MATH 391	Worksheet 02	F2019

Polynomial Interpolation - Lagrange and Newton Forms

01. The degree n Lagrange polynomials associated with the points $x_0 < x_1 < \cdots < x_n$, are defined by $$L_{n,j}(x) = \prod_{\substack{i=0 \\ i \neq j}}^{n} \frac{x - x_i}{x_j - x_i}, \quad j = 0, 1, \cdots, n.$$
Assuming the points to be equidistant, with $x_0 = 0$ and $x_n = 1$, use MATLAB to

a. sketch in the same plot the graph of $L_{1,0}(x)$ and $L_{1,1}(x)$

b. sketch in the same plot the graph of $L_{2,0}(x)$, $L_{2,1}(x)$ and $L_{2,2}(x)$

c. sketch in the same plot the graph of $L_{3,0}(x)$, $L_{3,1}(x)$, $L_{3,2}(x)$, and $L_{3,3}(x)$

02. Write Lagrange form of the polynomial that interpolates the given data points

a.

x	2	0	3
$f(x)$	11	7	28

b.

x	1	3	2	6
$f(x)$	-2	-22	-1	-37

03. Find the Lagrange form of the polynomial that interpolates $f(x) = \cos(x) + \sin(x)$ at $x_0 = 0$, $x_1 = \frac{1}{4}$, $x_2 = \frac{1}{2}$, and $x_3 = 1$. Find a bound for the absolute error in $[0, 1]$.

04. The degree 3 polynomial that interpolates $f(x) = \ln(x)$ at the points $x_0 = 0.4$, $x_1 = 0.5$, $x_2 = 0.7$, and $x_3 = 0.8$, is

$$P_3(x) = f(x_0)\, L_{3,0}(x) + f(x_1)\, L_{3,1}(x) + f(x_2)\, L_{3,2}(x) + f(x_3)\, L_{3,3}(x)$$

a. Sketch the graph of $f(x)$ and $P_3(x)$ in the same plot.

b. Show that
$$|f(x) - P_3(x)| \leq \frac{625}{64}\, |(x - x_0)(x - x_1)(x - x_2)(x - x_3)|, \qquad 0.4 \leq x \leq 0.8 \quad \cdots \quad (1)$$

c. Compare the left and right sides of (1) at $x = 0.6$. Comment.

05. The divided differences for a function $f(x)$ are listed in the table below

$x_0 = 0.0$	$f[x_0] =$		
$x_1 = 0.4$	$f[x_1] =$	$f[x_0, x_1] =$	
$x_2 = 0.7$	$f[x_2] = 6$	$f[x_1, x_2] = 10$	$f[x_0, x_1, x_2] = 50/7$

Find the missing entries.

06. Write Newton polynomial that interpolates the given data points

a.

x	2	0	3
$f(x)$	11	7	28

b.

x	1	3	2	6
$f(x)$	-2	-22	-1	-37

c.

x	1	0	-1	2
$f(x)$	3	1	-5	19

07. Let $P_3(x)$ be the polynomial that interpolates the points $(0,0)$, $(1/2,y)$, $(1,3)$ and $(2,2)$. If the coefficient of x^3 in $P_3(x)$ is equal to 6, find y.

08. Let $P_n(x)$ be the degree n polynomial that interpolates the function $f(x)$ at the pairwise distinct points $\left\{x_i\right\}_{0\le i\le n}$.

a. Show that $$f\left[x_0,x_1,\cdots,x_n\right]=\sum_{i=0}^{n}\frac{f(x_i)}{\prod\limits_{\substack{k=0\\k\neq i}}^{n}(x_i-x_k)}\quad\cdots\quad(*)$$

b. Use formula $(*)$ to write down $f\left[x_0,x_1\right]$, $f\left[x_0,x_1,x_2\right]$, and $f\left[x_0,x_1,x_2,x_3\right]$

c. What becomes of $(*)$ if the points are equidistant, i.e., $x_i=x_0+i\,h$, $h>0$?

09. Verify that the polynomial $p(x)=2-(x+1)+x\,(x+1)-2\,x\,(x+1)\,(x-1)$, interpolates the data points $\left\{(-1,2),\,(0,1),\,(1,2),\,(2,-7)\right\}$, then find the polynomial $q(x)$ that interpolates the same data points augmented with the point $(3,10)$

10. Show that if $Q(x)$ is a polynomial of degree n, then $Q\left[x_0,x\right]$, $x\neq x_0$ is a polynomial of degree at most $n-1$.

11. Let $P_{n-1}(x)$ be the degree $n-1$ polynomial that interpolates the data points $\left\{(x_i,y_i)\right\}_{0\le i\le n-1}$, and let $P_n(x)$ be the degree n polynomial that interpolates the data points $\left\{(x_i,y_i)\right\}_{0\le i\le n}$. Show that if $P_{n-1}(x_n)=y_n$, then $P_{n-1}(x)\equiv P_n(x)$.

12. Let $P_n(x)$ be the degree n polynomial that interpolates the function $f(x)$ at the pairwise distinct points $\left\{x_i\right\}_{0\le i\le n}$.

a. Show that $$f\left[x_0,x_1,\cdots,x_n\right]=\sum_{i=0}^{n}\frac{f(x_i)}{\prod\limits_{\substack{k=0\\k\neq i}}^{n}(x_i-x_k)}\quad\cdots\quad(*)$$

b. Use formula $(*)$ to write down $f\left[x_0,x_1\right]$, $f\left[x_0,x_1,x_2\right]$, and $f\left[x_0,x_1,x_2,x_3\right]$

c. What becomes of $(*)$ if the points $\left\{x_i\right\}_{0\le i\le n}$ are equidistant, i.e., $x_i=x_0+i\,h$, $h>0$?