Blue Book

INTERNAL ASSESSMENT BOOK

Name ....Ritik Hariani....

Subject...Linear Algebra Assignment... Class ....4 I....

SL.No.	PARTICULARS	Test Date	Page No.	Marks Awarded	Signature of Staff incharge
1	TEST - I				
2	TEST - II				
3	TEST - III				
4					
5					

CERTIFICATE

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Assignment - 2

PES1201801558
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A1. Let the given equation be ~~...~~ $A + Bx + Cx^2$

at $(1,1) \Rightarrow 1 = A + B + C$
at $(2,-1) \Rightarrow -1 = A + 2B + 4C$
at $(3,1) \Rightarrow 1 = A + 3B + 9C$

$$\underbrace{\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 3 & 9 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} A \\ B \\ C \end{bmatrix}}_{x} = \underbrace{\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}}_{b}$$

$$[A\ b] = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 4 & -1 \\ 1 & 3 & 9 & 1 \end{bmatrix}$$

$R_2 \to R_2 - R_1$
$R_3 \to R_3 - R_1$

$$\sim \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & -2 \\ 0 & 2 & 8 & 0 \end{bmatrix}$$

$R_3 \to R_3 - 2R_2$

$$\sim \left[\begin{array}{c|ccc} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & -2 \\ 0 & 0 & 2 & -4 \end{array}\right] = [U\ c]$$

PES1201801558

$2C = 4$

$\boxed{C = 2}$

$B + 3C = -2$

$\boxed{B = -8}$

$A + B + C = 1$

$\boxed{A = 7}$

$\therefore$ Equation of parabola is $y = 7 - 8x + 2x^2$

A2.

$$A = \begin{bmatrix} 2 & 5 & 2 & -5 \\ 4 & 12 & 3 & -14 \\ -10 & -29 & -5 & 38 \\ 10 & 21 & 21 & -6 \end{bmatrix}$$

$R_2 - 2R_1 \quad R_3 \rightarrow R_3 + 5R_1 \quad R_4 \rightarrow R_4 - 5R_1$

$$\sim \begin{bmatrix} 2 & 5 & 2 & -5 \\ 0 & 2 & -1 & -4 \\ 0 & -4 & 5 & 13 \\ 0 & -4 & 11 & 19 \end{bmatrix}$$

$R_3 \rightarrow R_3 + 2R_2 \quad R_4 \rightarrow R_4 + 2R_2$

$$\sim \begin{bmatrix} 2 & 5 & 2 & -5 \\ 0 & 2 & -1 & -4 \\ 0 & 0 & 3 & 5 \\ 0 & 0 & 9 & 11 \end{bmatrix}$$

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~~$R_4 \to R_4 + 5R_3$~~

$$2\begin{bmatrix} 2 & 5 & 2 & -5 \\ 0 & 2 & -3 & -4 \\ 0 & 0 & -1 & 5 \\ 0 & 0 & 0 & 36 \end{bmatrix}$$

$R_4 \to R_4 - 3R_3$

$$\begin{bmatrix} 2 & 5 & 2 & -5 \\ 0 & 2 & -1 & -4 \\ 0 & 0 & 3 & 5 \\ 0 & 0 & 0 & -4 \end{bmatrix} = U$$

$$L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ -5 & -2 & 1 & 0 \\ 5 & -2 & 3 & 1 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ -5 & -2 & 1 & 0 \\ 5 & -2 & 3 & 1 \end{bmatrix} \begin{bmatrix} 2 & 5 & 2 & -5 \\ 0 & 2 & -1 & -4 \\ 0 & 0 & 3 & 5 \\ 0 & 0 & 0 & -4 \end{bmatrix} = LU$$

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A3. Given linear transformation is -

$$T(x,y,z) = (x+2y-z,\ y+z,\ x+y-2z)$$

i) $T(1,0,0) = (1,0,1)$

$T(0,1,0) = (2,1,1)$

$T(0,0,1) = (-1,1,-2)$

Required transformation matrix is : $T = \begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 1 \\ 1 & 1 & -2 \end{bmatrix}$

ii) Performing GE on T : $R_3 \to R_3 - R_1$

$$T \sim \begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 1 \\ 0 & -1 & -1 \end{bmatrix}$$

$R_3 \to R_3 + R_2$

$$\sim \begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

Basis for $C(T) \Rightarrow (1,0,1), (2,1,1)$

Basis for $C(T^T) \Rightarrow (1,2,-1), (0,1,1)$

Now for $N(T)$,

DATE: / /

PES1201801558

$R_1 \to R_1 - 2R_2$

$$\sim \begin{bmatrix} 1 & 0 & -3 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

Basis for $N(T^T)$ is $(3, -1, 1)$

For $N(T^T)$

$$T^T = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 1 & 1 \\ -1 & 1 & -2 \end{bmatrix}$$

$R_2 \to R_2 - 2R_1 \quad R_3 \to R_3 + R_1$

$$\sim \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 1 & -1 \end{bmatrix}$$

$R_3 \to R_3 - R_2$

$$\sim \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

Basis for $N(T^T)$ is $(-1, 1, 1)$

For finding eigen value, characteristic equation is given by $|T - \lambda I| = 0$ PES1201801558

$$\begin{vmatrix} 1-\lambda & 2 & -1 \\ 0 & 1-\lambda & 1 \\ 1 & 1 & -2-\lambda \end{vmatrix}$$

Taking det,

$$[(1-\lambda)[(1-\lambda)(-2-\lambda) - 1] - 2(-1) - (\lambda-1)$$

$$[-\lambda^3 + 3\lambda] = 0$$

$$\lambda^3 - 3\lambda = 0$$

$$\lambda(\lambda^2 - 3) = 0$$

$$\lambda = 0, \sqrt{3}, -\sqrt{3}$$

Finding eigen vectors.

For $\lambda = -\sqrt{3}$

$$(T + \sqrt{3}I) = \begin{bmatrix} 1+\sqrt{3} & 2 & -1 \\ 0 & 1+\sqrt{3} & 1 \\ 1 & 1 & -2+\sqrt{3} \end{bmatrix}$$

$$\frac{x}{\begin{vmatrix} 2 & -1 \\ 1+\sqrt{3} & 1 \end{vmatrix}} = \frac{y}{\begin{vmatrix} 1+\sqrt{3} & -1 \\ 0 & 1 \end{vmatrix}} = \frac{z}{\begin{vmatrix} 1+\sqrt{3} & 2 \\ 0 & 1+\sqrt{3} \end{vmatrix}}$$

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PAGE No:
DATE: / /

PES1201801558

$$\frac{x}{2+(1+\sqrt{3})} = \frac{y}{-(1+\sqrt{3})} = \frac{z}{(1+\sqrt{3})^2}$$

$$\frac{x}{3+\sqrt{3}} = \frac{y}{-(1+\sqrt{3})} = \frac{z}{(4+2\sqrt{3})} = k_1$$

Eigen vector for $\lambda = -\sqrt{3}$ is $k_1(3+\sqrt{3}, -(1+\sqrt{3}), 4+2\sqrt{3})$

$$\Rightarrow k_1\left(\frac{-\sqrt{3}+3}{2}, \frac{-\sqrt{3}+1}{2}, 1\right)$$

(ii) For $\lambda = 0$

$$|T-\lambda I| = \begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 1 \\ 1 & 1 & -2 \end{bmatrix}$$

$$\frac{x}{2+1} = \frac{y}{-1} = \frac{z}{1} = k_2$$

Eigen vector is $k_2(3, -1, 1)$

(iii) For $\lambda = \sqrt{3}$

$$(T-\lambda I) = \begin{bmatrix} 1-\sqrt{3} & 2 & -1 \\ 0 & 1-\sqrt{3} & 1 \\ 1 & 1 & -2-\sqrt{3} \end{bmatrix}$$

PES1201801558

$$\frac{x}{2+(1-\sqrt{3})} = \frac{y}{-(1-\sqrt{3})} = \frac{z}{(1-\sqrt{3})^2}$$

$$\frac{x}{3-\sqrt{3}} = \frac{y}{\sqrt{3}-1} = \frac{z}{4-2\sqrt{3}} = k_3$$

$\therefore$ $k_3(3-\sqrt{3}, \sqrt{3}-1, 4-2\sqrt{3})$

Rationalising

$k_3\left(\frac{\sqrt{3}+3}{2}, \frac{\sqrt{3}+1}{2}, 1\right)$ //

iv) For $A = QR$ factorisation

$a = (1, 0, 1)$
$b = (2, 1, 1)$
$c = (-1, 1, -2)$

$$q_1 = \frac{a}{||a||} = \frac{(1, 0, 1)}{\sqrt{2}}$$

$$q_2 = \frac{B}{||B||}$$

$$B = b - (q^T b) q_1$$

$$= (2, 1, 1) - \left(\frac{2}{\sqrt{2}} + 0 + \frac{1}{\sqrt{2}}\right) \frac{(1, 0, 1)}{\sqrt{2}}$$

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PAGE No:
DATE: / /

PES1201801658

$= (2,1,1) - \frac{3}{2}(1,0,1)$

$= \left(\frac{1}{2}, 1, -\frac{1}{2}\right)$

$= \frac{1}{2}(1,2,-1)$ //

$\therefore q_2 = \frac{(1,2,-1)}{2\left(\sqrt{\frac{1}{4}+1+\frac{1}{4}}\right)} = \frac{(1,2,-1)}{\sqrt{6}}$

$q_3 = \frac{c}{\|c\|}$

$c = c - (q^T c) q_1 - (q_2^T c) q_2$

$= c - \left(\frac{-1}{\sqrt{2}} - \frac{2}{\sqrt{2}}\right)\left(\frac{1,0,1}{\sqrt{2}}\right) - \left(\frac{-1}{\sqrt{6}} + \frac{2}{\sqrt{6}} + \frac{2}{\sqrt{6}}\right)\left(\frac{1,2,-1}{\sqrt{6}}\right)$

$= (-1,1,-2) + \frac{3}{2}(1,0,1) - \frac{3}{6}(1,2,-1)$

$c = (0,0,0)$

$\Rightarrow q_3 = (0,0,0)$

$$R = \begin{bmatrix} q_1^T a & q_1^T b & q_1^T c \\ 0 & q_2^T b & q_2^T c \\ 0 & 0 & q_3^T c \end{bmatrix} = \begin{bmatrix} \sqrt{2} & 3/\sqrt{2} & -3/\sqrt{2} \\ 0 & 1/2 & 1/2 \\ 0 & 0 & 0 \end{bmatrix}$$

$\therefore A = QR = \begin{bmatrix} \frac{1}{\sqrt{2}} & 1/\sqrt{6} & 0 \\ 0 & 2/\sqrt{6} & 0 \\ 1/\sqrt{2} & -1/\sqrt{2} & 0 \end{bmatrix} \begin{bmatrix} \sqrt{2} & 3/\sqrt{2} & -3/\sqrt{2} \\ 0 & \sqrt{3/2} & \sqrt{3/2} \\ 0 & 0 & 0 \end{bmatrix}$

A4.

Given				
x	-4	1	2	3
y	4	6	10	8

Representing in matrix form

$$\begin{bmatrix} 1 & -4 \\ 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} \hat{c} \\ d \end{bmatrix} = \begin{bmatrix} 4 \\ 6 \\ 10 \\ 8 \end{bmatrix}$$

Normal equation is:

$$A^T A \hat{x} = A^T b$$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ -4 & 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & -4 \\ 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 4 & 2 \\ 2 & 30 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ -4 & 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 4 \\ 6 \\ 10 \\ 8 \end{bmatrix} = \begin{bmatrix} 28 \\ 34 \end{bmatrix}$$

The normal eq. becomes — PES1201801588

$$\begin{bmatrix} 4 & 2 \\ 2 & 30 \end{bmatrix} \begin{bmatrix} \hat{c} \\ \hat{d} \end{bmatrix} = \begin{bmatrix} 28 \\ 34 \end{bmatrix}$$

On solving,

$$\hat{c} = \frac{193}{29} \qquad \hat{d} = \frac{20}{29}$$

The required equation is $y = \frac{193}{29} + \frac{20}{29}x$.

A5) The matrix representing the plane is

$$A = \begin{bmatrix} 1 \\ 1 \\ 3 \\ 0 \\ 4 \end{bmatrix}$$

$$P = \frac{AA^T}{A^TA} = \frac{\begin{bmatrix} 1 \\ 1 \\ 3 \\ 0 \\ 4 \end{bmatrix} \begin{bmatrix} 1 & 1 & 3 & 0 & 4 \end{bmatrix}}{1+1+9+16}$$

$$= \frac{1}{27} \begin{bmatrix} 1 & 1 & 3 & 0 & 4 \\ 1 & 1 & 3 & 0 & 4 \\ 3 & 3 & 9 & 0 & 12 \\ 0 & 0 & 0 & 0 & 0 \\ 4 & 4 & 12 & 0 & 16 \end{bmatrix}$$

$Q = I - P$ PES1201801558

$$Q = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} - \frac{1}{27}\begin{bmatrix} 1 & 1 & 3 & 0 & 4 \\ 1 & 1 & 3 & 0 & 4 \\ 3 & 3 & 9 & 0 & 12 \\ 0 & 0 & 0 & 0 & 0 \\ 4 & 4 & 12 & 0 & 16 \end{bmatrix}$$

$$Q = \frac{1}{27}\begin{bmatrix} 26 & -1 & -3 & 0 & -4 \\ -1 & 26 & -3 & 0 & -4 \\ -3 & -3 & 18 & 0 & -12 \\ 0 & 0 & 0 & 1 & 0 \\ -4 & -4 & -12 & 0 & 11 \end{bmatrix}$$

A6.

i) $A = \begin{bmatrix} a & 2 & 2 \\ 2 & a & 2 \\ 2 & 2 & a \end{bmatrix}$

All sub determinants should be positive

1) $|a| > 0$, $a \in (0, \infty)$

2) $\begin{vmatrix} a & 2 \\ 2 & a \end{vmatrix} = a^2 - 4 > 0$, $a \in \cancel{(4, \infty)}$ $(-\infty, -2) \cup (2, \infty)$

3) $\begin{vmatrix} a & 2 & 2 \\ 2 & a & 2 \\ 2 & 2 & a \end{vmatrix}$ $(a+4)(a-2)^2 > 0 \Rightarrow a \in (-4, \infty)$

$\Rightarrow a \in (2, \infty)$

PAGE No:
DATE: / /

PES1201801558

(ii) $\begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix} \cdot \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$

$= a_{11}x_1^2 + a_{22}x_2^2 + a_{33}x_3^2 + 2a_{12}x_1x_2 + 2a_{13}x_1x_3$
$+ 2a_{23}x_2x_3$ — (1)

Given equation is $f = 2x_1^2 + 2x_2^2 + 2x_3^2 - 2x_1x_2 - 2x_2x_3$

Comparing with (1), we get:

$a_{11} = a_{22} = a_{33} = 2$

$a_{12} = -1$

$a_{13} = 0$

$a_{23} = -1$

$\therefore B = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$

A7. $A = \begin{bmatrix} -3 & 1 \\ 6 & -2 \\ 6 & -2 \end{bmatrix}$

$A^TA = \begin{bmatrix} -3 & 6 & 6 \\ 1 & -2 & -2 \end{bmatrix} \cdot \begin{bmatrix} -3 & 1 \\ 6 & -2 \\ 6 & -2 \end{bmatrix} = \begin{bmatrix} 81 & -27 \\ -27 & 9 \end{bmatrix} = B$

$|B - \lambda I| = 0$

$(81-\lambda)(9-\lambda) - (27)^2 = 0$

$\Rightarrow 729 - 90\lambda + \lambda^2 - 729 = 0$ PES1201801558

$\Rightarrow \lambda^2 - 90\lambda = 0$

$\lambda = 0, \lambda = 90$

Eigen values are 0, 90.

i) $\lambda = 0$, $A = \begin{bmatrix} 81 & -27 \\ -27 & 9 \end{bmatrix}$

$\Rightarrow 81x = 27y \Rightarrow \frac{x}{1} = \frac{y}{3}$

$\Rightarrow k_1(1, 3)$ is the ~~given~~ eigen vector

ii) $\lambda = 90$

$A = \begin{bmatrix} -9 & -27 \\ -27 & -81 \end{bmatrix}$

$\Rightarrow 9x = -27y \Rightarrow \frac{x}{-3} = \frac{y}{1}$

$\Rightarrow k_2(-3, 1)$ is the ~~g~~ eigen vector

Matrix $V = \begin{bmatrix} \frac{1}{\sqrt{10}} & -3/\sqrt{10} \\ 3/\sqrt{10} & 1/\sqrt{10} \end{bmatrix}$

For $\Sigma = AA^T = \begin{bmatrix} -3 & 1 \\ 6 & -2 \\ 6 & -2 \end{bmatrix} \begin{bmatrix} -3 & 6 & 6 \\ 1 & -2 & -2 \end{bmatrix}$

$= \begin{bmatrix} -10 & -20 & -20 \\ -20 & 40 & 40 \\ -20 & 40 & 40 \end{bmatrix}$

PAGE No:
DATE: / /

$$\left| (AA^T) - \lambda I \right| = 0$$

PES1201896558

$$(10-\lambda)\left((40-\lambda)^2 - 1600\right) + 20\left(-800 + 20\lambda + 800\right) - 20\left(-800 + 800 - 20\lambda\right) = 0$$

$$(10-\lambda)(\lambda^2 - 80\lambda) + 800\lambda = 0$$

$$\lambda^2(\lambda - 90) = 0$$

$\lambda_1 = 0$ $\lambda_2 = 90$

$\sigma_1 = 0$ $\sigma_2 = 3\sqrt{10}$

$$\Sigma = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 3\sqrt{10} \end{bmatrix}$$

For U: $AV = U\Sigma$

$$\begin{bmatrix} 0 & \sqrt{10} \\ 0 & -2\sqrt{10} \\ 0 & -2\sqrt{10} \end{bmatrix} = U \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 3\sqrt{10} \end{bmatrix}$$

$$\therefore U = \begin{bmatrix} 0 & 0 & 1/3 \\ 0 & 0 & -2/3 \\ 0 & 0 & -2/3 \end{bmatrix}$$

$$A = U \Sigma V^T$$

$$A = \begin{bmatrix} 0 & 0 & 1/3 \\ 0 & 0 & -2/3 \\ 0 & 0 & -2/3 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 3\sqrt{10} \end{bmatrix} \begin{bmatrix} 1/\sqrt{10} & 3/\sqrt{10} \\ -3/\sqrt{10} & 1/\sqrt{10} \end{bmatrix}$$