Student name:

1 Write down each statement *and* its converse. Are both, one or none true?

a If there are clouds, then there is rain.

b If you have broken your leg, then you are in pain.

c Suppose n is an integer. If n is odd, then $3n$ is odd.

d Suppose n is an integer. If $3n+4$ is even, then n is even.

e If $x+y=3$, then $x=1$ and $y=2$.

f Suppose m and n are integers. If m and n are both multiples of 4, then mn is a multiple of 16.

g Suppose x and y are real numbers. If $x>y$, then $x^2>y^2$.

2 Suppose n is a natural number. Prove the following statements:

a The number $(n+2)^2-n^2$ is divisible by 8 if and only if n is odd.

b The number $5n+3$ is even if and only if $n+3$ is even.

c The number n is even if and only if n^2+2n+1 is odd.

Answers to Chapter 6 Skillsheet 6C

1 Write down each statement *and* its converse. Are both, one or none true?

a Statement: If it there are clouds, then there is rain (false)

Converse: If there is rain, then there are clouds (true)

b Statement: If you have broken your leg, then you are in pain (true)

Converse: If you are in pain, then you have broken your leg (false)

c Statement: If n is odd, then $3n$ is odd (true)

Converse: If $3n$ is odd, then n is odd (true)

d Statement: If $3n+4$ is even, then n is even (true)

Converse: If n is even, then $3n+4$ is even (true)

e Statement: If $x+y=3$, then $x=1$ and $y=2$ (false)

Converse: If $x=1$ and $y=2$, then $x+y=3$ (true)

f Statement: If m and n are both multiples of 4, then mn is a multiple of 16 (true)

Converse: If mn is a multiple of 16, then m and n are both multiples of 4 (false)

g Statement: If $x>y$, then $x^2>y^2$ (false)

Converse: If $x^2>y^2$, then $x>y$ (false)

2 **a** $(\Rightarrow)$ If the number $(n+2)^2-n^2$ is divisible by 8 then for some integer k,

$$\begin{aligned}(n+2)^2-n^2&=8k\\ n^2+4n+4-n^2&=8k\\ 4n+4&=8k\\ n+1&=2k\\ n&=2k-1\end{aligned}$$

$(\Leftarrow)$ If n is odd then $n=2k+1$ for some integer k. Therefore,

$$\begin{aligned}(n+2)^2-n^2&=n^2+4n+4-n^2\\ &=4n+4\\ &=4(2k+1)+4\\ &=8k+8\\ &=8(k+1)\end{aligned}$$

Therefore, $(n+2)^2-n^2$ is divisible by 8, as required.

b $(\Rightarrow)$ If $5n+3$ is even then $5n+3=2k$ for some integer k. Therefore

$$\begin{aligned} 5n+3 &= 2k \\ n+4n+3 &= 2k \\ n+3 &= 2k-4n \\ n+3 &= 2(k-2n) \end{aligned}$$

so that $n+3$ is even, as required.

$(\Leftarrow)$ If $n+3$ is even then $n+3=2k$ for some integer k. Therefore

$$\begin{aligned} n+3 &= 2k \\ 4n+n+3 &= 4n+2k \\ 5n+3 &= 2(2n+k) \end{aligned}$$

so that $5n+3$ is even, as required.

c $(\Rightarrow)$ If n is even then $n=2k$ for some integer k. Therefore

$$\begin{aligned} n^2+2n+1 &= (n+1)^2 \\ &= (2k+1)^2 \\ &= 4k^2+4k+1 \\ &= 2(2k^2+2k)+1 \end{aligned}$$

so that n^2+2n+1 is odd, as required.

$(\Leftarrow)$ To prove this direction, we prove the contrapositive. That is, we need to show that if n is odd, then n^2+2n+1 is even. So we suppose that $n=2k+1$ for some integer k. Then,

$$\begin{aligned} n^2+2n+1 &= (n+1)^2 \\ &= (2k+1+1)^2 \\ &= (2k+2)^2 \\ &= 4(k+1)^2 \end{aligned}$$

so that n^2+2n+1 is even, as required.