Verification of Doppler Effect

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1 Theory and Physical Setup

To obtain consistent, repeatable results, the speeds and distances need to remain constant. As such, the source/receiver, depending on the experiment, was mounted on a spinning ceiling fan and the receiver/source mounted at a fixed position in accordance with the figure shown below

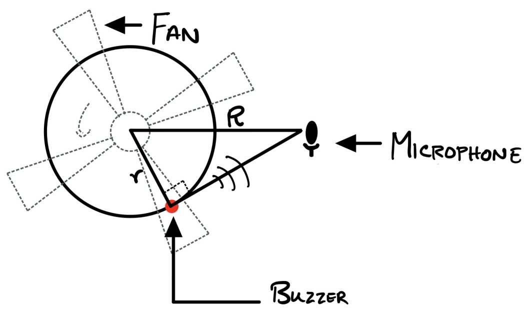

Figure 1: Bottom up view of the setup

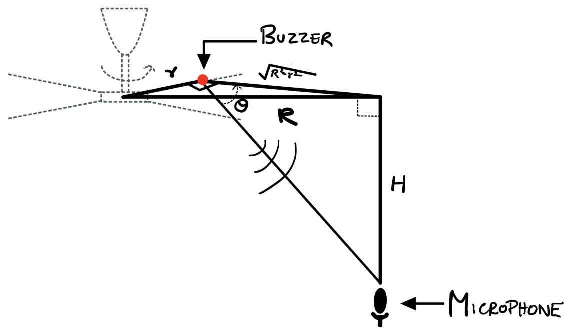

Figure 2: Side view of the setup

where

r is the radius of circle of the rotating device
R is the horizontal component of the distance between the center of rotation and the second device
H is the vertical component of the distance between the center of rotation and the second device

As such, the distance between the source and the observer is

$$d(\phi) = \sqrt{H^2 + (R + r\cos\phi)^2 + R^2 \sin^2 \phi}$$

where ϕ is an angle that describes the angular position of the fan mounted object.

As the fan rotates, the relative velocity between the two devices has a maxima and a minima when the device mounted on the fan has velocity tangential to the stationary device. This maximum velocity is given by

$$|v_{\text{max}}| = |v_{\text{min}}| = \omega r \cos\left(\arctan\left(\frac{H}{\sqrt{R^2 - r^2}}\right)\right)$$

This relative velocity causes a shift in the frequency of the sound due to the Doppler effect. This can be measured by calculating the instantaneous frequency of the recorded signal. However, as the distance between the two devices varies appreciably since the stationary device can only be kept so far away from the rotating one, Amplitude Modulation will be evident in our recorded signal since at closer points, the signal received will be stronger. In order to reduce the AM effect, the ratio $\frac{R}{r}$ should be made as large as possible. However, the amplitude of the recorded signal reduces and is thus more susceptible to noise. Another issue that is a result of a small ratio is that the wave will start becoming a skewed sine with a small fall time and a large rise time. This is because the tangents will start converging and there is a different angular distance that needs to be traversed between rising and lowering, and will be shown later qualitatively.

The AM effect however, does not affect the analysis much since it only causes intensity changes and doesn't affect the frequency attained. Also, the skewed sine waves also do not play a role since the maximum and minimum frequencies do not get impacted.

2 Calculation and Error Analysis

2.1 Code Explanation

The data parameters such as timestamps to select, bandwidth, central frequency are all enclosed in a JSON file. The code takes in the name of the corresponding JSON file (without the extension).

For the functioning of the code, the intensity of the data is assumed to not vary over the time sweep selected.

From the spectrum, the frequencies that have the highest deviation across the measurement time correspond to the maximally shifted frequencies. The period of the intensity of a given frequency can be calculated by finding the first peak in the Fourier transform of the intensity vs time data. The time period of the rotation of the fan is then used to calculate the velocity of the object, and thusly the velocity along the tangents.

The code outputs the calculated minimum and maximum observed frequencies as well as the rotation frequency of the fan. Two flags are defined in the code body which can be used: the `DEBUG` flag controls whether the intermediate plots are shown and the `SAVE` flag controls whether the intermediate plots are saved.

The maximum velocities and the corresponding frequency shift are then fitted to the equation of Doppler Effect using a separate code.

2.2 Source in Motion - r variation

2.2.1 Set 1

For this dataset, R and H were kept constant, while r was varied in increments of roughly 15 cm. For $r = 60$ cm, readings were taken at two different speeds of the fan to check for consistency between fan speeds.
Also, to check the consistency of the code, this data set was analysed manually. The setup had $R \approx 113$ cm and $H \approx 40$ cm.

For this dataset, the manual analysis of the dataset was done using Audacity. However, since the higher frequencies are not easily resolvable, the fundamental frequency was used for the analysis. The resulting data is as.

Filename	r (in cm)	Regulator Speed	Analysis Times		ω of fan (in rad/s)	Instantaneous Frequency	
			Start	Stop		Min	Max
60cm	60	3	27.523	27.908	16.320	2696	2853
60cm	60	5	2.44	2.77	19.04	2681	2878
45cm	45	3	40.982	41.366	16.362	2725	2838
30cm	30	3	0.036	0.421	16.320	2745	2827
15cm	15	3	34.624	0.381	16.491	2773	2821

Calculating the min and max relative velocity using the above data, and fitting this to the equation of Doppler effect, the results are

$$f_0 = (2782 \pm 4)\text{Hz}$$
$$c = (312 \pm 16)\text{m s}^{-1}$$

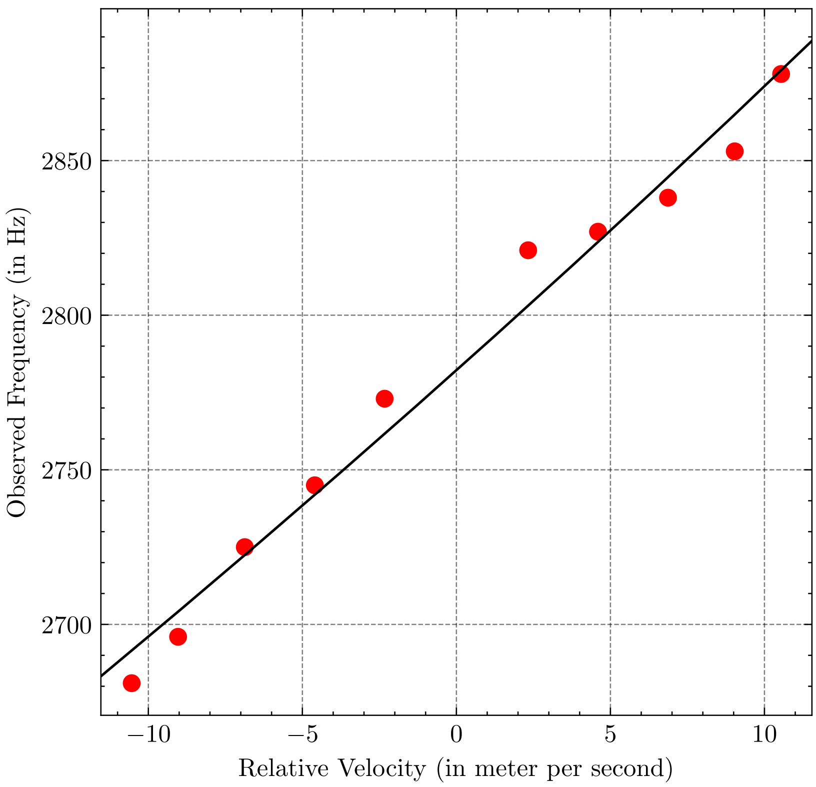

Figure 3: Observed frequency against relative velocity for varying r, set 1

A video was also taken to verify the ω, and the video results in a value of ω of 2.6 rotations per second, which is approximately 16.3 rad/s, which agrees well with the angular frequency returned from audacity.

2.2.2 Set 2

For this dataset, R and H were kept constant, while r was varied in increments of roughly 10 cm. Additionally, each set of distances was measured with two different regulator settings.
The fundamental frequency of the buzzer mounted on the fan was approximately 3516 Hz, and the 4^{th} harmonic corresponding to 12 600 Hz was used. The setup had $R \approx 225$ cm and $H \approx 100$ cm.

Filename	r (in cm)	Regulator Speed	ω of fan (in rad/s)	Instantaneous Frequency Min	Max
pos_1 speed_2	20	2	14.481	12480.46875	12667.96875
pos_1 speed_4	20	4	21.722	12398.4375	12679.6875
pos_2 speed_2	30	2	14.907	12480.46875	12726.5625
pos_2 speed_4	30	4	21.296	12375.0	12785.15625
pos_3 speed_2	40	2	14.481	12398.4375	12761.71875
pos_3 speed_4	40	4	21.296	12292.96875	12890.625

Calculating the min and max relative velocity using the above data, and fitting it to the equation of Doppler effect, the result is

$$f_0 = (12575 \pm 7)\text{Hz}$$
$$c = (351 \pm 14)\text{m}\,\text{s}^{-1}$$

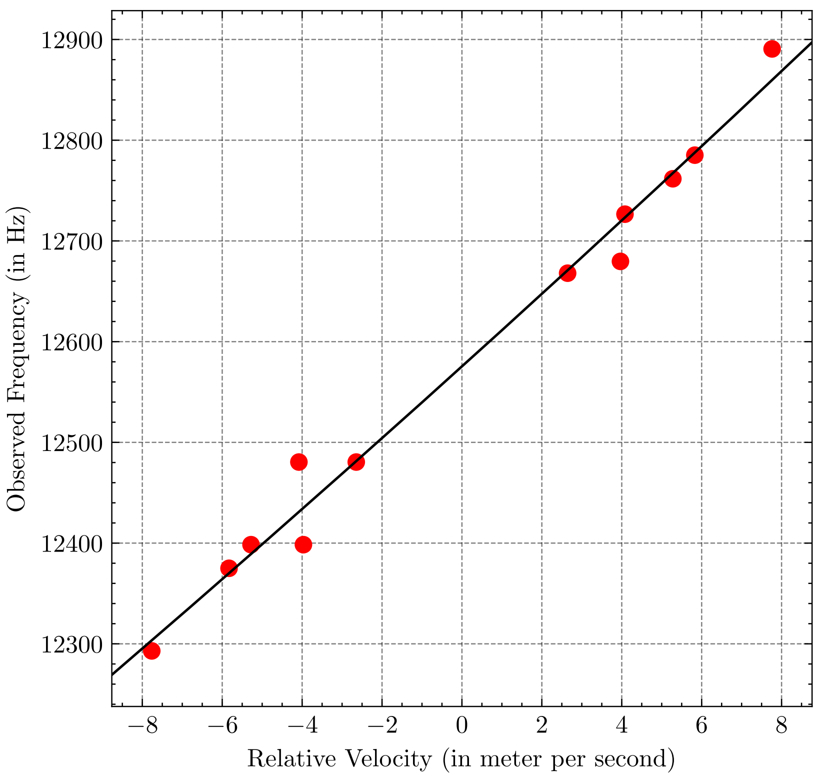

Figure 4: Observed frequency against relative velocity for varying r, set 2

2.3 Source in Motion - H variation

2.3.1 Set 1

For this dataset, r and R were kept constant, while H was varied in increments of roughly 30 cm. While the regulator could have been used the to vary the ω of the fan, only speed 1 on the regulator could be analyzed for this setup since anything beyond speed 1 was too fast and could not be resolved temporally.
The fundamental frequency of the buzzer mounted on the fan was approximately 3516 Hz, and the 4^{th} harmonic corresponding to 14 064 Hz was used. The setup had $r \approx 55$ cm and $R \approx 320$ cm.
For analysis, the size of FFT for spectrogram was set to $N = 4096$ and the default window from `scipy` was used. The obtained results are,

Filename	H	Analysis Times		ω of fan	Instantaneous Frequency	
	(in cm)	Start	Stop	(in rad/s)	Min	Max
H208	5	80	90	18.407	13734.375	14460.9375
H178	35	59	69	16.107	13792.96875	14425.78125
H148	65	75	85	16.107	13757.8125	14402.34375
H38	175	85	95	16.107	13816.40625	14440

The two highlight values have been corrected. The values generated from the code are $\omega = 24.160$ and $F_{\max} = 14261.71875$. These have been corrected manually. The corrected ω is accurate,but the correct $F_{\max}$ is just an eyeball guess.

Calculating the min and max relative velocity using the above data, and fitting it to the equation of Doppler effect, the result is

$$f_0 = (14096 \pm 9.57)\text{Hz}$$
$$c = (380 \pm 11)\text{m}\,\text{s}^{-1}$$

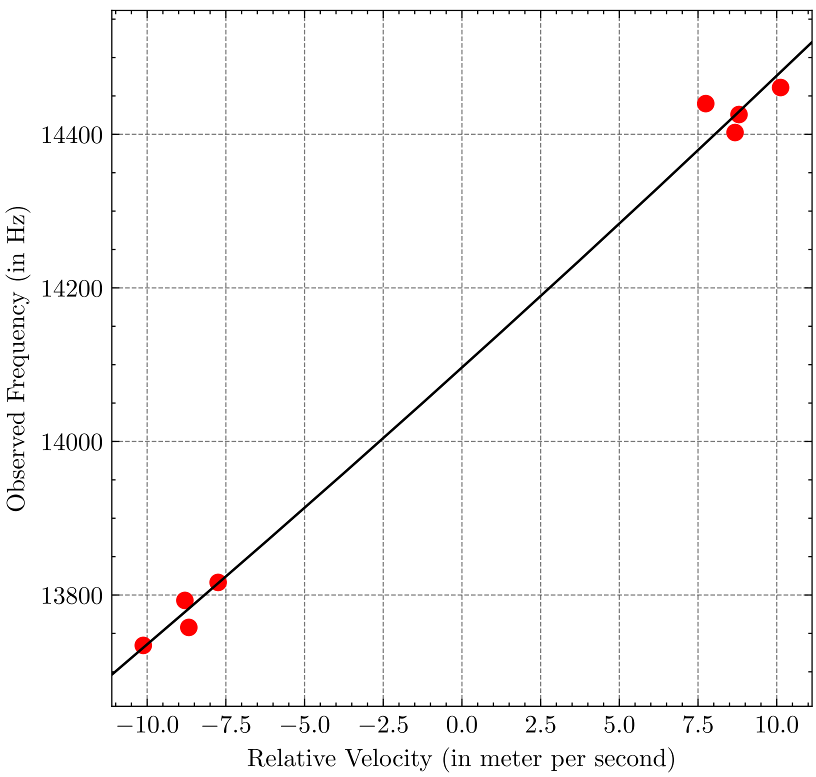

Figure 5: Observed frequency against relative velocity for varying H, set 1

2.3.2 Set 2

For this dataset, r and R were kept constant, while H was varied in increments of roughly 30 cm.
The fundamental frequency of the buzzer mounted on the fan was approximately 2916 Hz, and the 3^{rd} harmonic corresponding to 8750 Hz was used. The setup had $r \approx 46$ cm and $R \approx 164$ cm.
For analysis, the size of FFT for spectrogram was set to $N = 4096$ and the default window from `scipy` was used. All files were analyzes from $t = 40$ s to $t = 50$ s
The obtained results are,

Filename	H (in cm)	ω of fan (in rad/s)	Instantaneous Frequency Min	Max
0cm	0	20.708	8484.375	8906.25
30cm	30	20.708	8554.6875	8871.09375
60cm	60	21.475	8496.09375	8835.9375
90cm	90	20.708	8507.8125	8882.8125
120cm	120	20.708	8496.09375	8824.21875

Calculating the min and max relative velocity using the above data, and fitting it to the equation of Doppler effect, the result is

$$f_0 = (8682 \pm 10)\text{Hz}$$
$$c = (430 \pm 24)\text{m s}^{-1}$$

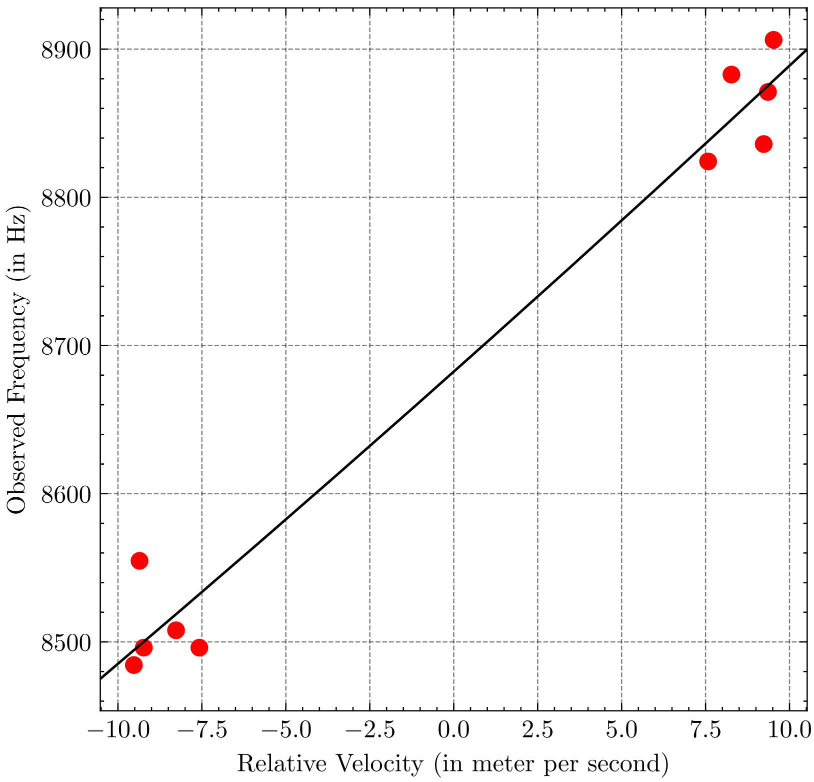

Figure 6: Observed frequency against relative velocity for varying H, set 2

2.4 Observer in Motion - r variation

The values of R and H were kept constant at approximately 2 m and 10 cm respectively and the receiver was mounted on the fan. Note that the maximum value of r is 55 cm, so the cosine factor in equation 1 differs from unity by 0.2%. Hence, value of velocity can be safely taken to be ωr. However, the fit is done with the mentioned parameters instead of any approximations.

In the analysis code, the size of the FFT for spectrogram was taken to be 2^{13} and the 'nuttall' window was used. The output generated by the code is summarised in the table below.

Filename	r (in cm)	ω of fan (in rad/s)	Instantaneous Frequency Min	Max
RJ_55	55	10.861	11056.640625	11414.0625
RJ_45	45	11.200	11074.21875	11390.625
RJ_35	35	12.139	11091.796875	11343.75
RJ_25	25	12.139	11121.09375	11296.875

Calculating the min and max relative velocity using the above data, and fitting it to the equation of Doppler effect, the result is

$$f_0 = (11223 \pm 5)\text{Hz}$$
$$c = (374 \pm 12)\text{m s}^{-1}$$

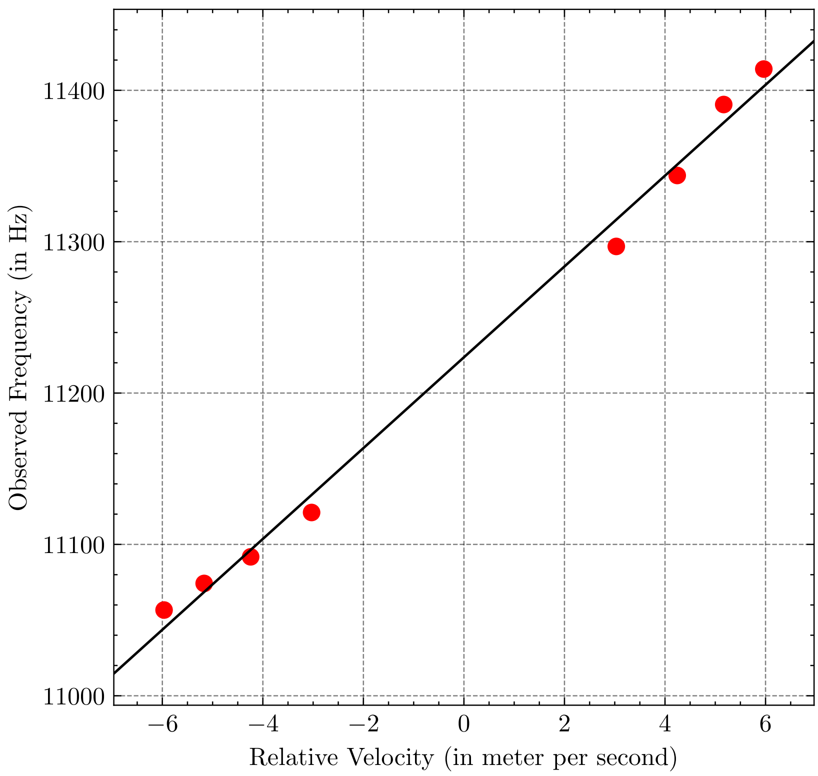

Figure 7: Observed frequency against relative velocity for varying r, with moving observer

2.5 Discussion

As noted before, the minimum and maximum frequencies occur when the velocity vector of the rotating body is along the line joining the source and the receiver; ergo, it is at the tangential points. Now the maximum relative velocity occurs when the angle between $\vec{R}$ and $\vec{r}$ is given by $\arctan\left(\frac{\sqrt{R^2-r^2}}{r}\right)$, which is $\pi/2$ only in the limit of $R \to \infty$.

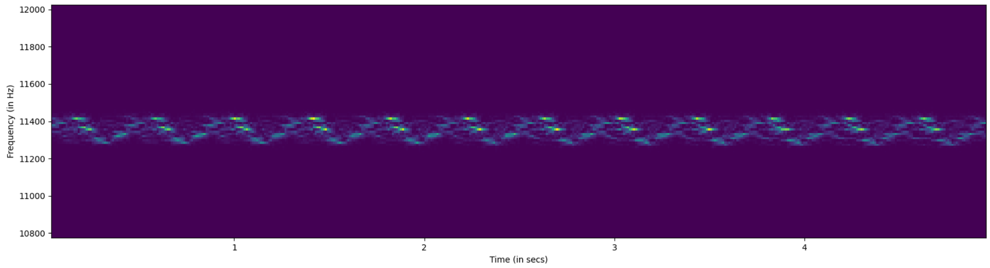

Figure 8: Spectrogram for $r = 15\,\mathrm{cm}$, from set 1

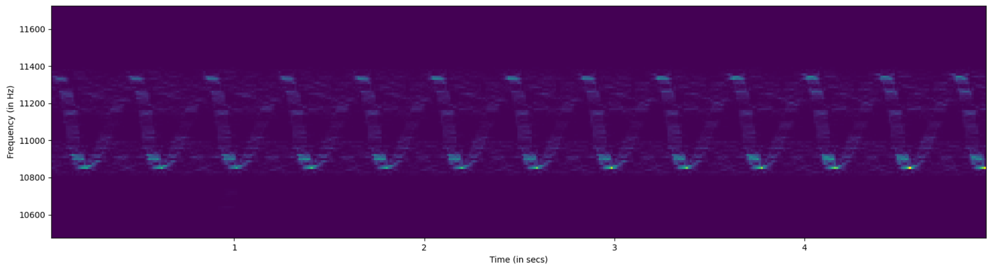

Figure 9: Spectrogram for $r = 60\,\mathrm{cm}$, from set 1

As expected, the separation between two consecutive maxima (or minima) is not twice the separation between a maximum and a minimum, so the plot looks like a skewed sine wave. This skewness increases (as in figure 9) as r becomes comparable to R and the skewness is smaller when r is smaller (as in figure 8).

3 Appendix

The drive link containing all the data, codes as well as the fit reports can be accessed here.