Linear Algebra

1. What is the rank of the given matrix A=$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 2 & 2 & 3 & 4 \\ 0 & 0 & 0 & 0 \\ 4 & 4 & 6 & 8 \end{bmatrix}$

    a. 3
    b. 2
    c. 1
    d. 4

Solution:

```
#Q1
A=matrix(c(0,0,0,0,2,2,3,4,0,0,0,0,4,4,6,8),nrow = 4,ncol = 4,byrow = TRUE)
print(A)
library(pracma)
Rank(A)
```

```
> Rank(A)
[1] 1
```

2. The number of independent rows or columns is known as the _____ of the matrix?
    a. Null space
    b. Rank
    c. Nullity

Solution:

Rank of a matrix refers to the number of linearly independent rows or columns of the matrix.

3. What is the dimension of the null space of the matrix A= $\begin{bmatrix} 0 & 1 & 1 \\ 1 & -1 & 0 \\ -1 & 0 & -1 \end{bmatrix}$

    a. 2
    b. 0
    c. 1
    d. 3

Solution:

Dimension of the null space is also called as Nullity.

According to the rank nullity theorem Rank(A)+Nullity(A)=Total number of attributes of A

Here Rank(A)= 2 and Total number of columns= 3. So, Nullity(A) =3,

Therefore, dimension of nullspace is 3-2=1

Nullity(A)=1

```
> A=matrix(c(0,1,1,1,-1,0,-1,0,-1),nrow = 3,ncol = 3,byrow = TRUE)
> print(A)
     [,1] [,2] [,3]
[1,]    0    1    1
[2,]    1   -1    0
[3,]   -1    0   -1
> library(pracma)
> Rank(A)
[1] 2
```

Below given is the data with 4 attributes and 3 samples, based on the data given below answer the questions 4 and 5.

p	q	r	s
77	28	93	254
84	38	85	363
84	99	1	193

4. How many attributes are independent of each other?
    a. 1
    b. 2
    c. 3
    d. 4

Solution:

```
> A=matrix(c(77,28,93,254,84,38,85,363,84,99,1,193),nrow = 3,ncol = 4,byrow = TRUE)
> print(A)
     [,1] [,2] [,3] [,4]
[1,]   77   28   93  254
[2,]   84   38   85  363
[3,]   84   99    1  193
> library(pracma)
> Rank(A)
[1] 3
```

5. The number of linear relations between attributes is
    a. 1
    b. 2
    c. 3
    d. 4

Solution:

Rank=3

Total number of attributes=4

Therefore, total number of relations= 4-3=1

6. Which of the following statement(s) is/are true about orthonormal vectors?
    a. All orthonormal vectors are not orthogonal
    b. All orthonormal vectors are orthogonal
    c. Two vectors are orthonormal to each other if their dot product is 0
    d. Orthonormal vectors are orthogonal vectors with unit magnitude

    Ans- orthonormal vectors are orthogonal vectors with unit magnitude and all orthonormal vectors are orthogonal

7. Let A and B be $n \times n$ real matrices. Then, $rank(A+B) \leq rank(A) + rank(B)$
    a. True
    b. False

8. The null space of A is equal to the null space of $A^T A$
    a. True
    b. False

9. The distance between two vectors say $V_1 = [10 \quad 5 \quad 2]^T$ and $V_2 = [5 \quad 3 \quad 1]^T$ is
    a. $\sqrt{10}$ units
    b. $\sqrt{20}$ units
    c. $\sqrt{30}$ units
    d. $\sqrt{40}$ units

$$Ans - \sqrt{(10-5)^2+(5-3)^2+(2-1)^2} = \sqrt{25+4+1} = \sqrt{30}\ \text{units}$$

10. The unit vector of a vector $[6\ \ 9]^T$ is

a. $\begin{bmatrix} 6/\sqrt{9} \\ 9/\sqrt{117} \end{bmatrix}$

b. $\begin{bmatrix} 6/\sqrt{117} \\ 9/\sqrt{9} \end{bmatrix}$

c. $\begin{bmatrix} \mathbf{6/\sqrt{117}} \\ \mathbf{9/\sqrt{117}} \end{bmatrix}$

d. $\begin{bmatrix} 9/\sqrt{117} \\ 117/\sqrt{6} \end{bmatrix}$

Ans-

$$\frac{6}{\sqrt{6^2+9^2}}, \frac{9}{\sqrt{6^2+9^2}}$$

$$\frac{6}{\sqrt{117}}, \frac{9}{\sqrt{117}}$$