MATH 391	**Worksheet 05**	**F2019**
	Numerical Integration	

01. Apply the Trapezoid Rule to the following integrals.

a. $\displaystyle\int_0^1 x^2\,\mathrm{d}x$ **b.** $\displaystyle\int_0^1 \mathrm{e}^{-x}\,\mathrm{d}x$ **c.** $\displaystyle\int_0^{\pi/4} \cos^2(x)\,\mathrm{d}x$

Find a bound for the error for each of the integrals above.

02. Repeat the problem above using the Midpoint Rule, then using Simpson's Rule.

03. Let $a < c < b$ and $f(x)$ a function defined in $[a\,,\,b]$. Using the degree-two Lagrange polynomial that interpolates $f(x)$ at the nodes $x_1 = a$, $x_2 = c$, $x_3 = b$, derive a three-point quadrature rule $\mathcal{Q}_3(f) = w_1\,f\big(x_1\big) + w_2\,f\big(x_2\big) + w_3\,f\big(x_3\big)$ that approximates $\displaystyle\int_a^b f(x)\,\mathrm{d}x$.

04. Given an interval $[a\,,\,b]$, let $h = \dfrac{b-a}{3}$, $x_1 = a + h$ and $x_2 = a + 2\,h$. Derive the 2-point open Newton-Cotes quadrature rule $\displaystyle\int_a^b f(x)\,\mathrm{d}x \approx w_1\,f\big(x_1\big) + w_2\,f\big(x_2\big) = \mathcal{Q}_2\big(f\big)$. What is the degree of precision of $\mathcal{Q}_2\big(f\big)$?

05. What is the degree of precision of the quadrature rule

$$\int_0^1 g(x)\,\mathrm{d}x \approx \frac{1}{24}\Big(11\,f(1/5) + f(2/5) + f(3/5) + 11\,f(4/5)\Big) = \mathcal{Q}_4\big(f\big)$$

Rewrite the above rule for the integral $\displaystyle\int_1^3 f(t)\,\mathrm{d}t$.

06. Given the table of values

x	1.8	2.0	2.2	2.4	2.6
$f(x)$	3.12014	4.42569	6.04241	8.03014	10.46675

List all the rules you know that can be applied to approximate $\displaystyle\int_{1.8}^{2.6} f(x)\,\mathrm{d}x$.

07. Write down each of the 3-interval composite rule over $[0\,,\,3]$ for each of the following rules

a. Trapezoid **b.** Midpoint **c.** Simpson's

08. Suppose the Composite Trapezoid Rule is used to approximate the integral $\displaystyle\int_0^1 e^{x^2}\,dx$.

a. How many intervals are needed to guarantee an error within 10^{-6}?

b. Repeat the question, assuming the Composite Simpson's Rule is used to approximate $\displaystyle\int_0^1 e^{x^2}\,dx$.

09. Use the 4-interval composite Simpson's rule to approximate the length L of the ellipse $4\,x^2 + 9\,y^2 = 36$. Compare with the exact value $L = 15.865\,439\,591\cdots$

Recall that the ellipse can be parametrized with $\begin{cases} x = 3\,\cos(t) \\ y = 2\,\sin(t) \end{cases} \quad 0 \le t \le 2\,\pi$

Hence the length is given by $L = \displaystyle\int_0^{2\,\pi} \sqrt{(x')^2 + (y')^2}\,dt = 4\int_0^{\pi/2} \sqrt{4 + 5\,\sin^2(t)}\,dt$