NPTEL DATA SCIENCE FOR ENGINEERS

ASSIGNMENT 5

1) It is intended to find the maxima of $\boldsymbol{f(x,y)=\mathbf{xy}}$ subject to the constraint $\boldsymbol{x+\mathbf{y}=6}$.

   The Lagrangian function is:-

   a) $L(\mathrm{x},\mathrm{y},\lambda)=xy$

   b) $L(\mathrm{x},\mathrm{y},\lambda)=\lambda(\mathrm{x}+\mathrm{y}-6)$

   c) $L(\mathrm{x},\mathrm{y},\lambda)=xy-\lambda(6-\mathrm{x}-\mathrm{y})$

   d) $\boldsymbol{L(\mathbf{x},\mathbf{y},\lambda)=xy+\lambda(6-\mathbf{x}-\mathbf{y})}$

   Ans-c

   Solution:-

   $\boldsymbol{f(x,y)=\mathbf{xy}}$

   $\boldsymbol{s.t.\ \ x+\mathbf{y}=6}$

   The Lagrangian function is modified version of the objective function with the constraints incorporated.

   $\boldsymbol{L(\mathbf{x},\mathbf{y},\lambda)=f(x,y)+\lambda(\mathbf{c}-g(x,y))}$, where $\boldsymbol{g(x,y)}$ is the constraint.

   $\boldsymbol{L(\mathbf{x},\mathbf{y},\lambda)=xy+\lambda(6-\mathbf{x}-\mathbf{y})}$

2) The necessary first order conditions for the objective function $\boldsymbol{f(x,y)=\mathbf{xy}}$ subject to the constraint $\boldsymbol{x+\mathbf{y}=6}$ is/are:-

   a) $\boldsymbol{\frac{\partial L}{\partial x}=y-\lambda=0}$

   b) $\boldsymbol{\frac{\partial L}{\partial y}=x-\lambda=0}$

   c) $\boldsymbol{\frac{\partial L}{\partial \lambda}=6-x-y=0}$

   d) $\frac{\partial L}{\partial y}=x+\lambda=0$

   Ans-a,b,c

Solution:-

$\nabla L = 0$
$L(\mathrm{x},\mathrm{y},\lambda) = xy + \lambda(6 - \mathrm{x} - \mathrm{y})$

$$\frac{\partial L}{\partial x} = y - \lambda = 0 \rightarrow 1$$

$$\frac{\partial L}{\partial y} = x - \lambda = 0 \rightarrow 2$$

$$\frac{\partial L}{\partial \lambda} = 6 - x - y = 0 \rightarrow 3$$

3) The value of the stationary point x^*, y^* and λ^* for the objective function $\boldsymbol{f}(\boldsymbol{x},\boldsymbol{y}) = \mathbf{xy}$ subject to the constraint $\boldsymbol{x} + \mathbf{y} = \mathbf{6}$ are:-

a) $x^* = 2, y^* = 1$ and $\lambda^* = 1$
b) $x^* = 2.73, y^* = 1.02$ and $\lambda^* = 5.46$
c) $x^* = y^* = \lambda^* = -1$
d) $\boldsymbol{x^* = y^* = \lambda^* = 3}$

By solving (1) (2) and (3) $\boldsymbol{x^* = y^* = \lambda^* = 3}$

4) The hessian matrix for the function $\boldsymbol{f}(\boldsymbol{x},\boldsymbol{y}) = \mathbf{xy}$ is:-

a) $\begin{bmatrix} \mathbf{0} & \mathbf{1} \\ \mathbf{1} & \mathbf{0} \end{bmatrix}$
b) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
c) $\begin{bmatrix} 1 & 0 \\ 3 & 8 \end{bmatrix}$
d) $\begin{bmatrix} 6 & 3 \\ 3 & 8 \end{bmatrix}$

Ans-a

The Hessian matrix of $f(x,y)$ is $\begin{bmatrix} \frac{\partial^2 f}{\partial x^2} & \frac{\partial f}{\partial x\,\partial y} \\ \frac{\partial f}{\partial y\,\partial x} & \frac{\partial^2 f}{\partial y^2} \end{bmatrix}$

$f = xy$	
$\frac{\partial f}{\partial x} = y$	$\frac{\partial f}{\partial y} = 1$
$\frac{\partial^2 f}{\partial x^2} = 0$	$\frac{\partial^2 f}{\partial y^2} = 0$
$\frac{\partial f}{\partial x\ \partial y} = 1$	$\frac{\partial f}{\partial y\ \partial x} = 1$

Hessian is given by $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

5) The eigen values for the hessian matrix obtained in Q4 are: -

   a) 0,1
   b) 0, -1
   c) 1,1
   d) 1, -1

   Ans- $\begin{bmatrix} -\lambda & 1 \\ 1 & -\lambda \end{bmatrix}$

   $|A - \lambda I| = \lambda^2 - 1 = 0$

   $\lambda = +1, -1$

Consider the objective function $\boldsymbol{max\ f(x) = xy}$ subject to $\boldsymbol{x + y^2 \leq 2}$ and $\boldsymbol{x, y} \geq 0$.

The lagrangian function is given by

$$\boldsymbol{L(x, y, \mu_1, \mu_2, \mu_3) = xy - \mu_1(x + y^2 - 2) - \mu_2(-x) - \mu_3(-y)}$$

Answer questions Q6-Q8 based on this input.

6) Which of the following is not an apt representation of the constraint?

   a) $x + y^2 \leq 2$
   b) $-x \geq 0$

c) $-x \leq 0$

Ans- b

Constraints are

$\mathbf{x + y^2 \leq 2}$

-$\boldsymbol{x} \leq \mathbf{0}$

-y$\leq \mathbf{0}$

7) The values of μ_1, μ_2 *and* μ_3 while evaluating the Karush-Kuhn-Tucker(KKT) condition with all the constraints being inactive are:-

a) $\mu_1 = \mu_2 = \mu_3 = 1$

b) $\boldsymbol{\mu_1 = \mu_2 = \mu_3 = 0}$

c) $\mu_1 = \mu_3 = 0,\ \mu_2 = 1$

d) $\mu_1 = \mu_2 = 0,\ \mu_3 = 1$

Ans- b

8) KKT conditions are used to verify that a given point provides an optimal solution

a) True

b) False

Ans- a