# Math 391

## November 25, 2019

# Worksheet 8

## 01.

Given that $\mathbf{A} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 3 & 1 & 0 \\ 4 & 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$, solve the system $\mathbf{A}x = \begin{bmatrix} 1 \\ 1 \\ 2 \\ 0 \end{bmatrix}$

$$\mathbf{A}x = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 3 & 1 & 0 \\ 4 & 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} x = \begin{bmatrix} 1 \\ 1 \\ 2 \\ 0 \end{bmatrix}$$

$$\mathbf{A}x = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 3 & 1 & 0 \\ 4 & 1 & 2 & 1 \end{bmatrix} (\mathbf{C}x) = \begin{bmatrix} 1 \\ 1 \\ 2 \\ 0 \end{bmatrix}, \text{ where C} = \begin{bmatrix} 2 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{cases} 1(\mathbf{C}x_1) = 1 \\ 1(\mathbf{C}x_2) = 1 \\ 1(\mathbf{C}x_1) + 3(\mathbf{C}x_2)1(\mathbf{C}x_3) = 2 \\ 4(\mathbf{C}x_1) + 1(\mathbf{C}x_2) + 2(\mathbf{C}x_3) + 1(\mathbf{C}x_4) = 2 \end{cases} \Rightarrow \begin{cases} (\mathbf{C}x_1) = 1 \\ (\mathbf{C}x_2) = 1 \\ (\mathbf{C}x_3) = -2 \\ (\mathbf{C}x_4) = -1 \end{cases}$$

$$\mathbf{C}x = \begin{bmatrix} 1 \\ 1 \\ -2 \\ -1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} x = \begin{bmatrix} 1 \\ 1 \\ -2 \\ -1 \end{bmatrix}$$

$$x = \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \end{bmatrix}$$

## 02.

For a) and b). try looking at the equation for each entry of AB.

$$\mathbf{AB}_{ij} = \sum \dots$$

For c) and d). use what you showed above.

## 04.

$Ax = b$ can be solved via back substitution if we decompose A such that $PA = LU$ which we can express our $Ax = b$ as $LUx = Pb$
First we solve $Ly = Pb$ for y.
then solve $Ux = y$ for x.

**06.b)**

$$L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ 3 & 4 & 1 & 0 \\ -1 & -3 & 0 & 1 \end{bmatrix}$$

$$U = \begin{bmatrix} 1 & 1 & 0 & 3 \\ 0 & -1 & -1 & -5 \\ 0 & 0 & 3 & 13 \\ 0 & 0 & 0 & -13 \end{bmatrix}$$