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Background of Markovitz mean-var optimization model:

Markovitz mean-var optimization model is a mathematical framework to find an optimal portfolio of securities or collection of assets, which try to balance the expected returns against risk associated with the assets.

The model is defined as follows:

$$\text{minimize } \boldsymbol{x}^{\mathrm{T}}\boldsymbol{C}\boldsymbol{x}$$
$$\text{subject to } \boldsymbol{\mu}^{\mathrm{T}}\boldsymbol{x} = r$$
$$\boldsymbol{e}^{\mathrm{T}}\boldsymbol{x} = 1, \text{ where } e = (1, \ldots, 1)^{\mathrm{T}}$$
$$\boldsymbol{x} \geq \boldsymbol{0}.$$

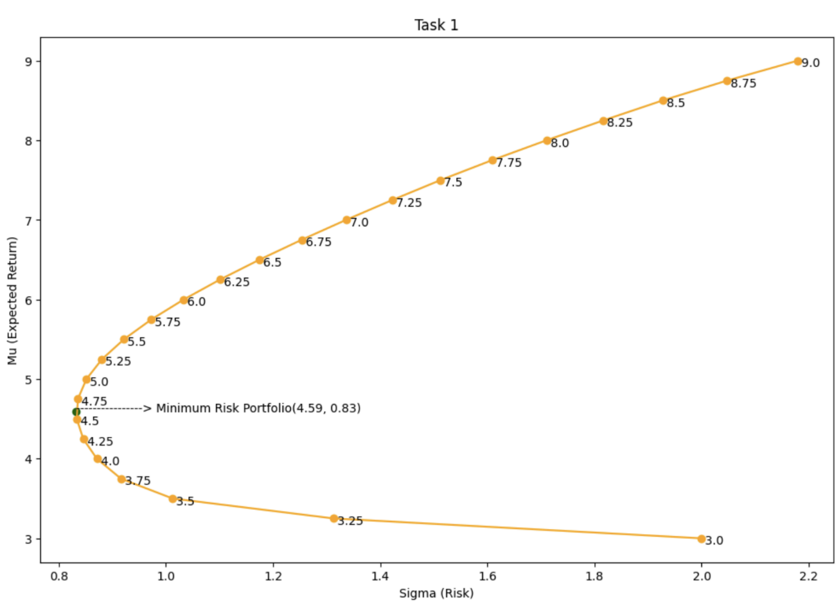
Where C is the Covariance matrix of assets which capture the risk associated, while $\boldsymbol{\mu}$ is the expected returns.

Aim: To find optimal set of $\boldsymbol{x}$ which contain the allocation of funds to 10 different assets.
Objective function: Min $\boldsymbol{x}^{\mathrm{T}}\boldsymbol{C}\boldsymbol{x}$ i.e. we try to reduce the risk, subject to certain constraints.
Constraints: $\boldsymbol{\mu}^{\mathrm{T}}\boldsymbol{x} = r$: This states that we need our return from portfolio to be r,
$\boldsymbol{e}^{\mathrm{T}}\boldsymbol{x} = 1$: This states all the funds should be invested,
$\boldsymbol{x} \geq \boldsymbol{0}$. : we will only hold long positions, i.e. short selling is not allowed.

In task 1 we are supposed to just run the Markovitz model for a range of return values. We get the following graph at right. The return at minimum risk of 0.8 is about 4.9. For returns from 2-2.75 the model was not able to find the solution. For returns above 5, it clearly shows the fundamental trade-off i.e High returns will have high risk.

There are some low return portfolio which also show high risk, this may be due to reasons such as:

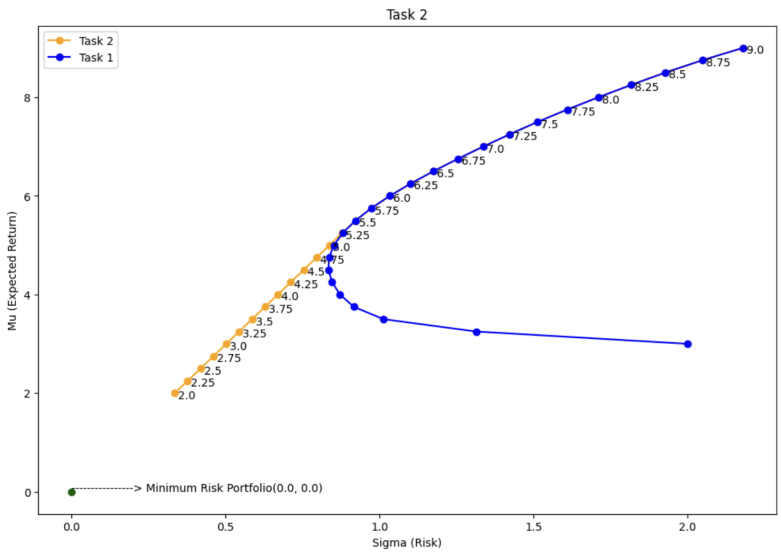
Inflexible constraints: such as fixed return r, no short selling, due to this model might find an optimal solution but it might allocate funds to high risk stocks.



In Task 2, we change the constraints and say not all capital needs to be invested and any capital which is not invested can be saved with no return and no risk. Due to this we get the minimum risk portfolio at zero, i.e. no capital is invested, so there is no risk and no return.

We see a relationship between return and risk to be kind of close to linear, this is due to relaxed constraint($\boldsymbol{e}^{\top}\boldsymbol{x} \leq 1$), and the solver is able to find better portfolios when compared to task 1.

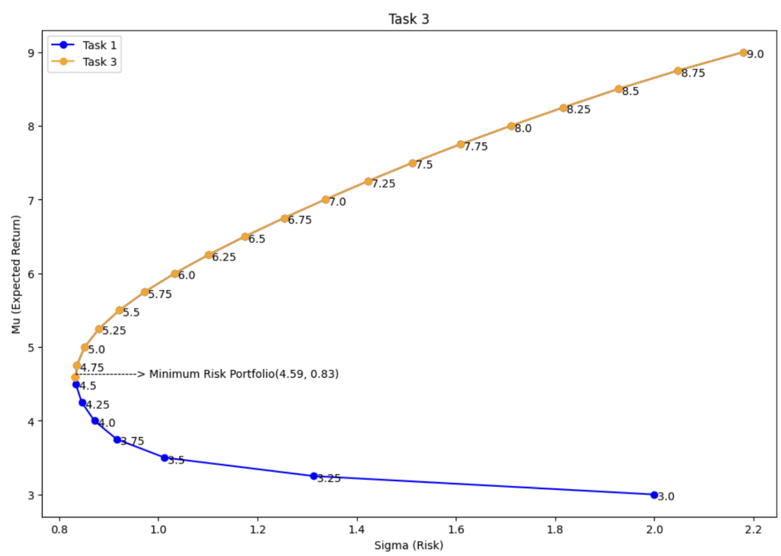
Given the flexibility where not all capital needs to be invested, it becomes possible to avoid high-risk stocks while achieving comparable returns as in task 1 but with lower overall risk. However, achieving high returns still typically requires accepting higher levels of risk. This model may help investors with low risk appetite.



In Task 3, we say $\boldsymbol{\mu}^{\top}\boldsymbol{x} \geq r$, Which means the portfolio should have some minimum return r, but can have more than return r as well. (Keeping $\boldsymbol{e}^{\top}\boldsymbol{x} = 1$ & $\boldsymbol{x} \geq \boldsymbol{0}$)

We have got minimum risk portfolio return at 4.8 with lowest risk of 0.8. So, for any portfolio with return values less than 5, the model assigns the same portfolio of minimum risk portfolio and is the reason why plot starts from 4.8 return.

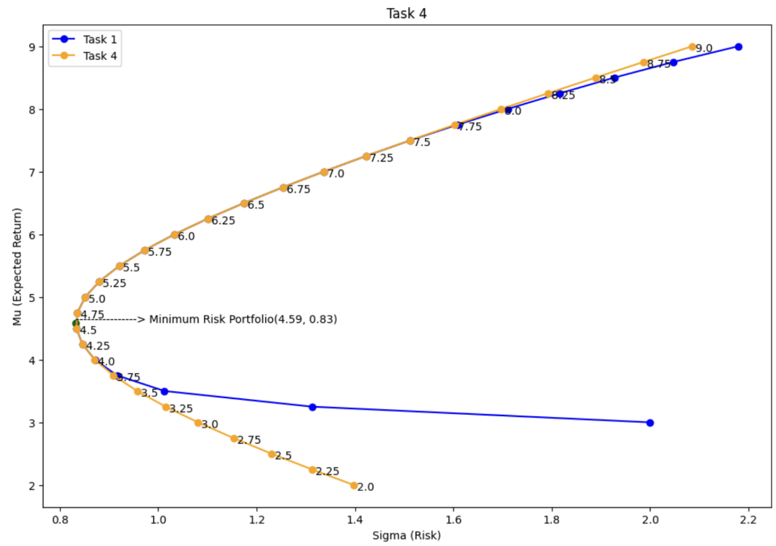
Same as task 1 for higher returns, the risk increases further.



In Task 4, we say short selling is possible($-\infty<x<\infty$, keeping $\boldsymbol{e}^{T}\boldsymbol{x} = 1$ & $\boldsymbol{\mu}^{T}\boldsymbol{x} = r$).
What this does is it gives more opportunities to allocate funds and is more flexible.

For instance, if a portfolio needs to achieve a return of 3.0, it might have to invest(long position) in a high-risk stock in task 1. In this case(Task 4), it could reduce the overall risk by shorting that high-risk stock.

This can be a possible reason why the plot, although similar to task 1, starts from relatively low risk.
The minimum risk portfolio stays the same as task 1. This model seems to be an improved version of task 1.



Conclusion:

We created a Markovitz mean-var optimization model and analyzed its returns vs risk in task 1. Further this model can be modified with change in constraints based on various situations and requirements of investors, such as not investing all the funds, shorting, flexibility to get higher returns, etc(Task 2-4). Changing the constraints will help in improving the model and analyzing various situations. Task 2 gave us the same returns at lower risk initially, this model can be used by risk averse investors. Task 3 introduced flexibility to get higher returns at lower risk for lower targeted returns. While task 4 introduced the concept of shorting the assets which gave for opportunities and which resulted in reducing the risk due to more opportunities for investing. Based on the rules, conditions and risk appetite of investors these models can be used to better assess the portfolio and balance the returns vs risk trade-off.

References:

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