MATH 391 **Worksheet 01** **F2019**

Polynomial Interpolation

01. Find the polynomial that interpolates the data points $\Big\{ (-1,-1),\ (0,3),\ (2,11),\ (3,27) \Big\}$

02. Show that the polynomial that interpolates the data

x	-2	-1	0	1	2	3
$f(x)$	1	4	11	16	13	-4

has degree 3.

Recall that if $P_n(x)$ is the degree n polynomial that interpolates $f(x)$ at the set of points $\Big\{x_i\Big\}_{0\le i\le n}$, and if $f(x)$ has $(n+1)$ continuous derivatives in the interval $\Big[\min\limits_{0\le i\le n} x_i\ ,\ \max\limits_{0\le i\le n} x_i\Big]$, then

$$f(x) - P_n(x) = \frac{f^{(n+1)}(\theta)}{(n+1)!}\,(x-x_0)\,(x-x_1)\,\cdots\,(x-x_n)$$

03. Let $f(x) = \cos(x) + \sin(x)$ and let $P_2(x)$ be the polynomial that interpolates $f(x)$ at the points $x_0 = 0,\ x_1 = 1/4,\ x_2 = 1/2$ and $x = 1$.

a. Find a bound for $\max\limits_{0\le x\le 1} |f(x) - P_2(x)|$

b. Use MATLAB to solve the system satisfied by the coefficients of $P_2(x)$ and compare the value of $|f(\pi/6) - P_2(\pi/6)|$ to the bound found in the previous question.

04. let $P_n(x)$ be the degree n polynomial that interpolates a function $f(x)$ at the points $\Big\{x_i\Big\}_{0\le i\le n}$. Suppose $x_i = x_0 + i\,h,\quad h > 0$, and that $f(x)$ has $(n+1)$ continuous derivatives in the interval $\big[x_0\ ,\ x_n\big]$.

a. Show that $\max\limits_{x_0\le x\le x_n} |(x-x_0)(x-x_1)\cdots(x-x_n)| = h^{n+1} \max\limits_{0\le t\le 1} |t(t-1)\cdots(t-n)|$

b. Compute $\max\limits_{0\le t\le 1} |t\,(t-1)\,\cdots\,(t-n)|,\quad$ for $n = 1,\,2,\,3$