# Predictive Modeling for Insurance Reserves

**Subtitle:** Aggregate Claims, Expected Costs, and VaR Calculation

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# Task Overview

- **Objective:** Estimate equalization reserves using predictive modeling.
- **Company Background:**
  - Mutual insurance association covering waste incinerator (WIS) and landfill (non-WIS) sites.
  - Coverage includes property damage, fire, and machinery breakdown.
- **Key Risks:**
  - Small but frequent losses.
  - Potential for large catastrophic losses (e.g., fire).
- **Approach:** Use statistical modeling to determine expected claims and Value at Risk (VaR).

# Claims Modeling Approach

- **Separate Models:** WIS and non-WIS sites modeled separately.
- **Frequency-Severity Model:**
  - **Frequency:** Poisson distribution (number of claims per year).
  - **Severity:** Pareto distribution (loss amount per claim).
- **Assumptions:**
  - Independent claims across sites.
  - Long-tailed severity distribution to account for large losses.

# Available Data & Adjustments

- **Historical Data:** 2016-2020 claims data.
- **Number of Sites:** 4 WIS, 2 non-WIS (constant exposure over time).
- **Inflation Adjustment:**
  - Claims are adjusted by 3% per year to reflect 2021 values.
  - Formula: $$Loss_{2021} = Loss_t \times (1.03)^{2021-t}$$

# Frequency Distribution Parameters

- **Poisson Distribution:** $P(N = k) = \frac{e^{-\lambda}\lambda^k}{k!}$
- **Estimation:**

$\lambda_{WIS}$ = Avg. number of WIS claims per site.

$\lambda_{non-WIS}$ = Avg. number of non-WIS claims per site.

# Severity Distribution Parameters

- **Pareto Distribution:** $f(x) = \frac{at^a}{x^{a+1}}, x \geq t$
- **Given:** Shape parameter (industry standard for property claims).
- **Scale Parameter:**
  - Estimated such that mean severity matches observed values.
  - Formula: $E[X] = \frac{ta}{a-1}$

# Expected Aggregate Loss Calculation

- Formula: $E[S] = E[N] \times E[X]$
- Expected Loss Per Site:
  - WIS: $4 \times \lambda_{WIS} \times E[X_{WIS}]$
  - Non-WIS: $2 \times \lambda_{non-WIS} \times E[X_{non-WIS}]$
- Total Expected Loss for 2021: Summing both site types.

# Variance and Standard Deviation of Aggregate Loss

- Formula: $Var[S] = E[N] \times Var[X] + Var[N] \times E[X]^2$
- Pareto Variance: $Var[X] = \frac{t^2 a}{(a-2)} - E[X]^2$
- Poisson Variance: $Var[N] = E[N]$

# Value at Risk (VaR) Calculation

- **Formula:**

$$VaR_{80\%} = E[S] + 0.84 \times \sigma(S)$$

- **Interpretation:** The reserve amount required to cover 80% of expected losses.

# Impact of Frequency Doubling

- **New Expected Aggregate Loss:**

$$E[S'] = 2 \times E[S]$$

- **Variance Adjustment:**

$$Var[S'] = 4 \times Var[S]$$

- **New VaR Calculation:** Adjusting for increased variance.

# Model Enhancements

- **Alternative Severity Distributions:** Lognormal, Gamma for better fit.
- **Bayesian Estimation:** Improving parameter uncertainty.
- **Machine Learning:**
  - Anomaly detection for fraudulent claims.
  - Predictive modeling for claim severity trends

# Confidence Interval Approximation

- **Formula:**

$$CI_{80\%} = E[S] \pm 1.28 \times \sigma(S)$$

- **Why This Matters:** Ensures reserve calculations are robust.

# Conclusion

- **Findings:**
  - Expected claims and VaR estimation for reserves.
  - Impact of frequency doubling.
  - Model enhancement suggestions.
- **Next Steps:**
  - Validate assumptions.
  - Refine modeling approach based on additional data.