[SINGLE ANSWER CORRECT TYPE]

1. If the sides a,b,c are the roots of the equation $x^3 - 18x^2 + 104x - 192 = 0$, then the value of $\frac{\cos A}{a}+\frac{\cos B}{b}+\frac{\cos C}{c}$ is equal to -

(A) $\frac{3}{64}$ (B) $\frac{29}{48}$ (C) $\frac{29}{96}$ (D) $\frac{3}{128}$

2. In a ΔABC, if $\tan A + 3\tan C = 0$, then angle B lies in -

(A) $\left(0,\frac{\pi}{6}\right]$ (B) $\left(\frac{\pi}{6},\frac{\pi}{2}\right)$ (C) $\left(\frac{\pi}{2},\frac{5\pi}{6}\right)$ (D) $\left[\frac{5\pi}{6},\pi\right)$

3. In ΔABC, if $a^2\cos 2A = 2b^2 + 2c^2 - a^2$, then A belongs to

(A) $\left(0,\frac{\pi}{6}\right)$ (B) $\left(\frac{\pi}{6},\frac{\pi}{4}\right)$ (C) $\left(\frac{\pi}{4},\frac{\pi}{2}\right)$ (D) $\left(\frac{\pi}{2},\pi\right)$

4. In ΔABC, if $\cos A + \sin A - \frac{2}{\cos B+\sin B} = 0$, then $\frac{a+b}{c}$ is equal to

(A) $\sqrt{2}$ (B) 1 (C) $\frac{1}{\sqrt{2}}$ (D) $2\sqrt{2}$

5. If sides of ΔABC are connected with relation $4a^2 + 9b^2 + 16c^2 = 6ab + 12bc + 8ac$, then cosA is equal to

(A) 0 (B) $-\frac{1}{2}$ (C) $\frac{6}{7}$ (D) $-\frac{11}{24}$

6. Two sides of a triangle are given by the roots of the equation $x^2 - 2\sqrt{3}x + 2 = 0$ and the angle between the sides is $\frac{\pi}{3}$. Then perimeter of the triangle is

(A) $6+\sqrt{3}$ (B) $2\sqrt{3}+\sqrt{6}$ (C) $2\sqrt{3}+\sqrt{10}$ (D) none of these

7. If in $\Delta ABC, \frac{\sin A}{3}=\frac{\sin B}{3}=\frac{\sin C}{2}$, then the value of cosA + cosB + cosC is equal to

(A) $\frac{13}{9}$ (B) $\frac{12}{13}$ (C) $\frac{14}{9}$ (D) $\frac{9}{13}$

8. In any triangle ABC, $\frac{a^2\sin(B-C)}{\sin B+\sin C}+\frac{b^2\sin(C-A)}{\sin C+\sin A}+\frac{c^2\sin(A-B)}{\sin A+\sin B}=$

(A) a + b + c (B) a + b – c (C) a – b + c (D) 0

[MULTIPLE ANSWER CORRECT TYPE]

9. In ΔABC, which of the following is/are possible (where notations have usual meaning)

(A) sinA : sinB : sinC = 1 : 2 : 3 (B) $\Delta=\frac{bc}{4}$

(C) (a + b + c)(a + b – c) = 3ab (D) $b^2 - c^2 = aR$

10. In a triangle if the length of two longer sides are 8 and 7 and its angles are in A.P., then smaller side can be

(A) 3 (B) 4 (C) 5 (D) 6

[SUBJECTIVE TYPE]

11. Given a = 13, b = 14, and c = 15, then find the sines of the angles.

12. Prove that: $a\cos\frac{B-C}{2}=(b+c)\sin\frac{A}{2}$

13. Prove that: $b^2\sin 2C + c^2\sin 2B = 2bc\sin A$

14. Prove that: $a(b\cos C - c\cos B) = b^2 - c^2$

15. Prove that: $\frac{\sin(B-C)}{\sin(B+C)}=\frac{b^2-c^2}{a^2}$

16. Prove that: $\frac{a+b}{a-b}=\tan\frac{A+B}{2}\cot\frac{A-B}{2}$

17. Prove that: $a^2 + b^2 + c^2 = 2(bc\cos A + ca\cos B + ab\cos C)$

18. If in any triangle the angles be to one another as 1 : 2 : 3 prove that the corresponding sides are as $1 : \sqrt{3} : 2$

19. In any triangle ABC, prove that : $\frac{1+\cos(A-B)\cos C}{1+\cos(A-C)\cos B}=\frac{a^2+b^2}{a^2+c^2}$

20. In any triangle ABC, prove that: $a\sin(B-C) + b\sin(C-A) + c\sin(A-B) = 0$

21. In any triangle ABC, prove that : $\frac{\cos 2A}{a^2}-\frac{\cos 2B}{b^2}=\frac{1}{a^2}-\frac{1}{b^2}$.

22. In any triangle ABC, prove that: $a(\sin B - \sin C) + b(\sin C - \sin A) + c(\sin A - \sin B) = 0$

23. If in a ΔABC, $\frac{\sin A}{\sin C}=\frac{\sin(A-B)}{\sin(B-C)}$, prove that a^2, b^2, c^2 are in A.P.

24. In any triangle ABC, prove that : $\frac{\sqrt{\sin A}-\sqrt{\sin B}}{\sqrt{\sin A}+\sqrt{\sin B}}=\frac{a+b-2\sqrt{ab}}{a-b}$.

25. In any triangle ABC, prove that : $b\cos B + c\cos C = a\cos(B-C)$

1. In ΔABC, if $\frac{(a+b)\cos C+(a+c)\cos B+(b+c)\cos A}{\sin A+\sin B+\sin C}=100$, then area of circumcircle of ΔABC is

(A) 2500π (B) 25000π (C) 1000π (D) 10000π

2. In ΔABC, if $\sin^2A + \sin^2B = \sin^2C$, then the triangle is -

(A) equilateral (B) isosceles (C) right angled (D) None of these

3. In ΔABC, $b = 4$, $c = 3$ & $\tan\left(\frac{B-C}{2}\right)=\frac{\sqrt{3}}{7}$, then area of ΔABC is

(A) $\sqrt{3}$ (B) $2\sqrt{3}$ (C) $3\sqrt{3}$ (D) $4\sqrt{3}$

4. In ΔABC with usual notations the value of $\sum(a-b)\cot\frac{C}{2}$ is

(A) 0 (B) $\sum\tan\frac{A}{2}$ (C) $(a^2+b^2+c^2)$ (D) 1

5. In any triangle ABC, $\frac{\tan\frac{A}{2}-\tan\frac{B}{2}}{\tan\frac{A}{2}+\tan\frac{B}{2}}=$

(A) $\frac{a-b}{a+b}$ (B) $\frac{a-b}{c}$ (C) $\frac{a-b}{a+b+c}$ (D) $\frac{c}{a+b}$

6. If the area of a triangle ABC is given by $\Delta = a^2 - (b - c)^2$, then $\tan\left(\frac{A}{2}\right)$ is equal to

(A) -1 (B) 0 (C) $\frac{1}{4}$ (D) $\frac{1}{2}$

7. If $\frac{\cos A}{a}=\frac{\cos B}{b}=\frac{\cos C}{c}$ and the side $a = 2$, then area of triangle is

(A) 1 (B) 2 (C) $\frac{\sqrt{3}}{2}$ (D) $\sqrt{3}$

8. The expression $\frac{(a+b+c)(b+c-a)(c+a-b)(a+b-c)}{4b^2c^2}$ is equal to

(A) $\cos^2 A$ (B) $\sin^2 A$ (C) cosA cosB cosC (D) None of these

9. If $b + c = 3a$, then the value of $\cot\left(\frac{B}{2}\right)\cot\left(\frac{C}{2}\right)$ is equal to

(A) 1 (B) 2 (C) $\sqrt{3}$ (D) $\sqrt{2}$

10. If in a triangle ABC, cos A cos B + sin A sin B sin C = 1, then the side are proportional to

(A) $1:1:\sqrt{2}$ (B) $1:\sqrt{2}:1$ (C) $\sqrt{2}:1:1$ (D) None of these

11. Let A, B, C are three angles such that cosA + cosB + cosC = 0 and if cosA . cosB . cosC = λ (cos 3A + cos 3B + cos 3C), then λ is equal to

(A) 1/3 (B) 1/6 (C) 1/9 (D)1/12

12. In any ΔABC, if $\cot\frac{A}{2}$, $\cot\frac{B}{2}$, $\cot\frac{C}{2}$ are in A.P. then a, b, c are in

(A) A.P. (B) G.P. (C) H.P. (D) none of these

13. In ΔABC, a cos (B – C) + b cos (C – A) + c cos (A – B) (where a, b, c are sides of Δ) equals

(A) $\frac{abc}{R^2}$ (B) $\frac{abc}{4R^2}$ (C) $\frac{4abc}{R^2}$ (D) None

14. In a ΔABC if $\angle A = 60°$, $\frac{b}{c}=\frac{\sqrt{3}+1}{2}$ then $\angle B - \angle C$ has value equal to

(A) 15° (B) 30° (C) 22.5° (D) 45°

15. In a triangle ABC, $bc\cos^2\frac{A}{2} + ca\cos^2\frac{B}{2} + ab\cos^2\frac{C}{2} =$

(A) $(s-a)^2$ (B) $(s-b)^2$ (C) $(s-c)^2$ (D) s^2

[SUBJECTIVE TYPE]

16. In ΔABC if $\frac{s-a}{2}=\frac{s-b}{3}=\frac{s-c}{4}$, then the value of $(140)s\left(\frac{\cos A}{a}+\frac{\cos B}{b}+\frac{\cos C}{c}\right)$ is

(where s is semi perimeter of ΔABC)

17. Let three sides of a triangle are three consecutive integers and largest angle is double of smallest angle, then length of largest side is equal to

18. With usual notations, if in a ΔABC we have $\frac{b+c}{11}=\frac{c+a}{12}=\frac{a+b}{13}$, then prove that $\frac{\cos A}{7}=\frac{\cos B}{19}=\frac{\cos C}{25}$.

19. In any ΔABC, prove that: $2\left(a\sin^2\frac{C}{2}+c\sin^2\frac{A}{2}\right)=a+c-b$

20. In any ΔABC, prove that: $4\left(bc\cos^2\frac{A}{2}+ca\cos^2\frac{B}{2}+ab\cos^2\frac{C}{2}\right)=(a+b+c)^2$

[STRAIGHT OBJECTIVE TYPE]

1. In an equilateral triangle, $R : r : r_2$ is equal to

(A) 1 : 1 : 1 (B) 1 : 2 : 3 (C) 2 : 1 : 3 (D) 3 : 2 : 4

2. If in a ΔABC, $a^2 + b^2 + c^2 = 8R^2$, where R = circumradius, then the triangle is

(A) equilateral (B) isosceles (C) right angled (D) none of these

3. If in equilateral triangle, in-radius is a rational number, then which of the following is not true ?

(A) circum radius is always rational (B) area is always irrational

(C) ex-radii are always rational (D) perimeter is always rational

4. In a triangle ABC, a : b : c = 4 : 5 : 6. The ratio of the radius of the circumcircle to that of the incircle is -

(A) 15/4 (B) 11/5 (C) 16/7 (D) 16/3

5. If the lengths of the sides of a triangle are 3, 4 and 5 units then R the circum radius is -

(A) 2.0 (B) 2.5 (C) 3.0 (D) 3.5

6. If the sides of a triangle are 3 : 7 : 8 then R : r =

(A) 2 : 7 (B) 7 : 2 (C) 3 : 7 (D) 7 : 3

7. In a triangle ABC, $\frac{a\cos A + b\cos B + c\cos C}{a+b+c}$ is equal to -

(A) $\frac{r}{R}$ (B) $\frac{R}{r}$ (C) $\frac{2r}{R}$ (D) $\frac{R}{2r}$

8. If r_1, r_2, r_3 in a triangle be in H.P. then the sides are in -

(A) A.P. (B) G.P. (C) H.P. (D) None of these

9. $\left(\frac{1}{r_1}+\frac{1}{r_2}\right)\left(\frac{1}{r_2}+\frac{1}{r_3}\right)\left(\frac{1}{r_3}+\frac{1}{r_1}\right) =$

(A) $\frac{64R^3}{abc}$ (B) $\frac{R^3}{4abc}$ (C) $\frac{64R^3}{a^2b^2c^2}$ (D) $\frac{R^3}{abc}$

10. If the sides be a, b, c then $\frac{b-c}{r_1} + \frac{c-a}{r_2} + \frac{a-b}{r_3} =$

(A) 5 (B) 4 (C) 0 (D) 1

11. $r_2 r_3 + r_3 r_1 + r_1 r_2 =$

(A) s^2 (B) Δ^2 (C) Δ/r^3 (D) R^2

[LINKED COMPREHENSION TYPE]

Paragraph for Question 12 to 14

Three sides a,b,c of ΔABC are in increasing A.P. and are the roots of the equation $x^3 - 12x^2 + px + q = 0$ where $p,q, \in R$ and a & c are prime numbers.

On the basis of above information, answer the following questions :

12. The value of $\tan A + \sin 2B + \cos 3C$ is -

(A) $\frac{172}{75}$ (B) $\frac{171}{100}$ (C) $\frac{161}{100}$ (D) $\frac{171}{75}$

13. The ratio of the radius of circumcircle and radius of incircle of ΔABC is -

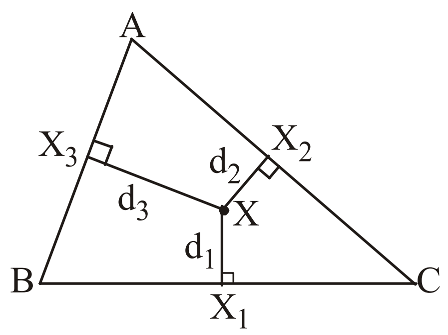
(A) $\frac{5}{2}$ (B) 2 (C) $\frac{3}{2}$ (D) Data insufficient

14. The number of solutions of the equation $a \sin x + b \cos x = c$ in $[-2\pi,\pi]$ are -

(A) 0 (B) 1 (C) 2 (D) 3

Paragraph for question nos. 15 to 17

In Δ ABC as shown, $XX_1 = d_1$; $XX_2 = d_2$; $XX_3 = d_3$ and X is the centre of the circumscribed circle around the ΔABC. a, b and c as usual are the sides BC, CA and AB respectively.



15. If $\lambda\left(\frac{a}{d_1}+\frac{b}{d_2}+\frac{c}{d_3}\right) = \frac{abc}{d_1d_2d_3}$, then the value of 'λ' is equal to

(A) 1 (B) 2 (C) 4 (D) 8

16. If R is the radius of the circumcircle of the ΔABC and $a(d_2 + d_3) + b(d_3 + d_1) + c(d_1 + d_2) = kR(a + b + c)$ then the value of 'k' is

(A) 1 (B) 1/2 (C) 1/3 (D) 2

17. Let h_a, h_b and h_c are the altitudes of the ΔABC from the angular points A, B and C respectively. If $(a^2 + b^2 + c^2) = t\,(h_ad_1 + h_bd_2 + h_cd_3)$ then 't' equals

(A) 1 (B) 2 (C) 3 (D) 4

[SUBJECTIVE TYPE]

18. In an equilateral ΔABC with each side of $\sqrt{3}$, with usual conventions, the value of $(r_1-r)\,(r_2-r)\,(r_3-r)$ is equal to

19. In an acute angled ΔABC, with usual conventions, the arithmetic mean of $\frac{1}{r_1},\frac{1}{r_2},\frac{1}{r_3}$ is k, then minimum value of $\sqrt{\frac{6abc\,k}{\Delta}}$ is

20. If r_1,r_2,r_3 are roots of $x^3 - 6x^2 + 11x - 6 = 0$ then the area of the triangle is $\frac{p}{\sqrt{q}}$, where p & q are coprimes then $(q - p)$ is equal to

1. If P_1, P_2 & P_3 are altitudes of ΔABC from its vertices A, B, C, then value of $\frac{1}{P_1}-\frac{1}{P_2}-\frac{1}{P_3}$, is -

(A) $-\frac{1}{r_1}$ (B) $\frac{1}{r_1}$ (C) $\frac{1}{r_3}$ (D) $-\frac{1}{r_2}$

2. If the area of right triangle ABC is 120 and the perimeter is 60 and BC is hypotenuse. If length of altitude corresponding to side BC can be expressed as $\frac{p}{q}$ where p & q are coprime, then (p + q) is equal to

(A) 123 (B) 133 (C) 143 (D) 153

3. For an equilateral triangle ABC, if α is the distance of orthocentre from any side of the triangle and β is the distance of incentre from any vertex of the triangle, then $\frac{\beta}{\alpha}$ is equal to -

(A) $\frac{2}{\sqrt{3}}$ (B) $\frac{1}{2}$ (C) 2 (D) $\frac{\sqrt{3}}{2}$

4. Let ABC be a triangle such that $\angle A = 45^\circ$, $\angle B = 75^\circ$ then $a + c\sqrt{2}$ is equal to -

(A) 0 (B) b (C) 2b (D) –b

5. In a ΔABC, $A = \frac{2\pi}{3}$, $b - c = 3\sqrt{3}$ cm and ar(ΔABC) = $\frac{9\sqrt{3}}{2}$ cm^2. Then a is –

(A) $6\sqrt{3}$ cm (B) 9 cm (C) 18 cm (D) None of these

6. In a triangle ABC, $2ac \sin\frac{1}{2}(A - B + C) =$

(A) $a^2 + b^2 - c^2$ (B) $c^2 + a^2 - b^2$ (C) $b^2 - c^2 - a^2$ (D) $c^2 - a^2 - b^2$

[MULTIPLE CORRECT TYPE]

7. In a triangle ABC, a = 7, b = 8, c = 9, BD is the median and BE is the altitude, then -

(A) $BE = 3\sqrt{5}$ (B) ED = 2 (C) cosA = 2/3 (D) $\Delta = 8\sqrt{5}$

8. If sides a, b and c of different lengths of triangle ABC satisfy $\frac{a^3+b^3+c^3}{a+b+c} = c^2$, then which of the following is/are always correct ?

(A) The angles of triangle are in A.P. (B) The sides of triangle are in A.P.

(C) $\tan\frac{C}{4} = \frac{1}{2+\sqrt{3}}$

(D) If $\angle A = 45^\circ$ then cotangent of one of the angle between median through vertex C and side AB is $\frac{\sqrt{3}-1}{2}$

9. If the sides of a triangle are in A.P. with common difference 1 and whose circumradius is $\frac{8}{\sqrt{15}}$, then which of the following can be side(s) of a triangle

(A) 2 (B) 3 (C) 4 (D) 5

10. If in ΔABC, (with usual notations) sides are rational & $2\log 3 = \log((a+b+c)) + \log\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$, then which of the following is/are irrational ?

(A) area of triangle (B) inradius of triangle
(C) circum radius of triangle (D) distance between circumcentre & centroid

[LINKED COMPREHENSION TYPE]

Paragraph for Question 11 to 13

In a ΔABC with $B = \frac{\pi}{3}$, sides a & c (c > a) are roots of the equation $\frac{2}{\sqrt{3}-1}x^2 - \left(3+\sqrt{3}\right)x + 2 = 0$ and altitude AD divides side BC in such a way that $\frac{\text{area of } \Delta ABD}{\text{area of } \Delta ADC} = \frac{2}{3}$.

On the basis of above information, answer the following questions :

11. tan $\angle DAC$ is equal to -

(A) $\frac{\sqrt{3}}{2}$ (B) $\frac{2}{\sqrt{3}}$ (C) $\frac{2}{3}$ (D) $\frac{3}{2}$

12. Area of triangle ABC is equal to -

(A) $\frac{\sqrt{3}(\sqrt{3}+1)}{4}$ (B) $\frac{\sqrt{3}}{2(\sqrt{3}-1)}$ (C) $\frac{\sqrt{3}}{2(\sqrt{3}+1)}$ (D) $\frac{\sqrt{3}(\sqrt{3}-1)}{2}$

13. If $\sum 2bc\cos A = p + q\sqrt{3}$, p, q $\in$ I, then (p + q) is equal to -

(A) 5 (B) 6 (C) 16 (D) 17

[MATRIX MATCH TYPE]

14. Let P be an interior point of acute angled ΔABC. Match the correct entries for the ratio of circumradii of circumcircles of the triangles, ΔPBC, ΔPCA, ΔPAB depending on the position of the point P with respect to ΔABC. (Notations used have standard meaning)

Column-I	Column-II
(A) If P divides the ΔABC into 3 triangles of equal areas	(P) $a\sec\frac{A}{2} : b\sec\frac{B}{2} : c\sec\frac{C}{2}$
(B) If P is equidistant from all the sides of ΔABC	(Q) a cosec2A : b cosec2B : c cosec2C
(C) If P is equidistant from all the vertices of ΔABC	(R) a cosecA : b cosecB : c cosecC
(D) If P divides externally the line joining centroid and circumcentre of ΔABC in the ratio 2 : 3	(S) 1 : 1 : 1
	(T) $\frac{a}{\sqrt{2b^2+2c^2-a^2}} : \frac{b}{\sqrt{2a^2+2c^2-b^2}} : \frac{c}{\sqrt{2a^2+2b^2-c^2}}$

15 In ΔABC, with usual conventions a = 13, b = 14, c = 15 and m,n are coprime numbers.

Column-I	Column-II
(A) If sinA + sinB + sinC = $\frac{m}{n}$, then	(P) $\lvert m - 2n\rvert = 9$
(B) If $\frac{r_1}{r_2} = \frac{m}{n}$, then	(Q) $\lvert m - 2n\rvert = 38$
(C) If $\frac{5\Delta}{abc} = \frac{m}{n}$, then	(R) $\lvert m - 2n\rvert = 13$
(D) If $\cot\frac{A}{2} + \cot\frac{B}{2} + \cot\frac{C}{2} = \frac{m}{n}$, then	(S) m + n = 15
	(T) m + n = 25

ANSWER KEY

RACE-1

1. (C) **2.** (A) **3.** (D) **4.** (A) **5.** (D) **6.** (B) **7.** (A) **8.** (D) **9.** (BCD)

10. (AC) **11.** $\frac{4}{5}, \frac{56}{65}$ and $\frac{12}{13}$

RACE-2

1. (A) **2.** (C) **3.** (C) **4.** (A) **5.** (B) **6.** (C) **7.** (D) **8.** (B) **9.** (B)

10. (A) **11.** (D) **12.** (A) **13.** (A) **14.** (B) **15.** (D) **16.** 330 **17.** 6

RACE-3

1. (C) **2.** (C) **3.** (D) **4.** (C) **5.** (B) **6.** (B) **7.** (A) **8.** (A) **9.** (C)

10. (C) **11.** (A) **12.** (B) **13.** (A) **14.** (C) **15.** (C) **16.** (A) **17.** (D) **18.** 1

19. 4 **20.** 5

RACE-4

1. (A) **2.** (B) **3.** (C) **4.** (C) **5.** (B) **6.** (B) **7.** (ABC) **8.** (ACD) **9.** (ABC)

10. (ABC) **11.** (A) **12.** (C) **13.** (B) **14.** A-T, B-P, C - Q, D - R **15.** A-Q, B-PS, C-S , D - QT