CAPACITANCE

Contents

JEE (ADVANCED) SYLLABUS

Capacitance ; Parallel plate capacitor with and without dielectrics; Capacitors in series and parallel; Energy stored in a capacitor.

JEE (MAIN) SYLLABUS

Capacitance ; Conductors and insulators, Dielectrics and electric polarization, capacitor, combination of capacitors in series and in parallel, capacitance of a parallel plate capacitor with and without dielectric medium between the plates, Energy stored in a capacitor.

CAPACITANCE

1. INTRODUCTION

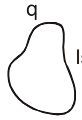
A capacitor can store energy in the form of potential energy in an electric field. In this chapter we'll discuss the capacity of conductors to hold charge and energy.

2. CAPACITANCE OF AN ISOLATED CONDUCTOR

When a conductor is charged its potential increases. It is found that for an isolated conductor (conductor should be of finite dimension, so that potential of infinity can be assumed to be zero) potential of the conductor is proportional to charge given to it.

q = charge on conductor

V = potential of conductor



$q \propto V$

$\Rightarrow$ $q = CV$

Where C is proportionality constant called capacitance of the conductor.

2.1 Definition of capacitance :

Capacitance of conductor is defined as charge required to increase the potential of conductor by one unit.

2.2 Important points about the capacitance of an isolated conductor :

(i) It is a scalar quantity.

(ii) Unit of capacitance is farad in SI units and its dimensional formula is $M^{-1} L^{-2} I^2 T^4$

(iii) 1 Farad : 1 Farad is the capacitance of a conductor for which 1 coulomb charge increases potential by 1 volt.

$$1 \text{ Farad} = \frac{1 \text{ Coulomb}}{1 \text{ Volt}}$$

$1\ \mu F = 10^{-6}$ F, 1nF = 10^{-9} F or 1 pF = 10^{-12} F

(iv) Capacitance of an isolated conductor depends on following factors :

(a) Shape and size of the conductor : On increasing the size, capacitance increases.

(b) On surrounding medium : With increase in dielectric constant K, capacitance increases.

(c) Presence of other conductors : When a neutral conductor is placed near a charged conductor, capacitance of conductors increases.

(v) Capacitance of a conductor do not depend on

(a) Charge on the conductor

(b) Potential of the conductor

(c) Potential energy of the conductor.

3. POTENTIAL ENERGY OR SELF ENERGY OF AN ISOLATED CONDUCTOR

Work done in charging the conductor to the charge on it against its own electric field or total energy stored in electric field of conductor is called self energy or self potential energy of conductor.

3.1 Electric potential energy (Self Energy) :

Work done in charging the conductor

$$W = \int_0^q \frac{q}{C} dq = \frac{q^2}{2C}$$

$$W = U = \frac{q^2}{2C} = \frac{1}{2} CV^2 = \frac{qV}{2}.$$

q = Charge on the conductor

V = Potential of the conductor

C = Capacitance of the conductor.

3.2 Self energy is stored in the electric field of the conductor with energy density (Energy per unit volume)

$\frac{dU}{dV} = \frac{1}{2} \in_0 E^2$ [The energy density in a medium is $\frac{1}{2} \in_0 \in_r E^2$]

where E is the electric field at that point.

3.3 In case of charged conductor energy stored is only out side the conductor but in case of charged insulating material it is outside as well as inside the insulator.

4. CAPACITANCE OF AN ISOLATED SPHERICAL CONDUCTOR

The capacitance of an isolated spherical conductor of radius R.

Let there is charge Q on sphere.

$\because$ Potential $V = \frac{KQ}{R}$

Hence by formula : Q = CV

$Q = \frac{CKQ}{R}$

$C = 4\pi\in_0 R$

Capacitance of an isolated spherical conductor

$C = 4\pi\in_0 R$

(i) If the medium around the conductor is vacuum or air.

$C_{Vacuum} = 4\pi\in_0 R$

R = Radius of spherical conductor. (may be solid or hollow.)

(ii) If the medium around the conductor is a dielectric of constant K from surface of sphere to infinity.

$C_{medium} = 4\pi\in_0 KR$

(iii) $\frac{C_{medium}}{C_{air/vaccum}}$ = K = dielectric constant.

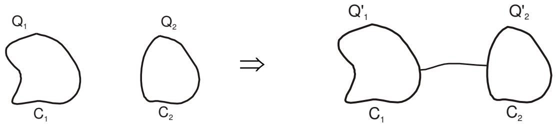
Solved Example

Example 1. Find out the capacitance of the earth ? (Radius of the earth = 6400 km)

Solution : $C = 4\pi\in_0 R = \frac{6400 \times 10^3}{9 \times 10^9} = 711\ \mu F$

5. SHARING OF CHARGES ON JOINING TWO CONDUCTORS (BY A CONDUCTING WIRE) :

Initially **Finally**



(i) Whenever there is potential difference, there will be movement of charge.

(ii) If released, charge always have tendency to move from **high potential energy** to **low potential energy .**

(iii) If released, positive charge moves from **high potential** to **low potential** [if only electric force act on charge].

(iv) If released, negative charge moves from **low potential** to **high potential** [if only electric force act on charge].

(v) The movement of charge will continue till there is potential difference between the conductors (finally potential difference = 0).

(vi) Formulae related with redistribution of charges :

Before connecting the conductors		
Parameter	**I^{st} Conductor**	**II^{nd} Conductor**
Capacitance	C_1	C_2
Charge	Q_1	Q_2
Potential	V_1	V_2

After connecting the conductors		
Parameter	**I^{st} Conductor**	**II^{nd} Conductor**
Capacitance	C_1	C_2
Charge	Q_1'	Q_2'
Potential	V	V

$$V = \frac{Q_1'}{C_1} = \frac{Q_2'}{C_2} \qquad \Rightarrow \qquad \frac{Q_1'}{Q_2'} = \frac{C_1}{C_2}$$

But, $Q_1' + Q_2' = Q_1 + Q_2$

$$\therefore \quad V = \frac{Q_1 + Q_2}{C_1 + C_2} = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}$$

$$\therefore \quad Q_1' = (Q_1 + Q_2)$$

and $Q_2' = \dfrac{C_1}{C_1 + C_2}(Q_1 + Q_2)$

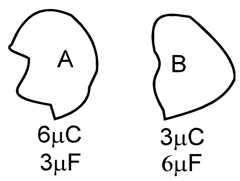
Heat loss during redistribution : $\Delta H = \dfrac{1}{2}\dfrac{C_1C_2}{C_1 + C_2}(V_1 - V_2)^2$

The loss of energy is in the form of Joule heating in the wire.

Note : Always put Q_1, Q_2, V_1 and V_2 with sign.

Solved Example

Example 2. A and B are two isolated conductors (that means they are placed at a large distance from each other). When they are joined by a conducting wire:



(i) Find out final charges on A and B ?

(ii) Find out heat produced during the process of flow of charges.

(iii) Find out common potential after joining the conductors by conducting wires?

Solution : (i) $Q_A' = \dfrac{3}{3+6}(6+3) = 3\mu C$

$Q_B' = \dfrac{6}{3+6}(6+3) = 6\mu C$

(ii) $\Delta H = \dfrac{1}{2} \cdot \dfrac{3\mu F.6\mu F}{(3\mu F + 6\mu F)} \cdot \left(2 - \dfrac{1}{2}\right)^2 = \dfrac{1}{2} \cdot (2\mu F) \cdot \left(\dfrac{3}{2}\right)^2 = \dfrac{9}{4}\mu J$

(iii) $V_C = \dfrac{3\mu C + 6\mu C}{3\mu F + 6\mu F} = 1\text{volt}.$

6. CAPACITOR :

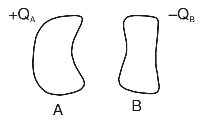
A capacitor or condenser consists of two conductors separated by an insulator or dielectric.

(i) When uncharged conductor is brought near to a charged conductor, the charge on conductors remains same but its potential decreases resulting in the increase of capacitance.

(ii) In capacitor two conductors have equal but opposite charges.

(iii) The conductors are called the plates of the capacitor. The name of the capacitor depends on the shape of the capacitor.

(iv) Formulae related with capacitors



(a) Q = CV

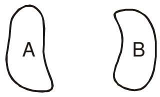
$$\Rightarrow \quad C = \frac{Q}{V} = \frac{Q_A}{V_A - V_B} = \frac{Q_B}{V_B - V_A}$$

Q = Charge of positive plate of capacitor.

V = Potential difference between positive and negative plates of capacitor

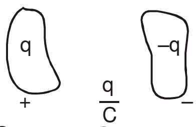
C = Capacitance of capacitor.

(b) Energy stored in the capacitor

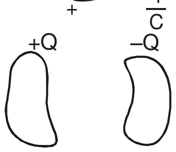


Initially charge = 0   0

Intermediate



Finally,



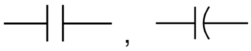
$$W = \int dW = \int_0^Q \frac{q}{C}\,dq = \frac{Q^2}{2C}$$

$\therefore$ Energy stored in the capacitor = U = $\frac{Q^2}{2C} = \frac{1}{2}CV^2 = \frac{1}{2}QV$.

This energy is stored inside the capacitor in its electric field with energy density

$$\frac{dU}{dV} = \frac{1}{2}\in E^2 \text{ or } \frac{1}{2}\in_0 \in_r E^2.$$

(v) The capacitor is represented as following:



(vi) Based on shape and arrangement of capacitor plates there are various types of capacitors.

(a) Parallel plate capacitor.   (b) Spherical capacitor.   (c) Cylindrical capacitor.

(vii) Capacitance of a capacitor depends on

(a) Area of plates.   (b) Distance between the plates.

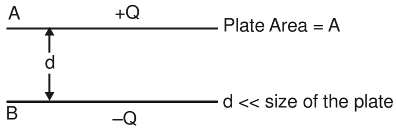
(c) Dielectric medium between the plates.

(viii) Electric field intensity between the plates of capacitors (air filled) $E = \sigma/\in_0 = V/d$

(ix) Force experienced by any plate of capacitor $F = q^2/2A\in_0$

Solved Example

Example 3. Find out the capacitance of parallel plate capacitor of plate area A and plate separation d.



Solution : $mE = \dfrac{Q}{A \in_0} \Rightarrow V_A - V_B = E.d. = \dfrac{Qd}{A \in_0} = \dfrac{Q}{C}$

$\Rightarrow \quad C = \dfrac{\in_0 A}{d}$

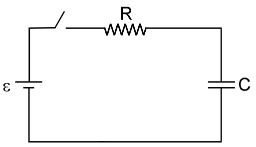
where A = area of the plates.

d = distance between plates.

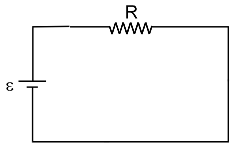
7. CIRCUIT SOLUTION FOR R–C CIRCUIT AT t = 0 (INITIAL STATE) AND AT t = ∞ (FINAL STATE)

Note : (i) Charge on the capacitor does not change instantaneously or suddenly if there is a resistance in the path (series) of the capacitor.

(ii) When an uncharged capacitor is connected with battery then its charge is zero initially hence potential difference across it is zero initially. At this time the capacitor can be treated as a conducting wire

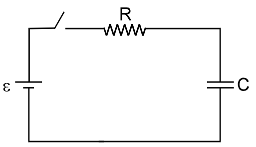


Before connection

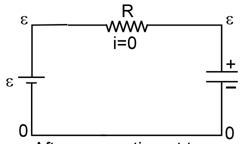


Just after connection at t = 0

(iii) The current will become zero finally (that means in steady state) in the branch which contains capacitor.



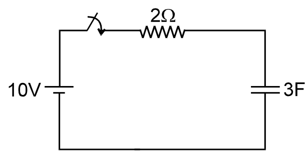
Before connection



After connection at t = ∞

Solved Examples

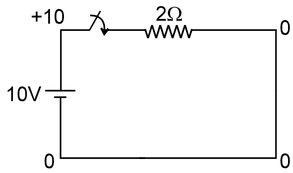
Example 4. Find out current in the circuit and charge on capacitor which is initially uncharged in the following situations.



(a) Just after the switch is closed.

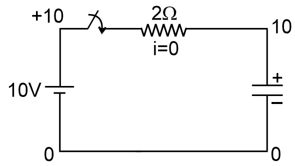
(b) After a long time when switch was closed.

Solution : **(a) For just after closing the switch :** potential difference across capacitor = 0



$\therefore \quad Q_C = 0 \qquad\qquad \therefore \quad i = \dfrac{10}{2} = 5A$

(b) After a long time

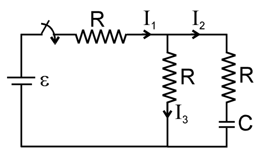


at steady state current i = 0

and potential difference across capacitor = 10 V

$\therefore \quad Q_C = 3 \times 10 = 30$ C

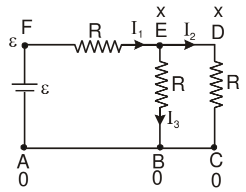
Example 5. Find out current I_1, I_2, I_3, charge on capacitor and $\frac{dQ}{dt}$ of capacitor in the circuit which is initially uncharged in the following situations.



(a) Just after the switch is closed

(b) After a long time when switch is closed.

Solution : **(a)** Initially the capacitor is uncharged so its behaviour is like a conductor. Let potential at A is zero so at B and C also zero and at F it is ε. Let potential at E is x so at D also x. Apply Kirchhoff's I[st] law at point E :



$$\frac{x-\varepsilon}{R} + \frac{x-0}{R} + \frac{x-0}{R} = 0 \quad \Rightarrow \quad \frac{3x}{R} = \frac{\varepsilon}{R}$$

$x = \frac{\varepsilon}{3}$; $Q_c = 0$

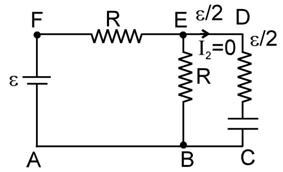
$$\therefore \quad I_1 = \frac{-\varepsilon/3+\varepsilon}{R} = \frac{2\varepsilon}{3R} \quad \Rightarrow I_2 = \frac{dQ}{dt} = \frac{\varepsilon}{3R} \text{ and } I_3 = \frac{\varepsilon}{3R}$$

Alternatively

$$i_1 = \frac{\varepsilon}{R_{eq}} = \frac{\varepsilon}{R+\frac{R}{2}} = \frac{2\varepsilon}{3R} \quad \Rightarrow \quad i_2 = i_3 = \frac{i_1}{2} = \frac{\varepsilon}{3R} \text{ and } \frac{dQ}{dt} = i_2 = \frac{\varepsilon}{3R}$$

(b) at t = ∞ (finally)

capacitor completely charged so their will be no current through it.



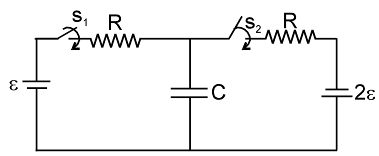
$I_2 = 0, \quad I_1 = I_3 = \frac{\varepsilon}{2R}$

$V_E - V_B = V_D - V_C = (\varepsilon/2R)R = \varepsilon/2$

$\Rightarrow Q_C = \frac{\varepsilon C}{2}$, $\quad \frac{dQ}{dt} = I_2 = 0$

Time	I_1	I_2	I_3	Q	dQ /dt
Time t = 0	$\frac{2\varepsilon}{3R}$	$\frac{\varepsilon}{3R}$	$\frac{\varepsilon}{3R}$	0	$\frac{\varepsilon}{3R}$
Finally t = ∞	$\frac{\varepsilon}{2R}$	0	$\frac{\varepsilon}{2R}$	$\frac{\varepsilon C}{2}$	0

Example 6. At t = 0 switch S_1 is closed and remains closed for a long time and S_2 remains open. Now S_1 is opened and S_2 is closed. Find out



(i) The current through the capacitor immediately after that moment

(ii) Charge on the capacitor long after that moment.

(iii) Total charge flown through the cell of emf 2ε after S_2 is closed.

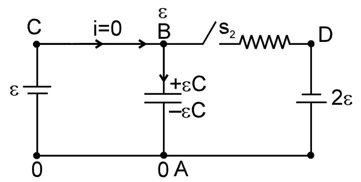
Solution : (i) Let Potential at point A is zero. Then at point B and C it will be ε (because current through the circuit is zero).

$V_B - V_A = (\varepsilon - 0)$

$\therefore$ Charge on capacitor = $C(\varepsilon - 0) = C\varepsilon$

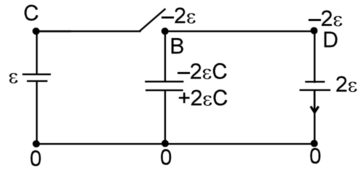
Now S_2 is closed and S_1 is open. (p.d. across capacitor and charge on it will not change suddenly)

Potential at A is zero so at D it is – 2ε.



$\therefore$ current through the capacitor = $\dfrac{\varepsilon-(-2\varepsilon)}{R} = \dfrac{3\varepsilon}{R}$ (B to D)

(ii) after a long time, i = 0



$V_B - V_A = V_D - V_A = -2\varepsilon$

$\therefore$ $Q = C(-2\varepsilon - 0) = -2\varepsilon C$

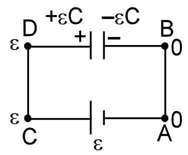
(iii) The charge on the lower plate (which is connected to the battery) changes from –εC to 2εC.

$\therefore$ this charge will come form the battery,

$\therefore$ charge flown from that cell is 3εC downward.

Example 7. A capacitor of capacitance C which is initially uncharged is connected with a battery. Find out heat dissipated in the circuit during the process of charging.

Solution : Final status



Let potential at point A is 0, so at B also 0 and at C and D it is ε. finally, charge on the capacitor

$Q_C = \varepsilon C$

$U_i = 0$

$U_f = \dfrac{1}{2} CV^2 = \dfrac{1}{2} C\varepsilon^2$

work done by battery = $\int Pdt$

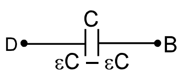
$W = \int \varepsilon i dt = \varepsilon \int i dt = \varepsilon \,.\, Q = \varepsilon.\, \varepsilon C = \varepsilon^2 C$

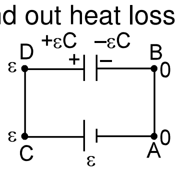
(Now onwards remember that w.d. by battery = εQ if Q has flown out of the cell from high potential and w.d. on battery is εQ if Q has flown into the cell through high potential)

Heat produced = $W - (U_f - U_i) = \varepsilon^2 C - \dfrac{1}{2}\varepsilon^2 C = \dfrac{C\varepsilon^2}{2}$.

Example 8. A capacitor of capacitance C which is initially charged upto a potential difference ε is connected with a battery of emf ε such that the positive terminal of battery is connected with positive plate of capacitor. Find out heat loss in the circuit during the process of charging.

Solution :





Since the initial and final charge on the capacitor is same before and after connection.

Here no charge will flow in the circuit so heat loss = 0

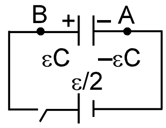
Example 9. A capacitor of capacitance C which is initially charged upto a potential difference ε is connected with a battery of emf $\varepsilon/2$ such that the positive terminal of battery is connected with positive plate of capacitor. After a long time

(i) Find out total charge flow through the battery

(ii) Find out total work done by battery

(iii) Find out heat dissipated in the circuit during the process of charging.

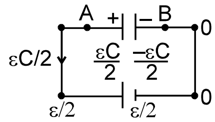
Solution : (i) Let potential of A is 0 so at B it is $\frac{\varepsilon}{2}$. So final charge on capacitor = $C\varepsilon/2$



Charge flow through the capacitor = $(C\varepsilon/2 - C\varepsilon) = -C\varepsilon/2$

So charge is entering into battery.

(ii) finally, Change in energy of capacitor = $U_{final} - U_{initial}$



$$= \frac{1}{2}C\left(\frac{\varepsilon}{2}\right)^2 - \frac{\varepsilon^2 C}{2} = \frac{1}{8}\varepsilon^2 C - \frac{1}{2}\varepsilon^2 C = -\frac{3}{8}\varepsilon^2 C$$

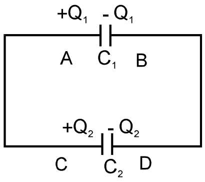
Work done by battery = $\frac{\varepsilon}{2} \times \left(-\frac{\varepsilon C}{2}\right) = -\frac{\varepsilon^2 C}{4}$

(iii) Work done by battery = Change in energy of capacitor + Heat produced

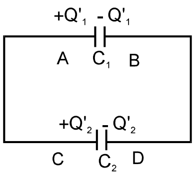
Heat produced = $\frac{3\varepsilon^2 C}{8} - \frac{\varepsilon^2 C}{4} = \frac{\varepsilon^2 C}{8}$

8. DISTRIBUTION OF CHARGES ON CONNECTING TWO CHARGED CAPACITORS:

When two capacitors are C_1 and C_2 are connected as shown in figure



Initially



Finally

Before connecting the capacitors		
Parameter	**Ist Capacitor**	**IInd Capacitor**
Capacitance	C_1	C_2
Charge	Q_1	Q_2
Potential	V_1	V_2

After connecting the capacitors		
Parameter	**Ist Capacitor**	**IInd Capacitor**
Capacitance	C_1	C_2
Charge	Q'_1	Q'_2
Potential	V	V

(a) Common potential : By charge conservation of plates A and C before and after connection.

$Q_1 + Q_2 = C_1V + C_2V$

$$\Rightarrow \; V = \frac{Q_1+Q_2}{C_1+C_2} = \frac{C_1V_1+C_2V_2}{C_1+C_2} = \frac{\text{Total charge}}{\text{Total capacitance}}$$

(b) $Q_1' = C_1V = \dfrac{C_1}{C_1+C_2}(Q_1 + Q_2) \quad \Rightarrow \quad Q_2' = C_2V = \dfrac{C_2}{C_1+C_2}(Q_1 + Q_2)$

(c) Heat loss during redistribution :

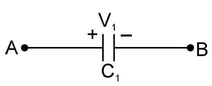
$$\Delta H = U_i - U_f = \frac{1}{2}\frac{C_1C_2}{C_1+C_2}(V_1 - V_2)^2$$

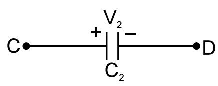
The loss of energy is in the form of Joule heating in the wire.

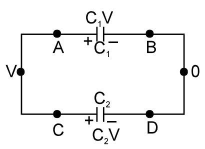
Note : (i) When plates of similar charges are connected with each other (+ with + and – with –) then put all values (Q_1, Q_2, V_1, V_2) with positive sign.

(ii) When plates of opposite polarity are connected with each other (+ with –) then take charge and potential of one of the plate to be negative.

Derivation of above formulae :







Let potential of B and D is zero and common potential on capacitors is V, then at A and C it will be V

$C_1V + C_2V = C_1V_1 + C_2V_2$

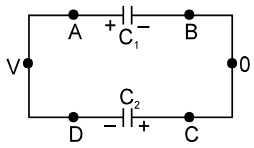
$$V = \frac{C_1V_1+C_2V_2}{C_1+C_2} \quad \Rightarrow \quad H = \frac{1}{2}C_1V_1^2 + \frac{1}{2}C_2V_2^2 - \frac{1}{2}(C_1 + C_2)V^2$$

$$= \frac{1}{2}C_1V_1^2 + \frac{1}{2}C_2V_2^2 - \frac{1}{2}\frac{(C_1V_1+C_2V_2)^2}{(C_1+C_2)}$$

$$= \frac{1}{2}\left[\frac{C_1^2V_1^2 + C_1C_2V_1^2 + C_2C_1V_2^2 + C_2^2V_2^2 - C_1^2V_1^2 - C_2^2V_2^2 - 2C_1C_2V_1V_2}{C_1+C_2}\right] = \frac{1}{2}\frac{C_1C_2}{C_1+C_2}(V_1 - V_2)^2$$

$$H = \frac{1}{2}\frac{C_1C_2}{C_1+C_2}(V_1 - V_2)^2$$

When oppositely charge terminals are connected then



$\therefore \; C_1V + C_2V = C_1V_1 - C_2V_2$

$$V = \frac{C_1V_1 - C_2V_2}{C_1+C_2} \text{ and } H = \frac{1}{2}\frac{C_1C_2}{C_1+C_2}(V_1 + V_2)^2$$

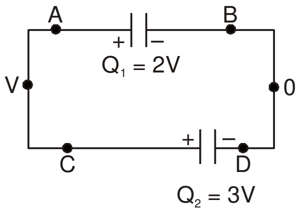
Solved Examples

Example 10 Find out the following if A is connected with C and B is connected with D.

(i) How much charge flows in the circuit.

(ii) How much heat is produced in the circuit.

Solution : **(i)**



Let potential of B and D is zero and common potential on capacitors is V, then at A and C it will be V.

By charge conservation, $3V + 2V = 40 + 30$

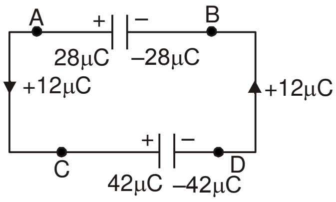
$5V = 70$

$V = 14$ volt

Charge flow = $40 - 28 = 12\ \mu C$

Now final charges on each plate is shown in the figure

(ii) Heat produced = $\frac{1}{2} \times 2 \times (20)^2 + \frac{1}{2} \times 3 \times (10)^2 - \frac{1}{2} \times 5 \times (14)^2$

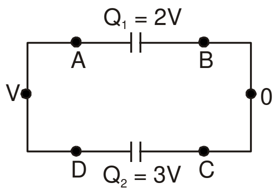


$= 400 + 150 - 490 = 550 - 490 = 60\ \mu J$

Note : (i) When capacitor plates are joined then the charge remains conserved.

(ii) We can also use direct formula of redistribution as given above.

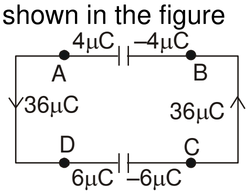
Example 11. Repeat above question if A is connected with D and B is connected with C.



Solution : Let potential of B and C is zero and common potential on capacitors is V, then at A and D it will be V

$2V + 3V = 10 \Rightarrow V = 2$ volt

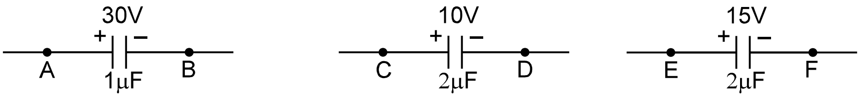
Now charge on each plate is shown in the figure



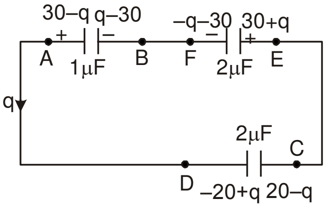
Heat produced = $400 + 150 - \frac{1}{2} \times 5 \times 4 = 550 - 10 = 540\ \mu J$

Note : Here heat produced is more. Think why?

Example 12. Three capacitors as shown of capacitance 1μF, 2μF and 2μF are charged upto potential difference 30 V, 10 V and 15 V respectively. If terminal A is connected with D, C is connected with E and F is connected with B. Then find out charge flow in the circuit and find the final charges on capacitors.



Solution : Let charge flow is q. Now applying kirchhoff's voltage low

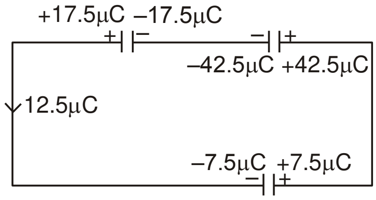


$$-\frac{(q-20)}{2}-\frac{(30+q)}{2}+\frac{30-q}{1}=0$$

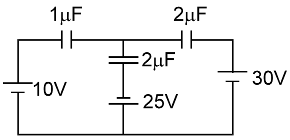
$-2q = -25$

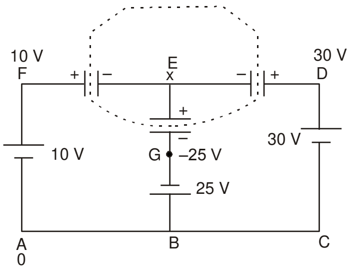
$q = 12.5\ \mu C$

Final charges on plates

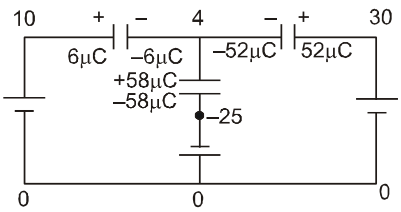


Example 13. In the given circuit find out the charge on each capacitor. (Initially they are uncharged)





Let potential at A is 0, so at D it is 30 V, at F it is 10 V and at point G potential is –25V and let potential at E is x. Now apply kirchhoff's I[st] law at point E. (Total charge of all the plates connected to 'E' must be same as before i.e. 0)



$\therefore \quad (x - 10) + (x - 30)2 + (x + 25)2 = 0$

$5x = 20$

$x = 4$ V

Final charges :

$Q_{2\mu F} = (30 - 4)2 = 52\mu C$

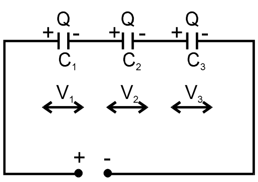
$Q_{1\mu F} = (10 - 4) = 6\mu C$

$Q_{2\mu F} = (4 - (-25))2 = 58\ \mu C$

9. COMBINATION OF CAPACITORS :

9.1 Series Combination :

(i) When initially uncharged capacitors are connected as shown then the combination is called series combination.



(ii) All capacitors will have same charge but different potential difference across them.

(iii) We can say that $V_1 = \frac{Q}{C_1}$

V_1 = potential across C_1

Q = charge on positive plate of C_1

C_1 = capacitance of capacitor similarly

$V_2 = \frac{Q}{C_2}$, $V_3 = \frac{Q}{C_3}$; ....

(iv) $V_1 : V_2 : V_3 = \frac{1}{C_1} : \frac{1}{C_2} : \frac{1}{C_3}$

We can say that potential difference across capacitor is inversely proportional to its capacitance in series combination.

$V \propto \frac{1}{C}$

Note : In series combination the smallest capacitor gets maximum potential.

(v) $$V_1 = \frac{\frac{1}{C_1}}{\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}+......}V$$

$$V_2 = \frac{\frac{1}{C_2}}{\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}+......}V$$

$$V_3 = \frac{\frac{1}{C_3}}{\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}+......}V$$

Where $V = V_1 + V_2 + V_3$

(vi) Equivalent Capacitance : Equivalent capacitance of any combination is that capacitance which when connected in place of the combination, stores same charge and energy that of the combination.

In series : $\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + ...$

Note : In series combination equivalent capacitance is always less the smallest capacitor of combination.

(vii) Energy stored in the combination $U_{combination} = \frac{Q^2}{2C_1} + \frac{Q^2}{2C_2} + \frac{Q^2}{2C_3}$

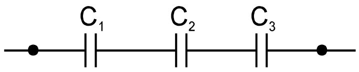
$U_{combination} = \frac{Q^2}{2C_{eq}}$

Energy supplied by the battery in charging the combination $U_{battery} = Q \times V = Q . \frac{Q}{C_{eq}} = \frac{Q^2}{C_{eq}}$

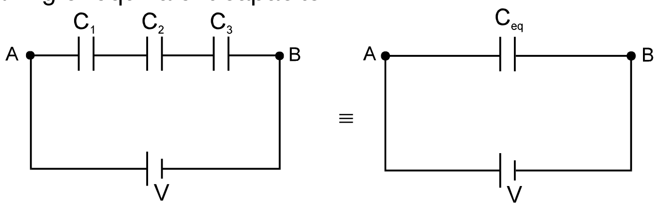
$\frac{U_{combination}}{U_{battery}} = \frac{1}{2}$

Note : Half of the energy supplied by the battery is stored in form of electrostatic energy and half of the energy is converted into heat through resistance. (if capacitors are initially uncharged)

Derivation of Formulae :

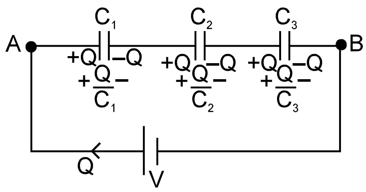


meaning of equivalent capacitor



$C_{eq} = \frac{Q}{V}$

Now, initially, the capacitor has no charge. Applying kirchhoff's voltage law



$$\frac{-Q}{C_1} + \frac{-Q}{C_2} + \frac{-Q}{C_3} + V = 0.$$

$$V = Q\left[\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}\right] \; ; \quad \frac{V}{Q} = \frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}$$

$$\Rightarrow \quad \frac{1}{C_{eq}} = \frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3} \text{ in general } \frac{1}{C_{eq}} = \sum_{n=1}^{n}\frac{1}{C_n}$$

Solved Examples

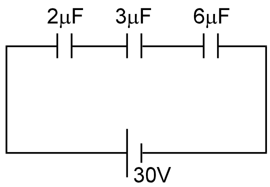
Example 14. Three initially uncharged capacitors are connected in series as shown in circuit with a battery of emf 30V. Find out following :

(i) charge flow through the battery,

(ii) potential energy in 3 μF capacitor.

(iii) U_{total} in capacitors

(iv) heat produced in the circuit



Solution : $\frac{1}{C_{eq}} = \frac{1}{2} + \frac{1}{3} + \frac{1}{6} = \frac{3+2+1}{6} = 1$

$C_{eq} = 1\mu F$.

(i) $Q = C_{eq} V = 30\mu C$.

(ii) charge on $3\mu F$ capacitor $= 30\mu C$

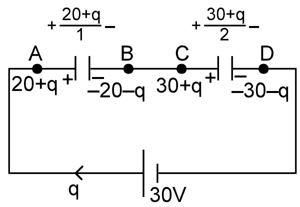
$$\text{energy} = \frac{Q^2}{2C} = \frac{30\times30}{2\times3} = 150\mu J$$

(iii) $U_{total} = \frac{30\times30}{2}\ \mu J = 450\ \mu J$

(iv) Heat produced = $(30\ \mu C)\ (30) - 450\ \mu J = 450\ \mu J$.

Example 15. Two capacitors of capacitance 1 μF and 2μF are charged to potential difference 20V and 15V as shown in figure. If now terminal B and C are connected together terminal A with positive of battery and D with negative terminal of battery then find out final charges on both the capacitor

Solution : Now applying kirchoff voltage law



$$\frac{-(20+q)}{1} - \frac{30+q}{2} + 30 = 0$$

$-40 - 2q - 30 - q = -60$

$3q = -10$

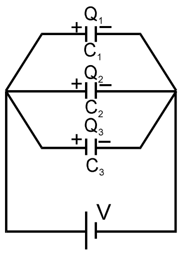
Charge flow = $-10/3\ \mu C$.

Charge on capacitor of capacitance $1\mu F = 20 + q = \frac{50}{3}\ \mu C$

Charge on capacitor of capacitance $2\mu F = 30 + q = \frac{80}{3}\ \mu C$

9.2 Parallel Combination :

(i) When one plate of each capacitors (more than one) is connected together and the other plate of each capacitor is connected together, such combination is called parallel combination.



(ii) All capacitors have same potential difference but different charges.

(iii) We can say that :

$Q_1 = C_1V$

Q_1 = Charge on capacitor C_1

C_1 = Capacitance of capacitor C_1

V = Potential across capacitor C_1

(iv) $Q_1 : Q_2 : Q_3 = C_1 : C_2 : C_3$

The charge on the capacitor is proportional to its capacitance

$Q \propto C$

(v) $Q_1 = \frac{C_1}{C_1+C_2+C_3}Q \quad \Rightarrow \quad Q_2 = \frac{C_2}{C_1+C_2+C_3}Q$

$Q_3 = \frac{C_3}{C_1+C_2+C_3}Q$

Where $Q = Q_1 + Q_2 + Q_3$ ......

Note : Maximum charge will flow through the capacitor of largest value.

(vi) Equivalent capacitance of parallel combination $C_{eq} = C_1 + C_2 + C_3$

Note : Equivalent capacitance is always greater than the largest capacitor of combination.

(vii) Energy stored in the combination :

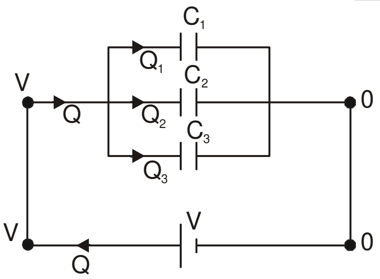
$V_{combination} = \frac{1}{2}C_1V^2 + \frac{1}{2}C_2V^2 + ... = \frac{1}{2}(C_1 + C_2 + C_3 .....)V^2 = \frac{1}{2}C_{eq}V^2$

$U_{battery} = QV = CV^2$

$$\frac{U_{combination}}{U_{battery}} = \frac{1}{2}$$

Note : Half of the energy supplied by the battery is stored in form of electrostatic energy and half of the energy is converted into heat through resistance. (If all capacitors are initially uncharged)

Formulae Derivation for parallel combination :



$Q = Q_1 + Q_2 + Q_3$

$\frac{Q}{V} = C_1V + C_2V + C_3V = V(C_1 + C_2 + C_3)$

$= C_1 + C_2 + C_3$

$C_{eq} = C_1 + C_2 + C_3$

In general $C_{eq} = \sum_{n=1}^{n} C_n$

Solved Example

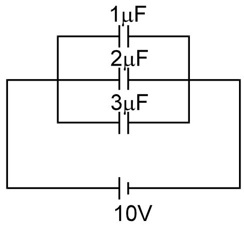
Example 16. Three initially uncharged capacitors are connected to a battery of 10 V is parallel combination find out following

(i) charge flow from the battery

(ii) total energy stored in the capacitors

(iii) heat produced in the circuit

(iv) potential energy in the 3μF capacitor.



Solution : (i) $Q = (30 + 20 + 10)\mu C = 60\ \mu C$

(ii) $U_{total} = \frac{1}{2} \times 6 \times 10 \times 10 = 300\ \mu J$

(iii) heat produced $= 60 \times 10 - 300 = 300\ \mu J$

(iv) $U_{3\mu F} = \frac{1}{2} \times 3 \times 10 \times 10 = 150\ \mu J$

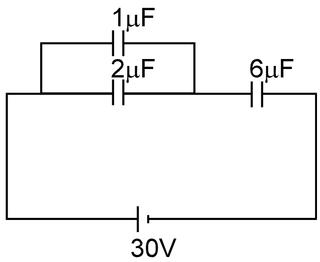
9.3 Mixed Combination :

The combination which contains mixing of series parallel combinations or other complex combinations fall in mixed category. There are two types of mixed combinations

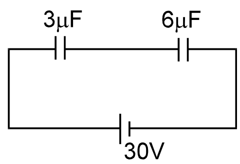
(i) Simple (ii) Complex.

Solved Example

Example 17. In the given circuit find out charge on 6μF and 1μF capacitor.



Solution : It can be simplified as $C_{eq} = \frac{18}{9} = 2\mu F$



charge flow through the cell $= 30 \times 2\ \mu C$

$Q = 60\ \mu C$

Now charge on 3μF = Charge on 6μF= 60 μC

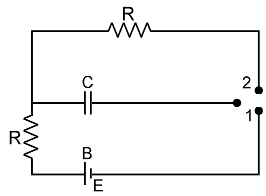
Potential difference across 3μF = 60/ 3= 20 V

$\therefore$ Charge on 1μF = 20 μC.

10. CHARGING AND DISCHARGING OF A CAPACITOR

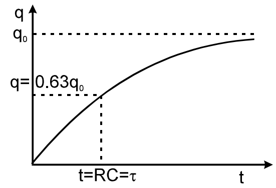
10.1 Charging of a condenser :

(i) In the following circuit. If key 1 is closed then the condenser gets charged. Finite time is taken in the charging process. The quantity of charge at any instant of time t is given by $q = q_0[1 - e^{-(t/RC)}]$



Where q_0 = maximum final value of charge at $t = \infty$.

According to this equations the quantity of charge on the condenser increases exponentially with increase of time.



(ii) If $t = RC = \tau$ then

$$q = q_0 [1 - e^{-(RC/RC)}] = q_0 \left[1 - \frac{1}{e}\right]$$

or $q = q_0 (1 - 0.37) = 0.63\, q_0 = 63\%$ of q_0

(iii) Time t = RC is known as time constant.

i.e. the time constant is that time during which the charge rises on the condenser plates to 63% of its maximum value.

(iv) The potential difference across the condenser plates at any instant of time is given by

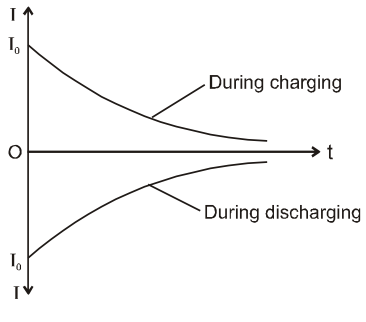
$V = V_0[1 - e^{-(t/RC)}]$ volt

(v) The potential curve is also similar to that of charge. During charging process an electric current flows in the circuit for a small interval of time which is known as the transient current. The value of this current at any instant of time is given by

$I = I_0[e^{-(t/RC)}]$ ampere

According to this equation the current falls in the circuit exponentially (Fig.).

(vi) If $t = RC = \tau$ = Time constant

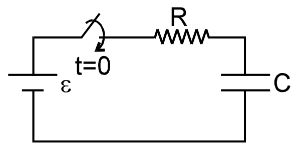


$$I = I_0 e^{(-RC/RC)} = \frac{I_0}{e} = 0.37\, I_0$$

$= 37\%$ of I_0

i.e. time constant is that time during which current in the circuit falls to 37% of its maximum value.

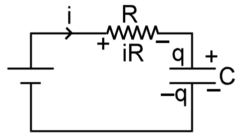
Derivation of formulae for charging of capacitor



it is given that initially capacitor is uncharged.

let at any time charge on capacitor is q

Applying kirchoff voltage law

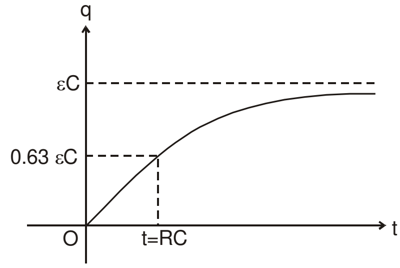


$$\varepsilon - iR - \frac{q}{C} = 0 \quad \Rightarrow \quad iR = \frac{\varepsilon C - q}{C}$$

$$i = \frac{\varepsilon C - q}{CR} \quad \Rightarrow \quad \frac{dq}{dt} = \frac{\varepsilon C - q}{CR}$$

$$\frac{dq}{dt} = \frac{\varepsilon C - q}{CR} \quad \Rightarrow \quad \frac{CR}{\varepsilon C - q}\,.\,dq = dt.$$

$$\int_O^q \frac{dq}{\varepsilon C - q} = \int_O^t \frac{dt}{RC} \quad \Rightarrow \quad -\ln(\varepsilon C - q) + \ln \varepsilon C = \frac{t}{RC}$$



$$\ln \frac{\varepsilon C}{\varepsilon C - q} = \frac{t}{RC} \quad ; \quad \varepsilon C - q = \varepsilon C\,.\,e^{-t/RC}$$

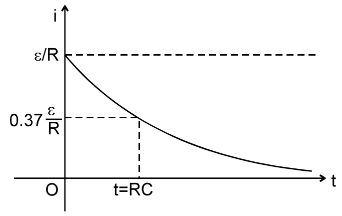
$q = \varepsilon C(1 - e^{-t/RC})$

RC = time constant of the RC series circuit.

After one time constant

$$q = \varepsilon C\left(1 - \frac{1}{e}\right) = \varepsilon C\,(1 - 0.37) = 0.63\,\varepsilon C.$$

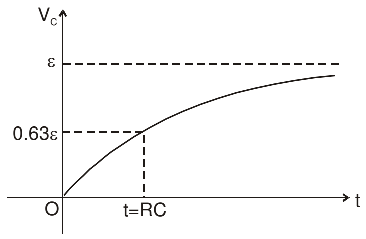
Current at any time t



$$i = \frac{dq}{dt} = \varepsilon C\left(-e^{-t/RC}\left(-\frac{1}{RC}\right)\right)$$

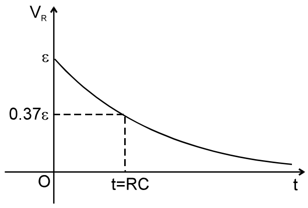
$$= \frac{\varepsilon}{R}e^{-t/RC}$$

Voltage across capacitor after one time constant V = 0.63 ε



$Q = CV$; $V_C = \varepsilon(1 - e^{-t/RC})$

Voltage across the resistor



$V_R = iR = \varepsilon e^{-t/RC}$

By energy conservation,

Heat dissipated = work done by battery – $\Delta U_{capacitor}$

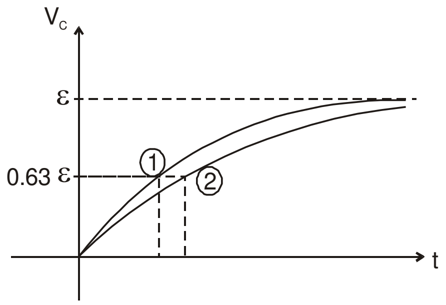
$$= C\varepsilon(\varepsilon) - \left(\frac{1}{2}C\varepsilon^2 - 0\right) = \frac{1}{2}C\varepsilon^2$$

Alternatively : Heat = H = $\int_0^{\infty} i^2 R\,dt$

$$= \int_0^{\infty} \frac{\varepsilon^2}{R^2} e^{-\frac{2t}{RC}} R\,dt = \frac{\varepsilon^2}{R}\int_0^{\infty} e^{-2t/RC}\,dt = \frac{\varepsilon^2}{R}\left[\frac{e^{-\frac{2t}{RC}}}{-2/RC}\right]_0^{\infty}$$

$$= -\frac{\varepsilon^2 RC}{2R}\left[e^{-\frac{2t}{RC}}\right]_0^{\infty} = \frac{\varepsilon^2 C}{2}$$

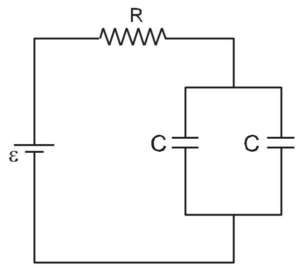
Note:



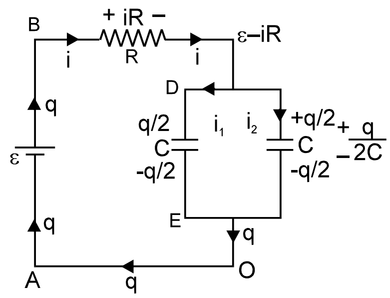
In the figure time constant of (2) is more than (1)

Solved Example

Example 18 Without using the formula of equivalent. Find out charge on capacitor and current in all the branches as a function of time.



Solution : Applying KVL in ABDEA



$$\varepsilon - iR = \frac{q}{2C}$$

$$i = \frac{\varepsilon}{R} - \frac{q}{2CR} = \frac{2C\varepsilon - q}{2CR}$$

$$\frac{dq}{2\varepsilon C - q} = \frac{dt}{2CR}$$

$$\int_0^q \frac{dq}{(2\varepsilon C - q)} = \frac{t}{2CR}$$

$$\frac{2\varepsilon C - q}{2\varepsilon C} = e^{-t/2RC}$$

$$q = 2\varepsilon C\,(1 - e^{-t/2RC})$$

$$q_1 = \frac{q}{2} = \varepsilon C\,(1 - e^{-t/2RC}) \Rightarrow i_1 = \frac{\varepsilon}{2R} e^{-t/2RC}$$

$$q_2 = \frac{q}{2} = \varepsilon C\,(1 - e^{-t/2RC}) \Rightarrow i_2 = \frac{\varepsilon}{2R} e^{-t/2RC}$$

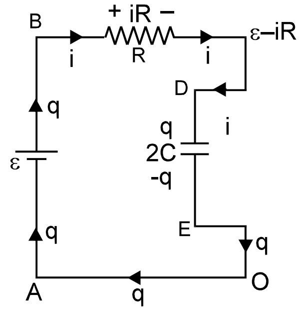
Alternate solution

by equivalent

Time constant of circuit = 2C × R = 2RC

maximum charge on capacitor = 2C × ε = 2Cε

Hence equations of charge and current are as given below



$q = 2\varepsilon C\,(1 - e^{-t/2RC})$

$q_1 = \dfrac{q}{2} = \varepsilon C\,(1 - e^{-t/2RC}) \quad \Rightarrow \quad i_1 = \dfrac{\varepsilon}{2R}\,e^{-t/2RC}$

$q_2 = \dfrac{q}{2} = \varepsilon C\,(1 - e^{-t/2RC}) \quad \Rightarrow \quad i_2 = \dfrac{\varepsilon}{2R}\,e^{-t/2RC}$

Example 19 A capacitor is connected to a 36 V battery through a resistance of 20Ω. It is found that the potential difference across the capacitor rises to 12.0 V in 2μs. Find the capacitance of the capacitor.

Solution : The charge on the capacitor during charging is given by $Q = Q_0(1 - e^{-t/RC})$.

Hence, the potential difference across the capacitor is $V = Q/C = Q_0/C\,(1 - e^{-t/RC})$.

Here, at t = 2 μs, the potential difference is 12V whereas the steady potential difference is

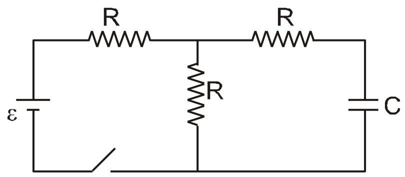
$Q_0/C = 36V$. So, $\Rightarrow \quad 12V = 36V(1 - e^{-t/RC})$

or, $1 - e^{-t/RC} = \dfrac{1}{3}$ or, $e^{-t/RC} = \dfrac{2}{3}$

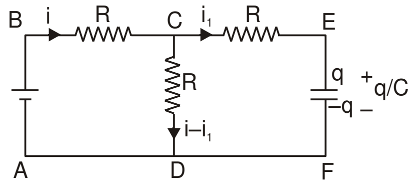
or, $\dfrac{t}{RC} = \ell n\left(\dfrac{3}{2}\right) = 0.405$ or, $RC = \dfrac{t}{0.405} = \dfrac{2\ \mu s}{0.45} = 4.936\ \mu s$

or, $C = \dfrac{4.936\mu s}{20\Omega} = 0.25\ \mu F$.

Example 20. Initially the capacitor is uncharged find the charge on capacitor as a function of time, if switch is closed at t = 0.



Solution : Applying KVL in loop ABCDA



$\varepsilon - iR - (i - i_1)\,R = 0$

$\varepsilon - 2iR + i_1R = 0 \qquad .....(i)$

Applying KVL in loop ABCEFDA

$\varepsilon - iR - i_1R - \dfrac{q}{C} = 0$

by eq (i) $\dfrac{2\varepsilon - \varepsilon - i_1R - 2i_1R}{2} = \dfrac{q}{C} \Rightarrow \varepsilon C - 3i_1RC = 2q$

$\varepsilon C - 2q = 3\,\dfrac{dq}{dt}.\,RC \quad \Rightarrow \int_0^q \dfrac{dq}{\varepsilon C - 2q} = \int_0^t \dfrac{dt}{3RC}$

$-\dfrac{1}{2}\ln\dfrac{\varepsilon C - 2q}{\varepsilon C} = \dfrac{t}{3RC} \Rightarrow q = \dfrac{\varepsilon C}{2}\left(1 - e^{-2t/3RC}\right)$

Method for objective :

In any circuit when there is only one capacitor then

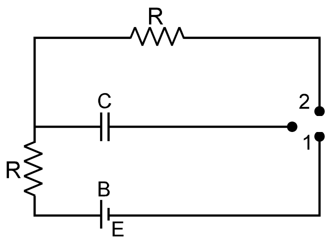
$q = Q_{st}\left(1-e^{-t/\tau}\right)$; Q_{st} = steady state charge on capacitor (has been found in article 6 in this sheet)

$\tau = R_{eff}.\ C$

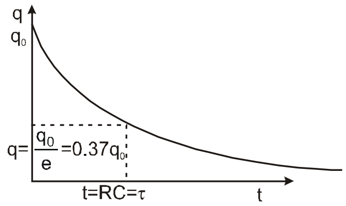
$R_{effective}$ is the resistance between the capacitor when battery is replaced by its internal resistance.

10.2 Discharging of a condenser :

(i) In the above circuit (in article 10.1) if key 1 is opened and key 2 is closed then the condenser gets discharged.



(ii) The quantity of charge on the condenser at any instant of time t is given by $q = q_0\ e^{-(t/RC)}$



i.e. the charge falls exponentially.

here q_0 = initial charge of capacitor

(iii) If $t = RC = \tau$ = time constant, then $q = \frac{q_0}{e} = 0.37q_0$ = 37% of q_0

i.e., the time constant is that time during which the charge on condenser plates in discharge process, falls to 37%

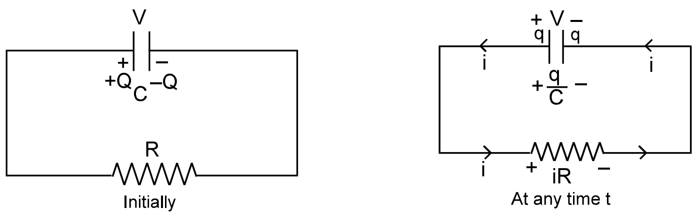
(iv) The dimensions of RC are those of time i.e. $M^0L^0T^1$ and the dimensions of $\frac{1}{RC}$ are those of frequency i.e. $M^0L^0T^{-1}$.

(v) The potential difference across the condenser plates at any instant of time t is given by $V = V_0e^{-(t/RC)}$ Volt.

(vi) The transient current at any instant of time is given by $I = -I_0e^{-(t/RC)}$ ampere.

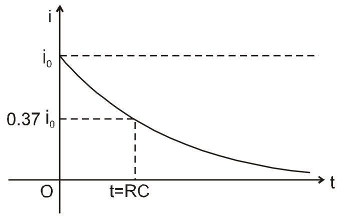
i.e. the current in the circuit decreases exponentially but its direction is opposite to that of charging current. (– ive only means that direction of current is opposite to that at charging current)

Derivation of equation of discharging circuit :



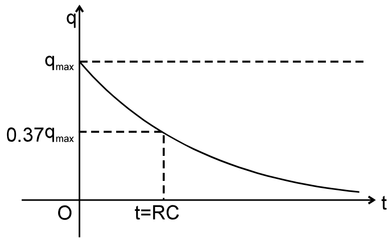
Applying K.V.L.

$$+\frac{q}{C} - iR = 0$$



$$i = \frac{q}{CR}$$

$$\int_Q^q \frac{-dq}{q} = \int_0^t \frac{dt}{CR}$$



$$-\ln\frac{q}{Q} = +\frac{t}{RC}$$

$$q = Q \,.\, e^{-t/RC}$$

$$i = -\frac{dq}{dt} = \frac{Q}{RC}e^{-t/RC} = i_0\, e^{-t/RC}$$

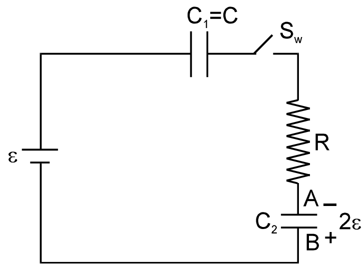
Solved Example

Example 21 At t = 0, switch is closed, if initially C_1 is uncharged and C_2 is charged to a potential difference 2ε then find out following (Given $C_1 = C_2 = C$)

(a) Charge on C_1 and C_2 as a function of time.

(b) Find out current in the circuit as a function of time.

(c) Also plot the graphs for the relations derived in part (a).



Solution : Let q charge flow in time 't' from the battery as shown. The charge on various plates of the capacitor is as shown in the figure. Now applying KVL

$$\varepsilon - \frac{q}{C} - iR - \frac{q - 2\varepsilon C}{C} = 0$$

$$\varepsilon - \frac{q}{C} - \frac{q}{C} + 2\varepsilon - iR = 0$$

$$3\varepsilon = \frac{2q}{C} + iR \quad \Rightarrow \quad 3\varepsilon - iR = \frac{2q}{C}$$

$$3\varepsilon C - iRC = 2q \quad \Rightarrow \quad \frac{dq}{dt}RC = 3\varepsilon C - 2q$$

$$\int_0^q \frac{dq}{3\varepsilon C - 2q} = \int_0^t \frac{dt}{RC} \quad \Rightarrow \quad -\frac{1}{2}\ell n\left(\frac{3C\varepsilon - 2q}{3C\varepsilon}\right) = \frac{t}{RC}$$

$$\ln\left(\frac{3\varepsilon C - 2q}{3\varepsilon C}\right) = -\frac{2t}{RC} \Rightarrow \quad 3\varepsilon C - 2q = 3\varepsilon C\; e^{-2t/RC}$$

$3\varepsilon C\,(1 - e^{-2t/RC}) = 2q \Rightarrow q = \frac{3}{2}\varepsilon C\;(1 - e^{-2t/RC})$

(charge on C, as function of time) **Ans.**

$i = \frac{dq}{dt} = \frac{3\varepsilon}{R}\,e^{-2t/RC}$ **Ans.**

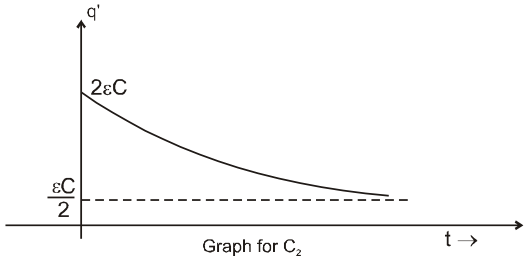
Charge on C_2 as function of time :

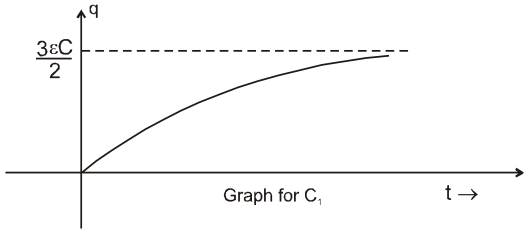
$q' = 2\varepsilon C - q$

$= 2\varepsilon C - \frac{3}{2}\varepsilon C + \frac{3}{2}\;\varepsilon C\,e^{-2t/RC}$

$= \frac{\varepsilon C}{2} + \frac{3}{2}\;\varepsilon C e^{-2t/RC}$

$= \frac{\varepsilon C}{2}\left[1 + 3e^{-2t/RC}\right]$

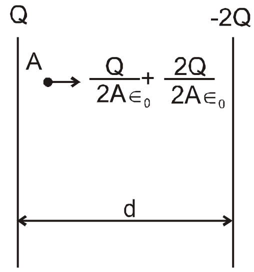




Solved Example

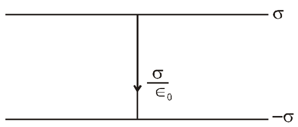
Example 22 Two parallel conducting plates of a capacitor of capacitance C containing charges Q and –2Q at a distance d apart. Find out potential difference between the plates of capacitors.

Solution : Capacitance = C



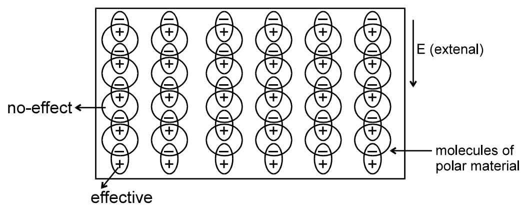
Electric field = $\frac{3Q}{2A\,\epsilon_0}$; $V = \frac{3Qd}{2A\,\epsilon_0}$ $\Rightarrow$ $V = \frac{3Q}{2C}$

11. CAPACITORS WITH DIELECTRIC



(i) In absence of dielectric $E = \frac{\sigma}{\epsilon_0}$

(ii) When a dielectric fills the space between the plates then molecules having dipole moment align themselves in the direction of electric field.



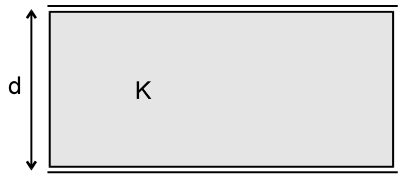
σ_b = induced charge density (called bound charge because it is not due to free electrons).

$*$ For polar molecules dipole moment $\neq 0$

$*$ For non-polar molecules dipole moment $= 0$

(iii) Capacitance in the presence of dielectric

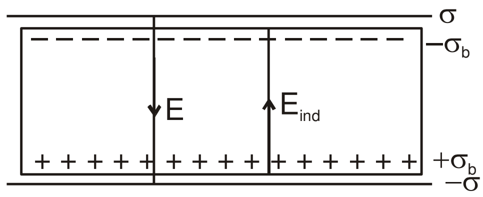
$$C = \frac{\sigma A}{V} = \frac{\sigma\ A}{\frac{\sigma}{K\in_0}.d} = \frac{AK\in_0}{d} = \frac{AK\in_0}{d}$$



Here capacitance is increased by a factor K.

$$C = \frac{AK\in_0}{d}$$

(iv) Polarisation of material : When nonpolar substance is placed in electric field then dipole moment is induced in the molecule. This induction of dipole moment is called polarisation of material. The induced charge also produce electric field.



σ_b = induced (bound) charge density.

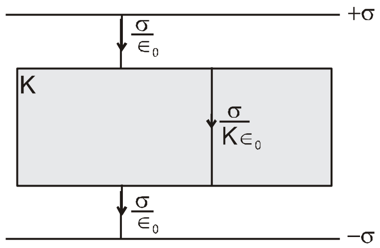
$$E_{in} = E - E_{ind} = \frac{\sigma}{\in_0} - \frac{\sigma_b}{\in_0}$$

It is seen the ratio of electric field between the plates in absence of dielectric and in presence of dielectric is constant for a material of dielectric. This ratio is called 'Dielectric constant' of that material. It is represented by ε_r or k.

$$E_{in} = \frac{\sigma}{K\in_0} \quad \Rightarrow \quad \sigma_b = \sigma\left(1 - \frac{1}{K}\right)$$

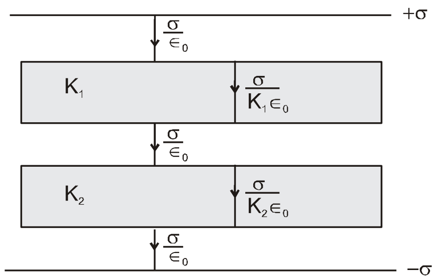
(v) If the medium does not filled between the plates completely then electric field will be as shown in figure

Case : (1) :

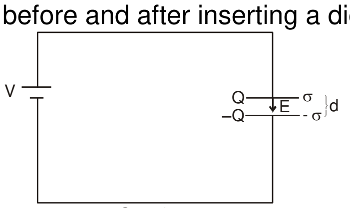


The total electric field produced by bound induced charge on the dielectric outside the slab is zero because they cancel each other.

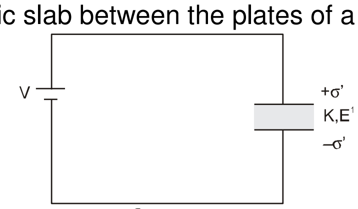
Case : (2)



(vi) Comparison of E (electric field), σ (surface charges density), Q (charge), C (capacitance) and before and after inserting a dielectric slab between the plates of a parallel plate capacitor.



Case I



Case II

$$C = \frac{\in_0 A}{d} \qquad\qquad C' = \frac{A\in_0 K}{d}$$

$Q = CV$

$E = \dfrac{\sigma}{\epsilon_0} = \dfrac{CV}{A\epsilon_0}$

$= \dfrac{V}{d}$

Here potential difference between the plates,

$Ed = V$

$E = \dfrac{V}{d}$

$\dfrac{V}{d} = \dfrac{\sigma}{\epsilon_0}$

$Q' = C'V$

$E' = \dfrac{\sigma}{K\epsilon_0} = \dfrac{CV}{A\epsilon_0}$

$E' = \dfrac{V}{d}$

Here potential difference between the plates

$E'd = V$

$E' = \dfrac{V}{d}$

$\dfrac{V}{d} = \dfrac{\sigma'}{K\epsilon_0}$

Equating both

$\dfrac{\sigma}{\epsilon_0} = \dfrac{\sigma'}{K\epsilon_0}$

$\sigma' = K\sigma$

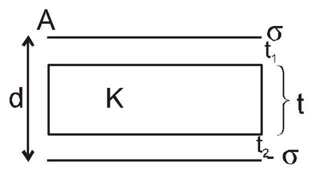
In the presence of dielectric, i.e. in case II capacitance of capacitor is more.

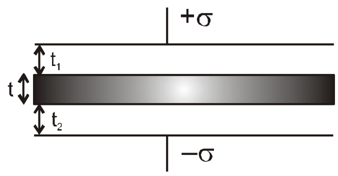
(vii) Energy density in a dielectric = $\dfrac{1}{2}\epsilon_0\epsilon_r E^2$

Solved Example

Example 23. If a dielectric slab of thickness t and area A is inserted in between the plates of a parallel plate capacitor of plate area A and distance between the plates d (d > t) then find out capacitance of system. What do you predict about the dependence of capacitance on location of slab?

Solution :





$C = \dfrac{Q}{v} = \dfrac{\sigma A}{v}$ $\quad V = \dfrac{\sigma t_1}{\epsilon_0} + \dfrac{\sigma t}{K\epsilon_0} + \dfrac{\sigma t_2}{\epsilon_0}$ $\quad (\because t_1 + t_2 = d - t)$

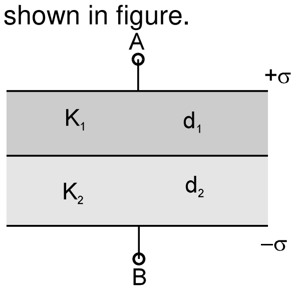
$= \dfrac{\sigma}{\epsilon_0}\left[t_1 + t_2 + \dfrac{t}{k}\right] \Rightarrow$ $\quad V = \dfrac{\sigma}{\epsilon_0}\left[d - t + \dfrac{t}{k}\right] = \dfrac{Q}{C} = \dfrac{\sigma A}{C}$ $\quad \Rightarrow C = \dfrac{\epsilon_0 A}{d - t + t/K}$

Note :

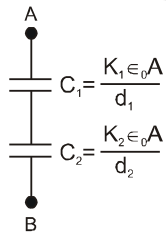
(i) Capacitance does not depend upon the position of dielectric (it can be shifted up or down still capacitance does not change).

(ii) If the slab is of metal then : $C = \dfrac{A\epsilon_0}{d - t}$ (for metal $k \to \infty$)

Example 24 Find out capacitance between A and B if two dielectric slabs of dielectric constant K_1 and K_2 of thickness d_1 and d_2 and each of area A are inserted between the plates of parallel plate capacitor of plate area A as shown in figure.



Solution : $C = \dfrac{\sigma A}{V}$; $V = E_1 d_1 + E_2 d_2 = \dfrac{\sigma d_1}{K_1 \in_0} + \dfrac{\sigma d_2}{K_2 \in_0} = \dfrac{\sigma}{\in_0}\left(\dfrac{d_1}{k_1} + \dfrac{d_2}{k_2}\right)$

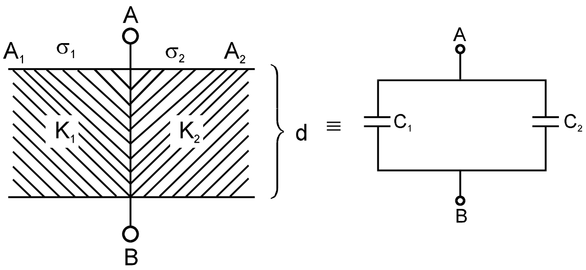


$\therefore \quad C = \dfrac{A \in_0}{\dfrac{d_1}{K_1} + \dfrac{d_2}{K_2}} \quad \Rightarrow \dfrac{1}{C} = \dfrac{d_1}{AK_1 \in_0} + \dfrac{d_2}{AK_2 \in_0}$

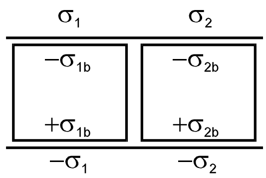
This formula suggests that the system between A and B can be considered as series combination of two capacitors.

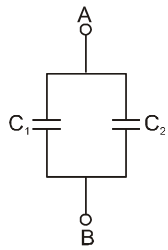
Example 25. Find out capacitance between A and B if two dielectric slabs of dielectric constant K_1 and K_2 of area A_1 and A_2 and each of thickness d are inserted between the plates of parallel plate capacitor of plate area A as shown in figure. $(A_1 + A_2 = A)$

Solution :



$C_1 = \dfrac{A_1K_1 \in_0}{d}$, $C_2 = \dfrac{A_2K_2 \in_0}{d}$





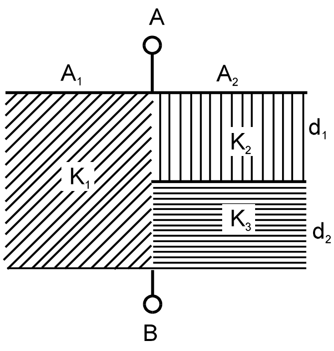
$E_1 = \dfrac{V}{d} = \dfrac{\sigma_1}{K_1 \in_0}$, $E_2 = \dfrac{V}{d} = \dfrac{\sigma_2}{K_2 \in_0}$

$\sigma_1 = \dfrac{K_1 \in_0 V}{d} \quad \sigma_2 = \dfrac{K_2 \in_0 V}{d}$

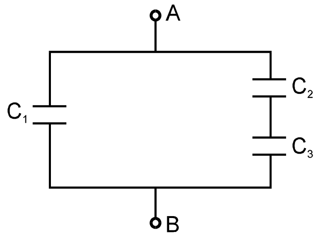
$C = \dfrac{Q_1 + Q_2}{V} = \dfrac{\sigma_1 A_1 + \sigma_2 A_2}{V} = \dfrac{K_1 \in_0 A_1}{d} + \dfrac{K_2 \in_0 A_2}{d}$

The combination is equivalent to : $C = C_1 + C_2$

Example 26. Find out capacitance between A and B if three dielectric slabs of dielectric constant K_1 of area A_1 and thickness d, K_2 of area A_2 and thickness d_1 and K_3 of area A_2 and thickness d_2 are inserted between the plates of parallel plate capacitor of plate area A as shown in figure. (Given distance between the two plates $d = d_1 + d_2$)



Solution : It is equivalent to $C = C_1 + \dfrac{C_2C_3}{C_2 + C_3}$

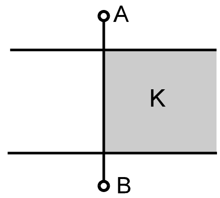


$$C = \frac{A_1K_2 \epsilon_0}{d_1 + d_2} + \frac{\frac{A_2K_2 \epsilon_0}{d_1} \cdot \frac{A_2K_3 \epsilon_0}{d_1}}{\frac{A_2K_2 \epsilon_0}{d_1} + \frac{A_2K_3 \epsilon_0}{d_2}}$$

$$= \frac{A_1K_1 \epsilon_0}{d_1 + d_2} + \frac{A_2^2K_2K_3 \epsilon_0}{A_2K_2 \epsilon_0 d_2 + A_2K_3 \epsilon_0 d_1}$$

$$= \frac{A_1K_2 \epsilon_0}{d_1 + d_2} + \frac{A_2^2K_2K_3 \epsilon_0}{K_2d_2 + K_3d_1}$$

Example 27. A dielectric of constant K is slipped between the plates of parallel plate condenser in half of the space as shown in the figure. If the capacity of air condenser is C, then new capacitance between A and B will be-



(A) $\dfrac{C}{2}$ (B) $\dfrac{C}{2K}$ (C) $\dfrac{C}{2}$ [1 + K] (D) $\dfrac{2[1+K]}{C}$

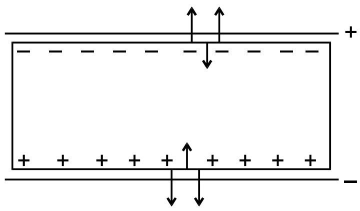
Solution : This system is equivalent to two capacitors in parallel with area of each plate $\dfrac{A}{2}$.

$$C' = C_1 + C_2 = \frac{\epsilon_0 A/2}{d} + \frac{\epsilon_0 (A/2)K}{d} = \frac{\epsilon_0 A}{2d} [1 + K] = \frac{C}{2} [1 + K]$$

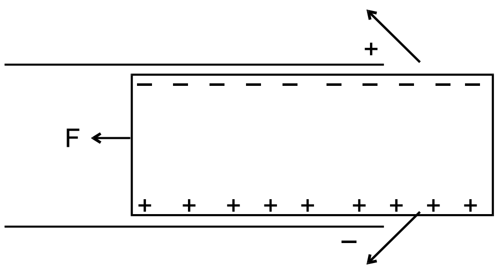
Hence the correct answer will be (C).

(viii) Force on a dielectric due to charged capacitor :

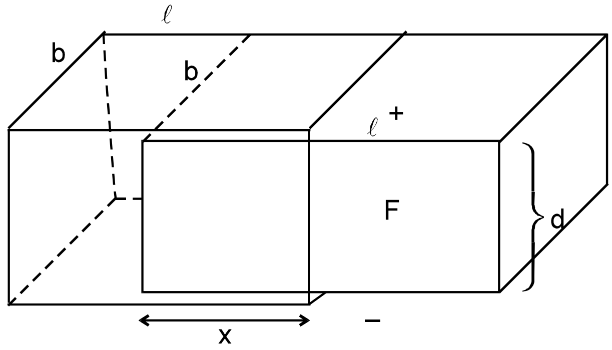
(a) If dielectric is completely inside the capacitor then force is equal to zero.



(b) If dielectric is not completely inside the capacitor.



Case-I : Voltage source remains connected



V = constant.

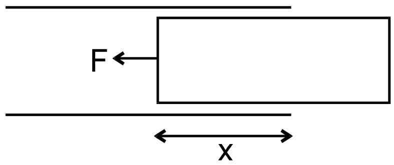
$U = \frac{1}{2} CV^2$

$F = \left(\frac{dU}{dx}\right) = \frac{V^2}{2}\frac{dC}{dx}$ where $C = \frac{xb \in_0 K}{d} + \frac{\in_0 (\ell - x)b}{d}$ $\Rightarrow C = \frac{\in_0 b}{d}$ $[Kx + \ell - x]$

$\frac{dC}{dx} = \frac{\in_0 b}{d}$ $(K - 1)$

$\therefore$ $F = \frac{\in_0 b(K-1)V^2}{2d}$ = constant (does not depend on x)

Case II : When charge on capacitor is constant



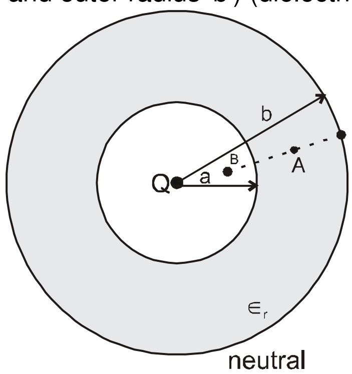
$C = \frac{xb \in_0 K}{d} + \frac{\in_0 (\ell - x)b}{d}$, $U = \frac{Q^2}{2C}$

$F = \left(\frac{dU}{dx}\right) = \frac{Q^2}{2C^2} \cdot \frac{dC}{dx}$ [where, $\frac{dC}{dx} = \frac{\in_0 b}{d}$ $(K - 1)$]

$= \frac{Q^2}{2C^2}.\frac{dC}{dx}$ (here force 'F' depends on x)

Solved Examples

Example 28. Find V and E at : (Q is a point charge kept at the centre of the non-conducting neutral thick sphere of inner radius 'a' and outer radius 'b') (dielectric constant = $\in_r$)



(i) $0 < r < a$ (ii) $a \leq r < b$ (iii) $r \geq b$

Solution : $-q$ and $+q$ charge will induce on inner and outer surface respectively

$$E(0 < r < a) = \frac{KQ}{r^2}$$

$$E\,(r \geq b) = \frac{KQ}{r^2}$$

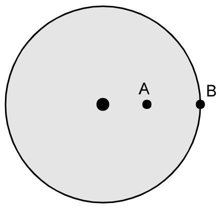
$$E\,(a \leq r < b) = \frac{KQ}{r^2} - \frac{Kq}{r^2} = \frac{KQ}{\epsilon_r\, r^2} \qquad \textbf{Ans.}$$

$$q = Q\,.\left(1 - \frac{1}{\epsilon_r}\right) \quad ; \quad V\,(r \geq b) = \frac{KQ}{r}$$

$$(a \leq r \leq b)\ V_A = V_P + \int_b^r \frac{KQ}{\epsilon_r\, r^2}(-dr) = \frac{kQ}{b} + \frac{kQ}{\epsilon_r}\left(\frac{1}{r} - \frac{1}{b}\right)$$

$$V\,(r \leq a) \qquad V_B = V_C + \int_a^r \frac{KQ}{r^2}(-dr) = \frac{kQ}{b} + \frac{kQ}{\epsilon_r}\left(\frac{1}{a} - \frac{1}{b}\right) + kQ\left(\frac{1}{r} - \frac{1}{a}\right)$$

Example 29. What is potential at a distance r (<R) in a dielectric sphere of uniform charge density ρ, radius R and dielectric constant ε_r.



Solution : $V_A = V_B + \frac{W_{B\to A}}{q}$

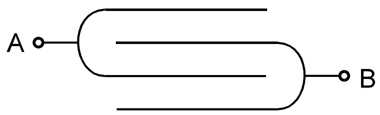
$$V = \frac{Q}{4\pi \epsilon_0 R} + \int_R^r \frac{\rho r}{3\epsilon_0 \epsilon_r}(-dr) = \frac{Q}{4\pi \epsilon_0 R} + \frac{\rho(R^2 - r^2)}{3\epsilon_0 \epsilon_r}$$

$$V_{outiside} = \frac{KQ}{r}$$

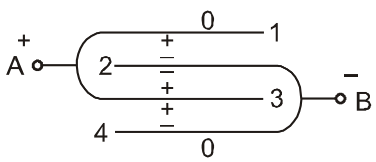
12. COMBINATION OF PARALLEL PLATES

Solved Examples

Example 30. Find out equivalent capacitance between A and B. (take each plate Area = A and distance between two conjugative plates is d)



Solution : Let numbers on the plates The charges will be as shown in the figure.

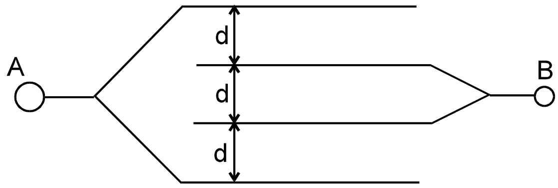


$V_{12} = V_{32} = V_{34}$

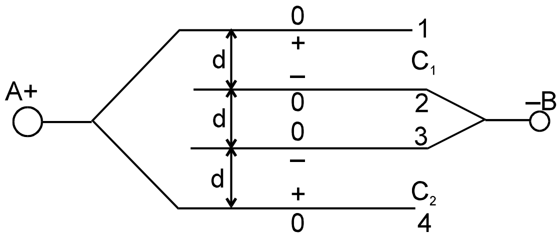
so all the capacitors are in parallel combination.

$$C_{eq} = C_1 + C_2 + C_3 = \frac{3A\epsilon_0}{d}$$

Example 31. Find out equivalent capacitance between A and B. (take each plate Area = A)

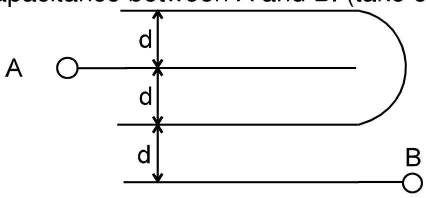


Solution :

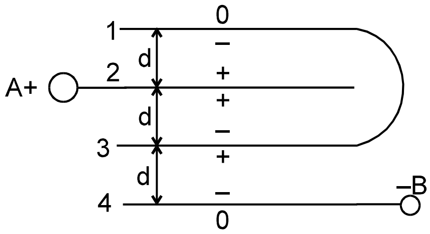


These are only two capacitors $C_{eq} = C_1 + C_2 = \frac{2A \in_0}{d}$

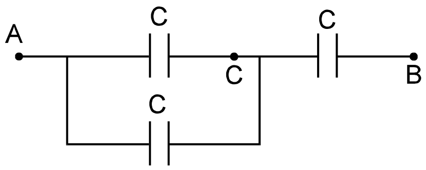
Example 32. Find out equivalent capacitance between A and B. (take each plate Area = A)



Solution :

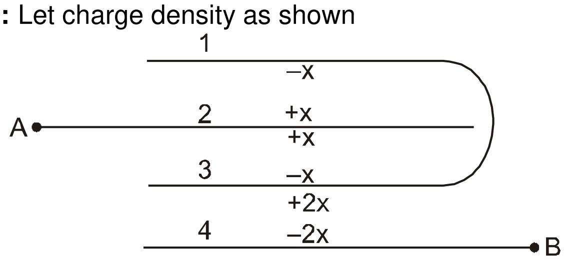


The modified circuit is



$C_{eq} = \frac{2C}{3} = \frac{2A \in_0}{3d}$

Other method : Let charge density as shown

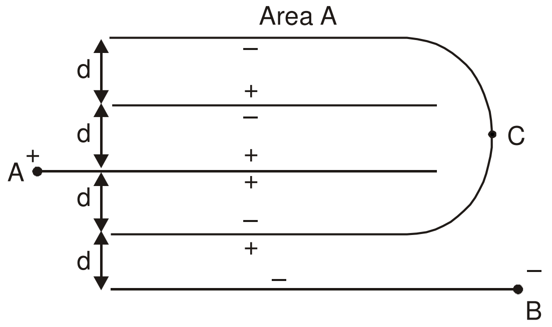


$C_{eq} = \frac{Q}{V} = \frac{2xA}{V}$

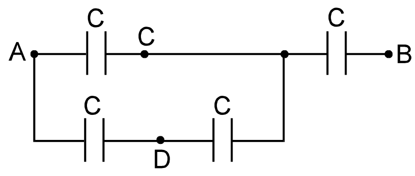
$V = V_2 - V_4 = (V_2 - V_3) + (V_3 - V_4) = \frac{xd}{\in_0} + \frac{2xd}{\in_0} = \frac{3xd}{\in_0}$

$\therefore \quad C_{eq} = \frac{2Ax \in_0}{3xd} = \frac{2A \in_0}{3d} = \frac{2C}{3}$.

Example 33. Find out equivalent capacitance between A and B.



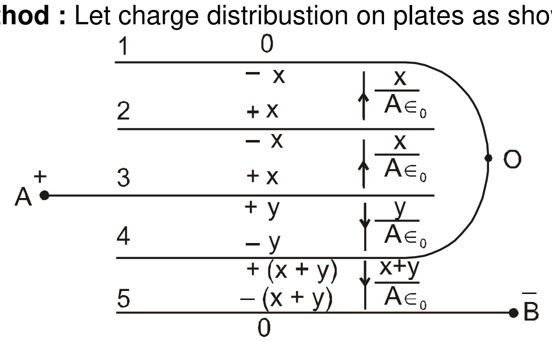
Solution : Let $C = \frac{A \epsilon_0}{d}$ Equivalent circuit :



$$\frac{1}{C_{eq}} = \frac{1}{C} + \frac{2}{3C} = \frac{5}{3C}$$

$$C_{eq} = \frac{3C}{5} = \frac{3A \epsilon_0}{5d}$$

Alternative Method : Let charge distribustion on plates as shown :



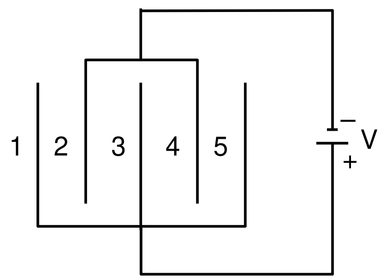
$$C = \frac{Q}{V} = \frac{x+y}{V_{AB}}$$

Potential of 1 and 4 is same

$$\frac{y}{A \epsilon_0} = \frac{2x}{A \epsilon_0} \qquad y = 2x$$

$$V = \left(\frac{2y + x}{A\varepsilon_0}\right)d \quad \Rightarrow \quad C = \frac{(x+2x)A \epsilon_0}{(5x)d} = \frac{3A \epsilon_0}{5d}.$$

Example 34. Five similar condenser plates, each of area A, are placed at equal distance d apart and are connected to a source of e.m.f. V as shown in the following diagram. The charge on the plates 1 and 4 will be-



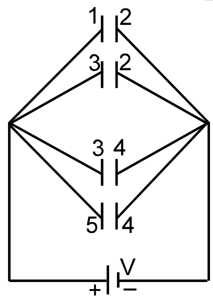
(A) $\frac{\epsilon_0 A}{d}, \frac{-2 \epsilon_0 A}{d}$

(B) $\frac{\epsilon_0 AV}{d}, \frac{-2 \epsilon_0 AV}{d}$

(C) $\frac{- \epsilon_0 AV}{d}, \frac{-3 \epsilon_0 AV}{d}$

(D) $\frac{\epsilon_0 AV}{d}, \frac{-4 \epsilon_0 AV}{d}$

Solution : by equivalent circuit diagram Charge on first plate Q = CV



$Q = \frac{\in_0 AV}{d}$

Charge on fourth plate Q´ = C(–V) $Q' = \frac{-\in_0 AV}{d}$

As plate 4 is repeated twice, hence charge on 4 will be Q´´ = 2Q´

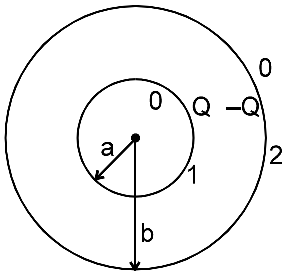
$Q'' = -\frac{2\in_0 AV}{d}$

Hence the correct answer will be (B).

13. OTHER TYPES OF CAPACITORS

Spherical capacitor :

This arrangement is known as spherical capacitor.



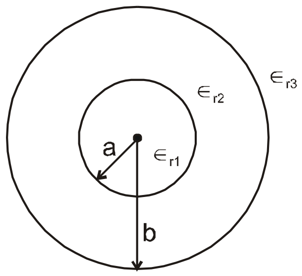
$V_1 - V_2 = \left[\frac{KQ}{a} - \frac{KQ}{b}\right] - \left[\frac{KQ}{b} - \frac{KQ}{b}\right] = \frac{KQ}{a} - \frac{KQ}{b}$

$C = \frac{Q}{V_1 - V_2} = \frac{Q}{\frac{KQ}{a} - \frac{KQ}{b}} = \frac{ac}{K(b-a)} = \frac{4\pi\in_0 ab}{b-a}$

$C = \frac{4\pi\in_0 ab}{b-a}$

If b >> a then

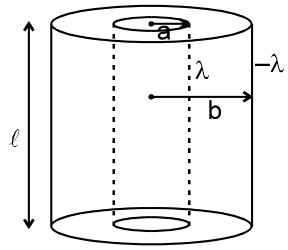
C = $4\pi\in_0 a$ (Like isolated spherical capacitor)



If dielectric mediums are filled as shown then : $C = \frac{4\pi\in_0\in_{r_2} ab}{b-a}$

Cylindrical capacitor

There are two co-axial conducting cylindrical surfaces where ℓ >> a and ℓ >> b, where a and b is radius of cylinders.



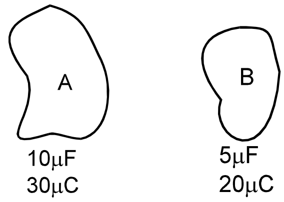
Capacitance per unit length $C = \frac{\lambda}{V}$

$= \frac{\lambda}{2K\lambda\ell n\frac{b}{a}} = \frac{4\pi\in_0}{2\ell n\frac{b}{a}} = \frac{2\pi\in_0}{\ell n\frac{b}{a}}$

Capacitance per unit length = $\frac{2\pi\in_0}{\ell n\frac{b}{a}}$ F/m

Miscellaneous Solved Example

Problem 1. When two isolated conductors A and B are connected by a conducting wire positive charge will flow from.



(A) A to B
(B) B to A
(C) will not flow
(D) can not say.

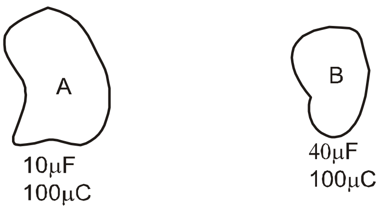
Solution : Charge always flows from higher potential body to lower potential body

Hence, $V_A = \dfrac{30}{10} = 3V \Rightarrow V_B = \dfrac{20}{5} = 4\ V$ As $V_B > V_A$ $\therefore$ (B) is correct Answer.

Problem 2. A conductor of capacitance 10μF connected to other conductor of capacitance 40 μF having equal charges 100 μC initially. Find out final voltage and heat loss during the process?

Answer : (i) V = 4V (ii) H = 225 μJ.

Solution :



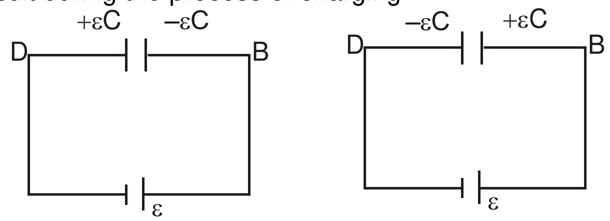
$C_1 = 10\mu F$ $C_2 = 40\mu F$
$Q_1 = 100\ \mu C$ $Q_2 = 100\mu C$
$V_1 = Q_1/C_1 = 10\ V$ $V_2 = Q_2/C_2 = 2.5$

Final voltage (V) $= \dfrac{C_1V_1 + C_2V_2}{(C_1 + C_2)} = \dfrac{Q_1 + Q_2}{C_1 + C_2} = \dfrac{200\mu C}{50\mu F} = 4V$

Heat loss during the process $= \dfrac{1}{2}\left[C_1V_1^2 + C_2V_2^2\right] - \dfrac{1}{2}V^2(C_1 + C_2)$

$= \dfrac{1}{2}\left[Q_1V_1 + Q_2V_2\right] - \dfrac{1}{2}V^2(C_1 + C_2) = \dfrac{1}{2}\times 100\mu[12.5] - \dfrac{1}{2}\times 16\ (50)\ \mu = 225\ \mu J$

Problem 3. A capacitor of capacitance C is charged from battery of e.m.f. ε and then disconnected. Now the positive terminal of the battery is connected with negative plate of capacitor. Find out heat loss in the circuit during the process of charging.



Initially finally

Net charge flow through battery = 2εC
Work done by battery = ε × 2εC = 2ε²C
Heat produced = $2\varepsilon^2 C$. **Ans.**

Solution : From figure
Net charge flow through
battery = $Q_{final} - Q_{initial} = \varepsilon C - (-\varepsilon C) = 2\varepsilon C$
$\therefore$ work done by battery (W) = $Q \times V = 2\varepsilon C \times \varepsilon = 2\varepsilon^2 C$
or Heat produced = $2\varepsilon^2 C$

Exercise-1

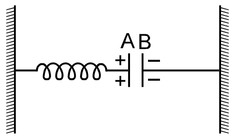
Marked Questions can be used as Revision Questions.

PART - I : SUBJECTIVE QUESTIONS

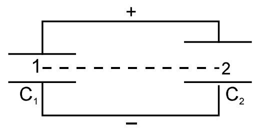
Section (A) : Definition of capacitance

A-1. When 30μC charge is given to an isolated conductor of capacitance 5μF. Find out the following
(i) Potential of the conductor
(ii) Energy stored in the electric field of conductor
(iii) If this conductor is now connected to another isolated conductor by a conducting wire (at very large distance) of total charge 50 μC and capacity 10 μF then
(a) find out the common potential of both the conductors.
(b) Find out the heat dissipated during the process of charge distribution.
(c) Find out the ratio of final charges on conductors.
(d) Find out the final charges on each conductor.

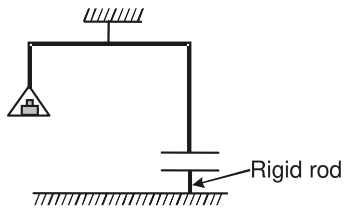
A-2. Plate A of a parallel air filled capacitor is connected to a nonconducting spring having force constant k and plate B is fixed. If a charge + q is placed on plate A and charge – q on plate B then find out extension in the spring in equilibrium. Assume area of plate is 'A'.



A-3. Two parallel plate capacitors with different distances between the plates are connected in parallel to a voltage source. A point positive charge Q is moved from a point 1 that is exactly in the middle between the plates of a capacitor C_1 to a point 2 (which lie in capacitor C_2) that lies at a distance from the negative plate of C_2 equal to half the distance between the plates of C_1. Is any work done in the process? If yes, calculate the work done by the field if potential at 1 and 2 are V_1 and V_2.



A-4. The lower plate of a parallel plate capacitor is supported on a rigid rod. The upper plate is suspended from one end of a balance. The two plates are joined together by a thin wire and subsequently disconnected. The balance is then counterpoised. Now a voltage V = 5000 volt is applied between the plates. The distance between the plates is d = 5 mm and the area of each plate is A = 100 cm^2. Then find out the additional mass placed to maintain balance. [All the elements other than plates are massless and nonconducting]

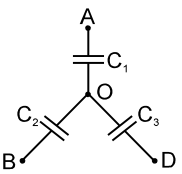


A-5. Each plate of a parallel plate air capacitor has an area S. What amount of work has to be performed by external agent to slowly increase the distance between the plates from x_1 to x_2 if:
(i) the charge of the capacitor, which is equal to q is kept constant in the process.
(ii) the voltage across the capacitor, which is equal to V is kept constant in the process.

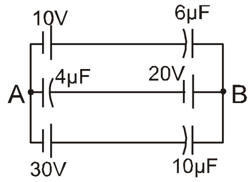
Section (B) : Circuits with capacitor and use of KCL and KVL

B-1. A capacitor of capacitance C, a resistor of resistance R and a battery of emf ε are connected in series at t = 0. What is the maximum value of
(a) the potential difference across the resistor. (b) the current in the circuit.
(c) the potential difference across the capacitor. (d) the energy stored in the capacitor.
(e) the power delivered by the battery. (f) the power converted into heat.

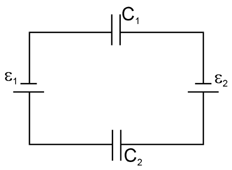
B-2. Three uncharged capacitors of capacitance C_1 = 1μF, C_2 = 2μF and C_3 = 3μF are connected as shown in the figure. The potential of point A, B and D are 10 volt, 25 volt and 20 volt respectively. Determine the potential at point O.



B-3. Find the potential difference between the points A and B ($V_A - V_B$) as shown in figure. (Initially all the capacitors are uncharged)

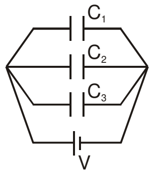


B-4._ In a circuit shown in the figure, find the potential difference between the left and right plates of each capacitor.



Section (C) : Combination of capacitors

C-1.(i) Find out the charges on the three capacitors connected to a battery as shown in figure. Take $C_1 = 1.0\ \mu F$, $C_2 = 2.0\ \mu F$, $C_3 = 3.0\ \mu F$ and V = 20 volt.



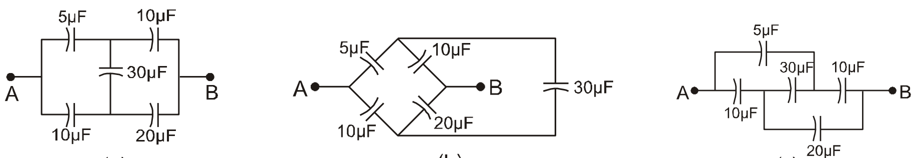
(ii) Find out the work done by the battery during the process of charging (initially all the capacitors are uncharged)

(iii) Find out the total energy stored in the capacitors.

C-2. If you have several 2.0 μF capacitors, each capable of withstanding 200 volts without breakdown, how would you assemble a combination having minimum number of capacitors and of given equivalent capacitance which capable of withstanding 1000 volts ;

(a) 0.40 μF (b) 1.2 μF

C-3. Find the capacitance between the point A and B of the given assemblies.

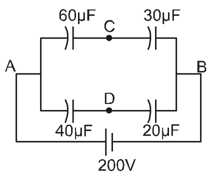


(a) (b) (c)

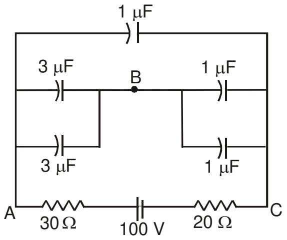
C-4. Take the potential of the point B as shown in the figure to be 100 V.

(a) Find the potentials at the point C and D.

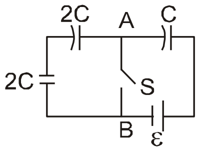
(b) If an uncharged capacitor is connected between C and D, then find the amount of charge that will appear on this capacitor



C-5. Find the potential difference between the points A and B and between the points B and C of figure in steady state.

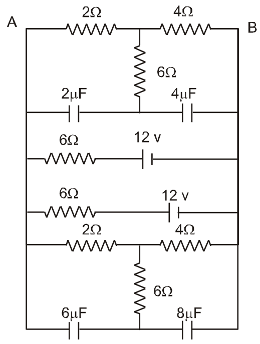


C-6. Consider the situation shown in the figure. The switch S is open for a long time and then closed and again steady state reached then



(a) Find the charge flown through the battery after the switch S is closed.
(b) Find the charge flown through the switch S from B to A.
(c) Find the work done by the battery after the switch S is closed.
(d) Find the change in energy stored in the system of capacitors.
(e) Find the heat developed in the system after the switch S is closed.

C-7. Find the final charges in steady state on the four capacitors of capacitance 2μF, 4μF, 6μF and 8μF as shown in figure. (Assuming initially they are uncharged). Also find the current through the wire AB at steady state.



Section (D) : Equation of charging and discharging

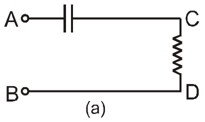
D-1. A capacitor is connected to a 12 V battery through a resistance of 10Ω. It is found that the potential difference across the capacitor rises to 4.0 V in 1μs. Find the capacitance of the capacitor.
(Given : $\ell n3 = 1.0986$, $\ell n2 = 0.693$)

D-2. A capacitor of capacity 1μF is connected in a closed series circuit with a resistance of 10^7 ohms, an open key and a cell of 2 V with negligible internal resistance:
(i) When the key is switched on at time t = 0, find;
(a) The time constant for the circuit.
(b) The charge on the capacitor at steady state.
(c) Time taken to deposit charge equal to half of charge that will deposit at steady state.
(ii) If after completely charging the capacitor, the cell is shorted by zero resistance at time t = 0, find the charge on the capacitor at t = 50 s. (Given : $e^{-5} = 6.73 \times 10^{-3}$, $\ell n2 = 0.693$)

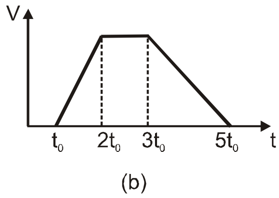
D-3. A capacitor of capacitance 200 μF is connected across a battery of emf 10.0 V through a resistance of 40 kΩ for 16.0 s. The battery is then replaced by a thick wire. What will be the charge on the capacitor 16.0 s after the battery is disconnected ? (Given : $e^{-2} = 0.135$)

D-4. A 5.0 μF capacitor having a charge of 20 μC is discharged through a wire of resistance 5.0 Ω. Find the heat dissipated in the wire between 25 to 50 μs after the connections are made. (Given : $e^{-2} = 0.135$)

D-5. A varying voltage is applied to the clamps AB (figure a) such that the voltage across the capacitor plates varies as shown in figure b.



(a)



(b)

Plot the time dependence of voltage across the clamps CD.

D-6. A capacitor of capacitance C is charged by charge q_0. At t = 0, it is connected to a battery of emf V and internal resistance r. Find the charge on the capacitor at time t (positive plate of capacitor connected with positive plate of battery).

Section (E) : Capacitor with dielectric

E-1. The two parallel plates of a capacitor have equal and opposite charges Q. The dielectric (which is filled between the capacitor plates) has a dielectric constant K and resistivity ρ. Show that the initially "leakage" current carried by the dielectric is given by the relationship $i = \dfrac{Q}{K \epsilon_0 \rho}$.

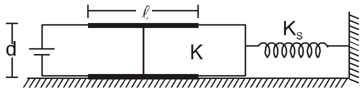
E-2. The parallel plates of a capacitor have an area 0.2 m^2 and are 10^{-2} m apart. The original potential difference between them is 3000 V, and it decreases to 1000 V when a sheet of dielectric is inserted between the plates filling the full space. Compute: ($\epsilon_0 = 9 \times 10^{-12}$ S. I. units)
(i) Original capacitance C_0.
(ii) The charge Q on each plate.
(iii) Capacitance C after insertion of the dielectric.
(iv) Dielectric constant K.
(v) Permittivity ϵ of the dielectric.
(vi) The original field E_0 between the plates.
(vii) The electric field E after insertion of the dielectric.

E-3. A parallel plate isolated condenser consists of two metal plates of area A and separation 'd'. A slab of thickness 't' and dielectric constant K is inserted between the plates with its faces parallel to the plates and having the same surface area as that of the plates. Find the capacitance of the system. If K = 2, for what value of t/d will the capacitance of the system be 3/2 times that of the condenser with air filling the full space? Calculate the ratio of the energy in the two cases and account for the energy change (assuming q charge on the plate to be constant).

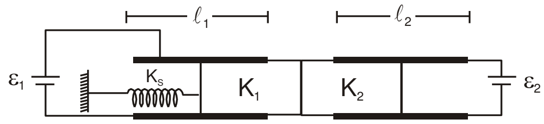
E-4. Two parallel plate air capacitors each of capacitance C were connected in series to a battery with e.m.f. ε. Then one of the capacitors was filled up with uniform dielectric with relative permittivity k. How many times did the electric field strength in that capacitor decrease? What amount of charge flows through the battery?

E-5. A parallel-plate capacitor of plate area A and plate separation d is charged by a ideal battery of e.m.f. V and then the battery is disconnected. A slab of dielectric constant 2k is then inserted between the plates of the capacitor so as to fill the whole space between the plates. Find the change in potential energy of the system in the process of inserting the slab.

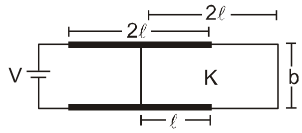
E-6. Consider the situation shown in figure. The width of each plate is b. The capacitor plates are rigidly clamped in the laboratory and connected to a battery of emf V. All surface are frictionless. Calculate the extension in the spring in equilibrium (spring is nonconducting).



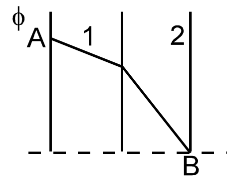
E-7. In figure shown, two parallel plate capacitors with fixed plates and connected to two batteries. The separation between the plates is same for the two capacitors. The plates are rectangular in shape with width b and lengths ℓ_1 and ℓ_2, the separation between plates is d. The left half of the dielectric slab has a dielectric constant K_1 and the right half K_2 ($K_2 > K_1$). EMF of the right battery is greater then left battery. Neglecting any friction, find the extension in spring in equilibrium (spring is nonconducting) ($\varepsilon_2 > \varepsilon_1$)



E-8. The plates of the parallel plate capacitor have plate area A and are clamped in the laboratory as shown in figure. The dielectric slab of mass m, length 2ℓ and width 2ℓ is released from rest with length ℓ inside the capacitor. Neglecting any effect of friction or gravity, show that the slab will execute periodic motion and find its time period. (plates of capacitor are square plates of side 2ℓ)



E-9. A parallel plate capacitor is filled with a dielectric up to one half of the distance between the plates. The manner in which the potential between the plates varies with distance is illustrated in the figure. Which half (1 or 2) of the space between the plates is filled with the dielectric and what will be the distribution of the potential after the dielectric is taken out of the capacitor provided that;
(a) The charges on the plates are conserved or
(b) The potential difference across the capacitor is constant.



E-10. Positive charge q is given to each plate of a parallel plate air capacitor having area of each plate A and seperation between them, d. Then find
(i) Capacitance of the system.
(ii) Charges appearing on each surface of plates
(iii) Electric field between the plates
(iv) Potential diffrence between the plates
(v) Energy stored between the plates

PART - II : ONLY ONE OPTIONS CORRECT TYPE

Section (A) : Definition of Capacitance

A-1. The radii of two metallic spheres are 5 cm and 10 cm and both carry equal charge of $75\mu C$. If the two spheres are shorted then charge will be transferred–
(A) 25 μC from smaller to bigger
(B) 25 μC from bigger to smaller
(C) 50 μC from smaller to bigger
(D) 50 μC from bigger to smaller

A-2. Two isolated charged metallic spheres of radii R_1 and R_2 having charges Q_1 and Q_2 respectively are connected to each other, then there is:
(A) No change in the electrical energy of the system
(B) An increase in the electrical energy of the system
(C) A decrease in the electrical energy of the system in any case
(D) A decrease in electrical energy of the system if $Q_1 R_2 \neq Q_2 R_1$

A-3. A parallel plate capacitor is charged up to a potential of 300 volts. Area of the plates is 100 cm^2 and spacing between them is 2 cm. If the plates are moved apart to a distance of 2.5 cm without disconnecting the power source, then ($\epsilon_0 = 9 \times 10^{-12}$ C^2 N^{-1} m^{-2}):
(i) Electric field inside the capacitor when distance is 2.5 cm :
(A) 15×10^2 V/m
(B) 3×10^3 V/m
(C) 12×10^3 V/m
(D) 6×10^3 V/m

(ii) Change in energy of the capacitor is :
(A) 6×10^{-8} J (B) -1215×10^{-10} J (C) 1215×10^{-10} J (D) -405×10^{-10} J

(iii) If the distance is increased after disconnecting the power source, then electric field inside the capacitor is :
(A) 6×10^3 V/m (B) 3×10^3 V/m (C) 12×10^3 V/m (D) 15×10^3 V/m

(iv) Change in energy of the capacitor in above case is :
(A) 303.75×10^{-9} J (B) -1215×10^{-10} J (C) 5.06×10^{-8} J (D) -303.75×10^{-9} J

A-4. A parallel plate capacitor is charged and then isolated. On increasing the plate separation–

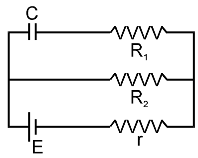
Charge	**Potential**	**Capacitance**
(A) remains constant	remains constant	decreases
(B) remains constant	increases	decreases
(C) remains constant	decreases	increases
(D) increases	increases	decreases

A-5. A parallel plate capacitor is charged and the charging battery is then disconnected. The plates of the capacitor are now moved, farther apart. The following things happen :
(A) The charge on the capacitor increases
(B) The electrostatics energy stored in the capacitor increases
(C) The voltage between the plates decreases
(D) The capacitance increases.

Section (B) : Circuits with capacitor and use of KCl and KVL

B-1. The work done against electric forces in increasing the potential difference of a condenser from 20V to 40V is W. The work done in increasing its potential difference from 40V to 50V will be (consider capacitance of capacitor remain constant)
(A) 4W (B) $\frac{3W}{4}$ (C) 2W (D) $\frac{W}{2}$

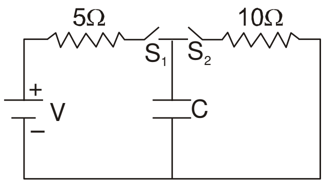
B-2. The magnitude of charge in steady state on either of the plates of condenser C in the adjoining circuit is



(A) CE (B) $\frac{CER_2}{(R_1+r)}$ (C) $\frac{CER_2}{(R_2+r)}$ (D) $\frac{CER_1}{(R_2+r)}$

B-3. The plate separation in a parallel plate condenser is d and plate area is A. If it is charged to V volt & battery is disconnected then the work done in increasing the plate separation to 2d will be –
(A) $\frac{3}{2}\frac{\epsilon_0 AV^2}{d}$ (B) $\frac{\epsilon_0 AV^2}{d}$ (C) $\frac{2\epsilon_0 AV^2}{d}$ (D) $\frac{\epsilon_0 AV^2}{2d}$

B-4. In the adjoining diagram, (assuming the battery to be ideal) the condenser C will be charged to potential V if -



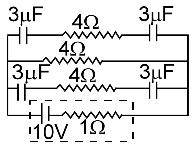
(A) S_1 and S_2 both are open (B) S_1 and S_2 both are closed
(C) S_1 is closed and S_2 is open (D) S_1 is open and S_2 is closed.

B-5. A parallel plate condenser of capacity C is connected to a battery and is charged to potential V. Another condenser of capacity 2C is connected to another battery and is charged to potential 2V. The charging batteries are removed and now the condensers are connected in such a way that the positive plate of one is connected to negative plate of another. The final energy of this system is –

(A) zero (B) $\frac{25CV^2}{6}$ (C) $\frac{3CV^2}{2}$ (D) $\frac{9CV^2}{2}$

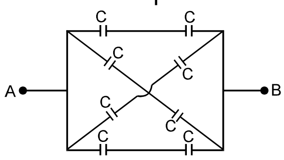
Section (C) : Combination of capacitors

C-1. In the following figure, the charge on each condenser in the steady state will be –



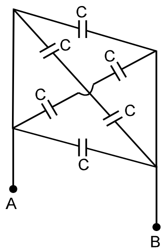
(A) 3μC (B) 6μC (C) 9μC (D) 12μC

C-2. In the adjoining circuit, the capacity between the points A and B will be -



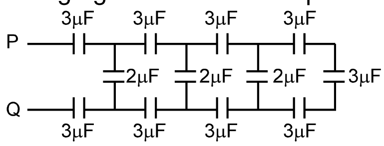
(A) C (B) 2C (C) 3C (D) 4C

C-3. The resultant capacity between the points A and B in the adjoining circuit will be -



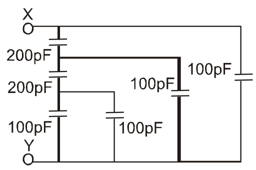
(A) C (B) 2C (C) 3C (D) 4C

C-4. The effective capacity in the following figure between the points P and Q will be –



(A) 3μF (B) 5μF (C) 2μF (D) 1μF

C-5. The equivalent capacitance between the terminals X and Y in the figure shown will be–

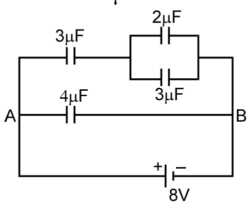


(A) 100 pF (B) 200 pF (C) 300 pF (D) 400 pF

C-6. The minimum number of condensers each of capacitance of 2μF, in order to obtain resultant capacitance of 5μF will be-

(A) 4 (B) 5 (C) 6 (D) 10

C-7. The charge on the condenser of capacitance $2\mu F$ in the following circuit will be –

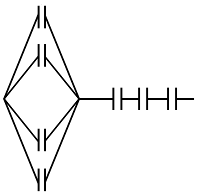
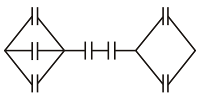
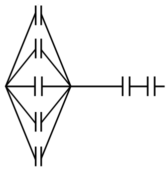
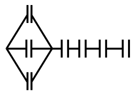


(A) $4.5\ \mu C$ (B) $6.0\ \mu C$ (C) $7\ \mu C$ (D) $30\ \mu C$

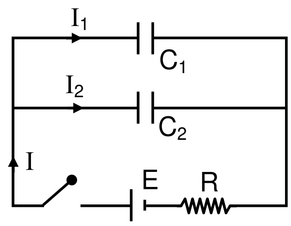
C-8. Two parallel plate condensers of capacity of $20\mu F$ and $30\mu F$ are charged to the potentials of 30V and 20V respectively. If likely charged plates are connected together then the common potential difference will be –

(A) 100 V (B) 50 V (C) 24 V (D) 10 V

C-9. How the seven condensers, each of capacity $2\mu F$, should be connected in order to obtain a resultant capacitance of $\frac{10}{11}\mu F$?

(A)  (B)  (C)  (D) 

C-10. In the circuit shown below the switch is closed at t = 0. For $0 < t < R(C_1 + C_2)$, the current I_1 in the capacitor C_1 in terms of total current I is **[Olympiad 2015 (stage-1)]**



(A) $\left(\frac{C_1}{C_2}\right)I$ (B) $\left(\frac{C_2}{C_1}\right)I$ (C) $\left(\frac{C_1}{C_1+C_2}\right)I$ (D) $\left(\frac{C_2}{C_1+C_2}\right)I$

C-11. A capacitor of capacitance C is charged to a potential difference V_0. The charging battery is disconnected and the capacitor is connected to a capacitor of unknown capacitance C_X. The potential difference across the combination is V. The value of C_x should be –

(A) $\frac{C(V_0 - V)}{V}$ (B) $\frac{C(V - V_0)}{V}$ (C) $\frac{CV}{V_0}$ (D) $\frac{CV_0}{V}$

Section (D) : Equation of charging and discharging

D-1. A 3 mega ohm resistor and an uncharged $1\ \mu F$ capacitor are connected in a single loop circuit with a constant source of 4 volt. At one second after the connection is made what are the rates at which;

(i) The charge on the capacitor is increasing.

(A) $4(1-e^{-1/3})\ \mu\ C/s$ (B) $4e^{-1/3}\ \mu\ C/s$ (C) $\frac{4}{3}e^{-1/3}\ \mu\ C/s$ (D) $\frac{4}{3}(1-e^{-1/3})\ \mu\ C/s$

(ii) Energy is being stored in the capacitor.

(A) $\frac{16}{3}(1-e^{-1/3})e^{-1/3}\ \mu J/s$ (B) $\frac{16}{3}(1-e^{-2/3})\ \mu J/s$

(C) $\frac{16}{3}e^{-2/3}\ \mu J/s$ (D) None of these

(iii) Joule heat is appearing in the resistor.

(A) $\frac{16}{3}e^{-1/3}\mu$ J/s (B) $\frac{1}{2}e^{-1/3}\mu$ J/s (C) $\frac{16}{3}(e^{-2/3})\,\mu$ J/s (D) $\frac{16}{3}(1-e^{-1/3})^2\,\mu$ J/s

(iv) Energy is being delivered by the source.

(A) $16\ (1-e^{-1/3})\,\mu$ J/s (B) 16μ J/s (C) $\frac{16}{3}e^{-1/3}\ \mu$ J/s (D) $\frac{16}{3}(1-e^{-1/3})\ \mu$ J/s

D-2. An uncharged capacitor of capacitance 8.0 μF is connected to a battery of emf 6.0 V through a resistance of 24 Ω, then

(i) The current in the circuit just after the connections are made is :

(A) 0.25 A (B) 0.5 A (C) 0.4 A (D) 0 A

(ii) The current in the circuit at one time constant after the connections are made is :

(A) 0.25 A (B) 0.09 A (C) 0.4 A (D) 0 A

D-3. An uncharged capacitor of capacitance 100μF is connected to a battery of emf 20V at t = 0 through a resistance 10Ω, then

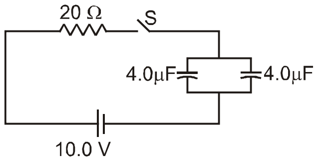
(i) the maximum rate at which energy is stored in the capacitor is :

(A) 10J/s (B) 20 J/s (C) 40J/s (D) 5J/s

(ii) time at which the rate has this maximum value is

(A) (4 ln 2) ms (B) (2 ln 2) ms (C) (ln 2) ms (D) (3 ln 2) ms

D-4. The charge on each of the capacitors 0.16 ms after the switch S is closed in figure is :



(A) 24 μC (B) 26.8 μC (C) 25.2 μC (D) 40 μC

D-5. The plates of a capacitor of capacitance 10 μF, charged to 60 μC, are joined together by a wire of resistance 10 Ω at t = 0, then

(i) the charge on the capacitor in the circuit at t = 0 is :

(A) 120 μC (B) 60 μC (C) 30 μC (D) 44 μC

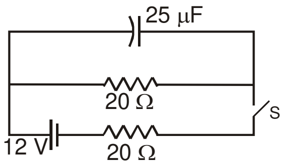
(ii) the charge on the capacitor in the circuit at t = 100 μs is :

(A) 120 μC (B) 60 μC (C) 22 μC (D) 18 μC

(iii) the charge on the capacitor in the circuit at t = 1.0 ms is : (take e^{10} = 20000)

(A) 0.003 μC (B) 60 μC (C) 44 μC (D) 18 μC

D-6. The switch S shown in figure is kept closed for a long time and then opened at t = 0, then the current in the middle 20 Ω resistor at t = 0.25 ms is :



(A) 0.629 A (B) 0.489 A (C) 0.189 A (D) 23 mA

Section (E) : Capacitor with dielectric

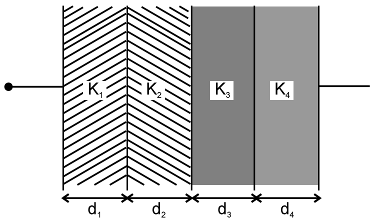
E-1. The distance between the plates of a parallel plate condenser is d. If a copper plate of same area but thickness d/2 is placed between the plates then the new capacitance will become -

(A) half (B) double (C) one fourth (D) unchanged

E-2. On placing a dielectric slab between the plates of an isolated charged condenser its –

	Capacitance	Charge	Potential Difference	Energy stored	Electric field
(A)	decreases	remains unchanged	decreases	increases	increases
(B)	increases	remains unchanged	increases	increases	decreases
(C)	increases	remains unchanged	decreases	decreases	decreases
(D)	decreases	remains unchanged	decreases	increases	remains unchanged

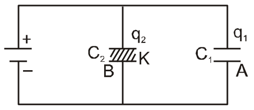
E-3. The effective capacitance of the system in adjoining figure will be -



(A) $C = \dfrac{\varepsilon_0 A}{\left[\dfrac{d_1}{K_1}+\dfrac{d_2}{K_2}+\dfrac{d_3}{K_3}+\dfrac{d_4}{K_4}\right]}$ (B) $C = \dfrac{\varepsilon_0 A}{4d}$

(C) $C = \dfrac{4d}{\varepsilon_0 A}$ (D) $C = \dfrac{K_1K_2K_4K_3}{4d}$

E-4. In the adjoining diagram two geometrically identical capacitors A and B are connected to a battery. Air is filled between the plates of C_1 and a dielectric is filled between the plates of C_2, then –



(A) $q_1 < q_2$ (B) $q_1 > q_2$ (C) $q_1 = q_2$ (D) None of these

E-5. A parallel plate condenser is connected to a battery of e.m.f. 4 volt. If a plate of dielectric constant 8 is inserted into it, then the potential difference on the condenser will be-
(A) 1/2 V (B) 2V (C) 4V (D) 32V

E-6. In the above problem if the battery is disconnected before inserting the dielectric, then potential difference will be-
(A) 1/2 V (B) 2V (C) 4V (D) 32V

E-7. A parallel plate condenser with plate separation d is charged with the help of a battery so that U_0 energy is stored in the system. A plate of dielectric constant K and thickness d is placed between the plates of condenser while battery remains connected. The new energy of the system will be-

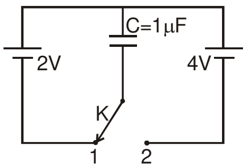
(A) KU_0 (B) K^2U_0 (C) $\dfrac{U_0}{K}$ (D) $\dfrac{U_0}{K^2}$

E-8. In the above problem if the battery is disconnected before placing the plate, then new energy will be –

(A) K^2U_0 (B) $\dfrac{U_0}{K^2}$ (C) $\dfrac{U_0}{K}$ (D) KU_0

PART - III : MATCH THE COLUMN

1. The circuit involves two ideal cells connected to a 1 μF capacitor via a key K. Initially the key K is in position 1 and the capacitor is charged fully by 2V cell. The key is then pushed to position 2. Column I gives physical quantities involving the circuit after the key is pushed from position 1. Column II gives corresponding results. Match the statements in Column I with the corresponding values in Column II.

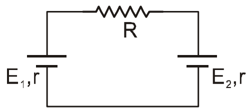


Column I	**Column II**
(A) The net charge crossing the 4 volt cell in μC is	(p) 2
(B) The magnitude of work done by 4 Volt cell in μJ is	(q) 6
(C) The gain in potential energy of capacitor in μJ is	(r) 8
(D) The net heat produced in circuit in μJ is	(s) 16

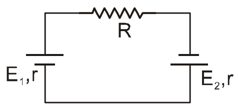
2. In each situation of column-I, a circuit involving two non-ideal cells of unequal emf E_1 and E_2 ($E_1 > E_2$) and equal internal resistance r are given. A resistor of resistance R is connected in all four situations and a capacitor of capacitance C is connected in last two situations as shown. Assume battery can supply infinity charge to the circuit (r, R $\neq$ 0, E_1, E_2 $\neq$ 0). Four statements are given in column-II. Match the situation of column-I with statements in column-II.

Column-I

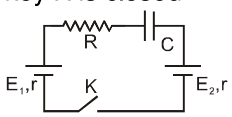
(A)



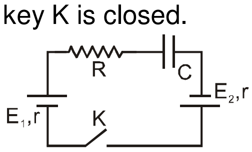
(B)



(C) The capacitor is initially uncharged. After the key K is closed



(D) The capacitor is initially uncharged. After the key K is closed.



Column-II

(p) magnitude of potential difference across both cells can never be same.

(q) cell of lower emf absorbs energy, that is, it gets charged up as long as current flows in circuit

(r) potential difference across cell of lower emf may be zero.

(s) current in the circuit can never be zero (even after steady state is reached).

Exercise-2

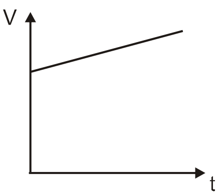
✍ **Marked Questions can be used as Revision Questions.**

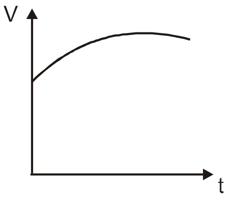
PART - I : ONLY ONE OPTION CORRECT TYPE

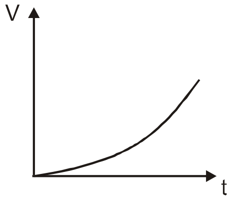
1. The plates of a parallel plate condenser are being moved away with a constant speed v. If the plate separation at any instant of time is d then the rate of change of capacitance with time is proportional to–

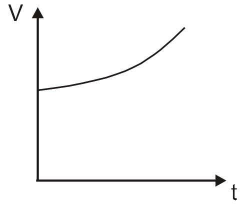
(A) $\frac{1}{d}$ (B) $\frac{1}{d^2}$ (C) d^2 (D) d

2. Choose Graph between potential and time for an isolated conductor of finite capacitance C, if its charge varies according to the formula $Q = (\alpha t + Q_0)$ coulomb, where Q_0 and α are positive constant.

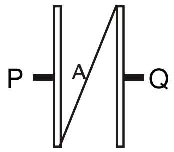
(A)


(B)


(C)


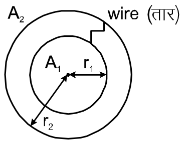
(D)


3. A parallel plate capacitor of capacitance C is as shown. A thin metal plate A is placed between the plates of the given capacitor in such a way that its edges touch the two plates as shown. The capacity a cross P and Q now becomes.



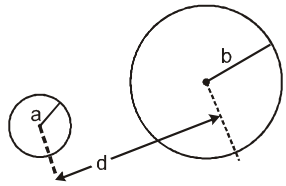
(A) 0 (B) 3C (C) 4C (D) ∞

4. Two spherical conductors A_1 and A_2 of radii r_1 and r_2 are placed concentrically in air. The two are connected by a copper wire as shown in figure. Then the equivalent capacitance of the system is:



(A) $\frac{4\pi \in_0 k\ r_1.r_2}{r_2 - r_1}$ (B) $4\pi\in_0 (r_1 + r_2)$ (C) $4\pi\in_0 r_2$ (D) $4\pi\in_0 r_1$

5. There are two conducting spheres of radius a and b (b > a) carrying equal and opposite charges. They are placed at a separation d (>>> a and b). The capacitance of system is

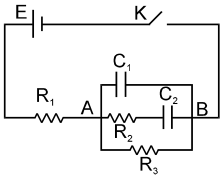


(A) $\frac{4\pi\varepsilon_0}{a-b-d}$ (B) $\frac{4\pi\varepsilon_0}{\frac{1}{a}-\frac{1}{b}-\frac{1}{d}}$ (C) $\frac{4\pi\varepsilon_0}{\frac{1}{a}+\frac{1}{b}-\frac{1}{d}}$ (D) $\frac{4\pi\varepsilon_0}{\frac{1}{a}+\frac{1}{b}-\frac{2}{d}}$

6.✍ A capacitor of capacitance C_0 is charged to a voltage V_0 and then isolated. An uncharged capacitor C is then charged from C_0, discharged and charged again ; the process is repeated n times. Due to this, potential of the C_0 is decreased to V, then value of C is :

(A) $C_0 [V_0/V]^{1/n}$ (B) $C_0[(V_0/V)^{1/n} - 1]$ (C) $C_0 [(V_0/V) - 1]$ (D) $C_0 [(V/V_0)^n + 1]$

7. A network of uncharged capacitors and resistances is as shown



Current through the battery immediately after key K is closed and after a long time interval is :

(A) $\frac{E}{R_1}, \frac{E}{R_1+R_3}$

(B) $\frac{E}{R_1+R_3}, \frac{E}{R_1+\frac{R_2 R_3}{R_2+R_3}}$

(C) Zero, $\frac{E}{R_1}$

(D) $\frac{E}{R_1+\frac{R_2 R_3}{R_2+R_3}}, \frac{E}{R_1}$

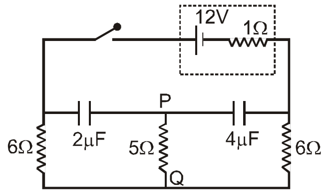
8. (i) A 3μF capacitor is charged up to 300 volt and 2μF is charged up to 200 volt. The capacitor are connected so that the plates of same polarity are connected together. The final potential difference between the plates of the capacitor after they are connected is :

(A) 220 V (B) 160 V (C) 280 V (D) 260 V

(ii) If instead of this, the plates of opposite polarity were joined together, then amount of charge that flows is :

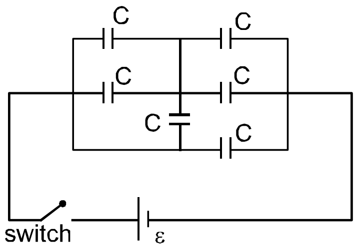
(A) 6×10^{-4} C (B) 1.5×10^{-4} C (C) 3×10^{-4} C (D) 7.5×10^{-4} C

9. In the circuit shown in figure the capacitors are initially uncharged. The current through resistor PQ just after closing the switch is :



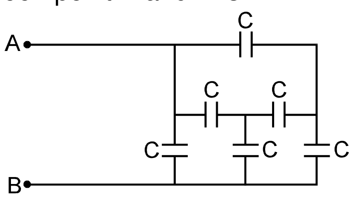
(A) 2A from P to Q (B) 2A from Q to P (C) 6A from P to Q (D) zero

10. Six capacitors each of capacitance 'C' is connected as shown in the figure and initially all the capacitors are uncharged. Now a battery of emf = ε is connected. How much charge will flow through the battery if the switch is on :



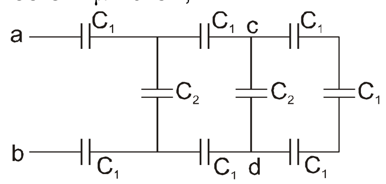
(A) $\frac{9C\varepsilon}{5}$ (B) $\frac{11\,C\varepsilon}{5}$ (C) $\frac{13\,C\varepsilon}{5}$ (D) $\frac{7\,C\varepsilon}{5}$

11. The equivalent capacitance between point A and B is



(A) C/4 (B) C/2 (C) C (D) 2C

12. In the arrangement of the capacitors shown in the figure, each C_1 capacitor has capacitance of $3\mu F$ and each C_2 capacitor has capacitance of $2\mu F$ then,



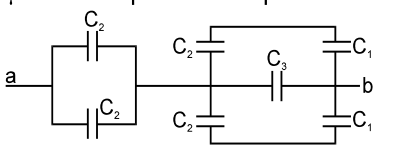
(i) Equivalent capacitance of the network between the points a and b is :

(A) $1\mu F$ (B) $2\mu F$ (C) $4\mu F$ (D) $\frac{3}{2}$ μF

(ii) If V_{ab} = 900 V, the charge on each capacitor nearest to the points 'a' and 'b' is :
(A) 300 μC (B) 600 μC (C) 450 μC (D) 900 μC

(iii) If V_{ab} = 900 V, then potential difference across points c and d is :
(A) 60 V (B) 100 V (C) 120 V (D) 200 V

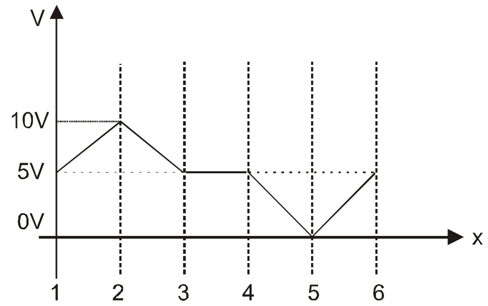
13. A combination arrangement of the capacitors is shown in the figure
(i) C_1 = 3 μF, C_2 = 6 μF and C_3 = 2 μF then equivalent capacitance between 'a' and 'b' is :



(A) 4 μF (B) 6 μF (C) 1 μF (D) 2 μF

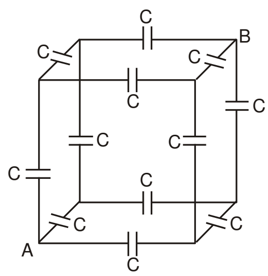
(ii) If a potential difference of 48 V is applied across points a and b, then charge on the capacitor C_3 at steady state condition will be :
(A) 8 μC (B) 16 μC (C) 32 μC (D) 64 μC

14. The V versus x plot for six identical metal plates of cross-sectional area A is as shown. The equivalent capacitance between 2 and 5 is (Adjacent plates are placed at a separation d) :



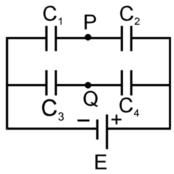
(A) $\frac{2\epsilon_0 A}{d}$ (B) $\frac{\epsilon_0 A}{d}$ (C) $\frac{3\epsilon_0 A}{d}$ (D) $\frac{\epsilon_0 A}{2d}$

15. Each edge of the cube contains a capacitance C. The equivalent capacitance between the points A and B will be –



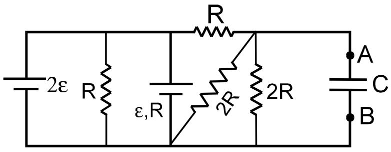
(A) $\frac{6C}{5}$ (B) $\frac{5C}{6}$ (C) $\frac{12C}{7}$ (D) $\frac{7C}{12}$

16. The potential difference between the points P and Q in the adjoining circuit will be-



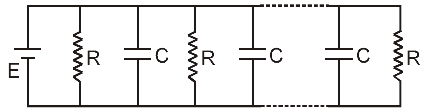
(A) $\frac{(C_1C_4 - C_2C_3)E}{(C_1 + C_3)(C_2 + C_4)}$ (B) $\frac{C_2C_3E}{C_1C_2(C_3 + C_4)}$ (C) $\frac{(C_2C_3 - C_1C_4)E}{(C_1 + C_2)(C_3 + C_4)}$ (D) $\frac{(C_2C_3 - C_1C_4)E}{(C_1 + C_2 + C_3 + C_4)}$

17. The time constant of the circuit shown is :



(A) $\frac{RC}{2}$ (B) $\frac{3RC}{5}$ (C) $\frac{RC}{3}$ (D) $\frac{RC}{4}$

18. n resistances each of resistance R are joined with capacitors of capacity C (each) and a battery of emf E as shown in the figure. In steady state condition ratio of charge stored in the first and last capacitor is:

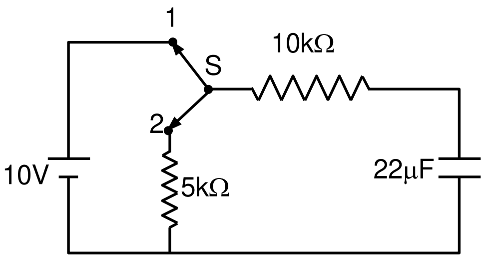


(A) n : 1 (B) (n – 1) : (n + 1) (C) (n^2 + 1) : (n^2 – 1) (D) 1 : 1

19. A fresh dry cell of 1.5 volt and two resistors of 10 kΩ each are connected in series. An analog voltmeter measures a voltage of 0.5 volt across each of the resistors. A 100μF capacitor is fully charged using the same source. The same voltmeter is now used to measure the voltage across it. The initial value of the current and the time in which the voltmeter reading falls to 0.5 volt are respectively. **[Olympiad 2014_stage-1]**

(A) 60μA, 11s (B) 120μA, 15s (C) 150μA, 15s (D) 150μA, 1.1 s

20. Refer to the circuit given below. Initially the switch S is in position 1 for 1.5 s. Then the switch is changed to position 2. After a time t (measured from the change over of the switch) the voltage across 5 kΩ resistance is found to be about 1.226 volt. Then, t is **[Olympiad 2014_stage-1]**

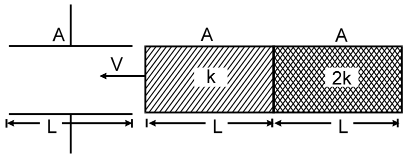


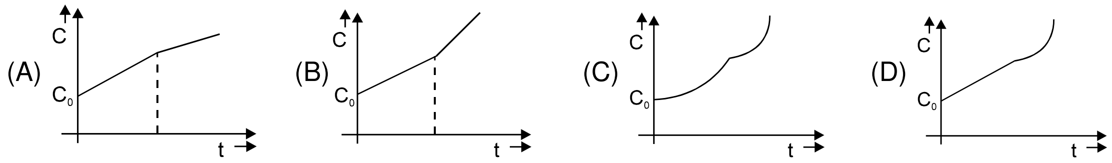
(A) 330 ms (B) 500 ms (C) 33 ms (D) data insufficient

21. The capacitance of a parallel plate condenser is C_0. If a dielectric of relative permittivity ε_r and thickness equal to one fourth the plate separation is placed between the plates, then its capacity becomes C. Then value of $\frac{C}{C_0}$ will be –

(A) $\frac{5\varepsilon_r}{4\varepsilon_r + 1}$ (B) $\frac{4\varepsilon_r}{3\varepsilon_r + 1}$ (C) $\frac{3\varepsilon_r}{2\varepsilon_r + 1}$ (D) $\frac{2\varepsilon_r}{\varepsilon_r + 1}$

22. A parallel plate capacitor without any dielectric has capacitance C_0. A dielectric slab is made up of two dielectric slabs of dielectric constants K and 2K and is of same dimensions as that of capacitor plates and both the parts are of equal dimensions arranged serially as shown. If this dielectric slab is introduced (dielectric K enters first) in between the plates at constant speed, then variation of capacitance with time will be best represented by :





23. An isolated metallic object is charged in vacuum to a potential V_0 using a suitable source, its electrostatic energy being W_0. It is then disconnected from the source and immersed in a large volume of dielectric with dielectric constant K. The electrostatic energy of the sphere in the dielectric is :

[Olympiad (Stage-1) 2017]

(A) $K^2 W_0$ (B) $K W_0$ (C) $\frac{W_0}{K^2}$ (D) $\frac{W_0}{K}$

24. Consider a parallel plate capacitor. When half of the space between the plates is filled with some dielectric material of dielectric constant K as shown in Fig. (1) below, the capacitance is C_1. However, if the same dielectric material fills half the space as shown in Fig. (2), the capacitance is C_2. Therefore, the ratio $C_1 : C_2$ is **[Olympiad (Stage-1) 2017]**

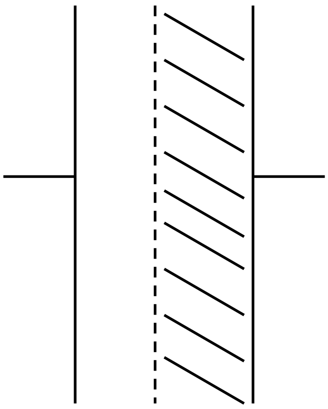


Fig. (1)

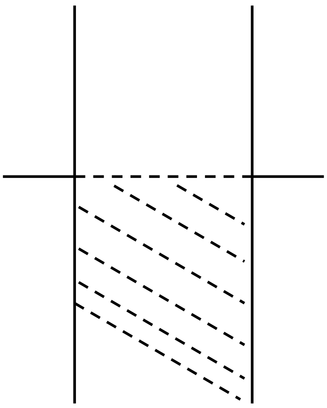
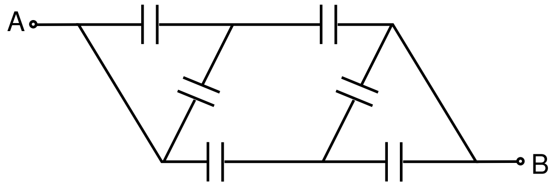


Fig. (2)

(A) 1 (B) $\frac{2K}{K+1}$ (C) $\frac{4K}{(K+1)^2}$ (D) $\frac{K+1}{2}$

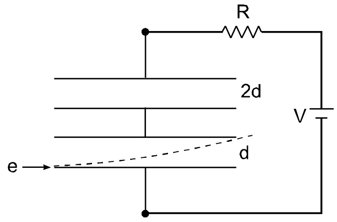
25. A network of six identical capacitors, each of capacitance C is formed as shown below. The equivalent capacitance between the point A and B is **[Olympiad (Stage-1) 2017]**



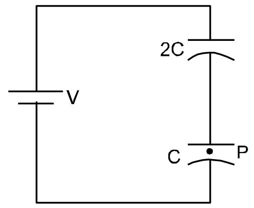
(A) 3C (B) 6C (C) 3C/2 (D) 4C/3

PART - II : SINGLE AND DOUBLE VALUE INTEGER TYPE

1. Both the capacitors shown in figure are made of square plates of edge a. The separations between the plates of the capacitors are d_1 and d_2 as shown in the figure. A battery of V volt and a resistance R are connected as shown in figure. At steady state an electron is projected between the plates of the lower capacitor from its lower plate along the plate as shown. Minimum speed should the electron be projected is given by $\frac{1}{\sqrt{n}}\left(\frac{Vea^2}{md^2}\right)^{1/2}$ so that it does not collide with any plate? Consider only the electric forces then find the value of n.

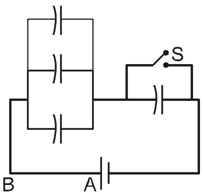


2. The particle P shown in the figure has a mass m and a charge –q. Each horizontal plate has a surface area A potential difference $V = n\left(\frac{mg\varepsilon_0 A}{2qc}\right)$ should be applied to the combination to hold the particle P in equilibrium then find the value of n.

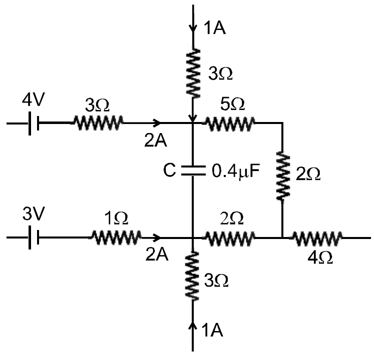


3. A capacitor of capacitance 2.0 μF is charged to a potential difference of 12 V. It is then connected to an uncharged capacitor of capacitance 4.0 μF. Find (a) the charge flow through connecting wire upto study strate on each of the two capacitors after the connection in μC (b) The total electrostatic energy stored in both capacitors in μJ (c) the heat produced during the charge transfer from one capacitor to the other in μJ.

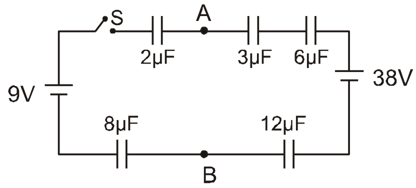
4. Four capacitors of capacitance 10 μF and a battery of 2V are arranged as shown. How much μC charge will flow through AB after the switch S is closed ?



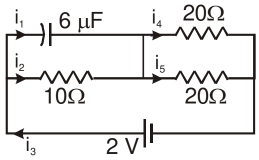
5. A part of circuit in a steady state along with the current flowing in the branches, the values of resistance etc., is shown in the figure. How much energy (μJ) stored in the capacitor C (0.4μF) **[1986, 4M]**



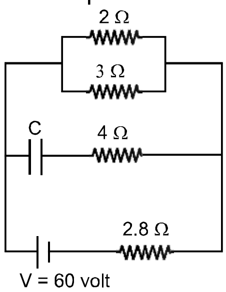
6. Five capacitors are connected as shown in the figure. Initially S is opened and all the capacitors are uncharged. When S is closed and steady state is obtained. Then find out potential difference between the points A and B in volt.



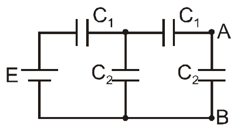
7. In steady state, find the charge on the capacitor in (μC) shown in figure.



8. Calculate the steady state current (in A) in the 2 Ω resistor shown in the circuit (see figure). The internal resistance of the battery is negligible and the capacitance of the condenser C is 0.2 μF **[JEE-1982; 2M]**

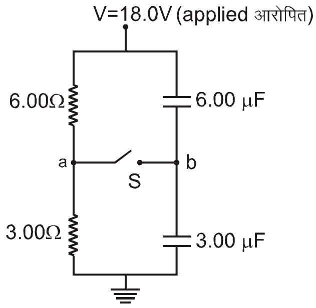


9. Find the potential difference between points A and B (in V) of the system shown in figure if the emf is equal to E = 110 V and the capacitance ratio $C_2/C_1 = \eta = 2.0$.

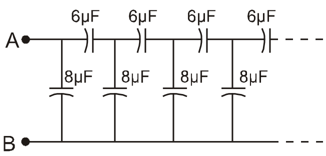


10. (i) What is the final potential (in V) of point b with respect to ground in steady state after switch S is closed ?

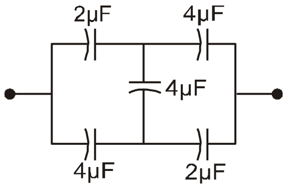
(ii) How much charge flows through switch S from b to a after it is closed in μC?



11. Find the equivalent capacitance in (μF) of the infinite ladder shown in the figure between the points A and B.



12. The equivalent capacitance of the combination shown in the figure between the indicated points is given by $\frac{n}{7}$ μF. then find the value of n.



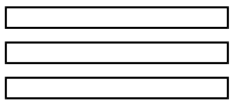
13. The electric field between the plates of a parallel–plate capacitance 2.0 μF drops to one third of its initial value in 4.4 μs when the plates are connected by a thin wire. Find the resistance of the wire in Ω. (Given : ℓn3 = 1.0986)

14. A capacitor of capacitance C charged by battery at V volt and then disconnected. At t = 0, it is connected to an uncharged capacitor of capacitance 2C through a resistance R. The charge on the second capacitor as a function of time is given by q = $\frac{\alpha CV}{3}\left(1 - e^{-\frac{3t}{\beta RC}}\right)$ then find the value of $\frac{\alpha}{\beta}$.

15. Hard rubber has a dielectric constant of 2.8 and a dielectric strength (maximum electric field) of 18×10^6 volt/meter. If it is used as the dielectric material filling the full space in a parallel plate capacitor. Minimum area may the plates of the capacitor have in order that the capacitance be 7.0×10^{-2} μF is equal to $\frac{\pi}{n}$ m². What should be the value of n if capacitor be able to withstand a potential difference of 4000 volts. ($\in_0 = \frac{10^{-9}}{36\pi}$ S.I unit)

16. Two square metal plates of side 1 m are kept 0.01 m apart like a parallel plate capacitor in air in such a way that one of their edges is perpendicular to an oil surface in a tank filled with an insulating oil. The plates are connected to a battery of 500 V. The plates are then lowered vertically into the oil at a speed of 0.001 ms^{-1}. The current $n \times 10^{-9}$A drawn from the battery during the process. Then find the value of n. (Dielectric constant of oil = 11], ($\in_0 = 8 \times 10^{-12}$ C² N^{-1} m^{-1}) **[JEE- 1994; 6M]**

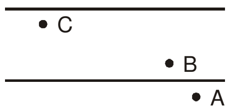
17. Three conducting plates of area 500 cm² area kept fixed as shown. Distance between adjacent plates is 8.85 mm. A charge of 1.0 nC is placed on the middle plate. (a) The charge on the outer surface of the upper plate is given by $n \times 10^{-11}$C then find the valu of n.(b) Find the potential difference (inV) developed between the upper and the middle plates.



18. Consider the arrangement of parallel plates of the previous problem. If 1.0nC charge is given to the upper plate instead of the middle, what will be the potential difference (in V) between (a) the upper and the middle plates and (b) the middle and the lower plates?

PART - III : ONE OR MORE THAN ONE OPTIONS CORRECT TYPE

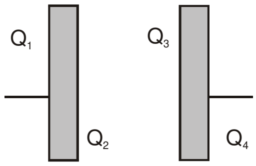
1. For a charged parallel plate capacitor shown in the figure, the force experienced by an alpha particle will be :



(A) maximum at C (B) zero at A (C) same at B and C (D) zero at C

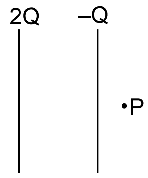
2. On a parallel plate capacitor following operations can be performed.
P - connect the capacitor to a battery of emf V
Q - disconnect the battery
R - reconnect the battery with polarity reversed
S - insert a dielectric slab in the capacitor
(A) In PQR (perform P, then Q, then R), the stored electric energy remains unchanged and no thermal energy is developed
(B) The charge appearing on the capacitor is greater after the action PSQ then after the action PQS
(C) The electric energy stored in the capacitor is greater after the action SPQ then after the action PQS
(D) The electric field in the capacitor after the action PS is the same as that after SP

3. In an isolated parallel plate capacitor of capacitance C the four surfaces have charges Q_1, Q_2, Q_3 and Q_4 as shown in the figure. The potential difference between the plates is :



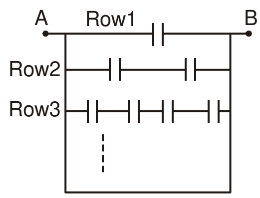
(A) $\frac{Q_1 + Q_2}{C}$ (B) $\left|\frac{Q_2}{C}\right|$ (C) $\left|\frac{Q_3}{C}\right|$ (D) $\frac{1}{C}[(Q_1 + Q_2) - (Q_3 - Q_4)]$

4. In the figure shown the plates of a parallel plate capacitor have unequal charges. Its capacitance is 'C'. P is a point outside the capacitor and close to the plate of charge –Q. The distance between the plates is 'd'.



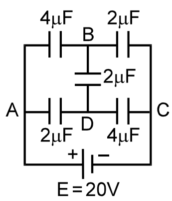
(A) A point charge at point 'P' will experience electric force due to capacitor
(B) The potential difference between the plates will be $\frac{3Q}{2C}$
(C) The energy stored in the electric field in the region between the plates is $\frac{9Q^2}{8C}$
(D) The force on one plate due to the other plate is $\frac{Q^2}{2\pi \epsilon_0 d^2}$

5. Rows of capacitors containing 1, 2, 4, 8, ...∞ capacitors, each of capacitance 2F, are connected in parallel as shown in figure. The potential difference across AB = 10 volt, then :



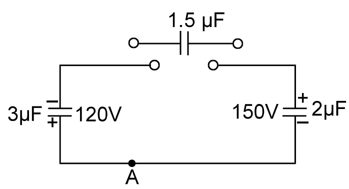
(A) Total capacitance across AB is 4F
(B) Charge of each capacitor will be same
(C) Charge on the capacitor in the first row is more than on any other capacitor
(D) Energy of all the capacitors is 50 J

6. The figure shows a diagonal symmetric arrangement of capacitors and a battery. If the potential of C is zero, then (All the capacitors are initially uncharged).



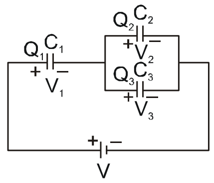
(A) $V_A = + 20$ V
(B) $4(V_A - V_B) + 2(V_D - V_B) = 2V_B$
(C) $2(V_A - V_D) + 2(V_B - V_D) = 4V_D$
(D) $V_A = V_B + V_D$

7. Two capacitors of 2 μF & 3 μF are charged to 150 volt & 120 volt respectively. The plates of a capacitor are connected as shown in the fig. A discharged capacitor of capacity 1.5 μF falls to the free ends of the wire and connected through the free ends of the wire. Then :



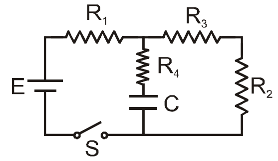
(A) Charge on the 1.5 μF capacitor will become 180 μC at steady state.
(B) Charge on the 2 μF capacitor will become 120 μC at steady state.
(C) Positive charge flows through point A from left to right.
(D) Positive charge flows through point A from right to left.

8. In the adjoining diagram all the capacitors are initially uncharged, they are connected with a battery as a shown in figure. Then



(A) $Q_1 = Q_2 + Q_3$ and $V = V_1 + V_2$
(B) $Q_1 = Q_2 + Q_3$ and $V = V_1 + \frac{V_2 + V_3}{2}$
(C) $Q_1 = Q_2 + Q_3$ and $V = V_1 + V_3$
(D) $Q_2 = Q_3$ and $V = V_2 + V_3$

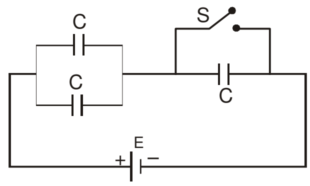
9. In the circuit shown in figure the switch S is closed at t = 0. A long time after closing the switch



(A) voltage drop across the capacitor is E
(B) current through the battery is $\frac{E}{R_1 + R_2 + R_3}$
(C) energy stored in the capacitor is $\frac{1}{2}C\left(\frac{(R_2 + R_3)E}{R_1 + R_2 + R_3}\right)^2$
(D) current through the resistance R_4 becomes zero

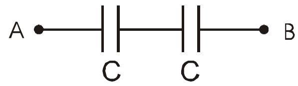
10. When a charged capacitor is connected with an uncharged capacitor, then which of the following is/are correct option/options.
(A) the magnitude of charge on the charged capacitor decreases.
(B) a steady state is obtained after which no further flow of charge occurs.
(C) the total potential energy stored in the capacitors remains conserved.
(D) the charge conservation is always true.

11. In the circuit shown in figure, each capacitor has a capacitance C. The emf of the cell is E and circuit already in steady state. If the switch S is closed.



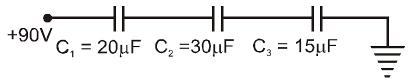
(A) some positive charge will flow out of the positive terminal of the cell
(B) some positive charge will enter the positive terminal of the cell
(C) the amount of charge flowing through the cell will be CE
(D) the amount of charge flowing through the cell will be $\left(\frac{4}{3}\right)CE$

12. Two similar condensers are connected in parallel and are charged to a potential V. Now these are separated out and are connected in series. Then



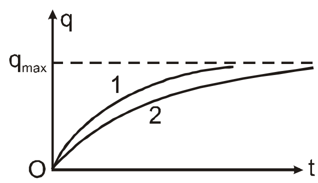
(A) the energy stored in the system increases
(B) the potential difference between end points may becomes zero.
(C) the potential difference between end points may becomes 2V.
(D) the charge on the plates mutually connected nullifies.

13. We have a combination as shown in following figure. Choose the correct options :



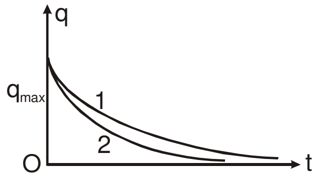
(A) The charge on each capacitor is 600 μC
(B) The potential difference between the plates of C_1 is 30 V
(C) The potential difference between the plates of C_2 is 20 V
(D) The potential difference between the plates of C_3 is 40 V

14. The charge on capacitor in two different RC circuits 1 and 2 are plotted as shown in figure.



Choose the correct statement(s) related to the two circuits.
(A) Both the capacitors are charged to the same magnitude of charge
(B) The emf's of cells in both the circuits are equal.
(C) The emf's of the cells may be different
(D) The emf E_1 is more than E_2

15. The instantaneous charge on capacitor in two discharging RC circuits is plotted with respect to time in figure. Choose the correct statement(s) (where E_1 and E_2 are emfs of two DC sources in two different charging circuits and capacitors are fully charged).



(A) $R_1C_1 > R_2C_2$ (B) $\frac{R_1}{R_2} < \frac{C_2}{C_1}$ (C) $R_1 > R_2$ if $E_1 = E_2$ (D) $C_2 > C_1$ if $E_1 = E_2$

16. Capacitor C_1 of the capacitance 1 microfarad and capacitor C_2 of capacitance 2 microfarad are separately charged fully by a common battery. The two capacitors are then separately allowed to discharge through equal resistors at time t = 0. **[JEE-IIT 1989]**
(A) the current in each of the two discharging circuits is zero at t = 0.
(B) the current in the two discharging circuits at t = 0 are equal but non zero.
(C) the current in the two discharging circuits at t = 0 are unequal
(D) capacitor C_1 loses 50% of its initial charge sooner than C_2 loses 50% of its initial charge

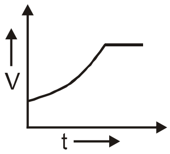
17. The terminals of a battery of emf V are connected to the two plates of a parallel plate capacitor. If the space between the plates of the capacitor is filled with an insulator of dielectric constant K, then :
(A) the electric field in the space between the plates does not change
(B) the capacitance of the capacitor increases
(C) the charge stored in the capacitor increases
(D) the electrostatic energy stored in the capacitor decreases

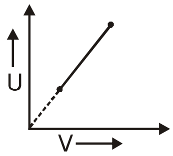
18. A parallel plate capacitor of plate area A & plate separation d is charged to a potential difference V & then the battery disconnected. A slab of dielectric constant K is then inserted between the plates of the capacitor so as to fill the space between the plates. If Q, E and W denote respectively, the magnitude of the charge on each plate, the magnitude of the electric field between the plates (after the slab is inserted) & the magnitude of the work done on the system, in the process of inserting the slab, then : **[JEE 1997, 2M]**

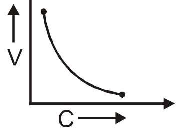
(A) $Q = \frac{\epsilon_0 AV}{d}$ (B) $Q = \frac{\epsilon_0 KAV}{d}$ (C) $E = \frac{V}{Kd}$ (D) $W = \frac{\epsilon_0 AV^2}{2d}\left(1-\frac{1}{K}\right)$

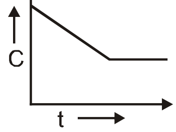
19. The plates of a parallel plate capacitor with no dielectric are connected to a voltage source. Now a dielectric of dielectric constant K is inserted to fill the whole space between the plates with voltage source remaining connected to the capacitor.
(A) the energy stored in the capacitor will become K–times
(B) the electric field inside the capacitor will decrease to K-times
(C) the force of attraction between the plates will increase to K^2 – times
(D) the charge on the capacitor will increase to K-times

20. A parallel plate capacitor has a dielectric slab in it. The slab just fills the space inside the capacitor. The capacitor is charged by a battery and the battery is disconnected. Now the slab is started to pull out uniformly at t = 0. If at time t, capacitance of the capacitor is C, potential difference across plate is V, and energy stored in it is U, then which of the following graphs are correct ?

(A) 

(B) 

(C) 

(D) 

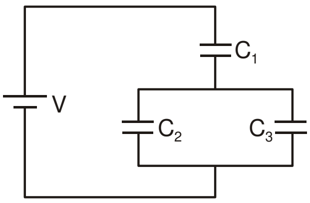
21. A parallel plate air capacitor is connected to a battery. The quantities charge, electric field and energy associated with this capacitor are given by Q_0, V_0, E_0 and U_0 respectively. A dielectric slab is now introduced to fill the space between the plates with the battery still in connection. The corresponding quantities now given by Q, V, E and U are related to the previous one as ; **[1985; 2M]**

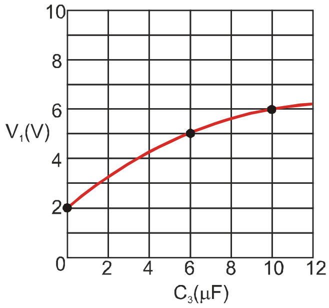
(A) $Q > Q_0$ (B) $V > V_0$ (C) $E > E_0$ (D) $U > U_0$

PART - IV : COMPREHENSION

Comprehension-1

Capacitor C_3 in the circuit is a variable capacitor (its capacitance can be varied). Graph is plotted between potential difference V_1 (across capacitor C_1) versus C_3. Electric potential V_1 approaches on asymptote of 10 V as $C_3 \to \infty$.





1. EMF of the battery is equal to :

(A) 10 V (B) 12 V (C) 16 V (D) 20 V

2. The capacitance of the capacitor C_1 has value :

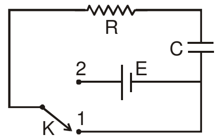
(A) 2 μF (B) 6 μF (C) 8 μF (D) 12 μF

3. The capacitance of C_2 is equal to :

(A) 2 μF (B) 6 μF (C) 8 μF (D) 12 μF

Comprehension-2

In the shown circuit involving a resistor of resistance R Ω, capacitor of capacitance C farad and an ideal cell of emf E volts, the capacitor is initially uncharged and the key is in position 1. At t = 0 second the key is pushed to position 2 for t_0 = RC seconds and then key is pushed back to position 1 for t_0 = RC seconds. This process is repeated again and again. Assume the time taken to push key from position 1 to 2 and vice versa to be negligible.



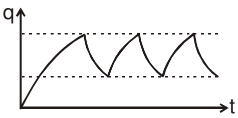
4. The charge on capacitor at t = 2RC second is

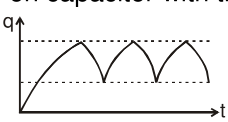
(A) CE (B) $CE\left(1-\frac{1}{e}\right)$ (C) $CE\left(\frac{1}{e}-\frac{1}{e^2}\right)$ (D) $CE\left(1-\frac{1}{e}+\frac{1}{e^2}\right)$

5. The current through the resistance at t = 1.5 RC seconds is

(A) $\frac{E}{e^2R}(1-\frac{1}{e})$ (B) $\frac{E}{eR}(1-\frac{1}{e})$ (C) $\frac{E}{R}(1-\frac{1}{e})$ (D) $\frac{E}{\sqrt{e}R}(1-\frac{1}{e})$

6. Then the variation of charge on capacitor with time is best represented by

(A)


(B)


(C)

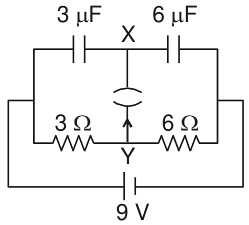

(D)


Exercise-3

✍ **Marked Questions can be used as Revision Questions.**
*** Marked Questions may have more than one correct option.**

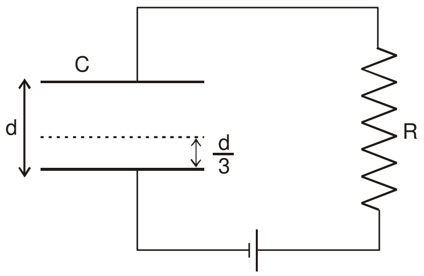
PART - I : JEE (ADVANCED) / IIT-JEE PROBLEMS (PREVIOUS YEARS)

1. A circuit is connected as shown in the figure with the switch S open. When the switch is closed, the total amount of charge that flows from Y to X is **[JEE 2007' 3/81]**



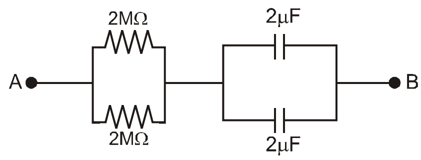
(A) 0 (B) 54 μC (C) 27 μC (D) 81 μC

2. A parallel plate capacitor C with plates of unit area and separation d is filled with a liquid of dielectric constant K = 2. The level of liquid is $\frac{d}{3}$ initially. Suppose the liquid level decreases at a constant speed V, the time constant as a function of time t is **[JEE' 2008 ; 3/163]**
Figure :

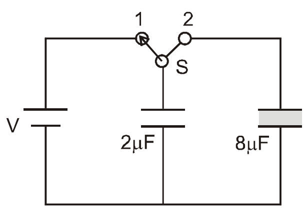


(A) $\frac{6\varepsilon_0 R}{5d+3Vt}$ (B) $\frac{(15d+9Vt)\varepsilon_0 R}{2d^2-3dVt-9V^2t^2}$ (C) $\frac{6\varepsilon_0 R}{5d-3Vt}$ (D) $\frac{(15d-9Vt)\varepsilon_0 R}{2d^2+3dVt-9V^2t^2}$

3. At time t = 0, a battery of 10 V is connected across points A and B in the given circuit. If the capacitors have no charge initially, at what time (in seconds) does the voltage across them become 4 V?
[Take : ℓn 5 = 1.6, ℓn 3 = 1.1] **[JEE 2010 ; 3/163]**

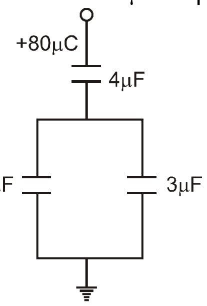


4. A 2μF capacitor is charged as shown in figure. The percentage of its stored energy dissipated after the switch S is turned to position 2 is **[JEE 2010 ; 3/160, –1]**



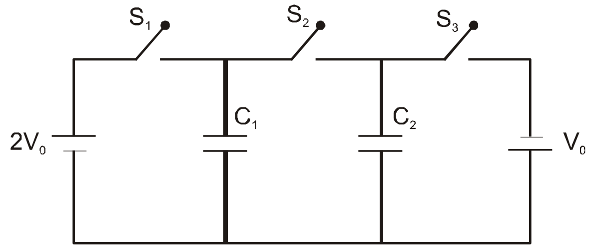
(A) 0% (B) 20% (C) 75% (D) 80%

5. In the given circuit, a charge of +80 μC is given to the upper plate of the 4μF capacitor. Then in the steady state, the charge on the upper plate of the 3μF capacitor is : **[IIT-JEE-2012, Paper-2; 3/66, –1]**



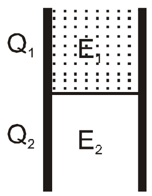
(A) +32 μC (B) +40 μC (C) +48 μC (D) +80 μC

6.* In the circuit shown in the figure, there are two parallel plate capacitors each of capacitance C. The switch S_1 is pressed first to fully charge the capacitor C_1 and then released. The switch S_2 is then pressed to charge the capacitor C_2. After some time, S_2 is released and then S_3 is pressed. After some time. **[JEE (Advanced) 2013, 3/60]**



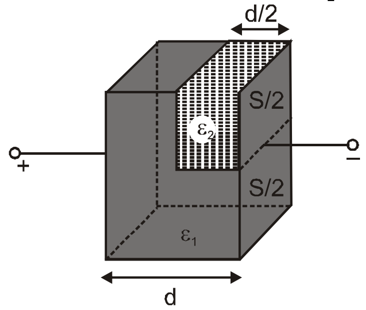
(A) the charge on the upper plate of C_1 is $2CV_0$ (B) the charge on the upper plate of C_1 is CV_0
(C) the charge on the upper plate of C_2 is 0 (D) the charge on the upper plate of C_2 is $–CV_0$

7.* A parallel plate capacitor has a dielectric slab of dielectric constant K between its plates that covers 1/3 of the area of its plates, as shown in the figure. The total capacitance of the capacitor is C while that of the portion with dielectric in between is C_1. When the capacitor is charged, the plate area covered by the dielectric gets charge Q_1 and the rest of the area gets charge Q_2. Choose the correct option/options, igonoring edge effects. **[JEE (Advanced) 2014,P-1, 3/60]**



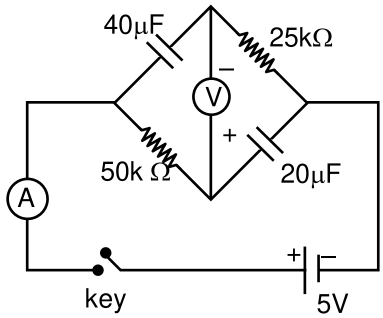
(A) $\frac{E_1}{E_2} = 1$ (B) $\frac{E_1}{E_2} = \frac{1}{K}$ (C) $\frac{Q_1}{Q_2} = \frac{3}{K}$ (D) $\frac{C}{C_1} = \frac{2+K}{K}$

8. A parallel plate capacitor having plates of area S and plate separation d, has capacitance C_1 in air. When two dielectrics of different relative permittivities (ε_1 = 2 and ε_2 = 4) are introduced between the two plates as shown in the figure, the capacitance becomes C_2. The ratio C_2/C_1 is
[JEE (Advanced) 2015 ; P-2, 4/88, –2]



(A) 6/5 (B) 5/3 (C) 7/5 (D) 7/3

9.* In the circuit shown below, the key is pressed at time t = 0. Which of the following statement(s) is (are) true? **[JEE (Advanced) 2016 ; P-2, 4/62, –2]**



(A) The voltmeter displays –5 V as soon as the key is pressed, and displays +5 V after a long time

(B) The voltmeter will display 0 V at time t = ℓn 2 seconds

(C) The current in the ammeter becomes 1/e of the initial value after 1 second

(D) The current in the ammeter becomes zero after a long time

PARAGRAPH -1

Consider a simple RC circuit as shown in figure 1.

Process 1 : In the circuit the switch S is closed at t = 0 and the capacitor is fully charged to voltage V_0 (i.e., charging continues for time T>> RC). In the process some dissipation (E_D) occurs across the resistance R. The amount of energy finally stored in the fully charged capacitor is E_C.

Process 2 : In a different process the voltage is first set to $\frac{V_0}{3}$ and maintained for a charging time T >>RC. Then the voltage is raised to $\frac{2V_0}{3}$ without discharging the capacitor and again maintained for a time T >> RC. The process is repeated one more time by raising the voltage to V_0 and the capacitor is charged to the same final voltage V_0 as in Process 1.

These two processes are depicted in figure 2.

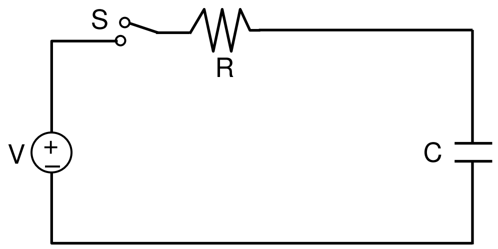


Figure 1

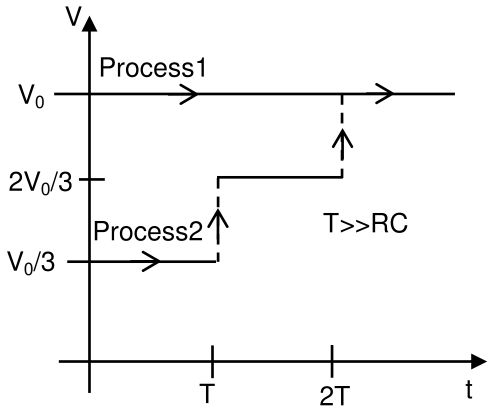


Figure 2

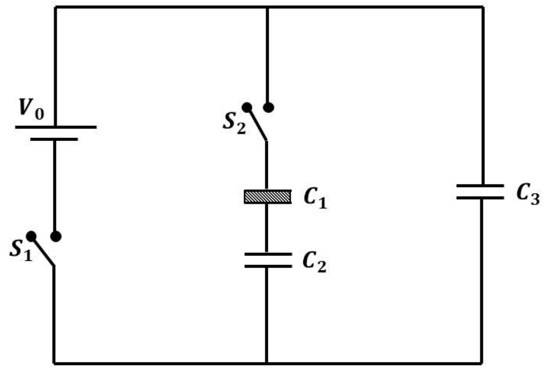
10. In process 1, the energy stored in the capacitor E_C and heat dissipated across resistance E_D are related by : **[JEE(Advanced)-2017 ; P-2, 3/61]**

(A) $E_C = \frac{1}{2}E_D$ (B) $E_C = E_D \ln 2$ (C) $E_C = 2E_D$ (D) $E_C = E_D$

11. In process 2, total energy dissipated across the resistance E_D is : **[JEE(Advanced)-2017 ; P-2, 3/61]**

(A) $E_D = 3\left(\frac{1}{2}CV_0^2\right)$ (B) $E_D = \frac{1}{3}\left(\frac{1}{2}CV_0^2\right)$ (C) $E_D = 3CV_0^2$ (D) $E_D = \frac{1}{2}CV_0^2$

12. Three identical capacitors C_1, C_2 and C_3 have a capacitance of 1.0 μF each and they are uncharged initially. They are connected in a circuit as shown in the figure and C_1 is then filled completely with a dielectric material of relative permittivity ε_r. The cell electromotive force (emf) V_0 = 8V. First the switch S_1 is closed while the switch S_2 is kept open. When the capacitor C_3 is fully charged, S_1 is opened and S_2 is closed simultaneously. When all the capacitors reach equilibrium, the charge on C_3 is found to be 5μC. The value of ε_r =____________. **[JEE (Advanced) 2018 ; P-1, 3/60]**



PART - II : JEE (MAIN) / AIEEE PROBLEMS (PREVIOUS YEARS)

1. A battery is used to charge a parallel plate capacitor till the potential difference between the plates becomes equal to the electromotive force of the battery. The ratio of the energy stored in the capacitor and the work done by the battery will be **[AIEEE-2007, 3/120]**

(1) 1 (2) 2 (3) 1/4 (4) 1/2

2. A parallel plate condenser with a dielectric of dielectric constant K between the plates has a capacity C and is charged to a potential V volts. The dielectric slab is slowly removed from between the plates and then reinserted. The net work done by the system in this process is **[AIEEE-2007, 3/120]**

(1) $1/2\ (K-1)CV^2$ (2) $CV^2(K-1)/K$ (3) $(K-1)CV^2$ (4) zero

3. A parallel plate capacitor with air between the plates has a capacitance of 9 pF. The separation between its plates is 'd'. The space between the plates is now filled with two dielectrics. One of the dielectrics has dielectric constant k_1 = 3 and thickness d/3 while the other one has dielectric constant k_2 = 6 and thickness 2d/3. Capacitance of the capacitor is now : **[AIEEE-2008, 3/105]**

(1) 45 pF (2) 40.5 pF (3) 20.25 pF (4) 1.8 pF

4. Let C be the capacitance of a capacitor discharging through a resistor R. Suppose t_1 is the time taken for the energy stored in the capacitor to reduce to half its initial value and t_2 is the time taken for the charge to reduce to one-fourth its initial value. Then the ratio t_1/t_2 will be **[AIEEE-2010, 4/144]**

(1) 1 (2) 1/2 (3) 1/4 (4) 2

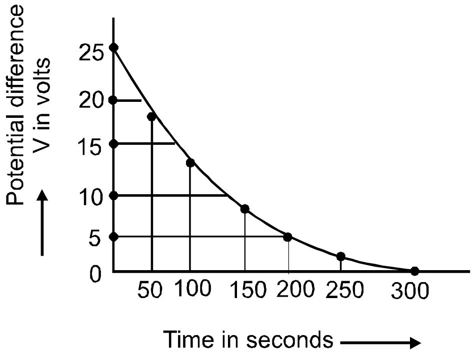
5. A resistor 'R' and 2μF capacitor in series is connected through a switch to 200 V direct supply. Across the capacitor is a neon bulb that lights up at 120 V. Calculate the value of R to make the bulb light up 5s after the switch has been closed. ($\log_{10}2.5 = 0.4$) **[AIEEE - 2011, 4/120, –1]**

(1) $1.3 \times 10^4\ \Omega$ (2) $1.7 \times 10^5\ \Omega$ (3) $2.7 \times 10^6\ \Omega$ (4) $3.3 \times 10^7\ \Omega$

6. Combination of two identical capacitors, a resistor R and a dc voltage source of voltage 6V is used in an experiment on a (C – R) circuit. It is found that for a parallel combination of the capacitor the time in which the voltage of the fully charged combination reduces to half its original voltage is 10 second. For series combination the time needed for reducing the voltage of the fully charged series combination by half is : **[AIEEE 2011, 11 May; 4/120, –1]**

(1) 10 second (2) 5 second (3) 2.5 second (4) 20 second

7. The figure shows an experimental plot discharging of a capacitor in an RC circuit. The time constant τ of this circuit lies between : **[AIEEE 2012 ; 4/120, –1]**

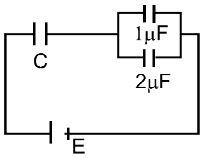


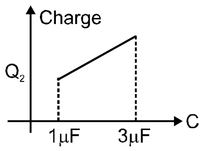
(1) 150 sec and 200 sec (2) 0 and 50 sec (3) 50 sec and 100 sec (4) 100 sec and 150 sec

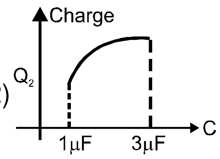
8. Two capacitors C_1 and C_2 are charged to 120 V and 200 V respectively. It is found that by connecting them together the potential on each one can be made zero. Then : **[JEE(Main) 2013 ; 4/120, –1]**
(1) $5C_1 = 3C_2$ (2) $3C_1 = 5C_2$ (3) $3C_1 + 5C_2 = 0$ (4) $9C_1 = 4C_2$

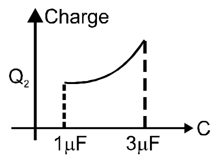
9. A parallel plate capacitor is made of two circular plates separated by a distance of 5 mm and with a dielectric of dielectric constant 2.2 between them. When the electric field in the dielectric is 3×10^4 V/m, the charge density of the positive plate will be close to : **[JEE(Main) 2014 ; 4/120, –1]**
(1) 6×10^{-7}C/m^2 (2) 3×10^{-7}C/m^2 (3) 3×10^4C/m^2 (4) 6×10^4C/m^2

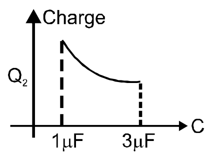
10. In the given circuit, charge Q_2 on the 2μF capacitor changes as C is varied from 1μF to 3μF. Q_2 as a function of 'C' is given properly by : (figures are drawn schematically and are not to scale) **[JEE (Main) 2015; 4/120, –1]**



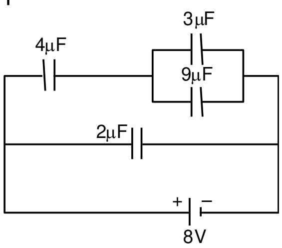
(1) 

(2) 

(3) 

(4) 

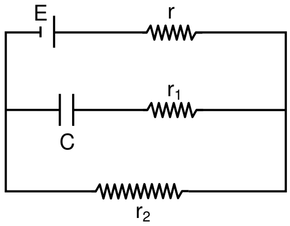
11. A combination of capacitors is set up as shown in the figure. The magnitude of the electric field, due to a point charge Q (having a charge equal to the sum of the charges on the 4μF and 9μF capacitors), at a point distance 30 m from it, would equal : **[JEE (Main) 2016; 4/120, –1]**



(1) 360 N/C (2) 420 N/C (3) 480 N/C (4) 240 N/C

12. A capacitance of 2μF is required in an electrical circuit across a potential difference of 1.0 kV. A large number of 1μF capacitors are available which can withstand a potential difference of not more than 300 V. The minimum number of capacitors required to achieve this is : **[JEE (Main) 2017; 4/120, –1]**
(1) 32 (2) 2 (3) 16 (4) 24

13. In the given circuit diagram when the current reaches steady state in the circuit, the charge on the capacitor of capacitance C will be : **[JEE (Main) 2017; 4/120, –1]**



(1) $CE\dfrac{r_1}{(r_1+r)}$ (2) CE (3) $CE\dfrac{r_1}{(r_2+r)}$ (4) $CE\dfrac{r_2}{(r+r_2)}$

14. A parallel plate capacitor of capacitance 90 pF is connected to a battery of emf 20V. If a dielectric material of dielectric constant K = 5/3 is inserted between the plates, the magnitude of the induced charge will be : **[JEE (Main) 2018; 4/120, –1]**

(1) 2.4 n C (2) 0.9 n C (3) 1.2 n C (4) 0.3 n C

Answers

EXERCISE-1

PART - I

Section (A) :

A-1. (i) 6 V (ii) 90 μJ

(iii) (a) $\dfrac{16}{3}$ V (b) $\dfrac{5}{3}$ μJ

(c) $\dfrac{Q_{5\mu F}}{Q_{10\mu F}} = \dfrac{1}{2}$

(d) $Q_{5\mu F} = \dfrac{80}{3}\mu C$ $Q_{10\mu F} = \dfrac{160}{3}\mu C$

A-2. $\dfrac{q^2}{2k \in_0 A}$

A-3. Work done by the field = Q ($V_1 - V_2$)

A-4. $\dfrac{\in_0}{2g}\times 10^{10}$ kg = 4.425 g

A-5. (i) $\dfrac{q^2(x_2 - x_1)}{2\in_0 S}$

(ii) $(-)\dfrac{\in_0 SV^2\left(\frac{1}{x_2}-\frac{1}{x_1}\right)}{2}$

Section (B) :

B-1. (a) ε (b) $\dfrac{\varepsilon}{R}$ (c) ε

(d) $\dfrac{1}{2}$ $C\varepsilon^2$ (e) $\dfrac{\varepsilon^2}{R}$ (f) $\dfrac{\varepsilon^2}{R}$

B-2. V_o = 20 V **B-3.** –22 V

B-4. $V_1 = \dfrac{(\varepsilon_2 - \varepsilon_1)}{\left(1+\dfrac{C_1}{C_2}\right)}$, $V_2 = \dfrac{(\varepsilon_1 - \varepsilon_2)}{\left(1+\dfrac{C_2}{C_1}\right)}$

Section (C) :

C-1. (i) 20 μC, 40 μC, 60 μC

(ii) 2400 μJ (iii) 1200 μJ

C-2. (a) five 2 μF capacitors in series

(b) 3 parallel rows ; each consisting of five 2.0 μF capacitors in series

C-3. (a) 10 μF (b) 10μF (c) 10μF

C-4. (a) 700/3 V at each point (b) zero

C-5. 25 V and 75 V.

C-6. (a) Cε/2, (b) –Cε, (c) $C\varepsilon^2/2$

(d) $C\varepsilon^2/4$ (e) $C\varepsilon^2/4$

C-7. 4 μC, 16 μC, 12 μC and 32 μC, 1 A.

Section (D) :

D-1. $\dfrac{10^{-7}}{\ln(3/2)}$ F = 0.25 μF

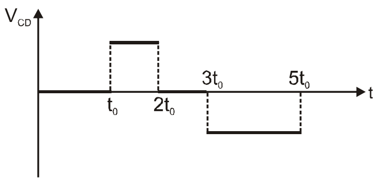
D-2. (i) (a) 10 s (b) 2 μC (c) 10 ln 2 = 6.93 sec.

(ii) q = (2 e^{-5})μC = 1.348 x 10^{-8} C

D-3. q = 20 × 10^{-4} (1 – e^{-2})e^{-2} = 233.55 μC

D-4. 40 (1 – e^{-2})e^{-2} μJ = 4.7 μJ.

D-5.



D-6. CV(1 – $e^{-t/Cr}$) + $q_0e^{-t/Cr}$

Section (E) :

E-2. (i) $20\epsilon_0 = 180$ pF (ii) 5.4×10^{-7} C
(iii) 540 pF (iv) 3
(v) $27 \times 10^{-12}\ C^2 N^{-1} m^{-2}$ (vi) 3×10^5 V/m
(vii) 1×10^5 V/m

E-3. $C = \dfrac{\epsilon_0 A}{d - t + \frac{t}{k}}$, $\dfrac{t}{d} = \dfrac{2}{3}$, $\dfrac{U_i}{U_F} = \dfrac{3}{2}$,

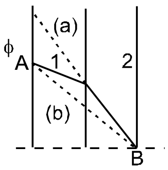
$\Delta U = -\dfrac{q^2 d}{6 \epsilon_0 A}$

E-4. $\dfrac{E_i}{E_f} = \dfrac{1}{2}\ (1 + k)$, $\Delta q = \dfrac{1}{2}\ C\varepsilon \dfrac{k-1}{k+1}$

E-5. $\dfrac{\epsilon_0 AV^2}{2d}\left(\dfrac{1}{2K} - 1\right)$ **E-6.** $\dfrac{\epsilon_0 bv^2(K-1)}{2dK_S}$

E-7. $\dfrac{\epsilon_0 b}{2dK_S}\left[(K_2 - 1)\varepsilon_2^{\,2} - (K_1 - 1)\ \ \varepsilon_1^{\,2}\right]$

E-8. $4.\sqrt{\dfrac{2bm}{\epsilon_0 V^2 (K-1)}}$ **E-9.**



E-10. (i) $C = \dfrac{\varepsilon_0 A}{d}$
(ii) on outer surfaces charge = q
on inner surfaces charge = 0
(iii) E = 0 (iv) $\Delta V = 0$ (v) U = 0

PART - II

Section (A) :

A-1.	(A)	**A-2.**	(D)
A-3.	(i) (C) (ii) (D) (iii) (D) (iv) (C)		
A-4.	(B)	**A-5.**	(B)

Section (B) :

B-1.	(B)	**B-2.**	(C)	**B-3.**	(D)
B-4.	(C)	**B-5.**	(C)		

Section (C) :

C-1.	(D)	**C-2.**	(B)	**C-3.**	(C)
C-4.	(D)	**C-5.**	(B)	**C-6.**	(A)
C-7.	(B)	**C-8.**	(C)	**C-9.**	(C)
C-10.	(C)	**C-11.**	(A)		

Section (D) :

D-1. (i) (C) (ii) (A) (iii) (C) (iv) (C)
D-2. (i) (A) (ii) (B)
D-3. (i) (A) (ii) (C)
D-4. (C) **D-5.** (i) (B) (ii) (C) (iii) (A)
D-6. (C)

Section (E) :

E-1.	(B)	**E-2.**	(C)	**E-3.**	(A)
E-4.	(A)	**E-5.**	(C)	**E-6.**	(A)
E-7.	(A)	**E-8.**	(C)		

PART - III

1. (A) – p ; (B) – r ; (C) – q ; (D) – p
2. (A) – p, q, s; (B) – p, r, s; (C) – p, q ;(D) – p, r

EXERCISE-2

PART - I

1.	(B)	**2.**	(A)	**3.**	(D)
4.	(C)	**5.**	(D)	**6.**	(B)
7.	(A)	**8.**	(i) (D) (ii) (A)		
9.	(D)	**10.**	(B)	**11.**	(D)
12.	(i) (A)	(ii) (D) (iii) (B)		**13.**	(i) (A) (ii) (D)
14.	(B)	**15.**	(A)	**16.**	(C)
17.	(A)	**18.**	(D)	**19.**	(D)
20.	(A)	**21.**	(B)	**22.**	(B)
23.	(D)	**24.**	(C)	**25.**	(D)

PART - II

1.	6			**2.**	3
3.	(a) 16	(b) 48	(c) 96	**4.**	45
5.	45	**6.**	24	**7.**	6
8.	9	**9.**	10		
10.	(i) 6	(ii) 54		**11.**	12
12.	20	**13.**	2.0.	**14.**	1
15.	5	**16.**	4	**17.**	(a) 50 (b) 10
18.	(a) 10	(b) 10			

PART - III

1.	(BC)	**2.**	(BCD)	**3.**	(BC)
4.	(ABC)	**5.**	(AC)	**6.**	(ABCD)
7.	(ABC)	**8.**	(ABC)	**9.**	(BCD)
10.	(ABD)	**11.**	(AD)	**12.**	(BC)
13.	(ABCD)	**14.**	(AC)	**15.**	(AC)
16.	(BD)	**17.**	(ABC)	**18.**	(ACD)
19.	(ACD)	**20.**	(ABCD)	**21.**	(AD)

PART - IV

1.	(A)	**2.**	(C)	**3.**	(A)
4.	(C)	**5.**	(D)	**6.**	(C)

EXERCISE-3

PART - I

1.	(C)	**2.**	(A)	**3.**	2
4.	(D)	**5.**	(C)	**6.**	(BD)
7.	(AD)	**8.**	(D)	**9.**	(ABCD)
10.	(D)	**11.**	(B)	**12.**	1.50

PART - II

1.	(4)	**2.**	(4)	**3.**	(2)
4.	(3)	**5.**	(3)	**6.**	(3)
7.	(4)	**8.**	(2)	**9.**	(1)
10.	(2)	**11.**	(2)	**12.**	(1)
13.	(4)	**14.**	(3)		

High Level Problems (HLP)

Marked Questions can be used as Revision Questions.

SUBJECTIVE QUESTIONS

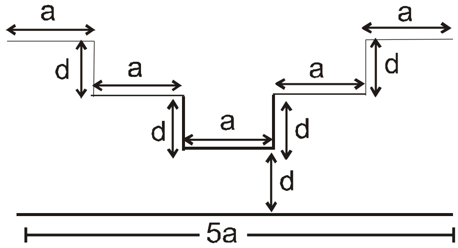
1. A capacitor having a capacitance of 200 μF is charged to a potential difference of 20V. The charging battery is disconnected and the capacitor is connected to another battery of emf 10V with the positive plate of the capacitor joined with the positive terminal of the battery.
(a) Find the charges on the capacitor before and after the reconnection in steady state.
(b) Find the net charge flown through the 10 V battery
(c) Is work done by the battery or is it done on the battery? Find its magnitude.
(d) Find the decrease in electrostatic field energy.
(e) Find the heat developed during the flow of charge after reconnection.

2. Six 1 μF capacitors are so arranged that their equivalent capacitance is 0.70 μF. If a potential difference of 600 volt is applied to the combination, what charge will appear on each capacitor?

3. A battery of 10 volt is connected to a capacitor of capacity 0.1 F. The battery is now removed and this capacitor is connected to a second uncharged capacitor. If the charge distributes equally on these two capacitors, find the total energy stored in the two capacitors. Further, compare this energy with the initial energy stored in the first capacitor. **[REE - 1996, 5]**

4. The circular plates A and B of a parallel plate air capacitor have a diameter of 0.1 m and are 2×10^{-3} m apart. The plates C and D of a similar capacitor have a diameter of 0.12 m and are 3×10^{-3} m apart. Plate A is earthed. Plates B and D are connected together. Plate C is connected to the positive pole of a 120 volt battery whose negative is earthed. Calculate **[REE - 1998, 5]**
(i) The combined capacitance of the arrangement and
(ii) The energy stored in it.

5. A capacitor is made of a flat plate of area A and a second plate having a stair-like structure as shown in figure. The width of each stair is a and the height is d. Both plates have the same width perpendicular to plane of paper. Find the capacitance of the assembly.

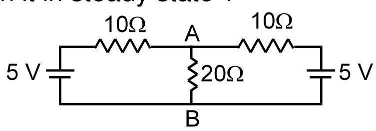


6. Calculate the capacitance of a parallel plate condenser, with plate area A and distance between plates d, when filled with a dielectric whose dielectric constant varies as; **[REE 2000,6]**

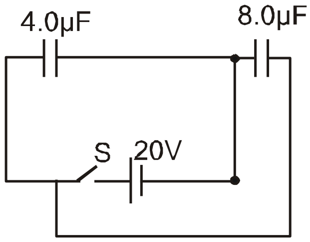
$$K(x) = 1 + \frac{\beta x}{\epsilon_0} \quad 0 < x < \frac{d}{2}\ ; \qquad K(x) = 1 + \frac{\beta}{\epsilon_0}(d - x) \quad \frac{d}{2} < x < d.$$

For what value of β would the capacity of the condenser be twice that when it is without any dielectric?

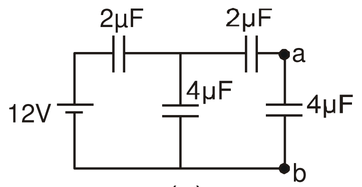
7. (a) Find the current in the 20 Ω resistor shown in figure.
(b) If a capacitor of capacitance 4μF is joined between the points A and B, what would be the electrostatic energy stored in it in steady state ?



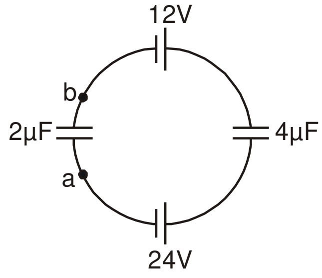
8. (i) Find the total charge flown through the battery in the arrangement shown in figure after switch S is closed (initially all the capacitors are uncharged).
(ii) Find out final charge on each capacitor.



9. Find the potential difference $V_a - V_b$ between the points a and b shown in each part of the figure.

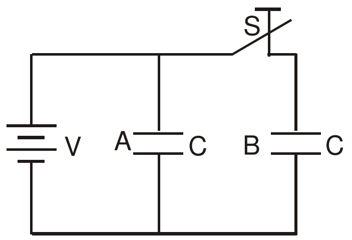


(a)

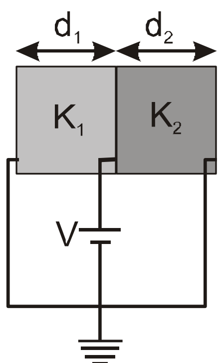


(b)

10. The figure shows two identical parallel plate capacitors connected to a battery with the switch S closed. The switch is now opened and the free space between the plates of the capacitors is filled with a dielectric of dielectric constant (or relative permittivity) 3. Find the ratio of the total electrostatic energy stored in both capacitors before and after the introduction of the dielectric. **[JEE 1983; 6M]**

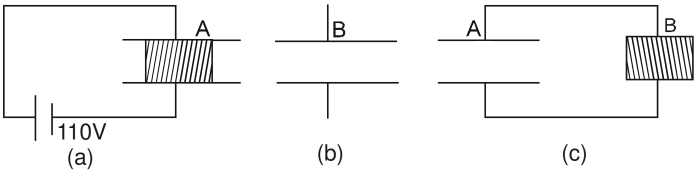


11. A capacitor is composed of three parallel conducting plates. All three plates are of the same area A. The first pair of plates are kept a distance d_1 apart, and the space between them is filled with a medium of a dielectric K_1. The corresponding data for the second pair are d_2 and K_2, respectively. The middle plate is connected to the positive terminal of a constant voltage source V and the external plates are connected the other terminal of V.



(a) Find the capacitance of the system.
(b) What is the surface charge density on the middle plate ?
(c) Compute the energy density in the medium K_1.

12. Two parallel plate capacitors A and B have the same separation d = 8.85 × 10^{-4} m between the plates. The plate areas of A and B are 0.04 m^2 and 0.02 m^2 respectively. A slab of dielectric constant (relative permittivity K = 9) has dimensions such that it can exactly fill the space between the plates of capacitor B. **[JEE 1993, 2+3+2=7 Marks]**

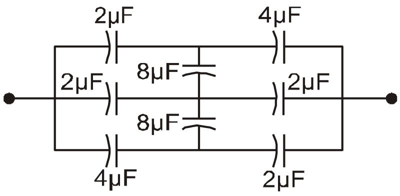


(i) The dielectric slab is placed inside A as shown in the figure (a). A is charged to potential difference of 110 V. Calculate the capacitance of A and energy stored in it :

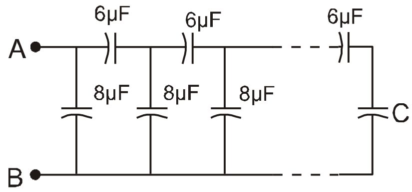
(ii) The battery is disconnected and then the dielectric also slab is moved from A. Find the work done by the external agency in removing the slab from A.

(iii) The same dielectric slab is now placed inside B, filling it completely. The two capacitors A and B are then connected as shown in the figure (c). Calculate the energy stored in the system.

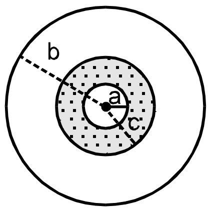
13. Find the equivalent capacitance of the combinations shown in the figure between the indicated points.



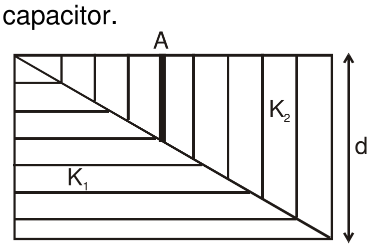
14. A finite ladder circuit is constructed by connecting several sections of 6 μF, 8 μF capacitor combinations as shown in the figure. Circuit is terminated by a capacitor of capacitance C. Find the value of C, such that the equivalent capacitance of the ladder between the points A and B becomes independent of the number of sections in between?



15. A spherical capacitor is made of two conducting spherical shells of radii a and b = 3a. The space between the shells is filled with a dielectric of dielectric constant K = 3 upto a radius c = 2a as shown. If the capacitance of given arrangement is n times the capacitance of an isolated spherical conducting shell of radius a. Then find value of n.



16. The capacitance of a parallel plate capacitor with plate area A and separation d, is C. The space between the plates is filled with two wedges of dielectric constant K_1 and K_2 respectively (figure). Find the capacitance of the resulting capacitor. **[JEE-1996; 2M/100]**



HLP Answers

1. (a) $4000\mu C$, $2000\mu C$ (b) $2000\ \mu C$ (c) work is done on the battery, 20 mJ
(d) 30 mJ (e) 10 mJ

2. $420\mu C$ on one, $180\ \mu C$ on two, $60\mu C$ on remaining 3 capacitors

3. $\frac{5}{2}$ J, $\frac{U_{initial}}{U_{final}} = \frac{5}{2.5} = 2$ **4.** (i) $\frac{30\pi \in_0}{49} \approx 17pF$ (ii) 122.4 nJ **5.** $\frac{8 \in_o A}{15d}$

6. $C = \frac{A\beta}{2\ell n\left(1+\frac{\beta d}{2\in_0}\right)}$, $\beta d = 4\in_0 \ell n\left(1+\frac{\beta d}{2\in_0}\right)$. Solution of this equation gives required value of β.

7. (a) $\frac{1}{5}$ A (b) $32\ \mu J$ **8.** (i) $240\mu C$, (ii) $Q_{4\mu F} = 80\ \mu C$, $Q_{8\mu F} = 160\ \mu C$

9. (a) $\frac{12}{11}$ V (b) – 8 V **10.** $\frac{3}{5}$

11. (a) $\in_0 A\left(\frac{K_1}{d_1}+\frac{K_2}{d_2}\right)$, (b) $\frac{Q_1}{A} = \left(\frac{K_1 \in_0 A}{d_1}\right)\frac{V}{A} = \frac{K_1 \in_0}{d_1}V$ and $\frac{Q_2}{A} = \left(\frac{K_2 \in_0 A}{d_2}\right)\frac{V}{A} = \frac{K_2 \in_0}{d_2}V$ (c) $\frac{\in_0 K_1 V^2}{2d_1^2}$

12. (i) 2nF, $12.1\mu J$, (ii) $48.4\mu J$, (iii) $11\mu J$ **13.** $\frac{27}{7}\ \mu F$ **14.** $12\ \mu F$

15. n = 3 **16.** $C_R = \frac{CK_1K_2}{K_2 - K_1}\ \ell n\frac{K_2}{K_1}$ where $C = \frac{\in_0 A}{d}$