Math 211 - Week 4 - Determinants

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Week 4 - Question 5 (a)

If A is a 3x3 matrix with a zero determinant, then one column must be a multiple of some other column. *True or False?*

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$$\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix} = 0 \qquad \begin{vmatrix} 1 & 0 & -2 \\ 0 & 1 & 2 \\ 2 & 2 & 0 \end{vmatrix} =$$

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False, $\text{Det}(A) = 0$ if one column is a linear combination of up to (n-1) columns

This would be the same if the question asked about rows.

Week 4 - Question 5 (b)

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True, Quick elementary column operation to get a column of 0s, expand over that column

What is a elementary column operation??

For A and B two *nxn* matrices, $\det(A+B) = \det(A)+\det(B)$
True or False?

Week 4 - Question 5 (c)

For A and B two nxn matrices, $\det(A+B) = \det(A)+\det(B)$
True or False?

Take $A = B = I_n$
$\det(A + B) = \det(2A) = \det(2I_n) = 2^n$
$\det(A) + \det(B) = \det(I_n) + \det(I_n) = 2\det(I_n) = 2$

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False

Week 4 - Question 5 (d)

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For A an nxn matrix, $\det(3A) = 3\det(A)$ *True or False?*

False , $\det(3A) = 3^n \det(A)$, Take $A = I_2$ to confirm

Week 4 - Question 5 (e)

If A^{-1} exists then $det(A^{-1}) = det(A)^{-1}$ *True or False?*

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$A^{-1}A = I_n$
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$\det(A^{-1})det(A) = 1$

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True

Week 4 - Question 5 (f)

If B is obtained by multiplying a single row of A by 4 then $\det(B) = 4\det(A)$ *True or False?*

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True

Week 4 - Question 5 (g)

For A an nxn matrix, det(-A)$=(-1)^n$ det(A) *True or False?*

Week 4 - Question 5 (g)

For A an nxn matrix, det(-A)$=(-1)^n$ det(A) *True or False?*

True

Week 4 - Question 5 (h)

If A is a real nxn matrix, then $det(A^T A) \geq 0$ *True or False?*

Week 4 - Question 5 (h)

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True $det(A^TA) = det(A^T)det(A) = det(A)det(A) = det(A)^2 \geq 0$

Week 4 - Question 5 (i)

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Cramers rule

For a system:

$Ax = b$

where nxn matrix A has a nonzero determinant, and the vector $x = (x_1, \ldots, x_n)^{\mathrm{T}}$ is the column vector of the variables.

$x_i = \frac{\det(A_i)}{\det(A)} \qquad i = 1, \ldots, n$

where A_i is the matrix formed by replacing the i-th column of A by the column vector b.

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