CENTER OF MASS

Contents

JEE (ADVANCED) SYLLABUS

Center of Mass : Systems of particles; Center of mass and its motion; Impulse; Elastic and inelastic collisions. Conservation of linear momentum.

JEE (MAIN) SYLLABUS

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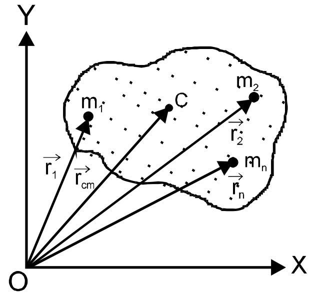
Every physical system has associated with it a certain point whose motion characterises the motion of the whole system. When the system moves under some external forces, then this point moves as if the entire mass of the system is concentrated at this point and also the external force is applied at this point for translational motion. This point is called the center of mass of the system.

CENTER OF MASS OF A SYSTEM OF 'N' DISCRETE PARTICLES

Consider a system of N point masses m_1, m_2, m_3, ... m_n whose position vectors from origin O are given by $\vec{r}_1$, $\vec{r}_2$, $\vec{r}_3$,............... respectively. Then the position vector of the center of mass C of the system is given by

$$\vec{r}_{cm} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + ... + m_n\vec{r}_n}{m_1 + m_2 + ... + m_n} \quad ; \quad \vec{r}_{cm} = \frac{\sum_{i=1}^{n} m_i\vec{r}_i}{\sum_{i=1}^{n} m_i}$$



$$\vec{r}_{cm} = \frac{1}{M}\sum_{i=1}^{n} m_i\vec{r}_i$$

where, $m_i\vec{r}_i$ is called the moment of mass of the particle w.r.t O.

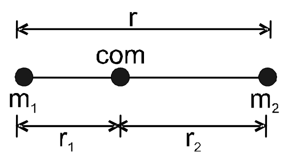
$M = \left(\sum_{i=1}^{n} m_i\right)$ is the total mass of the system.

Note: If the origin is taken at the center of mass then $\sum_{i=1}^{n} m_i\vec{r}_i = 0$. Hence, the COM is the point about which the sum of “mass moments” of the system is zero.

POSITION OF COM OF TWO PARTICLES

Center of mass of two particles of masses m_1 and m_2 separated by a distance r lies in between the two particles. The distance of center of mass from any of the particle (r) is inversely proportional to the mass of the particle (m)

i.e. $r \propto 1/m$



or $\frac{r_1}{r_2} = \frac{m_2}{m_1}$

or $m_1r_1 = m_2r_2$

or $r_1 = \left(\frac{m_2}{m_2 + m_1}\right)r$ and $r_2 = \left(\frac{m_1}{m_1 + m_2}\right)r$

Here, r_1 = distance of COM from m_1

and r_2 = distance of COM from m_2

From the above discussion, we see that

$r_1 = r_2 = 1/2$ if $m_1 = m_2$, i.e., COM lies midway between the two particles of equal masses.

Similarly, $r_1 > r_2$ if $m_1 < m_2$ and $r_1 < r_2$ if $m_2 < m_1$, i.e., COM is nearer to the particle having larger mass.

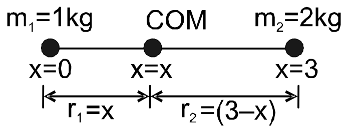
Solved Example

Example 1. Two particles of mass 1 kg and 2 kg are located at x = 0 and x = 3m. Find the position of their center of mass.

Solution : Since, both the particles lies on x-axis, the COM will also lie on x-axis. Let the COM is located at x = x, then

r_1 = distance of COM from the particle of mass 1kg = x

and r_2 = distance of COM from the particle of mass 2 kg = (3 – x)



Using $\frac{r_1}{r_2} = \frac{m_2}{m_1}$ or $\frac{x}{3-x} = \frac{2}{1}$ or x = 2 m

Thus, the COM of the two particles is located at x = 2m

Example 2. The position vector of three particles of masses m_1 = 1 kg, m_2 = 2 kg and m_3 = 3kg are $\vec{r}_1 = (\hat{i}+4\hat{j}+\hat{k})$m, $\vec{r}_2 = (\hat{i}+\hat{j}+\hat{k})$m and $\vec{r}_3 = (2\hat{i}-\hat{j}-2\hat{k})$m respectively. Find the position vector of their center of mass.

Solution : The position vector of COM of the three particles will be given by $\vec{r}_{COM} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3}{m_1+m_2+m_3}$

Substituting the values, we get

$$\vec{r}_{COM} = \frac{(1)(\hat{i}+4\hat{j}+\hat{k})+(2)(\hat{i}+\hat{j}+\hat{k})+(3)(2\hat{i}-\hat{j}-2\hat{k})}{1+2+3} = \frac{1}{2}(3\hat{i}+\hat{j}-\hat{k})\text{m} \qquad \textbf{Ans.}$$

Example 3. Four particles of mass 1 kg, 2 kg, 3 kg and 4 kg are placed at the four vertices A, B, C and D of a square of side 1 m. Find the position of center of mass of the particles.

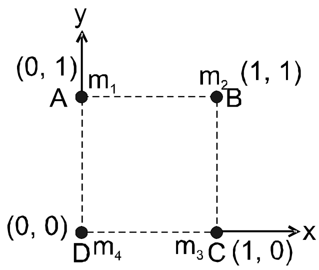
Solution : Assuming D as the origin, DC as x-axis and DA as y-axis, we have

m_1 = 1 kg, (x_1, y_1) = (0, 1m)

m_2 = 2 kg, (x_2, y_2) = (1m, 1m)

m_3 = 3 kg, (x_3, y_3) = (1m, 0)

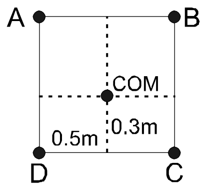
and m_4 = 4 kg, (x_4, y_4) = (0, 0)



Co-ordinates of their COM are

$$x_{COM} = \frac{m_1x_1 + m_2x_2 + m_3m_3 + m_4x_4}{m_1+m_2+m_3+m_4}$$

$$= \frac{(1)(0)+2(1)+3(1)+4(0)}{1+2+3+4} = \frac{5}{10} = \frac{1}{2}\text{ m} = 0.5\text{ m}$$



Similarly, $$y_{COM} = \frac{m_1y_1 + m_2y_2 + m_3y_3 + m_4y_4}{m_1+m_2+m_3+m_4}$$

$$= \frac{(1)(1)+2(1)+3(0)+4(0)}{1+2+3+4} = \frac{3}{10} = 0.3\text{ m}$$

$\therefore$ (x_{COM}, y_{COM}) = (0.5 m, 0.3 m) **Ans.**

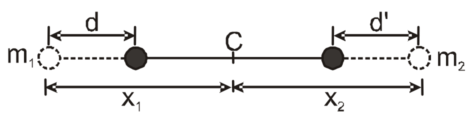
Thus, position of COM of the four particles is as shown in figure.

Example 4. Consider a two-particle system with the particles having masses m_1 and m_2. If the first particle is pushed towards the center of mass through a distance d, by what distance should the second particle be moved so as to keep the center of mass at the same position?

Solution : Consider figure. Suppose the distance of m_1 from the center of mass C is x_1 and that of m_2 from C is x_2. Suppose the mass m_2 is moved through a distance d′ towards C so as to keep the center of mass at C.

Then, $m_1x_1 = m_2x_2$ ....(i)

and $m_1(x_1 - d) = m_2(x_2 - d')$ ....(ii)



Subtracting (ii) from (i)

$m_1d = m_2d'$ or, $d' = \frac{m_1}{m_2}d$

CENTER OF MASS OF A CONTINUOUS MASS DISTRIBUTION

For continuous mass distribution the center of mass can be located by replacing summation sign with an integral sign. Proper limits for the integral are chosen according to the situation

$$x_{cm} = \frac{\int x\,dm}{\int dm},\quad y_{cm} = \frac{\int y\,dm}{\int dm},\quad z_{cm} = \frac{\int z\,dm}{\int dm}$$

$\int dm = M$ (mass of the body)

$$\vec{r}_{cm} = \frac{1}{M}\int \vec{r}\,dm$$

Note: If an object has symmetric mass distribution about x axis then y coordinate of COM is zero and vice-versa

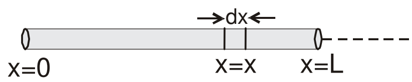
CENTER OF MASS OF A UNIFORM ROD

Suppose a rod of mass M and length L is lying along the x-axis with its one end at x = 0 and the other at x = L. Mass per unit length of the rod = $\frac{M}{L}$

Hence, dm, (the mass of the element dx situated at x = x is) = $\frac{M}{L}dx$

The coordinates of the element dx are (x, 0, 0). Therefore, x-coordinate of COM of the rod will be

$$x_{COM} = \frac{\int_0^L x\,dm}{\int dm} = \frac{\int_0^L (x)\left(\frac{M}{L}dx\right)}{M} = \frac{1}{L}\int_0^L x\,dx = \frac{L}{2}$$



The y-coordinate of COM is $y_{COM} = \frac{\int y\,dm}{\int dm} = 0$. Similarly, $z_{COM} = 0$

i.e., the coordinates of COM of the rod are $\left(\frac{L}{2}, 0, 0\right)$, i.e. it lies at the center of the rod.

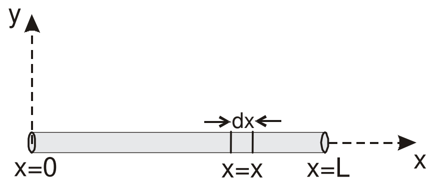
Solved Example

Example 5. A rod of length L is placed along the x-axis between x = 0 and x = L. The linear density (mass/length) λ of the rod varies with the distance x from the origin as $\lambda = Rx$. Here, R is a positive constant. Find the position of center of mass of this rod.

Solution : Mass of element dx situated at x = x is $dm = \lambda\,dx = Rx\,dx$

The COM of the element has coordinates (x, 0, 0)

Therefore, x-coordinate of COM of the rod will be x_{COM}



$$= \frac{\int_0^L x\,dm}{\int dm} = \frac{\int_0^L (x)(Rx)dx}{\int_0^L (Rx)dx} = \frac{R\int_0^L x^2 dx}{R\int_0^L x\,dx} = \frac{\left[\frac{x^3}{3}\right]_0^L}{\left[\frac{x^2}{2}\right]_0^L} = \frac{2L}{3}$$

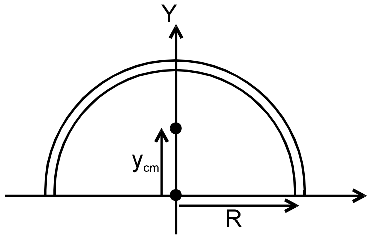
The y-coordinate of COM of the rod is $y_{COM} = \frac{\int y\,dm}{\int dm} = 0$ (as y = 0)

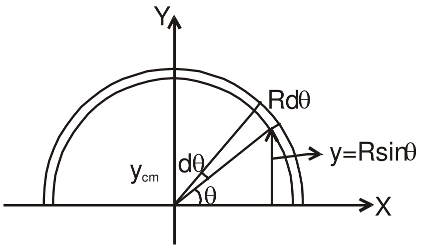
Similarly, $z_{COM} = 0$

Hence, the center of mass of the rod lies at $\left[\frac{2L}{3}, 0,\ 0\right]$ **Ans.**

CENTER OF MASS OF A SEMICIRCULAR RING

Figure shows the object (semi circular ring). By observation we can say that the x-coordinate of the center of mass of the ring is zero as the half ring is symmetrical about y-axis on both sides of the origin. Only we are required to find the y-coordinate of the center of mass.





To find y_{cm} we use $y_{cm} = \dfrac{1}{M}\int dmy$ .....(i)

Here for dm we consider an elemental arc of the ring at an angle θ from the x-direction of angular width $d\theta$. If radius of the ring is R then its y coordinate will be R $\sin\theta$, here dm is given as

$$dm = \frac{M}{\pi R} \times R\, d\theta$$

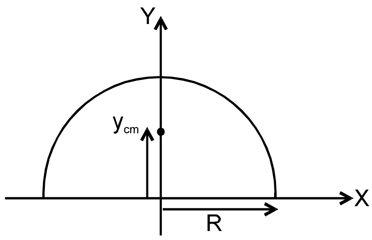
So from equation ....(i), we have

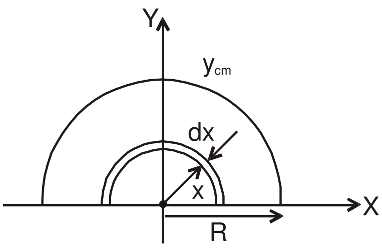
$$y_{cm} = \frac{1}{M}\int_0^{\pi} \frac{M}{\pi R} R d\theta\ (R \sin\theta) = \frac{R}{\pi}\int_0^{\pi} \sin\theta d\theta$$

$$y_{cm} = \frac{2R}{\pi} \qquad .....(ii)$$

CENTER OF MASS OF SEMICIRCULAR DISC

Figure shows the half disc of mass M and radius R. Here, we are only required to find the y-coordinate of the center of mass of this disc as center of mass will be located on its half vertical diameter. Here to find y_{cm}, we consider a small elemental ring of mass dm of radius x on the disc (disc can be considered to be made up such thin rings of increasing radii) which will be integrated from 0 to R. Here dm is given as $dm = \dfrac{2M}{\pi R^2}(\pi\, x)dx$





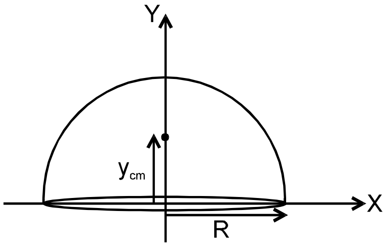
Now the y-coordinate of the element is taken as $\dfrac{2x}{\pi}$, as in previous section, we have derived that the center of mass of a semi circular ring is concentrated at $\dfrac{2R}{\pi}$

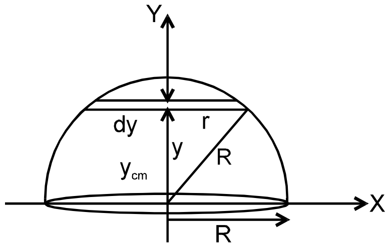
Here y_{cm} is given as $y_{cm} = \dfrac{1}{M}\displaystyle\int_0^R dm\ \frac{2x}{\pi} = \frac{1}{M}\int_0^R \frac{4M}{\pi R^2} x^2 dx$

$$y_{cm} = \frac{4R}{3\pi}$$

CENTER OF MASS OF A SOLID HEMISPHERE

The hemisphere is of mass M and radius R. To find its center of mass (only y-coordinate), we consider an element disc of width dy, mass dm at a distance y from the center of the hemisphere. The radius of this elemental disc will be given as r = $\sqrt{R^2 - y^2}$





The mass dm of this disc can be given as dm = $\dfrac{3M}{2\pi R^3} \times \pi r^2 dy = \dfrac{3M}{2R^3}\ (R^2 - y^2)dy$

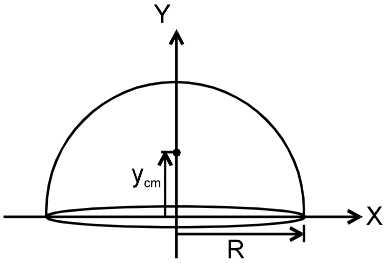
y_{cm} of the hemisphere is given as $y_{cm} = \dfrac{1}{M}\int_0^R dmy\ = \dfrac{1}{M}\int_0^R \dfrac{3M}{2R^3}\ (R^2 - y^2)\ dy\ y = \dfrac{3}{2R^3}\int_0^R (R^2 - y^2)ydy$

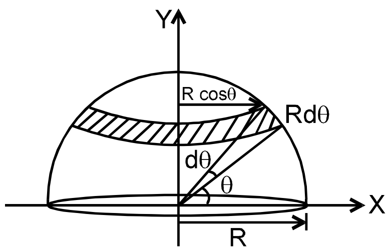
$$y_{cm} = \frac{3R}{8}$$

CENTER OF MASS OF A HOLLOW HEMISPHERE

A hollow hemisphere of mass M and radius R. Now we consider an elemental circular strip of angular width $d\theta$ at an angular distance θ from the base of the hemisphere. This strip will have an area.

$$dS = 2\pi R \cos\theta\ Rd\theta$$





Its mass dm is given as

$$dm = \frac{M}{2\pi R^2}\ 2\pi R \cos\theta\ Rd\theta$$

Here y-coordinate of this strip of mass dm can be taken as R $\sin\theta$. Now we can obtain the center of mass of the system as.

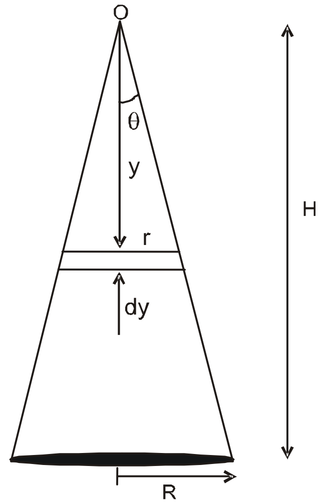
$$y_{cm} = \frac{1}{M}\int_0^{\frac{\pi}{2}} dmR\sin\theta\ = \frac{1}{M}\int_0^{\frac{\pi}{2}}\left(\frac{M}{2\pi R^2}2\pi R^2\cos\theta d\theta\right)R\ \sin\theta$$

$$= R\int_0^{\frac{\pi}{2}}\sin\theta\cos\theta d\theta \quad \Rightarrow \qquad y_{cm} = \frac{R}{2}$$

CENTER OF MASS OF A SOLID CONE

A solid cone has mass M, height H and base radius R. Obviously the center of mass of this cone will lie somewhere on its axis, at a height less than H/2. To locate the center of mass we consider an elemental disc of width dy and radius r, at a distance y from the apex of the cone. Let the mass of this disc be dm, which can be given as

$$dm = \frac{3M}{\pi R^2 H} \times \pi r^2\, dy$$

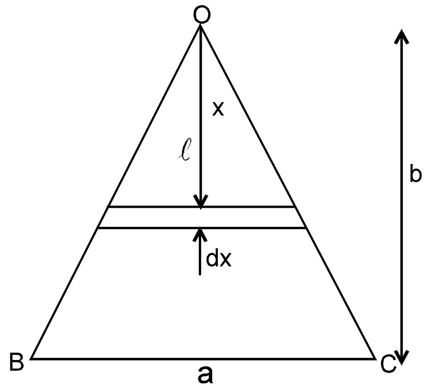


here y_{cm} can be given as $y_{cm} = \frac{1}{M}\int_0^H y\,dm = \frac{1}{M}\int_0^H \left(\frac{3M}{\pi R^2 H}\pi\left(\frac{Ry}{H}\right)^2 dy\right) y = \frac{3}{H^3}\int_0^H y^3 dy = \frac{3H}{4}$

Solved Example

Example 6. Find out the center of mass of an isosceles triangle of base length a and altitude b. Assume that the mass of the triangle is uniformly distributed over its area.

Solution : To locate the center of mass of the triangle, we take a strip of width dx at a distance x from the vertex of the triangle. Length of this strip can be evaluated by similar triangles as $\ell = x.\,(a/b)$

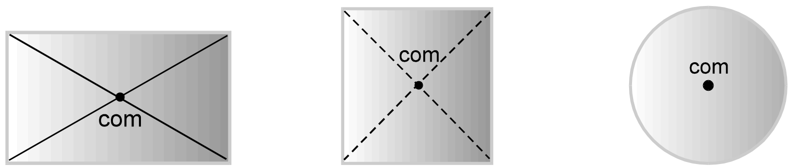


Mass of the strip is $dm = \frac{2M}{ab}\ell dx$

Distance of center of mass from the vertex of the triangle is $x_{CM} = \frac{1}{M}\int x\,dm = \int_0^b \frac{2x^2}{b^2}dx = \frac{2}{3}b$

Proceeding in the similar manner, we can find the COM of certain rigid bodies. Center of mass of some well known rigid bodies are given below :

1. Center of mass of a uniform rectangular, square or circular plate lies at its center. Axis of symmetry plane of symmetry.



2. For a laminar type (2-dimensional) body with uniform negligible thickness the formulae for finding the position of center of mass are as follows :

$$\vec{r}_{COM} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + ...}{m_1 + m_2 + ...} = \frac{\rho A_1 t\vec{r}_1 + \rho A_2 t\vec{r}_2 + ...}{\rho A_1 t + \rho A_2 t + ...} \qquad (\because\ m = \rho At)$$

or $\vec{r}_{COM} = \frac{A_1\vec{r}_1 + A_2\vec{r}_2 + ...}{A_1 + A_2 + ...}$ Here, A stands for the area,

3. If some mass of area is removed from a rigid body, then the position of center of mass of the remaining portion is obtained from the following formulae :

(i) $\vec{r}_{COM} = \dfrac{m_1\vec{r}_1 - m_2\vec{r}_2}{m_1 - m_2}$ or $\vec{r}_{COM} = \dfrac{A_1\vec{r}_1 - A_2\vec{r}_2}{A_1 - A_2}$

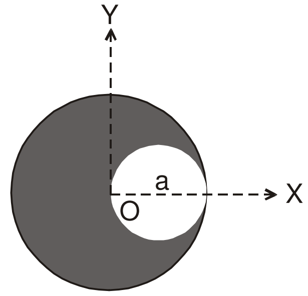
(ii) $x_{COM} = \dfrac{m_1x_1 - m_2x_2}{m_1 - m_2}$ or $x_{COM} = \dfrac{A_1x_1 - A_2x_2}{A_1 - A_2}$

$y_{COM} = \dfrac{m_1y_1 - m_2y_2}{m_1 - m_2}$ or $y_{COM} = \dfrac{A_1y_1 - A_2y_2}{A_1 - A_2}$

and $z_{COM} = \dfrac{m_1z_1 - m_2z_2}{m_1 - m_2}$ or $z_{COM} = \dfrac{A_1z_1 - A_2z_2}{A_1 - A_2}$

Here, m_1, A_1, $\vec{r}_1$, x_1, y_1 and z_1 are the values for the whole mass while m_2, A_2, $\vec{r}_2$, $\vec{x}_2$, y_2 and z_2 are the values for the mass which has been removed. Let us see two examples in support of the above theory.

Example 7. Find the position of center of mass of the uniform lamina shown in figure.



Solution : Here, A_1 = area of complete circle = πa^2

A_2 = area of small circle = $\pi\left(\dfrac{a}{2}\right)^2 = \dfrac{\pi a^2}{4}$

(x_1, y_1) = coordinates of center of mass of large circle = (0, 0)

and (x_2, y_2) = coordinates of center of mass of small circle = $\left(\dfrac{a}{2}, 0\right)$

Using $x_{COM} = \dfrac{A_1x_1 - A_2x_2}{A_1 - A_2}$

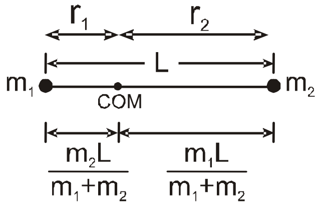
we get $x_{COM} = \dfrac{-\dfrac{\pi a^2}{4}\left(\dfrac{a}{2}\right)}{\pi a^2 - \dfrac{\pi a^2}{4}} = \dfrac{-\left(\dfrac{1}{8}\right)}{\left(\dfrac{3}{4}\right)} a = -\dfrac{a}{6}$ and $y_{COM} = 0$ as y_1 and y_2 both are zero.

Therefore, coordinates of COM of the lamina shown in figure are $\left(-\dfrac{a}{6}, 0\right)$ **Ans.**

CENTER OF MASS OF SOME COMMON SYSTEMS

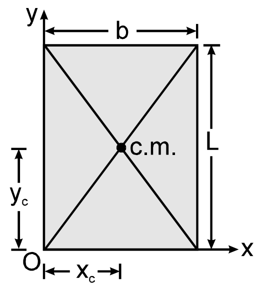
⇒ A system of two point masses $m_1 r_1 = m_2 r_2$

The center of mass lies closer to the heavier mass.



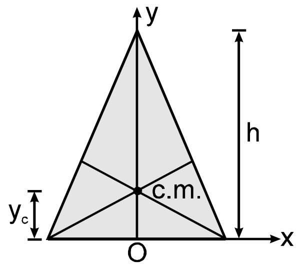
⇒ Rectangular plate (By symmetry)

$x_c = \frac{b}{2}$ $\quad y_c = \frac{L}{2}$



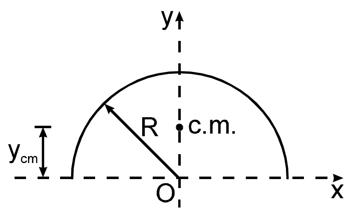
⇒ A triangular plate (By qualitative argument)

at the centroid : $y_c = \frac{h}{3}$



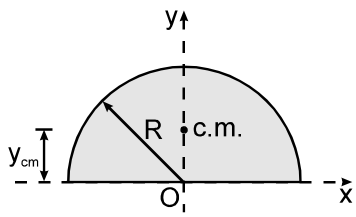
⇒ A semi-circular ring

$y_c = \frac{2R}{\pi}$ $\quad x_c = 0$



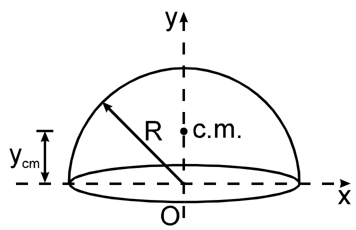
⇒ A semi-circular disc

$y_c = \frac{4R}{3\pi}$ $\quad x_c = 0$



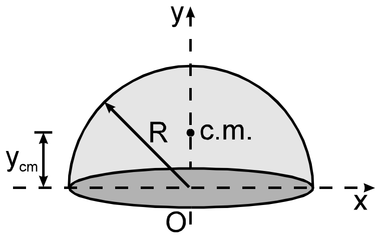
⇒ A hemispherical shell

$y_c = \frac{R}{2}$ $\quad x_c = 0$



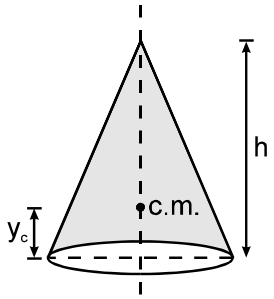
⇒ A solid hemisphere

$y_c = \frac{3R}{8} \qquad x_c = 0$



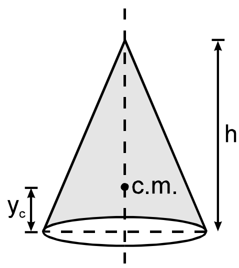
⇒ A circular cone (solid)

$y_c = \frac{h}{4}$



⇒ A circular cone (hollow)

$y_c = \frac{h}{3}$



MOTION OF CENTER OF MASS AND CONSERVATION OF MOMENTUM :

Velocity of center of mass of system

$$\vec{v}_{cm} = \frac{m_1 \frac{d\vec{r}_1}{dt} + m_2 \frac{d\vec{r}_2}{dt} + m_3 \frac{d\vec{r}_3}{dt} .... + m_n \frac{d\vec{r}_n}{dt}}{M} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3 \ .... + m_n\vec{v}_n}{M}$$

Here numerator of the right hand side term is the total momentum of the system i.e., summation of momentum of the individual component (particle) of the system

Hence velocity of center of mass of the system is the ratio of momentum of the system to the mass of the system.

$\therefore \quad \vec{P}_{System} = M\,\vec{v}_{cm}$

Acceleration of center of mass of system

$$\vec{a}_{cm} = \frac{m_1 \frac{d\vec{v}_1}{dt} + m_2 \frac{d\vec{v}_2}{dt} + m_3 \frac{d\vec{v}_3}{dt} .... + m_n \frac{d\vec{v}_n}{dt}}{M} = \frac{m_1\vec{a}_1 + m_2\vec{a}_2 + m_3\vec{a}_3 .... + m_n\vec{a}_n}{M}$$

$$= \frac{\text{Net force on system}}{M} = \frac{\text{Net External Force} + \text{Net internal Force}}{M} = \frac{\text{Net External Force}}{M}$$

($\because$ action and reaction both of an internal force must be within the system. Vector summation will cancel all internal forces and hence net internal force on system is zero)

$\therefore \quad \vec{F}_{ext} = M\vec{a}_{cm}$

where $\vec{F}_{ext}$ is the sum of the 'external' forces acting on the system. The internal forces which the particles exert on one another play absolutely no role in the motion of the center of mass.
If no external force is acting on a system of particles, the acceleration of center of mass of the system will be zero. If $a_c = 0$, it implies that v_c must be a constant and if v_{cm} is a constant, it implies that the total momentum of the system must remain constant. It leads to the principal of conservation of momentum in absence of external forces.
If $\vec{F}_{ext} = 0$ then = $\vec{v}_{cm}$ constant

"If resultant external force is zero on the system, then the net momentum of the system must remain constant".

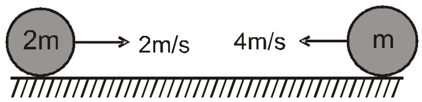
Motion of COM in a moving system of particles :

(1) COM at rest :

If $F_{ext} = 0$ and $V_{cm} = 0$, then COM remains at rest. Individual components of the system may move and have non-zero momentum due to mutual forces (internal), but the net momentum of the system remains zero.

(i) All the particles of the system are at rest.

(ii) Particles are moving such that their net momentum is zero.
Example:



(iii) A bomb at rest suddenly explodes into various smaller fragments, all moving in different directions then, since the explosive forces are internal & there is no external force on the system for explosion therefore, the COM of the bomb will remain at the original position and the fragment fly such that their net momentum remains zero.

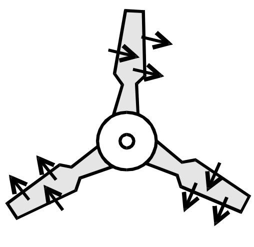
(iv) Two men standing on a frictionless platform, push each other, then also their net momentum remains zero because the push forces are internal for the two men system.

(v) A boat floating in a lake, also has net momentum zero if the people on it changes their position, because the friction force required to move the people is internal of the boat system.

(vi) Objects initially at rest, if moving under mutual forces (electrostatic or gravitation)also have net momentum zero.

(vii) A light spring of spring constant k kept compressed between two blocks of masses m_1 and m_2 on a smooth horizontal surface. When released, the blocks acquire velocities in opposite directions, such that the net momentum is zero.

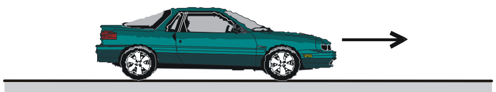
(viii) In a fan, all particles are moving but COM is at rest



(2) COM moving with uniform velocity :

If $F_{ext} = 0$, then V_{cm} remains constant therefore, net momentum of the system also remains conserved. Individual components of the system may have variable velocity and momentum due to mutual forces (internal), but the net momentum of the system remains constant and COM continues to move with the initial velocity.

(i) All the particles of the system are moving with same velocity.
e.g.: A car moving with uniform speed on a straight road, has its COM moving with a constant velocity.



(ii) Internal explosions / breaking does not change the motion of COM and net momentum remains conserved. A bomb moving in a straight line suddenly explodes into various smaller fragments, all moving in different directions then, since the explosive forces are internal & there is no external force on the system for explosion therefore, the COM of the bomb will continue the original motion and the fragment fly such that their net momentum remains conserved.

(iii) Man jumping from cart or buggy also exert internal forces therefore net momentum of the system and hence, Motion of COM remains conserved.

(iv) Two moving blocks connected by a light spring on a smooth horizontal surface. If the acting forces is only due to spring then COM will remain in its motion and momentum will remain conserved.

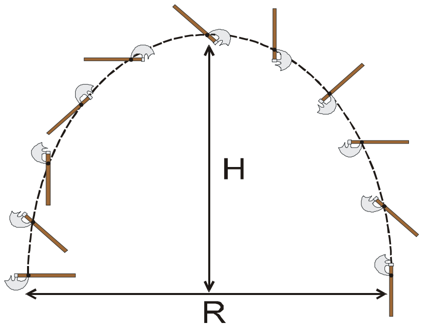
(v) Particles colliding in absence of external impulsive forces also have their momentum conserved.

(3) COM moving with acceleration :

If an external force is present then COM continues its original motion as if the external force is acting on it, irrespective of internal forces.

Example:

Projectile motion : An axe thrown in air at an angle θ with the horizontal will perform a complicated motion of rotation as well as parabolic motion under the effect of gravitation



The motion of axe is complicated but the COM is moving in a parabolic motion.

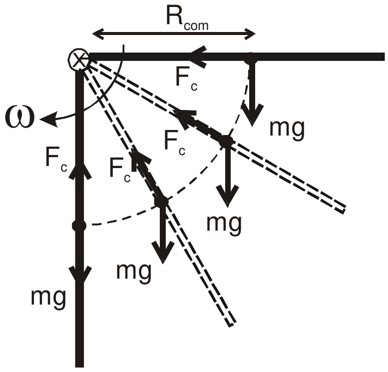
$$H_{com} = \frac{u^2 \sin^2 \theta}{2g}$$

$$R_{com} = \frac{u^2 \sin 2\theta}{g} \qquad T = \frac{2u \sin\theta}{g}$$

Example :

Circular Motion : A rod hinged at an end, rotates, than its COM performs circular motion. The centripetal force (F_c) required in the circular motion is assumed to be acting on the COM.

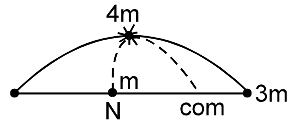
$F_c = m\omega^2 R_{COM}$



Solved Example

Example 8. A projectile is fired at a speed of 100 m/s at an angle of 37º above the horizontal. At the highest point, the projectile breaks into two parts of mass ratio 1 : 3, the lighter piece coming to rest. Find the distance from the launching point to the point where the heavier piece lands.

Solution : Internal force do not effect the motion of the center of mass, the center of mass hits the ground at the position where the original projectile would have landed. The range of the original projectile is,



$$x_{COM} = \frac{2u^2 \sin\theta\cos\theta}{g} = \frac{2\times 10^4 \times \frac{3}{5}\times\frac{4}{5}}{10} \text{ m}$$

= 960 m

The center of mass will hit the ground at this position. As the smaller block comes to rest after breaking, it falls down vertically and hits the ground at half of the range, i.e., at x = 480 m. If the heavier block hits the ground at x_2, then

$$x_{COM} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2}$$

$$960 = \frac{(m)(480) + (3m)(x_2)}{(m + 3m)}$$

x_2 = 1120 m **Ans.**

Momentum Conservation :

The total linear momentum of a system of particles is equal to the product of the total mass of the system and the velocity of its center of mass. $\vec{P} = M\vec{v}_{cm}$

$$\vec{F}_{ext} = \frac{d\vec{P}}{dt}$$

If $\vec{F}_{ext} = 0 \Rightarrow \frac{d\vec{P}}{dt} = 0$; $\vec{P}$ = constant

When the vector sum of the external forces acting on a system is zero, the total linear momentum of the system remains constant.

$\vec{P}_1 + \vec{P}_2 + \vec{P}_3 + ... + \vec{P}_n$ = constant.

Solved Examples

Example 9. A shell is fired from a cannon with a speed of 100 m/s at an angle 60º with the horizontal (positive x-direction). At the highest point of its trajectory, the shell explodes into two equal fragments. One of the fragments moves along the negative x-direction with a speed of 50 m/s. What is the speed of the other fragment at the time of explosion.

Solution : As we know in absence of external force the motion of center of mass of a body remains unaffected. Thus, here the center of mass of the two fragments will continue to follow the original projectile path. The velocity of the shell at the highest point of trajectory is

$v_M = u\cos\theta = 100 \times \cos 60^\circ = 50$ m/s

Let v_1 be the speed of the fragment which moves along the negative x-direction and the other fragment has speed v_2. Which must be along positive x-direction. Now from momentum conservation, we have

$$mv = \frac{-m}{2} v_1 + \frac{m}{2} v_2 \qquad \text{or} \qquad 2v = v_2 - v_1$$

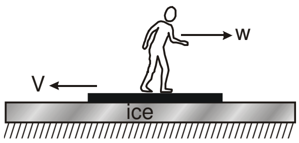
or $v_2 = 2v + v_1 = (2 \times 50) + 50 = 150$ m/s

Example 10. A man of mass m is standing on a platform of mass M kept on smooth ice. If the man starts moving on the platform with a speed v relative to the platform, with what velocity relative to the ice does the platform recoil ?

Solution : Consider the situation shown in figure. Suppose the man moves at a speed w towards right and the platform recoils at a speed V towards left, both relative to the ice. Hence, the speed of the man relative to the platform is V + w. By the question,

$V + w = v$, or $w = v - V$ .....(i)

Taking the platform and the man to be the system, there is no external horizontal force on the system. The linear momentum of the system remains constant. Initially both the man and the platform were at rest. Thus,

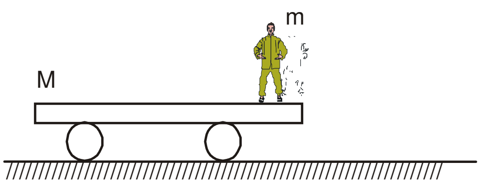


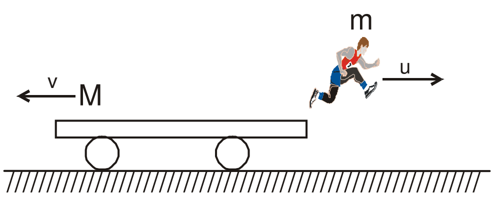
$0 = MV - mw$ or $MV = m(v - V)$ [Using (i)]

or, $V = \dfrac{mv}{M+m}$

Example 11. A flat car of mass M is at rest on a frictionless floor with a child of mass m standing at its edge. If child jumps off from the car towards right with an initial velocity u, with respect to the car, find the velocity of the car after its jump.

Solution : Let car attains a velocity v, and the net velocity of the child with respect to earth will be u – v, as u is its velocity with respect to car.





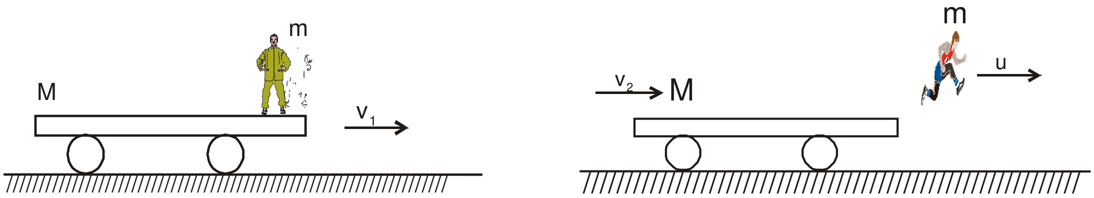
Initially, the system was at rest, thus according to momentum conservation, momentum after jump must be zero, as

$m(u - v) = Mv$

$v = \dfrac{mu}{m+M}$

Example 12. A flat car of mass M with a child of mass m is moving with a velocity $v_1$on a friction less surface. The child jumps in the direction of motion of car with a velocity u with respect to car. Find the final velocities of the child and that of the car after jump.

Solution : This case is similar to the previous example, except now the car is moving before jump. Here also no external force is acting on the system in horizontal direction, hence momentum remains conserved in this direction. After jump car attains a velocity v_2 in the same direction, which is less than v_1, due to backward push of the child for jumping. After jump child attains a velocity $u + v_2$ in the direction of motion of car, with respect to ground.

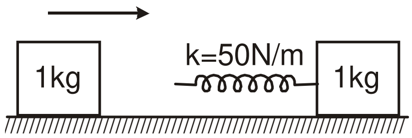


According to momentum conservation $(M + m)v_1 = Mv_2 + m\ (u + v_2)$

Velocity of car after jump is $v_2 = \dfrac{(M+m)v_1 - mu}{M+m}$

Velocity of child after jump is $u + v_2 = \dfrac{(M+m)v_1 + (M)u}{M+m}$

Example 13. Each of the blocks shown in figure has mass 1 kg. The rear block moves with a speed of 2 m/s towards the front block kept at rest. The spring attached to the front block is light and has a spring constant 50 N/m. Find the maximum compression of the spring. Assume, on a friction less surface



Solution : Maximum compression will take place when the blocks move with equal velocity. As no net external horizontal force acts on the system of the two blocks, the total linear momentum will remain constant. If V is the common speed at maximum compression, we have,

(1 kg) (2 m/s) = (1 kg)V + (1 kg)V   or,   V = 1 m/s.

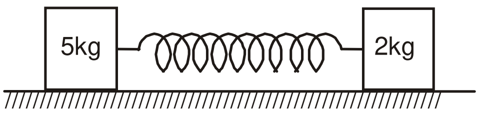
Initial kinetic energy = $\frac{1}{2}$ (1 kg) $(2\ m/s)_2$ = 2 J.

Final kinetic energy = $\frac{1}{2}$ (1 kg) $(1 m/s)^2$ + $\frac{1}{2}$ (1 kg) $(1\ m/s)^2$ = 1 J

The kinetic energy lost is stored as the elastic energy in the spring.

Hence, $\frac{1}{2}$ (50 N/m) x^2 = 2J – 1J = 1 J   or,   x = 0.2 m

Example 14. Figure shows two blocks of masses 5 kg and 2 kg placed on a frictionless surface and connected with a spring. An external kick gives a velocity 14 m/s to the heavier block towards the lighter one. Deduce (a) velocity gained by the center of mass and (b) the separate velocities of the two blocks with respect to center of mass just after the kick.



Solution : (a) Velocity of center of mass is $v_{cm} = \dfrac{5 \times 14 + 2 \times 0}{5+2}$ = 10 m/s

(b) Due to kick on 5 kg block, it starts moving with a velocity 14 m/s immediately, but due to inertia 2 kg block remains at rest, at that moment. Thus, velocity of 5 kg block with respect to the center of mass is v_1 = 14 – 10 = 4 m/s and the velocity of 2 kg block w.r.t. to center of mass is v_2 = 0 – 10 = –10 m/s

Example 15. A light spring of spring constant k is kept compressed between two blocks of masses m and M on a smooth horizontal surface. When released, the blocks acquire velocities in opposite directions.The spring loses contact with the blocks when it acquires natural length. If the spring was initially compressed through a distance x, find the final speeds of the two blocks.

Solution : Consider the two blocks plus the spring to be the system. No external force acts on this system in horizontal direction. Hence, the linear momentum will remain constant. Suppose, the block of mass M moves with a speed v_1 and the other block with a speed v after losing contact with the spring. From conservation of linear momentum in horizontal direction we have

$$Mv_1 - mv_2 = 0 \quad \text{or} \quad v_1 = \frac{m}{M}v_2, \qquad .....(i)$$

Initially, the energy of the system = $\frac{1}{2}kx^2$

Finally, the energy of the system = $\frac{1}{2}mv_2^2 + \frac{1}{2}Mv_1^2$

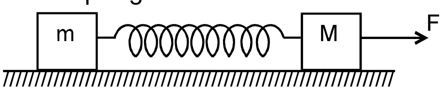
As there is no friction, mechanical energy will remain conserved.

Therefore, $\frac{1}{2}mv_2^2 + \frac{1}{2}Mv_1^2 = \frac{1}{2}kx^2 \qquad .....(ii)$

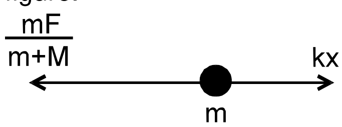
Solving Eqs. (i) and (ii), we get

or, $\quad v_2 = \left[\frac{kM}{m(M+m)}\right]^{1/2} x \quad$ and $\quad v_1 = \left[\frac{km}{M\ (M+m)}\right]^{1/2} x \qquad$ **Ans.**

Example 16. A block of mass m is connected to another block of mass M by a massless spring of spring constant k. The blocks are kept on a smooth horizontal plane and are at rest. The spring is unstretched when a constant force F starts acting on the block of mass M to pull it. Find the maximum extension of the spring.



Solution : We solve the situation in the reference frame of center of mass. As only F is the external force acting on the system, due to this force, the acceleration of the center of mass is F/(M + m). Thus with respect to center of mass there is a Pseudo force on the two masses in opposite direction, the free body diagram of m and M with respect to center of mass (taking center of mass at rest) is shown in figure.



Taking center of mass at rest, if m moves maximum by a distance x_1 and M moves maximum by a distance x_2, then the work done by external forces (including Pseudo force) will be

$$W = \frac{mF}{m+M} \cdot x_1 + \left(F - \frac{MF}{m+M}\right) \cdot x_2 = \frac{mF}{m+M} \cdot (x_1 + x_2)$$

This work is stored in the form of potential energy of the spring as U = $\frac{1}{2}k(x_1 + x_2)^2$

Thus on equating we get the maximum extension in the spring, as after this instant the spring starts contracting.

$$\frac{1}{2}k(x_1 + x_2)^2 = \frac{mF}{m+M} \cdot (x_1 + x_2)$$

$$x_{max} = x_1 + x_2 = \frac{2mF}{k(m+M)}$$

IMPULSE

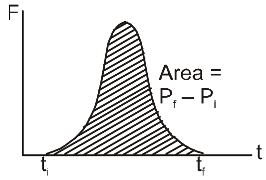
Impulse of a force $\vec{F}$ acting on a body for the time interval t = t_1 to t = t_2 is defined as :

$$\vec{I} = \int_{t_1}^{t_2} \vec{F}dt \quad \Rightarrow \quad \vec{I} = \int \vec{F}dt = \int m\frac{d\vec{v}}{dt}dt = \int md\vec{v}$$

$$\vec{I} = m(\vec{v}_2 - \vec{v}_1) = \Delta\vec{P} = \text{change in momentum due to force}$$

Also, $\vec{I}_{Res} = \int_{t_1}^{t_2} \vec{F}_{Res}dt = \Delta\vec{P}$ **(impulse - momentum theorem)**

Note : Impulse applied to an object in a given time interval can also be calculated from the area under force time (F-t) graph in the same time interval.



Instantaneous Impulse :

There are many cases when a force acts for such a short time that the effect is instantaneous, e.g., a bat striking a ball. In such cases, although the magnitude of the force and the time for which it acts may each be unknown but the value of their product (i.e., impulse) can be known by measuring the initial and final momenta. Thus, we can write.

$$\vec{I} = \int \vec{F}\ dt = \Delta\vec{P} = \vec{P}_f - \vec{P}_i$$

Important Points :

(1) It is a vector quantity.
(2) Dimensions = [MLT^{-1}]
(3) SI unit = kg m/s
(4) Direction is along change in momentum.
(5) Magnitude is equal to area under the F-t. graph.
(6) $\vec{I} = \int \vec{F}dt = \vec{F}_{av}\int dt = \vec{F}_{av}\Delta t$
(7) It is not a property of a particle, but it is a measure of the degree to which an external force changes the momentum of the particle.

Solved Example

Example 17. The hero of a stunt film fires 50 g bullets from a machine gun, each at a speed of 1.0 km/s. If he fires 20 bullets in 4 seconds, what average force does he exert against the machine gun during this period.

Solution : The momentum of each bullet

= (0.050 kg) (1000 m/s) = 50 kg-m/s.

The gun has been imparted this much amount of momentum by each bullet fired. Thus, the rate of change of momentum of the gun

$$= \frac{(50\text{kg} - \text{m/s}) \times 20}{4\text{s}} = 250 \text{ N}.$$

In order to hold the gun, the hero must exert a force of 250 N against the gun.

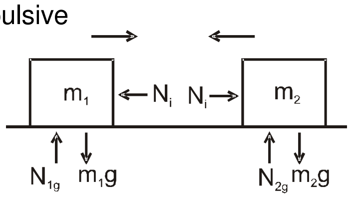
Impulsive force :

A force, of relatively higher magnitude and acting for relatively shorter time, is called impulsive force.
An impulsive force can change the momentum of a body in a finite magnitude in a very short time interval. **Impulsive force** is a relative term. There is no clear boundary between an impulsive and Non-Impulsive force.
Note : Usually colliding forces are impulsive in nature.
Since, the application time is very small, hence, very little motion of the particle takes place.

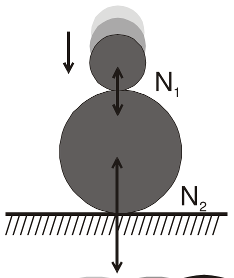
Important points :

1. Gravitational force and spring force are always non-Impulsive.
2. Normal, tension and friction are case dependent.
3. An impulsive force can only be balanced by another impulsive force.

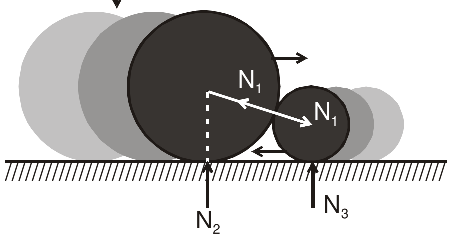
1. Impulsive Normal : In case of collision, normal forces at the surface of collision are always impulsive

eg. 

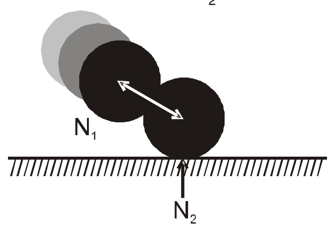
N_i = Impulsive; N_g = Non-impulsive



Both normals are Impulsive

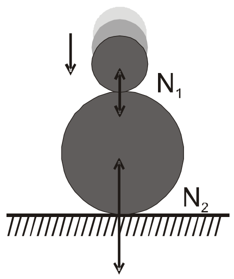


N_1, N_3 = Impulsive; N_2 = non-impulsive

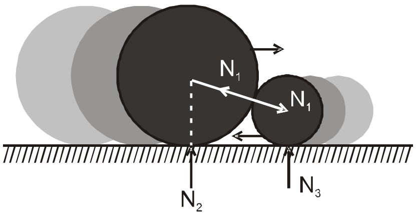


Both normals are Impulsive

2. Impulsive Friction : If the normal between the two objects is impulsive, then the friction between the two will also be impulsive.

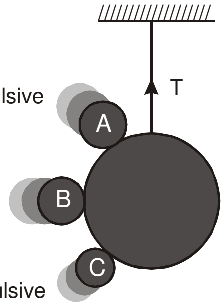


Friction at both surfaces is impulsive



Friction due to N_2 is non-impulsive and due to N_3 and N_1 are impulsive.

3. **Impulsive Tensions :** When a string jerks, equal and opposite tension act suddenly at each end. Consequently equal and opposite impulses act on the bodies attached with the string in the direction of the string. There are two cases to be considered.
   - **(a) One end of the string is fixed :** The impulse which acts at the fixed end of the string cannot change the momentum of the fixed object there. The object attached to the free end however will undergo a change in momentum in the direction of the string. The momentum remains unchanged in a direction perpendicular to the string where no impulsive forces act.
   - **(b) Both ends of the string attached to movable objects :** In this case equal and opposite impulses act on the two objects, producing equal and opposite changes in momentum. The total momentum of the system therefore remains constant, although the momentum of each individual object is changed in the direction of the string. Perpendicular to the string however, no impulse acts and the momentum of each particle in this direction is unchanged.

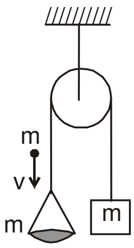


All normal are impulsive but tension
T is impulsive only for the ball A

For this example : In case of rod, Tension is always impulsive and in case of spring, Tension is always non-impulsive.

Solved Example

Example 18. A block of mass m and a pan of equal mass are connected by a string going over a smooth light pulley. Initially the system is at rest when a particle of mass m falls on the pan and sticks to it. If the particle strikes the pan with a speed v, find the speed with which the system moves just after the collision.



Solution : Let the required speed is V.

Further, let J_1 = impulse between particle and pan

and J_2 = impulse imparted to the block and the pan by the string

Using, impulse = change in momentum

For particle $J_1 = mv - mV$ .....(i)

For pan $J_1 - J_2 = mV$ .....(ii)

For block $J_2 = mV$ .....(iii)

Solving, these three equation, we get $V = \frac{v}{3}$ **Ans.**

Alternative Solution :

Applying conservation of linear momentum along the string;

$mv = 3mV$

we get, $V = \frac{v}{3}$ **Ans.**

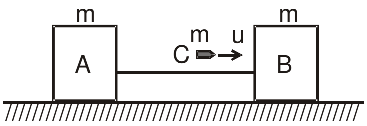
Example 19. Two identical block A and B, connected by a massless string are placed on a frictionless horizontal plane. A bullet having same mass, moving with speed u strikes block B from behind as shown. If the bullet gets embedded into the block B then find :

(a) The velocity of A, B, C after collision.

(b) Impulse on A due to tension in the string

(c) Impulse on C due to normal force of collision.

(d) Impulse on B due to normal force of collision.



Solution : (a) By Conservation of linear momentum $v = \frac{u}{3}$

(b) $\int T dt = \frac{mu}{3}$

(c) $\int N \ dt = m\left(\frac{u}{3} - u\right) = \frac{-2mu}{3}$

(d) $\int (N - T) \ dt = \int N dt - \int T \ dt = \frac{mu}{3}$

$\int N \ dt = \frac{2mu}{3}$

COLLISION OR IMPACT

Collision is an event in which an impulsive force acts between two or more bodies for a short time, which results in change of their velocities.

Note :

(a) In a collision, particles may or may not come in physical contact.

(b) The duration of collision, Δt is negligible as compared to the usual time intervals of observation of motion.

(c) In a collision the effect of external non impulsive forces such as gravity are not taken into a account as due to small duration of collision (Δt) average impulsive force responsible for collision is much larger than external forces acting on the system.

The collision is infact a redistribution of total momentum of the particles. Thus, law of conservation of linear momentum is indispensable in dealing with the phenomenon of collision between particles.

Line of Impact

The line passing through the common normal to the surfaces in contact during impact is called line of impact. The force during collision acts along this line on both the bodies.

Direction of Line of impact can be determined by:

(a) Geometry of colliding objects like spheres, discs, wedge etc.

(b) Direction of change of momentum.

If one particle is stationary before the collision then the line of impact will be along its motion after collision.

Classification of collisions

(a) On the basis of line of impact

(i) Head-on collision : If the velocities of the colliding particles are along the same line before and after the collision.

(ii) Oblique collision : If the velocities of the colliding particles are along different lines before and after the collision.

(b) On the basis of energy :

(i) Elastic collision : In an elastic collision, the colliding particles regain their shape and size completely after collision. i.e., no fraction of mechanical energy remains stored as deformation potential energy in the bodies. Thus, kinetic energy of system after collision is equal to kinetic energy of system before collision. Thus in addition to the linear momentum, kinetic energy also remains conserved before and after collision.

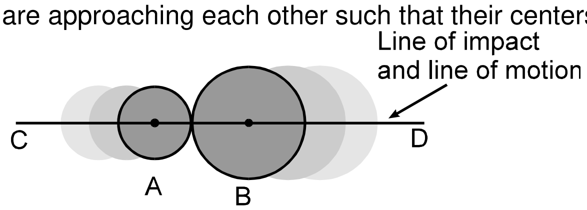
(ii) Inelastic collision : In an inelastic collision, the colliding particles do not regain their shape and size completely after collision. Some fraction of mechanical energy is retained by the colliding particles in the form of deformation potential energy. Thus, the kinetic energy of the particles after collision is not equal to that of before collision. However, in the absence of external forces, law of conservation of linear momentum still holds good.

(iii) Perfectly inelastic : If velocity of separation along the line of impact just after collision becomes zero then the collision is perfectly inelastic. Collision is said to be **perfectly inelastic** if both the particles stick together after collision and move with same velocity,

Note : Actually collision between all real objects are neither perfectly elastic nor perfectly inelastic, its inelastic in nature.

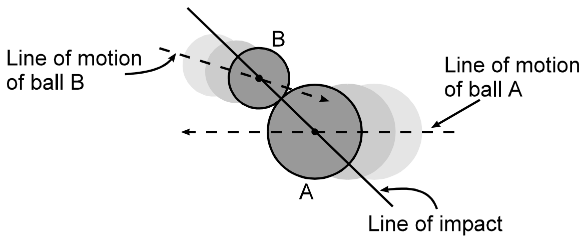
Examples of line of impact and collisions based on line of impact

(i) Two balls A and B are approaching each other such that their centers are moving along line CD.



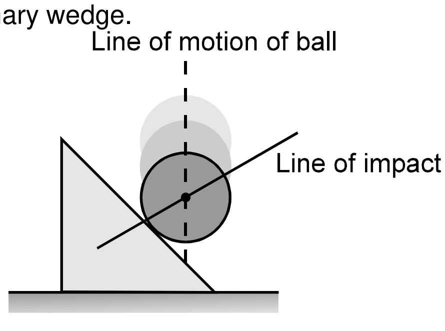
Head on Collision

(ii) Two balls A and B are approaching each other such that their center are moving along dotted lines as shown in figure.



Oblique Collision

(iii) Ball is falling on a stationary wedge.



COEFFICIENT OF RESTITUTION (e)

The coefficient of restitution is defined as the ratio of the impulses of reformation and deformation of either body.

$$e = \frac{\text{Impulse of reformation}}{\text{Impulse of deformation}} = \frac{\int F_r dt}{\int F_d dt}$$

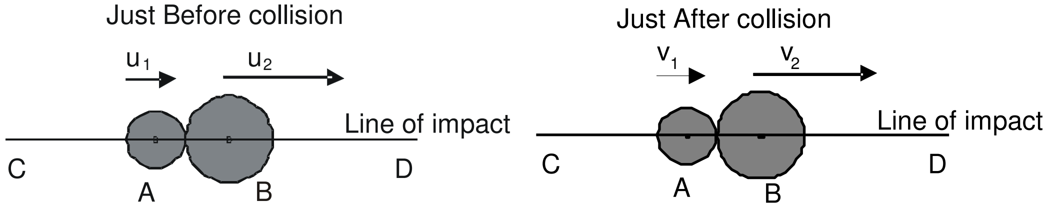
$$= \frac{\text{Velocity of seperation along line of impact}}{\text{Velocity of approach along line of impact}}$$

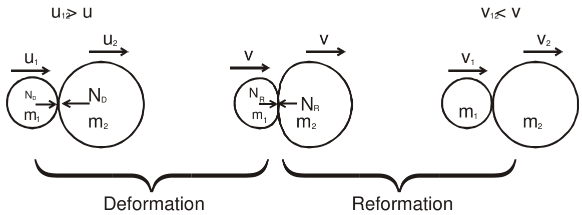
The most general expression for coefficient of restitution is

$$e = \frac{\text{velocity of separation of points of contact along line of impact}}{\text{velocity of approach of point of contact along line of impact}}$$

Example for calculation of e

Two smooth balls A and B approaching each other such that their centers are moving along line CD in absence of external impulsive force. The velocities of A and B just before collision be u_1 and u_2 respectively. The velocities of A and B just after collision be v_1 and v_2 respectively.





$\because$ $F_{ext} = 0$ momentum is conserved for the system.

$\Rightarrow$ $m_1u_1 + m_2 u_2 = (m_1 + m_2)v = m_1v_1 + m_2v_2$

$$\Rightarrow \quad v = \frac{m_1u_1 + m_2u_2}{m_1 + m_2} = \frac{m_1v_1 + m_2v_2}{m_1 + m_2} \qquad ....(1)$$

Impulse of Deformation :

J_D = change in momentum of any one body during deformation.

$= m_2 (v - u_2)$ for m_2

$= m_1 (-v + u_1)$ for m_1

Impulse of Reformation :

J_R = change in momentum of any one body during Reformation.

$= m_2 (v_2 - v)$ for m_2

$= m_1 (v - v_1)$ for m_1

$$e = \frac{\text{Impulse of Reformation}(\vec{J}_R)}{\text{Impulse of Deformation}(\vec{J}_D)} = \frac{v_2 - v_1}{u_1 - u_2} = \frac{\text{Velocity of separation along line of impact}}{\text{Velocity of approach along line of impact}}$$

Note : e is independent of shape and mass of object but depends on the material. The coefficient of restitution is constant for a pair of materials.

(a) e = 1 Impulse of Reformation = Impulse of Deformation
Velocity of separation = Velocity of approach
Kinetic energy of particles after collision may be equal to that of before collision.
Collision is elastic.

(b) e = 0 Impulse of Reformation = 0
Velocity of separation = 0
Kinetic energy of particles after collision is not equal to that of before collision.
Collision is perfectly inelastic.

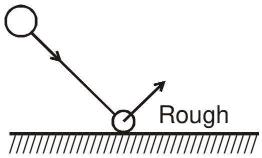
(c) 0 < e < 1 Impulse of Reformation < Impulse of Deformation
Velocity of separation < Velocity of approach
Kinetic energy of particles after collision is not equal to that of before collision.
Collision is Inelastic.

Note : In case of contact collisions e is always less than unity.

$\therefore \quad 0 \le e \le 1$

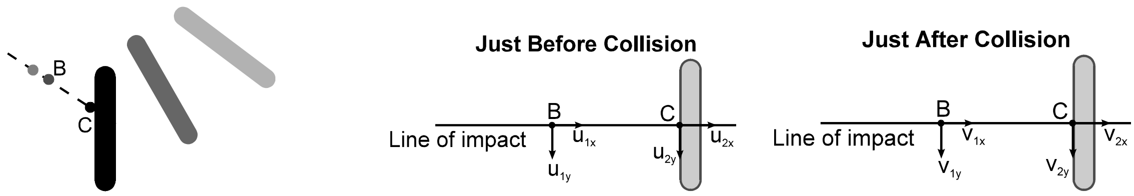
Important Point :

In case of elastic collision, if rough surface is present then $k_f < k_i$ (because friction is impulsive).
Where, k is Kinetic Energy.



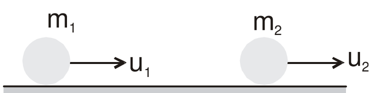
A particle 'B' moving along the dotted line collides with a rod also in state of motion as shown in the figure. The particle B comes in contact with point C on the rod.

To write down the expression for coefficient of restitution e, we first draw the line of impact. Then we resolve the components of velocities of points of contact of both the bodies along line of impact just before and just after collision.

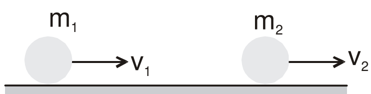


Then $\quad e = \dfrac{v_{2x} - v_{1x}}{u_{1x} - u_{2x}}$

Collision in one dimension (Head on)



(a)
Before Collision



(b)
After Collision

$e = \dfrac{v_2 - v_1}{u_1 - u_2} \Rightarrow \quad (u_1 - u_2)e = (v_2 - v_1)$

By momentum conservation, $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

$v_2 = v_1 + e(u_1 - u_2)$ and $v_1 = \dfrac{m_1u_1 + m_2u_2 - m_2e(u_1 - u_2)}{m_1 + m_2}$

$v_2 = \dfrac{m_1u_1 + m_2u_2 + m_1e(u_1 - u_2)}{m_1 + m_2}$

Special Case :

(1) $e = 0$

$\Rightarrow$ $v_1 = v_2$

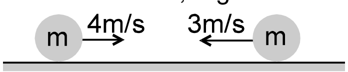
$\Rightarrow$ for perfectly inelastic collision, both the bodies, move with same vel. after collision.

(2) $e = 1$

and $m_1 = m_2 = m$,

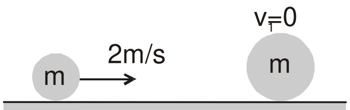
we get $v_1 = u_2$ and $v_2 = u_1$

i.e., when two particles of equal mass collide elastically and the collision is head on, they exchange their velocities., e.g.



Before Collision

After Collision



Before Collision

After Collision

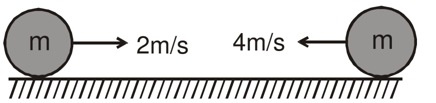
(3) $m_1 >> m_2$

$m_1 + m_2 = m_1$ and $\frac{m_2}{m_1} = 0$

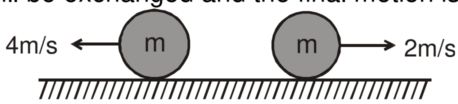
$\Rightarrow$ $v_1 = u_1$ No change and $v_2 = u_1 + e(u_1 - u_2)$

Solved Example

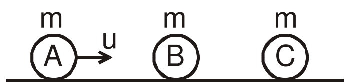
Example 20. Two identical balls are approaching towards each other on a straight line with velocity 2 m/s and 4 m/s respectively. Find the final velocities, after elastic collision between them.



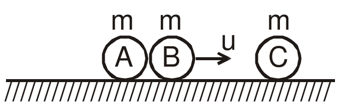
Solution : The two velocities will be exchanged and the final motion is reverse of initial motion for both.



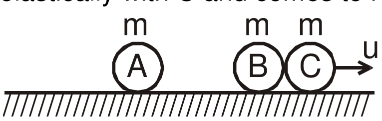
Example 21. Three balls A, B and C of same mass 'm' are placed on a frictionless horizontal plane in a straight line as shown. Ball A is moved with velocity u towards the middle ball B. If all the collisions are elastic then, find the final velocities of all the balls.



Solution : A collides elastically with B and comes to rest but B starts moving with velocity u



After a while B collides elastically with C and comes to rest but C starts moving with velocity u

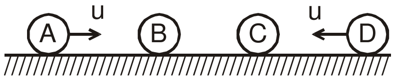


$\therefore$ Final velocities

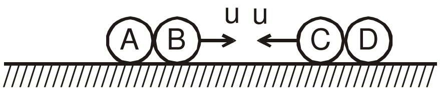
$V_A = 0$;

$V_B = 0$ and $V_C = u$ **Ans.**

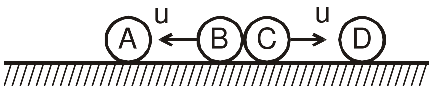
Example 22. Four identical balls A, B, C and D are placed in a line on a frictionless horizontal surface. A and D are moved with same speed 'u' towards the middle as shown. Assuming elastic collisions, find the final velocities.



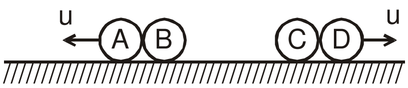
Solution : A and D collides elastically with B and C respectively and come to rest but B and C starts moving with velocity u towards each other as shown



B and C collides elastically and exchange their velocities to move in opposite directions



Now, B and C collides elastically with A and D respectively and come to rest but A and D starts moving with velocity u away from each other as shown



$\therefore$ Final velocities $V_A = u\ (\leftarrow)$; $V_B = 0$; $V_C = 0$ and $V_D = u\ (\rightarrow)$ **Ans.**

Example 23. Two particles of mass m and 2m moving in opposite directions on a frictionless surface collide elastically with velocity v and 2v respectively. Find their velocities after collision, also find the fraction of kinetic energy lost by the colliding particles.

Solution : Let the final velocities of m and 2m be v_1 and v_2 respectively as shown in the figure:

By conservation of momentum : $m(2v) + 2m(-v) = m(v_1) + 2m\ (v_2)$

or $0 = mv_1 + 2mv_2$

or $v_1 + 2v_2 = 0$ .....(1)

and since the collision is elastic:

$v_2 - v_1 = 2v - (-v)$

or $v_2 - v_1 = 3v$ .....(2)

Solving the above two equations, we get,

$v_2 = v$ and $v_1 = -2v$ **Ans.**

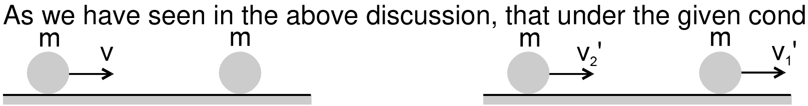
i.e., the mass 2m returns with velocity v while the mass m returns with velocity 2v in the direction shown in figure:

The collision was elastic therefore, no kinetic energy is lost, KE loss = $KE_i - KE_f$

or, $\left(\frac{1}{2}m(2v)^2 + \frac{1}{2}(2m)(-v)^2\right) - \left(\frac{1}{2}m(-2v)^2 + \frac{1}{2}(2m)v^2\right) = 0$

Example 24. On a frictionless surface, a ball of mass m moving at a speed v makes a head on collision with an identical ball at rest. The kinetic energy of the balls after the collision is 3/4th of the original. Find the coefficient of restitution.

Solution : As we have seen in the above discussion, that under the given conditions :



Before Collision After Collision

By using conservation of linear momentum and equation of e, we get,

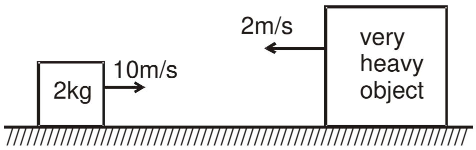
$v_1' = \left(\frac{1+e}{2}\right)v$ and $v_2' = \left(\frac{1-e}{2}\right)v$

Given that $K_f = \frac{3}{4}K_i$ or $\frac{1}{2}mv_1'^2 + \frac{1}{2}mv_2'^2 = \frac{3}{4}\left(\frac{1}{2}mv^2\right)$

Substituting the value, we get

$\left(\frac{1+e}{2}\right)^2 + \left(\frac{1-e}{2}\right)^2 = \frac{3}{4}$ or $e = \frac{1}{\sqrt{2}}$ **Ans.**

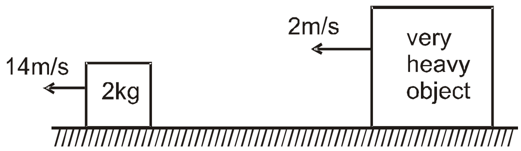
Example 25. A block of mass 2 kg is pushed towards a very heavy object moving with 2 m/s closer to the block (as shown). Assuming elastic collision and frictionless surfaces, find the final velocities of the blocks.



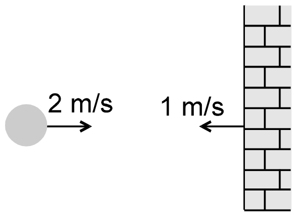
Solution : Let v_1 and v_2 be the final velocities of 2kg block and heavy object respectively then,

$v_1 = u_1 + 1\ (u_1 - u_2) = 2u_1 - u_2 = -14$ m/s

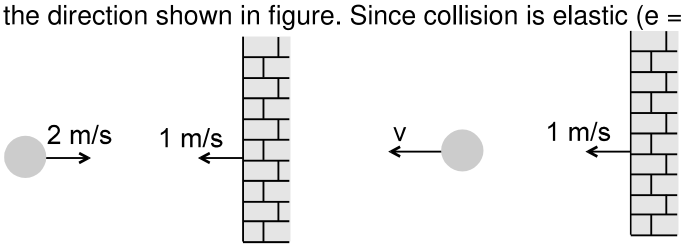
$v_2 = -2$m/s



Example 26. A ball is moving with velocity 2 m/s towards a heavy wall moving towards the ball with speed 1m/s as shown in fig. Assuming collision to be elastic, find the velocity of the ball immediately after the collision.



Solution : The speed of wall will not change after the collision. So, let v be the velocity of the ball after collision in the direction shown in figure. Since collision is elastic (e = 1),



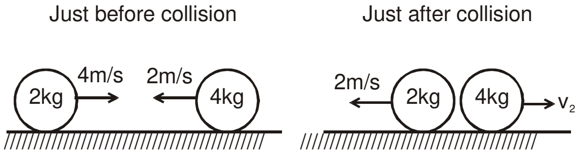
Before Collision After Collision

separation speed = approach speed

or $v - 1 = 2 + 1$

or $v = 4$ m/s **Ans.**

Example 27. Two balls of masses 2 kg and 4 kg are moved towards each other with velocities 4 m/s and 2 m/s respectively on a frictionless surface. After colliding the 2 kg ball returns back with velocity 2m/s.



Then find:

(a) Velocity of 4 kg ball after collision

(b) Coefficient of restitution e.

(c) Impulse of deformation J_D.

(d) Maximum potential energy of deformation.

(e) Impulse of reformation J_R.

Solution : (a) By momentum conservation, $2(4) - 4(2) = 2(-2) + 4(v_2)$ $\Rightarrow$ $v_2 = 1$ m/s

(b) $e = \dfrac{\text{velocity of separation}}{\text{velocity of approach}} = \dfrac{1-(-2)}{4-(-2)} = \dfrac{3}{6} = 0.5$

(c) At maximum deformed state, by conservation of momentum, common velocity is v = 0.

$J_D = m_1(v - u_1) = m_2(v - u_2) = 2(0 - 4) = -8$ N – s $= 4(0 - 2) = -8$ N - s

or $= 4(0 - 2) = -8$ N - s

(d) Potential energy at maximum deformed state U = loss in kinetic energy during deformation.

or $U = \left(\dfrac{1}{2}m_1u_1^2 + \dfrac{1}{2}m_2u_2^2\right) - \dfrac{1}{2}(m_1 + m_2)v^2 = \left(\dfrac{1}{2}2(4)^2 + \dfrac{1}{2}4(2)^2\right) - \dfrac{1}{2}(2 + 4)(0)^2$

or U = 24 Joule

(e) $J_R = m_1(v_1 - v) = m_2(v - v_2) = 2(-2 - 0) = -4$ N-s

or $= 4(0 - 1) = -4$ N-s

or $e = \dfrac{J_R}{J_D}$

$\Rightarrow$ $J_R = eJ_D = (0.5)(-8) = -4$ N-s

Collision in two dimension (oblique)

1. A pair of equal and opposite impulses act along common normal direction. Hence, linear momentum of individual particles do change along common normal direction. If mass of the colliding particles remain constant during collision, then we can say that linear velocity of the individual particles change during collision in this direction.

2. No component of impulse act along common tangent direction. Hence, linear momentum or linear velocity of individual particles (if mass is constant) remain unchanged along this direction.

3. Net impulse on both the particles is zero during collision. Hence, net momentum of both the particles remain conserved before and after collision in any direction.

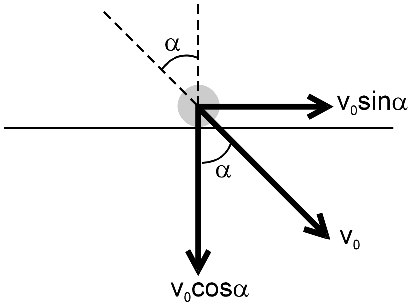
4. Definition of coefficient of restitution can be applied along common normal direction, i.e., along common normal direction we can apply

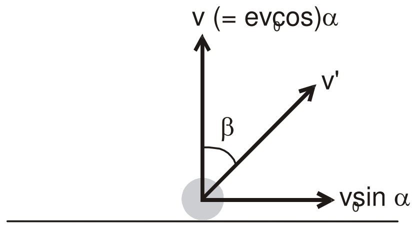
Relative speed of separation = e (relative speed of approach)

Solved Example

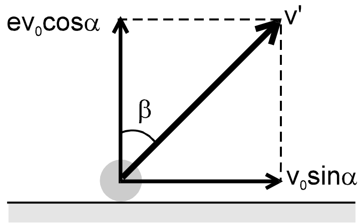
Example 28. A ball of mass m hits a floor with a speed v_0 making an angle of incidence a with the normal. The coefficient of restitution is e. Find the speed of the reflected ball and the angle of reflection of the ball.

Solution : The component of velocity v_0 along common tangential direction $v_0 \sin \alpha$ will remain unchanged. Let v be the component along common normal direction after collision. Applying,
Relative speed of separation = e (Relative speed of approach) along common normal direction, we get $v = ev_0 \cos \alpha$





Thus, after collision components of velocity v' are $v_0 \sin \alpha$ and $ev_0 \cos \alpha$



$\therefore \quad v' = \sqrt{(v_0 \sin\alpha)^2 + (ev_0 \cos\alpha)^2}$ **Ans.**

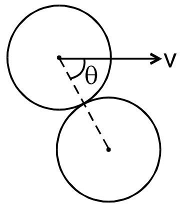
and $\tan \beta = \dfrac{v_0 \sin\alpha}{ev_0 \cos\alpha}$ or $\tan \beta = \dfrac{\tan\alpha}{e}$ **Ans.**

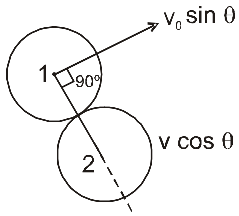
Note : For elastic collision, e = 1

$\therefore \quad v' = v_0$ and $\beta = \alpha$

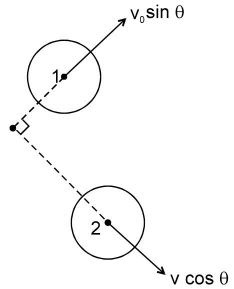
Example 29. A ball of mass m makes an elastic collision with another identical ball at rest. Show that if the collision is oblique, the bodies go at right angles to each other after collision.

Solution : In head on elastic collision between two particles, they exchange their velocities. In this case, the component of ball 1 along common normal direction, v cos θ





Before collision



After collision

becomes zero after collision, while that of 2 becomes v cos θ. While the components along common tangent direction of both the particles remain unchanged. Thus, the components along common tangent and common normal direction of both the balls in tabular form are given below.

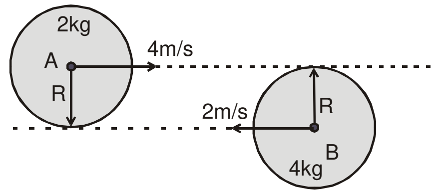
Ball	Component along common tangent direction		Component along common normal direction	
	Before collision	After collision	Before collision	After collision
1	v sin θ	v sin θ	v cos θ	0
2	0	0	0	v cos θ

From the above table and figure, we see that both the balls move at right angle after collision with velocities v sin θ and v cos θ.

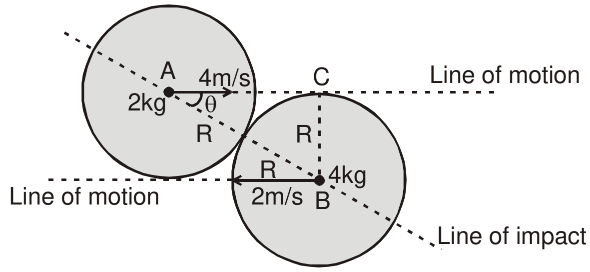
Note : When two identical bodies have an oblique elastic collision, with one body at rest before collision, then the two bodies will go in ⊥ directions.

Example 30. Two spheres are moving towards each other. Both have same radius but their masses are 2 kg and 4 kg. If the velocities are 4 m/s and 2 m/s respectively and coefficient of restitution is e = 1/3, find.

(a) The common velocity along the line of impact.
(b) Final velocities along line of impact.
(c) Impulse of deformation.
(d) impulse of reformation.
(e) Maximum potential energy of deformation.
(f) Loss in kinetic energy due to collision.

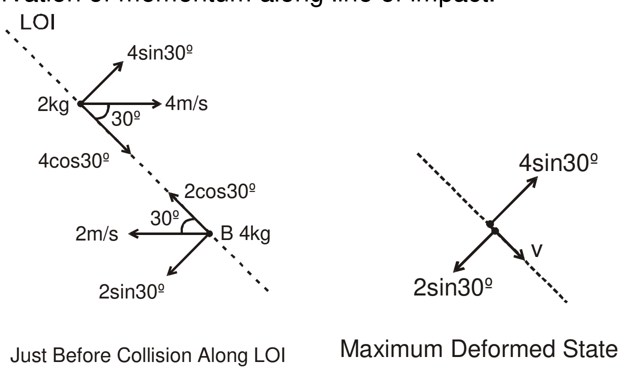


Solution :



In ΔABC $\sin\theta = \dfrac{BC}{AB} = \dfrac{R}{2R} = \dfrac{1}{2}$ or $\theta = 30^\circ$

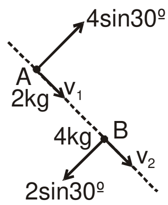
(a) By conservation of momentum along line of impact.



$2(4\cos 30^\circ) - 4(2\cos 30^\circ) = (2 + 4)v$

or $v = 0$ (common velocity along LOI)

(b)



Just After Collision Along LOI

Let v_1 and v_2 be the final velocity of A and B respectively then, by conservation of momentum along line of impact,

$2(4\cos 30^\circ) - 4(2\cos 30^\circ) = 2(v_1) + 4(v_2)$

or $0 = v_1 + 2v_2$ ....(1)

By coefficient of restitution,

$$e = \frac{\text{velocity of separation along LOI}}{\text{velocity of approach along LOI}}$$

or $\frac{1}{3} = \frac{v_2 - v_1}{4\cos 30^\circ + 2\cos 30^\circ}$ or $v_2 - v_1 = \sqrt{3}$ .... (2)

from the above two equations,

$v_1 = \frac{-2}{\sqrt{3}} m/s$ and $v_2 = \frac{1}{\sqrt{3}} m/s$

(c) $J_D = m_1(v - u_1) = 2(0 - 4\cos 30^\circ) = -4\sqrt{3}$ N-s

(d) $J_R = eJ_D = \frac{1}{3}(-4\sqrt{3}) = -\frac{4}{\sqrt{3}}$ N-s

(e) Maximum potential energy of deformation is equal to loss in kinetic energy during deformation upto maximum deformed state,

$U = \frac{1}{2} m_1(u_1 \cos\theta)^2 + \frac{1}{2} m_2(u_2 \cos\theta)^2 - \frac{1}{2}(m_1 + m_2)v^2$

$= \frac{1}{2} 2(4\cos 30^\circ)^2 + \frac{1}{2} 4(-2\cos 30^\circ)^2 - \frac{1}{2}(2 + 4)(0)^2$ or $U = 18$ Joule

(f) Loss in kinetic energy, $\Delta KE = \frac{1}{2} m_1(u_1 \cos\theta)^2 + \frac{1}{2} m_2(u_2 \cos\theta)^2 - \left(\frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2\right)$

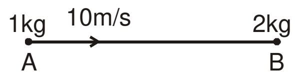
$= \frac{1}{2} 2(4\cos 30^\circ)^2 + \frac{1}{2} 4(-2\cos 30^\circ)^2 - \left(\frac{1}{2}2\left(\frac{2}{\sqrt{3}}\right)^2 + \frac{1}{2}4\left(\frac{1}{\sqrt{3}}\right)^2\right)$

$\Delta KE = 16$ Joule

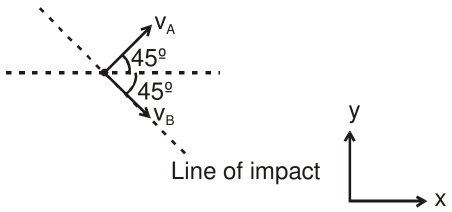
Example 31. Two point particles A and B are placed in line on a frictionless horizontal plane. If particle A (mass 1 kg) is moved with velocity 10 m/s towards stationary particle B (mass 2 kg) and after collision the two move at an angle of 45º with the initial direction of motion, then find :

(a) Velocities of A and B just after collision.

(b) Coefficient of restitution.



Solution : The very first step to solve such problems is to find the line of impact which is along the direction of force applied by A on B, resulting the stationary B to move. Thus, by watching the direction of motion of B, line of impact can be determined. In this case line of impact is along the direction of motion of B. i.e. 45º with the initial direction of motion of A.



(a) By conservation of momentum, along x direction: $m_A u_A = m_A v_A \cos 45^\circ + m_B v_B \cos 45^\circ$

or $1(10) = 1(v_A \cos 45^\circ) + 2(v_B \cos 45^\circ)$

or $v_A + 2v_B = 10\sqrt{2}$ .....(1)

along y direction

$0 = m_A v_A \sin 45^\circ + m_B v_B \sin 45^\circ$

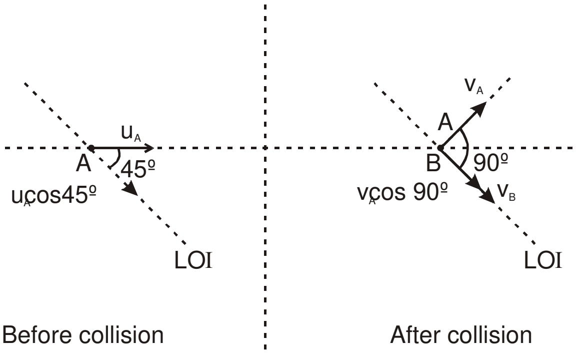
or $0 = 1(v_A \sin 45^\circ) - 2(v_B \sin 45^\circ)$

or $v_A = 2v_B$ .....(2)

solving the two equations,

$v_A = \frac{10}{\sqrt{2}}$ m/s and $v_B = \frac{5}{\sqrt{2}}$ m/s

(b) $e = \dfrac{\text{velocity of separation along line of impect}}{\text{veloctiy of approach along line of impect}}$



or $e = \dfrac{v_B - v_A\cos 90º}{u_A\cos 45º} = \dfrac{\frac{5}{\sqrt{2}} - 0}{\frac{10}{\sqrt{2}}} = \dfrac{1}{2}$ **Ans.**

Example 32. A smooth sphere of mass m is moving on a horizontal plane with a velocity $3\hat{i} + \hat{j}$ when it collides with a vertical wall which is parallel to the vector $\hat{j}$. If the coefficient of restitution between the sphere and the wall is $\frac{1}{2}$, find

(a) the velocity of the sphere after impact,

(b) the loss in kinetic energy caused by the impact.

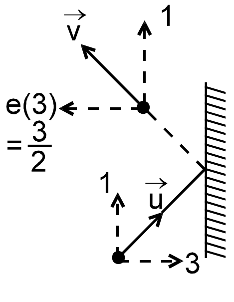
(c) the impulse $\vec{J}$ that acts on the sphere.

Solution : Let $\vec{v}$ be the velocity of the sphere after impact.

To find $\vec{v}$ we must separate the velocity components parallel and perpendicular to the wall.

Using the law of restitution the component of velocity parallel to the wall remains unchanged while component perpendicular to the wall becomes e times in opposite direction.

Thus, $\vec{v} = -\frac{3}{2}\hat{i} + \hat{j}$



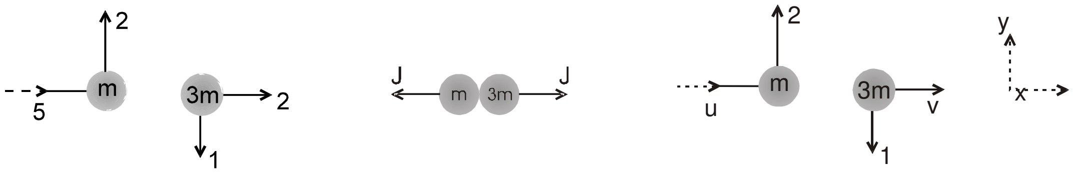
(a) Therefore, the velocity of the sphere after impact is $= -\frac{3}{2}\hat{i} + \hat{j}$ **Ans.**

(b) The loss in K.E. $= \frac{1}{2}mv^2 - \frac{1}{2}mu^2 = \frac{1}{2}m(3^2 + 1^2) - \frac{1}{2}m\left(\left\{\frac{3}{2}\right\}^2 + 1^2\right) = \frac{27}{8}m$ **Ans.**

(c) $\vec{J} = \Delta\vec{P} = \vec{P}_f - \vec{P}_i = m(\vec{v}) - m(\vec{u}) = m\left(-\frac{3}{2}\hat{i} + \hat{j}\right) - m\left(3\hat{i} + \hat{j}\right) = -\frac{9}{2}m\hat{i}$ **Ans.**

Example 33. Two smooth spheres, A and B, having equal radii, lie on a horizontal table. A is of mass m and B is of mass 3m. The spheres are projected towards each other with velocity vector $5\hat{i}+2\hat{j}$ and $2\hat{i}-\hat{j}$ respectively and when they collide the line joining their centers is parallel to the vector $\hat{i}$. If the coefficient of restitution between A and B is $\frac{1}{3}$, find the velocities after impact and the loss in kinetic energy caused by the collision. Find also the magnitude of the impulses that act at the instant of impact.

Solution : The line of centers at impact, is parallel to the vector , the velocity components of A and B perpendicular to are unchanged by the impact.



Applying conservation of linear momentum and the law of restitution, we have

in x direction $5m + (3m)(2) = mu + 3\,mv$ .....(i)

and $\frac{1}{3}(5-2) = v - u$ .....(ii)

Solving these equations, we have $u = 2$ and $v = 3$

The velocities of A and B after impact are therefore,

$2\hat{i}+2\hat{j}$ and $3\hat{i}-\hat{j}$ respectively **Ans.**

Before impact the kinetic energy of A is $\frac{1}{2}m(5^2+2^2) = \frac{29}{2}m$

and of B is $\frac{1}{2}(3m)(2^2+1^2) = \frac{15}{2}m$

After impact the kinetic energy of A is $\frac{1}{2}m(2^2+2^2) = 4m$

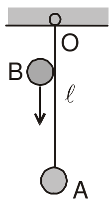
and of B is $\frac{1}{2}(3m)(3^2+1^2) = 15\,m$

Therefore, the loss in K.E. at impact is $\frac{29}{2}m + \frac{15}{2}m - 4m - 15m = 3m$ **Ans.**

To find value of J, we consider the change in momentum along $\hat{i}$ for one sphere only.

For sphere B $J = 3m(3-2)$ or $J = 3m$ **Ans.**

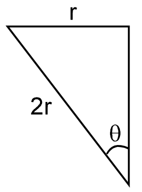
Example 34. A small steel ball A is suspended by an inextensible thread of length $\ell = 1.5$ from O. Another identical ball is thrown vertically downwards such that its surface remains just in contact with thread during downward motion and collides elastically with the suspended ball. If the suspended ball just completes vertical circle after collision, calculate the velocity of the falling ball just before collision. ($g = 10\ ms^{-2}$)

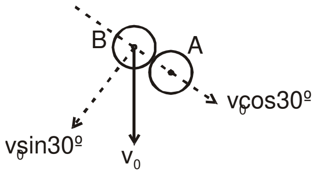


Solution : Velocity of ball A just after collision is $\sqrt{5gl}$

Let radius of each ball be r and the joining centers of the two balls makes an angle θ with the vertical at the instant of collision, then

$\sin\theta = \frac{r}{2r} = \frac{1}{2}$ or $\theta = 30º$





Let velocity of ball B (just before collision) be v_0. This velocity can be resolved into two components, (i) $v_0 \cos 30º$, along the line joining the center of the two balls and (ii) $v_0 \sin 30º$ normal to this line. Head -on collision takes place due to $v_0 \cos 30º$ and the component $v_0 \sin 30º$ of velocity of ball B remains unchanged.

Since, ball A is suspended by an inextensible string, therefore, just after collision, it can move along horizontal direction only. Hence, a vertically upward impulse is exerted by thread on the ball A. This means that during collision two impulses act on ball A simultaneously. One is impulsive interaction J between the balls and the other is impulsive reaction J' of the thread.

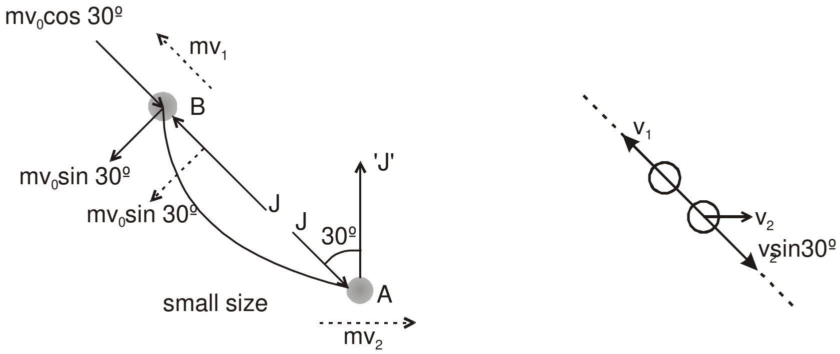
Velocity v_1 of ball B along line of collision is given by

$J - mv_0 \cos 30º = mv_1$

or $v_1 = \frac{J}{m} - v_0 \cos 30º$ .....(i)

Horizontal velocity v_0 of ball A is given by $J \sin 30º = mv_2$

or $v_2 = \frac{J}{2m}$ .....(ii)



Since, the balls collide elastically, therefore, coefficient of restitution is e = 1.

Hence, $e = \frac{v_2 \sin 30º - (-v_1)}{v_0 \cos 30º - 0} = 1$ .....(iii)

Solving Eqs. (i), (ii), and (iii), $J = 1.6\, mv_0 \cos 30º$

$\therefore$ $v_1 = 0.6\, v_0 \cos 30º$ and $v_2 = 0.8\, v_0 \cos 30º$

Since, ball A just completes vertical circle, therefore $v_2 = \sqrt{5g\ell}$

$\therefore$ $0.8 v_0 \cos 30º = \sqrt{5g\ell}$ or $v_0 = 12.5\ ms^{-1}$ **Ans.**

VARIABLE MASS SYSTEM :

If a mass is added or ejected from a system, at rate μ kg/s and relative velocity $\vec{v}_{rel}$ (w.r.t. the system), then the force exerted by this mass on the system has magnitude $\mu|\vec{v}_{rel}|$.

Thrust Force ($\vec{F}_t$)

$$\vec{F}_t = \vec{v}_{rel}\left(\frac{dm}{dt}\right)$$

Suppose at some moment t = t mass of a body is m and its velocity is $\vec{v}$. After some time at t = t + dt its mass becomes (m – dm) and velocity becomes $\vec{v}+d\vec{v}$. The mass dm is ejected with relative velocity $\vec{v}_r$. Absolute velocity of mass 'dm' is therefore ($\vec{v}$ + $\vec{v}_r$). If no external forces are acting on the system, the linear momentum of the system will remain conserved, or $\vec{P}_i = \vec{P}_f$

or $m\vec{v} = (m - dm)(\vec{v} + d\vec{v}) + dm(\vec{v} + \vec{v}_r)$

or $m\vec{v} = m\vec{v} + md\vec{v} - (dm)\vec{v} - (dm)(d\vec{v}) + (dm)\vec{v} + \vec{v}_r\,dm$

The term (dm) ($d\vec{v}$) is too small and can be neglected.

$\therefore$ $md\vec{v} = -\vec{v}_r\,dm$ or $m\left(\frac{d\vec{v}}{dt}\right) = \vec{v}_r\left(-\frac{dm}{dt}\right)$

Here, $m\left(-\frac{d\vec{v}}{dt}\right)$ = thrust force $\left(\vec{F}_t\right)$

and $-\frac{dm}{dt}$ = rate at which mass is ejecting

or $\vec{F}_t = \vec{v}_r\left(\frac{dm}{dt}\right)$

Problems related to variable mass can be solved in following four steps

1. Make a list of all the forces acting on the main mass and apply them on it.

2. Apply an additional thrust force $\vec{F}_t$ on the mass, the magnitude of which is $\left|\vec{v}_r\left(\pm\frac{dm}{dt}\right)\right|$ and direction is given by the direction of $\vec{v}_r$ in case the mass is increasing and otherwise the direction of $-\vec{v}_r$ if it is decreasing.

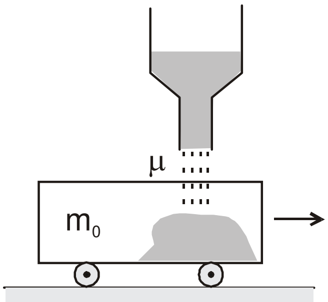
3. Find net force on the mass and apply $\vec{F}_{net} = m\frac{d\vec{v}}{dt}$ (m = mass at the particular instant)

4. Integrate it with proper limits to find velocity at any time t.

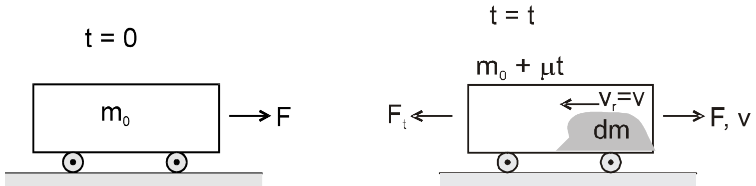
Note : Problems of one-dimensional motion (which are mostly asked in JEE) can be solved in easier manner just by assigning positive and negative signs to all vector quantities. Here are few example in support of the above theory.

Solved Examples

Example 35. A flat car of mass m_0 starts moving to the right due to a constant horizontal force F. Sand spills on the flat car from a stationary hopper. The rate of loading is constant and equal to μ kg/s. Find the time dependence of the velocity and the acceleration of the flat car in the process of loading. The friction is negligibly small.



Solution : Initial velocity of the flat car is zero. Let v be its velocity at time t and m its mass at that instant. Then



At t = 0, v = 0 and m = m_0 at t = t, v = v and m = $m_0 + \mu t$

Here, $v_r = v$ (backwards)

$$\frac{dm}{dt} = \mu$$

$\therefore$ $F_t = v_r \frac{dm}{dt} = \mu v$ (backwards)

Net force on the flat car at time t is $F_{net} = F - F_t$

or $m\frac{dv}{dt} = F - \mu v$ ....(i)

or $(m_0 + \mu t)\frac{dv}{dt} = F - \mu v$

or $\int_0^v \frac{dv}{F - \mu v} = \int_0^t \frac{dt}{m_0 + \mu t}$

$\therefore$ $-\frac{1}{\mu}[\ell n (F - \mu v)]_0^v = \frac{1}{\mu}[\ell n (m_0 + \mu t)]_0^t$

$\Rightarrow$ $\ell n\left(\frac{F}{F - \mu v}\right) = \ell n\left(\frac{m_0 + \mu t}{m_0}\right)$

$\therefore$ $\frac{F}{F - \mu v} = \frac{m_0 + \mu t}{m_0}$ or $v = \frac{Ft}{m_0 + \mu t}$ **Ans.**

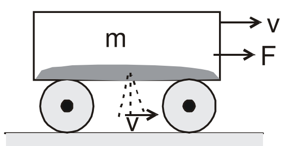
From Eq. (i), $\frac{dv}{dt}$ = acceleration of flat car at time t

or $= \frac{F - \mu v}{m}$

$a = \left(\frac{F - \frac{F\mu t}{m_0 + \mu t}}{m_0 + \mu t}\right)$ or $a = \frac{Fm_0}{(m_0 + \mu t)^2}$ **Ans.**

Example 36. A cart loaded with sand moves along a horizontal floor due to a constant force F coinciding in direction with the cart's velocity vector. In the process sand spills through a hole in the bottom with a constant rate μkg/s. Find the acceleration and velocity of the cart at the moment t, if at the initial moment t = 0 the cart with loaded sand had the mass m_0 and its velocity was equal to zero. Friction is to be neglected.

Solution : In this problem the sand spills through a hole in the bottom of the cart. Hence, the relative velocity of the sand v_r will be zero because it will acquire the same velocity as that of the cart at the moment.



$v_r = 0$

Thus, $F_t = 0$ $\left(\text{as } F_t = v_r\frac{dm}{dt}\right)$

and the net force will be F only.

$\therefore$ $F_{net} = F$

or $m\left(\frac{dv}{dt}\right) = F$ ....(i)

But here $m = m_0 - \mu t$

$\therefore$ $(m_0 - \mu t)\frac{dv}{dt} = F$ or $\int_0^v dv = \int_0^t \frac{F\ dt}{m_0 - \mu t}$

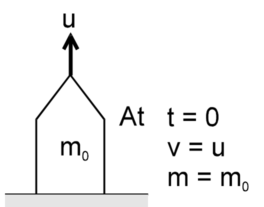
$\therefore$ $v = \frac{F}{-\mu}\left[\ell n(m_0 - \mu t)\ \right]_0^t$ or $v = \frac{F}{\mu}\ln\left(\frac{m_0}{m_0 - \mu t}\right)$ **Ans.**

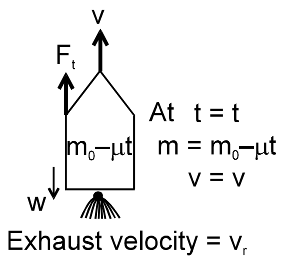
From eq. (i), acceleration of the cart

$a = \frac{dv}{dt} = \frac{F}{m}$ or $a = \frac{F}{m_0 - \mu t}$ **Ans.**

Rocket propulsion :

Let m_0 be the mass of the rocket at time t = 0. m its mass at any time t and v its velocity at that moment. Initially, let us suppose that the velocity of the rocket is u.





Further, let $\left(\frac{-dm}{dt}\right)$ be the mass of the gas ejected per unit time and v_r the exhaust velocity of the gases with respect to rocket. Usually $\left(\frac{-dm}{dt}\right)$ and v_r are kept constant throughout the journey of the rocket. Now, let us write few equations which can be used in the problems of rocket propulsion. At time t = t,

1. Thrust force on the rocket $F_t = v_r\left(\frac{-dm}{dt}\right)$ (upwards)
2. Weight of the rocket $W = mg$ (downwards)
3. Net force on the rocket $F_{net} = F_t - W$ (upwards)

   or $F_{net} = v_r\left(\frac{-dm}{dt}\right) - mg$

4. Net acceleration of the rocket $a = \frac{F}{m}$

   or $\frac{dv}{dt} = \frac{v_r}{m}\left(\frac{-dm}{dt}\right) - g$

   or $dv = \frac{v_r}{m}(-dm) - g\,dt$

   or $\int_u^v dv = v_r\int_{m_0}^{m}\frac{-dm}{m} - g\int_0^t dt$

   Thus, $v = u - gt + v_r\,\ell n\left(\frac{m_0}{m}\right)$ ...(i)

Note :

1. $F_t = v_r\left(-\frac{dm}{dt}\right)$ is upwards, as v_r is downwards and $\frac{dm}{dt}$ is negative.

2. If gravity is ignored and initial velocity of the rocket u = 0, Eq. (i) reduces to $v = v_r \ln\left(\frac{m_0}{m}\right)$

Solved Examples

Example 37. A rocket, with an initial mass of 1000 kg, is launched vertically upwards from rest under gravity. The rocket burns fuel at the rate of 10 kg per second. The burnt matter is ejected vertically downwards with a speed of 2000 ms^{-1} relative to the rocket. If burning stops after one minute. Find the maximum velocity of the rocket. (Take g as at 10 ms^{-2})

Solution : Using the velocity equation $v = u - gt + v_r \ln\left(\frac{m_0}{m}\right)$

Here u = 0, t = 60s, g = 10 m/s^2, v_r = 2000 m/s, m_0 = 1000 kg

and $m = 1000 - 10 \times 60 = 400$ kg

We get $v = 0 - 600 + 2000 \ln\left(\frac{1000}{400}\right)$

or $v = 2000 \ln 2.5 - 600$

The maximum velocity of the rocket is $200(10 \ln 2.5 - 3) = 1232.6\ ms^{-1}$ **Ans.**

LINEAR MOMENTUM CONSERVATION IN PRESENCE OF EXTERNAL FORCE.

$\vec{F}_{ext} = \frac{d\vec{P}}{dt} \quad \Rightarrow \quad \vec{F}_{ext}\, dt = d\vec{P} \quad \Rightarrow \quad d\vec{P} = \vec{F}_{ext}\,)_{impulsive}\, dt$

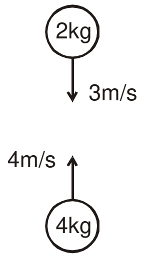
$\therefore$ If $\vec{F}_{ext}\,)_{impulsive} = 0 \quad \Rightarrow \quad d\vec{P} = 0$

or $\vec{P}$ is constant

Note : Momentum is conserved if the external force present is non-impulsive. eg. gravitation or spring force.

Solved Examples

Example 38. Two balls are moving towards each other on a vertical line collides with each other as shown. Find their velocities just after collision.

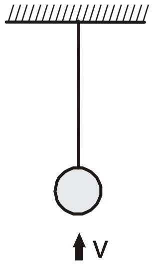


Solution : Let the final velocity of 4 kg ball just after collision be v. Since, external force is gravitational which is non - impulsive, hence, linear momentum will be conserved.

Applying linear momentum conservation :

$2(-3) + 4(4) = 2(4) + 4(v)$ or $v = \frac{1}{2}$ m/s

Example 39. A bullet of mass 50g is fired from below into the bob of mass 450g of a long simple pendulum as shown in figure. The bullet remains inside the bob and the bob rises through a height of 1.8 m. Find the speed of the bullet. Take g = 10 m/s^2.



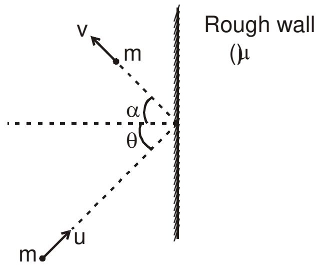
Solution : Let the speed of the bullet be v. Let the common velocity of the bullet and the bob, after the bullet is embedded into the bob, is V. By the principle of conservation of the linear momentum,

$$V = \frac{(0.05\ \text{kg})\ v}{0.45\ \text{kg} + 0.05\ \text{kg}} = \frac{v}{10}$$

The string becomes loose and the bob will go up with a deceleration of g = 10 m/s^2. As it comes to rest at a height of 1.8 m, using the equation $v^2 = u^2 + 2ax$,

$$1.8\ \text{m} = \frac{(v/10)^2}{2 \times 10\ \text{m/s}^2} \qquad \text{or,} \qquad v = 60\ \text{m/s}.$$

Example 40. A small ball of mass m collides with a rough wall having coefficient of friction μ at an angle θ with the normal to the wall. If after collision the ball moves with angle α with the normal to the wall and the coefficient of restitution is e then find the reflected velocity v of the ball just after collision.



Solution : $mv \cos\alpha - (m(-u\cos\theta)) = \int N dt$

$mv \sin\alpha - mu \sin\theta = -\mu \int N dt$

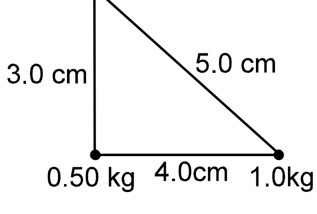
and $e = \dfrac{v\cos\alpha}{u\cos\theta} \qquad \Rightarrow \qquad v\cos\alpha = eu\cos\theta$

or $\quad mv \sin\alpha - mu\sin\theta = -\mu(mv\cos\alpha + mu\cos\theta)$

or $\quad v = \dfrac{u}{\sin\alpha}\,[\sin\theta - \mu\cos\theta\,(e+1)]$ **Ans.**

Solved Miscellaneous Problems

Problem 1. Three particles of masses 0.5 kg, 1.0 kg and 1.5 kg are placed at the three corners of a right angled triangle of sides 3.0 cm, 4.0 cm and 5.0 cm as shown in figure. Locate the center of mass of the system.

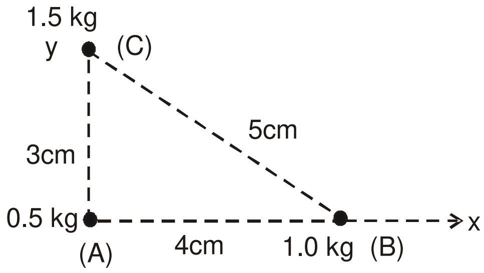


Solution : taking x and y axes as shown.

coordinates of body A = (0,0)

coordinates of body B = (4,0)

coordinates of body C = (0,3)



x-coordinate of c.m. = $\dfrac{m_A x_A + m_B x_B + M_C r_C}{m_A + m_B + m_C}$

$= \dfrac{0.5\times 0 + 1.0\times 4 + 1.5\times 0}{0.5+1.0+1.5} = \dfrac{4}{3}\ \dfrac{\text{kg}-\text{cm}}{\text{kg}}$ = 1.33cm

similarly y - coordinate of c.m. = $\dfrac{0.5\times 0 + 1.0\times 0 + 1.5\times 3}{0.5+1.0+1.5} = \dfrac{4.5}{3}\ \dfrac{\text{kg}-\text{cm}}{\text{kg}}$ = 1.5 cm

So, certer of mass is 1.33 cm right and 1.5 cm above particle A.

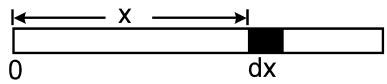
Problem 2. The linear mass density of a straight rod of length L varies as ρ = A + Bx where x is the distance from the left end. Locate the center of mass from left end.

Solution : Let take a strip of width \`dx\` at distance x from one end.

dm = mass of \`dx\` strip = f dx

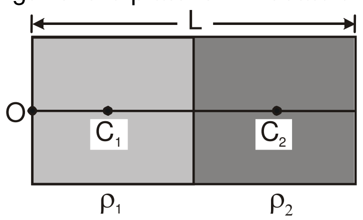
dm = (A+Bx)dx ......(1)

By definition $X_{com} = \dfrac{\int_O^L x\,dm}{\int_O^L dm}$

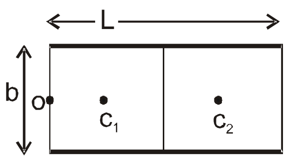


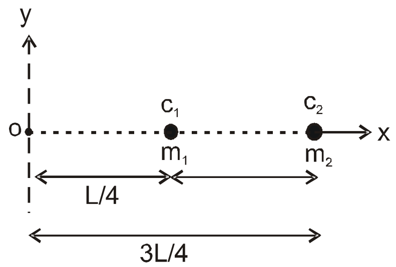
from eq (1) $\Rightarrow X_{com} = \dfrac{\int_O^L x(A+Bx)dx}{\int_O^L (A+Bx)dx} = \dfrac{\int_O^L (Ax+Bx^2)dx}{\int_O^L (A+Bx)dx} = \dfrac{\dfrac{AL^2}{2}+\dfrac{BL^3}{3}}{AL+\dfrac{BL^2}{2}} = \dfrac{3AL+2BL^2}{3(2A+BL)}$ **Ans.**

Problem 3. Half of the rectangular plate shown in figure is made of a material of density ρ_1 and the other half of density ρ_2. The length of the plate is L. Locate the center of mass of the plate.



Solution :





Replacing half of rectangular plate by a point mass at its center of mass as shown in figure.

$M_1 = \dfrac{\rho_1 L}{2} b \qquad M_2 = \dfrac{\rho_2 L}{2} b$

$X_{com} = \dfrac{m_1 \frac{L}{4} + m_2\, 3\frac{L}{4}}{m_1+m_2} = \dfrac{\left(\rho_1.\dfrac{L}{2}.b\right)\dfrac{L}{4}+\left(\rho_2.\dfrac{L}{2}.b\right).\dfrac{3L}{4}}{\left(\rho_1.\dfrac{L}{2}.b\right)+\left(\rho_2.\dfrac{L}{2}.b\right)} = \dfrac{(\rho_1+3\rho_2)}{4(\rho_1+\rho_2)}$ L from point O.

Problem 4. In a boat of mass 4 M and length ℓ on a frictionless water surface. Two men A (mass = M) and B(mass 2M) are standing on the two opposite ends. Now A travels a distance $\ell/4$ relative to boat towards its center and B moves a distance $3\ell/4$ relative to boat and meet A. Find the distance travelled by the boat on water till A and B meet.

Solution : Let x is distance travelled by boat.

Initial position of center of mass

$$= \frac{M_{Boat}X_{Boat} + M_A X_A + M_B X_B}{M_{Boat} + M_A + M_B} = \frac{4M\frac{\ell}{2} + M.0 + 2M.\ell}{4M + M + 2M} = \frac{4M\ell}{7M} = \frac{4}{7}\ell$$

Final position of center of mass $= \dfrac{4M\left\{\frac{\ell}{2}+x\right\} + M\left\{x+\frac{\ell}{4}\right\} + 2M\left\{x+\frac{\ell}{4}\right\}}{7M}$

$$= \frac{2M\ell + \frac{M\ell}{4} + \frac{M\ell}{2} + 7\ Mx}{7M} = \frac{\frac{11\,M\ell}{4} + 7\ Mx}{7M} = \frac{\frac{11\,M\ell}{4} + 7x}{7}$$

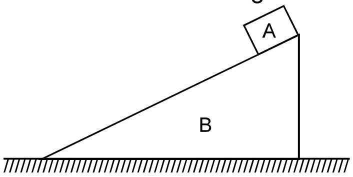
since there is no horizontal force, position of center of mass remains unchanged.

center of mass initially = center of mass finally

$$\Rightarrow \quad \frac{4}{7}\ell = \frac{\frac{11\,\ell}{4} + 7x}{7}$$

$$4\ell = \frac{11\,\ell}{4} + 7x \Rightarrow \quad x = \frac{5\ell}{28}$$

Problem 5. A block A (mass = 4M) is placed on the top of a wedge B of base length l (mass = 20 M) as shown in figure. When the system is released from rest. Find the distance moved by the wedge B till the block A reaches at lowest end of wedge. Assume all surfaces are frictionless.



Solution : Initial position of center of mass

$$= \frac{X_B M_B + X_A M_A}{M_B + M_B} = \frac{X_B.20\ \ M + \ell.4M}{24M} = \frac{5X_B + \ell}{6}$$

Final position of center of mass $= \dfrac{(X_B + x)20M + 4Mx}{24M} = \dfrac{5(X_B + x) + x}{6}$

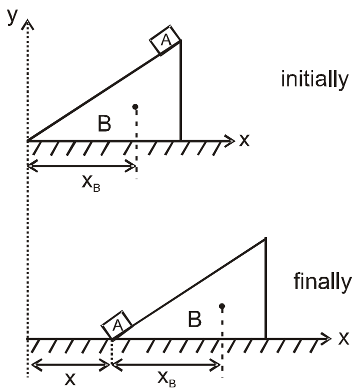
since there is no horizontal force on system

center of mass initially = center of mass finally.

$5X_B + \ell = \ 5X_B + 5x + x$

$\ell \ = \ 6x$

$$\boxed{x = \frac{\ell}{6}}$$



Problem 6. An isolated particle of mass m is moving in a horizontal xy plane, along x-axis. At a certain height above ground, it suddenly explodes into two fragments of masses m/4 and 3m/4. An instant later, the smaller fragment is at y = + 15 cm. Find the position of heavier fragment at this instant.

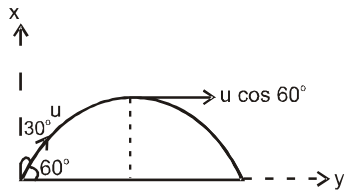
Solution : As particle is moving along x-axis, so, y-coordinate of COM is zero.

$$Y_M\, M = Y_{\frac{M}{4}}\left(\frac{M}{4}\right) + Y_{\frac{3M}{4}}\left(\frac{3M}{4}\right) \quad \Rightarrow \quad 0 \times M = 15\left(\frac{M}{4}\right) + Y_{\frac{3M}{4}}\left(\frac{3M}{4}\right)$$

$$\boxed{Y_{\frac{3M}{4}} = -5\text{cm}}$$

Problem 7. A shell is fired from a cannon with a speed of 100 m/s at an angle 30º with the vertical (y-direction). At the highest point of its trajectory, the shell explodes into two fragments of masses in the ratio 1 : 2. The lighter fragment moves vertically upwards with an initial speed of 200 m/s. What is the speed of the heavier fragment at the time of explosion.

Solution : Velocity at highest point = u cosθ = 100 cos 60 = 50m/s.
taking, x and y axes as shown in figure,



Velocity of m = $50\hat{i}$ m / sec

m explodes into $\frac{m}{3}$ and $\frac{2m}{3}$, velocity of $\frac{m}{3} = 200\hat{j}$ m/sec

Let, velocity of $\frac{2m}{3} = \vec{V}$

Applying law of conservation of momentum

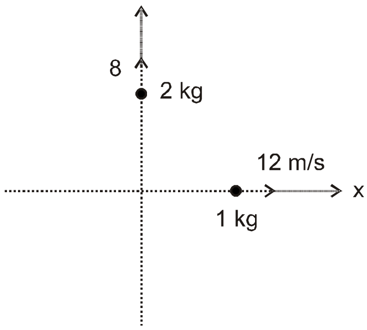
$$m \times 50\hat{i} = \frac{m}{3} \times 200\hat{j} + \frac{2m}{3}\vec{V} \quad \Rightarrow \quad 50\hat{i} - \frac{200}{3}\hat{j} = \frac{2}{3}\vec{V}$$

so $\vec{V} = 75\hat{i} - 100\hat{j}$

Speed = $|\vec{V}| = \sqrt{(75)^2 + (100)^2} = 25\sqrt{9+16} = 125$ m/sec **Ans.**

Problem 8. A shell at rest at origin explodes into three fragments of masses 1 kg, 2 kg and m kg. The fragments of masses 1 kg and 2 kg fly off with speeds 12 m/s along x-axis and 8 m/s along y-axis respectively. If m kg flies off with speed 40 m/s then find the total mass of the shell.

Solution : As initial velocity $\vec{V} = 0$, Initial momentum
= (1 + 2 + m) × 0 = 0



Finally, let velocity of M = $\vec{V}$. We know $|\vec{V}|$ = 40 m/s.
Initial momentum = final momentum.

$$0 = 1 \times 12\hat{i} + 2 \times 8\hat{j} + m\vec{V}$$

$$\Rightarrow \quad \vec{V} = -\frac{(12\hat{i} + 16\hat{j})}{m}$$

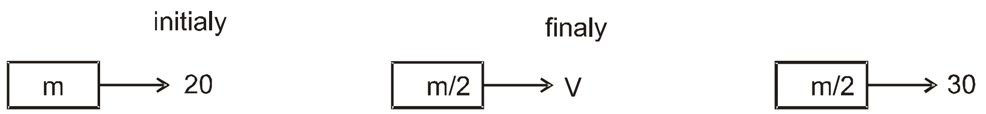
$$|\vec{V}| = \sqrt{\frac{(12)^2 + (16)^2}{m^2}} = \frac{1}{m}\sqrt{(12)^2 + (16)^2} = 40 \text{ \{given\}}$$

$$m = \sqrt{\frac{(12)^2 + (16)^2}{40}} = 0.5 \text{ kg}$$

Total mass = 1 + 2 + 0.5 = 3.5 kg

Problem 9. A block moving horizontally on a smooth surface with a speed of 20 m/s bursts into two equal parts continuing in the same direction. If one of the parts moves at 30 m/s, with what speed does the second part move and what is the fractional change in the kinetic energy of the system.

Solution :



Applying momentum conservation ;

$m \times 20 = \frac{m}{2} V + \frac{m}{2} \times 30 \quad \Rightarrow \quad 20 = \frac{V}{2} + 15$

So, $V = 10$ m/s

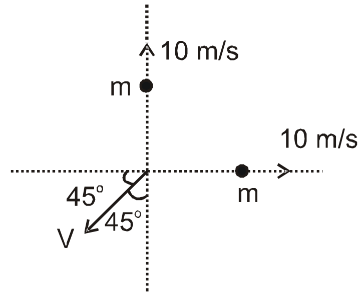
initial kinetic energy $= \frac{1}{2} m \times (20)^2 = 200$ m

final kinetic energy $= \frac{1}{2} . \frac{m}{2} . (10)^2 + \frac{1}{2} \times \frac{m}{2} (30)^2 = 25\text{ m} + 225\text{ m} = 250$ m

fractional change in kinetic energy $= \frac{(\text{final K. E}) - (\text{initial K. E})}{\text{initial K.E}} = \frac{250\text{m} - 200\text{m}}{200\text{m}} = \frac{1}{4}$

Problem 10. A block at rest explodes into three equal parts. Two parts starts moving along X and Y axes respectively with equal speeds of 10 m/s. Find the initial velocity of the third part.

Solution : Let total mass = 3 m, initial linear momentum = 3m × 0



Let velocity of third part = $\vec{V}$

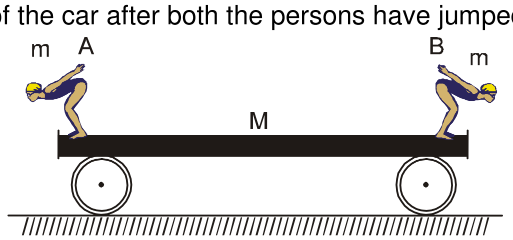
Using conservation of linear momentum :

$m \times 10\hat{i} + m \times 10\hat{j} + m\vec{V} = 0$

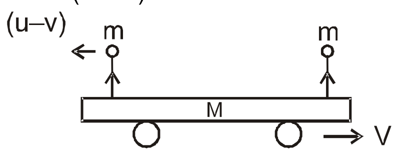
So, $\vec{V} = (-10\hat{i} - 10\hat{j})$ m/sec.

$|\vec{V}| = \sqrt{(10)^2 + (10)^2} = 10\sqrt{2}$, making angle 135° below x-axis

Problem 11. Two persons A and B, each of mass m are standing at the two ends of rail-road car of mass M. The person A jumps to the left with a horizontal speed u with respect to the car. Thereafter, the person B jumps to the right, again with the same horizontal speed u with respect to the car. Find the velocity of the car after both the persons have jumped off.



Solution : When person A jumps, let car achieve velocity V in forward direction. So, velocity of 'A' w.r.t. ground = (u – v) in backward direction.



Applying momentum conservation.

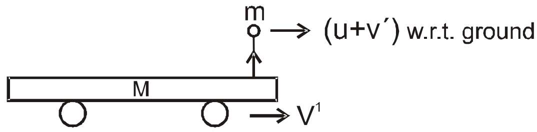
$(M + m)v - m(u - v) = 0$

$(M + m)v + mv = mu$

$$V = \frac{m}{M+2m}u$$

When person B jumps, let velocity of car becomes V´ in forward direction.

So velocity of 'B' w.r.t ground = u + v´



Applying momentum conservation :

$$(m + M)V = MV' + m(u + V')$$

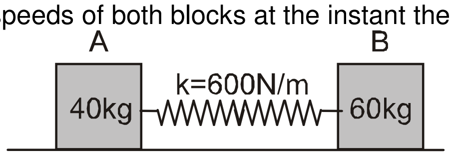
$$\frac{(m+M)\ mu}{M+2m} = (m + M)V' + mu$$

$$V' = \frac{-m^2u}{(M+m)(M+2m)}$$

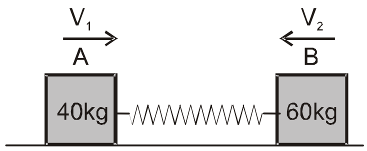
$$V' = \frac{-m^2u}{(M+m)(M+2m)}$$ ' backward' {–ve sign signifies direction}

$$= \frac{m^2u}{(M+2m)(M+m)}$$ **Ans.**

Problem 12. Blocks A and B have masses 40 kg and 60 kg respectively. They are placed on a smooth surface and the spring connected between them is stretched by 1.5m. If they are released from rest, determine the speeds of both blocks at the instant the spring becomes unstretched.



Solution :



Let, both block start moving with velocity V_1 and V_2 as shown in figure

Since no horizontal force on system so, applying momentum conservation

$0 = 40\ V_1 - 60\ V_2$ $\boxed{2V_1 = 3V_2}$ ....(1)

Now applying energy conservation, Loss in potential energy = gain in kinetic energy

$$\frac{1}{2}kx^2 = \frac{1}{2}m_1V_1^2 + \frac{1}{2}m_2V_2^2$$

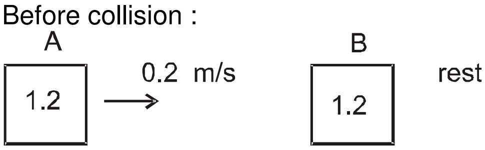
$$\frac{1}{2} \times 600 \times (1.5)^2 = \frac{1}{2} \times 40 \times V_1^2 + \frac{1}{2} \times 60 \times V_2^2 \quad ....(2)$$

Solving equation (1) and (2) we get,

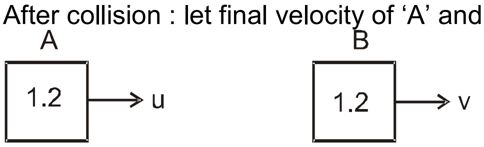
$V_1 = 4.5$ m/s, $V_2 = 3$ m/s.

Problem 13. A block of mass 1.2 kg moving at a speed of 20 cm/s collides head-on with a similar block kept at rest. The coefficient of restitution is 3/5. Find the loss of the kinetic energy during the collision

Solution : Before collision :



After collision : let final velocity of 'A' and 'B' be u and V respectively



Applying momentum conservation :

$$1.2 \times 0.2 = 1.2\ u + 1.2\ V$$

$$0.24 = 1.2\ (u + V) \quad .....(1)$$

Coefficient of restitution

$$e = \frac{V-u}{0.2} = \frac{3}{5} \quad .....(2)$$

Applying momentum conservation :

$1.24 + 1.2\ V = 0.2u \ \ u + V - 0.2$ m/s .....(2)

Solving equation (1) and (2) we have ,

$$V = \frac{4}{25}\text{ m/sec} \qquad \text{and } u = \frac{1}{25}\text{ m/sec}$$

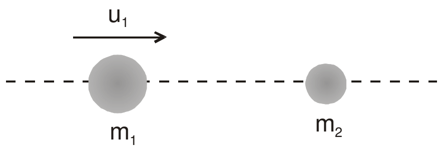
Loss of kinetic energy = (initial kinetic energy) – (final kinetic energy)

$$= \frac{1}{2} \times 1.2 \times (0.2)^2 - \left\{\frac{1}{2}\times 1.2\times\left(\frac{4}{25}\right)^2 + \frac{1}{2}\times(1.2)\times\left(\frac{1}{25}\right)^2\right\}$$

$$= 0.6\left\{0.04 - \frac{16}{625} - \frac{1}{625}\right\} = 0.6\ \{0.04 \times 0.0256 \times 0.0016\}$$

$$= 0.6\ \{0.0128\} = 0.00768\text{ J} = 7.7 \times 10^{-3}\text{ J}$$

Problem 14. The sphere of mass m_1 travels with an initial velocity u_1 directed as shown and strikes the stationary sphere of mass m_2 head on. For a given coefficient of restitution e, what condition on the mass ratio $\frac{m_1}{m_2}$ ensures that the final velocity of m_2 is greater than u_1?



Solution : Let velocity of m_1 & m_2 after collision be u & V respectively.

$$\text{Coefficient of restitution} = e = \frac{\text{velocity of separation}}{\text{velocity of approch}} = \frac{V-u}{u_1} \quad ....(1)$$

applying momentum conservation,

initial momentum = final momentum

$$m_1u_1 = m_1u + m_2V \quad ....(2)$$

$$m_1u_1 = m_1\ (V - eu_1) + m_2V$$

$$\frac{V}{u_1} = \frac{m_1(1+e)}{m_1+m_2} = \frac{(1+e)}{1+\frac{m_2}{m_1}}$$

$$\frac{(1+e)}{1+\frac{m_2}{m_1}} = \frac{V}{u_1} > 1\ \{\text{given}\} \qquad \boxed{\frac{m_1}{m_2} > \frac{1}{e}}\ \ \textbf{Ans.}$$

Problem 15. Find the mass of the rocket as a function of time, if it moves with a constant acceleration a, in absence of external forces. The gas escapes with a constant velocity u relative to the rocket and its initial mass was m_0.

Solution : Using, $F_{net} = V_{rel}\left(\frac{-dm}{dt}\right)$

$$F_{net} = -u\frac{dm}{dt} \quad ......(1)$$

$$F_{net} = ma \quad ......(2)$$

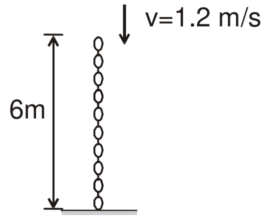
Solving equation (1) and (2)

$$ma = -\ u\frac{dm}{dt}$$

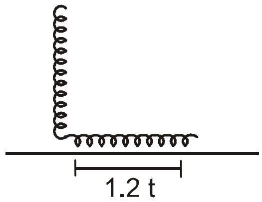
$$\int_{m_o}^{m}\frac{dm}{m} = \int_{o}^{t}\frac{-adt}{u} \qquad \ell n\frac{m}{m_o} = \frac{-at}{u}$$

$$\frac{m}{m_o} = e^{-at/u} \qquad \boxed{m = m_0e^{-\frac{at}{u}}}\ \ \textbf{Ans.}$$

Problem 16. If the chain is lowered at a constant speed v = 1.2 m/s, determine the normal reaction exerted on the floor as a function of time. The chain has a mass of 80 kg and a total length of 6 m.



Solution : after time 't'



Linear mass density of chain = $\frac{80}{6} = \frac{40}{3}$ kg/m = λ

Weight of portion of chain lying on ground = mg

= $\frac{40}{3} \times 1.2\, t \frac{40}{3} \times 1.2\, t = 16\, t$, N

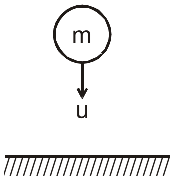
Thrust force = F_t = $V \times \frac{dm}{dt}$

= $1.2 \times \lambda\, v = 1.2\, \lambda\, v$

= $1.2 \times \frac{40}{3} \times 1.2 = 1.2 \times 40 \times 0.4 = 19.2$N

Normal reaction exerted on floor = W + F_t = (16t + 19.2) N vertically downward

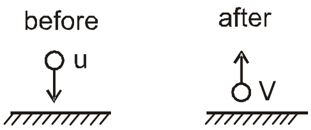
Problem 17. A ball is approaching to ground with speed u. If the coefficient of restitution is e then find out:



(a) the velocity just after collision.

(b) the impulse exerted by the normal due to ground on the ball.

Solution :



$e = \frac{\text{velocity of separation}}{\text{velocity of approah}} = \frac{v}{u}$

(a) velocity after collision = V = eu ....(1)

(b) Impulse exerted by the normal due to ground on the ball = change in momentum of ball.

= {final momentum} – {initial momentum}

= {m v} – {– mu}

= mv + mu

= m {u + eu}

= mu {1 + e} **Ans.**

Exercise-1

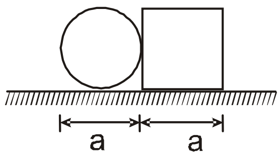
✍ Marked Questions can be used as Revision Questions.

PART - I : SUBJECTIVE QUESTIONS

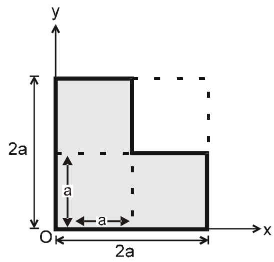
Section (A) : Calculation of centre of mass

A-1. Three particles of mass 1 kg, 2 kg and 3 kg are placed at the corners A, B and C respectively of an equilateral triangle ABC of edge 1 m. Find the distance of their centre of mass from A.

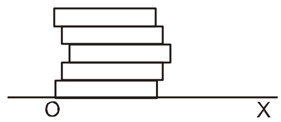
A-2. A square plate of edge 'a' and a circular disc of same diameter are placed touching each other at the midpoint of an edge of the plate as shown in figure. If mass per unit area for the two plates are same then find the distance of centre of mass of the system from the centre of the disc.



A-3.✍ Find the position of centre of mass of the uniform planner sheet shown in figure with respect to the origin (O)

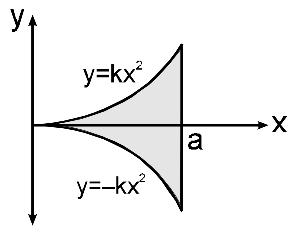


A-4. Five homogeneous bricks, each of length L, are arranged as shown in figure. Each brick is displaced with respect to the one in contact by L/5. Find the x-coordinate of the centre of mass relative to the origin O shown.



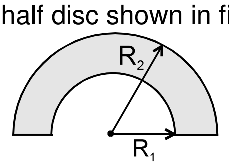
A-5. A uniform disc of radius R is put over another uniform disc of radius 2R made of same material having same thickness. The peripheries of the two discs touches each other. Locate the centre of mass of the system taking center of large disc at origin.

A-6.✍ A thin uniform sheet of metal of uniform thickness is cut into the shape bounded by the line x = a and $y = \pm k x^2$, as shown. Find the coordinates of the centre of mass.



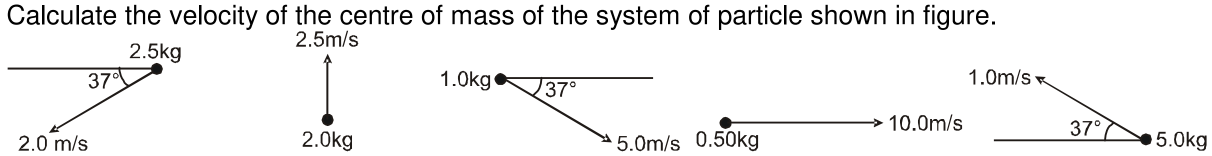
A-7. A disc of radius R is cut out from a larger uniform disc of radius 2R in such a way that the edge of the hole touches the edge of the disc. Locate the centre of mass of **remaining** part.

A-8.✍ Find the centre of mass of an annular half disc shown in figure.



Section (B) : Motion of centre of mass

B-1.✍ Calculate the velocity of the centre of mass of the system of particle shown in figure.



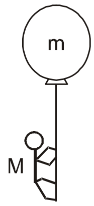
B-2. Two blocks of masses 10 kg and 30 kg are placed along a vertical line. The first block is raised through a height of 7 cm. By what distance should the second mass be moved to raise the centre of mass by 1 cm ?

B-3. A projectile is fired from a gun at an angle of 45º with the horizontal and with a speed of 20 m/s relative to ground. At the highest point in its flight the projectile explodes into two fragments of equal mass. One fragment comes at rest just after explosion. How far from the gun does the other fragment land, assuming a horizontal ground ? Take g = 10 m/s^2?

B-4. A mercury thermometer is placed in a gravity free hall without touching anything. As temperature rises mercury expands and ascend in thermometer. If height ascend by mercury in thermometer is h then by what height centre of mass of " mercury and thermometer" system descend?

B-5. Two men 'A' and 'B' are standing on opposite edge of a 6m long platform which is further kept on a smooth floor. They starts moving towards each other and finally meet at the midpoint of platform. Find the displacement of platform if mass of A, B and platform are 40kg, 60kg and 50kg respectively.

B-6 A man of mass M hanging with a light rope which is connected with a balloon of mass m. The system is at rest and equilibrium in air. When man rises a distance h with respect to balloon Find.
(a) The distance raised by man
(b) The distance descended by balloon



Section (C) : Conservation of linear momentum

C-1. A nucleus (mass no. 238) initially at rest decay into another nucleus (mass no 234) emitting an α particle (mass no - 4) with the speed of 1.17×10^7 m/sec. Find the recoil speed of the remaining nucleus.

C-2. A stone of mass 5 kg is thrown upwards with a speed of 36 m/sec. With what speed earth recoil. Mass of the earths 6×10^{24} kg (assuming that there is no external force on the system)

C-3. In a process a neutron which is initially at rest, decays into a proton, an electron and an antineutrino. The ejected electron has a momentum of $p_1 = 2.4 \times 10^{-26}$ kg-m/s and the antineutrino has $p_2 = 7.0 \times 10^{-27}$ kg-m/s. Find the recoil speed of the proton if the electron and the antineutrino are ejected (a) along the same direction. (b) in mutually perpendicular directions. (Mass of the proton $m_p = 1.67 \times 10^{-27}$ kg.)

C-4. Three particles of mass 20 g, 30 g and 40 g are initially moving along the positive direction of the three coordinate axes x, y and z respectively with the same velocity of 20 cm/s. If due to their mutual interaction, the first particle comes to rest, the second acquires a velocity $10\hat{i} + 20\hat{k}$. What is the velocity (in cm/s) of the third particle?

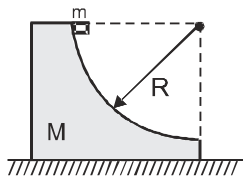
C-5. A truck of mass M is at rest on a frictionless road. When a monkey of mass m starts moving on the truck in forward direction, the truck recoils with a speed v backward on the road, with what velocity is the monkey moving with respect to truck?

C-6. A boy of mass 60 kg is standing over a platform of mass 40 kg placed over a smooth horizontal surface. He throws a stone of mass 1 kg with velocity v = 10 m/s at an angle of 45º with respect to the ground. Find the displacement of the platform (with boy) on the horizontal surface when the stone lands on the ground. (g = 10 m/s^2)

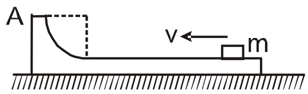
C-7. A (trolley + child) of total mass 200 kg is moving with a uniform speed of 36 km/h on a frictionless track. The child of mass 20 kg starts running on the trolley from one end to the other (10 m away) with a speed of 10 ms^{-1} relative to the trolley in the direction of the trolley's motion and jumps out of the trolley with the same relative velocity. What is the final speed of the trolley? How much has the trolley moved from the time the child begins to run and just before jump?

C-8. A bullet of mass 0.01kg and travelling at a speed of 500 m/s strikes and passes horizontally through a block of mass 2kg which is suspended by a string of length 5m. The centre of gravity of the block is found to raise a vertical distance of 0.1 m. What is the speed of the bullet after it emerges from the block. [g = 9.8m/s^2] (Time of passing of bullet is negligible)

C-9. A small cube of mass 'm' slides down a circular path of radius 'R' formed from a large block of mass 'M' as shown in figure 'M' rests on a table and both blocks move without friction. The blocks are initially at rest and 'm' starts from the top of the path. Find the velocity 'v' of the cube as it leaves the block. Initially the line joining m and the centre is horizontal.



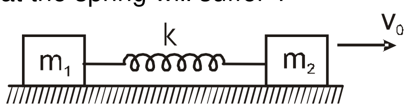
C-10. A small block of mass m moving with speed 'V' on a smooth horizontal part of a bigger block of mass M which is further kept on smooth floor. The curved part of the surface shown is one fourth of a circle. Find the speed of the bigger block when the smaller block reaches the point A of the surface.



Section (D) : spring - mass system

D-1. Two masses m_1 and m_2 are connected by a spring of spring constant k and are placed on a smooth horizontal surface. Initially the spring is stretched through a distance 'd' when the system is released from rest. Find the distance moved by the two masses when spring is compressed by a distance 'd'.

D-2. Two block of masses m_1 and m_2 are connected with the help of a spring of spring constant k initially the spring in its natural length as shown. A sharp impulse is given to mass m_2 so that it acquires a velocity v_0 towards right. If the system is kept on smooth floor then find (a) the velocity of the centre of mass, (b) the maximum elongation that the spring will suffer ?



D-3. Two blocks A and B of mass m_A and m_B are connected together by means of a spring and are resting on a horizontal frictionless table. The blocks are then pulled apart so as to stretch the spring and then released. Show that the ratio of their kinetic energies at any instant is in the inverse ratio of their masses.

Section (E) : Impulse

E-1. Velocity of a particle of mass 2 kg varies with time t according to the equation $\vec{v} = (2t\hat{i} + 4\hat{j})m/s$. Here t is in seconds. Find the impulse imparted to the particle in the time interval from t = 0 to t = 2 s.

E-2. A ball of mass 100 g moving with a speed of 4 m/sec strikes a horizontal surface at an angle of 30º from the surface. The ball is reflected back with same speed and same angle of reflection find (a) The impulse imparted to the ball (b) change in magnitude of momentum of the ball.

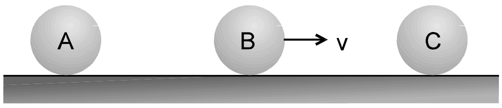
E-3. During a heavy rain, hailstones of average size 1.0 cm in diameter fall with an average speed of 20 m/s. Suppose 2000 hailstones strike every square meter of a 10 m × 10 m roof perpendicularly in one second and assume that the hailstones do not rebound. Calculate the average force exerted by the falling hailstones on the roof. Density of hailstones is 900 kg/m^3, take (π = 3.14)

E-4. A steel ball of mass 0.5 kg is dropped from a height of 4 m on to a horizontal heavy steel slab. The ball strike the slab and rebounds to its original height. (Take g = 10 m/sec^2)
(a) Calculate the impulse delivered to the ball during impact.
(b) If the ball is in contact with the slab for 0.002s, find the average reaction force on the ball during impact.

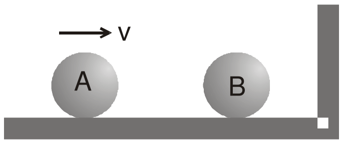
Section (F) : Collision

F-1. A particle moving with kinetic energy K makes a head on elastic collision with an identical particle at rest. Find the maximum elastic potential energy of the system during collision.

F-2. Three balls A, B and C are placed on a smooth horizontal surface. Given that $m_A = m_C = 4m_B$. Ball B collides with ball C with an initial velocity v as shown in figure. Find the total number of collisions between the balls. All collisions are elastic.



F-3. Two balls shown in figure are identical. Ball A is moving towards right with a speed v and the second ball is at rest. Assume all collisions to be elastic. Show that the speeds of the balls remain unchanged after all the collisions have taken place. (Assume frictionless surface)



F-4. A ball of mass m moving at a speed v makes a head on collision with an identical ball at rest. The kinetic energy of the balls after the collision is 3/4 of the original kinetic energy. Calculate the coefficient of restitution.

F-5. A particle of mass m moving with a speed v hits elastically another stationary particle of mass 2m in a fixed smooth horizontal circular tube of radius r. Find the time when the next collision will take place?

F-6. A block of mass 1 kg moving at a speed of 2.5 m/s collides with another block of mass 0.5 kg. If both the blocks come to rest after collision what was the velocity of the 0.5 kg block before the collision?

F-7. A 3kg block 'A ' moving with 4 m/sec on a smooth table collides inelastically and head on with an 8kg block 'B' moving with speed 1.5 m/sec towards 'A '. Given e = 1/2
(a) What is final velocities of both the blocks
(b) Find out the impulse of reformation and deformation
(c) Find out the maximum potential energy of deformation
(d) Find out loss in kinetic energy of system.

Section (G) : Variable mass

G-1. A rocket of mass m = 20 kg has M = 180kg fuel. The uniform exhaust velocity of the fuel is v_r = 1.6 km/s.
(i) Calculate the minimum rate of consumption of fuel so that the rocket may rise from the ground.
(ii) Also calculate the maximum vertical speed gained by the rocket when the rate of consumption of fuel μ is (g = 10 m/s^2) & (ℓn 10 = 2.30)
(a) 2 kg/s (b) 20 kg/s

G-2. Sand drops from a stationary hopper at the rate of 5 kg/s falling on a conveyor belt moving with a constant speed of 2 m/s. What is the force required to keep the belt moving and what is the power delivered by the motor moving the belt?

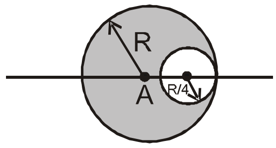
PART - II : ONLY ONE OPTION CORRECT TYPE

Section (A) : Calculation of centre of mass

A-1. The centre of mass of a body :

(A) Lies always at the geometrical centre (B) Lies always inside the body
(C) Lies always outside the body (D) Lies within or outside the body

A-2. The centre of mass of the shaded portion of the disc is : (The mass is uniformly distributed in the shaded portion) :

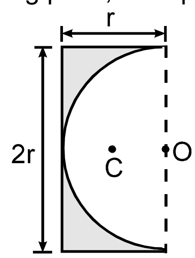


(A) $\frac{R}{20}$ to the left of A (B) $\frac{R}{12}$ to the left of A
(C) $\frac{R}{20}$ to the right of A (D) $\frac{R}{12}$ to the right of A

A-3. A thin uniform wire is bent to form the two equal sides AB and AC of triangle ABC, where AB = AC = 5 cm. The third side BC, of length 6cm, is made from uniform wire of twice the linear mass density of the first. The distance of centre of mass from A is :

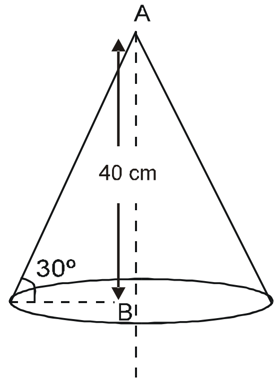
(A) $\frac{34}{11}$ cm (B) $\frac{11}{34}$ cm (C) $\frac{34}{9}$ cm (D) $\frac{11}{45}$ cm

A-4. A semicircular portion of radius 'r' is cut from a uniform rectangular plate as shown in figure. The distance of centre of mass 'C' of remaining plate, from point 'O' is :



(A) $\frac{2r}{(3-\pi)}$ (B) $\frac{3r}{2(4-\pi)}$ (C) $\frac{2r}{(4+\pi)}$ (D) $\frac{2r}{3(4-\pi)}$

A-5. A uniform solid cone of height 40 cm is shown in figure. The distance of centre of mass of the cone from point B (centre of the base) is :



(A) 20 cm (B) 10/3 cm (C) 20/3 cm (D) 10 cm

A-6. The centre of mass of a system of particles is at the origin. From this we conclude that

(A) The number of particles on positive x-axis is equal to the number of particles on negative x-axis
(B) The total mass of the particles on positive x-axis is same as the total mass on negative x-axis
(C) The number of particles on X-axis may be equal to the number of particles on Y-axis.
(D) If there is a particle on the positive X-axis, there must be at least one particle on the negative X-axis.

A-7. All the particles of a system are situated at a distance r from the origin. The distance of the centre of mass of the system from the origin is

(A) = r (B) ≤ r (C) > r (D) ≥ r

Section (B) : Motion of centre of mass

B-1. Two particles of mass 1 kg and 0.5 kg are moving in the same direction with speed of 2m/s and 6m/s respectively on a smooth horizontal surface. The speed of centre of mass of the system is :

(A) $\frac{10}{3}$ m/s (B) $\frac{10}{7}$ m/s (C) $\frac{11}{2}$ m/s (D) $\frac{12}{3}$ m/s

B-2. Two particles of equal mass have initial velocities $2\hat{i}$ ms^{-1} and $2\hat{j}$ ms^{-1}. First particle has a constant acceleration $(\hat{i}+\hat{j})$ ms^{-2} while the acceleration of the second particle is always zero. The centre of mass of the two particles moves in

(A) Circle (B) Parabola (C) Ellipse (D) Straight line

B-3. Two particles having mass ratio n : 1 are interconnected by a light inextensible string that passes over a smooth pulley. If the system is released, then the acceleration of the centre of mass of the system is :

(A) $(n-1)^2$ g (B) $\left(\frac{n+1}{n-1}\right)^2$ g (C) $\left(\frac{n-1}{n+1}\right)^2$ g (D) $\left(\frac{n+1}{n-1}\right)$ g

B-4. A bomb travelling in a parabolic path under the effect of gravity, explodes in mid air. The centre of mass of fragments will:

(A) Move vertically upwards and then downwards
(B) Move vertically downwards
(C) Move in irregular path
(D) Move in the parabolic path which the unexploded bomb would have travelled.

B-5. If a ball is thrown upwards from the surface of earth then initially (assuming that there is no external force on the system):

(A) The earth remains stationary while the ball moves upwards
(B) The ball remains stationary while the earth moves downwards
(C) The ball and earth both move towards each other
(D) The ball and earth both move away from each other

B-6. Internal forces in a system can change

(A) Linear momentum only
(B) Kinetic energy only
(C) Both kinetic energy and linear momentum
(D) Neither the linear momentum nor the kinetic energy of the system.

B-7. Two balls of different masses are thrown in air with different velocities. While they are in air acceleration of centre of mass of the system. (Neglect air resistance)

(A) Depends on the direction of the motion of two balls
(B) Depends on the masses of the two balls
(C) Depends on the magnitude of velocities of the two balls
(D) Is equal to g

B-8. There are two particles of same mass. If one of the particles is at rest always and the other has an acceleration $\vec{a}$. Acceleration of centre of mass is

(A) zero (B) $\frac{1}{2}\vec{a}$ (C) $\vec{a}$

(D) centre of mass for such a system can not be defined.

B-9. A body of mass 1 kg moving in the x-direction, suddenly explodes into two fragments of mass $\frac{1}{8}$ kg and $\frac{7}{8}$ kg. An instant later, the smaller fragment is 0.14 m above the x-axis. The position of the heavier fragment is - **[REE - 1995]**

(A) $\frac{1}{50}$ m above x-axis (B) $\frac{1}{50}$ m below x-axis (C) $\frac{7}{50}$ m below x-axis (D) $\frac{7}{50}$ m above x-axis

[REE - 1995]

B-10. Two particles of equal mass m are projected form the ground with speed v_1 and v_2 at angle θ_1 and θ_2 (θ_1, $\theta_2 \neq 0$, 180º) as shown in figure. The centre of mass of the two particles

(A) will move in a parabolic path for any values of v_1, v_2, θ_1 and θ_2
(B) can move in a vertical line
(C) can move in a horizontal line
(D) will move in a straight line for any value of v_1, v_2, θ_1 and θ_2

Section (C) : Conservation of linear momentum

C-1. Two particles A and B initially at rest move towards each other under a mutual force of attraction. The speed of centre of mass at the instant when the speed of A is v and the speed of B is 2v is **[JEE - 1989]**

(A) v (B) Zero (C) 2 v (D) 3 v /2

C-2. If the KE of a particle becomes four times of its initial value, then the new momentum will be more than its initial momentum by

(A) 50% (B) 100% (C) 125% (D) 150%

C-3. A particle of mass 4m which is at rest explodes into three fragments. Two of the fragments each of mass m are found to move with a speed 'v' each in mutually perpendicular directions. The minimum energy released in the process of explosion is

(A) (2/3) mv^2 (B) (3/2) mv^2 (C) (4/3) mv^2 (D) (3/4) mv^2

C-4. A 500 kg boat has an initial speed of 10 ms^{-1} as it passes under a bridge. At that instant a 50 kg man jumps straight down into the boat from the bridge. The speed of the boat after the man and boat attaing a common speed is

(A) $\frac{100}{11}$ ms^{-1} (B) $\frac{10}{11}$ ms^{-1} (C) $\frac{50}{11}$ ms^{-1} (D) $\frac{5}{11}$ ms^{-1}

C-5. A man of mass 'm' climbs on a rope of length L suspended below a balloon of mass M. The balloon is stationary with respect to ground. If the man begins to climb up the rope at a speed v_{rel} (relative to rope). In what direction and with what speed (relative to ground) will the balloon move?

(A) downwards, $\frac{mv_{rel}}{m+M}$ (B) upwards, $\frac{Mv_{rel}}{m+M}$

(C) downwards, $\frac{mv_{rel}}{M}$ (D) downwards, $\frac{(M+m)v_{rel}}{M}$

C-6. A shell is fired from a canon with a velocity V at an angle θ with the horizontal direction. At the highest point in its path, it explodes into two pieces of equal masses. One of the pieces come to rest. The speed of the other piece immediately after the explosion is

(A) 3V cosθ (B) 2V cosθ (C) $\frac{3}{2}$ V cosθ (D) V cosθ

C-7. Two masses of 1g and 4g are moving with equal kinetic energy. The ratio of the magnitude of their linear momentum is - **[REE - 1989]**

(A) 1 : 1 (B) 1 : 2 (C) 1 : 3 (D) 1 : 4

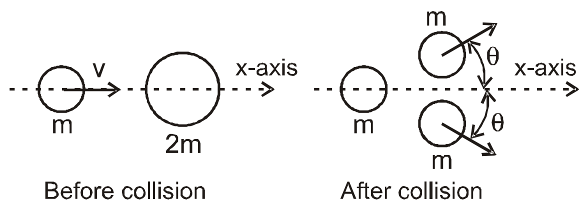
C-8. The spacecraft of mass M moves with velocity V in free space at first, then it explodes breaking into two pieces. If after explosion a piece of mass m comes to rest, the other piece of space craft will have a velocity:

(A) MV/(M – m) (B) MV/(M + m) (C) mV/(M – m) (D) mV/(M + m)

C-9. Two particles approach each other with different velocities. After collision, one of the particles has a momentum $\vec{p}$ in their center of mass frame. In the same frame, the momentum of the other particle is **[REE - 1998]**

(A) 0 (B) $-\vec{p}$ (C) $-\vec{p}/2$ (D) $-2\vec{p}$

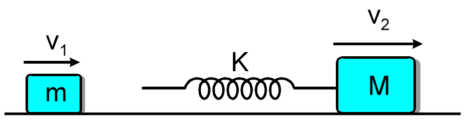
C-10. A particle of mass m is moving along the x-axis with speed v when it collides with a particle of mass 2m initially at rest. After the collision, the first particle has come to rest, and the second particle has split into two equal-mass pieces that are shown in the figure. Which of the following statements correctly describes the speeds of the two pieces ? ($\theta > 0$)



(A) Each piece moves with speed v.
(B) Each piece moves with speed v/2.
(C) One of the pieces moves with speed v/2, the other moves with speed greater than v/2
(D) Each piece moves with speed greater than v/2.

Section (D) : spring - mass system

D-1. Two blocks of masses m and M are moving with speeds v_1 and v_2 ($v_1 > v_2$) in the same direction on the frictionless surface respectively, M being ahead of m.

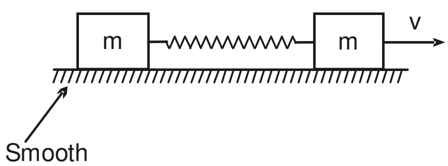


An ideal spring of force constant k is attached to the backside of M (as shown). The maximum compression of the spring when the block collides is :

(A) $v_1\sqrt{\frac{m}{k}}$ (B) $v_2\sqrt{\frac{M}{k}}$

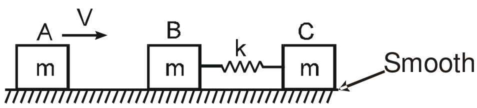
(C) $(v_1 - v_2)\sqrt{\frac{mM}{(M+m)K}}$ (D) None of above is correct.

D-2. Two masses are connected by a spring as shown in the figure. One of the masses was given velocity v = 2 k, as shown in figure where 'k' is the spring constant. Then maximum extension in the spring will be (initially spring is in natural length)



(A) 2 m (B) m (C) $\sqrt{2mk}$ (D) $\sqrt{3mk}$

D-3. Mass A hits B inelastically (e = 0) while moving horizontally with some velocity along the common line of centres of the three equal masses each of same mass. Initially mass B and C are stationary and the spring is unstretched. Then which is incorrect.



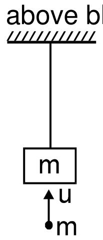
(A) compression will be maximum when blocks have same velocity
(B) velocity of C is maximum when (A + B) is at rest
(C) velocity of C is maximum when spring is undeformed.
(D) velocity of C is minimum when spring is undeformed.

Section (E) : impulse

E-1. A ball of mass 50 gm is dropped from a height h = 10 m. It rebounds losing 75 percent of its kinetic energy. If it remains in contact with the ground for Δt = 0.01 sec., the impulse of the impact force is : (take g = 10 m/s^2)
(A) 1.3 N–s (B) 1.06 N-s (C) 1300 N–s (D) 105 N–s

E-2. The area of F-t curve is A, where 'F' is the force acting on one mass due to the other. If one of the colliding bodies of mass M is at rest initially, its speed just after the collision is :
(A) A/M (B) M/A (C) AM (D) $\sqrt{\frac{2A}{M}}$

E-3. A bullet of mass m moving vertically upwards instantaneously with a velocity 'u' hits the hanging block of mass 'm' and gets embedded in it, as shown in the figure. The height through which the block rises after the collision. (Assume sufficient space above block) is :

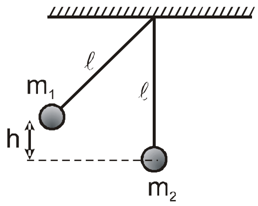


(A) $u^2/2g$ (B) u^2/g (C) $u^2/8g$ (D) $u^2/4g$

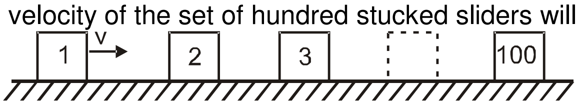
Section (F) : Collision

F-1. A bullet of mass m = 50 gm strikes ($\Delta t \approx 0$) a sand bag of mass M = 5 kg hanging from a fixed point, with a horizontal velocity $\vec{v}_p$. If bullet sticks to the sand bag then just after collision the ratio of final & initial kinetic energy of the bullet is :
(A) 10^{-2} (B) 10^{-3} (C) 10^{-6} (D) 10^{-4}

F-2. In the arrangement shown, the pendulum on the left is pulled aside. It is then released and allowed to collide with other pendulum which is at rest. A perfectly inelastic collision occurs and the system rises to a height h/4. The ratio of the masses (m_1 / m_2) of the pendulum is :
(A) 1 (B) 2 (C) 3 (D) 4



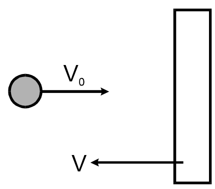
F-3. There are hundred identical sliders equally spaced on a frictionless track as shown in the figure. Initially all the sliders are at rest. Slider 1 is pushed with velocity v towards slider 2. In a collision the sliders stick together. The final velocity of the set of hundred stucked sliders will be :



(A) $\frac{v}{99}$ (B) $\frac{v}{100}$ (C) zero (D) v

F-4. A solid iron ball A of radius r collides head on with another stationary solid iron ball B of radius 2r. The ratio of their speeds just after the collision (e = 0.5) is :
(A) 3 (B) 4 (C) 2 (D) 1

F-5. A particle of mass m moves with velocity v_0 = 20 m/sec towards a large wall that is moving with velocity v = 5 m/sec. towards the particle as shown. If the particle collides with the wall elastically, the speed of the particle just after the collision is :



(A) 30 m/s (B) 20 m/s (C) 25 m/s (D) 22 m/s

F-6. Two perfectly elastic balls of same mass m are moving with velocities u_1 and u_2. They collide head on elastically n times. The kinetic energy of the system finally is :

(A) $\frac{1}{2}\frac{m}{n}u_1^2$ (B) $\frac{1}{2}\frac{m}{n}(u_1^2+u_2^2)$ (C) $\frac{1}{2}m(u_1^2+u_2^2)$ (D) $\frac{1}{2}mn(u_1^2+u_2^2)$

F-7. A massive ball moving with speed v collides head-on with a tiny ball at rest having a mass very less than the mass of the first ball. If the collision is elastic, then immediately after the impact, the second ball will move with a speed approximately equal to :

(A) v (B) 2v (C) v/2 (D) ∞

F-8. A sphere of mass m moving with a constant velocity hits another stationary sphere of the same mass. If e is the coefficient of restitution, then ratio of speed of the first sphere to the speed of the second sphere after head on collision will be :

(A) $\left(\frac{1-e}{1+e}\right)$ (B) $\left(\frac{1+e}{1-e}\right)$ (C) $\left(\frac{e+1}{e-1}\right)$ (D) $\left(\frac{e-1}{e+1}\right)$

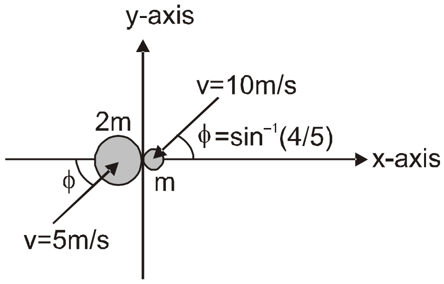
F-9. A ball of mass 'm', moving with uniform speed, collides elastically with another stationary ball. The incident ball will lose maximum kinetic energy when the mass of the stationary ball is **[REE - 1996]**

(A) m (B) 2m (C) 4m (D) infinity

F-10. Ball 1 collides head on with an another identical ball 2 at rest. Velocity of ball 2 after collision becomes two times to that of ball 1 after collision. The coefficient of restitution between the two balls is :

(A) e = 1/3 (B) e = 1/2 (C) e = 1/4 (D) e = 2/3

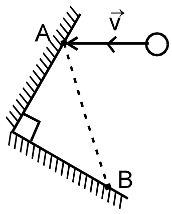
F-11. Two smooth spheres made of identical material having masses 'm' and 2m undergoes an oblique impact as shown in figure. The initial velocities of the masses are also shown. The impact force is along the line joining their centres along the x-axis. The coefficient of restitution is $\frac{5}{9}$. The velocities of the masses after the impact and the approximate percentage loss in kinetic energy.



(A) $\frac{10}{3}\hat{i}+8\hat{j}$; $\frac{5}{3}\hat{i}+4\hat{j}$, 15% (B) $\frac{5}{3}\hat{i}-8\hat{j}$; $\frac{-5}{3}\hat{i}+4\hat{j}$, 20%

(C) $\frac{10}{3}\hat{i}-8\hat{j}$; $\frac{-5}{3}\hat{i}+4\hat{j}$, 25% (D) $\frac{10}{3}\hat{i}-8\hat{j}$; $\frac{-5}{3}\hat{i}+4\hat{j}$, 20%

F-12. AB is an L shaped obstacle fixed on a horizontal smooth table. A ball strikes it at A, gets deflected and restrikes it at B. If the velocity vector before collision is $\vec{v}$ and coefficient of restitution of each collision is 'e', then the velocity of ball after its second collision at B is



(A) $e^2\vec{v}$ (B) $-e^2\vec{v}$ (C) $-e\vec{v}$ (D) data insufficient

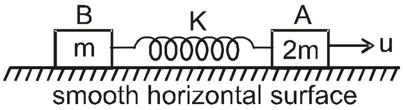
Section (G) : Variable mass

G-1. If the thrust force on a rocket which is ejecting gases with a relative velocity of 300 m/s, is 210 N. Then the rate of combustion of the fuel will be :

(A) 10.7 kg/sec (B) 0.07 kg/sec (C) 1.4 kg/sec (D) 0.7 kg/sec

PART - III : MATCH THE COLUMN

1. Two blocks A and B of mass 2m and m respectively are connected by a massless spring of spring constant K. This system lies over a smooth horizontal surface. At t = 0 the block A has velocity u towards right as shown while the speed of block B is zero, and the length of spring is equal to its natural length at that instant. In each situation of column I, certain statements are given and corresponding results are given in column II. Match the statements in column I corresponding to results in column II.



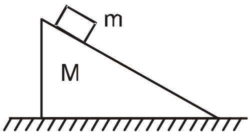
Column I	Column II
(A) The velocity of block A	(p) can never be zero
(B) The velocity of block B	(q) may be zero at certain instants of time
(C) The kinetic energy of system of two blocks	(r) is minimum at maximum compression of spring
(D) The potential energy of spring	(s) is maximum at maximum extension of spring

2. In each situation of column-I, a system involving two bodies is given. All strings and pulleys are light and friction is absent everywhere. Initially each body of every system is at rest. Consider the system in all situation of column I from rest till any collision occurs. Then match the statements in column-I with the corresponding results in column-II

Column-I | **Column-II**

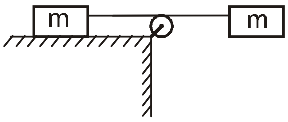
(A) The block plus wedge system is placed over smooth horizontal surface. After the system is released from rest, the centre of mass of system.

(p) Shifts towards right



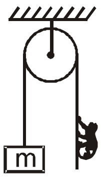
(B) The string connecting both the blocks of mass m is horizontal. Left block is placed over smooth horizontal table as shown. After the two block system is released from rest, the centre of mass of system

(q) Shifts downwards



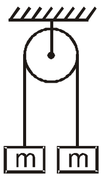
(C) The block and monkey have same mass. The monkey starts climbing up the rope. After the monkey starts climbing up, the centre of mass of monkey + block system.

(r) Shifts upwards



(D) Both block of mass m are initially at rest. The left block is given initial velocity u downwards. Then, the centre of mass of two block system afterwards.

(s) Does not shift



(t) Shifts towards left

Exercise-2

✍ **Marked Questions can be used as Revision Questions.**

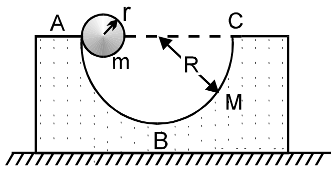
PART - I : ONLY ONE OPTION CORRECT TYPE

1. A uniform sphere is placed on a smooth horizontal surface and a horizontal force F is applied on it at a distance h above the centre. The acceleration of the centre of mass of the sphere
(A) is maximum when h = 0
(B) is maximum when h = R
(C) is maximum when h = R/2
(D) is independent of h

2. A ball moves horizontally in a closed box making several collisions with the walls. The box is kept on a smooth horizontal surface. During the motion of the ball, the velocity of the centre of mass :
(A) of the box remains constant
(B) of the box plus the ball system remains constant
(C) depends on value of e
(D) of the ball relative to the box remains constant

3. A ring of mass m and a particle of same mass are fixed on a disc of same mass such that centre of mass of the system lies at centre of the disc. The system rotates such that centre of mass of the disc moves in a circle of radius R with a constant angular velocity ω. From this we conclude that
(A) An external force $m\omega^2R$ must be applied to central particle
(B) An external force $m\omega^2R$ must be applied to the ring
(C) An external force $3m\omega^2R$ must be applied to central particle
(D) An external force $3m\omega^2R$ must be applied any where on the system

4.✍ A block of mass M with a semicircular track of radius R rests on a horizontal frictionless surface. A uniform cylinder of radius r and mass m is released from rest from the top point A. The cylinder slips on the semicircular frictionless track. The distance travelled by the block when the cylinder reaches the point B is :

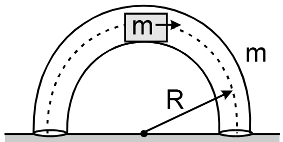


(A) $\frac{M(R-r)}{M+m}$ (B) $\frac{m(R-r)}{M+m}$ (C) $\frac{(M+m)R}{M}$ (D) none

5.✍ In the above question, the velocity of the block when the cylinder reaches point (B) is : **[JEE - 1983]**
(A) $M\sqrt{\frac{2g(R-r)}{M(M+m)}}$ (B) $m\sqrt{\frac{2g(R-r)}{m(M+m)}}$ (C) $m\sqrt{\frac{2g(R-r)}{M(M+m)}}$ (D) $M\sqrt{\frac{2g(R+r)}{M(M+m)}}$

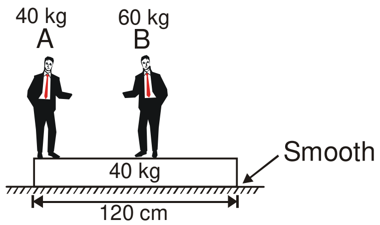
6.✍ A uniform thin rod of mass M and Length L is standing vertically along the y-axis on a smooth horizontal surface, with its lower end at the origin (0, 0). A slight disturbance at t = 0 causes the lower end to slip on the smooth surface along the positive x-axis, and the rod starts falling. The acceleration vector of centre of mass of the rod during its fall is : [$\vec{R}$ is reaction from surface] **[JEE - 1993]**
(A) $\vec{a}_{CM} = \frac{M\vec{g}+\vec{R}}{M}$ (B) $\vec{a}_{CM} = \frac{M\vec{g}-\vec{R}}{M}$ (C) $\vec{a}_{CM} = M\vec{g}-\vec{R}$ (D) None of these

7. In a vertical plane inside a smooth hollow thin tube a block of same mass as that of tube is released as shown in figure. When it is slightly disturbed it moves towards right. By the time the block reaches the right end of the tube, displacement of the tube will be (where 'R' is mean radius of tube). Assume that the tube remains in vertical plane.



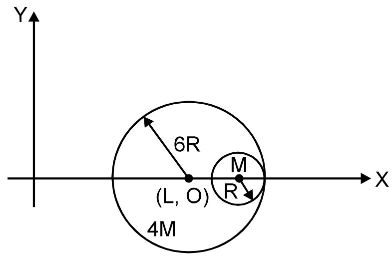
(A) $\frac{2R}{\pi}$ (B) $\frac{4R}{\pi}$ (C) $\frac{R}{2}$ (D) R

8. Two men 'A' and 'B' are standing on a plank. 'B' is at the middle of the plank and 'A' is at the left end of the plank. Bottom surface of the plank is smooth. System is initially at rest and masses are as shown in figure. 'A' and 'B' start moving such that the position of 'B' remains fixed with respect to ground and 'A' meets 'B'. Then the point where A meets B is located at :



(A) the middle of the plank (B) 30 cm from the left end of the plank
(C) the right end of the plank (D) None of these

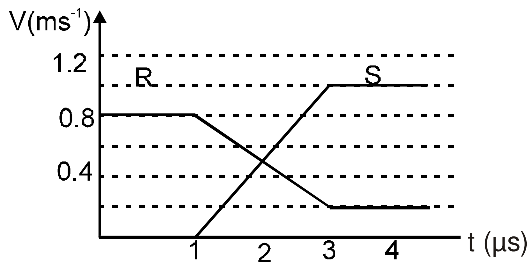
9. A small sphere of radius R is held against the inner surface of a larger sphere of radius 6R. The masses of large and small spheres are 4M and M respectively. This arrangement is placed on a horizontal table as shown. There is no friction between any surfaces of contact. The small sphere is now released. The coordinates of the centre of the large sphere when the smaller sphere reaches the other extreme position is: **[JEE - 1996]**



(A) (L – 2R, 0) (B) (L + 2R, 0) (C) (2R, 0) (D) (2R – L, 0)

10. An isolated particle of mass m is moving in a horizontal (x – y) plane along the x-axis, at a certain height above the ground. It suddenly explodes into two fragments of masses $\frac{m}{4}$ & $\frac{3m}{4}$. An instant later, the smaller fragment is at y = +15 cm. The larger fragment at this instant is at – **[JEE- 1997, 1]**
(A) y = –5 cm (B) y = +20 cm (C) y = +5 cm (D) y = –20 cm

11. The diagram shows the velocity - time graph for two masses R and S that collided head on elastically. Which of the following statements is true?

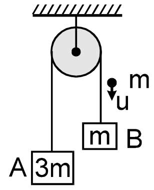


I. R and S moved in the same direction after the collision.
II. The velocities of R and S were equal at the mid time of the collision.
III. The mass of R was greater than mass of S.
(A) I only (B) II only (C) I and II only (D) I, II and III

12. A stationary body explodes into two fragments of masses m_1 and m_2. If momentum of one fragment is p, the minimum energy of explosion is
(A) $\frac{p^2}{2(m_1+m_2)}$ (B) $\frac{p^2}{2\sqrt{m_1m_2}}$ (C) $\frac{p^2(m_1+m_2)}{2m_1m_2}$ (D) $\frac{p^2}{2(m_1-m_2)}$

13. A ball of mass = 100 gm is released from a height h_1 = 2.5 m from the ground level and then rebounds to a height h_2 = 0.625 m. The time of contact of the ball and the ground is Δt = 0.01 sec. The impulsive (impact) force offered by the ball on the ground is :
(A) 105 N (B) 1.05 N (C) 2.08 N (D) 208 N

14. A system of two blocks A and B are connected by an inextensible massless strings as shown. The pulley is massless and frictionless. Initially the system is at rest when, a bullet of mass 'm' moving with a velocity 'u' as shown hits the block 'B' and gets embedded into it. The impulse imparted by tension force to the block of mass 3m is :



(A) $\frac{5mu}{4}$ (B) $\frac{4mu}{5}$ (C) $\frac{2mu}{5}$ (D) $\frac{3mu}{5}$

15. A stationary body explodes into four identical fragments such that three of them fly off mutually perpendicular to each other, each with same K.E., E_0. The minimum energy of explosion will be :

(A) $6E_0$ (B) $\frac{4E_0}{3}$ (C) $4E_0$ (D) $8E_0$

16. A super-ball is to bounce elastically back and forth between two rigid walls at a distance d from each other. Neglecting gravity and assuming the velocity of super-ball to be v_0 horizontally, the average force (in large time interval) being exerted by the super-ball on one wall is :

(A) $\frac{1}{2}\frac{mv_0^2}{d}$ (B) $\frac{mv_0^2}{d}$ (C) $\frac{2mv_0^2}{d}$ (D) $\frac{4mv_0^2}{d}$

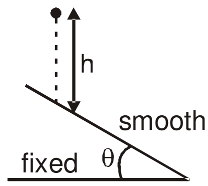
17. A ball is bouncing down a set of stairs. The coefficient of restitution is e. The height of each step is d and the ball bounces one step at each bounce. After each bounce the ball rebounds to a height h above the next lower step. Neglect width of each step in comparison to h and assume the impacts to be effectively head on. Which of the following relation is correct ? (Given that h > d)

(A) $\frac{h}{d} = 1 - e^2$ (B) $\frac{h}{d} = 1 - e$ (C) $\frac{h}{d} = \frac{1}{1-e^2}$ (D) $\frac{h}{d} = \frac{1}{1-e}$

18. A ball collides with a smooth and fixed inclined plane of inclination θ after falling vertically through a distance h. If it moves horizontally just after the impact, the coefficient of restitution is :

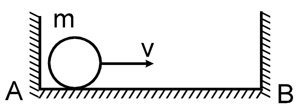
(A) $\tan^2\theta$ (B) $\cot^2\theta$ (C) $\tan\theta$ (D) $\cot\theta$

19. A ball of mass m strikes the fixed inclined plane after falling through a height h. If it rebounds elastically, the impulse on the ball is :



(A) $2m\cos\theta\sqrt{2gh}$ (B) $2m\cos\theta\sqrt{gh}$ (C) $\frac{2m\sqrt{2gh}}{\cos\theta}$ (D) $2m\sqrt{2gh}$

20. A small ball on a frictionless horizontal surface moves towards right with a velocity V. It collides with the wall and returns back and continues to and fro motion. If the average speed for first to and fro motion of the ball is $\left(\frac{2}{3}\right)$V, then the coefficient of restitution of impact is :



(A) 0.5 (B) 0.8 (C) 0.25 (D) 0.75

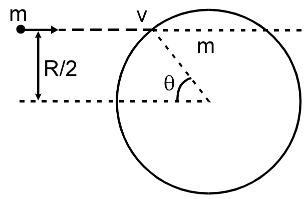
21. A sphere of mass m_1 = 2kg collides with a sphere of mass m_2 = 3kg which is at rest. Mass m_1 will move at right angle to the line joining centres at the time of collision, if the coefficient of restitution is :

(A) $\frac{4}{9}$ (B) $\frac{1}{2}$ (C) $\frac{2}{3}$ (D) $\sqrt{\frac{2}{3}}$

22. Two identical billiard balls are in contact on a smooth table. A third identical ball strikes them symmetrically and comes to rest after impact. The coefficient of restitution is :

(A) $\frac{2}{3}$ (B) $\frac{1}{3}$ (C) $\frac{1}{6}$ (D) $\frac{\sqrt{3}}{2}$

23. A particle of mass m strikes elastically with a disc of radius R, with a velocity $\vec{v}$ as shown in the figure. If the mass of the disc is equal to that of the particle and the surface of the contact is smooth, then the velocity of the disc just after the collision is :



(A) $\frac{2v}{3}$ (B) $\frac{v}{2}$ (C) $\frac{\sqrt{3}v}{2}$ (D) v

24. A wagon filled with sand has a hole so that sand leaks through the bottom at a constant rate λ. An external force $\vec{F}$ acts on the wagon in the direction of motion. Assuming instantaneous velocity of the wagon to be $\vec{v}$ and initial mass of system to be m_0, the force equation governing the motion of the wagon is :

(A) $\vec{F} = m_0 \frac{d\vec{v}}{dt} + \lambda \vec{v}$ (B) $\vec{F} = m_0 \frac{d\vec{v}}{dt} - \lambda \vec{v}$ (C) $\vec{F} = (m_0 - \lambda t) \frac{d\vec{v}}{dt}$ (D) $\vec{F} = (m_0 - \lambda t) \frac{d\vec{v}}{dt} + \lambda \vec{v}$

25. A balloon having mass ' m ' is filled with gas and is held in hands of a boy. Then suddenly it get released and gas starts coming out of it with a constant rate. The velocity of the ejected gases is also constant 2 m/s with respect to the balloon. Find out the velocity of the balloon when the mass of gas is reduced to half. (Neglect gravity & Bouyant force),

(A) ℓn 2 (B) 2 ℓn 4 (C) 2 ℓn 2 (D) none of these

26. A shell of mass 100 kg is fired horizontally from a cannon having a mass of 5 × 10^3 kg. The kinetic energy of the shell at the end of the barrel is 7.5 × 10^6 J. The kinetic energy imparted to the cannon due to recoil is :

(A) 2 × 10^5 J (B) 7.5 × 10^6 J (C) 1.5 × 10^5 J (D) 10^5 J

27. A skater of mass m standing on ice throws a stone of mass M with a velocity v in a horizontal direction. The distance, skater will move back is (the coefficient of friction between the skater and the ice is μ) :

(A) $\frac{M^2v^2}{2m\mu g}$ (B) $\frac{Mv^2}{2m^2\mu g}$ (C) $\frac{M^2v^2}{2m^2\mu g}$ (D) $\frac{M^2v^2}{2m^2\mu^2 g}$

28. A ball of mass m hits directly another ball of mass M at rest and is brought to rest by the impact. One third of the kinetic energy of the ball is lost due to collision. The coefficient of restitutions is **[Olympiad (Stage-1) 2017]**

(A) 1/3 (B) ½ (C) 2/3 (D) $\sqrt{\frac{2}{3}}$

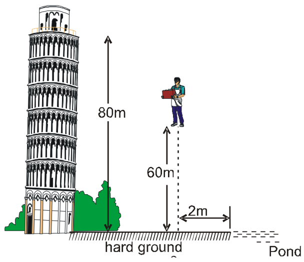
29. Two particles A and B of equal masses have velocities $\vec{V}_A = 2\hat{i} + \hat{j}$ and $\vec{V}_B = -\hat{i} + 2\hat{j}$. The particles move with accelerations $\vec{a}_A = -4\hat{i} - \hat{j}$ and $\vec{a}_B = -2\hat{i} + 3\hat{j}$ respectively. The centre of mass of the two particles move along **[Olympiad (Stage-1) 2017]**

(A) a straight line (B) a parabola (C) a circle (D) an ellipse

PART - II : SINGLE AND DOUBLE VALUE INTEGER TYPE

1. A train of mass M = πkg is moving on a circular track of radius ' R ' with constant speed V = 2m/s. The length of the train is half of the perimeter of the track. The linear momentum of the trian will be

2. A man of mass 60 kg start falling from building of height 80 m with a bag of 2 kg in his hand. After falling through a distance 20 m he throw the bag horizontally with respect to him so that he fall in a pond 2m away from the vertical lines of fall. Calculate horizontal distance of bag from the vertical line of fall where it lands. (take g = 10 m/s^2)

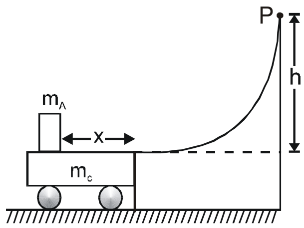


3. A railway flat car has an artillery gun installed on it and contains shells for firing. The combined system excluding shell has a mass M and moves with a velocity v_0. The barrel of the gun makes an angle α = 60° with the horizontal. A shell of mass m leaves the barrel at a speed u = 40m/s relative to initial state of barrel in the forward direction. The speed of the flat car so that it may stop after the firing is: (neglect friction) Given : M = 10m

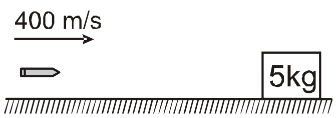
4. A car with a gun mounted on it is kept on horizontal friction less surface. Total mass of car, gun and shell is 50 kg. Mass of each shell is 1 kg. If shell is fired horizontally with relative velocity 100 m/sec with respect to gun. What is the recoil speed of car after second shot in nearest integer ?

5. A large stone of mass $\frac{M_e}{2}$ is released when centre of mass of the stone is at a height h (h <<R_e). Find speed of stone when it is at a height of $\frac{h}{2}$. M_e and R_e are mass and radius of earth. Given h = $\frac{3}{20}$ m.

6. A block A having a mass m_A = 3kg is released from rest at the position P shown and slides freely down the smooth fixed inclined ramp. When it reaches the bottom of the ramp it slides horizontally onto the surface of a cart of mass m_c = 2kg for which the coefficient of friction between the cart and the block is $\mu = \frac{2}{5}$. If h = 6m be the initial height of A, determine the position 'x' (with respect to cart) of the box on the cart after it comes to rest relative to cart. (The cart moves on smooth horizontal surface.)

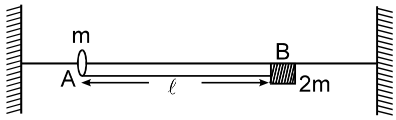


7. A bullet fired horizontally with a speed of 400 m/sec. It strikes a wooden block of mass 5 kg initially at rest placed on a horizontal floor as shown in the figure. It emerges with a speed of 200 m/sec and the block slides a distance 20 cm before coming to rest. If the coefficient of friction between block and the surface is λ/50 then find λ. Mass of bullet is 20 gm. (Take g = 10 m/s^2)

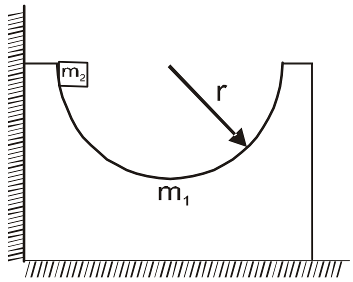


8. A bullet fired horizontally with a speed of 400 m/sec. It strikes a wooden block of mass 2 kg hanging vertically with the help of long string. After striking with bullet, block rises a height of 20 cm. If speed with which bullet emerges out from block is 10 λ then find λ. Mass of bullet is 20 gm. (take g = 10 m/s^2)

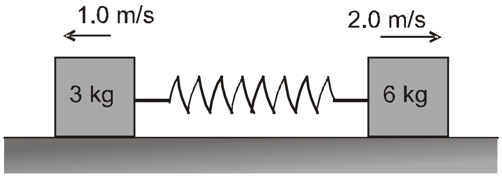
9. A small ring A of mass m = $\frac{1}{2}$ kg is attached at an end of a light string the other end of which is tied to a block B of mass 2m. The ring is free to move on a fixed smooth horizontal rod as shown. Find tension in the string when it becomes vertical.



10. A symmetric block of mass m_1 = 1kg with a groove of hemispherical shape of radius r = 5m rests on a smooth horizontal surface in contact with the wall as shown in the figure. A small block of mass m_2 = 1kg slides without friction from the initial position. Find the maximum velocity of the block m_1.

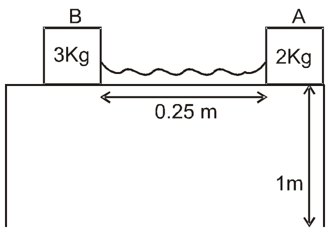


11. Two blocks of mass 3 kg and 6 kg respectively are placed on a smooth horizontal surface. They are connected by a light spring of force constant k = 200 N/m. Initially the spring is unstretched and the indicated velocities are imparted to the blocks. Find maximum extension of the spring in cm ?

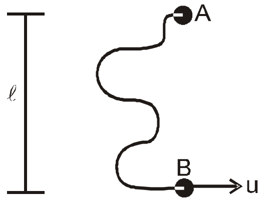


12. Two blocks of masses 10kg and 4kg are connected by a spring of negligible mass and are placed on a frictionless horizontal surface. An impulse gives a speed of 14 ms^{-1} to the heavier block in the direction of the lighter block. Then, find velocity of the centre of mass ? **[JEE 2002 Scr., 2/105]**

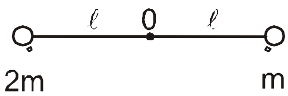
13. A particle A of mass 2kg lies on the edge of a table of height 1m. It is connected by a light inelastic string of length 0.7m to a second particle B of mass 3kg which is lying on the table 0.25m from the edge (line joining A & B is perpendicular to the edge). If A is pushed gently so that it starts falling from table. After some time string becomes tight. If the impulse of the tension in the string at that moment is 3λ/5 then find λ. Assume all contacts are smooth. g = 10 m/s^2



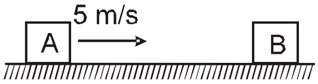
14. Two particles A and B each of mass m are attached by a light inextensible string of length 2ℓ. The whole system lies on a smooth horizontal table with B initially at a distance ℓ from A. The particle at end B is projected across the table with speed u = $4\sqrt{3}$ m/s perpendicular to AB. Find velocity of ball A just after the jerk ?



15. Two balls having masses m and 2m are fastened to two light strings of same length = 1m. The other ends of the strings are fixed at O. Both the balls are moved away such that strings become horizontal and ball are on different sides of the fixed point as shown in the figure.
Now both the balls are released and they collide elastically. Find height raised by ball of mass m after collision (Assume string does not break).



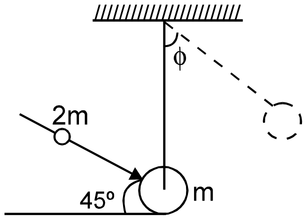
16. Two blocks of equal masses are placed on a horizontal surface. The surface of A is smooth but that of B has a friction coefficient of 0.2 with the floor. Block A is given a speed of 5 m/s, towards B which is kept at rest.



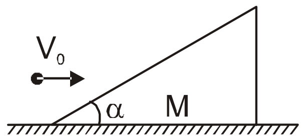
If the distance travelled by B is $\frac{5}{4}\lambda$ then find λ if the collision is perfectly elastic. Take g = 10 m/s^2.

17. A block of mass 2 kg moving at 2.0 m/s collides head on with another block of equal mass kept at rest. If the loss in kinetic energy of system is half of the maximum possible loss of kinetic energy of system. If the coefficient of restitution $\frac{\lambda}{\sqrt{2}}$. Then find λ.

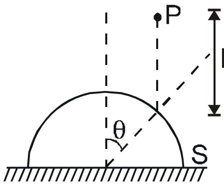
18. A ball of mass 'm' is suspended by a massless string of length 'ℓ' from a fixed point. A ball of mass 2m strikes in the direction of θ = 45º from horizontal & sticks to it. If the initial velocity of 2m is $x\sqrt{g\ell}$ so that system deflects by $\phi = \frac{\pi}{2}$ and if string is cut at ϕ = 60º, and the velocity at highest point of trajectory is $\frac{\sqrt{g\ell}}{y}$, then find x + y :



19. A wedge (free to move) of mass 'M' has one face making an angle α with horizontal and is resting on a smooth rigid floor. A particle of mass 'm' hits the inclined face of the wedge with a horizontal velocity v_0. It is observed that the particle rebounds in vertical direction after impact. Neglect friction between particle and the wedge & take M = 2m, v_0 = 10m/s, tanα = 2, g = 10m/s^2. Assume that the inclined face of the wedge is sufficiently long so that the particle hits the same face once more during its downward motion. Calculate the time elapsed between the two impacts.



20. A hemisphere S and a particle P are of same mass m = $\sqrt{2}$ kg. P is dropped from a height 'h'. S is kept on a smooth horizontal surface. The friction between P and S is also absent. P collides elastically with S at the point shown in the figure. After collision the velocity of the particle becomes horizontal. Find ratio of impulse of ground on hemisphere to speed of hemisphere after collision ?

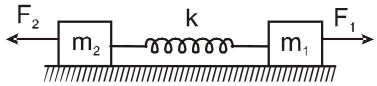


21. A particle of mass 2m is projected at an angle of 45º with horizontal with a velocity of $20\sqrt{2}$ m/s. After 1 s explosion takes place and the particle is broken into two equal pieces. As a result of explosion one part comes to rest. Find the maximum height attained from the ground by the other part. Take g = 10 m/s^2.

22. A projectile of mass m is fired with a speed v = 20 m/s at an angle θ = 45° from a smooth horizontal field. The coefficient of restitution of collision between the projectile and the field is e = 1/2. How far from the starting point, does the projectile make its third collision with the field?

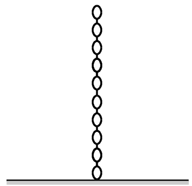
23. A body of mass 5kg moves along the x-axis with a velocity 2m/s. A second body of mass 10kg moves along the y-axis with a velocity of $\sqrt{3}$ m/s. They collide at the origin and stick together. If the amount of heat liberated in the collision is $\frac{\lambda}{3}$ then find λ? **[REE - 1996]**

24. Two blocks initially at rest having masses m_1 and m_2 are connected by spring of spring constant $k = 2/3$ N/m (as shown in the figure). The block of mass m_1 is pulled by a constant force $F_1 = 4N$ and the other block is pulled by a constant force $F_2 = 2N$ Find the maximum elongation of the spring (the spring is initially relaxed) Assuming $m_2 = 2m_1$

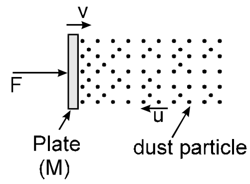


25. A ball released from rest collide elastically to a fixed inclined plane of inclination $\alpha = 30°$ after falling through a height h = 3m. Find the distance between the points along the incline where it strike the incline.

26. A uniform chain of mass m = 1kg and length ℓ = 1m hangs on a thread and touches the surface of a table by its lower end. Find the force exerted by the table on the chain when x = 0.5m length of chain has fallen on the table. The fallen part does not form heap.



27. A plate of mass M is moved with constant velocity v = 40m/s against powder of dust particles moving with constant velocity u = 40m/s in opposite direction stick to plate as shown. The density of the dust is $\rho = 10^{-3}$ kg/m^3 and plate area is A = 10m^2 Find the force F required to keep the plate moving uniformly.



PART - III : ONE OR MORE THAN ONE OPTIONS CORRECT TYPE

1. A system of particles has its centre of mass at the origin. The x-coordinates of all the particles
(A) may be positive
(B) may be negative
(C) may be non-negative
(D) may be non-positive

2. In which of the following cases the centre of mass of a system is certainly not at its centre ?
(A) A rod whose density continuously increases from left to right
(B) A rod whose density continuously decreases from left to right
(C) A rod whose density decreases from left to right upto the centre and then increases
(D) A rod whose density increases from left to right upto the centre and then decreases

3. If the net external force acting on a system is zero, then the centre of mass
(A) must not move
(B) must not accelerate
(C) may move
(D) may accelerate

4. In an elastic collision in absence of external force, which of the following is/are correct : **[REE - 1995]**
(A) The linear momentum is conserved
(B) The potential energy is conserved in collision
(C) The final kinetic energy is less than the initial kinetic energy
(D) The final kinetic energy is equal to the initial kinetic energy

5. A small ball collides with a heavy ball initially at rest. In the absence of any external impulsive force, it is possible that
(A) Both the balls come to rest
(B) Both the balls move after collision
(C) The moving ball comes to rest and the stationary ball starts moving
(D) The stationary ball remains stationary, the moving ball changes its velocity.

6. A block moving in air explodes in two parts then just after explosion (neglect change in momentum duet to gravity)
(A) The total momentum of two parts must be equal to the momentum of the block before explosion.
(B) The total kinetic energy of two parts must be equal as that of block before explosion.
(C) The total momentum must change
(D) The total kinetic energy must increase

7. Two bodies of same mass collide head on elastically then
(A) Their velocities are interchanged
(B) Their speeds are interchanged
(C) Their momenta are interchanged
(D) The faster body slows down and the slower body speeds up.

8. External force $\vec{F}(\vec{F} \neq 0)$ acts on a system of particles. The velocity and the acceleration of the centre of mass are found to be v_{cm} and a_{cm} at an instant, then it is possible that
(A) $v_{cm} = 0$, $a_{cm} = 0$ (B) $v_{cm} = 0$, $a_{cm} \neq 0$ (C) $v_{cm} \neq 0$, $a_{cm} = 0$ (D) $v_{cm} \neq 0$, $a_{cm} \neq 0$

9. A bag of mass M hangs by a long thread and a bullet (mass m) comes horizontally with velocity v and gets caught in the bag. Then for the combined system (bag + bullet) :
(A) Momentum is mMv/(M + m) (B) KE is (1/2) Mv^2
(C) Momentum is mv (D) KE is $m^2v^2/2(M + m)$

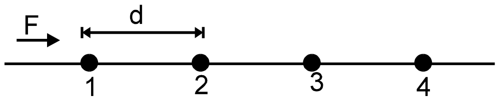
10. A set of n-identical cubical blocks lie at rest along a line on a smooth horizontal surface. The separation between any two adjacent blocks is L. The block at one end is given a speed V towards the next one at time t = 0. All collisions are completely inelastic, then **[JEE - 1995]**
(A) The last block starts moving at $t = n(n-1)\frac{L}{2V}$
(B) The last block starts moving at $t = (n-1)\frac{L}{V}$
(C) The centre of mass of the system will have a final speed v/n
(D) The centre of mass of the system will have a final speed v

11. A particle strikes a horizontal smooth floor with a velocity u making an angle θ with the floor and rebounds with velocity v making an angle ϕ with the floor. If the coefficient of restitution between the particle and the floor is e, then :
(A) the impulse delivered by the floor to the body is $mu(1 + e)\sin\theta$.
(B) $\tan\phi = e\tan\theta$.
(C) $v = u\sqrt{1-(1-e^2)\sin^2\theta}$
(D) the ratio of the final kinetic energy to the initial kinetic energy is $(\cos^2\theta + e^2\sin^2\theta)$

12. A ball of mass m moving with a velocity v hits a massive wall moving towards the ball with a velocity u. An elastic impact lasts for a time Δt.
(A) The average elastic force acting on the ball is $\frac{m(u+v)}{\Delta t}$
(B) The average elastic force acting on the ball is $\frac{2m(u+v)}{\Delta t}$
(C) The kinetic energy of the ball increases by 2mu (u + v)
(D) The kinetic energy of the ball remains the same after the collision.

13. Two blocks A and B each of mass 'm' are connected by a massless spring of natural length L and spring constant k. The blocks are initially resting on a smooth horizontal plane. A third block C also of mass m moves on the plane with a speed 'v' along the line joining A and B and collides elastically with A then which of the following is/are correct : **[JEE - 1993]**
(A) K E of the AB system at maximum compression of the spring is zero
(B) The KE of AB system at maximum compression is (1/4) mv^2
(C) The maximum compression of spring is $v\sqrt{m/k}$
(D) The maximum compression of spring is $v\sqrt{m/2k}$

14. The figure shows a string of equally spaced beads of mass m, separated by distance d. The beads are free to slide without friction on a thin wire.



A constant force F acts on the first bead initially at rest till it makes collision with the second bead. The second bead then collides with the third and so on. Suppose all collisions are elastic, then :

(A) speed of the first bead immediately before and immediately after its collision with the second bead is $\sqrt{\frac{2Fd}{m}}$ and zero respectively.

(B) speed of the first bead immediately before and immediately after its collision with the second bead is $\sqrt{\frac{2Fd}{m}}$ and $\frac{1}{2}\sqrt{\frac{2Fd}{m}}$ respectively.

(C) speed of the second bead immediately after its collision with third bead is zero.

(D) the average speed of the first bead is $\frac{1}{2}\sqrt{\frac{2Fd}{m}}$.

15. A shell explodes in a region of negligible gravitational field, giving out n fragments of equal mass m. Then its total **[REE - 1997]**

(A) Kinetic energy is smaller than that before the explosion

(B) Kinetic energy is greater than that before the explosion

(C) Momentum and kinetic energy depend on n

(D) Momentum is equal to that before the explosion.

16. A man of mass m is at rest on a stationary flat car. The car can move without friction along horizontal rails. The man starts walking with velocity v relative to the car, work done by him

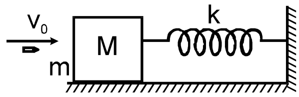
(A) is less than $\frac{1}{2}$ mv^2 if he walks along the rails

(B) is equal to $\frac{1}{2}$ mv^2 if he walks normal to the rails

(C) can never be less than $\frac{1}{2}mv^2$

(D) is greater than $\frac{1}{2}mv^2$ if he walks along the rails

17. A bullet of mass m = 1kg strikes a block of mass M = 2kg connected to a light spring of stiffness k = 3N/m with a speed V_0 = 3m/s. If the bullet gets embedded in the block then.



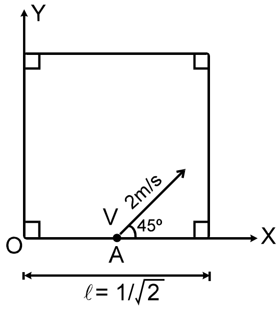
(A) linear momentum of bullet and block system is not conserve during impact because spring force is impulsive.

(B) linear momentum of bullet and block system isconserve during impact because spring force is nonimpulsive.

(C) Maximum compression in the spring is 2m.

(D) The maximum compression in the spring is 1m.

18. A striker is shot from a square carrom board from a point A exactly at midpoint of one of the walls with a speed 2 m/sec at an angle of 45° with the x-axis as shown. The collisions of the striker with the walls of the fixed carrom are perfectly elastic. The coefficient of kinetic friction between the striker and board is 0.2.



(A) x coordinate of the striker when it stops (taking point O to be the origin and neglect friction between wall and striker) is $\frac{1}{2\sqrt{2}}$.

(B) y coordinate of the striker when it stops (taking point O to be the origin and neglect friction between wall and striker) is $\frac{1}{\sqrt{2}}$.

(C) x coordinate of the striker when it stops (taking point O to be the origin and neglect friction between wall and striker) is $\frac{1}{\sqrt{2}}$.

(D) y coordinate of the striker when it stops (taking point O to be the origin and neglect friction between wall and striker) is $\frac{1}{2\sqrt{2}}$.

19. Three identical balls of mass m and radius R are placed on frictionless horizontal x-y plane. Ball A at (0, 0), Ball B at ($\sqrt{2}$ R, – $\sqrt{2}$ R). Ball A is suddenly given an impulse $\vec{P} = \sqrt{2}mV\hat{i}$. If collision between balls A and B is perfectly elastic while between B and C is head on and perfectly inelastic, then.

(A) The y componant of relative velocity of ball A with respect to ball C after a long time is $\frac{3v}{2\sqrt{2}}$.

(B) The y componant of relative velocity of ball A with respect to ball C after a long time is $\frac{v}{2\sqrt{2}}$.

(C) The x componant of relative velocity of ball A with respect to ball C after a long time is $\frac{v}{2\sqrt{2}}$.

(D) The x componant of relative velocity of ball A with respect to ball C after a long time is $\frac{3v}{2\sqrt{2}}$.

20. A projectile is thrown horizontally from top of a tower of height 20 m with a velocity 10 m/sec. It strikes the smooth ground whose co-efficient of restitution is 0.5. [g = 10 m/s^2] (Neglect friction):

(A) The time elapsed (in seconds) after projection when it strikes the ground 1st time will be 4sec.

(B) The time elapsed (in seconds) after projection when it strikes the ground 1st time will be 2sec.

(C) The time elapsed (in seconds) after projection when it strikes the ground 2nd time will be 3sec.

(D) The time elapsed (in seconds) after projection when it strikes the ground 2nd time will be 4sec.

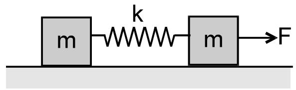
21. Two balls A and B moving in the same direction collide. The mass of B is p times that of A. Before the collision the velocity of A was q times that of B. After the collision A comes to rest. If e be the coefficient of restitution then which of the following conclusion/s is/are correct ? **[Olympiad (Stage-1) 2017]**

(A) $e = \frac{p+q}{pq-p}$ (B) $e = \frac{p+q}{pq+q}$ (C) $p \geq \frac{q}{q-2}$ (D) $p \geq 1$

PART - IV : COMPREHENSION

Comprehension-1

Two blocks of equal mass m are connected by an unstretched spring and the system is kept at rest on a frictionless horizontal surface. A constant force F is applied on the first block pulling it away from the other as shown in figure. (Initially system is at rest)



1. Then the displacement of the centre of mass in time t is

(A) $\frac{Ft^2}{2m}$ (B) $\frac{Ft^2}{3m}$ (C) $\frac{Ft^2}{4m}$ (D) $\frac{Ft^2}{m}$

2. If the extension of the spring is x_0 at time t, then the displacement of the right block at this instant is :

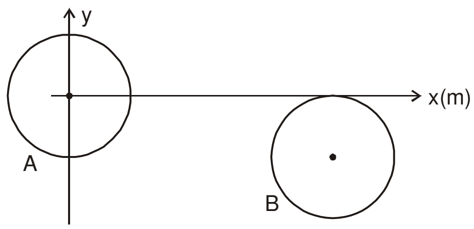
(A) $\frac{1}{2}\left(\frac{Ft^2}{2m}+x_0\right)$ (B) $-\frac{1}{2}\left(\frac{Ft^2}{2m}+x_0\right)$ (C) $\frac{1}{2}\left(\frac{Ft^2}{2m}-x_0\right)$ (D) $\left(\frac{Ft^2}{2m}+x_0\right)$

3. If the extension of the spring is x_0 at time t, then the displacement of the left block at this instant is :

(A) $\left(\frac{Ft^2}{2m}-x_0\right)$ (B) $\frac{1}{2}\left(\frac{Ft^2}{2m}+x_0\right)$ (C) $\frac{1}{2}\left(\frac{2Ft^2}{m}-x_0\right)$ (D) $\frac{1}{2}\left(\frac{Ft^2}{2m}-x_0\right)$

Comprehension-2

Two smooth balls A and B, each of mass m and radius R, have their centres at (0, 0, R) and at (5R, – R, R) respectively, in a coordinate system as shown. Ball A, moving along positive x axis, collides with ball B. Just before the collision, speed of ball A is 4 m/s and ball B is stationary. The collision between the balls is elastic.



4. Velocity of the ball A just after the collision is :

(A) (i + $\sqrt{3}$ j) m/s (B) (i – $\sqrt{3}$ j) m/s (C) (2i + $\sqrt{3}$ j) m/s (D) (2i + 2j) m/s

5. Impulse of the force exerted by A on B during the collision, is equal to

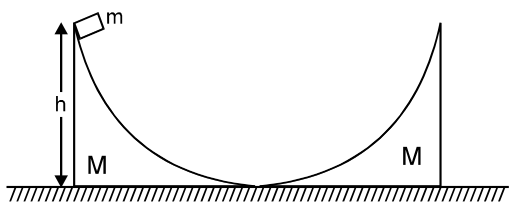
(A) $(\sqrt{3}mi+3mj)\ \frac{kgm}{s}$ (B) $\left(\frac{\sqrt{3}}{2}mi-\sqrt{3}mj\right)\frac{kgm}{s}$ (C) $(3mi-\sqrt{3}mj)\ \frac{kgm}{s}$ (D) $(2\sqrt{3}mi+3j)\ \frac{kgm}{s}$

6. Coefficient of restitution during the collision is changed to $\frac{1}{2}$, keeping all other parameters unchanged. What is the velocity of the ball B after the collision ?

(A) $\frac{1}{2}$ $(3\sqrt{3}\,i + 9\,j)$ m/s (B) $\frac{1}{4}$ $(9\,i - 3\sqrt{3}\,j)$m/s (C) $(6\,i + 3\sqrt{3}\,j)$ m/s (D) $(6\,i - 3\sqrt{3}\,j)$ m/s

Comprehension-3

The inclined surfaces of two moveable wedges of the same mass M = 2kg are smoothly placed just in contact with each other and placed on the horizontal plane as shown in the figure. A small block of mass m = 1kg slides down the left wedge from a height h = 9m. Neglect the friction and both wedges can move independently.



7.✍ Find velocity of left wedge just after when block leave the wedge

(A) $\sqrt{30}$ m/s (B) $\sqrt{50}$ m/s (C) $\sqrt{25}$ m/s (D) $\sqrt{45}$ m/s

8.✍ To what maximum height will the block rise along the right wedge?

(A) 5m (B) 4m (C) 2m (D) 3m

9.✍ Find velocity of right wedge when samller bolck is at maximum height on right wedge.

(A) $\sqrt{\frac{40}{3}}$ (B) $\sqrt{\frac{20}{3}}$ (C) $\sqrt{\frac{15}{3}}$ (D) $\sqrt{\frac{42}{3}}$

Exercise-3

✍ **Marked Questions can be used as Revision Questions.**

* **Marked Questions may have more than one correct option.**

PART - I : JEE (ADVANCED) / IIT-JEE PROBLEMS (PREVIOUS YEARS)

1. **STATEMENT-1 :** In an elastic collision between two bodies, the relative speed of the bodies after collision is equal to the relative speed before the collision. **[JEE-2007, 3/162]**

because

STATEMENT-2 : In an elastic collision, the linear momentum of the system is conserved

(A) Statement-1 is True, Statement-2 is True; Statement-2 **is** a correct explanation for Statement-1

(B) Statement-1 is True,Statement-2 is True; Statement-2 **is NOT** a correct explanation for Statement-1

(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True.

(E) Statement-1 is False, Statement-2 is False

2*. Two balls, having linear momenta $\vec{p}_1 = p\hat{i}$ and $\vec{p}_2 = -p\hat{i}$, undergo a collision in free space. There is no external force acting on the balls. Let $\vec{p}'_1$ and $\vec{p}'_2$ be their final momenta. Which of the following option(s) is(are) **NOT ALLOWED** for any non-zero value of p, a_1, a_2, b_1, b_2, c_1 and c_2.**[JEE-2008, 3/163]**

(A) $\vec{p}'_1 = a_1\hat{i} + b_1\hat{j} + c_1\hat{k}$
$\vec{p}'_2 = a_2\hat{i} + b_2\hat{j}$

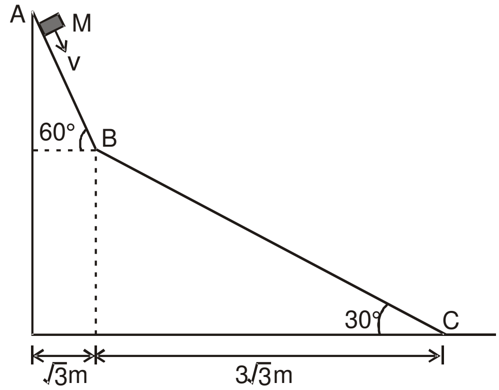
(B) $\vec{p}'_1 = c_1\hat{k}$
$\vec{p}'_2 = c_2\hat{k}$

(C) $\vec{p}'_1 = a_1\hat{i} + b_1\hat{j} + c_1\hat{k}$
$\vec{p}'_2 = a_2\hat{i} + b_2\hat{j} - c_1\hat{k}$

(D) $\vec{p}'_1 = a_1\hat{i} + b_1\hat{j}$
$\vec{p}'_2 = a_2\hat{i} + b_1\hat{j}$

Paragraph

A small block of mass M moves on a frictionless surface of an inclined plane, as shown in figure. The angle of the incline suddenly changes from 60° to 30° at point B. The block is initially at rest at A. Assume that collisions between the block and the incline are totally inelastic (g = 10 m/s^2). Figure : **[JEE-2008, 12/163]**



3.✍ The speed of the block at point B immediately after it strikes the second incline is

(A) $\sqrt{60}$ m/s (B) $\sqrt{45}$ m/s (C) $\sqrt{30}$ m/s (D) $\sqrt{15}$ m/s

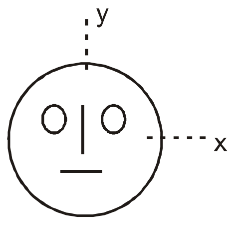
4.✍ The speed of the block at point C, immediately before it leaves the second incline is

(A) $\sqrt{120}$ m/s (B) $\sqrt{105}$ m/s (C) $\sqrt{90}$ m/s (D) $\sqrt{75}$ m/s

5.✍ If collision between the block and the incline is completely elastic, then the vertical (upward) component of the velocity of the block at point B, immediately after it strikes the second incline is

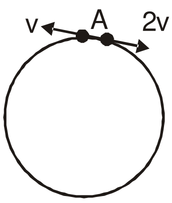
(A) $\sqrt{30}$ m/s (B) $\sqrt{15}$ m/s (C) 0 (D) – $\sqrt{15}$ m/s

6. Look at the drawing given in the figure which has been drawn with ink of uniform line-thickness. The mass of ink used to draw each of the two inner circles, and each of the two line segments is m. The mass of the ink used to draw the outer circle is 6m. The coordinates of the centres of the different parts are: outer circle (0, 0), left inner circle (–a, a), right inner circle (a, a), vertical line (0, 0) and horizontal line (0, –a). The y-coordinate of the centre of mass of the ink in this drawing is **[JEE-2009, 3/160, –1]**



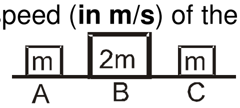
(A) $\frac{a}{10}$ (B) $\frac{a}{8}$ (C) $\frac{a}{12}$ (D) $\frac{a}{3}$

7. Two small particles of equal masses start moving in opposite directions from a point A in a horizontal circular orbit. Their tangential velocities are v and 2v, respectively, as shown in the figure. Between collisions, the particles move with constant speeds. After making how many elastic collisions, other than that at A, these two particles will again reach the point A? **[JEE-2009, 3/160, –1]**



(A) 4 (B) 3 (C) 2 (D) 1

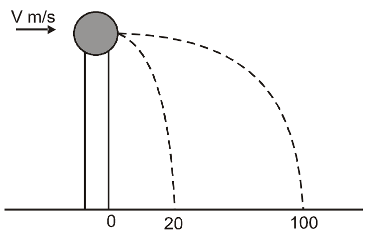
8. Three objects A,B and C are kept in a straight line on a frictionless horizontal surface. These have masses m, 2m and m, respectively. The object A moves towards B with a speed 9 m/s and makes an elastic collision with it. Thereafter, B makes completely inelastic collision with C. All motions occur on the same straight line. Find the final speed (**in m/s**) of the object C. **[JEE-2009, 4/160,–1]**



9*. A point mass of 1kg collides elastically with a stationary point mass of 5 kg. After their collision, the 1 kg mass reverses its direction and moves with a speed of 2 ms^{-1}. Which of the following statement(s) is/are correct for the system of these two masses ? **[JEE-2010, 3/163]**

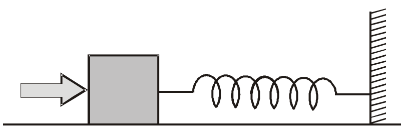
(A) Total momentum of the system is 3 kg ms^{-1} (B) Momentum of 5 kg mass after collision is 4 kg ms^{-1}
(C) Kinetic energy of the centre of mass is 0.75J (D) Total kinetic energy of the system is 4 J

10. A ball of mass 0.2 kg rests on a vertical post of height 5 m. A bullet of mass 0.01 kg, traveling with a velocity V m/s in a horizontal direction, hits the centre of the ball. After the collision, the ball and bullet travel independently. The ball hits the ground at a distance of 20 m and the bullet at a distance of 100 m from the foot of the post. The initial velocity V of the bullet is **[JEE-2011, 3/160, –1]**



(A) 250 m/s (B) 250$\sqrt{2}$ m/s (C) 400 m/s (D) 500 m/s

11.✍ A block of mass 0.18 kg is attached to a spring of force-constant 2 N/m. The coefficient of friction between the block and the floor is 0.1. Initially the block is at rest and the spring is un-stretched. An impulse is given to the block as shown in the figure.



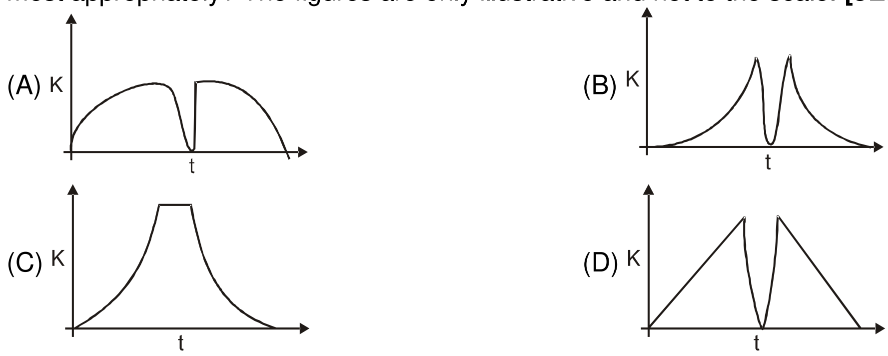
The block slides a distance of 0.06 m and comes to rest for the first time. The initial velocity of the block in m/s is V= N/10. Then N is **[JEE-2011, 4/160]**

12. A bob of mass m, suspended by a string of length l_1, is given a minimum velocity required to complete a full circle in the vertical plane, At the highest point, it collides elastically with another bob of mass m suspended by a string of length l_2, which is initially at rest. Both the strings are mass-less and inextensible. If the second bob, after collision acquires the minimum speed required to complete a full circle in the vertical plane, the ratio $\frac{l_1}{l_2}$ is : **[JEE (Advanced) 2013; 4/60]**

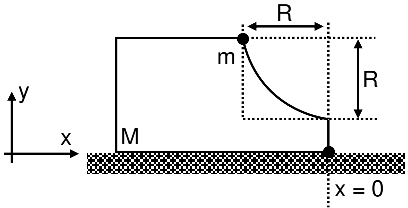
13. A particle of mass m is projected from the ground with an initial speed u_0 at an angle α with the horizontal. At the highest point of its trajectory, it makes a completely inelastic collision with another identical particle, which was thrown vertically upward from the ground with the same initial speed u_0. The angle that the composite system makes with the horizontal immediately after the collision is :
[JEE (Advanced) 2013, 3/60, –1]

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{4}+a$ (C) $\frac{\pi}{4}-a$ (D) $\frac{\pi}{4}+\frac{a}{2}$

14. A tennis ball is dropped on a horizontal smooth surface. It bounces back to its original position after hitting the surface. The force on the ball during the collision is proportional to the length of compression of the ball. Which one of the following sketches describes the variation of its kinetic energy K with time t most appropriately? The figures are only illustrative and not to the scale. **[JEE(Advanced) 2014,3/60,–1]**



15*. A block of mass M has a circular cut with a frictionless surface as shown. The block rests on the horizontal frictionless surface of a fixed table. Initially the right edge of the block is at x = 0, in a coordinate system fixed to the table. A point mass m is released from rest at the topmost point of the path as shown and its slides down. When the mass loses contact with the block, its position is x and the velocity is v. At the instant, which of the following options is/are correct ?



(A) The velocity of the point mass m is : v = $\sqrt{\frac{2gR}{1+\frac{m}{M}}}$ **[JEE(Advanced) 2017; P-1, 4/61, –2]**

(B) The x component of displacement of the center of mass of the block M is : $-\frac{mR}{M+m}$

(C) The position of the point mass is : x = $-\sqrt{2}\frac{mR}{M+m}$

(D) The velocity of the block M is : V = $-\frac{m}{M}\sqrt{2gR}$

16*. A flat plate is moving normal to its plane through a gas under the action of a constant force F. The gas is kept at very low pressure. The speed of the plate v is much less than the average speed u of the gas molecules. Which of the following options is/are true ? **[JEE (Advanced) 2017 ; P-1, 4/61, –2]**
(A) The pressure difference between the leading and trailing faces of the plate is proportional to uv.
(B) At a later time the external force F balances the resistive force
(C) The resistive force experienced by the plate is proportional to v
(D) The plate will continue to move with constant non-zero acceleration, at all times

PART - II : JEE (MAIN) / AIEEE PROBLEMS (PREVIOUS YEARS)

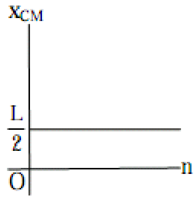
1. A circular disc of radius R is removed from a bigger circular disc of radius 2R such that the circumferences of the discs coincide. The centre of mass of the new disc is αR distance from the centre of the bigger disc. The value of α is : **[AIEEE 2007, 3/120]**

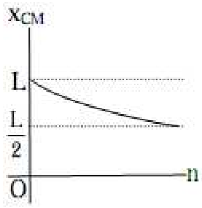
(1) 1/3 (2) 1/2 (3) 1/6 (4) 1/4

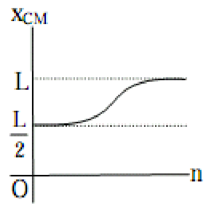
2. A block of mass 0.50 kg is moving with a speed of 2.00 ms^{-1} on a smooth surface. It strikes another mass of 1.00 kg and then they move together as a single body. The energy loss during the collision is : **[AIEEE 2008, 3/105]**

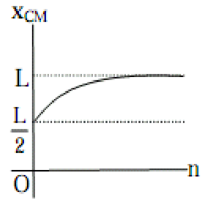
(1) 1.00 J (2) 0.67 J (3) 0.34 J (4) 0.16 J

3. A thin rod of length 'L' is lying along the x-axis with its ends at x = 0 and x = L. Its linear density (mass/length) varies with x as $k\left(\frac{x}{L}\right)^n$, where n can be zero or any positive number. If the position x_{CM} of the centre of mass of the rod is plotted against 'n', which of the following graphs best approximates the dependence of x_{CM} on n ? **[AIEEE 2008, 3/105]**

(1)


(2)


(3)


(4)


4. **Statement-1 :** Two particles moving in the same direction do not lose all their energy in a completely inelastic collision. **[AIEEE 2010, 4/144]**

Statement-2 : Principle of conservation of momentum holds true for all kinds of collisions.

(1) Statement-1 is true, Statement-2 is true; Statement-2 is the correct explanation of Statement-1.
(2) Statement-1 is true, Statement-2 is true; Statement-2 is not the correct explanation of Statement-1
(3) Statement-1 is false, Statement-2 is true.
(4) Statement-1 is true, Statement-2 is false.

5. This question has statement I and Statement II. Of the four choices given after the Statements, choose the one that best describes the two Statements.

Statement-I : A point particle of mass m moving with speed υ collides with stationary point particle of mass M. If the maximum energy loss possible is given as $f\left(\frac{1}{2}m\upsilon^2\right)$ then f = $\left(\frac{m}{M+m}\right)$.

Statement-II : Maximum energy loss occurs when the particles get stuck together as a result of the collision. **[JEE (Main) 2013, 4/120]**

(1) Statement -I is true, Statment -II is true, Statement -II is the correct explanation of Statement -I.
(2) Statement -I is true, Statment - II is true, Statement - II is not the correct explanation of Statement - I.
(3) Statement -I is true, Statment - II is false.
(4) Statement -I is false, Statment - II is true.

6. A particle of mass m moving in the x direction with speed 2v is hit by another particle of mass 2m moving in the y direction with speed v. If the collision is perfectly inelastic, the percentage loss in the energy during the collision is close to : **[JEE (Main) 2015; 4/120, –1]**

(1) 44% (2) 50% (3) 56% (4) 62%

7. Distance of the centre of mass of a solid uniform cone from its vertex is z_0. If the radius of its base is R and its height is h then z_0 is equal to **[JEE (Main) 2015; 4/120, –1]**

(1) $\frac{h^2}{4R}$ (2) $\frac{3h}{4}$ (3) $\frac{5h}{8}$ (4) $\frac{3h^2}{8R}$

8. It is found that if a neutron suffers an elastic collinear collision with deuterium at rest, fractional loss of its energy is p_d, while for its similar collision with carbon nucleus at rest, fractional loss of energy is p_c. The values of p_d and p_c are respectively : **[JEE (Main) 2018; 4/120, –1]**

(1) (0, 0) (2) (0, 1) (3) (.89, .28) (4) (.28, .89)

9. The mass of a hydrogen molecule is 3.32×10^{-27} kg. If 10^{23} hydrogen molecules strike, per second, a fixed wall of area $2cm^2$ at an angle of 45° to the normal, and rebound elastically with a speed of 10^3 m/s, then the pressure on the wall is nearly : **[JEE (Main) 2018; 4/120, –1]**

(1) 2.35×10^2 N/m^2 (2) 4.70×10^2 N/m^2 (3) 2.35×10^3 N/m^2 (4) 4.70×10^3 N/m^2

10. In a collinear collision, a particle with an initial speed v_0 strikes a stationary particle of the same mass. If the final total kinetic energy is 50% greater than the original kinetic energy, the magnitude of the relative velocity between the two particles, after collision, is : **[JEE (Main) 2018; 4/120, –1]**

(1) $\frac{v_0}{2}$ (2) $\frac{v_0}{\sqrt{2}}$ (3) $\frac{v_0}{4}$ (4) $\sqrt{2}v_0$

Answers

EXERCISE - 1

PART - I

Section (A) :

A-1. $\frac{\sqrt{19}}{6}$ m

A-2. $\frac{4a}{(4+\pi)}$ right of the disc centre

A-3. (5a/6, 5a/6) **A-4.** $\frac{33L}{50}$

A-5. At R/5 from the centre of the bigger disc towards the centre of the smaller disk.

A-6. $\left(\frac{3}{4}a, O\right)$

A-7. At R/3 from the centre of the original disc away from the centre of the hole.

A-8. $\frac{4(R_2^3 - R_1^3)}{3\pi(R_2^2 - R_1^2)}$

Section (B) :

B-1. $\frac{\sqrt{5}}{11}$ m/s at an angle $\tan^{-1}(2)$ above the direction towards right.

B-2. 1 cm downward. **B-3.** 60m

B-4. zero **B-5.** 0.4 m

B-6 (a) $\frac{mh}{m+M}$ (b) $\frac{Mh}{m+M}$

Section (C) :

C-1. 2.0×10^5 m/s **C-2.** 3.0×10^{-23} m/s

C-3. (a) $\frac{p_1+p_2}{m_p}$ = 18.6 m/s

(b) $\frac{\sqrt{p_1^2+p_2^2}}{m_p}$ = 15.0 m/sec

C-4. $2.5\hat{i}+15\hat{j}+5\hat{k}$ **C-5.** $\left(1+\frac{M}{m}\right)v$

C-6. 10 cm **C-7.** 9m/s, 9m

C-8. 220 m/s **C-9.** $v = \sqrt{\frac{2gR}{1+\frac{m}{M}}}$

C-10. $\frac{mv}{M+m}$

Section (D) :

D-1. $\frac{2m_2d}{m_1+m_2}, \frac{2m_1d}{m_1+m_2}$

D-2. (a) $\frac{m_2v_0}{m_1+m_2}$ (b) $v_0\left[\frac{m_1m_2}{(m_1+m_2)k}\right]^{1/2}$

Section (E) :

E-1. $8\hat{i}$ m/s

E-2. (a) 0.4 kg-m/s (b) zero

E-3. $\rho\left(\frac{4}{3}\pi\frac{d^3}{8}\right)vNA$ =1884 N

E-4. (a) $4\sqrt{5}$ Ns (b) $2000\sqrt{5}$ N

Section (F) :

F-1. K/2.

F-2. number of collisions = 2

F-4. $e = \frac{1}{\sqrt{2}}$ **F-5.** $t = \frac{2\pi r}{v}$

F-6. 5 m/s opposite to the direction of motion of the first ball.

F-7. (a) V_A = +2m/s, $V_B = +\frac{3}{4}$ m/s,

(b) 6Ns, 12 Ns, (c) 33J (d) $\frac{99}{4}$ J

Section (G) :

G-1. (i) $\frac{(m+M)g}{v_r}$ = 1.25 kg/s,

(ii) $v = v_r \ell n\left(\frac{m+M}{m}\right) - g\left(\frac{M}{\mu}\right)$

(a) 2.8 km/s, (b) 3.6 km/s.

G-2. F_{ext} = 10N; P = 20 watt.

PART - II

Section (A) :

A-1.	(D)	**A-2.**	(A)	**A-3.**	(A)
A-4.	(D)	**A-5.**	(D)	**A-6.**	(C)
A-7.	(B)				

Section (B) :

B-1.	(A)	**B-2.**	(D)	**B-3.**	(C)
B-4.	(D)	**B-5.**	(D)	**B-6.**	(B)
B-7.	(D)	**B-8.**	(B)	**B-9.**	(B)
B-10.	(B)				

Section (C) :

C-1.	(B)	**C-2.**	(B)	**C-3.**	(B)
C-4.	(A)	**C-5.**	(A)	**C-6.**	(B)
C-7.	(B)	**C-8.**	(A)	**C-9.**	(B)
C-10.	(D)				

Section (D) :

D-1.	(C)	**D-2.**	(C)	**D-3.**	(B)

Section (E) :

E-1.	(B)	**E-2.**	(A)	**E-3.**	(C)

Section (F) :

F-1.	(D)	**F-2.**	(A)	**F-3.**	(B)
F-4.	(C)	**F-5.**	(A)	**F-6.**	(C)
F-7.	(B)	**F-8.**	(A)	**F-9.**	(A)
F-10.	(A)	**F-11.**	(C)	**F-12.**	(C)

Section (G) :

G-1.	(D)

PART - III

1. (A) $\rightarrow$ p ; (B) $\rightarrow$ q ; (C) $\rightarrow$ p, r ; (D) $\rightarrow$ q, s

2. (A) $\rightarrow$ q ; (B) $\rightarrow$ p, q ; (C) $\rightarrow$ r ; (D) $\rightarrow$ s

EXERCISE - 2

PART - I

1.	(D)	**2.**	(B)	**3.**	(D)
4.	(B)	**5.**	(C)	**6.**	(A)
7.	(C)	**8.**	(C)	**9.**	(B)
10.	(A)	**11.**	(D)	**12.**	(C)
13.	(A)	**14.**	(D)	**15.**	(A)
16.	(B)	**17.**	(C)	**18.**	(A)
19.	(A)	**20.**	(A)	**21.**	(C)
22.	(A)	**23.**	(C)	**24.**	(C)
25.	(C)	**26.**	(C)	**27.**	(C)
28.	(C)	**29.**	(B)		

PART – II

1.	4	**2.**	60	**3.**	2
4.	4	**5.**	1	**6.**	6
7.	8	**8.**	20	**9.**	70
10.	10	**11.**	30	**12.**	10
13.	6	**14.**	3	**15.**	2
16.	5	**17.**	1	**18.**	5
19.	3	**20.**	2	**21.**	35
22.	70	**23.**	35	**24.**	10
25.	12	**26.**	15	**27.**	64

PART - III

1.	(CD)	**2.**	(AB)	**3.**	(BC)
4.	(AD)	**5.**	(BC)	**6.**	(AD)
7.	(ABCD)	**8.**	(BD)	**9.**	(CD)
10.	(AC)	**11.**	(ABCD)	**12.**	(BC)
13.	(BD)	**14.**	(ACD)	**15.**	(BD)
16.	(AB)	**17.**	(BD)	**18.**	(AB)
19.	(AC)	**20.**	(BD)	**21.**	(ACD)

PART - IV

1.	(C)	**2.**	(A)	**3.**	(D)
4.	(A)	**5.**	(C)	**6.**	(B)
7.	(A)	**8.**	(B)	**9.**	(A)

EXERCISE - 3

PART - I

1.	(D)	**2.**	(AD)	**3.**	(B)
4.	(B)	**5.**	(C)	**6.**	(A)
7.	(C)	**8.**	4	**9.**	(AC)
10.	(D)	**11.**	4	**12.**	5
13.	(A)	**14.**	(B)	**15.**	(AB)
16.	(ABC)				

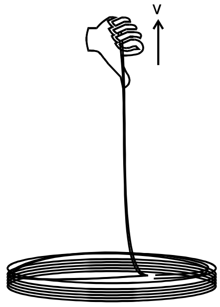
PART - II

1.	(1)	**2.**	(2)	**3.**	(4)
4.	(1)	**5.**	(4)	**6.**	(3)
7.	(2)	**8.**	(3)	**9.**	(3)
10.	(4)				

High Level Problems (HLP)

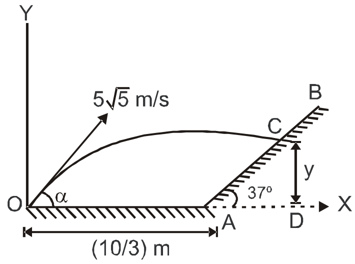
SUBJECTIVE QUESTIONS

1. A uniform rope of linear mass density λ and length ℓ is coiled on a smooth horizontal surface. One end is pulled up with constant velocity v. Then find average power applied by the external agent in pulling the entire rope just off the ground ?

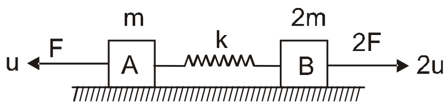


2. In the above question the maximum power delivered by the agent in pulling up the rope is

3. A particle is projected from point O on the ground with velocity $u = 5\sqrt{5}$ m/s at angle $\alpha = \tan^{-1}(0.5)$. It strikes at a point C on a fixed smooth plane AB having inclination of 37º with horizontal as shown in figure. If the particle does not rebound, calculate.
(a) coordinates of point C in reference to coordinate system as shown in the figure.
(b) maximum height from the ground to which the particle rises. ($g = 10$ m/s^2)



4. Two blocks A & B of mass 'm' & 2m respectively are joined to the ends of an undeformed massless spring of spring constant ' k '. They can move on a horizontal smooth surface. Initially A & B have velocities ' u ' towards left and '2 u' towards right respectively.
Constant forces of magnitudes F and 2 F are always acting on A and B respectively in the directions shown. Find the maximum extension in the spring during the motion.



5. Two identical buggies 1 and 2 with one man in each move without friction due to inertia along the parallel rails toward each other. When the buggies got opposite to each other, the men exchange their places by jumping in the direction perpendicular to the direction of motion. As a consequence, buggy 1 stops and buggy 2 keeps moving in the same direction, with its velocity becoming equal to v. Find the initial velocities of the buggies v_1 and v_2 if the mass of each buggy (without a man) equals M and the mass of each man is m.

6. Two identical buggies move one after the other due to inertia (without friction) with the same velocity v_0. A man of mass m rides the rear buggy. At a certain moment the man jumps into the front buggy with a velocity u relative to his buggy. Knowing that the mass of each buggy is equal to M, find the velocities with which the buggies will move after that.

7. Two men, each of mass m, stand on the edge of a stationary buggy of mass M. Assuming the friction to be negligible, find the velocity of the buggy after both men jump off with the same horizontal velocity u relative to the buggy: (i) simultaneously; (ii) one after the other. In what case will the velocity of the buggy be greater and how many times?

8. A stationary pulley carries a rope whose one end supports a ladder with a man and the other end the counterweight of mass M. The man of mass m climbs up a distance ℓ' with respect to the ladder and then stops. Neglecting the mass of the rope and the friction in the pulley axle, find the displacement ℓ of the centre of inertia of this system.

9. A particle of mass m_1 experienced a perfectly elastic collision with a stationary particle of mass m_2. What fraction of the kinetic energy does the striking particle lose, if
(a) it recoils at right angles to its original motion direction;
(b) the collision is a head-on one?

10. Body 1 experiences a perfectly elastic collision with a stationary Body 2. Determine their mass ratio, if
(a) after a head-on collision the particles fly apart in the opposite directions with equal, velocities;
(b) the particles fly apart symmetrically relative to the initial motion direction of particle 1 with the angle of divergence $\theta = 60°$.

11. A ball moving transitionally collides with another stationary ball of the same mass. At the moment of impact the angle between the straight line passing through the centres of the balls and the direction of the initial motion of the striking ball is equal to $\alpha = 45°$. Assuming the balls to be smooth, find the fraction η of the kinetic energy of the striking ball that turned into potential energy at the moment of the maximum deformation.

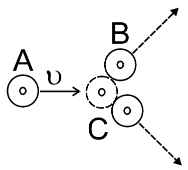
12. Particle 1 moving with velocity v = 10 m/s experienced a head-on collision with a stationary particle 2 of the same mass. As a result of the collision, the kinetic energy of the system decreased by $\eta = 1.0\%$. Find the magnitude and direction of the velocity of particle 1 after the collision.

13. A particle of mass m having collided with a stationary particle of mass M deviated by an angle $\pi/2$ whereas the particle M start moving at an angle $\theta = 30°$ to the direction of the initial motion of the particle m. How much (in percent) and in what way has the kinetic energy of this system changed after the collision, if M / m = 5.0 ?

14. A closed system consists of two particles of masses m_1 and m_2 which move at right angles to each other with velocities v_1 and v_2 Find:
(a) the momentum of each particle and
(b) the total kinetic energy of the two particles in the reference frame fixed to their centre of inertia.

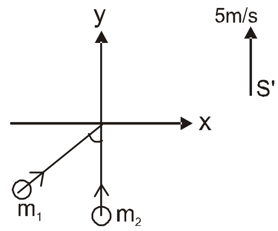
15. A particle of mass m_1 collides elastically with a stationary particle of mass m_2 ($m_1 > m_2$). Find the maximum angle through which the striking particle may deviate as a result of the collision.

16. Three identical discs A, B, and C as shown in figure rest on a smooth horizontal plane. The disc A is set in motion with velocity v after which it experiences an elastic collision simultaneously with discs B and C. The distance between the centres of the latter discs prior to the collision is η times the diameter of each disc Find the velocity of the disc A after the collision. At what value of η will the disc A recoil after the collision; stop; move on?

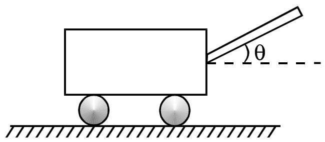


17. A spaceship of mass m_0 moves in the absence of external ' forces with a constant velocity v_0. To change the motion direction a jet engine is switched on. It starts ejecting a gas jet with velocity u, which is constant relative to the spaceship and directed at right angles to the spaceship motion. The engine is shut down when the mass of the spaceship decreases to m. Through what angle α did the motion direction of the spaceship deviate due to the jet engine operation?

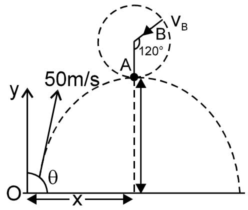
18. Two smooth spheres of the same radius, but which have different masses m_1 & m_2 collide inelastically. Their velocities before collision are 13 m/s & 5 m/s respectively along the directions shown in the figure in which $\cot\theta = 5/12$. An observer S′ moving parallel to the positive y - axis with a constant speed of 5m/s observes this collision. He finds the final velocity of m_1 to be 5m/s along the y-direction and the total loss in the kinetic energy of the system to be 1/72 of its initial value. Determine;
(a) The ratio of the masses
(b) The velocity of m_2 after collision with (respect to observer)



19. A cart of total mass M_0 is at rest on a rough horizontal road. It ejects bullets at rate of λ kg/s at an angle θ with the horizontal and at velocity 'u' (constant) relative to the cart. The coefficient of friction between the cart and the ground is μ. Find the velocity of the cart in terms of time ' t '. The cart moves with sliding.

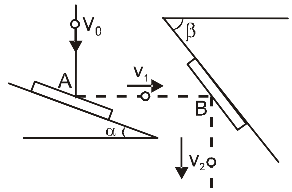


20. A bullet of mass M is fired with a velocity 50 m/s at an angle with the horizontal. At the highest point of its trajectory, it collides head on with a bob of mass 3M suspended by a massless rod of length $\frac{10}{3}$m and gets embedded in the bob. After the collision the rod moves through an angle 120°. Find



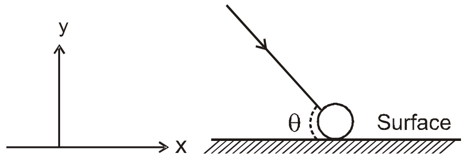
(a) The angle of projection.
(b) The vertical and horizontal coordinates of the initial position of the bob with respect to the point of firing of the bullet. (g = 10m/s^2)

21. A steel ball falling vertically strikes a fixed rigid plate A with velocity v_0 and rebounds horizontally. The ball then strikes a second fixed rigid plate B and rebounds vertically as shown. Assuming smooth surface and the effect of gravity on motion of ball is to be neglected. Determine



(a) The required angles α and β.
(b) The magnitude of the velocity v_1 & v_2. Consider coefficient of restitution for both plates as e.

22. A small spherical ball undergoes an elastic collision with a rough horizontal surface. Before the collision, it is moving at an angle θ to the horizontal (see Fig). You may assume that the frictional force obeys the law $f = \mu N$ during the contact period, where N is the normal reaction on the ball and μ is the coefficient of friction.



(a) Obtain θ_m (μ) so that the subsequent horizontal range of the ball after leaving the horizontal surface is maximized.
(b) Find the allowed range for θ_m.

23. Two skaters (A and B), each of mass 70 kg, are approaching each another on a frictionless surface, each with a speed of 1 ms^{-1}. Skater A carries a ball of mass 10 kg. Both skaters can toss the ball at 5 ms^{-1} relative to themselves such that when A tosses the ball at t = 0 s to B then the ball leaves at 6 ms^{-1} with respect to the ground.

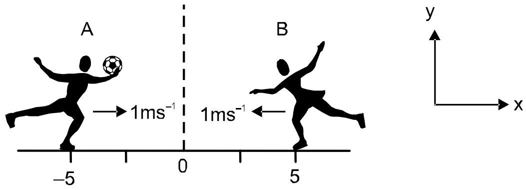


Figure-1

Further, they start (t = 0s) tossing the ball back and forth when they are 10m apart (see Fig. (1)). Assume that the motion is one dimensional, all collisions are completely inelastic and that the time delay between receiving the ball and tossing it back is 1s.

(a) State initial momenta of skaters (just before t = 0 s).

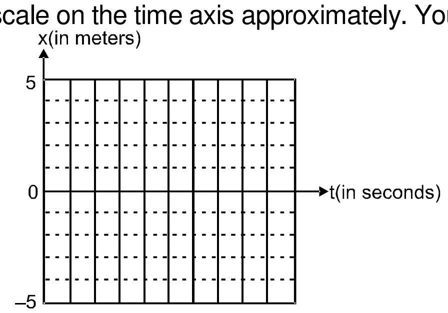
$\vec{P}_A$ = ; $\vec{P}_B$ =

(b) At t = 0 s skater A tosses the ball to skater B. State momenta of both the skaters immediately after B catches the ball.

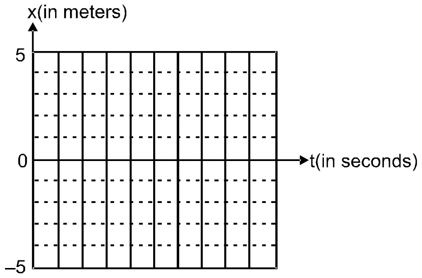
$\vec{P}_A$ = ; $\vec{P}_B$ =

(c) Indicates the minimum number of tosses by each skater required to avoid collision.
Number of tosses by A = ; Number of tosses by B =

(d) Indicate motion of each skater on the following x–t plot if no tosses are made. [Note : For this and next part you must select the scale on the time axis approximately. You may use a pencil for sketching].



(e) Indicate motion of each skater on the following x–t plot from t = 0 s till just after one round trip by the ball (from A to B and back to A).

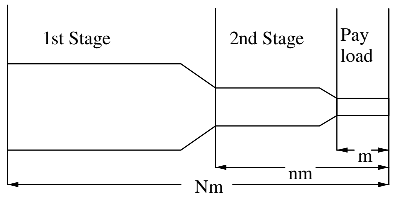


24. A neutron is scattered through (≡ deviation from its original direction) θ degree in an elastic collision with an initially stationary deutron. If the neutron loses $\frac{2}{3}$ of its initial K.E. to the deutron then find the value of θ. (In atomic mass unit, the mass of a neutron is 1u and mass of a deutron is 2u).

25. A shell flying with velocity v = 500 m/s bursts into three identical fragments so that the kinetic energy of the system increases η = 1.5 times. What maximum velocity can one of the fragments obtain?

26. A particle moves along a closed trajectory in a central field of force where the particle's potential energy $U = kr^2$ (k is positive constant, r is the distance of the particle from the centre O of the field). Find the mass of the particle if its minimum distant; from the point O equals r_1 and its velocity at the point farthest from O equals v_2-

27. This problem is designed to illustrate the advantage that can be obtained by the use of multiple-staged instead of single-staged rockets as launching vehicles. Suppose that the payload (e.g., a space capsule) has mass m and is mounted on a two-stage rocket (see figure). The total mass (both rockets fully fuelled, plus the payload) is Nm.



The mass of the second-stage rocket plus the payload, after first-stage burnout and separation, is nm. In each stage the ratio of container mass to initial mass (container plus fuel) is r, and the exhaust speed is V, constant relative to the engine. Note that at the end of each state when the fuel is completely exhausted, the container drops off immediately without affecting the velocity of rocket. Ignore gravity.

(a) Obtain the velocity v of the rocket gained from the first-stage burn, starting from rest in terms of {V,N,n,r}
(b) Obtain a corresponding expression for the additional velocity u gained from the second stage burn.
(c) Adding v and u, you have the payload velocity w in terms of N, n, and r. Taking N and r as constants, find the value of n for which w is a maximum. For this maximum condition obtain u/v.
(d) Find an expression for the payload velocity w_s of a single-stage rocket with the same values of N, r, and V
(e) Suppose that it is desired to obtain a payload velocity of 10 km/s, using rockets which V = 2.5km/s and r = 0.1. Using the maximum condition of part (c) obtain the value of N if the job is to be done with a two-stage rocket. **[Physics Olympiad (STAGE-2) - 2016]**

HLP Answers

1. $\frac{\lambda \ell v g}{2} + \lambda v^3$ **2.** $\lambda \ell g v + v^3 \lambda$ **3.** (a) (5m, 1.25m) (b) 4.45 m.

4. $x_{max} = \frac{4F + \sqrt{16F^2 + 54mu^2k}}{3k}$ **5.** $v_1 = -mv/(M-m)$, $v_2 = Mv/(M-m)$

6. $v_{rear} = v_0 - \frac{\mathbf{m}}{\mathbf{M+m}}u$; $v_{front} = v_0 + \frac{\mathbf{mMu}}{\mathbf{(M+m)^2}}$

7. (i) $v_1 = \frac{-2m\vec{u}}{(M+2m)}$ (ii) $v_2 = \frac{-m(2M+3m)\vec{u}}{(M+m)(M+2m)}$, $v_2 > v_1$ by a factor of $\frac{(2M+3m)}{(2M+2m)}$

8. $\ell = \ell' m / 2M$ **9.** (a) $\eta = 2m_1/(m_1 + m_2)$; (b) $\eta = \frac{4\, m_1\, m_2}{(m_1 + m_2)^2}$

10. (a) $\frac{m_1}{m_2} = 1/3$; (b) $\frac{m_1}{m_2} = 1 + 2\cos\theta = 2.0$ **11.** $\eta = \frac{1}{2}\cos^2\alpha = 0.25$

12. Will continue moving in the same direction, although this time with the velocity $v' = (1 - \sqrt{1-2\eta})v/2$. For $\eta << 1$ the velocity $v' = \eta v/2 = 5$ cm/s.

13. $\Delta T/T = (1 + m/M)\tan^2\theta + m/M - 1 = -40\%$

14. (a) $p = \mu\sqrt{v_1^2 + v_2^2}$; (b) $T = 1/2\mu(v_1^2 + v_2^2)$. Here $\mu = m_1m_2/(m_1 + m_2)$

15. $\sin\theta_{max} = m_2/m_1$

16. $v' = -v(2 - \eta^2)/(6 - \eta^2)$. Respectively at smaller η, equal, or greater than $\sqrt{2}$

17. $\alpha = (u/u_0)\ \ln(m_0/m)$ **18.** (a) $\frac{m_1}{m_2} = \frac{9}{13}$ (b) 9 m/s

19. $v = (u\cos\theta - \mu u\sin\theta)\ell n\left(\frac{M_0}{(M_0 - \lambda t)}\right) - \mu gt$ **20.** (a) 37º, (b) x = 120 m and y = 45 m

21. $\tan\alpha = \sqrt{e}$, $v_1 = \sqrt{e}\,v_0$, $\cot\beta = \sqrt{e}$, $v_2 = ev_0$

22. From fig. $v_y = v\sin\theta$

Note that the surface is rough and there is frictional force along the x-direction. Hence elastic collision does not constrain the velocity along the x-direction. it implies that the y-component of the velocity $v\sin\theta$ changes only in sign. From Newton's second law along vertical y-direction, change in momentum is given by the linear impulse, which yields :

$$2mv\sin\theta = \int N dt \qquad ...(i)$$

From newton's second law along x-direction, we have

$$mv\cos\theta - \mu\int N dt = mv_x \qquad ...(ii)$$

Inserting Eq. (i) in Eq. (ii) we obtain

$$v_x = v(\cos(\theta) - 2\mu\sin(\theta))$$

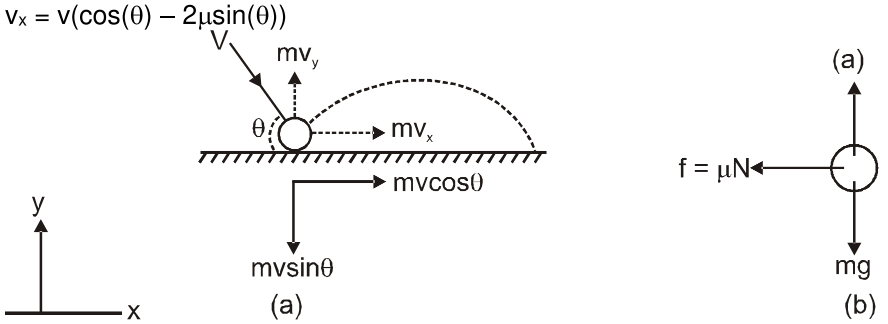


Figure : Free body diagram of the ball in contact with the floor.

Range = v_x × time of flight = $v_x \times \frac{2v_y}{g}$

$$R(\theta) = \frac{2v^2}{g}f(\theta)$$

where $f(\theta) = \sin\theta(\cos\theta - 2\mu\sin\theta)$

To maximize R, set $f'(\theta_m) = 0$

which yields $\theta_m = \frac{1}{2}\cot^{-1}(2\mu)$

(b) Possible range of θ_m : $\theta_m \in]0, \pi/4[$

23. (a) $\vec{P}_A = 80\,\hat{i}$ kg. m.s^{-1} or $\vec{P}_a = 70\,\hat{i}$ kg.m.s^{-1} ; $\vec{P}_b = -70\,\hat{i}$ kg.m.s^{-1}

(b) $\vec{P}_A = 20\,\hat{i}$ kg.m.s^{-1} ; $\vec{P}_b = -10\,\hat{i}$ kg.m.s^{-1} or $\vec{P}_b = -70/8\,\hat{i}$ kg.m.s^{-1}

(c) A = 1, B = 1

(d) See Fig. (1)

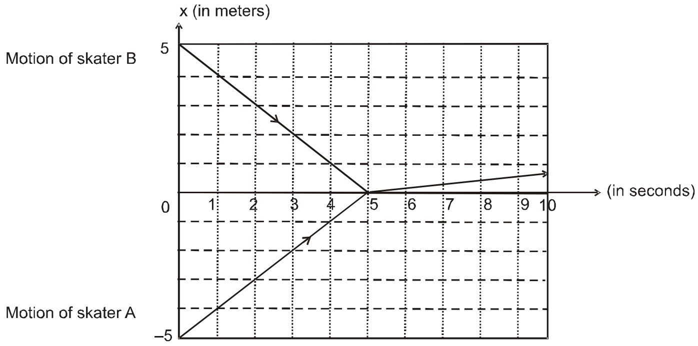


Figure 1:

(e) See Fig. (2)

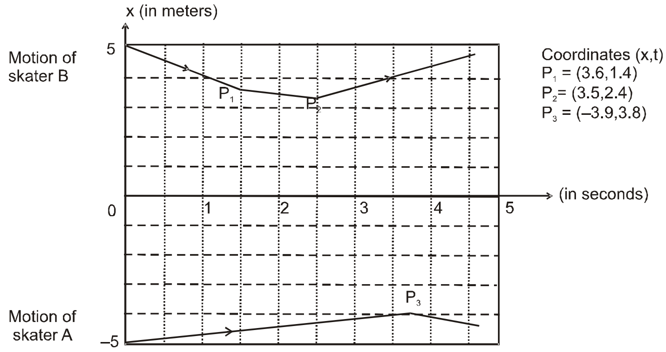


Figure 2:

24. 90 **25.** $v_{max} = v(1 + \sqrt{2(\eta - 1)}) = 1.0$ km per second **26.** $m = 2kr_1^2/v_2^2$.

27. (a) Variable mass equation gives $m\frac{d\vec{v}}{dt} = \vec{F}_{external} + V_{relative}\frac{dm}{dt}$

No gravity hence $\vec{F}_{external} = 0$, $\vec{v}_{relative} = v$. Solving rocket equation

$$= v = V\ln\frac{m_i}{m_f} \quad .....(1)$$

Here initially mass m_i = Nm .....(2)

Final mass m_f = [Nr + n(1 – r)]m .....(3)

(b) Now m_i = nm, m_f = m(nr + 1 – r). Equation (1) yields

$$u = V\ln\frac{n}{nr+1-r} \quad .....(4)$$

(c) From equation (3 to 4)

$$\omega = V\ln\frac{N_n}{[Nr+n(1-r)]\,[nr+1-r]}$$

V ln f(n)

Maximizing ω is equivalent to maximizing f(n). Differentiating and setting equal to zero, we obtain

$$n = \sqrt{N} \Rightarrow \frac{u}{v} = \frac{\ln[\sqrt{N}/\{r\sqrt{N}+(1-r)\}]}{\ln[N/Nr+\sqrt{N}(1-r)\}]} = 1 \quad .....(5)$$

where we have used equation (1 and 4)

(d) Here m_i = Nm and m_f = m + r(Nm – m). Using equation $w_s = V\ln\frac{N}{Nr+1-r}$

(e) Payload velocity $w = u + v = 2V\ln\frac{\sqrt{N}}{r\sqrt{N}+1-r}$

For the desired value of w, N = 649.4. Answer should be an integer number for the number of state. Hence N = 650.