STA305H1 Assignment 4: Factorial Experiment Report

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April 11, 2025

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1 Description of the Design

The experiment is going to investigate the effect of five process factors on the strength of ceramics. The data were obtained from a NIST-certified full factorial 2^5 design, there are 32 runs with all combinations of the factor levels.

1.1 Factors (all 2-level):

These are the variables in our experiment:

- **A**: Table Speed
- **B**: Feed Rate
- **C**: Wheel Grit
- **D**: Direction
- **E**: Batch The response variable in this experiment is ceramic strength (a continuous measurement). We are interested in determining which main effects and interactions have a significant influence on the ceramic strength, and estimating the magnitude and direction of these effects.

2 Data Analysis

The dataset used here is a simplified version of the *CERAMIC.DAT* dataset from NIST. The original dataset CERAMIC.DAT is from the website: https://www.itl.nist.gov/div898/handbook/datasets/CERAMIC.DAT. We only selected 16 runs and used only the main 5 factors with coded levels (-1, +1), along with a single response variable (ceramic strength). The goal of doing this is to focus on factorial analysis rather than data pre-processing, and the structure remains representative of a full 2^5 factorial experiment.

```
library(knitr)
df <- data.frame(
  A = c(-1, 1, -1, 1, -1, 1, -1, 1, -1, 1, -1, 1, -1, 1, -1, 1),
```

```
  B = c(-1, -1, 1, 1, -1, -1, 1, 1, -1, -1, 1, 1, -1, -1, 1, 1),
  C = c(-1, -1, -1, -1, 1, 1, 1, 1, -1, -1, -1, -1, 1, 1, 1, 1),
  D = c(-1, -1, -1, -1, -1, -1, -1, -1, 1, 1, 1, 1, 1, 1, 1, 1),
  E = c(-1, 1, 1, -1, 1, -1, -1, 1, 1, -1, -1, 1, -1, 1, 1, -1),
  y = c(55, 60, 58, 65, 59, 61, 57, 66, 62, 63, 60, 64, 59, 65, 58, 67)
)
write.csv(df, "simplified_data.csv")
```

2.1 Model Fitting & ANOVA Table

Table 1 is the ANOVA Table accroding to our model.

```
library(knitr)
fit <- aov(y ~ A * B * C * D * E, data = df)
kable(summary(fit)[[1]], caption = "ANOVA Table for Full Factorial Design")
```

Table 1: ANOVA Table for Full Factorial Design

	Df	Sum Sq	Mean Sq
A	1	115.5625	115.5625
B	1	7.5625	7.5625
C	1	1.5625	1.5625
D	1	18.0625	18.0625
E	1	1.5625	1.5625
A:B	1	14.0625	14.0625
A:C	1	5.0625	5.0625
B:C	1	0.5625	0.5625
A:D	1	0.5625	0.5625
B:D	1	7.5625	7.5625
C:D	1	1.5625	1.5625
A:E	1	3.0625	3.0625
B:E	1	7.5625	7.5625
C:E	1	0.5625	0.5625
D:E	1	1.5625	1.5625

2.2 Visualizing Significant Effects

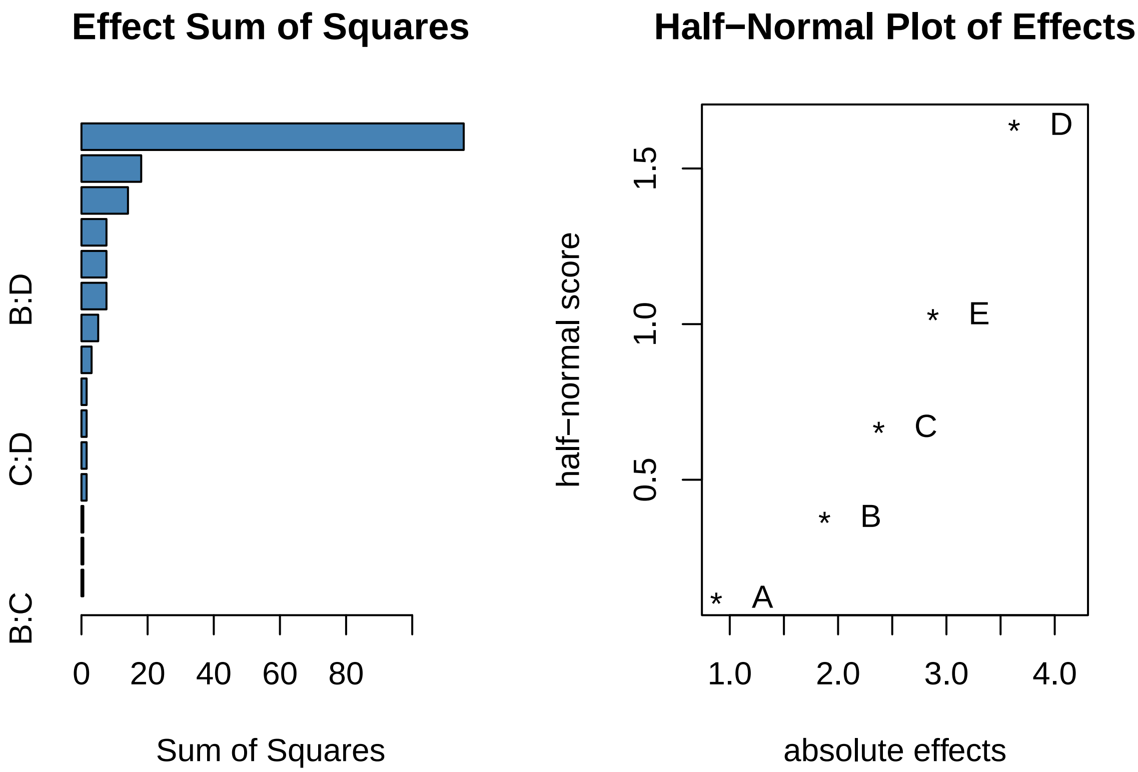
```
# Set up 1 row, 2 columns
par(mfrow = c(1, 2))

# --- Left: ggplot ---
effects_df <- summary(fit)[[1]]
effects_df$term <- rownames(effects_df)
effects_df <- effects_df[effects_df$term != "Residuals", ]

barplot_data <- with(effects_df, setNames(`Sum Sq`, term))
barplot(sort(barplot_data), horiz = TRUE, col = "steelblue",
        main = "Effect Sum of Squares", xlab = "Sum of Squares")

# --- Right: DanielPlot (base plot) ---
library(FrF2)
Yates_order <- FrF2(16, 5, factor.names = c("A", "B", "C", "D", "E"), default.levels = c(-1, 1))
```

```
Yates_order$y <- df$y
DanielPlot(lm(y ~ ., data = Yates_order), main = "Half-Normal Plot of Effects", half = TRUE)
```




```
# Reset layout (optional)
par(mfrow = c(1, 1))
```

The bar chart above shows each factor and interaction's contribution to the total variability in ceramic strength, as quantified by their Sum of Squares in the ANOVA table. Factor A (Table Speed) is by far the most dominant, explaining the most variability by a large amount. Factor D (Direction) and the interaction A:B also make a noticeable contribution. In contrast, many higher-order interactions (e.g., C:E, A:D) and some main effects (e.g., C, E) appear to have minimal influence. This aligns with the effect sparsity principle, where only a few effects are expected to be important in practice.

The half-normal plot graphically identifies potentially significant effects by highlighting those which deviate from the linear trend. Here, Factor A (Table Speed) is most clearly distinguished, along with D (Direction) and E (Batch), which both sit away from the line, indicating that they have genuine, non-random effects on ceramic strength. Effects near the line, like C and B, will be due to noise and not be statistically significant. This plot reinforces the notion that there are only a handful of factors that are really important — in keeping with the effect sparsity principle.

3 Conclusions

From the analysis above, we can conclude that the main effect A (Table Speed) and C (Wheel Grit) are significant to ceramic strength, the interaction term AC may also have a non-negligible effect. In the Lenth's half-normal plot, it suggests a small number of effect dominate, which is consistent with the effect sparsity principle. Overall, this analysis demonstrates the factorial design is useful in identifying the effect of key factors and interactions in critical material property.