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Automatic aircraft cargo load planning

S Limbourg[1], M Schyns[1*] and G Laporte[2]
[1]*University of Liège, Liège, Belgium; and* [2]*HEC Montréal, Montréal, Canada*

The goal of this paper is the development of a new mixed integer linear program designed for optimally loading a set of containers and pallets into a compartmentalised cargo aircraft. It is based on real-world problems submitted by a professional partner. This model takes into account strict technical and safety constraints. In addition to the standard goal of optimally positioning the centre of gravity, we also propose a new approach based on the moment of inertia. This double goal implies an increase in aircraft efficiency and a decrease in fuel consumption. Cargo loading generally remains a manual, or at best a computer-assisted, and time-consuming task. A fully automatic software was developed to quickly compute optimal solutions. Experimental results show that our approach achieves better solutions than manual planning, within only a few seconds.

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1. Introduction

The goal of this paper is the development of a mixed integer linear program for the optimal loading of a set of *unit load devices* (ULDs) into a cargo aircraft. A ULD is an assembly of components consisting of a container or of a pallet covered with a net, whose purpose is to provide standardised size units for individual pieces of baggage or cargo, and to allow for rapid loading and unloading. This system is broadly used. Airbus, Boeing, Lockheed and McDonnell-Douglas propose different versions of cargo aircraft built to transport ULDs. This problem is of crucial importance to airline companies for at least two reasons. First, aircraft loading is subject to strict safety constraints. Indeed, the stress imposed on the structure of an improperly loaded aircraft can result in the destruction of valuable equipment and ultimately in the loss of lives. Second, improper loading decreases the efficiency of an aircraft with respect to its altitude, manoeuvrability, rate of climb, and speed. An inappropriate load could even prevent the flight from being safely completed or even from starting. At the opposite, an optimal load should yield a lesser fuel consumption and, consequently, lead to a decrease in costs and environmental impact.

The problem considered in this paper is the optimal loading of ULDs of different types, contours and shapes in an aircraft. The set of ULDs to be loaded as well as the set of available positions are known before planning starts. The solution should be such that the *centre of gravity* (CG) of the loaded plane should be as close as possible to a recommended position determined by safety and fuel economy considerations. A brief description of these concepts may be found in Sabre (2007). In addition, the loading should be concentrated or 'packed' around this central position. This is particularly important when the aircraft is not fully loaded. We propose an original approach for handling this feature, based on the moment of inertia. In addition to these basic constraints, a feasible solution must also satisfy other requirements. Each position can only accept some specific types of ULDs, depending on their contour, type and weight, the plane must be balanced longitudinally and transversally, the total weight concentrated at each inch of the aircraft length and for each deck must be less than given thresholds (combined weight constraints), the cumulative weight at each inch from the front up to the middle of the plane must also be less than another threshold function (cumulative forward constraint), and a two-level threshold is also applied to the aft part (cumulative aft constraints). Because of the weight and CG constraints, the loading problem is sometimes called the *weight and balance* problem.

We take under consideration the minimal set of constraints that must be satified by any operator, and other common ones. Every different company may have to deal with additional constraints related to the nature of the shipment, to the destinations, or simply company policies. Since our approach is based on a general mathematical model, such constraints can be readily handled if they can be represented by linear equations. The software we

*Correspondence: *M Schyns, QuantOM, HEC Management School, University of Liège, Rue Louvrex 14 (N1), Liège 4000, Belgium.*
E-mail: M.Schyns@ulg.ac.be

propose is fully interactive. It combines the freedom of the classical manual approach based on the loadmaster's knowledge of practical constraints with the power of the optimiser.

The aircraft cargo loading problem is normally solved by *loadmasters* who use interactive graphical tools with drag and drop capabilities. This means the loadmaster can generate several potential solutions whose quality is assessed by a set of indicators. This works well in practice but is time consuming. For example, an experienced loadmaster can load 40 ULDs on a Boeing 747 in about 15 min. The solution typically satisfies the basic constraint but may be suboptimal. Load planning is often executed at the last possible moment before the plane departure, which means that fine tuning is not always a practical option. This is particularly true for express delivery companies such as FedEx, TNT and DHL whose business model relies on timely operations. On any given day, the cargo loading problem is solved tens of thousands of times worldwide (ICAO, 2009).

The scientific literature on aircraft cargo load planning contains a number of mathematical models and heuristics. There exist several variations of the aircraft cargo load planning problem. We have identified three main cases. First, several papers consider how to optimise the loading of freight inside ULDs (Chan and Kumar, 2006; Yan *et al*, 2008; Li *et al*, 2009; Tang and Chang, 2010; Wu, 2010). A second important question is how to select the ULDs or items to load in an aircraft or a fleet of aircraft. Papers on this subject are split between military (Ng, 1992; Heidelberg *et al*, 1998; Guéret *et al*, 2003; Nance *et al*, 2011) and commercial applications (Mongeau and Bès, 2003; Fok and Chun, 2004; Souffriau *et al*, 2008; Tian *et al*, 2008). Finally, as we do, some authors optimise the location of ULDs in an aircraft (see eg Amiouny *et al*, 1992; Mongeau and Bès, 2003; Fok and Chun, 2004; Souffriau *et al*, 2008). Note that these cases are not exhaustive and some papers fall within two categories. This literature also varies on at least four other dimensions: the precise definition of the objective function, the nature of the shipments, the constraints taken into account, and the solution algorithm.

In the field of ULD location problems, the papers by Mongeau and Bès (2003) and Souffriau *et al* (2008) are the closest to our work. Even if this is not their first goal, they consider the ULD locations in the aircraft and their impact on the CG. Moreover, as in this paper, they do not attempt to fill the aircraft continuously by prohibiting empty spaces between the items (see eg Amiouny *et al*, 1992) but they try to allocate the ULDs into predefined positions. Hence they deviate from bin packing approaches encountered in some models. Another common point is that they work with standardised ULDs. Other papers deal with passengers (see eg Tian *et al*, 2008), bulk freight (see eg Amiouny *et al*, 1992) or military items with specific properties. Finally, Mongeau and Bès (2003) and Souffriau *et al* (2008) propose exact methods, whereas all other papers we have examined describe heuristics (see eg Fok and Chun, 2004). However, these two papers differ from our work on two important aspects. The first relates to the set of constraints they consider, even if their model is clearly extensible. We have incorporated more realistic weight and balance constraints including some that are specific to the Boeing 747, one of the most common cargo aircraft. The second and most important difference lies in the definition of the objective function. Their main goal before location is a selection problem. They try, using different criteria, to determine an optimal subset of ULDs to load in the aircraft and leave the remaining ones for a future yet undefined flight. Mongeau and Bès (2003) optimise the mass of goods loaded while Souffriau *et al* (2008) maximise the total cargo value. This implies that the aircraft is nearly always loaded at full capacity. However, there are often far fewer ULDs to load than what the aircraft is capable to carry (see IATA, 2010). This last situation is not a simple subcase of the first one. Indeed, when the aircraft is not loaded at full capacity, extra caution must be taken. In particular, it is best to pack the shipments around the CG. We propose in this paper an original approach based on the moment of inertia to tackle this problem. Note that the fact that we do not consider the selection problem is not necessarily a weakness even when the ULD list to load exceeds the aircraft capacity. Indeed, in practice, loading is generally a two-step process based on different requirements. The commercial department selects the ULDs to be shipped immediately, based on commercial priority criteria, and then provides the reduced list to the loadmaster so that he may optimise the loading. Our proposed system is therefore independent of the commercial dimension but is valid whatever the commercial constraints are.

The scientific contribution of this paper is the development of an integer linear programming model for the aircraft load planning problem. The model can handle all constraints of the problem. We have also developed a software that takes as input all of the problem's data and feeds them to the CPLEX integer linear programming solver through the proposed model. Tests were carried out on a set of real instances suggested by our partner, CHAMP Cargosystems. It is shown that feasible and optimal solutions can be reached within a few seconds only. Our solutions were compared to those obtained by an experienced loadmaster and shown to be at least as good in every respect. Moreover, the software allows the loadmaster to accept the solution or to iteratively restart the optimisation after restricting any ULDs to specific positions in order to take into account more complex problems.

The remainder of this paper is organised as follows. The next section contains an exhaustive description of our mathematical model. This is followed by some case studies and by conclusions.

2. Mathematical model

Before presenting our mathematical model, we first discuss its various components.

2.1. Main variables and parameters

The model works with a set $\mathbb{U}$ of ULDs. The dimensions of a ULD may be retrieved from its code. The three first characters of the IATA identification code (9 or 10 characters) provide the category, the base dimensions and the contour of the ULD. We will also use the weight w_i of each ULD i. We consider that this weight is uniformly distributed inside the ULD.

The second main component of the problem is the set $\mathbb{P}$ of positions. An aircraft is divided into predefined marked positions on one or several decks. Each position j may receive at most one ULD and allows only some ULD types. The longitudinal location of a position, called the *arm*, is expressed in inches from a virtual point called *datum*, generally arbitrarily set in front of the aircraft nose. Each position is defined by two values: the forward arm and the aft arm. We also define the central arm value a_j of position j as the mean of these two values, that is the point where the ULD weight will be concentrated. Laterally, only three cases may occur: either a position is on the left-hand side, on the right-hand side or it covers the whole section. We denote by P_L (resp. P_R) the set of positions on the left-hand (resp. right-hand) side.

At the start of the optimisation process, the loadmaster receives the list of ULDs with their code and their weight, as well as the list of positions with their exact location and the list of ULD types that may fit in each position. The goal is to assign each ULD to one position, subject to some constraints. A natural way to model this problem is to define binary variables x_{ij} equal to 1 if and only if the ULD i is allocated to position j.

2.2. Allowable positions

Because of their size, not all ULDs fit in all positions, which means that the corresponding x_{ij} variables can be set equal to zero. To simplify the presentation of the model, we do not remove these variables. Infeasible assignments are of course not considered in our software.

A first set of constraints states that a position can accept at most one ULD. These constraints are

$$\sum_{i \in \mathbb{U}} x_{ij} \leqslant 1 \quad \forall j \in \mathbb{P}. \tag{1}$$

Finally, in some cases, larger ULDs cannot fit in classical positions. In this case, the loadmaster may combine, exactly or partially, several positions into a larger one. The set of possible new larger *overlying* positions are defined in advance and considered as independent ones in our model. However, when a ULD is loaded in such a position, the underlying positions cannot be allocated to other ULDs and, conversely, when a ULD is loaded in a basic position, the overlying position is no longer available:

$$x_{ij} + x_{i'j'} \leqslant 1 \quad \forall i, i' \in \mathbb{U}, \forall j \in \mathbb{P}, \forall j' \in \mathbb{O}_j, \tag{2}$$

where $\mathbb{O}_j$ denotes the set of position indices underlying position P_j. This set is empty for most of the positions and does not give rise to a constraint.

2.3. Full load

In our problem, the airline company has already decided which ULDs have to be loaded and a position must be found for each of them:

$$\sum_{j \in \mathbb{P}} x_{ij} = 1 \quad \forall i \in \mathbb{U}. \tag{3}$$

2.4. Centre of gravity, moment of inertia and objective function

Balance control refers to the location of the CG of an aircraft. This is of primary importance to aircraft stability, which determines in-flight safety and helps control fuel consumption. The CG is the point at which the total weight of the aircraft is assumed to be concentrated. It must be located within specific limits for safe flight. Both lateral and longitudinal balances are important, but the prime concern is longitudinal balance, that is the location of the CG along the longitudinal or lengthwise axis. If the CG is too far aft, the aircraft will be unstable, and will have difficulty to recover from a stall. If the CG is too far forward, the downward tail load will have to be increased to maintain level flight. This increased tail load has the same effect as carrying additional weight; the aircraft will have to fly at a higher angle of attack, and drag will increase. In practice, the CG must lie within a specific area defined by the aircraft manufacturer. Within this range, the airline companies and the pilots have some degrees of freedom. They may decide to set the CG more to the aft to reduce fuel consumption. They can also locate the CG in a more forward position for manoeuvrability reasons. Each company has his own policies about this and it is beyond the scope of this paper to determine the best value for every situation (aircraft type, commercial agreements, etc). Solutions can be highly sensitive to small constraint variations. As stated by Mongeau and Bès (2003), 'to give an idea of the relevance of the problem: a displacement of the center of gravity of less than 75 cm in a long-range aircraft yields, over a 10 000 km flight, a saving of 4000 kg of fuel'.

The CG is a function of the ULD weights and locations. The ULD location is given by the arm of the assigned position a_j. The deviation from an *index datum* (*ID*) value representing the requested *CG* is given by

$$\sum_{i\in\mathbb{U}}\sum_{j\in\mathbb{P}} w_i(a_j - ID)x_{ij}/W, \tag{4}$$

where $W = \sum_{i\in\mathbb{U}} w_i$ is the total weight of the load. Minimising the absolute value of (4) could be a good candidate for the objective function. However, we propose a different approach. Since we know that the optimal requested value of (4) is zero, this objective may be integrated as a constraint. Moreover, as mentioned above, companies may define different *ID* values for practical reasons. Slight deviations ε only have little impact and certainly do not endanger the flight. This yields the constraint

$$-\varepsilon \leqslant \sum_{i\in\mathbb{U}}\sum_{j\in\mathbb{P}} w_i(a_j - ID)x_{ij}/W \leqslant \varepsilon. \tag{5}$$

There is another reason to use this formulation. Experienced loadmasters start their planning by loading first ULDs as close as possible to the CG and gradually move away from it. This results in a load 'packed' around the CG. Note that constraint (5) does not ensure a good load distribution. It could yield feasible solutions with a balanced weight at each extremity and nothing at the CG, which would exert an excessive stress on the aircraft structure. Indeed, aircraft have a flexible structure and an inadequate load distribution may cause fuselage deformation. While this problem may be reduced by defining weight constraints on the different sections of the aircraft (see Sections 2.7 and 2.8), we have adopted a different and original approach. In addition to constraints (5), we define a 'packing' objective function as a variation of (4). Each ULD U_i, weighted by w_i, should be set in a position P_j as close as possible to the *ID*:

$$\min \sum_{i\in\mathbb{U}}\sum_{j\in\mathbb{P}} w_i(a_j - ID)^2 x_{ij}. \tag{6}$$

In other words, we optimise the moment of inertia under CG constraints. This is possible since the objective function is clearly not conflicting with constraints (5).

2.5. Lateral balance

The basic aircraft design assumes that lateral symmetry exists. For each unit of weight added to the left of the centreline of the aircraft (also known as *buttock line zero*, or BL-0), there is generally an equal weight at a corresponding location on the right. The position of the lateral CG is not normally computed for an aircraft, but the pilot must be aware of the adverse effects that will result from a laterally unbalanced condition. This is corrected by using the aileron trim tab until enough fuel has been used from the tank on the heavy side to balance the aircraft. The trim tab deflects the aileron to produce additional lift on the heavy side, but this also produces additional drag and the aircraft flies inefficiently. We must therefore ensure that the lateral imbalance lies within reasonable limits. The threshold, called *delta weight* $\bar{D}$, is a simple function of W. The lateral imbalance constraint is then given by

$$-\bar{D} \leqslant \sum_{i\in\mathbb{U}} w_i \left(\sum_{j\in\mathbb{P}_\mathbb{R}} x_{ij} - \sum_{j\in\mathbb{P}_\mathbb{L}} x_{ij} \right) \leqslant \bar{D}. \tag{7}$$

2.6. Feasibility envelopes

Based on aircraft maximum structural capability, the aircraft manufacturer defines certified limits for the CG and the operating weights. These are represented graphically by feasibility envelopes (Figure 1). The vertical axis represents weight. The CG is measured on the horizontal axis. Two equivalent notations exist for it. These are either based on an index or as a percentage of the *mean aerodynamic chord* (MAC). The MAC is the chord of an imaginary airfoil that has all of the aerodynamic characteristics of the actual airfoil. In the sequel, we will work with the %MAC (the dashed lines with corresponding values at the top) but we will also provide for information the index values (on the horizontal axis). The shaded envelopes are those for a *zero fuel weight* (ZFW) indicator. The ZFW is the weight of the empty aircraft including the staff, plus the weight of the cargo. One should ensure that the ZFW lies within some allowed weight limits:

$$ZFW_{\min} \leqslant ZFW \leqslant ZFW_{\max}, \tag{8}$$

while constraint (5) forces the corresponding CG to lie within the horizontal boundaries of its envelope.

Since the ULD positions do not appear directly in the ZFW definition, this is not a relevant constraint for the optimisation process. We just have to check beforehand that the ULD set to load is not too heavy and therefore that the ZFW lies within its boundaries. However, the ZFW also appears indirectly in the model. Indeed, other indicators are based on the ZFW and must lie for safety reasons in specific envelopes at some operating stages. In particular, the *takeoff weight* (TOW) is defined as the ZFW plus the total fuel weight and less the taxi fuel. The TOW must either lie in the takeoff envelope or in the dashed smaller one when the ULD allocation cannot satisfy some constraints (see Section 2.9) or when the ZFW lies in the extended ZFW zone. This does not apply to all aircraft types but it does to the largely operated Boeing 747.

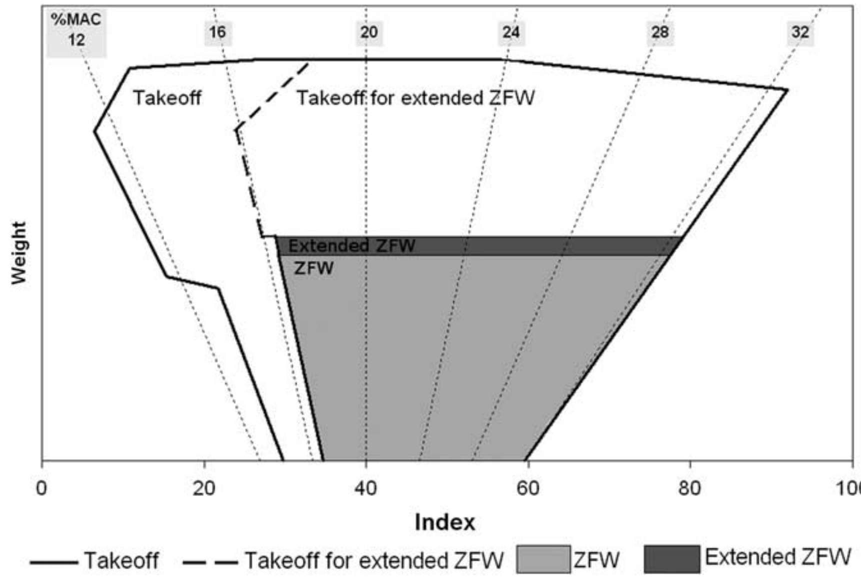

Figure 1 Feasibility envelope.

2.7. Combined load limits

For structural design reasons, it is not allowed to put too much weight on given sections of the aircraft. Different compartments are defined on each deck, and each has a total weight limit. There is also a combined weight limit over all decks. These combined limits vary according to the location in the aircraft, that is more weight is allowed in the middle, and less at the extremities. The limit is a piecewise linear function of the distance with respect to the datum (see Figure 2). These weight limitations are expressed in kg per inch. It is not sufficient to ensure that the total weight over one section is under the limit. One must also load the ULDs in such a way that at each inch of one section, the limit per inch is not exceeded, that is the weight limit must be uniformly distributed over the section.

This last consideration complicates the model significantly. Applied directly, it would mean that one constraint must be defined for each inch segment of the aircraft and for each deck, which represents about 7000 constraints for a Boeing 747. Fortunately, since we have made the assumption that the weight is uniformly distributed within each ULD, the load weight is a piecewise linear function with breakpoints defined at each P_j limit. By sorting all position limits, aft and forward, by distance from the datum, we can easily define a set of areas starting at each limit location and ending at the next one. This should be done for the main deck, the lower deck and both decks together, and hence we distinguish the three cases by the index D. For deck D, the kth area is denoted by O_k^D. Within each of these areas, for the three cases, the weight is known to be

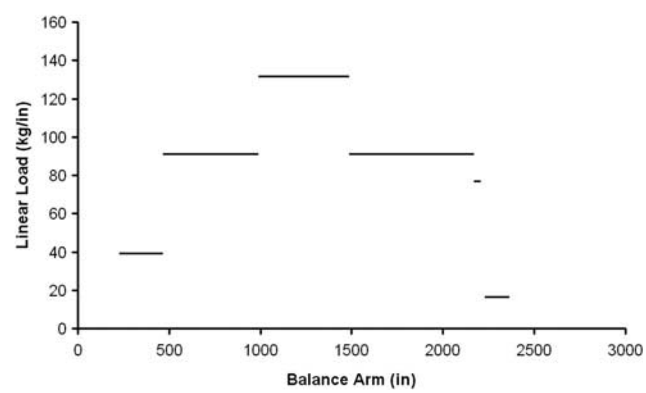

Figure 2 Feasible regions according to the maximum combined load limits.

uniformly distributed, even when considering the two decks together. Therefore, the 'max kg per inch' constraints can be transformed into a set of 'max total weight per area O_k^D' constraints which are far less numerous. Note that the breakpoints of the weight limit function may not coincide with the frontiers of the areas O_k^D. This implies that the weight limit is not always uniformly distributed over each O_k^D. We take care of this by adding the locations of the breakpoints to the list of positions limits before sorting. As a result, we are no longer working with positions or complete ULDs, but with areas often containing fractions of ULDs.

With P_j interpreted as an area of the aircraft, by defining and computing beforehand the values $\bar{O}_k^D$ as the maximal weight for area O_k^D, and o_{ijk}^D as the proportion of w_i falling in $\{O_k^D \cap P_j\}$ when ULD_i is set in position P_j, we can impose the constraints

$$\sum_{i \in \mathbb{U}} \sum_{j \in \mathbb{P} | P_j \cap O_k^D \neq \emptyset} x_{ij} o_{ijk}^D \leqslant \bar{O}_k^D \quad \forall D \in \mathbb{D}, \forall k \in \mathbb{O}^D. \tag{9}$$

2.8. Cumulative load limits

Instead of applying weight limits to consecutive slices of the aircraft, we now define limits over overlaying areas. The problem is split into two parts: the aft body and the forward body. The cumulative weight distribution from the nose to the centre of the aircraft must lie below a forward piecewise linear limit function, and the cumulative weight distribution from the tail to the centre of the aircraft must lie below an aft piecewise linear limit function. This constraint is also called *fuselage shear load* on some aircraft. In practice, these constraints are often satisfied when the combined ones are enforced. Note also that the combined and cumulative constraints are more restrictive at the extremities of the aircraft than close to the centre. They are therefore fully compatible with the minimisation of the moment of inertia. When the aircraft is loaded close to its maximal capacity, they act as the objective function, but they are not sufficient when this is not.

To model the cumulative load limits, the approach is similar to the one defined for the combined weight limits. For the forward constraint, we construct as before a sorted list with each position limit and the breakpoints of the forward cumulative limit function. We consider both decks together but only the positions in the forward section. Each area is defined as before, using the successive locations in the list. The same procedure is applied to the aft section but by sorting in reverse order, from the tail to the centre of the aircraft. We denote by F_k (resp. T_k) the consecutive forward (resp. aft) areas. Again, the weight is uniformly distributed in each of them. The variable f_{ijk} (resp. t_{ijk}) is the proportion of w_i falling in $\{F_k \cap P_j\}$ (resp. $\{T_k \cap P_j\}$) when ULD_i is set in position P_j. If $\bar{F}_k$ (resp. $\bar{T}_k$) denotes the maximal cumulative allowable weight for the section starting at the nose (resp. the tail) and ending with F_k (resp. T_k), then we impose the constraints

$$\sum_{i \in \mathbb{U}} \sum_{j \in \mathbb{P} | P_j \cap \bigcup_{c=1}^{k} F_c \neq \emptyset} \sum_{l=1}^{k} x_{ij} f_{ijl} \leqslant \bar{F}_k \quad \forall k \in \mathbb{F} \tag{10}$$

and

$$\sum_{i \in \mathbb{U}} \sum_{j \in \mathbb{P} | P_j \cap \bigcup_{c=1}^{k} T_c \neq \emptyset} \sum_{l=1}^{k} x_{ij} t_{ijl} \leqslant \bar{T}_k \quad \forall k \in \mathbb{T}. \tag{11}$$

Note that since each weight limit area includes the preceding ones, these constraints may be efficiently constructed using a recursive procedure.

2.9. Restricted aft body cumulative load limit

Another original feature of our work is the implementation of a restricted version of the aft body cumulative load limit. This is particularly useful for the Boeing 747. When all the preceding constraints are satisfied, the aircraft may theoretically take off. However, it is preferable to load the aft section so as to satisfy a more restrictive cumulative aft limit (see Figure 3). When this is not possible, the envelope for the takeoff weight is also reduced to the takeoff envelope for the extended ZFW (see Figure 1).

This restricted version of the cumulative aft limit is defined in the aircraft manuals. The breakpoints of the load limit function are the same as those for the extended one. We define the new limit values by $\bar{R}_k$ instead of $\bar{T}_k$ (with $\bar{R}_k \leqslant \bar{T}_k$). Basically, the constraints can be expressed as in (11) by simply replacing $\bar{T}_k$ with $\bar{R}_k$. The difficulty is that these constraints could be too strict. They should not be applied if they make the problem infeasible. We define a new binary variable y expressing whether or not constraint (11) is applied for each area k. The constraint is applied when $y=0$ and relaxed otherwise. The relaxation is achieved by increasing the weight limit by a large unreachable value. A good and strong candidate is W, the total weight of the load. The modified constraints are therefore

$$\sum_{i \in \mathbb{U}} \sum_{j \in \mathbb{P} | P_j \cap \bigcup_{c=1}^{k} T_c \neq \emptyset} \sum_{l=1}^{k} x_{ij} t_{ijl} - Wy \leqslant \bar{R}_k \quad \forall k \in \mathbb{T}. \tag{12}$$

Finally, we must guarantee that y takes the value zero whenever possible. This is obtained by adding the penalty term $L^2 Wy$ in the objective function:

$$\text{minimise} \sum_{i \in \mathbb{U}} \sum_{j \in \mathbb{P}} w_i (a_j - ID)^2 x_{ij} + L^2 Wy. \tag{13}$$

The coefficient $L^2 W$, where L denotes the total length of the aircraft in inches, ensures that the penalty term will be larger than any value for the moment of inertia term whenever $y=1$. Priority is therefore given to solutions

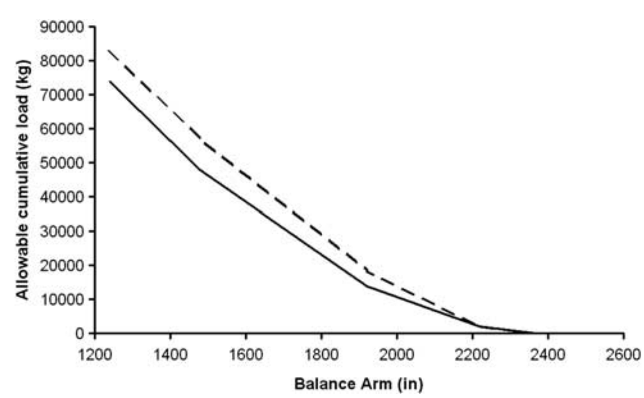

Figure 3 Aft cumulative load limits.

satisfying the restricted cumulative aft limits, before minimising the moment of inertia.

We could be even more restrictive by applying the restricted cumulative limit separately to each T_k area. This means that instead of defining y, we would need to define one variable y_k for each area T_k. The objective function would then be adapted by taking the sum of the y_k instead of simply y. However, what matters in practice is to decide whether we have to consider the extended zone of the envelope or not. This is a binary decision based on the satisfaction of the limit for the entire deck, without taking into account the sub-areas. Moreover, with an array of y_k, the problem becomes much larger and its execution time could be excessive.

2.10. Summary of the model

In summary, the core of the optimisation model can now be expressed by the following mixed integer programming *CargoOpt* model:

$$\text{minimise} \sum_{i\in\mathbb{U}} \sum_{j\in\mathbb{P}} w_i(a_j - ID)^2 x_{ij} + L^2 W y$$

subject to

$$x_{ij} = 0 \quad \forall i \in \mathbb{U}, \forall j \in \mathbb{P} | U_i \text{ does not fit in } P_j$$

$$\sum_{i\in\mathbb{U}} x_{ij} \leqslant 1 \quad \forall j \in \mathbb{P}$$

$$x_{ij} + x_{i'j'} \leqslant 1 \quad \forall i, i' \in \mathbb{U}, \forall j \in \mathbb{P}, \forall j' \in \mathbb{O}_j$$

$$\sum_{j\in\mathbb{P}} x_{ij} = 1 \quad \forall i \in \mathbb{U}$$

$$-\varepsilon \leqslant \sum_{i\in\mathbb{U}} \sum_{j\in\mathbb{P}} w_i(a_j - ID)x_{ij}/W \leqslant \varepsilon$$

$$-\bar{D} \leqslant \sum_{i\in\mathbb{U}} w_i \left(\sum_{j\in\mathbb{P}_\mathbb{R}} x_{ij} - \sum_{j\in\mathbb{P}_\mathbb{L}} x_{ij} \right) \leqslant \bar{D}$$

$$\sum_{i\in\mathbb{U}} \sum_{j\in\mathbb{P}|P_j\cap O_k^D\neq\emptyset} x_{ij} o_{ijk}^D \leqslant \bar{O}_k^D \quad \forall D \in \mathbb{D}, \forall k \in \mathbb{O}^D$$

$$\sum_{i\in\mathbb{U}} \sum_{j\in\mathbb{P}|P_j\cap\bigcup_{c=1}^{k} F_c\neq\emptyset} \sum_{l=1}^{k} x_{ij} f_{ijl} \leqslant \bar{F} \quad \forall k \in \mathbb{F}$$

$$\sum_{i\in\mathbb{U}} \sum_{j\in\mathbb{P}|P_j\cap\bigcup_{c=1}^{k} T_c\neq\emptyset} \sum_{l=1}^{k} x_{ij} t_{ijl} \leqslant \bar{T}_k \quad \forall k \in \mathbb{T}$$

$$\sum_{i\in\mathbb{U}} \sum_{j\in\mathbb{P}|P_j\cap\bigcup_{c=1}^{k} T_c\neq\emptyset} \sum_{l=1}^{k} x_{ij} t_{ijl} - Wy \leqslant \bar{R}_k \quad \forall k \in \mathbb{T}$$

$$x_{ij} \in \{0, 1\} \quad \forall i \in \mathbb{U}, \forall j \in \mathbb{P}$$

$$y \in \{0, 1\}.$$

2.11. Tuning options

One of our main objectives is to reach a solution with a CG close to that required by the loadmaster, the level precision being controlled by the constant ε. In the simulations, we set this value at 0.01, which is rather strict. We could increase the precision by providing a smaller value. Alternatively, in order to make comparisons with problems in which the objective is to minimise the deviation between the CG obtained and an ideal value, we could define ε as a positive variable and add the term $L^2 W\varepsilon$ to the objective function. A second possible direct adjustment to the model concerning the restricted cumulative aft body constraint has already been mentioned in Section 2.9.

It turns out that these two modifications do not improve the quality of the results but only increase computation time. They will therefore not be integrated in our model even if the first one is implemented in Section 3.2 to demonstrate the superiority of the inertia approach.

3. Case studies

We have tested our mathematical model on a set of real-world data provided by our industrial partner CHAMP Cargosystems. The challenge is to find a feasible and optimal position for every ULD within a minimal amount of time. By feasible, we mean a solution satisfying all the constraints presented above. By optimal, we mean a solution where the CG is at the location requested by the loadmaster and packed as much as possible. We will therefore present in the subsequent sections, for a set of realistic cases, the computation times, comparisons of execution times between our automatic approach and a manual one, the deviation of the CG with respect to the value requested by the loadmaster, the moment of inertia of the solution, and the weight and balance quality.

In order to generate these results, we have written a software in Java. The role of this software is to prepare the data, to call the professional optimisation library IBM ILOG CPLEX and to analyse the results. It has been compiled and tested under Windows XP and under Linux (Ubuntu 10.04). The optimisation steps were performed on a personal laptop computer (Windows XP, Dual-Core 2.5GHz, 2.8GB of RAM) and with CPLEX 12. Since we must solve a mixed integer linear program, we have used the classical branch-and-cut CPLEX solver with the default parameters.

We first solve a complex real-world case and analyse its solution. We then compare the minimisation of the moment of inertia to the minimisation of the centre of gravity. The third section contains a short presentation of other real-word cases which shows that our results can be generalised. Considering Boeing aircraft in the simulations is not restrictive since the ULD system is used in a similar way in other aircraft built by Airbus, McDonnel-Douglas and Lockheed, for example.

3.1. The case of a Boeing 747

Our main case study contains a large number of ULDs (42 to be precise) and a high capacity and largely operated aircraft, that is a Boeing 747. This case is used during the training of loadmasters by our partner. A Boeing 747 is divided into 67 basic positions, plus 10 larger ones overlaying some of the basic positions. We know the exact location and dimensions of each position, as well as the list of ULD types that each may contain. The positions are represented by boxes in Figure 4. Some positions are on the main deck (first row) and others are on the lower deck (second row). Note that the scale is respected for the length but not for the width. Each position is identified by a code on the side of the box. This code gives an indication on the location. The final letter ‘R’, ‘L’ or ‘P’ means respectively, the right-hand side, the left-hand side, or an overlay. All aircraft parameters and limits were taken from the manufacturer’s manual.

The combined weight limit function for all decks is shown by the horizontal lines in the third row of Figure 4. The horizontal axis of the figure corresponds to the aircraft longitudinal axis and uses the same scale as the positions. The vertical axis represents the maximal number of kg per inch starting from zero, up to the maximal value written above the left-upper corner. The same is done for the cumulative limits in the fourth row of Figure 4. This time, however, the limit is a continuous increasing function for the forward body and a continuous decreasing function for the aft body. The restricted aft version is also plotted below the aft extended one.

Figure 4 also illustrates the solution obtained by the software. Each shaded box is a ULD with its type and weight. All constraints of the model are satisfied. The total load weight on each side of the aircraft and for each deck is given on the right-hand side. The aircraft has a lateral weight delta of only 1875 kg. This is of the same order of magnitude as the lightest ULDs and far below the threshold. The aircraft can therefore be considered laterally balanced. In the third row of the figure, the shaded area provides the level of the load weight at each inch. It strictly remains below the thresholds and is very far from the limit at the centre of the plane. At the opposite, we can observe that the heaviest ULDs are close to the CG. Since there is no lower deck there, it is not possible to put more weight and reach the limit. The cumulative weight from the nose to the centre and the cumulative weight from the tail to the

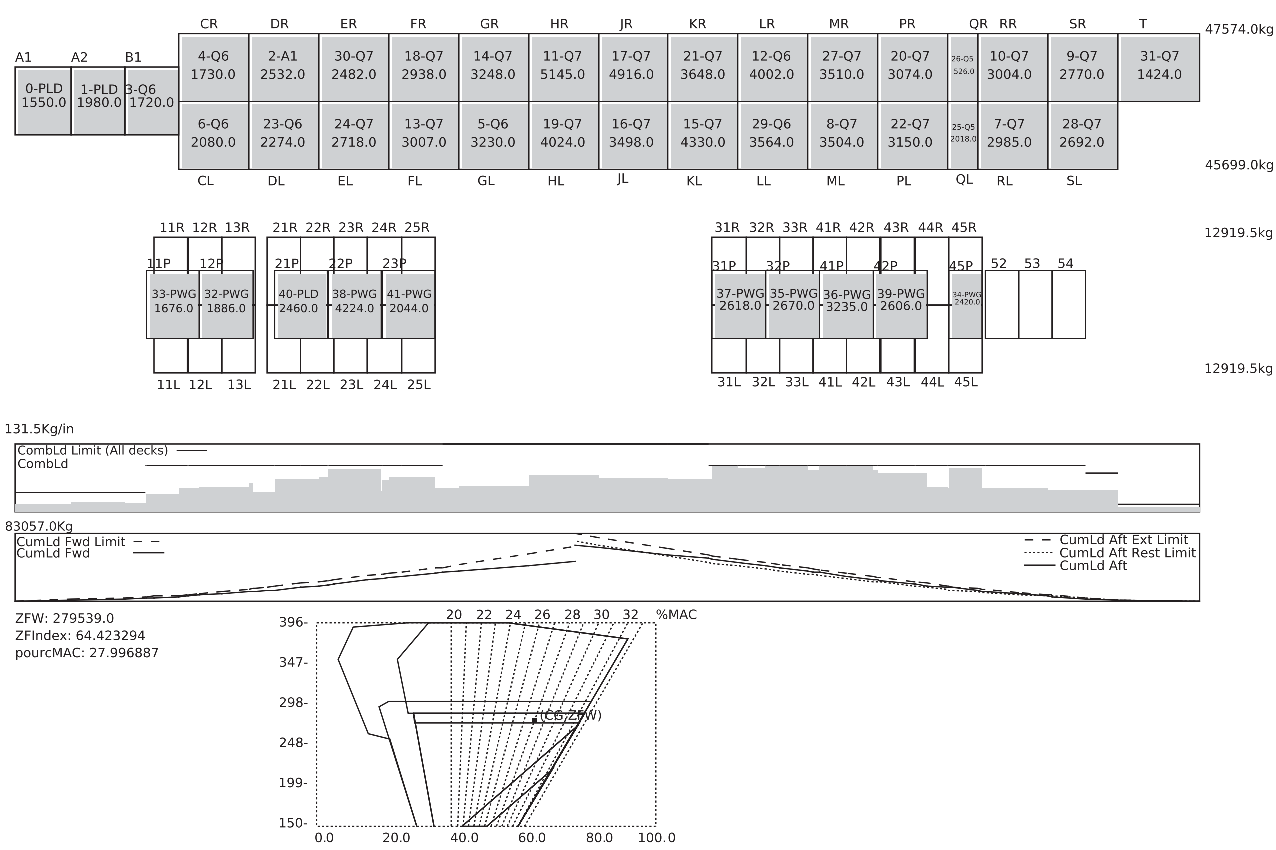

Figure 4 Optimal load obtained by the CargoOpt model.

centre yield, respectively, an increasing line and a decreasing line in the fourth row. These lines remain below the upper curves showing the cumulative limits. However, for this difficult case, it is impossible to obtain a solution for which the aft body cumulative weight remains inferior to the restricted threshold. This is a soft constraint, which means that we have to work with an extended version of the envelope, but there is no problem with this and the solution is perfectly acceptable. The only negative consequence is that the solver has spent a few milliseconds of execution time just to know that the restricted constraint should not be applied. The last constraint is about the envelope. The ZFW and the $\%MAC$ are given at the bottom of Figure 4 and it can be verified that the corresponding point lies within the envelope.

Concerning the quality of the solution, we may measure the deviation between the CG obtained and that initially requested by the loadmaster. We would also like to check whether the load is packed around the CG, which is measured by the moment of inertia. The quality of the packing is difficult to evaluate since the aircraft is loaded at full capacity and is therefore naturally packed for any feasible solution. Nonetheless, as mentioned above, we can observe that the heaviest ULDs are close to the CG, which is already something usually done in practice and corresponds to a good solution. In this case, the location of the CG requested by the loadmaster is expressed as a percentage of the MAC value and is equal to 28, that is the CG must lie at 28% from the front (LEMAC) of the MAC. To give an order of magnitude, the Boeing 747 MAC is about 328 feet long. The ε precision required is 0.01. This is a very restrictive value since it yields in this case a deviation of less than one centimeter with respect to the required CG. With a value of 27.997, the goal is achieved.

The third performance measure is computation time which is extremely good. Less than two seconds were required to solve this instance. Computation time is therefore not a barrier to overcome before being able to consider possible extensions to the model. This result may look surprising given that such mixed integer problems are known to be extremely difficult to solve, and heuristics are usually the only way to compute feasible solutions in a limited amount of time. We believe that this is not the case here for at least two reasons. First, the size of real-life instances is limited by the capacity of the largest aircraft. The Boeing 747 is one of the largest ones but it does not yield an intractable instance. Second, the problem is well conditioned. As stated in the next section, the branch-and-cut process is very efficiently driven by the inertia term in the objective function. Since optimal solutions can be computed within only a few seconds, there was no need for us to develop heuristics.

Finally, we also provide a typical result obtained by a trained loadmaster working by hand. The comparison is not entirely fair since some discrepancies may arise due to the software and parameters used, for example we consider the position T on the right side while the manual and other computer-assisted tools locate it slightly shifted to the centre. Moreover, the loadmaster had to enforce one more constraint in this case: the final destination of five ULDs was different from that of the 37 remaining ones and the loadmaster had to place them close to each other in the aircraft. This constraint was not implemented in our approach but our solution happens to satisfy it. All results are summarised in Table 1 and in Figure 5. Our solution is always at least as good as the loadmaster's solution and was reached more quickly, with less risk of error.

Table 1 Loading 42 ULDs into a boeing 747 aircraft: main results

	Load master	*CargoOpt*
#ULDs	42	42
ZFW	279 539 kg	279 539 kg
% MAC	27.601	27.997
Inertia	3.1E10	3.1E10
Time	1200 s	2 s
Weight delta	5693 kg	1875 kg
Weight constraints	Satisfied	Satisfied
Restricted aft constraint	No	No

3.2. Moment of inertia versus centre of gravity

In the previous results, it is difficult to measure the quality of the moment of inertia optimisation. Indeed, the aircraft is loaded close to its maximal capacity and all feasible solutions are therefore naturally packed. However, the number of ULDs to load is sometimes more limited. According to the IATA's quarterly report (IATA, 2010), the monthly average of the freight load factor varied in 2010 between 52% and 58%. We present in this section a case where we have to allocate only 26 ULDs in the 77 available positions of the aircraft. In reality, 32 positions of the lower deck are inoperative for the case under consideration for two reasons. First they are too small for the ULDs that have to be loaded. Second, these positions are overlaid by 10 larger ones which are necessary for some large ULDs. If we remove the last three positions of the lower deck that can only be used for bulk items, we must load 26 ULDs in an aircraft with a real capacity of 42 positions. Other cases are considered in the next section.

When minimising the moment of inertia, a feasible optimal solution is again obtained very quickly within less than one second. The resulting load is plotted in Figure 6. All ULDs are packed around the CG and the heaviest ones are located in the middle. The moment of inertia is equal to 5.3E9. To give an order of magnitude, with respect to the definition of the moment of inertia moment in Equation (6), the total load weight W is 63 810 kg and the length L of the Boeing 747 is about 2 365 feet.

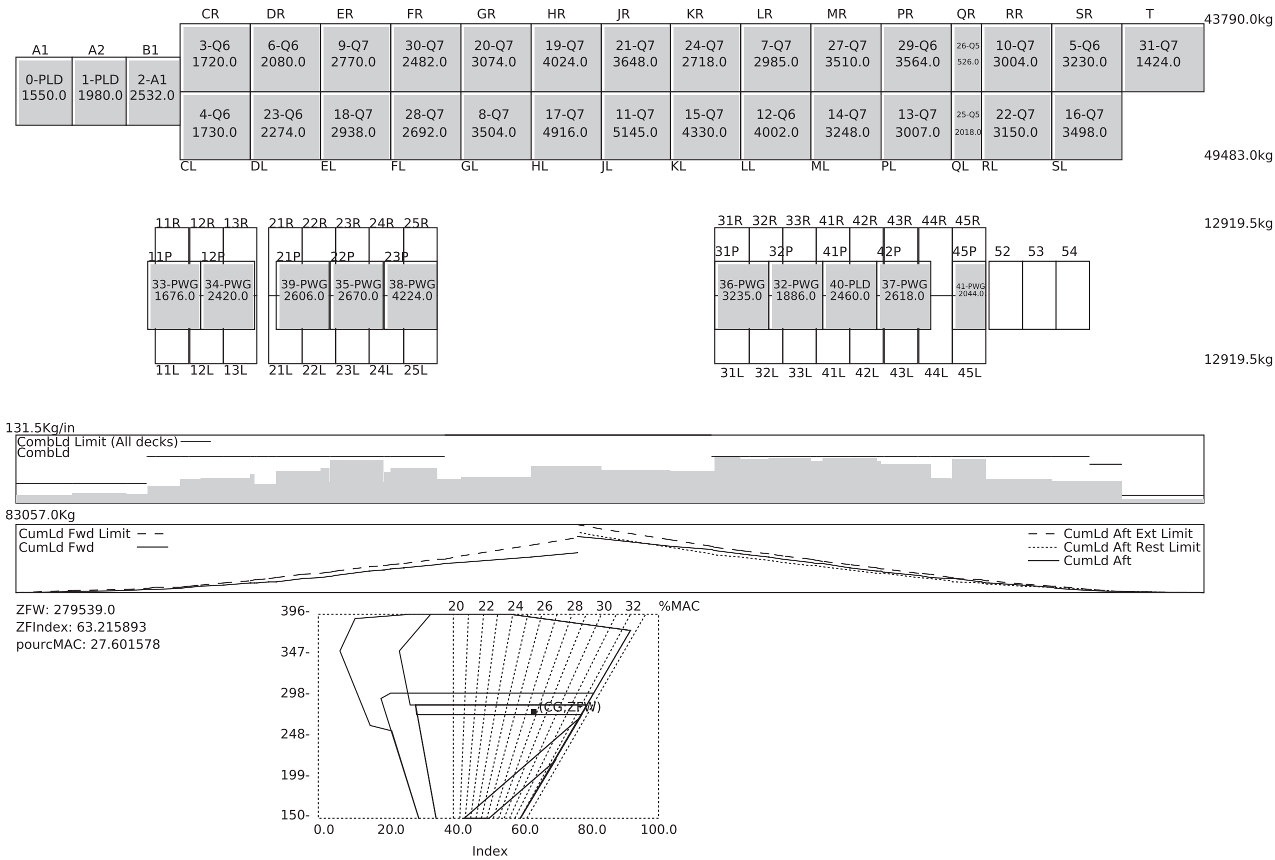

Figure 5 Solution proposed by the loadmaster.

One can wonder what would have happened if we had not minimised the moment of inertia, but the mean absolute deviation from the requested CG, which is a more obvious choice. This comparison can easily be made without modifying the constraints of the model CargoOpt, but by adapting its objective function. We only need to remove the moment of inertia term from the objective function and minimise ε instead. Note that this last term is optional in our model because constraint (5) ensures an adequate precision ε of 0.01. The result of this test is given in Figure 7. One can see that the load is no longer packed. Also, the moment of inertia is now equal to 1.74E10, more than three times the value obtained when minimising this objective. The first solution clearly reduces the stress on the structure but also, since an object with a small moment of inertia is by definition easier to rotate, this physical principle can be interpreted as an aircraft with a higher degree of manoeuvrability and efficiency.

Another initially unexpected result is the time required to compute these two solutions. While the computing time is very fast in both cases, minimising the moment of inertia is 16 times faster. It seems that the moment of inertia term in the objective function efficiently drives the optimisation process, in contrast to the CG model which provides less information.

3.3. Additional cases

An important question is whether the good results presented above are representative. To provide a partial answer, we have solved other real-world cases of different sizes. We still consider the same aircraft but with different loads. The zero fuel weights are of the same order of magnitude as before and all the weight constraints, with the exception of the restricted aft body for case E, are satisfied. Computational results are summarised in Table 2, where columns B and E correspond to the cases presented above with some additional information.

The MAC percentage achieved with CargoOpt is close to the one required and is more precise than the one obtained manually by the loadmaster. All weight and balance constraints are satisfied. In particular, the lateral imbalance weight is small. The sixth line of the table gives the optimised moment of inertia. The next one is the moment of inertia when we only minimise the deviation between the %MAC and the %MAC required, and not the moment of inertia. As mentioned, the difference between these two values may be large when the number of ULDs to load is small with respect to the number of available positions. Visually, we always observe in the first case that the ULDs are packed around the CG, with the heaviest

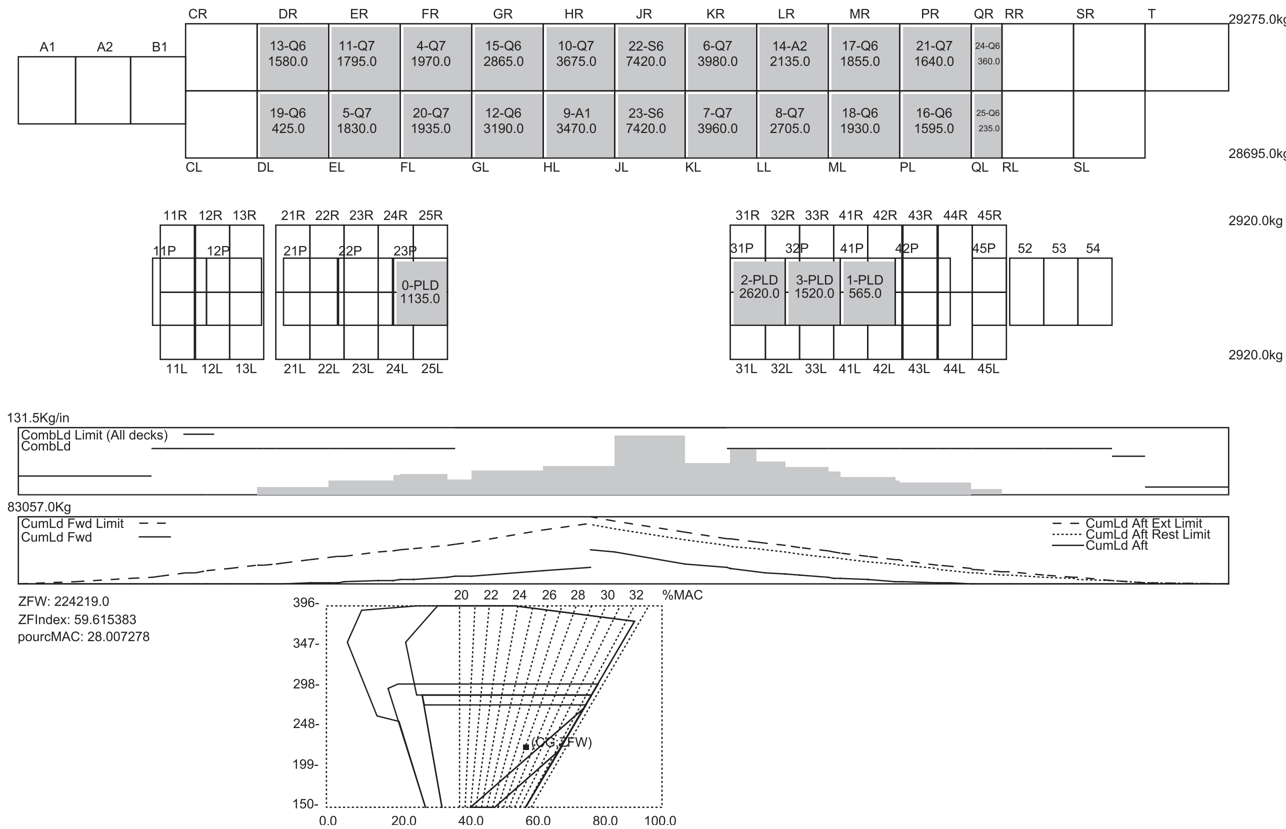

Figure 6 Optimal load planning when minimising the moment of inertia.

ones in the central positions. It is not clear whether one can do better. At the opposite, when minimising only the CG, some ULDs may be distributed all along the aircraft with empty spaces between them. When the number of ULDs is large, the aircraft is packed in a natural way and there are very few differences between the two approaches.

Some observations about computation times are worth making. When minimising the moment of inertia, as we suggest in our model, we always obtain optimal solutions within a few seconds. Minimising only the CG deviation, as it could have been alternatively done, is slower, especially when the number of ULDs is very small or very large. We believe that the slow execution times when the number of ULDs is large are due not directly to the number of ULDs, but to the total weight that must be loaded. In these extreme cases, the cumulative aft body restricted constraint is tighter and the inertia term again has an even more important positive effect on the execution time. In case of E the latter constraint cannot be satisfied.

These six real-world cases were provided by our industrial partner and were not specifically designed for our experimentations. In order to check the results further, we have also randomly generated loads of different sizes and types. Since these experiments did not provide different results and insights, we have decided not to include them in this paper.

3.4. Interactive software

The software we propose is a fully interactive graphical tool. It computes optimal allocations and displays the solution graphically. Moreover, the loadmaster can restrict ULDs to specific positions by drag and drop according to his personal experience and to the real requirements to be fulfilled. This is an iterative process. At each step, the loadmaster defines the restricted set of ULDs and initiates the optimisation process for the others. At the end of each step, he can accept all or part of the solution, move some ULDs, restrict new ones to specific positions, and finally restart the optimisation process until a suitable solution has been found. A major advantage of this approach is that it allows the consideration of additional constraints, for example, a specific ULD that should be loaded close to a door for quick unloading, and it relies on the loadmaster's experience for constraints that cannot be handled by the model. This interactive approach is possible because of the short computation times needed to solve the model.

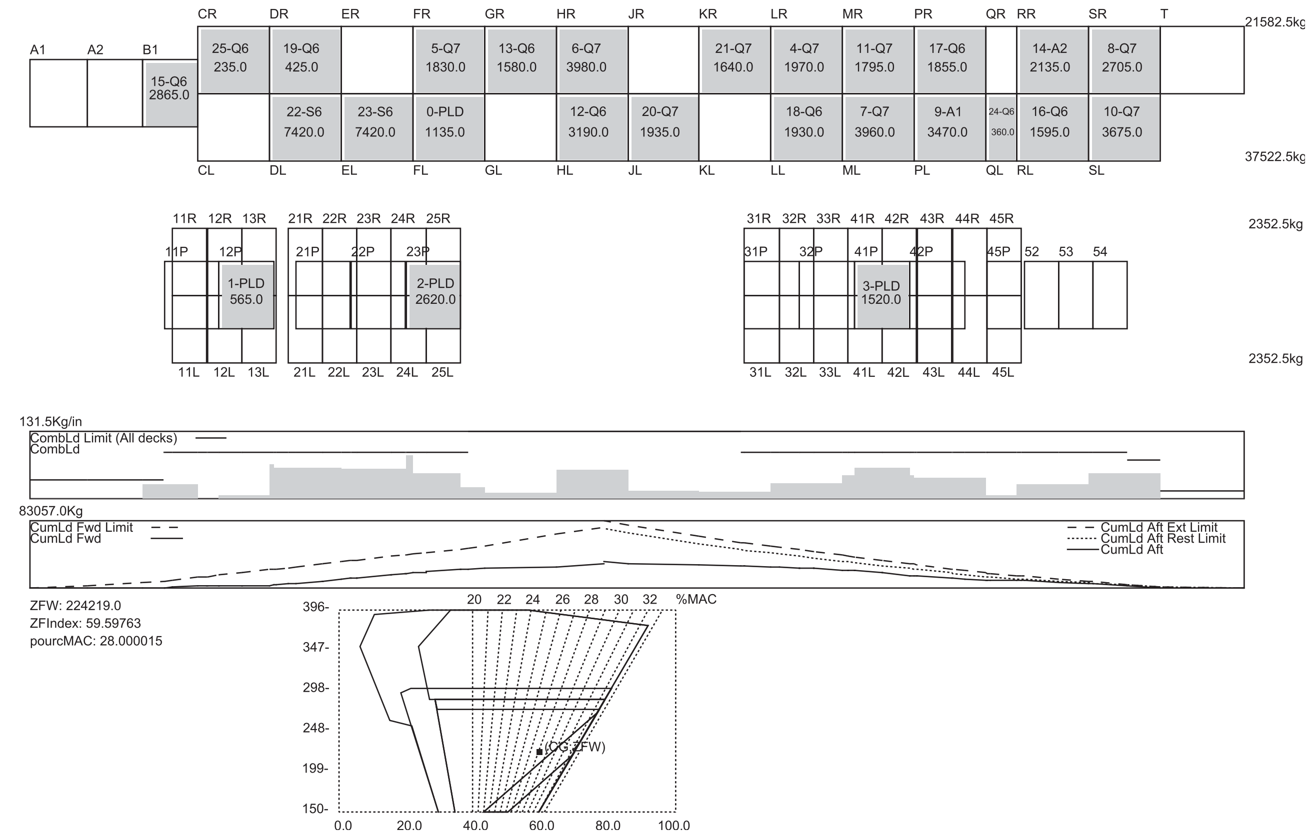

Figure 7 Optimal load planning when minimising the centre of gravity.

Table 2 Optimal load planning of a boeing 747 aircraft for different real-world cases

	A	*B*	*C*	*D*	*E*	*F*
#ULDs	23	26	30	42	42	45
W	60 418 kg	63 810 kg	59 360 kg	103 975 kg	120 112 kg	107 674 kg
% MAC (CargoOpt)	27.992	28.007	28.000	27.996	27.997	27.998
% MAC (LM)	26.1	27.5	27.3	28.1	27.601	28
Inertia (CargoOpt)	4.4E9	5.3E9	7.3E9	1.8E10	3.1E10	2.5E10
Inertia (CG optim)	1.6E10	1.7E10	1.4E10	2.6E10	3.3E10	2.6E10
Time (CargoOpt)	1.4 s	0.8 s	1.0 s	1.5 s	2.0 s	2.9 s
Time (CG optim)	116.6 s	13.0 s	1.9 s	441.9 s	1.2 s	155.7 s
Weight delta	1 990 kg	580 kg	2 135 kg	1 025 kg	1 875 kg	666 kg

4. Conclusions

Our goal was the development of a mixed integer linear program for the optimal loading of a set of containers and pallets into a compartmentalised cargo aircraft. We had three goals in mind. First, the model should integrate a set of realistic constraints ensuring that the aircraft is allowed to take off and fly safely. Control of the longitudinal balance, lateral balance, feasibility envelopes, combined weight and cumulative weight have been successfully included. Particularly, we have also considered a restricted version of the cumulative aft body weight constraint which is important for the case of the Boeing 747 aircraft. This complicates the model with a disjunctive constraint and a new variable.

Second, we looked for a feasible and optimal solution. The aircraft must be loaded so as to set the CG as close as possible to the CG requested by the loadmaster. This is really important for the aircraft stability but also for the reduction of cost and environmental impact. Indeed, the CG is the main parameter available at this level to control the fuel consumption. This is achieved by our software with a very high precision. Another very important concept we have proposed is the minimisation of the moment of inertia, which implies that the load will be packed around the CG. We have shown empirically that our approach leads to a reduction of the stress on the aircraft structure and to a significant improvement of the aircraft manoeuvrability.

Manual load planning is time consuming and costly, especially for express courier operators, but it remains a common practice. Our third objective was therefore to propose a way to accomplish this task efficiently. The software we have developed provides an optimal solution within a few seconds. Our software only requires the list of ULDs and the aircraft parameters to compute a solution. This contrasts with most current systems which are mainly interactive.

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