2. Lorentz Transformation

Assumption:

The transformation is linear and of the form:

① $x = \gamma(x' + Vt')$, $y = y'$, $z = z'$

$\boxed{\text{Assumption: } t \neq t'}$

From the principle of relativity,

$$x' = \gamma(x - Vt) \quad ②$$

Suppose a light pulse leaves the common origin of S and S', at $t = t' = 0$. Therefore, after a time t it will travel $x = ct$ or $x' = ct'$.

$$\Rightarrow \quad ct = \gamma(x' + Vt') = \gamma(ct' + Vt') = \gamma(c+V)t' \quad ③$$

$$ct' = \gamma(x - Vt) = \gamma(ct - Vt) = \gamma(c-V)t \quad ④$$

$$t' = \frac{\gamma(c-V)t}{c} \quad ⑥ \qquad \text{or} \quad t = \frac{\gamma(c+V)t'}{c} \quad ⑦$$

$\Rightarrow$ Put ⑥ into ③.

$$ct = \gamma(c+V) \cdot \frac{\gamma(c-V)t}{c}$$

$$\Rightarrow \boxed{\gamma = \frac{1}{\sqrt{1 - \frac{V^2}{c^2}}}} \quad ⑧$$

$\Rightarrow$ Put ⑧ into ①, ②, and ⑦

$$\boxed{x_1 = \gamma(x - Vt)}$$

$$\boxed{t = \gamma\left(t' + \frac{Vx'}{c^2}\right)} \qquad \boxed{x = \gamma(x' + Vt')}$$