GEOMETRICAL OPTICS

Contents

JEE (ADVANCED) SYLLABUS

Rectilinear propagation of light; Reflection and refraction at plane and spherical surfaces; Total internal reflection; Deviation and dispersion of light by a prism; Thin lenses; Combinations of mirrors and thin lenses; Magnification.

JEE (MAIN) SYLLABUS

Reflection and refraction of light at plane and spherical surface, mirror formula, Total internal reflection and its applications, Deviation and dispersion of light by a prism. Lens Formula, Magnification, Power of a Lens. Combination of thin lenses in contact, Microscope and Astronomical Telescope (Reflecting and refracting and their magnifying powers.

GEOMETRICAL OPTICS

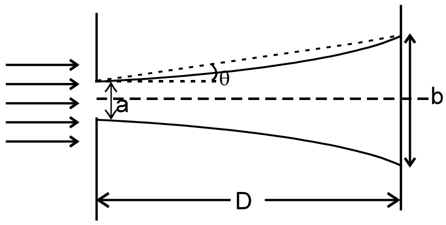
INTRODUCTION :

Blue lakes, ochre deserts, green forest, and multicolored rainbows can be enjoyed by anyone who has eyes with which to see them. But by studying the branch of physics called **optics**, which deals with the behaviour of light and other electromagnetic waves, we can reach a deeper appreciation of the visible world. A knowledge of the properties of light allows us to understand the blue color of the sky and the design of optical devices such as telescopes, microscopes, cameras, eyeglasses, and the human eyes. The same basic principles of optics also lie at the heart of modern developments such as the laser, optical fibers, holograms, optical computers, and new techniques in medical imaging.

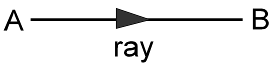
1. CONDITION FOR RECTILINEAR PROPAGATION OF LIGHT

(Only for information not in IIT-JEE syllabus)

Some part of the optics can be understood if we assume that light travels in a straight line and it bends abruptly when it suffers reflection or refraction.
The assumption that the light travels in a straight line is correct if
(i) the medium is isotropic, i.e. its behaviour is same in all directions and (ii) the obstacle past which the light moves or the opening through which the light moves is not very small.
Consider a slit of width 'a' through which monochromatic light rays pass and strike a screen, placed at a distance D as shown.



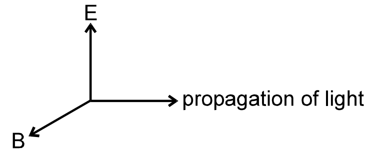
It is found that the light strikes in a band of width 'b' more than 'a'. This bending is called **diffraction**.
Light bends by (b-a)/2 on each side of the central line .It can be shown by wave theory of light that $\sin\theta = \frac{\lambda}{a}$ ....(A), where θ is shown in figure.
This formula indicates that the **bending is considerable only when a $\simeq$ λ** . Diffraction is more pronounced in sound because its wavelength is much more than that of light and it is of the order of the size of obstacles or apertures. Formula (A) gives $\frac{b-a}{2D} \approx \frac{\lambda}{a}$.
It is clear that the bending is negligible if $\frac{D\lambda}{a} << a$ or $a >> \sqrt{D\lambda}$. If this condition is fulfilled, light is said to move rectilinearly. In most of the situations including geometrical optics the conditions are such that we can safely assume that light moves in straight line and bends only when it gets reflected or refracted.
Thus geometrical optics is an approximate treatment in which the light waves can be represented by straight lines which are called rays. A **ray** of light is the straight line path of transfer of light energy. Arrow represents the direction of propagation of light.
Figure shows a ray which indicates light is moving from A to B.



2. PROPERTIES OF LIGHT

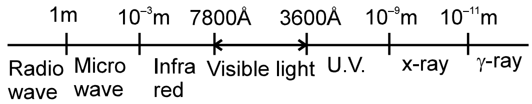
(i) Speed of light in vacuum, denoted by c, is equal to 3×10^8 m/s approximately.

(ii) Light is electromagnetic wave (proposed by Maxwell). It consists of varying electric field and magnetic field.



(iii) Light carries energy and momentum.

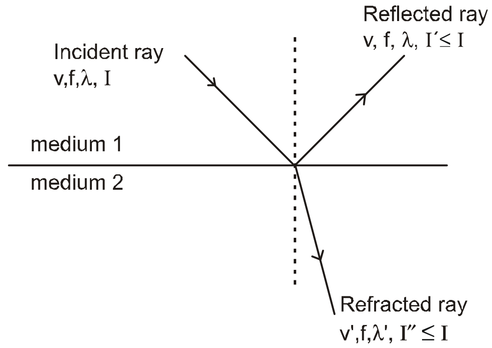
(iv) The formula $v = f\lambda$ is applicable to light.



Electromagnetic spectrum

(v) When light gets reflected in same medium, it suffers no change in frequency, speed and wavelength.

(vi) Frequency of light remains unchanged when it gets reflected or refracted.

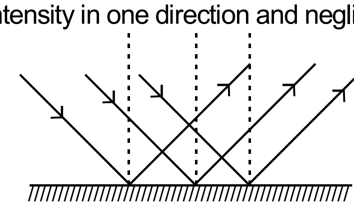


3. REFLECTION OF LIGHT

When light rays strike the boundary of two media such as air and glass, a part of light is turned back into the same medium. This is called Reflection of Light.

(a) Regular Reflection:

When the reflection takes place from a perfect plane surface it is called **Regular Reflection**. In this case the reflected light has large intensity in one direction and negligibly small intensity in other directions.



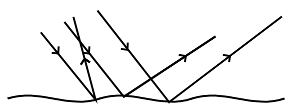
Regular Reflection

(b) Diffused Reflection

When the surface is rough, we do not get a regular behaviour of light. Although at each point light ray gets reflected irrespective of the overall nature of surface, difference is observed because even in a narrow beam of light there are many rays which are reflected from different points of surface and it is quite possible that these rays may move in different directions due to irregularity of the surface. This process enables us to see an object from any position.

Such a reflection is called as **diffused reflection**.

For example reflection from a wall, from a news paper etc. This is why you can not see your face in news paper and in the wall.

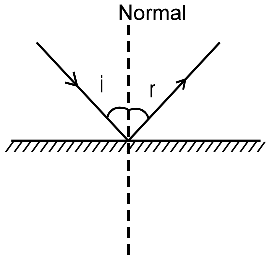


Diffused Reflection

3.1 Laws of Reflection

(a) The incident ray, the reflected ray and the normal at the point of incidence lie in the same plane. This plane is called the **plane of incidence (or plane of reflection)**. This condition can be expressed mathematically as $\vec{R}\,.\,(\vec{I}\times\vec{N}) = \vec{N}\,.\,(\vec{I}\times\vec{R}) = \vec{I}\,.(\vec{N}\times\vec{R}) = 0$ where $\vec{I}$, $\vec{N}$ and $\vec{R}$ are vectors of any magnitude along incident ray, the normal and the reflected ray respectively.

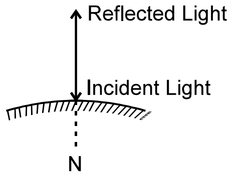
(b) The angle of incidence (the angle between normal and the incident ray) and the angle of reflection (the angle between the reflected ray and the normal) are equal, i.e.,



$\angle$**i** = $\angle$**r**

Special Cases :

Normal Incidence : In case light is incident normally,

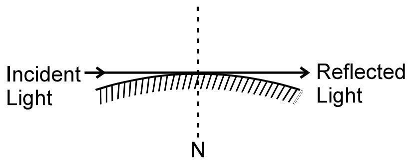


i = r = 0

δ = 180º

Note : We say that the ray has retraced its path.

Grazing Incidence : In case light strikes the reflecting surface tangentially,



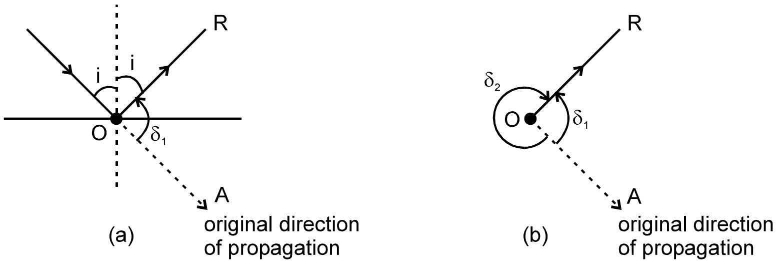
i = r = 90 ; deviation,δ = 0º or 360º

Note : In case of reflection speed (magnitude of velocity) of light remains unchanged but in grazing incidence velocity remains unchanged.

Solved Example

Example 1. Show that for a light ray incident at an angle 'i' on getting reflected the angle of deviation is $\delta = \pi - 2i$ or $\pi + 2i$.

Solution :



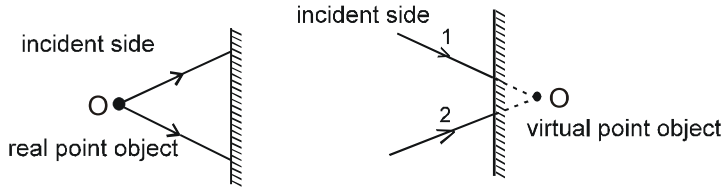
From figure (b) it is clear that light ray bends either by δ_1 anticlockwise or by δ_2 (= $2\pi - \delta_1$) clockwise.

From figure (a) $\delta_1 = \pi - 2i$.

$\therefore \quad \delta_2 = \pi + 2i$.

3.2 Object and Image

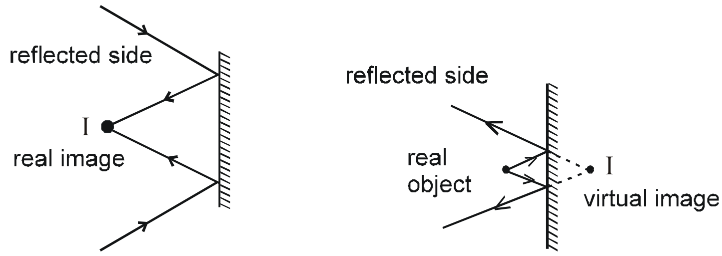
(a) Object (O) : Object is defined as point of intersection of **incident** rays.



Let us call the side in which incident rays are present as incident side and the side in which reflected (refracted) rays are present, as reflected (refracted) side.

Note : An object is called **real** if it lies on incident side otherwise it is called **virtual**. (In case of plane mirror only)

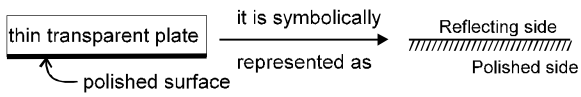
(b) Image (I) : Image is defined as point of intersection of **reflected** rays (in case of reflection) or **refracted** rays (in case of refraction).



Note : An image is called **real** if it lies on reflected or refracted side otherwise it is called **virtual.**

4. PLANE MIRROR

Plane mirror is formed by polishing one surface of a plane thin glass plate .It is also said to be silvered on one side.

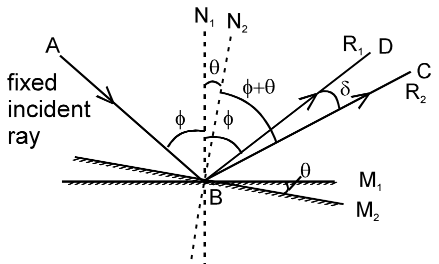


PLANE MIRROR

A beam of parallel rays of light, incident on a plane mirror will get reflected as a beam of parallel reflected rays.

Solved Example

Example 2. For a fixed incident light ray, if the mirror be rotated through an angle θ (about an axis which lies in the plane of mirror and perpendicular to the plane of incidence), show that the reflected ray turns through an angle 2θ in same sense.



Solution : See figure M_1, N_1 and R_1 indicate the initial position of mirror, initial normal and initial direction of reflected light ray respectively. M_2, N_2 and R_2 indicate the final position of mirror, final normal and final direction of reflected light ray respectively. From figure it is clear that ABC = $2\phi + \delta = 2(\phi + \theta)$ or $\delta = 2\theta$.

Note : Keeping the mirror fixed if the incident ray is rotated by angle θ about the normal then reflected ray rotates by same angle in the same direction of rotation

4.1 Point object

Characteristics of image due to reflection by a plane mirror :

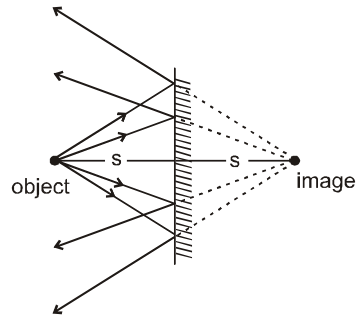
(i) Distance of object from mirror = Distance of image from the mirror.

(ii) All the incident rays from a point object will meet at a single point after reflection from a plane mirror which is called image.

(iii) The line joining a point object and its image is normal to the reflecting surface.

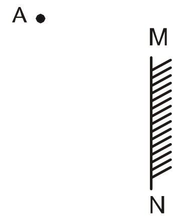
(iv) For a real object the image is virtual and for a virtual object the image is real

(v) The region in which observer's eye must be present in order to view the image is called **field of view**.

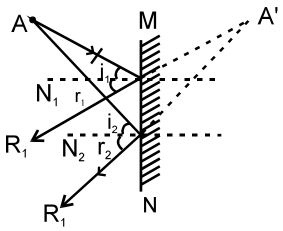


Solved Example

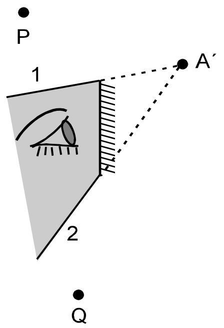
Example 3. Figure shows a point object A and a plane mirror MN. Find the position of image of object A, in mirror MN, by drawing ray diagram. Indicate the region in which observer's eye must be present in order to view the image. (This region is called **field of view**).



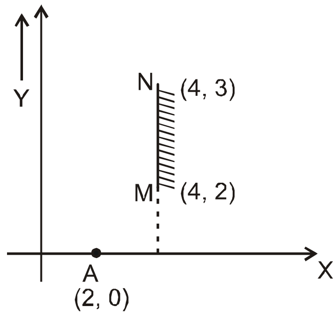
Solution See figure, consider any two rays emanating from the object. N_1 and N_2 are normals ; $i_1 = r_1$ and $i_2 = r_2$



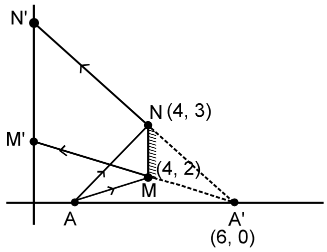
The meeting point of reflected rays R_1 and R_2 is image A'. Though only two rays are considered it must be understood that all rays from A reflect from mirror such that their meeting point is A´. To obtain the region in which reflected rays are present, join A´ with the ends of mirror and extend. The following figure shows this region as shaded. In figure, there are no reflected rays beyond the rays 1 and 2, therefore the observers P and Q cannot see the image because they do not receive any reflected ray.



Example 4. Find the region on Y axis in which reflected rays are present. Object is at A (2, 0) and MN is a plane mirror, as shown.

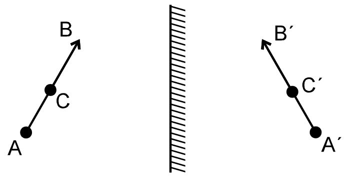


Solution : The image of point A, in the mirror is at A’ (6, 0).
Join A’ M and extend to cut Y axis at M’ (Ray originating from A which strikes the mirror at M gets reflected as the ray MM’ which appears to come from A’). Join A’N and extend to cut Y axis at N’ (Ray originating from A which strikes the mirror at N gets reflected as the ray NN’ which appears to come from A’).
From geometry.
M’ ≡ (0, 6)
N’ ≡ (0, 9). M’N’ is the region on Y axis in which reflected rays are present.



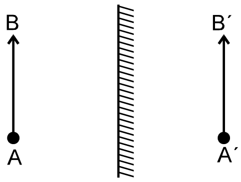
4.2 Extended object :

An extended object like AB shown in figure is a combination of infinite number of point objects from A to B. Image of every point object will be formed individually and thus infinite images will be formed. A´ will be image of A, C´ will be image of C, B´ will be image of B etc. All point images together form extended image. Thus extended image is formed of an extended object.

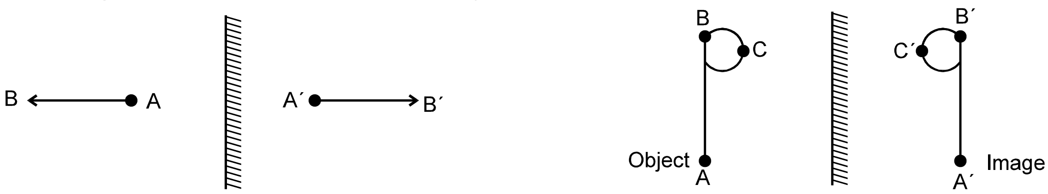


Properties of image of an extended object, formed by a plane mirror :

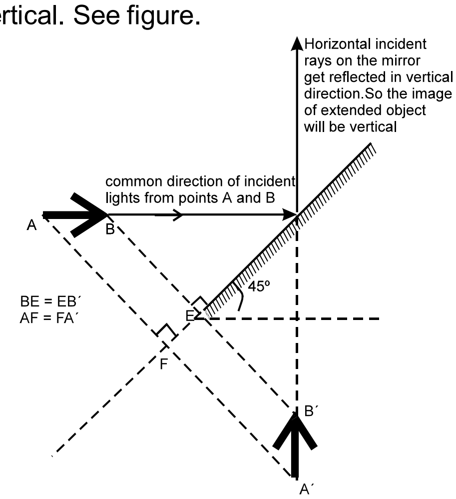
(1) Size of extended object = Size of extended image.
(2) The image is erect, if the extended object is placed parallel to the mirror.



(3) The image is inverted if the extended object lies perpendicular to the plane mirror.



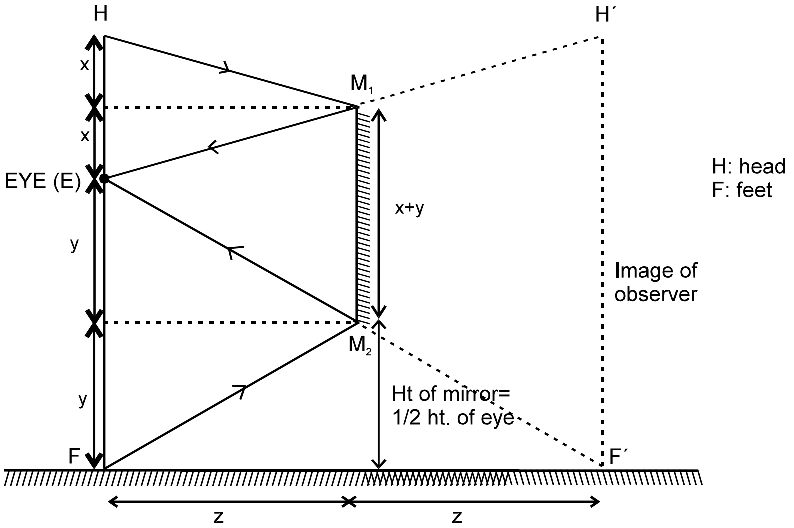
(4) If an extended horizontal object is placed infront of a mirror inclined 45° with the horizontal, the image formed will be vertical. See figure.



Solved Example

Example 5. Show that the minimum size of a plane mirror, required to see the full image of an observer is half the size of that observer.

Solution : See the following figure. It is self explanatory if you consider lengths 'x' and 'y' as shown in figure.



Alter :

ΔEM_1M_2 and $\Delta E\ H'F'$ are similar

$\therefore \quad \dfrac{M_1M_2}{H'F'} = \dfrac{z}{2z}$

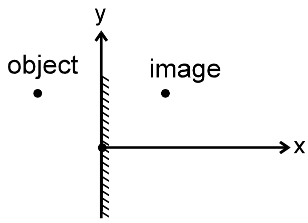
or $\quad M_1 M_2 = H'F' / 2 = HF / 2$

4.3 Relation between velocity of object and image :

From mirror property :

$x_{im} = -x_{om}$, $y_{im} = y_{om}$ and $z_{im} = z_{om}$

Here x_{im} means 'x' coordinate of image with respect to mirror. Similarly others have corresponding meaning.



Differentiating w.r.t time, we get

$v_{(im)x} = -v_{(om)x}$; $v_{(im)y} = v_{(om)y}$; $v_{(im)z} = v_{(om)z}$,

$\Rightarrow$ For x axis $v_{iG} - v_{mG} = -(v_{oG} - v_{mG})$

but For y axis and z axis $v_{iG} - v_{mG} = (v_{oG} - v_{mG})$ or $v_{iG} = v_{oG}$.

here: v_{iG} = velocity of image with respect to ground.

Solved Example

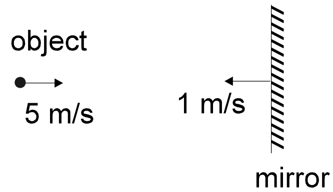
Example 6. An object moves with 5 m/s towards right while the mirror moves with 1m/s towards the left as shown. Find the velocity of image.

Solution : Take $\longrightarrow$ as + direction. $v_i - v_m = v_m - v_0$

$v_i - (-1) = (-1) - 5$

$\therefore \quad v_i = -7$m/s.

$\Rightarrow$ 7 m/s and direction towards left.



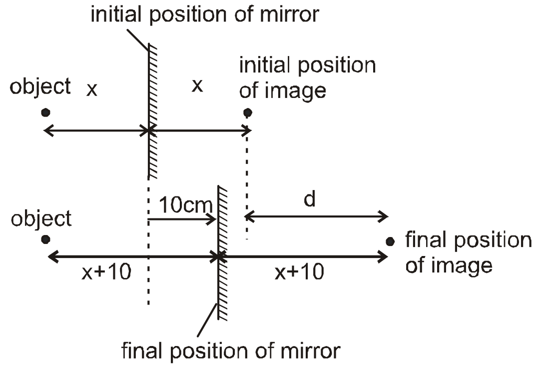
Example 7. There is a point object and a plane mirror. If the mirror is moved by 10 cm away from the object find the distance which the image will move.

Solution : We know that $x_{im} = -x_{om}$ or $x_i - x_m = x_m - x_o$

or $\Delta x_i - \Delta x_m = \Delta x_m - \Delta x_o$.

In this question $\Delta x_o = 0$; $\Delta x_m = 10$ cm.

Therefore $\Delta x_i = 2\Delta x_m - \Delta x_o = 20$ cm.

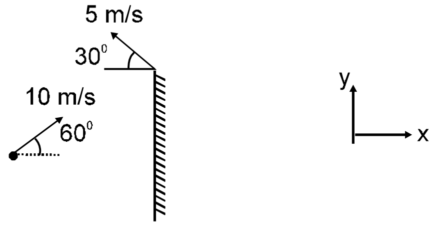


Alter :

$2(x + 10) = 2x + d$

$\therefore$ d = 20 cm

Example 8. In the situation shown in figure, find the velocity of image.



Solution : Along x direction, applying $v_i - v_m = -(v_0 - v_m)$

$v_i - (-5\cos 30^o) = -(10\cos 60^o - (-5\cos 30^o))$

$\therefore\ v_i = -5(1+\sqrt{3})$ m/s

Along y direction $v_0 = v_i$

$\therefore\ v_i = 10\sin 60° = 5\sqrt{3}$ m/s

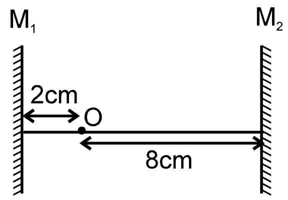
$\therefore$ Velocity of the image $= -5(1+\sqrt{3})\,\hat{i} + 5\sqrt{3}\,\hat{j}$ m/s.

4.4 Images formed by two plane mirrors :

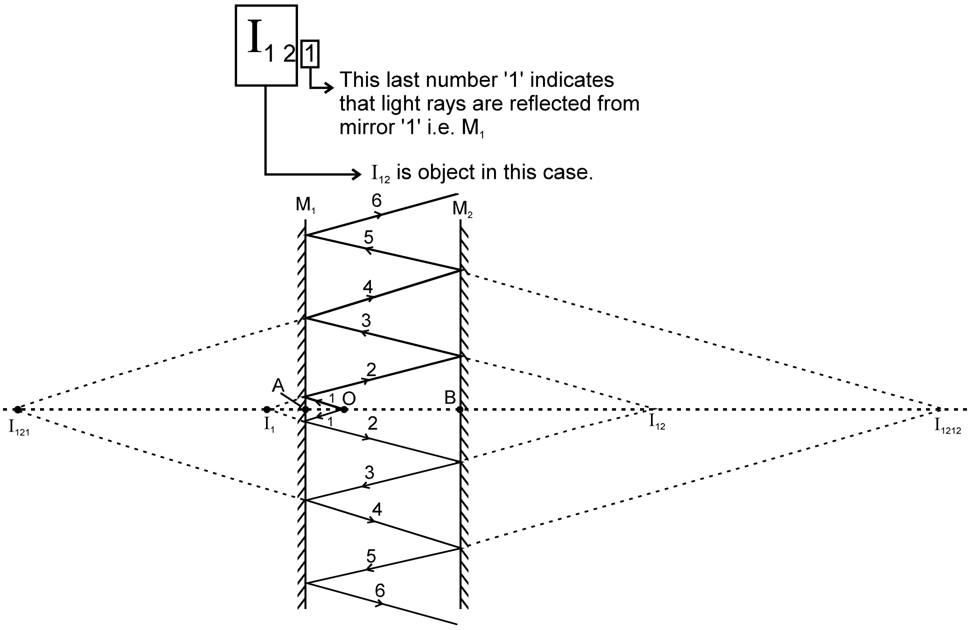
If rays after getting reflected from one mirror strike second mirror, the image formed by first mirror will function as an object for second mirror, and this process will continue for every successive reflection.

Solved Example

Example 9. Figure shows a point object placed between two parallel mirrors. Its distance from M_1 is 2 cm and that from M_2 is 8 cm. Find the distance of images from the two mirrors considering reflection on mirror M_1 first.



Solution : To understand how images are formed see the following figure and table. You will require to know what symbols like I_{121} stands for. See the following diagram.

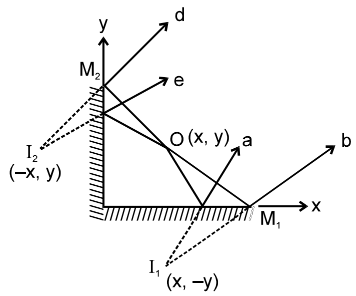


Incident rays	Reflected by	Reflected rays	Object	Image	Object distance	Image distance
Rays 1	M_1	Rays 2	O	I_1	AO = 2cm	AI_1 = 2 cm
Rays 2	M_2	Rays 3	I_1	I_{12}	BI_1 = 12 cm	BI_{12} = 12 cm
Rays 3	M_1	Rays 4	I_{12}	I_{121}	AI_{12} = 22cm	AI_{121} = 22cm
Rays 4	M_2	Rays 5	I_{121}	I_{1212}	BI_{121} =32cm	BI_{1212}=32cm

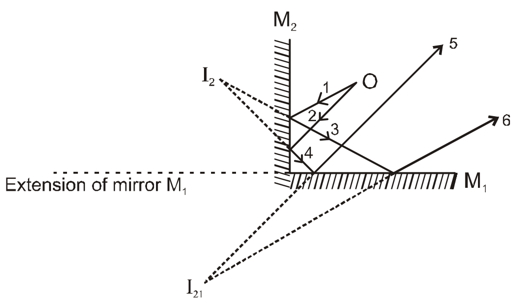
Similarly images will be formed by the rays striking mirror M_2 first. Total number of images = ∞.

Example 10. Consider two perpendicular mirrors. M_1 and M_2 and a point object O. Taking origin at the point of intersection of the mirrors and the coordinate of object as (x, y), find the position and number of images.

Solution Rays 'a' and 'b' strike mirror M_1 only and these rays will form image I_1 at (x, –y), such that O and I_1 are equidistant from mirror M_1. These rays do not form further image because they do not strike any mirror again. Similarly rays 'd' and 'e' strike mirror M_2 only and these rays will form image I_2 at (–x, y), such that O and I_2 are equidistant from mirror M_2.

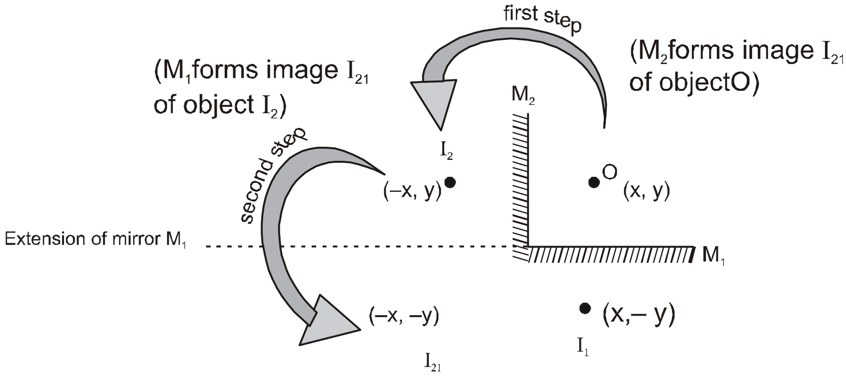


Now consider those rays which strike mirror M_2 first and then the mirror M_1.



For incident rays 1, 2 object is O, and reflected rays 3, 4 form image I_2.
Now rays 3, 4 incident on M_1 (object is I_2) which reflect as rays 5, 6 and form image I_{21}. Rays 5, 6 do not strike any mirror, so image formation stops.

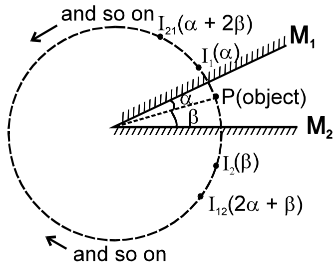
I_2 and I_{21}, are equidistant from M_1. To summarize see the following figure



For rays reflecting first from M_1 and then from M_2, first image I_1 (at (x, –y)) will be formed and this will function as object for mirror M_2 and then its image I_{12} (at (–x, –y)) will be formed.
I_{12} and I_{21} coincide.
$\therefore$ Three images are formed

4.5 Locating all the images formed by two plane mirrors:

Consider two plane mirrors M_1 and M_2 inclined at an angle $\theta = \alpha + \beta$ as shown in figure.
Point P is an object kept such that it makes angle α with mirror M_1 and angle β with mirror M_2. Image of object P formed by M_1, denoted by I_1, will be inclined by angle α on the other side of mirror M_1. This angle is written in bracket in the figure besides I_1. Similarly image of object P formed by M_2, denoted by I_2, will be inclined by angle β on the other side of mirror M_2. This angle is written in bracket in the figure besides I_2.



Now I_2 will act as an object for M_1 which is at an angle $(\alpha + 2\beta)$ from M_1. Its image will be formed at an angle $(\alpha + 2\beta)$ on the opposite side of M_1. This image will be denoted as I_{21}, and so on. Think when this will process stop.
Hint : The virtual image formed by a plane mirror must not be in front of the mirror or its extension.

Number of images formed by two inclined mirrors

(i) If $\frac{360^\circ}{\theta}$ = even number ; number of image = $\frac{360^\circ}{\theta} - 1$

(ii) If $\frac{360^\circ}{\theta}$ = odd number ; number of image = $\frac{360^\circ}{\theta} - 1$, if the object is placed on the angle bisector.

(iii) If $\frac{360^\circ}{\theta}$ = odd number ; number of image = $\frac{360^\circ}{\theta}$, if the object is not placed on the angle bisector.

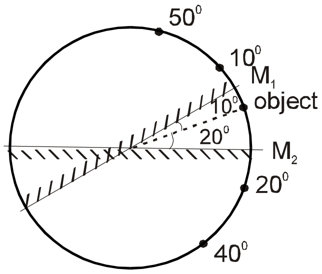
(iv) If $\frac{360^\circ}{\theta} \neq$ integer, then count the number of images as explained above.

Solved Example

Example 11. Two mirrors are inclined by an angle 30°. An object is placed making 10° with the mirror M_1. Find the positions of first two images formed by each mirror. Find the total number of images using (i) direct formula and (ii) counting the images.

Solution : Figure is self explanatory.

Number of images



(i) Using direct formula : $\dfrac{360^\circ}{30^\circ}$ = 12 (even number)

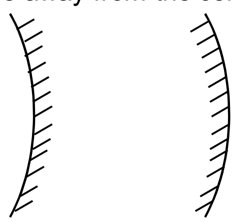
$\therefore$ number of images = 12 – 1 = 11

(ii) By counting. See the following table

Image formed by Mirror M_1 (angles are measured from the mirror M_1.)		Image formed by Mirror M_2 (angles are measured from the mirror M_2.)
10^0	$+30^0$	20^0
50^0	$+30^0$	40^0
70^0	$+30^0$	80^0
110^0	$+30^0$	100^0
130^0	$+30^0$	140^0
170^0		160^0
Stop because next angle will be more than 180º		Stop because next angle will be more than 180º
To check whether the final images made by the two mirrors coincide or not : add the last angles and the angle between the mirrors. If it comes out to be exactly 360º, it implies that the final images formed by the two mirrors coincide. Here last angles made by the mirrors + the angle between the mirrors = 160º + 170º + 30º = 360º. Therefore in this case the last images coincide. Therefore the number of images = number of images formed by mirror M_1 + number of images formed by mirror M_2 -1 (as the last images coincide)= 6 + 6 –1 = 11.		

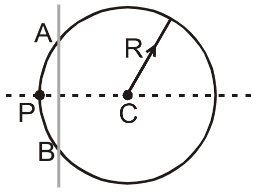
5. SPHERICAL MIRRORS

Spherical Mirror is formed by polishing one surface of a part of sphere. Depending upon which part is shining the spherical mirror is classified as (a) Concave mirror, if the side towards center of curvature is shining and (b) Convex mirror if the side away from the center of curvature is shining.



Concave mirror Convex mirror

5.1 Important terms related with spherical mirrors :



A spherical shell with the center of curvature, pole aperture and radius of curvature identified

(a) Center of Curvature (C) :

The center of the sphere from which the spherical mirror is formed is called the center of curvature of the mirror. It is represented by C and is indicated in figure.

(b) Pole (P) :

The center of the mirror is called as the Pole. It is represented by the point P on the mirror APB in figure.

(c) Principal Axis :

The Principal Axis is a line which is perpendicular to the plane of the mirror and passes through the pole. The Principal Axis can also be defined as the line which joins the Pole to the Center of Curvature of the mirror.

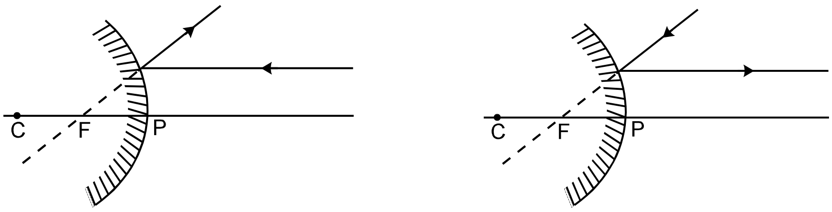
(d) Aperture (A) :

The aperture is the segment or area of the mirror which is available for reflecting light. In figure. APB is the aperture of the mirror.

(e) Principle focus (F) :

It is the point of intersection of all the reflected rays for which the incident rays strike the mirror (with small aperture) parallel to the principal axis. In concave mirror it is real and in the convex mirror it is virtual. The distance from pole to focus is called **focal length.**

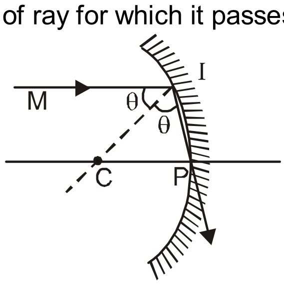
Concave mirror



Convex mirror

Solved Example

Example 12. Find the angle of incidence of ray for which it passes through the pole, given that MI || CP.



Solution : $\angle$ MIC = $\angle$CIP = θ

MI || CP $\angle$MIθ = $\angle$ICP = θ

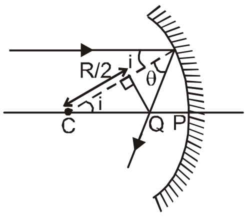
CI = CP

$\angle$CIP = $\angle$CPI = θ

$\therefore$ In ΔCIP all angle are equal

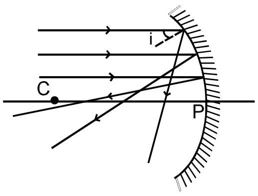
$3\theta = 180^\circ \Rightarrow \theta = 60^\circ$

Example 13. Find the distance CQ if incident light ray parallel to principal axis is incident at an angle i. Also find the distance CQ if i $\to$ 0.

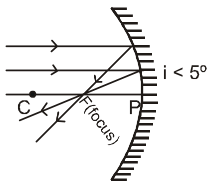


Solution : $\cos i = \dfrac{R}{2CQ} \Rightarrow CQ = \dfrac{R}{2\cos i}$

As i increases cos i decreases.
Hence CQ increases



If i is a small angle cos i $\approx$ 1
$\therefore$ CQ = R/2

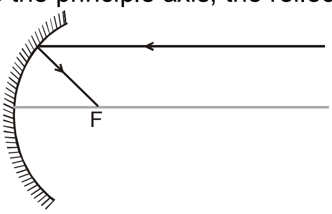


So, paraxial rays meet at a distance equal to R / 2 from center of curvature, which is called focus.

5.1 Ray tracing :

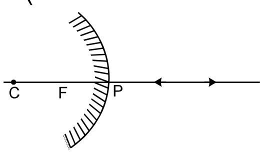
Following facts are useful in ray tracing.

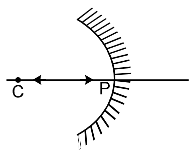
(i) If the incident ray is parallel to the principle axis, the reflected ray passes through the focus.



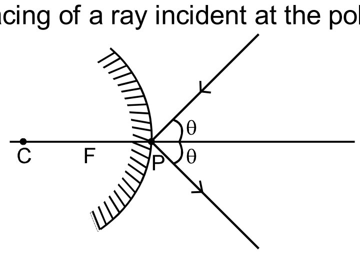
(ii) If the incident ray passes through the focus, then the reflected ray is parallel to the principle axis.

(iii) Incident ray passing through centre of curvature will be reflected back through the centre of curvature (because it is a normally incident ray).





(iv) It is easy to make the ray tracing of a ray incident at the pole as shown in below.



5.2 Sign Convention

We are using co–ordinate sign convention.

(i) Take origin at pole (in case of mirror) or at optical centre (in case of lens). Take X axis along the Principal Axis, taking **positive direction along the incident light.**
u, v, R and **f** indicate the x coordinate of object, image, centre of curvature and focus respectively.

(ii) y-coordinates are taken positive above Principle Axis and negative below Principle Axis' $\mathbf{h_1}$ and $\mathbf{h_2}$ denote the y coordinates of object and image respectively.

Note :

- This sign convention is used for reflection from mirror, reflection through flat or curved surfaces or lens.

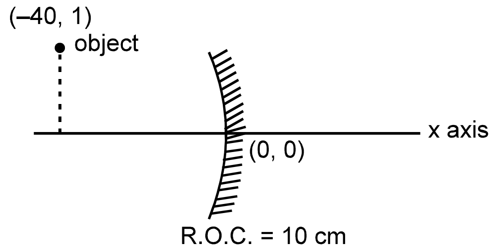
5.3 Formula for Reflection from spherical mirrors :

(a) Mirror formula : $\frac{1}{v}+\frac{1}{u}=\frac{2}{R}=\frac{1}{f}$

X-coordinate of centre of curvature and focus of concave mirror are negative and those for convex mirror are positive. In case of mirrors since light rays reflect back in X-direction, therefore **-ve sign of v indicates real image and +ve sign of v indicates virtual image.**

Solved Example

Example 14. Figure shows a spherical concave mirror with its pole at (0, 0) and principal axis along x axis. There is a point object at (–40 cm, 1cm), find the position of image.



Solution : According to sign convention,

u = –40 cm

h_1 = +1 cm

f = – 5 cm.

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f} \Rightarrow \frac{1}{v}+\frac{1}{-40}=\frac{1}{-5}\ ;\ v=\frac{-40}{7}\text{cm.}\ ;\ \frac{h_2}{h_1}=\frac{-v}{u}$$

$$\Rightarrow h_2 = -\frac{-v}{u} \times h_1 = \frac{-\left(-\frac{40}{7}\right)\times 1}{-40} = -\frac{1}{7}\text{ cm.}$$

$\therefore$ The position of image is $\left(\frac{-40}{7}\text{cm},-\frac{1}{7}\text{cm}\right)$

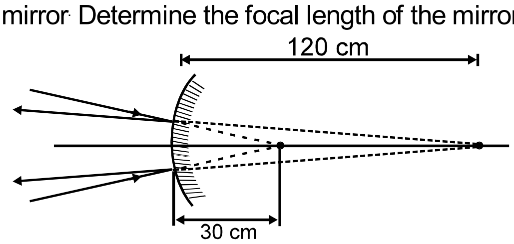
Example 15. Converging rays are incident on a convex spherical mirror so that their extensions intersect 30 cm behind the mirror on the optical axis. The reflected rays form a diverging beam so that their extensions intersect the optical axis 1.2 m from the mirror. Determine the focal length of the mirror.

Solution : In this case u = + 30

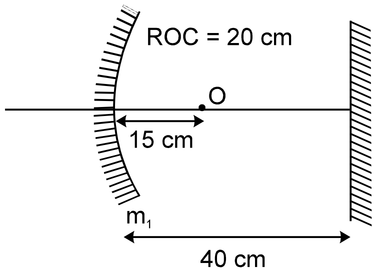
$\Rightarrow$ v = + 120

$$\therefore \frac{1}{f}=\frac{1}{v}+\frac{1}{u}=\frac{1}{120}+\frac{1}{30}$$

f = 24 cm



Example 16. Find the position of final image after three successive reflections taking first reflection on m_1.



Solution : **I reflection :** Focus of mirror = – 10 cm ⇒ u = – 15 cm

Applying mirror formula : $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$ ⇒ v = – 30 cm.

For II reflection on plane mirror : u = – 10 cm

∴ v = 10 cm

For III reflection on curved mirror again : u = – 50 cm ; f = – 10 cm

Applying mirror formula : $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$; v = – 12.5 cm.

(b) **Lateral magnification** (or transverse magnification) denoted by m is defined as $m = \frac{h_2}{h_1}$ and is related as $m = -\frac{v}{u}$. From the definition of m, positive sign of m indicates erect image and negative sign indicates inverted image.

(c) In case of successive reflection from mirrors, the overall lateral magnification is given by $m_1 \times m_2 \times m_3$ ......, where m_1, m_2 etc. are lateral magnifications produced by individual mirrors.
h_1 and h_2 denote the y coordinate of object and image respectively.

Note :

- Using **5.3(a)** and **5.3(b)** the following conclusions can be made **(check yourself)**.

Nature of Object	**Nature of Image**	**Inverted or erect**
Real	Real	Inverted
Real	Virtual	Erect
Virtual	Real	Erect
Virtual	Virtual	Inverted

From **5.3(a)** and **5.3(b)**; we get $m=\frac{f}{f-u}=\frac{f-v}{f}$ .... (just a time saving formula)

Solved Example

Example 17. An extended object is placed perpendicular to the principal axis of a concave mirror of radius of curvature 20 cm at a distance of 15 cm from pole. Find the lateral magnification produced.

Solution u = – 15 cm f = – 10 cm

Using $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$ we get, v = – 30 cm

∴ $m = -\frac{v}{u} = -2$.

Aliter : $m = \frac{f}{f-u} = \frac{-10}{-10-(-15)} = -2$

Example 18. A person looks into a spherical mirror. The size of image of his face is twice the actual size of his face. If the face is at a distance 20 cm then find the nature and radius of curvature of the mirror.

Solution : Person will see his face only when the image is virtual. Virtual image of real object is erect. Hence m = 2

$\therefore \quad \frac{-v}{u} = 2 \qquad \Rightarrow \qquad v = 40$ cm

Applying $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$; f = – 40 cm or R = –80 cm (concave)

$\therefore$ R.O.C. = 80 cm

Alter : $m = \frac{f}{f-u} \qquad \Rightarrow \qquad 2 = \frac{f}{f-(-20)}$

$\Rightarrow$ f = – 40 cm or R = –80cm (concave)

$\therefore$ R.O.C. = 80 cm

Example 19. An image of a candle on a screen is found to be double its size. When the candle is shifted by a distance 5 cm then the image become triple its size. Find the nature and ROC of the mirror.

Solution : Since the images formed on screen it is real. Real object and real image implies concave mirror.

Applying $\quad m = \frac{f}{f-u} \quad$ or $\quad -2 = \frac{f}{f-(u)}$ .....(1)

After shifting $\quad -3 = \frac{f}{f-(u+5)}$ .....(2)

[Why u + 5 ?, why not u – 5 : In a concave mirror, the size of real image will increase, only when the real object is brought closer to the mirror. In doing so, its x coordinate will increase]

From (1) & (2) we get,

f = – 30 cm or R = –60 cm (concave) and R.O.C. = 60cm

(d) Velocity of image

(i) Object moving perpendicular to principal axis : From the relation in 5.3.(b) we have

$\frac{h_2}{h_1} = -\frac{v}{u}$ or $h_2 = -\frac{v}{u} . h_1$

If a point object moves perpendicular to the principal axis, x coordinate of both the object & the image become constant. On differentiating the above relation w.r.t. time, we get,

$\frac{dh_2}{dt} = -\frac{v}{u}\frac{dh_1}{dt}$

Here, $\frac{dh_1}{dt}$ denotes velocity of object perpendicular to the principal axis and $\frac{dh_2}{dt}$ denotes velocity of image perpendicular to the principal axis.

(ii) Object moving along principal axis : On differentiating the mirror formula with respect to time we get $\frac{dv}{dt} = -\frac{v^2}{u^2}\frac{du}{dt}$, where $\frac{dv}{dt}$ is the velocity of image along principal axis and $\frac{du}{dt}$ is the velocity of object along principal axis. Negative sign implies that the image, in case of mirror, always moves in the direction opposite to that of object. This discussion is for velocity with respect to mirror and along the x axis.

(iii) Object moving at an angle with the principal axis : Resolve the velocity of object along and perpendicular to the principal axis and find the velocities of image in these directions separately and then find the resultant.

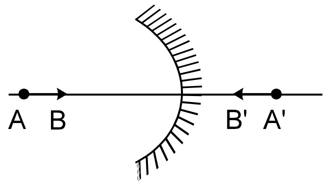
(e) Optical power of a mirror (in Dioptre) = $-\frac{1}{f}$

f = focal length with sign and in meters.

(f) If object lying along the principal axis is not of very small size, the **longitudinal magnification** = $\frac{v_2 - v_1}{u_2 - u_1}$ (it will always be inverted)

(g) If the size object is very small as compared to its distance from Pole then

On differentiating the mirror formula we get $\frac{dv}{du} = -\frac{v^2}{u^2}$: Mathematically 'du' implies small change in position of object and 'dv' implies corresponding small change in position of image. If a small object lies along principal axis, du may indicate the size of object and dv the size of its image along principal axis (Note that the focus should not lie in between the initial and final points of object). In this case $\frac{dv}{dt}$ is called longitudinal magnification. **Negative sign indicates inversion of image irrespective of nature of image and nature of mirror.**



Solved Example

Example 20. A point object is placed 60 cm from pole of a concave mirror of focal length 10 cm on the principle axis. Find
(a) the position of image
(b) If object is shifted 1 mm towards the mirror along principle axis find the shift in image. Explain the result.

Solution : (a) u = – 60 cm
f = – 10cm

$$v = \frac{fu}{u-f} = \frac{-10\ (-60)}{-60-(-10)} = \frac{600}{-50} = -12 \text{ cm.}$$

(b) $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

Differentiating, we get $dv = -\frac{v^2}{u^2}du = -\left(\frac{-12}{-60}\right)^2$ [1 mm] $= -\frac{1}{25}$ mm

[$\because$ du = 1mm; sign of du is + because it is shifted in +ve direction defined by sign convention.]

(a) –ve sign of dv indicates that the image will shift towards negative direction.

(b) The sign of v is negative. Which implies the image is formed on negative side of pole.

(a) and (b) together imply that the image will shift away from pole.

Note that differentials dv and du denote small changes only.

(h) Newton's Formula: **XY = f ^{2}**

X and Y are the distances (along the principal axis) of the object and image respectively from the principal focus. This formula can be used when the distances are mentioned or asked from the focus.

6. REFRACTION OF LIGHT

When the light changes its medium, some changes occurs in its properties, the phenomenon is known as refraction.

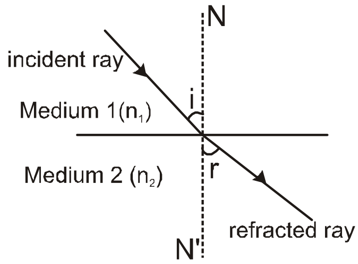
- If the light is incident at an angle (0º < i < 90º) then it deviates from its actual path. It is due to change in speed of light as light passes from one medium to another medium.
- If the light is incident normally then it goes to the second medium without bending, but still it is called refraction.
- Refractive index of a medium is defined as the factor by which speed of light reduces as compared to the speed of light in vacuum $\mu = \frac{c}{v} = \frac{\text{speed of light in vacuum}}{\text{speed of light in medium}}$.

More (less) refractive index implies less (more) speed of light in that medium, which therefore is called optical denser (rarer) medium.

6.1 Laws of Refraction

(a) The incident ray, the normal to any refracting surface at the point of incidence and the refracted ray all lie in the same plane called the plane of incidence or plane of refraction.

(b) $\frac{\text{Sin i}}{\text{Sin r}}$ = Constant for any pair of media and for light of a given wave length. This is known as *Snell's* Law.



Also, $\frac{\text{Sin i}}{\text{Sin r}} = \frac{n_2}{n_1} = \frac{v_1}{v_2} = \frac{\lambda_1}{\lambda_2}$

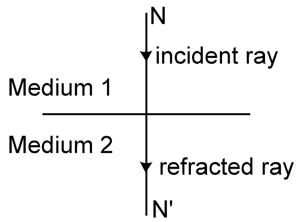
For applying in problems remember **$n_1$sini = $n_2$sinr**

$\frac{n_2}{n_1} = {}_1n_2$ = Refractive Index of the second medium with respect to the first medium.

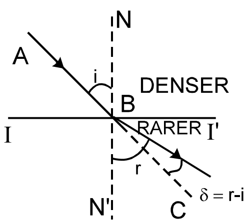
c = speed of light in air (or vacuum) = 3 x 10^8 m/s.

Special cases :

- Normal incidence : i = 0 ; from Snell's law : r = 0

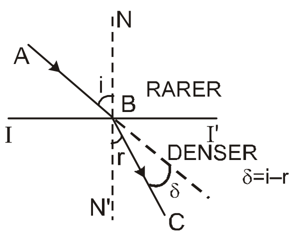


- When light moves from denser to rarer medium it bends away from normal.



r > i

- When light moves from rarer to denser medium it bends towards the normal.

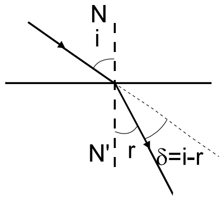


Note :
- Higher the value of R.I., denser (optically) is the medium.
- Frequency of light does not change during refraction.
- Refractive index of the medium relative to vacuum = $\sqrt{\mu_r \in_r}$

$n_{vacuum} = 1$; $n_{air} \gtrsim 1$; n_{water} (average value) = 4/3 ; n_{glass} (average value)= 3/2

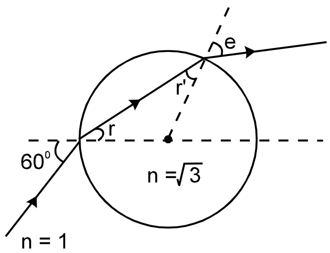
6.2 Deviation of a Ray Due to Refraction

Deviation (δ) of ray incident at $\angle$ i and refracted at $\angle$ r is given by $\delta = |i - r|$.



Solved Example

Example 21. A light ray is incident on a glass sphere at an angle of incidence 60^0 as shown. Find the angles r, r', e and the total deviation after two refractions.



Solution : Applying Snell's law $1\sin 60° = \sqrt{3}\ \sin r$

$\Rightarrow$ r = 30^0

From symmetry r' = r = 30°.

Again applying Snell's law at second surface $1\sin e = \sqrt{3}\ \sin r$

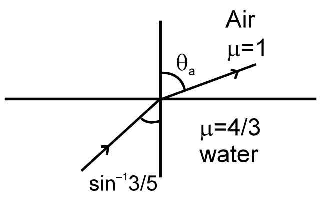
$\Rightarrow$ e = 60^0

Deviation at first surface = i – r = $60^0 - 30^0 = 30^0$

Deviation at second surface = e – r' = $60^0 - 30^0 = 30^0$

Therefore total deviation = 60^0.

Example 22 : Find the angle θ_a made by the light ray when it gets refracted from water to air, as shown in figure.



Solution : Snell's Law

$\mu_W \sin\theta_W = \mu_a \sin\theta_a \quad \Rightarrow \quad \frac{4}{3} \times \frac{3}{5} = 1\ \sin\theta_a$

$\sin\theta_a = \frac{4}{5} \qquad \theta_a = \sin^{-1}\frac{4}{5}$

Example 23. Find the speed of light in medium 'a' if speed of light in medium 'b' is $\frac{c}{3}$ where c = speed of light in vacuum and light refracts from medium 'a' to medium 'b' making 45º and 60º respectively with the normal.

Solution : Snell's Law $\mu_a \sin\theta_a = \mu_b \sin\theta_b$

$$\frac{c}{v_a}\sin\theta_a = \frac{c}{v_b}\sin\theta_b.$$

$$\frac{c}{v_a}\sin 45^\circ = \frac{c}{c/3}\sin 60^\circ.$$

$$v_a = \frac{\sqrt{2}c}{3\sqrt{3}}$$

6.3 Principle of Reversibility of Light Rays

(a) A ray travelling along the path of the reflected ray is reflected along the path of the incident ray.
(b) A refracted ray reversed to travel back along its path will get refracted along the path of the incident ray. Thus the incident and refracted rays are mutually reversible.

7. REFRACTION THROUGH A PARALLEL SLAB

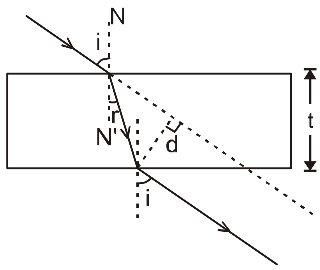
When light passes through a parallel slab, having same medium on both sides, then

(a) Emergent ray is parallel to the incident ray.

Note :

- Emergent ray will not be parallel to the incident ray if the medium on both the sides of slab are different.

(b) Light is shifted laterally, given by (students should be able to derive it)



d = $\frac{t\sin(i-r)}{\cos r}$; t = thickness of slab

Solved Example

Example 24. Find the lateral shift of light ray while it passes through a parallel glass slab of thickness 10 cm placed in air. The angle of incidence in air is 60º and the angle of refraction in glass is 45º.

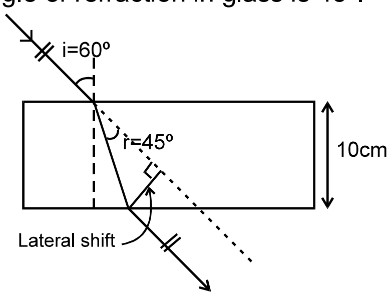
Solution :

$$d = \frac{t\sin(i-r)}{\cos r}$$

$$= \frac{10\sin(60^\circ - 45^\circ)}{\cos 45^\circ}$$

$$= \frac{10\sin 15^\circ}{\cos 45^\circ}$$

$$= 10\sqrt{2}\sin 15^\circ \text{cm}$$



7.1 Apparent Depth and shift of Submerged Object

At near normal incidence (small angle of incidence i) apparent depth (d′) is given by:

$$\mathbf{d' =} \frac{d}{n_{relative}} \quad \text{and} \quad v' = \frac{v}{n_{relative}}$$

where $n_{relative} = \dfrac{n_i \text{ (R.I. of medium of incidence)}}{n_r \text{ (R.I. of medium of refraction)}}$

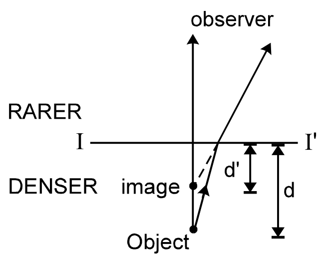
d = distance of object from the interface = real depth

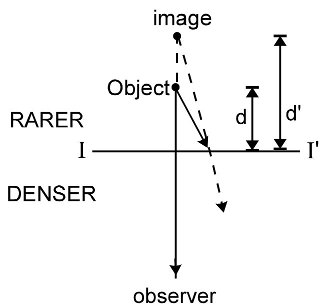
d' = distance of image from the interface = apparent depth

v = velocity of object perpendicular to interface relative to surface.

v′ = velocity of image perpendicular to interface relative to surface.

This formula can be easily derived using Snell's law and applying the condition of nearly normal incidence.... (try it or see in text book).





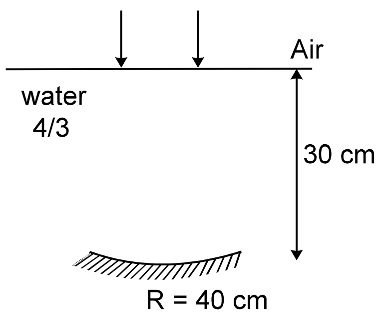
Apparent shift = d $\left(1 - \dfrac{1}{n_{rel}}\right)$

Solved Example

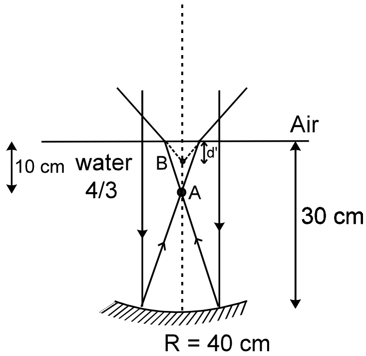
Example 25. An object lies 100 cm inside water. It is viewed from air nearly normally. Find the apparent depth of the object.

Solution : $d' = \dfrac{d}{n_{relative}} = \dfrac{100}{\frac{4/3}{1}} = 75 \text{ cm}$

Example 26. A concave mirror is placed inside water with its shining surface upwards and principal axis vertical as shown. Rays are incident parallel to the principal axis of concave mirror. Find the position of final image.



Solution: The incident rays will pass undeviated through the water surface and strike the mirror parallel to its principal axis. Therefore for the mirror, object is at ∞. Its image A (in figure) will be formed at focus which is 20 cm from the mirror.

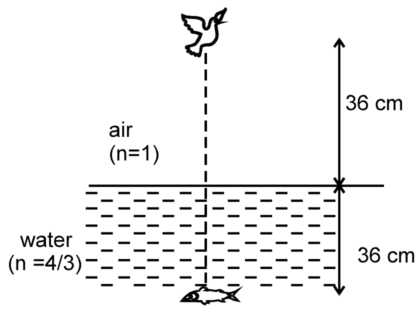


Now for the interface between water and air, d = 10 cm.

$\therefore \quad d' = \dfrac{d}{\left(\dfrac{n_w}{n_a}\right)} = \dfrac{10}{\left(\dfrac{4/3}{1}\right)} = 7.5$ cm.

Example 27. See the figure

(i) Find apparent height of the bird.
(ii) Find apparent depth of fish.
(iii) At what distance will the bird appear to the fish?
(iv) At what distance will the fish appear to the bird?
(v) If the velocity of bird is 12 cm/sec downward and the fish is 12 cm/sec in upward direction, then find out their relative velocities with respect to each other.



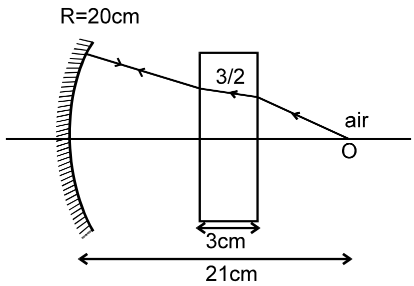
Solution (i) $d'_B = \dfrac{36}{\dfrac{1}{\left(\dfrac{4}{3}\right)}} = \dfrac{36}{3/4} = 48$ cm (ii) $d'_F = \dfrac{36}{4/3} = 27$ cm

(iii) For fish : $d_B = 36 + 48 = 84$ cm ; $d_B = 36 + 48 = 84$ cm
(iv) For bird : $d_F = 27 + 36 = 63$ cm. ; $d_F = 27 + 36 = 63$ cm.

(v) Velocity of fish with respect to bird = $12 + \left(\dfrac{12}{4/3 / 1/1}\right) = 21$ cm/sec.

Velocity of bird with respect to fish = $12 + \left(\dfrac{12}{3/4 / 1/1}\right) = 28$ cm/sec.

Example - 28 See the figure. Find the distance of final image formed by mirror



Solution : Shift = $3\left(1 - \dfrac{1}{3/2}\right)$

For mirror object is at a distance

$= 21 - 3\left(1 - \dfrac{1}{3/2}\right) = 20$ cm

$\therefore$ Object is at the centre of curvature of mirror. Hence the light rays will retrace and image will be formed on the object itself.

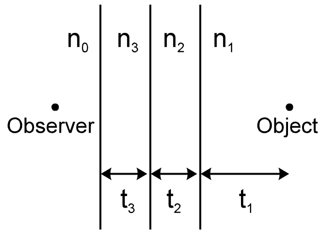
7.2 Refraction through a composite slab (or refraction through a number of parallel media, as seen from a medium of R. I. n_0)

Apparent depth (distance of final image from final surface)

$$= \frac{t_1}{n_{1\,rel}} + \frac{t_2}{n_{2\,rel}} + \frac{t_3}{n_{3\,rel}} + \ldots\ldots + \frac{t_n}{n_{n\ \ rel}}$$

Apparent shift

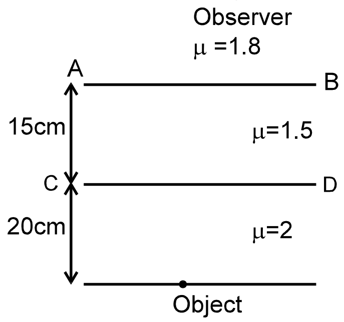
$$= t_1\left[1-\frac{1}{n_{1\,rel}}\right] + t_2\left[1-\frac{1}{n_{2\,rel}}\right] + \ldots\ldots + \left[1-\frac{n}{n_{n\ rel}}\right]t_n$$



Where ' t ' represents thickness and ' n ' represents the R.I. of the respective media, relative to the medium of observer. (i.e. $n_{1rel} = n_1/n_0$, $n_{2\,rel} = n_2/n_0$ etc.)

Solved Example

Example 29. See the figure. Find the apparent depth of object seen below surface AB.



Solution : $D_{app} = \sum\frac{d}{\mu} = \frac{20}{\left(\frac{2}{1.8}\right)} + \frac{15}{\left(\frac{1.5}{1.8}\right)} = 18 + 18 = 36$ cm.

8. CRITICAL ANGLE AND TOTAL INTERNAL REFLECTION (T. I. R.)

Critical angle is the angle made in denser medium for which the angle of refraction in rarer medium is 90°. When angle in denser medium is more than critical angle, then the light ray reflects back in denser medium following the laws of reflection and the interface behaves like a perfectly reflecting mirror.

In the figure

O = Object

NN′ = Normal to the interface

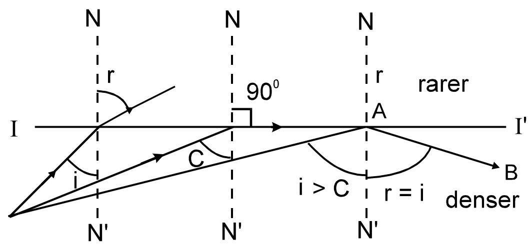
II′ = Interface

C = Critical angle;

AB = reflected ray due to T. I. R.

When i = C then r = 90°

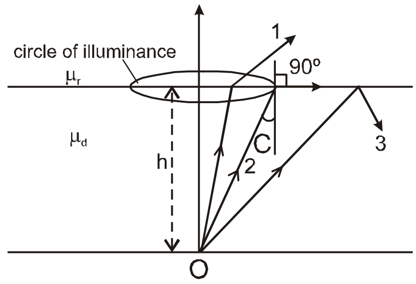
$\therefore$ **C =** $\sin^{-1}\frac{n_r}{n_d}$



8.1 Conditions of T. I. R.

(a) light is incident on the interface from denser medium.

(b) Angle of incidence should be greater than the critical angle (i > C). Figure shows a luminous object placed in denser medium at a distance h from an interface separating two media of refractive indices μ_r and μ_d. Subscript r & d stand for rarer and denser medium respectively.



In the figure, ray 1 strikes the surface at an angle less than critical angle C and gets refracted in rarer medium. Ray 2 strikes the surface at critical angle and grazes the interface. Ray 3 strikes the surface making an angle more than critical angle and gets internally reflected. The locus of points where ray strikes at critical angle is a circle, called **circle of illuminance**. All light rays striking inside the circle of illuminance get refracted in rarer medium. If an observer is in rarer medium, he/she will see light coming out only from within the circle of illuminance. If a circular opaque plate covers the circle of illuminance, no light will get refracted in rarer medium and then the object can not be seen from the rarer medium. Radius of C.O.I can be easily found.

Solved Example

Example 30. Find the maximum angle that can be made in glass medium (μ = 1.5) if a light ray is refracted from glass to vacuum.

Solution : 1.5 sin C = 1 sin 90º, where C = critical angle.

sin C = 2/3

C = $\sin^{-1}$ 2/3

Example 31. Find the angle of refraction in a medium (μ = 2) if light is incident in vacuum, making angle equal to twice the critical angle.

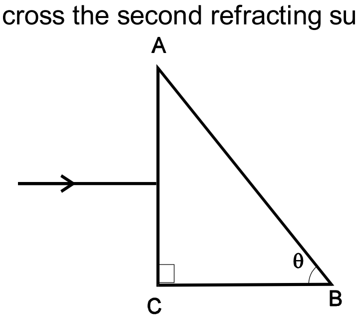
Solution : Since the incident light is in rarer medium. Total Internal Reflection can not take place.

$C = \sin^{-1}\dfrac{1}{\mu} = 30^\circ \qquad \therefore \qquad i = 2C = 60^\circ$

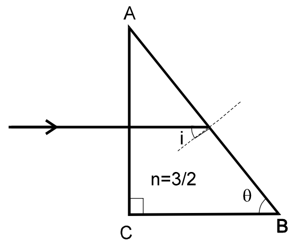
Applying Snell's Law. $\qquad$ 1 sin 60º = 2 sin r

$\sin r = \dfrac{\sqrt{3}}{4} \qquad \Rightarrow \qquad r = \sin^{-1}\left(\dfrac{\sqrt{3}}{4}\right).$

Example 32. What should be the value of angle θ so that light entering normally through the surface AC of a prism (n = 3/2) does not cross the second refracting surface AB?



Solution : Light ray will pass the surface AC without bending since it is incident normally. Suppose it strikes the surface AB at an angle of incidence i.



$i = 90 - \theta$

For the required condition : $90° - \theta > c$

or $\sin(90° - \theta) > \sin c$ or $\cos\theta > \sin c = \frac{1}{3/2} = \frac{2}{3}$

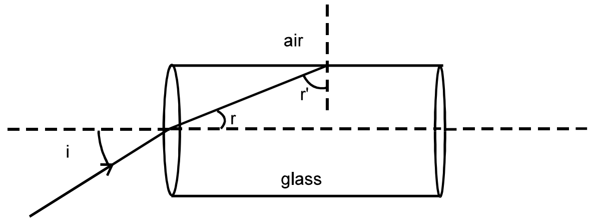
or $\theta < \cos^{-1}. \frac{2}{3}$

Example 33. What should be the value of refractive index n of a glass rod placed in air, so that the light entering through the flat surface of the rod does not cross the curved surface of the rod?

Solution : It is required that all possible r' should be more than critical angle. This will be automatically fulfilled if minimum r' is more than critical angle .....(A)

Angle r' is minimum when r is maximum i.e. C (why ?). Therefore the minimum value of r' is 90 – C.

From condition (A) :



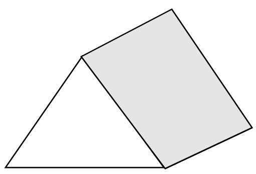
$90° - C > C$ or $C < 45°$

$\sin C < \sin 45°$; $\frac{1}{n} < \frac{1}{\sqrt{2}}$ or $n > \sqrt{2}$.

9. CHARACTERISTICS OF A PRISM

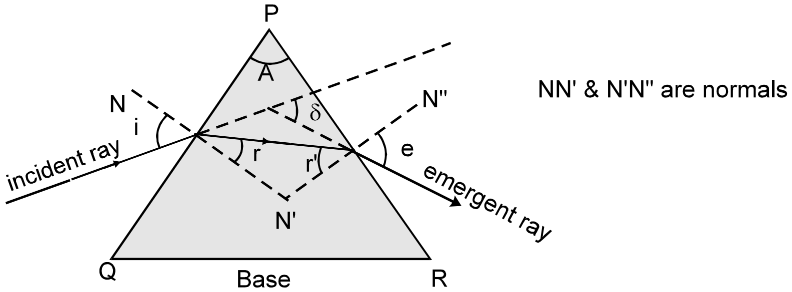
(a) A homogeneous solid transparent and refracting medium bounded by two plane surfaces inclined at an angle is called a prism :

3-D view



Refraction through a prism:

View from one side



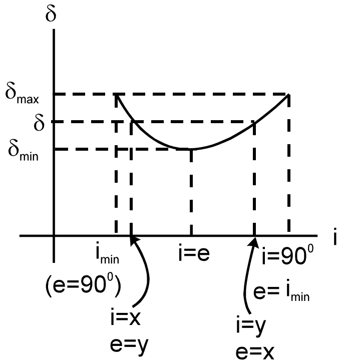
(b) PQ and PR are refracting surfaces.

(c) $\angle QPR = A$ is called refracting angle or the angle of prism (also called Apex angle).

(d) δ = angle of deviation

(e) For refraction of a monochromatic (single wave length) ray of light through a prism;
$\delta = (i + e) - (r_1 + r_2)$ and $r_1 + r_2 = A$
$\therefore$ **$\delta = i + e - A$.**

(f) Variation of δ versus i (shown in diagram).
For one δ (except δ min) there are two values of angle of incidence. If i and e are interchanged then we get the same value of δ because of reversibility principle of light



Note :
(i) For application of above result medium on both sides of prism must be same.
(ii) Based on above graph we can also derive following result, which says that i and e can be interchanged for a particular deviation in other words there are two angle of incidence for a given deviation (except minimum deviation).

i	r_1	r_2	e	δ
θ_1	θ_2	θ_3	θ_4	θ_5
θ_4	θ_3	θ_2	θ_1	θ_5

(g) There is one and only one angle of incidence for which the angle of deviation is minimum.

(h) When $\delta = \delta_{min}$, the angle of minimum deviation, then i = e and $r_1 = r_2$, the ray passes symmetrically w.r.t. the refracting surfaces. We can show by simple calculation that
$\delta_{min} = 2i_{min} - A$
where i_{min} = angle of incidence for minimum deviation, and r = A/2.

$$\therefore \quad n_{rel} = \frac{\sin\left[\frac{A + \delta_m}{2}\right]}{\sin\left[\frac{A}{2}\right]}, \text{ where } n_{rel} = \frac{n_{prism}}{n_{surroundings}}$$

Also $\delta_{min} = (n - 1) A$ (for small values of $\angle A$)

(i) For a thin prism ($A \le 10^o$) and for small value of i, all values of

$$\delta = (n_{rel} - 1) A \quad \text{where } n_{rel} = \frac{n_{prism}}{n_{surrounding}}$$

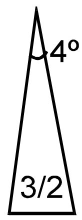
Solved Example

Example 34. Refracting angle of a prism A = 60º and its refractive index is, n = 3/2, what is the angle of incidence i to get minimum deviation? Also find the minimum deviation. Assume the surrounding medium to be air (n = 1).

Solution : For minimum deviation, $r_1 = r_2 = \frac{A}{2} = 30^o$.
applying Snell's law at I surface

$$1 \times \sin i = \frac{3}{2} \sin 30^o \quad \Rightarrow \quad i = \sin^{-1}\left(\frac{3}{4}\right) \quad \Rightarrow \quad \delta_{min} = 2\sin^{-1}\left(\frac{3}{4}\right) - 60^o$$

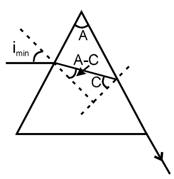
Example 35. See the figure. Find the deviation caused by a prism having refracting angle 4º and refractive index $\frac{3}{2}$.



Solution : $\delta = (\frac{3}{2} - 1) \times 4° = 2°$

Example 36. For a prism, A = 60º, n = $\sqrt{\frac{7}{3}}$. Find the minimum possible angle of incidence, so that the light ray is refracted from the second surface. Also find δ_{max}.

Solution : In minimum incidence case the angles will be as shown in figure Applying snell's law :



$1 \times \sin i_{min} = \sqrt{\frac{7}{3}}\ \sin(A - C)$

$= \sqrt{\frac{7}{3}}\ (\sin A \cos C - \cos A \sin C) = \sqrt{\frac{7}{3}}\left(\sin 60\sqrt{1-\frac{3}{7}} - \cos 60\sqrt{\frac{3}{7}}\right) = \frac{1}{2}$

$\therefore\ i_{min} = 30º \qquad \therefore\ \delta_{max} = i_{min} + 90° - A = 30° + 90° - 60° = 60°.$

Example 37. Show that if A > A_{max} (= 2 C), then total internal reflection occurs at second refracting surface PR of the prism for any value of 'i'.

Solution : For T.I.R. at second surface

r' > C $\Rightarrow$ (A – r) > C or A > (C + r)

The above relation will be fulfilled if

or A > C + r_{max} or A > C + C or A > 2C

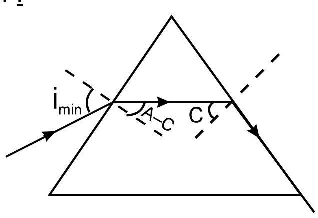
(j) On the basis of above example and similar reasoning, it can be shown that (you should try the following cases (ii) and (iii) yourself.)

(i) If A > 2C, all rays are reflected back from the second surface.

(ii) If A $\leq$ C, no rays are reflected back from the second surface i.e. all rays are refracted from second surface.

(iii) If 2C $\geq$ A > C, some rays are reflected back from the second surface and some rays are refracted from second surface, depending on the angle of incidence.

(k) δ is maximum for two values of i



$\Rightarrow$ i_{min} (corresponding to e = 90º) and *i* = 90º (corresponding to e_{min}).

For i_{min} : $n_s \sin i_{min} = n_p \sin(A - C)$

If i < i_{min} then T.I.R. takes place at second refracting surface PR.

10. DISPERSION OF LIGHT

The angular splitting of a ray of white light into a number of components and spreading in different directions is called Dispersion of Light. [It is for whole Electro Magnetic Wave in totality]. This phenomenon is because waves of different wavelength move with same speed in vacuum but with different speeds in a medium.

Therefore, the refractive index of a medium depends slightly on wavelength also. This variation of refractive index with wavelength is given by Cauchy's formula.

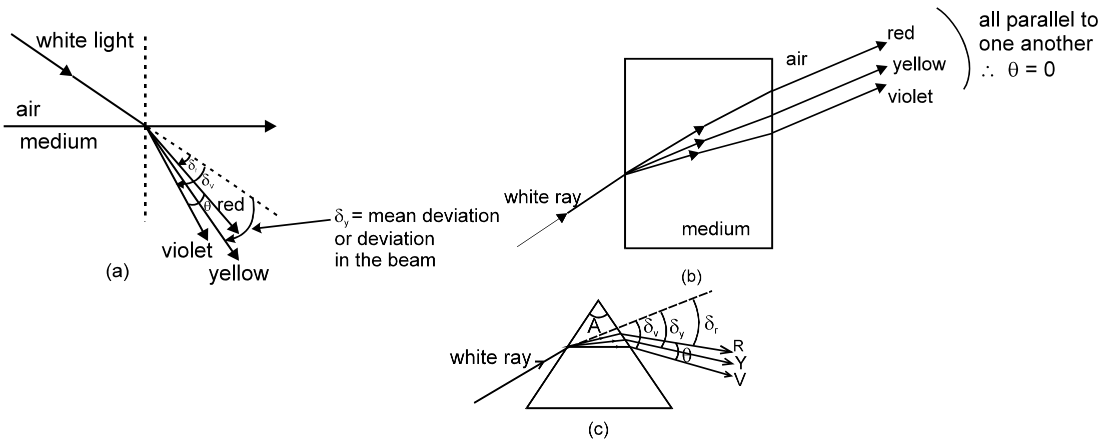
Cauchy's formula $n(\lambda) = a + \frac{b}{\lambda^2}$ where a and b are positive constants of a medium.

Note :

- Such phenomenon is not exhibited by sound waves.

  Angle between the rays of the extreme colours in the refracted (dispersed) light is called **angle of dispersion.** $\theta = \delta_v - \delta_r$ (Fig. (a))

  Fig (a) and (c) represents dispersion, whereas in fig. (b) there is no dispersion.



For prism of small 'A' and with small 'i' :

$\theta = \delta_v - \delta_r = (n_v - n_r) A$

Deviation of beam (also called mean deviation)

$\delta = \delta_y = (n_y - 1) A$

n_v, n_r and n_y are R. I. of material for violet, red and yellow colours respectively.

Solved Example

Example 38. The refractive indices of flint glass for red and violet light are 1.613 and 1.632 respectively. Find the angular dispersion produced by a thin prism of flint glass having refracting angle 5°.

Solution : Deviation of the red light is $\delta_r = (\mu_r - 1)A$ and deviation of the violet light is $\delta_v = (\mu_v - 1)A$.

The dispersion $= \delta_v - \delta_r = (\mu_v - \mu_r)A = (1.632 - 1.613) \times 5^0 = 0.095^0$.

Note :

- Numerical data reveals that if the average value of μ is small $\mu_v - \mu_r$ is also small and if the average value of μ is large $\mu_v - \mu_r$ is also large. Thus, larger the mean deviation, larger will be the angular dispersion.

  Dispersive power (ω) of the medium of the material of prism is given by : $\omega = \dfrac{n_v - n_r}{n_y - 1}$

- ω is the property of a medium.

  For small angled prism ($A \le 10^o$) with light incident at small angle i :

  $$\frac{n_v - n_r}{n_y - 1} = \frac{\delta_v - \delta_r}{\delta_y} = \frac{\theta}{\delta_y} = \frac{\text{angular dispersion}}{\text{deviation of mean ray (yellow)}}$$

  [$n_y = \dfrac{n_v + n_r}{2}$ if n_y is not given in the problem]

- $n - 1$ = refractivity of the medium for the corresponding colour.

Example 39. Refractive index of glass for red and violet colours are 1.50 and 1.60 respectively. Find
(a) the refractive index for yellow colour, approximately
(b) Dispersive power of the medium.

Solution : (a) $\mu_y \simeq \dfrac{\mu_V + \mu_R}{2} = \dfrac{1.50 + 1.60}{2} = 1.55$ (b) $\omega = \dfrac{\mu_V - \mu_R}{\mu_y - 1} = \dfrac{1.60 - 1.50}{1.55 - 1} = 0.18.$

10.1 Dispersion without deviation (Direct Vision Combination)

The condition for direct vision combination is :

$$\left[n_y - 1\right] A = \left[n'_y - 1\right] A' \Leftrightarrow \left[\frac{n_v + n_r}{2} - 1\right] A = \left[\frac{n'_v + n'_r}{2} - 1\right] A'$$

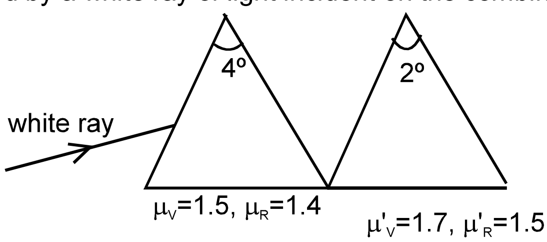
Two or more prisms can be combined in various ways to get different combination of angular dispersion and deviation.

10.2 Deviation without dispersion (Achromatic Combination)

Condition for achromatic combination is: $(n_v - n_r) A = (n'_v - n'_r) A'$

Solved Example

Example 40. If two prisms are combined, as shown in figure, find the total angular dispersion and angle of deviation suffered by a white ray of light incident on the combination.



Solution : Both prisms will turn the light rays towards their bases and hence in same direction. Therefore turnings caused by both prisms are additive.

Total angular dispersion = $\theta + \theta' = (\mu_V - \mu_R)A + (\mu'_V - \mu'_R)A'$

$= (1.5 - 1.4)\,4^o + (1.7 - 1.5)2^o = 0.8°$

Total deviation = $\delta + \delta'$

$= \left(\frac{\mu_V + \mu_R}{2} - 1\right)A + \left(\frac{\mu'_V + \mu'_R}{2} - 1\right)A' = \left(\frac{1.5 + 1.4}{2} - 1\right)0.4^o + \left(\frac{1.7 + 1.5}{2} - 1\right)0.2^o$

$= (1.45 - 1)\,0.4^o + (1.6 - 1)\,0.2^o = 0.45 \times 0.4^o + 0.6 \times 0.2^o = 1.80 + 1.2 = \mathbf{3.0^o}$ **Ans.**

Example 41. Two thin prisms are combined to form an achromatic combination. For I prism A = 4º, $\mu_R = 1.35$, $\mu_Y = 1.40$, $\mu_V = 1.42$ for II prism $\mu'_R = 1.7$, $\mu'_Y = 1.8$ and $\mu'_V = 1.9$ find the prism angle of II prism and the net mean deviation.

Solution : Condition for achromatic combination. $\theta = \theta'$

$(\mu_V - \mu_R)A = (\mu'_V - \mu'_R)A'$ $\therefore$ $A' = \frac{(1.42 - 1.35)4^o}{1.9 - 1.7} = 1.4°$

$\delta_{Net} = \delta - \delta' = (\mu_Y - 1)A - (\mu'_Y - 1)A' = (1.40 - 1)\,4^o - (1.8 - 1)\,1.4° = 0.48^o$.

Example 42. A crown glass prism of angle 5° is to be combined with a flint prism in such a way that the mean ray passes without deviation. Find (a) the apex angle of the flint glass prism needed and (b) the angular dispersion produced by the given combination when white light goes through it. Refractive indices for red, yellow and violet light are 1.5, 1.6 and 1.7 respectively for crown glass and 1.8, 2.0 and 2.2 for flint glass.

Solution : The deviation produced by the crown prism is $\delta = (\mu - 1)A$ and by the flint prism is :

$\delta' = (\mu' - 1)A'$.

The prisms are placed with their angles inverted with respect to each other. The deviations are also in opposite directions. Thus, the net deviation is :

$D = \delta - \delta' = (\mu - 1)A - (\mu' - 1)A'$. ......(1)

(a) If the net deviation for the mean ray is zero,

$(\mu - 1)A = (\mu' - 1)A'$ or, $A' = \frac{(\mu - 1)}{(\mu' - 1)}A = \frac{1.6 - 1}{2.0 - 1} \times 5° = 3°$

(b) The angular dispersion produced by the crown prism is : $\delta_v - \delta_r = (\mu_v - \mu_r)A$

and that by the flint prism is, $\delta'_v - \delta'_r = (\mu'_v - \mu'_r)A$

The net angular dispersion is, $(\mu_v - \mu_r)A - (\mu'_v - \mu'_r)A = (1.7 - 1.5) \times 5^0 - (2.2 - 1.8) \times 3° = -0.2°$.

The angular dispersion has magnitude 0.2°.

11. SPECTRUM

(Only for your knowledge and not of much use for JEE)

Ordered pattern produced by a beam emerging from a prism after refraction is called *Spectrum*. Types of spectrum:

11.1 Types of spectrum:

(a) Line spectrum : Due to source in atomic state.

(b) Band spectrum : Due to source in molecular state.

(c) Continuous spectrum : Due to white hot solid.

11.2 In Emission spectrum

Bright colours or lines, emitted from source are observed.

The spectrum emitted by a given source of light is called emission spectrum. It is a wavelength-wise distribution of light emitted by the source. The emission spectra are given by incandescent solids, liquids and gases which are either burnt directly as a flame (or a spark) or burnt under low pressure in a discharge tube.

11.3 In Absorption spectrum

Dark lines indicates frequencies absorbed.

When a beam of light from a hot source is passed through a substance (at a lower temperature), a part of the light is transmitted but rest of it is absorbed. With the help of a spectrometer, we can know the fraction of light absorbed corresponding to each wavelength. The distribution of the wavelength absorption of light by a substance is called an absorption spectrum. Every substance has its own characteristic absorption spectrum.

11.4 Spectrometer

Consists of a collimator (to collimate light beam), prism and telescope. It is used to observe the spectrum and also measure deviation.

12. REFRACTION AT SPHERICAL SURFACES

For paraxial rays incident on a spherical surface separating two media:

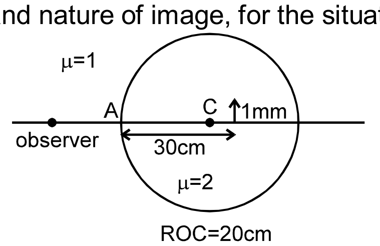
$$\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R} \text{ .... (A)}$$

where light moves from the medium of refractive index n_1 to the medium of refractive index n_2.

Transverse magnification (m) (of dimension perpendicular to principal axis) due to refraction at spherical surface is given by $\mathbf{m} = \frac{v-R}{u-R} = \left(\frac{v/n_2}{u/n_1}\right)$

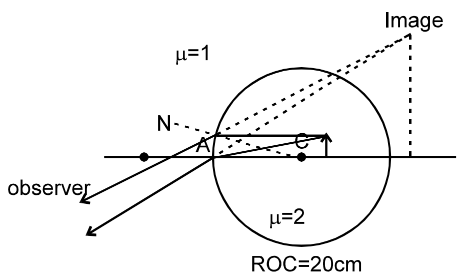
Solved Example

Example 43. Find the position, size and nature of image, for the situation shown in figure. Draw ray diagram.



Solution : For refraction near point A, u = – 30 ; R = – 20; $n_1 = 2$; $n_2 = 1$.

Applying refraction formula $\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}$



$$\frac{1}{v} - \frac{2}{-30} = \frac{1-2}{-20}$$

v = – 60 cm

$$m = \frac{h_2}{h_1} = \frac{n_1 v}{n_2 u} = \frac{2(-60)}{1(-30)} = 4$$

$\therefore$ $h_2 = 4$ mm.

Special case:
Refraction at plane Surfaces

Putting R = ∞ in the formula $\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}$, we get ; $\mathbf{v} = \frac{n_2 u}{n_1}$

The same sign of v and u implies that the object and the image are always on the same side of the interface separating the two media. If we write the above formula as $v = \frac{u}{n_{rel}}$,

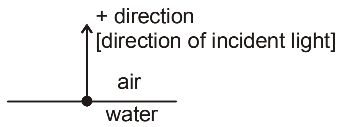
it gives the relation between the apparent depth and real depth, as we have seen before.

Solved Example

Example 44. Using formula of spherical surface or otherwise, find the apparent depth of an object placed 10 cm below the water surface, if seen near normally from air.

Solution : Put R = ∞ in the formula of the Refraction at Spherical Surfaces we get,

$v = \frac{un_2}{n_1}$



$u = -10$ cm

$n_1 = \frac{4}{3}$

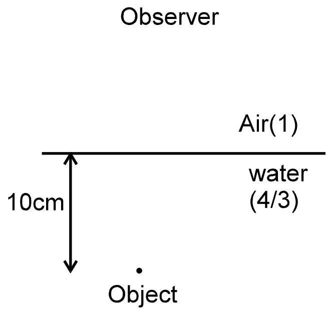
$n_2 = 1$

$v = -\frac{10 \times 1}{4/3} = -7.5$ cm

Negative sign implies that the image is formed in water.

Alter :

$d_{app} = \frac{d_{real}}{\mu_{rel}} = \frac{10}{4/3} = \frac{30}{4} = 7.5$ cm.



13. THIN LENS

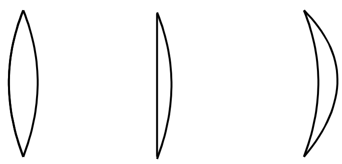
A thin lens is called convex if it is thicker at the middle and it is called concave if it is thicker at the ends. One surface of a convex lens is always convex. Depending on the other surface a convex lens is categorized as

(a) biconvex or convexo convex, if the other surface is also convex,

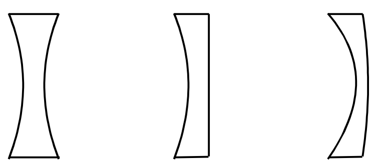
(b) Plano convex if the other surface is plane and

(c) Concavo convex if the other surface is concave.

Similarly concave lens is categorized as concavo-concave or biconcave, plano-concave and convexo-concave.



Bi convex Plano convex Concavo convex



Bi concave Plano concave Convexo concave

For a spherical, thin lens having the same medium on both sides:

$$\frac{1}{v} - \frac{1}{u} = (n_{rel} - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \qquad ....(a),$$

where $n_{rel} = \frac{n_{lens}}{n_{medium}}$ and R_1 and R_2 are x coordinates of the centre of curvature of the 1st surface and 2nd surface respectively.

$$\frac{1}{f} = (n_{rel} - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \text{ Lens Maker's formula} \qquad ....(b)$$

From (a) and (b)

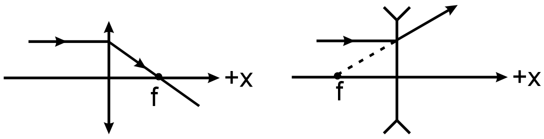
$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

Lens has two Focii :

If $u = \infty$, then $\frac{1}{v} - \frac{1}{\infty} = \frac{1}{f}$ $\Rightarrow$ $v = f$

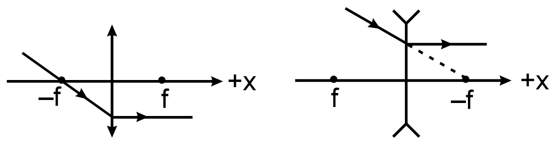
$\Rightarrow$ If incident rays are parallel to principal axis then its refracted ray will cut the principal axis at 'f'. It is called 2nd focus.

In case of converging lens it is positive and in case of diverging lens it is negative.



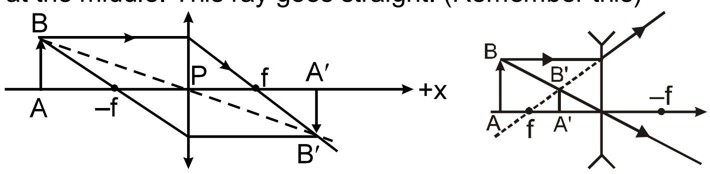
If $v = \infty$ that means $\frac{1}{\infty} - \frac{1}{u} = \frac{1}{f}$ $\Rightarrow$ $u = -f$

$\Rightarrow$ If incident rays cuts principal axis at $-f$ then its refracted ray will become parallel to the principal axis. It is called 1st focus. In case of converging lens it is negative ($\because$ f is positive) and in the case of diverging lens it positive ($\because$ f is negative)



use of $-f$ & $+f$ is in drawing the ray diagrams.

Notice that the point B, its image B′ and the pole P of the lens are collinear. It is due to parallel slab nature of the lens at the middle. This ray goes straight. (Remember this)



From the relation $\frac{1}{f} = (n_{rel} - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$ it can be seen that the second focal length depends on two factors.

(A) The factor $\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$ is

- **(i)** **Positive** for all types of **convex** lenses and
- **(ii)** **Negative** for all types of **concave** lenses.

(B) The factor $(n_{rel} - 1)$ is

- **(i)** **Positive** when **surrounding medium is rarer than the medium of lens.**
- **(ii)** **Negative** when **surrounding medium is denser than the medium of lens.**

(C) So a lens is **converging** if **f is positive** which happens when **both the factors (A) and (B) are of same sign**.

(D) And a lens is **diverging** if **f is negative** which happens when the **factors (A) and (B) are of opposite signs**.

(E) Focal length of the lens depends on medium of lens as well as surrounding.

(F) It also depends on wavelength of incident light. Incapability of lens to focus light rays of various wavelengths at single point is known as **chromatic aberration**.

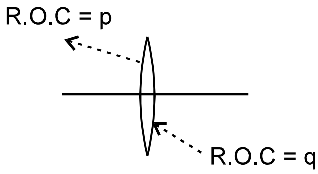
Solved Example

Example 45. Find the behaviour of a concave lens placed in a rarer medium.

Solution : Factor (A) is **negative**, because the lens is **concave**.
Factor (B) is **positive**, because the **lens is placed in a rarer medium**.
Therefore the focal length of the lens, which depends on the product of these factors, is negative and hence the lens will behave as diverging lens.

Example 46. Show that the factor $\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$ (and therefore focal length) does not depend on which surface of the lens light strike first.

Solution : Consider a convex lens of radii of curvature p and q as shown.

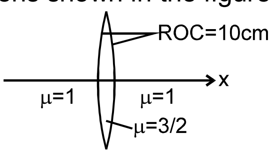


CASE 1 : Suppose light is incident from left side and strikes the surface with radius of curvature p, first. Then $R_1 = +p$; $R_2 = -q$ and $\left(\frac{1}{R_1}-\frac{1}{R_2}\right) = \left(\frac{1}{p}-\frac{1}{-q}\right)$

CASE 2 : Suppose light is incident from right side and strikes the surface with radius of curvature q, first. Then $R_1 = +q$; $R_2 = -p$ and $\left(\frac{1}{R_1}-\frac{1}{R_2}\right) = \left(\frac{1}{q}-\frac{1}{-p}\right)$

Though we have shown the result for biconvex lens, it is true for every lens.

Example 47. Find the focal length of the lens shown in the figure.

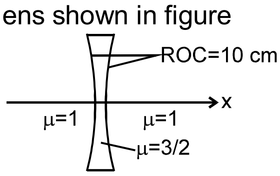


converging lens

Solution : $\because \ \frac{1}{f} = (n_{rel} - 1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$

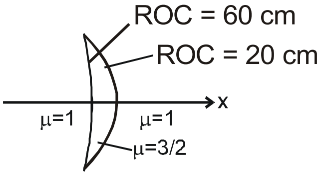
$\Rightarrow \ \frac{1}{f} = (3/2 - 1)\left(\frac{1}{10}-\frac{1}{(-10)}\right) \Rightarrow \frac{1}{f} = \frac{1}{2} \times \frac{2}{10} \Rightarrow f = +10$ cm.

Example 48. Find the focal length of the lens shown in figure



Solution : $\frac{1}{f} = (n_{rel} - 1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right) = \left(\frac{3}{2}-1\right)\left(\frac{1}{-10}-\frac{1}{10}\right)$; f = – 10 cm

Example 49. Find the focal length of the lens shown in figure



(a) If the light is incident from left side. (b) If the light is incident from right side.

Solution : (a) $\frac{1}{f} = (n_{rel} - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \left(\frac{3}{2} - 1\right)\left(\frac{1}{-60} - \frac{1}{-20}\right)$; f = 60 cm

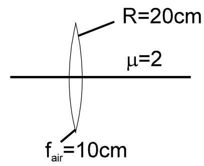
(b) $\frac{1}{f} = (n_{rel} - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \left(\frac{3}{2} - 1\right)\left(\frac{1}{20} - \frac{1}{60}\right)$; f = 60 cm

Example 50. Point object is placed on the principal axis of a thin lens with parallel curved boundaries i.e., having same radii of curvature. Discuss about the position of the image formed.

Solution : $\frac{1}{f} = (n_{rel} - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = 0$ [$\because R_1 = R_2$]

$\frac{1}{v} - \frac{1}{u} = 0$ or $v = u$ i.e. rays pass without appreciable bending.

Example 51. Focal length of a thin lens in air, is 10 cm. Now medium on one side of the lens is replaced by a medium of refractive index $\mu = 2$. The radius of curvature of surface of lens, in contact with the medium, is 20 cm. Find the new focal length.



Solution : Let radius of I surface be R_1 and refractive index of lens be μ . Let parallel rays be incident on the lens. Applying refraction formula at first surface

$$\frac{\mu}{v_1} - \frac{1}{\infty} = \frac{\mu - 1}{R_1} \qquad .....(1)$$

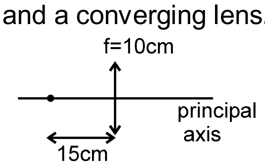
At II surface $\frac{2}{v} - \frac{\mu}{v_1} = \frac{2-\mu}{-20}$ .....(2)

Adding (1) and (2)

$$\frac{\mu}{v_1} - \frac{1}{\infty} + \frac{2}{v} - \frac{\mu}{v_1} = \frac{\mu-1}{R_1} + \frac{2-\mu}{-20}$$

$$= (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{-20}\right) - \frac{\mu-1}{20} - \frac{2-\mu}{20} = \frac{1}{f} \text{ (in air)} + \frac{1}{20} - \frac{2}{20} \Rightarrow v = 40 \text{ cm} \Rightarrow f = 40 \text{ cm}$$

Example - 52 Figure shows a point object and a converging lens. Find the final image formed.

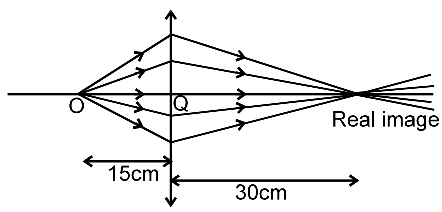


Solution : $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

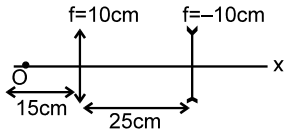
$\frac{1}{v} - \frac{1}{-15} = \frac{1}{10}$

$\frac{1}{v} = \frac{1}{10} - \frac{1}{15} = \frac{1}{30}$

$v = +30$ cm



Example 53. See the figure Find the position of final image formed.



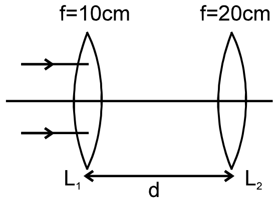
Solution : For converging lens

$u = -15$ cm, $f = 10$ cm $v = \frac{fu}{f+u} = 30$ cm

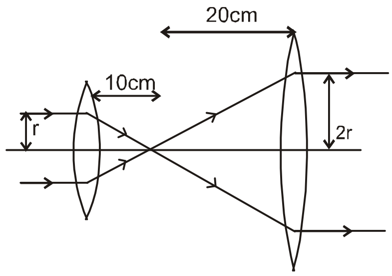
For diverging lens, $u = 5$ cm

$f = -10$ cm $v = \frac{fu}{f+u} = 10$ cm

Example 54. Figure shows two converging lenses. Incident rays are parallel to principal axis. What should be the value of d so that final rays are also parallel.



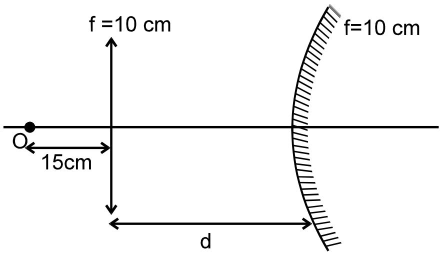
Solution : Final rays should be parallel. For this the II focus of L_1 must coincide with I focus of L_2.



d = 10 + 20 = 30 cm

Here the diameter of ray beam becomes wider.

Example 55. See the figure Find the position of final image formed.



Solution : For lens, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$\frac{1}{v} - \frac{1}{-15} = \frac{1}{10}$ $\Rightarrow$ v = + 30 cm

Hence it is object for mirror u = – 15 cm

$\frac{1}{v} + \frac{1}{-15} = \frac{1}{10}$ $\Rightarrow$ v = – 30 cm

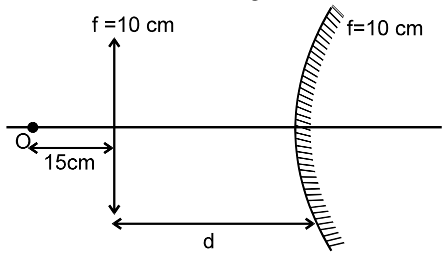
Now for second time it again passes through lens u = – 15 cm

v = ? ; f = 10 cm

$\frac{1}{v} - \frac{1}{-15} = \frac{1}{10}$ $\Rightarrow$ v = + 30

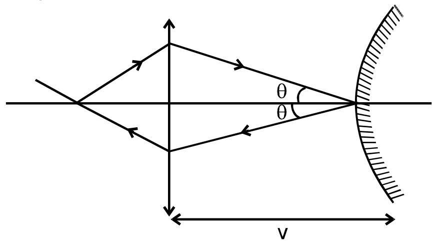
Hence final image will form at a distance 30 cm from the lens towards left.

Example 56. What should be the value of d so that image is formed on the object itself.

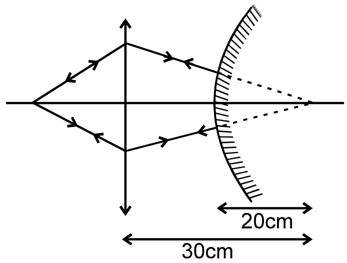


Solution : For lens : $\frac{1}{v} - \frac{1}{-15} = \frac{1}{10}$ $\quad$ v = + 30 cm

Case I : If d = 30, the object for mirror will be at pole and its image will be formed there itself.



Case II : If the rays strike the mirror normally, they will retrace and the image will be formed on the object itself



$\therefore$ d = 30 – 20 = 10 cm

13.2 Transverse magnification (m)

Transverse magnification (m) (perpendicular to principal axis) is given by $m = \frac{v}{u}$. If the lens is thick or/and the medium on both sides is different, then we have to apply the formula given for refraction at spherical surfaces step by step.

Solved Example

Example 57. An extended real object of size 2 cm is placed perpendicular to the principal axis of a converging lens of focal length 20 cm. The distance between the object and the lens is 30cm.
(i) Find the lateral magnification produced by the lens.
(ii) Find the height of the image.
(iii) Find the change in lateral magnification, if the object is brought closer to the lens by 1 mm along the principal axis.

Solution : Using $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ and $m = \frac{v}{u}$ we get $m = \frac{f}{f+u}$ ......(A)

$\therefore$ $m = \frac{+20}{+20+(-30)} = \frac{+20}{-10} = -2$

–ve sign implies that the image is inverted.

(ii) $\frac{h_2}{h_1} = m$ $\quad \therefore \quad$ $h_2 = mh_1 = (-2)(2) = -4$ cm

(iii) Differentiating (A) we get

$$dm = \frac{-f}{(f+u)^2}\,du \quad = \frac{-(20)}{(-10)^2}\,(0.1) = \frac{-2}{100} = -.02$$

Note that the method of differential is valid only when changes are small.

Alternate method : u (after displacing the object) = –(30 + 0.1) = – 29.9 cm

Applying the formula $m = \dfrac{f}{f+u}$

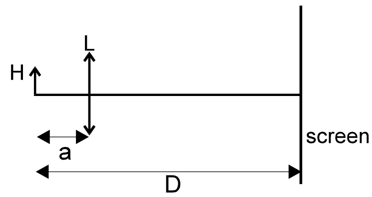
$$m = \frac{20}{20+(-29.9)} = -2.02$$

∴ change in 'm' = – 0.02.

Since in this method differential is not used, this method can be used for any changes, small or large.

13.3 Displacement Method to find Focal length of Converging Lens :

Fix an object of small height H and a screen at a distance D from object (as shown in figure). Move a converging lens from the object towards the screen. Let a sharp image forms on the screen when the distance between the object and the lens is 'a'. From lens formula we have



$$\frac{1}{D-a} - = \frac{1}{f} \quad \text{or} \quad a^2 - Da + fD = 0 \quad ...(A)$$

This is quadratic equation and hence two values of 'a' are possible. Call them a_1 and a_2. Thus a, and a_2 are the roots of the equation. From the properties of roots of a quadratic equation,

∴ $a_1 + a_2 = D \Rightarrow a_1a_2 = fD$

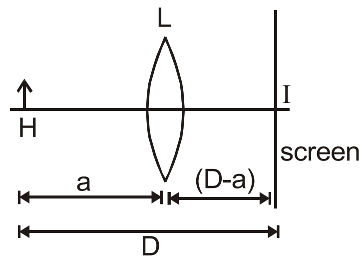
Also $(a_1 - a_2) = \sqrt{(a_1+a_2)^2 - 4a_1a_2} = \sqrt{D^2 - 4fD}$ = d (suppose).

'd' physically means the separation between the two position of lens.

The focal length of lens in terms of D and d.

so, $a_1 - a_2 = \sqrt{(a_1+a_2)^2 - 4a_1a_2}$

$$\sqrt{D^2 - 4fd} = d \Rightarrow \boxed{f = \frac{D^2 - d^2}{4D}}$$



condition, d = 0, i.e. the two position coincide

$$f = \frac{D^2}{4D}$$

∴ $\boxed{D = 4f}$

Roots of the equation $a^2 - Da + fD = 0$, become imaginary if

$b^2 - 4ac < 0$.

$= D^2 - 4fD < 0$

$= D(D - 4f) < 0$.

$= \boxed{D - 4f < 0}$

for real value of a in equation $a^2 - Da + fD = 0$

$b^2 - 4ac \geq 0. \quad = D^2 - 4fD \geq 0.$

so, $D \geq 4f \quad \Rightarrow \quad D_{min} = 4f$

Lateral magnification in displacement method:

if m_1 and m_2 be two magnifications in two positions (In the displacement method)

$$m_1 = \frac{v_1}{u_1} = \frac{(D-a_1)}{-a_1} \qquad m_2 = \frac{v_2}{u_2} = \frac{D-a_2}{-a_2} = \frac{a_1}{-(D-a_1)}$$

So $m_1 m_2 = \dfrac{(D-a_1)}{-a_1} \times \dfrac{a_1}{-(D-a_1)} = 1.$

If image length are h_1 and h_2 in the two cases,

then $m_1 = -\dfrac{h_1}{H}$ $\Rightarrow$ $m_2 = -\dfrac{h_2}{H}$ $\Rightarrow$ $m_1 m_2 = 1$

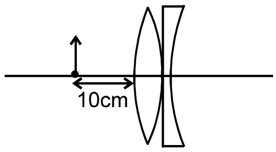
$\therefore$ $\dfrac{h_1 h_2}{H^2} = 1$ $\Rightarrow$ $h_1 h_2 = H^2$ $\Rightarrow$ $H = \sqrt{h_1\ h_2}$

14. COMBINATION OF LENSES

The equivalent focal length of thin lenses in contact is given by $\dfrac{1}{F} = \dfrac{1}{f_1} + \dfrac{1}{f_2} + \dfrac{1}{f_3}$ ...., where f_1, f_2, f_3 are focal lengths of individual lenses. If two lenses are separated by a distance d and the incident light rays are parallel to the common principal axis, then the combination behaves like a single lens of focal length given by the relation $\dfrac{1}{F} = \dfrac{1}{f_1} + \dfrac{1}{f_2} - \dfrac{d}{f_1 f_2}$ and the position of equivalent lens is $\dfrac{-dF}{f_1}$ with respect to 2nd lens.

Solved Example

Example 58. Find the lateral magnification produced by the combination of lenses shown in the figure.

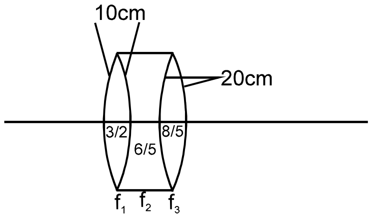


Solution : $\dfrac{1}{f} = \dfrac{1}{f_1} + \dfrac{1}{f_2} = \dfrac{1}{10} - \dfrac{1}{20} = \dfrac{1}{20}$ $\Rightarrow$ $f = +20$

$\therefore$ $\dfrac{1}{v} - \dfrac{1}{-10} = \dfrac{1}{20}$ $\Rightarrow$ $\dfrac{1}{v} = \dfrac{1}{20} - \dfrac{1}{10} = \dfrac{-1}{20} = -20$ cm

$\therefore$ $m = \dfrac{-20}{-10} = 2$

Example 59. Find the focal length of equivalent system.



Solution : $\dfrac{1}{f_1} = \left(\dfrac{3}{2} - 1\right)\left(\dfrac{1}{10} + \dfrac{1}{10}\right) = \dfrac{1}{2} \times \dfrac{2}{10} = \dfrac{1}{10}$

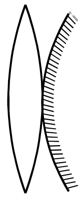
$\dfrac{1}{f_2} = \left(\dfrac{6}{5} - 1\right)\left(\dfrac{-1}{10} - \dfrac{1}{20}\right) = \dfrac{1}{5} \times \left(\dfrac{-30}{10 \times 20}\right) = \dfrac{-3}{100}$

$\dfrac{1}{f_3} = \left(\dfrac{8}{5} - 1\right)\left(\dfrac{1}{20} + \dfrac{1}{20}\right) = \dfrac{3}{50}$

$\dfrac{1}{f} = \dfrac{1}{f_1} + \dfrac{1}{f_2} + \dfrac{1}{f_3} = \dfrac{1}{10} + \dfrac{-3}{100} + \dfrac{3}{50}$ $f = \dfrac{100}{13}$ **Ans.**

15. COMBINATION OF LENS AND MIRROR

The combination of lens and mirror behaves like a mirror of focal length 'f' given by



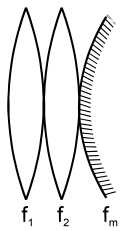
$$\frac{1}{f} = \frac{1}{F_m} - \frac{2}{F_\ell}$$

If lenses are more than one, 'f' is given by

$$\frac{1}{f} = \frac{1}{F_m} - 2\left(\sum \frac{1}{f_\ell}\right)$$

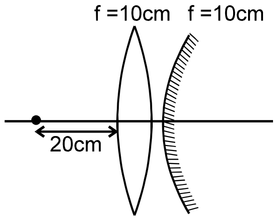
For the following figure, '

f' is given by $\frac{1}{f} = \frac{1}{F_m} - 2\left(\frac{1}{f_1} + \frac{1}{f_2}\right)$



Solved Example

Example 60. Find the position of final image formed. (The gap shown in figure is of negligible width)



Solution : $\frac{1}{f_{eq}} = \frac{1}{10} - \frac{2}{10} = \frac{-1}{10}$

$\Rightarrow$ $f_{eq} = -10$ cm

$$\frac{1}{v} + \frac{1}{-20} = \frac{1}{-10}$$

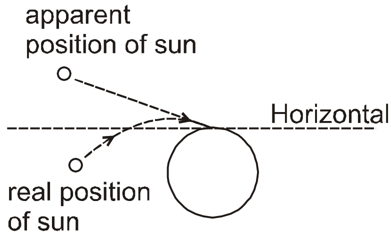
$\Rightarrow$ v = – 20 cm

Hence image will be formed on the object itself

Some interesting facts about light :

(1) THE SUN RISES BEFORE IT ACTUALLY RISES AND SETS AFTER IT ACTUALLY SETS :

The atmosphere is less and less dense as its height increase, and it is also known that the index of refraction decrease with a decrease in density. So, there is a decrease of the index of refraction with height. Due to this the light rays bend as they move in the earth's atmosphere



(2) THE SUN IS OVAL SHAPED AT THE TIME OF ITS RISE AND SET :

The rays diverging from the lower edge of the sun have to cover a greater thickness of air than the rays from the upper edge. Hence the former are refracted more than the latter, and so the vertical diameter of the sun appears to be a little shorter than the horizontal diameter which remains unchanged.

(3) THE STARS TWINKLE BUT NOT THE PLANETS :

The refractive index of atmosphere fluctuates by a small amount due to various reasons. This causes slight variation in bending of light due to which the apparent position of star also changes, producing the effect of twinkling.

(4) GLASS IS TRANSPARENT, BUT ITS POWDER IS WHITE :

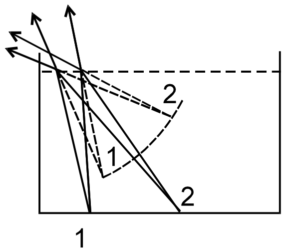
When powdered, light is reflected from the surface of innumerable small pieces of glass and so the powder appears white. Glass transmits most of the incident light and reflects very little hence it appears transparent.

(5) GREASED OR OILED PAPER IS TRANSPARENT, BUT PAPER IS WHITE :

The rough surface of paper diffusely reflects incident light and so it appears white. When oiled or greased, very little reflection takes place and most of the light is allowed to pass and hence it appears transparent.

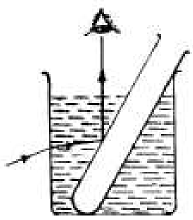
(6) AN EXTENDED WATER TANK APPEARS SHALLOW AT THE FAR END :

This is due to Total internal reflection



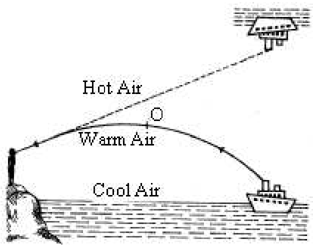
(7) A TEST TUBE OR A SMOKED BALL IMMERSED IN WATER PEARS SILVERY WHITE WHEN VIEWED FROM THE TOP :

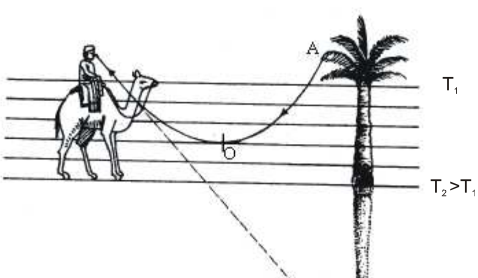
This is due to Total internal reflection



(8) SHIPS HANG INVERTED IN THE AIR IN COLD COUNTRIES AND TREES HANG INVERTED UNDERGROUND IN DESERTS:

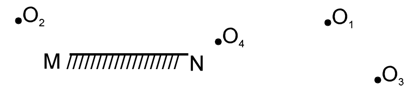
This is due to Total internal reflection



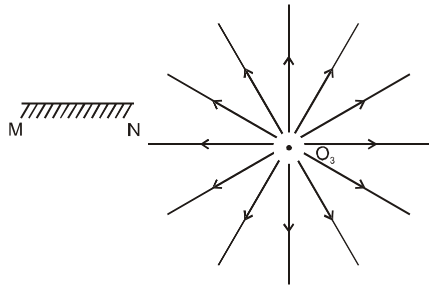


Solved Miscellaneous Problems

Problem 1. See the following figure. Which of the object(s) shown in figure will not form its image in the mirror.

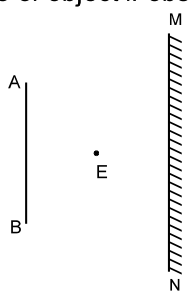


Solution :



No ray from O_3 is incident on reflecting surface of the mirror, so its image is not formed.

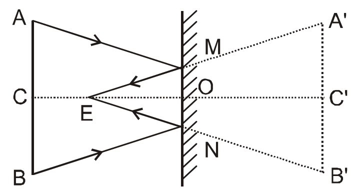
Problem 2. Figure shows an object AB and a plane mirror MN placed parallel to object. Indicate the mirror length required to see the image of object if observer's eye is at E.



Solution : Required length of mirror = MN.

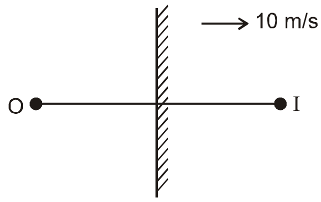
Δ MNE & ΔA'B'E are similar

$$\frac{MN}{OE} = \frac{A'B'}{C'E} \quad \Rightarrow \quad MN = \frac{A'B'}{2} = \frac{AB}{2}.$$



Problem 3. An object is kept fixed in front of a plane mirror which is moved by 10 m/s away from the object, find the velocity of the image.

Solution :



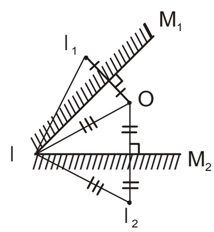
$$\vec{V}_{IM} = -\vec{V}_{OM}$$

$$\vec{V}_{I,G} - \vec{V}_{M,G} = -\vec{V}_{O,G} + \vec{V}_{M,G}$$

$$\Rightarrow \quad \vec{V}_{M,G} = \frac{\vec{V}_{I,G} + \vec{V}_{O,G}}{2} = \frac{\vec{V}_{I,G}}{2} \quad (\because \vec{V}_{O,G} = 0)$$

$$\vec{V}_{I,G} = 10\,\hat{i} \text{ m/s} = 20\,\hat{i} \text{ m/s}$$

Problem 4. Figure shows two inclined plane mirrors M_1 and M_2 and an object O. Its images formed in mirrors M_1 and M_2 individually are I_1 and I_2 respectively. Show that I_1 and I_2 and O lie on the circumference of a circle with centre at I. [This result can be extended to show that all the images will also lie on the same circle. Note that this result is independent of the angle of inclination of mirrors.]. I is the point of intersection of the mirrors.

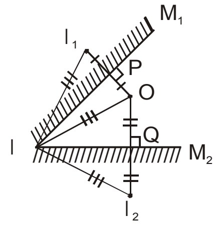


Solution : Clearly, ΔIOQ and $\Delta I I_2Q$ are congruent and ΔIOP and ΔII_1P are congruent

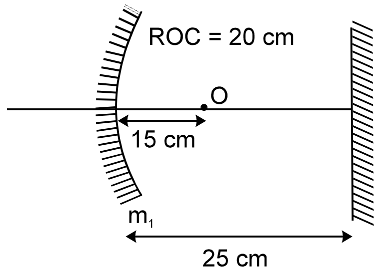
So, $II_1 = IO$ and $IO = II_2$

Hence, $II_1 = IO = II_2$

So, I_1 and I_2 and O lie on the circumference of a circle with centre I.



Problem 5. Find the position of final image after three successive reflections taking first reflection on m_1



Solution : **1st reflection at m_1**

u = – 15cm

f = –10 cm

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f} = \frac{-3+2}{30}=-\frac{1}{30}$$

v = – 30 cm

2nd reflection at plane mirror :

u = 5 cm

v = – 5 cm

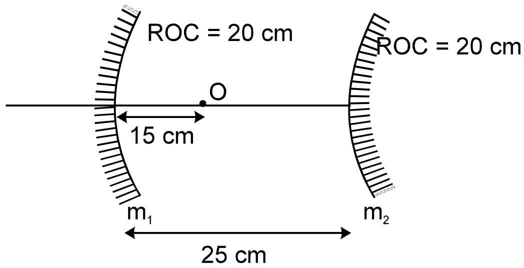
For III reflection on curved mirror again :

u = – 20 cm

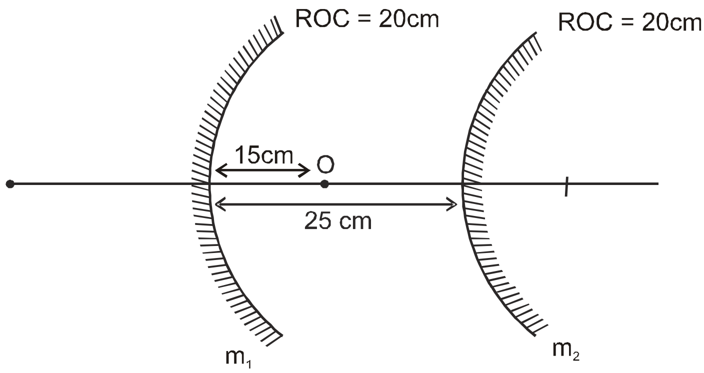
$$v = \frac{uf}{u-f} = \frac{(-20)\times(-10)}{-20+10} = \frac{200}{-10} = -20 \text{ cm}$$

Image is 20 cm right of m_1

Problem 6. Find the position of final image after three successive reflections taking first reflection on m_1.



Solution :



1^{st} reflection at mirror m_1 : u = –15 cm, f = – 10cm

$$\frac{1}{v} = \frac{1}{f} - \frac{1}{u}$$

$$\therefore \quad v = \frac{uf}{u-f} = \frac{(-15)\times(-10)}{(-15)+10} = \frac{150}{-5} \text{ cm} = -30 \text{ cm.}$$

Thus, image is formed at a point 5 cm right of m_2 which will act as an object for the reflection at m_2

For 2^{nd} reflection at m_2

u = 5 cm, f = 10 cm

$$v = \frac{uf}{u-f} = \frac{5\times10}{5-10} = \frac{50}{-5} = -10 \text{ cm.}$$

3^{rd} reflection at m_1 again.

u = –15 cm f = – 10 cm

$$v = \frac{uf}{u-f} = \frac{-15\times(-10)}{(-15)+10} = -30 \text{ cm.}$$ **Ans.**

Image is formed at 30 cm right of m_1

Problem 7. A coin is placed 10 cm in front of a concave mirror. The mirror produces a real image that has diameter 4 times that of the coin. What is the image distance.

Solution : $m = \frac{d_2}{d_1} = -\frac{v}{u}$

$\Rightarrow \quad -4 = -\frac{v}{u} \quad \Rightarrow \quad v = 4u$

= 4 × (–10) = – 40 cm

Problem 8. A small statue has a height of 1 cm and is placed in front of a spherical mirror. The image of the statue is inverted and is 0.5cm tall and located 10 cm in front of the mirror. Find the focal length and nature of the mirror.

Solution : We have $m = \frac{h_2}{h_1} = -\frac{0.5}{1} = -0.5$

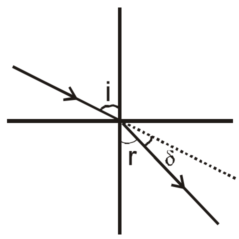
v = – 10 cm (real image)

But $m = \frac{f-v}{f} \quad -0.5 = \frac{f+10}{f} \quad \Rightarrow \quad f = \frac{-20}{3}$ cm

so, concave mirror. **Ans.**

Problem 9. A light ray deviates by 30° (which is one third of the angle of incidence) when it gets refracted from vacuum to a medium. Find the refractive index of the medium.

Solution : $\delta = i - r$



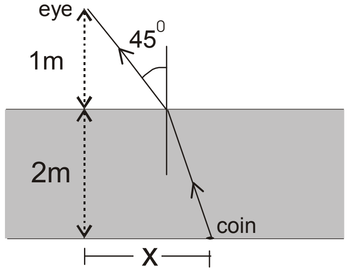
$\Rightarrow \quad \frac{i}{3} = i - r = 30^o \quad \Rightarrow \quad i = 90^o$

$\Rightarrow \quad 2i = 3r$

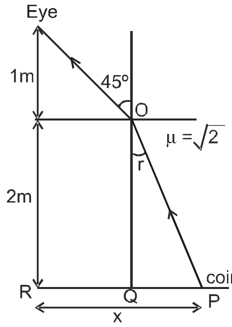
$\therefore \quad r = \frac{2i}{3} = 60^o$

So, $\mu = \frac{\sin 90^o}{\sin 60^o} = \frac{1}{\sqrt{3}/2} = \frac{2}{\sqrt{3}}$ **Ans.**

Problem 10. A coin lies on the bottom of a lake 2m deep at a horizontal distance x from the spotlight (a source of thin parallel beam of light) situated 1 m above the surface of a liquid of refractive index $\mu = \sqrt{2}$ and height 2m. Find x.



Solution : $\sqrt{2} = \frac{\sin 45^\circ}{\sin r}$



$\Rightarrow \sin r = \frac{1}{2}$

$\Rightarrow r = 30^\circ$

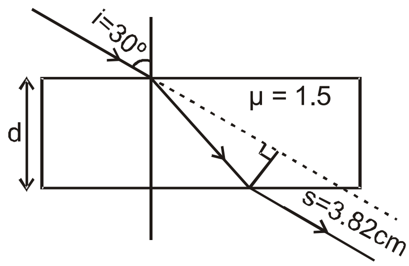
x = RQ + QP = 1m + 2tan30° m

$= \left(1 + \frac{2}{\sqrt{3}}\right)$ m **Ans.**

Problem 11. A ray of light falls at an angle of 30º onto a plane-parallel glass plate and leaves it parallel to the initial ray. The refractive index of the glass is 1.5. What is the thickness d of the plate if the distance between the rays is 3.82 cm? [Given : $\sin^{-1}\left(\frac{1}{3}\right) = 19.5^\circ$; $\cos 19.5^\circ = 0.94$; $\sin 10.5^\circ = 0.18$]

Solution : Using $s = \frac{d\sin(i-r)}{\cos r}$

$\Rightarrow \quad d = \frac{3.82 \times \cos r}{\sin(30^\circ - r)}$ .....(1)



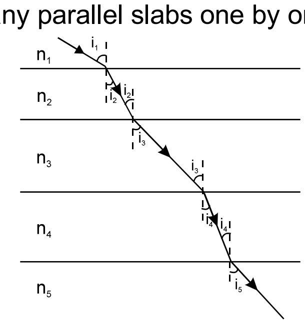
Also, $1.5 = \frac{\sin 30^\circ}{\sin r} \Rightarrow \sin r = \frac{1}{3}$

so, $r = 19.5^\circ$

So, $d = \frac{3.82 \times \cos 19.5^\circ}{\sin(30^\circ - 19.5^\circ)} = \frac{3.82 \times 0.94}{\sin 10.5^\circ}$

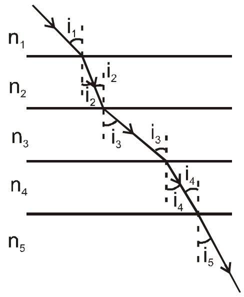
$= \frac{3.82 \times 0.94}{0.18} = 19.948 \text{ cm} \approx 0.2 \text{ m}$

Problem 12. A light passes through many parallel slabs one by one as shown in figure.



Prove that $n_1 \sin i_1 = n_2 \sin i_2 = n_3 \sin i_3 = n_4 \sin i_4$ =............**[Remember this].** Also prove that if $n_1 = n_4$ then light rays in medium n_1 and in medium n_4 are parallel.

Solution : We have, $\frac{\sin i_1}{\sin i_2} = \frac{n_2}{n_1}$



$\Rightarrow$ $n_1 \sin i_1 = n_2 \sin i_2$ ....(i)

Similarly $n_2 \sin i_2 = n_3 \sin i_3$

so on

so, $n_1 \sin i_1 = n_2 \sin i_2 = n_3 \sin i_3 = \ldots\ldots\ldots\ldots.$

$n_1 \sin i_1 = n_4 \sin i_4 \Rightarrow$ $\sin i_1 = \sin i_4$ $(\because n_1 = n_2)$

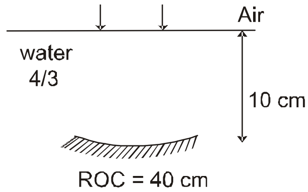
so, $i_1 = i_4$

Hence, light rays in medium n_1 and in medium n_4 are parallel.

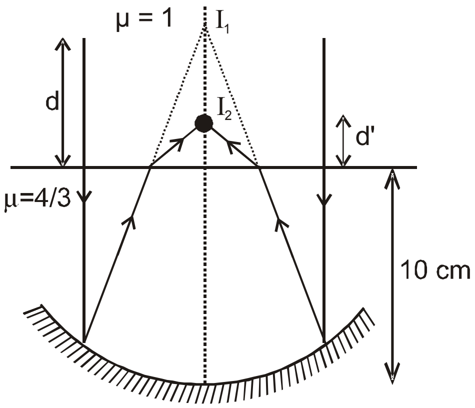
Problem 13. An object lies 90 cm in air above water surface .It is viewed from water nearly normally. Find the apparent height of the object.

Solution : $d' = \frac{d}{n_{rel}} = \frac{d}{n_i / n_r} = \frac{90 \quad cm}{\frac{1}{4/3}} = \frac{90 \times 4}{3}$ cm = 120 cm **Ans.**

Problem 14. A concave mirror is placed inside water with its shining surface upwards and principal axis vertical as shown. Rays are incident parallel to the principal axis of concave mirror. Find the position of final image.



Solution :



We have,

$u = -\infty$, $f = -20$ cm

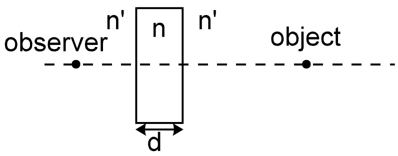
So, $v = -20$ cm

So, d = 10 cm

$\therefore$ $d' = \frac{d}{\mu_{rel}} = \frac{10cm}{4/3} = \frac{30}{4}$ cm = 7.5 cm **Ans.**

Problem 15. Prove that the shift in position of object due to parallel slab is given by shift = $d\left(1 - \frac{1}{n_{rel}}\right)$

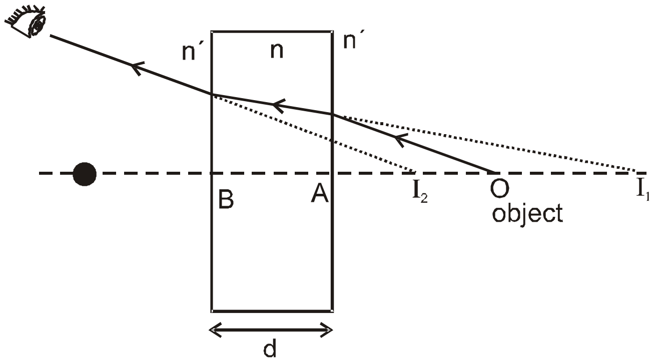
where $n_{rel} = \frac{n}{n'}$.



Solution : Because of the ray refraction at the first surface, the image of O is formed at I_1. For this refraction, the real depth is AO = x and apparent depth is AI_1.

Thus : $AI_1 = \dfrac{AO}{n_i/n_r} = \dfrac{AO}{n'/n} = \dfrac{n(AO)}{n'}$.

The point I_1 acts as the object for the refraction of second surface. Due to this refraction, the image of I_1 is formed at I_2. Thus,

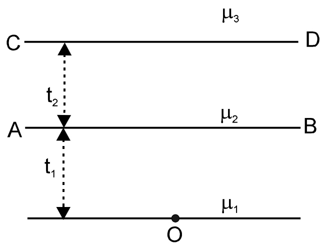


$BI_2 = \dfrac{(BI_1)}{(n/n')} = \dfrac{n'}{n}$ (BI_1) = n'/n (AB + AI_1) = $\dfrac{n'}{n}\left[d + \dfrac{n}{n'}(AO)\right] = \dfrac{n'}{n}d$ + AO.

Net shift = OI_2 = BO – BI_2 = d + (AO) – $\dfrac{n'}{n}$d – AO

= $d\left(1 - \dfrac{n'}{n}\right) = d\left(1 - \dfrac{1}{n_{rel}}\right)$ where $n_{rel} = \dfrac{n}{n'}$. **Ans.**

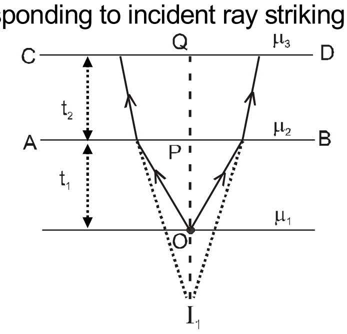
Problem 16. Find the apparent depth of object O below surface AB, seen by an observer in medium of refractive index μ_2



Solution : $d_{app.} = \dfrac{t_1}{\mu_1/\mu_2}$

Problem 17. In above question what is the depth of object corresponding to incident rays striking on surface CD in medium μ_2.

Solution : Depth of the object corresponding to incident ray striking on the surface CD in medium $\mu_2 = t_2 + PI_1$

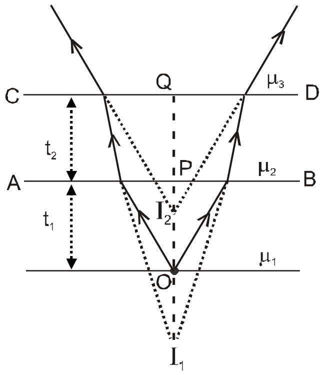


$= t_2 + \dfrac{t_1}{\mu_1/\mu_2}$

Problem 18. In above question if observer is in medium μ_3, what is the apparent depth of object seen below surface CD.

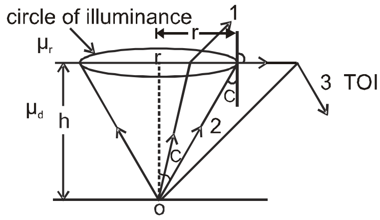
Solution : If the observer is in medium μ_3. Apparent depth below surface CD = QI_2.

$$= \sum \frac{t_i}{(n_{rel})_i} = \frac{t_2}{\mu_2/\mu_3} + \frac{t_1}{\mu_1/\mu_3}$$



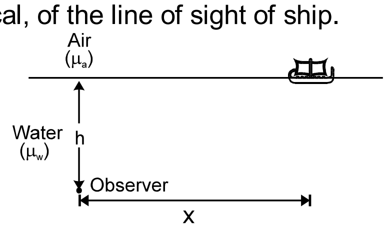
Problem 19. Find the radius of circle of illuminance, if a luminous object is placed at a distance h from the interface in denser medium.

Solution : $\tan C = \frac{r}{h}$. $\quad\therefore\quad r = h \tan C. \quad$ But $\quad C = \sin^{-1}\frac{1}{(\mu_d/\mu_r)}$

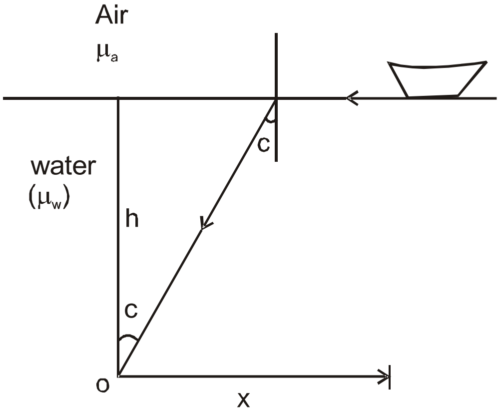


so, $r = h \tan\left[\sin^{-1}\frac{1}{(\mu_d/\mu_r)}\right] = h.\frac{\mu_r}{\sqrt{\mu_d^2 - \mu_r^2}}$

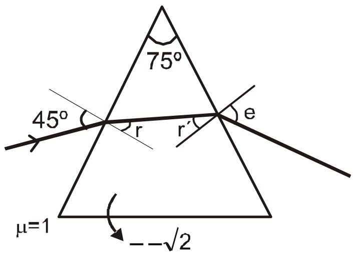
Problem 20. A ship is sailing in river. An observer is situated at a depth h in water (μ_w). If x >> h, find the angle made from vertical, of the line of sight of ship.



Solution : $C = \sin^{-1}\left(\frac{\mu_a}{\mu_w}\right)$



Problem 21. Find r, r´, e, δ for the case shown in figure.



Solution : Here $\theta = 180° - 75° = 105°$

$\sin 45° = \sqrt{2} \sin r$

$\therefore \quad r = \sin^{-1} \frac{1}{2} = 30°.$

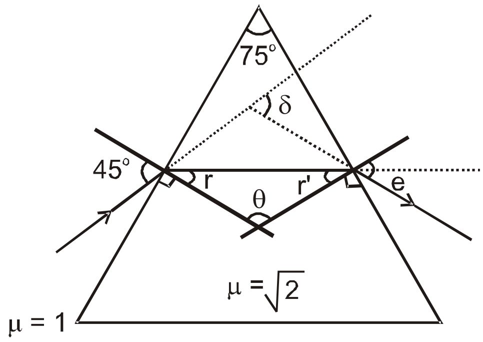
$r' = 180° - (r + \theta) = 180° - 30° - 105° = 45°$

$\sin e = \sqrt{2} \sin r'$

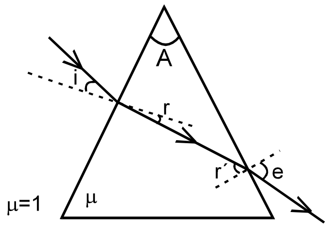
$\therefore \quad \sin e = \sqrt{2} \times \sin 45° = 1$

$\therefore \quad \boxed{e = 90°}$

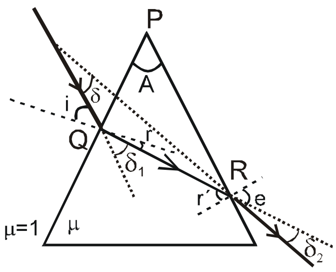
So, $\delta = i + e - A = 45° + 90° - 75° = 60°$.



Problem 22. For the case shown in figure prove the relations $r' - r = A$ and $\delta = |(i - e) + A|$ (do not try to remember these relations because the prism is normally not used in this way).



Solution :



In Δ PQR, $A + \angle PQR + \angle QRP = 180°$

$= A + r + 90^o + 90^o - r' = 180°$

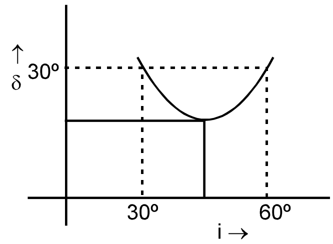
$\therefore \quad \boxed{r' - r = A}$

Deviation after Ist refraction $\delta_1 = (i - r)$ (anticlock wise)

Deviation after IInd refraction $\delta_2 = (e - r')$ (clock wise)

Hence net deviation $\delta = \delta_1 - \delta_2 = (i - r) - (e - r') = i - e + A$

Problem 23. From the graph of angle of deviation δ versus angle of incidence i, find the prism angle



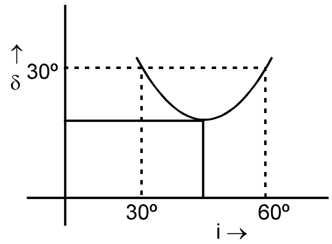
Solution : From the graph ;

$\delta = i + e - A.$

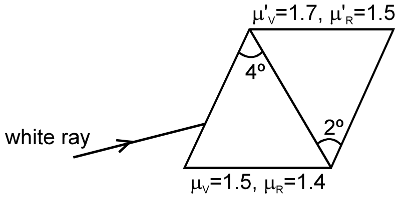
$30° = 30° + 60° - A$

$\therefore \boxed{A = 60°}$

(use the result : If i and e are interchanged then we get same value of δ)



Problem 24. If two prisms are combined, as shown in figure, find the net angular dispersion and angle of deviation suffered by a white ray of light incident on the combination.



Solution : Net angular dispersion = $(\delta_v - \delta_r) - (\delta'_v - \delta'_r)$

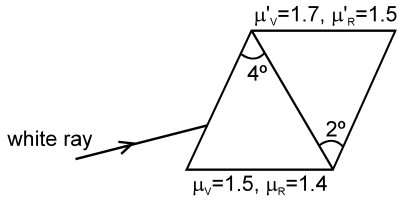
$= (\mu_v - \mu_r) A - (\mu'_v - \mu'_r) A'$

$= (1.5 - 1.4)\times 4° - (1.7 - 1.5)\times 2° = 0$

Angle of deviation

$$= \left(\frac{\mu_v + \mu_r}{2} - 1\right) A - \left(\frac{\mu'_v + \mu'_r}{2} - 1\right) A'$$

$$= \left(\frac{1.5 + 1.4}{2} - 1\right) \times 4° - \left(\frac{1.7 + 1.5}{2} - 1\right) \times 2° = 0.6°$$



Problem 25. The dispersive powers of crown and flint glasses are 0.03 and 0.05 respectively. The refractive indices for yellow light for these glasses are 1.517 and 1.621 respectively. It is desired to form an achromatic combination of prisms of crown and flint glasses which can produce a deviation of 1° in the yellow ray. Find the refracting angles of the two prisms needed.

Solution : $\omega_c = 0.03 = \dfrac{n_v - n_r}{n_y - 1}$

$\therefore (n_v - n_r) = 0.03\ (1.5\ 17 – 1) = 0.0155$

and, $\omega_f = 0.05 = \dfrac{n'_v - n'_r}{n'_y - 1}$

$\therefore n'_v - n'_r = 0.05 \times (1.6\ 21 - 1) = 0.031$

$\theta = (n_v - n_r) A - (n'_v - n'_r) A'$

$= 0.0155 A - 0.031 A'$ ......(1)

But $\delta_{net} = 1$

So, $(n_y - 1) A - (n'_y - 1) A' = 1$

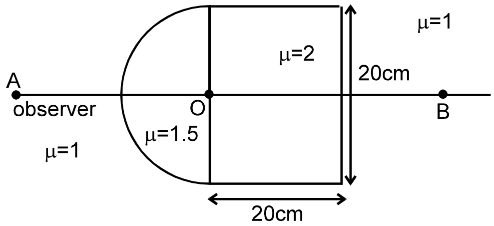
$= 0.517 A - 0.621 A' = 1$ ......(2)

$\therefore A = 4.8°$ and $A' = 2.4°$

Problem 26. See the situation shown in figure

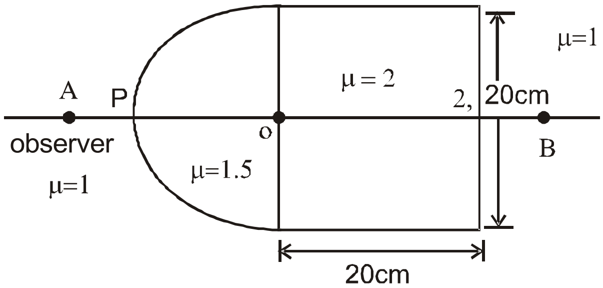
(1) Find the position of image as seen by observer A.

(2) Find the position of image as seen by observer B.



Solution : (i) As seen by observer A .

R = – 10 cm, u = – 10 cm.



So, $\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}$, $\frac{1}{v} - \frac{1.5}{(-10)} = \frac{1-1.5}{(-10)}$

v = –10 cm (at O)

(ii) As seen by observer B

R = ∞

u = – 20 cm

$\frac{1}{v} - \frac{2.0}{-20} = \frac{1-2.0}{\infty}$

v = – 10 cm image will be formed 10 cm right of O.

Problem 27. Find the focal length of a double-convex lens with R_1 = 15 cm and R_2 = –25 cm. The refractive index of the lens material n = 1.5.

Solution : $\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = (1.5-1)\left(\frac{1}{15} + \frac{1}{25}\right) = 0.5\left(\frac{10+6}{150}\right) = \frac{8}{150}$.

$f = \frac{150}{8} = 18.75$ cm

Problem 28. Find the focal length of a plano-convex lens with R_1 = 15 cm and R_2 = ∞. The refractive index of the lens material n = 1.5.

Solution : $\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = (1.5-1)\left(\frac{1}{15} - \frac{1}{\infty}\right) = 0.5 \times \frac{1}{15}$

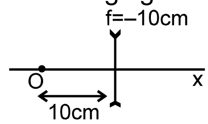
∴ f = 30 cm .

Problem 29. Find the focal length of a concavo-convex lens (positive meniscus) with R_1 = 15 cm and R_2 = 25 cm. The refractive index of the lens material n = 1.5.

Solution : $\frac{1}{f} = (1.5 - 1)\left(\frac{1}{15} - \frac{1}{25}\right) = 0.5\left(\frac{10-6}{150}\right)$.

$\therefore\ f = \frac{300}{4} = 75$ cm

Problem 30. Figure shows a point object and a diverging lens.



Find the final image formed.

Solution : $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$\frac{1}{v} = \frac{1}{-10} + \frac{1}{(-10)} = -\frac{2}{10}$ $\Rightarrow v = -5$ cm

Problem 31. An extended real object is placed perpendicular to the principal axis of a concave lens of focal length –10 cm, such that the image found is half the size of object.

(a) Find the object distance from the lens

(b) Find the image distance from the lens and draw the ray diagram

(c) Find the lateral magnification if object is moved by 1 mm along the principal axis towards the lens.

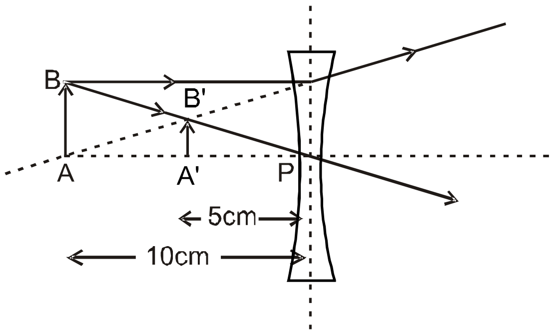
Solution : (a) We have, $f = -10$ cm.

$m = \frac{h_2}{h_1} = 0.5 = \frac{v}{u}$

So, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ gives.

$\Rightarrow \frac{1}{0.5u} - \frac{1}{u} = \frac{1}{-10}$ $\therefore\ u = -10$cm

(b)



$\frac{v}{u} = 0.5$.

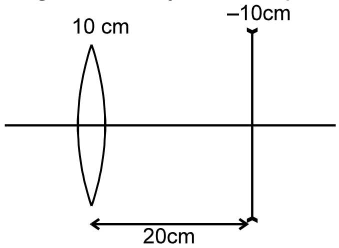
$\therefore\ v = 0.5 \times (-10)$ cm $= -5$ cm

Ray diagram :

(c) $m = \frac{f}{f+u} \Rightarrow dm = \frac{-f}{(f+u)^2}\,du = \frac{(+10)}{(-10-10)^2}\,(0.1) = 0.0025$ cm

so, final lateral magnification (m + dm) = 0.5025 cm **Ans.**

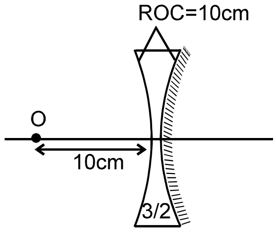
Problem 32. Find the equivalent focal length of the system for paraxial rays parallel to axis.



Solution : $\frac{1}{f_{eq}} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2} = \frac{1}{10} + \frac{1}{-10} - \frac{20}{10(-10)} = \frac{1}{5}$

$\Rightarrow$ f_{eq} = 5 cm

Problem 33. See the figure. Find the equivalent focal length of the combination shown in the figure and position of image.



Solution : For the concave lens $\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

$= \left(\frac{3}{2} - 1\right)\left(\frac{1}{-10} - \frac{1}{10}\right) = -\frac{1}{2} \times \frac{2}{10} = \frac{1}{10}$

And, $f_m = \frac{R}{2} = \frac{10}{2} = 5$ cm

$\therefore \frac{1}{f_{eq}} = \frac{1}{f_m} - 2\frac{1}{f} = \frac{1}{5} + 2 \times \frac{1}{10} = \frac{2}{5}$ $\boxed{f_{eq} = 2.5\text{cm}}$ **Ans.**

HANDOUT OPTICAL INSTRUMENTS

Optical Instruments

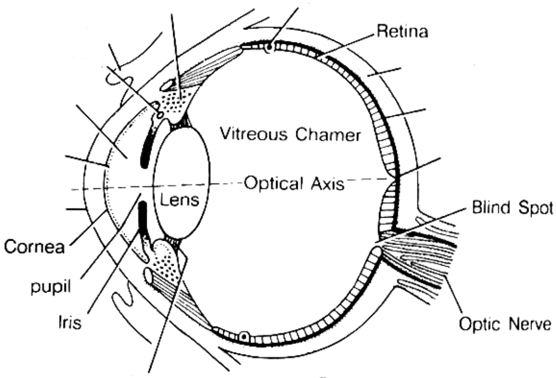
1 Human Eye

1.1 Structure of Eye

Light enters the eye through a curved front surface, the cornea. It passes through the pupil which is the central hole in the iris. The size of the pupil can change under control of muscles. The light is further focussed by the eye-lens on the retina. The retina is a film of nerve fibres covering the curved back surface of the eye. The retina contains rods and cones which sense light intensity and colour, respectively, and transmit electrical signals via the optic nerve to the brain which finally processes this information. The shape (curvature) and therefore the focal length of the lens can be modified somewhat by the ciliary muscles. For example, when the muscle is relaxed, the focal length is about 2.5 cm and (for a normal eye) objects at infinity are in sharp focus on the retinas.

When the object is brought closer to the eye, in order to maintain the same image-lens distance (2.5 cm), the focal length of the eye-lens becomes shorter by the action of the ciliary muscles. This property of the eye in called **accommodation**.

If the object is too close to the eye, the lens cannot curve enough to focus the image on to the retina, and the image is blurred.



Human Eye

The closest distance for which the lens can focus light on the retina is called the **least distance of distinct vision or the near point.** The standard value (for normal vision) taken here is 25 cm (the near point is given the symbol D.)

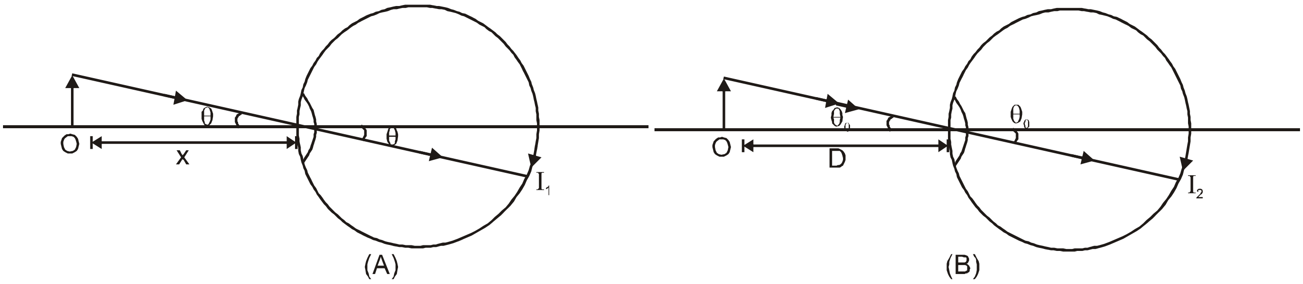
When the image is situated at infinity the ciliary muscles are least strained to focus the final image on the retina, this situation is known as **normal adjustment**.

1.2 Regarding Eye:

1. In eye convex eye-lens forms real inverted and diminished image at the retina by changing its convexity (the distance between eye lens and retina is fixed)

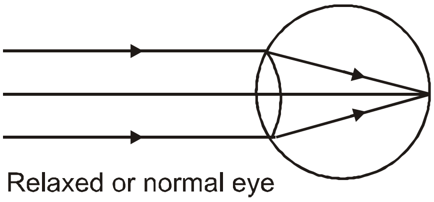
2. The human eye is most sensitive to yellow green light having wavelength 5550 Å and least to violet (4000Å) and red (7000 Å)

3. The size of an object as perceived by eye depends on its visual-angle when object is distant its visual angle θ and hence image I_1 at retina is small (it will appear small) and as it is brought near to the eye its visual angle θ_0 and hence size of image I_2 will increase.

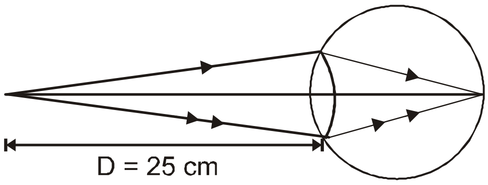


4. The far and near point for normal eye are usually taken to be infinity and 25 cm respectively i.e.., normal eye can see very distant object clearly but near objects only if they are at distance greater than 25 cm from the eye. The ability of eye to see objects from infinite distance to 25 cm from it is called **Power of accommodation.**

5. If object is at infinity i.e. parallel beam of light enters the eye is least strained and said to be relaxed or unstrained. However, if the object is at least distance of distinct vision (L.D.D.V) i.e., D (=25 cm) eye is under maximum strain and visual angle is maximum.



Relaxed or normal eye
(A)



Maximum strained eye
(B)

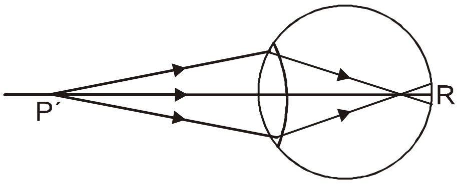
6. The limit of resolution of eye is one minute i.e. two object will not be visible distinctly to the eye if the angle subtended by them on the eye is lesser than one minute.

7. The persistence of vision is (1/10) sec i.e., If time interval between two consecutive light pulses is lesser than 0.1 sec eye cannot distinguish them separately. This fact is taken into account in motion pictures.

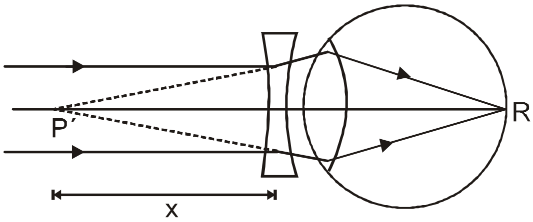
1.3 Defects of vision

1. Myopia [or short-sightendness or near - sightendness]

In it distant objects are not clearly visible. The far point for a myopic eye is much nearer than infinity.



Defective eye
(A)



Corrected eye
(B)

If P´ is far point for a myopic eye, then the image of an object placed at the point P´ will be formed on the retina as shown in the figure (A).

The myopic eye will get cured against this defect, if it is able to see the objects at infinity clearly. In order to correct the eye for this defect, a concave lens of suitable focal length is placed close to the eye, so that the parallel ray of light from point P´ of the myopic eye as shown in figure (B).

If x is the distance of the far point from the eye, then for the concave lens placed before the eye:

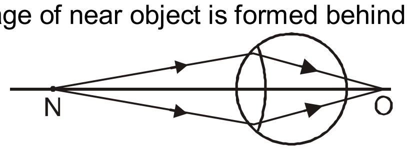
$u = \infty$ and $v = -x$

solving, $f = -x$

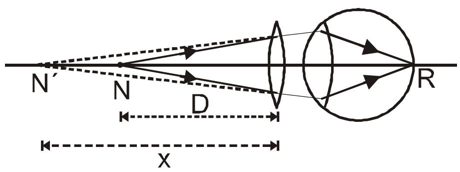
Thus, myopic eye is cured against the defect by using a concave lens of focal length equal to the distance of its far point from the eye.

2 Hypermetropia [Or Long-sightendness or far-sightendness]

In it near object are not clearly visible i.e., Near Point is at a distance greater than 25 cm and hence image of near object is formed behind the retina.



Defective Eye
(A)



Corrected Eye
(B)

In case of a hypermetropic eye, when the object lies at the point N (at the near point for a normal eye), its image is formed behind the retina as shown in figure (A).

The near point N´ for hypermetropic eye is farther than N, the near point for a normal eye.

Such defect will get cured, if the eye can see an object clearly, when place at the near point N for the normal eye. To correct this defect, a convex lens of suitable focal length is placed close to the eye so that the rays of light form an object placed at the point N after refraction through the lens appear to come from the near point N´ of the hypermetropic eye as shown in figure (B).

Let x be the distance of the near point N´ from the eye and D, the least distance of distinct vision i.e. the distance of near point N for the normal eye. Then, for the convex lens placed before the eye,

$u = -D$

and $v = -x$

If f is the focal length of the required convex lens, then from the lens formula, we have

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{-x} - \frac{1}{-D} = \frac{1}{f}$$

$$f = \frac{xD}{x - D} \qquad \text{...(1)}$$

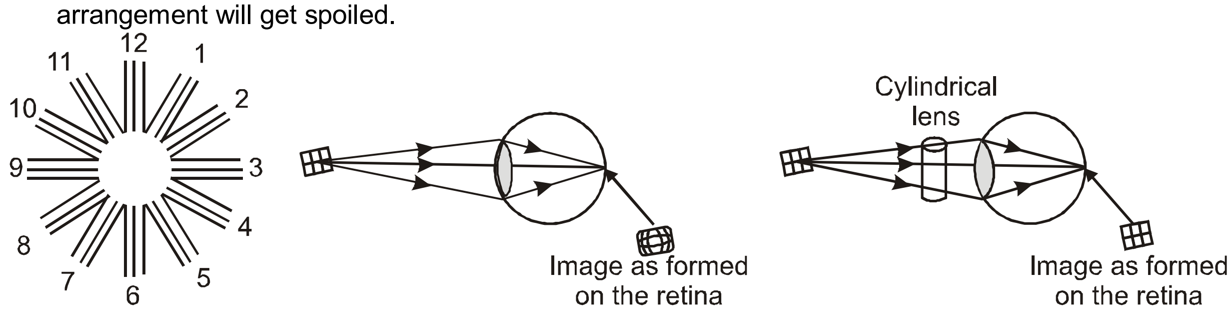
Thus, an eye suffering from hypermetropia can be cured against the defect by using a convex lens of focal length given by equation (1).

3 Presbyopia

In this both near and far objects are not clearly visible i.e., far point is lesser than infinity and near point greater than 25 cm. It is an old age disease as at old age ciliary muscles lose their elasticity and so can not change the focal length of eye-lens effectively and hence eye loses its power of accommodation.

4 Astigmatism

In it due to imperfect spherical nature of eye-lens, the focal length of eye lens is two orthogonal directions becomes different and so eye cannot see object in two orthogonal directions clearly simultaneously. This defect is directional and is remedied by using cylindrical lens in particular direction. If in the spectacle of a person suffering from astigmatism, the lens is slightly rotated the arrangement will get spoiled.



Example 1. A person cannot see objects clearly beyond 50 cm. What should be the power of corrective lens used ?

Solution :

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

for correcting for point $u = -\infty$, $v = -50$ cm

$$-\frac{1}{50} + \frac{1}{\infty} = \frac{1}{f}$$

$f = -50$ cm

$$P = \frac{1}{f} = \frac{1}{-0.5} = -2D$$

Example 2. A certain myopic person has a far point of 150 cm. (a) What power a corrective lens must have to allow him to see distant objects clearly ? (b) If he is able to read a book at 25 cm, while wearing the glasses, is his near point less than 25 cm ?

Solution : (a) Here, the distance of the far point, x = 150 cm

The defect can be corrected by using concave lens of focal length,

f = –x = –150 cm = –1.5 m

The power of the lens is given by

$P = \frac{1}{f} = \frac{1}{-1.5}$ = 0.67 D

(b) here, u = –25 cm ; f = –150 cm

From the lens equation, we have

$v = \frac{uf}{u+f} = \frac{(-25)\times(-150)}{(-25)+(-150)}$ = 21.43 cm

Therefore, the near point will be at a distance of 21.43 cm i.e. less than 25cm.

Optical instruments used primarily to assist the eye in viewing an object.

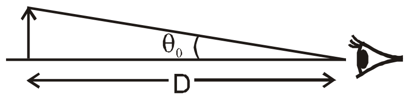
1. Microscope

It is an optical instrument used to increase the visual angle of neat objects which are too small to be seen by naked eye.

1.1 Simple Microscope

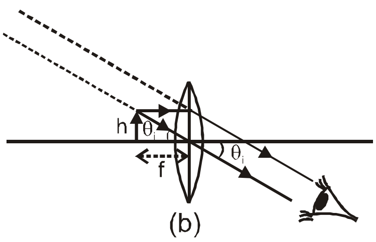
The normal human eye can focus a sharp image of an object on the retina if the object is located anywhere from infinity to a certain point called the **near point (D)**. If you move the object closer to the eye than the near point, the perceived retinal image becomes fuzzy. For an average viewer the near point, D = 25 cm from the eye. When the object is at the eye's near point, its image on the retina is as large as it can be and still be in focus.

The apparent size of an object is determined by the size of its image on the retina. If the eye is unaided, this size depends on the angle θ_o subtended by the object at the eye, called its **angular size** as shown in figure (a).



unaided eye

(a)



(b)

To look closely at a small object, such as an insect or a crystal, you bring it close to your eye, making the subtended angle and the retinal image as large as possible. But your eye cannot focus sharply on objects that are closer than the near point, so the angular size of an object is greatest (that is, it subtends the largest possible viewing angle) when it is placed at the near point.

A converging lens can be used to form a virtual image that is larger and farther from the eye than the object itself, as shown in figure (b). Then the object can be moved closer to the eye, and the angular size of the image may be substantially larger than the angular size of the object at 25 cm without the lens. A lens used in this way is called a **simple microscope,** otherwise known as a magnifying glass.

The usefulness of the magnifier is given by its angular magnification.

(i) **If the image is formed at infinity (normal adjustment):** The virtual image is most comfortable to view when it is placed at infinity, so that the ciliary muscle of the eye is relaxed; this means that the object is placed at the focal point of the magnifier. In this case we find angular magnification.

Angular magnification or magnifying power (M) is defined as the ratio of the angle subtended by the image (situated at infinity) at the eye to the angle subtended by the object seen directly at the eye when situated at near point D.

In figure (a) the object is at the near point, where it subtends an angle θ_o at the eye.

$$\theta_o \approx \tan\theta_o \approx \frac{h}{D} \qquad ...(1)$$

In figure (b) a magnifier in front of the eye forms an image at infinity, and the angle subtended at the magnifier is θ_i

$$\theta_i \approx \tan\theta_i \approx \frac{h}{f} \qquad ...(2)$$

Angular magnification, $\quad M = \frac{\theta_i}{\theta_o} = \frac{D}{f} \qquad ...(3)$

(ii) **If the image is at formed at near point, D:** The linear magnification 'm', for the image formed at the near point D, by a simple microscope can be obtained by using the relation

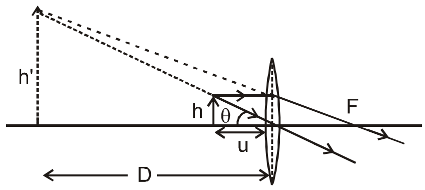
$$m = \frac{h'}{h} = \frac{v}{u} = \left(1 - \frac{v}{f}\right) \qquad ...(4)$$

h is the size of the object and h' is the size of the image

$$m = \left(1 + \frac{D}{f}\right) \qquad ...(5) \qquad (v = -D)$$

Angular magnification or magnifying power(M) is defined as the ratio of the angle subtended by the image at the eye to the angle subtended by the object seen directly at the eye when both lie at near point D.

In figure (c) a magnifier in front of the eye forms an image at D, and the angle subtended at the magnifier is θ_i



(c)

Example 3. A man with normal near point (25 cm) reads a book with small print using a magnifying thin convex lens of focal length 5 cm. (a) What is the closest and farthest distance at which he can read the book when viewing through the magnifying glass? (b) What is the maximum and minimum magnifying power possible using the above simple microscope?

Solution : (a) As for normal eye far and near point are and 25 cm respectively, so for magnifier $v_{max} = -\infty$ and $v_{min} = -25\,cm$. However, for a lens as

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \qquad \text{i.e.,} \qquad u = \frac{f}{\left(\frac{f}{v}\right) - 1}$$

So u will be minimum when $v_{min} = -25\,cm$

i.e., $(u)_{min} = \dfrac{5}{\left(\frac{-5}{25}\right) - 1} = -\dfrac{25}{6} = -4.17\,cm$

And u will be maximum when $v_{max} = \infty$

So, the closest and farthest distance of the book from the magnifier (or eye) for clear viewing are 4.17 cm and 5 cm respectively.

(b) An in case of simple magnifier $MP = \left(\frac{D}{u}\right)$. So MP will be minimum when u_{max} = 5 cm

i.e. $(MP)_{min} = \dfrac{-25}{-5} = 5 \qquad \left[= \dfrac{D}{f}\right]$

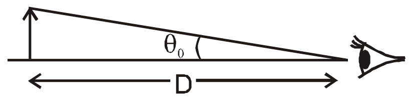
And MP will be maximum when $u_{min} = \left(\dfrac{25}{6}\right)$cm $\qquad$ i.e., $\qquad (MP)_{max} = \dfrac{-25}{-\left(\frac{25}{6}\right)} = 6\left[= 1 + \dfrac{D}{f}\right]$

1.2 Compound Microscope

When we need greater magnification than what we can get with a simple magnifier, the instrument that we usually use is a compound microscope.
The essential parts of a compound microscope are two convex lenses of different focal length placed coaxially. These lenses are referred to as:
(a) Objective lens or objective: It is a lens of small aperture and small focal length placed facing the object.
(b) Eye piece: It is a lens of large aperture and small focal length placed facing the object.

The object O to be viewed is placed just beyond the first focal point of the **objective** lens that forms a real and enlarged image I′ as shown in figure. In a properly designed instrument this image lies just inside the first focal point of the **eyepiece**. The eyepiece acts as a simple magnifier, and forms a final virtual image of I. The position of may be anywhere between the near and far points of the eye.

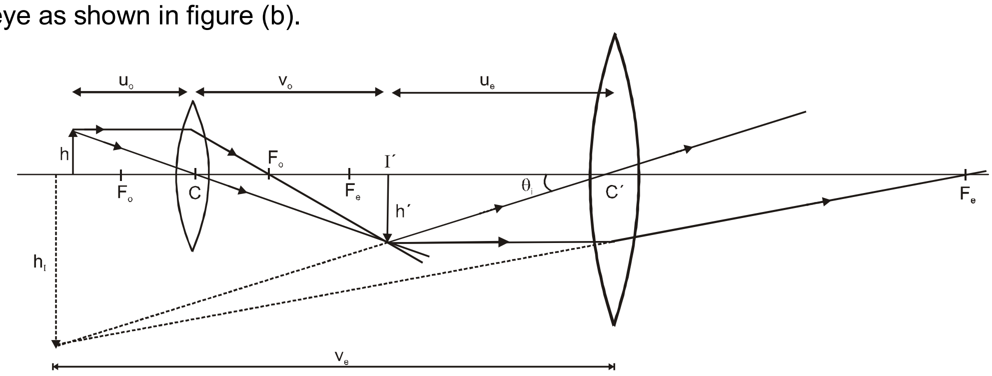
In figure (a) the object is at the near point, where it subtends an angle θ_o at the eye.



unaided eye
(a)

$$\theta_o \approx \tan\theta_o \approx \frac{h}{D} \qquad ...(1)$$

(i) When image is formed at near point, D: Let θ_i be the angle subtended by the final image at the eye as shown in figure (b).



(b)

Angular magnification or magnifying power (M) is defined as the ratio of the angle subtended by the final image at the eye to the angle subtended by the object seen directly at the eye when both lie at near point D.
The angular magnification produced is,

$$M = \frac{\theta_i}{\theta_o} \approx \frac{\tan\theta_i}{\tan\theta_o} \qquad ...(2)$$

$$\theta_i \approx \tan\theta_i = \frac{h_i}{v_e} = \frac{h_i}{D} \qquad (\because v_e = D \text{ in magnitude})$$

$$\theta_o \approx \tan\theta_o = \frac{h}{D}$$

$$M = \frac{h_i}{h} \qquad ...(3)$$

Linear magnification, $m = \frac{h_i}{h} = m_o \times m_e$ ...(4)

$M = m_o m_e$ (from eq. (3) and eq. (4))

where m_o = linear magnification produced by objective lens = $\frac{v_o}{u_o}$ ...(5)

m_e = linear magnification produced by eye piece = $\frac{v_e}{u_e}$

using lens formula for eye piece, $\frac{1}{v_e} - \frac{1}{u_e} = \frac{1}{f_e}$

$$m_e = \frac{v_e}{u_e} = 1 - \frac{v_e}{f_e} = 1 + \frac{D}{f_e} \qquad \text{...(6)} \qquad (\because v_e = -D)$$

From equations (3), (4) and (6) we have

$$M = \frac{v_o}{u_o}\left(1 + \frac{D}{f_e}\right), \qquad \text{...(7)}$$

In practice, the focal length of objective lens is very small and the object is just placed outside the focus of the objective lens,

$u_o = -f_o$

Since the focal length of the eye lens is also small, the distance of image I' from objective lens is nearly equal to the length of the microscope tube, L

$v_o = L$

substituting in equation (7),

$$M = -\frac{L}{f_o}\left(1 + \frac{D}{f_e}\right)$$

This equation shows that a compound microscope will have high magnifying power, if the objective lens and the eye piece both have small focal length. The negative sign shows that final image will be inverted w.r.t. object.

(ii) **When image is formed at infinity**: The magnifying power of compound microscope is given by

$M = m_o \times m_e$

Magnification produced by objective lens, $m_o = \frac{v_o}{u_o}$

The eye lens produces the final image at infinity. Then,

$m_e = \frac{D}{f_e}$ (as discussed in case of simple microscope)

Therefore, $M = \frac{v_o}{u_o}\frac{D}{f_e}$,

$$M = -\frac{L}{f_o}\frac{D}{f_e}$$

Example 4. The focal length of the objective and eyepiece of a microscope are 2 cm and 5 cm respectively and the distance between them is 20 cm. Find the distance of object from the objective, when the final image seen by the eye is 25 cm from the eyepiece. Also find the magnifying power.

Solution : Given f_0 = 2 cm, f_e = 5 cm

$|v_o| + |u_e|$ = 20 cm

$\therefore$ v_e = – 25 cm

From lens formula $\frac{1}{f_e} = \frac{1}{v_o} - \frac{1}{u_e}$

$$\frac{1}{u} = \frac{1}{v_e} - \frac{1}{f_e} = -\frac{1}{25} - \frac{1}{5}$$

$\therefore$ $u_e = -\frac{25}{6}$ cm

Distance of real image from objective

$$v_o = 20 - |u_e| = 20 - \frac{25}{6} = \frac{120-25}{6} = \frac{95}{6} \text{ cm}$$

Now $\quad \frac{1}{f_o} = \frac{1}{v_o} - \frac{1}{u_o}$

given $\quad \frac{1}{u_o} = \frac{1}{v_o} - \frac{1}{f_o} = \frac{1}{(95/6)} - \frac{1}{2} \quad$ i.e., $\frac{1}{u_o} = \frac{6}{95} - \frac{1}{2} = \frac{12-95}{190} = -\frac{83}{190}$

$\therefore \quad u_o = -\frac{190}{83} = -2.3$ cm

Magnifying power $M = -\frac{v_o}{u_o}\left(1+\frac{D}{f_e}\right) = -\frac{95/6}{(190/83)}\left(1+\frac{25}{3}\right) = -41.5$

2. Telescope

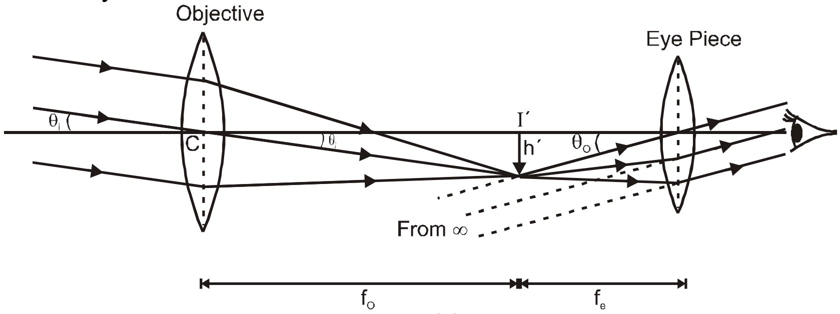
2.1 Astronomical Telescope

It is an optical instrument used to increase the visual angle of distant large objects such as a star a planet or a cliff etc. Astronomical telescope consists of two converging lens. The one facing the object is called objective and has large focal length and aperture. Other lens is called eye piece. It has small aperture and is of small focal length. The distance between the two lenses is adjustable.

The objective forms a real and inverted image at its focal plane of the distant object. The distance of the eye piece is adjusted, till the final image is formed at the near point, D. In case, the position of the eye piece is so adjusted that final image is formed at infinity, the telescope is said to be in normal adjustment.

(i) When the image is formed at infinity (Normal adjustment)

When a parallel beam of light rays from a distant object falls on objective, its real and inverted image I' is formed on the other side of the objective and at a distance f_o. If the position of the eye piece is adjusted, so that the image I' lies at its focus, then the final highly magnified image will be formed at infinity.



(a)

Angular magnification or magnifying power (M) here is defined as the ratio of the angle subtended by the final image at the eye as seen through the telescope to the angle subtended by the object, seen directly at the eye when both the object and the image lie at infinity.

$$M = \frac{\theta_i}{\theta_o} = \frac{\tan\theta_i}{\tan\theta_o} \qquad \text{(for small angle } \tan\theta \approx \theta)$$

From figure (a), $\quad \tan\theta_i = \frac{h'}{CI'}$, $\tan\theta_o = \frac{h'}{C'I'}$

$$M = \frac{CI'}{C'I'} = -\frac{f_o}{f_e} \qquad (CI' = f_o,\ C'I' = -f_e)$$

If f_o is large and f_e is small, the magnification will be high. In normal adjustment the length of tube

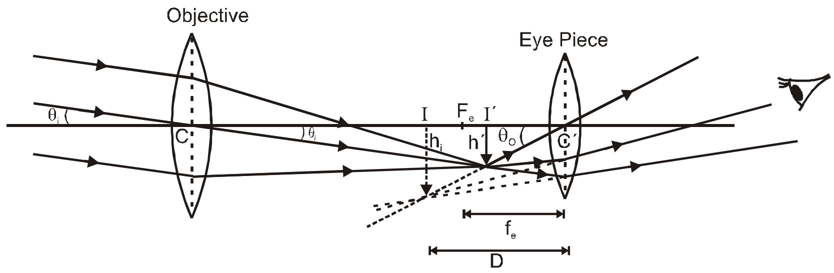
$$L = (f_0 + u_e)$$

(ii) If the final image is formed at D (near point)

When a parallel beam of light rays from a distant object falls on objective, its real and inverted image I' is formed on the other side of the objective and at a distance f_o. If the position of the eye piece is adjusted, so that the final image I is formed at near point D.

Angular magnification or magnifying power (M) here is defined as the ratio of the angle subtended by the final image formed at near point at the eye to the angle subtended by the object lying at infinity seen directly at the eye.

$M = \dfrac{\theta_i}{\theta_o} = \dfrac{\tan\theta_i}{\tan\theta_o}$ (for small angle $\tan\theta \approx \theta$)



(b)

From figure (b), $\tan\theta_o = \dfrac{h'}{CI'}$, $\tan\theta_i = \dfrac{h'}{C'I'}$

$M = \dfrac{CI'}{C'I'} = -\dfrac{f_o}{u_e}$

For eye lens, $\dfrac{1}{v_e} - \dfrac{1}{u_e} = \dfrac{1}{f_e}$,

$\dfrac{1}{u_e} = \dfrac{1}{f_e} + \dfrac{1}{D}$ ($\because$ $u_e = -u_e$, $v_e = -D$)

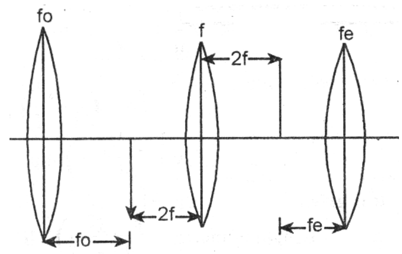
$M = -\dfrac{f_o}{f_e}\left(1 + \dfrac{f_e}{D}\right)$

If f_o is large and f_e is small, the magnification will be high.

2.2 Terrestrial Telescope

Uses a third lens in between objective and eyepieces so as to form final image erect. This lens simply invert the image formed by objective without affecting the magnification.

Length of tube $L = f_0 + f_e + 4f$



Example 5. A telescope consists of two convex lens of focal length 16 cm and 2 cm. What is angular magnification of telescope for relaxed eye? What is the separation between the lenses? If object subtends an angle of 0.5º on the eye, what will be angle subtended by its image ?

Solution : Angular magnification

$M = \dfrac{\alpha}{\beta} = \dfrac{F}{f} = \dfrac{16}{2} = 8$ cm

Separation between lenses = F + f = 16 + 2 = 18 cm

Here $\alpha = 0.5º$

$\therefore$ Angular subtended by image

$\beta = M\,\alpha = 8 \times 0.5º = 4º$

Example 6. The magnifying power of the telescope if found to be 9 and the separation between the lenses is 20 cm for relaxed eye. What are the focal lengths of component lenses ?

Solution : Magnification $M = \frac{F}{f}$

Separation between lenses

$d = F + f$

Given $\frac{F}{f} = 9$ i.e., $F = 9f$ ......(1)

and $F + f = 20$ ......(2)

Putting value of F from (1) in (2), we get

$9f + f = 20 \Rightarrow 10f = 20 \Rightarrow f = \frac{20}{10} = 2\text{cm}$

$\therefore$ $F = 9f = 9 \times 2 = 18$ cm

$\therefore$ $F = 18$ cm, $f = 2$ cm

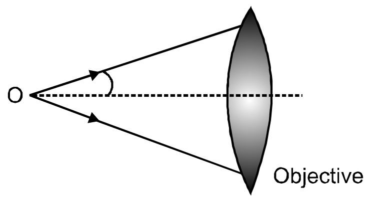
Comparison between Compound - Microscope & Astronomical - Telescope

S.No.	Compound - Microscope	Astronomical - Telescope
1.	It is used to increase visual angle of near tiny object.	It is used to increase visual angle of distant large objects.
2.	In it field and eye lens both are convergent, of short focal length and aperture.	In it field lens is of large focal length and aperture while eye lens of short focal length and aperture and both are convergent.
3.	Final image is inverted. virtual and enlarged and at a distance D to ∞ from the eye.	Final image in inverted, virtual and enlarged at a distance D to ∞ from the eye
4.	MP does not change appreciably if field and eye lens are interchanged [MP ~ $(LD/f_0 f_e)$]	MP becomes $(1/m^2)$ times of its initial value if field and eye-lenses are interchanged as MP ~$[f_0/f_e]$
5.	MP is increased by decreasing the focal of length of both the lenses viz. find and length eye lens.	MP is increased by increasing the focal length field of field lens (and decreasing the focal of eye lens.)
6.	RP is increased by decreasing the wavelength of light used.	RP is increased by increasing the aperture of objective.

RESOLVING POWER (R.P.)

(1) Microscope : In reference to a microscope, the minimum distance between two lines at which they are just distinct is called Resolving limit (RL) and it's reciprocal is called Resolving power (RPO)

$$\text{R.L.} = \frac{\lambda}{2\mu\sin\theta} \text{ and R.P.} = \frac{2\mu\sin\theta}{\lambda} \Rightarrow \text{R.P.} \propto \frac{1}{\lambda}$$



λ = Wavelength of light used to illuminate the object

μ = Refractive index of the medium between object and objective.

θ = Half angle of the cone of light from the point object, $\mu \sin\theta$ = Numerical aperture.

(2) Telescope : Smallest angular separations ($d\theta$) between two distant object, whose images are separated in the telescope is called resolving limit. So resolving limit $d\theta = \frac{1.22\lambda}{a}$ and resolving power

$$(\text{RP}) = \frac{1}{d\theta} = \frac{a}{1.22\lambda} \Rightarrow \text{R.P.} \propto \frac{1}{\lambda}$$ where a = aperture of objective.

Example 7 Find the half angular width of the central bright maximum in the Fraunhofer diffraction pattern of a slit of width 12 × 10^{-5} cm when the slit is illuminated by monochromatic light of wavelength 6000Å.

Solution. Here $\sin\theta = \frac{\lambda}{a}$

Where θ is half angular width of the central maximum.

A = 12 × 10^{-5} cm, λ = 6000Å = 6 × 10^{-5}cm.

$\therefore \quad \sin\theta = \frac{\lambda}{a} = \frac{6\times10^{-5}}{12\times10^{-5}} = 0.50$

or $\quad \theta = 30°$

Example 8 In Fraunhofer diffraction due to a narrow slit a screen is placed 2 m away from the lens to obtain the pattern. If the slit width is 0.2 mm and the first minima lie 5 mm on either side of the central maximum, find the wavelength of light.

Solution. In the case of Fraunhofer diffraction at a narrow rectangular aperature,

a sin θ = nλ

n = 1

$\therefore \quad$ a sin θ = λ

$\sin\theta = \frac{x}{D}$

$\therefore \quad \frac{ax}{D} = \lambda$

$\lambda = \frac{ax}{D}$

Here a = 0.2 mm = 00.2cm

x = 5 mm = 0.5 cm

D = 2m = 200 cm

$\therefore \quad \lambda = \frac{0.02\times0.5}{200}$

λ = 5 × 10^{-5} cm

L = 5000Å

Example 9. Light of wavelength 6000Å is incident on a slit of width 0.30 nm. The screen is placed 2m from the slit. Find : (a) the position of the first dark fringe and (b) the width of the central bright fringe.

Solution. The first dark fringe is on either side of the central bright fringe.

Here, n = ±1, D = 2m

λ = 6000Å = 6 × 10^{-7}m

$\sin\theta = \frac{x}{D}$

a = 0.30 mm = 3 × 10^{-4} m

a sin θ = nλ

(a) $\quad x = \frac{n\lambda D}{a}$

$x = \pm\left[\frac{1\times6\times10^{-7}\times2}{3\times10^{-4}}\right]$

x = ±4 × 10^{-3}m

The positive and negative signs correspond to the dark fringes on either side of the central bright fringe.

(b) The width of the central bright fringe,

y = 2x

= 2 × 4 × 10^{-3}

= 8 × 10^{-3} m = 8 mm

Example 10. A signal slit of width 0.14 mm is illuminated normally by monochromatic light and diffraction bands are observed on a screen 2m away. If the centre of the second dark band is 1.6 cm from the middle of the central bright band, deduce the wavelength of light used.

Solution. In the case of Fraunhofer diffraction at a narrow rectangular slit,

$a \sin \theta = n\lambda$

Here θ gives the directions of the minimum

$n = 2$, $\lambda = ?$

$a = 0.14 \text{ mm} = 0.14 \times 10^{-3} \text{ m}$

$D = 2 \text{ m}$

$x = 1.6 \text{ cm} = 1.6 \times 10^{-2} \text{ m}$

$$\sin \theta = \frac{x}{D} = \frac{n\lambda}{a} \quad \therefore \quad \lambda = \frac{xa}{nD}$$

$$= \frac{1.6 \times 10^{-2} \times 0.14 \times 10^{-3}}{2.2} = 5.6 \times 10^{-7} \text{ m} = 5600\text{Å}$$

Example 11. A screen is placed 2m away from a narrow slit which is illuminated with light of wavelength 6000Å. If the first minimum lies 50 mm on either side of the central maximum, calculate the slit width.

Solution. In the case of Fraunhofer diffraction at a narrow slit,

$a \sin \theta = n\lambda$

$$\sin \theta = \frac{x}{D} \quad \therefore \quad \frac{ax}{D} = n\lambda$$

Here width of the slit = a = ?

$x = 5 \text{ mm} = 5 \times 10^{-3} \text{ m}$

$D = 2\text{m}$

$\lambda = 6000\text{Å} = 6 \times 10^{-7}\text{m}$

$n = 1$

$$a = \left(\frac{n\lambda D}{x}\right)$$

$$a = \left(\frac{1 \times 6 \times 10^{-7} \times 2}{5 \times 10^{-3}}\right)$$

$a = 2.4 \times 10^{-4} \text{ m}$

$a = 0.24 \text{ mm}$

Example 12. Find the angular width of the central bright maximum in the Fraunhofer diffraction pattern of a slit of width 12×10^{-5} cm when the slit is illuminated by monochromatic light of wavelength 6000Å.

Solution. Here $\sin\theta = \dfrac{\lambda}{a}$

where θ is the half angular width of the central maximum

$a = 12 \times 10^{-5} \text{ cm} = 12 \times 10^{-7} \text{ m}$

$\lambda = 6000\text{Å} = 6 \times 10^{-7} \text{ m}$

$$\sin\theta = \frac{6 \times 10^{-7}}{12 \times 10^{-7}} = 0.5$$

$\theta = 30°$

Angular width of the central maximum.

$2\theta = 60°$

Example 13. Diffraction pattern of a signal slit of width 0.5 cm is formed by a lens of focal length 40 cm. Calculate the distance between the first dark and the next bright fringe from the axis. Wave length = 4890Å.

Solution. For minimum intensity

$a \sin \theta_n = n\lambda$

$\sin \theta_n = \frac{x_1}{f'} \Rightarrow n = 1$

$\frac{x_1}{f} = \frac{\lambda}{a}$

Here λ = 4890Å = 4890×10^{-10} m

a = 0.5 cm = 5×10^{-3} m

f = 40 cm = 0.4 m

$x_1 = \frac{f\lambda}{a}$

$x_1 =$

$x_1 = 3.912 \times 10^{-5}$ m

For secondary maximum

$a \sin \theta_n = \frac{0.4 \times 4890 \times 10^{-10}}{5 \times 10^{-3}}$

For the first secondary maximum

n = 1

$\sin \theta_n = \frac{(2n+1)\lambda}{2}$

$\frac{x_2}{f} = \frac{3\lambda}{2a}$

$x_2 = \frac{3\lambda f}{2a}$

$x_2 = \frac{3 \times 4890 \times 10^{-10} \times 0.4}{2 \times 5 \times 10^{-5}}$

$x_2 = 5.868 \times 10^{-5}$ m

Difference,

$x_2 - x_1 = 5.868 \times 10^{-5} - 3.912 \times 10^{-5}$

$= 1.956 \times 10^{-5}$ m

$= 1.596 \times 10^{-2}$ mm

Example 14. Find the separation of two points on the moon that can be resolved by a 500 cm telescope. The distance of the moon is 3.8×10^5 km. The eye is most sensitive to light of wavelength 5500 Å.

Solution. The limit of resolution of a telescope is given by

$d\theta = \frac{1.22\lambda}{a}$

Here $\lambda = 5500 \times 10^{-8}$cm, a = 500 cm

$\therefore \quad d\theta = \frac{1.22 \times 5500 \times 10^{-8}}{500}$

$\therefore \quad d\theta = 13.42 \times 10^{-8}$ radian

Let the distance between the two point be x

$\therefore \quad d\theta = \frac{x}{R}$

Here R = 3.8×10^{10} cm

x = R.dθ

$= 3.8 \times 10^{10} \times 13.42 \times 10^{-8}$

$= 50.996 \times 10^2$ cm = 50.996 meters

Example 15. Calculate the aperture of the objective of a telescope which may by used to resolve stars separated by 4.88×10^{-6} radian for light of wavelength 6000Å

Solution. Here λ = 6000Å = 6×10^{-5} cm, $\theta = 4.88 \times 10^{-6}$ radian

D = ?

$$\theta = \frac{1.22\lambda}{\theta}$$

or $$D = \frac{1.22\lambda}{\theta} = \frac{1.22 \times 6 \times 10^{-5}}{4.88 \times 10^{-8}} = 15\text{cm}$$

Example 16. Two pin holes 1.5 mm apart are placed in front of a source of light of wavelength 5.5×10^{-5} cm and seen through a telescope with its objectives stopped down to a diameter of 0.4 cm. Find the maximum distance from the telescope at which the pin holes can be resolved.

Solution. Here, $\lambda = 5.5 \times 10^{-5}$ cm

a = 0.4 cm

$$d\theta = \frac{1.22\lambda}{a}$$

Also $$d\theta = \frac{x}{d}$$

x = 1.5 mm = 0.15 cm

$$\therefore \quad \frac{x}{d} = \frac{1.22\lambda}{a}$$

$$d = \frac{xa}{1.22\lambda}$$

$$d = \frac{0.15 \times 0.4}{1.22 \times 5.5 \times 10^{-5}}\text{cm}$$

d = 894.2 cm = 8.942 m

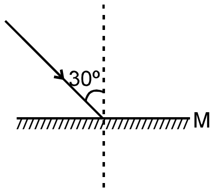
Exercise-1

✍ **Marked Questions can be used as Revision Questions.**

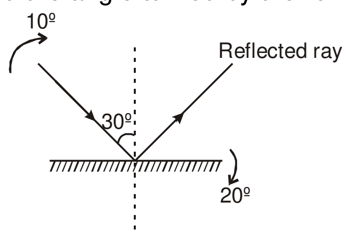
PART - I : SUBJECTIVE QUESTIONS

SECTION (A) : PLANE MIRROR

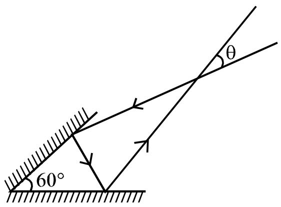
A-1. Find the angle of deviation (both clockwise and anticlockwise) suffered by a ray incident on a plane mirror, (as shown in figure) at an angle of incidence 30º.



A-2. Figure shows a plane mirror on which a light ray is incident. If the incident light ray is turned by 10º and the mirror by 20º, as shown, find the angle turned by the reflected ray.

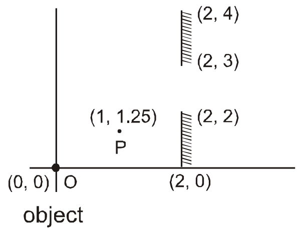


A-3. A light ray is incident on a plane mirror, which after getting reflected strikes another plane mirror, as shown in figure. The angle between the two mirrors is 60º. Find the angle 'θ' shown in figure.

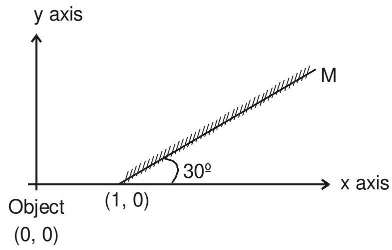


A-4. Sun rays are incident at an angle of 24° with the horizon. How can they be directed parallel to the horizon using a plane mirror?

A-5. Two plane mirrors are placed as shown in the figure and a point object 'O' is placed at the origin
(a) How many images will be formed.
(b) Find the position(s) of image(s).
(c) Will the incident ray passing through a point 'P' (1, 1.25) take part in image formation.



A-6. A point object is placed at (0, 0) and a plane mirror 'M' is placed, inclined 30º with the x axis.
(a) Find the position of image.
(b) If the object starts moving with velocity 1 $\hat{i}$ m/s and the mirror is fixed find the velocity of image.



SECTION (B) : SPHERICAL MIRROR

B-1. A rod of length 5 cm lies along the principal axis of a concave mirror of focal length 10 cm in such a way that the end farther from the pole is 15 cm away from it. Find the length of the image.

B-2. A point source is at a distance 35 cm on the optical axis from a spherical concave mirror having a focal length 25 cm. At what distance measured along the optical axis from the concave mirror should a plane mirror (perpendicular to principal axis) be placed for the image it forms (due to rays falling on it after reflection from the concave mirror) to coincide with the point source?

B-3. Find the diameter of the image of the moon formed by a spherical concave mirror of focal length 11.4 m. The diameter of the moon is 3450 km and the distance between the earth and the moon is 3.8×10^5 km.

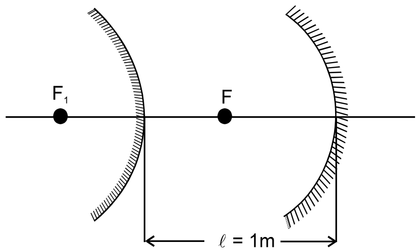
B-4. The radius of curvature of a convex spherical mirror is 1.2 m. How far away from the mirror is an object of height 1.2 cm if the distance between its virtual image and the mirror is 0.35 m? What is the height of the image? [Apply formula for paraxial rays]

B-5. A converging beam of light rays is incident on a concave spherical mirror whose radius of curvature is 0.8 m. Determine the position of the point on the optical axis of the mirror where the reflected rays intersect, if the extensions of the incident rays intersect the optical axis 40 cm from the mirror's pole.

B-6. A point object is placed on the principal axis at 60 cm in front of a concave mirror of focal length 40 cm on the principal axis. If the object is moved with a velocity of 10 cm/s (a) along the principal axis, find the velocity of image (b) perpendicular to the principal axis, find the velocity of image at that moment.

B-7. A man uses a concave mirror for shaving. He keeps his face at a distance of 20 cm from the mirror and gets an image which is 1.5 times enlarged. Find the focal length of the mirror.

B-8. Two spherical mirrors (convex and concave) having the same focal length of 36 cm are arranged as shown in figure so that their optical axes coincide. The separation between the mirrors is 1 m. At what distance from the concave mirror should an object be placed so that its images formed by the concave and convex mirrors independently are identical in size?

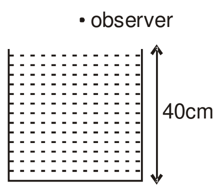


SECTION (C) : REFRACTION IN GENERAL, REFRACTION AT PLANE SURFACE AND T.I.R.

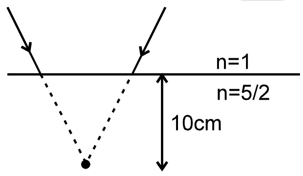
C-1. A light ray falling at an angle of 60° with the surface of a clean slab of ice of thickness 1.00 m is refracted into it at an angle of 15°. Calculate the time taken by the light rays to cross the slab. Speed of light in vacuum = 3×10^8 m/s.

C-2. A light ray is incident at 45° on a glass slab. The slab is 3 cm thick, and the refractive index of the glass is 1.5. What will the lateral displacement of the ray be as a result of its passage through the slab? At what angle will the ray emerge from the slab?

C-3. In the given figure an observer in air (n = 1) sees the bottom of a beaker filled with water (n = 4/3) upto a height of 40 cm. What will be the depth felt by this observer.

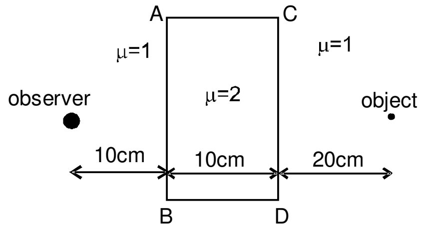


C-4. In the given figure rays incident on an interface would converge 10 cm below the interface if they continued to move in straight lines without bending. But due to refraction, the rays will bend and meet some where else. Find the distance of meeting point of refracted rays below the interface, assuming the rays to be making small angles with the normal to the interface.

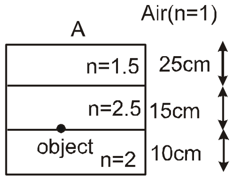


C-5. A fish is rising up vertically inside a pond with velocity 4 cm/s, and notices a bird, which is diving vertically downward along the same vertical line as that of fish and its velocity appears to be 16 cm/s (to the fish). What is the real velocity of the diving bird, if refractive index of water is 4/3?

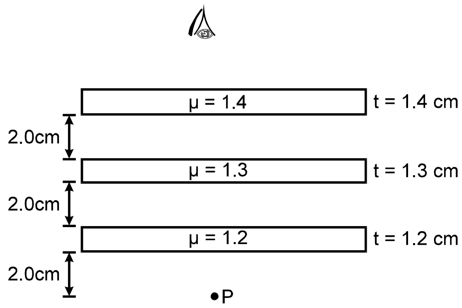
C-6. Find the apparent distance between the observer and the object shown in the figure and shift in the position of object.



C-7. Find the apparent depth of the object seen by observer A (in the figure shown)



C-8. Locate the image of the point P as seen by the eye in the figure.

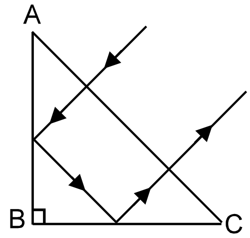


C-9. A small object is placed at the centre of the bottom of a cylindrical vessel of radius 3 cm and height $3\sqrt{3}$ cm filled completely with a liquid. Consider the ray leaving the vessel through a corner. Suppose this ray and the ray along the axis of the vessel are used to trace the image. Find the apparent depth of the image. Refractive index of liquid = $\sqrt{3}$.

C-10. A point source is placed at a depth h below the surface of water (refractive index = μ).The medium above the surface of water is air (μ =1).Find the area on the surface of water through which light comes in air from water.

C-11. Light is incident from glass (μ = 3/2) side on interface of glass and air. Find the angle of incidence for which the angle of deviation is 90º.

C-12. At what values of the refractive index of a rectangular prism can a ray travel as shown in figure. The section of the prism is an isosceles triangle and the ray is normally incident onto the face AC.

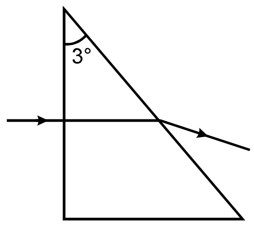


SECTION D : REFRACTION BY PRISM

D-1. A prism (n = 2) of apex angle 90° is placed in air (n = 1). What should be the angle of incidence so that light ray strikes the second surface at an angle of incidence 60º.

D-2. The cross section of a glass prism has the form of an equilateral triangle. A ray is incident onto one of the faces perpendicular to it. Find the angle θ between the incident ray and the ray that leaves the prism. The refractive index of glass is $\mu = 1.5$.

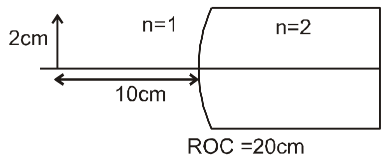
D-3. Find the angle of deviation suffered by the light ray shown in figure for following two conditions The refractive index for the prism material is $\mu = 3/2$.
(i) When the prism is placed in air ($\mu = 1$)
(ii) When the prism is placed in water ($\mu = 4/3$)



D-4. The refractive index of a prism is μ. Find the maximum angle of the prism for which a ray incident on it will be transmitted through other face without total internal reflection.

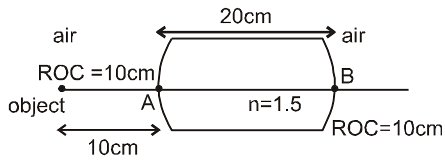
SECTION (E) : REFRACTION BY SPHERICAL SURFACE

E-1. An extended object of size 2 cm is placed at a distance of 10 cm in air (n = 1) from pole, on the principal axis of a spherical curved surface. The medium on the other side of refracting surface has refractive index n = 2. Find the position, nature and size of image formed after single refraction through the curved surface.

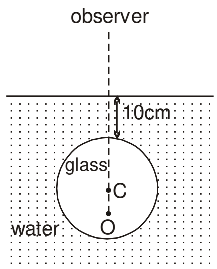


E-2. A point object lies inside a transparent solid sphere of radius 20 cm and of refractive index n = 2. When the object is viewed from air through the nearest surface it is seen at a distance 5 cm from the surface. Find the apparent distance of object when it is seen through the farthest curved surface.

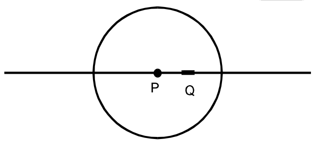
E-3. An object is placed 10 cm away from a glass piece (n = 1.5) of length 20 cm bounded by spherical surfaces of radii of curvature 10 cm. Find the position of final image formed after two refractions at the spherical surfaces.



E-4. There is a small air bubble inside a glass sphere ($\mu = 1.5$) of radius 5 cm. The bubble is at 'O' at 7.5 cm below the surface of the glass. The sphere is placed inside water ($\mu = 4/3$) such that the top surface of glass is 10 cm below the surface of water. The bubble is viewed normally from air. Find the apparent depth of the bubble.

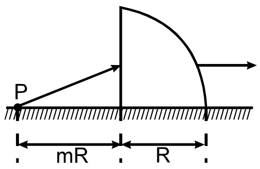


E-5. A small object Q of length 1 mm lies along the principal axis of a spherical glass of radius R = 10 cm and refractive index is 3/2. The object is seen from air along the principal axis from left. The distance of object from the centre P is 5 cm. Find the size of the image. Is it real, inverted?



E-6. A narrow parallel beam of light is incident paraxially on a solid transparent sphere of radius r kept in air. What should be the refractive index if the beam is to be focused
(a) at the farther surface of the sphere, (b) at the centre of the sphere.

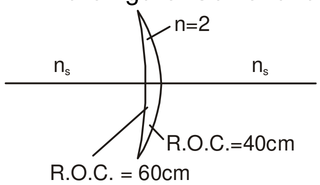
E-7. A quarter cylinder of radius R and refractive index 1.5 is placed on a table. A point object P is kept at a distance of mR from it. Find the value of m for which a ray from P will emerge parallel to the table as shown in the figure.



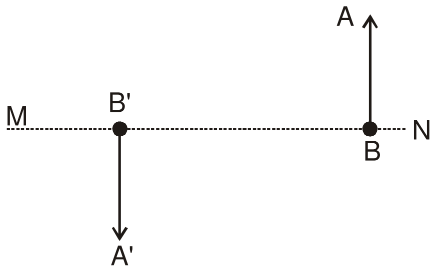
SECTION (F) : LENS

F-1. Lenses are constructed by a material of refractive index 2. The magnitude of the radii of curvature are 20 cm and 30 cm. Find the focal lengths of the possible lenses with the above specifications.

F-2. Find the focal length of lens shown in the figure. Solve for three cases $n_s = 1.5$, $n_s = 2.0$, $n_s = 2.5$.



F-3. Given an optical axis MN and the positions of a real object AB and its image A 'B', determine diagrammatically the position of the lens (its optical centre O) and its foci. Is it a converging or diverging lens? Is the image real or virtual?



F-4. A thin lens made of a material of refractive index μ_2 has a medium of refractive index μ_1 on one side and a medium of refractive index μ_3 on the other side. The lens is biconvex and the two radii of curvature has equal magnitude R. A beam of light travelling parallel to the principal axis is incident on the lens. Where will the image be formed if the beam is incident from (a) the medium μ_1 and (b) from the medium μ_3?

F-5. Two glasses with refractive indices of 1.5 & 1.7 are used to make two identical double-convex lenses.
(i) Find the ratio of their focal lengths.
(ii) How will each of these lenses act on a ray parallel to its optical axis if the lenses are submerged into a transparent liquid with a refractive index of 1.6?

F-6. An object of height 1 cm is set at right angles to the optical axis of a double convex lens of optical power 5 D and 25 cm away from the lens. Determine the focal length of the lens, the position of the image, the linear magnification of the lens, and the height of the image formed by it.

F-7. A lens placed between a candle and a fixed screen forms a real triply magnified image of the candle on the screen. When the lens is moved away from the candle by 0.8 m without changing the position of the candle, a real image one-third the size of the candle is formed on the screen. Determine the focal length of the lens.

F-8. A pin of length 1 cm lies along the principal axis of a converging lens, the centre being at a distance of 5.5 cm from the lens. The focal length of the lens is 3 cm. Find the size of the image.

F-9. The radius of the sun is 0.75×10^9 m and its distance from the earth is 1.5×10^{11} m. Find the diameter of the image of the sun formed by a lens of focal length 40 cm.

F-10. A 2.5 dioptre lens forms a virtual image which is 4 times the object placed perpendicularly on the principal axis of the lens. Find the required distance of the object from the lens.

F-11. A diverging lens of focal length 20 cm is placed coaxially 5 cm towards left of a converging mirror of focal length 10 cm. Where should an object be placed towards left of the lens so that a real image is formed at the object itself ?

F-12. A convex lens and a convex mirror are placed at a separation of 15 cm. The focal length of the lens is 25 cm and radius of curvature of the mirror is 80 cm. Where should a point source be placed between the lens and the mirror so that the light, after getting reflected by the mirror and then getting refracted by the lens, comes out parallel to the principal axis?

F-13. A point object is placed on the principal axis of a converging lens of focal length 15 cm at a distance of 30 cm from it. A glass plate (μ = 1.50) of thickness 3 cm is placed on the other side of the lens perpendicular to the axis. Find the position of the image of the point object.

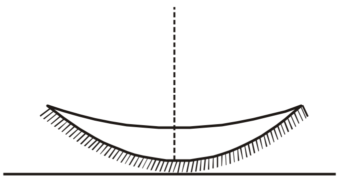
F-14. A converging lens of focal length 10 cm and a diverging lens of focal length 5 cm are placed 5 cm apart with their principal axes coinciding. A beam of light travelling parallel to the principal axis and having a beam diameter 5.0 mm, is incident on the combination. Show that the emergent beam is parallel to the incident one. Find the beam diameter of the emergent beam. Also find out the ratio of emergent and incident intensities.

SECTION (G) : COMBINATION OF LENSES/LENS AND MIRRORS.

G-1. Two identical thin converging lenses brought in contact so that their axes coincide are placed 12.5 cm from an object. What is the optical power of the system and each lens, if the real image formed by the system of lenses is four times as large as the object?

G-2. A point object is placed at a distance of 15 cm from a convex lens. The image is formed on the other side at a distance of 30 cm from the lens. When a concave lens is placed in contact with the convex lens, the image shifts away further by 30 cm. Calculate the focal lengths of the two lenses.

G-3. The convex surface of a thin concavo-convex lens of glass of refractive index 1.5 has a radius of curvature 20 cm. the concave surface has a radius of curvature 60 cm. The convex side is silvered and placed on a horizontal surface as shown in figure. (a) Where should a pin be placed on the axis so that its image is formed at the same place? (b) If the concave part is filled with water (μ = 4/3), find the distance through which the pin should be moved so that the image of the pin again coincides with the pin.

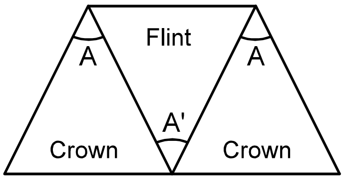


SECTION (H) : DISPERSION OF LIGHT

H-1. A certain material has refractive indices 1.53, 1.60 and 1.68 for red, yellow and violet light respectively. (a) Calculate the dispersive power. (b) Find the angular dispersion produced by a thin prism of angle 6° made of this material.

H-2. A flint glass prism and a crown glass prism are to be combined in such a way that the deviation of the mean ray is zero. The refractive index of flint and crown glasses for the mean ray are 1.6 and 1.9 respectively. If the refracting angle of the flint prism is 6°, what would be the refracting angle of crown prism?

H-3. Three thin prisms are combined as shown in figure. The refractive indices of the crown glass for red, yellow and violet rays are μ_r, μ_y and μ_v respectively and those for the flint glass are μ_r', μ_y' and μ_v' respectively. Find the ratio A'/A for which (a) system produces deviation without dispersion (achromatic combination) and (b) system produces dispersion without deviation (direct vision arrangement).



H-4. The focal lengths of a convex lens for red, yellow and violet rays are 100 cm, 99 cm and 98 cm respectively. Find the dispersive power of the material of the lens.

H-5. A thin prism of angle 5.0°, $\omega = 0.07$ and $\mu_y = 1.30$ is combined with another thin prism having $\omega' = 0.08$ and $\mu'_y = 1.50$. The combination produces no deviation in the mean ray. (a) Find the angle of the second prism. (b) Find the net angular dispersion produced by the combination when a beam of white light passes through it. (c) If the prisms are similarly directed, what will be the deviation in the mean ray? (d) Find the angular dispersion in the situation described in (c).

SECTION (I) : FOR JEE MAIN

I-1. A small telescope has an objective lens of focal length 144 cm and an eye-piece of focal length 6.0 cm. What is the magnifying power of the telescope? What is the separation between the objective and the eye-piece ?

I-2. An angular magnification (magnifying power) of 30 X is desired using an objective of focal length 1.25cm and an eye-piece of focal length 5 cm. How will you set up the compound microscope for normal adjustment (Final image at ∞)?

I-3. A compound microscope consists of an objective lens of focal 2.0 cm and an eye-piece of focal length 6.25 cm separated by a distance of 15 cm. How far from the objective should an object be placed in order to obtain the final image at (a) least distance of distinct vision (25 cm), (b) infinity? What is the magnifying power of the microscope in each case ?

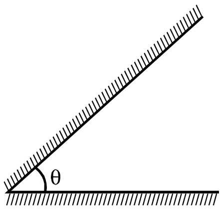
PART - II : ONLY ONE OPTION CORRECT TYPE

SECTION (A) : PLANE MIRROR

A-1. Two plane mirrors are inclined to each other at an angle 60°. If a ray of light incident on the first mirror is parallel to the second mirror, it is reflected from the second mirror

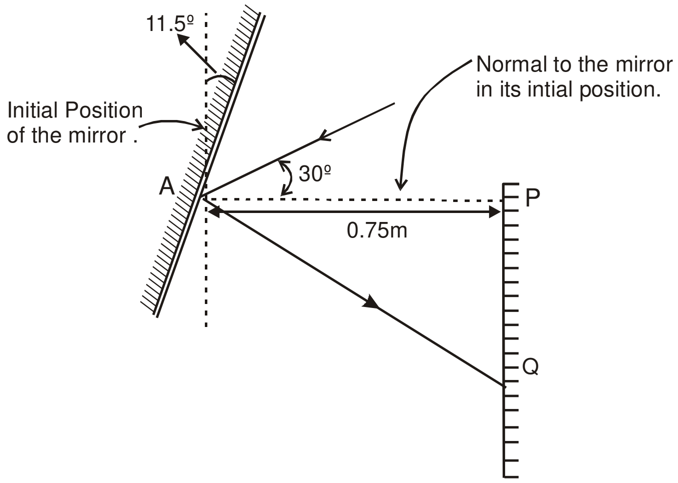
(A) Perpendicular to the first mirror
(B) Parallel to the first mirror
(C) Parallel to the second mirror
(D) Perpendicular to the second mirror

A-2. Two mirrors are inclined at an angle θ as shown in the figure. Light ray is incident parallel to one of the mirrors. Light will start retracing its path after third reflection if :



(A) $\theta = 45°$ (B) $\theta = 30°$ (C) $\theta = 60°$ (D) all three

A-3. The view in the figure is from above a plane mirror suspended by a thread connected to the centre of the mirror at point A. A scale is located 0.75 m (the distance from point A to point P) to the right of the centre of the mirror. Initially, the plane of the mirror is parallel to the side of the scale; and the angle of incidence of a light ray which is directed at the centre of the mirror is 30º. A small torque applied to the thread causes the mirror to turn 11.5° away from its initial position. The reflected ray then intersects the scale at point Q.



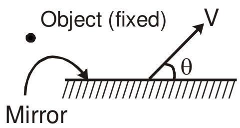
The distance from point P to point Q on the scale is :
(A) 1.00 m (B) 0.56 m (C) 1.02 m (D) 0.86 m.

A-4. A point object is kept in front of a plane mirror. The plane mirror is performing SHM of amplitude 2 cm. The plane mirror moves along the x-axis and x- axis is normal to the mirror. The amplitude of the mirror is such that the object is always infront of the mirror. The amplitude of SHM of the image is
(A) zero (B) 2 cm (C) 4 cm (D) 1 cm

A-5. A person's eye is at a height of 1.5 m. He stands infront of a 0.3m long plane mirror which is 0.8 m above the ground. The length of the image he sees of himself is:
(A) 1.5m (B) 1.0m (C) 0.8m (D) 0.6m

A-6. An unnumbered wall clock shows time 04: 25: 37, where 1st term represents hours, 2nd represents minutes and the last term represents seconds. What time will its image in a plane mirror show.
(A) 08: 35: 23 (B) 07: 35: 23 (C) 07: 34: 23 (D) none of these

A-7. An object and a plane mirror are as shown in figure. Mirror is moved with velocity V as shown. The velocity of image is :



(A) 2 V sinθ (B) 2 V (C) 2V cosθ (D) none of these

A-8. A plane mirror is moving with velocity $4\hat{i}+5\hat{j}+8\hat{k}$. A point object in front of the mirror moves with a velocity $3\hat{i}+4\hat{j}+5\hat{k}$. Here $\hat{k}$ is along the normal to the plane mirror and facing towards the object. The velocity of the image is :
(A) $-3\hat{i}-4\hat{j}+5\hat{k}$ (B) $3\hat{i}+4\hat{j}+11\hat{k}$ (C) $-3\hat{i}-4\hat{j}+11\hat{k}$ (D) $7\hat{i}+9\hat{j}+11\hat{k}$

A-9. Two plane mirrors are parallel to each other and spaced 20 cm apart. An object is kept in between them at 15 cm from A. Out of the following at which point(s) image(s) is/are not formed in mirror A (distance measured from mirror A):
(A) 15 cm (B) 25 cm (C) 45 cm (D) 55 cm

SECTION (B) : SPHERICAL MIRROR

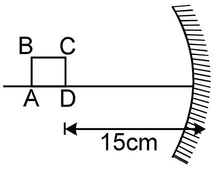
B-1. An object of height 1 cm is kept perpendicular to the principal axis of a convex mirror of radius of curvature 20 cm. If the distance of the object from the mirror is 20 cm then the distance (in cm) between heads of the image and the object will be:

(A) $\sqrt{\frac{6404}{9}}$ (B) $\sqrt{\frac{6414}{9}}$ (C) $\frac{40}{3}$ (D) none of these

B-2. A point object is kept between a plane mirror and a concave mirror facing each other. The distance between the mirrors is 22.5 cm. Plane mirror is placed perpendicular to principal axis of concave mirror. The radius of curvature of the concave mirror is 20 cm. What should be the distance of the object from the concave mirror so that after two successive reflections the final image is formed on the object itself? (Consider first reflection from concave mirror)

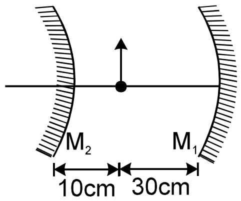
(A) 5 cm (B) 15 cm (C) 10 cm (D) 7.5 cm

B-3. A square ABCD of side 1mm is kept at distance 15 cm infront of the concave mirror as shown in the figure. The focal length of the mirror is 10 cm. The length of the perimeter of its image will be(nearly):



(A) 8 mm (B) 2 mm (C) 12 mm (D) 6 mm

B-4. In the figure shown find the total magnification after two successive reflections first on M_1 and then on M_2 .



(A) + 1 (B) – 2 (C) + 2 (D) – 1

B-5. A luminous point object is moving along the principal axis of a concave mirror of focal length 12 cm towards it. When its distance from the mirror is 20 cm its velocity is 4 cm/s. The velocity of the image in cm/s at that instant is

(A) 6, towards the mirror (B) 6, away from the mirror
(C) 9, away from the mirror (D) 9, towards the mirror.

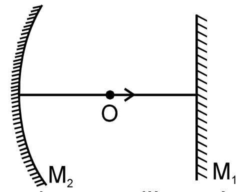
B-6. A particle is moving towards a fixed spherical mirror. The image:

(A) must move away from the mirror
(B) must move towards the mirror
(C) may move towards the mirror
(D) will move towards the mirror, only if the mirror is convex.

B-7. A point object on the principal axis at a distance 15 cm in front of a concave mirror of radius of curvature 20 cm has velocity 2 mm/s perpendicular to the principal axis. The magnitude of velocity of image at that instant will be:

(A) 2 mm/s (B) 4 mm/s (C) 8 mm/s (D) 16 mm/s

B-8. In the figure M_1 and M_2 are two fixed mirrors as shown. If the object ' O ' moves towards the plane mirror, then the image I (which is formed after two successive reflections from M_1 & M_2 respectively) will move



(A) towards right (B) towards left
(C) with zero velocity (D) cannot be determined

B-9. A point object at 15 cm from a concave mirror of radius of curvature 20 cm is made to oscillate along the principal axis with amplitude 2 mm. The amplitude of its image will be
(A) 2 mm (B) 4 mm (C) 8 mm (D) 16 mm

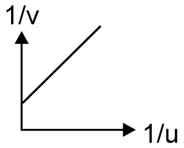
B-10. The distance of an object from the focus of a convex mirror of radius of curvature ' a ' is ' b '. Then the distance of the image from the focus is:
(A) $b^2 / 4a$ (B) a / b^2 (C) $a^2 / 4b$ (D) $4b / a^2$

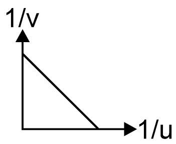
B-11. The largest distance of the image of a real object from a convex mirror of focal length 20 cm can be:
(A) 20 cm (B) infinite
(C) 10 cm (D) depends on the position of the object

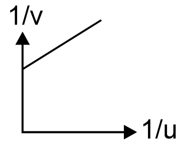
B-12. Which of the following can form erect, virtual, diminished image?
(A) plane mirror (B) concave mirror (C) convex mirror (D) none of these

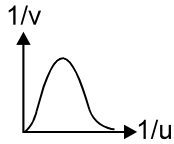
B-13. I is the image of a point object O formed by spherical mirror, then which of the following statements is **incorrect** :
(A) If O and I are on same side of the principal axis, then they have to be on opposite sides of the mirror.
(B) If O and I are on opposite side of the principal axis, then they have to be on same side of the mirror.
(C) If O and I are on opposite side of the principal axis, then they can be on opposite side of the mirror as well.
(D) If O is on principal axis then I has to lie on principal axis only.

B-14. An object is placed at a distance u from a concave mirror and its real image is received on a screen placed at a distance of v from the mirror. If f is the focal length of the mirror, then the graph between 1/v versus 1/u is

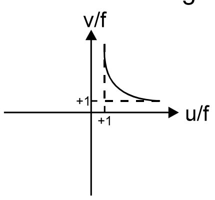
(A) 

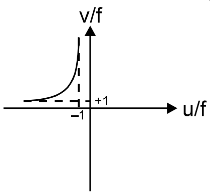
(B) 

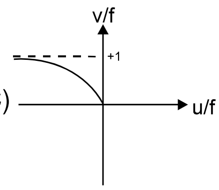
(C) 

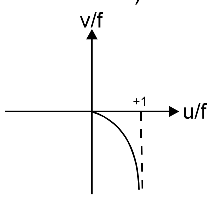
(D) 

B-15. A real inverted image in a concave mirror is represented by graph (u, v, f are coordinates)

(A) 

(B) 

(C) 

(D) 

B-16. When observed from the earth the angular diameter of the sun is 0.5 degree. The diameter of the image of the sun when formed in a concave mirror of focal length 0.5 m will be about
(A) 3.0 mm (B) 4.4 mm (C) 5.6 mm (D) 8.8 mm

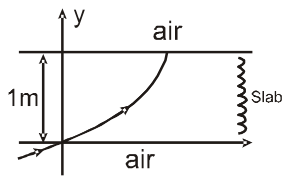
SECTION (C) : LAWS OF REFRACTION, REFRACTION AT PLANE SURFACE AND T.I.R.

C-1. The wavelength of light in vacuum is 6000 Aº and in a medium it is 4000 Aº. The refractive index of the medium is:
(A) 2.4 (B) 1.5 (C) 1.2 (D) 0.67

C-2. A ray of light passes from vacuum into a medium of refractive index n. If the angle of incidence is twice the angle of refraction, then the angle of incidence is:
(A) $\cos^{-1}(n/2)$ (B) $\sin^{-1}(n/2)$ (C) $2\cos^{-1}(n/2)$ (D) $2\sin^{-1}(n/2)$

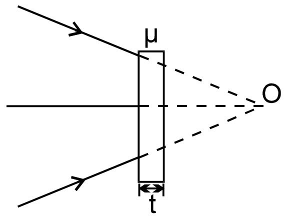
C-3. A ray of light is incident on a parallel slab of thickness t and refractive index n. If the angle of incidence θ is small, then the displacement in the incident and emergent ray will be:
(A) $\frac{t\theta(n-1)}{n}$ (B) $\frac{t\theta}{n}$ (C) $\frac{t\theta n}{n-1}$ (D) none of these

C-4. A ray of light travelling in air is incident at grazing incidence on a slab with variable refractive index, $n(y) = [k y^{3/2} + 1]^{1/2}$ where $k = 1\ m^{-3/2}$ and follows path as shown in the figure. What is the total deviation produced by slab when the ray comes out.



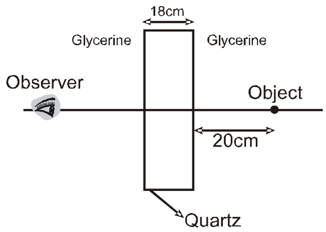
(A) 60º (B) 53º (C) $\sin^{-1}(4/9)$ (D) no deviation at all

C-5. A beam of light is converging towards a point. A plane parallel plate of glass of thickness t, refractive index μ is introduced in the path of the beam as shown in the figure. The convergent point is shifted by (assume near normal incidence):



(A) $t\left(1-\frac{1}{\mu}\right)$ away (B) $t\left(1+\frac{1}{\mu}\right)$ away
(C) $t\left(1-\frac{1}{\mu}\right)$ nearer (D) $t\left(1+\frac{1}{\mu}\right)$ nearer

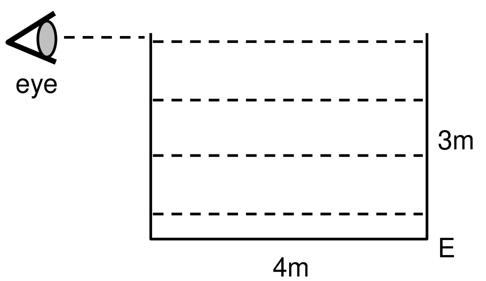
C-6. Given that, velocity of light in quartz = 1.5×10^8 m/s and velocity of light in glycerine = $(9/4) \times 10^8$ m/s. Now a slab made of quartz is placed in glycerine as shown. The shift of the object produced by slab is



(A) 6 cm (B) 3.55 cm (C) 9 cm (D) 2 cm

C-7. The critical angle of light going from medium A to medium B is θ. The speed of light in medium A is v. The speed of light in medium B is:
(A) $\frac{v}{\sin\theta}$ (B) $v\sin\theta$ (C) $v\cot\theta$ (D) $v\tan\theta$

C-8. A rectangular metal tank filled with a certain liquid is as shown in the figure. The observer, whose eye is in level with the top of the tank can just see the corner E of the tank. Therefore, the minimum refractive index of the liquid is **[Olympiad-2016; Stage-I]**



(A) 1.67 (B) 1.50 (C) 1.33 (D) 1.25

C-9. A point source of light is viewed through a plate of glass of thickness t and of refractive index 1.5. The source appears **[Olympiad 2017 (Stage–I)]**
(A) closer by a distance 2t/3
(B) closer by a distance t/3
(C) farther by a distance t/3
(D) farther by a distance 2t/3

SECTION (D) : REFRACTION BY PRISM

D-1. A ray of monochromatic light is incident on one refracting face of a prism of angle 75^0. It passes through the prism and is incident on the other face at the critical angle. If the refractive index of the material of the prism is $\sqrt{2}$, the angle of incidence on the first face of the prism is
(A) 30^0 (B) 45^0 (C) 60^0 (D) 0^0

D-2. A prism having refractive index $\sqrt{2}$ and refracting angle 30º, has one of the refracting surfaces polished. A beam of light incident on the other refracting surface will retrace its path if the angle of incidence is:
(A) 0º (B) 30º (C) 45º (D) 60º

D-3. A ray of light is incident at angle i on a surface of a prism of small angle A and emerges normally from the opposite surface. If the refractive index of the material of the prism is μ, the angle of incidence i is nearly equal to :
(A) A/μ (B) $A/(2\mu)$ (C) μA (D) $\mu A/2$

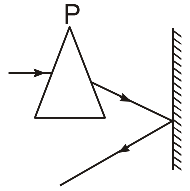
D-4. A prism of refractive index $\sqrt{2}$ has refracting angle 60º. Answer the following questions
(a) In order that a ray suffers minimum deviation it should be incident at an angle :
(A) 45^0 (B) 90^0 (C) 30^0 (D) none of these

(b) Angle of minimum deviation is :
(A) 45^0 (B) 90^0 (C) 30^0 (D) none of these

(c) Angle of maximum deviation is :
(A) 45^0
(B) $\sin^{-1}(\sqrt{2}\sin 15^\circ)$
(C) $30^\circ + \sin^{-1}(\sqrt{2}\sin 15^\circ)$
(D) none of these

D-5. The maximum refractive index of a material, of a prism of apex angle 90º, for which light may be transmitted is:
(A) $\sqrt{3}$ (B) 1.5 (C) $\sqrt{2}$ (D) None of these

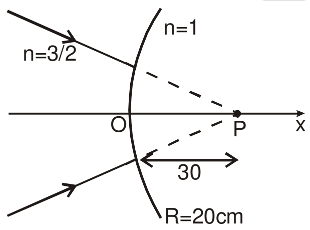
D-6. A prism having an apex angle of 4° and refractive index of 1.50 is located in front of a vertical plane mirror as shown in the figure. A horizontal ray of light is incident on the prism. The total angle through which the ray is deviated is
(A) 4^0 clockwise
(B) 178^0 clockwise
(C) 2^0 clockwise
(D) 8^0 clockwise



SECTION (E) : REFRACTION BY SPHERICAL SURFACE

E-1. There is a small black dot at the centre C of a solid glass sphere of refractive index μ. When seen from outside, the dot will appear to be located:
(A) away from C for all values of μ
(B) at C for all values of μ
(C) at C for $\mu = 1.5$, but away from C for $\mu \neq 1.5$
(D) at C only for $\sqrt{2} \leq \mu \leq 1.5$.

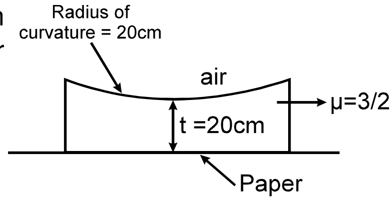
E-2. The image for the converging beam after refraction through the curved surface (in the given figure) is formed at:



(A) x = 40 cm (B) x = $\frac{40}{3}$ cm

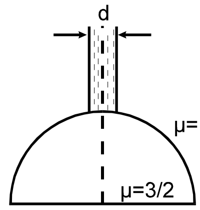
(C) x = $-\frac{40}{3}$ cm (D) x = $\frac{180}{7}$ cm

E-3. In the given figure a plano-concave lens is placed on a paper on which a flower is drawn. How far above its actual position does the flower appear to be ?



(A) 10 cm (B) 15 cm
(C) 50 cm (D) none of these

E-4. A beam of diameter ' d ' is incident on a glass hemisphere as shown in the figure. If the radius of curvature of the hemisphere is very large in comparison to d, then the diameter of the beam at the base of the hemisphere will be:



(A) $\frac{3}{4}$d (B) d

(C) $\frac{d}{3}$ (D) $\frac{2}{3}$d

SECTION (F) : LENS

F-1. A convexo - concave diverging lens is made of glass of refractive index 1.5 and focal length 24 cm. Radius of curvature for one surface is double that of the other. Then radii of curvature for the two surfaces are (in cm):
(A) 6, 12 (B) 12, 24 (C) 3, 6 (D) 18, 36

F-2. Two symmetric double convex lenses A and B have same focal length, but the radii of curvature differ so that, $R_A = 0.9\ R_B$. If $n_A = 1.63$, find n_B.
(A) 1.7 (B) 1.6 (C) 1.5 (D) 4/3

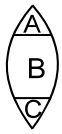
F-3. When a lens of power P (in air) made of material of refractive index μ is immersed in liquid of refractive index μ_0. Then the power of lens is:
(A) $\frac{\mu-1}{\mu-\mu_0}$P (B) $\frac{\mu-\mu_0}{\mu-1}$P (C) $\frac{\mu-\mu_0}{\mu-1}\cdot\frac{P}{\mu_0}$ (D) none of these

F-4. A lens behaves as a converging lens in air and a diverging lens in water. The refractive index of the material is (refractive index of water = 1.33)
(A) equal to unity (B) equal to 1.33
(C) between unity and 1.33 (D) greater than 1.33

F-5. The diameter of the sun subtends an angle of 0.5^0 at the surface of the earth. A converging lens of focal length 100 cm is used to provide an image of the sun on to a screen. The diameter (in mm) of the image formed is nearly
(A) 1 (B) 3 (C) 5 (D) 9

F-6. A thin lens of focal length f and its aperture diameter d, forms a real image of intensity I. Now the central part of the aperture upto diameter (d/2) is blocked by an opaque paper. The focal length and image intensity would change to :
(A) f/2, I/2 (B) f, I/4 (C) 3f/4, I/2 (D) f, 3I/4

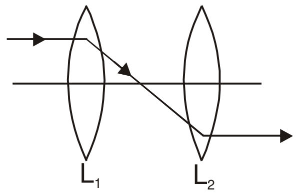
F-7. A thin symmetrical double convex lens of power P is cut into three parts, as shown in the figure. Power of A is:



(A) 2 P (B) $\frac{P}{2}$

(C) $\frac{P}{3}$ (D) P

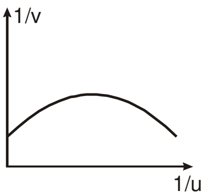
F-8. In the figure given below, there are two convex lens L_1 and L_2 having focal length of f_1 and f_2 respectively. The distance between L_1 and L_2 will be :

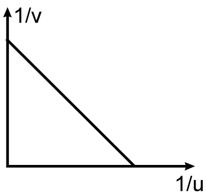


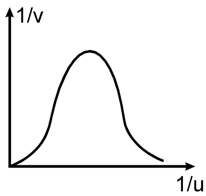
(A) f_1 (B) f_2

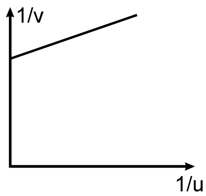
(C) $f_1 + f_2$ (D) $f_1 - f_2$

F-9. An object is placed at a distance u from a **converging lens** and its real image is received on a screen placed at a distance of v from the lens. If f is the focal length of the lens, then the graph between 1/v versus 1/u is:

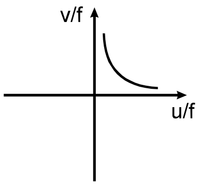
(A) 

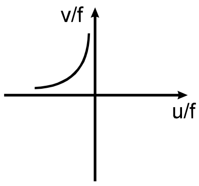
(B) 

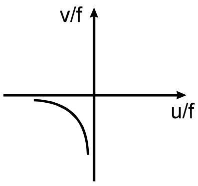
(C) 

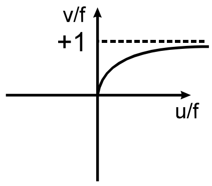
(D) 

F-10. A virtual erect image by a **diverging lens** is represented by (u, v, f are coordinates)

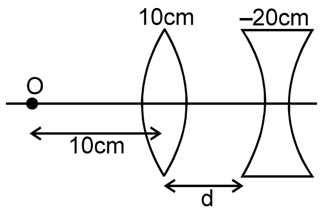
(A) 

(B) 

(C) 

(D) 

F-11. What should be the value of distance d so that final image is formed on the object itself? (Focal lengths of the lenses are as given in the figure).



(A) 10 cm
(B) 20 cm
(C) 5 cm
(D) none of these

F-12. A thin linear object of size 1 mm is kept along the principal axis of a convex lens of focal length 10 cm. The object is at 15 cm from the lens. The length of the image is:

(A) 1 mm (B) 4 mm (C) 2 mm (D) 8 mm

F-13. A biconvex lens is used to project a slide on screen. The slide is 2 cm high and placed at 10 cm from the lens. The image is 18 cm high. What is the focal length of the lens?

(A) 9 cm (B) 18 cm (C) 4.5 cm (D) 20 cm

F-14. The minimum distance between a real object and its real image formed by a thin converging lens of focal length f is

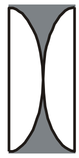
(A) 4f (B) 2f (C) f (D) f/2

F-15. A small fish, 4cm below the surface of a lake, is viewed through a thin converging lens of focal length 30 cm held 2 cm above the water surface. Refractive index of water is 1.33. The image of the fish from the lens is at a distance of

(A) 10 cm (B) 8 cm (C) 6 cm (D) 4 cm

SECTION (G) : COMBINATION OF THIN LENS/LENS AND MIRRORS.

G-1. Two plano-convex lenses each of focal length 10 cm & refractive index 3/2 are placed as shown in the figure. In the space left, water $\left(R.I. = \frac{4}{3}\right)$ is filled. The whole arrangement is in air. The optical power of the system is (in dioptre):



(A) 6.67 (B) – 6.67 (C) 33.3 (D) 20

G-2. A plano-convex lens, when silvered at its plane surface is equivalent to a concave mirror of focal length 28 cm. When its curved surface is silvered and the plane surface not silvered, it is equivalent to a concave mirror of focal length 10 cm, then the refractive index of the material of the lens is:
(A) 9/14 (B) 14/9 (C) 17/9 (D) none of these

G-3. In the above question the radius of curvature of the curved surface of plano-convex lens is :
(A) $\frac{280}{9}$ cm (B) $\frac{180}{7}$ cm (C) $\frac{39}{3}$ cm (D) $\frac{280}{11}$ cm

G-4. The focal length of a plano-concave lens is –10 cm, then its focal length when its plane surface is polished is (n = 3/2):
(A) 20 cm (B) – 5 cm (C) 5 cm (D) none of these

G-5. A convex lens of focal length 25 cm and a concave lens of focal length 20 cm are mounted coaxially separated by a distance d cm. If the power of the combination is zero, d is equal to
(A) 45 (B) 30 (C) 15 (D) 5

SECTION (H) : DISPERSION OF LIGHT

H-1. The dispersion of light in a medium implies that :
(A) lights of different wavelengths travel with different speeds in the medium
(B) lights of different frequencies travel with different speeds in the medium
(C) the refractive index of medium is different for different wavelengths
(D) all of the above.

H-2. Critical angle of light passing from glass to air is minimum for
(A) red (B) green (C) yellow (D) violet

H-3. A plane glass slab is placed over various coloured letters. The letter which appears to be raised the least is:
(A) violet (B) yellow (C) red (D) green

H-4. A medium has $n_v = 1.56$, $n_r = 1.44$. Then its dispersive power is:
(A) 3/50 (B) 6/25 (C) 0.03 (D) none of these

H-5. All the listed things below are made of flint glass. Which one of these have greatest dispersive power (ω).
(A) prism (B) glass slab (C) biconvex lens (D) all have same ω

H-6. Light of wavelength 4000 Å is incident at small angle on a prism of apex angle 4º. The prism has $n_v = 1.5$ & $n_r = 1.48$. The angle of dispersion produced by the prism in this light is:
(A) 0.2º (B) 0.08º (C) 0.192º (D) None of these

SECTION (I) : FOR JEE MAIN

I-1. A simple microscope has a focal length of 5 cm. The magnification at the least distance of distinct vision is-
(A) 1 (B) 5 (C) 4 (D) 6

I-2. In a compound microscope, the intermediate image is -
(A) virtual, erect and magnified (B) real, erect and magnified
(C) real, inverted and magnified (D) virtual, erect and reduced

I-3. A Galileo telescope has an objective of focal length 100 cm & magnifying power 50. The distance between the two lenses in normal adjustment will be
(A) 150 cm (B) 100 cm (C) 98 cm (D) 200 cm

I-4. The convex lens is used in-
(A) Microscope (B) Telescope (C) Projector (D) All of the above

I-5. The magnifying power of a simple microscope can be increased if an eyepiece of :
(A) shorter focal length is used (B) longer focal length is used
(C) shorter diameter is used (D) longer diameter is used

I-6. The focal length of the objective of a microscope is
(A) arbitrary (B) less than the focal length of eyepiece
(C) equal to the focal length of eyepiece (D) greater than the focal length of eyepiece

I-7. An astronomical telescope has an eyepiece of focal-length 5 cm. If the angular magnification in normal adjustment is 10, when final image is at least distance of distinct vision (25cm) from eye piece, then angular magnification will be :
(A) 10 (B) 12 (C) 50 (D) 60

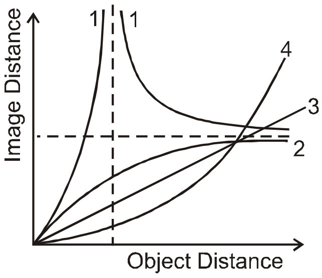
I-8. A person with a defective sight is using a lens having a power of +2D. The lens he is using is
(A) concave lens with f = 0.5 m (B) convex lens with f = 2.0 m
(C) concave lens with f = 0.2 m (D) convex lens with f = 0.5 m

I-9. The focal lengths of the objective and eye-lens of a microscope are 1 cm and 5 cm respectively. If the magnifying power for the relaxed eye is 45, then the length of the tube is :
(A) 30 cm (B) 25 cm (C) 15 cm (D) 12 cm

I-10. If the focal length of objective and eye lens are 1.2 cm and 3 cm respectively and the object is put 1.25 cm away from the objective lens and the final image is formed at infinity. The magnifying power of the microscope is :
(A) 150 (B) 200 (C) 250 (D) 400

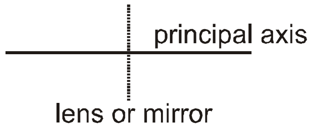
PART - III : MATCH THE COLUMN

1. A small particle is placed at the pole of a concave mirror and then moved along the principal axis to a large distance. During the motion, the distance between the pole of the mirror and the image is measured. The procedure is then repeated with a convex mirror, a concave lens and a convex lens. The graph is plotted between image distance versus object distance. Match the curves shown in the graph with the mirror or lens that is corresponding to it. (Curve 1 has two segments)



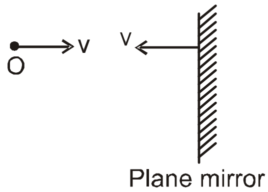
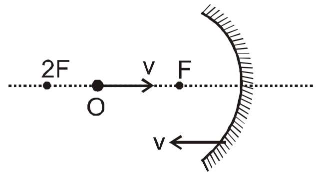
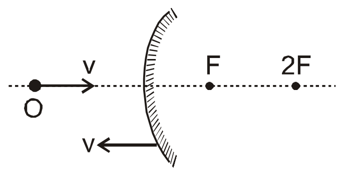
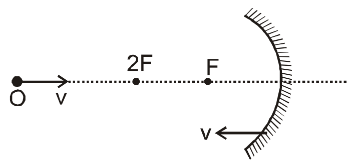
Lens/Mirror	Curve
(A) Converging lens	(p) 1
(B) Converging Mirror	(q) 2
(C) Diverging Lens	(r) 3
(D) Diverging Mirror	(s) 4

2. Column-I gives certain situations regarding a point object and its image formed by an optical instrument. The possible optical instruments are diverging and conveging mirrors or lenses as given in Column-II. Same side of principal axis means both image and object should either be above the principal axis or both should be below the principal axis as shown in figure. Same side of optical instrument means both image and object should be either left of the optical instrument or both should be on right of the optical instrument as shown in figure. Match the statements in column-I with the corresponding statements in column-II.



Column I	Column II
(A) If point object and its image are on same side of principal axis and opposite sides of the optical instrument then the optical instrument is	(p) Concave mirror
(B) If point object and its image are on opposite side of principal axis and same sides of the optical instrument then the optical instrument is	(q) Convex mirror
(C) If point object and its image are on same side of principal axis and same sides of the optical instrument then the optical instrument is	(r) Diverging lens
(D) If point object and its image are on opposite side of principal axis and opposite sides of the optical instrument then the optical instrument is	(s) Converging lens

3. Column–I shows velocity of a point object 'O' (along principal axis in case of convex or concave mirror) and mirrors with respect to ground. Here speed of mirror and object 'O' is v and F is the focus of mirror. Match the Column -I and Column-II for given instant.

Column - I	**Column - II**
(A) 	(p) Speed of image with respect to mirror is same as speed of object with respect to mirror.
(B) 	(q) Speed of image with respect to mirror is greater than as speed of object with respect to mirror.
(C) 	(r) Speed of image with respect to mirror is less than as speed of object with respect to mirror.
(D) 	(s) Distance between image and mirror decreases

Exercise-2

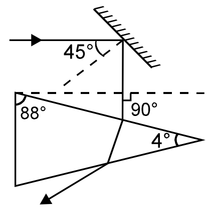
Marked Questions can be used as Revision Questions.

PART - I : ONLY ONE OPTION CORRECT TYPE

1. An object is placed 30 cm (from the reflecting surface) in front of a block of glass 10 cm thick having its farther side silvered. The final image is formed at 23.2 cm behind the silvered face. The refractive index of glass is :

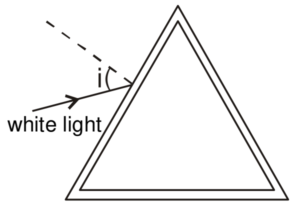
(A) 1.41 (B) 1.46 (C) 200/ 132 (D) 1.61

2. A ray of light strikes a plane mirror at an angle of incidence 45º as shown in the figure. After reflection, the ray passes through a prism of refractive index 1.50, whose apex angle is 4º. The angle through which the mirror should be rotated if the total deviation of the ray is to be 90º is:



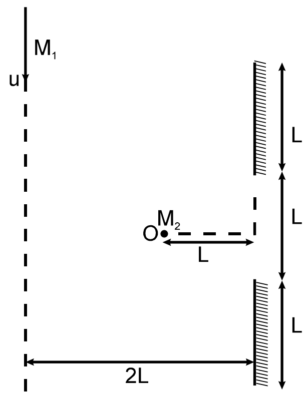
(A) 1^0 clockwise (B) 1^0 anticlockwise
(C) 2^0 clockwise (D) 2^0 anticlockwise

3. A beam of white light is incident on hollow prism of glass as shown in figure. Then :



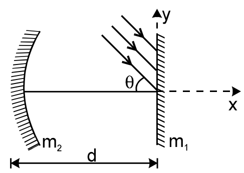
(A) the light emerging from prism gives no dispersion
(B) the light emerging from prism gives spectrum but the bending of all colours is away from base.
(C) the light emerging from prism gives spectrum, all the colours bend towards base, the violet the most and red the least.
(D) the light emerging from prism gives spectrum, all the colours bend towards base, the violet the least and red the most.

4. Two plane mirrors of length L are separated by distance L and a man M_2 is standing at distance L from the connecting line of mirrors as shown in figure. A man M_1 is walking in a straight line at distance 2 L parallel to mirrors at speed u, then man M_2 at O will be able to see image of M_1 for time:



(A) $\frac{4L}{u}$ (B) $\frac{3L}{u}$ (C) $\frac{6L}{u}$ (D) $\frac{9L}{u}$

5. In the figure shown a thin parallel beam of light is incident on a plane mirror m_1 at small angle 'θ'. m_2 is a concave mirror of focal length 'f'. After three successive reflections of this beam the x and y coordinates of the image is

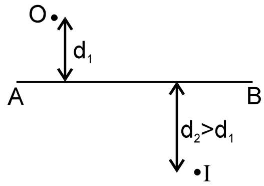


(A) $x = f - d, y = f\theta$ (B) $x = d + f , y = f\theta$ (C) $x = f - d, y = -f\theta$ (D) $x = d - f , y = -f\theta$

6. The distance between an object and its doubly magnified image by a concave mirror is: [Assume f = focal length]

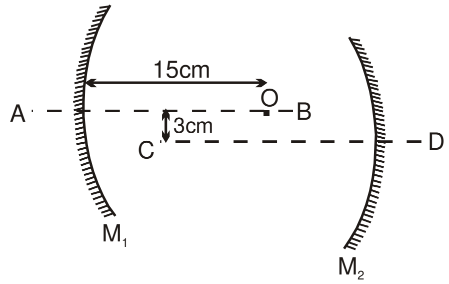
(A) 3 f/2 (B) 2 f/3
(C) 3 f (D) depends on whether the image is real or virtual.

7. In the figure shown, the image of a real object is formed at point I. AB is the principal axis of the mirror. The mirror must be:



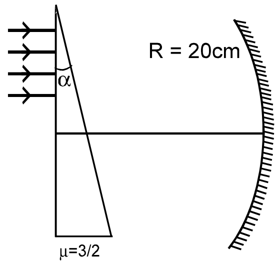
(A) concave and placed towards right of I
(B) concave and placed towards left of O
(C) convex and placed towards right of I
(D) convex and placed towards left of I.

8. In the shown figure M_1 and M_2 are two concave mirrors of the same focal length 10 cm. AB and CD are their principal axes respectively. A point object O is kept on the line AB at a distance 15 cm from M_1. The distance between the mirrors is 20 cm. Considering two successive reflections first on M_1 and then on M_2. The distance of final image from the line AB is:



(A) 3 cm (B) 1.5 cm (C) 4.5 cm (D) 1 cm

9. In the given figure a parallel beam of light is incident on the upper part of a prism of angle 1.8º and R.I. 3/2. The light coming out of the prism falls on a concave mirror of radius of curvature 20 cm. The distance of the point (where the rays are focused after reflection from the mirror) from the principal axis is: [use π = 3.14]

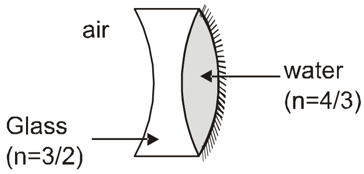


(A) 9 cm (B) 1.5 7 mm (C) 3.14 mm (D) none of these

10. For a prism kept in air, of apex angle 45^0, it is found that the angle of emergence is 45^0 for grazing incidence. Calculate the refractive index of the prism.

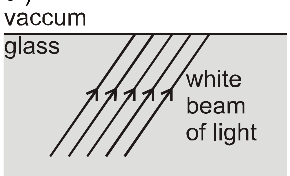
(A) $(2)^{1/2}$ (B) $(3)^{1/2}$ (C) 2 (D) $(5)^{1/2}$

11. In the figure shown the radius of curvature of the left & right surface of the concave lens are 10 cm & 15 cm respectively. The radius of curvature of the mirror is 15 cm. equivalent focal length of the combination is :



(A) the system behaves like a convex mirror of focal length 18cm
(B) the system behaves like a concave mirror of focal length 18cm
(C) the system behaves like a convex mirror of focal length 36cm
(D) the system behaves like a concave mirror of focal length 36cm

12. **STATEMENT – 1:** A thin white parallel beam of light is incident on a plane glass- vacuum interface as shown. The beam may not undergo dispersion after suffering deviation at the interface (The beam is not incident normally on the interface.)

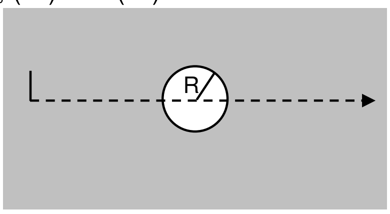


STATEMENT – 2: Vacuum has same refractive index for all colours of white light.
(A) Statement-1 is True, Statement-2 is True; Statement-2 **is** a correct explanation for Statement-1
(B) Statement-1 is True, Statement-2 is True; Statement-2 **is NOT** a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True.

13. Two identical lenses made of the same material of refractive index 1.5 have the focal length 12 cm. These lenses are kept in contact and immersed in a liquid of refractive index 1.35. The combination behaves as **[Olympiad stage–I 2016]**
(A) convex lens of focal length 27 cm
(B) concave lens of focal length 6 cm
(C) convex lens of focal length 9 cm
(D) convex lens of focal length 6 cm

14. In cases of real images formed by a thin convex lens, the linear magnification is (I) directly proportional to the image distance, (II) inversely proportional to the object distance, (III) directly proportional to the distance of image from the nearest principal focus, (IV) inversely proportional to the distance of the object from the nearest principal focus. From these the correct statements are : **[Olympiad 2017 (Stage–I)]**
(A) (I) and (II) only.
(B) (III) and (IV) only
(C) (I), (II),(III) and (IV) all.
(D) None of (I), (II), (III) and (IV).

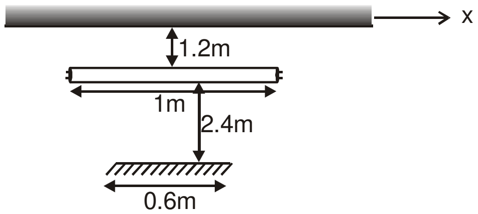
15. Rays from an object immersed in water (μ = 1.33) traverse a spherical air bubble of radius R. If the object is located far away from the bubble, its image as seen by the observer located on the other side of the bubble will be **[Olympiad 2017 (Stage–I)]**



(A) virtual, erect and diminished
(B) real, inverted and magnified
(C) virtual, erect and magnified
(D) real, inverted and diminished

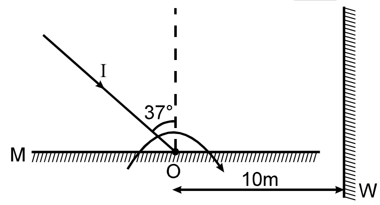
PART - II : SINGLE AND DOUBLE VALUE INTEGER TYPE

1. A fluorescent lamp of length 1 m is placed horizontally at a depth of 1.2 m below a ceiling. A plane mirror of length 0.6 m is placed below the lamp parallel to and symmetric to the lamp at a distance 2.4 m from it as shown in figure. Find the length in meters (distance between the extreme points of the visible region along x-axis) of the reflected patch of light on the ceiling.



2. A plane mirror 50 cm long, is hung on a vertical wall of a room, with its lower edge 50 cm above the ground. A man stands infront of the mirror at a distance 2 m away from the mirror. If his eyes are at a height 1.8 m above the ground, then the length (distance between the extreme points of the visible region perpendicular to the mirror) of the floor visible to him due to reflection from the mirror is $\frac{x}{26}$ m. Find the value of x.

3. A light ray I is incident on a plane mirror M. The mirror is rotated in the direction as shown in the figure by an arrow at frequency $9/\pi$ rps. The light reflected by the mirror is received on the wall W at a distance 10 m from the axis of rotation. When the angle of incidence becomes 37º the speed of the spot (a point) on the wall is V × 10^2 m/s. Find the value of V.



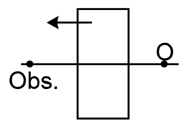
4. A burning candle is placed in front of a concave spherical mirror on its principal optical axis at a distance of (4/3)F from the pole of the mirror (here F is the focal length of the mirror). The candle is arranged at right angle to the axis. The image of the candle in the concave mirror impinges upon a convex mirror of focal length 2 F. The distance between the mirrors is 3F and their axes coincide. The image of the candle in the first mirror plays the part of a virtual object with respect to the second mirror and gives a real image arranged between the two mirrors, Find the total linear magnification (magnitude only) of the system.

5. A concave mirror forms the real image of a point source lying on the optical axis at a distance of 50 cm from the mirror. The focal length of the mirror is 25 cm. The mirror is cut into two halves and its halves are drawn a distance of 1 cm apart (from each other) in a direction perpendicular to the optical axis. Find the distance (in cm) between the two images formed by the two halves of the mirror.

6. A convex mirror and a concave mirror each of focal length 10 cm are placed coaxially. They are separated by 40cm and their reflecting surfaces face each other. A point object is kept on the principle axis at a distance x cm from the concave mirror such that final image after two reflections, first on the concave mirror, is on the object itself. Find the integer next to x.

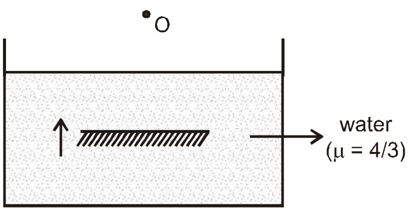
7. The x-y plane is the boundary between two transparent media. Medium-1 with z > 0 has refractive index $\sqrt{2}$ and medium 2 with z < 0 has a refractive index $\sqrt{3}$. A ray of light in medium-1 given by the vector $\vec{A} = 6\sqrt{3}\,\hat{i} + 8\sqrt{3}\,\hat{j} - 10\hat{k}$ is incident on the plane of separation. If the unit vector in the direction of refracted ray in medium – 2 is $\frac{1}{5}\left(a\hat{i} + b\hat{j} - \frac{5}{\sqrt{2}}\hat{k}\right)$ then find the value of ab.

8. (a) In the figure shown a slab of refractive index 3/2 is moved towards a stationary observer with speed 6 cm/s. A point 'O' is observed by the observer with the help of paraxial rays through the slab. Both 'O' and observer lie in air. Find the velocity (in cm/s) with which the image will appear to move to observer.

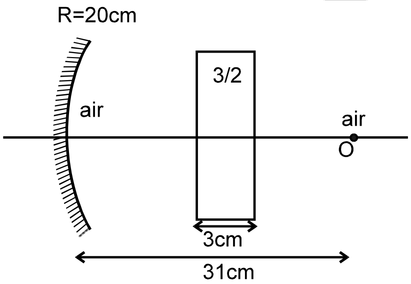


(b) In the previous question if the object moves towards right with a velocity of 6 cm/s and then the velocity of the final image (in cm/s) as seen by observer :

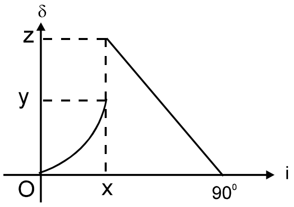
9. Mirror in the arrangement shown in figure is moving up with speed 4 cm/sec. Find the speed of final image of object O (in cm/s) formed after two refraction and one reflection.



10. A point object is placed on principal axis of a concave mirror of radius of curvature 20 cm at a distance 31 cm from poll of the mirror. A glass slab of thikness 3 cm and refractive index 1.5 is placed between object and mirror as shown in the figure.
Find the distance (in cm) of final image formed by the system from the mirror .

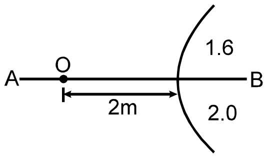


11. Light is incident from glass to air. The variation of the angle of deviation δ with the angle of incidence i for $0 < i < 90°$ is shown. The refractive index of glass is $\frac{2}{\sqrt{3}}$. If the value of (x+y+z) is $\frac{n\pi}{6}$ then find value of n.

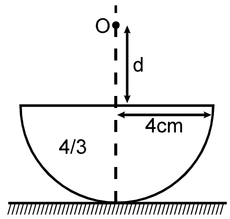


12. A hemispherical portion of the surface of a solid glass sphere (μ = 1.5) of radius 10 cm (surrounding is air) is silvered to make the inner side reflecting. An object is placed on the axis of the hemisphere at a distance 30cm from the centre of the sphere. The light from the object is refracted at the unsilvered part, then reflected from the silvered part and again refracted at the unsilvered part. What is distance (in cm) of final image from pole of reflecting surface.

13. In the figure shown a point object O is placed in air. A spherical boundary of radius of curvature 1.0 m separates two media. AB is principal axis. The refractive index above AB is 1.6 and below AB is 2.0. Find the separation between the images (in m) formed due to refraction at spherical surface.

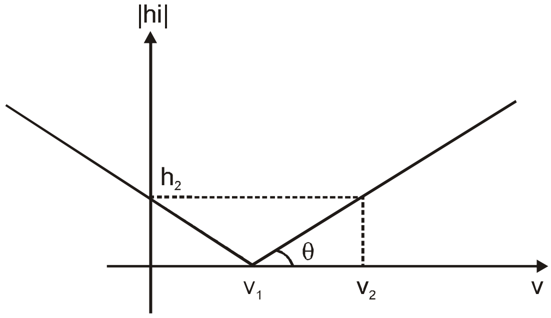


14. A glass hemisphere of refractive index 4/3 and of radius 4 cm is placed on a plane mirror. A point object is placed on axis of this sphere at a distance 'd' from O as shown in the figure. If the final image is formed at infinity, then find the value of ' d ' in mm.

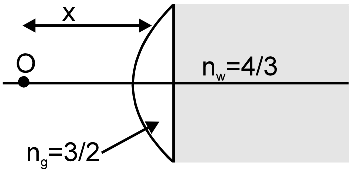


15. A converging lens of focal length 15 cm and a converging mirror of focal length 10 cm are placed 50 cm apart with common principal axis. A point source is placed in between the lens and the mirror at a distance of 40 cm from the lens. Find the distance (in cm) between the final two images formed.

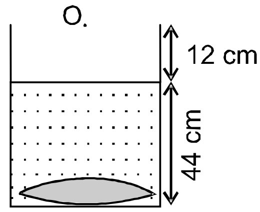
16. An object of height h_0 = 1 cm is moved along principal axis of a convex lens of focal length f = 10 cm. Figure shows variation of magnitude of height of image with image distance (v). Find $v_2 - v_1$ in cm.



17. In the given figure an object ' O ' is kept in air in front of a thin plano convex lens of radius of curvature 10 cm. It's refractive index is 3/2 and the medium towards right of plane surface is water of refractive index 4/3. What should be the distance ' x ' (in cm) of the object so that the rays become parallel finally.



18. An object O is kept in air and a lens of focal length 10 cm (in air) is kept at the bottom of a container which is filled upto a height 44 cm by water. The refractive index of water is 4/3 and that of glass is 3/2. The bottom of the container is closed by a thin glass slab of refractive index 3/2. Find the distance (in cm) of the final image formed by the system from bottom of container (refer to figure shown).



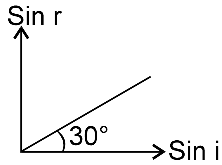
19. The dispersive power of the material of a lens is 0.04 and the focal length of the lens is 10 cm. Find the difference in the focal length (in mm) of the lens for violet and red colour.

PART - III : ONE OR MORE THAN ONE OPTIONS CORRECT TYPE

1. The image (of a real object) formed by a concave mirror is twice the size of the object. The focal length of the mirror is 20 cm. The distance of the object from the mirror is (are)
(A) 10 cm (B) 30 cm (C) 25 cm (D) 15 cm

2. Which of the following statements are **incorrect** for spherical mirrors.
(A) a concave mirror forms only virtual images for any position of real object
(B) a convex mirror forms only virtual images for any position of a real object
(C) a concave mirror forms only a virtual diminished image of an object placed between its pole and the focus
(D) a convex mirror forms a virtual enlarged image of an object if it lies between its pole and the focus.

3. A ray of monochromatic light is incident on the plane surface of separation between two media x and y with angle of incidence ‘i’ in the medium x and angle of refraction ‘r’ in the medium y. The graph shows the relation between sin r and sin i.

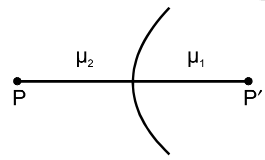


(A) the speed of light in the medium y is $(3)^{1/2}$ times than in medium x.
(B) the speed of light in the medium y is $(1/3)^{1/2}$ times than in medium x.
(C) the total internal reflection can take place when the incidence is in x.
(D) the total internal reflection can take place when the incidence is in y.

4. For the refraction of light through a prism kept in air
(A) For every angle of deviation there are two angles of incidence.
(B) The light travelling inside an isosceles prism is necessarily parallel to the base when prism is set for minimum deviation.
(C) There are two angles of incidence for maximum deviation.
(D) Angle of minimum deviation will increase if refractive index of prism is increased keeping the outside medium unchanged.

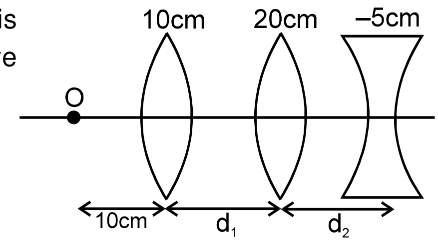
5. An equilateral prism deviates a ray through 40° for two angles of incidence differing by 20°. The possible angles of incidences are:
(A) 40° (B) 50° (C) 20° (D) 60°

6. Two refracting media are separated by a spherical interface as shown in the figure. P P′ is the principal axis, μ_1 and μ_2 are the refractive indices of medium of incidence and medium of refraction respectively. Then:



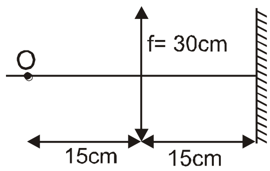
(A) if $\mu_2 > \mu_1$, then there cannot be a real image of real object
(B) if $\mu_2 > \mu_1$, then there cannot be a real image of virtual object
(C) if $\mu_1 > \mu_2$, then there cannot be a virtual image of virtual object
(D) if $\mu_1 > \mu_2$, then there cannot be a real image of real object

7. The values of d_1 & d_2 for final rays to be parallel to the principal axis are : (focal lengths of the lenses are written above the respective lenses in the given figure)



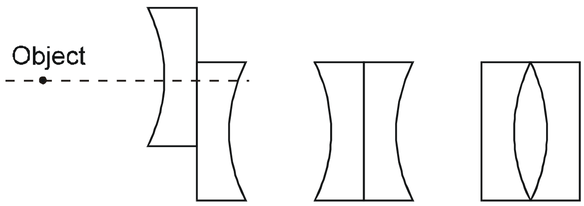
(A) d_1 = 10 cm, d_2 = 15 cm
(B) d_1 = 20 cm, d_2 = 15 cm
(C) d_1 = 30 cm, d_2 = 15 cm
(D) None of these

8. An object O is kept infront of a converging lens of focal length 30 cm behind which there is a plane mirror at 15 cm from the lens as shown in the figure.



(A) the final image is formed at 60 cm from the lens towards right of it
(B) the final image is at 60 cm from lens towards left of it
(C) the final image is real
(D) the final image is virtual.

9. If a symmetrical biconcave thin lens is cut into two identical halves. They are placed in different ways as shown:



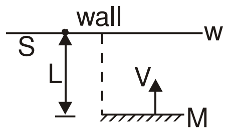
Fig(i) Fig(ii) Fig(iii)

(A) three images will be formed in case (i)
(B) two images will be formed in the case (i)
(C) the ratio of focal lengths in (ii) & (iii) is 1
(D) the ratio of focal lengths in (ii) & (iii) is 2

10. A narrow beam of white light goes through a slab having parallel faces
(A) The light never splits in different colours
(B) The emergent beam is white
(C) The light inside the slab is split into different colours
(D) The light inside the slab is white

11. By properly combining two prisms made of different materials, it is possible to
(A) have dispersion without average deviation
(B) have deviation without dispersion
(C) have both dispersion and average deviation
(D) have neither dispersion nor average deviation

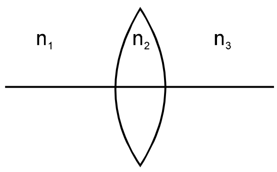
12. A flat mirror M is arranged parallel to a wall W at a distance L from it as shown in the figure. The light produced by a point source S kept on the wall is reflected by the mirror and produces a light patch on the wall. The mirror moves with velocity v towards the wall.



(A) The patch of light will move with the speed v on the wall.
(B) The patch of light will not move on the wall.
(C) As the mirror comes closer the patch of light will become larger and shift away from the wall with speed larger than v.
(D) The width of the light patch on the wall remains the same.

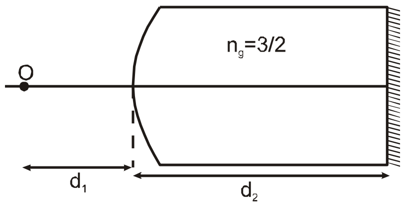
13. A man wants to photograph a white donkey as a Zebra after fitting a glass with black streaks onto the lens of his camera.
(A) The image will look like a white donkey on the photograph.
(B) The image will look like a Zebra on the photograph
(C) The image will be more intense compared to the case in which no such glass is used.
(D) The image will be less intense compared to the case in which no such glass is used.

14. An equiconvex lens of refractive index n_2 is placed such that the refractive index of the surrounding media is as shown. Then the lens :



(A) must be diverging if n_2 is less than the arithmetic mean of n_1 and n_3
(B) must be converging if n_2 is greater than the arithmetic mean of n_1 and n_3
(C) may be diverging if n_2 is less than the arithmetic mean of n_1 and n_3
(D) will neither be diverging nor converging if n_2 is equal to arithmetic mean of n_1 and n_3

15. In the figure shown a point object O is placed in air on the principal axis. The radius of curvature of the spherical surface is 60 cm. I_f is the final image formed after all the refractions and reflections.



(A) If d_1 = 120 cm, then the 'I_f' is formed on 'O' for any value of d_2.
(B) If d_1 = 240 cm, then the 'I_f' is formed on 'O' only if d_2 = 360 cm.
(C) If d_1 = 240 cm, then the 'I_f' is formed on 'O' for all values of d_2.
(D) If d_1 = 240 cm, then the 'I_f' cannot be formed on 'O'.

16. An object is kept on the principal axis of a convex mirror of focal length 10 cm at a distance of 10 cm from the pole. The object starts moving at a velocity 20 mm/sec towards the mirror at angle 30º with the principal axis. What will be the speed of its image and direction with the principal axis at that instant.

(A) speed = $5\frac{\sqrt{7}}{4}$ mm/sec (B) speed = $\frac{5\sqrt{7}}{2}$ mm/sec

(C) $\tan^{-1}\left(\frac{2}{\sqrt{3}}\right)$ with the principal axis (D) none of these

17. A particle is moving towards a fixed convex mirror. The image also moves. If V_i = speed of image and V_O = speed of the object, then
(A) $V_i < V_O$ if $|u| < |F|$ (B) $V_i > V_O$ if $|u| > |F|$ (C) $V_i < V_O$ if $|u| > |F|$ (D) $V_i = V_O$ if $|u| = |F|$

18. A small air bubble is trapped inside a transparent cube of size 12 cm. When viewed from one of the vertical faces, the bubble appears to be at 5 cm from it. When viewed from opposite face, it appears at 3 cm from it.
(A) The distance of the air bubble from the first face is 7.5 cm.
(B) The distance of the air bubble from the first face is 9 cm.
(C) Refractive index of the material of the cube is 2.0.
(D) Refractive index of the material of the cube is 1.5.

19. A parallel beam of light is incident normally on the flat surface of a hemisphere of radius 6 cm and refractive index 1.5, placed in air as shown in figure (i). Assume paraxial ray approximation.

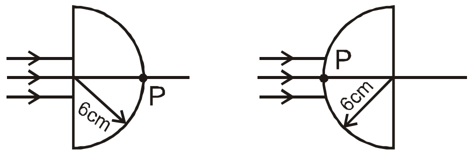


figure (i) figure (ii)

(A) The rays are focused at 12 cm from the point P to the right, in the situation as shown in figure (i)
(B) The rays are focused at 16 cm from the point P to the right, in the situation as shown in figure (i)
(C) If the rays are incident at the curved surface (figure (ii)) then these are focused at distance 18 cm from point P to the right.
(D) If the rays are incident at the curved surface (figure (ii)) then these are focused at distance 14 cm from point P to the right.

20. A ray is incident on a refracting surface of RI μ at an angle of incidence i and the corresponding angle of refraction is r. The deviation of the ray after refraction is given by δ = i-r. Then, one may conclude that **[Olympiad 2017 (Stage–I)]**

(A) r increases with I (B) δ increases with i
(C) δ decreases with I (D) the maximum value of δ is $\cos^{-1}\left(\frac{1}{\mu}\right)$

21. A convex lens and concave lens are kept in contact and the combination is used for the formation of image of a body by keeping it at different places on the principal axis. The image formed by this combination of lenses can be : **[Olympiad 2017 (Stage –I)]**

(A) Magnified, inverted and real (B) Diminished, inverted and real
(C) Diminished, erect and virtual (D) Magnified, erect and virtual

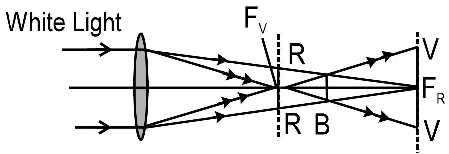
PART - IV : COMPREHENSION

COMPREHENSION-1

Chromatic Aberration

The image of a white object in white light formed by a lens is usually coloured and blurred. This defect of image is called chromatic aberration and arises due to the fact that focal length of a lens is different for different colours. As R.I. μ of lens is maximum for violet while minimum for red, violet is focused nearest to the lens while red farthest from it as shown in figure.

As a result of this, in case of convergent lens if a screen is placed at F_V centre of the image will be violet and focused while sides are red and blurred. While at F_R, reverse is the case, i.e., centre will be red and focused while sides violet and blurred. The difference between f_V and f_R is a measure of the longitudinal chromatic aberration (L.C.A), i.e.,



L.C.A. = $f_R - f_V = -$ df with df = $f_V - f_R$ ....(1)

However, as for a single lens,

$$\frac{1}{f} = (\mu - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right] \quad ....(2)$$

$$\Rightarrow \quad -\frac{df}{f^2} = d\mu\left[\frac{1}{R_1} - \frac{1}{R_2}\right] \quad ....(3)$$

Dividing Eqn. (3) by (2) ;

$$-\frac{df}{f} = \frac{d\mu}{(\mu - 1)} = \omega \qquad \left[\omega = \frac{d\mu}{(\mu - 1)}\right] = \text{dispersive power} ....(4)$$

And hence, from Eqns. (1) and (4),

L.C.A. = $-df = \omega f$

Now, as for a single lens neither f nor ω can be zero, we cannot have a single lens free from chromatic aberration.

Condition of Achromatism :

In case of two thin lenses in contact

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} \qquad \text{i.e.,} \qquad -\frac{dF}{F^2} = -\frac{df_1}{f_1^2} - \frac{df_2}{f_2^2}$$

The combination will be free from chromatic aberration if dF = 0

i.e., $\frac{df_1}{f_1^2} + \frac{df_2}{f_2^2} = 0$

which with the help of Eqn. (4) reduces to

$$\frac{\omega_1 f_1}{f_1^2} + \frac{\omega_2 f_2}{f_2^2} = 0 \qquad \text{i.e.,} \qquad \frac{\omega_1}{f_1} + \frac{\omega_2}{f_2} = 0 \qquad ....(5)$$

This condition is called condition of achromatism (for two thin lenses in contact) and the lens combination which satisfies this condition is called achromatic lens**,** from this condition, i.e., from Eqn. (5) it is clear that in case of achromatic doublet :

(1) The two lenses must be of different materials.

Since, if $\omega_1 = \omega_2$, $\frac{1}{f_1} + \frac{1}{f_2} = 0$ i.e., $\frac{1}{F} = 0$ or $F = \infty$

i.e., combination will not behave as a lens, but as a plane glass slab.

(2) As ω_1 and ω_2 are positive quantities, for equation (5) to hold, f_1 and f_2 must be of opposite nature, i.e. if one of the lenses is converging the other must be diverging.

(3) If the achromatic combination is convergent,

$$f_C < f_D \quad \text{and as} \quad -\frac{f_C}{f_D} = \frac{\omega_C}{\omega_D}, \qquad \omega_C < \omega_D$$

i.e., in a convergent achromatic doublet, convex lens has lesser focal length and dispersive power than the divergent one.

1. Chromatic aberration in the formation of images by a lens arises because :
(A) of non-paraxial rays.
(B) the radii of curvature of the two sides are not same.
(C) of the defect in grinding.
(D) the focal length varies with wavelength.

2. Chromatic aberration of a lens can be corrected by :
(A) providing different suitable curvatures of its two surfaces.
(B) proper polishing of its two surfaces.
(C) suitably combining it with another lens.
(D) reducing its aperture.

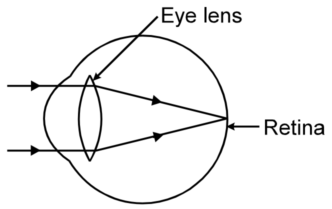
3. A combination is made of two lenses of focal lengths f and f' in contact ; the dispersive powers of the materials of the lenses are ω and ω'. The combination is achromatic when :
(A) $\omega = \omega_0$, $\omega' = 2\omega_0$, $f' = 2f$
(B) $\omega = \omega_0$, $\omega' = 2\omega_0$, $f' = f/2$
(C) $\omega = \omega_0$, $\omega' = 2\omega_0$, $f' = -f/2$
(D) $\omega = \omega_0$, $\omega' = 2\omega_0$, $f' = -2f$

4. The dispersive power of crown and flint glasses are 0.02 and 0.04 respectively. An achromatic converging lens of focal length 40 cm is made by keeping two lenses, one of crown glass and the other of flint glass, in contact with each other. The focal lengths of the two lenses are :
(A) 20 cm and 40 cm
(B) 20 cm and –40 cm
(C) –20cm and 40 cm
(D) 10 cm and –20cm

5. Chromatic aberration in a spherical concave mirror is proportional to :
(A) f
(B) f^2
(C) $1/f$
(D) None of these

COMPREHENSION-2

The ciliary muscles of eye control the curvature of the lens in the eye and hence can alter the effective focal length of the system. When the muscles are fully relaxed, the focal length is maximum. When the muscles are strained the curvature of lens increases (that means radius of curvature decreases) and focal length decreases. For a clear vision the image must be on retina. The image distance is therefore fixed for clear vision and it equals the distance of retina from eye-lens. It is about 2.5 cm for a grown-up person (Refer the figure below).



A person can theoretically have clear vision of objects situated at any large distance from the eye. The smallest distance at which a person can clearly see is related to minimum possible focal length. The ciliary muscles are most strained in this position. For an average grown-up person minimum distance of object should be around 25 cm.

A person suffering for eye defects uses spectacles (eye glass). The function of lens of spectacles is to form the image of the objects within the range in which person can see clearly. The image of the spectacle-lens becomes object for eye-lens and whose image is formed on retina.

The number of spectacle-lens used for the remedy of eye defect is decided by the power of the lens required and the number of spectacle-lens is equal to the numerical value of the power of lens with sign. For example power of lens required is +3D (converging lens of focal length $\frac{100}{3}$ cm) then number of lens will be + 3.

For all the calculations required you can use the lens formula and lens maker's formula. Assume that the eye lens is equiconvex lens. Neglect the distance between eye lens and the spectacle lens.

6. Minimum focal length of eye lens of a normal person is

(A) 25 cm (B) 2.5 cm (C) $\frac{25}{9}$ cm (D) $\frac{25}{11}$ cm

7. Maximum focal length of eye lens of normal person is

(A) 25 cm (B) 2.5 cm (C) $\frac{25}{9}$ cm (D) $\frac{25}{11}$ cm

8. A nearsighted man can clearly see object only upto a distance of 100 cm and not beyond this. The number of the spectacles lens necessary for the remedy of this defect will be.

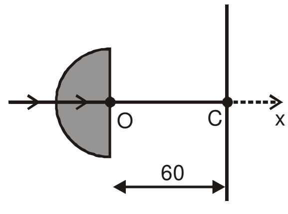
(A) +1 (B) – 1 (C) + 3 (D) – 3

9. A farsighted man cannot see object clearly unless they are at least 100 cm from his eyes. The number of the spectacles lens that will make his range of clear vision equal to an average grown up person :

(A) + 1 (B) – 1 (C) + 3 (D) – 3

COMPREHENSION-3

Figure shows a solid transparent semi cylinder of radius 10 cm. A screen is placed at a distance 60 cm from O. A narrow beam is incident along x-axis at O. If cylinder starts rotating about O in clockwise direction with angular speed 6 rad/s then spot formed on screen will move upward (Refractive index of material of cylinder = $\frac{5}{3}$)



10. What is initial angular velocity of ray refracted from plane surface.

(A) 2 rad/s (B) 10 rad/s (C) 16 rad/s (D) 4 rad/s

11. At what distance from C bright spot on screen will disappear.

(A) 100 cm (B) 80 cm (C) 120 cm (D) 100 cm

Exercise-3

✍ **Marked Questions can be used as Revision Questions.**

PART - I : JEE (ADVANCED) / IIT-JEE PROBLEMS (PREVIOUS YEARS)

*** Marked Questions may have more than one correct option.**

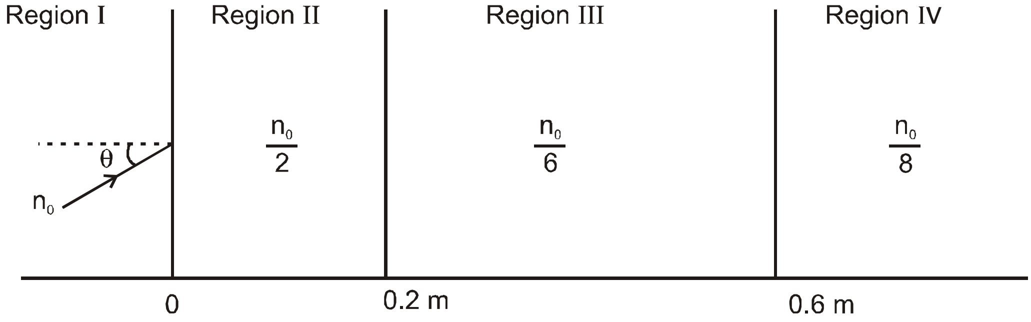
1. Two beams of red and violet colours are made to pass separately through a prism (angle of the prism is 60º). In the position of minimum deviation, the angle of refraction will be **[JEE' 2008_, 3/163]**

(A) 30º for both the colours (B) greater for the violet colour

(C) greater for the red colour (D) equal but not 30º for both the colours

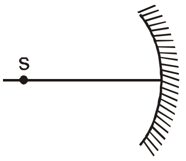
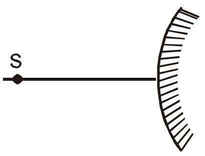
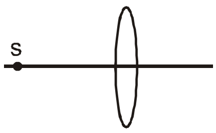
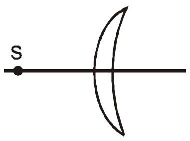
2. A light beam is traveling from Region I to Region IV (Refer Figure). The refractive index in Regions I, II, III and IV are n_0, $\frac{n_0}{2}$, and $\frac{n_0}{8}$, respectively. The angle of incidence θ for which the beam just misses entering Region IV is **[JEE' 2008, 3/163]**

Figure :



(A) $\sin^{-1}\left(\frac{3}{4}\right)$ (B) $\sin^{-1}\left(\frac{1}{8}\right)$ (C) $\sin^{-1}\left(\frac{1}{4}\right)$ (D) $\sin^{-1}\left(\frac{1}{3}\right)$

3.✍ An optical component and an object S placed along its optic axis are given in **Column I**. The distance between the object and the component can be varied. The properties of images are given in **Column II**. Match all the properties of images from **Column II** with the appropriate components given in **Column I**. **[JEE' 2008, 6/163, –1]**

Column I	**Column II**
(A) 	(p) Real image
(B) 	(q) Virtual image
(C) 	(r) Magnified image
(D) 	(s) Image at infinity

4. A ball is dropped from a height of 20 m above the surface of water in a lake. The refractive index of water is 4/3. A fish inside the lake, in the line of fall of the ball, is looking at the ball. At an instant, When the ball is 12.8 m above the water surface, the fish sees the speed of ball as [Take g = 10 m/s^2]

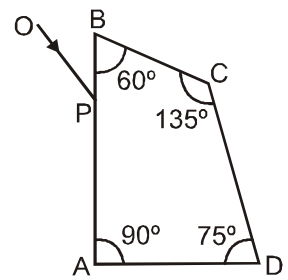
[JEE' 2009; 3/160, –1]

(A) 9 m/s (B) 12 m/s (C) 16 m/s (D) 21.33 m/s

5.* A student performed the experiment of determination of focal length of a concave mirror by u-v method using an optical bench of length 1.5 meter. The focal length of the mirror used is 24 cm. The maximum error in the location of the image can be 0.2 cm. The 5 sets of (u, v) values recorded by the student (in cm) are : (42, 56), (48, 48), (60, 40), (66, 33), (78, 39). The data set(s) that **cannot** come from experiment and is (are) incorrectly recorded, is (are) **[JEE' 2009; 4/160, –1]**

(A) (42, 56) (B) (48, 48) (C) (66, 33) (D) (78, 39)

6*. A ray OP of monochromatic light is incident on the face AB of prism ABCD near vertex B at an incident angle of 60º (see figure). If the refractive index of the material of the prism is $\sqrt{3}$, which of the following is (are) correct ? **[JEE' 2010; 3/163]**



(A) The ray gets totally internally reflected at face CD
(B) The ray comes out through face AD
(C) The angle between the incident ray and the emergent ray is 90º
(D) The angle between the incident ray and the emergent ray is 120º

7. The focal length of a thin biconvex lens is 20cm. When an object is moved from a distance of 25cm in front of it to 50cm, the magnification of its image changes from m_{25} to m_{50}. The ratio $\frac{m_{25}}{m_{50}}$ is :

[JEE 2010; 3/163]

8. A biconvex lens of focal length 15 cm is in front of a plane mirror. The distance between the lens and the mirror is 10 cm. A small object is kept at a distance of 30 cm from the lens. The final image is
(A) Virtual and at a distance of 16 cm from mirror
(B) Real and at distance of 16 cm from the mirror
(C) Virtual and at a distance of 20 cm form the mirror
(D) Real and at a distance of 20 cm from the mirror **[JEE' 2010; 5/163, –2]**

9. Image of an object approaching a convex mirror of radius of curvature 20 m along its optical axis is observed to move from $\frac{25}{3}$ m to $\frac{50}{7}$ m in 30 seconds. What is the speed of the object in km per hour.

[JEE' 2010; 3/163]

10. A large glass slab (μ = 5/3) of thickness 8 cm is placed over a point source of light on a plane surface. It is seen that light emerges out of the top surface of the slab from a circular area of radius R cm. What is the value of R? **[JEE' 2010; 3/163]**

11. Two transparent media of refractive indices μ_1 and μ_3 have a solid lens shaped transparent material of refractive index μ_2 between them as shown in figures in **column** **II.** A ray traversing these media is also shown in the figures. In **Column** **I** different relationships between μ_1, μ_2 and μ_3 are given. Match them to the ray diagrams shown in **Column** **II.** **[JEE' 2010; 8/163]**

Column I

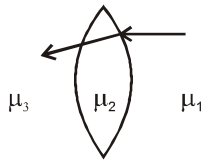
(A) $\mu_1 < \mu_2$

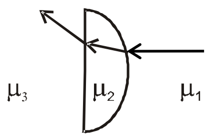
(B) $\mu_1 > \mu_2$

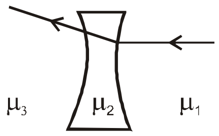
(C) $\mu_2 = \mu_3$

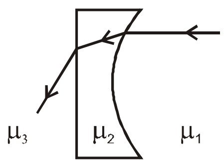
(D) $\mu_2 > \mu_3$

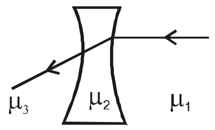
Column II

(p)


(q)


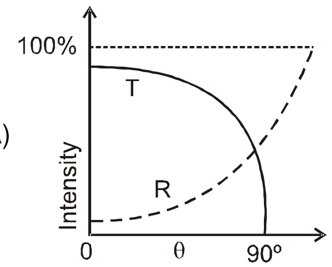
(r)


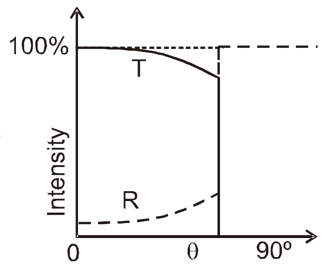
(s)


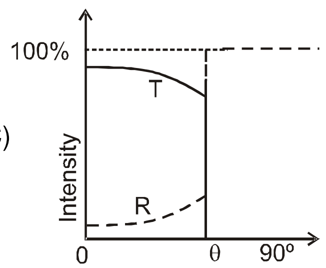
(t)


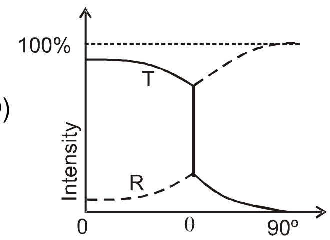
12. A light ray traveling in glass medium is incident on glass-air interface at an angle of incidence θ. The reflected (R) and transmitted (T) intensities, both as function of θ, are plotted. The correct sketch is

[JEE' 2011; 3/160, –1]

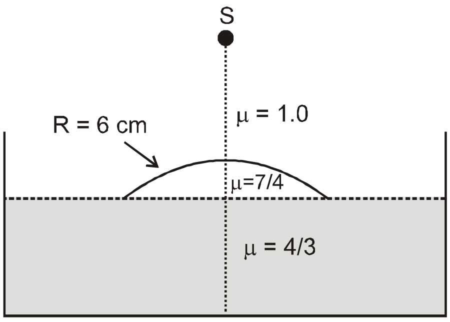
(A)


(B)


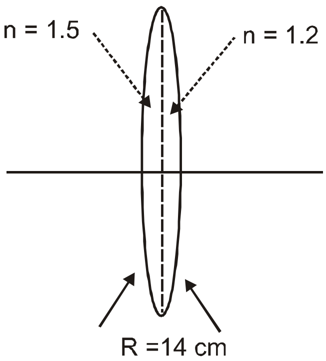
(C)


(D)


13. Water (with refractive index = 4/3) in a tank is 18 cm deep. Oil of refractive index 7/4 lies on water making a convex surface of radius of curvature 'R = 6 cm' as shown. Consider oil to act as a thin lens. An object 'S' is placed 24 cm above water surface. The location of its image is at 'x' cm above the bottom of the tank. Then 'x' is **[JEE' 2011; 4/160]**



14. A bi-convex lens is formed with two thin plano-convex lenses as shown in the figure. Refractive index n of the first lens is 1.5 and that of the second lens is 1.2. Both the curved surfaces are of the same radius of curvature R = 14 cm. For this bi-convex lens, for an object distance of 40 cm, the image distance will be **[IIT-JEE-2012; Paper-1 : 3/70, –1]**



(A) –280.0 cm (B) 40.0 cm (C) 21.5 cm (D) 13.3 cm

Paragraph for Question 15 and 16

Most materials have the refractive index, n > 1. So, when a light ray from air enters a naturally occurring material, then by Snells' law, $\frac{\sin\theta_1}{\sin\theta_2} = \frac{n_2}{n_1}$, it is understood that the refracted ray bends towards the normal. But it never emerges on the same side of the normal as the incident ray. According to electromagnetism, the refractive index of the medium is given by the relation, $n = \left(\frac{c}{v}\right) = \pm\sqrt{\varepsilon_r \mu_r}$ where c is the speed of electromagnetic waves in vacuum, v its speed in the medium, ε_r and μ_r are negative, one must choose the negative root of n. Such negative refractive index materials can now be artificially prepared and are called meta-materials. They exhibit significantly different optical behavior, without violating any physical laws. Since n is negative, it results in a change in the direction of propagation of the refracted light. However, similar to normal materials, the frequency of light remains unchanged upon refraction even in meta-materials. **[IIT-JEE-2012, Paper-2 : 3/66, –1]**

15. Choose the correct statement.

(A) The speed of light in the meta-material is $v = c|n|$

(B) The speed of light in the meta-material is $v = \frac{c}{|n|}$

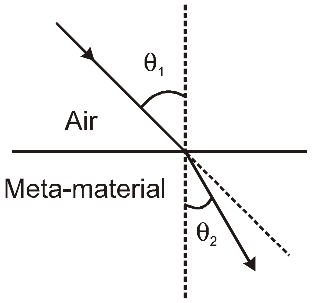
(C) The speed of light in the meta-material is v = c.

(D) The wavelength of the light in the meta-material (λ_m) is given by $\lambda_m = \lambda_{air}|n|$, where λ_{air} is the wavelength of the light in air.

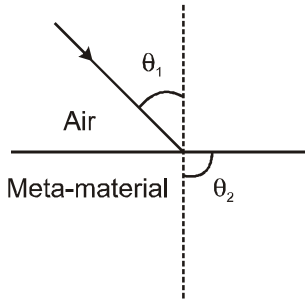
16. For light incident from air on a meta-material, the appropriate ray diagram is :

[IIT-JEE-2012, Paper-2 : 3/66, –1]

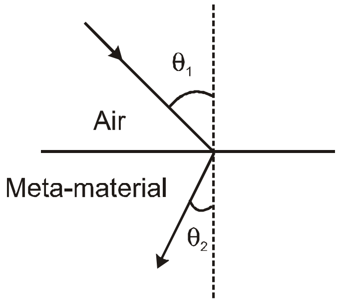
(A)



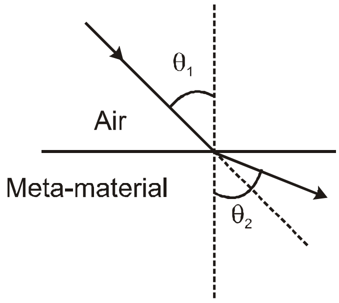
(B)



(C)



(D)



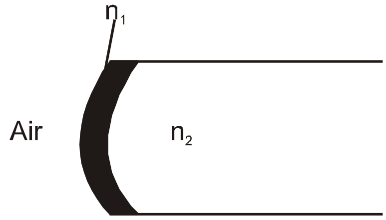
17. The image of an object, formed by a plano-convex lens at a distance of 8 m behind the lens, is real and is one-third the size of the object. The wavelength of light inside the lens is 2/3 times the wavelength in free space. The radius of the curved surface of the lens is : **[JEE-2013 (Advanced); 3/60, –1]**

(A) 1 m (B) 2 m (C) 3 m (D) 6 m

18. A ray of light travelling in the direction $\frac{1}{2}\left(\hat{i}+\sqrt{3}\hat{j}\right)$ is incident on a plane mirror. After reflection, it travels along the direction $\frac{1}{2}\left(\hat{i}-\sqrt{3}\hat{j}\right)$. The angle of incidence is : **[JEE-2013 (Advanced); 3/60, –1]**

(A) 30º (B) 45º (C) 60º (D) 75º

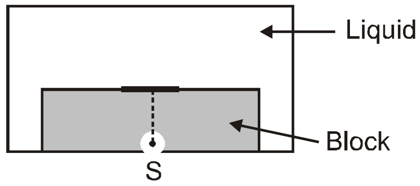
19.* A transparent thin film of uniform thickness and refractive index n_1 = 1.4 is coated on the convex spherical surface of radius R at one end of a long solid glass cylinder of refractive index n_2 = 1.5. as shown in the figure. Rays of light parallel to the axis of the cylinder traversing through the film from air to glass get focused at distance f_1 from the film, while rays of light traversing from glass to air get focused at distance f_2 from the film. Then



[JEE (Advanced)-2014,P-1, 3/60]

(A) $|f_1|$ = 3R (B) $|f_1|$ = 2.8R
(C) $|f_2|$ = 2R (D) $|f_2|$ = 1.4R

20. A point source S is placed at the bottom of a transparent block of height 10 mm and refractive index 2.72. It is immersed in a lower refractive index liquid as shown in the figure. It is found that the light emerging from the block to the liquid forms a circular bright spot of diameter 11.54 mm on the top of the block. The refractive index of the liquid is **[JEE (Advanced)-2014, 3/60, –1]**



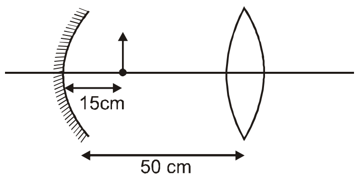
(A) 1.21 (B) 1.30 (C) 1.36 (D) 1.42

21. Four combinations of two thin lenses are given in List-I. The radius of curvature of all curved surface is r and the refractive index of all lenses is 1.5. Match lens combinations in List-I with their focal length in List-II and select the correct answer using the code given below the lists. **[JEE (Advanced) 2014, 3/60, –1]**

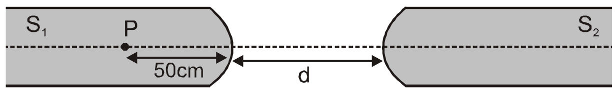
List-I		**List-II**	
P.		**1.**	2r
Q.		**2.**	r/2
R.		**3.**	–r
S.		**4.**	r

Code :
(A) P-1, Q-2, R-3, S-4 (B) P-2, Q-4, R-3, S-1 (C) P-4, Q-1, R-2, S-3 (D) P-2, Q-1, R-3, S-4

22. Consider a concave mirror and a convex lens (refractive index = 1.5) of focal length 10 cm each, separated by a distance of 50 cm in air (refractive index = 1) as shown in the figure. An object is placed at a distance of 15 cm from the mirror. Its erect image formed by this combination has magnification M_1. When the set-up is kept in a medium of refractive index 7/6, the magnification becomes M_2. The magnitude $\left|\frac{M_2}{M_1}\right|$ **[JEE (Advanced) 2015 ; P-1,4/88]**

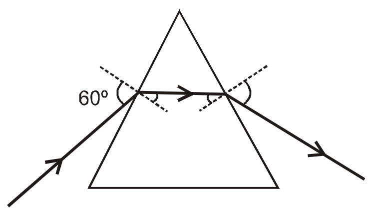


23. Two idenctical glass rods S_1 and S_2 (refractive index = 1.5) have one convex end of radius of curvature 10 cm. They are placed with the curved surfaces at a distance d as shown in the figure, with their axes (shown by the dashed line) aligned. When a point source of light P is placed inside rod S_1 on its axis at a distance of 50 cm from the curved face, the light rays emanating from it are found to be parallel to the axis inside S_2. The distance d is : **[JEE(Advanced) 2015 ; P-1,4/88, –2]**



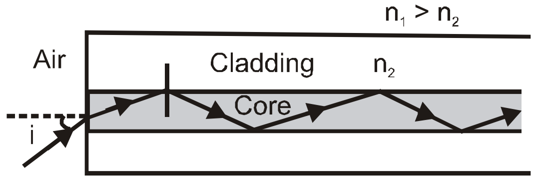
(A) 60 cm (B) 70 cm (C) 80 cm (D) 90 cm

24. A monochromatic beam of light is incident at 60º on one face of an equilateral prism of refractive index n and emerges from the opposite face making an angle θ(n) with the normal (see the figure). For $n = \sqrt{3}$ the value of θ is 60º and $\frac{d\theta}{dn}$ = m. The value of m is : **[JEE(Advanced) 2015 ; P-2,4/88]**



Paragraph for Question 25 and 26

Light guidance in an optical fiber can be understood by considering a structure comprising of thin solid glass cylinder of refractive index n_1 surrounded by a medium of lower refractive index n_2. The light guidance in the structure takes place due to successive total internal reflections at the interface of the media n_1 and n_2 as shown in the figure. All rays with the angle of incidence i less than a particular value i_m are confined in the medium of refractive index n_1. The numerical aperture (NA) of the structure is defined as sin i_m.



25.* For two structures namely S_1 with $n_1 = \sqrt{45}/4$ and $n_2 = 3/2$, and S_2 with $n_1 = 8/5$ and $n_2 = 7/5$ and taking the refractive index of water to be 4/3 and that of air to be 1, the correct option(s) is (are)

[JEE (Advanced) 2015 ; P-2,4/88, –2]

(A) NA of S_1 immersed in water is the same as that of S_2 immersed in a liquid of refractive index $\frac{16}{3\sqrt{15}}$

(B) NA of S_1 immersed in liquid of refractive index $\frac{6}{\sqrt{15}}$ is that as that of S_2 immersed in water

(C) NA of S_1 placed in air is the same as that of S_2 immersed in liquid of refractive index $\frac{4}{\sqrt{15}}$

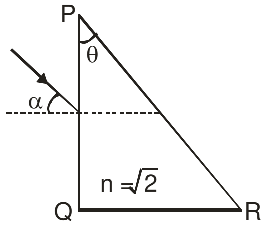
(D) NA of S_1 placed in air is the same as that of S_2 placed in water

26. If two structures of same cross-sectional area, but different numerical apertures NA_1 and NA_2 ($NA_2 < NA_1$) are joined longitudinally, the numerical aperture of the combined structure is

[JEE (Advanced) 2015 ; P-2,4/88, –2]

(A) $\frac{NA_1\ NA_2}{NA_1 + NA_2}$ (B) $NA_1 + NA_2$ (C) NA_1 (D) NA_2

27. A parallel beam of light is incident from air at an angle α on the side PQ of a right angled triangular prism of refractive index $n = \sqrt{2}$. Light undergoes total internal reflection in the prism at the face PR when α has a minimum value of 45º. The angle θ of the prism is : **[JEE Advanced 2016; P-1, 3/62, –1]**



(A) 15º (B) 22.5º (C) 30º (D) 45º

28.* A plano-convex lens is made of a material of refractive index n. When a small object is placed 30 cm away in from of the curved surface of the lens, an image of double the size of the object is produced. Due to reflection from the convex surface of the lens, another faint image is observed at a distance of 10 cm away from the lens. Which of the following statement (s) is(are) true?

(A) The refractive index of the lens is 2.5 **[JEE Advanced 2016 ; P-1, 4/62, –2]**

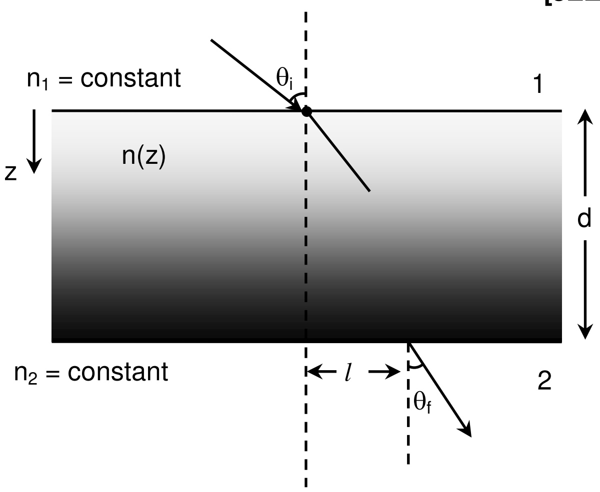
(B) The radius of curvature of the convex surface is 45 cm

(C) The faint image is erect and real

(D) The focal length of the lens is 20 cm

29.* A transparent slab of thickness d has a refractive index n(z) that increases with z. Here z is the vertical distance inside the slab, measured from the top. The slab is placed between two media with uniform refractive indices n_1 and n_2 (> n_1), as shown in the figure. A ray of light is incident with angle θ_i from medium 1 and emerges in medium 2 with refraction angle θ_f with a lateral displacement l :

[JEE Advanced 2016 ; P-1, 4/62, –2]



Which of the following statement(s) is (are) true ?

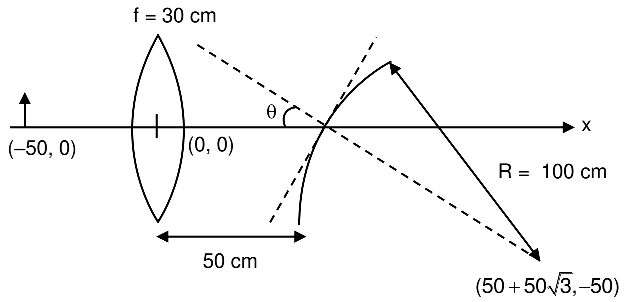
(A) $n_1 \sin\theta_i = n_2 \sin\theta_f$

(B) $n_1 \sin\theta_i = (n_2 - n_1)\sin\theta_f$

(C) l is independent of n_2

(D) l is dependent of n(z)

30. A smaller object is placed 50 cm to the left of a thin convex lens of focal length 30 cm. A convex spherical mirror of radius of curvature 100 cm is placed to the right of the lens at a distance of 50 cm. The mirror is tilted such that the axis of the mirror is at an angle θ = 30º to the axis of the lens, as shown in the figure. **[JEE Advanced 2016; P-2, 3/62, –1]**



If the origin of the coordinate system is taken to be at the centre of the lens, the coordinates (in cm) of the point (x, y) at which the image is formed are :

(A) $(125/3,\ 25/\sqrt{3})$ (B) $(25,\ 25\sqrt{3})$ (C) $(50-25\sqrt{3},\ 25)$ (D) (0, 0)

31. For an isosceles prism of angle A and refractive index μ, it is found that the angle of minimum deviation δ_m = A. Which of the following options is/are correct ? **[JEE Advanced 2017; P-1, 4/61, –2]**

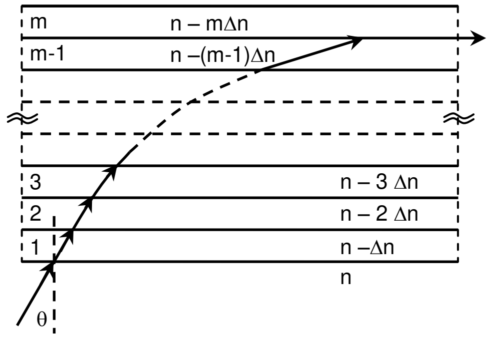
(A) At minimum deviation, the incident angle i_1 and the refracting angle r_1 at the first refracting surface are related by $r_1 = \left(\frac{i_1}{2}\right)$

(B) For this prism, the refractive index μ and the angle of prism A are related as $A = \frac{1}{2}\cos^{-1}\left(\frac{\mu}{2}\right)$

(C) For the angle of incidence i_1 = A, the ray inside the prism is parallel to the base of the prism

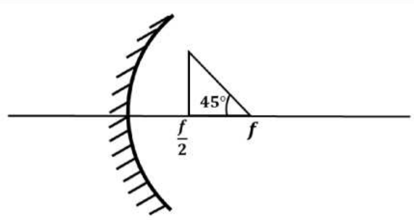
(D) For this prism, the emergent ray at the second surface will be tangential to the surface when the angle of incidence at the first surface is $i_1 = \sin^{-1}\left[\sin A\sqrt{4\cos^2\frac{A}{2}-1} - \cos A\right]$

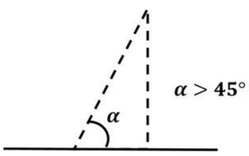
32. A monochromatic light is travelling in a medium of refractive index n = 1.6. It enters a stack of glass layers from the bottom side at an angle $\theta = 30°$. The interfaces of the glass layers are parallel to each other. The refractive indices of different glass layers are monotonically decreasing as $n_m = n - m\Delta n$, where n_m is the refractive index of the m^{th} slab and $\Delta n = 0.1$ (see the figure). The ray is refracted out parallel to the interface between the $(m - 1)^{th}$ and m^{th} slabs from the right side of the stack. What is the value of m ? **[JEE Advanced 2017; P-1, 3/61]**

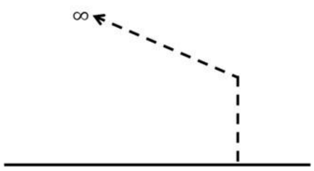


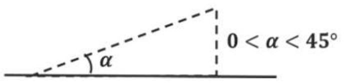
33. Sunlight of intensity 1.3 kWm^{-2} is incident normally on a thin convex lens of focal length 20 cm. Ignore the energy loss of light due to the lens and assume that the lens aperture size is much smaller than its focal length. The average intensity of light, in kWm^{-2}, at a distance 22 cm from the lens on the other side is __________. **[JEE (Advanced) 2018, P-1, 3/60]**

34*. A wire is bent in the shape of a right angled triangle and is placed in front of a concave mirror of focal length f, as shown in the figure. Which of the figures shown in the four options qualitatively represent(s) the shape of the image of the bent wire? (These figures are not to scale.) **[JEE (Advanced) 2018, P-2, 4/60, –2]**



(A) 

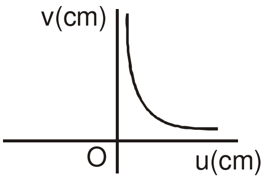
(B) 

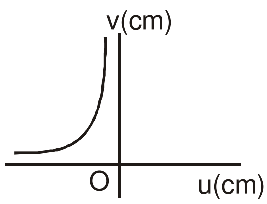
(C) 

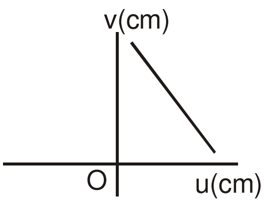
(D) 

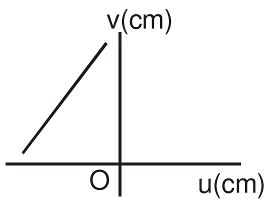
PART - II : JEE (MAIN) / AIEEE PROBLEMS (PREVIOUS YEARS)

1. A student measures the focal length of a convex lens by putting an object pin at a distance 'u' from the lens and measuring the distance 'v' of the image pin. The graph between 'u' and 'v' plotted by the student should look like - **[AIEEE-2008, 3/105]**

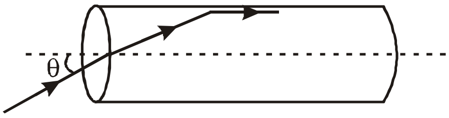
(1) 

(2) 

(3) 

(4) 

2. A transparent solid cylindrical rod has a refractive index of $\frac{2}{\sqrt{3}}$. It is surrounded by air. A light ray is incident at the mid-point of one end of the rod as shown in the figure. **[AIEEE-2009, 4/144]**



The incident angle (θ) for which the light ray grazes along the wall of the rod is:

(1) $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$ (2) $\sin^{-1}\left(\frac{2}{\sqrt{3}}\right)$ (3) $\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$ (4) $\sin^{-1}\left(\frac{1}{2}\right)$

3. In an optics experiment, with the position of the object fixed, a student varies the position of a convex lens and for each position, the screen is adjusted to get a clear image of the object. A graph between the object distance u and the image distance v, from the lens, is plotted using the same scale for the two axes. A straight line passing through the origin and making an angle of 45° with the x-axis meets the experimental curve at P. The coordinates of P will be: **[AIEEE-2009, 4/144]**

(1) $\left(\frac{f}{2}, \frac{f}{2}\right)$ (2) (f, f) (3) (4f, 4f) (4) (2f, 2f)

4. A car is fitted with a convex side–view mirror of focal length 20 cm. A second car 2.8 m behind the first car is overtaking the first car at a relative speed of 15 m/s. The speed of the image of the second car as seen in the mirror of the first one is : **[AIEEE - 2011, 4/120, –1]**

(1) $\frac{1}{10}$ m/s (2) $\frac{1}{15}$ m/s (3) 10 m/s (4) 15 m/s

5. Let the x - y plane be the boundary between two transparent media. Medium 1 in $z \geq 0$ has refractive index of $\sqrt{2}$ and medium 2 with $z < 0$ has a refractive index of $\sqrt{3}$. A ray of light in medium 1 given by the vector $\vec{A} = 6\sqrt{3}\ \hat{i} + 8\sqrt{3}\ \hat{j} - 10\ \hat{k}$ in incident on the plane of separation. The angle of refraction in medium 2 is : **[AIEEE - 2011, 4/120, –1]**

(1) 30º (2) 45º (3) 60º (4) 75º

6. A beaker contains water up to a height h_1 and kerosene of height h_2 above water so that the total height of (water + kerosene) is ($h_1 + h_2$). Refractive index of water is μ_1 and that of kerosene is μ_2. The apparent shift in the position of the bottom of the beaker when viewed from above is :

[AIEEE 2011, 11 MAY; 4/120, –1]

(1) $\left(1+\frac{1}{\mu_1}\right) h_1 - \left(1+\frac{1}{\mu_2}\right) h_2$ (2) $\left(1-\frac{1}{\mu_1}\right) h_1 + \left(1-\frac{1}{\mu_2}\right) h_2$

(3) $\left(1+\frac{1}{\mu_1}\right) h_2 - \left(1+\frac{1}{\mu_2}\right) h_1$ (4) $\left(1-\frac{1}{\mu_1}\right) h_2 + \left(1-\frac{1}{\mu_2}\right) h_1$

7. When monochromatic red light is used instead of blue light in a convex lens, its focal length will :

[AIEEE 2011, 11 MAY; 4/120, –1]

(1) increase (2) decrease
(3) remain same (4) does not depend on colour of light

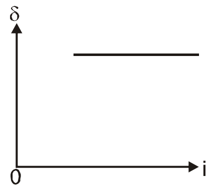
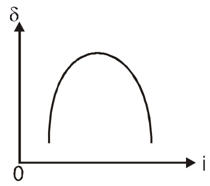
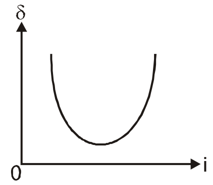
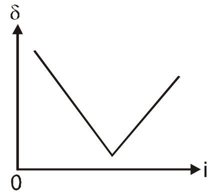
8. An object 2.4 m in front of a lens forms a sharp image on a film 12 cm behind the lens. A glass plate 1 cm thick, of refractive index 1.50 is interposed between lens and film with its plane faces parallel to film. At what distance (from lens) should object shifted to be in sharp focus on film ?

[AIEEE 2012 ; 4/120, –1]

(1) 7.2 m (2) 2.4 m (3) 3.2 m (4) 5.6 m

9. Diameter of a plano - convex lens is 6 cm and thickness at the centre is 3 mm. If speed of light in material of lens is 2×10^8 m/s, the focal length of the lens is : **[JEE(Main) 2013, 4/120, –1]**
(1) 15 cm (2) 20 cm (3) 30 cm (4) 10 cm

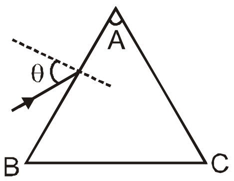
10. The graph between angle of deviation (δ) and angle of incidence (i) for a triangular prism is represented by : **[JEE (Main) 2013; 4/120, –1]**

(1) 
(2) 
(3) 
(4) 

11. A thin convex lens made from crown glass $\left(\mu = \frac{3}{2}\right)$ has focal length f. When it is measured in two different liquids having refractive indices $\frac{4}{3}$ and $\frac{5}{3}$, it has the focal lengths f_1 and f_2 respectively. The correct relation between the focal length is : **[JEE (Main) 2014, 4/120, –1]**
(1) $f_1 = f_2 < f$ (2) $f_1 > f$ and f_2 becomes negative
(3) $f_2 > f$ and f_1 becomes negative (4) f_1 and f_2 both become negative

12. White light is incident from the water to the air - water interface at the critical angle (θ) for green light. Select the correct statement. **[JEE (Main) 2014; 4/120, –1]**
(1) The entire spectrum of visible light will come out of the water at an angle of 90º to the normal.
(2) The spectrum of visible light whose frequency is less than that of green light will come out to the air medium.
(3) The spectrum of visible light whose frequency is more than that of green light will come out to the air medium.
(4) The entire spectrum of visible light will come out of the water at various angles to the normal.

13. Monochromatic light is incident on a glass prism of angle A. If the refractive index of the material of the prism is μ, a ray, incident at an angle θ, on the face AB would get transmitted through the face AC of the prism provided: **[JEE (Main)-2015; 4/120, –1]**



(1) $\theta > \sin^{-1}\left[\mu \sin\left(A - \sin^{-1}\left(\frac{1}{\mu}\right)\right)\right]$ (2) $\theta < \sin^{-1}\left[\mu \sin\left(A - \sin^{-1}\left(\frac{1}{\mu}\right)\right)\right]$
(3) $\theta > \cos^{-1}\left[\mu \sin\left(A + \sin\left(\frac{1}{\mu}\right)\right)\right]$ (4) $\theta < \cos^{-1}\left[\mu \sin\left(A + \sin\left(\frac{1}{\mu}\right)\right)\right]$

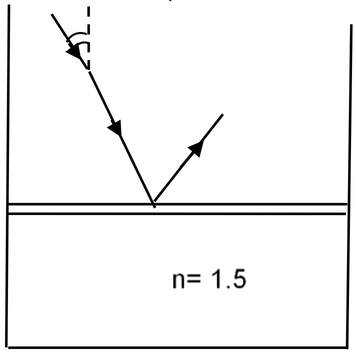
14. An observer looks at a distant tree of height 10 m with a telescope of magnifying power of 20. To the observer the tree appears: **[JEE (Main)-2016; 4/120, –1]**
(1) 10 times nearer (2) 20 times taller (3) 20 times nearer (4) 10 times taller

15. In an experiment for determination of refractive index of glass of a prism by $i - \delta$, plot, it was found that a ray incident at angle 35°, suffers a deviation of 40° and that it emerges at angle 79°. In that case which of the following is closest to the maximum possible value of the refractive index ? **[JEE (Main)-2016; 4/120, –1]**
(1) 1.6 (2) 1.7 (3) 1.8 (4) 1.5

16. A diverging lens with magnitude of focal length 25cm is placed at a distance of 15 cm from a converging lens of magnitude of focal length 20 cm. A beam of parallel light falls on the diverging lens. The final image formed is : **[JEE Main 2017, 4/120, –1]**

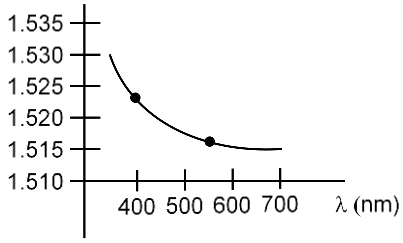
(1) real and at a distance of 6 cm from the convergent lens
(2) real and at a distance of 40 cm from convergent lens.
(3) virtual and at a distance of 40 cm from convergent lens
(4) real and at distance of 40 cm from the divergent lens.

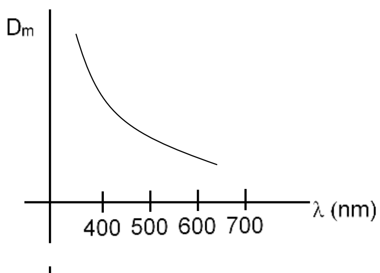
17. Consider a tank made of glass (refractive index 1.5) with a thick bottom. It is filled with a liquid of refractive index μ. A student finds that, irrespective of what the incident angle i (see figure) is for a beam of light entering the liquid, the light reflected from the liquid glass interface is never completely polarized. For this to happen, the minimum value of μ is : **[JEE Main 2019, 4/120, –1]**

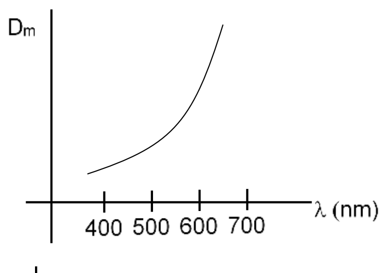


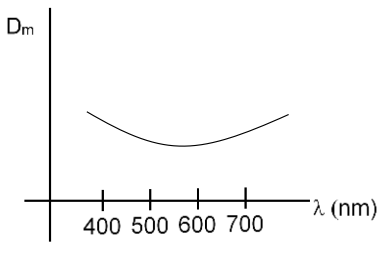
(1) $\frac{5}{\sqrt{3}}$ (2) $\frac{4}{3}$ (3) $\frac{3}{\sqrt{5}}$ (4) $\sqrt{\frac{5}{3}}$

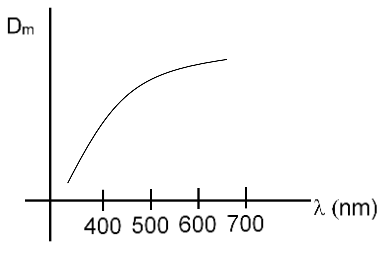
18. The variation of refractive index of a crown glass thin prism with wavelength of the incident light is shown. Which of the following graphs is the correct one, if D_m is the angle of minimum deviation ?
[JEE Main 2019, 4/120, –1]



(1)


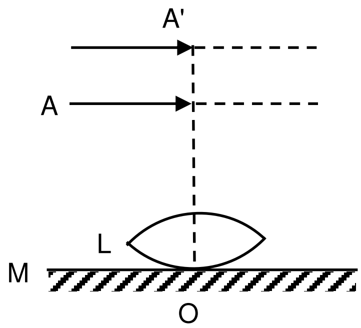
(2)


(3)


(4)


19. A thin convex lens L (refractive index = 1.5) is placed on a plane mirror M. When a pin is placed at A, such that OA = 18 cm, its real inverted image is formed at A itself, as shown in figure. When a liquid of refractive index μ_ℓ is put between the lens and the mirror, the pin has to be moved to A' such that OA' = 27 cm, to get its inverted real image A' itself. The value of μ_ℓ will be :

[JEE Main 2019 April, 4/120, –1]



(1) $\frac{4}{3}$ (2) $\sqrt{2}$ (3) $\sqrt{3}$ (4) $\frac{3}{4}$

Answers

EXERCISE-1

PART - I

SECTION (A) :

A-1. 120º anticlockwise and 240º clockwise.

A-2. 30º clockwise.

A-3. 60º

A-4. Mirror should be placed on the path of the rays at an $\angle$ of 78º or 12° to the horizontal

A-5. (a) 1 ; (b) (4, 0) ; (c) No

A-6. (a) Position of image

$= (1\cos 60^\circ\,\hat{i}\,, -1\sin 60^\circ\,\hat{j})$

(b) Velocity of image

$= (1\cos 60^\circ, +1\sin 60^\circ)$ m/s.

SECTION (B) :

B-1. Infinitely large.

B-2. $\frac{245}{4}$ cm = 61.25 cm

B-3. 10.35 cm = $\frac{3933}{380}$ cm

B-4. 84 cm, 0.5 cm

B-5. 0.2 m from the mirror

B-6. (a) 40 cm/s opposite to the velocity of object., (b) 20 cm/s opposite to the velocity of object.

B-7. 60 cm **B-8.** 86 cm

SECTION (C) :

C-1. $2/3 \times 10^{-8}$ sec

C-2. $3\left(\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{7}}\right)$cm = 9.9 mm, 45°

C-3. 30 cm **C-4.** 25 cm.

C-5. 9 cm/s **C-6.** 35 cm, Shift = 5 cm.

C-7. $\frac{68}{3}$ cm **C-8.** 0.9 cm above P

C-9. $\sqrt{3}$ cm **C-10.** $\frac{\pi h^2}{\mu^2 - 1}$

C-11. 45° **C-12.** n > $\sqrt{2}$

SECTION (D) :

D-1. 90º **D-2.** θ = 60°

D-3. (i) 1.5°, (ii) $\frac{3^\circ}{8}$ **D-4.** $2\sin^{-1}\frac{1}{\mu}$

SECTION (E) :

E-1. 40 cm from pole in the medium of refractive index 1, virtual, erect and 4 cm in size.

E-2. 80 cm **E-3.** 50 cm right of B.

E-4. $\frac{27}{2}$ = 13.5 cm below the surface of water

E-5. 8/3 mm, virtual at v = –20, no inversion

E-6. (a) 2, (b) not possible, it will focus close to the centre if the refractive index is large

E-7. m = 4/3

SECTION (F) :

F-1. ± 12 cm, ± 60 cm

F-2. 360 cm; ∞ ; – 600 cm

F-3. Converging ; real

F-4. (a) $\frac{\mu_3 R}{2\mu_2 - \mu_1 - \mu_3}$ (b) $\frac{\mu_1 R}{2\mu_2 - \mu_1 - \mu_3}$

F-5. **(i)** 7/5

(ii) In this liquid the 1st lens will be diverging & the 2nd a converging one

F-6. 20 cm, 1 m, –4, 4 cm **F-7.** 0.3 m

F-8. 1.5 cm **F-9.** 0.4 cm

F-10. 30 cm

F-11. 60 cm from the lens further away from the mirror

F-12. $\frac{5}{3}$ cm from the lens

F-13. 31 cm from the lens

F-14. 1.0 cm if the light is incident from the side of concave lens and 2.5 mm if it is incident from the side of the convex lens and the corresponding ratio of intensities are 1/4 and 4.

SECTION (G) :

G-1. 10 D, Optical power of each lens = 5 D.

G-2. 10 cm for convex lens and 60 cm for concave lens

G-3. (a) 15 cm from the lens on the axis (b) 1.14 cm towards the lens

SECTION (H) :

H-1. (a) 1/4 = 0.25 (b) 0.90°

H-2. 4°

H-3. (a) $\frac{2(\mu_v - \mu_r)}{\mu_v' - \mu_r'}$, (b) $\frac{2(\mu_y - 1)}{\mu_y' - 1}$

H-4. $\frac{99}{4900}$

H-5. (a) 3° (b) 0.015º (c) 3º (d) 0.225°

SECTION (I) :

I-1. 24; 150 cm

I-2. u_0 = – 1.45, v_0 = 8.75, L = $v_0 + f_e$ = 13.75

I-3. (a) $v_e = -2.5$ cm and $f_e = 6.25$ cm give $u_e = -5$ cm ; $v_0 = (15 - 5)$ cm = 10 cm.
$f_0 = u_0 = -2.5$ cm;
Magnifying power = 10/2.5 × 25/5 = 20
(b) $u_e = -6.25$ cm, $v_0 = (15 - 6.25)$ cm = 8.75, $f_0 = 2.0$ cm. Therefore,
$u_0 = -(70/27) = -2.59$ cm.
Magnifying power = $v_0/|u_0| \times (25/6.25)$ = 27/8 × 4 = 13.5

PART - II

SECTION (A) :

A-1.	(B)	**A-2.**	(B)	**A-3.**	(A)
A-4.	(C)	**A-5.**	(D)	**A-6.**	(C)
A-7.	(A)	**A-8.**	(B)	**A-9.**	(C)

SECTION (B) :

B-1.	(A)	**B-2.**	(B)	**B -3.**	(C)
B-4.	(C)	**B-5.**	(C)	**B-6.**	(C)
B-7.	(B)	**B-8.**	(A)	**B-9.**	(C)
B-10.	(C)	**B-11.**	(A)	**B-12.**	(C)
B-13.	(C)	**B-14.**	(B)	**B-15.**	(A)
B-16.	(B)				

SECTION (C) :

C-1.	(B)	**C-2.**	(C)	**C-3.**	(A)
C-4.	(D)	**C-5.**	(A)	**C-6.**	(A)
C-7.	(A)	**C-8.**	(D)	**C-9.**	(B)

SECTION (D) :

D-1 .	(B)	**D-2.**	(C)	**D-3.**	(C)	
D-4.	**(a)**	(A)	**(b)**	(C)	**(c)**	(C)
D-5.	(C)	**D-6.**	(B)			

SECTION (E) :

E-1.	(B)	**E-2.**	(A)	**E-3.**	(A)
E-4.	(D)				

SECTION (F) :

F-1.	(A)	**F-2.**	(A)	**F-3.**	(C)
F-4.	(C)	**F-5.**	(D)	**F-6.**	(D)
F-7.	(D)	**F-8.**	(C)	**F-9.**	(B)
F-10.	(D)	**F-11.**	(A)	**F-12.**	(B)
F-13.	(A)	**F-14.**	(A)	**F-15.**	(C)

SECTION (G) :

G-1.	(A)	**G-2.**	(B)	**G-3.**	(A)
G-4.	(C)	**G-5.**	(D)		

SECTION (H) :

H-1.	(D)	**H-2.**	(D)	**H-3.**	(C)
H-4.	(B)	**H-5.**	(D)	**H-6.**	(D)

SECTION (I) :

I-1.	(D)	**I-2.**	(C)	**I-3.**	(C)
I-4.	(D)	**I-5.**	(A)	**I-6.**	(B)
I-7.	(B)	**I-8.**	(D)	**I-9.**	(C)
I-10.	(B)				

PART - III

1. (A) – p ; (B) – p ; (C) – q ; (D) – q
2. (A) – p, q ; (B) – p,q ; (C) – r, s ; (D) – r, s
3. (A) – p, s ; (B) – q ; (C) – r, s ; (D) – r

EXERCISE-2

PART - I

1.	(C)	**2.**	(B)	**3.**	(A)
4.	(C)	**5.**	(D)	**6.**	(A)
7.	(B)	**8.**	(B)	**9.**	(B)
10.	(B)	**12.**	(B)	**13.**	(A)
14.	(B)	**15.**	(A)		

PART - II

1.	3	**2.**	45	**3.**	10
4.	6	**5.**	2	**6.**	13
7.	6	**8.**	(a) 0,	(b) 6	
9.	6	**10.**	16	**11.**	5
12.	0	**13.**	12	**14.**	30
15.	9	**16.**	10	**17.**	20
18.	90	**19.**	4		

PART - III

1.	(AB)	**2.**	(ACD)	**3.**	(BD)
4.	(CD)	**5.**	(AD)	**6.**	(AC)
7.	(ABC)	**8.**	(BC)	**9.**	(AC)
10.	(BC)	**11.**	(ABC)	**12.**	(BD)
13.	(AD)	**14.**	(ABD)	**15.**	(AB)
16.	(BC)	**17.**	(AC)	**18.**	(AD)
19.	(AD)	**20.**	(ABD)	**21.**	(ABCD)

PART - IV

1.	(D)	**2.**	(C)	**3.**	(D)
4.	(B)	**5.**	(D)	**6.**	(D)
7	(B)	**8.**	(B)	**9.**	(C)
10.	(D)	**11.**	(B)		

EXERCISE-3

PART – I

1. (A) **2.** (B)
3. (A) → (p,q,r,s); (B) → (q);
(C) → (p,q,r,s); (D) → (p,q,r,s)

4.	(C)	**5**	(CD)	**6.**	(ABC)
7.	6	**8.**	(B)	**9.**	3
10.	6				

11. (A) – p, r ; (B) –q,s,t ; (C) – p,r,t ; (D) – q, s

12.	(C)	**13.**	2	**14.**	(B)
15.	(B)	**16.**	(C)	**17.**	(C)
18.	(A)	**19.**	(AC)	**20.**	(C)
21.	(B)	**22.**	7	**23.**	(B)
24.	2	**25.**	(AC)	**26.**	(D)
27.	(A)	**28.**	(AD)	**29.**	(ACD)
30.	(B)	**31.**	(ACD)	**32.**	(8)
33.	130.00		**34.**	(D)	

PART - II

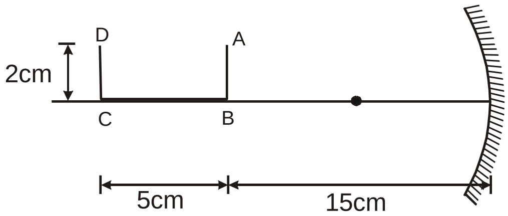
1.	(1)	**2.**	(3)	**3.**	(4)
4.	(2)	**5.**	(2)	**6.**	(2)
7.	(1)	**8.**	(4)	**9.**	(3)
10.	(3)	**11.**	(2)	**12.**	(2)
13.	(1)	**14.**	(3)	**15.**	(4)
16.	(2)	**17.**	(3)	**18.**	(1)
19.	(1)				

High Level Problems (HLP)

Marked Questions can be used as Revision Questions.

SUBJECTIVE QUESTIONS

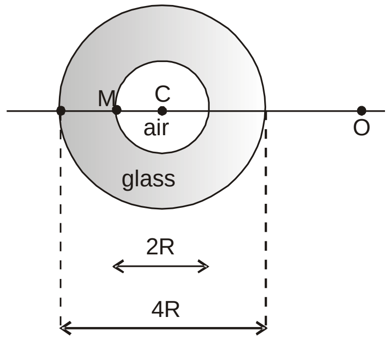
1. A U-shaped wire is placed before a concave mirror having radius of curvature 20 cm as shown in figure. Find the total length of the image.



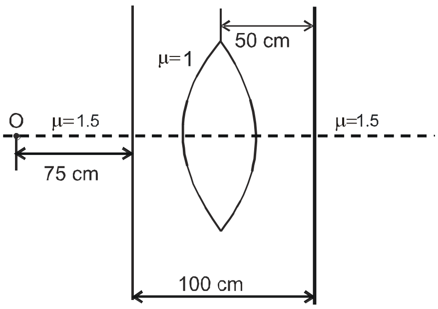
2. (i) A paper weight of refractive index n = 3/2 in the form of a hemisphere of radius 3.0 cm is used to hold down a printed page. An observer looks at the page vertically through the paperweight. At what height above the page will the printed letters near the centre appear to the observer?

(ii) Solve the previous problem if the paperweight is inverted at its place so that the spherical surface touches the paper.

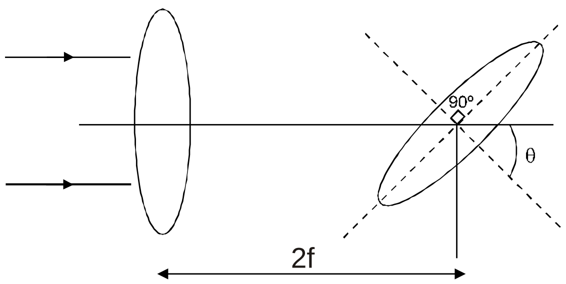
3. In the given figure, a hollow sphere of glass of refractive index n has a small mark M on its interior surface which is observed by an observer O from a point outside the sphere. C is centre of the sphere. The inner cavity (air) is concentric with the external surface and thickness of the glass is everywhere equal to the radius of the inner surface. Find the distance by which the mark will appear nearer than it really is, in terms of n and R assuming paraxial rays.



4. Two media each of refractive index 1.5 with plane parallel boundaries are separated by 100 cm. A convex lens of focal length 60 cm is placed midway between them with its principal axis normal to the boundaries. A luminous point object O is placed in one medium on the axis of the lens at a distance 125 cm from it. Find the position of its image formed as a result of refraction through the system.



5. Two converging lenses of the same focal length f are separated by distance 2 f as shown in figure. The axis of the second lens is inclined at small angle θ with respect to the axis of the first lens. A parallel paraxial beam of light is incident from left side on the lens. Find the coordinates of the final image with respect to the origin of the first lens.

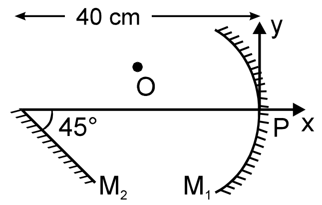


6. Two plane mirrors form an angle of 120°. The distance between the two images of a point source formed in them is 20 cm. Determine the distance from the light source to the point where the mirrors touch if it lies on the bisector of angle formed by the mirrors.

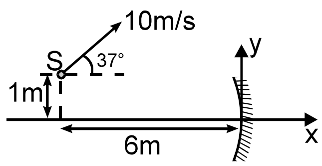
7. A kid of height 1.1 ft is sleeping straight between focus and centre of curvature along the principal axis of a concave mirror of small aperture. His head is towards the mirror and is 0.5 ft from the focus of the mirror. How a plane mirror should be placed so that the image formed by it due to reflected light from concave mirror looks like a person of height 5.5 ft standing vertically. Draw the ray diagram. Find the focal length of the concave mirror.

8. The average size of an Indian face is 24 × 16 cm$^{2.}$ Find the minimum size of a plane mirror required to see the face completely by:
(i) one eyed man **(ii)** two eyed man. (Distance between eyes is = 4 cm)

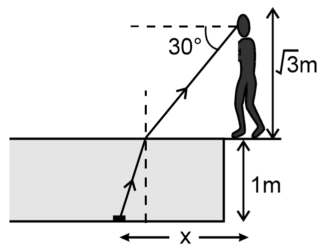
9. As shown in the figure, an object O is at the position (– 10, 2) with respect to the origin P. The concave mirror M_1 has radius of curvature 30 cm. A plane mirror M_2 is kept at a distance 40 cm infront of the concave mirror. Considering first reflection on the concave mirror M_1 and second on the plane mirror M_2. Find the coordinates of the second image w.r.t. the origin P.



10. A point source S is moving with a speed of 10 m/s in x-y plane as shown in the figure. The radius of curvature of the concave mirror is 4m. Determine the velocity vector of the image formed by paraxial rays.



11. A man is standing at the edge of a 1m deep swimming pool, completely filled with a liquid of refractive index $\sqrt{3/2}$. The eyes of the man are $\sqrt{3}$ m above the ground. A coin located at the bottom of the pool appears to be at an angle of depression of 30^0 with reference to the eye of man. Then find horizontal distance (represented by x in the figure) of the coin from the eye of the man.

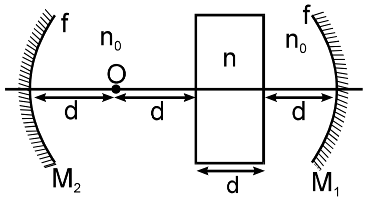


12. An object lies in front of a thick parallel glass slab, the bottom of which is polished. If the distance between first two images formed by bottom surface is 4cm then find the thickness of the slab.
[Assume n_{glass} = 3/2 and paraxial rays]

13. A beam of parallel rays of diameter ' b ' propagates in glass at an angle θ to its plane. Find the diameter of the beam when it goes to air through this face. (n_{glass} = n)

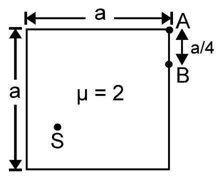
14. A small ball is thrown from the edge of one bank of a river of width 100 m to just reach the other bank. The ball was thrown in the vertical plane (which is also perpendicular to the banks) at an angle 37° to the horizontal. Taking the starting point as the origin O, vertically upward direction as positive y-axis and the horizontal line passing through the point O and perpendicular to the bank as x-axis find:
(a) equation of trajectory of the image formed by refraction by the water surface
(water surface is at the level y = 0)
(b) instantaneous velocity of the image formed due to refraction. [Use g = 10 m/s^2, R. I. of water =4/3]

15. Two concave mirrors each of focal length 'f' are placed infront of each other co-axially at a distance of 4d in a medium of refractive index n_0. A plane glass slab of refractive index 'n' & thickness 'd' is placed at a distance of 'd' from M_1. A point object O is placed at a distance of 'd' from M_2 as shown in the figure. Consider first reflection by M_2, then refraction on slab and then reflection by M_1. Determine the distance of this image after reflection from M_1.

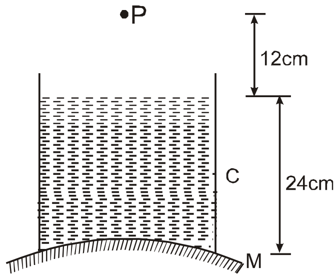


16. An observer observes a fish moving upwards in a cylindrical container of cross section area 1 m^2 filled with water up to a height of 5 m. A hole is present at the bottom of the container having cross section area 1/1000 m^2. Find out the speed of the image of fish observed by observer when the bottom hole is just opened. (Given: The fish is moving with the speed of 6 m/s towards the observer, μ of water = 4/3)

17. The figure shows the square front face (of side 'a') of a transparent cuboidal block. The thickness or the third dimension of the block is negligible in comparison to 'a'. The block has uniform refractive index μ equal to 2. A point source S which can emit light in all directions can move inside the block. It is desired that no light of 'S' should pass through AB. Sketch the region in which S should be present to satisfy this condition.

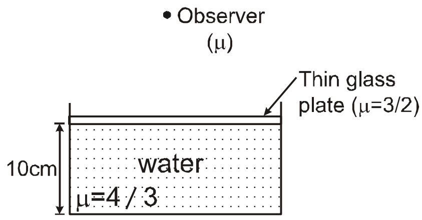


18. An insect at point 'P' sees its two images in the water-mirror system as shown in the figure. One image is formed due to direct reflection from water surface and the other image is formed due to refraction, reflection & again refraction by water mirror system in order. Find the separation between the two images. M has focal length 60 cm. (n_w = 4/3)

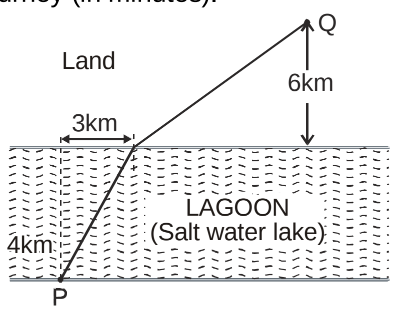


19. A ray of light is incident on a surface in a direction given by vector $\vec{A} = 2\hat{i} - 2\hat{j} + \hat{k}$. The normal to that surface passing through the point of incidence is along the vector $\vec{N} = \hat{j} - 2\hat{k}$. The unit vector in the direction of reflected ray is given by $\vec{R} = a\hat{i} + b\hat{j} + c\hat{k}$. Find three equations in terms of a, b, c using which we can find the values of a, b & c.

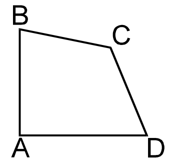
20. In the given figure if observer sees the bottom of vessel at 8 cm, find the refractive index of the medium in which observer is present.



21. A man starting from point P crosses a 4 km wide lagoon and reaches point Q in the shortest possible time by the path shown in the figure. If the person swims at a speed of 3 km/hr and walks at a speed of 4 km/hr, then find his time of journey (in minutes).



22. In the given figure, the faces of prism ABCD made of glass with a refractive index n form dihedral angles $\angle A = 90°$, $\angle B = 75°$, $\angle C = 135°$ & $\angle D = 60°$ (The Abbe's prism). A beam of light falls on face AB & after total internal reflection from face BC escapes through face AD. Find the range of n and angle of incidence α of the beam onto face AB, if a beam that has passed through the prism in this manner is perpendicular to the incident beam.



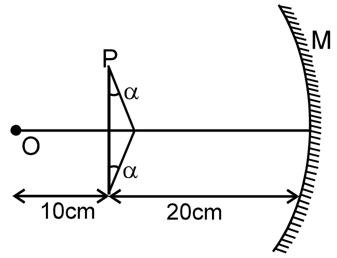
23. A point source of light is placed at a distance h below the surface of a large deep lake.

(a) Show that the fraction f of the light energy that escapes directly from the water surface is independent of h and is given by $f = \frac{1}{2} - \frac{1}{2n}\sqrt{n^2 - 1}$ where n is the index of refraction of water.
(**Note:** Absorption within the water and reflection at the surface; except where it is total, have been neglected)

(b) Evaluate this ratio for n = 4/3.

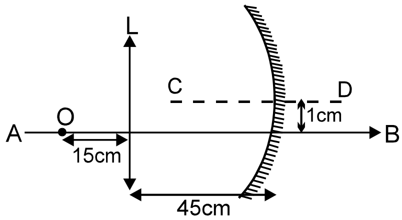
24. A glass prism with a refracting angle of 60° has a refractive index 1.52 for red and 1.6 for violet light. A parallel beam of white light is incident on one face at an angle of incidence, which gives minimum deviation for red light. Find :
(a) the angle of incidence
(b) angular width of the spectrum
(c) the length of the spectrum if it is focussed on a screen by lens of focal length 100 cm.
[**Use:** sin (49.7º) = 0.760; sin (31.6º) = 0.520 ; sin (28.4º) = 0.475; sin (56º) = 0.832 ; = 22/7]

25. In the given figure, O is a point object kept on the principal axis of a concave mirror M of radius of curvature 20 cm. P is a prism of angle $\alpha = 1.8°$. Light falling on the prism (at small angle of incidence) get refracted through the prism and then fall on the mirror. Refractive index of prism is 3/2. Find the distance between the images formed by the concave mirror due to this light.



26. Light travelling in air falls at an incidence angle of 2° on one refracting surface of a prism of refractive index 1.5 and angle of refraction 4º. The medium on the other side is water (n = 4/3). Find the deviation produced by the prism.

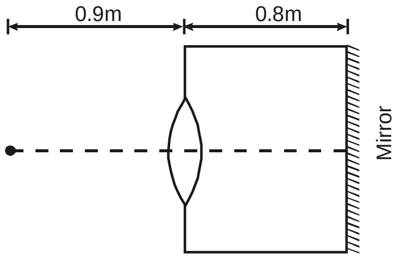
27. In the figure shown L is a converging lens of focal length 10cm and M is a concave mirror of radius of curvature 20cm. A point object O is placed in front of the lens at a distance 15cm. AB and CD are optical axes of the lens and mirror respectively. Find the distance of the final image formed by this system from the optical centre of the lens. The distance between CD & AB is 1 cm.



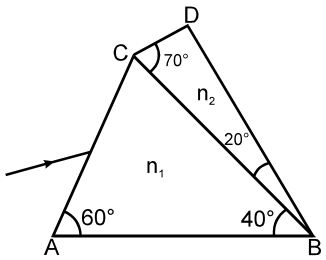
28. An object is kept at rest on the principal axis of a lens. Initially the object is at a distance three times the focal length ' f ' of the lens. The lens runs towards the object at a constant speed u, until the distance between the object and its real image becomes 4 f. If the image always forms on a moving screen then express the velocity of the screen as a function of time.

29. A convex lens produces an image of a candle flame upon a screen whose distance from candle is D. When the lens is displaced through a distance x, (the distance between the candle and the screen is kept constant), it is found that a sharp image is again produced upon the screen. Find the focal length of the lens in terms of D and x.

30. A thin equiconvex lens of glass of refractive index $\mu = 3/2$ & of focal length 0.3 m in air is sealed into an opening at one end of a tank filled with water ($\mu = 4/3$). On the opposite side of the lens, a mirror is placed inside the tank on the tank wall perpendicular to the lens axis, as shown in figure. The separation between the lens and the mirror is 0.8 m. A small object is placed outside the tank infront of the lens at a distance of 0.9 m from the lens along its axis. Find the position (relative to the lens) of the image of the object formed by the system.



31. A prism of refractive index n_1 and another prism of refractive index n_2 are stuck together without a gap as shown in the figure. The angles of the prisms are as shown. n_1 and n_2 depend on λ, the wavelength of light according to $n_1 = 1.20 + \frac{10.8 \times 10^4}{\lambda^2}$ and $n_2 = 1.45 + \frac{1.80 \times 10^4}{\lambda^2}$ where λ is in nm.

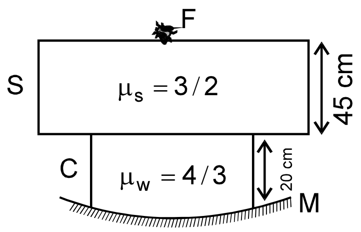


(i) Calculate the wavelength λ_0 for which rays incident at any angle on the interface BC pass through without bending at that interface.

(ii) For light of wavelength λ_0, find the angle of incidence i on the face AC such that the deviation produced by the combination of prisms is minimum.

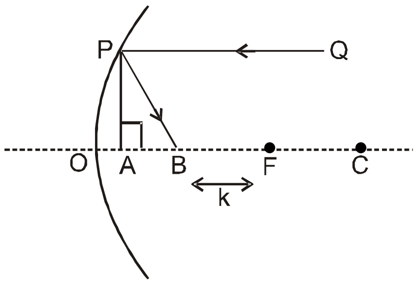
32. A pole of length 2.00 m stands half dipped in a swimming pool with water level 1 m higher than the bed (bottom). The refractive index of water is 4/3 and sunlight is coming at an angle of 37° with the vertical. Find the length of the shadow of the pole on the bed.
Use $\sin^{-1}(0.45) = 26.8°$, $\tan(26.8°) = 0.5$

33. A fly F is sitting on a glass slab S 45cm thick & of refractive index 3/2. The slab covers the top of a container C containing water (R.I. 4/3) upto a height of 20 cm. Bottom of container is closed by a concave mirror M of radius of curvature 40 cm. Locate the final image formed by all refractions & reflection assuming paraxial rays.



34. A glass porthole is made at the bottom of a ship for observing sea life. The hole diameter D is much larger than the thickness of the glass. Determine the area S of the field of vision at the sea bottom for the porthole if the refractive index of water is μ_w and the sea depth is h.

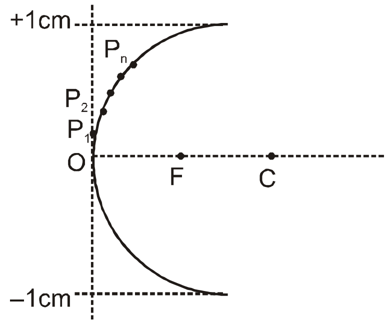
35. The figure below depicts a concave mirror with center mirror with center of curvature C focus F, and a horizontally drawn OFC as the optic axis. The radius of curvature is R (OC = R) and OF = R/2). A ray of ligth QP, parallel to the optical axis and at a perpendicular distance w($w \leq R/2$) from it, is incident on the mirror at P. It is reflected to the point B on the optical axis, such that BF = k. Here k is a measure of lateral aberration.



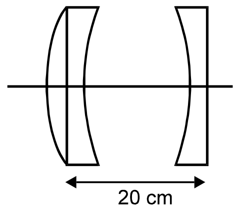
(a) Express k in terms of {w, R}. k =

(b) Sketch k vs w for $w \in [0, R/2]$

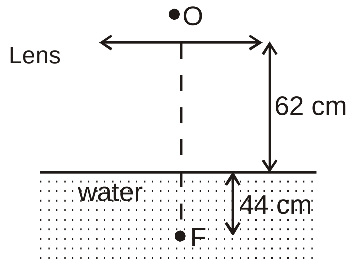
(c) Consider points P_1, P_2 ......P_n on the concave mirror which are increasingly further away from the optic centre O and approximately equidistant from each other(see figure below). Rays parallel to the optic axis are incident at P_1, P_2,.......P_n and reflected to points on the optic axis. Consider the points where these rays reflected from P_n, P_{n-1}, ...P_2 intersect the rays reflected from P_{n-1}, P_{n-2}, ... P_1 respectively. Qualitatively sketch the locus of these points in figure below for a mirror (shown with solid line) with radius of curvature 2 cm.



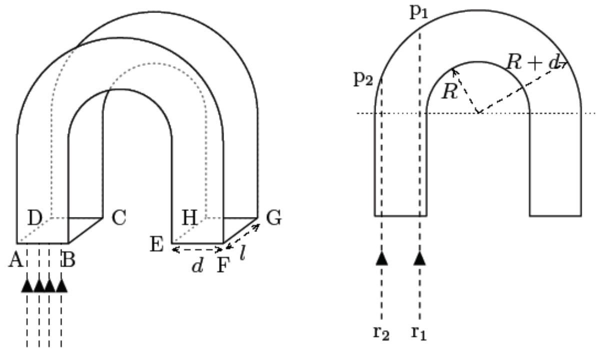
36. A symmetrical converging convex lens of focal length 10 cm & diverging concave symmetrical lens of focal length –20 cm are cut from the middle and perpendicularly and symmetrically to their principal axis. The parts thus obtained are arranged as shown in the figure. Find the focal length (in cm) of this arrangement



37. In the given figure, a stationary observer O looking at a fish F (in water of, $\mu = 4/3$) through a converging lens of focal length 90 cm. The lens is allowed to fall freely from a height 62.0 cm with its axis vertical. The fish and the observer are on the principal axis of the lens. The fish moves up with constant velocity 100 cm/s. Initially it was at a depth of 44.0 cm. The velocity with which the fish appears to move to the observer at t = 0.2 sec is($x+\frac{3}{4}$)m/s. Find the value of x. ($g = 10\ m/s^2$)



38. A glass rod of refractive index 1.50 of rectangular cross section {$d \times \ell$} is bent into a "U" shape see Fig. (A). The cross sectional view of this rod is shown in Fig.(B).



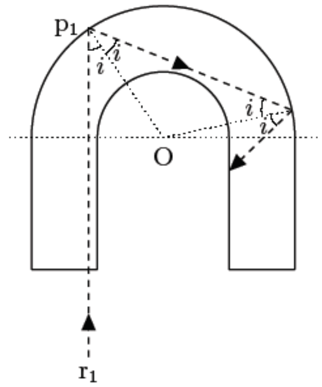
(A) (B)

Bent portion of the rod is semi-circular with inner and outer radii R and R + d respectively. Parallel monochromatic beam of light is incident normally on face ABCD. **[Olympiad-2016, Stage-2]**

(a) Consider two monochromatic rays r_1 and r_2 in Fig. (B). State whether the following statements are True or False.

Statement	True/ False
If r_1 is total internally reflected from the semi circular section at the point p_1 then r_2 will necessarily be total internally reflected at the point p_2.	True
If r_2 is total internally reflected from the semi circular section at the point p_2 then r_1 will necessarily be total internally reflected at the point p_1	False

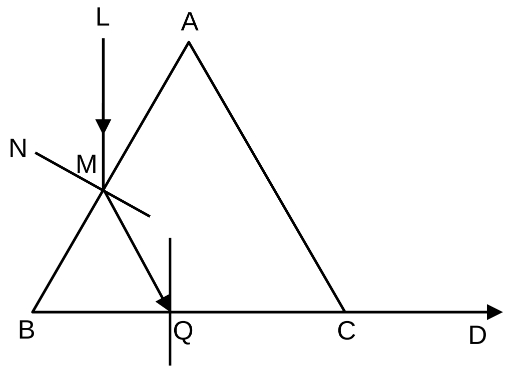
(b) Consider the ray r_1 whose point of incidence is very close to the edge BC. Assume it undergoes total internal reflection at p_1. In cross sectional view below, draw the trajectory of this reflected ray beyond the next glass-air boundary that it encounters.



(c) Obtain the minimum value of the ratio R/d for which any light ray entering the glass normally through the face ABCD undergoes at least one total internal reflection.

(d) A glass rod with the above computed minimum ratio of R/d, is fully immersed water of refractive index 1.33. What fraction of light flux entering the glass through the plane surface ABCD undergoes at least one total internal reflection?

39. The following figure shows the section ABC of an equilateral triangular prism. A ray of light enters the prism along LM and emerges along QD. If the refractive index of the material of the prism is 1.6, angle LMN is **[Olympiad 2017 (Stage–I)]**



HLP Answers

1. 16 cm

2. (i) No shift is observed (ii) 1 cm

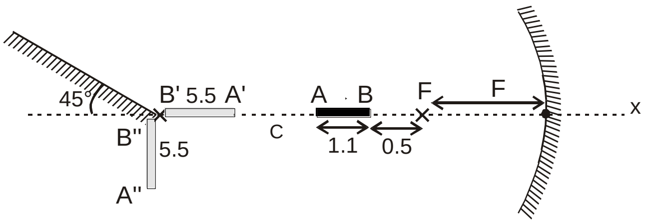
3. $(n - 1)R/(3n - 1)$

4. 200 cm, right of the lens

5. $\frac{f(1-2\cos\theta)}{1-\cos\theta}$, 0

6. 11.5 cm = $\frac{20}{\sqrt{3}}$

7. The plane mirror should be placed at an angle of 45º with negative x-axis; f = 2ft.



8. (i) 12×8 cm^2 (ii) 12×6 cm^2

9. Coordinates of I_2 w.r.t. P = (– 46, – 70)

10. $\vec{V}_i = V_{ix}\hat{i} + V_{iy}\hat{j} = -2\hat{i} - 4\hat{j}$

11. d = 4000 mm

12. t = 2 cm

13. CD = $\frac{b \cdot \sqrt{1-n^2\cos^2\theta}}{\sin\theta}$

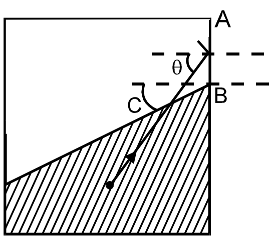
14. (a) $\left(x - \frac{x^2}{100}\right)$

(b) $\vec{v}\,(t)_{image} = 20\sqrt{\frac{5}{3}}\,\hat{i} + 20\left[\sqrt{\frac{5}{3}} - \frac{2}{3}t\right]\hat{j}$

15. $\left|\frac{\left(\frac{df}{f-d}-d\left(1-\frac{n_0}{n}\right)+4d\right)f}{\frac{df}{d-f}+d\left(1-\frac{n_0}{n}\right)-4d+f}\right|$

16. 4.4975 m/s

17.



18. 24 cm.

19. $a^2 + b^2 + c^2 = 1$; $3a + 4b + 2c = 0$; $b - 2c = 4/3$

20. $\frac{16}{15}$

21. 250

22. $\sqrt{2} < n \le 2$, and $45^\circ < \alpha \le 90^\circ$

23. **(b)** $(4 - \sqrt{7})/8$

24. (a) 49.7º, (b) 56º – 49.7º = 6.3º (c) $f\theta$ = 11cm

25. $\frac{\pi}{20}$ cm.

26. 1°

27. $6\sqrt{26}$ cm

28. $v_i = u\left[1 - \left\{\frac{f}{ut-2f}\right\}^2\right]$

29. $f = (D^2 - x^2)/4D$

30. 90 cm from the lens towards right

31. **(i)** λ_0 = 600 nm, n = 1.5

(ii) $i = \sin^{-1}(0.75) = 48.59^\circ$

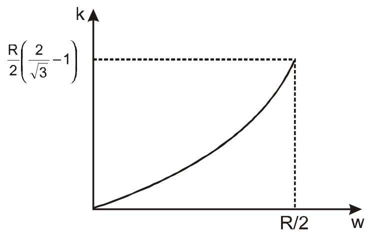
32. 1.25 m

33. $\left(\frac{135}{6}\right)$ cm = 22.5 cm below the upper surface of the glass slab

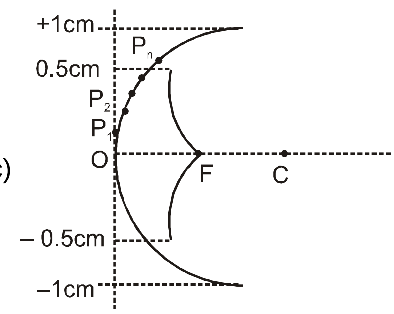
34. $\pi\left[\frac{h}{\sqrt{\mu_w^2-1}} + \frac{D}{2}\right]^2$

35. (a) $k = \frac{R}{2}\left[\frac{R}{(R^2-\omega^2)^{1/2}} - 1\right]$

(b)



(c)



36. 80 cm **37.** 22

38. For total internal reflection

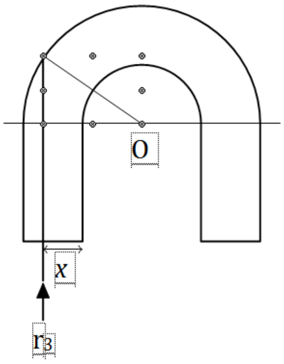
$\sin i \geq \sin\theta_c$

where is the incidence angle of the ray on bent portion of the rod (see figure in part (b)) and is the critical angle for glass-air boundary. For a light ray close to edge BC

$$\frac{R}{R+d} \geq \frac{1}{n_{glass}}$$

For refractive index $n_{glass} = 1.5$ $R \geq 2d$

Minimum value of R/d = 2



Let the intensity of beam be I_0. Flux entering through glass slab will be dlI_0. Assume that any light ray up to distance x from the edge BC undergoes at least one total internal reflection. Then the flux going through at least one total internal reflection will be $(d - x)lI_0$. Also

$$\frac{R+x}{R+d} = \frac{n_{water}}{n_{glass}}$$

For R/ d = 2 and $n_{water} = 1.33$, x = 2d/ 3.

Fraction of light = 0.33

39. 35.6°