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Q. Let $A_{n\times n}$ over F. Suppose rows of A are linearly independent set of vectors in F. Thus A is invertible

let $\alpha_1, \alpha_2, \dots \alpha_n$ be row vectors of A

let W be subspace of F^n spanned by $\alpha_1, \alpha_2, \dots \alpha_n$

Since $\alpha_1, \alpha_2, \dots \alpha_n$ are linearly independent $\therefore \dim(W) = n$

By corollary, If W is a proper subspace of a finite dimensional vector space V, then W is finite dimensional & $\dim W \leq \dim V$

$\hookrightarrow$ Proof By th^m

If W is a subspace of a finite dimensional vector space V, every linearly independent subset of W is finite & is also a part of finite basis for W

Hence $\boxed{\dim W \leq \dim V}$

Since W is proper subspace $\therefore$

Hence $\dim W < \dim V$

$\therefore W = F^n$ Hence there exist B_{ij} in F s.t $\epsilon_i = \sum_{j=1}^{n} B_{ij}\alpha_j$ $\quad 1 \leq i \leq n$ & $B_{ij} \in F$

where $\{\epsilon_1, \epsilon_2, \dots \epsilon_n\}$ is standard basis of F^n.

Thus for matrix B, with entries B_{ij} $BA = I$ Hence A is invertible.

Q. If W_1 & W_2 are finite dimensional subspaces of V then $W_1 + W_2$ is finite dimensional & $\dim W_1 + \dim W_2 = \dim(W_1 \cap W_2) + \dim(W_1 + W_2)$

Proof

$W_1 \cap W_2$ is subspace of W_1 & W_2 $\therefore$ By th^m W is a subspace of a finite dimensional vector space V.

every linearly independent subset of W is finite & is part of finite basis of W

Hence $W_1 \cap W_2$ has finite basis $\{\alpha_1, \dots \alpha_k\}$

which is part of basis $\{\alpha_1, \alpha_2, \dots \alpha_k, \beta_1, \dots \beta_m\}$ for W_1

$\{\alpha_1, \dots \alpha_k, \gamma_1, \dots \gamma_n\}$ for W_2

By th^m $\sum_{i=1}^{n} W_i$ is subspace which is spanned by vector set formed by $\bigcup_{i=1}^{k} W_i$

$\therefore$ Subspace $W_1 + W_2$ is spanned by $\{\alpha_1, \dots \alpha_k, \beta_1, \dots \beta_m, \gamma_1, \dots \gamma_n\}$

$\sum x_i \alpha_i + \sum y_j \beta_j + \sum z_r \gamma_r = 0$

$\sum x_i \alpha_i + \sum y_j \beta_j = \sum x_i \alpha_i + \sum y_j \beta_j$
$- \sum z_r \gamma_r$

By property of addition and because W_1 is subspace

$\sum z_r \gamma_r$ belong to W_1 & also belongs to W_2

Hence $\sum z_r \gamma_r \in W_1 \cap W_2$

$\therefore \sum z_r \gamma_r = \sum c_i \alpha_i$ for some $c_1, \ldots c_k$

Because $\{\alpha_1, \ldots \alpha_k, \gamma_1, \ldots \gamma_n\}$ is basis of W_2

$\therefore$ It is independent Hence each of scalar $z_r = 0$

Thus

$\sum x_i \alpha_i + \sum y_j \beta_j = 0$ $\therefore$ each $x_i = 0$ & each $y_j = 0$

Also $\{\alpha_1, \ldots \alpha_k, \beta_1, \ldots \beta_m\}$ is also independent set

$\{\alpha_1, \ldots \alpha_k, \beta_1 \ldots \beta_m, \ldots \gamma_1, \ldots \gamma_n\}$ is basis for $W_1 + W_2$ becau

Hence it is linearly independent & spans $(W_1 + W_2)$

Hence finally $\dim W_1 + \dim W_2 = (k+m) + (k+n)$

$= k + (m + k + n)$

$= \dim(W_1 \cap W_2) + \dim(W_1 + W_2)$

Hence proved