

Why sample variance is $n-1$?

Population

$$\mu = \frac{\sum_{i=1}^N x_i}{N}$$

$$\sigma^2 = \frac{\sum_{i=1}^N (x_i - \mu)^2}{N}$$

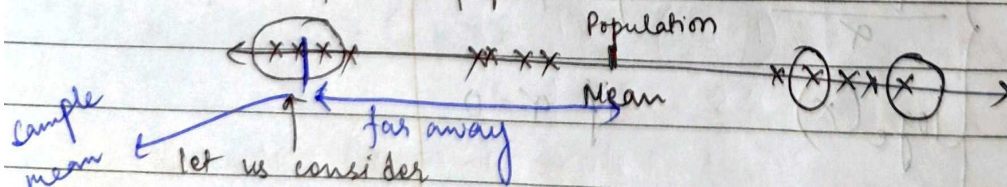
Sample

$$\bar{x} = \frac{\sum_{i=1}^n x_i}{n}$$

$$s^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n-1} \rightarrow \text{Unbiased estimation}$$

$$s^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n} \rightarrow \text{It is a biased estimation}$$

let us consider, from a population data we have some sample points



we have seen, selected

this data points, then my sample mean will also be located there

In order to overcome the biased estimate, we divide by $(n-1)$

i) when divide by n :-

→ estimate $\bar{x} < \mu$

Sample variance $<<$ Population variance

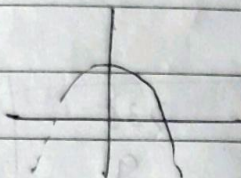
ii) when divide by $(n-1)$:-

$\bar{x} \approx \mu$

iii) when divide by $(n-2)$:-

$\bar{x} >> \mu$

$n-3$
 $\bar{x} >>> \mu$



When dividing by $(n-1)$, the sample variance tends to approach to the true population variance.