The data collected in running the cost of which is Rs 60,000

Year	Resale Value	cost of Spare	cost of labour	operating cost	Cummulative	C-S	TC	A(n)
1	42,000	4000	14000	18,000	18000	18,000	36000	36,000
2	30,000	4270	16000	20,270	38270	30,000	68270	34135
3	20,400	4880	18000	22880	61150	39600	100750	33583
4	14400	5700	21000	26700	87850	45600	133450	33362.50
5	9650	6800	25,000	31800	119650	50350	170,000	34000

60,000

1803. *The data collected in running a machine, the cost of which is Rs. 60,000, are given below :*

Year		*1*	*2*	*3*	*4*	*5*
Resale value (Rs.)	:	42,000	30,000	20,400	14,400	9,650
Cost of spares (Rs.)	:	4,000	4,270	4,880	5,700	6,800
Cost of labour (Rs.)	:	14,000	16,000	18,000	21,000	25,000

Determine the optimum period for replacement of the machine.

Solution. The operating or maintenance cost of machine in successive years is as follows :

Year	:	*1*	2	*3*	*4*	5
Operating cost (Rs.)	:	18,000	20,270	22,880	26,700	31,800

(The cost of spares and labour together determine operating or running or maintenance cost.)

The average total annual cost is computed below :

Year of service n	*Operating cost (Rs.)* $f(n)$	*Cum. operating cost (Rs.)* $\Sigma f(n)$	*Resale value (Rs.)* S	*Depreciaction cost (Rs.)* $C-S$	*Total cost (Rs.)* TC	*Average cost (Rs.)* $A(n)$
1	18,000	18,000	42,000	18,000	36,000	36,000.00
2	20,270	38,270	30,000	30,000	68,270	34,135.00
3	22,880	61,150	20,400	39,600	1,00,750	33,583.30
4	26,700	87,850	14,400	45,600	1,33,450	**33,362.50**
5	31,800	1,19,650	9,650	50,350	1,70,000	34,000.00

The calculations in the above table show that the average cost is ***lowest*** during the ***fourth year***. Hence the machine should be replaced after every fourth year.

1812. A new tempo costs Rs. 80,000 and may be sold at the end of any year at the following prices :

Year (end)	:	*1*	2	*3*	*4*	*5*	*6*
Selling price (in Rs.) (at present value)	:	50,000	33,000	20,000	11,000	6,000	1,000

The corresponding annual operating costs are :

Year (end)	:	*1*	2	*3*	*4*	*5*	*6*
Cost/year (in Rs.) (at present value)	:	10,000	12,000	15,000	20,000	30,000	50,000

It is not only possible to sell the tempo after use but also to buy a second hand tempo. It may be cheaper to do so than to replace by a new tempo.

Age of tempo	:	*0*	*1*	2	*3*	*4*	*5*
Purchase price (in Rs.) (at present value)	:	80,000	58,000	40,000	26,000	16,000	10,000

What is the age to buy and to sell tempo so as to minimize average annual cost?

[Osmania M.B.A. 1990]

1813. (*a*) A transport manager finds from his past records that the costs per year of running a truck whose purchase price is Rs. 6,000 are as given below :

Year	:	*1*	2	*3*	*4*	*5*	*6*	*7*	*8*
Running cost (Rs.)	:	1,000	1,200	1,400	1,800	2,300	2,800	3,400	4,000
Resale value (Rs.)	:	3,000	1,500	750	375	200	200	200	200

Determine at what age is replacement due?

[IAS 1993; Kerala M.Com. 1991; Madras B.E. (Comp. Sc.) 1988; Jodhpur M.Sc. (Math.) 1992; Bangalore B.E. (Mech.) 1980]

(*b*) Let the owner of a fleet have three trucks, two of which are two years old and the third one year old. The cost price, running cost and resale vale of these trucks are same as given in (*a*). Now he is considering a new type of truck with 50% more capacity than one of the old ones at a unit price of Rs. 8,000. He estimates that the running costs and resale price for the truck will be as follows :

Year	:	1	2	3	4	5	6	7	8
Running costs (Rs.)	:	1,200	1,500	1,800	2,400	3,100	4,000	5,000	6,100
Resale price (Rs.)	:	4,000	2,000	1,000	500	300	300	300	300

Assuming that the loss of flexibility due to fewer trucks is of no importance, and that he will continue to have sufficient work for three of the old trucks, what should his policy be?

[Poona M.B.A. 1992]

1814. Machine *A* costs Rs. 3,600. Annual operating costs are Rs. 40 for the first year and then increase by Rs. 360 every year. Assuming that machine *A* has no resale value, determine the best replacement age.

Another machine *B*, which is similar to machine *A*, costs Rs. 4,000. Annual running costs are Rs. 200 for the first year and then increase by Rs. 200 every year. It has resale value of Rs. 1,500, Rs. 1,000 and Rs. 500 if replaced at the end of first, second and third years respectively. It has no resale value during fourth year and onwards.

Which machine would you prefer to purchase? Future costs are not to be discounted.

[Guwahati M.C.A. 1992]

1815. Machine *A* costs Rs. 45,000 and the operating costs are estimated at Rs. 1,000 for the first year, increasing by Rs. 10,000 per year in the second and subsequent years. Machine *B* costs Rs. 50,000 and operating costs are Rs. 2,000 for the first year, increasing by Rs. 4,000 in the second and subsequent years. If we now have a machine of type *A*, should we replace it with *B*? If so when? Assume that both machines have no resale value and future costs are not discounted.

[AIMA (Dip. in Mgtt), May 1990]

18 : 2.2 Replacement Policy when Value of Money changes with time

When the time value of money is taken into consideration, we shall assume that (*i*) the equipment in question has no salvage value, and (*ii*) the maintenance costs are incurred in the beginning of the different time periods.

Since it is assumed that the maintenance cost increases with time and each cost is to be paid just in the start of the period, let the money carry a rate of interest r per year. Thus a rupee invested now will be worth $(1+r)$ after a year, $(1+r)^2$ after two years, and so on. In this way a rupee invested today will be worth $(1+r)^n$, n years hence, or, in other words, if we have to make a payment of one rupee in n years time, it is equivalent to making a payment of $(1+r)^{-n}$ rupees today. The quantity $(1+r)^{-n}$ is called the *present worth* factor (*Pwf*) of one rupee spent in n years time from now onwards. The expression $(1+r)^n$ is known as the *payment compound amount factor (Caf)*. of one rupee spent in n years time.

Let the initial cost of the equipment be C and let R_n be the operating cost in year n. Let v be the rate of interest in such a way that $v=(1+r)^{-1}$ is the discount rate (present worth factor). Then the present value of all future discounted costs V_n associated with a policy of replacing the equipment at the end of each n years is given by

$$V_n = \{(C+R_0)+vR_1+v^2R_2+\ldots v^{n-1}R_{n-1}\}+\{(C+R_0)v_n+v^{n+1}R_1+v^{n+2}R_2+\ldots+v^{2n-1}R_{n-1}\}+\ldots$$

$$=\left[C+\sum_{k=0}^{n-1} v^k R_k\right]\times\sum_{k=0}^{\infty}(v^n)^k=\left[C+\sum_{k=0}^{n-1} v^k R_k\right]\Big/(1-v^n)^{-1}$$

Now, V_n will be a minimum for that value of n, for which

$$V_{n+1}-V_n > 0 \quad \text{and} \quad V_{n-1}-V_n > 0.$$

For this, we write

$$V_{n+1}-V_n=\left[C+\sum_{k=0}^{n} v^k R_k\right](1-v^{n+1})^{-1}-V_n$$

$$= v^n\,[R_n-(1-v)\,V_n]\,/\,(1-v^{n+1})$$

and similarly

$$V_n-V_{n-1} = V^{n-1}\,[R_{n-1}-(1-v)\,V_n]\,/\,(1-v^{m-1})$$

Since v is the depreciation value of money, it will always be less than 1 and therefore $1-v$ will always be positive. This implies that $v^n/(1-v^{n+1})$ will always be positive.

Hence, $V_{n+1} - V_n > 0 \Rightarrow R_n > (1-v) V_n$ and $V_n - V_{n-1} < 0 \Rightarrow R_{n-1} < (1-v) V_n$

Thus,
$$R_{n-1} < (1-v) V_n < R_n$$

or
$$R_{n-1} < \frac{C + R_0 + vR_1 + v^2R_2 + \ldots + v^{n-1}R_{n-1}}{1 + v + v^2 + \ldots + v^{n-1}} < R_n,$$

since $(1-v^n)(1-v)^{-1} = \sum_{n=0}^{n-1} v^k.$

The expression which lies between R_{n-1} and R_n is called the "*weighted average cost*" of all the previous n years with weights $1, v, v^2, \ldots, v^{n-1}$ respectively.

Hence, the optimal replacement policy of the equipment after n periods is :

(*a*) Do not replace the equipments if the next period's operating cost is less than the weighted average of previous costs.

(*b*) Replace the equipments if the next period's operating cost is greater than the weighted average of previous costs.

Remark. Procedure for determining the weighted average of costs (annualized cost) may be summarized in the following steps :

Step 1. Find the present value of the maintenance cost for each of the years, *i.e.*, $\Sigma R v^{n-1}$ $(n = 1, 2, \ldots)$; where $v = (1+r)^{-1}$

Step 2. Calculate cost plus the accumulated present values obtained in *step 1*, *i.e.*, $C + \Sigma R v^{n-1}$.

Step 3. Find the cumulative present value factor up to each of the years 1, 2, 3, ..., *i.e.*, Σv^{n-1}.

Step 4. Determine the annualized cost $W(n)$, by dividing the entries obtained in *step 2* by the corresponding entries obtained in *step 3*, *i.e.*, $[C + \Sigma v^{n-1}] / \Sigma v^{n-1}$.

Corollary. When the time value of money is not taken into consideration, the rate of interest becomes zero and hence v approaches unity. Therefore, as $v \to 1$, we get

$$R_{n-1} < \frac{C + R_0 + R_1 + \ldots + R_{n-1}}{1 + 1 + \ldots + n \text{ times}} < R_n$$

or
$$R_{n-1} < W(n) < R_n$$

Note. It may be noted that the above result is in complete agreement with the result that was obtained in 18 : 2.1.

Selection of the Best Equipment Amongst Two

Following is the procedure for determining a policy for the selection of an economically best item amongst the available equipments :

Step 1. Considering the case of two equipments, say A and B, we first find the best replacement age for both the equipments by making use of

$$R_{n-1} < (1-v) V_n < R_n.$$

Let the optimum replacement age for A and B comes out to be n_1 and n_2 respectively.

Step 2. Next, compute the fixed annual payment (or weighted average cost) for each equipment by using the formula

$$W(n) = \frac{C + R_0 + vR_1 + v^2R_2 + \ldots + v^{n-1}R_{n-1}}{1 + v + \ldots + v^{n-1}}$$

and substitute $n = n_1$ for equipment A and $n = n_2$ for equipment B in it.

Step 3. (*i*) If $W(n_1) < W(n_2)$, choose equipment A.

(*ii*) If $W(n_1) > W(n_2)$, choose equipment B.

(*iii*) If $W(n_1) = W(n_2)$, both equipments are equally good.