## Replacement Policy when Value of Money Changes with time

When value of money changes with time then we shall assume that

(1) The equipment in question has no salvage value (Resale value)

(2) The maintenance cost incurred in the beginning of the different time periods

Let the money carrying a rate of interest $r$ per year.

A rupee invested now will be worth $(1+r)$ after a year, $(1+r)^2$ after two years, and so on. In this way a rupee invested now will be worth $(1+r)^n$, $n$ years after.

If we have to make a payment of one rupee in $n$ years time, it is equivalent to making a payment of $(1+r)^{-n}$ rupees today.

The quantity $(1+r)^{-n}$ is called the present worth factor (Pwf) of one rupee spent in $n$ years time from now onwards.

The expression $(1+r)^n$ is known as the payment compound amount factor (Caf) of one rupee spent in $n$ years time.

Let $C$ be the initial cost of equipment & $R_n$ be the operating cost in year $n$.

$$\nu = (1+r)^{-1} = \frac{1}{1+r} = \text{Discount rate}$$

$r =$ rate of interest

Since $\nu$ is the depreciation value of money, it will always be less than 1

$$R_{n-1} < \frac{C + R_0 + \nu R_1 + \nu^2 R_2 + \cdots + \nu^{n-1} R_{n-1}}{1 + \nu + \nu^2 + \cdots + \nu^{n-1}} < R_n$$

Weighted average cost of all the previous $n$ years with weights $1, \nu, \nu^2, \ldots \nu^{n-1}$ respectively

Replacement Policy

(a) Do not replace the equipments if the next period's operating cost is less than the weighted average of the previous costs

(b) Replace the equipments if the next period's operating cost is greater than the weighted average of previous costs (*b*) Let the owner of a fleet have three trucks, two of which are two years old and the third one year old. The cost price, running cost and resale vale of these trucks are same as given in (*a*). Now he is considering a new type of truck with 50% more capacity than one of the old ones at a unit price of Rs. 8,000. He estimates that the running costs and resale price for the truck will be as follows :

| Year | : | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|---|
| Running costs (Rs.) | : | 1,200 | 1,500 | 1,800 | 2,400 | 3,100 | 4,000 | 5,000 | 6,100 |
| Resale price (Rs.) | : | 4,000 | 2,000 | 1,000 | 500 | 300 | 300 | 300 | 300 |

Assuming that the loss of flexibility due to fewer trucks is of no importance, and that he will continue to have sufficient work for three of the old trucks, what should his policy be?

*[Poona M.B.A. 1992]*

**1814.** Machine $A$ costs Rs. 3,600. Annual operating costs are Rs. 40 for the first year and then increase by Rs. 360 every year. Assuming that machine $A$ has no resale value, determine the best replacement age.

Another machine $B$, which is similar to machine $A$, costs Rs. 4,000. Annual running costs are Rs. 200 for the first year and then increase by Rs. 200 every year. It has resale value of Rs. 1,500, Rs. 1,000 and Rs. 500 if replaced at the end of first, second and third years respectively. It has no resale value during fourth year and onwards.

Which machine would you prefer to purchase? Future costs are not to be discounted.

*[Guwahati M.C.A. 1992]*

**1815.** Machine $A$ costs Rs. 45,000 and the operating costs are estimated at Rs. 1,000 for the first year, increasing by Rs. 10,000 per year in the second and subsequent years. Machine $B$ costs Rs. 50,000 and operating costs are Rs. 2,000 for the first year, increasing by Rs. 4,000 in the second and subsequent years. If we now have a machine of type A, should we replace it with $B$? If so when? Assume that both machines have no resale value and future costs are not discounted.

*[AIMA (Dip. in Mgtt), May 1990]*

### 18 : 2.2 Replacement Policy when Value of Money changes with time

When the time value of money is taken into consideration, we shall assume that (*i*) the equipment in question has no salvage value, and (*ii*) the maintenance costs are incurred in the beginning of the different time periods.

Since it is assumed that the maintenance cost increases with time and each cost is to be paid just in the start of the period, let the money carry a rate of interest $r$ per year. Thus a rupee invested now will be worth $(1+r)$ after a year, $(1+r)^2$ after two years, and so on. In this way a rupee invested today will be worth $(1+r)^n$, $n$ years hence, or, in other words, if we have to make a payment of one rupee in $n$ years time, it is equivalent to making a payment of $(1+r)^{-n}$ rupees today. The quantity $(1+r)^{-n}$ is called the *present worth factor (Pwf)* of one rupee spent in $n$ years time from now onwards. The expression $(1+r)^n$ is known as the *payment compound amount factor (Caf).* of one rupee spent in $n$ years time.

Let the initial cost of the equipment be $C$ and let $R_n$ be the operating cost in year $n$. Let $v$ be the rate of interest in such a way that $v=(1+r)^{-1}$ is the discount rate (present worth factor). Then the present value of all future discounted costs $V_n$ associated with a policy of replacing the equipment at the end of each $n$ years is given by

$$V_n = \{(C+R_0)+vR_1+v^2R_2+\dots v^{n-1}R_{n-1}\}+\{(C+R_0)v_n+v^{n+1}R_1+v^{n+2}R_2+\dots+v^{2n-1}R_{n-1}\}+\dots$$

$$= \left[C+\sum_{k=0}^{n-1} v^k R_k\right]\times\sum_{k=0}^{\infty}(v^n)^k = \left[C+\sum_{k=0}^{n-1} v^k R_k\right]\Big/(1-v^n)^{-1}$$

Now, $V_n$ will be a minimum for that value of $n$, for which

$$V_{n+1}-V_n > 0 \quad \text{and} \quad V_{n-1}-V_n > 0.$$

For this, we write

$$V_{n+1}-V_n = \left[C+\sum_{k=0}^{n} v^k R_k\right](1-v^{n+1})^{-1}-V_n$$

$$= v^n\,[R_n-(1-v)\,V_n]\,/\,(1-v^{n+1})$$

and similarly

$$V_n-V_{n-1} = V^{n-1}\,[R_{n-1}-(1-v)\,V_n]\,/\,(1-v^{n-1})$$

Since $v$ is the depreciation value of money, it will always be less than 1 and therefore $1 - v$ will always be positive. This implies that $v^n/(1 - v^{n+1})$ will always be positive.

Hence, $V_{n+1} - V_n > 0 \Rightarrow R_n > (1 - v) V_n$ and $V_n - V_{n-1} < 0 \Rightarrow R_{n-1} < (1 - v) V_n$

Thus,
$$R_{n-1} < (1 - v) V_n < R_n$$

or
$$R_{n-1} < \frac{C + R_0 + vR_1 + v^2R_2 + \ldots + v^{n-1}R_{n-1}}{1 + v + v^2 + \ldots + v^{n-1}} < R_n,$$

since $(1 - v^n)(1 - v)^{-1} = \sum_{n=0}^{n-1} v^k$.

The expression which lies between $R_{n-1}$ and $R_n$ is called the "*weighted average cost*" of all the previous $n$ years with weights $1, v, v^2, \ldots, v^{n-1}$ respectively.

Hence, the optimal replacement policy of the equipment after $n$ periods is :

(*a*) Do not replace the equipments if the next period's operating cost is less than the weighted average of previous costs.

(*b*) Replace the equipments if the next period's operating cost is greater than the weighted average of previous costs.

***Remark.*** Procedure for determining the weighted average of costs (annualized cost) may be summarized in the following steps :

*Step 1.* Find the present value of the maintenance cost for each of the years, *i.e.*, $\Sigma Rv^{n-1}$ ($n = 1, 2, \ldots$); where $v = (1 + r)^{-1}$

*Step 2.* Calculate cost plus the accumulated present values obtained in *step 1*, *i.e.*, $C + \Sigma Rv^{n-1}$.

*Step 3.* Find the cumulative present value factor up to each of the years 1, 2, 3, ..., *i.e.*, $\Sigma v^{n-1}$.

*Step 4.* Determine the annualized cost $W(n)$, by dividing the entries obtained in *step 2* by the corresponding entries obtained in *step 3*, *i.e.*, $[C + \Sigma v^{n-1}] / \Sigma v^{n-1}$.

***Corollary.*** When the time value of money is not taken into consideration, the rate of interest becomes zero and hence $v$ approaches unity. Therefore, as $v \to 1$, we get

$$R_{n-1} < \frac{C + R_0 + R_1 + \ldots + R_{n-1}}{1 + 1 + \ldots + n \text{ times}} < R_n$$

or
$$R_{n-1} < W(n) < R_n$$

***Note.*** It may be noted that the above result is in complete agreement with the result that was obtained in 18 : 2.1.

## Selection of the Best Equipment Amongst Two

Following is the procedure for determining a policy for the selection of an economically best item amongst the available equipments :

*Step 1.* Considering the case of two equipments, say $A$ and $B$, we first find the best replacement age for both the equipments by making use of

$$R_{n-1} < (1 - v) V_n < R_n.$$

Let the optimum replacement age for $A$ and $B$ comes out to be $n_1$ and $n_2$ respectively.

*Step 2.* Next, compute the fixed annual payment (or weighted average cost) for each equipment by using the formula

$$W(n) = \frac{C + R_0 + vR_1 + v^2R_2 + \ldots + v^{n-1}R_{n-1}}{1 + v + \ldots + v^{n-1}}$$

and substitute $n = n_1$ for equipment $A$ and $n = n_2$ for equipment $B$ in it.

*Step 3.* (*i*) If $W(n_1) < W(n_2)$, choose equipment $A$.

(*ii*) If $W(n_1) > W(n_2)$, choose equipment $B$.

(*iii*) If $W(n_1) = W(n_2)$, both equipments are equally good.

## SAMPLE PROBLEMS

**1816.** *Let the value of money be assumed to be 10% per year and suppose that machine A is replaced after every 3 years whereas machine B is replaced after every six years. The yearly costs of both the machines are given below :*

| *Year* | : | *1* | *2* | *3* | *4* | *5* | *6* |
|---|---|---|---|---|---|---|---|
| *Machine A* | : | 1,000 | 200 | 400 | 1,000 | 200 | 400 |
| *Machine B* | : | 1,700 | 100 | 200 | 300 | 400 | 500 |

*[Bharathidasan B.Com. 1999]*

*Determine which machine should be purchased.*

**Solution.** Since the money carries the rate of interest, the present worth of the money to be spent over in a period of one year is

$$v = \frac{100}{100+10} = \frac{10}{11} = 0.9091$$

$\therefore$ The total discounted cost (present worth) of $A$ for 3 years is

$$1000 + 200 \times (0.9091) + 400 \times (0.9091)^2 = \text{Rs. } 1512 \text{ approx.}$$

Again, the total discounted cost of $B$ for six years is

$$1{,}700 + 100 \times (0.9091) + 200 \times (0.9091)^2 + 300 \times (0.9091)^3 + 400 \times (0.9091)^4 + 500 \times (0.9091)^5 = \text{Rs. } 2{,}765.$$

Average yearly cost of machine $A$ = Rs. 1,512/3 = Rs. 504.

Average yearly cost of machine $B$ = Rs. 2,765/6 = Rs. 461.

This shows that the apparent advantage is with machine $B$. But, the comparison is unfair since the periods for which the costs are considered are different. So, if we consider 6 years period for machine $A$ also, then the total discounted cost of $A$ will be

$$1{,}000 + 200 \times (0.9091) + 400 \times (0.9091)^2 + 1{,}000 \times (0.9091)^3 + 200 \times (0.9091)^4 + 400 \times (0.9091)^5.$$

After simplification this comes out to be Rs. 2,647 which is Rs. 118 less costlier than machine $B$ over the same period.

Hence machine $A$ should be purchased.

**1817.** *A pipeline is due for repairs. It will cost Rs. 10,000 and last for 3 years. Alternatively, a new pipeline can he laid at a cost of Rs. 30,000 and lasts for 10 years. Assuming cost of capital to be 10% and ignoring salvage value, which alternative should be chosen?*

**Solution.** Consider the two types of pipeline for infinite replacement cycles of 10 years for the new pipeline, and 3 years for the existing pipeline.

Since, the discount rate of money per year is 10%, therefore the present worth of money to be spent over in a period of one year is

$$v = \frac{100}{100+10} = 0.9091$$

Let $k_n$ denote the discounted value of all future costs associated with a policy of replacing the equipment after $n$ years. Then, if we designate the initial outlay by $C$,

$$k_n = C + Cv^n + Cv^{2n} + \ldots + \infty = C(1 + v^n + v^{2n} + \ldots + \infty) = C/(1 - v^n)$$

Making use of values of $C$, $v$ and $n$ for two types of pipelines, the discounted value, therefore, yields

$$k_3 = \frac{10{,}000}{1 - (0.9091)^3} = \text{Rs. } 4{,}021$$

for the existing pipeline, and

$$k_{10} = \frac{30{,}000}{1 - (0.9091)^{10}} = \frac{30{,}000}{1 - 0.3855} = \text{Rs. } 48{,}820$$

for the new pipeline.

Since $k_3 < k_{10}$, the existing pipeline should be continued. Alternatively, the comparison may be made over $3 \times 10 = 30$ years.

**1818.** *The cost of a new machine is Rs. 5,000. The maintenance cost of nth year is given by $C_n = 500(n-1)$; $n = 1, 2, \ldots$ Suppose that the discount rate per year is 0.5. After how many years it will be economical to replace the machine by a new one?*

*[Madras M.B.A. 1996; Meerut M.Sc. (Math.) 1996]*

**Solution.** Since the discount rate of money per year is 0.05, the present worth of the money to be spent over a period of one year is

$$v = (1 + 0.05) = 0.9523.$$