{n}$; $V = \lambda f \qquad \Rightarrow \qquad V = \frac{2}{n} f$

$f = \frac{330}{2} \times n$

For destructive interference

$K\Delta x = (2n + 1)\pi$

$\frac{2\pi}{\lambda} . 2 = (2n + 1)\pi$

$\frac{1}{\lambda} = \frac{(2n+1)}{4}$; $f = \frac{V}{\lambda} = \frac{330 \times (2n+1)}{4}$

Example 3. Light from two sources, each of same frequency and travelling in same direction, but with intensity in the ratio 4 : 1 interfere. Find ratio of maximum to minimum intensity.

Solution : $\frac{I_{max}}{I_{min}} = \left(\frac{\sqrt{I_1} + \sqrt{I_2}}{\sqrt{I_1} - \sqrt{I_2}}\right)^2 = \left(\frac{\sqrt{\frac{I_1}{I_2}} + 1}{\sqrt{\frac{I_1}{I_2}} - 1}\right)^2 = \left(\frac{2+1}{2-1}\right)^2 = 9 : 1.$

2. WAVEFRONTS

Consider a wave spreading out on the surface of water after a stone is thrown in. Every point on the surface oscillates. At any time, a photograph of the surface would show circular rings on which the disturbance is maximum. Clearly, all points on such a circle are oscillating in phase because they are at the same distance from the source. Such a locus of points which oscillate in phase is an example of a wavefront.

A wavefront is defined as a surface of constant phase. The speed with which the wavefront moves outwards from the source is called the wave speed. The energy of the wave travels in a direction perpendicular to the wavefront.

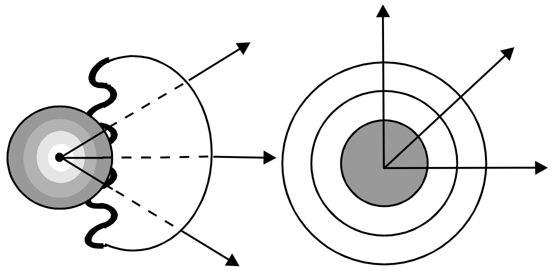
Figure (2.1a) shows light waves from a point source forming a spherical wavefront in three dimensional space. The energy travels outwards along straight lines emerging from the source. i.e., radii of the spherical wavefront. These lines are the rays. Notice that when we measure the spacing between a pair of wavefronts along any ray, the result is a constant. This example illustrates two important general principles which we will use later:

(i) Rays are perpendicular to wavefronts.

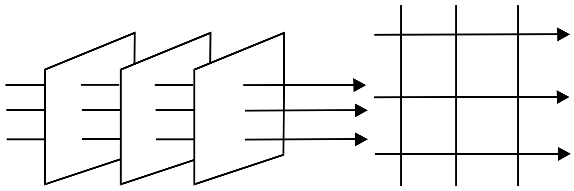
(ii) The time taken by light to travel from one wavefront to another is the same along any ray.

If we look at a small portion of a spherical wave, far away from the source, then the wavefronts are like parallel planes. The rays are parallel lines perpendicular to the wavefronts. This is called a plane wave and is also sketched in Figure (2.1b)

A linear source such as a slit illuminated by another source behind it will give rise to cylindrical wavefronts. Again, at larger distance from the source, these wavefronts may be regarded as planar.



(a)



(b)

Figure : 2.1 : Wavefronts and the corresponding rays in two cases: (a) diverging spherical wave. (b) plane wave. The figure on the left shows a wave (e.g.. light) in three dimensions. The figure on the right shows a wave in two dimensions (a water surface).

3. COHERENCE :

Two sources which vibrate with a fixed phase difference between them are said to be coherent. The phase differences between light coming form such sources does not depend on time.

In a conventional light source, however, light comes from a large number of individual atoms, each atom emitting a pulse lasting for about 1 ns. Even if atoms were emitting under similar conditions, waves from different atoms would differ in their initial phases. Consequently light coming from two such sources have a fixed phase relationship for about 1ns, hence interference pattern will keep changing every billionth of a second. The eye can notice intensity changes which lasts at least one tenth of a second. Hence we will observe uniform intensity on the screen which is the sum of the two individual intensities. Such sources are said to be incoherent. Light beam coming from two such independent sources do not have any fixed phase relationship and they do not produce any stationary interference pattern. For such sources, resultant intensity at any point is given by

$$I = I_1 + I_2 \qquad ...... (3.1)$$

4. YOUNG'S DOUBLE SLIT EXPERIMENT (Y.D.S.E.)

In 1802 Thomas Young devised a method to produce a stationary interference pattern. This was based upon division of a single wavefront into two; these two wavefronts acted as if they emanated from two sources having a fixed phase relationship. Hence when they were allowed to interfere, stationary interference pattern was observed.

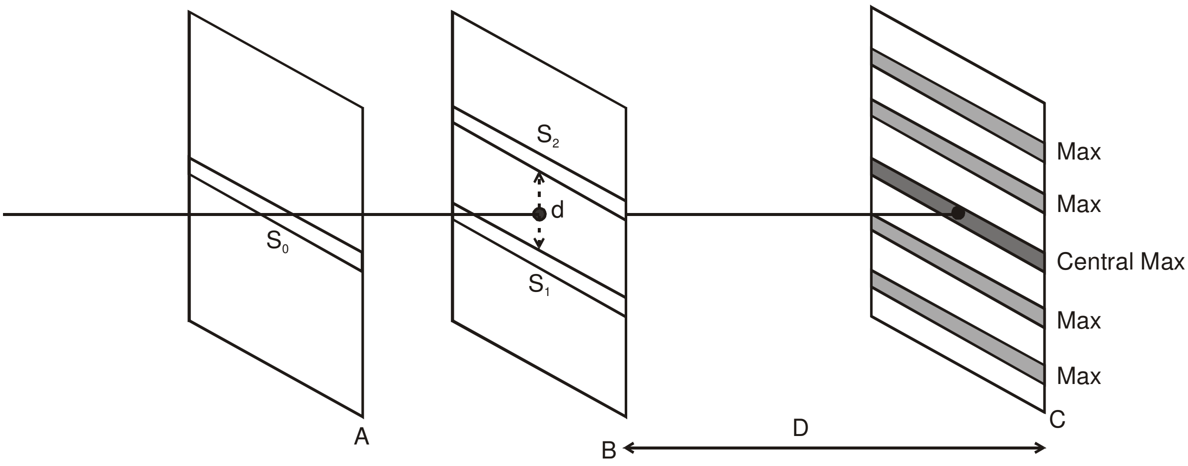


Figure : 4.1 : Young's Arrangement to produce stationary interference pattern by division of wave front S_0 into S_1 and S_2

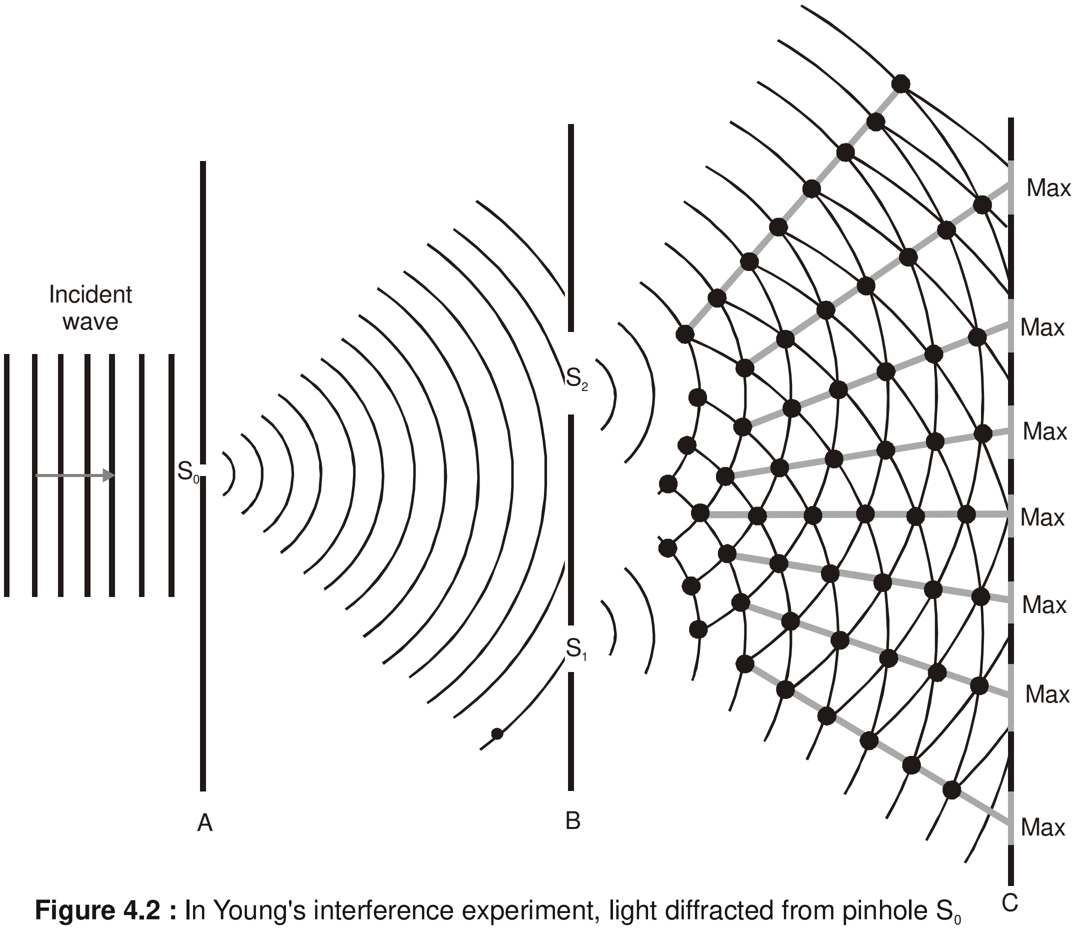


Figure 4.2 : In Young's interference experiment, light diffracted from pinhole S_0 excounters pinholes S1 and S2 in screen B. Light diffracted from these two pinholes overlaps in the region between screen B and viewing screen C, producing an interference pattern on screen C.

4.1 Analysis of Interference Pattern

We have insured in the above arrangement that the light wave passing through S_1 is in phase with that passing through S_2. However the wave reaching P from S_2 may not be in phase with the wave reaching P from S_1, because the latter must travel a longer path to reach P than the former. We have already discussed the phase-difference arising due to path difference. If the path difference is equal to zero or is an integral multiple of wavelengths, the arriving waves are exactly in phase and undergo constructive interference.

If the path difference is an odd multiple of half a wavelength, the arriving waves are out of phase and undergo fully destructive interference. Thus, it is the path difference Δx, which determines the intensity at a point P.

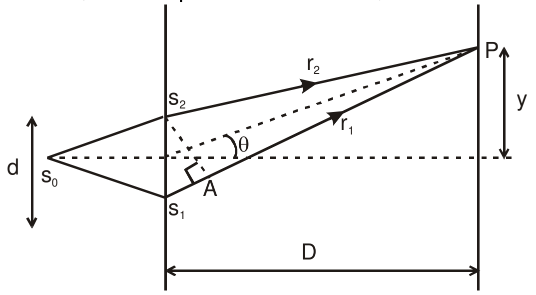


Figure : 4.3

Path difference $\Delta p = S_1P - S_2P = \sqrt{\left(y+\frac{d}{2}\right)^2 + D^2} - \sqrt{\left(y-\frac{d}{2}\right)^2 + D^2}$ .....(4.1)

Approximation I : For D >> d, we can approximate rays $_1$ and $_2$ as being approximately parallel, at angle θ to the principle axis.

Now, $S_1P - S_2P = S_1A = S_1S_2 \sin\theta$

$\Rightarrow$ path difference = d sin θ .....(4.2)

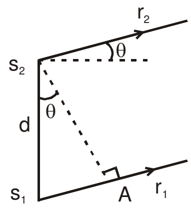


Figure : 4.4

Approximation II : Further if θ is small, i.e., y << D, $\sin\theta = \tan\theta = \frac{y}{D}$

and hence, path difference = $\frac{dy}{D}$ .....(4.3)

for maxima (constructive interference),

$\Delta p = \frac{d.y}{D} = n\lambda \quad \Rightarrow \quad y = \frac{n\lambda D}{d}$, n = 0, ± 1, ± 2, ± 3 .....(4.4)

Here n = 0 corresponds to the central maxima

n = ±1 correspond to the 1st maxima

n = ±2 correspond to the 2nd maxima and so on.

for minima (destructive interference).

$$\Delta p = \pm\frac{\lambda}{2}, \pm\frac{3\lambda}{2} \pm\frac{5\lambda}{2} \quad \Rightarrow \quad \Delta p = \begin{cases} (2n-1)\frac{\lambda}{2} & n = 1, 2, 3............ \\ (2n+1)\frac{\lambda}{2} & n = -1, -2, -3........ \end{cases}$$

consequently, $$y = \begin{cases} (2n-1)\frac{\lambda D}{2d} & n = 1, 2, 3............ \\ (2n+1)\frac{\lambda D}{2d} & n = -1, -2, -3....... \end{cases}$$ ...... (4.5)

Here n = ± 1 corresponds to first minima,

n = ± 2 corresponds to second minima and so on.

4.2 Fringe width :

It is the distance between two maxima of successive order on one side of the central maxima. This is also equal to distance between two successive minima.

fringe width $\beta = \dfrac{\lambda D}{d}$ ...... (4.6)

- Notice that it is directly proportional to wavelength and inversely proportional to the distance between the two slits.

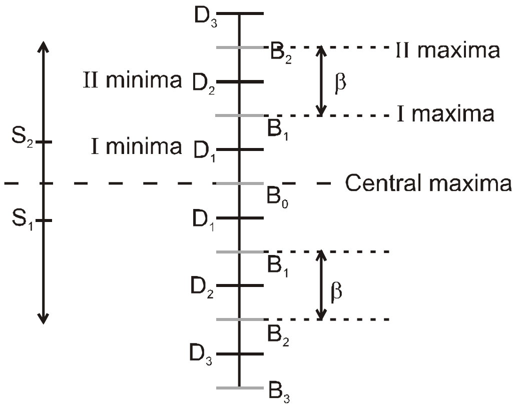


Figure : 4.5 fringe pattern in YDSE

4.3 Maximum order of Interference Fringes :

In section 4.1 we obtained, $y = \dfrac{n\lambda D}{d}$, n = 0, ± 1, ± 2 .....

for interference maxima, but n cannot take infinitely large values, as that would violate the approximation (II) i.e., θ is small or $y << D$

$\Rightarrow \quad \dfrac{y}{D} = \dfrac{n\lambda}{d} << 1$

Hence the above formula (4.4 & 4.5) for interference maxima/minima are applicable when

$n << \dfrac{d}{\lambda}$

when n becomes comparable to $\dfrac{d}{\lambda}$ path difference can no longer be given by equation (4.3) but by (4.2)

Hence for maxima

$\Delta p = n\lambda \qquad \Rightarrow \quad d\sin\theta = n\lambda \qquad \Rightarrow \quad n = \dfrac{d\sin\theta}{\lambda}$

Hence highest order of interference maxima, $n_{max} = \left[\dfrac{d}{\lambda}\right]$ ..... (4.7)

where [ ] represents the greatest integer function.
Similarly highest order of interference minima,

$n_{min} = \left[\dfrac{d}{\lambda} + \dfrac{1}{2}\right]$ .... (4.8)

Alter

$\Delta p = S_1P - S_2P$

$\Delta p \le d \quad \Rightarrow \quad \Delta p_{max} = d$

(3rd side of a triangle is always greater than the difference in length of the other two sides)

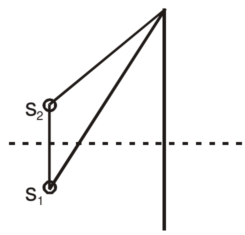


Figure : 4.6

4.4 Intensity :

Suppose the electric field components of the light waves arriving at point P(in the Figure : 4.3) from the two slits S_1 and S_2 vary with time as

$E_1 = E_0 \sin \omega t$ and $E_2 = E_0 \sin (\omega t + \phi)$

Here $\phi = k\Delta x = \dfrac{2\pi}{\lambda} \Delta x$

and we have assumed that intensity of the two slits S_1 and S_2 are same (say I_0); hence waves have same amplitude E_0.

then the resultant electric field at point P is given by,

$E = E_1 + E_2 = E_0 \sin \omega t + E_0 \sin (\omega t + \phi) = E_0' \sin (\omega t + \phi')$

where $E_0'^2 = E_0^2 + E_0^2 + 2E_0 . E_0 \cos \phi = 4 E_0^2 \cos^2 \phi/2$

Hence the resultant intensity at point P,

$$I = 4I_0 \cos^2 \frac{\phi}{2} \qquad ....(4.9)$$

$I_{max} = 4I_0$ when $\dfrac{\phi}{2} = n\pi$, $\quad n = 0, \pm1, \pm2,.......,$

$I_{min} = 0$ when $\dfrac{\phi}{2} = \left(n - \dfrac{1}{2}\right)\pi \quad n = 0, \pm1, \pm 2 ....$

Here $\phi = k\Delta x = \dfrac{2\pi}{\lambda} \Delta x$

If $D >> d$, $\quad \phi = \dfrac{2\pi}{\lambda} d \sin \theta$

If $D >> d$ & $y << D$, $\phi = \dfrac{2\pi}{\lambda} d \dfrac{y}{D}$

However if the two slits were of different intensities I_1 and I_2,

say $E_1 = E_{01} \sin \omega t$ and $E_2 = E_{02} \sin (\omega t + \phi)$

then resultant field at point P,

$E = E_1 + E_2 = E_0 \sin (\omega t + \phi)$

where $E_0^2 = E_{01}^2 + E_{O2}^2 + 2E_{01} E_{02} \cos \phi$

Hence resultant intensity at point P,

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos \phi \qquad ...... (4.10)$$

Solved Example

Example 4. In a YDSE, D = 1m, d = 1mm and λ = 1/2 mm

(i) Find the distance between the first and central maxima on the screen.

(ii) Find the no of maxima and minima obtained on the screen.

Solution : (i) $D >> d$

Hence $\Delta P = d \sin \theta \qquad \dfrac{d}{\lambda} = 2,$

clearly, $n << \dfrac{d}{\lambda} = 2$ is not possible for any value of n.

Hence $\Delta p = \dfrac{dy}{D}$ cannot be used

for Ist maxima,

$\Delta p = d \sin \theta = \lambda$

$\Rightarrow \sin \theta = \dfrac{\lambda}{d} = \dfrac{1}{2}$

$\Rightarrow \theta = 30º$

Hence, $y = D \tan \theta = \dfrac{1}{\sqrt{3}}$ meter

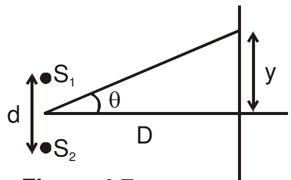


Figure 4.7

(ii) Maximum path difference $\Delta P_{max} = d = 1$ mm

$\Rightarrow$ Highest order maxima, $n_{max} = \left[\frac{d}{\lambda}\right] = 2$

and highest order minima $n_{min} = \left[\frac{d}{\lambda} + \frac{1}{2}\right] = 2$

Total no. of maxima = $2n_{max} + 1^* = 5$ *(central maxima).
Total no. of minima = $2n_{min} = 4$

Example 5. Monochromatic light of wavelength 5000 Aº is used in Y.D.S.E., with slit-width, d = 1mm, distance between screen and slits, D = 1m. If intensity at the two slits are, $I_1 = 4I_0$, $I_2 = I_0$, find
(i) fringe width β
(ii) distance of 5th minima from the central maxima on the screen
(iii) Intensity at $y = \frac{1}{3}$ mm
(iv) Distance of the 1000th maxima from the central maxima on the screen.
(v) Distance of the 5000th maxima from the central maxima on the screen.

Solution : (i) $\beta = \frac{\lambda D}{d} = \frac{5000 \times 10^{-10} \times 1}{1 \times 10^{-3}} = 0.5$ mm

(ii) $y = (2n - 1)\frac{\lambda D}{2d}$, n = 5

$\Rightarrow$ y = 2.25 mm

(iii) At $y = \frac{1}{3}$ mm, $y << D$ Hence $\Delta p = \frac{d.y}{D}$

$\Delta\phi = \frac{2\pi}{\lambda}\Delta p = 2\pi\frac{dy}{\lambda D} = \frac{4\pi}{3}$

Now resultant intensity

$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\Delta\phi = 4I_0 + I_0 + 2\sqrt{4I_0^2}\cos\Delta\phi = 5I_0 + 4I_0\cos\frac{4\pi}{3} = 3I_0$

(iv) $\frac{d}{\lambda} = \frac{10^{-3}}{0.5 \times 10^{-6}} = 2000$

n = 1000 is not << 2000

Hence now $\Delta p = d \sin\theta$ must be used

Hence, $d\sin\theta = n\lambda = 1000\lambda \Rightarrow \sin\theta = 1000\frac{\lambda}{d} = \frac{1}{2} \Rightarrow \theta = 30^\circ$

$y = D\tan\theta =$ meter

(v) Highest order maxima

$n_{max} = \left[\frac{d}{\lambda}\right] = 2000$. Hence, n = 5000 is not possible.

5. SHAPE OF INTERFERENCE FRINGES IN YDSE

We discuss the shape of fringes when two pinholes are used instead of the two slits in YDSE.

Fringes are locus of points which move in such a way that its path difference from the two slits remains constant.

$S_2P - S_1P = \Delta =$ constant ....(5.1)

If $\Delta = \pm\frac{\lambda}{2}$, the fringe represents 1st minima.

If $\Delta = \pm\frac{3\lambda}{2}$ it represents 2nd minima

If $\Delta = 0$ it represents central maxima,

If $\Delta = \pm\lambda$, it represents 1st maxima etc.

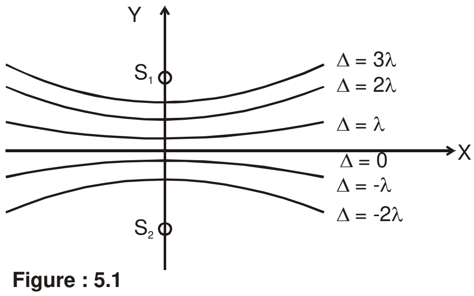


Figure : 5.1

Equation (5.1) represents a hyperbola with its two foci at S_1 and S_2

The interference pattern which we get on screen is the section of hyperboloid of revolution when we revolve the hyperbola about the axis S_1S_2.

A. If the screen is perpendicular to the X axis, i.e. in the YZ plane, as is generally the case, fringes are hyperbolic with a straight central section.

B. If the screen is in the XY plane, again fringes are hyperbolic.

C. If screen is perpendicular to Y axis (along S_1S_2), ie in the XZ plane, fringes are concentric circles with center on the axis S_1S_2; the central fringe is bright if $S_1S_2 = n\lambda$ and dark if $S_1S_2 = (2n-1)\dfrac{\lambda}{2}$.

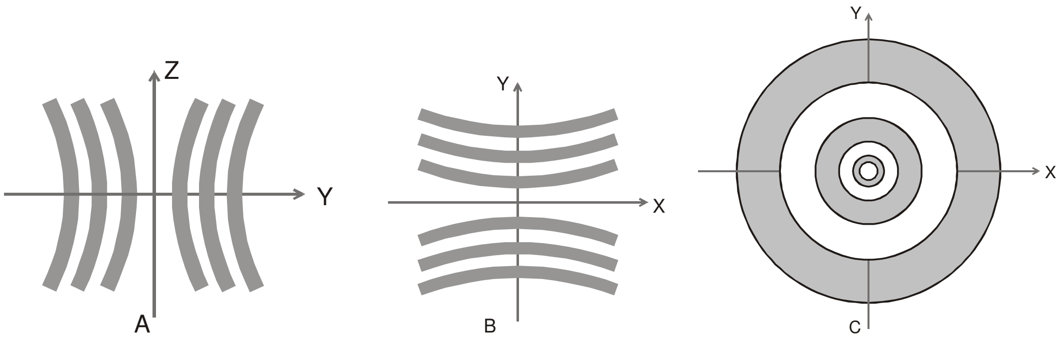


Figure : 5.2

6. YDSE WITH WHITE LIGHT

The central maxima will be white because all wavelengths will constructively interference here. However slightly below (or above) the position of central maxima fringes will be coloured. For example if P is a point on the screen such that $S_2P - S_1P = \dfrac{\lambda_{violet}}{2} = 190$ nm,

completely destructive interference will occur for violet light. Hence we will have a line devoid of violet colour that will appear reddish. And if $S_2P - S_1P = \dfrac{\lambda_{red}}{2}$; 350 nm,

completely destructive interference for red light results and the line at this position will be violet. The coloured fringes disappear at points far away from the central white fringe; for these points there are so many wavelengths which interfere constructively, that we obtain a uniform white illumination. for example if

$S_2P - S_1P = 3000$ nm,

then constructive interference will occur for wavelengths $\lambda = \dfrac{3000}{n}$ nm. In the visible region these wavelength are 750 nm (red), 600 nm (yellow), 500 nm (greenish–yellow), 428.6 nm (violet). Clearly such a light will appear white to the unaided eye.

Thus with white light we get a white central fringe at the point of zero path difference, followed by a few coloured fringes on its both sides, the color soon fading off to a uniform white.

In the usual interference pattern with a monochromatic source, a large number of identical interference fringes are obtained and it is usually not possible to determine the position of central maxima. Interference with white light is used to determine the position of central maxima in such cases.

Solved Example

Example 6. A beam of light consisting of wavelengths 6000Å and 4500Å is used in a YDSE with D = 1m and d = 1 mm. Find the least distance from the central maxima, where bright fringes due to the two wavelengths coincide.

Solution: $\beta_1 = \dfrac{\lambda_1 D}{d} = \dfrac{6000\times10^{-10}\times1}{10^{-3}} = 0.6$ mm ; $\beta_2 = \dfrac{\lambda_2 D}{d} = 0.45$ mm

Let n_1th maxima of λ_1 and n_2th maxima of λ_2 coincide at a position y.

then, $y = n_1\ \beta_1 = n_2\ \beta_2$ = LCM of β_1 and β_2

$\Rightarrow$ y = LCM of 0.6 mm and 0.45 mm

y = 1.8 mm **Ans.**

At this point 3rd maxima for 6000 Å & 4th maxima for 4500 Å coincide

Example 7. White light is used in a YDSE with D = 1m and d = 0.9 mm. Light reaching the screen at position y = 1 mm is passed through a prism and its spectrum is obtained. Find the missing lines in the visible region of this spectrum.

Solution : $\Delta p = \frac{yd}{D} = 9 \times 10^{-4} \times 1 \times 10^{-3}$ m = 900 nm

for minima $\Delta p = (2n - 1)\lambda/2$

$\Rightarrow \lambda = \frac{2\Delta P}{(2n-1)} = \frac{1800}{(2n-1)} = \frac{1800}{1}, \frac{1800}{3}, \frac{1800}{5}, \frac{1800}{7}$ ....

of these 600 nm and 360 nm lie in the visible range. Hence these will be missing lines in the visible spectrum.

7. GEOMETRICAL PATH & OPTICAL PATH

Actual distance travelled by light in a medium is called geometrical path (Δx). Consider a light wave given by the equation

$E = E_0 \sin(\omega t - kx + \phi)$

If the light travels by Δx, its phase changes by $k\Delta x = \frac{\omega}{v}\Delta x$, where ω, the frequency of light does not depend on the medium, but v, the speed of light depends on the medium as $v = \frac{c}{\mu}$.

Consequently, change in phase $\Delta\phi = k\Delta x = \frac{\omega}{c}(\mu\Delta x)$

It is clear that a wave travelling a distance Δx in a medium of refractive index μ suffers the same phase change as when it travels a distance $\mu\Delta x$ in vacuum. i.e., a path length of Δx in medium of refractive index μ is equivalent to a path length of $\mu\Delta x$ in vacuum.

The quantity $\mu\Delta x$ is called the optical path length of light, Δx_{opt}. And in terms of optical path length, phase difference would be given by,

$$\Delta\phi = \frac{\omega}{c}\Delta x_{opt} = \frac{2\pi}{\lambda_0}\Delta x_{opt} \qquad .... (7.1)$$

where λ_0 = wavelength of light in vacuum. However in terms of the geometrical path length Δx,

$$\Delta\phi = \frac{\omega}{c}(\mu\Delta x) = \frac{2\pi}{\lambda}\Delta x \qquad .....(7.2)$$

where λ = wavelength of light in the medium ($\lambda = \frac{\lambda_0}{\mu}$).

7.1 Displacement of fringe : On introduction of a glass slab in the path of the light coming out of the slits– On introduction of the thin glass-slab of thickness t and refractive index μ, the optical path of the ray S_1P increases by $t(\mu - 1)$. Now the path difference between waves coming form S_1 and S_2 at any point P is

$\Delta p = S_2P - (S_1P + t(\mu - 1)) = (S_2P - S_1P) - t(\mu - 1)$

$\Rightarrow \Delta p = d\sin\theta - t(\mu - 1)$ if d << D

and $\Delta p = \frac{yd}{D} - t(\mu - 1)$ If y << D as well.

for central bright fringe, $\Delta p = 0 \Rightarrow \frac{yd}{D} = t(\mu - 1)$.

$\Rightarrow y = OO' = (\mu - 1)t\frac{D}{d} = (\mu - 1)\, t \,.\, \frac{\beta}{\lambda}$

The whole fringe pattern gets shifted by the same distance

$\Delta = (\mu - 1)\,.\,\frac{D}{d} = (\mu - 1)t\,\frac{\beta}{\lambda}$.

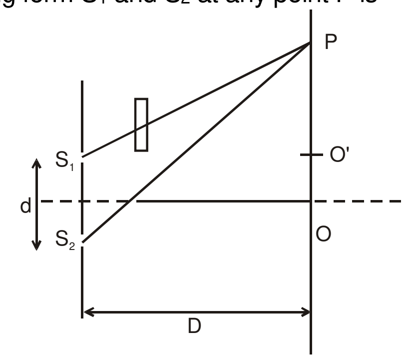


Figure : 7.1

* Notice that this shift is in the direction of the slit before which the glass slab is placed. If the glass slab is placed before the upper slit, the fringe pattern gets shifted upwards and if the glass slab is placed before the lower slit the fringe pattern gets shifted downwards.

Solved Examples

Example 8. In a YDSE with d = 1mm and D = 1m, slabs of (t = 1μm, μ = 3) and (t = 0.5 μm, μ = 2) are introduced in front of upper and lower slit respectively. Find the shift in the fringe pattern.

Solution : Optical path for light coming from upper slit S_1 is

$S_1P + 1\mu m\,(2-1) = S_2P + 0.5\ \mu m$

Similarly optical path for light coming from S_2 is

$S_2P + 0.5\ \mu m\,(2-1) = S_2P + 0.5\ \mu m$

Path difference : $\Delta p = (S_2P + 0.5\ \mu m) - (S_1P + 2\mu m) = (S_2P - S_1P) - 1.5\ \mu m. = \frac{yd}{D} - 1.5\ \mu m$

for central bright fringe $\Delta p = 0$

$\Rightarrow \quad y = \frac{1.5\mu m}{1mm} \times 1m = 1.5\ mm.$

The whole pattern is shifted by 1.5 mm upwards. **Ans.**

8. YDSE WITH OBLIQUE INCIDENCE

In YDSE, ray is incident on the slit at an inclination of θ_0 to the axis of symmetry of the experimental set-up for points above the central point on the screen, (say for P_1)

$\Delta p = d\sin\theta_0 + (S_2P_1 - S_1P_1)$

$\Rightarrow \quad \Delta p = d\sin\theta_0 + d\sin\theta_1 \qquad$ (If d << D)

and for points below O on the screen, (say for P_2)

$\Delta p = |(d\sin\theta_0 + S_2P_2) - S_1P_2| = |d\sin\theta_0 - (S_1P_2 - S_2P_2)|$

$\Rightarrow \quad \Delta p = |d\sin\theta_0 - d\sin\theta_2| \qquad$ (if d << D)

We obtain central maxima at a point where, $\Delta p = 0$.

$(d\sin\theta_0 - d\sin\theta_2) = 0 \qquad$ or $\qquad \theta_2 = \theta_0$.

This corresponds to the point O' in the diagram. Hence we have finally for path difference.

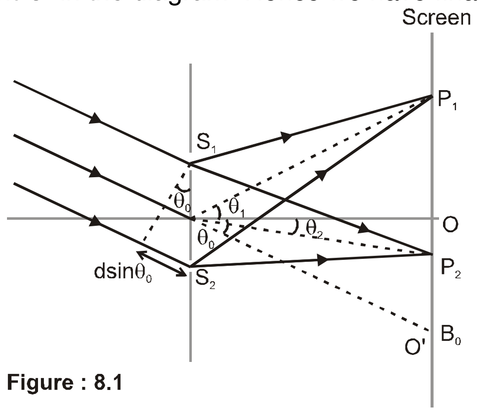


Figure : 8.1

$$\Delta p = \begin{cases} d(\sin\theta_0 + \sin\theta) - \text{for points above O} \\ d(\sin\theta_0 - \sin\theta) - \text{for points between O\&O'} \\ d(\sin\theta - \sin\theta_0) - \text{for points below O'} \end{cases} \qquad ...\ (8.1)$$

Solved Example

Example 9. In YDSE with D = 1m, d = 1mm, light of wavelength 500 nm is incident at an angle of 0.57º w.r.t. the axis of symmetry of the experimental set up. If centre of symmetry of screen is O as shown.

(i) find the position of central maxima

(ii) Intensity at point O in terms of intensity of central maxima I_0.

(iii) Number of maxima lying between O and the central maxima.

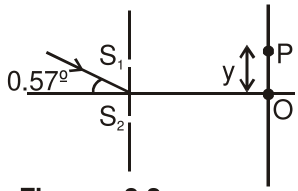


Figure : 8.2

Solution : (i) $\theta = \theta_0 = 0.57^{\circ}$

$\Rightarrow$ $y = -D \tan\theta - D\theta = -1 \text{ meter} \times \left(\frac{0.57}{57}\text{rad}\right)$

$\Rightarrow$ $y = -1\text{cm}.$

(ii) for point 0, $\theta = 0$

Hence, $\Delta p = d \sin\theta_0$; $d\theta_0 = 1 \text{ mm} \times (10^{-2} \text{ rad}) = 10{,}000 \text{ nm} = 20 \times (500 \text{ nm})$

$\Rightarrow$ $\Delta p = 20\,\lambda$

Hence point O corresponds to 20th maxima

$\Rightarrow$ intensity at O = I_0

(iii) 19 maxima lie between central maxima and O, excluding maxima at O and central maxima.

9. THIN-FILM INTERFERENCE

In YDSE we obtained two coherent source from a single (incoherent) source by division of wave-front. Here we do the same by division of Amplitude (into reflected and refracted wave).

When a plane wave (parallel rays) is incident normally on a thin film of uniform thickness d then waves reflected from the upper surface interfere with waves reflected from the lower surface.

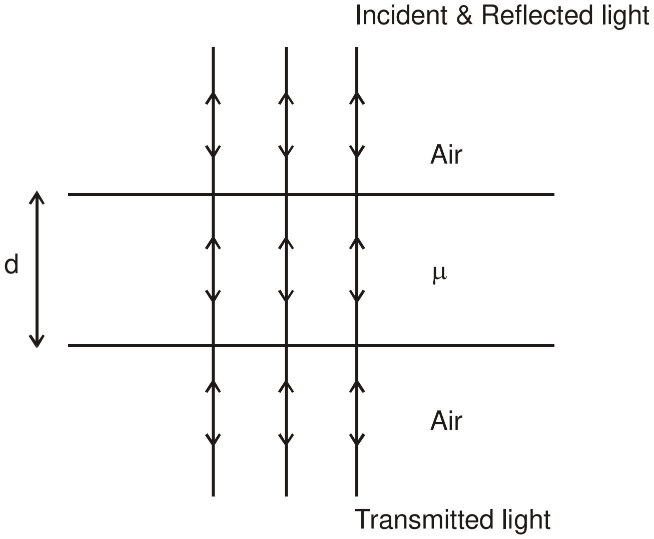


Figure : 9.1

Clearly the wave reflected from the lower surface travel an extra optical path of $2\mu d$, where μ is refractive index of the film.

Further if the film is placed in air the wave reflected from the upper surface (from a denser medium) suffers a sudden phase change of π, while the wave reflected from the lower surface (from a rarer medium) suffers no such phase change.

Consequently condition for constructive and destructive interference in the reflected light is given by,

$2\mu d = n\lambda$ for destructive interference

and $2\mu d = (n +)\lambda$ for constructive interference ....(9.1)

where $n = 0, 1, 2$ ....,

and λ = wavelength in free space.

Interference will also occur in the transmitted light and here condition of constructive and destructive interference will be the reverse of (9.1)

$$\text{i.e. } 2\mu d = \begin{cases} n\lambda & \text{for constructive interference} \\ (n + \frac{1}{2}\lambda) & \text{for destructive interference} \end{cases} \qquad ....(9.2)$$

This can easily be explained by energy conservation (when intensity is maximum in reflected light it has to be minimum in transmitted light) However the amplitude of the directly transmitted wave and the wave transmitted after one reflection differ substantially and hence the fringe contrast in transmitted light is poor. It is for this reason that thin film interference is generally viewed only in the reflected light.

In deriving equation (9.1) we assumed that the medium surrounding the thin film on both sides is rarer compared to the medium of thin film.
If medium on both sides are denser, then there is no sudden phase change in the wave reflected from the upper surface, but there is a sudden phase change of π in waves reflected from the lower surface. The conditions for constructive and destructive interference in reflected light would still be given by equation 9.1.
However if medium on one side of the film in denser and that on the other side is rarer, then either there is no sudden phase in any reflection, or there is a sudden phase change of π in both reflection from upper and lower surface. Now the condition for constructive and destructive interference in the reflected light would be given by equation 9.2 and not equation 9.1.

Solved Example

Example 10. White light, with a uniform intensity across the visible wavelength range 430–690 nm, is perpendicularly incident on a water film, of index of refraction μ = 1.33 and thickness d = 320 nm, that is suspended in air. At what wavelength λ is the light reflected by the film brightest to an observer?

Solution : This situation is like that of Figure (9.1), for which equation (9.1) gives the interference maxima. Solving for λ and inserting the given data, we obtain

$$\lambda = \frac{2\mu d}{m+1/2} = \frac{(2)(1.33)(320\text{nm})}{m+1/2} = \frac{851\text{nm}}{m+1/2}$$

for m = 0, this give us λ = 1700 nm, which is in the infrared region. For m = 1, we find λ = 567 nm, which is yellow-green light, near the middle of the visible spectrum. For m = 2, λ = 340 nm, which is in the ultraviolet region. So the wavelength at which the light seen by the observer is brightest is λ = 567 nm. **Ans.**

Example 11. A glass lens is coated on one side with a thin film of magnesium fluoride (MgF_2) to reduce reflection from the lens surface (figure). The index of refraction of MgF_2 is 1.38; that of the glass is 1.50. What is the least coating thickness that eliminates (via interference) the reflections at the middle of the visible spectrum (λ = 550 nm)? Assume the light is approximately perpendicular to the lens surface.

Solution : The situation here differs from figure (9.1) in that $n_3 > n_2 > n_1$. The reflection at point a still introduces a phase difference of π but now the reflection at point b also does the same (see figure 9.2). Unwanted reflections from glass can be, suppressed (at a chosen wavelength) by coating the glass with a thin transparent film of magnesium fluoride of a properly chosen thickness which introduces a phase change of half a wavelength. For this, the path length difference 2L within the film must be equal to an odd number of half wavelengths:

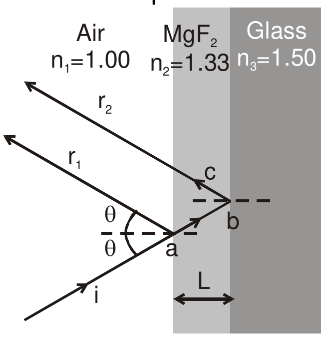


Figure : 9.2

$2L = (m + 1/2)\lambda_{n2}$, or, with $\lambda_{n2} = \lambda/n_2$, $2n_2 L = (m + 1/2)\lambda$.

We want the least thickness for the coating, that is, the smallest L. Thus we choose m = 0, the smallest value of m. Solving for L and inserting the given data, we obtain

$$L = \frac{\lambda}{4n_2} = \frac{550\text{nm}}{(4)(1.38)} = 96.6 \text{ nm}$$ **Ans.**

10 HUYGENS' CONSTRUCTION

Huygens, the Dutch physicist and astronomer of the seventeenth century, gave a beautiful geometrical description of wave propagation. We can guess that he must have seen water waves many times in the canals of his native place Holland. A stick placed in water and oscillated up and down becomes a source of waves. Since the surface of water is two dimensional, the resulting wavefronts would be circles instead of spheres. At each point on such a circle, the water level moves up and down. Huygens' idea is that we can think of every such oscillating point on a wavefront as a new source of waves. According to Huygens' principle, what we observe is the result of adding up the waves from all these different sources. These are called secondary waves or wavelets.

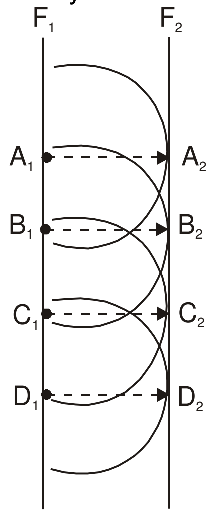


Figure : 10.1

Huygens' principle is illustrated in (Figure : 10.1) in the simple case of a plane wave.

(i) At time t = 0, we have a wavefront F_1, F_1 separates those parts of the medium which are undisturbed from those where the wave has already reached.

(ii) Each point on F_1 acts like a new source and sends out a spherical wave. After a time 't' each of these will have radius vt. These spheres are the secondary wavelets.

(iii) After a time t, the disturbance would now have reached all points within the region covered by all these secondary waves. The boundary of this region is the new wavefront F_2. Notice that F_2 is a surface tangent to all the spheres. It is called the forward envelope of these secondary wavelets.

(iv) The secondary wavelet from the point A_1 on F_1 touches F_2 at A_2. Draw the line connecting any point A_1 on F_1 to the corresponding point A_2 on F_2. According to Huygens, A_1 A_2 is a ray. It is perpendicular to the wavefronts F_1 and F_2 and has length vt. This implies that rays are perpendicular to wavefronts. Further, the time taken for light to travel between two wavefronts Is the same along any ray. In our example, the speed 'v' of the wave has been taken to be the same at all points in the medium. In this case, we can say that the distance between two wavefronts is the same measured along any ray.

(v) This geometrical construction can be repeated starting with F_2 to get the next wavefront F_3 a time t later, and so on. This is known as Huygens' construction.

Huygens' construction can be understood physically for waves in a material medium, like the surface of water. Each oscillating particle can set its neighbors into oscillation, and therefore acts as a secondary source. But what if there is no medium, as for light travelling in vacuum? The mathematical theory, which cannot be given here, shows that the same geometrical construction works in this case as well.

10.1 REFLECTION AND REFRACTION.

We can use a modified form of Huygens' construction to understand reflection and refraction of light. Figure (10.2a) shows an incident wavefront which makes an angle 'i' with the surface separating two media, for example, air and water. The phase speeds in the two media are v_1 and v_2. We can see that when the point A on the incident wavefront strikes the surface, the point B still has to travel a distance BC = AC sin i, and this takes a time t = BC/v_1 = AC (sin i) / v_1. After a time t, a secondary wavefront of radius v_2t with A as centre would have travelled into medium 2. The secondary wavefront with C as centre would have just started, i.e. would have zero radius. We also show a secondary wavelet originating from a point D in between A and C. Its radius is less than v_2t. The wavefront in medium 2 is thus a line passing through C and tangent to the circle centred on A. We can see that the angle r' made by this refracted wavefront with the surface is given by AE = v_2t = AC sin r'. Hence, t = AC (sin r'} / v_2. Equating the two expressions for 't' gives us the law of refraction in the form sin i/sin r' = v_1/v_2. A similar picture is drawn in Fig. (10.2 b) for the reflected wave which travels back into medium 1. In this case, we denote the angle made by the reflected wavefront with the surface by r, and we find that i = r. Notice that for both reflection and refraction, we use secondary wavelets starting at different times. Compare this with the earlier application (Fig.10.1) where we start them at the same time.

The preceding argument gives a good physical picture of how the refracted and reflected waves are built up from secondary wavelets. We can also understand the laws of reflection and refraction using the concept that the time taken by light to travel along different rays from one wavefront to another must be the same. (Fig.) Shows the incident and reflected wavefronts when a parallel beam of light falls on a plane surface. One ray POQ is shown normal to both the reflected and incident wavefronts. The angle of incidence i and the angle of reflection r are defined as the angles made by the incident and reflected rays with the normal. As shown in Fig. (c), these are also the angles between the wavefront and the surface.

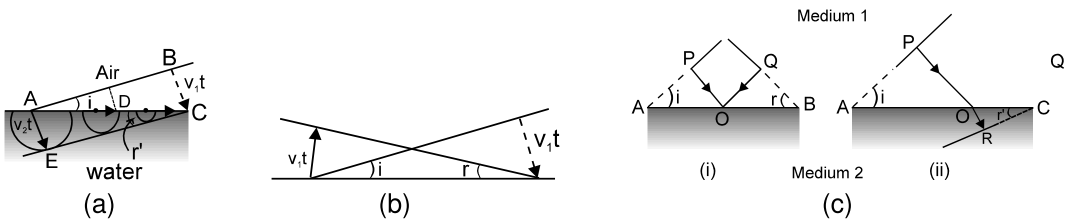


Figure : 10.2

(Fig.) (a) Huygens' construction for the (a) refracted wave. (b) Reflected wave. (c) Calculation of propagation time between wavefronts in (i) reflection and (ii) refraction.

We now calculate the total time to go from one wavefront to another along the rays. From Fig. (c), we have, we have Total time for light to reach from P to Q

$$= \frac{PO}{v_1} + \frac{OQ}{v_1} = \frac{AO\sin i}{v_1} + \frac{OB\sin r}{v_1} = \frac{OA\sin i + (AB - OA)\sin r}{v_1} = \frac{AB\sin r + OA(\sin i - \sin r)}{v_1}$$

Different rays normal to the incident wavefront strike the surface at different points O and hence have different values of OA. Since the time should be the same for all the rays, the right side of equation must actually be Independent of OA. The condition, for this to happen is that the coefficient of OA in Eq. (should be zero, i.e., sin i = sin r. We, thus have the law of reflection, i = r. Figure also shows refraction at a plane surface separating medium 1 (speed of light v_1) from medium 2 (speed of light v_2). The incident and refracted wavefronts are shown, making angles i and r' with the boundary. Angle r' is called the angle of refraction. Rays perpendicular to these are also drawn. As before, let us calculate the time taken to travel between the two wavefronts along any ray.

Time taken from P to R = $\frac{PO}{v_1} + \frac{OR}{v_2}$

$$= \frac{OA\sin i}{v_1} + \frac{(AC - OA)\sin r'}{v_2} = \frac{AC\sin\ r'}{v_2} + OA\left(\frac{\sin i}{v_1} - \frac{\sin r'}{v_2}\right)$$

This time should again be independent of which ray we consider. The coefficient of OA in Equation is, therefore, zero,. That is, $\frac{\sin i}{\sin r'} = \frac{v_1}{v_2} = n_{21}$

where n_{21} is the refractive index of medium 2 with respect to medium 1. This is the Snell's law of, refraction that we have already dealt with from Eq. n_{21} is the ratio of speed of light in the first medium

(v_1) to that in the second medium (v_2). Equation is, known as the Snell's law of refraction. If the first medium is vacuum, we have $\frac{\sin i}{\sin r'} = \frac{c}{v_2} = n_2$

where n_2 is the refractive index of medium 2 with respect to vacuum, also called the absolute refractive index of the medium. A similar equation defines absolute refractive index n_1 of the first medium. From Eq. we then get $n_{21} = \frac{v_1}{v_2} = \left(\frac{c}{n_1}\right) / \left(\frac{c}{n_2}\right) = \frac{n_2}{n_1}$

The absolute refractive index of air is about 1.0003, quite close to 1. Hence, for all practical purposes, absolute refractive index of a medium may be taken with respect to air. For water, n_1 = 1.33, which means $v_1 = \frac{c}{1.33}$, i.e. about 0.75 times the speed of light in vacuum. The measurement of the speed of light in water by Foucault (1850) confirmed this prediction of the wave theory.

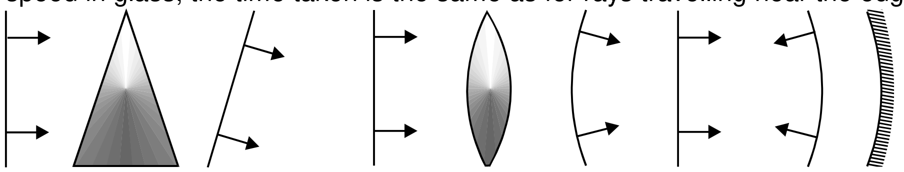
Once we have the laws of reflection and refraction, the behaviour of prisms. lenses, and mirrors can be understood. These topics are discussed in detail in the previous Chapter. Here we just describe the behaviour of the wavefronts in these three cases (Fig.)

(i) Consider a plane wave passing through a thin prism. Clearly, the portion of the incoming wavefront which travels through the greatest thickness of glass has been delayed the most. Since light travels more slowly in glass. This explains the tilt in the emerging wavefront.

(ii) Similarly, the central part of an incident plane wave traverses the thickest portion of a convex lens and is delayed the most. The emerging wavefront has a depression at the centre. It is spherical and converges to a focus,

(iii) A concave mirror produces a similar effect. The centre of the wavefront has to travel a greater distance before and after getting reflected, when compared to the edge. This again produces a converging spherical wavefront.

(iv) Concave lenses and convex mirrors can be understood from time delay arguments in a similar manner. One interesting property which is obvious from the pictures of wavefronts is that the total time taken from a point on the object to the corresponding point on the image is the same measured along any ray (Fig.). For example, when a convex lens focuses light to form a real image, it may seem that rays going through the centre are shorter. But because of the slower speed in glass, the time taken is the same as for rays travelling near the edge of the lens.



(a) (b) (c)

Figure : 10.3

11. RESOLVING POWER (R.P.)

A large number of images are formed as consequence of light diffraction of from a source. If two sources are separated such that their central maxima do not overlap, their images can be distinguished and are said to be resolved R.P. of an optical instrument is its ability to distinguish two neighbouring points.

Linear R.P. $d / \lambda D$ here D = Observed distance

Angular R.P. d / λ d = Distance between two points,

(1) Microscope : In reference to a microscope, the minimum distance between two lines at which they are just distinct is called Resolving limit (RL) and it's reciprocal is called Resolving power (RPO)

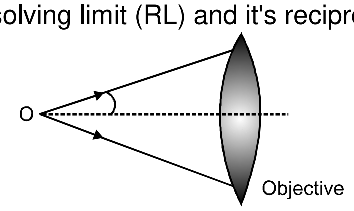


Figure : 11.1

R.L. = $\frac{\lambda}{2\mu\sin\theta}$ and R.P. = $\frac{2\mu\sin\theta}{\lambda}$ $\Rightarrow$ R.P. $\propto \frac{1}{\lambda}$

λ = Wavelength of light used to illuminate the object

μ = Refractive index of the medium between object and objective.

θ = Half angle of the cone of light from the point object, $\mu \sin\theta$ = Numerical aperture.

(2) Astronomical telescope : The ability of an astronomical telescope to form separate and magnified image of two neighbouring astronomical objects is called its resolving power.

The least distance between two neighbouring objects for which telescope can form separate images is called the limit of resolution.

The angular limit of resolution of a telescope is given by $\theta = \frac{1.22\lambda}{d}$

where λ = wavelength of light used

d = diameter of aperture of objective lens.

Resolving power is the reciprocal of limit of resolution.

$\therefore$ Resolving power = $\frac{d}{1.22\lambda}$

(i) As resolving power $\propto$ d. Therefore, resolving power of the telescope increases on increasing diameter of the aperture of the objects lens

(ii) As resolving power $\propto \frac{1}{\lambda}$ therefore, resolving power of the telescope decreases on increasing the wavelength of the light used

(iii) Resolving power of a telescope is independent of the focal length of the objective lens. Hence, on increasing the focal length of the objective lens, resolving power remains unchanged.

Solved Miscellaneous Problems

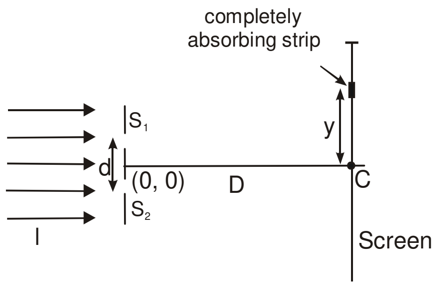
Problem 1. Two light waves are given by, $E_1 = 2\sin(100\pi t - kx + 30^\circ)$ and $E_2 = 3\cos(200\pi t - k'x + 60^\circ)$. The ratio of intensity of first wave to that of second wave is :

(A) $\frac{2}{3}$ (B) $\frac{4}{9}$ (C) $\frac{1}{9}$ (D) $\frac{1}{3}$

Answer : **(B)**

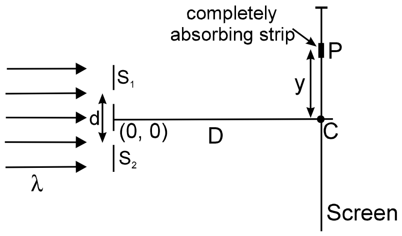
Solution : $I \propto A^2$ $\therefore$ $\frac{I_1}{I_2} = \frac{2^2}{3^2} = 4/9$

Problem 2. Figure shows two, identical narrow slits S_1 and S_2. A very small completely absorbing strip is placed at distance 'y' from the point C. 'C' is the point on the screen equidistant from S_1 and S_2. Assume $\lambda << d << D$ where λ, d and D have usual meaning. When S_2 is covered the force due to light acting on strip is 'f' and when both slits are opened the force acting on strip is 2f. Find the minimum positive 'y' (<< D) coordinate of the strip in terms of λ, d and D.



Solution : Let I_0 be the intensity at P due to individual slits S_1 or S_2. When waves due to both the sources superpose at P, the net intensity at P is

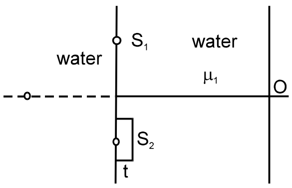
$I = 4I_0 \cos^2 \frac{\pi y}{\beta}$ where $\beta = \frac{\lambda D}{d}$. Since force on strip at P is twice due to individual source.



Hence $I = 2I_0$

$\therefore \quad \cos^2 \frac{\pi y}{\beta} = \frac{1}{2}$ or $\frac{\pi y}{\beta} = \frac{\pi}{4}$ or $y = \frac{\beta}{4} = \frac{\lambda D}{4d}$ **Ans.**

Problem 3. A Young's double slit experiment is conducted in water (μ_1) as shown in the figure, and a glass plate of thickness t and refractive index μ_2 is placed in the path of S_2. The magnitude of the phase difference at O is : (Assume that 'λ' is the wavelength of light in air). O is symmetrical w.r.t. S_1 and S_2.



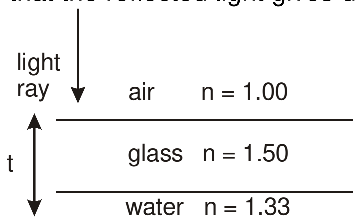
(A) $\left|\left(\frac{\mu_2}{\mu_1} - 1\right)t\right| \frac{2\pi}{\lambda}$ (B) $\left|\left(\frac{\mu_1}{\mu_2} - 1\right)t\right| \frac{2\pi}{\lambda}$ (C) $|((\mu_2 - \mu_1)t| \frac{2\pi}{\lambda}$ (D) $|(\mu_2 - 1)t| \frac{2\pi}{\lambda}$

Answer : **(C)**

Solution : Here path difference will be : $\Delta x = (\mu_2 - \mu_1)t \quad \Rightarrow \quad \delta = \frac{2\pi}{\lambda}(\mu_2 - \mu_1)t$

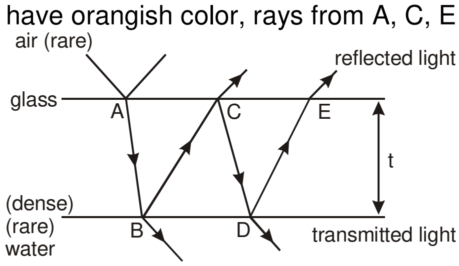
Hence (C)

Problem 4. A light ray is incident normal to a thin layer of glass. In the given figure, what is the minimum thickness of the glass so that the reflected light gives destructive interference (λ_{air} = 600 nm) ?



(A) 50 nm (B) 100 nm (C) 150 nm (D*) 200 nm

Solution : For reflected light to have orangish color, rays from A, C, E must be out of phase for λ = 600



or $\delta = (2n - 1)\frac{\lambda}{2}$ or $2\mu_g t - \frac{\lambda}{2} = (2n - 1)\frac{\lambda}{2}$

or $t = \frac{n\lambda}{2\mu_g}$ or $t_{min} = \frac{\lambda}{2\mu_g} = 200$ nm

Problem 5. Plane of transmission of two polarids are perpendicular to each other. If one of them is rotated through 60º, then what percentage of the incident unpolarised light will be transmitted at final emergence by the polaroids ?

(A) 50% (B) 62.5 % (C) 37.5% (D) 12.5 %

Answer : **(C)**

Solution : Initially the polaroids are crossed to each other, that is the angle between their polarising directions is 90º. When one is rotated through 60º, then the angle between their polarising directions directions will become 30º.

Let the intensity of the incident unpolarised light = I_0

This light is plane polarised and passes through the second polaroid.

The intensity of light emerging form the second polaroid is $I_2 = I_1 \cos^2\theta$

θ = the angle between the polarising directions of the two polaroids.

$I_1 = \frac{1}{2}I_0$ and $\theta = 30º$ so $I_2 = I_1 \cos^2 30º \Rightarrow \frac{I_2}{I_0} = \frac{3}{8}$

$\therefore$ transmission percentage = $\frac{I_2}{I_0} \times 100 = \frac{3}{8} \times 100 = 37.5\%$

Problem 6. **(a)** The diameter of pupil of eye varies from person to person and also depends on illumination. However, assuming its average value to be 2 mm, find the angular limit of resolution of human eye for a wavelength 550 nm. Also assume that resolution is limited only by diffraction.

(b) Using the information obtained above, find the maximum distance at which the eye will be able to resolve the two headlights of an approaching car which are 1.5 m apart.

Solution : **(a)** For diffraction at a circular aperture, angular limit of resolution can be expressed as,

$\theta_R = 1.22 \frac{\lambda}{d}$ [d is the (diameter) of the aperture]

Here $\lambda = 550 \text{ nm} = 550 \times 10^{-9}$ m

$d = 2\text{mm} = 2 \times 10^{-3}$ m

$\therefore \quad \theta_R = 1.22 \times \frac{550 \times 10^{-9}}{2 \times 10^{-3}}$ radian $= 3.35 \times 10^{-4}$ radian $= 1.92 \times 10^{-2}$ degree

$\therefore \quad \theta_R = 1.15'$ (minute)

(b) Angle subtended at the eye by the separation between the headlights,

$\theta \approx \frac{1.5}{D}$ [for small θ]

The headlights cannot be resolved if

$\theta < \theta_R$

or if $\frac{1.5}{D} < 3.35 \times 10^{-4}$

[$\theta_R = 3.35 \times 10^{-4}$ radian. In SI system, angle has to be measured in radian.]

$\therefore \quad D > \frac{1.5}{3.35 \times 10^{-4}} \quad \Rightarrow \quad D > 4477.6$ m

Hence maximum distance, for which resolution is possible is 4477.6 m.

Problem 7. A laser beam of 10 mW power and wavelength 7000Å has aperture 3mm. If it is focused by a lens of focal length 5 cm, calculate the area and intensity of image.

Solution : According to theory of diffraction at circular aperture

$$\theta = 1.22\frac{\lambda}{d} = 1.22 \times \frac{7 \times 10^{-7}}{3 \times 10^{-3}} = 2.85 \times 10^{-4} \text{rad}$$

Now, if r is the radius of image formed by the lens at its focus, $\theta = (r/f)$

$r = f\theta = (5 \times 10^{-2}) \times (2.85 \times 10^{-4}) = 14.25 \times 10^{-6}$ m

and so, $A = \pi r^2 = 3.14\,(14.25 \times 10^{-6})^2 \simeq 6.4 \times 10^{-10}$ m^2

and so, $I = \frac{E}{St} = \frac{P}{S} = \frac{10 \times 10^{-3}}{6.4 \times 10^{-10}} = 15.6 \frac{\text{MW}}{\text{m}^2} = 1.56 \frac{\text{kW}}{\text{cm}^2}$

Problem 8. A mixture of plane polarized and unpolarised light falls normally on a polarizing sheet. On rotating the polarizing sheet about the direction of the incident beam, the (transmitted intensity) varies by a factor of 4. Find the ratio of the intensities I_p and I_0, repectively, of the polarized and unpolarised components in the incident beam. Next the axis of the polarising sheet is fixed at an angle of 45° with the direction when the transmitted intensity is maximum. Then obtain the total intensity of the transmitted beam in terms of I_0.

Solution : The transmitted intensity of unpolarised light will be constant for all the orientations of the polarsised sheet whereas intensity of polarised light will be expressed by the law of Malus,

$I'_p = I_p.\cos^2\theta$

Let us consider the intensity of transmitted, polarized and unpolarised component to be I'_p and I'_0 respectively.

For $\theta = \pi/2$, $I'_p = 0$ and for $\theta = 0°$, $I'_p = I_p$ while for all orientations $I'_0 = \frac{I_0}{2}$

From given condition, $I_{max} = I_P + \frac{I_0}{2}$ when $\theta = 0°$ and $I_{min} = \frac{I_0}{2}$ when $\theta = \pi/2$

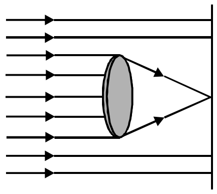
$\therefore \quad I_P + \frac{I_0}{2} = 4 \cdot \frac{I_0}{2}$ or $I_P = \frac{3I_0}{2}$ i.e., $\frac{I_P}{I_0} = \frac{3}{2}$

For $\theta = 45°$, $I = I_P \cos^2 45° + \frac{I_0}{2} = \frac{3I_0}{2} \times \frac{1}{2} + \frac{I_0}{2} = \frac{5I_0}{4}$ **Ans.**

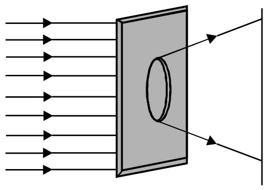
HANDOUT DIFFRACTION AND POLARISATION OF LIGHT

DIFFRACTION OF LIGHT

Bending of light rays from sharp edges of an opaque obstacle or aperture and its spreading in the eometricle shaddow region is defined as diffraction of light or deviation of light from its rectilinear propogation tendency is defined as diffraction of light.



diffraction from obstacle

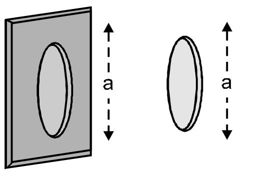


diffraction from aperture

- Diffraction was discovered by Grimaldi
- Theoritically explained by Fresnel
- Diffraction is possible in all type of waves means in mechanical or electromagnetic waves shows diffraction.

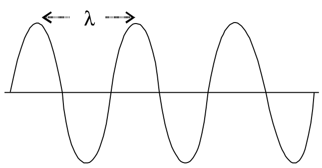
Diffraction depends on two factors :

(i) Size of obstacles or aperture



aperture obstacle

(ii) Wave length of the wave



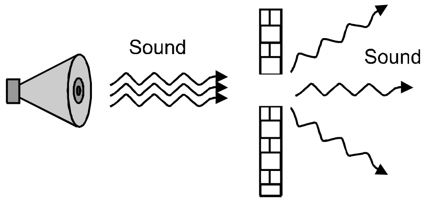
Condition of diffraction. Size of obstacle or aperture should be nearly equal to the wave length of light

$$\lambda \simeq a \qquad \frac{a}{\lambda} \simeq 1$$

If size of obstacle is much greater then wave length of light, the rectilinear motion of light is observed.

- It is practically observed when size of aperture or obstacle is greater than 50 λ then obstacle or aperture does not shows diffraction.
- Wave length of light is in the order 10^{-7}m. In general obstacle of this wave length is not present so light rays does not show diffraction and it appears to travel in straight line Sound wave shows more diffraction as compare to light rays because wavelength of sound is high (16 mm to 16m). So it is generally diffracted by the objects in our daily life.

- Diffraction of ultrasonic wave is also not observed as easily as sound wave because their wavelength is of the order of about 1 cm. Diffraction of radio waves is very conveniently observed because of its very large wavelength (2.5 m to 250 m). X-ray can be diffracted easily by crystel. It was discovered by Lave.



diffraction of sound from a window

TYPES OF DIFFRACTION

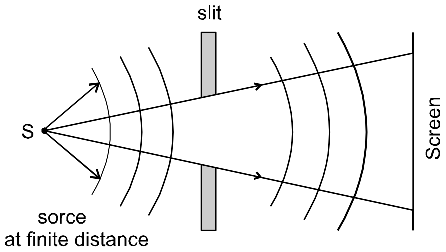
(i) There are two type of diffraction of light :

(a) Fresnel's diffraction (b) Fraunhofer's's diffraction

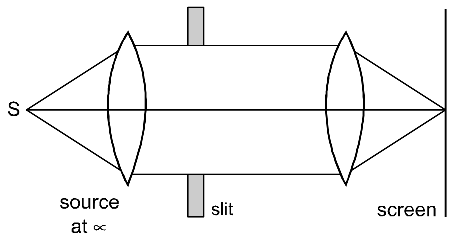
(a) Fresnel diffraction

If either source or screen or both are at finite from the diffracting device (obstacle or aperture), the diffraction is called fresnel diffraction and the pattern is the shadow of the diffracting device modified by diffraction effect.

Example :- Diffraction at a straight edge, small opaque disc, narrow wire are examples of fresnel diffraction.



Fresnel's diffraction



Fraunhofer's diffraction

(b) Fraunhofer diffraction

Fraunhofer diffraction is a particular limiting case of fresnel diffraction.

In this case, both source and screen are effectively at infinite distance from the diffracting device and pattern is the image of source modified by diffraction effects.

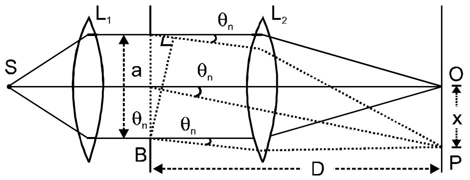
Example :- Diffraction at single slit, double slit and diffraction grating are the examples of fraunhofer diffraction.

Comparison between fresnel and fraunhofer diffraction

	Fresnel Diffraction	Fraunhofer Diffraction
(a)	Source and screen both are at finite distance from the diffractor	Source and screen both are at infinite distance from the diffractor
(b)	Incident and diffracted wave fronts are spherical or cylinderical	Incident and diffracted wavefronts are plane due to infinite distance from source
(c)	Mirror or lennses are not used for obtaining the diffraction pattern	Lens are used in this diffraction pattern
(d)	Centre of diffraction pattern is sometime bright and sometime dark depending on size of diffractor and distance of observation point.	Centre of diffraction is always bright
(e)	Amplitude of wave coming from different half period zones are different due to difference of obliquity	Amplitude of waves coming from different half period zones are same due to same obliquity

FRAUNHOFER DIFFRACTION DUE TO SINGLE SLIT

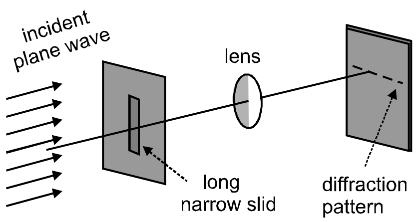
Ab is single slit of width a, Plane wavefront is incident on a slit AB. Secondary wavelets coming from every part of AB reach the axial point P in same phase forming the central maxima. The intensity of central maxima is maximum in this diffrection. Where θ_n represents direction of n^{th} minima Path difference BB' = a sin θ_n\



for n^{th} minima a sin $\theta_n = n\lambda$ $\therefore$ $\sin\theta_n \approx \theta_n = \frac{n\lambda}{a}$ (if θ_n is small)

- When path difference between the secondary wavelets coming from A and B is $n\lambda$ or $2n\left[\frac{\lambda}{2}\right]$

or even multiple of $\frac{\lambda}{2}$ then minima occurs



For minima $a \sin\theta_n = 2n\left[\frac{\lambda}{2}\right]$

- When path difference between the secondary wavelets coming from A and B is $(2n + 1)\frac{\lambda}{2}$

or odd multiple of $\frac{\lambda}{2}$ then maxima occurs

For maxima $a \sin\theta_n = (2n + 1)\frac{\lambda}{2}$ where n = 1, 2, 3 ......

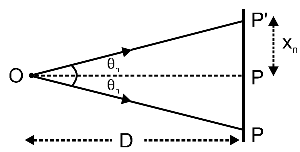
n = 1 $\rightarrow$ first maxima and n = 2 $\rightarrow$ second maxima.

- In alternate order minima and maxima occurs on both sides of central maxima.

For n^{th} minima

If distance of n^{th} minima from central maxima = x_n

distance of slit from screen = D, width of silt = a



Path difference $\delta = a \sin\theta_n = \frac{2n\lambda}{2} \Rightarrow \sin\theta_n = \frac{n\lambda}{a}$

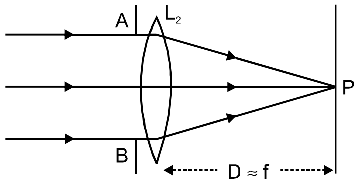
In Δ POP' $\tan\theta_n = \frac{x_n}{D}$ If θ_n is small $\Rightarrow \sin\theta_n \approx \tan\theta_n \approx \theta_n$

$X_n = \frac{n\lambda D}{a} \Rightarrow \theta_n = \frac{x_n}{D} = \frac{n\lambda}{a}$ First minima occurs both sides on central maxima.

For first minima $x = \frac{D\lambda}{a}$ and $\theta = \frac{x}{D} = \frac{\lambda}{a}$

- Linear width of central maxima $w_x = 2x \Rightarrow w_x = \frac{2d\lambda}{a}$
- Angular width of central maxima $w_\theta = 2\theta = \frac{2\lambda}{a}$

SPECIAL CASE



Lens L_2 is shifted very near to slit AB.
In this case distance between slit and screen will be nearly equal to the focal lenght of lense L_2
(i.e. $D \approx f$)

$$\theta_n = \frac{x_n}{f} = \frac{n\lambda}{a} \qquad \Rightarrow \qquad x_n = \frac{n\lambda f}{a}$$

$$w_x = \frac{2\lambda f}{a}$$

and angular width of cnetral maxima $w_B = \frac{2x}{f} = \frac{2x}{a}$

Fringe width

Distance between two consecutive maxima (bright fringe) or minima (dark fringe) is known as fringe width.

$$\beta = x_{n+1} - x_n = (n+1)\frac{\lambda D}{a} - \frac{n\lambda D}{a} = \frac{\lambda D}{a}$$

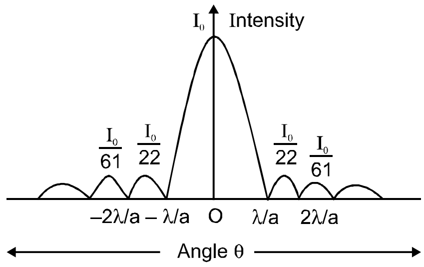
Intensity curve of Fraunhofer's diffraction

Intensity of maxima in Fraunhofer's diffrection is determined by $I = \left[\frac{2}{(2n+1)\pi}\right]^2 I_0$

I_0 = intensity of central maxima
n = order of maxima

intensity of first maxima $I_1 = d\theta \frac{4}{9\pi^2} I_0 \approx \frac{I_0}{22}$

intensity of second maxima $I_2 = \frac{4}{25\pi^2} I_0 \approx \frac{I_0}{61}$



- Diffraction occurs in slit is always fraunhofer diffraction as diffraction pattern obtained from the cracks between the fingers, when viewed a distant tubelight and in YDSE experiment are fraunhofer diffraction

GOLDEN KEY POINTS

- The width of central maxima $\propto \lambda$, that is, more for red colour and less for blue.
  i.e., $w_x \propto \lambda$
  as $\lambda_{blue} < \lambda_{red} \Rightarrow w_{blue} < w_{red}$

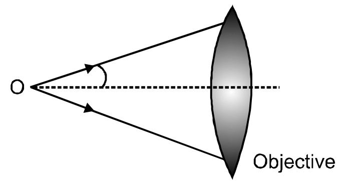
- For obtaining the fraunhofer diffraction, focal length of second lens (L_2) is used.
  $w_x \propto \lambda \propto f \propto 1/a$
  width will be more for narrow slit
- By decreasing linear width of slit, the width of central maxima increase.

RESOLVING POWER (R.P.)

A large number of images are formed as consequence of light diffraction of from a source. If two sources are separated such that their central maxima do not overlap, their images can be distinguished and are said to be resolved R.P. of an optical instrument is its ability to distinguish two neighbouring points.

Linear R.P. $d / \lambda D$ here D = Observed distance
Angular R.P. d / λ d = Distance between two points,

(1) **Microscope :** In reference to a microscope, the minimum distance between two lines at which they are just distinct is called Resolving limit (RL) and it's reciprocal is called Resolving power (RPO)



$$\text{R.L.} = \frac{\lambda}{2\mu \ \sin\theta} \text{ and R.P.} = \frac{2\mu\sin\theta}{\lambda} \Rightarrow \text{R.P.} \propto \frac{I}{\lambda}$$

λ = Wavelength of light used to illuminate the object

μ = Refractive index of the medium between object and objective.

θ = Half angle of the cone of light from the point object, $\mu \sin \theta$ = Numerical aperture.

(2) **Telescope :** Smallest angular separations ($d\theta$) between two distant object, whose images are separated in the telescope is called resolving limit. So resolving limit $d\theta = \frac{1.22\lambda}{a}$ and resolving power

$$(\text{RP}) = \frac{1}{d\theta} = \frac{a}{1.22\lambda} \Rightarrow \text{R.P.} \propto \frac{1}{2}$$ where a = aperture of objective.

DIFFERENCE BETWEEN INTERFERENCE AND DIFFRACTION :

	Interference		Diffraction
(1)	It is the phenomenon of superposition of two waves coming from two different coherent sources	(1)	It is the phenomenon of superposition of two waves coming from two different parts of the same wave front.
(2)	In intrference pattern, all bright lines are equally bright and equally spaced	(2)	All bright lines are not equally bright and equally wide. Brightness and width goes on decreasing with the angle of diffraction.
(3)	All dark lines are totaally dark	(3)	Dark lines are not perfectly dark. Their contrast with bright lines and width goes on decreasing with angle of diffraction.
(4)	In interference bands are large in number	(4)	In diffraction bands are a few in number

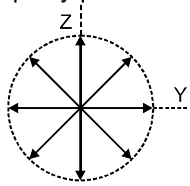
POLARISATION

Experiments on interference and diffraction have shown that light is a form of wave motion. These effects do not tell us about the type of wave motion i.e. whether the light waves are longitudinal or transverese. The phenomenon of polarization has helped to establish beyond doubt that light waves are transverse waves.

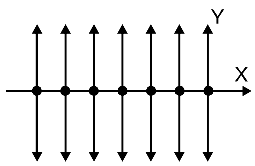
UNPOLARISED LIGHT

An ordinary beam of light consists of a large number of waves emitted by the atoms of the light source.

Each atom produces a wave with its own orientation of electric vector $\vec{E}$ so all direction of vibration of $\vec{E}$ are equally probable.



unpolarised light propagating along. X-axis



unpolarised light

The resultant electromagnetic wave is a super position of waves produced by the individual atomic sources and it is called unpolarised light. In ordinary or unpolarised light, the vibrations of the electric vector occur symmetrically in all possible directions in a plane perpendicular to the direction of propagation of light.

POLARISATION

The phenomenon of restricting the vibration of light (electric vector) in a particular direction perpendicular to the direction of propagation of wave is called polarisation of light.

In polarised light, the vibration of the electric vector occur in a plane perpendicular to the direction of propagation of light and are confined to a single direction in the plane (do not occur symmetrically in all possible directions.)

After polarisation the vibrations become asymmetrical about the direction of propagation of light.

POLARISER

Tourmaline crystal

When light is passed through a tomaline crystal cut parallel to its optic axis, the vibrations of the light carrying out of the tourmaline crystal are confined only to one direction in a plane perpendicular to the direction of propagation of light. The emergent light from the crystal is said to be plane polarised light.

Nicol Prism

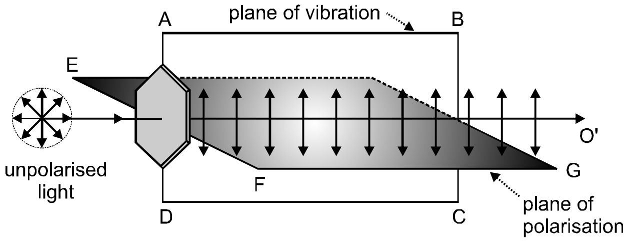
A nicol prism is an optical device which can be used for the production and detection of plane polarised light. It was invented by William Nicol in 1828.

Polaroid

A polaroid is a thin commercial sheet in the form of circular disc which makes use of the property of selective absorption to produce an intense beam of plane polarised light.

PLANE OF POLARISATION AND PLANE OF VIBRATION :

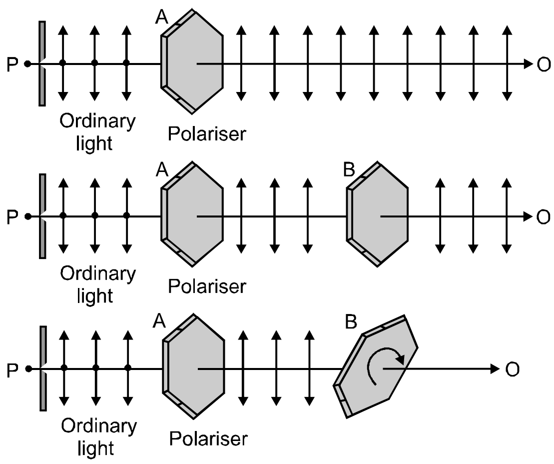
The plane in which vibrations of light vector and the direction of propogation lie is known as plane of vibration A plane normal to the plane of vibration and in which no vibration takes place is known as plane of polarisation.



EXPERIMENTAL DEMONSTRATION OF POLARISATION OF LIGHT

Take two tourmaline crystals cut parallel to their crystallographic axis (optic axis)

First hold the crystal A normally to the path of a beam of colour light. The emergent beam will be slightly coloured.



Rotate the crystal A about PO. No change in the intensity or the colour of the emergent beam of light. Take another crystal B and hold it in the path of the emergent beam of so that its axis is parallel to the axis of the crystal A. The beam of light passes through both the crystals and outcoming light appears coloured.

Now, rotate the crystal B about the axis PO. It will be seen that the intensity of the emergent beam decreases and when the axes of both the crystals are at right angles to each other no light comes out of the crystal B.

If the crystal B is further rotated light reappears and intensity becomes maximum again when their axes are parallel. This occurs after a further rotation of B through 90º

This experiment confirms that the light waves are transverse in nature.

The vibrations in light waves are perpendicular to the direction of propogation of the wave.

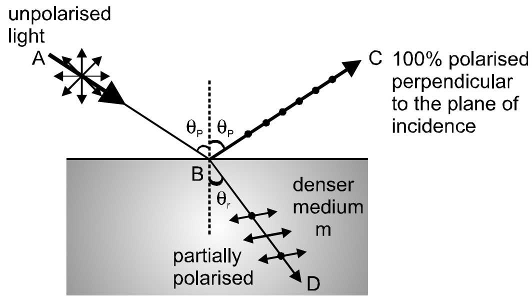
First crystal A polarises the light so it is called polariser.

Second crystal B, analyses the light whether it is polarised or not, so it is called analyser.

METHODS OF OBTAINING PLANE POLARISED LIGHT

Polarisation of reflection

The simplest method to produce plane polarised light is by reflection. This method was discovered by Malus in 1808. When a beam of ordinary light is reflected from a surface, the reflected light is partially polarised. The degree of polarisation of the polarised light in the reflected beam is greatest when it is incident at an angle called polarising angle or Brewster's angle.



Polarising angle

Polarising angle is that angle of incidence at which the reflected light is completely plane polarisation.

Brewster's Law

When unpolarised light strikes at polarising angle θ_P on a interface separating a rare medium from a denser medium of refractive index μ, such that $\mu = \tan\theta_P$ then the reflected light (light in rare medium) is completely polarised. Also reflected and refracted and refracted rays are normal to each other.

This relation is known as Brewster's Law.

The law state that the tangent of the polarising angle of incidence of a transparent medium is equal to tis refractive index $\mu = \tan\theta_P$

In case of polarisation by reflection :

(i) For $i = \theta_P$ refracted light is partially polarised.

(ii) For $i = \theta_P$ reflected and refracted rays are perpendicular to each other.

(iii) For $i < \theta_P$ or $i > \theta_P$ both reflected and refracted light become partially polarised.

According to snell's law $\mu = \dfrac{\sin\theta_P}{\sin\theta_r}$ ....(i)

But according to Brewster's law $\mu = \tan\theta_O = \dfrac{\sin\theta_P}{\cos\theta_P}$ ....(ii)

From equation (i) and (ii) $\dfrac{\sin\theta_P}{\sin\theta_r} = \dfrac{\sin\theta_P}{\cos\theta_P} \Rightarrow \sin\theta_r = \cos\theta_P$

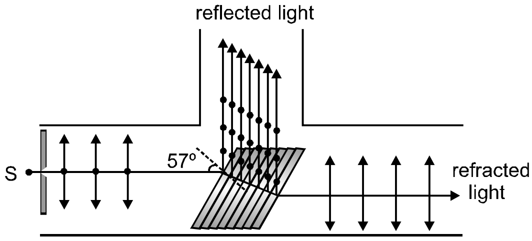
$\therefore$ $\sin\theta_r = \sin(90º - \theta_P) \Rightarrow \theta_r = 90º - \theta_P$ or $\theta_P + \theta_r = 90º$

Thus reflected and refracted rays are mutually perpendicular

By Refraction

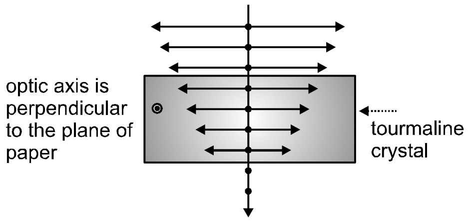
In this method, a pile of glass plates is formed by taking 20 to 30 microscope slides and light is made to be incident a polarising angle 57º. According Brewster law, the reflected light will be plane polarised with vibrations perpendicualr to the plane of incidence and the transmitted light will be partially polarised.

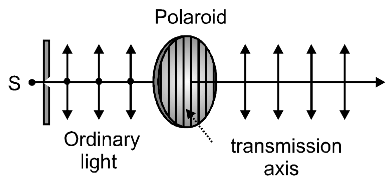
Since in one reflection about 15% of the light with vibration perpendicular to plane of paper is reflected therefore after passing through a number of plates emerging light will become plane polarised with vibrations in the plane of paper.



By Dichroism

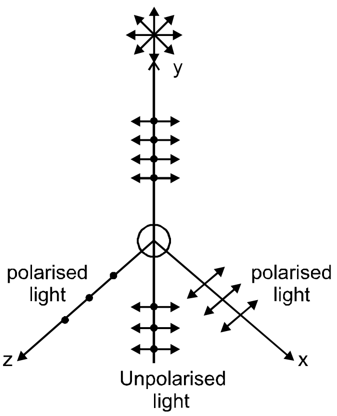
Some crystals such as tourmaline and sheets of iodosulphate of quinone have the property of strongly absorbing the light with vibrations perpendicular of a specific direction (called transmision axis) and transmitting the light with vibration parallel to it. This selective absorption of light is called dichroism. So if unpolarised light passes through proper thickness of these, the transmitted light will plane polarised withvibrations parallel to transmission axis. Polaroids work onthis principle.





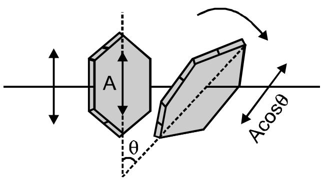
By scattering :

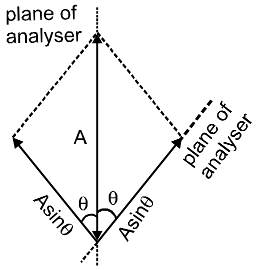
When light is incident on small particles of dust, air molecule etc. (having smaller size as compared to the wavelength of light,) it is absorbed by the electrons and is re-radiated in all directions. The phenomenon is called as scattering. Light scattered in a direction at right angles to the incident light is always plane-polarised.



Law of Malus

When a completely plane polarised light beam light beam is incident analyser, then intensity intensity of emergent light varies as the square of cosine of the angle between the planes of transmision of the analyser and the polarizer. $I \propto \cos^2\theta \Rightarrow I = I_0 \cos^2\theta$





(i) If $\theta = 0^\circ$ then $I = I_0$ maximum value (Parallel arrangement)

(ii) If $\theta = 90^\circ$ then, $I = 0$ minimum value (Crossed arrangement)

If plane polarised light of intensity I_0 (= KA^2) is incident on a polaroid and its vibrations of amplitude A make angle θ with transmission axis, then the component of vibrations parallel to transmission axis will be $A \cos\theta$ while perpendicular to it will be be $A \sin\theta$.

Polaroid will pass only those vibrations which are parallel to transmission axis i.e. $A\cos\theta$,

$\because \quad I_0 \propto A^2$

So the intensity of emergent light $\quad I = K (A\cos\theta)^2 = KA^2\cos^2\theta$

If an unpolarised light is converted into plane polarised light its intensity becomes half.

If light of light emerging from the second polaroid is :

$I_2 = I_1 \cos^2\theta \qquad \theta$ = angle between the transmission axis of the two polaroids.

Optical Activity

When plane polarised light passes through certain substances, the plane of polarisation of the emergent light is rotated about the direction of propagation of light through a certain angle. This phenomenon is optical rotation.

The substance which rotate the plane of polarision rotates the plane of polarisation is known as optical active substance. Ex. Sugar solution, sugar crystal, solidum chlorate etc.

Optical activity of a substance is measured with the help of polarimeter in terms of specific rotation which is defined as the rotaion produced by a solution of length 10 cm (1dm) and of unit concentration (1g/cc) for a given wave length of light at a given temp.

specific rotation $\left[\alpha\right]_{t^\circ C}^{\lambda} = \dfrac{\theta}{L \times C}$ θ = rotation in length L at concentration

Types of optically active substances

(a) Dextro rotatory substances

Those substance which rotate the plane of polarisation in clockwise direction are called dextro rotatory of right handed substances.

(b) Laveo rotatory substances

These substance which rotate the plane of polarisation in the anticlockwise direction are calledlaveo rotatory or left handed substances.

The amount of optical rotation depends upon the thickness and density of the crsytal or concentration in case of solutions, the temperature and the wavelength of light used.

Rotation varies inversely as the square of the wavelength of light.

APPLICATIONS AND USES OF POLARISATION

- By determining the polarising and and using Brewster's law $\mu = \tan\theta_P$ refractive index of dark transparent substance can be determined.
- In calculators and watches, numbers and letters are formed by liquid crystals through polarisation of light called liquid crystal display (L.C.D)
- In CD player polarised laser beam acts as needle for producing sound from compact disc.
- It has also been used in recording and reproducing three dimensional pictures.
- Polarised light is used in optical stress analysis known as photoelasticity.
- Polarisation is also used to study asymmetries in molecules and crystals through the phenomenon of optical activity.

Some Questions with their solutions

1. Light of wavelength 6000Å is incident normally on a slit of width 24×10^{-5} cm. Find out the angular position of second minimum from central maximum ?

2. Light of wavelength 6328Å is incident normally on a slit of width 0.2 mm. Calculate the angular width of central maximum on a screen distance 9 m ?

3. Light of wavelength 5000Å is incident on a slit of width 0.1 mm. Find out the width of the central bright line on a screen distance 2m from the slit ?

4. The fraunhofer diffraction pattern of single slit is formed at the focal plane of a lens of focal length 1m. The width of the slit is 0.3 mm. If the thrid minimum is formed at a distance of 5 mm from the central maximum then calculate the wavelength of light.

5. Find the half angular width of the central bright maximum in the Fraunhofer diffraction pattern of a slit of width 12×10^{-5} cm when the slit is illuminated by monochromatic light of wavelength 6000Å.

6. Light of wavelength 6000 Å is incident on a slit of width 0.30 mm. The screen is placed 2 m from the slit. Find (a) the position of the first dark fringe and (b) the width of the central bright fringe.

7. A Slit of width a is illuminated by monochromatic light of wavelength 650 nm at normal incidence. Calculate the value of a when :
(a) the first minimum falls at an angle of diffraction of 30º
(b) the first maximum falls at an angles of diffraction of 30º

8. Red light of wavelength 6500 Å from a distance source falls on a slit 0.50 mm wide. What is the distance between the first two dark bands on each side of the central bright of the diffraction pattern observed on a screen placed 1.8 m. from the slit.

9. In a single slit diffraction experiment first minimum for λ_1 = 660 nm coincides with first maxima for wavelength λ_2. Calculate λ_2.

10. Two polaroids are crossed to each other. When one of them is rotated through 60º, then what percentage of the incident unpolarised light will be transmitted by the polaroids ?

11. At what angle of incidence will the light reflected from water (μ = 1.3) be completely polarised ?

12. If light beam is incident at polarising angle (56.3º) on air-glass interface, then what is the angle of refraction in glass ?

13. A polariser and an analyser are oriented so that maximum light is transmitted, what will be the intensity of outcoming light when analyer is rotated through 60º.

Solutions

Sol.1 a $\sin\theta = 2\lambda$ given $\lambda = 6 \times 10^{-7}$ m, $a = 24 \times 10^{-5} \times 10^{-2}$ m

$\sin\theta = \frac{2\lambda}{a} = \frac{2\times6\times10^{-7}}{24\times10^{-7}}$ $\therefore$ $\theta = 30º$

Sol.2. given $\lambda = 6.328 \times 10^{-7}$ m, $a = 0.2 \times 10^{-3}$ m

$w_\theta = \frac{2\lambda}{a} = \frac{2\times6.328\times10^{-7}}{2\times10^{-4}}$ radian $= \frac{6.328\times10^{-3}\times180}{3.14} = 0.36\equiv$

Sol.3 $w_x = \frac{2\lambda}{a} = \frac{2\times2\times5\times10^{-7}}{10^{-4}} = 20$ mm

Sol.4 $x_n = \frac{nf\lambda}{a} \Rightarrow \lambda = \frac{ax_n}{fn} = \frac{3\times10^{-4}\times5\times10^{-3}}{3\times1} = 5000$Å [$\because$ n = 3]

Sol.5 $\because$ $\sin\theta = \frac{\lambda}{a}$ θ = half angular width of the central maximum.

$a = 12 \times 10^{-5}$ cm, $\lambda = 6000$ Å $= 6 \times 10^{-5}$ cm $\therefore \sin\theta = \frac{\lambda}{a} = \frac{6\times10^{-5}}{12\times10^{-5}} = 0.50 \Rightarrow \theta = 30º$

Sol.6 The first fringe is on either side of the central bright fringe.

here $n = \pm 1$, D = 2m, $\lambda = 6000$ Å = 6 j 10^{-7} m

$\therefore$ $\sin\theta = \frac{x}{D}$ $\Rightarrow$ $a = 0.30$ mm $= 3 \times 10^{-4}$ m $\Rightarrow a \sin\theta = n\lambda \Rightarrow \frac{ax}{D} = n\lambda$

(a) $x = \frac{n\lambda D}{a}$ $\Rightarrow$ $x = \pm\left[\frac{1\times6\times10^{-7}\times2}{3\times10^{-4}}\right] = \pm4 \times 10^{-3}$ m

The positive and negative signs corresponds to the dark fringes on either side of the central bright fringe.

(b) The width of the central bright fringe $y = 2x = 2 \times 4 \times 10^{-3} = 8 \times 10^{-3}$ m = 8 mm

Sol.7 (a) for first minimum $\sin\theta_1 = \frac{\lambda}{a}$

$\therefore$ $a = \frac{\lambda}{\sin\theta_1} = \frac{650\times10^{-9}}{\sin\ 30º} = \frac{650\times10^{-9}}{0.5} = 1.3 \times 10^{-6}$ m

(b) for first maximum $\sin\theta_1 = \frac{3\lambda}{2a}$, $\therefore$ $a = \frac{3\lambda}{2\sin\theta} = \frac{3\times650\times10^{-9}}{2\times0.5} = 1.95 \times 10^{-6}$ m

Sol.8 Given $\lambda = 6500$Å $= 65 \times 10^{-8}$ m, a = 0.5 mm $= 0.5 \times 10^{-3}$ m., D = 1.8 mm

Sol.9 For minima in diffraction pattern $d \sin\theta = n\lambda$

For first minima $d \sin\theta_1 = (1)\lambda_1 \Rightarrow \sin\theta_1 = \frac{\lambda_1}{d}$

for first maxima $d \sin\theta_2 = \frac{3}{2}\lambda_2 \Rightarrow \sin\theta_2 = \frac{3\lambda_2}{2d}$

Two will coincide if, $\theta_1 = \theta_2$ or $\sin\theta_1 = \sin\theta_2$

$\therefore$ $\frac{\lambda_1}{d} = \frac{3\lambda_2}{2d} \Rightarrow \lambda_2 = \frac{2}{3}\lambda_1 = \frac{2}{3} \times 660$ nm = 440 nm.

Sol.10 Initially the polaroids are crossed to each other, that is the angle between their polarising directions is 90º. When one is rotated through 60º, then the angle between their polarising directions directions will become 30º.

Let the intensity of the incident unpolarised light = I_0

This light is plane polarised and passes through the second polaroid.

The intensity of light emerging form the second polaroid is $I_2 = I_1 \cos^2\theta$

θ = the angle between the polarising directions of the two polaroids.

$I_1 = \frac{1}{2}I_0$ and $\theta = 30º$ so $O_2 = I_1 \cos^2 30º \Rightarrow \frac{I_2}{I_0} = \frac{3}{8}$

$\therefore$ transmission percentage = $\frac{I_2}{I_0} \times 100 = \frac{3}{8} \times 100 = 37.5\%$

Sol.11 $\mu = 1.3$, From Brewster's law $\tan \theta_P = \mu = 1.3$ $\Rightarrow \theta = \tan^{-1} 1.3 = 53º$

Sol.12 $\because i_P + r_P = 90º$ $\therefore r_P = 90º - i_P = 90º - 56.3º = 33.7º$

Sol.13 According to Malus Law $I = I = I_0 \cos^2\theta = I_0 \cos^2 60º = I_0\left[\frac{1}{2}\right]^2 = \frac{I_0}{4}$

Exercise-1

✍ **Marked Questions can be used as Revision Questions.**

PART - I : SUBJECTIVE QUESTIONS

SECTION (A) : PRINCIPLE OF SUPERPOSITION, PATH DIFFERENCE, WAVEFRONTS, AND COHERENCE

A-1. Two sources of intensity I & 4I are used in an interference experiment. Find the intensity at points where the waves from the two sources superimpose with a phase difference of **[REE 1991, 3]**

(a) zero (b) $\frac{\pi}{2}$ & (c) They meet at phase difference of π.

A-2.✍ An electromagnetic wave travelling through a transparent medium is given by

$E_x(y,t) = E_{ox}\sin 2\pi\left[\frac{y}{5\times10^{-7}} - 3\times10^{14}t\right]$ in SI units. Then what is the refractive index of the medium?

SECTION (B) : YDSE WITH MONOCHROMATIC LIGHT

B-1.✍ What is the effect on the fringe width of interference fringes in a Young's double slit experiment due to each of the following operations :
(a) The screen is moved away from the plane of the slits.
(b) The (monochromatic) source is replaced by another (monochromatic) source of shorter wavelength.
(c) The separation between the two slits is increased.
(d) The width of two slits are slightly increased.
[In each operation, take all parameters, other than the one specified to remain unchanged]

B-2. Two slits separated by a distance of 1 mm, are illuminated with red light of wavelength 6.5×10^{-7} m. The interference fringes are observed on a screen placed 1 m from the slits. Find the distance between the third dark fringe and the fifth bright fringe on the same side of the central maxima.

B-3. In a Young's double slit experiment, the fringe width is found to be 0.4 mm. If the whole apparatus is immersed in water of refractive index (4/3), without disturbing the geometrical arrangement, what is the new fringe width?

B-4. Find the angular fringe width in a Young's double slits experiment with blue-green light of wavelength 6000 Å. The separation between the slits is 3.0×10^{-3} m.

SECTION (C) : YDSE WITH POLYCHROMATIC LIGHT

C-1.✍ A source emitting two light waves of wavelengths 580 nm and 700 nm is used in a young's double slit interference experiment. The separation between the slits is 0.20 mm and the interference is observed on a screen placed at 150 cm from the slits. Find the linear separation between the first maximum (next to the central maximum) corresponding to the two wavelengths.

SECTION (D) : YDSE WITH GLASS SLAB, OPTICAL PATH

D-1.✍ A flint glass and a crown glass are fitted on the two slits of a double slit apparatus. The thickness of the strips is 0.40 mm and the separation between the slits is 0.12 cm. The refractive index of flint glass and crown glass are 1.62 and 1.52 respectively for the light of wavelength 480 nm which is used in the experiment. The interference is observed on a screen a distance one meter away. (a) What would be the fringe-width? (b) At what distance from the geometrical centre will the nearest maximum be located?

D-2. Find the thickness of a plate which will produce a change in optical path equal to one fourth of the wavelength λ of the light passing through it normally. The refractive index of the plate is μ.

SECTION (E) : YDSE WITH OBLIQUE INCIDENCE AND OTHER MODIFICATIONS IN EXPERIMENTAL SETUP OF YDSE

E-1. A parallel beam of monochromatic light of wavelength λ is used in a Young's double slit experiment. The siits are separated by a distance d and the screen is placed parallel to the plane of the slits. The incident beam makes an angle $\theta = \sin^{-1}\left(\frac{\lambda}{2d}\right)$ with the normal to the plane of the slits. A transparent sheet of refractive index. 'μ' and thickness $t = \frac{\lambda}{2(\mu-1)}$ is introduced infront of one of the slit. Find the intensity at the geometrical centre.

SECTION (F) : THIN FILM INTERFERENCE

F-1. A soap film of thickness 0.3 μm appears dark when seen by the refracted light of wavelength 580 nm. What is the index of refraction of the soap solution, if it is known to be between 1.3 and 1.5?

F-2. A parallel beam of light of wavelength 560 nm falls on a thin film of oil (refractive index = 1.4). What should be the minimum thickness of the film so that it weakly transmits the light?

SECTION (G) : FOR JEE (MAIN)

G-1. A slit of width 'a' is illuminated by light of wavelength 6000 A^0 for what value of 'a' will the :
(i) First maximum fall at an angle of diffraction of 30° ?
(ii) First minimum fall at an angle of diffraction 30° ?

PART - II : ONLY ONE OPTION CORRECT TYPE

SECTION (A) : PRINCIPLE OF SUPERPOSITION, PATH DIFFERENCE, WAVEFRONTS, AND COHERENCE

A-1. Ratio of intensities of two light waves is given by 4 : 1. The ratio of the amplitudes of the waves is :
(A) 2 : 1 (B) 1 : 2 (C) 4 : 1 (D) 1 : 4

A-2. Two coherent monochromatic light beams of intensities I and 4I are superimposed; the maximum and minimum possible intensities in the resulting beam are :
(A) 5I and I (B) 5I and 3I (C) 9I and I (D) 9I and 3I

SECTION (B) : YDSE WITH MONOCHROMATIC LIGHT

B-1. The contrast in the fringes in any interference pattern depends on :
(A) Fringe width (B) Wavelength
(C) Intensity ratio of the sources (D) Distance between the sources

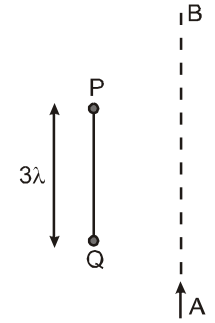
B-2. Initially interference is observed with the entire experimental set up inside a chamber filled with air. Now the chamber is evacuated. With the same source of light used, a careful observer will find that
(A) The interference pattern is almost absent as it is very much diffused
(B) There is no change in the interference pattern
(C) The fringe width is slightly decreased
(D) The fringe width is slightly increased

B-3. Yellow light emitted by sodium lamp in Young's double slit experiment is replaced by monochromatic blue light of the same intensity :
(A) fringe width will decrease. (B) fringe width will increase.
(C) fringe width will remain unchanged. (D) fringes will become less intense.

B-4. In a YDSE: D = 1 m, d = 1 mm and λ = 500 nm. The distance of 1000^{th} maxima from the central maxima is:
(A) 0.5 m (B) 0.577 m (C) 0.495 m (D) does not exist

B-5. In a Young's double slit experiment, d = 1 mm, λ = 6000 Å & D = 1 m. The slits produce same intensity on the screen. The minimum distance between two points on the screen having 75 % intensity of the maximum intensity is:
(A) 0.45 mm (B) 0.40 mm (C) 0.30 mm (D) 0.20mm

B-6. Two coherent light sources each of wavelength λ are separated by a distance 3 λ. The total number of minima formed on line AB which runs from $-\infty$ to $+\infty$ is:



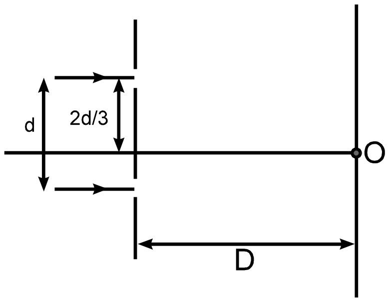
(A) 2 (B) 4 (C) 6 (D) 8

B-7. In a Young's double slit experiment the intensity at a point I where the corresponding path difference is one sixth of the wavelength of light used. If I_0 denotes the maximum intensity, the ratio $\frac{I}{I_0}$ is equal to **[Olympiad 2015 (stage-1)]**

(A) $\frac{1}{4}$ (B) $\frac{1}{2}$ (C) $\frac{\sqrt{3}}{2}$ (D) $\frac{3}{4}$

SECTION (C) : YDSE WITH POLYCHROMATIC LIGHT

C-1. In the figure shown if a parallel beam of white light is incident on the plane of the slits then the distance of the nearest white spot on the screen from O is: [assume d << D, λ << d]



(A) 0 (B) d/2 (C) d/3 (D) d/6

C-2. The Young's double slit experiment is performed with blue and with green light of wavelengths 4360 Å and 5460 Å respectively. If X is the distance of 4th maximum from the central one, then :

(A) X(blue) = X(green) (B) X(blue) > X(green) (C) X(blue) < X(green) (D) $\frac{X(\text{blue})}{X(\text{green})} = \frac{5460}{4360}$

SECTION (D) : YDSE WITH GLASS SLAB, OPTICAL PATH

D-1. A two slit Young's interference experiment is done with monochromatic light of wavelength 6000 Å. The slits are 2 mm apart. The fringes are observed on a screen placed 10 cm away from the slits. Now a transparent plate of thickness 0.5 mm is placed in front of one of the slits and it is found that the interference pattern shifts by 5 mm. The refractive index of the transparent plate is : **[REE 1985]**

(A) 1.2 (B) 0.6 (C) 2.4 (D) 1.5

D-2. In a YDSE both slits produce equal intensities on the screen. A 100 % transparent thin film is placed in front of one of the slits. Now the intensity of the geometrical centre of system on the screen becomes 75 % of the previous intensity. The wavelength of the light is 6000Å and μ_{glass} = 1.5. The thickness of the film cannot be:

(A) 0.2 μm (B) 1.0 μm (C) 1.4 μm (D) 1.6 μm

SECTION(E) : YDSE WITH OBLIQUE INCIDENCE AND OTHER MODIFICATIONS IN EXPERIMENTAL SETUP OF YDSE

E-1. In a YDSE experiment, thin films of thickness t_1 and t_2 are placed infront of slits S_1 and S_2 as shown in figure-1 and figure-2. It is observed that first minima and second maxima are produced at point 'O' in first and second experiment respectively. Point 'O' and 'S' are symmetrical with respect to S_1 and S_2. Both the films have same refractive index if $\frac{t_2}{t_1} = \frac{x}{25}$, then calculate 'x' :

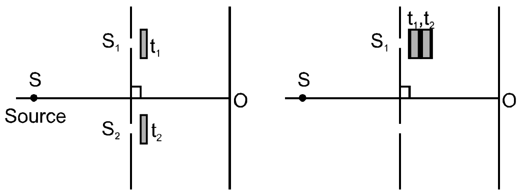
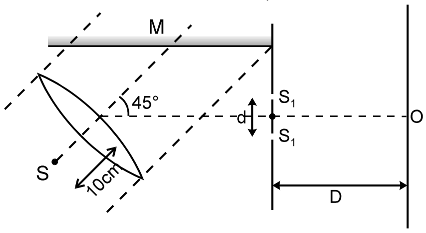


Figure-1 Figure-2

(A) 3 (B) 9 (C) 12 (D) 15

E-2. In Youngs double slit experiment, distance between the slits is d and that between the slits and screen is D. Angle between principle axis of lens and perpendicular bisector of S_1 and S_2 is 45°. The point source S is placed at the focus of lens and aperture of lens is much larger than d. Assuming only the reflected light from plane mirror M is incident on slits, distance of central maxima from O will be :



(A) D (B) $\frac{D}{\sqrt{3}}$ (C) $D\sqrt{3}$ (D) $\frac{D}{\sqrt{4}}$

SECTION (F) : THIN FILM INTERFERENCE

F-1. White light is incident normally on a glass plate (in air) of thickness 500 nm and refractive index of 1.5. The wavelength (in nm) in the visible region (400 nm - 700nm) that is strongly reflected by the plate is:
(A) 450 (B) 600 (C) 400 (D) 500

SECTION (G) : FOR JEE (MAIN)

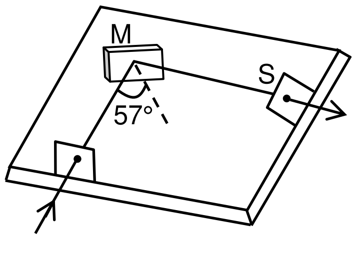
G-1. A slit of size 0.15 cm is placed at 2.1 m from a screen. On illuminated it by a light of wavelength 5 x 10^{-5} cm. The width of diffraction pattern will be:-
(A) 70 mm (B) 0.14 mm (C) 1.4 cm (D) 0.14 cm

G-2. The diameter of objective of a telescope is 1m. Its resolving limit for the light of wave length 4538 Å, will be
(A) 5.54×10^{-7} rad (B) 2.54×10^{-4} rad (C) 6.54×10^{-7} rad (D) None of these

G-3. When an unpolarized light of intensity I_0 is incident on a polarizing sheet, the intensity of the light which does not get transmitted is :
(A) $\frac{1}{2}I_0$ (B) $\frac{1}{4}I_0$ (C) zero (D) I_0

G-4. A single slit diffraction pattern is obtained using a beam of red light. What happens if the red light is replaced by blue light?
(A) There is no change in the diffraction pattern
(B) Diffraction fringes become narrower and crowded together
(C) Diffraction fringes become boarder and crowded together
(D) The diffraction pattern disappears

G-5. Shows a glass plate placed vertically on a horizontal table with a beam of unpolarised light falling on its surface at 57° with the normal. The electric vectors in the reflected light on the screen S will vibrate with respect to the plane of incidence :
(A) in a vertical plane
(B) in a horizontal plane
(C) in a plane making an angle of 45° with the vertical
(D) in a plane making an angle of 57° with the horizontal



G-6. Two point white dots are 1 mm apart on a black paper. They are viewed by eye of pupil diameter 3 mm. Approximately, what is the maximum distance at which these dots can be resolved by the eye?
[Take wavelength of light = 500 nm]
(A) 6m (B) 3m (C) 5m (D) 1m

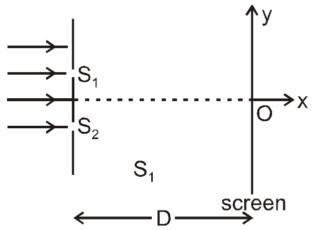
G-7. Visible light passing through a circular hole form a diffraction disc of radius 0.1 mm on a screen. If X-ray is passed through the same set-up, the radius of the diffraction disc will be :
(A) zero (B) < 0.1 mm (C) 0.1 mm (D) > 0.1 mm

G-8. The resolving power of a telescope is more when its objective lens has
(A) greater focal length (B) smaller focal length (C) greater diameter (D) smaller diameter

G-9. Resolving power of a microscope depends upon
(A) the focal length and aperture of the eye lens (B) the focal lengths of the objective and the eye lens
(C) the apertures of the objective and the eye lens (D) the wavelength of light illuminating the object

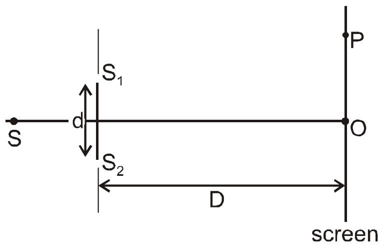
PART - III : MATCH THE COLUMN

1. A monochromatic parallel beam of light of wavelength λ is incident normally on the plane containing slits S_1 and S_2. The slits are of unequal width such that intensity only due to one slit on screen is four times that only due to the other slit. The screen is placed perpendicular to x-axis as shown. The distance between slits is d and that between screen and slit is D. Match the statements in column-I with results in column-II. (S_1S_2 << D and λ << S_1S_2)



Column-I	Column-II
(A) The distance between two points on screen having equal intensities, such that intensity at those points is $\frac{1}{9}$th of maximum intensity.	(p) $\frac{D\lambda}{3d}$
(B) The distance between two points on screen having equal intensities, such that intensity at those points is $\frac{3}{9}$th of maximum intensity.	(q) $\frac{D\lambda}{d}$
(C) The distance between two points on screen having equal intensities, such that intensity at those points is $\frac{5}{9}$th of maximum intensity.	(r) $\frac{2D\lambda}{d}$
(D) The distance between two points on screen having equal intensities, such that intensity at those points is $\frac{7}{9}$th of maximum intensity.	(s) $\frac{3D\lambda}{d}$

2. In a typical Young's double slit experiment, S_1 and S_2 are identical slits and equidistant from a point monochromatic source S of light having wavelength λ. The distance between slits is represented by d and that between slits and screen is represented by D. P is a fixed point on the screen at a distance $y = \frac{\lambda D_o}{2d_o}$ from central order bright on the screen: where D_o, d_o are initial values of D and d respectively. In each statement of column-I some changes are made to above mentioned situation.



The distance between the slits and the source is very large. The effect of corresponding changes is given in column-II. Match the statements in column-I with resulting changes in column-II.

Column-I	Column-II
(A) The distance d between the slits is doubled keeping distance between slits and screen fixed	(p) fringe width increases.
(B) The distance D between slit and screen is doubled by shifting screen to right	(q) Magnitude of optical path difference between interfering waves at P will decrease.
(C) The width of slit S_1 is decreased (such that intensity of light due to slit S_1 on screen decreases) and the distance D between slit and screen is doubled by shifting screen to right	(r) Magnitude of optical path difference between interfering waves at P will increase.
(D) The whole setup is submerged in water of refractive index 4/3. (Neglecting absorption in medium)	(s) The intensity at P will increase

Exercise-2

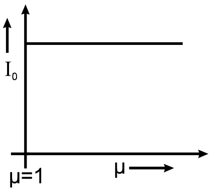
✍ Marked Questions can be used as Revision Questions.

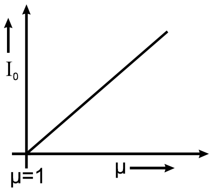
PART - I : ONLY ONE OPTION CORRECT TYPE

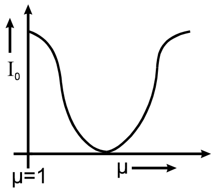
1. If the ratio of the intensity of two coherent sources is 4 then the visibility $[(I_{max} - I_{min})/(I_{max} + I_{min})]$ of the fringes is

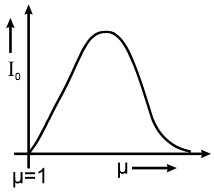
(A) 4 (B) 4/5 (C) 3/5 (D) 9

2.✍ In a YDSE experiment if a slab whose refractive index can be varied is placed in front of one of the slits then the variation of resultant intensity at mid-point of screen with 'µ' will be best represented by ($\mu \geq 1$). [Assume slits of equal width and there is no absorption by slab]

(A)


(B)


(C)


(D)


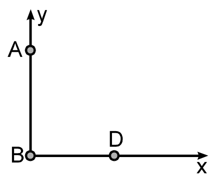
3. In a Young's double slit experiment the slit is illuminated by a source having two wavelengths of 400 nm and 600 nm. If distance between slits, d = 1mm, and distance between the plane of the slit and screen, D = 10 m then the smallest distance from the central maximum where there is complete darkness is :

(A) 2mm (B) 3mm (C) 12 mm (D) there is no such point

4.✍ If the first minima in a Young's slit experiment occurs directly infornt of one of the slits. (Distance between slit & screen D = 12 cm and distance between slits d = 5 cm) then the wavelength of the radiation used is :

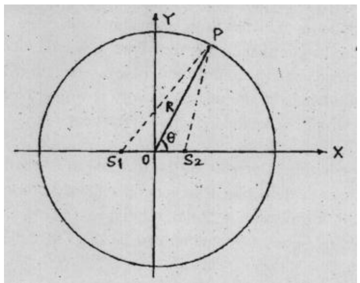
(A) 2 cm only (B) 4 cm only (C) 2m, $\frac{2}{3}$ cm, $\frac{2}{5}$ cm (D) 4cm, $\frac{4}{3}$ cm, $\frac{4}{5}$ cm

5. An interference is observed due to two coherent sources 'A' & 'B' having phase constant zero separated by a distance 4 λ along the y-axis where λ is the wavelength of the source. A detector D is moved on the positive x-axis. The number of points on the x-axis excluding the points, x = 0 & x = ∞ at which maximum will be observed is



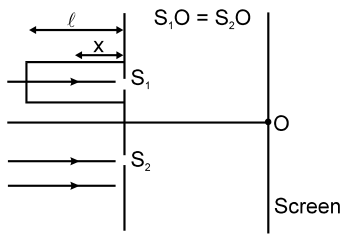
(A) three (B) four (C) two (D) infinite

6. Two coherent sources of light S_1 and S_2, equidistant from the origin, are separated by a distance 2λ as shown. They emit light of wavelength λ. Interference is observed on a screen placed along the circle of large radius R. Point is seen to be a point of constructive interference. Then angle θ (other than 0° and 90°) is



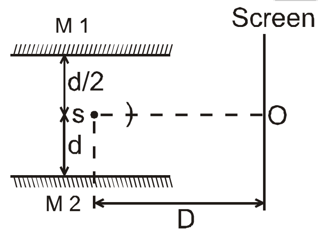
(A) 45°
(B) 30°
(C) 60°
(D) Not possible in the first quadrant

7.✍ In the figure shown, a parallel beam of light is incident on the plane of the slits of a Young's double slit experiment. Light incident on the slit, S_1 passes through a medium of variable refractive index $\mu = 1 + ax$ (where 'x' is the distance from the plane of slits as shown), upto a distance 'ℓ' before falling on S_1. Rest of the space is filled with air. If at 'O' a minima is formed, then the minimum value of the positive constant a (in terms of ℓ and wavelength 'λ' in air) is :



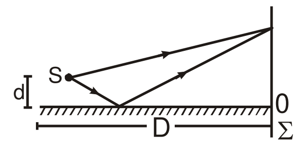
(A) $\frac{\lambda}{\ell}$ (B) $\frac{\lambda}{\ell^2}$ (C) $\frac{\ell^2}{\lambda}$ (D) None of these

8. M_1 and M_2 are two plane mirrors which are kept parallel to each other as shown. There is a point 'O' on perpendicular screen just infront of 'S'. What should be the wavelength of light coming from monchromatic source 'S'. So that a maxima is formed at 'O' due to interference of reflected light from both the mirrors. [Consider only 1st reflection]. [$D >> d$, $d >> \lambda$]



(A) $\frac{3d^2}{D}$ (B) $\frac{3d^2}{2D}$ (C) $\frac{d^2}{D}$ (D) $\frac{2d^2}{D}$

9. A long narrow horizontal slit lies 1 mm above a plane mirror. The interference pattern produced by the slit and its image is viewed on a screen Σ distant 1m from the slit. The wavelength of light is 600 nm. Then the distance of the first maxima above the mirror is equal to ($d << D$):



(A) 0.30 mm (B) 0.15 mm (C) 60 mm (D) 7.5 mm

10. A parallel monochromatic beam of light is incident normally on a narrow slit. A diffraction pattern is formed on a screen placed perpendicular to the direction of the incident beam. At the first minimum of the diffraction pattern, the phase difference between the rays coming from the two edges of the slit is: [Diffraction – Not in JEE syllabus now] **[JEE 1998, 2]**

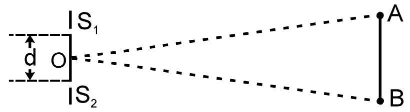
(A) 0 (B) $\pi/2$ (C) π (D) 2π

11. A slit of width a is illuminated by parallel monochromatic light of wavelength λ. The value of a at which the first minimum of the diffraction pattern will from at $\theta = 30°$ is **[Olympiad (State-1) 2017]**

(A) $\lambda/2$ (B) λ (C) 2λ (D) 3λ

PART - II : SINGLE AND DOUBLE VALUE INTEGER TYPE

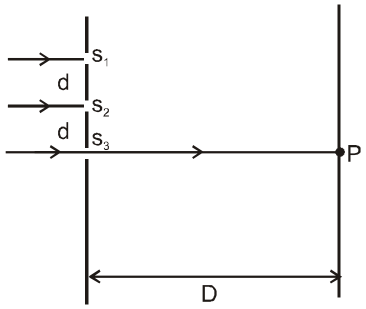
1. Figure shows two coherent sources S_1–S_2 vibrating in same phase. AB is a straight wire lying at a far distance from the sources S_1 and S_2. Let $\frac{\lambda}{d} = 10^{-3}$. $\angle$ BOA = 0.12°. How many bright spots will be seen on the wire, including points A and B.



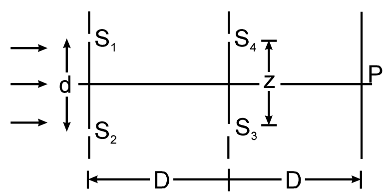
2. In the figure shown three slits s_1, s_2 and s_3 are illuminated with light of wavelength λ. $\lambda << d$ and $D >> d$. Each slit produces same intensity I on the screen. If resultant intensity at the point on screen directly infront of s_2 is 3I then the maximum value of λ is $\frac{nd^2}{2D}$

(i) Find value of n.

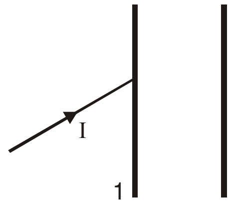
(ii) Also find intensity at point P if λ is of part (i)



3. Parallel monochromatic beam is falling normally on two slits S_1 and S_2 separated by d as shown in figure. By some mechanism, the separation between the slits S_3 and S_4 can be changed. The intensity is measured at the point P which is at the common perpendicular bisector of S_1S_2 and S_3S_4. When $z = \frac{D\lambda}{2d}$, the intensity measured at P is I and when $z = \frac{4D\lambda}{d}$, Intensity is xI. Find x.



4. A narrow monochromatic beam of light of intensity I is incident on a glass plate as shown in figure. Another identical glass plate is kept close to the first one & parallel to it. Each glass plate reflects 25 % of the light incident on it & transmits the remaining. The ratio of the minimum & the maximum intensities in the interference pattern formed by the two beams obtained after one reflection at each plate is 1 : n find value of n. **[JEE 1990, 7]**

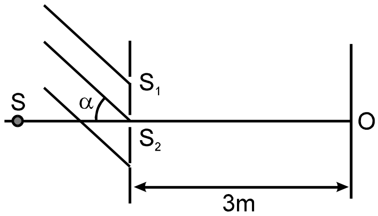


5.✍ In a Young's experiment, the upper slit is covered by a thin glass plate of refractive index 1.4 while the lower slit is covered by another glass plate having the same thickness as the first one but having refractive index 1.7. Interference pattern is observed using light of wavelength 5400 Å. It is found that the point P on the screen where the central maximum fell before the glass plates were inserted now has $(3/4)^{th}$ the original intensity. It is further observed that what used to be the 5th maximum earlier, lies below the point O while the 6th minimum lies above O. The thickness of the glass plate is $n \times 10^{-7}$m. Find the value of n. (Absorption of light by glass plate may be neglected) **[JEE 1997, 5/100]**

PART - III : ONE OR MORE THAN ONE OPTIONS CORRECT TYPE

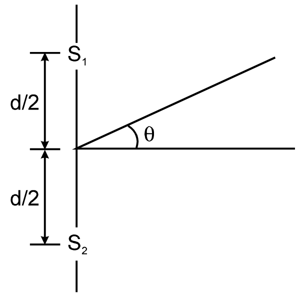
1. In Young's double slit experiment, the interference pattern is found to have an intensity ratio between bright and dark fringes as 9. This implies : **[JEE 1982]**
(A) the intensities at the screen due to the two slits are 5 and 4 units
(B) the intensities at the screen due to the two slits are 4 and 1 units
(C) the amplitude ratio of the individual waves is 3
(D) the amplitude ratio of the individual waves is 2

2.✍ A parallel beam of light (λ = 5000 Å) is incident at an angle α = 30° with the normal to the slit plane in a young's double slit experiment. Assume that the intensity due to each slit at any point on the screen is I_0. Point O is equidistant from S_1 & S_2.The distance between slits is 1mm.
(A) the intensity at O is $4I_0$
(B) the intensity at O is zero
(C) the intensity at a point on the screen 4m from O is $4I_0$
(D) the intensity at a point on the screen 4m from O is zero



3. A Young's double slit experiment is performed with white light:
(A) The maxima next to the central will be red. (B) The central maxima will be white
(C) The maxima next to the central will be violet (D) There will not be a completely dark fringe.

4.✍ In an interfrence arrangement similar to Young's double-slit experiment, the slits S_1 & S_2 are illuminated with coherent microwave sources, each of frequency 10^6 Hz. The sources are synchronized to have zero phase difference. The slits are separated by a distance d = 150.0 m and screen is at very large distance from slits. The intensity $I(\theta)$ is measured as a function of θ, where θ is defined as shown. Screen is at a large distance. If I_0 is the maximum intensity then $I(\theta)$ for $0 \le \theta \le 90º$ is given by: **[JEE 1995, 1 + 2]**



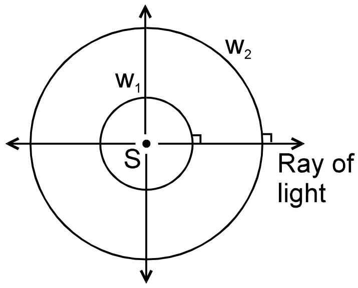
(A) $I(\theta) = \frac{I_0}{2}$ for θ = 30º (B) $I(\theta) = \frac{I_0}{4}$ for θ = 90º
(C) $I(\theta) = I_0$ for θ = 0º (D) $I(\theta)$ is constant for all values of θ.

5. White light is used to illuminate the two silts in a Young's double slit experiment. The separation between the slits is b and the screen is at a distance d (>> b) from the slits. At a point on the screen directly in front of one of the slits, certain wavelengths are missing. Some of these missing wavelengths are : **[JEE 1984]**
(A) $\lambda = \frac{b^2}{d}$ (B) $\lambda = \frac{2b^2}{d}$ (C) $\lambda = \frac{b^2}{3d}$ (D) $\lambda = \frac{2b^2}{3d}$

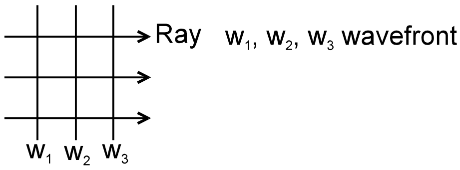
PART - IV : COMPREHENSION

Comprehension-1

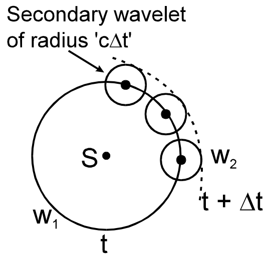
Huygen was the first scientist who proposed the idea of wave theory of light. He said that the light propagates in form of wavefronts. A wavefront is an imaginary surface at every point of which waves are in the same phase. For example the wavefronts for a point source of light is collection of concentric spheres which have centre at the origin. w_1 is a wavefront. w_2 is another wavefront.



The radius of the wavefront at time 't' is 'ct' in this case where 'c' is the speed of light. The direction of propagation of light is perpendicular to the surface of the wavefront. The wavefronts are plane wavefronts in case of a parallel beam of light.



Huygen also said that every point of the wavefront acts as the source of secondary wavelets. The tangent drawn to all secondary wavelets at a time is the new wavefront at that time. The wavelets are to be considered only in the forward direction (i.e. the direction of propagation of light) and not in the reverse direction. If a wavefront w_1 at time t is given, then to draw the wavefront at time t + Δt take some points on the wavefront w_1 and draw spheres of radius 'cΔt'. They are called secondary wavelets.

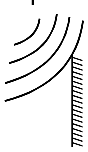


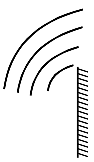
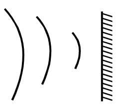
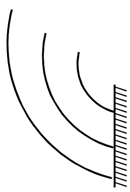
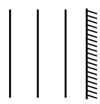
Draw a surface w_2 which is tangential to all these secondary wavelets. w_2 is the wavefront at time 't + Δt'. Huygen proved the laws of reflection and laws of refraction using concept of wavefronts.

1. A point source of light is placed at origin, in air. The equation of wave front of the wave at time t, emitted by source at t = 0, is (take refractive index of air as 1)

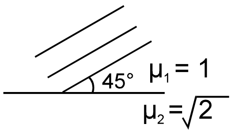
(A) $x + y + z = ct$ (B) $x^2 + y^2 + z^2 = t^2$ (C) $xy + yz + zx = c^2t^2$ (D) $x^2 + y^2 + z^2 = c^2t^2$

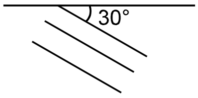
2. Spherical wave fronts shown in figure, strike a plane mirror. Reflected wave fronts will be as shown in

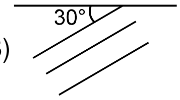


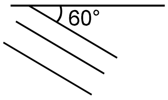
(A)  (B)  (C)  (D) 

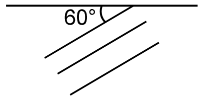
3. Wavefronts incident on an interface between the media are shown in the figure. The refracted wavefronts will be as shown in



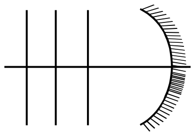
(A) 

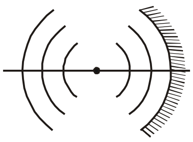
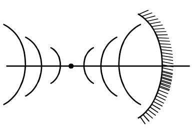
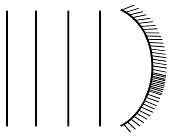
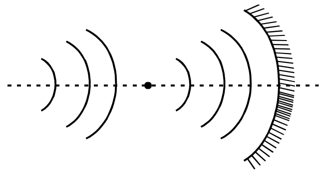
(B) 

(C) 

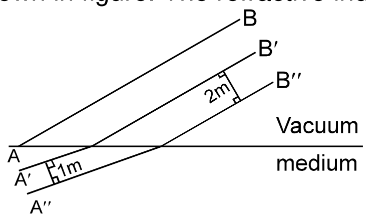
(D) 

4. Plane wavefronts are incident on a spherical mirror as shown. The reflected wavefronts will be



(A) 
(B) 
(C) 
(D) 

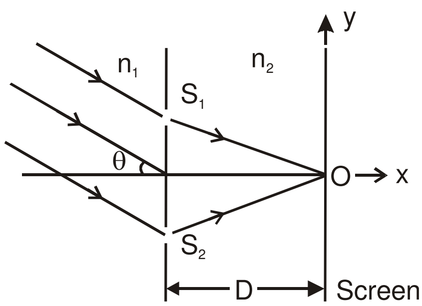
5. Certain plane wavefronts are shown in figure. The refractive index of medium is



(A) 2 (B) 4 (C) 1.5 (D) Cannot be determined

Comprehension-2

In the figure an arrangement of young's double slit experiment is shown. A parallel beam of light of wavelength 'λ' (in medium n_1) is incident at an angle 'θ' as shown. Distance $S_1O = S_2O$. Point 'O' is the origin of the coordinate system. The medium on the left and right side of the plane of slits has refractive index n_1 and n_2 respectively. Distance between the slits is d. The distance between the screen and the plane of slits is D. Using D = 1m, d = 1mm, θ = 30°, λ = 0.3mm, $n_1 = \frac{4}{3}$, $n_2 = \frac{10}{9}$, answer the following



6. The y-coordinate of the point where the total phase difference between the interefering waves is zero, is

(A) y = 0 (B) $y = +\frac{3}{4}$ m (C) $y = -\frac{3}{4}$ m (D) $-\frac{1}{\sqrt{3}}$ m

7. If the intensity due to each light wave at point 'O' is I_0 then the resultant intensity at point 'O' will be -

(A) Zero (B) $2I_0\left(1+\cos\frac{40\pi}{9}\right)$ (C) $3I_0$ (D) I_0

8. y-coordinate of the nearest maxima above 'O' will be -

(A) $\frac{150}{\sqrt{154}}$ cm (B) 24 cm (C) $\frac{100}{\sqrt{99}}$ cm (D) None of these

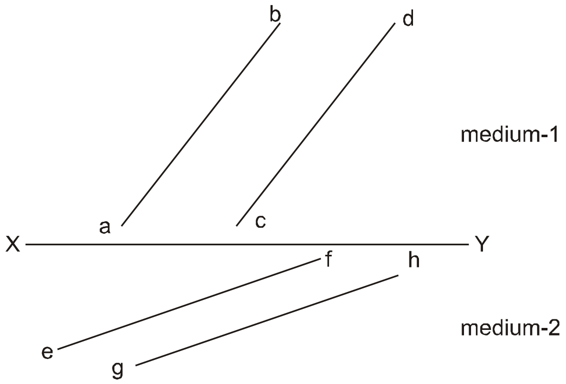
Exercise-3

✍ **Marked Questions can be used as Revision Questions.**
*** Marked Questions may have more than one correct option.**

PART - I : JEE (ADVANCED) / IIT-JEE PROBLEMS (PREVIOUS YEARS)

Paragraph of question no. 1 to 3

The figure shows surface XY separating two transparent media, medium–1 and medium–2. The lines ab and cd represent wavefronts of a light wave travelling in medium–1 and incident on XY. The lines ef and gh represent wavefronts of the light wave in medium–2 after refraction. **[JEE 2007, 4+4+4/162]**



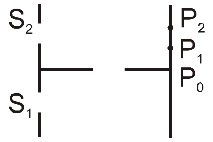
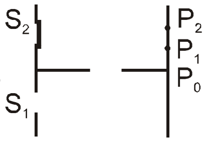
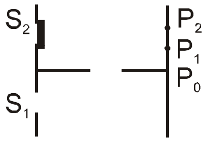
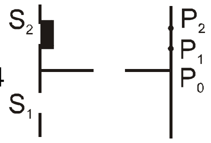
1. Light travels as a
(A) parallel beam in each medium
(B) convergent beam in each medium
(C) divergent beam in each medium
(D) divergent beam in one medium and convergent beam in the other medium

2. The phases of the light wave at c, d, e and f are ϕ_c, ϕ_d, ϕ_e and ϕ_f respectively. It is given that $\phi_c \neq \phi_f$:
(A) ϕ_c cannot be equal to ϕ_d
(B) ϕ_d can be equal to ϕ_e
(C) $(\phi_d - \phi_f)$ is equal to $(\phi_c - \phi_e)$
(D) $(\phi_d - \phi_c)$ is not equal to $(\phi_f - \phi_e)$

3. Speed of light is
(A) the same in medium–1 and medium–2
(B) larger in medium–1 than in medium–2
(C) larger in medium–2 than in medium–1
(D) different at b and d

4*.✍ In a Young's double slit experiment, the separation between the two slits is d and the wavelength of the light is λ. The intensity of light falling on slit 1 is four times the intensity of light falling on slit 2. Choose the correct choice(s). **[JEE 2008, 3/163]**
(A) If $d = \lambda$, the screen will contain only one maximum
(B) If $\lambda < d < 2\lambda$, at least one more maximum (besides the central maximum) will be observed on the screen
(C) If the intensity of light falling on slit 1 is reduced so that it becomes equal to that of slit 2, the intensities of the observed dark and bright fringes will increase
(D) If the intensity of light falling on slit 2 is increased so that it becomes equal to that of slit 1, the intensities of the observed dark and bright fringes will increase

5. **Column I** shows four situations of standard Young's double slit arrangement with the screen placed far away from the slits S_1 and S_2. In each of these cases $S_1P_0 = S_2P_0$, $S_1P_1 - S_2P_1 = \lambda/4$ and $S_1P_2 - S_2P_2 = \lambda/3$, where λ is the wavelength of the light used. In the cases B, C and D, a transparent sheet of refractive index μ and thickness t is pasted on slit S_2. The thicknesses of the sheets are different in different cases. The phase difference between the light waves reaching a point P on the screen from the two slits is denoted by $\delta(P)$ and the intensity by $I(P)$. Match each situation given in **Column-I** with the statement(s) in **Column-II** valid for that situation. **[JEE 2009, 8/240]**

Column–I		Column–II	
(A)		(p)	$\delta(P_0) = 0$
(B)	$(\mu - 1)t = \lambda/4$ 	(q)	$\delta(P_1) = 0$
(C)	$(\mu - 1)t = \lambda/2$ 	(r)	$I(P_1) = 0$
(D)	$(\mu - 1)t = 3\lambda/4$ 	(s)	$I(P_0) > I(P_1)$
		(t)	$I(P_2) > I(P_1)$

6. Young's double slit experiment is carried out by using green, red and blue light, one color at time. The fringe widths recorded are β_G, β_R and β_B, respectively. Then **[IIT-JEE-2012, Paper-1; 3/70, –1]**

(A) $\beta_G > \beta_B > \beta_R$ (B) $\beta_B > \beta_G > \beta_R$ (C) $\beta_R > \beta_B > \beta_G$ (D) $\beta_R > \beta_G > \beta_B$

7. In the Young's double slit experiment using a monochromatic light of wavelength λ, the path difference (in terms of an integer n) corresponding to any point having half the peak intensity is :

[JEE (Advanced) 2013 ; P-1, 2/60]

(A) $(2n+1)\dfrac{\lambda}{2}$ (B) $(2n+1)\dfrac{\lambda}{4}$ (C) $(2n+1)\dfrac{\lambda}{8}$ (D) $(2n+1)\dfrac{\lambda}{16}$

8.✍ Using the expression $2d \sin \theta = \lambda$, one calculates the values of d by measuring the corresponding angles θ in the range 0 to 90º. The wavelength λ is exactly knowns and the error in θ is constant for all values of θ. As θ increases from 0º : **[JEE (Advanced) 2013 ; P-2, 3/60, –1]**

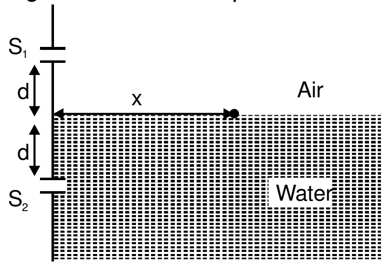
(A) the absolute error in d remains constant. (B) the absolute error in d increases.
(C) the fractional error in d remains constant. (D) the fractional error in d decreases.

9*. A light source, which emits two wavelengths λ_1 = 400 nm and λ_2 = 600 nm, is used in a Young's double slit experiment. If recorded fringe widths for λ_1 and λ_2 are β_1 and β_2 and the number of fringes for them within a distance y on one side of the central maximum are m_1 and m_2, respectively, then

[JEE (Advanced) 2014, P-1, 3/60]

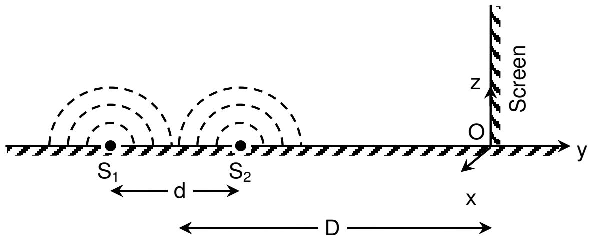
(A) $\beta_2 > \beta_1$
(B) $m_1 > m_2$
(C) From the central maximum, 3rd maximum of λ_2 overlaps with 5th minimum of λ_1
(D) The angular separation of fringes for λ_1 is greater than λ_2

10.✍ A young's double slit interference arrangement with slits S_1 and S_2 is immersed in water (refractive index = 4/3) as shown in the figure. The positions of maxima on the surface of water are given by $x^2 = p^2m^2\lambda^2 - d^2$, where λ is the wavelength of light in air (refractive index = 1), 2d is the separation between the slits and m is an integer. The value of p is **[JEE (Advanced) 2015 ; P-1, 4/88]**



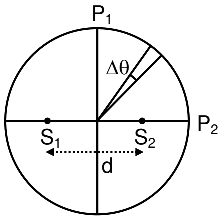
11*. While conducting the Young's double slit experiment, a student replaced to two slits with a large opaque plate in the x-y plane containing two small holes that act as two coherent point sources (S_1, S_2) emitting light of wavelength 600 nm. The student mistakenly placed the screen parallel to the x-z plane (for z > 0) at a distance D = 3 m from the mid-point of S_1S_2, as shown schematically in the figure. The distance between the sources d = 0.6003 mm. The origin O is at the intersection of the screen and the line joining S_1S_2. Which of the following is(are) true of the intensity pattern on the screen ?

[JEE (Advanced) 2016, P-2, 4/62, –2]



(A) Semi circular bright and dark bands centered at point O
(B) Hyperbolic bright and dark bands with foci symmetrically placed about O in the x-direction
(C) The region very close to the point O will be dark
(D) Straight bright and dark bands parallel to the x-axis

12.* Two coherent monochromatic point sources S_1 and S_2 of wavelength λ = 600 nm are placed symmetrically on either side of the center of the circle as shown. The sources are separated by a distance d = 1.8 mm. This arrangement produces interference fringes visible as alternate bright and dark spots on the circumference of the circle. The angular separation between two consecutive bright spots is $\Delta\theta$. Which of the following options is/are correct ? **[JEE (Advanced) 2017, P-2, 4/61, –2]**



(A) The total number of fringes produced between P_1 and P_2 in the first quadrant is close to 3000
(B) A dark spot will be formed at the point P_2
(C) At P_2 the order of the fringe will be maximum
(D) The angular separation between two consecutive bright spots decreases as we move from P_1 to P_2 along the first quadrant

PART - II : JEE (MAIN) / AIEEE PROBLEMS (PREVIOUS YEARS)

1. In a Young's double slit experiment the intensity at a point where the path difference is $\frac{\lambda}{6}$ (λ being the wavelength of the light used) is I. If I_0 denotes the maximum intensity, I/I_0 is equal to:

[AIEEE 2007 ; 3/120, –1]

(1) $\frac{1}{\sqrt{2}}$ (2) $\frac{\sqrt{3}}{2}$ (3) $\frac{1}{2}$ (4) $\frac{3}{4}$

Direction : Questions number 2 – 4 are based on the following paragraph.

An initially parallel cylindrical beam travels in a medium of refractive index $\mu(I) = \mu_0 + \mu_2 I$, where μ_0 and μ_2 are positive constants and I is the intensity of the light beam. The intensity of the beam is decreasing with increasing radius. **[AIEEE-2010, 4/144, – 1]**

2. How does beam travell after entering in medium ?
(1) diverge
(2) converge
(3) diverge near the axis and converge near the periphery
(4) travel as a cylindrical beam

3. The initial shape of the wavefront of the beam is :
(1) convex
(2) concave
(3) convex near the axis and concave near the periphery
(4) planar

4. The speed of light in the medium is
(1) minimum on the axis of the beam (2) the same everywhere in the beam
(3) directly proportional to the intensity I (4) maximum on the axis of the beam

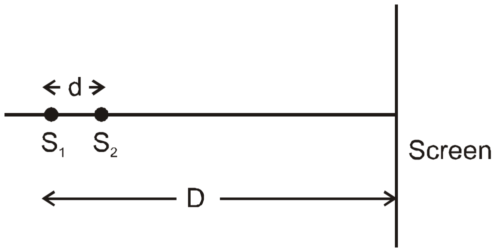
5. At two points P and Q on a screen in Young's double slit experiment, waves from slits S_1 and S_2 have a path difference of 0 and $\lambda/4$ respectively. The ratio of intensities at P and Q will be :
[AIEEE 2011, 11 May; 4/120, –1]
(1) 2 : 1 (2) $\sqrt{2}$: 1 (3) 4 : 1 (4) 3 : 2

6.✍ In a Young's double slit experiment, the two slits act as coherent sources of waves of equal amplitude A and wavelength λ. In another experiment with the same arrangement the two slits are made to act as incoherent sources of waves of same amplitude and wavelength. If the intensity at the middle point of the screen in the first case is I_1 and in the second case is I_2, then the ratio $\frac{I_1}{I_2}$ is :
[AIEEE 2011, 11 May ; 4/120, –1]
(1) 2 (2) 1 (3) 0.5 (4) 4

7.✍ **Statement-1 :** On viewing the clear blue portion of the sky through a Calcite Crystal, the intensity of transmitted light varies as the crystal is rotated.
Statement-2 : The light coming from the sky is polarized due to scattering of sun light by particles in the atmosphere. The scattering is largest for blue light **[AIEEE 2011, 11 May; 4/120, –1]**
(1) Statement-1 is true, statement-2 is false.
(2) Statement-1 is true, statement-2 is true, statement-2 is the correct explanation of statment-1
(3) Statement-1 is true, statement-2 is true, statement-2 is not the correct explanation of statement-1
(4) Statement-1 is false, statement-2 is true.

8. **Direction :** The question has a paragraph followed by two statements, Statement-1 and Statement-2. Of the given four alternatives after the statements, choose the one that describes the statements.
A thin air film is formed by putting the convex surface of a plane-convex lens over a plane glass plate. With monochromatic light, this film gives an interference pattern due to light reflected from the top (convex) surface and the bottom (glass plate) surface of the film. **[AIEEE - 2011, 4/120, –1]**
Statement-1 : When light reflects from the air-glass plate interface, the reflected wave suffers a phase change of π
Statement-2 : The centre of the interference pattern is dark.
(1) Statement-1 is true, statement-2 is false.
(2) Statement-1 is true, Statement-2 is true, Statement-2 is the correct explanation of Statement-1
(3) Statement-1 is true, Statement-2 is true, Statement-2 is not the correct explanation of Statement-1
(4) Statement-1 is false, Statement-2 is true

9. Two coherent point sources S_1 and S_2 are separated by a small distance 'd' as shown. The fringes obtained on the screen will be : **[JEE (Main) 2013, 4/120, –1]**



(1) points (2) straight lines (3) semi-circles (4) concentric circles

10. Tow beams, A and B, of plane polarized light with mutually perpendicular planes of polarization are seen through a polaroid. From the position when the beam A has maximum intensity (and beam B has zero intensity), a rotation of polaroid through 30º makes the two beams appear equally bright. If the initial intensities of the two beams are I_A and I_B respectively, then $\frac{I_A}{I_B}$ equals **[JEE (Main) 2014; 4/120, –1]**

(1) 3 (2) $\frac{3}{2}$ (3) 1 (4) $\frac{1}{3}$

11. On a hot summer night, the refractive index of air is smallest near the ground and increases with height from the ground. When a light beam is directed horizontally, the Huygens' principle leads us to conclude that as it travels, the light beam : **[JEE (Main) 2015; 4/120, –1]**

(1) becomes narrower (2) goes horizontally without any deflection
(3) bends downwards (4) bends upwards

12. Assuming human pupil to have a radius of 0.25 cm and a comfortable viewing distance of 25 cm, the minimum separation between two objects that human eye can resolve at 500 nm wavelength is :

[JEE (Main) 2015; 4/120, –1]

(1) 1 μm (2) 30 μm (3) 100 μm (4) 300 μm

13. The box of a pin hole camera, of length L, has hole of radius a. it is assumed that when the hole is illuminated by a parallel beam of light of wavelength λ the spread of the spot (obtained on the opposite wall of the camera) is the sum of its geometrical spread and the spread due to diffraction. The spot would then have its minimum size (say b_{min}) when : **[JEE (Main) 2016; 4/120, –1]**

(1) $a = \sqrt{\lambda L}$ and $b_{min} = \left(\frac{2\lambda^2}{L}\right)$ (2) $a = \sqrt{\lambda L}$ and $\sqrt{4\lambda L}$

(3) $a = \frac{\lambda^2}{L}$ and $b_{min} = \sqrt{4\lambda L}$ (4) $a = \frac{\lambda^2}{L}$ and $b_{min} = \left(\frac{2\lambda^2}{L}\right)$

14. In a Young's double slit experiment, slits are separated by 0.5 mm, and the screen is placed 150 cm away. A beam of light consisting of two wavelengths, 650 nm and 520 nm, is used to obtain interference fringes on the screen. The least distance from the common central maximum to the point where the bright fringes due to both the wavelengths coincide is : **[JEE (Main) 2017 ; 4/120, –1]**

(1) 15.6 mm (2) 1.56 mm (3) 7.8 mm (4) 9.75 mm

15. Unpolarized light of intensity I passes through an ideal polarizer A. Another identical polarizer B is placed behind A. The intensity of light beyond B is found to be $\frac{I}{2}$. Now another identical polarizer C is placed between A and B. The intensity beyond B is now found to be $\frac{I}{8}$. The angle between polarizer A and C is : **[JEE (Main) 2018 ; 4/120, –1]**

(1) 45° (2) 60° (3) 0° (4) 30°

16. The angular width of the central maximum in a single slit diffraction pattern is 60°. The width of the slit is 1 μm. The slit is illuminated by monochromatic plane waves. If another slit of same width is made near it, Young's fringes can be observed on a screen placed at a distance 50 cm from the slits. If the observed fringe width is 1 cm, what is slit separation distance? (i.e. distance between the centres of each slit.) **[JEE (Main) 2018; 4/120, –1]**

(1) 75 μm (2) 100 μm (3) 25 μm (4) 50 μm

Exercise-1

EXERCISE-1

PART-I

SECTION (A) :

A-1. (a) 9I (b) 5I (c) I

A-2. 2

SECTION (B) :

B-1. (a) Angular separation of the fringes remains constant ($= \lambda$ / d).The actual separation of the fringe increases in proportion to the distance of the screen from the plane of the two slits.
(b) The separation of the fringes (and also angular separation) decrease.
(c) The separation of the fringe (and also angular separation) decreases.
(d) By slightly increasing the width of the slits, we are only increasing the intensity of incident beam. Again no change in λ, D, d. so β unchanged but sharpness of the fringe increase.

B-2. 1.625 mm **B-3.** 0.30 mm

B-4. $\frac{180}{\pi} \times 2 \times 10^{-4}$ degree = 0.011º

SECTION (C) :

C-1. 0.9 mm

SECTION (D) :

D-1. (a) $\beta = 4.0 \times 10^{-4}$ m (b) $\frac{\beta}{3}$ and $\frac{2\beta}{3}$

D-2. $\frac{\lambda}{4(\mu-1)}$

SECTION (E) :

E-1. Maximum

SECTION (F) :

F-1. 1.45 **F-2.** 100 nm

SECTION (G) :

G-1. (i) 1.8 μm (ii) 1.2 μm

PART-II

SECTION (A) :

A-1. (A) **A-2.** (C)

SECTION (B) :

B-1. (C) **B-2.** (D) **B-3.** (A)
B-4. (B) **B-5.** (D) **B-6.** (C)
B-7. (D)

SECTION (C) :

C-1. (D) **C-2.** (C)

SECTION (D) :

D-1. (A) **D-2.** (D)

SECTION (E) :

E-1. (D) **E-2.** (A)

SECTION (F) :

F-1. (B)

SECTION (G) :

G-1. (D) **G-2.** (A) **G-3.** (A)
G-4. (B) **G-5.** (A) **G-6.** (C)
G-7. (B) **G-8.** (C) **G-9.** (D)

PART-III

1. (A)– q,r,s ; (B)–p,q,r,s ; (C)–q,r,s ; (D) – p,q,r,s

2. (A) – r, s;(B)–p, q, s; (C) – p, q, s ; (D) – r, s

EXERCISE-2

PART-I

1. (B) **2.** (C) **3.** (D)
4. (A) **5.** (A) **6.** (C)
7. (B) **8.** (B) **9.** (B)
10. (D) **11.** (C)

PART-II

1. 3 **2.** (i) 3 (ii) 3 I
3. 2 **4.** 49 **5.** 93

PART-III

1. (BD) **2.** (AC) **3.** (BCD)
4. (AC) **5.** (AC)

PART-IV

1. (D) **2.** (C) **3.** (B)
4. (A) **5.** (A) **6.** (C)
7. (D) **8.** (A)

EXERCISE-3

PART-I

1. (A) **2.** (C) **3.** (B)
4. (AB)
5. (A) $\rightarrow$ p, s; (B) $\rightarrow$ q; (C) $\rightarrow$ t ; (D) $\rightarrow$ r, s, t
6. (D) **7.** (B) **8.** (D)
9. (ABC) **10.** (3) **11.** (AC)
12. (AC)

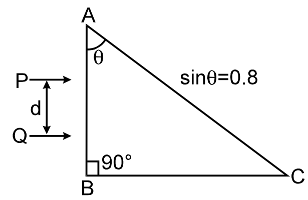
PART-II

1. (4) **2.** (2) **3.** (4)
4. (1) **5.** (1) **6.** (1)
7. (2) **8.** (3) **9.** (4)
10. (4) **11.** (4) **12.** (2)
13. (2) **14.** (3) **15.** (1)
16. (3)

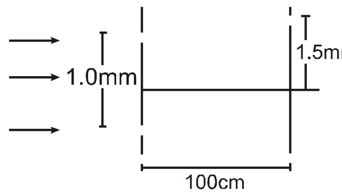
High Level Problems (HLP)

SUBJECTIVE QUESTIONS

1. Two parallel beams of light P & Q (separation d) containing radiations of wavelengths 4000 Å & 5000 Å (which are mutually coherent in each wavelength separately) are incident normally on a prism as shown in figure. The refractive index of the prism as a function of wavelength is given by the relation, $\mu(\lambda) = 1.20 + \frac{b}{\lambda^2}$, where λ is in Å & b is a positive constant. The value of b is such that the condition for total reflection at the face AC is just satisfied for one wavelength & is not satisfied for the other. A convergent lens is used to bring these transmitted beams into focus. If the intensities of the upper & the lower beams immediately after transmission from the face AC, are 4I & I respectively, find the resultant intensity at the focus. **[JEE 1991, 8]**

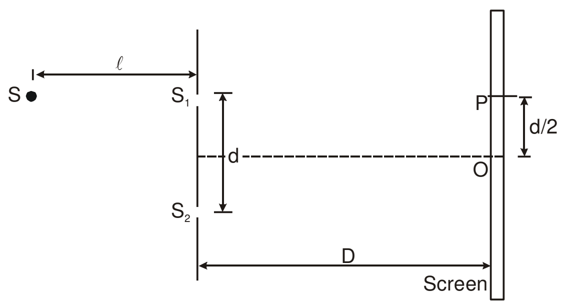


2. White coherent light (400 nm-700 nm) is sent through the slits of a Young's double slit experiment (as shown in the figure). The separation between the slits is 1 mm and the screen is 100 cm away from the slits. There is a hole in the screen at a point 1.5 mm away (along the width of the fringes) from the central line. (a) For which wavelength(s) there will be minima at that point ? (b) which wavelength(s) will have a maximum intensity?



3. A beam of light consisting of two wavelengths, 6500 Å and 5200 Å is used in double slit experiment (1 Å = 10^{-10} m). The distance between the slits is 2.0 mm and the distance between the plane of the slits and the screen is 120 cm. (a) Find the distance of the third bright fringe on the screen from the central maximum for the wavelength 6500 Å. (b) What is the least distance from the central maximum where the bright fringes due to both the wavelengths coincide?

4. A source S is kept directly behind the slit S_1 in a double–slit apparatus. Find the phase difference at a point O which is equidistant from S_1 & S_2. What will be the phase difference at P if a liquid of refraction index μ is filled; (wavelength of light in air is λ due to the source). Assume same intensity due to S_1 and S_2 on screen and position at liquid. ($\lambda << d$, $d << D$, $\ell >> d$)



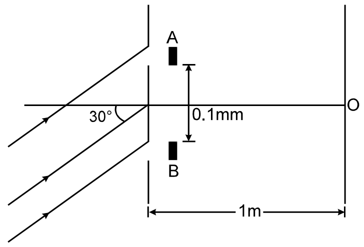
(a) between the screen and the slits.

(b) between the slits & the source S. In this case find the minimum distance between the points on the screen where the intensity is half the maximum intensity on the screen.

5. A monochromatic light of λ = 5000 Å is incident on two slits separated by a distance of 5×10^{-4} m. The interference pattern is seen on a screen placed at a distance of 1 m from the slits. A thin glass plate of thickness 1.5×10^{-6} m & refractive index $\mu = 1.5$ is placed between one of the slits & the screen. Find the intensity at the centre of the screen, if the intensity there is I_0 in the absence of the plate. Also find the lateral shift of the central maximum. **[REE 1993, 4]**

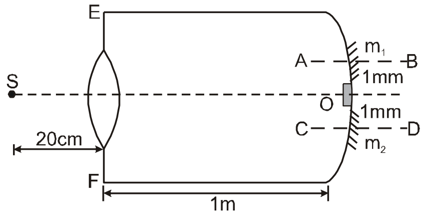
6. In a YDSE experiment, the distance between the slits & the screen is 100 cm. For a certain distance between the slits, an interference pattern is observed on the screen with the fringe width 0.25 mm. When the distance between the slits is increased by Δd = 1.2 mm, the fringe width decreased to n = 2/3 of the original value. In the final position, a thin glass plate of refractive index 1.5 is kept in front of one of the slits & the shift of central maximum is observed to be 20 fringe width. Find the thickness of the plate & wavelength of the incident light.

7. In a YDSE a parallel beam of light of wavelength 6000 Å is incident on slits at angle of incidence 30º. A & B are two thin transparent films each of R.I. 1.5. Thickness of A is 20.4 μm. Light coming through A & B have intensities I and 4I respectively on the screen. Intensity at point O which is symmetric relative to the slits is 3 I. The central maxima is above O.



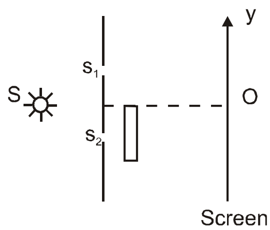
(a) What is the maximum thickness of B to do so.
Assuming thickness of B to be that found in part (a) answer the following parts.
(b) Find fringe width, maximum intensity & minimum intensity on screen.
(c) Distance of nearest minima from O.
(d) Intensity at 5 cm on either side of O.

8. An equi convex lens of focal length 10 cm (in air) and R.I. 3/2 is put at a small opening on a tube of length 1 m fully filled with liquid of R.I. 4/3. A concave mirror of radius of curvature 20 cm is cut into two halves m_1 and m_2 and placed at the end of the tube. m_1 & m_2 are placed such that their principal axis AB and CD respectively are separated by 1 mm each from the principal axis of the lens.



A slit S placed in air illuminates the lens with light of frequency 7.5 $\times$ 10^{14} Hz. The light reflected from m_1 and m_2 forms interference pattern on the left end EF of the tube. O is an opaque substance to cover the hole left by m_1 & m_2. Find :
(a) the position of the image formed by lens water combination.
(b) the distance between the images formed by m_1 & m_2.
(c) width of the fringes on EF.

9. The Young's double slit experiment is done in a medium of refractive index 4/3. A light of 600 nm wavelength is falling on the slits having 0.45 mm separation. The lower slit S_2 is covered by a thin glass sheet of thickness 10.4 μm and refractive index 1.5. The interference pattern is observed on a screen placed 1.5 m form the slits as shown. **[JEE 1999 (Main), 5+3+2/200]**



(a) Find the location of central maximum (bright fringe with zero path difference) on the y-axis.
(b) Find the light intensity at point O relative to the maximum fringe intensity.
(c) Now if 600 nm light is replaced by white light of range 400 nm to 700 nm, find the wavelengths of the light that form maxima exactly at point O.
[All wavelengths in the problem are for the given medium of refractive index 4/3. Ignore dispersion]

10. A glass plate of refractive index 1.5 is coated with a thin layer of thickness t and refractive index 1.8. Light of wavelength λ travelling in air is incident normally on the layer. It is partly reflected at the upper and the lower surface of the layer and the two reflected rays interfere. Write the condition for their constructive interference. If λ = 648 nm, obtain the least value of t for which the rays interfere constructively. **[JEE 2000 (Main), 4/100]**

HLP Answers

1. 9I

2. (a) 428 nm, 600 nm, (b) 500 nm

3. (a) 1.17 mm. (b) 1.56 mm

4. (a) $\Delta\phi = \left(\frac{1}{\ell} + \frac{\mu}{D}\right)\frac{\pi d^2}{\lambda}$ (b) $\Delta\phi = \left(\frac{\mu}{\ell} + \frac{1}{D}\right)\frac{\pi d^2}{\lambda}$; $D_{min} = \frac{\beta}{2} = \frac{\lambda D}{2d}$

5. 0, 1.5 mm

6. λ = 600 nm , t = 24 μm

7. (a) t_B = 120 μm (b) β = 6 mm ; I_{max} = 9I , I_{min} = I (c) β/6 = 1 mm
(d) I (at 5 cm above O) = 9I, I (at 5 cm below O) = 3I

8. (a) 80 cm behind the lens (b) 4 mm (c) β = 60 μm

9. (a) y = – 4.33 mm (b) $I_0 = \frac{3}{4} I_{max}$ (c) λ = 650 nm, 433.3 nm

10. $2\mu t = \left(n + \frac{1}{2}\right)\lambda$ with μ = 1.8 and n = 0,1,2,3....... ; 90 nm