

# Course Outline 2023 Year 11 Mathematics Methods Units 1 & 2

### Unit 1

Contains the three topics:

- · Counting and probability
- Functions and graphs
- Trigonometric functions.

Unit 1 begins with the study of probability and statistics with a review of the fundamentals of probability, and the introduction of the concepts of conditional probability and independence. A review of the basic algebraic concepts and techniques required for a successful introduction to the study of functions and calculus is covered. Simple relationships between variable quantities are reviewed, and these are used to introduce the key concepts of a function and its graph. The study of the trigonometric functions begins with a consideration of the unit circle using degrees and the trigonometry of triangles and its application. Radian measure is introduced, and the graphs of the trigonometric functions are examined and their applications in a wide range of settings are explored.

## Unit 2

Contains the three topics:

- Exponential functions
- · Arithmetic and geometric sequences and series
- Introduction to differential calculus.

In Unit 2, exponential functions are introduced and their properties and graphs examined. Arithmetic and geometric sequences and their applications are introduced and their recursive definitions applied. Rates and average rates of change are introduced and this is followed by the key concept of the derivative as an 'instantaneous rate of change'. These concepts are reinforced numerically (by calculating difference quotients), geometrically (as slopes of chords and tangents), and algebraically. This first calculus topic concludes with derivatives of polynomial functions, using simple applications of the derivative to sketch curves, calculate slopes and equations of tangents, determine instantaneous velocities, and solve optimisation problems.

| Week/s | •       |  | Resources  | Skills /<br>General<br>Capabilities | Assessment |
|--------|---------|--|--|-------------------------------------|------------|
|        |         | Term 1   | itv  |                                     |            |
| 1-2    | Combir  |  | Topics  Term 1  1.1 Counting and probability  Ind the notion of a combination as a set of $r$ aken from a set of $n$ distinct objects taken from a set of $n$ objects taken from a sistinct objects of the expansion of for small positive integers $n$ mits and sets  The concepts and language of outcomes, spaces, and events, as sets of outcomes anguage and notation for events, including: for $A'$ for the complement of an event $A$ and $A \cup B \cup C$ for the intersection and union of the three events $A$ , $B$ and $A \cup B$ of the intersection and of the three events $A$ , $B$ and $A \cup B$ or expectively gnise mutually exclusive events. A and set operations elementates of probability are events and set operations elementates of probability and if $A$ is a certainty the probability and if $A$ is a certainty the rules: $P(A) = 1 - P(A)$ and $P(A \cup B) = P(A) = 1 - P(A)$ and the formula $P(A \cap B) = P(A)P(B)$ for denote the formula $P(A \cap B) = P(A)P(B)$ for denote of independence of an event $A$ are an use the formula $P(A \cap B) = P(A)P(B)$ for denote of independence of an event $A$ are defined by $A \cap A $ |                                     |            |
|        | 1.1.1   | understand the notion of a combination as a set of $r$ objects taken from a set of $n$ distinct objects  |  | # -<br>× =                          | Week 3     |
|        | 1.1.2   | use the notation $\binom{n}{r}$ and the formula $\binom{n}{r} = \frac{n!}{r!(n-r)!}$ for the number of combinations of $r$ objects taken from a  |  | 21CLD<br>Study Skills:              |            |
|        | 1.1.3   | set of $n$ distinct objects investigate Pascal's triangle and its properties to link $\binom{n}{r}$ to the binomial coefficients of the expansion of $(x+y)^n$ for small positive integers $n$ |  |                                     |            |
| 3-5    | Langua  | ge of events and sets  | Chapter 9  |                                     |            |
|        | 1.1.4   | review the concepts and language of outcomes, sample spaces, and events, as sets of outcomes   | Onapier 5  | +-<br>× +                           |            |
|        | 1.1.5   | use set language and notation for events, including:   |  |                                     |            |
|        |         | a. $\bar{A}$ (or $A'$ ) for the complement of an event $A$   |  |                                     |            |
|        |         | b. $A \cap B$ and $A \cup B$ for the intersection and union of events $A$ and $B$ respectively   |  |                                     |            |
|        |         | c. $A \cap B \cap C$ and $A \cup B \cup C$ for the intersection and union of the three events $A, B$ and $C$ respectively  |  |                                     |            |
|        |         | d. recognise mutually exclusive events.  |  |                                     |            |
|        | 1.1.6   | use everyday occurrences to illustrate set descriptions and representations of events and set operations   |  |                                     |            |
|        | Review  | of the fundamentals of probability   |  |                                     |            |
|        | 1.1.7   | review probability as a measure of 'the likelihood of occurrence' of an event  |  |                                     |            |
|        | 1.1.8   | review the probability scale: $0 \le P(A) \le 1$ for each event $A$ , with $P(A) = 0$ if $A$ is an impossibility and $P(A) = 1$ if $A$ is a certainty  |  |                                     |            |
|        | 1.1.9   | review the rules: $P(\overline{A}) = 1 - P(A)$ and $P(A \cup B) = P(A) + P(B) - P(A \cap B)$   |  |                                     |            |
|        | 1.1.10  | use relative frequencies obtained from data as estimates of probabilities  |  |                                     |            |
|        | Conditi | onal probability and independence  |  |                                     |            |
|        | 1.1.11  | understand the notion of a conditional probability and recognise and use language that indicates conditionality  |  |                                     |            |
|        | 1.1.12  | use the notation $P(A B)$ and the formula $P(A \cap B) =$  |  |                                     |            |
|        | 1.1.13  | P(A B)P(B) understand the notion of independence of an event $A$ from an event $B$ , as defined by $P(A B) = P(A)$   |  |                                     |            |
|        | 1.1.14  | establish and use the formula $P(A \cap B) = P(A)P(B)$ for independent events $A$ and $B$ , and recognise the  |  |                                     |            |
|        | 1.1.15  | use relative frequencies obtained from data as estimates of conditional probabilities and as indications of possible independence of events  |  |                                     |            |

| Week/s | Topics  | Resources                                    | Skills /<br>General<br>Capabilities              | Assessment                |  |  |  |
|--------|---|--|--|---------------------------|--|--|--|
|        | 1.2 Functions and graphs  |  |  |                           |  |  |  |
| 5-6    | Lines and linear relationships 1.2.1 recognise features of the graph of $y = mx + c$ , including its linear nature, its intercepts and its slope or gradient  | Review of<br>Chapter 1<br>and 2<br>Chapter 3 | *=<br>*=<br>*=                                   |                           |  |  |  |
|        | 1.2.2 determine the equation of a straight line given sufficient information; including parallel and perpendicular lines  | Onapier 3                                    | 21CLD<br>Graphs –<br>recognise<br>features and   |                           |  |  |  |
|        | Quadratic relationships   |  | determine equations.                             |                           |  |  |  |
|        | 1.2.3 examine examples of quadratically related variables<br>1.2.4 recognise features of the graphs of $y = x^2$ , $y = a(x - b)^2 + c$ , and $y = a(x - b)(x - c)$ , including their parabolic nature, turning points, axes of symmetry and intercepts |  | RealWorld<br>Problem<br>Solving21CL<br>D Level 3 |                           |  |  |  |
|        | <ul><li>1.2.5 solve quadratic equations, including the use of quadratic formula and completing the square</li><li>1.2.6 determine the equation of a quadratic given sufficient</li></ul>  |  |  |                           |  |  |  |
|        | information  1.2.7 determine turning points and zeros of quadratics and understand the role of the discriminant   |  |  |                           |  |  |  |
|        | 1.2.8 recognise features of the graph of the general quadratic $y = ax^2 + bx + c$  |  |  |                           |  |  |  |
| 7      | Inverse proportion  | Chapter 5                                    | <b>X</b>   |                           |  |  |  |
|        | <ul><li>1.2.9 examine examples of inverse proportion</li><li>1.2.10 recognise features and determine equations of the</li></ul>   | ≻5B  | ^ -  |                           |  |  |  |
|        | graphs of $y = \frac{1}{x}$ and $y = \frac{a}{x-b}$ , including their hyperbolic shapes and their asymptotes.   | Chapter 4<br>➤ 4A                            |  |                           |  |  |  |
| 8      | <b>Powers and polynomials</b> 1.2.11 recognise features of the graphs of $y = x^n$ for $n \in N$ , $n = -1$ and $n = \frac{1}{2}$ , including shape, and behaviour as $x \to \infty$ and $x \to -\infty$  | Chapter 7                                    | <b> ★ ★ ★ ★ ★ ★ ★ ★ ★ </b>                       | Test 1<br>Week7<br>Fri p1 |  |  |  |
|        | 1.2.12 identify the coefficients and the degree of a polynomial   |  |  |                           |  |  |  |
|        | 1.2.13 expand quadratic and cubic polynomials from factors 1.2.14 recognise features and determine equations of the graphs of $y = x^3$ , $y = a(x - b)^3 + c$ and $y = k(x - a)(x - b)(x - c)$ , including shape, intercepts and                       |  |  |                           |  |  |  |
|        | behaviour as $x \to \infty$ and $x \to -\infty$<br>1.2.15 factorise cubic polynomials in cases where all roots are given or easily obtained from the graph  |  |  |                           |  |  |  |
|        | 1.2.16 solve cubic equations using technology, and algebraically in cases where all roots are given or easily obtained from the graph   |  |  |                           |  |  |  |
| 9      | Graphs of relations   | Chapter 4                                    |  |                           |  |  |  |
|        | 1.2.17 recognise features and determine equations of the graphs of $x^2 + y^2 = r^2$ and $(x-a)^2 + (y-b)^2 = r^2$ , including their circular shapes, their centres and their radii   | > 4B to 4E                                   |  |                           |  |  |  |
|        | 1.2.18 recognise features of the graph of $y^2 = x$ , including its parabolic shape and its axis of symmetry  |  |  |                           |  |  |  |

| Week/s |          | Topics  | Resources  | Skills /<br>General<br>Capabilities   | Assessment       |
|--------|----------|---|------------|---------------------------------------|------------------|
| 10     | Function | ons   | Chapter 6  | +-<br>×÷                              |                  |
|        | 1.2.19   | understand the concept of a function as a mapping<br>between sets and as a rule or a formula that defines<br>one variable quantity in terms of another              |            | 21CLD<br>Study Skills:<br>Reflection. |                  |
|        | 1.2.20   | use function notation; determine domain and range; recognise independent and dependent variables  |            | Test 2<br>Reflection                  |                  |
|        | 1.2.21   | understand the concept of the graph of a function   |            |                                       |                  |
|        | 1.2.22   | examine translations and the graphs of $y = f(x) + a$<br>and $y = f(x - b)$   |            |                                       |                  |
|        | 1.2.23   | examine dilations and the graphs of $y = cf(x)$ and $y = f(dx)$   |            |                                       |                  |
|        | 1.2.24   | recognise the distinction between functions and relations and apply the vertical line test  |            |                                       |                  |
|        |          | Term 2  |            |                                       |                  |
| 1      | Cosino   | 1.3 Trigonometric functions and sine rules  | Chapter 13 |                                       | TEST 2           |
|        | 1.3.1    | review sine, cosine and tangent as ratios of side lengths in right-angled triangles   |            | × =                                   | Week 2<br>Fri p1 |
|        | 1.3.2    | understand the unit circle definition of $\cos\theta$ , $\sin\theta$ and $\tan\theta$ and periodicity using degrees   |            |                                       |                  |
|        | 1.3.3    | examine the relationship between the angle of inclination of a line and the gradient of that line   |            |                                       |                  |
|        | 1.3.4    | establish and use the cosine and sine rules, including consideration of the ambiguous case and the formula $Area = \frac{1}{2}bc \sin A$ for the area of a triangle |            |                                       |                  |
| 2-3    | Circula  | ar measure and radian measure   | Chapter 12 |                                       |                  |
|        | 1.3.5    | define and use radian measure and understand its relationship with degree measure   |            |                                       |                  |
|        | 1.3.6    | use radian measure to calculate lengths of arcs and areas of sectors and segments in a circle   |            |                                       |                  |
|        | Trigon   | ometric functions   |            |                                       |                  |
|        | 1.3.7    | understand the unit circle definition of $\sin\theta$ , $\cos\theta$ and $\tan\theta$ and periodicity using radians   |            |                                       |                  |
|        | 1.3.8    | recognise the exact values of $\sin\theta$ , $\cos\theta$ and $\tan\theta$ at integer multiples of $\frac{\pi}{6}$ and $\frac{\pi}{4}$                              |            |                                       |                  |
|        | 1.3.9    | recognise the graphs of $y = \sin x$ , $y = \cos x$ , and $y = \tan x$ on extended domains  |            |                                       |                  |
|        | 1.3.10   | examine amplitude changes and the graphs of $y = a \sin x$ and $y = a \cos x$   |            |                                       |                  |
|        | 1.3.11   | examine period changes and the graphs of $y = \sin bx$ , $y = \cos bx$ and $y = \tan bx$  |            |                                       |                  |
|        | 1.3.12   | examine phase changes and the graphs of $y = \sin(x - c)$ , $y = \cos(x - c)$ and $y = \tan(x - c)$   |            |                                       |                  |

|       | •                  |   |                           |                     |                              |
|-------|--------------------|---|---------------------------|---------------------|------------------------------|
|       | 1.3.13             | examine the relationships $\sin\left(x + \frac{\pi}{2}\right) = \cos x$ and   |                           |                     |                              |
|       |                    | $\cos\left(x - \frac{\pi}{2}\right) = \sin x$   |                           |                     |                              |
|       | 1.3.14             | prove and apply the angle sum and difference identities   |                           |                     |                              |
|       | 1.3.15             | identify contexts suitable for modelling by trigonometric functions and use them to solve practical problems  |                           |                     |                              |
|       | 1.3.16             | solve equations involving trigonometric functions using technology, and algebraically in simple cases   |                           |                     |                              |
| 4     |                    | Revision  |                           |                     |                              |
| 5 – 6 |                    | SEMESTER 1 EXAMS  |                           |                     | EXAM                         |
|       |                    | 2.3 Introduction to differential cal  | culus                     |                     |                              |
| 7-8   | <b>Rates</b> 2.3.1 | of change interpret the difference quotient $\frac{f(x+h)-f(x)}{h}$ as the  | Chapter 17                |                     |                              |
|       | 2.3.2              | average rate of change of a function $f$ use the Leibniz notation $\delta x$ and $\delta y$ for changes or increments in the variables $x$ and $y$  | > 17 A to C               |                     |                              |
|       | 2.3.3              | use the notation $\frac{\delta y}{\delta x}$ for the difference quotient $\frac{f(x+h)-f(x)}{h}$ where $y=f(x)$                                     |                           |                     |                              |
|       | 2.3.4              | interpret the ratios $\frac{f(x+h)-f(x)}{h}$ and $\frac{\delta y}{\delta x}$ as the slope or gradient of a chord or secant of the graph of $y=f(x)$ |                           | 21CLD               |                              |
|       | The co             | encept of the derivative  |                           | ICT 4               |                              |
|       | 2.3.5              | examine the behaviour of the difference quotient $\frac{f(x+h)-f(x)}{h}$ as $h \to 0$ as an informal introduction to the                            |                           | Learning<br>Level 2 |                              |
|       |                    | concept of a limit  |                           |                     |                              |
|       | 2.3.6              | define the derivative $f'(x)$ as $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$  |                           |                     |                              |
|       | 2.3.7              | use the Leibniz notation for the derivative: $\frac{dy}{dx} = \lim_{\delta x \to 0} \frac{\delta y}{\delta x}$ and the correspondence               |                           |                     |                              |
|       |                    | $\frac{dy}{dx} = f'(x)$ where $y = f(x)$  |                           |                     |                              |
|       | 2.3.8              | interpret the derivative as the instantaneous rate of change  |                           |                     |                              |
|       | 2.3.9              | interpret the derivative as the slope or gradient of a tangent line of the graph of $y = f(x)$  |                           |                     |                              |
| 9-10  | -                  | utation of derivatives estimate numerically the value of a derivative for simple power functions  | Chapter 17<br>> 17 D to F |                     | Inv 2<br>Week 10<br>Thurs p1 |
|       | 2.3.11             | examine examples of variable rates of change of non-<br>linear functions  |                           |                     |                              |
|       | 2.3.12             | establish the formula $\frac{d}{dx}(x^n) = nx^{n-1}$ for non-negative integers $n$ expanding $(x+h)^n$ or by factorising $(x+h)^n - x^n$            |                           |                     |                              |
|       | Proper             | ties of derivatives   |                           |                     |                              |
|       | -                  | understand the concept of the derivative as a function  |                           |                     |                              |
|       |                    | identify and use linearity properties of the derivative   |                           |                     |                              |
|       |                    | calculate derivatives of polynomials  |                           |                     |                              |
| L     |                    |   | l .                       |                     |                              |

| Week/s |         | Topics   | Resources              | Skills /<br>General<br>Capabilities | Assessment       |
|--------|---------|--|------------------------|-------------------------------------|------------------|
|        |         | Term 3 2.3 Introduction to differential ca   | alculus                |                                     |                  |
| 1-3    | Applic  | ations of derivatives  | Chapter 18             |                                     |                  |
|        | 2.3.16  | determine instantaneous rates of change  |                        |                                     |                  |
|        | 2.3.17  | determine the slope of a tangent and the equation of the tangent   |                        |                                     |                  |
|        | 2.3.18  | construct and interpret position-time graphs with velocity as the slope of the tangent   |                        |                                     |                  |
|        | 2.3.19  | recognise velocity as the first derivative of displacement with respect to time  |                        |                                     |                  |
|        | 2.3.20  | sketch curves associated with simple polynomials, determine stationary points, and local and global maxima and minima, and examine behaviour as $x \to \infty$ and $x \to -\infty$ |                        |                                     |                  |
|        | 2.3.21  | solve optimisation problems arising in a variety of contexts involving polynomials on finite interval domains  |                        |                                     |                  |
| 3-4    | Anti-de | erivatives   | Chapter 17             |                                     | TEST 3           |
|        | 2.3.22  | calculate anti-derivatives of polynomial functions   | > 17 G                 |                                     | Week 4<br>Fri p1 |
|        |         | 2.1 Exponential functions  |                        |                                     |                  |
| 5-7    | Indice  | s and the index laws   | Chapter 14             |                                     |                  |
|        | 2.1.1   | review indices (including fractional and negative indices) and the index laws  |                        | <b>4</b> ΔΔ                         |                  |
|        | 2.1.2   | use radicals and convert to and from fractional indices  |                        |                                     |                  |
|        | 2.1.3   | understand and use scientific notation and significant figures   |                        |                                     |                  |
|        | Expon   | ential functions   |                        |                                     |                  |
|        | 2.1.4   | establish and use the algebraic properties of exponential functions  |                        |                                     |                  |
|        | 2.1.5   | recognise the qualitative features of the graph of $y = a^x$ ( $a > 0$ ), including asymptotes, and of its translations ( $y = a^x + b$ and $y = a^{x-c}$ )                        |                        |                                     |                  |
|        | 2.1.6   | identify contexts suitable for modelling by exponential functions and use them to solve practical problems   |                        |                                     |                  |
|        | 2.1.7   | solve equations involving exponential functions using technology, and algebraically in simple cases  |                        |                                     |                  |
|        |         | 2.2 Arithmetic and geometric sec   | _                      |                                     |                  |
| 7-9    | Arithm  | netic sequences  | Chapter 15 > 15 A to C |                                     |                  |
|        | 2.2.1   | recognise and use the recursive definition of an arithmetic sequence: $t_{n+1}=t_n+d$  | 7 10 10 0              | <b>↓</b> ΔΔ                         |                  |
|        | 2.2.2   | develop and use the formula $t_n=t_1+(n-1)d$ for the general term of an arithmetic sequence and recognise its linear nature  |                        | •                                   |                  |

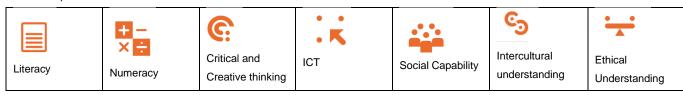
| 2.2.3 | use arithmetic sequences in contexts involving   |  |  |  |
|-------|--|--|--|--|
|       | •  |  |  |  |
|       | discrete linear growth or decay, such as simple  |  |  |  |
|       | interest   |  |  |  |
| 2.2.4 | establish and use the formula for the sum of the   |  |  |  |
|       | first $n$ terms of an arithmetic sequence  |  |  |  |
| Geom  | etric sequences  | Chapter 15   |  |  |
| 2.2.5 | recognise and use the recursive definition of a  | > 15 D to E  |  |  |
|       | geometric sequence: $t_{n+1} = t_n r$  |  |  |  |
| 2.2.6 | develop and use the formula $t_n = t_1 r^{n-1}$ for the general term of a geometric sequence and recognise its exponential nature                    |  |  |  |
| 2.2.8 | establish and use the formula $S_n = t_1 \frac{r^{n-1}}{r^n}$ for the  |  |  |  |
|       | sum of the first $n$ terms of a geometric sequence   |  |  |  |
| 2.2.9 | use geometric sequences in contexts involving geometric growth or decay, such as compound interest   |  |  |  |
|       | Term 4   |  |  |  |
| Geom  | etric sequences  | Chapter 15   |  |  |
| 2.2.7 | understand the limiting behaviour as $n\to\infty$ of the terms $t_n$ in a geometric sequence and its dependence on the value of the common ratio $r$ | > 15 F   |  |  |
|       | Revision   |  |  |  |
|       | SEMESTER 2 EXAMS   |  |  |  |
|       | Units 1 and 2  |  |  |  |
|       | Geom. 2.2.5 2.2.6 2.2.8 2.2.9  | <ul> <li>first n terms of an arithmetic sequence</li> <li>Geometric sequences</li> <li>2.2.5 recognise and use the recursive definition of a geometric sequence: t<sub>n+1</sub> = t<sub>n</sub>r</li> <li>2.2.6 develop and use the formula t<sub>n</sub> = t<sub>1</sub>r<sup>n-1</sup> for the general term of a geometric sequence and recognise its exponential nature</li> <li>2.2.8 establish and use the formula S<sub>n</sub> = t<sub>1</sub> r<sup>n-1</sup>/r-1 for the sum of the first n terms of a geometric sequence</li> <li>2.2.9 use geometric sequences in contexts involving geometric growth or decay, such as compound interest</li> <li>Term 4</li> <li>Geometric sequences</li> <li>2.2.7 understand the limiting behaviour as n → ∞ of the terms t<sub>n</sub> in a geometric sequence and its dependence on the value of the common ratio r</li> <li>Revision</li> </ul> | 2.2.4 establish and use the formula for the sum of the first $n$ terms of an arithmetic sequence  Geometric sequences  2.2.5 recognise and use the recursive definition of a geometric sequence: $t_{n+1} = t_n r$ 2.2.6 develop and use the formula $t_n = t_1 r^{n-1}$ for the general term of a geometric sequence and recognise its exponential nature  2.2.8 establish and use the formula $S_n = t_1 \frac{r^{n-1}}{r-1}$ for the sum of the first $n$ terms of a geometric sequence  2.2.9 use geometric sequences in contexts involving geometric growth or decay, such as compound interest  Term 4  Geometric sequences  2.2.7 understand the limiting behaviour as $n \to \infty$ of the terms $t_n$ in a geometric sequence and its dependence on the value of the common ratio $r$ Revision  SEMESTER 2 EXAMS | 2.2.4 establish and use the formula for the sum of the first $n$ terms of an arithmetic sequence  Geometric sequences  2.2.5 recognise and use the recursive definition of a geometric sequence: $t_{n+1} = t_n r$ 2.2.6 develop and use the formula $t_n = t_1 r^{n-1}$ for the general term of a geometric sequence and recognise its exponential nature  2.2.8 establish and use the formula $S_n = t_1 \frac{r^{n-1}}{r-1}$ for the sum of the first $n$ terms of a geometric sequence  2.2.9 use geometric sequences in contexts involving geometric growth or decay, such as compound interest  Term 4  Geometric sequences  2.2.7 understand the limiting behaviour as $n \to \infty$ of the terms $t_n$ in a geometric sequence and its dependence on the value of the common ratio $r$ Revision  SEMESTER 2 EXAMS |

### Icon Key

#### Habits of Mind



#### **General Capabilities**



# Cross Curricular Priorities

