Physics and Simulation Model for CUDA-Based Rock-into-Cloth System

1. Overview

This document describes the complete physical model, mathematical formulation, and simulation architecture for a CUDA-based simulation of spherical rigid-body rocks falling into a deformable cloth mesh. The model uses:

- A mass-spring-damper representation for cloth
- A discrete element method (DEM) for rocks and contact
- Penalty-based collision detection and friction
- Velocity Verlet integration
- Real-time visualization via CUDA-OpenGL interoperability

2. Cloth Model

The cloth is modeled as a 2D grid of mass points connected by springs:

2.1 Mass Points

Each node i has:

- Position $\mathbf{x}_i \in \mathbb{R}^3$
- Velocity $\mathbf{v}_i$
- Accumulated force $\mathbf{F}_i$

2.2 Spring Forces (Hooke's Law)

For a spring between nodes i and j with rest length L_0:

$$\mathbf{F}_{ij}^{\text{spring}} = -k_s \left(\|\mathbf{x}_j - \mathbf{x}_i\| - L_0 \right) \cdot \hat{\mathbf{d}}_{ij}$$

$$\hat{\mathbf{d}}_{ij} = \frac{\mathbf{x}_j - \mathbf{x}_i}{\|\mathbf{x}_j - \mathbf{x}_i\|}$$

2.3 Damping Forces

$$\mathbf{F}_{ij}^{\text{damp}} = -k_d \left[(\mathbf{v}_j - \mathbf{v}_i) \cdot \hat{\mathbf{d}}_{ij} \right] \hat{\mathbf{d}}_{ij}$$

2.4 Gravity

$$\mathbf{F}_i^{\text{gravity}} = m \cdot \mathbf{g}$$

2.5 Plasticity

If the spring strain exceeds a plastic limit $\varepsilon_{\text{plastic}}$, update the rest length:

$$\frac{\|\mathbf{x}_j - \mathbf{x}_i\| - L_0}{L_0} > \varepsilon_{\text{plastic}} \quad \Rightarrow \quad L_0 \leftarrow \|\mathbf{x}_j - \mathbf{x}_i\|$$

3. Rigid Rocks (DEM)

Each rock is a rigid sphere represented by:

- Position $\mathbf{x}_r$
- Velocity $\mathbf{v}_r$
- Angular velocity $\boldsymbol{\omega}_r$
- Radius R, mass M

3.1 Equations of Motion

Linear:

$$M \cdot \frac{d\mathbf{v}_r}{dt} = \sum \mathbf{F}_{\text{contact}} + M \cdot \mathbf{g}$$

Angular:

$$I \cdot \frac{d\boldsymbol{\omega}_r}{dt} = \sum \boldsymbol{\tau}$$

3.2 Inertia

$$I = \frac{2}{5} M R^2$$

4. Rock–Cloth Contact Forces (DEM)

4.1 Collision Detection

A contact occurs when:

$$\delta = R - \|\mathbf{x}_i - \mathbf{x}_r\| > 0$$

4.2 Normal Force (Penalty)

$$\hat{n} = \frac{\mathbf{x}_i - \mathbf{x}_r}{\|\mathbf{x}_i - \mathbf{x}_r\|}$$

$$\mathbf{F}_n = k_n \delta \hat{n} - c_n (\mathbf{v}_{\text{rel}} \cdot \hat{n}) \hat{n}$$

4.3 Friction

$$\mathbf{v}_{\text{rel}} = \mathbf{v}_i - (\mathbf{v}_r + \boldsymbol{\omega}_r \times (\mathbf{x}_i - \mathbf{x}_r))$$

$$\mathbf{v}_t = \mathbf{v}_{\text{rel}} - (\mathbf{v}_{\text{rel}} \cdot \hat{n}) \hat{n}$$

$$\mathbf{F}_t = -\min(\mu \|\mathbf{F}_n\|, c_t \|\mathbf{v}_t\|) \cdot \frac{\mathbf{v}_t}{\|\mathbf{v}_t\|}$$

4.4 Torque

$$\boldsymbol{\tau}_r = (\mathbf{x}_i - \mathbf{x}_r) \times \mathbf{F}_t$$

5. Time Integration (Velocity Verlet)

Position:

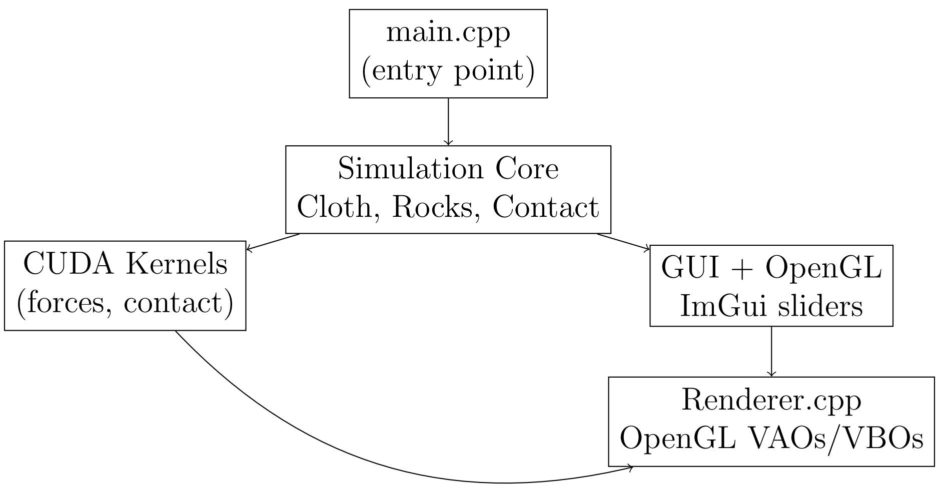
$$\mathbf{x}_{t+\Delta t} = \mathbf{x}_t + \mathbf{v}_t \Delta t + \frac{1}{2} \mathbf{a}_t \Delta t^2$$

Force recomputation $\Rightarrow$ compute $\mathbf{a}_{t+\Delta t}$

Velocity:

$$\mathbf{v}_{t+\Delta t} = \mathbf{v}_t + \frac{1}{2} (\mathbf{a}_t + \mathbf{a}_{t+\Delta t}) \Delta t$$

6. Simulation System Diagram



7. Summary

This simulation combines mass-spring cloth mechanics, DEM-based rigid-body rock modeling, and penalty-based contact to create a CUDA-parallel simulation with high visual fidelity. It is modular, extensible, and designed for both scientific correctness and interactive visualization.