Q Let the ~~Money~~ Value of money be assumed to be 10% per year and suppose that machine A is replaced after ~~year~~ every 3 years whereas machine B is replaced after every six years. The yearly costs of both Machine are given below

year	1	2	3	4	5	6
Machine A	1000	200	400	1000	200	400
Machine B	1700	100	200	300	400	500

Soln

Present worth of money to be spent over in a period of one years

$$= \frac{1}{1+r} = \frac{100}{100+10} = \frac{10}{11} = 0.9091$$

Total discounted cost (Present worth) of A for 3 years

$$= 1000 + 200(0.9091) + 400(0.9091)^2$$

$$= 1512 ₹$$

Total discounted cost (Present worth) of B for 6 years

$$= 1700 + 100(0.9091) + 200(0.9091)^2 + \cdots + 500(0.909$$

$$= 2765$$

Average yearly cost of A = $\frac{1512}{3}$ = 504 ₹

Average yearly cost of B = $\frac{2765}{6}$ = 461 ₹

This show that B is advantagious

But we have take time period of A 3 years & B 6 years. If we consider time period 6 years for A Also then total discounted cost of A is

$= 1000 + 200(0.901) + \cdots + 400(0.9091)^5$

= 2647 ₹

Now average yearly cost for machine A is = 718 ₹ 441

hence A machine is advantagious.

100 —10%→ 110

110 —— 100 (10%)

1 —— $\frac{100}{110} = \frac{10}{11} = 0.9091$

200 —— 200 × 0.9091

SAMPLE PROBLEMS

1816. *Let the value of money be assumed to be 10% per year and suppose that machine A is replaced after every 3 years whereas machine B is replaced after every six years. The yearly costs of both the machines are given below :*

Year	:	*1*	*2*	*3*	*4*	*5*	*6*
Machine A	:	1,000	200	400	1,000	200	400
Machine B	:	1,700	100	200	300	400	500

Determine which machine should be purchased. ***[Bharathidasan B.Com. 1999]***

Solution. Since the money carries the rate of interest, the present worth of the money to be spent over in a period of one year is

$$v = \frac{100}{100+10} = \frac{10}{11} = 0.9091$$

$\therefore$ The total discounted cost (present worth) of A for 3 years is

$$1000 + 200 \times (0.9091) + 400 \times (0.9091)^2 = \text{Rs. } 1512 \text{ approx.}$$

Again, the total discounted cost of B for six years is

$$1{,}700 + 100 \times (0.9091) + 200 \times (0.9091)^2 + 300 \times (0.9091)^3 + 400 \times (0.9091)^4 + 500 \times (0.9091)^5 = \text{Rs. } 2{,}765.$$

Average yearly cost of machine A = Rs. 1,512/3 = Rs. 504.

Average yearly cost of machine B = Rs. 2,765/6 = Rs. 461.

This shows that the apparent advantage is with machine B. But, the comparison is unfair since the periods for which the costs are considered are different. So, if we consider 6 years period for machine A also, then the total discounted cost of A will be

$$1{,}000 + 200 \times (0.9091) + 400 \times (0.9091)^2 + 1{,}000 \times (0.9091)^3 + 200 \times (0.9091)^4 + 400 \times (0.9091)^5.$$

After simplification this comes out to be Rs. 2,647 which is Rs. 118 less costlier than machine B over the same period.

Hence machine A should be purchased.

Now since the running cost of 9th year is Rs. 1,600 and that of 10th year is Rs. 1,800 and since 1,800 > 1,751.72, it is better to replace the machine A after 9th year.

Similarly, for machine B since 1,800 > 1,680.23, it is better to replace the machine B after 8th year.

Further since the weighted average cost in 9 years of machine A is Rs. 1751.72 and the weighted average cost in 8 years of machine B is Rs. 1,680.23, it is advisable to purchase machine B.

PROBLEMS

1820. Let $v = 0.9$ and initial price is Rs. 5,000. Running cost varies as follows :

Year	:	*1*	*2*	*3*	*4*	*5*	*6*	*7*
Running cost (in Rs.)	:	400	500	700	1,000	1,300	1,700	2,100

What would be the optimum replacement interval?

1821. The initial cost of an item is Rs. 15,000 and maintenance or running costs for different years are given below :

Year	:	*1*	*2*	*3*	*4*	*5*	*6*	*7*
Running cost (in Rs.)	:	2,500	3,000	4,000	5,000	6,500	8,000	10,000

What is the replacement policy to be adopted if the capital is worth 10% and there is no salvage value?

1822. The yearly cost of 2 machines A and B when the money value is neglected is as follows :

Year	:	*1*	*2*	*3*	*4*	*5*
Machine *A*	:	1,800	1,200	1,400	1,600	1,000
Machine *B*	:	2,800	200	1,400	1,100	600

Find their cost patterns if money value is 10% per year and hence find which machine is most economical. ***[Madras B.E. (Mech.) 1999]***

1823. A manual stamper currently valued at Rs. 1,000 is expected to last 2 years and costs Rs. 4,000 per year to operate. An automatic stamper which can be purchased for Rs. 3,000 will last 4 years and can be operated at an annual cost of Rs. 3,000. If money carries the rate of interest 10% per annum, determine which stamper should be purchased.

1824. A manufacturer is offered two machines A and B. A is priced at Rs. 5,000 and running costs are estimated at Rs. 800 for each of the first five years, increasing by Rs. 200 per year in the sixth and subsequent years. Machine B, which has the same capacity as A, costs Rs. 2,500 but will have running costs of Rs. 1,200 per year for six years, increasing by Rs. 200 per year thereafter.

If money is worth 10% per year, which machine should be purchased? (Assume that the machines will eventually be sold for scrap at a negligible price.) ***[Madras B.E. 1999]***

1804. End of 6^{th} year.

1805. End of 7^{th} year.

1806. End of 5^{th} year.

1807. End of 4^{th} year.

1808. End of 5^{th} year.

1809. After 6 years.

1810. End of 3^{rd} year,

1811. (*i*) End of 5th year, (*ii*) After 2 years.

1812. Four years, net saving will be Rs. 1,250.

1813. (*a*) End of 5^{th} year. (*b*) All the three old trucks should be replaced after two years.

1814. Replace *A* at the end of 5^{th} year, and *B* at the end of 6^{th} year; machine *B* should be purchased.

1815. Machine *A* should be replaced by machine *B* after two years.

1820. Six years.

1821. End of 5^{th} year.

1822. Machine *B* must be purchased.

1823. Automatic stamper should be purchased.

1824. Machine *A* is to be replaced after 9 years and machine *B* after 8 years.
Machine *B* should be purchased.

1825. Equipment *B* should be purchased after 6 years.

1826. End of first year.

1829. Group replacement after a week.

1830. Group replacement is better.

1831. Group replacement after second week.

1832. Individual replacement policy is better.

1833. (*a*) Group replacement after 3 months. Group replacement is superior.
(*b*) Group replacement is still superior, but optimum replacement interval is six months.

1834. Preventive group replacement by the end of 4^{th} year.

1835. Group replacement after 3 months.

1836. Group replacement after 6^{th} week.

1837. Group replacement after 4 months.

1839. $p_1 = 0.6, p_2 = 0.12, p_3 = 0.24, p_4 = 0.18, p_5 = 0.12, p_6 = 0.09, p_7 = 0.06, p_8 = 0.06,$
$p_9 = 0.04, p_{10} = 0.03.$

1840. 12 chemists every year; promotion of new entrants will start by the end of 5th year.

1841. (*a*) 96 employees. (*b*) At age 31, clerks are to be promoted to manager at the age of 52 years.

1845. Replacement is due when $Z > X$, where z is the running cost with $0 \leq x \leq X$ and X is the maximum value of x.

1847. Increase in reliability = .03407.

1848. MTBF = 208933 hours.