3. Deriving Length Contraction

Let an object of length l_0 be at rest on the x axis in S frame. The coordinates of its two endpoints are x_1 and x_2, $l_0 = x_2 - x_1$

The coordinates of its two end points in S' frame are x_1' and x_2'. The length is $l = x_2' - x_1'$

$$x = \gamma(x' + vt')$$

$$x_2 - x_1 = \gamma(x_2' + vt_2') - \gamma(x_1' + vt_1')$$

$$l_0 = \gamma l$$

$$\Rightarrow \boxed{l = \frac{l_0}{\gamma}}$$

$$\boxed{t_2' = t_1'}$$

An observer in S' measure this length by measuring x_2' and x_1' at the same time.

4. Deriving time dilation

The proper time means the time interval between two events that happes at the same location in space $\Rightarrow \boxed{x_2' = x_1'}$

$\Delta t = t_2 - t_1$ S

$\Delta t_0 = t_2' - t_1'$ S'

$$t = \gamma(t' + \frac{vx'}{c^2})$$

$$t_2 - t_1 = \gamma(t_2' + \frac{vx_2'}{c^2}) - \gamma(t_1' + \frac{vx_1'}{c^2})$$

$$t_2 - t_1 = \gamma t_2' - \gamma t_1'$$

$$\boxed{\Delta t = \gamma \Delta t_0}$$