WORK, POWER & ENERGY

Contents

JEE (ADVANCED) SYLLABUS

Work, Power & Energy : Work, Power, Kinetic Energy, Potential Energy, Conservation of Mechanical Energy.

JEE (MAIN) SYLLABUS

Work, Power & Energy : Work done by a constant force and a variable force; kinetic energy, work energy theorem, power. Notion of potential energy, potential energy of a spring, conservative forces; conservation of mechanical energy (kinetic and potential energies); non-conservative forces; motion in a vertical circle, elastic and inelastic collisions in one and two dimensions.

WORK, POWER & ENERGY

INTRODUCTION :

The term 'work' as understood in everyday life has a different meaning in scientific sense. If a coolie is carrying a load on his head and waiting for the arrival of the train, he is not performing any work in the scientific sense. In the present study, we shall have a look into the scientific aspect of this most commonly used term i.e., work.

WORK DONE BY CONSTANT FORCE:

The physical meaning of the term work is entirely different from the meaning attached to it in everyday life. In everyday life, the term 'work' is considered to be synonym of 'labour', 'toil', effort' etc. In physics, there is a specific way of defining work.

Work is said to be done by a force when the force produces a displacement in the body on which it acts in any direction except perpendicular to the direction of the force.

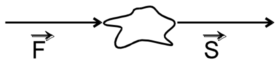
For work to be done, following two conditions must be fulfilled.

(i) A force must be applied.

(ii) The applied force must produce a displacement in any direction except perpendicular to the direction of the force.

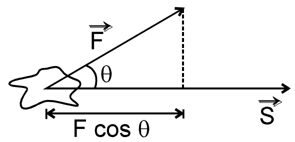
Suppose a force $\vec{F}$ is applied on a body in such a way that the body suffers a displacement $\vec{S}$ in the direction of the force. Then the work done is given by

W = FS



However, the displacement does not always take place in the direction of the force. Suppose a constant force $\vec{F}$, applied on a body, produces a displacement $\vec{S}$ in the body in such a way that $\vec{S}$ is inclined to $\vec{F}$ at an angle θ. Now the work done will be given by the dot product of force and displacement.

W = $\vec{F}.\vec{S}$



Since work is the dot product of two vectors therefore it is a scalar quantity.

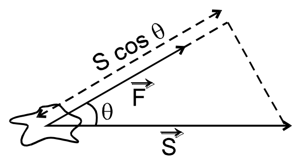
W = FS cos θ or W = (F cos θ)S

$\therefore$ W = component of force in the direction of displacement × magnitudes of displacement.

So work is the product of the component of force in the direction of displacement and the magnitude of the displacement.

Also, W = F (S cos θ)

or work is product of the component of displacement in the direction of the force and the magnitude of the displacement.



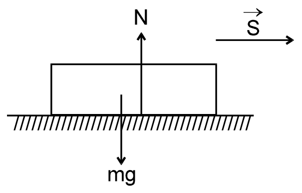
Special Cases :

Case (I) : When θ = 90º, then W = FS cos 90º = 0.

So, work done by a force is zero if the body is displaced in a direction perpendicular to the direction of the force.

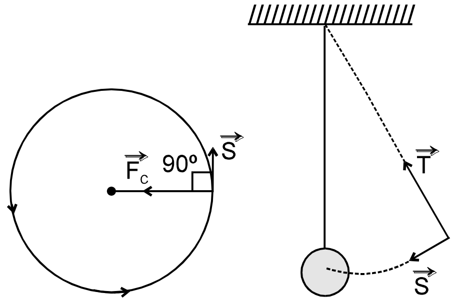
Examples :

1. Consider a body sliding over a horizontal surface. The work done by the force of gravity and the reaction of the surface will be zero. This is because both the force of gravity and the reaction act normally to the displacement.



The same argument can be applied to a man carrying a load on his head and walking on a railway platform.

2. Consider a body moving in a circle with constant speed. At every point of the circular path, the centripetal force and the displacement are mutually perpendicular (Figure). So, the work done by the centripetal force is zero. The same argument can be applied to a satellite moving in a circular orbit. In this case, the gravitational force is always perpendicular to displacement. So, work done by gravitational force is zero.



3. The tension in the string of a simple pendulum is always perpendicular to displacement. (Figure). So, work done by the tension is zero.

Case (II) : When S = 0, then W = 0.

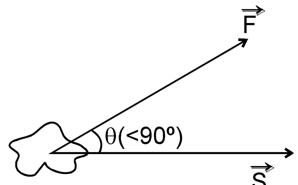
So, work done by a force is zero if the body suffers no displacement on the application of a force.

Example : A person carrying a load on his head and standing at a given place does no work.

Case (III) : When 0º ≤ θ < 90º [Figure], then cos θ is positive. Therefore.

W (= FS cos θ) is positive.

Work done by a force is said to be positive if the applied force has a component in the direction of the displacement.



Positive work

Examples :

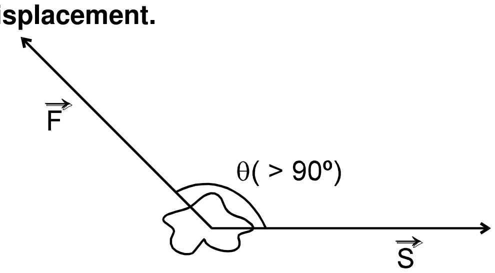
1. When a horse pulls a cart, the applied force and the displacement are in the same direction. So, work done by the horse is positive.

2. When a load is lifted, the lifting force and the displacement act in the same direction. So, work done by the lifting force is positive.

3. When a spring is stretched, both the stretching force and the displacement act in the same direction. So, work done by the stretching force is positive.

Case (IV) : When 90º < θ ≤ 180º (Figure), then cosθ is negative. Therefore W (= FS cos θ) is negative.

Work done by a force is said to be negative if the applied force has component in a direction opposite to that of the displacement.



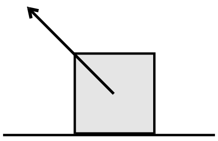
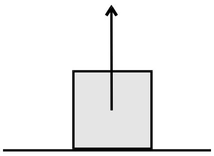
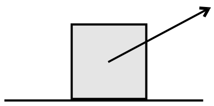
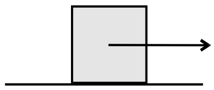
Negative work

Examples :

1. When brakes are applied to a moving vehicle, the work done by the braking force is negative. This is because the braking force and the displacement act in opposite directions.
2. When a body is dragged along a rough surface, the work done by the frictional force is negative. This is because the frictional force acts in a direction opposite to that of the displacement.
3. When a body is lifted, the work done by the gravitational force is negative. This is because the gravitational force acts vertically downwards while the displacement is in the vertically upwards direction.

Solved Example

Example 1. Figure shows four situations in which a force acts on a box while the box slides rightward a distance d across a frictionless floor. The magnitudes of the forces are identical, their orientations are as shown. Rank the situations according to the work done on the box during the displacement, from most positive to most negative.

(A)  (B)  (C)  (D) 

Answer : D, C, B, A

Explanation : In (D) $\theta = 0^\circ$, $\cos\theta = 1$ (maximum value). So, work done is maximum.

In (C) $\theta < 90^\circ$, $\cos\theta$ is positive. Therefore, W is positive.

In (B) $\theta = 90^\circ$, $\cos\theta$ is zero. W is zero.

In (A) θ is obtuse, $\cos\theta$ is negative. W is negative.

WORK DONE BY MULTIPLE FORCES :

If several forces act on a particle, then we can replace $\vec{F}$ in equation $W = \vec{F}.\vec{S}$ by the net force $\Sigma\vec{F}$ where

$\Sigma = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + ...$

$\therefore \quad W = [\Sigma\vec{F}].\vec{S}$ ...(i)

This gives the work done by the net force during a displacement of the particle.

We can rewrite equation (i) as :

$W = \vec{F}_1.\vec{S} + \vec{F}_2\vec{S} + \vec{F}_3\vec{S} + . \quad . \quad . \quad .$

or $\quad W = W_1 + W_2 + W_3 + ...$

So, the work done on the particle is the sum of the individual works done by all the forces acting on the particle.

Important points about work :

1. Work is defined for an interval or displacement. There is no term like instantaneous work similar to instantaneous velocity.
2. For a particular displacement, work done by a force is independent of type of motion i.e. whether it moves with constant velocity, constant acceleration or retardation etc.
3. For a particular displacement work is independent of time. Work will be same for same displacement whether the time taken is small or large.
4. When several forces act, work done by a force for a particular displacement is independent of other forces.
5. A force is independent from reference frame. Its displacement depends on frame so work done by a force is frame dependent therefore work done by a force can be different in different reference frame.
6. Effect of work is change in kinetic energy of the particle or system.
7. Work is done by the source or agent that applies the force.

Units of work :

1. Unit of work :

I. In cgs system, the unit of work is erg.
One erg of work is said to be done when a force of one dyne displaces a body through one centimetre in its own direction.
$\therefore$ 1 erg = 1 dyne × 1 cm = 1g cm s^{-2} × 1 cm = 1 g cm^2 s^{-2}
Note : Erg is also called dyne centimetre.

II. In SI i.e., International System of units, the unit of work is joule (abbreviated as J). It is named after the famous British physicist James Personal Joule (1818 – 1869).
One joule of work is said to be done when a force of one Newton displaces a body through one metre in its own direction.
1 joule = 1 Newton × 1 metre = 1 kg ×1 m/s^2 × 1 m = 1kg m^2 s^{-2}
Note : Another name for joule is Newton metre.

Relation between joule and erg

1 joule = 1 Newton × 1 metre ; 1 joule = 10^5 dyne × 10^2 cm = 10^7 dyne cm
1 joule = 10^7 erg ; 1 erg = 10^{-7} joule

DIMENSIONS OF WORK :

[Work] = [Force] [Distance] = [MLT^{-2}] [L] = [ML^2T^{-2}]
Work has one dimension in mass, two dimensions in length and '–2' dimensions in time,
On the basis of dimensional formula, the unit of work is kg m^2 s^{-2}.
Note that 1 kg m^2 s^{-2} = (1 kg m s^{-2}) m = 1 N m = 1 J.

Solved Examples

Example 2. There is an elastic ball and a rigid wall. Ball is thrown towards the wall. The work done by the normal reaction exerted by the wall on the ball is -
(A) +ve (B) – ve (C) zero (D) None of these
Answer : **(C)**
Solution : As the point of application of force does not move, the w.d by normal reaction is zero.

Example 3. Work done by the normal reaction when a person climbs up the stairs is -
(A) +ve (B) – ve (C) zero (D) None of these
Answer : **(C)**
Solution : As the point of application of force does not move, the w.d by normal reaction is zero.

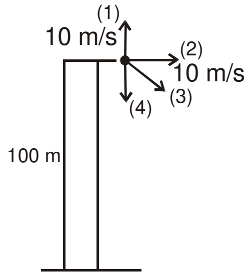
Example 4. Work done by static friction force when a person starts running is ________.
Solution : As the point of application of force does not move, the w.d by static friction is zero.

WORK DONE BY VARIOUS REAL FORCES

Work done by gravity Force.

Solved Example

Example 5. The mass of the particle is 2 kg. It is projected as shown in four different ways with same speed of 10 m/s. Find out the work done by gravity by the time the stone falls on ground.

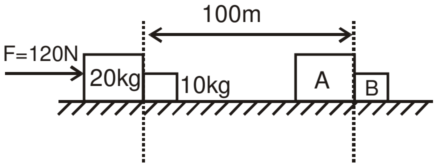


Solution : W = $|\vec{F}||\vec{S}|\cos\theta$ = 2000 J in each case.

Work done by normal reaction.

Solved Example

Example 6.



(i) Find work done by force F on A during 100 m displacement.

(ii) Find work done by force F on B during 100 m displacement.

(iii) Find work done by normal reaction on B and A during the given displacement.

(iv) Find out the kinetic energy of block A & B finally.

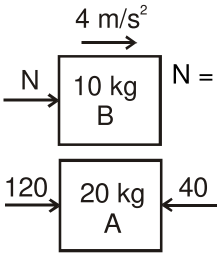
Solution : (i) $(W_F)_{\text{on A}} = F\Delta S \cos\theta$

$= 120 \times 100 \times \cos 0° = 12000$ J

(ii) $(W_F)_{\text{on B}} = 0$

F does not act on B

(iii)



$N = 10 \times 4 = 40$ N

$(W_N)_{\text{on B}} = 40 \times 100 \times \cos 0° = 4000$ J

$(W_N)_{\text{on A}} = 40 \times 100 \times \cos 180° = -4000$ J

(iv) $v^2 = u^2 + 2as$ $\quad u = 0$

$\therefore \quad v^2 = 2 \times 4 \times 100 \Rightarrow v = 20\sqrt{2}$ m/s

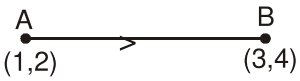
$\therefore \quad KE_A = \frac{1}{2} \times 20 \times 800 = 8000$ J

$KE_B = \frac{1}{2} \times 10 \times 800 = 4000$ J

W.D. by normal reaction on system of A & B is zero. i.e. w.d. by internal reaction on a rigid system is zero.

Example 7. A particle is displaced from point A (1, 2) to B(3, 4) by applying force $\vec{F} = 2\hat{i} + 3\hat{j}$. Find the work done by $\vec{F}$ to move the particle from point A to B.

Solution : $W = \vec{F}\cdot\Delta\vec{s}$



$\Delta S = (3-1)\hat{i} + (4-2)\hat{j} = (2\hat{i} + 3\hat{j}) \cdot (2\hat{i} + 2\hat{j}) = 2 \times 2 + 3 \times 2 = 10$ units

ENERGY :

Definition: Energy is defined as internal capacity of doing work. When we say that a body has energy we mean that it can do work.

Energy appears in many forms such as mechanical, electrical, chemical, thermal (heat), optical (light), acoustical (sound), molecular, atomic, nuclear etc., and can change from one form to the other.

KINETIC ENERGY :

Definition : Kinetic energy is the internal capacity of doing work of the object by virtue of its motion. Kinetic energy is a scalar property that is associated with state of motion of an object. An aero-plane in straight and level flight has kinetic energy of translation and a rotating wheel on a machine has kinetic energy of rotation. If a particle of mass m is moving with speed 'v' much less than the speed of the light than the kinetic energy 'K' is given by $K = \frac{1}{2}mv^2$

Important Points for K.E.

1. As mass m and v^2 ($\vec{v}.\vec{v}$) are always positive, kinetic energy is always positive scalar i.e, kinetic energy can never be negative.

2. The kinetic energy depends on the frame of reference,

$K = \frac{p^2}{2m}$ and $P = \sqrt{2mK}$; P = linear momentum

The speed v may be acquired by the body in any manner. The kinetic energy of a group of particles or bodies is the sum of the kinetic energies of the individual particles. Consider a system consisting of n particles of masses $m_1, m_2, ..., m_n$. Let $\vec{v_1}$, $\vec{v_2}$, ... , $\vec{v_n}$ be their respective velocities. Then, the total kinetic energy E_k of the system is given by

$$E_k = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 + ... + \frac{1}{2}m_nv_n^2$$

If m is measured in gram and v in cm s^{-1}, then the kinetic energy is measured in erg. If m is measured in kilogram and v in m s^{-1}, then the kinetic energy is measured in joule. It may be noted that the units of kinetic energy are the same as those of work. Infect, this is true of all forms of energy since they are inter-convertible.

Typical kinetic energies (K) :

S.No.	Object	Mass (kg)	Speed (m s^{-1})	K(J)
1	Air molecule	$\approx 10^{-26}$	500	$\approx 10^{-21}$
2	Rain drop at terminal speed	3.5×10^{-5}	9	1.4×10^{-3}
3	Stone dropped from 10 m	1	14	10^{2}
4	Bullet	5×10^{-5}	200	10^{3}
5	Running athlete	70	10	3.5×10^{3}
6	Car	2000	25	6.3×10^{5}

RELATION BETWEEN MOMENTUM AND KINETIC ENERGY :

Consider a body of mass m moving with velocity v. Linear momentum of the body, p = mv

Kinetic energy of the body, $E_k = \frac{1}{2}mv^2$

$E_k = \frac{1}{2m}(m^2v^2)$ or $E_k = \frac{p^2}{2m}$ or $p = \sqrt{2mE_k}$

Solved Example

Example 8. The kinetic energy of a body is increased by 21%. What is the percentage increase in the magnitude of linear momentum of the body?

Solution : $E_{k2} = \frac{121}{100}E_{k1}$ or $\frac{1}{2}m\,v_2^2 = \frac{121}{100}\frac{1}{2}mv_1^2$ or $v_2 = \frac{11}{10}v_1$

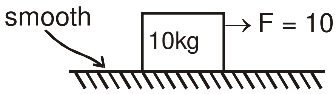
or $mv_2 = \frac{11}{10}mv_1$ or $p_2 = \frac{11}{10}p_1$

or $\frac{p_2}{p_1} - 1 = \frac{11}{10} - 1 = \frac{1}{10}$

or $\frac{p_2 - p_1}{p_1} \times 100 = \frac{1}{10} \times 100 = 10$

So, the percentage increase in the magnitude of linear momentum is 10%.

Example 9.



Force shown acts for 2 seconds. Find out w.d. by force F on 10 kg in 3 seconds.

Solution : $w = \vec{F} . \Delta\vec{S} \Rightarrow w = \vec{F} . \Delta\vec{S} . \cos 0° \Rightarrow w = 10\,\Delta\vec{S}$

Now $10 = 10\,a \quad \therefore \quad a = 1\ m/s^2 \Rightarrow S = \frac{1}{2}at^2 = \frac{1}{2} \times 1 \times 2^2 = 2\ m$

$w = 10 \times 2 = 20\ J$

Example 10. Find Kinetic energy after 2 seconds.

Solution : $V = 0 + at \Rightarrow V = 1 \times 2 = 2m/s$

$\therefore \quad KE = \frac{1}{2} \times 10 \times 2^2 = 20\ J.$

WORK DONE BY A VARIABLE FORCE :

When the magnitude and direction of a force vary in three dimensions, it can be expressed as a function of the position. For a variable force work is calculated for infinitely small displacement and for this displacement force is assumed to be constant

$$dW = \vec{F}.d\vec{s}$$

The total work done will be sum of infinitely small work

$$W_{A \to B} = \int_A^B \vec{F} \cdot d\vec{s} = \int_A^B (F\cos\theta)ds$$

In terms of rectangular components,

$$\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k} \quad \Rightarrow \quad d\vec{s} = dx\,\hat{i} + dy\,\hat{j} + dz\,\hat{k}$$

$$W_{A\to B} = \int_{x_A}^{x_B} F_x dx + \int_{y_A}^{y_B} F_y dy + \int_{z_A}^{z_B} F_z dz$$

Solved Example

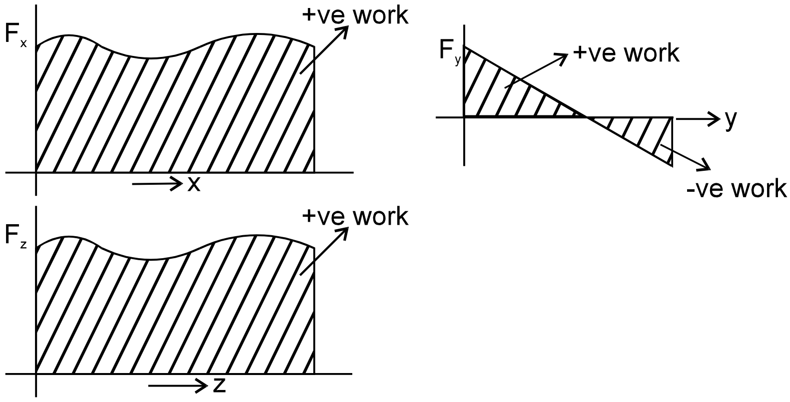
Example 11. An object is displaced from position vector $\vec{r}_1 = (2\hat{i} + 3\hat{j})m$ to $\vec{r}_2 = (4\hat{i} + 6\hat{j})m$ under the action of a force $\vec{F} = (3x^2\hat{i} + 2y\hat{j})N$. Find the work done by this force.

Solution : $W = \int_{\vec{r}_i}^{\vec{r}_f} \vec{F} \bullet d\vec{r} = \int_{\vec{r}_1}^{\vec{r}_2} (3x^2\hat{i} + 2y\hat{j}) \bullet (dx\hat{i} + dy\hat{j} + dz\hat{k})$

$= \int_{\vec{r}_1}^{\vec{r}_2} (3x^2dx + 2ydy) = [x^3 + y^2]_{(2,\ 3)}^{(4,\ 6)} = 83\ J$ **Ans.**

AREA UNDER FORCE DISPLACEMENT CURVE :

Graphically area under the force-displacement is the work done

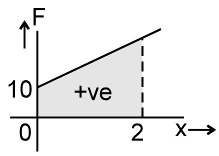


The work done can be positive or negative as per the area above the x-axis or below the x-axis respectively.

Solved Example

Example 12. A force F = 0.5x + 10 acts on a particle. Here F is in Newton and x is in metre. Calculate the work done by the force during the displacement of the particle from x = 0 to x = 2 metre.

Solution : Small amount of work done dW in giving a small displacement $d\vec{x}$ is given by



$dW = \vec{F} . d\vec{x}$

or $dW = Fdx \cos 0^{\circ}$

or $dW = Fdx$ $\quad [\because \cos 0^{\circ} = 1]$

Total work done, $W = \int_{x=0}^{x=2} Fdx = \int_{x=0}^{x=2} (0.5\,x + 10)dx$

$= \int_{x=0}^{x=2} 0.5x \quad dx + \int_{x=0}^{x=2} 10\,dx = 0.5 \left| \frac{x^2}{2} \right|_{x=0}^{x=2} + 10 \left| \quad x \right|_{x=0}^{x=2}$

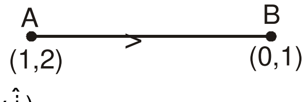
$= \frac{0.5}{2}\ [2^2 - 0^2] + 10[2 - 0] = (1 + 20) = 21\ J$

Work done by Variable Force $W = \int dW = \int \vec{F} \cdot \overrightarrow{ds}$

Solved Example

Example 13. An object is displaced from point A(1, 2) to B(0, 1) by applying force $\vec{F} = x\hat{i} + 2y\hat{j}$. Find out work done by $\vec{F}$ to move the object from point A to B.

Solution : $dW = \vec{F} . d\vec{s}$



$dW = (x\hat{i} + 2y\hat{j})\ (dx\ \hat{i} + dy\hat{j})$

$dW = \int_1^0 x\,dx + \int_2^1 2y\,dy$

$\therefore W = -3.5\ J$

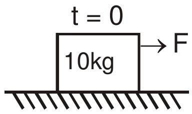
Example 14. The linear momentum of a body is increased by 10%. What is the percentage change in its kinetic energy?

Solution : Percentage increase in kinetic energy = 21%]

$$\left[\ \text{Hint}.mv_2 = \frac{110}{100}mv_1,\ \ v_2 = \frac{11}{10}v_1, \frac{E_2}{E_1} = \left(\frac{11}{10}\right)^2 = \frac{121}{100}\right.$$

$$\left.\text{Percentage increase in kinetic energy} = \frac{E_2 - E_1}{E_1} \times 100\right] = 21\%$$

Example 15. A time dependent force F = 10 t is applied on 10 kg block as shown in figure.



Find out the work done by F in 2 seconds.

Solution : $dW = \vec{F}.d\vec{s}$

dW = 10 t. dx

dW = 10 t v dt .... (1) dx = vdt

also $10 \times \dfrac{dv}{dt} = 10\ t$

$\therefore\ \int_0^v dv = \int_0^t t\,dt \Rightarrow v = \dfrac{t^2}{2}$ .... (2)

from (1) & (2)

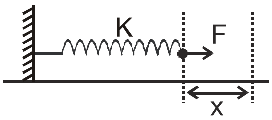
$dW = 10\ t \cdot \left(\dfrac{t^2}{2}\right) dt\ ;\ dW = 5t^3dt\ ;\ W = \dfrac{5}{4}\left[t^4\right]_0^2 = 20\ J$

Aliter : $\Delta K.E. = \dfrac{1}{2} \times 10\ (2^2 - 0) = 0$

WORK DONE BY SPRING FORCE

Solved Example

Example 16.



Initially spring is relaxed. A person starts pulling the spring by applying a variable force F. Find out the work done by F to stretch it slowly to a distance by x.

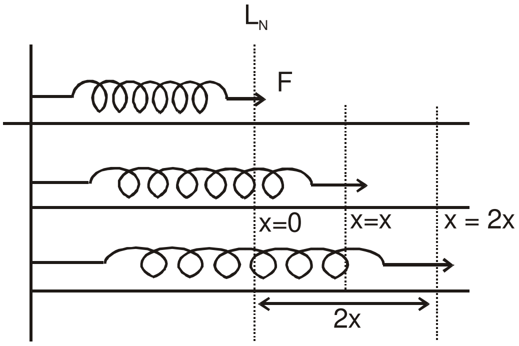
Solution : $\int dW = \int F \bullet ds = \int_0^x Kxdx \Rightarrow \quad W = \left(\dfrac{Kx^2}{2}\right)_0^x = \dfrac{Kx^2}{2}$

Example 17. In the above example
(i) Where has the work gone ?
(ii) Work done by spring on wall is zero. Why?
(iii) Work done by spring force on man is _______ .

Solution : (i) It is stored in the form of potential energy in spring.
(ii) Zero, as displacement is zero.
(iii) $-\dfrac{1}{2}Kx^2$

Example 18. Find out work done by applied force to slowly extend the spring from x to 2x.

Solution : Initially the spring is extended by x



$W = \vec{F}.d\vec{s}$

$W = \int_x^{2x} Kx.dx$

$W = \left[\frac{Kx^2}{2}\right]_x^{2x} = \frac{3}{2}Kx^2$

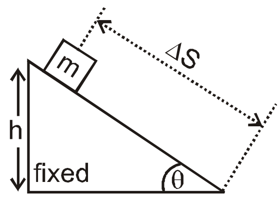
It can also found by difference of PE.

i.e., $U_f = \frac{1}{2}K(2x)^2 = 2kx^2 \Rightarrow U_i = \frac{1}{2}kx^2 \Rightarrow U_f - U_i = \frac{3}{2}kx^2$

WORK DONE BY OTHER CONSTANT FORCES

Solved Example

Example 19. A block of mass m is released from top of a smooth fixed inclined plane of inclination θ.



Find out work done by normal reaction & gravity during the time block comes to bottom.

Solution : $W_N = 0$ as $F \perp \Delta S$

$W_g = \vec{F}.\Delta\vec{S} = mg . \Delta S . \cos(90 - \theta) = mg\,\Delta S \sin\theta = mgh$

Example 20. Find out the speed of the block at the bottom and its kinetic energy.

Solution : $V^2 = u^2 + 2as$

$V^2 = 0 + 2(g\sin\theta)\ \frac{h}{\sin\theta}$

$\Rightarrow\ V^2 = 2gh$

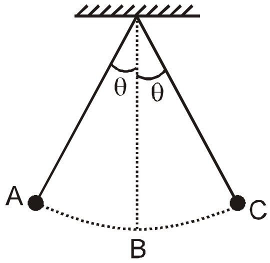
$\Rightarrow\ V = \sqrt{2gh}$

$KE = \frac{1}{2}mv^2 = mgh$

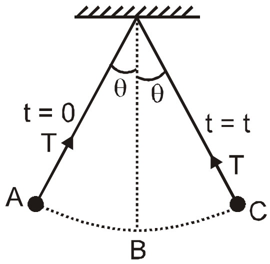
WORK DONE BY TENSION

Solved Example

Example 21. A bob of pendulum is released at rest from extreme position as shown in figure. Find work done by tension from A to B, B to C and C to A.



Solution : Zero because $F_T \perp dS$ at all time.



Example 22. In the above question find out work done by gravity from A to B and B to C.

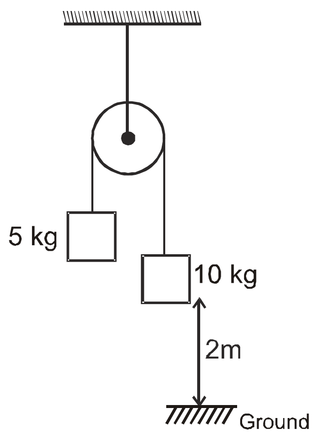
Solution : $W_g = \vec{F} . \Delta\vec{S}$

$= mg\ \Delta S \cos\theta$

$W_g = mg\ (\ell - \ell \cos\theta)$ for A to B

$W_g = -\ mg\ (\ell - \ell \cos\theta)$ for B to C

Example 23.



The system is released from rest. When 10 kg block reaches at ground then find :
(i) Work done by gravity on 10 kg (ii) Work done by gravity on 5 kg
(iii) Work done by tension on 10 kg (iv) Work done by tension on 5 kg.

Solution : (i) $(W_g)_{10\,kg} = 10\ g \times 2 = 200$ J

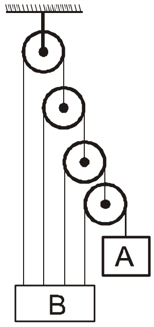
(ii) $(W_g)_{5kg} = 5\ g \times 2 \times \cos 180° = -100$ J

(iii) $(W_T)_{10\,kg} = \frac{200}{3} \times 2 \times \cos 180° = \frac{-400}{3}$ J

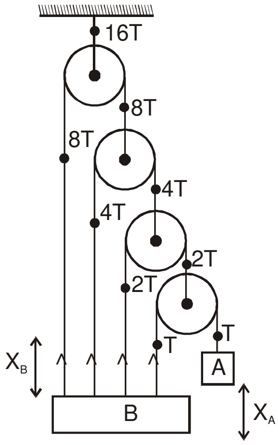
(iv) $(W_T)_{5\,kg} = \frac{200}{3} \times 2 \times \cos 0° = \frac{400}{3}$ J

Net w.d. by tension is zero. Work done by internal tension i.e. (tension acting within system) on the system is always zero if the length remains constant.

Example 24. The velocity block A of the system shown in figure is V_A at any instant. Calculate velocity of bock B at that instant.



Solution : Work done by internal tension is zero.

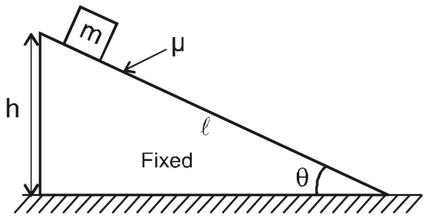


$\therefore$ $15\ T \times X_B - T \times X_A = 0$

$X_A = 15\ X_B$

$\therefore$ $V_A = 15\ V_B$

Example 25. A block of mass m is released from top of an incline plane of inclination θ. The coefficient of friction between the block and incline surface is μ ($\mu < \tan\theta$). Find work done by normal reaction, gravity & friction, when block moves from top to the bottom.



Solution : $W_N = 0$ $\therefore$ $F_N \perp \Delta S$

$W_g = mg\ \ell \sin\theta$

$W_f = -\mu mg \cos\theta.\ell$

Example 26. What is kinetic energy of block of mass m at bottom in above problem ?

Solution : $V^2 = u^2 + 2as$

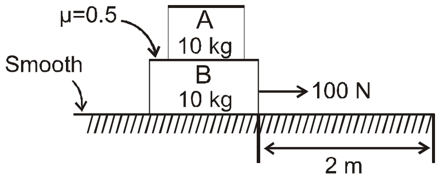
$V^2 = 2(g\sin\theta - \mu\, g\cos\theta)\,(\ell)$

$\therefore \quad KE = m\, 2\frac{1}{2}\,(g\sin\theta - \mu g\cos\theta)\,\ell = mg\,\ell\,(\sin\theta - \mu\cos\theta)$

WORK DONE BY FRICTION

Solved Example

Example 27. In the given figure



(i) Find work done by applied force during displacement 2m.

(ii) Find work done by frictional force on B by A during the displacement.

Solution : (i) $100 \times 2 \times \cos 0° = 200$ J

(ii) $f_{smax} = \mu mg = 0.5 \times 10\, g = 50$ N

Assuming they move together.

$100 = 20\, a \Rightarrow a = 5$ m/s^2

Check Friction on A ; $\quad f = 10 \times 5 = 50$ N

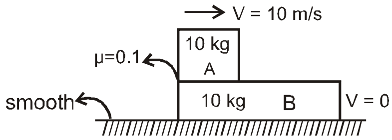
$f_{reqd} = f_{available}$ $\quad \therefore$ They move together

Hence $(W_f)_{on\,B} = -100$ J

$(W_f)_{on\,A} = 100$J. Net zero

i.e. w.d. by internal static friction is zero.

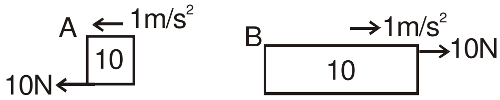
Example 28. A block of mass 10 kg is projected with speed 10 m/s on the surface of the plank of mass 10 kg, kept on smooth ground as shown in figure.



(i) Find out the velocity of two blocks when frictional force stops acting.

(ii) Find out displacement of A & B till velocity becomes equal.

Solution : (i)



$V_A = 10 - 1t \quad \Rightarrow \quad V_B = 1t$

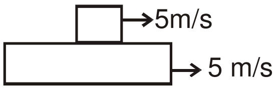
Frictional force stops acting when

$V_A = V_B \quad \Rightarrow \quad 10 - t = t$

$10 = 2t \quad \Rightarrow \quad t = 5$ sec.

$V_B = V_A = 5$m/s

Situation becomes



(ii) $S_A = 10 \times 5 - \frac{1}{2} \times 1 \times 5^2 = 37.5$ m

$S_B = \frac{1}{2} \times 1 \times 5^2 = 12.5$ m

Example 29. In the above question find work done by kinetic friction on A & B.

Solution : $(W_{KF})_{on}$ A = 10 × 37.5 cos 180° = – 375 J

$(W_{KF})_{on}$ B = 10 × 12.5 cos 0 = 125 J

work done by KF on system of A & B = – 375 + 125 = – 250 J

Work done by KF on a system is always negative.

Heat generated = – (W_{KF}) on system

(W_{KF}) on system = – $(f_K \times S_{relative})$ = – 10 × 25 = – 250 J

True / False :

Example 30. Work done by kinetic friction on a body is never zero.

Answer : False

Example 31. Work done by kinetic friction on a system is always negative.

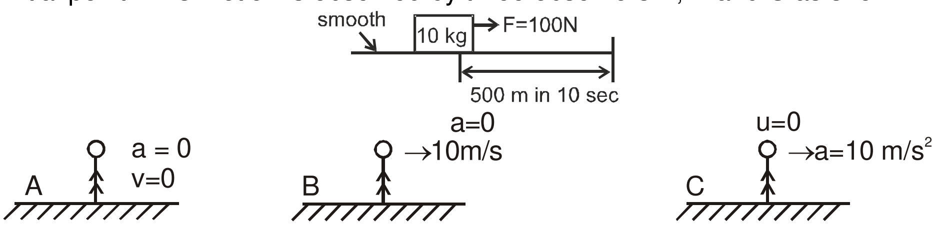
Answer : True

WORK DONE BY PSEUDO FORCE

Kinetic Energy of a body frame dependent as velocity is a frame dependent quantity. Therefore pseudo force work has to be considered.

Solved Example

Example 32. A block of mass 10 kg is pulled by force F = 100 N. It covers a distance 500 m in 10 sec. From initial point. This motion is observed by three observers A, B and C as shown in figure.



Find out work done by the force F in 10 seconds as observed by A, B & C.

Solution : $(W_F)_{\text{on block w.r.t A}}$ = 100 × 500 J = 50,000 J

$(W_F)_{\text{on block w.r.t B}}$ = 100 [500 – 10 × 10] = 40,000 J

$(W_F)_{\text{on block w.r.t C}}$ = 100 [500 – 500] = 0

WORK DONE BY INTERNAL FORCE

$F_{AB} = - F_{BA}$ i.e. sum of internal forces is zero.

But it is not necessary that work done by internal force is zero. There must be some deformation or reformation between the system to do internal work. In case of a rigid body work done by internal force is zero.

Work-Energy Theorem :

According to work-energy theorem, the work done by all the forces on a particle is equal to the change in its kinetic energy.

$W_C + W_{NC} + W_{PS} = \Delta K$

Where, W_C is the work done by all the conservative forces.

W_{NC} is the work done by all non-conservative forces.

W_{PS} is the work done by all psuedo forces.

Modified Form of Work-Energy Theorem :

We know that conservative forces are associated with the concept of potential energy, that is

$W_C = - \Delta U$

So, Work-Energy theorem may be modified as

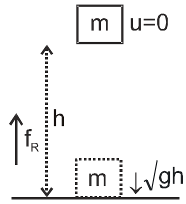
$W_{NC} + W_{PS} = \Delta K + \Delta U$

$W_{NC} + W_{PS} = \Delta E$

Solved Examples

Example 33. A body of mass m when released from rest from a height h, hits the ground with speed $\sqrt{gh}$. Find work done by resistive force.

Solution : Identify initial and final state and identify all forces.



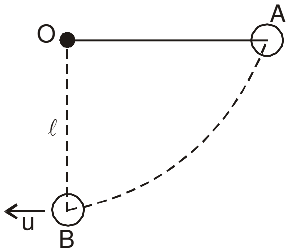
$$W_g + W_{air\ res.} + W_{int\ force} = \Delta K$$

$$mgh + W_{air\ res} + 0 = \frac{1}{2} m \left(\sqrt{gh}\right)^2 - 0$$

$$\Rightarrow \quad W_{air\ res.} = -\frac{mgh}{2}$$

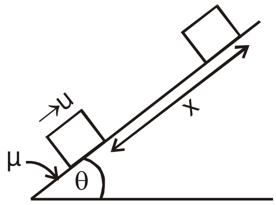
Example 34. The bob of a simple pendulum of length l is released when the string is horizontal. Find its speed at the bottom.

Solution : $W_g + W_T = \Delta K$



$$mg\ell + 0 = \frac{1}{2} mu^2 - 0 \ ; \ u = \sqrt{2g\ell}$$

Example 35. A block is given a speed u up the inclined plane as shown.

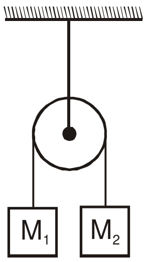


Using work energy theorem find out x when the block stops moving.

Solution : $W_g + W_f + W_N = \Delta K$

$$-\ mg\ x \sin\theta - \mu\ mgx \cos\theta + 0 = 0 - \frac{1}{2} mu^2 \qquad \Rightarrow \qquad x = \frac{u^2}{2g(\sin\theta + \mu\cos\theta)}$$

Example 36. The masses M_1 and M_2 ($M_2 > M_1$) are released from rest.



Using work energy theorem find out velocity of the blocks when they move a distance x.

Solution : $(W_{all\ F})_{system} = (\Delta K)_{system}$

$(W_g)_{sys} + (W_T)_{sys} = (\Delta K)_{sys}$ as $(W_T)_{sys} = 0$

$$M_2gx - M_1gx = \frac{1}{2}(M_1 + M_2)V^2 - 0 \qquad \text{..........} \ (1)$$

$$V = \sqrt{\frac{2(M_2 - M_1)gx}{M_1 + M_2}}$$

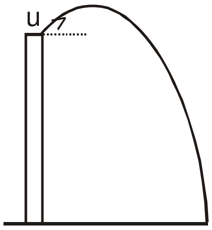
Example 37. In the above question find out acceleration of blocks.

Solution : $(M_2g - M_1g) = \frac{1}{2}(M_1 + M_2)\, 2v\frac{dv}{dx}$ [Differentiating equation (1) above]

$\Rightarrow \left(\frac{M_2 - M_1}{M_1 + M_2}\right)g = v\frac{dv}{dx} = a$

Example 38. A stone is projected with initial velocity u from a building of height h. After some time the stone falls on ground. Find out speed with it strikes the ground.

Solution : $W_{\text{all forces}} = \Delta K$



$W_g = \Delta K$

$mgh = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$

$v = \sqrt{u^2 + 2gh}$

Power :

Power is defined as the time rate of doing work.

When the time taken to complete a given amount of work is important, we measure the power of the agent of doing work.

The average power ($\bar{P}$ or p_{av}) delivered by an agent is given by

$\bar{P}$ or $p_{av} = \frac{W}{t}$ where W is the amount of work done in time t.

Power is the ratio of two scalars- work and time. So, power is a scalar quantity. If time taken to complete a given amount of work is more, then power is less. For a short duration dt, if P is the power delivered during this duration, then

$$P = \frac{\vec{F}\cdot d\vec{S}}{dt} = \vec{F}\cdot\frac{d\vec{S}}{dt} = \vec{F}\cdot\vec{v}$$

This is instantaneous power. It may be +ve, –ve or zero.

By definition of dot product,

$$P = Fv\cos\theta$$

where θ is the smaller angle between $\vec{F}$ and $\vec{v}$.

This P is called as instantaneous power if dt is very small.

Solved Example

Example 39. A block moves in uniform circular motion because a cord tied to the block is anchored at the centre of a circle. Is the power of the force exerted on the block by the cord is positive, negative or zero?

Answer : Zero

Explanation. $\vec{F}$ and $\vec{v}$ are perpendicular.

$\therefore$ Power $= \vec{F}\cdot\vec{v} = Fv\cos 90^{\circ} = $ Zero.

Unit of Power :

A unit power is the power of an agent which does unit work in unit time.

The power of an agent is said to be one watt if it does one joule of work in one second.

1 watt = 1 joule/second = 10^7 erg/second

Also, $\quad 1 \text{ watt} = \dfrac{1\text{newton} \times 1\text{metre}}{1\text{second}} = 1\text{Nms}^{-1}$.

Dimensional formula of power

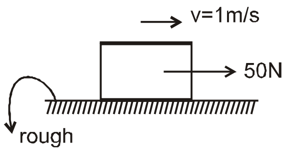
$$[\text{Power}] = \frac{[\text{Work}]}{[\text{Time}]} = \frac{[ML^2T^{-2}]}{[T]} = [ML^2T^{-3}]$$

Power has 1 dimension in mass, 2 dimensions in length and – 3 dimensions in time.

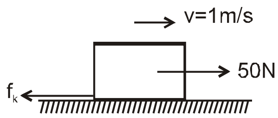
S.No.	Human Activity	Power (W)
1	Heart beat	1.2
2	Sleeping	83
3	Sitting	120
4	Riding in a car	140
5	Walking (4.8 km h^{-1})	265
6	Cycling (15 km h^{-1})	410
7	Playing Tennis	440
8	Swimming (breaststroke, 1.6 km h^{-1})	475
9	Skating	535
10	Climbing Stairs (116 steps min^{-1})	685
11	Cycling (21.3 km h^{-1})	700
12	Playing Basketball	800
13	Tube light	40
14	Fan	60

Solved Example

Example 40 A block moves with constant velocity 1 m/s under the action of horizontal force 50 N on a horizontal surface. What is the power of external force and friction?



Solution : Since a = 0 i.e. f_k = 50 N



$P_{ext} = 50 \times 1 = 50$ W

$P_f = -50 \times 1 = -50$ W

Power is also the rate at which energy is supplied.

Net power = $P_1 + P_2 + P_3$ ....

$$P_{net} = \frac{dW_1}{dt} + \frac{dW_2}{dt} \text{......} \quad \Rightarrow \quad P_{net} = \left(\frac{dW_1 + dW_2 + \text{..........}}{dt}\right)$$

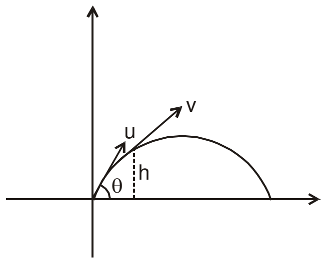
$$P_{net} = \frac{dK}{dt} \qquad \because W_{all} = \Delta K$$

$\therefore$ Rate of change of kinetic energy is also power.

Solved Example

Example 41 A stone is projected with velocity at an angle θ with horizontal. Find out
(i) Average power of the gravity during time t.
(ii) Instantaneous power due to gravitational force at time t where t is time of flight.
(iii) When is average power zero ?
(iv) When is P_{inst} zero ?
(v) When is P_{inst} negative ?
(vi) When is P_{inst} positive ?

Solution : (i) $<P> = \frac{w}{T} = -\frac{mgh}{t} = -\frac{mg\left[u\sin\theta t - \frac{1}{2}gt^2\right]}{t}$



$<P> = mg\left[\frac{gt}{2} - u\sin\theta\right]$

(ii) Instantaneous power

$P = \vec{F}\cdot\vec{v} = (-mg\,\hat{j})\,[u\cos\theta\,\hat{i} + (u\sin\theta - gt))\,\hat{j}]$

$= -mg(u\sin\theta - gt)$

(iii) $\frac{gt}{2} = u\sin\theta \Rightarrow \quad t = \frac{2u\sin\theta}{g}$, i.e. time of flight.

(iv) When $\vec{F}$ & $\vec{V}$ are $\perp$ i.e. at $t = \frac{u\sin\theta}{g}$ which is at the highest point.

(v) From base to highest point.
(vi) From highest point to base.

POTENTIAL ENERGY

Energy : It is the internal capacity to do work.

Kinetic Energy : It is internal capacity to do work by virtue of relative motion.

Potential Energy : It is the internal capacity to do work by virtue of relative position.
Example : Gravitational Potential Energy , Spring PE etc.

Potential Energy
Definition:
Potential energy is the internal capacity of doing work of a system by virtue of its configuration.
In case of conservative force (field) potential energy is equal to negative of work done by the conservative force in shifting the body from some reference position to given position.
Therefore, in case of conservative force

$$\int_{U_1}^{U_2} dU = -\int_{r_1}^{r_2} \vec{F}\cdot d\vec{r} \qquad \text{i.e.} \qquad U_2 - U_1 = -\int_{r_1}^{r_2} \vec{F}\cdot d\vec{r} = -W$$

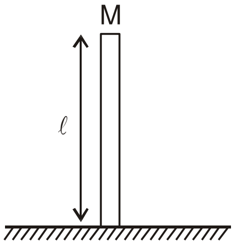
Whenever and wherever possible, we take the reference point at ∞ and assume potential energy to be zero there, i.e., If we take $r_1 = \infty$ and $U_1 = 0$ then

$$U = -\int_{\infty}^{r} \vec{F}\cdot d\vec{r} = -W$$

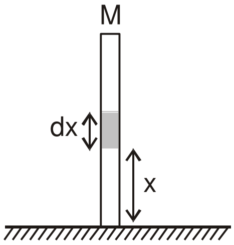
(a) Gravitation Potential Energy :
U = mgh for a particle at a height h above reference level.

Solved Example

Example 42 Calculate potential energy of a uniform vertical rod of mass M and length ℓ.



Solution : dU = (dm) gx



$$\int_0^U dU = \int_0^{\ell}\left(\frac{M}{\ell}dx\right)gx$$

$$U = \frac{Mg\ell}{2}$$

(b) Spring potential energy :

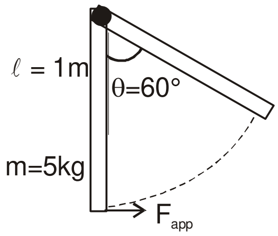
$$U = \frac{1}{2}Kx^2$$

Where x is change in length from its natural length.

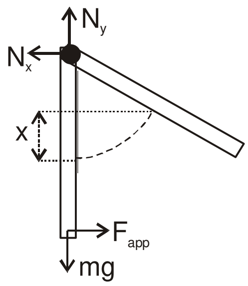
Note : Gravitational potential energy can be +ve , –ve or zero but spring potential energy will always be +ve.

Solved Example

Example 43 In the given figure, a uniform rod of mass m and length l is hinged at one end. Find the work done by applied force in slowly bringing the rod to the inclined position.



Solution : $W_{ALL} = \Delta K$ by work energy theorem



$$x = \frac{\ell}{2} - \frac{\ell}{2}\cos 60° = \frac{1}{4} m = 0.25 \text{ m}$$

$$W_{N_x} + W_{N_y} + W_g + W_{Fapp} = \Delta K \qquad (W_{N_x} = W_{N_y} = 0)$$

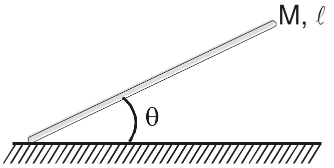
$\therefore$ 0 – mg(0.25) + W_{Fapp} = 0 – 0

$\because$ ΔK = 0 as slowly brought

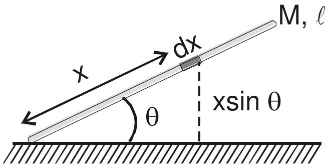
$\therefore$ W_{Fapp} = 5 × 9.8 × 0.25

It can be seen that W_{Fapp} = mgh = 5 × 9.8 × 0.25 J.

Example 44 A uniform rod of mass M and length ℓ is held in position shown in the figure. Find the potential energy of the rod.



Solution : dU = dm.g.h

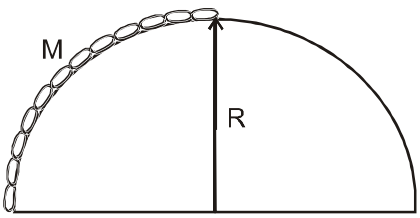


$$\int dU = \int_0^{\ell} \frac{M}{\ell}\, dx \,.\, g \,.\, x \sin\theta$$

$$\therefore \quad U = \frac{Mg\sin\theta\ell}{2}$$

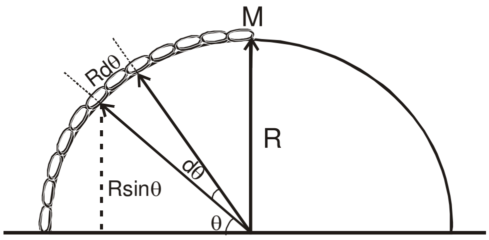
Note that centre of mass of the rod is at height $\left(\frac{\ell}{2}\sin\theta\right)$ from the ground.

Example 45



A chain of mass M is kept on a hemisphere as shown. Find potential energy of the chain.

Solution :



We know that $\frac{\text{arc}}{\text{Radius}} = \theta$

$\therefore$ elemental length = $Rd\theta$

$$\therefore \quad dm = \frac{M}{\pi/2} d\theta = \frac{2M}{\pi} d\theta$$

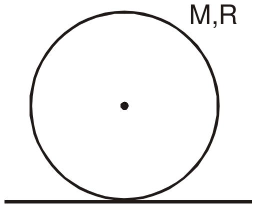
$$\text{Now } dU = dmgh = \left(\frac{2M}{\pi} d\theta\right)(g)(R\sin\theta)$$

$$\therefore \quad \int_0^U dU = \frac{2M}{\pi} Rg \int_0^{\pi/2} \sin\theta\, d\theta$$

$$U = \frac{2MgR}{\pi}\left(-\cos\theta\right)_0^{\pi/2}$$

$$\Rightarrow \quad U = Mg\left(\frac{2R}{\pi}\right)$$ [Note that $\left(\frac{2R}{\pi}\right)$ is the height of COM]

Example 46 A uniform solid sphere of mass M and radius R is kept on the horizontal surface.



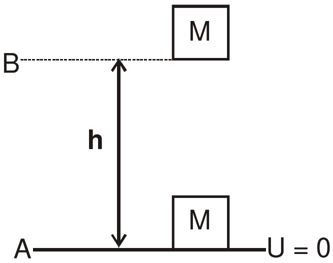
Find potential energy of the solid sphere.

Solution : $U = Mgh_{cm}$

for symmetrical body COM is the geometrical centre of the body.

$\therefore \quad U = MgR$

Example 47 Find work done by gravity in moving the block



(i) from A to B (ii) from B to A (iii) Calculate $U_A - U_B$ (iv) Calculate $U_B - U_A$

Solution : $(W_g)_{A \text{ to } B} = -\,mgh$

$(W_g)_{B \text{ to } A} = mgh$

$U_A - U_B = -\,mgh$

$U_B - U_A = mgh$

It can be said

$\mathbf{W_g = -\,\Delta U}$

Work done by gravity is the -ve of the change in PE.

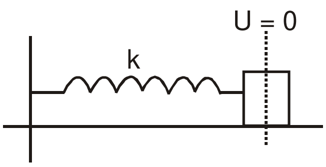
i.e. $W_g = -\,[U_f - U_i] \quad \Rightarrow \quad W_g = U_i - U_f$

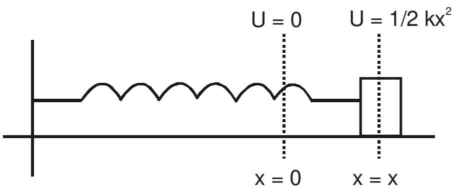
Important Points for P.E. :

1. Potential energy can be defined only for conservative forces. It has no relevance for non-conservative forces.
2. Potential energy can be positive or negative, depending upon choice of frame of reference.
3. Potential energy depends on frame of reference but change in potential energy is independent of reference frame.
4. Potential energy should be considered to be a property of the entire system, rather than assigning it to any specific particle.
5. It is a function of position and does not depend on the path.

Work done by spring force

As above $W_{SP} = -\,\Delta U$





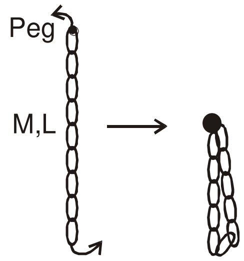
$\therefore \quad W_{SPF} = -\,\Delta U$

$W_{SPF} = U_i - U_F$

$W_{SPF} = 0 - \frac{1}{2}kx^2 = -\frac{1}{2}kx^2$

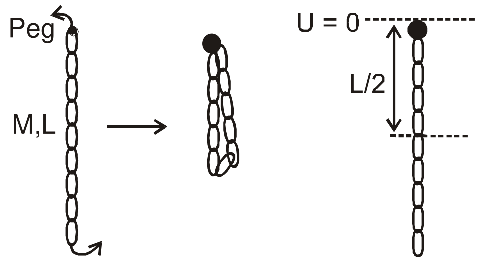
Solved Example

Example 48

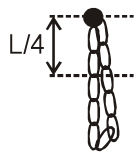


Find out work done by external agent to slowly hang the lower end of the chain to the peg.

Solution :



Initially $U_i = -Mg\frac{L}{2}$



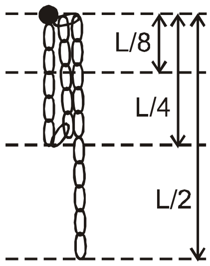
$U_f = -Mg\ \frac{L}{4}$

$\therefore\ W_g = U_i - U_f = (-Mg\frac{L}{2}) - (-Mg\frac{L}{4}) = -Mg\frac{L}{4}$

Using work energy theorem, $W_g + W_{ext} = \Delta K = 0 \quad \Rightarrow \quad W_{ext} = -W_g = Mg\frac{L}{4}$

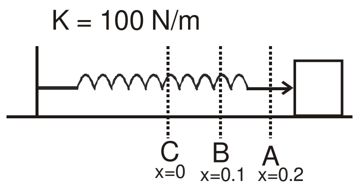
Example 49 In above example find out the work done by external agent to slowly hang the middle link to peg.

Solution : $U_i = -Mg\frac{L}{2}$; $U_f = \left(-\left(\frac{M}{2}\right)g\frac{L}{8}\right) - \left(\frac{M}{2}g\frac{L}{4}\right)$



$\therefore\ W_{ext} = U_f - U_i = \frac{5}{16}MgL$

Example 50



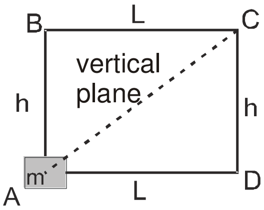
Find out the work done by spring force from A to B and from B to C. x = 0 is position of natural length.

Solution : $(W_{spring})_{A\to B} = U_i - U_f = \frac{1}{2}K\,(0.2)^2 - \frac{1}{2}K(0.1)^2$

$\therefore\ (W_{spring})_{A\to B} = \frac{3}{2}\,J$

Similarly $(W_{spring})_{BC} = \frac{1}{2}\,J$

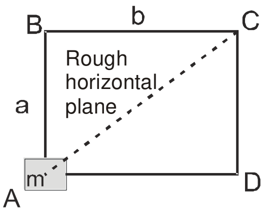
Example 51 **(a)** The mass m is moved from A to C along three different paths



(i) ABC (ii) ADC (iii) AC

Find out work done by gravity in the three cases.

(b) The block is moved from A to C along three different paths. Applied force is horizontal. Find work done by friction force in path



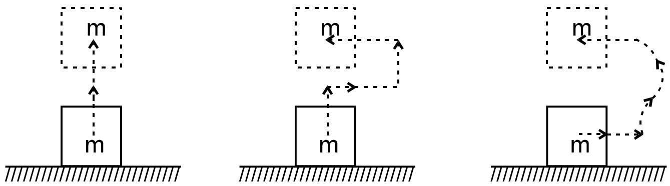
(i) ABC (ii) ADC (iii) AC

Solution : **(a)** (i) $-mgh$ (ii) $-mgh$ (iii) $-mgh$

(b) (i) $W_{ABC} = -\mu mg\,(a+b)$ (ii) $W_{ADC} = -\mu mg\,(a+b)$ (iii) $W_{AC} = -\mu mg\,(\sqrt{a^2+b^2})$

CONSERVATIVE FORCES

A force is said to be conservative if work done by or against the force in moving a body depends only on the initial and final positions of the body and not on the nature of path followed between the initial and final positions.



(a) (b) (c)

Consider a body of mass m being raised to a height h vertically upwards as show in above figure. The work done is *mgh*. Suppose we take the body along the path as in (b). The work done during horizontal motion is zero. Adding up the works done in the two vertical parts of the paths, we get the result *mgh* once again. Any arbitrary path like the one shown in (c) can be broken into elementary horizontal and vertical portions. Work done along the horizontal parts is zero. The work done along the vertical parts add up to *mgh.* Thus we conclude that the work done in raising a body against gravity is independent of the path taken. It only depends upon the initial and final positions of the body. We conclude from this discussion that the force of gravity is a conservative force.

Examples of Conservative forces.

(i) Gravitational force, not only due to the Earth but in its general form as given by the universal law of gravitation, is a conservative force.

(ii) Elastic force in a stretched or compressed spring is a conservative force.

(iii) Electrostatic force between two electric charges is a conservative force.

(iv) Magnetic force between two magnetic poles is a conservative forces.

In fact, all fundamental forces of nature are conservative in nature.

Forces acting along the line joining the centres of two bodies are called central forces. Gravitational force and Electrostatic forces are two important examples of central forces. Central forces are conservative forces.

PROPERTIES OF CONSERVATIVE FORCES

(i) Work done by or against a conservative force depends only on the initial and final positions of the body.

(ii) Work done by or against a conservative force does no depend upon the nature of the path between initial and final positions of the body.

If the work done a by a force in moving a body from an initial location to a final location is independent of the path taken between the two points, then the force is conservative.

(iii) Work done by or against a conservative force in a round trip is zero.

If a body moves under the action of a force that does no total work during any round trip, then the force is conservative; otherwise it is non-conservative.
The concept of potential energy exists only in the case of conservative forces.

(iv) The work done by a conservative force is completely recoverable.

Complete recoverability is an important aspect of the work of a conservative force.

NON-CONSERVATIVE FORCES

A force is said to be non-conservative if work done by or against the force in moving a body depends upon the path between the initial and final positions.

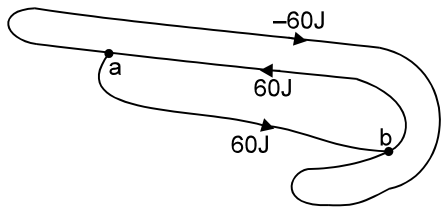
The frictional forces are non-conservative forces. This is because the work done against friction depends on the length of the path along which a body is moved. It does not depend only on the initial and final positions. Note that the work done by frictional force in a round trip is not zero.

The velocity-dependent forces such as air resistance, viscous force etc., are non conservative forces.

S.No.	Conservative forces	Non-Conservative forces
1	Work done does not depend upon path	Work done depends on path.
2	Work done in round trip is zero.	Work done in a round trip is not zero.
3	Central in nature.	Forces are velocity-dependent and retarding in nature.
4	When only a conservative force acts within a systrem, the kinetic enrgy and potential energy can change. However their sum, the mechanical energy of the system, does not change.	Work done against a non-conservative force may be dissipated as heat energy.
5	Work done is completely recoverable.	Work done in not completely recoverable.

Solved Example

Example 52 The figure shows three paths connecting points a and b. A single force F does the indicated work on a particle moving along each path in the indicated direction. On the basis of this information, is force F conservative?



Answer : No

Explanation : For a conservative force, the work done in a round trip should be zero.

Example 53 Find the work done by a force $\vec{F} = x\hat{i} + y\hat{j}$ acting on a particle to displace it from point A(0, 0) to B(2, 3).

Solution : $dW = \vec{F}.ds = (x\hat{i} + y\hat{j}).(dx\hat{i} + dy\hat{j})$

$$W = \int_0^2 xdx + \int_0^3 ydy = \left[\frac{x^2}{2}\right]_0^2 + \left[\frac{y^2}{2}\right]_0^3 = \frac{13}{2} \text{ units}$$

True or False

Example 54. In case of a non conservative force work done along two different paths will always be different.

Answer : False

Example 55. In case of non conservative force work done along two different paths may be different.

Answer : True

Example 56. In case of non conservative force work done along all possible paths cannot be same.

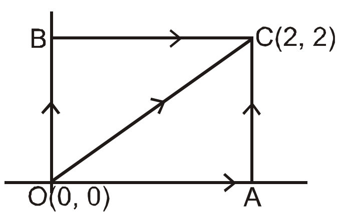
Answer : True

Example 57. Find work done by a force $\vec{F} = x\hat{i} + xy\hat{j}$ acting on a particle to displace it from point O(0, 0) to C(2, 2).

Solution : $\int dW = \int_0^2 xdx \quad + \quad \int_0^2 xydy$

can be found — cannot be found until x is known in terms of y i.e. until equation of path is known.

Example 58. Find the work done by $\vec{F}$ from O to C for above example if paths are given as below.



Solution : OAC $\Rightarrow$ OA + AC

for OA y = 0 $\therefore$ dy = 0

$\therefore \int dW_{OA} = \int_0^2 xdx + 0$ $\therefore$ $W_{OA} = 2$ J

for AC x = 2 dx = 0

$\int dW_{AC} = 0 + 2\int_0^2 ydy$ $\therefore$ $W_{AC} = 4J$

$W_{OAC} = W_{OA} + W_{AC} = 2 + 4 = 6J$

(ii) OBC $\Rightarrow$ OB + BC

for OB x = 0 dx = 0 $\therefore$ $W_{OB} = O$

for BC y = 2 dy = 0

$\therefore \int dW = \int xdx$ $\therefore$ $W = \left[\frac{x^2}{2}\right]_0^2 = 2$ J

$\therefore$ $W_{OAC} \neq W_{OBC}$

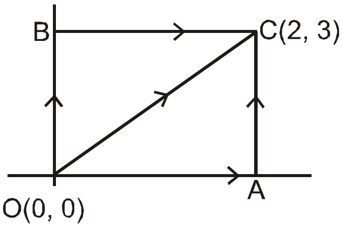
Hence the force is non-conservative.

(iii) For W_{OC} dW = xdy + xydx

for OC x = y dx = dy

$dW = \int_0^2 xdx + \int_0^2 y^2dy$ $W = \frac{14}{3}$ unit

Example 59 Find out work done by the force $\vec{F} = y\hat{i} + x\hat{j}$ to displace the particle from point O to C along the given paths. Decide whether the force is conservative or non–conservative.



Solution : (i) OAC OA + AC

for OA y = 0 dy = 0

$\therefore$ dW = 0 $W_{OA} = 0$

for AC x = 2 dx = 0

$\int dW = 2\int_0^3 dy$ $\Rightarrow$ W = 6 J $\Rightarrow$ $W_{OAC} = 6$ units

(ii) OBC $\Rightarrow$ OB + BC

for OB x = 0 dx = 0 $\therefore$ dW = 0

for BC y = 3 dy = 0

$\int dW = \int_0^2 3dx$ $\Rightarrow$ W = 6 units $\Rightarrow$ $W_{OBC} = 6$ units

(iii) OC

for OC $y = \frac{3}{2}x$ $dy = \frac{3}{2}dx$

$\therefore \int dW = \int_0^2 \frac{3}{2}xdx + \int_0^2 \frac{3}{2}xdx \Rightarrow \int dW = 3\int_0^2 x\,dx \Rightarrow$ $W_{OC} = 6$ units

Above force seems conservative but cannot be confirmed yet unless we can integrate it without the knowledge of path. Again we had dw = xdy + ydx & xdy + ydx can be written as dxy

$\therefore \int dW = \int dxy$ $\Rightarrow$ $W = \int_{0,0}^{2,3} dxy = [xy]_{0,0}^{2,3} = 6J$

Hence knowledge of path was not required to integrate the above so F is conservative.

POTENTIAL ENERGY AND CONSERVATIVE FORCE :

$F_S = -\ \partial U/\partial s$,

i.e., the projection of the field force, the vector **F**, at a given point in the direction of the displacement dr equals the derivative of the potential energy U with respect to a given direction, taken with the opposite sign. The designation of a partial derivative $\partial/\partial s$ emphasizes the fact of deriving with respect to a definite direction.

So, having reversed the sign of the partial derivatives of the function U with respect to x, y, z, we obtain the projection F_x, F_y and F_z of the vector **F** on the unit vectors **i**, **j** and **k**. Hence, one can readily find the vector itself : $F = F_x\mathbf{i} + F_y\mathbf{j} + F_z\mathbf{k}$, or

When conservative force does positive work then PE decreases

$$dU = -\ dw$$

$$dU = -\ F.ds$$

$$dU = -\ (F_x\hat{i} + F_y\hat{j} + F_z\hat{k})\ .\ (dx\hat{i} + dy\hat{j} + dz\hat{k})$$

$$dU = -\ F_x dx - F_y dy - F_z dz$$

if y & z are constants then dy = 0 dz = 0

$$dU = -F_x dx$$

$\therefore\quad F_x = -\ \dfrac{dU}{dx}$ if y & z are constant

$$\equiv F_x = \frac{-\partial U}{\partial x}$$

Similarly $F_y = \dfrac{-\partial u}{\partial y}$; $F_z = \dfrac{-\partial u}{\partial z}$

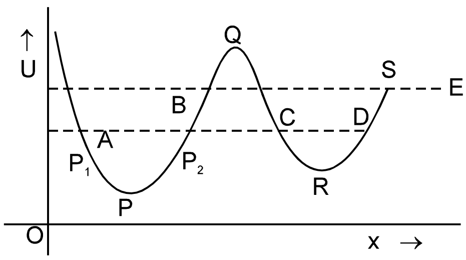
$$F = -\left(\frac{\partial U}{\partial x}\mathbf{i} + \frac{\partial U}{\partial y}\mathbf{j} + \frac{\partial U}{\partial z}\mathbf{k}\right).$$

The quantity in parentheses is referred to as the scalar gradient of the function U and is denoted by grad U or ∇U. We shall use the second, more convenient, designation where ∇ ("nabla") signifies the symbolic vector or operator

$$\nabla = \mathbf{i}\frac{\partial}{\partial x} + \mathbf{j}\frac{\partial}{\partial y} + \mathbf{k}\frac{\partial}{\partial z}$$

Potential Energy curve :

- A graph plotted between the PE a particle and its displacement from the centre of force field is called PE curve.



- Using graph, we can predict the rate of motion of a particle at various positions.
- Force on the particle is $F_{(x)} = -\dfrac{dU}{dx}$

Case-I : On increasing x, if U increases, force is in (–) ve x direction i.e. attraction force.

Case-II : On increasing x, if U decreases, force is in (+) ve x-direction i.e. repulsion force.

Solved Examples

Example 60. The potential energy of spring is given by $U = \frac{1}{2}kx^2$, where x is extension spring. Find the force associated with this potential energy.

Solution : $F_x = \frac{-\partial u}{\partial x} = -kx \qquad F_y = 0 \qquad F_z = 0.$

Example 61. The potential energy of a particle in a space is given by $U = x^2 + y^2$. Find the force associated with this potential energy.

Solution : $F_x = \frac{-\partial u}{\partial x} = -[2x + 0] = -2x$

$F_y = \frac{-\partial u}{\partial y} = -(2y + 0) = -2y\ ;\ \vec{F} = -2x\hat{i} - 2y\hat{j}$

Example 62. Find out the potential energy of given force $\vec{F} = -2x\hat{i} - 2y\hat{j}$.

Solution : $dU = -dW$

$\int dU = \int -(-2x\hat{i} - 2y\hat{j}) \bullet (dx\hat{i} + dy\hat{j})$

$\int dU = \int 2xdx + \int 2ydy \qquad \therefore \qquad \mathbf{U = x^2 + y^2 + C}$

Example 63 Find out the potential energy of the force $F = y\hat{i} + x\hat{j}$.

Solution : $dU = -dW$

$dU = -(y\hat{i} + x\hat{j}) \cdot (dx\hat{i} + dy\hat{j})$

$\int dU = \int -ydx + \int -xdy$

$\int dU = -\int dxy \qquad \Rightarrow \qquad U = -xy + c$

Example 64 Find out the force for which potential energy U = – xy.

Solution : $\vec{F} = -\left[\frac{\partial U}{\partial x}\hat{i} + \frac{\partial U}{\partial y}\hat{j}\right] \qquad \Rightarrow \qquad \vec{F} = -\left[\frac{\partial(-xy)}{\partial x}\hat{i} + \frac{\partial(-xy)}{\partial y}\hat{j}\right]$

$\vec{F} = y\hat{i} + x\hat{j}$ Hence verifying the previous example.

EQUILIBRIUM OF A PARTICLE

Different positions of a particle :

Position of equilibrium : If net force acting on a body is zero, it is said to be in equilibrium. For equilibrium $\frac{dU}{dx} = 0$. Points P, Q & R are the states of equilibrium positions.

Types of equilibrium :

- **Stable equilibrium :** When a particle is displaced slightly from a position and a force acting on it brings it back to the initial position, it is said to be in stable equilibrium position.

  Necessary conditions : $-\frac{dU}{dx} = 0$, and $\frac{d^2U}{dx^2} = +ve$

- **Unstable Equilibrium :** When a particle is displaced slightly from a position and force acting on it tries to displace the particle further away from the equilibrium position, it is said to be in unstable equilibrium.

  Condition : $-\dfrac{dU}{dx} = 0$ potential energy is maximum i.e. $= \dfrac{d^2U}{dx^2} = -$ ve

- **Neutral equilibrium :** In the neutral equilibrium potential energy is constant. When a particle is displaced from its position it does not experience any force acting on it and continues to be in equilibrium in the displaced position. This is said to be neutral equilibrium.

  A particle is in equilibrium if the acceleration of the particle is zero. As acceleration is frame dependent quantity therefore equilibrium depends on motion of observer also.

Solved Examples

Example 65 The potential energy between two atoms in a molecule is given by, $U_{(x)} = \dfrac{a}{x^{12}} - \dfrac{b}{x^6}$, where a and b are positive constants and x is the distance between the atoms. The system is in stable equilibrium when -

(A) $x = 0$ (B) $x = \dfrac{a}{2b}$ (C) $x = \left(\dfrac{2a}{b}\right)^{1/6}$ (D) $x = \left(\dfrac{11a}{5b}\right)$

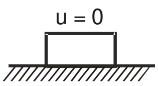
Answer: **(C)**

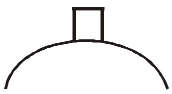
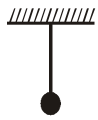
Solution : Given that, $U(x) = \dfrac{a}{x^{12}} - \dfrac{b}{x^6}$

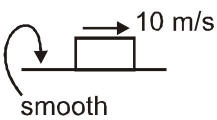
We, know $F = -\dfrac{du}{dx} = (-12)\, a\, x^{-13} - (-6b)\, x^{-7} = 0$

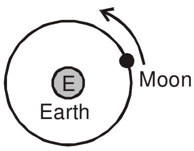
or $\dfrac{-6b}{x^7} = \dfrac{12a}{x^{13}}$ or $x^6 = 12a/6b = 2a/b$ or $x = \left(\dfrac{2a}{b}\right)^{1/6}$

$\dfrac{d^2U}{dx^2} = +\text{ve}$ at $x = \left(\dfrac{2a}{b}\right)^{1/6}$

Example 66. (A) 

(B) 
(C) 
(D) 

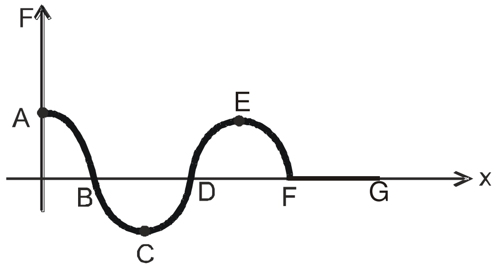
(E) 

(F) 

of the cases above which is not a case of equilibrium.

Solution : (F) as moon is always accelerated. It has centripetal acceleration or it is changing its velocity all the time.

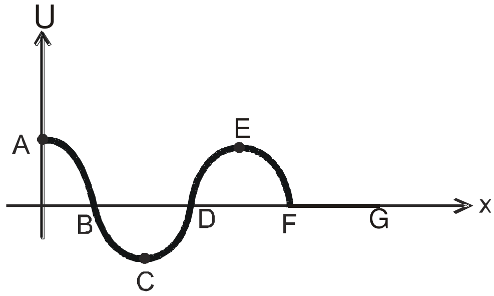
Example 67.



Find out positions of equilibrium and determine whether they are stable, unstable or neutral.

Solution : Equilibrium is at B, D, F as force is zero here.
For checking type of equilibrium displace slightly.
We have B as stable equilibrium
D as unstable equilibrium and F as neutral equilibrium

Example 68.



Identify the points of equilibrium and discuss their nature.

Solution : C, E, F are points of equilibrium because $F = \frac{-\partial U}{\partial x}$
When slope of U - x curve is zero then F is zero.
Check stability through slopes at near by points.
If we move right then slope should be positive for stable equilibrium and vice versa. In short it is like a hill and plateau.

MECHANICAL ENERGY :

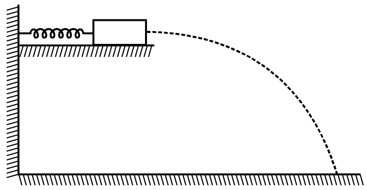
Definition: Mechanical energy 'E' of an object or a system is defined as the sum of kinetic energy 'K' and potential energy 'U', i.e., $E = K + U$

Important Points for M.E.:

1. It is a scalar quantity having dimensions $[ML^2T^{-2}]$ and SI units joule.
2. It depends on frame of reference.
3. A body can have mechanical energy without having either kinetic energy or potential energy. However, if both kinetic and potential energies are zero, mechanical energy will be zero. The converse may or may not be true, i.e., if E = 0 either both PE and KE are zero or PE may be negative and KE may be positive such that KE + PE = 0.
4. As mechanical energy E = K + U, i.e., E - U = K. Now as K is always positive, $E - U \geq 0$, i.e., for existence of a particle in the field, $E \geq U$.
5. As mechanical energy E = K + U and K is always positive, so, if 'U' is positive 'E' will be positive. However, if potential energy U is negative, 'E' will be positive if $K > |U|$ and E will be negative if $K < |U|$ i.e., mechanical energy of a body or system can be negative, and negative mechanical energy means that potential energy is negative and in magnitude it is more than kinetic energy. Such a state is called bound state, e.g., electron in an atom or a satellite moving around a planet are in bound state.

Solved Examples

Example 69 As shown in figure there is a spring block system. Block of mass 500 g is pressed against a horizontal spring fixed at one end to compress the spring through 5.0 cm. The spring constant is 500 N/m. When released, the block moves horizontally till it leaves the spring. Calculate the distance where it will hit the ground 4 m below the spring?



Solution : When block released, the block moves horizontally with speed V till it leaves the spring.

By energy conservation $\frac{1}{2}kx^2 = \frac{1}{2}mv^2$

$V^2 = \frac{kx^2}{m} \quad \Rightarrow \quad V = \sqrt{\frac{kx^2}{m}}$

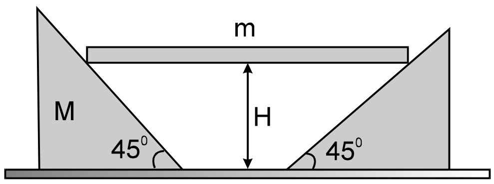
Time of flight $t = \sqrt{\frac{2H}{g}}$

So, horizontal distance travelled from the free end of the spring is V × t

$= \sqrt{\frac{kx^2}{m}} \times \sqrt{\frac{2H}{g}} = \sqrt{\frac{500 \times (0.05)^2}{0.5}} \times \sqrt{\frac{2 \times 4}{10}} = 2 \text{ m}$

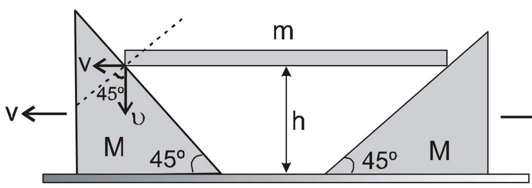
So, At a horizontal distance of 2 m from the free end of the spring.

Example 70 A rigid body of mass m is held at a height H on two smooth wedges of mass M each of which are themselves at rest on a horizontal frictionless floor. On releasing the body it moves down pushing aside the wedges. The velocity of recede of the wedges from each other when rigid body is at a height h from the ground is



(A) $\sqrt{\frac{2mg(H-h)}{m+2M}}$ (B) $\sqrt{\frac{2mg(H-h)}{2m+M}}$ (C) $\sqrt{\frac{8mg(H-h)}{m+2M}}$ (D) $\sqrt{\frac{8mg(H-h)}{2m+M}}$

Solution :



Let speed of the wedge and the rigid body be V and υ respectively.

Then applying wedge constraint we get

$V \cos 45^\circ = \upsilon \cos 45^\circ$

$\therefore \quad V = \upsilon$ ...(i)

Using energy conservation,

$$mg(H - h) = 2\left(\frac{1}{2}MV^2\right) + \frac{1}{2}m\upsilon^2 \qquad ...(ii)$$

From equation (i) and (ii)

$$V = \sqrt{\frac{2mg(H-h)}{m+2M}}$$

$\therefore$ The velocity of recede of wedges from each other = $2 \times V = \sqrt{\frac{8mg(H-h)}{m+2M}}$

So, answer is **(C)**

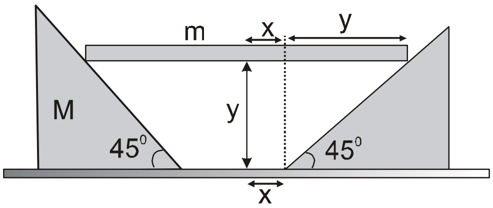
Alter : Length of rod = ℓ

$$x + y = \frac{\ell}{2}$$

$$\frac{dx}{dt} + \frac{dy}{dt} = 0$$

velocity of block = velocity of rod

decrease in potential energy = increase in kinetic energy



$$mg\,(H - h) = \frac{1}{2}mV^2 + \frac{1}{2}MV^2 + \frac{1}{2}MV^2$$

$$\therefore \quad V = \sqrt{\frac{2mg(H-h)}{2M+m}}$$

$$\therefore \quad 2V = \sqrt{\frac{8mg(H-h)}{2M+m}}$$

Exercise-1

✍ **Marked Questions can be used as Revision Questions.**

PART - I : SUBJECTIVE QUESTIONS

Section (A) : Work done by constant force

A-1. A block of mass m is pulled on a rough horizontal surface which has a friction coefficient μ. A horizontal force F is applied which is capable of moving the body uniformly with speed v. Find the work done on the block in time t by (a) weight of the block, (b) Normal reaction by surface on the block, (c) friction, (d) F.

A-2. A gardener pulls a lawn roller along the ground through a distance of 20 m. If he applies a force of 20 kg wt in a direction inclined at 60º to the ground, find the work done by him. (Take g = 10 m/s^2)

A-3. Calculate the work done against gravity by a coolie in carrying a load of mass 10 kg on his head when he moves uniformly a distance of 5 m in the (i) horizontal direction (ii) upwards vertical direction. (Take g = 10 m/s^2)

A-4. A body is constrained to move in the y-direction. It is subjected to a force $(-2\hat{i} + 15\hat{j} + 6\hat{k})$ Newton. What is the work done by this force in moving the body through a distance of 10 m in positive y-direction ?

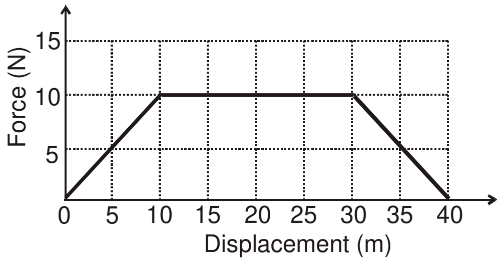
A-5. A block of mass 500 g slides down on a rough incline plane of inclination 53° with a uniform speed. Find the work done against the friction as the block slides through 2 m. [g = 10 m/s^2]

A-6. A block of mass 20 kg is slowly slid up on a smooth incline of inclination 53° by a person. Calculate the work done by the person in moving the block through a distance of 4 m, if the driving force is (a) parallel to the incline and (b) in the horizontal direction. [g = 10 m/s^2]

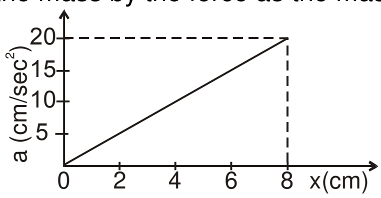
Section (B) : Work done by A variable force

B-1. A particle moves along the x-axis from x = 0 to x = 5 m under the influence of a force F (in N) given by $F = 3x^2 - 2x + 7$. Calculate the work done by this force.

B-2. Adjacent figure shows the force-displacement graph of a moving body, what is the work done by this force in displacing body from x = 0 to x = 35 m ?



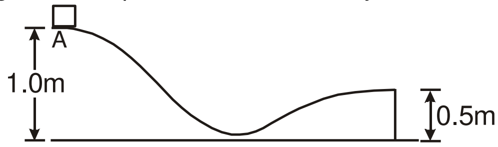
B-3. A 10 kg mass moves along x-axis. Its acceleration as function of its position is shown in the figure. What is the total work done on the mass by the force as the mass moves from x = 0 to x = 8 cm?



B-4. A chain of length ℓ and mass m is slowly pulled at constant speed up over the edge of a table by a force parallel to the surface of the table. Assuming that there is no friction between the table and chain, calculate the work done by force till the chain reaches to the horizontal surface of the table.

Section (C) : Work Energy Theorem

C-1. Figure shows a particle sliding on a frictionless track which terminates in a straight horizontal section. If the particle starts slipping from the point A, how far away from the track will the particle hit the ground ?



C-2. A bullet of mass 20 g is found to pass two points 30 m apart in a time interval of 4 second. Calculate the kinetic energy of the bullet if it moves with constant speed.

C-3. In a ballistics demonstration, a police officer fires a bullet of mass 50.0 g with speed 200 m s^{-1} on soft plywood of thickness 2.00 cm. The bullet emerges with only 10% of its initial kinetic energy. What is the emergent speed of the bullet ?

C-4. It is well known that a raindrop or a small pebble falls under the influence of the downward gravitational force and the opposing resistive force. The resistive force is known to be proportional to the speed of the drop. Consider a drop or small pebble of 1 g falling (from rest) from a cliff of height 1.00 km. It hits the ground with a speed of 50.0 m s^{-1}. What is the work done by the unknown resistive force?

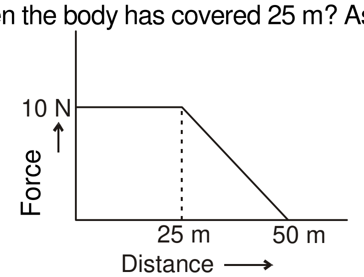
C-5. A bullet of mass 20 g is fired from a rifle with a velocity of 800 ms^{-1}. After passing through a mud wall 100 cm thick, velocity drops to 100 m s^{-1}. What is the average resistance of the wall ? (Neglect friction due to air and work of gravity)

C-6. A force of 1000 N acts on a particle parallel to its direction of motion which is horizontal. Its velocity increases from 1 m s^{-1} to 10 m s^{-1}, when the force acts through a distance of 4 metre. Calculate the mass of the particle. Given : a force of 10 Newton is necessary for overcoming friction.

C-7. A rigid body of mass 5 kg initially at rest is moved by a horizontal force of 20 N on a frictionless table. Calculate the work done by the force on the body in 10 second and prove that this equals the change in kinetic energy of the body.

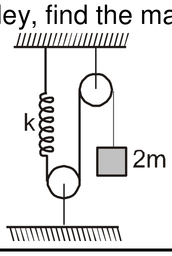
C-8. A rigid body of mass 2 kg initially at rest moves under the action of an applied horizontal force of 7 N on a table with coefficient of kinetic friction = 0.1. Calculate the
(a) work done by the applied force on the body in 10 s.
(b) work done by friction on the body in 10 s.
(c) work done by the net force on the body in 10 s.
(d) change in kinetic energy of the body in 10 s.

C-9. A body of mass 5 kg is acted upon by a variable force. The force varies with the distance covered by the body. What is the speed of the body when the body has covered 25 m? Assume that the body starts from rest.

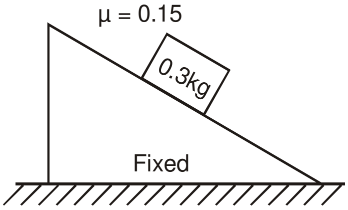


C-10. A block of mass m moving at a speed v compresses a spring through a distance x before its speed becomes one fourth. Find the spring constant of the spring.

C-11. Consider the situation shown in figure. Initially the spring is undeformed when the system is released from rest. Assuming no friction in the pulley, find the maximum elongation of the spring.

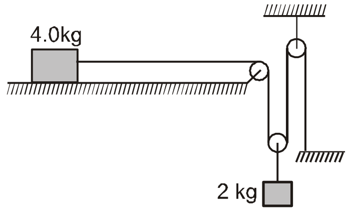


C-12. A rigid body of mass 0.3 kg is taken slowly up an inclined plane of length 10 m and height 5 m (assuming the applied force to be parallel to the inclined plane), and then allowed to slide down to the bottom again. The co-efficient of friction between the body and the plane is 0.15. Using g = 9.8 m/s^2 find the



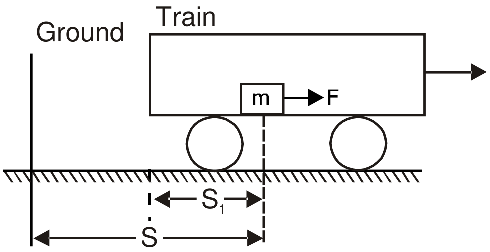
(a) work done by the gravitational force over the round trip.
(b) work done by the applied force over the upward journey
(c) work done by frictional force over the round trip.
(d) kinetic energy of the body at the end of the trip?

C-13. As shown in figure, there is pulley block system. The system is released from rest and the block of mass 2kg is found to have a speed 0.3 m/s after it has descended through a distance of 2m. Find the coefficient of kinetic friction between the block and the table. (g = 10 m/s^2)



C-14. A block of mass 200 g is moving with a speed of 4 m/s at the highest point in a closed circular tube of radius 10 cm kept fixed in a vertical plane. The cross-section of the tube is such that the block just fits in it. The block makes several oscillations inside the tube and finally stops at the lowest point. Find the work done by the tube on the block during the process. (g = 10 m/s^2)

C-15. A block of mass m sits at rest on a frictionless table in a train that is moving with speed v_c (w.r.t. ground) along a straight horizontal track (fig.) A person in the train pushes on the block with a net horizontal force F for a time t in the direction of the car's motion.



(i) What is the final speed of the block according to a person in the train?
(ii) What is the final speed of the block according to a person standing on the ground outside the train?
(iii) How much did kinetic energy of the block change according to the person in the car?
(iv) How much did kinetic energy of the block change according to the person on the ground?
(v) In terms of F, m & t how far did the force displace the object according to the person in car?
(vi) According to the person on the ground?
(vii) How much work does each say the force did?
(viii) Compare the work done to the KE gain according to each person.
(ix) What can you conclude from this computation?

C-16. A block having mass 500 g slides on a rough horizontal table, if the friction coefficient between block and table is 0.2 and initial speed of the block is 60 cm/s. Then calculate :
(i) Work done by frictional force in bringing the block to rest.
(ii) How far does the block move before coming to rest. (g = 10 m/s^2)

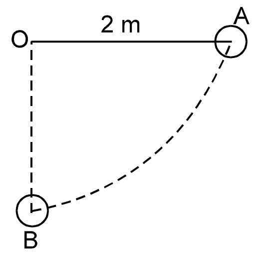
Section (D) : Potential energy and mechanical energy conservation

D-1. A projectile is fired from the top of a 40 m high cliff with an initial speed of 50 m/s at an unknown angle. Find its speed when it hits the ground. (g = 10 m/s^2)

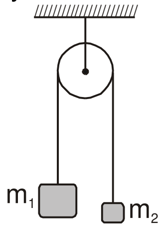
D-2. A rain drop of radius 2 mm falls from a height of 250 m above the ground. What is the work done by the gravitational force on the drop? (Density of water = 1000 kg/m^3)

D-3. Calculate the velocity of the bob of a simple pendulum at its mean position if it is able to rise to a vertical height of 10 cm. Given : g = 980 cm s^{-2}.

D-4. The bob of a pendulum is released from a horizontal position A as shown in figure. If the length of the pendulum is 2 m, what is the speed with which the bob arrives at the lowermost point B, given that it dissipated 10% of its initial potential energy w.r.t. B point against air resistance? ($g = 10\ m/s^2$)

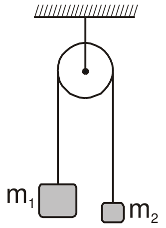


D-5. The heavier block in an Atwood machine has a mass twice that of the lighter one. The tension in the string is 16.0 N when the system is set into motion. Find the decrease in the gravitational potential energy during the first second after the system is released from rest.



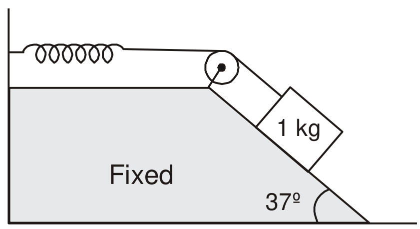
Atwood Machine

D-6. The two blocks in an Atwood machine have masses 2.0 kg and 3.0 kg. Find the work done by gravity during the fourth second after the system is released from rest. ($g = 10\ m/s^2$)



Atwood Machine

D-7. A 1 kg block situated on a rough inclined plane is connected to a spring of spring constant 100 N m^{-1} as shown in figure. The block is released from rest with the spring in the unstretched position. The block moves 10 cm along the incline before coming to rest. Find the coefficient of friction between the block and the incline assume that the spring has negligible mass and the pulley is frictionless. Take $g = 10\ ms^{-2}$.



Section (E) : Power

E-1. An elevator of mass 500 kg is to be lifted up at a constant velocity of 0.4 m s^{-1}. What should be the minimum horse power of the motor to be used? (Take $g = 10\ m\ s^{-2}$ and 1 hp = 750 watts).

E-2. A lift is designed to carry a load of 4000 kg in 10 seconds through 10 floors of a building averaging 6 metre per floor. Calculate the horse power of the lift. (Take $g = 10\ m\ s^{-2}$ and 1 hp = 750 watts).

E-3. A labourer lifts 100 stones to a height of 6 metre in two minute. If mass of each stone be one kilogram, calculate the average power. Given : $g = 10\ ms^{-2}$.

E-4. A motor is capable of raising 400 kg of water in 5 minute from a well 120 m deep. What is the power developed by the motor? [$g = 10\ m/sec^2$]

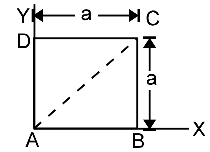
E-5. A man of mass 70 kg climbs up a vertical staircase at the rate of 1 ms^{-1}. What is the power developed by the man? [$g = 10\ m/sec^2$]

E-6. The power of a pump motor is 2 kilowatt. How much water per minute can it raise to a height of 10 metre? Given : $g = 10\ ms^{-2}$.

E-7. An engine develops 10 kW of power. How much time will it take to lift a mass of 200 kg through a height of 40 m? Given : $g = 10\ ms^{-2}$.

Section (F) : Conservative and nonconservative forces and equilibrium

F-1. A force **F** = x^2y^2**i** + x^2y^2**j** (N) acts on a particle which moves in the XY plane.



(a) Determine F is conservative or not and

(b) find the work done by **F** as it moves the particle from A to C (fig.) along each of the paths ABC, ADC, and AC.

F-2. Calculate the forces F(y) associated with the following one-dimensional potential energies:

(a) $U = -\omega y$ (b) $U = ay^3 - by^2$ (c) $U = U_0 \sin \beta y$

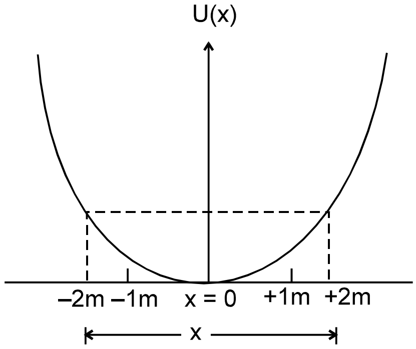
F-3. The potential energy function of a particle in a region of space is given as U = ($2x^2 + 3y^3 + 2z$) J. Here x, y and z are in metres. Find the force acting on the particle at point P(1m, 2m, 3m).

F-4. Force acting on a particle in a conservative force field is :

(i) $\vec{F} = (2\hat{i} + 3\hat{j})$ (ii) $\vec{F} = (2x\hat{i} + 2y\hat{j})$ (iii) $\vec{F} = (y\hat{i} + x\hat{j})$

Find the potential energy function, if it is zero at origin.

F-5. The potential energy function for a particle executing linear simple harmonic motion is given by $U_{(x)} = \frac{1}{2}kx^2$, where k is the force constant. For k = 0.5 N m^{-1}, the graph of U(x) versus x is shown in figure. Show that a particle of total energy 1 J moving under this potential 'turns back' when it reaches x = ± 2m.



PART - II : ONLY ONE OPTION CORRECT TYPE

Section (A) : Work done by constant force

A-1. A rigid body of mass m is moving in a circle of radius r with a constant speed v. The force on the body $\frac{mv^2}{r}$ is and is directed towards the centre. What is the work done by this force in moving the body over half the cirumference of the circle.

(A) $\frac{mv^2}{\pi r^2}$ (B) Zero (C) $\frac{mv^2}{r^2}$ (D) $\frac{\pi r^2}{mv^2}$

A-2. If the unit of force and length each be increased by four times, then the unit of work is increased by

(A) 16 times (B) 8 times (C) 2 times (D) 4 times

A-3. A man pushes wall and fails to displace it. He does

(A) Negative work (B) Positive but not maximum work

(C) No work at all (D) Maximum work

A-4. A rigid body moves a distance of 10 m along a straight line under the action of a force of 5 N. If the work done by this force on the body is 25 joules, the angle which the force makes with the direction of motion of the body is

(A) 0º (B) 30º (C) 60º (D) 90º

A-5. A rigid body of mass m kg is lifted uniform velocity by a man to a height of one metre in 30 sec. Another man lifts the same mass with uniform velocity to the same height in 60 sec. The work done on the body against gravitation by them are in ratio

(A) 1 : 2 (B) 1 : 1 (C) 2 : 1 (D) 4 : 1

A-6. The work done in slowly pulling up a block of wood weighing 2 kN for a length of 10m on a smooth plane inclined at an angle of 15º with the horizontal by a force parallel to the incline is

(A) 4.36 kJ (B) 5.17 kJ (C) 8.91 kJ (D) 9.82 kJ

A-7. A 50 kg man with 20 kg load on his head climbs up 20 steps of 0.25 m height each. The work done by the man on the block during climbing is

(A) 5 J (B) 350 J (C) 1000 J (D) 3540 J

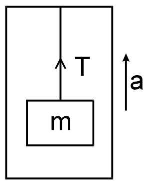
A-8. A particle moves from position $\vec{r}_1 = 3\hat{i} + 2\hat{j} - 6\hat{k}$ to position $\vec{r}_2 = 14\hat{i} + 13\hat{j} + 9\hat{k}$ under the action of force $4\hat{i} + \hat{j} + 3\hat{k}$ N. The work done by this force will be

(A) 100 J (B) 50 J (C) 200 J (D) 75 J

A-9. A ball is released from the top of a tower. The ratio of work done by force of gravity in first, second and third second of the motion of the ball is

(A) 1 : 2 : 3 (B) 1 : 4 : 9 (C) 1 : 3 : 5 (D) 1 : 5 : 3

A-10. A block of mass m is suspended by a light thread from an elevator. The elevator is accelerating upward with uniform acceleration a. The work done by tension on the block during t seconds is (u = 0) :



(A) $\frac{m}{2}(g + a)\, at^2$ (B) $\frac{m}{2}(g - a)at^2$ (B) $\frac{m}{2}\, gat^2$ (D) 0

A-11. Work done by force of kinetic friction on the system.

(A) must be zero (B) must be positive (C) must be negative (D) None of these

A-12. **Statement-1 :** A person walking on a horizontal road with a load on his head does no work on the load against gravity.

Statement-2 : No work is said to be done, if directions of force and displacement of load are perpendicular to each other.

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

Section (B) : Work done by A variable force

B-1. A particle moves under the effect of a force F = Cx from x = 0 to x = x_1. The work done in the process is

(A) Cx_1^2 (B) $\frac{1}{2}Cx_1^2$ (C) Cx_1 (D) Zero

B-2. Two springs have their force constant as k_1 and k_2($k_1 > k_2$). When they are stretched indivisually by the same constant force up to equilibrium -
(A) No work is done by this force in case of both the springs
(B) Equal work is done by this force in case of both the springs
(C) More work is done by this force in case of second spring
(D) More work is done by this force in case of first spring

B-3. A rigid body is acted upon by a horizontal variable force which is inversely proportional to the distance covered from its initial position 's'. The work done by this force will be proportional to :
(A) s (B) s^2 (C) $\sqrt{s}$ (D) None of these

B-4. The work done by the frictional force on a surface in drawing a circle of radius r on the surface by a pencil of negligible mass with a normal pressing force N (coefficient of friction μ_k) is :
(A) $4\pi r^2 \mu_K N$ (B) $-2\pi r^2 \mu_K N$ (C) $-2\pi r \mu_K N$ (D) zero

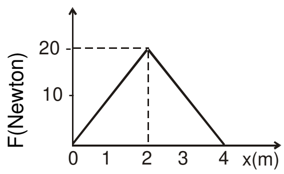
B-5. A force acting on a particle varies with the displacement x as $F = ax - bx^2$. Where a = 1 N/m and b = 1 N/m^2. The work done by this force for the first one meter (F is in newtons, x is in meters) is :
(A) $\frac{1}{6}$ J (B) $\frac{2}{6}$ J (C) $\frac{3}{6}$ J (D) None of these

Section (C) : Work Energy Theorem

C-1. The kinetic energy of a body of mass 2 kg and momentum of 2 Ns is
(A) 1 J (B) 2J (C) 3 J (D) 4 J

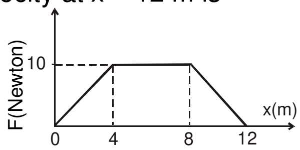
C-2. A particle of mass m at rest is acted upon by (only) force F for a time t. Its kinetic energy after an interval t is :
(A) $\frac{F^2t^2}{m}$ (B) $\frac{F^2t^2}{2m}$ (C) $\frac{F^2t^2}{3m}$ (D) $\frac{Ft}{2m}$

C-3. The graph between the magnitude of resistive force F acting on a body and the position of the body travelling in a straight line is shown in the figure. The mass of the body is 25 kg and initial velocity is 2 m/s. When the distance covered by the body is 4m, its kinetic energy would be (not other force acts on it)



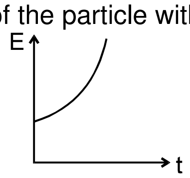
(A) 50 J (B) 40 J (C) 20 J (D) 10 J

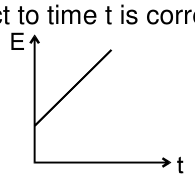
C-4. A particle of mass 0.1 kg is subjected to a force which varies with distance as shown in figure. If it starts its journey from rest at x = 0, its velocity at x = 12 m is

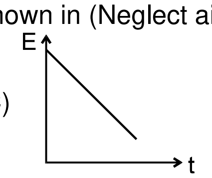


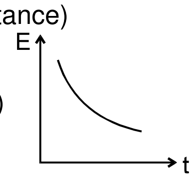
(A) 0 m/s (B) $20\sqrt{2}$ m/s (C) $20\sqrt{3}$ m/s (D) 40 m/s

C-5. A particle is projected horizontally from a height h. Taking g to be constant every where, kinetic energy E of the particle with respect to time t is correctly shown in (Neglect air resistance)

(A) 

(B) 

(C) 

(D) 

C-6. If v, p and E denote the megnitude of velocity, momentum and kinetic energy of the particle, then :
(A) p = dE/dv (B) p = dE/dt (C) p = dv/dt (D) None of these

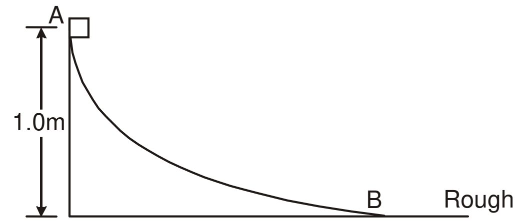
C-7. A heavy stone is thrown from a cliff of height h with a speed v. The stone will hit the ground with maximum speed if it is thrown
(A) vertically downward (B) vertically upward
(C) horizontally (D) the speed does not depend on the initial direction.

C-8. A body moving at 2 m/s can be stopped over a distance x. If its kinetic energy is doubled, how long will it go before coming to rest, if the retarding force remains unchanged ?
(A) x (B) 2x (C) 4x (D) 8x

C-9. A retarding force is applied to stop a train. The train stops after 80 m. If the speed is doubled, then the distance travelled when same retarding force is applied is
(A) The same (B) Doubled (C) Halved (D) Four times

C-10. A particle moves in a straight line with retardation proportional to its displacement. Its loss of kinetic energy for any displacement x is proportional to
(A) x^2 (B) e^x (C) x (D) $\log_e x$

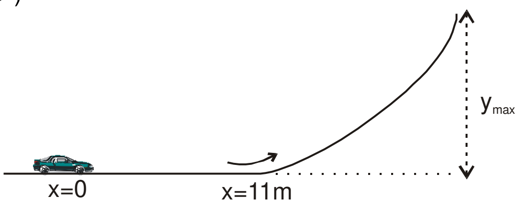
C-11. A block weighing 10 N travles down a smooth curved track AB joined to a rough horizontal surface (figure). The rough surface has a friction coefficient of 0.20 with the block. If the block starts slipping on the track from a point 1.0 m above the horizontal surface, the distance it will move on the rough surface is :

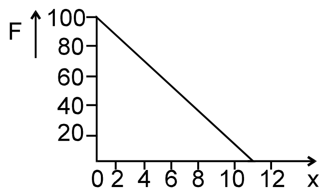


(A) 5.0 m (B) 10.0 m (C) 15.0 m (D) 20.0 m

C-12. A small mass slides down an inclined plane of inclination θ with the horizontal. The co-efficient of friction is $\mu = \mu_0 x$ where x is the distance through which the mass slides down and μ_0 is a constant. Then the distance covered by the mass before it stops is:
(A) $\frac{2}{\mu_0}\tan\theta$ (B) $\frac{4}{\mu_0}\tan\theta$ (C) $\frac{1}{2\mu_0}\tan\theta$ (D) $\frac{1}{\mu_0}\tan\theta$

C-13. A toy car of mass 5 kg starts from rest and moves up a ramp under the influence of force F (F is applied in the direction of velocity) plotted against displacement x. The maximum height attained is given by ($g = 10\ m/s^2$)





(A) y_{max} = 20 m (B) y_{max} = 15 m (C) y_{max} = 11 m (D) y_{max} = 5 m

C-14 A body of 5 kg mass is raised vertically to a height of 10 m by a force of 120 N. Find the final velocity of the body :
(A) $\sqrt{280}$ m/s (B) $\sqrt{200}$ m/s (C) 20 m/s (D) None of these

C-15. The ratio of work done by the internal forces of a car in order to change its speed from 0 to V and from V to 2V is (Assume that the car moves on a horizontal road) –
(A) 1 (B) 1/2 (C) 1/3 (D) 1/4

C-16. A body of mass 4 kg moves under the action of a force $\vec{F} = \left(4\hat{i} + 12t^2\hat{j}\right)N$, where t is the time in second. The initial velocity of the particle is $\left(2\hat{i} + \hat{j} + 2\hat{k}\right)ms^{-1}$. If the force is applied for 1 s, work done is :
[Olympiad (Stage-1) 2017]
(A) 4 J (B) 8 J (C) 12 J (D) 16 J

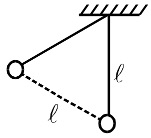
Section (D) : Mechanical Energy conservation

D-1. The negative of the work done by the conservative internal forces on a system equals the change in its
(A) total energy (B) kinetic energy (C) potential energy (D) none of these

D-2. A body is dropped from a certain height. When it loses U amount of its energy it acquires a velocity 'v'. The mass of the body is :
(A) $2U/v^2$ (B) $2v/U^2$ (C) $2v/U$ (D) $U^2/2v$

D-3. A stone is projected vertically up with a velocity u, reaches upto a maximum height h. When it is at a height of 3h/4 from the ground, the ratio of KE and PE at that point is : (consider PE = 0 at the point of projection)
(A) 1 : 1 (B) 1 : 2 (C) 1 : 3 (D) 3 : 1

D-4. A bob hangs from a rigid support by an inextensible string of length ℓ. If it is displaced through a distance ℓ (from the lowest position) keeping the string straight & then released. The speed of the bob at the lowest position is :

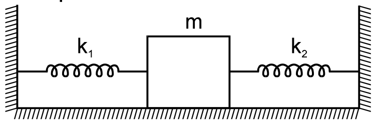


(A) $\sqrt{g\ell}$ (B) $\sqrt{3g\ell}$ (C) $\sqrt{2g\ell}$ (D) $\sqrt{5g\ell}$

D-5. Two springs A and B ($k_A = 2k_B$) are stretched by applying forces of equal magnitudes at the four ends. If the energy stored in A is E, then in B is (assume equilibrium):
(A) E/2 (B) 2E (C) E (D) E/4

D-6. When a spring is stretched by 2 cm, it stores 100 J of energy. If it is stretched further by 2 cm, the stored energy will be increased by
(A) 100 J (B) 200 J (C) 300 J (D) 400 J

D-7. A block of mass m is attached to two unstretched springs of spring constants k_1 and k_2 as shown in figure. The block is displaced towards right through a distance x and is released. Find the speed of the block as it passes through the mean position shown.

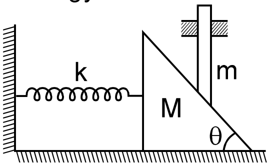


(A) $\sqrt{\frac{k_1+k_2}{m}}\,x$ (B) $\sqrt{\frac{k_1k_2}{m(k_1+k_2)}}\,x$ (C) $\sqrt{\frac{k_1^2k_2^2}{m(k_1^2+k_2^2)}}\,x$ (D) $\sqrt{\frac{k_1^3k_2^3}{m(k_1^3+k_2^3)}}$

D-8. A spring when stretched by 2 mm its potential energy becomes 4 J. If it is stretched by 10 mm, its potential energy is equal to
(A) 4 J (B) 54 J (C) 415 J (D) 100 J

D-9. A spring of spring constant k placed horizontally on a rough horizontal surface. It is compressed against a block of mass m which is placed on rough surface, so as to store maximum energy in the spring. If the coefficient of friction between the block and the surface is μ, the potential energy stored in the spring is : (block does not slide due to force of spring.)
(A) $\frac{\mu^2m^2g^2}{k}$ (B) $\frac{2\mu m^2g^2}{k}$ (C) $\frac{\mu^2m^2g^2}{2k}$ (D) $\frac{3\mu^2mg^2}{k}$

D-10. A wedge of mass M fitted with a spring of stiffness 'k' is kept on a smooth horizontal surface. A rod of mass m is kept on the wedge as shown in the figure. System is in equilibrium and at rest Assuming that all surfaces are smooth, the potential energy stored in the spring is :



(A) $\frac{mg^2\tan^2\theta}{2K}$ (B) $\frac{m^2g\tan^2\theta}{2K}$ (C) $\frac{m^2g^2\tan^2\theta}{2K}$ (D) $\frac{m^2g^2\tan^2\theta}{K}$

D-11. A body of mass m dropped from a certain height strikes a light vertical fixed spring of stiffness k. The height of its fall before touching the spring if the maximum compression of the spring is equal to $\frac{3mg}{k}$ is :

(A) $\frac{3mg}{2k}$ (B) $\frac{2mg}{k}$ (C) $\frac{3mg}{4K}$ (D) $\frac{mg}{4K}$

D-12. A running man has half the kinetic energy of that of a boy of half of his mass. The man speeds up by 1 m/s so as to have same kinetic energy as that of the boy. The original speed of the man will be

(A) $\sqrt{2}$ m/s (B) ($\sqrt{2}$ – 1)m/s (C) $\frac{1}{(\sqrt{2}-1)}$ m/s (D) $\frac{1}{\sqrt{2}}$ m/s

D-13. Two equal masses are attached to the two ends of a spring of spring constant k. The masses are pulled out symmetrically to stretch the spring by a length x over its natural length. The work done by the spring on each mass during the above streching is

(A) $\frac{1}{2}kx^2$ (B) $-\frac{1}{2}kx^2$ (C) $\frac{1}{4}kx^2$ (D) $-\frac{1}{4}kx^2$

D-14. A rod of length 1m and mass 0.5 kg hinged at one end, is initially hanging vertical. The other end is now raised slowly until it makes an angle 60º with the vertical. The required work is : (use g = 10 m/s^2)

(A) $\frac{5}{2}$ J (B) $\frac{5}{4}$ J (C) $\frac{17}{8}$ J (D) $\frac{5\sqrt{3}}{4}$ J

D-15. A block of mass 250 g is kept (does not sticks to spring) on a vertical spring of spring constant 100 N/m fixed from below (block is in equilibrium). The spring is now compressed to have a length 10 cm shorter than its natural length and the system is released from this position. How high does the block rise from this position? Take g = 10 m/s^2.

(A) 20 cm (B) 30 cm (C) 40 cm (D) 50 cm

Section (E) : Power

E-1. A car of mass ‘m’ is driven with a constant acceleration ‘a’ along a straight level road against a constant external resistive force ‘R’. When the velocity of the car is ‘V’, the rate at which the engine of the car is doing work will be

(A) RV (B) maV (C) (R + ma)V (D) (ma – R)V

E-2. The average power required to lift a 100 kg mass through a height of 50 metres in approximately 50 seconds would be

(A) 50 J/s (B) 5000 J/s (C) 100 J/s (D) 980 J/s

E-3. A block of mass m is moving with a constant acceleration 'a' on a rough horizontal plane. If the coefficient of friction between the block and plane is µ.The power delivered by the external agent at a time t from the beginning is equal to :

(A) ma^{2}t (B) µmgat (C) µm(a + µg) gt (D) m(a + µg) at

E-4. A particle moves with a velocity $\vec{v} = (5\hat{i} - 3\hat{j} + 6\hat{k})$ m/s under the influence of a constant force $\vec{F} = (10\hat{i} + 10\hat{j} + 20\hat{k})$N. The instantaneous power applied to the particle is :

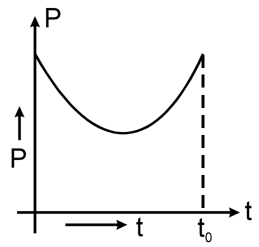
(A) 200 J/s (B) 40 J/s (C) 140 J/s (D) 170 J/s

E-5. An electric motor creates a tension of 4500 N in hoisting cable and reels it at the rate of 2 m/s. What is the power of electric motor ?

(A) 9 W (B) 9 KW (C) 225 W (D) 9000 H.P.

E-6. A man M_1 of mass 80 kg runs up a staircase in 15 s. Another man M_2 also of mass 80 kg runs up the stair case in 20 s. The ratio of the power developed by them (P_1 / P_2) will be :
(A) 1 (B) 4/3 (C) 16/9 (D) None of the above

E-7. Power versus time graph for a given force is given below. Work done by the force upto time t ($\leq t_0$).



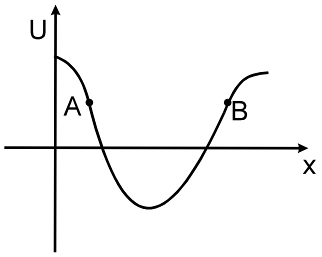
(A) First decreases then increases (B) First increases then decreases
(C) Always increases (D) Always decreases

E-8. An engine pumps up 1000 kg of coal from a mine 100 m deep in 50 sec. The pump is running with diesel and efficiency of diesel engine is 25%. Then its power consumption will be (g = 10m/sec^2) :
(A) 10 kW (B) 80 kW (C) 20 kW (D) 24 kW

Section (F) : Conservative & nonconservative forces and equilibrium

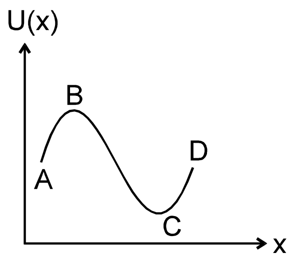
F-1. The potential energy of a particle in a field is $U = \frac{a}{r^2} - \frac{b}{r}$, where a and b are constant. The value of r in terms of a and b where force on the particle is zero will be :
(A) $\frac{a}{b}$ (B) $\frac{b}{a}$ (C) $\frac{2a}{b}$ (D) $\frac{2b}{a}$

F-2. Potential energy v/s displacement curve for one dimensional conservative field is shown. Force at A and B is respectively.



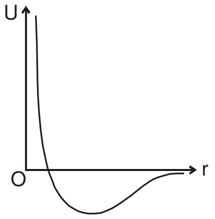
(A) Positive, Positive (B) Positive, Negative (C) Negative, Positive (D) Negative, Negative

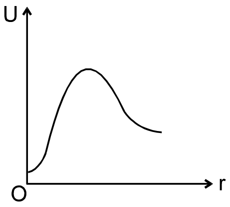
F-3. The potential energy of a particle varies with distance x as shown in the graph.

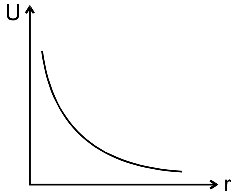


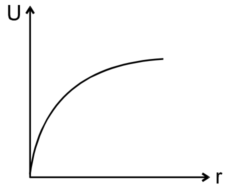
The force acting on the particle is zero at
(A) C (B) B (C) B and C (D) A and D.

F-4. The diagrams represent the potential energy U as a function of the inter-atomic distance r. Which diagram corresponds to stable molecules found in nature.

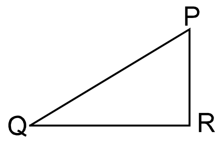
(A) 

(B) 

(C) 

(D) 

F-5. For the path PQR in a conservative force field (fig.), the amount of work done in carrying a body from P to Q & from Q to R are 5 J & 2 J respectively . The work done in carrying the body from P to R will be –



(A) 7 J (B) 3 J (C) $\sqrt{21}$ J (D) zero

F-6. The potential energy for a force field $\vec{F}$ is given by U(x, y) = sin (x + y). The force acting on the particle of mass m at $\left(0, \frac{\pi}{4}\right)$ is

(A) 1 (B) $\sqrt{2}$ (C) $\frac{1}{\sqrt{2}}$ (D) 0

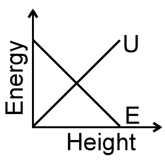
F-7. A particle is taken from point A to point B under the influence of a force. Now it is taken back from B to A and it is observed that the work done in taking the particle from A to B is not equal to the work done in taking it from B to A. If W_{nc} and W_c is the work done by non-conservative forces and conservative forces present in the system respectively, ΔU is the change in potential energy, Δk is the change in kinetic energy, then choose the incorrect option.

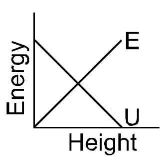
(A) $W_{nc} - \Delta U = \Delta k$ (B) $W_c = -\Delta U$ (C) $W_{nc} + W_c = \Delta k$ (D) $W_{nc} - \Delta U = -\Delta k$

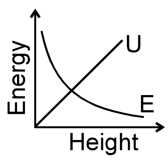
F-8. The potential energy of a system of two particles is given by $U(x) = a/x^2 - b/x$. Find the minimum potential energy of the system, where x is the distance of separation and a, b are positive constants.

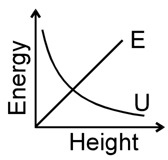
(A) $-\frac{b^2}{4a}$ (B) $\frac{b^2}{4a}$ (C) $\frac{2a}{b}$ (D) $-\frac{2a}{b}$

F-9. Which of the following graphs is correct for kinetic energy (E) and potential energy (U) (with height (h) measured from the ground) for a particle thrown vertically upward from a horizontal ground (h << R_E and U = 0 at h = 0)

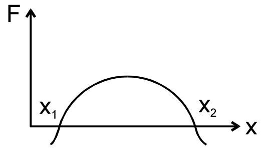
(A) 

(B) 

(C) 

(D) 

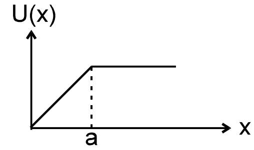
F-10. The force acting on a body moving along x-axis varies with the position of the particle as shown in the figure.

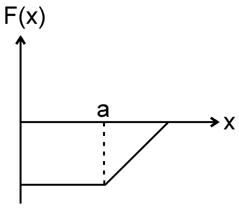


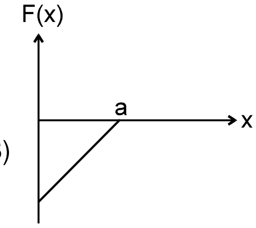
The body is in stable equilibrium at

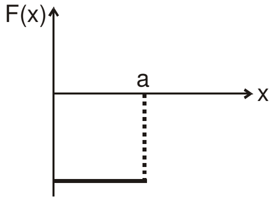
(A) $x = x_1$ (B) $x = x_2$ (C) both x_1 and x_2 (D) neither x_1 nor x_2

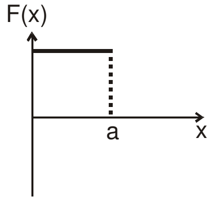
F-11. The potential energy of a system is represented in the first figure, the force acting on the system will be represented by



(A) 

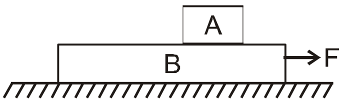
(B) 

(C) 

(D) 

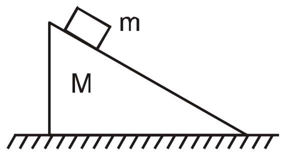
PART - III : MATCH THE COLUMN

1. A block A of mass m kg lies on block B of mass m kg. B in turn lies on smooth horizontal plane. The coefficient of friction between A and B is μ. Both the blocks are initially at rest. A horizontal force F is applied to lower block B at t = 0 such that there is relative motion between A and B.
In the duration from t = 0 second till the lower block B undergoes a displacement of magnitude L, match the statements in column-I with results in column-II.



Column-I	Column-II
(A) Work done by friction force on block A is	(p) positive
(B) Work done by friction force on block B is	(q) negative
(C) Work done by friction on block A plus work done by friction on block B is	(r) less than μ mgL in magnitude
(D) Work done by force F on block B is	(s) equal to μ mgL in magnitude

2. A block of mass m lies on wedge of mass M. The wedge in turn lies on smooth horizontal surface. Friction is absent everywhere. The wedge block system is released from rest. All situation given in column-I are to be estimated in duration the block undergoes a vertical displacement 'h' starting from rest (assume the block to be still on the wedge). Match the statement in column-I with the results in column-II. (g is acceleration due to gravity)



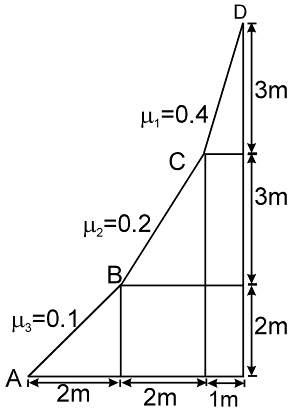
Column I	Column II
(A) Work done by normal reaction acting on the block is	(p) positive
(B) Work done by normal reaction (exerted by block) acting on wedge is	(q) negative
(C) The sum of work done by normal reaction on block and work done by normal reaction (exerted by block) on wedge is	(r) zero
(D) Net work done by all forces on block is	(s) less than mgh in magnitude

Exercise-2

✍ **Marked Questions can be used as Revision Questions.**

PART - I : ONLY ONE OPTION CORRECT TYPE

1. As shown in figure a body of mass 1 kg is shifted from A to D slowly on inclined planes by applying a force parallel to incline plane, such that the block is always in contact with the plane surfaces. Neglecting the jerk experienced at points C and B, total work done by the force is :

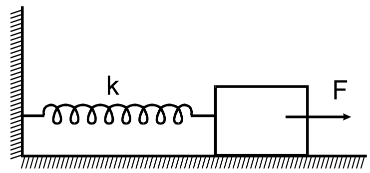


(A) 90 J (B) 56 J (C) 180 J (D) 0 J

2.✍ A small block of mass m is kept on a rough inclined surface of inclination θ fixed in a elevator. The elevator goes down with a uniform velocity v and the block does not slide on the wedge. The work done by the force of friction on the block with respect to ground in time t will be:
(A) zero (B) $-mgvt\cos^2\theta$ (C) $-mgvt\sin^2\theta$ (D) $mgvt\sin2\theta$

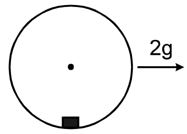
3. A force $\vec{F} = \left(3t\,\hat{i} + 5\,\hat{j}\right)$ N acts on a body due to which its position varies as $\vec{s} = \left(2t^2\,\hat{i} - 5\,\hat{j}\right)$. Work done by this force in first two seconds is:
(A) 23 J (B) 32 J (C) zero (D) can't be obtained

4.✍ A block attached to a spring, pulled by a constant horizontal force, is kept on a smooth surface as shown in the figure. Initially, the spring is in the natural state. Then the maximum positive work that the applied force F can do is : [Given that spring does not break]



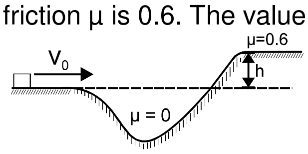
(A) $\frac{F^2}{K}$ (B) $\frac{2F^2}{K}$ (C) ∞ (D) $\frac{F^2}{2K}$

5. A block of mass m is placed inside a smooth hollow cylinder of radius R whose axis is kept horizontally. Initially system was at rest. Now cylinder is given constant acceleration 2g in the horizontal direction by external agent. The maximum angular displacement of the block with the vertical is :



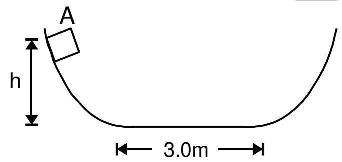
(A) $2\tan^{-1}2$ (B) $\tan^{-1}2$ (C) $\tan^{-1}1$ (D) $\tan^{-1}\left(\frac{1}{2}\right)$

6. In the figure a block slides along a track from one level to a higher level, by moving through an intermediate valley. The track is frictionless until the block reaches the higher level. There a frictional force stops the block in a distance d. The block's initial speed v_0 is 6m/s, the height difference h is 1.1 m and the coefficient of kinetic friction μ is 0.6. The value of d is :



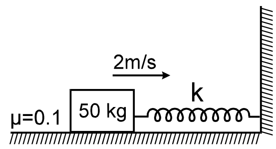
(A) 1.17 m (B) 1.71 m (C) 7.11 m (D) 11.7 m

7. A small particle slides along a track with elevated ends and a flat central part, as shown in figure. The flat part has a length 3m. The curved portions of the track are frictionless, but for the flat part the coefficient of kinetic friction is μ = 0.2. The particle is released at point A, which is at a height h = 1.5 m above the flat part of the track. The position where the particle finally come to rest is:



(A) left to mid point of the flat part (B) right to the mid point of the flat part
(C) Mid point of the flat part (D) None of these

8. A block of mass 50 kg is projected horizontally on a rough horizontal floor. The coefficient of friction between the block and the floor is 0.1. The block strikes a light spring of stiffness k = 100 N/m with a velocity 2m/s. The maximum compression of the spring is :



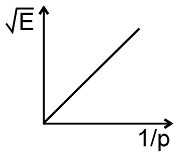
(A) 1 m (B) 2 m (C) 3 m (D) 4 m

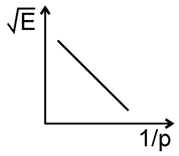
9. A car of mass m starts moving so that its velocity varies according to the law v = $\beta\sqrt{s}$, where β is a constant, and s is the distance covered. The total work performed by all the forces which are acting on the car during the first t seconds after the beginning of motion is
(A) $m\beta^4 t^2/8$ (B) $m\beta^2 t^4/8$ (C) $m\beta^4 t^2/4$ (D) $m\beta^2 t^4/4$

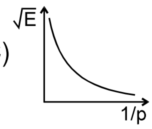
10. An open knife edge of mass 'm' is dropped from a height 'h' on a wooden floor. If the knife penetrates upto depth 'd' into the wood, the average resistance offered by the wood to the knife edge is

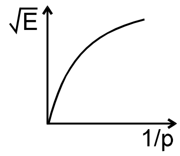
(A) mg (B) $mg\left(1-\frac{h}{d}\right)$ (C) $mg\left(1+\frac{h}{d}\right)$ (D) $mg\left(1+\frac{h}{d}\right)^2$

11. The graph between $\sqrt{E}$ and $\frac{1}{p}$ is (E = kinetic energy and p = momentum)

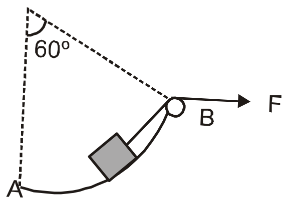
(A)


(B)


(C)


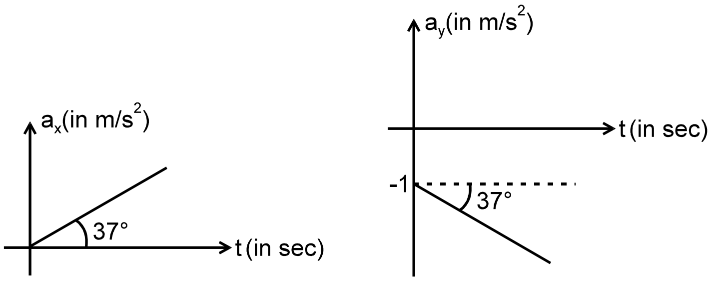
(D)


12. A 10 kg small block is pulled in the vertical plane along a frictionless surface in the form of an arc of a circle of radius 10 m. The applied force is of 200 N as shown in the figure. If the block started from rest at A, the speed at B would be: (g = 10 m/s^2)



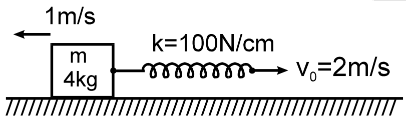
(A) $\sqrt{3}$ m/s (B) $10\sqrt{3}$ m/s
(C) $100\sqrt{3}$ m/s (D) None of these

13. In the figure the variation of components of acceleration of a particle of mass 1 kg is shown w.r.t. time. The initial velocity of the particle is $\vec{u} = (-3\hat{i} + 4\hat{j})$ m/s. The total work done by the resultant force on the particle in time interval from t = 0 to t = 4 seconds is :



(A) 22.5 J (B) 10 J (C) 0 (D) None of these

14. The spring block system lies on a smooth horizontal surface. The free end of the spring is being pulled towards right with constant speed v_0 = 2m/s. At t = 0 sec, the spring of constant k = 100 N/cm is unstretched and the block has a speed 1 m/s to left. The maximum extension of the spring is



(A) 2 cm (B) 4 cm (C) 6 cm (D) 8 cm

15. The potential energy of a particle of mass m free to move along x-axis is given by U = $1/2kx^2$ for x < 0 and U = 0 for x ≥ 0 (x denotes the x-coordinate of the particle and k is a positive constant). If the total mechanical energy of the particle is E, then its speed at $x = -\sqrt{\frac{2E}{k}}$ is

(A) zero (B) $\sqrt{\frac{2E}{m}}$ (C) $\sqrt{\frac{E}{m}}$ (D) $\sqrt{\frac{E}{2m}}$

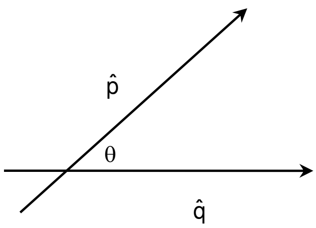
16. Force acting on a particle moving in a straight line varies with the velocity v of the particle as F = K/v, where K is a constant. The work done by this force in time t is

(A) $\frac{K}{v^2}t$ (B) 2Kt (C) Kt (D) $\frac{2Kt}{v^2}$

17. A force $\vec{F} = -K(y\,\hat{i} + x\,\hat{j})$ where K is a positive constant, acts on a particle moving in the x-y plane. Starting from the origin, the particle is taken along the positive x-axis to the point (a, 0) and then parallel to the y-axis to the point (a, a). The total work done by the force $\vec{F}$ on the particle is **[JEE 1998]**

(A) $-2Ka^2$ (B) $2Ka^2$ (C) $-Ka^2$ (D) Ka^2

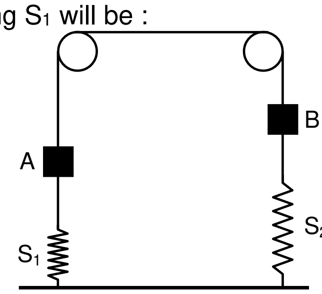
18._ Motion of a particle in a plane is described by the non-orthogonal set of coordinates (p, q) with unit vectors $(\hat{p}, \hat{q})$ inclined at an angle θ as shown in the diagram. If the mass of the particle is m, its kinetic energy is given by $\left(\dot{x} = \frac{dx}{dt}\right)$ **[Olympiad (Stage-1) 2017]**



(A) $\frac{1}{2}m(\dot{p}^2 + \dot{q}^2 + \dot{p}\dot{q}\cos\theta)$ (B) $\frac{1}{2}m(\dot{p}^2 + \dot{q}^2 - \dot{p}\dot{q}(1 - \sin\theta)$

(C) $\frac{1}{2}m(\dot{p}^2 + \dot{q}^2 + 2\dot{p}\dot{q}\cos\theta)$ (D) $\frac{1}{2}m(\dot{p}^2 + \dot{q}^2 + \dot{p}\dot{q}\cot\theta)$

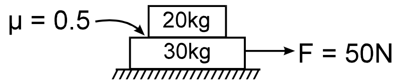
19._ In the figure shown below masses of blocks A and B are 3 kg and 6 kg respectively. The force constants of springs S_1 and S_2 are 160 N/m and 40 N/m respectively. Length of the light string connecting the blocks is 8 m. The system is released from rest with the springs at their natural lengths. The maximum elongation of spring S_1 will be : **[Olympiad (Stage-1) 2017]**



(A) 0.294 m (B) 0.490 m (C) 0.588 m (D) 0.882 m

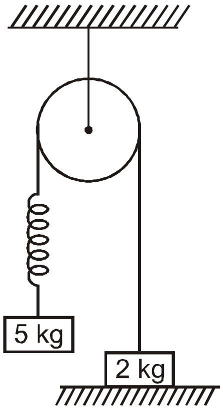
PART - II : SINGLE AND DOUBLE VALUE INTEGER TYPE

1. A small block of mass 20 kg rests on a bigger block of mass 30 kg, which lies on a smooth horizontal plane. Initially the whole system is at rest. The coefficient of friction between the blocks is 0.5. A horizontal force F = 50 N is applied on the lower block. Find the work done (in J) by frictional force on upper block in t = 2sec.



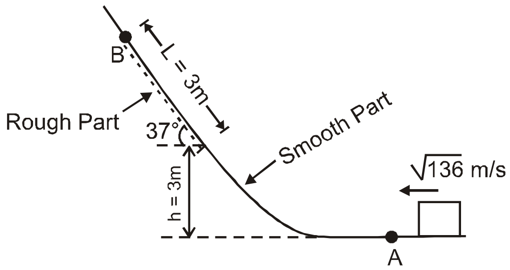
2. A uniform chain of length ℓ and mass m overhangs on a rough horizontal table with its 3/4 part on the table. The friction coefficient between the table and the chain is μ. Find the magnitude of work (in joules) done by the friction during the period the chain slips off the table (take μ = 0.2, g = 10m/s^2, L = 2m, m = 16 kg).

3. The system as shown in the figure is released from rest. The pulley, spring and string are ideal & friction is absent everywhere. If speed of 5 kg block when 2 kg block leaves the contact with ground is $2\sqrt{x}$ m/s, then value of x is : (spring constant k = 40 N/m & g = 10 m/s^2)

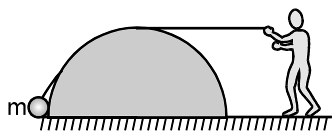


4. A spring (k = 100 Nm^{-1}) is suspended in vertical position having one end fixed at top & other end joined with a 2kg block. When the spring is in non deformed shape, the block is given initial velocity 2 m/s in downward direction. The maximum elongation of the spring is $\left(\frac{\sqrt{3}+1}{n}\right)$ meter. Find n:

5. A small block slides along a path that is without friction until the block reaches the section L = 3m, which begins at height h = 3m on a flat incline of angle 37°, as shown. In that section, the coefficient of kinetic friction is 0.50. The block passes through point A with a speed of $\sqrt{136}$ m/s. Find the speed (in m/s) of the block as it passes through point B where the friction ends, (Take g = 10 m/s^2)

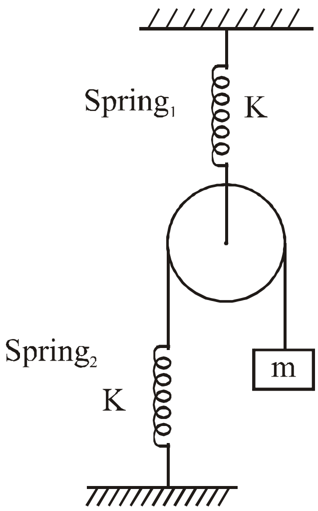


6. As shown in the figure a person is pulling a mass ' m ' from ground on a fixed rough hemispherical surface upto the top of the hemisphere with the help of a light inextensible string. Find the work done (in Joules) by tension in the string on mass m if radius of hemisphere is R and friction coefficient is μ. Assume that the block is pulled with negligible velocity (take μ = 0.1, m = 1kg, g = 10m/s^2, R = 1m).

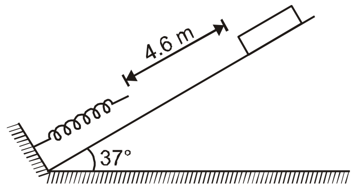


7. Two blocks of masses m_1 and m_2 are connected by a spring of stiffness k. The coefficient of friction between the blocks and the surface is μ. Find the minimum constant horizontal force F (in Newton) to be applied to m_1 in order to slide the mass m_2. (Initally spring is in its natural length). (Take m_1 = 3 kg, m_2 = 5kg, g = 10 m/s^2, μ = 0.2)

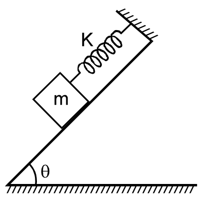
8. All springs, string and pulley shown in figure are light. Initially when both the springs were in their natural state, the system was released from rest. The maximum displacement of block m is $x \times \left(\frac{5mg}{k}\right)$. Calculate x.



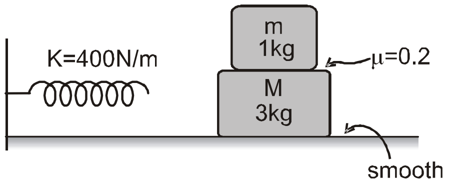
9. As shown in the figure a spring fixed at the bottom end of an incline plane of inclination 37°. A block of mass 4 kg starts slipping down the incline from a point 4.6 m away from the spring. The block compresses the spring by 40 cm, stops momentarily and then rebounds through a distance of 3 m up the incline. If the spring constant of the spring is $\frac{10^3 x}{8}$ N/m, then value of x is. Take g = 10 m/s^2.



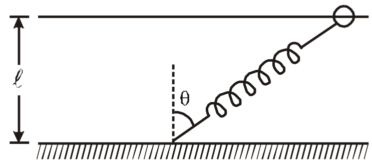
10. In the figure shown, a spring of spring constant K is fixed at one end and the other end is attached to the mass 'm'. The coefficient of friction between block and the inclined plane is 'μ'. The block is released when the spring is in its natural length. Find the maximum speed of the block during the motion. (θ = 45°, μ =0.2, m= 20 kg, k = 10 N/m, g = 10m/s^2)



11. As shown in the figure, there is no friction between the horizontal surface and the lower block (M = 3 kg) but friction coefficient between both the blocks is 0.2. Both the blocks move together with initial speed V towards the spring, compresses it and due to the force exerted by the spring, moves in the reverse direction of the initial motion. What can be the maximum value of V (in cm/s) so that during the motion, there is no slipping between the blocks (use g = 10 m/s^2).

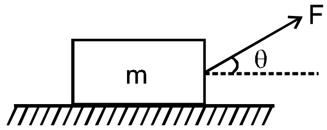


12. One end of a spring of natural length ℓ and spring constant k is fixed at the ground and the other is fitted with a smooth ring of mass m which is allowed to slide on a horizontal rod fixed at a height ℓ (figure). Initially, the spring makes an angle of θ with the vertical when the system is released from rest. If the speed of the ring when the spring becomes vertical is $(2\ell/3)\sqrt{\frac{k}{m}}$ m/s then find the value of angle θ (in degree):



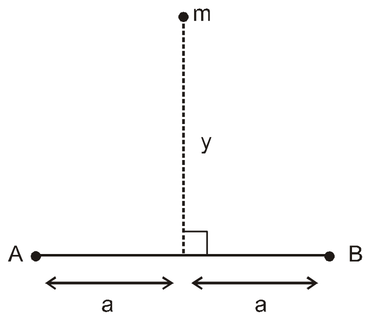
13. A particle of mass 'M' is moved rectilinearly under constant power P_0. At some instant after the start, its speed is v and at a later instant, the speed is 2v. Neglecting friction, distance travelled (in m) by the particle as its speed increases from v to 2v is 7x. Find x (take P_0 = 4 watt, M = 12 kg, v = 3m/s):

14. A block of mass m = 2kg is pulled along a rough horizontal surface by applying a constant force at an angle $\theta = \tan^{-1} 2$ with the horizontal as shown in the figure. The friction coefficient between the block and the surface is $\mu = 0.5$. If the block travels at a uniform velocity v = 5 m/s then calculate the average power (Watt) of the applied force. (Take acceleration due to gravity g = 10 m/s^2)



15. A particle of mass 2 kg is subjected to a two dimensional conservative force given by, $F_x = -2x + 2y$, $F_y = 2x - y^2$. (x, y in m and F in N). If the particle has kinetic energy of 8/3 J at point (2, 3), find the speed (in m/s) of the particle when it reaches (1, 2).

16. Potential energy of a particle of mass m, depends on distance y from line AB according to given relation $U = \frac{K}{\sqrt{y^2 + a^2}}$, where K is a positive constant. A particle of mass m is projected from $y = \sqrt{3}a$ towards line AB (perpendicular to it) then minimum velocity so that it cannot return to its initial point is $\sqrt{\frac{K}{aNm}}$, calculate N.

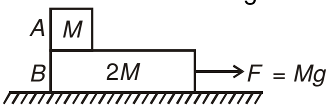


17. The potential energy (in SI units) of a particle of mass 2 kg in a conservative field is U = 6x – 8y. If the initial velocity of the particle is $\vec{u} = -1.5\hat{i} + 2\hat{j}$ then find the total distance (in meter) travelled by the particle in first two seconds.

PART - III : ONE OR MORE THAN ONE OPTIONS CORRECT TYPE

1. No work is done by a force on an object if
(A) the force is always perpendicular to its velocity
(B) the force is always perpendicular to its acceleration
(C) the object has no motion but the point of application of the force moves on the object
(D) the object moves in such a way that the point (of the body) of application of the force remains fixed.

2. In the figure shown, there is no friction between B and ground and $\mu = 2/3$ between A and B.



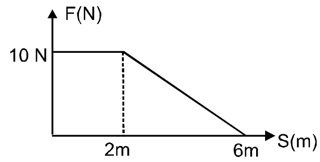
(A) The net work done on block A with respect to B is zero

(B) The net work done on block A with respect to ground for a displacement 'S' is $\frac{MgS}{3}$

(C) The net work done on block B with respect to ground for a displacement 'S' is $\frac{2MgS}{3}$

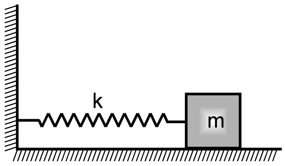
(D) The work done by friction with respect to ground on A and B is equal and opposite in sign.

3. A body of constant mass m = 1 kg moves under variable force F as shown. If at t = 0, S = 0 and velocity of the body is $\sqrt{20}$ m/s and the force is always along direction of velocity, then choose the incorrect options



(A) velocity of the particle will increase upto S = 2m and then decrease.
(B) the final velocity at S = 6 m is 10 m/s
(C) the final velocity at S = 6 m is $4\sqrt{5}$ m/s
(D) the acceleration is constant up to S = 2m and then it is negative.

4. A block of mass 'm' is attached to one end of a massless spring of spring constant ' k '. The other end of the spring is fixed to a wall. The block can move on a horizontal rough surface. The coefficient of friction between the block and the surface is μ. The block is released when the spring has a compression $\frac{2\mu mg}{k}$ of then choose the incorrect option(s) :



(A) The maximum speed of the block is $\mu g\sqrt{\frac{m}{k}}$

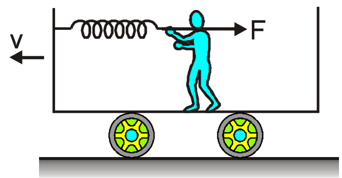
(B) The maximum speed of the block is $2\mu g\sqrt{\frac{m}{k}}$

(C) The block will have velocity towards left during its motion.

(D) The extension in the spring at the instant the velocity of block become zero for the first time after being released is $\frac{\mu mg}{k}$.

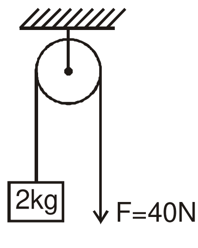
5. The kinetic energy of a particle continuously increases with time
(A) the resultant force on the particle must be parallel to the velocity at all instants.
(B) the resultant force on the particle must be at an angle less than 90° with the velocity all the time
(C) its height above the ground level must continuously decrease
(D) the magnitude of its linear momentum is increasing continuously

6. A man applying a force F upon a stretched spring is stationary in a compartment moving with constant speed v. The compartment covers a distance L in some time t.



(A) The man acting with force F on spring does the work w = – FL.
(B) The total work performed by man on the compartment with respect to ground is zero.
(C) The work done by friction acting on man with respect to ground is, w = – FL.
(D) The total work done by man with respect to ground is, w = – FL.

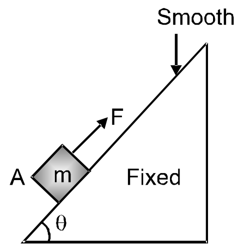
7. A block of mass 2 kg is hanging over a smooth and light pulley through a light string. The other end of the string is pulled by a constant force F = 40 N. At t = 0 the system is at rest as shown. Then in the time interval from t = 0 to t = $\frac{2}{\sqrt{10}}$ seconds, pick up the correct statement (s) : (g = 10 m/s^2)



(A) tension in the string is 40 N
(B) work done by gravity is – 20 J
(C) work done by tension on block is 80J
(D) None of these

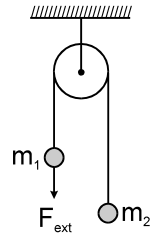
8. One end of a light spring of spring constant k is fixed to a wall and the other end is tied to a block placed on a smooth horizontal surface. In a displacement, the work done by the spring is $\frac{1}{2}$ kx^2. The possible cases are
(A) the spring was initially compressed by a distance x and was finally in its natural length
(B) it was initially stretched by a distance x and finally was in its natural length
(C) it was initially in its natural length and finally in a compressed position
(D) it was initially in its natural length and finally in a stretched position

9. A force F (power of F is p = constant) is applied on block A of mass m as shown in figure, F is parallel to the inclined plane. Then :



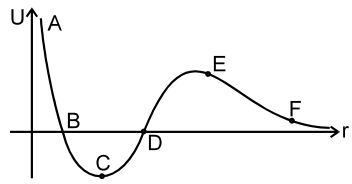
(A) The maximum speed of block A is $\frac{P}{mg\sin\theta}$
(B) The maximum speed of block A is $\frac{P}{mg\cos\theta}$
(C) The speed of block A first increases and then becomes constant
(D) Speed of block A continuously increases

10. Two bodies of mass m_1 and m_2 ($m_2 > m_1$) are connected by a light inextensible string which passes through a smooth fixed pulley as shown. Then choose the correct option(s)



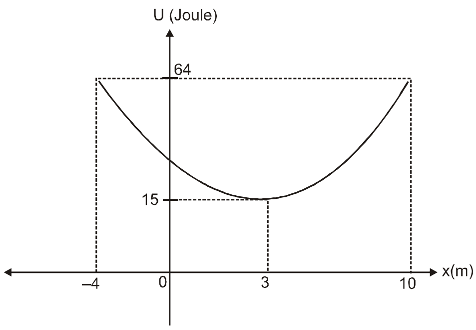
(A) The instantaneous power delivered by an external agent to pull m_1 with constant velocity v is $(m_2 - m_1)gv$
(B) The instantaneous power delivered by an external agent to pull m_1 with constant velocity v is $(m_2 + m_1)gv$
(C) The instantaneous power delivered by an external agent to pull m_1 with constant acceleration a at any instant t, starting from rest, is $[m_2(g + a) - m_1(g - a)]$ at
(D) The instantaneous power delivered by an external agent to pull m_1 with constant acceleration a at any instant t, starting from rest, is $[m_2(g + a) + m_1(g - a)]$ at

11. The given plot shows the variation of U, the potential energy of interaction between two particles with the distance separating them is r. Then which of the following statements is / are correct. :



(A) B and D are equilibrium points
(B) C is a point of stable equilibrium
(C) The force of interaction between the two particles is attractive between points C and D and repulsive between points D and E on the curve.
(D) The force of interaction between the particles is repulsive between points E and F on the curve.

12. A single conservative force F(x) acts on a particle that moves along the x-axis. The graph of the potential energy with x is given. At x = 5m, the particle has a kinetic energy of 50J and its potential energy is related to position 'x' as $U = 15 + (x - 3)^2$ Joule, where x is in meter.



(A) The mechanical energy of system is 69 J.
(B) The mechanical energy of system is 19J.
(C) At x = 3, the kinetic energy of particle is minimum
(D) The maximum value of kinetic energy is 54 J.

13. A body of mass 1.0 kg moves in X-Y plane under the influence of a conservative force. Its potential energy is given by U = 2x + 3y where (x, y) denote the coordinates of the body. The body is at rest at (2, –4) initially. All the quantities have SI units. Therefore, the body **[Olympiad (Stage-1) 2017]**
(A) moves along a parabolic path
(B) moves with a constant acceleration
(C) never crosses the X axis
(D) has a speed of $2\sqrt{13}$ m/s at time t = 2s.

PART - IV : COMPREHENSION

Comprehension-1 :

A block having mass 4 kg is pushed down along an inclined plane of inclination 53° with a force of 40 N acting parallel to the incline. It is found that the block moves on the incline with an acceleration of 10 m/s^2. The initial velocity of block is zero (take g = 10m/s^2).

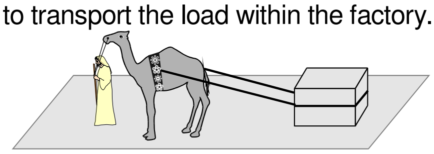
1. Find the work done by the applied force in the 2 seconds from starting of motion,
(A) 800 J (B) – 800 J (C) 640 J (D) – 640 J

2. Find the work done by the weight of the block in the 2 seconds from starting of motion,
(A) 800 J (B) – 800 J (C) 640 J (D) – 640 J

3. Find the work done by the frictional force acting on the block in the 2 seconds from starting of motion.
(A) 800 J (B) – 800 J (C) 640 J (D) – 640 J

Comprehension-2

Ram and Ali are two friends. Both work in a factory. Ali uses a camel to transport the load within the factory. Due to low salary & degradation in health of camel, Ali becomes worried and meets his friend Ram and discusses his problem. Ram collected some data & with some assumptions concluded the following.



(i) The load used in each trip is 1000 kg and has friction coefficient $\mu_k = 0.1$ and $\mu_s = 0.2$.
(ii) Mass of camel is 500 kg.
(iii) Load is accelerated for first 50 m with constant acceleration, then it is pulled at a constant speed of 5m/s for 2 km and at last stopped with constant retardation in 50 m. (String used for pulling load is almost horizontal).

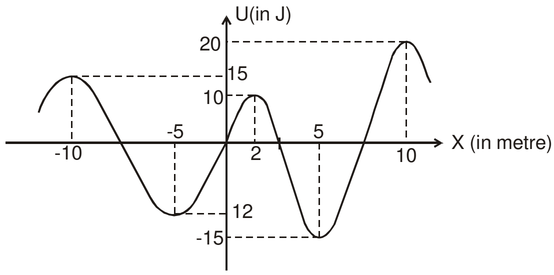
4. Sign of work done by the camel on the load during parts of motion, accelerated motion, uniform motion and retarted motion respectively are:
(A) +ve, + ve, +ve (B) +ve, +ve, – ve (C) +ve, zero, – ve (D) +ve, zero, +ve

5. The ratio of magnitude of work done by camel on the load during accelerated motion to retarded motion is :
(A) 3 : 5 (B) 2.2 : 1 (C) 1 : 1 (D) 5 : 3

6. Maximum power transmitted by the camel to load is :
(A) 6250 J/s (B) 5000 J/s (C) 10^5 J/s (D) 1250 J/s

Comprehension-3

In the figure the variation of potential energy of a particle of mass m = 2kg is represented w.r.t. its x-coordinate. The particle moves under the effect of this conservative force along the x-axis.



7. If the particle is released at the origin then :
(A) it will move towards positive x-axis.
(B) it will move towards negative x-axis.
(C) it will remain stationary at the origin.
(D) its subsequent motion cannot be decided due to lack of information.

8. If the particle is released at x = 2 + Δ where $\Delta \to 0$ (it is positive) then its maximum speed in subsequent motion will be :
(A) $\sqrt{10}$ m/s (B) 5 m/s (C) $5\sqrt{2}$ (D) 7.5 m/s

9. x = – 5 m and x = 10 m positions of the particle are respectively of
(A) neutral and stable equilibrium. (B) neutral and unstable equilibrium.
(C) unstable and stable equilibrium. (D) stable and unstable equilibrium.

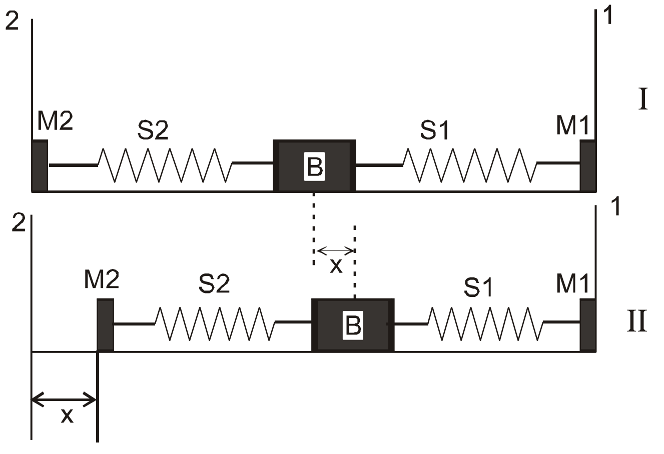
Exercise-3

✍ **Marked Questions can be used as Revision Questions.**

PART - I : JEE (ADVANCED) / IIT-JEE PROBLEMS (PREVIOUS YEARS)

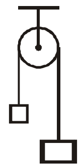
1.✍ **STATEMENT-1 :** A block of mass m starts moving on a rough horizonntal surface with a velocity v. It stops due to friction between the block and the surface after moving through a certain distance. The surface is now tilted to an angle of 30º with the horizontal and the same block is made to go up on the surface with the same initial velocity v. The decrease in the mechanical energy in the second situation is smaller than that in the first situation.
Because
STATEMENT-2 : The coefficient of friction between the block and the surface decreases with the increase in the angle of inclination. **[JEE 2007' 3/184]**
(A) Statement-1 is True, Statement-2 is True; Statement-2 **is** a correct explanation for Statement-1
(B) Statement-1 is True, Statement-2 is True; Statement-2 **is NOT** a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True.

2. A block (B) is attached to two unstretched springs S1 and S2 with spring constants k and 4k, respectively (see figure I). The other ends are attached to identical supports M1 and M2 not attached to the walls. The springs and supports have negligible mass. There is no friction anywhere. The block B is displaced towards wall 1 by a small distance x (figure II) and released. The block returns and moves a maximum distance y towards wall 2. Displacements x and y are measured with respect to the equilibrium position of the block B. The ratio $\frac{y}{x}$ is. Figure : **[JEE 2008, 3/163]**



(A) 4 (B) 2 (C) $\frac{1}{2}$ (D) $\frac{1}{4}$

3.✍ A light inextensible string that goes over a smooth fixed pulley as shown in the figure connects two blocks of masses 0.36 kg and 0.72 kg. Taking g = 10 m/s^2, find the work done (**in joules**) by the string on the block of mass 0.36 kg during the first second after the system is released from rest. **[JEE 2009, 4/160, –1]**

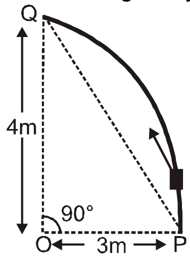


4. The work done on a particle of mass m by a force, $K\left[\frac{x}{\left(x^2+y^2\right)^{3/2}}\hat{i}+\frac{y}{\left(x^2+y^2\right)^{3/2}}\hat{j}\right]$ (K being a constant of appropriate dimensions), when the particle is taken from the point (a, 0) to the point (0, a) along a circular path of radius a about the origin in the x-y plane is : **[JEE-2013 ; 4/60]**

(A) $\frac{2K\pi}{a}$ (B) $\frac{K\pi}{a}$ (C) $\frac{K\pi}{2a}$ (D) 0

5. A particle of mass 0.2 kg is moving in one dimension under a force that delivers a constant power 0.5 W to the particle. If the initial speed (in ms^{-1}) of the particle is zero, the speed (in ms^{-1}) after 5s is : **[JEE-2013 ; 4/60]**

6. Consider an elliptically shaped rail PQ in the vertical plane with OP = 3m and OQ= 4m. A block of mass 1kg is pulled along the rail from P to Q with a force of 18 N, Which is always parallel to line PQ (see the figure given). Assuming no frictional losses, the kinetic energy of the block when it reaches Q is (n × 10) joules. The value of n is (take acceleration due to gravity = 10 ms^{-2})**[JEE (Advanced) 2014 ; P-1, 3/60]**



7*. A particle of mass m is initially at rest at the origin. It is subjected to a force and starts moving along the x-axis. Its kinetic energy K changes with time as $dK/dt = \gamma t$ where λ is a positive constant of appropriate dimensions. Which of the following statements is (are) true? **[JEE (Advanced) 2018 ; P-2, 4/60, –2]**
(A) The force applied on the particle is constant
(B) The speed of the particle is proportional to time
(C) The distance of the particle from the origin increases linearly with time
(D) The force is conservative

PART - II : JEE (MAIN) / AIEEE PROBLEMS (PREVIOUS YEARS)

1. A particle of mass 100 g is thrown vertically upwards with a speed of 5 m/s. the work done by the force of gravity during the time the particle goes up is **[AIEEE 2006, 1.5/180, –1]**
(1) – 0.5 J (2) –1.25 J (3) +1.25 J (4) 0.5 J

2. ✍ A ball of mass 0.2 kg is thrown vertically upwards by applying a force by hand. If the hand moves 0.2 m while applying the force and the ball goes upto 2 m height further, find the magnitude of the force. Consider g = 10 m/s^2 **[AIEEE 2006, 3/180, –1]**
(1) 22 N (2) 4 N (3) 16 N (4) 20 N

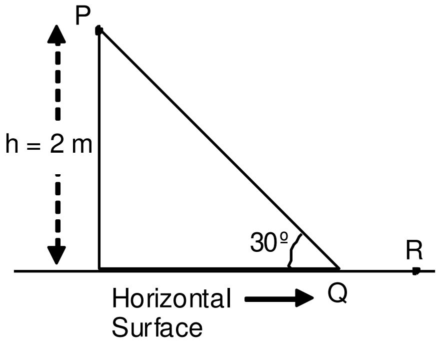
3. ✍ A particle is projected at 60° to the horizontal with a kinetic energy K. The kinetic energy at the highest point is **[AIEEE 2007; 3/120, –1]**
(1) K (2) zero (3) K/4 (4) K/2

4. An athlete in the Olympic Games covers a distance of 100 m in 10 s. His kinetic energy can be estimated to be in the range **[AIEEE 2008 ; 3/105, –1]**
(1) 2×10^5 J – 3×10^5 J (2) 20,000 J – 50,000 J
(3) 2,000 J – 5,000 J (4) 200 J – 500 J

5. At time t = 0s a particle starts moving along the x-axis. If its kinetic energy increases uniformly with time 't', the net force acting on it must be proportional to : **[AIEEE 2011 (11-05-2011) 3/120, –1]**
(1) constant (2) t (3) $\frac{1}{\sqrt{t}}$ (4) $\sqrt{t}$

6. When a rubber-band is stretched by a distance x, it exerts a restoring force of magnitude $F = ax + bx^2$ where a and b are constants. The work done in stretching the unstretched rubber-band by L is : **[JEE (Main) 2014, 3/120, –1]**
(1) $aL^2 + bL^3$ (2) $\frac{1}{2}(aL^2 + bL^3)$ (3) $\frac{aL^2}{2}+\frac{bL^3}{3}$ (4) $\frac{1}{2}\left(\frac{aL^2}{2}+\frac{bL^3}{3}\right)$

7. A point particle of mass m, moves along the uniformly rough track PQR as shown in the figure. The coefficient of friction, between the particle and the rough track equals μ. The particle is released, from rest, from the point P and it comes to rest at a point R. The energies, lost by the ball, over the parts, PQ and QR, of the track, are equal to each other, and no energy is lost when particle changes direction from PQ to QR. The values of the coefficient of friction μ and the distance x(= QR), are, respectively close to : **[JEE (Main) 2016; 4/120, – 1]**



(1) 0.2 and 3.5 m (2) 0.29 and 3.5 m (3) 0.29 and 6.5 m (4) 0.2 and 6.5 m

8. A person trying to lose weight by burning fat lifts a mass of 10 kg upto a height of 1 m 1000 times. Assume that the potential energy lost each time he lowers the mass is dissipated. How much fat will he use up considering the work done only when the weight is lifted up ? Fat supplies 3.8×10^7 J of energy per kg which is converted to mechanical energy with a 20% efficiency rate. Take g = 9.8 ms^{-2}

[JEE (Main) 2016; 4/120, –1]

(1) 6.45×10^{-3} kg (2) 9.89×10^{-3} kg (3) 12.89×10^{-3} kg (4) 2.45×10^{-3} kg

9. A time dependent force F = 6t acts on a particle of mass 1kg. If the particle starts from rest, the work done by the force during the first 1 sec. will be : **[JEE (Main) 2017; 4/120, –1]**

(1) 18 J (2) 4.5 J (3) 22 J (4) 9 J

10. A body of mass $m = 10^{-2}$ kg is moving in a medium and experiences a frictional force $F = -kv^2$. Its initial speed is $v_0 = 10\ ms^{-1}$. If after 10 s, its energy is $\frac{1}{8}mv_0^2$, the value of k will be :

[JEE (Main) 2017; 4/120, –1]

(1) 10^{-1}Kg $m^{-1}s^{-1}$ (2) 10^{-3}Kg m^{-1} (3) 10^{-3}Kg s^{-1} (4) 10^{-4}Kg m^{-1}

11. A particle is moving in a circular path of radius a under the action of an attractive potential $U = -\frac{k}{2r^2}$. Its total energy is : **[JEE (Main) 2018; 4/120, –1]**

(1) zero (2) $-\frac{3}{2}\frac{k}{a^2}$ (3) $-\frac{k}{4a^2}$ (4) $\frac{k}{2a^2}$

Answers

EXERCISE-1

PART - I

Section (A) :

A-1. (a) zero, (b) zero, (c) $-\mu mgvt$, (d) $\mu mgvt$

A-2. 2000 J **A-3.** (i) Zero, (ii) 500J

A-4. 150 J **A-5.** 8 J

A-6. (a) 640 J (b) 640J

Section (B) :

B-1. 135 J. **B-2.** $\frac{575}{2}$ J = 287.5 J

B-3. 8×10^{-2} J **B-4.** $\frac{mg\ell}{2}$

Section (C) :

C-1. At a horizontal distance of 1 m from the end of the track.

C-2. $\frac{9}{16} = 0.5625$ J

C-3. $v_f = 20\sqrt{10} = 63.2\ ms^{-1}$

C-4. $-\frac{35}{4}$ J = – 8.75 J

C-5. F = 6300 N **C-6.** 80 kg

C-7. 4000 J

C-8. (a) 875 Joule (b) –250 joule (c) 625 joule.
(d) 625 joule, Change in kinetic energy of the body is equal to the work done by the net force in 10 second. This is in accordance with work-energy theorem.

C-9. 10 ms^{-1} **C-10.** $\frac{15mv^2}{16x^2}$

C-11. 4mg/k

C-12. (a) Since the gravitational force is a conservative force therefore the work done in round trip is zero.
(b) $w_F = (9.8)\ (0.3)\ (1/2)\ (1+0.15\sqrt{3})\ (10)$J
$\cong 18.519$J
(c) $-0.15 \times 0.3 \times 9.8 \times (\sqrt{3}/2) \times 20$ J
$\cong -7.638$ J
(d) $0.3 \times 9.8 \times (10/2)\ (1 - 0.15 \times \sqrt{3})$
$\cong 10.880$ J

C-13. $\mu = 0.245$ **C-14.** – 2 J

C-15. (i) $a_1 = F/m$, so $v_1 = a_1t = Ft/m$.
(ii) Since velocities add, $v = v_c + v_1 = v_c + Ft/m$
(iii) $\Delta K_1 = m(v_1)^2/2 = F^2t^2/2m$
(iv) $\Delta K = m\ (v_c + v_1)^2/2 - mv_c^2/2$
(v) s_1 is $a_1t^2/2 = Ft^2/2m$
(vi) $s_1 + v_ct$
(vii) $W_g = F\ [V_c t + \frac{1}{2}\frac{Ft^2}{m}]$, $W_t = F\ [\frac{1}{2}\frac{F}{m}t^2]$
(viii) Compare W and W_1 with ΔK and ΔK_1, they are respectively equal.
(ix) The work-energy theorem holds for moving observers.

C-16. (i) –0.09 J (ii) 9 cm

Section (D) :

D-1. $10\sqrt{33}$ m/s **D-2.** 0.082 J

D-3. $\frac{7}{5}\ ms^{-1} = 1.40\ ms^{-1}$ **D-4.** 6 ms^{-1}

D-5. 2g = 19.6 J **D-6.** $0.7g^2 = 70$ J

D-7. $\frac{1}{8}$

Section (E) :

E-1. $\frac{8}{3}$ hp **E-2.** 320 hp **E-3.** 50 W

E-4. 1600W **E-5.** 700 W **E-6.** 1200 kg

E-7. 8 second

Section (F) :

F-1. (a) No
(b) $W_{ABC} = W_{ADC} = \frac{a^5}{3}$ (J), $W_{AC} = \frac{2a^5}{5}$ (J)

F-2. (a) $F = -\frac{dU}{dy} = \omega$;
(b) $F = -\frac{dU}{dy} = -3ay^2 + 2by$;
(c) $F = -\frac{dU}{dy} = -\beta U_0 \cos\beta y$

F-3. $\vec{F} = -(4\hat{i} + 36\hat{j} + 2\hat{k})$N

F-4. (i) U(x, y, z) = (–2x – 3y)
(ii) $U(x, y, z) = -(x^2 + y^2)$
(iii) U(x, y, z) = – xy.

PART - II

Section (A) :

A-1.	(B)	**A-2.**	(A)	**A-3.**	(C)
A-4.	(C)	**A-5.**	(B)	**A-6.**	(B)
A-7.	(C)	**A-8.**	(A)	**A-9.**	(C)
A-10.	(A)	**A-11.**	(C)	**A-12.**	(A)

Section (B) :

B-1.	(B)	**B-2.**	(C)	**B-3.**	(D)
B-4.	(D)	**B-5.**	(A)		

Section (C) :

C-1.	(A)	**C-2.**	(B)	**C-3.**	(D)
C-4.	(D)	**C-5.**	(A)	**C-6.**	(A)
C-7.	(D)	**C-8.**	(B)	**C-9.**	(D)
C-10.	(A)	**C-11.**	(A)	**C-12.**	(A)
C-13.	(C)	**C-14**	(A)	**C-15.**	(C)
C-16.	(D)				

Section (D) :

D-1.	(C)	**D-2.**	(A)	**D-3.**	(C)
D-4.	(A)	**D-5.**	(B)	**D-6.**	(C)
D-7.	(A)	**D-8.**	(D)	**D-9.**	(C)
D-10.	(C)	**D-11.**	(A)	**D-12.**	(C)
D-13.	(D)	**D-14.**	(B)	**D-15.**	(A)

Section (E) :

E-1.	(C)	**E-2.**	(D)	**E-3.**	(D)
E-4.	(C)	**E-5.**	(B)	**E-6.**	(B)
E-7.	(C)	**E-8.**	(B)		

Section (F) :

F-1.	(C)	**F-2.**	(B)	**F-3.**	(C)
F-4.	(A)	**F-5.**	(A)	**F-6.**	(A)
F-7.	(D)	**F-8.**	(A)	**F-9.**	(A)
F-10.	(B)	**F-11.**	(C)		

PART - III

1. (A) $\rightarrow$ p,r ;(B) $\rightarrow$ q,s ; (C) $\rightarrow$ q,r ; (D) $\rightarrow$ p

2. (A) $\rightarrow$ q,s ; (B) $\rightarrow$ p,s ; (C) $\rightarrow$ r,s ;(D) $\rightarrow$ p,s

EXERCISE-2

PART - I

1.	(A)	**2.**	(C)	**3.**	(B)
4.	(B)	**5.**	(A)	**6.**	(A)
7.	(C)	**8.**	(A)	**9.**	(A)
10.	(C)	**11.**	(C)	**12.**	(B)
13.	(B)	**14.**	(C)	**15.**	(A)
16.	(C)	**17.**	(C)	**18.**	(C)
19.	(A)				

PART - II

1.	40	**2.**	18	**3.**	2
4.	5	**5.**	4	**6.**	11
7.	11	**8.**	2	**9.**	9
10.	8	**11.**	20	**12.**	53
13.	27	**14.**	25	**15.**	2
16.	1	**17.**	15		

PART - III

1.	(ACD)	**2.**	(ABCD)	**3.**	(ACD)
4.	(BCD)	**5.**	(BD)	**6.**	(ABC)
7.	(AC)	**8.**	(AB)	**9.**	(AC)
10.	(AC)	**11.**	(BD)	**12.**	(AD)
13.	(BCD)				

PART - IV

1.	(A)	**2.**	(C)	**3.**	(D)
4.	(A)	**5.**	(D)	**6.**	(A)
7.	(B)	**8.**	(B)	**9.**	(D)

EXERCISE-3

PART - I

1.	(C)	**2.**	(C)	**3.**	8
4.	(D)	**5.**	5	**6.**	5
7.	(ABD)				

PART – II

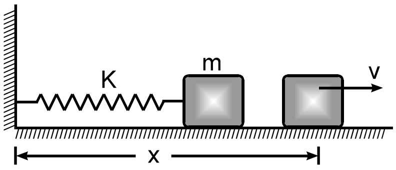
1.	(2)	**2.**	(1)	**3.**	(3)
4.	(3)	**5.**	(3)	**6.**	(3)
7.	(2)	**8.**	(3)	**9.**	(2)
10.	(4)	**11.**	(1)		

High Level Problems (HLP)

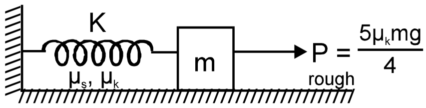
Marked Questions can be used as Revision Questions.

SUBJECTIVE QUESTIONS

1. A block of mass ' m ' is pushed against a spring of spring constant ' k ' fixed at one end to a wall. The block can slide on a frictionless table as shown in the figure. The natural length of the spring is L_0 and it is compressed to one-fourth of natural length and the block is released. Find its velocity as a function of its distance (x) from the wall and maximum velocity of the block. The block is not attached to the spring.



2. A block of mass m rests on a rough horizontal plane having coefficient of kinetic friction μ_k and coefficient of static friction μ_s. The spring is in its natural length, when a constant force of magnitude $P = \frac{5\mu_k mg}{4}$ starts acting on the block.

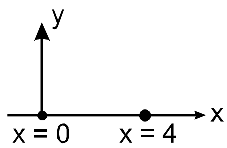


The spring force F is a function of extension x as $F = kx^3$. (Where k is spring constant)

(a) Comment on the relation between μ_s and μ_k for the motion to start.

(b) Find the maximum extension in the spring (Assume the force P is sufficient to make the block move).

3. A particle of mass m = 1 kg lying on x-axis experiences a force given by law $\vec{F} = x(3x-2)\,\hat{i}$ Newton, where x is the x-coordinate of the particle in meters.

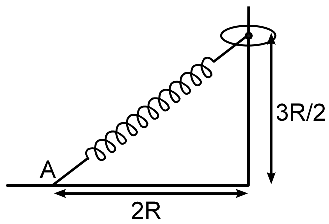


(a) Locate the points on x-axis where the particle is in equilibrium.

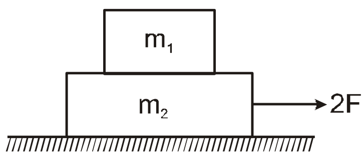
(b) Draw the graph of variation of force F (y-axis) with x-coordinate of the particle (x-axis). Hence or otherwise indicate at which positions the particle is in stable or unstable equilibrium.

(c) What is the minimum speed to be imparted to the particle placed at x = 4 meters such that it reaches the origin.

4. A ring of mass ' m ' can slide along a fixed rough vertical rod as shown in fig. The ring is connected by a spring of spring constant $k = \frac{4mg}{R}$ where 2 R is the natural length of spring. The other end of spring is fixed to the ground at point A at a horizontal distance of 2 R from the base of the rod. If the ring is released from a height of **3R/2** & it reaches the ground with a speed $\sqrt{3gR}$, find co-efficient of friction between the rod & ring.



5. A block of mass m_1 is kept over another block of mass m_2 and the system rests on a horizontal surface (as shown in figure). A constant horizontal force 2F acting on the lower block produces an acceleration $\frac{F}{(m_1+m_2)}$ in the system, the two blocks always move together.



(a) Find the coefficient of kinetic friction between the bigger block and the horizontal surface.

(b) Find the frictional force acting on the smaller block.

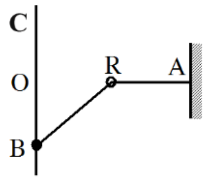
(c) Find the work done by the force of friction on the smaller block by the bigger block during a displacement x of the system.

6. A box having mass 400 kg is to be slowly slide through 10 m on a horizontal straight track having friction coefficient 0.2 with the box.

(a) Find the work done by the person pulling the box with a rope at an angle θ with the horizontal.

(b) Find the work when the person has chosen a value of θ which ensures him the minimum magnitude of the force. [$g = 10\ m/s^2$]

7. A small bead 'B' of mass m is free to slide on a fixed smooth vertical wire, as indicated in the diagram. One end of a light elastic string, of unstreched length a and force constant 2mg/a is attached to B. The string passes through a smooth fixed ring R and the other end of the string is attached to the fixed point A, AR being horizontal. The point O on the wire is at same horizontal level as R and AR = RO = a.



(i) In the equilibrium position, find OB.
(ii) The bead B is raised to a point C of the wire above O, where OC = a and is released from rest. Find the speed of the bead as it passes O and find the greatest depth below O of the bead in the subsequent motion.

8. A particle of mass m approaches a region of force starting from r = + ∞. The potential energy function in terms of distance r from the origin is given by,

$$U(r) = \frac{K}{2a^3}(3a^2 - r^2) \text{ for, } 0 \le r \le a$$

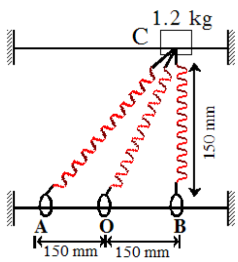
$$= K/r \quad \text{for, } r \ge a$$

where K > 0 (positive constant)
(a) Derive the force F(r) and determine whether it is repulsive or attractive.
(b) With what velocity should the particle start at r = ∞ to cross over to other side of the origin.
(c) If the velocity of the particle at r = ∞ is $\sqrt{\frac{2K}{am}}$ towards the origin describe the motion.

9. A uniform string of mass 'M' and length 2a, is placed symmetrically over a smooth and small pulley and has particles of masses 'm' and 'm′' attached to its ends; show that when the string runs off the peg its velocity is $\sqrt{\left\{\frac{M+2(m-m')}{M+m+m'}ag\right\}}$. Assume that m > m′.

10. A single conservative force F(x) acts on a 1.0 kg particle that moves along the x-axis. The potential energy U(x) is given by : $U(x) = 20 + (x - 2)^2$ where x is in meters. At x = 5.0 m the particle has a kinetic energy of 20 J.
(i) What is the mechanical energy of the system ?
(ii) Make a plot of U (x) as a function of x for – 10 m < x < 10m, and on the same graph draw the line that represents the mechanical energy of the system. Use part (ii) to determine
(iii) The least value of x and
(iv) The greatest value of x between which the particle can move.
(v) The maximum kinetic energy of the particle and
(vi) The value of x at which it occurs.
(vii) Determine the equation for F (x) as a function of x.
(viii) For what value of x does F(x) = 0 ?

11. A 1.2 kg collar C may slide without friction along a fixed smooth horizontal rod. It is attached to three springs each of constant K = 400 N/m and 150 mm undeformed length. Knowing that the collar is released from rest in the position shown. Determine the maximum velocity it will reach in its motion. [Here A, O, B are fixed points.]



12. A block of mass 4 kg is moved from rest on a smooth inclined plane of inclination 53° by applying a constant force of 40 N parallel to the incline. The force acts for two seconds. (a) Show that the work done by the applied force is not less than 160 J. (b) Find the work done by the force of gravity in that two seconds if the work done by the applied force is 160 J. (c) Find the kinetic energy of the block at the instant the force ceases to act in case (b). [Take g = 10 m/s²]

13. There is a vertically suspended spring, mass system. When block of mass 10 kg is suspended from lower end of the spring, it is stretched by 20 cm under the load of block at equilibrium position. When an upward speed of 4 m/s is imparted to the block by giving a sharp impulse from below, how much high will it rise from equilibrium position.

14. A certain spring is found not to obey Hooke's law, it exerts a restoring force $F(x) = -\alpha x - \beta x^2$ if it is stretched or compressed, where $\alpha = 48$ N/m and $\beta = 24$ N/m^2. The mass of the spring is negligible. An object with mass 1 kg on a frictionless, horizontal surface is attached to the spring, pulled a distance 1m to the right to stretch the spring and released. The speed of the object when it is 0.5m to the right of the x = 0 equilibrium position is

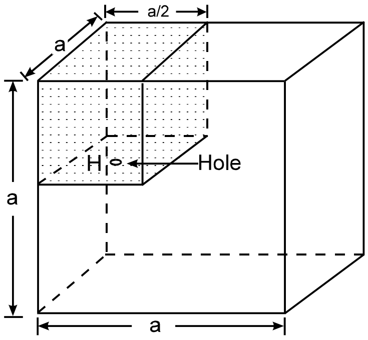
15. Wind entering in a wind mill with a velocity of 20 m/sec facing area of the windmill is 10m^2 and density of air is 1.2 kg/m^3. If wind energy is converted into electrical energy with 33.3% efficiency, then find electrical power produced by the wind mill in kw.

16. An engine can pull 4 coaches at a maximum speed of 20 m/s. Mass of the engine is twice the mass of every coach. Assuming resistive forces to be proportional to the weight, approximate maximum speeds of the engine when it pulls 12 and 6 coaches are (power of engine remains constant) :

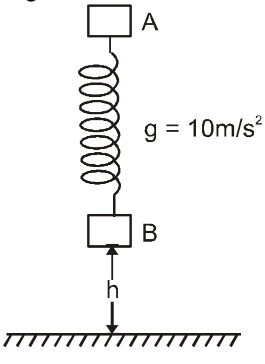
17. A pump motor is used to deliver water at a certain rate from a given pipe. To obtain "n" times water from the same pipe in the same time, the factor by which the power of the motor should be increased is: **[REE 1998]**

18. A chain of mass M and length ℓ is held vertically such that its bottom end just touches the surface of a horizontal table. The chain is released from rest. Assume that the portion of chain on the table does not form a heap. The momentum of the portion of the chain above the table after the top end of the chain falls down by a distance $\frac{\ell}{8}$.

19. The figure shows a hollow cube of side 'a' of volume V. There is a small chamber of volume $\frac{V}{4}$ in the cube as shown. This chamber is completely filled by m kg of water. Water leaks through a hole H and spreads in the whole cube. Then the work done by gravity in this process assuming that the complete water finally lies at the bottom of the cube is :



20. From what minimum height 'h' in metre must the system be released when spring is in its natural length as shown in the figure. So that after perfectly inelastic collision. (e = 0), of block B, with ground, B may be lifted off ground. (Take k = 40 N/m, g = 10 m/s^2, m_A = 2kg, m_B = 4kg)

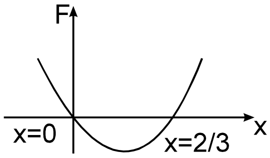


HLP Answers

1. $v = \sqrt{\frac{k}{m}\left[\left(\frac{3L_0}{4}\right)^2 - (L_0 - x)^2\right]}$ when $x < L_0$; $v_{max} = \frac{3L_0}{4}\sqrt{\frac{k}{m}}$ when $x \ge L_0$

2. (a) $5\mu_k > 4\mu_s$; (b) $x = \left(\frac{\mu_K mg}{K}\right)^{1/3}$

3. (a) $x = 0$ and $x = \frac{2}{3}$ m (b)



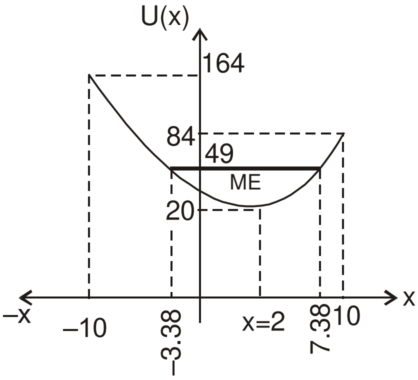
The particle is in stable equilibrium at x = 0 metre and unstable equilibrium at $x = \frac{2}{3}$ metre (c) $v = \sqrt{\frac{2600}{27}}$ m/s

4. $\frac{1}{8(3 - 4\ell n2)}$ **5.** (a) $\frac{F}{(m_1 + m_2)g}$ (b) $\frac{m_1F}{(m_1 + m_2)}$ (c) $\frac{m_1Fx}{(m_1 + m_2)}$

6. (a) $\frac{40000}{5 + \tan\theta}$ J (b) 7692.31 J 7690 J **7.** (i) OB = a/2 (ii) $v = \sqrt{4ag}$, d = 2a

8. (a) repulsive (b) $v > \sqrt{\frac{3k}{am}}$ (c) stops at r = a & then reaches to $r = \infty$.

10. (i) 49 J (ii)



(iii) $-\sqrt{29} + 2 \approx -3.38$ m

(iv) $-\sqrt{29} + 2 \approx 7.38$ m (v) 29 J (vi) x = 2m (vii) F = 2 (2 – x)

(viii) x = 2

11. $\left[\frac{15}{2}\left\{(\sqrt{5} - 1)^2 + (\sqrt{2} - 1)^2\right\}\right]^{1/2}$ m/s = 3.189 m/s **12.** (b) – 128 J (c) 32 J

13. 0.56 m = 56 cm **14.** $5\sqrt{2}$ m/s **15.** 16

16. 8.5 m/s and 15 m/s respectively **17.** n^3 **18.** $\frac{7}{16} M\sqrt{g\ell}$

19. $\frac{5}{8}$ mga **20.** 2