Given $a+b=b+c$

T.P $a=c$

Proof $a = a+0$ } (Definity of additive identity)

$0 = b+(-b)$ } Additive inverse

$\Rightarrow a = a+b+(-b) \Rightarrow a = \underbrace{b+c+(-b)}_{a+b=b+c \text{ (Given)}} = c+0$ } since $b+(-b)=0$

$\Rightarrow \boxed{a=c}$ H/P

Given $a \cdot b = b \cdot c$

T.P $a=c$

Proof $a = a \cdot 1$ } (Def. of multiplicative identity)

$1 = b \cdot (b^{-1})$ } multiplicative inverse

(Commutativity)

$\Rightarrow a = a \cdot b \cdot b^{-1} = \underbrace{b \cdot c \cdot b^{-1}}_{ab=bc \text{ (Given)}} = c \cdot b \cdot b^{-1} = c \cdot 1 = c$

$\Rightarrow \boxed{a=c}$ H/P

Q ~~Sub~~ Any/every subfield of $(\mathbb{C}, +, *)$ must contain every rational no.

firstly we shows that any subfield contains all integers

~~F = C~~ $F' =$ subfield of $(\mathbb{C}, +, *)$

$\{0, 1 \in F'\}$

↳ multiplactive & additive identity

thus any natural no. ~~can~~ must be included since ~~a~~ $a \in \mathbb{N} \Rightarrow a \cdot 1 = \underbrace{1+1+1+1 \cdots}_{a \text{ times}}$

There F' contains all natural no

Now by the property of additive inverse

$\forall\ a \in N_0$ $\boxed{-a \in F'}$

$a \in F'$

thus F' contains all integers

Now, proof by contradiction

let p/q be a natural which does not belong to F'

$p, q \in Z$

$q \in F'$ } already proved

thus $\frac{1}{q} \in F'$ } multipliative identity

$p \in F'$ $\Rightarrow$ $\left(p \cdot \frac{1}{q}\right) \in F'$ } closure

$\Rightarrow$ $\boxed{\frac{p}{q} \in F'}$ Contradiction

{ Hence all rational numbers must belong to F' }

H/P A.