Answers

Chapter 1

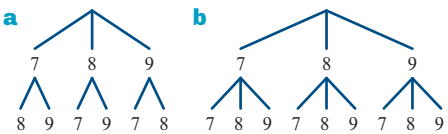
Exercise 1A

1 45

2 8

3 120

4 a **b**



5 a 27 **b** 6

6 30

7 a 6 **b** 18 **c** 20 **d** 15

8 BB, BR, BG, RB, RG, GB, GR, GG

9 12

10 9

11 a 6 **b** 13

12 16

Exercise 1B

1 1, 1, 2, 6, 24, 120, 720, 5040, 40 320, 362 880, 3 628 800

2 a 5 **b** 90 **c** 66 **d** 161 700

3 a $n+1$ **b** $n+2$ **c** $n(n-1)$ **d** $\dfrac{n+2}{(n+1)!}$

4 1, 4, 12, 24, 24

5 DOG, DGO, ODG, OGD, GOD, GDO

6 120

7 362 880

8 FR, FO, FG, RF, RO, RG, OF, OR, OG, GF, GR, GO

9 a 720 **b** 720 **c** 360

10 a 120 **b** 120 **c** 60

11 20 160

12 a 125 **b** 60

13 a 120 **b** 360 **c** 720

14 60

15 a 17 576 000 **b** 11 232 000

16 a 384 **b** 3072

17 $(m, n) = (6, 0), (6, 1), (5, 3)$

18 $(n^2 - n) \cdot (n-2)! = n \cdot (n-1) \cdot (n-2)! = n!$

19 30

Exercise 1C

1 a 120 **b** 72 **c** 24 **d** 96

2 a 120 **b** 48 **c** 72 **d** 12

3 a 360 **b** 144 **c** 144 **d** 72

4 a 1152 **b** 1152

5 a 600 **b** 108 **c** 431 **d** 52

6 a 720 **b** 48 **c** 144 **d** 96 **e** 48

7 a 900 **b** 900

8 84

9 32

10 a 480 **b** 192

11 144

Exercise 1D

1 35 **2** 34 650

3 4 989 600 **4** 56

5 27 720

6 a 420 **b** 105 **c** 90 **d** 12 **e** 105

7 35

8 a 15 **b** $\dfrac{(m+n)!}{m! \cdot n!}$

9 a 52! **b** $\dfrac{104!}{(2!)^{52}}$ **c** $\dfrac{(52n)!}{(n!)^{52}}$

10 4900

11 89

ISBN 978-1-009-25961-3

Exercise 1E

1 1, 5, 10, 10, 5, 1

2 **a** 7 **b** 6 **c** 66 **d** 56 **e** 100
f 499 500

3 **a** n **b** $\frac{n(n-1)}{2}$ **c** n **d** $n+1$
e $\frac{(n+2)(n+1)}{2}$ **f** $\frac{n(n+1)}{2}$

4 **a** 720 **b** 120

5 2 598 960

6 **a** 10 **b** 45 **c** 45 **d** 10

7 45 379 620

8 56

9 **a** 45 **b** 16

10 15

11 $${}^nC_{n-r} = \frac{n!}{(n-r)!\,(n-(n-r))!}$$
$$= \frac{n!}{(n-r)!\,r!} = {}^nC_r$$

12 Each diagonal is obtained by choosing 2 vertices from n vertices. This can be done in ${}^nC_2$ ways. But n of these choices define a side of the polygon, not a diagonal. Therefore there are ${}^nC_2 - n$ diagonals.

13 There are ${}^{10}C_5$ ways to choose 5 students for team A. The remaining 5 students will belong to team B. However, the labelling of the teams does not matter, so we must divide by 2.

14 462

15 $${}^{n-1}C_{r-1} + {}^{n-1}C_r$$
$$= \frac{(n-1)!}{(r-1)!\,(n-1-(r-1))!} + \frac{(n-1)!}{r!\,(n-1-r)!}$$
$$= \frac{(n-1)!}{(r-1)!\,(n-r)!} + \frac{(n-1)!}{r!\,(n-r-1)!}$$
$$= \frac{(n-1)!}{(r-1)!\,(n-r-1)!}\left(\frac{1}{n-r} + \frac{1}{r}\right)$$
$$= \frac{(n-1)!}{(r-1)!\,(n-r-1)!} \cdot \frac{n}{r(n-r)}$$
$$= \frac{n!}{r!\,(n-r)!} = {}^nC_r$$

16 **a** 2300 **b** 152 **c** 2148

Exercise 1F

1 153 **2** 126
3 1176 **4** 140
5 **a** 1716 **b** 700 **c** 980 **d** 1568
6 **a** 25 200 **b** 4200
7 **a** 1 392 554 592 **b** 5 250 960
8 **a** 15 504 **b** 10 800 **c** 15 252
9 **a** 21 **b** 10 **c** 11
10 2100
11 **a** 204 490 **b** 7 250 100
12 **a** 48 **b** 210
13 1440 **14** 3600
15 14 400 **16** 150
17 3744

Exercise 1G

1 ${}^7C_2 = 21$, ${}^6C_2 = 15$, ${}^6C_1 = 6$

2 1, 7, 21, 35, 35, 21, 7, 1; ${}^7C_2 = 21$, ${}^7C_4 = 35$

3 1, 8, 28, 56, 70, 56, 28, 8, 1;
${}^8C_4 = 70$, ${}^8C_6 = 28$

4 $2^6 = 64$

5 $2^5 = 32$

6 $2^{10} = 1024$

7 $2^6 - 1 = 63$

8 $2^8 - {}^8C_1 - {}^8C_0 = 247$

9 $2^8 = 256$

10 $2^4 - 1 = 15$

11 **a** 128 **b** 44

Exercise 1H

1 4

2 Label 26 holes from A to Z. Put each of the 27 words into the hole labelled by its first letter. Some hole contains at least two words.

3 Label 4 holes by 0, 1, 2, 3. Put each of the 5 numbers into the hole labelled by its remainder when divided by 4. Some hole contains at least two numbers.

4 **a** 3 **b** 5 **c** 14

5 Divide [0, 1] into 10 subintervals: [0, 0.1), [0.1, 0.2), . . . , [0.9, 1]. Some interval contains at least two of the 11 numbers.

6 Divide into 4 equilateral triangles of side length 1 unit as shown. Some triangle contains at least two of the 5 points.

7 Divide the rectangle into squares of size 2×2. There are 12 squares and 13 points, so some square contains at least two points.
The distance between two points in the same square cannot exceed the length of the square's diagonal, $\sqrt{2^2 + 2^2} = 2\sqrt{2}$.

8 **a** For two-digit numbers, the possible digital sums are $1, 2, \ldots, 18$. Since $19 > 18$, some digital sum occurs at least twice.
b For three-digit numbers, the possible digital sums are $1, 2, \ldots, 27$. Since $82 = 3 \times 27 + 1$, some digital sum occurs at least 4 times.

9 Label 4 holes by 0, 1, 2, 3. Place each number into the hole labelled by its remainder when divided by 4. Since $13 = 3 \times 4 + 1$, some hole contains at least 4 numbers.

Cambridge Senior Maths for Western Australia
Mathematics Specialist 1&2
ISBN 978-1-009-25961-3

10 Two teams can be chosen in ${}^8C_2 = 28$ ways. Since there are 29 games, some pair of teams play each other at least twice.

11 At least 26 students. To show that 26 numbers suffice, label 25 holes by (1 or 49), (2 or 48), ..., (24 or 26), (25). To show that 25 numbers do not, consider $1, 2, 3, \ldots, 25$.

12 Label the chairs $1, 2, \ldots, 14$. There are 14 groups of three consecutive chairs:

$\{1, 2, 3\}, \{2, 3, 4\}, \ldots, \{13, 14, 1\}, \{14, 1, 2\}$

Each of the 10 people belongs to 3 groups, so there are 30 people to be allocated to 14 groups. Since $30 \geq 2 \times 14 + 1$, some group contains at least 3 people.

13 Draw a diameter through one of the 4 points. This creates 2 half circles. One half circle contains at least two of the 3 remaining points (and the chosen point).

14 There are 195 possible sums: $3, 4, \ldots, 197$. There are ${}^{35}C_2 = 595$ ways to choose a pair of players. Since $595 \geq 3 \times 195 + 1$, at least 4 pairs have the same sum.

15 Label the chairs $1, 2, \ldots, 12$. There are 6 pairs of opposite seats:

$\{1, 7\}, \{2, 8\}, \{3, 9\}, \{4, 10\}, \{5, 11\}, \{6, 12\}$

Some pair contains two of the 7 boys.

16 Label n holes by $0, 1, 2, \ldots, n - 1$. Place each guest in the hole labelled by the number of hands they shake. The first or last hole must be empty. (If a guest shakes 0 hands, then no guest shakes n hands. If a guest shakes n hands, then no guest shakes 0 hands.) This leaves $n - 1$ holes, so some hole contains at least two guests.

Chapter 1 review

Short-answer questions

1 **a** 20 **b** 190 **c** 300 **d** 4950

2 11

3 **a** 27 **b** 6

4 120 **5** 60

6 18 **7** 31

8 10 **9** 3

Extended-response questions

1 **a** 120 **b** 360 **c** 72 **d** 144

2 **a** 20 **b** 80 **c** 60

3 **a** 210 **b** 84 **c** 90 **d** 195

4 **a** 420 **b** 15 **c** 105 **d** 12

5 **a** **i** 20 **ii** 10 **iii** 64
b 8

6 **a** 210 **b** 100 **c** 10 **d** 80

7 **a** 676 **b** 235 **c** 74

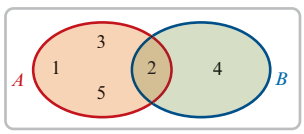
8 **a** 924
b There are at least $365 \times 3 = 1095$ days in three years and there are 924 different paths, so some path is taken at least twice.
c **i** 6 **ii** 70 **iii** 420

9 196

Chapter 2

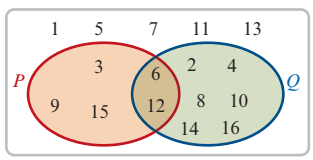
Exercise 2A

1 ξ



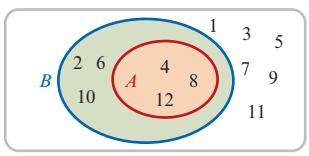
a $\{4\}$ **b** $\{1, 3, 5\}$ **c** $\{1, 2, 3, 4, 5\} = \xi$
d $\varnitothing$ **e** $\varnitothing$

2 ξ



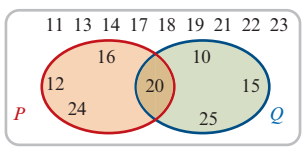
a $\{1, 2, 4, 5, 7, 8, 10, 11, 13, 14, 16\}$
b $\{1, 3, 5, 7, 9, 11, 13, 15\}$
c $\{2, 3, 4, 6, 8, 9, 10, 12, 14, 15, 16\}$
d $\{1, 5, 7, 11, 13\}$ **e** $\{1, 5, 7, 11, 13\}$

3 ξ



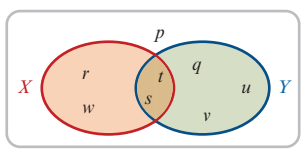
a $\{1, 2, 3, 5, 6, 7, 9, 10, 11\}$
b $\{1, 3, 5, 7, 9, 11\}$ **c** $\{2, 4, 6, 8, 10, 12\}$
d $\{1, 3, 5, 7, 9, 11\}$ **e** $\{1, 3, 5, 7, 9, 11\}$

4 ξ



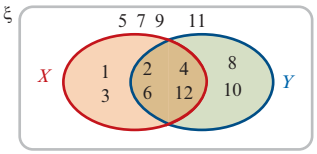
a $\{10, 11, 13, 14, 15, 17, 18, 19, 21, 22, 23, 25\}$
b $\{11, 12, 13, 14, 16, 17, 18, 19, 21, 22, 23, 24\}$
c $\{10, 12, 15, 16, 20, 24, 25\}$
d $\{11, 13, 14, 17, 18, 19, 21, 22, 23\}$
e $\{11, 13, 14, 17, 18, 19, 21, 22, 23\}$

5 ξ

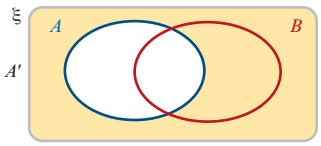


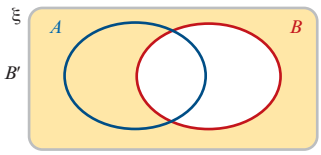
a $\{p, q, u, v\}$ **b** $\{p, r, w\}$ **c** $\{p\}$
d $\{p, q, r, u, v, w\}$ **e** $\{q, r, s, t, u, v, w\}$ **f** $\{p\}$

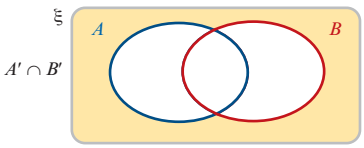
Cambridge Senior Maths for Western Australia Mathematics Specialist 1&2
ISBN 978-1-009-25961-3

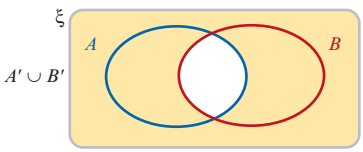
6 

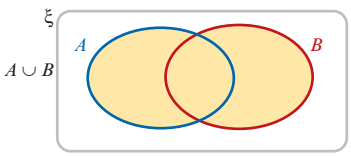
a {5, 7, 8, 9, 10, 11} **b** {1, 3, 5, 7, 9, 11}
c {1, 3, 5, 7, 8, 9, 10, 11}
d {1, 3, 5, 7, 8, 9, 10, 11}
e {1, 2, 3, 4, 6, 8, 10, 12} **f** {5, 7, 9, 11}

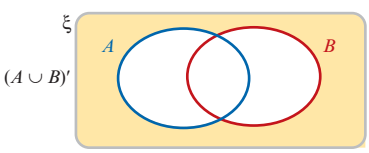
7 a 

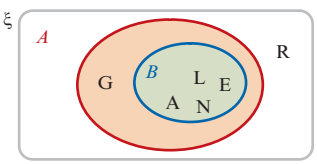
b 

c 

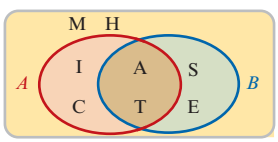
d 

e 

f 

8 

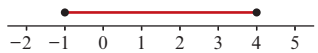
a {R} **b** {G, R} **c** {L, E, A, N}
d {A, N, G, E, L} **e** {R} **f** {G, R}

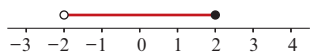
9 

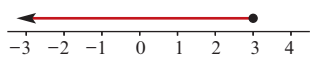
a {E, H, M, S} **b** {C, H, I, M}
c {A, T} **d** {H, M} **e** {C, E, H, I, M, S}
f {H, M}

Exercise 2B

1 a Yes **b** Yes **c** Yes
2 a No **b** No **c** No
3 a $\frac{9}{20}$ **b** $\frac{3}{11}$ **c** $\frac{3}{25}$ **d** $\frac{2}{7}$ **e** $\frac{4}{11}$ **f** $\frac{2}{9}$
4 a $0.\dot{2}85714\dot{}$ **b** $0.4\dot{5}$ **c** 0.35
d $0.\dot{3}07692\dot{}$ **e** $0.0588235294117647\dot{}$

5 a 

b 

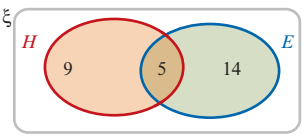
c 

d 

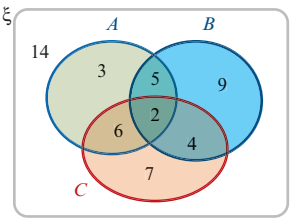
e 

6 a $(-\infty, 3)$ **b** $[-3, \infty)$ **c** $(-\infty, -3]$
d $(5, \infty)$ **e** $[-2, 3)$ **f** $[-2, 3]$
g $(-2, 3]$ **h** $(-5, 3)$

Exercise 2C

1 a 

b i 19 **ii** 9 **iii** 23

2 a 

b i 23 **ii** 37 **iii** 9

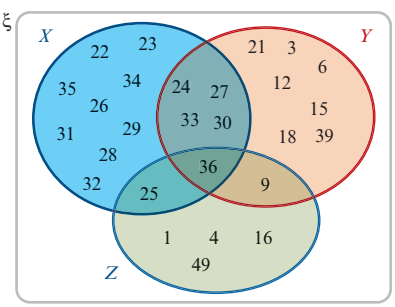
3 20%

4 7

5 a 5 **b** 10

6 45

7 a $x = 5$ **b** 16 **c** 0

8 a 

b i $X \cap Y \cap Z = \{36\}$ **ii** $|X \cap Y| = 5$

Cambridge Senior Maths for Western Australia
Mathematics Specialist 1&2
ISBN 978-1-009-25961-3

9 31 students; 15 black, 12 green, 20 red
10 $|M \cap F| = 11$ **11** 1
12 $x = 6$; 16 students **13** 102 students

Exercise 2D

1 a {1, 3, 4} **b** {1, 3, 4, 5, 6} **c** {4}
d {1, 2, 3, 4, 5, 6} **e** 3
f ∅, {4}, {5}, {6}, {4, 5}, {4, 6}, {5, 6}, {4, 5, 6}
2 36
3 4
4 150
5 a 64 **b** 32
6 a 72 **b** 72 **c** 36 **d** 108
7 a 12 **b** 38
8 88 **9** 80
10 4
11 a 756 **b** 700 **c** 360 **d** 1096
12 1 452 555 **13** 3417
14 5

Chapter 2 review

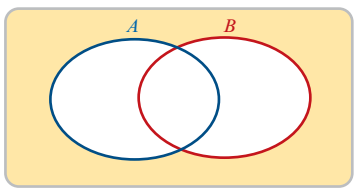
Short-answer questions

1 a $\frac{7}{90}$ **b** $\frac{5}{11}$ **c** $\frac{1}{200}$
d $\frac{81}{200}$ **e** $\frac{4}{15}$ **f** $\frac{6}{35}$
2 a 15 **b** 15
3 a 1 **b** 22 **c** 22
4 5 **5** 2 cm^2
6 a 57 **b** 3 **c** 32
7 12
8 192

Extended-response questions

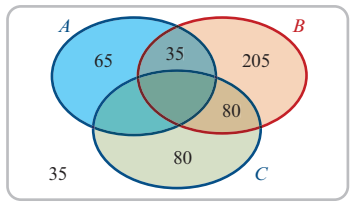
2 a i Region 8
ii Male, red hair, blue eyes
iii Male, not red hair, blue eyes
b i 5 **ii** 182
3 a i Students shorter than or equal to 180 cm
ii Students who are female or taller than 180 cm
iii Students who are male and shorter than or equal to 180 cm

b



$(A \cup B)' = A' \cap B'$ is shaded

4 a



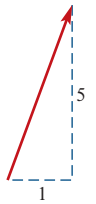
$|A \cap C| = 0$

b 160 **c** 65 **d** 0
5 a 24 **b** 4 **c** 24 **d** $\frac{3}{4}$

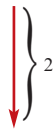
Chapter 3

Exercise 3A

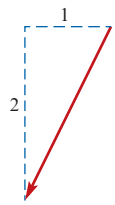
1 a



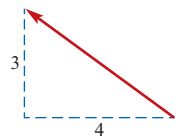
b



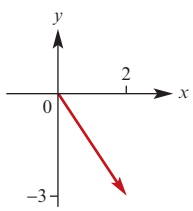
c



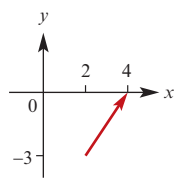
d



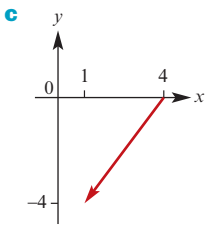
2 $a = 5,\ b = 1$
3 $a = 3,\ b = -15$
4 a $\begin{bmatrix} 1 \\ -2 \end{bmatrix}$ **b** $\begin{bmatrix} 2 \\ 2 \end{bmatrix}$ **c** $\begin{bmatrix} -1 \\ -3 \end{bmatrix}$ **d** $\begin{bmatrix} -2 \\ 3 \end{bmatrix}$ **e** $\begin{bmatrix} 1 \\ 3 \end{bmatrix}$
5 a i $\begin{bmatrix} 2 \\ -1 \end{bmatrix}$ **ii** $\begin{bmatrix} -5 \\ 0 \end{bmatrix}$ **iii** $\begin{bmatrix} 4 \\ -2 \end{bmatrix}$
b $\boldsymbol{a} + \boldsymbol{b} = -\boldsymbol{c}$
6 a



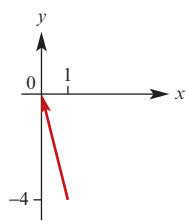
b



c

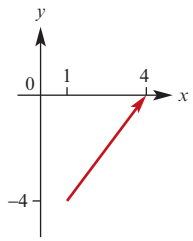


d

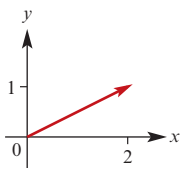


Cambridge Senior Maths for Western Australia
Mathematics Specialist 1&2
ISBN 978-1-009-25961-3

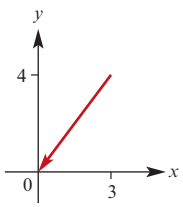
e



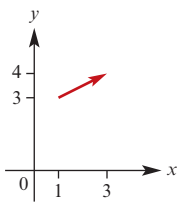
7 a



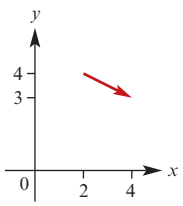
b



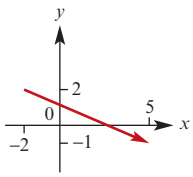
c



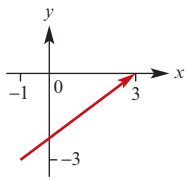
d



e

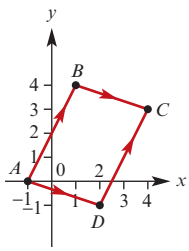


f



8 a and **c**

9 a b



d Parallelogram

10 $m = -11,\ n = 7$

11 a i $\boldsymbol{b} - \frac{1}{2}\boldsymbol{a}$ **ii** $\boldsymbol{b}$

b $\overrightarrow{MN} = \overrightarrow{AD}$

12 a $\overrightarrow{CB} = \boldsymbol{a} - \boldsymbol{b},\ \overrightarrow{MN} = \frac{1}{2}(\boldsymbol{b} - \boldsymbol{a})$

b $\overrightarrow{CB} = -2\overrightarrow{MN}$

13 a $\boldsymbol{a}$ **b** $\boldsymbol{b}$ **c** $2\boldsymbol{a}$ **d** $2\boldsymbol{b}$

e $-\boldsymbol{a}$ **f** $\boldsymbol{b} - \boldsymbol{a}$ **g** $\boldsymbol{a} + \boldsymbol{b}$

14 a $\boldsymbol{a}$ **b** $-\boldsymbol{b}$ **c** $\boldsymbol{a} + \boldsymbol{b}$

d $-\boldsymbol{a} - \boldsymbol{b}$ **e** $\boldsymbol{b} - \boldsymbol{a}$

15 a $\boldsymbol{a} - \boldsymbol{b}$ **b** $\frac{1}{3}(\boldsymbol{b} - \boldsymbol{a})$ **c** $\frac{1}{3}(\boldsymbol{a} + 2\boldsymbol{b})$

d $\frac{1}{9}(\boldsymbol{a} + 2\boldsymbol{b})$ **e** $\frac{1}{9}(4\boldsymbol{a} - \boldsymbol{b})$

16 a $\boldsymbol{u} + \boldsymbol{v}$ **b** $\boldsymbol{v} + \boldsymbol{w}$ **c** $\boldsymbol{u} + \boldsymbol{v} + \boldsymbol{w}$

17 a $\overrightarrow{OB} = \boldsymbol{u} + \boldsymbol{v},\ \overrightarrow{OM} = \boldsymbol{u} + \frac{1}{2}\boldsymbol{v}$ **b** $\boldsymbol{u} - \frac{1}{2}\boldsymbol{v}$

c $\frac{2}{3}\left(\boldsymbol{u} - \frac{1}{2}\boldsymbol{v}\right)$

d $\overrightarrow{OP} = \frac{2}{3}(\boldsymbol{u} + \boldsymbol{v}) = \frac{2}{3}\overrightarrow{OB}$ **e** 2 : 1

Exercise 3B

1 $2\boldsymbol{i} - 7\boldsymbol{j}$

2 a $5\boldsymbol{i} + 6\boldsymbol{j}$ **b** $-5\boldsymbol{i} + 6\boldsymbol{j}$ **c** $5\boldsymbol{i} - 6\boldsymbol{j}$

3 a 5 **b** 2 **c** 5 **d** 13

4 a 13 **b** $x = 2,\ y = -7$

5 $7\boldsymbol{i} + \frac{5}{2}\boldsymbol{j}$

6 a i $\frac{2}{5}\boldsymbol{i}$ **ii** $\frac{-2}{5}\boldsymbol{i} + \boldsymbol{j}$ **iii** $\frac{1}{6}\left(-\frac{2}{5}\boldsymbol{i} + \boldsymbol{j}\right)$

iv $\frac{1}{3}\boldsymbol{i} + \frac{1}{6}\boldsymbol{j}$ **v** $2\boldsymbol{i} + \boldsymbol{j}$

b i $\overrightarrow{ON} = \frac{1}{6}\overrightarrow{OA}$ **ii** 1 : 5

7 $4\sqrt{2}$ units

8 a $k = \frac{3}{2},\ \ell = \frac{1}{2}$ **b** $x = 6,\ y = 2$

c $x = 3, y = 3$ **d** $k = -\frac{1}{3},\ \ell = -\frac{5}{3}$

9 $3\boldsymbol{i} - 2\boldsymbol{j},\ \sqrt{13}$

10 a $-2\boldsymbol{i} + 4\boldsymbol{j}$ **b** $-6\boldsymbol{i} + \boldsymbol{j}$ **c** 5

11 a $D(-6, 3)$ **b** $F(4, -3)$ **c** $G\left(\frac{3}{2}, -\frac{3}{2}\right)$

12 $A(-1, -4),\ B(-2, 2),\ C(0, 10)$

13 a i $2\boldsymbol{i} - \boldsymbol{j}$ **ii** $-5\boldsymbol{i} + 4\boldsymbol{j}$ **iii** $\boldsymbol{i} + 7\boldsymbol{j}$

iv $6\boldsymbol{i} + 3\boldsymbol{j}$ **v** $6\boldsymbol{i} + 3\boldsymbol{j}$

b $D(8, 2)$

14 a $\overrightarrow{OP} = 12\boldsymbol{i} + 5\boldsymbol{j},\ \overrightarrow{PQ} = 6\boldsymbol{i} + 8\boldsymbol{j}$ **b** 13, 10

15 a i $\sqrt{29}$ **ii** $\sqrt{116}$ **iii** $\sqrt{145}$

b $(\sqrt{29})^2 + (\sqrt{116})^2 = (\sqrt{145})^2$

16 a i $-\boldsymbol{i} - 3\boldsymbol{j}$ **ii** $4\boldsymbol{i} + 2\boldsymbol{j}$ **iii** $-3\boldsymbol{i} + \boldsymbol{j}$

b i $\sqrt{10}$ **ii** $2\sqrt{5}$ **iii** $\sqrt{10}$

17 a i $-3\boldsymbol{i} + 2\boldsymbol{j}$ **ii** $7\boldsymbol{j}$

iii $-3\boldsymbol{i} - 5\boldsymbol{j}$ **iv** $\frac{1}{2}(-3\boldsymbol{i} - 5\boldsymbol{j})$

b $M\left(\frac{-3}{2}, \frac{9}{2}\right)$

18 a $\frac{1}{5}(3\boldsymbol{i} + 4\boldsymbol{j})$ **b** $\frac{1}{\sqrt{10}}(3\boldsymbol{i} - \boldsymbol{j})$

c $\frac{1}{\sqrt{2}}(-\boldsymbol{i} + \boldsymbol{j})$ **d** $\frac{1}{\sqrt{2}}(\boldsymbol{i} - \boldsymbol{j})$

e $\frac{6}{\sqrt{13}}\left(\frac{1}{2}\boldsymbol{i} + \frac{1}{3}\boldsymbol{j}\right)$ **f** $\frac{1}{\sqrt{13}}(3\boldsymbol{i} - 2\boldsymbol{j})$

Exercise 3C

1 a 17 **b** 13 **c** 8 **d** −10

e −4 **f** 3 **g** −58

Cambridge Senior Maths for Western Australia
Mathematics Specialist 1&2
ISBN 978-1-009-25961-3

2 **a** 5 **b** 13 **c** 8 **d** -5 **e** 13

3 **a** $15\sqrt{2}$ **b** $-15\sqrt{2}$

4 **a** $|\boldsymbol{a}|^2 + 4|\boldsymbol{b}|^2 + 4\boldsymbol{a} \cdot \boldsymbol{b}$ **b** $4\boldsymbol{a} \cdot \boldsymbol{b}$
c $|\boldsymbol{a}|^2 - |\boldsymbol{b}|^2$ **d** $|\boldsymbol{a}|$

5 **a** $-3\boldsymbol{i} + \boldsymbol{j}$ **b** $\sqrt{10}$ **c** 116.57°

6 $\sqrt{66}$

7 **a** $-\frac{11}{2}$ **b** $\frac{10}{3}$ **c** -1 **d** $\frac{-2 \pm \sqrt{76}}{6}$

8 **a** $-\boldsymbol{a} + q\boldsymbol{b}$ **b** $\frac{22}{29}$ **c** $\left(\frac{44}{29}, \frac{110}{29}\right)$

9 **a** 139.40° **b** 71.57° **c** 26.57° **d** 126.87°

11 **a** $\frac{3}{2}\boldsymbol{i}$ **b** 45° **c** 116.57°

12 **a** **i** $\frac{3}{2}\boldsymbol{i} + 2\boldsymbol{j}$ **ii** $\frac{1}{2}\boldsymbol{i} + 3\boldsymbol{j}$ **b** 27.41° **c** 55.30°

Exercise 3D

1 **a** $\frac{1}{\sqrt{10}}(\boldsymbol{i} + 3\boldsymbol{j})$ **b** $\frac{1}{\sqrt{2}}(\boldsymbol{i} + \boldsymbol{j})$ **c** $\frac{1}{\sqrt{2}}(\boldsymbol{i} - \boldsymbol{j})$

2 **a** **i** $\frac{1}{5}(3\boldsymbol{i} + 4\boldsymbol{j})$ **ii** $\sqrt{2}$
b $\frac{\sqrt{2}}{5}(3\boldsymbol{i} + 4\boldsymbol{j})$

3 **a** **i** $\frac{1}{5}(3\boldsymbol{i} + 4\boldsymbol{j})$ **ii** $\frac{1}{13}(5\boldsymbol{i} + 12\boldsymbol{j})$
b $\frac{1}{\sqrt{65}}(4\boldsymbol{i} + 7\boldsymbol{j})$

4 **a** $-\frac{11}{17}(\boldsymbol{i} - 4\boldsymbol{j})$ **b** $\frac{13}{17}(\boldsymbol{i} - 4\boldsymbol{j})$ **c** $4\boldsymbol{i}$

5 **a** 2 **b** $\frac{1}{\sqrt{5}}$ **c** $\frac{2\sqrt{3}}{\sqrt{7}}$ **d** $\frac{-1 - 4\sqrt{5}}{\sqrt{17}}$

6 **a** $\boldsymbol{a} = \boldsymbol{u} + \boldsymbol{w}$ where $\boldsymbol{u} = 2\boldsymbol{i}$ and $\boldsymbol{w} = \boldsymbol{j}$
b $\boldsymbol{a} = \boldsymbol{u} + \boldsymbol{w}$ where $\boldsymbol{u} = 2\boldsymbol{i} + 2\boldsymbol{j}$ and $\boldsymbol{w} = \boldsymbol{i} - \boldsymbol{j}$
c $\boldsymbol{a} = \boldsymbol{u} + \boldsymbol{w}$ where $\boldsymbol{u} = \boldsymbol{0}$ and $\boldsymbol{w} = -\boldsymbol{i} + \boldsymbol{j}$

7 **a** $2\boldsymbol{i} + 2\boldsymbol{j}$ **b** $\frac{1}{\sqrt{2}}(-\boldsymbol{i} + \boldsymbol{j})$

8 **a** $\frac{3}{2}(\boldsymbol{i} - \boldsymbol{j})$ **b** $\frac{5}{2}(\boldsymbol{i} + \boldsymbol{j})$ **c** $\frac{5\sqrt{2}}{2}$

9 **a** **i** $\boldsymbol{i} - \boldsymbol{j}$ **ii** $\boldsymbol{i} - 5\boldsymbol{j}$
b $\frac{3}{13}(\boldsymbol{i} - 5\boldsymbol{j})$ **c** $\frac{2\sqrt{26}}{13}$ **d** 2

Exercise 3E

1 **a** **i** $\frac{4}{5}\boldsymbol{p}$ **ii** $\frac{1}{5}\boldsymbol{p}$ **iii** $-\boldsymbol{p}$ **iv** $\frac{1}{5}(\boldsymbol{q} - \boldsymbol{p})$ **v** $\frac{1}{5}\boldsymbol{q}$
b RS and OQ are parallel
c $ORSQ$ is a trapezium
d 120 cm^2

2 **a** **i** $\frac{1}{3}\boldsymbol{a} + \frac{2}{3}\boldsymbol{b}$ **ii** $\frac{k}{7}\boldsymbol{a} + \frac{6}{7}\boldsymbol{b}$
b **i** 3 **ii** $\frac{7}{2}$

3 **a** **i** $\overrightarrow{OD} = 2\boldsymbol{i} - 0.5\boldsymbol{j}$, $\overrightarrow{OE} = \frac{15}{4}\boldsymbol{i} + \frac{9}{4}\boldsymbol{j}$
ii $\frac{\sqrt{170}}{4}$
b **i** $p\left(\frac{15}{4}\boldsymbol{i} + \frac{9}{4}\boldsymbol{j}\right)$
ii $(q + 2)\boldsymbol{i} + (4q - 0.5)\boldsymbol{j}$
c $p = \frac{2}{3}$, $q = \frac{1}{2}$

5 **a** **i** $\overrightarrow{AB} = \boldsymbol{c}$ **ii** $\overrightarrow{OB} = \boldsymbol{a} + \boldsymbol{c}$ **iii** $\overrightarrow{AC} = \boldsymbol{c} - \boldsymbol{a}$
b $|\boldsymbol{c}|^2 - |\boldsymbol{a}|^2$

6 **a** $\boldsymbol{r} + \boldsymbol{t}$ **b** $\frac{1}{2}(\boldsymbol{s} + \boldsymbol{t})$

Chapter 3 review

Short-answer questions

1 **a** $\frac{12}{7}$ **b** ± 9

2 $A(2, -1)$, $B(5, 3)$, $C(3, 8)$, $D(0, 4)$

3 $p = \frac{1}{6}$

4 **a** $\sqrt{26}$ **b** $\frac{1}{\sqrt{26}}(\boldsymbol{i} - 5\boldsymbol{j})$

5 6

6 **a** $\frac{1}{5}(4\boldsymbol{i} + 3\boldsymbol{j})$ **b** $\frac{16}{25}(4\boldsymbol{i} + 3\boldsymbol{j})$

7 **a** **i** $\boldsymbol{a} + \boldsymbol{b}$ **ii** $\frac{1}{3}(\boldsymbol{a} + \boldsymbol{b})$ **iii** $\boldsymbol{b} - \boldsymbol{a}$
iv $\frac{1}{3}(2\boldsymbol{a} - \boldsymbol{b})$ **v** $\frac{2}{3}(2\boldsymbol{a} - \boldsymbol{b})$
b $\overrightarrow{TR} = 2\overrightarrow{PT}$, so P, T and R are collinear

8 **a** $s = -2$, $t = 5$
b $\sqrt{29}$

9 $\sqrt{109}$ units

10 **a** $(-1, 10)$ **b** $h = 3$, $k = -2$

11 $m = 2$, $n = 1$

12 **a** $\boldsymbol{b} = \boldsymbol{a} + \boldsymbol{c}$ **b** $\boldsymbol{b} = \frac{2}{5}\boldsymbol{a} + \frac{3}{5}\boldsymbol{c}$

13 **a** 13 **b** 10 **c** 8 **d** -11
e -9 **f** 0 **g** -27

15 **a** $\frac{6}{5}$ **b** $\pm\frac{3}{\sqrt{2}}$ **c** $\frac{7}{3}$

16 **a** **i** $\overrightarrow{AB} = -\boldsymbol{i}$ **ii** $\overrightarrow{AC} = -5\boldsymbol{j}$
b $\boldsymbol{0}$ **c** 1

Extended-response questions

1 **a** $\begin{bmatrix} -31 \\ -32 \end{bmatrix}$ **b** $\begin{bmatrix} -15 \\ -20 \end{bmatrix}$ **c** $|\overrightarrow{OR}| = 25$

2 **a** $\sqrt{34}$ **b** $\sqrt{10} - \sqrt{20}$ **c** $\boldsymbol{r} = \boldsymbol{i} - 9\boldsymbol{j}$

3 **a** $(25, -7)$, $\begin{bmatrix} 7 \\ 24 \end{bmatrix}$ **b** $\begin{bmatrix} -20 \\ 15 \end{bmatrix}$

4 **a** $(12, 4)$ **b** $\begin{bmatrix} k - 12 \\ -4 \end{bmatrix}$
c $\sqrt{160}$, k, $\sqrt{(k - 12)^2 + 16}$, $k = \frac{40}{3}$
d 34.7°

Cambridge Senior Maths for Western Australia Mathematics Specialist 1&2 ISBN 978-1-009-25961-3

Chapter 4

Exercise 4A

1 $T_1 = 3$ kg wt, $T_2 = 7$ kg wt

2 $T_1 = T_2 = \frac{5\sqrt{2}}{2}$ kg wt

3 90°

4 $T_1 = 14.99$ kg wt, $T_2 = 12.10$ kg wt

5 28.34 kg wt, W48.5°S

6 $T = 40$ kg wt, $N = 96$ kg wt

7 $F = 6.39$ kg wt

8 **a** No **b** Yes

9 146.88°, 51.32°, 161.8°

10 **a** 7.5 kg wt **b** 9.64 kg wt **c** 7.62 kg wt

11 32.97 kg wt, 26.88 kg wt, 39.29 kg wt, $W = 39.29$ kg

Exercise 4B

1 13.05 kg wt

2 5.74 kg wt

3 3.73 kg wt, 8.83 kg wt

4 4.13 kg wt

5 6.93 kg wt

6 31.11 kg, 23.84 kg wt

7 44.10 kg, 22.48° to the vertical

8 6.43 kg wt, 7.66 kg wt, 11.92 kg

9 3.24 kg wt

Chapter 4 review

Short-answer questions

1 9 kg wt, 12 kg wt

2 $10\sqrt{3}$ kg wt, 150° to the 10 kg wt

3 $14\sqrt{5}$ kg wt, $28\sqrt{5}$ kg wt

4 $5\sqrt{3}$ kg wt

5 $-\frac{7}{8}$

6 $\frac{40\sqrt{3}}{3}$ kg wt

7 $\frac{15\sqrt{2}}{2}$ kg wt

8 28 kg, $14\sqrt{3}$ kg wt

9 $4\sqrt{3}$ kg wt

Chapter 5

Short-answer questions

1 24

2 360

3 **a** 125 **b** 60

4 **a** 9 **b** 25

5 **a** 24 **b** 30 **c** 28 **d** 45

6 **a** 120 **b** 120

7 **a** 120 **b** 36

8 **a** 96 **b** 24 **c** 72 **d** 60

9 10

10 **a** 20 **b** 325 **c** 210 **d** 56

11 **a** 28 **b** 21 **c** $2^8 = 256$

12 60

13 120

14 7

15 **a** 3 **b** 12 **c** 8

16 **a** 13 **b** 13 **c** 13 **d** −13
e 5 **f** 0 **g** −13

17 **a** $m = \frac{46}{11}$, $n = -\frac{18}{11}$ **b** $p = -48$
c $p = 3, 5$

18 $F = 7$ kg wt, $\cos\theta = \frac{-31}{49}$

19 $\cos\theta = \frac{-5}{8}$

20 $Q^2 = 100 - 48\sqrt{2}$

21 **a** $T = 5$ kg wt, $N = 5\sqrt{3}$ kg wt
b $T = \frac{10\sqrt{3}}{3}$ kg wt, $N = \frac{20\sqrt{3}}{3}$ kg wt

22 $T = 10$ kg wt, $\tan\theta = \frac{3}{4}$

23 $\frac{50}{13}$ kg wt, $\frac{120}{13}$ kg wt

Extended-response questions

1 **a** 2160 **b** 360 **c** 900 **d** 1260

2 **a** 70 **b** 30 **c** 15 **d** 55

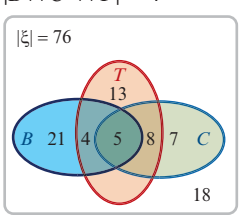
3 **a** 20 **b** 4 **c** 68

4 **a** 420 **b** 60 **c** 120 **d** 24

5 **a** 300 **b** 10 and 15

6 **a** $|B' \cap C' \cap T| = |C \cap T|$,
$|B \cap C' \cap T'| = 3|B' \cap C \cap T'|$,
$|B \cap C' \cap T| = 4$

b



c **i** 5 **ii** 0

7 **a** $\overrightarrow{AE} = \frac{1}{t+1}(2\boldsymbol{a} + t\boldsymbol{b})$

b $\overrightarrow{AE} = \frac{1}{8}(7\boldsymbol{a} + \overrightarrow{AF})$ **d** $t = \frac{9}{7}$

8 **b** $(n-1)\boldsymbol{a} - n\boldsymbol{b} + \boldsymbol{c}$

Cambridge Senior Maths for Western Australia Mathematics Specialist 1&2 ISBN 978-1-009-25961-3

9 a $\overrightarrow{AB} = \boldsymbol{b} - \boldsymbol{a}$, $\overrightarrow{PQ} = \frac{-3}{10}\boldsymbol{a} + \frac{1}{2}\boldsymbol{b}$

b i $n\left(\frac{-3}{10}\boldsymbol{a} + \frac{1}{2}\boldsymbol{b}\right)$ **ii** $\left(k + \frac{1}{2}\right)\boldsymbol{b} - \frac{1}{2}\boldsymbol{a}$

c $n = \frac{5}{3}$, $k = \frac{1}{3}$

10 a $4\sqrt{2}$ km/h blowing from the south-west

b $\sqrt{5}$ km/h; 200 m downstream

c 43.1 km/h at bearing 80° **d** 222°

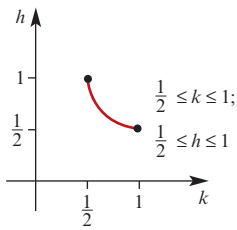
11 b ii $\overrightarrow{ZG} = \frac{1}{3h}\overrightarrow{ZH} + \frac{1}{3k}\overrightarrow{ZK}$

iii $\frac{1}{h} + \frac{1}{k} = 3$ **iv** $h = \frac{2}{3}$; similarity

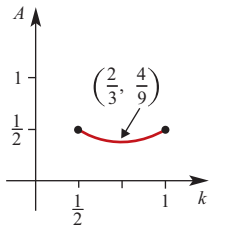
v $\frac{4}{9}$ cm^2

vi $h = \frac{1}{2}$; H is midpoint of ZX, $K = Y$

vii



viii



Chapter 6

See solutions supplement

Chapter 7

Exercise 7A

1 a $x = 100$, $y = 50$

b $x = 126$, $y = 252$, $z = 54$

c $y = 145$, $z = 290$

d $x = 180$, $y = 90$

e $x = 45$, $y = 90$, $z = 270$

2 a $x = 68$, $y = 121$ **b** $x = 112$, $y = 87$

c $x = 50$, $y = 110$

3 110°, 110°, 140°

4 $\angle ABC = 98°$, $\angle BCD = 132°$, $\angle CDE = 117°$, $\angle DEA = 110°$, $\angle EAB = 83°$

7 60° or 120°

8 $\angle P = 78°$, $\angle Q = 74°$, $\angle R = 102°$, $\angle S = 106°$

Exercise 7B

1 a $x = 73$, $y = 81$ **b** $x = 57$, $q = 57$

c $x = 53$, $y = 74$, $z = 53$

d $x = 60$, $y = 60$, $z = 20$, $w = 100$

e $w = 54$, $x = 54$, $y = 72$, $z = 54$

2 a 40° **b** 40° **c** 80°

3 32° and 148°

4 $\angle ACB = 40°$, $\angle ABC = 70°$, $\angle BAT = 40°$

Exercise 7C

1 a 10 cm **b** 6 cm

2 7 cm

3 $5\sqrt{6}$ cm

Chapter 7 review

Short-answer questions

1 $\angle MCN = 18°$

2 a $x = 110$, $y = 70$ **b** $x = 35$, $y = 35$

c $x = 47$, $y = 53$, $z = 100$

d $x = 40$, $y = 40$, $z = 70$

6 a $x = 66$ **b** $x = 116$ **c** $x = 66$, $y = 114$

8 3 cm

Extended-response questions

5 b 24 cm^2

Chapter 8

Exercise 8A

1 a 4π **b** 3π **c** $-\frac{5\pi}{2}$ **d** $\frac{\pi}{12}$

e $-\frac{\pi}{18}$ **f** $-\frac{7\pi}{4}$

2 a 225° **b** −120° **c** 105° **d** −330°

e 260° **f** −165°

3 a 0 **b** −1 **c** 1 **d** −1

4 a −1 **b** −1 **c** 1 **d** 1

Exercise 8B

1 a $-\frac{1}{\sqrt{2}}$ **b** $-\frac{1}{\sqrt{2}}$ **c** 1 **d** 1 **e** $\frac{1}{\sqrt{2}}$

f $\frac{1}{\sqrt{2}}$ **g** 0 **h** $\frac{\sqrt{3}}{2}$ **i** 0 **j** 0

k 1 **l** 0 **m** $-\frac{1}{2}$ **n** −1 **o** −1

2 a $\frac{1}{\sqrt{2}}$ **b** $\frac{1}{2}$ **c** $\frac{\sqrt{3}}{2}$ **d** $-\frac{1}{2}$

e $\frac{1}{\sqrt{2}}$ **f** $\frac{\sqrt{3}}{2}$

3 a 0.6 **b** 0.6 **c** 0.3 **d** −0.3

e −0.3 **f** 0.6 **g** −0.6 **h** −0.3

Cambridge Senior Maths for Western Australia Mathematics Specialist 1&2 ISBN 978-1-009-25961-3

4 $\cos x = -\frac{\sqrt{3}}{2}, \tan x = -\frac{1}{\sqrt{3}}$

5 $\sin x = -\frac{\sqrt{51}}{10}, \tan x = \frac{\sqrt{51}}{7}$

6 $\cos x = -\frac{\sqrt{3}}{2}, \tan x = \frac{1}{\sqrt{3}}$

7 $\cos x = \frac{\sqrt{91}}{10}, \tan x = -\frac{3\sqrt{91}}{91}$

Exercise 8C

1 $2\pi - a,\ 2\pi - b,\ 2\pi - c,\ 2\pi - d$

2 a $\frac{4\pi}{3}, \frac{5\pi}{3}$ **b** $\frac{2\pi}{3}, \frac{5\pi}{6}, \frac{5\pi}{3}, \frac{11\pi}{6}$

c $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$ **d** $\frac{5\pi}{6}, \frac{3\pi}{2}$

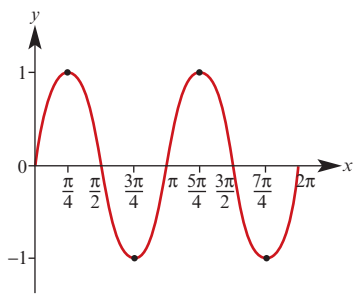
e $0, \frac{\pi}{3}, \pi, \frac{4\pi}{3}, 2\pi$ **f** $\frac{\pi}{2}, \frac{2\pi}{3}, \frac{3\pi}{2}, \frac{5\pi}{3}$

3 a $-\frac{5\pi}{6}, -\frac{\pi}{6}$ **b** $0, -\frac{2\pi}{3}, -\frac{\pi}{3}, \frac{\pi}{3}, \frac{2\pi}{3}, -\pi, \pi$

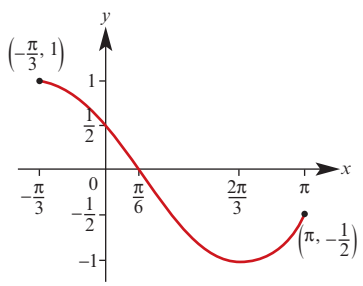
c 0 **d** $0, -\frac{2\pi}{3}$ **e** $-\frac{5\pi}{6}, -\frac{\pi}{2}, \frac{\pi}{6}, \frac{\pi}{2}$

Exercise 8D

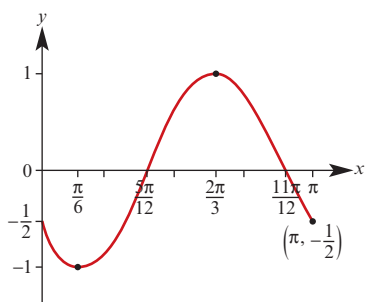
1 a



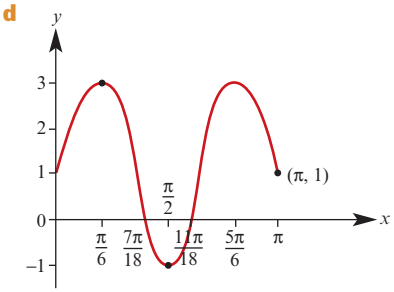
b



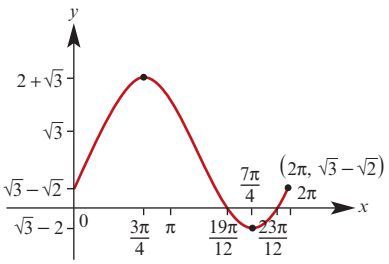
c



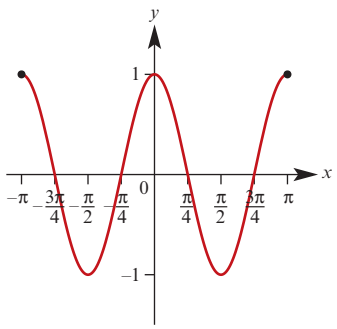
d



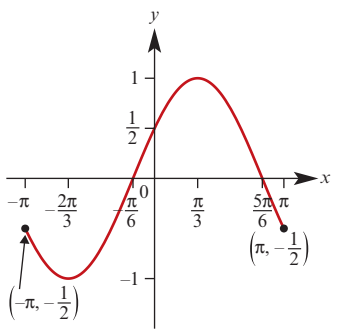
e



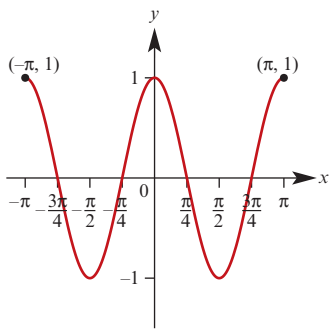
f



2 a

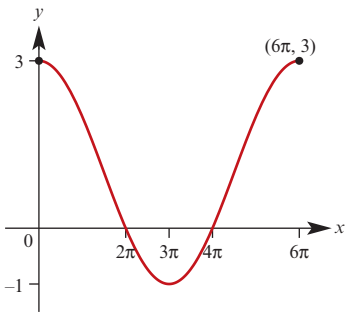


b

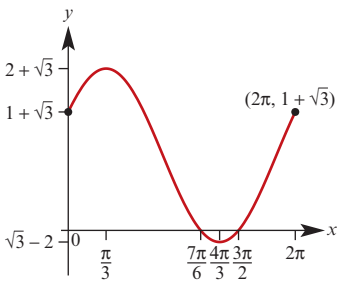


Cambridge Senior Maths for Western Australia
Mathematics Specialist 1&2
ISBN 978-1-009-25961-3

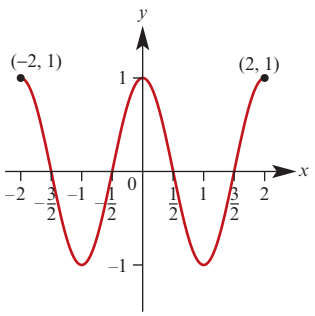
c



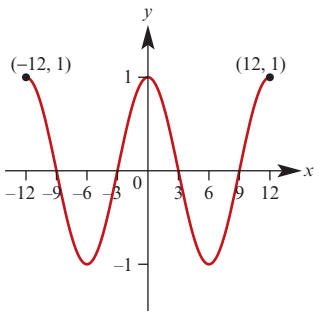
d



e



f



Exercise 8E

1 **a** 1 **b** $\sqrt{3}$ **c** $\frac{1}{\sqrt{3}}$

2 **a** $\sqrt{3}$ **b** $\frac{1}{\sqrt{3}}$ **c** -1

3 **a** $\frac{-\sqrt{17}}{17}$ **b** $\frac{-4\sqrt{17}}{17}$ **c** $\frac{-1}{4}$ **d** $\frac{-1}{4}$

4 **a** $\frac{\sqrt{21}}{7}$ **b** $\frac{-2\sqrt{7}}{7}$ **c** $\frac{\sqrt{3}}{2}$ **d** $\frac{-\sqrt{3}}{2}$

5 **a** $\frac{3\pi}{4}, \frac{7\pi}{4}$ **b** $\frac{\pi}{3}, \frac{4\pi}{3}$ **c** $\frac{\pi}{6}, \frac{7\pi}{6}$

d $-\frac{7\pi}{8}, -\frac{3\pi}{8}, \frac{\pi}{8}, \frac{5\pi}{8}$ **e** $-\frac{5\pi}{6}, -\frac{\pi}{3}, \frac{\pi}{6}, \frac{2\pi}{3}$

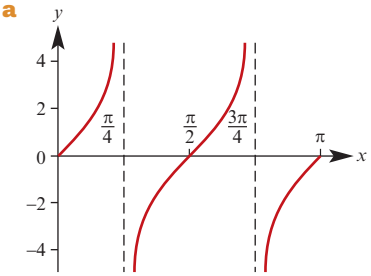
f $-\frac{7\pi}{12}, -\frac{\pi}{12}, \frac{5\pi}{12}, \frac{11\pi}{12}$

6 **a** $\frac{3\pi}{8}, \frac{7\pi}{8}, \frac{11\pi}{8}, \frac{15\pi}{8}$

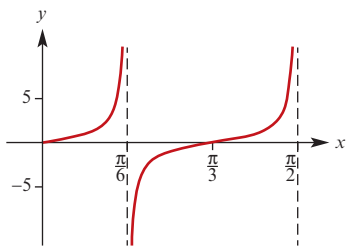
b $-\frac{7\pi}{8}, -\frac{3\pi}{8}, \frac{\pi}{8}, \frac{5\pi}{8}$

c $-\frac{13\pi}{18}, -\frac{7\pi}{18}, -\frac{\pi}{18}, \frac{5\pi}{18}, \frac{11\pi}{18}, \frac{17\pi}{18}$ **d** $-\frac{\pi}{6}$

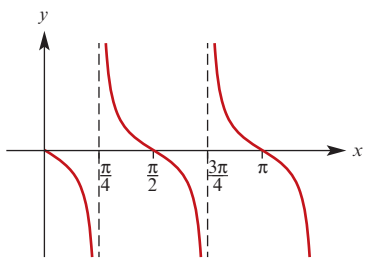
7 **a**



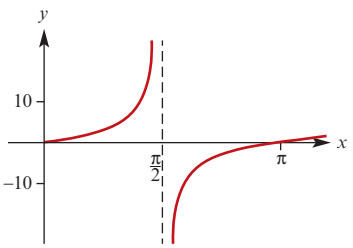
b



c

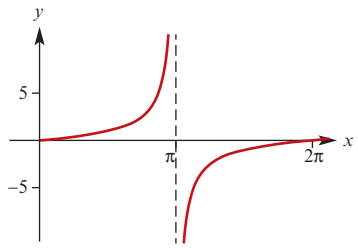


d

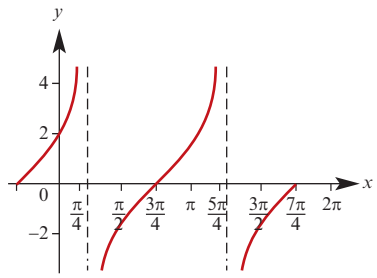


Cambridge Senior Maths for Western Australia Mathematics Specialist 1&2 ISBN 978-1-009-25961-3

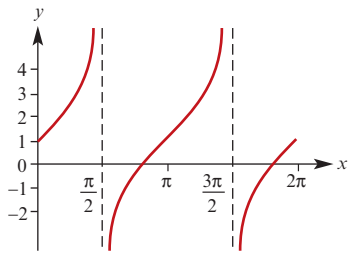
e



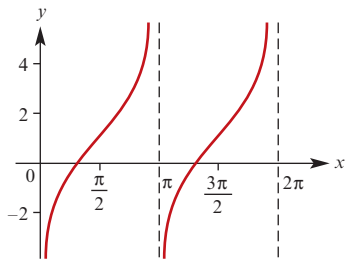
f



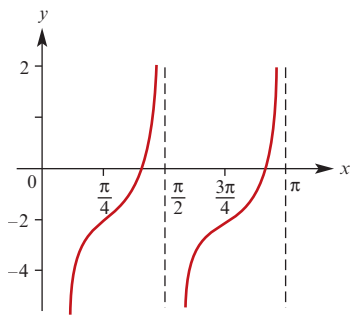
g



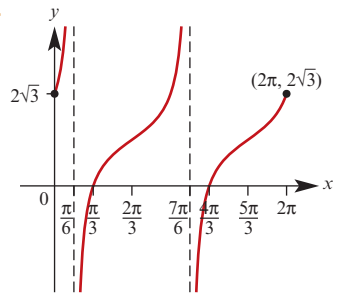
h



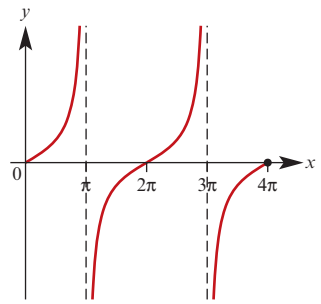
i



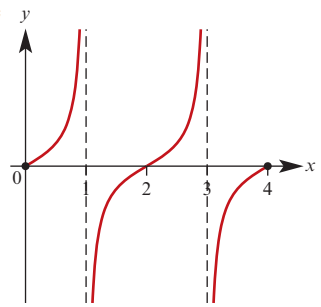
8 a



b



c



Exercise 8F

1 a i 2π **ii** 4π **iii** -4π

b i $\frac{4\pi}{3}, \frac{8\pi}{3}$ **ii** $\frac{14\pi}{3}, \frac{10\pi}{3}$ **iii** $\frac{-14\pi}{3}, \frac{-10\pi}{3}$

2 a $2n\pi \pm \frac{\pi}{6}, n \in \mathbb{Z}$

b $\frac{2n\pi}{3} + \frac{\pi}{9}$ or $\frac{2n\pi}{3} + \frac{2\pi}{9}, n \in \mathbb{Z}$

c $n\pi + \frac{\pi}{3}, n \in \mathbb{Z}$

3 a $\frac{\pi}{6}, \frac{5\pi}{6}$ **b** $\frac{\pi}{12}, \frac{11\pi}{12}$ **c** $\frac{\pi}{3}, \frac{5\pi}{6}$

4 $\frac{-11\pi}{6}, \frac{-7\pi}{6}, \frac{\pi}{6}, \frac{5\pi}{6}$

5 $\frac{-\pi}{3}, \frac{\pi}{3}, \frac{5\pi}{3}$

ISBN 978-1-009-25961-3

6 a $x = n\pi - \frac{\pi}{6}$ or $x = n\pi - \frac{\pi}{2}, n \in \mathbb{Z}$

b $x = \frac{n\pi}{2} - \frac{\pi}{12}, n \in \mathbb{Z}$

c $x = 2n\pi + \frac{5\pi}{6}$ or $x = 2n\pi - \frac{\pi}{2}, n \in \mathbb{Z}$

7 $x = \frac{(4n-1)\pi}{4}$ or $x = n\pi, n \in \mathbb{Z}$;
$\left\{\frac{-5\pi}{4}, -\pi, \frac{\pi}{4}, 0, \frac{3\pi}{4}, \pi, \frac{7\pi}{4}\right\}$

8 $x = \frac{n\pi}{3}, n \in \mathbb{Z}$; $\left\{-\pi, \frac{-2\pi}{3}, \frac{-\pi}{3}, 0\right\}$

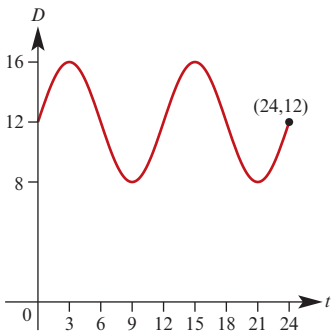
9 $x = \frac{6n-1}{12}$ or $x = \frac{3n+2}{6}, n \in \mathbb{Z}$;
$\left\{\frac{-2}{3}, \frac{-7}{12}, \frac{-1}{6}, \frac{-1}{12}, \frac{1}{3}, \frac{5}{12}, \frac{5}{6}, \frac{11}{12}\right\}$

Exercise 8G

1 a i 0.00 hours **ii** 24.00 hours

b 13 February ($t = 1.48$),
24 October ($t = 9.86$)

2 a



b $t \in [0, 6] \cup [12, 18]$ **c** 15.9 m

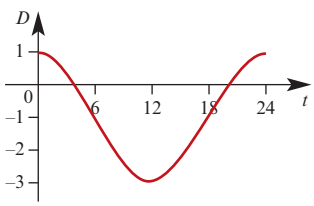
3 a 7 metres **b** 1 metre

c $t = \frac{1}{4}$ or $t = \frac{5}{4}$ **d** $t = \frac{3}{4}$ or $t = \frac{7}{4}$

e Particle oscillates between $x = 1$ and $x = 7$

4 a 19.5°C **b** $D = -1 + 2\cos\left(\frac{\pi t}{12}\right)$

c



d $\{t : 4 < t < 20\}$

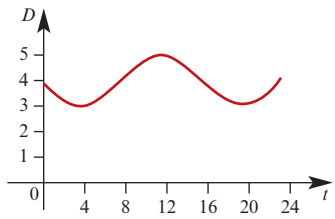
5 a 2 a.m. **b** 8 a.m. and 8 p.m.

6 a i $\frac{3}{2}$ **ii** 12 **iii** $d(t) = \frac{7}{2} - \frac{3}{2}\cos\left(\frac{\pi}{6}t\right)$

iv 1.5 m

b Between 9 p.m. and 3 a.m. and between 9 a.m. and 3 p.m. each day

7 a



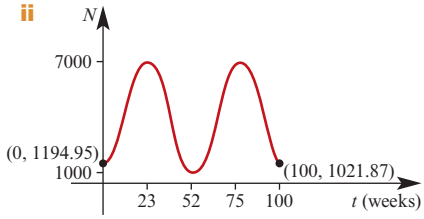
b The boat can enter at 8 a.m. and must leave by 4 p.m.

c The boat can enter at 6:40 a.m. and must leave by 5:20 p.m.

8 a i 52 weeks **ii** 3000 **iii** [1000, 7000]

b i $N(0) = 1194.95$, $N(100) = 1021.87$

ii



c i $t = 23, 75$ **ii** 49

d $\left(14\frac{1}{3}, 31\frac{2}{3}\right) \cup \left(66\frac{1}{3}, 83\frac{2}{3}\right)$

e $d = 25\,000$, $a = 15\,000$, $b = 10$, $c = 5$

Chapter 8 review

Short-answer questions

1 a $\frac{13\pi}{6}$ **b** $\frac{14\pi}{3}$ **c** $\frac{37\pi}{6}$ **d** $\frac{71\pi}{12}$ **e** $\frac{11\pi}{12}$
f $\frac{5\pi}{2}$ **g** $\frac{7\pi}{3}$ **h** $\frac{13\pi}{6}$ **i** $\frac{2\pi}{9}$

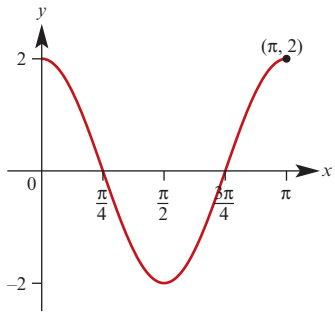
2 a 330° **b** 765° **c** 405° **d** 105°
e 1530° **f** −495° **g** −225° **h** −585°
i 1035°

3 a $\frac{1}{\sqrt{2}}$ **b** $-\frac{1}{\sqrt{2}}$ **c** −1 **d** 0 **e** $\frac{\sqrt{3}}{2}$
f −1 **g** $-\sqrt{3}$ **h** 1

4 a 4, 4π **b** 5, $\frac{\pi}{3}$ **c** $\frac{1}{3}, \frac{\pi}{2}$ **d** 2, $\frac{2\pi}{5}$
e 7, 8 **f** $\frac{2}{3}$, 3

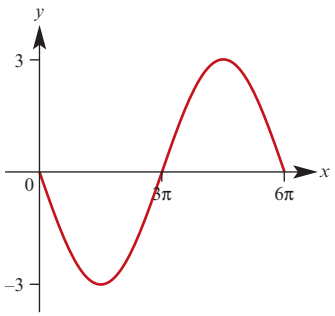
5 a 5, 1 **b** 9, −1

6 a $y = 2\cos(2x)$

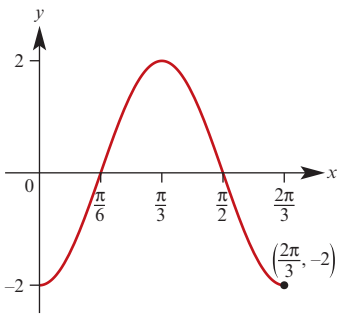


Cambridge Senior Maths for Western Australia
Mathematics Specialist 1&2
ISBN 978-1-009-25961-3

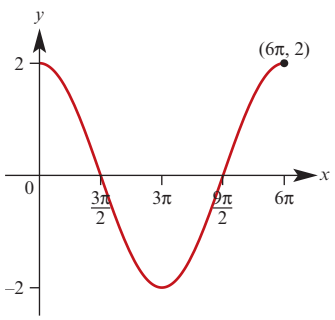
b $y = -3\sin\left(\frac{x}{3}\right)$



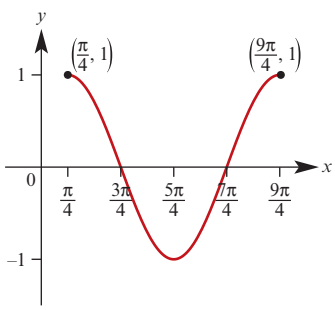
c $y = -2\cos(3x)$



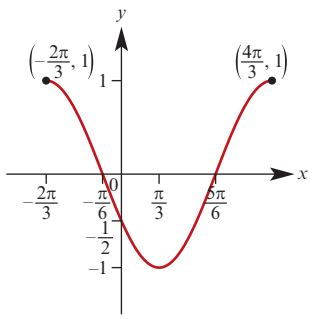
d $y = 2\cos\left(\frac{x}{3}\right)$



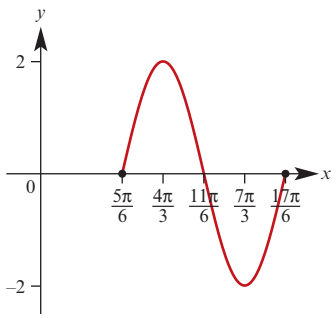
e $y = \cos\left(x - \frac{\pi}{4}\right)$



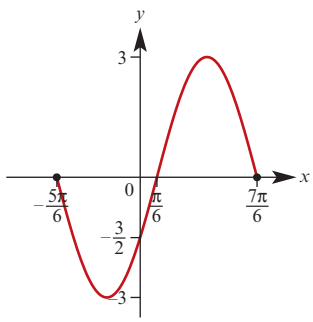
f $y = \cos\left(x + \frac{2\pi}{3}\right)$



g $y = 2\sin\left(x - \frac{5\pi}{6}\right)$



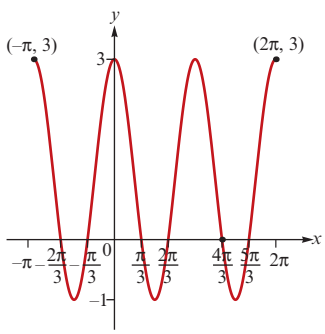
h $h = -3\sin\left(x + \frac{5\pi}{6}\right)$



7 a $-\frac{5\pi}{6}, \frac{5\pi}{6}$ **b** $-\frac{7\pi}{12}, -\frac{5\pi}{12}, \frac{5\pi}{12}, \frac{7\pi}{12}$

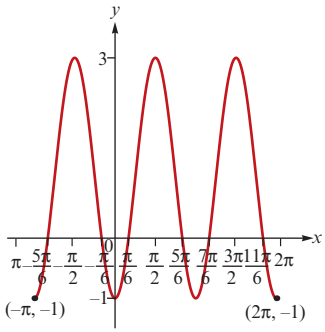
c $\pi, \frac{5\pi}{3}$ **d** $\frac{2\pi}{3}$ **e** $\pi, \frac{5\pi}{3}$

8 a $f(x) = 2\cos(2x) + 1$

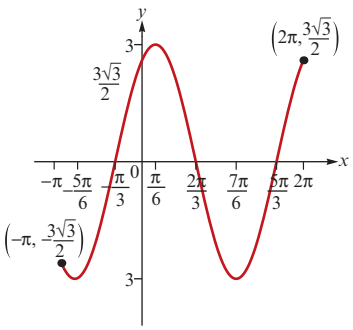


Cambridge Senior Maths for Western Australia
Mathematics Specialist 1&2
ISBN 978-1-009-25961-3

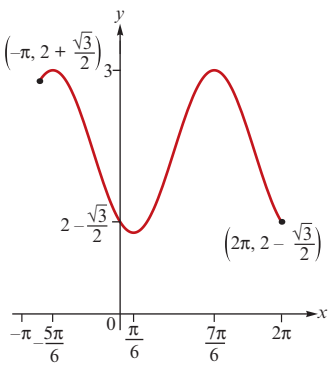
b $f(x) = 1 - 2\cos(2x)$



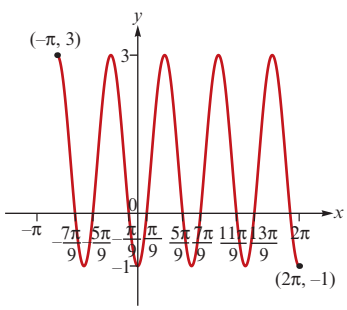
c $f(x) = 3\sin\left(x + \frac{\pi}{3}\right)$



d $f(x) = 2 - \sin\left(x + \frac{\pi}{3}\right)$



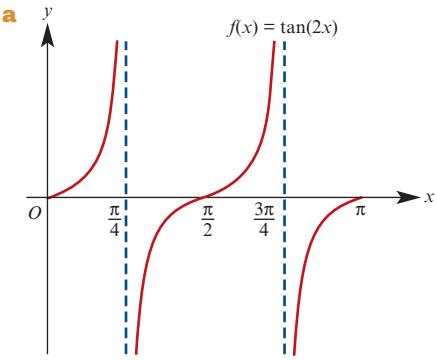
e $f(x) = 1 - 2\cos(3x)$



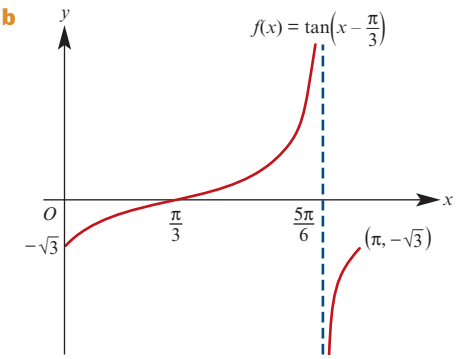
9 a $\frac{2\pi}{3}, \frac{5\pi}{3}$ **b** $\frac{\pi}{9}, \frac{4\pi}{9}, \frac{7\pi}{9}, \frac{10\pi}{9}, \frac{13\pi}{9}, \frac{16\pi}{9}$

c $\frac{3\pi}{2}$ **d** $\frac{\pi}{8}, \frac{5\pi}{8}, \frac{9\pi}{8}, \frac{13\pi}{8}$

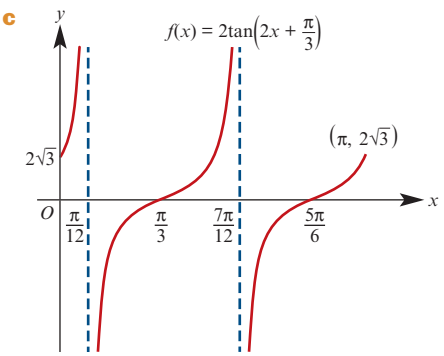
10 a



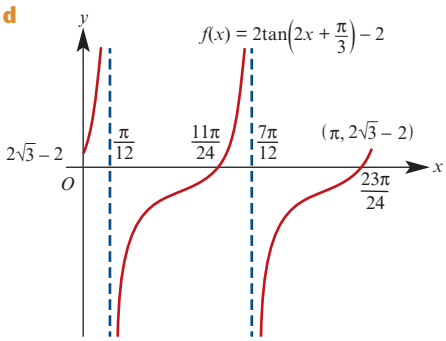
b



c



d



11 a $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$ **b** $\frac{\pi}{12}, \frac{5\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12}$

c $\frac{\pi}{18}, \frac{11\pi}{18}, \frac{13\pi}{18}, \frac{23\pi}{18}, \frac{25\pi}{18}, \frac{35\pi}{18}$

d $\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

12 0°, 60°, 180°, 300°, 360°

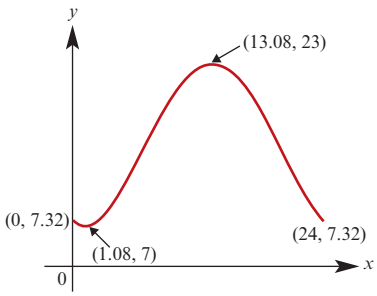
Cambridge Senior Maths for Western Australia Mathematics Specialist 1&2 ISBN 978-1-009-25961-3

13 a $n\pi - \frac{\pi}{4}, n \in \mathbb{Z}$ **b** $\frac{2n\pi}{3}, n \in \mathbb{Z}$
c $-\frac{\pi}{4} + n\pi, n \in \mathbb{Z}$

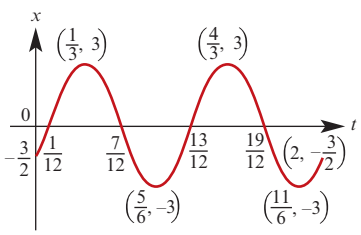
Extended-response questions

1 a i $a = 13.4$ **ii** $b = 2$ **iii** $k = 12$
b 3 a.m., 9 a.m., 3 p.m., 9 p.m.
c $2 < t < 10,\ 14 < t < 22$

2 a 7.3°C **b** Min = 7°C; Max = 23°C
c Between 9:40 a.m. and 4:30 p.m.
d



3 a $a = \frac{\pi}{6}$
b



c 3 m **d** $\frac{5}{6}$ s **e** 1 s **f** $\frac{1}{4}$ s
g i 24 m **ii** 30 m

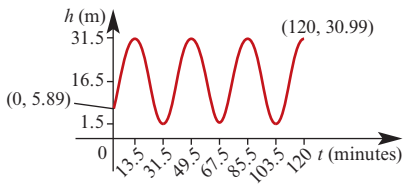
4 a $p = 6,\ q = 4.2$ **b** 3 a.m., 3 p.m.
c 6 m **d** 7 a.m., 11 a.m., 7 p.m., 11 p.m.
e 8 hours

5 a i $-1 < k < 1$
ii $k = -1$ or $k = 3$
iii $k < -1$ or $k > 3$
b A translation of 1 unit in the negative direction of the y-axis, followed by a dilation of factor $\frac{1}{2}$ from the x-axis and a dilation of factor 3 from the y-axis
c i $h = \frac{\pi}{2}$ **ii** $h = \frac{\pi}{6}$

6 a A translation of $\frac{\pi}{2}$ units in the positive direction of the x-axis
b A dilation of factor $\frac{1}{2}$ from the y-axis, followed by a translation of $\frac{\pi}{4}$ units in the negative direction of the x-axis, and then a dilation of factor $\frac{1}{4}$ from the x-axis
c i $y = -\sin\left(\frac{\pi x}{2}\right) + 4$
ii Range = [3, 5]; Period = 4

7 a For N: Max = 7000, occurs in April ($t = 4$); Min = 1000, occurs in October ($t = 10$).
For M: Max = 8500, occurs in June ($t = 6$); Min = 2500, occurs at end of January and November ($t = 1$ and $t = 11$)
b $t = 4.31$ (April), population is 6961; $t = 0.24$ (January), population is 2836
c 145 556 in May ($t = 5.19$)
d $t = 7.49$ (July)

8 a



b 5.89 m **c** 27.51 s **d** 6 times **e** 20 times
f 4.21 m **g** 13.9 m

Chapter 9

Exercise 9A

1 a -1 **b** $-\sqrt{2}$ **c** $\frac{-2}{\sqrt{3}} = \frac{-2\sqrt{3}}{3}$ **d** 1
e -2 **f** 2 **g** $\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$ **h** 2

2 a -1 **b** $\frac{-2}{\sqrt{3}} = \frac{-2\sqrt{3}}{3}$ **c** 1
d $\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$ **e** $-\sqrt{2}$ **f** $\frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$
g -1 **h** $\frac{-2}{\sqrt{3}} = \frac{-2\sqrt{3}}{3}$ **i** $\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

3 a $\frac{\pi}{6}, \frac{5\pi}{6}$ **b** $\frac{\pi}{6}, \frac{7\pi}{6}$ **c** $\frac{3\pi}{4}, \frac{5\pi}{4}$ **d** $\frac{\pi}{4}, \frac{5\pi}{4}$

4 a $-\frac{8}{17}$ **b** $\frac{15}{17}$ **c** $-\frac{15}{8}$

5 $\cos\theta = \frac{24}{25},\ \sin\theta = -\frac{7}{25}$

6 $-\frac{\sqrt{29}}{5}$

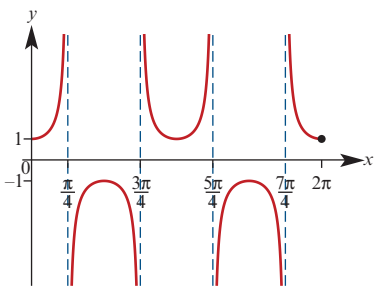
7 $\frac{8}{31}$

8 $\frac{15}{4(9+\sqrt{5})} = \frac{15(6-\sqrt{5})}{124}$

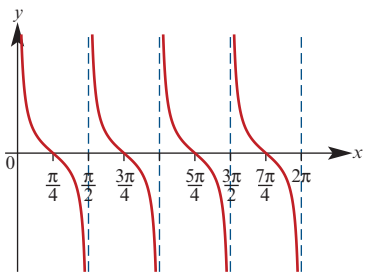
Cambridge Senior Maths for Western Australia Mathematics Specialist 1&2
ISBN 978-1-009-25961-3

Exercise 9B

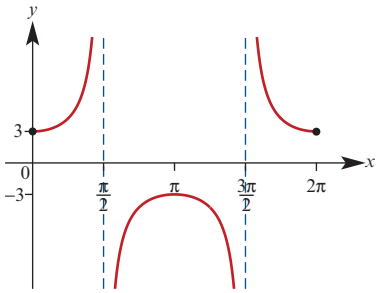
1 a



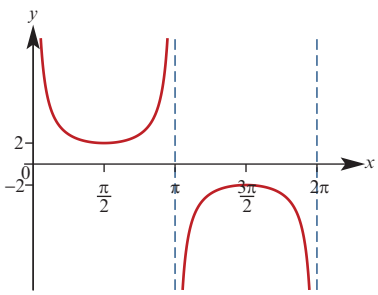
b



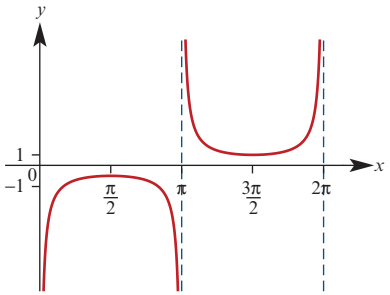
c



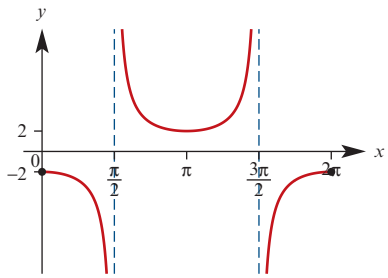
d



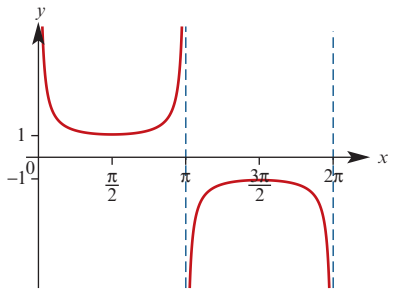
e



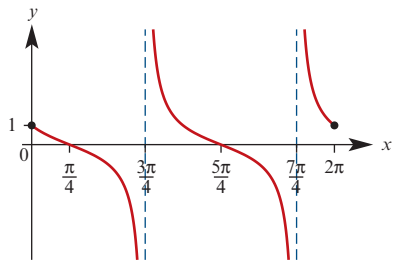
f



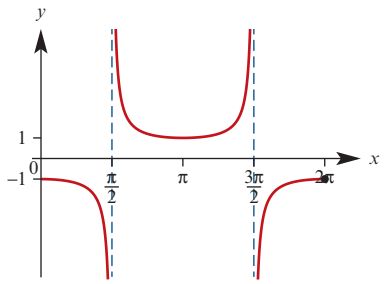
2 a



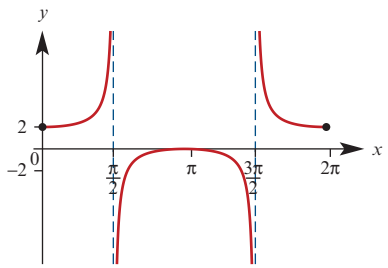
b



c

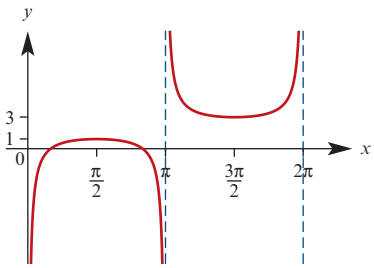


d

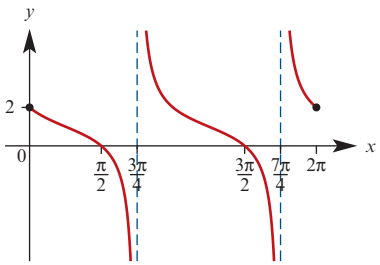


Cambridge Senior Maths for Western Australia
Mathematics Specialist 1&2
ISBN 978-1-009-25961-3

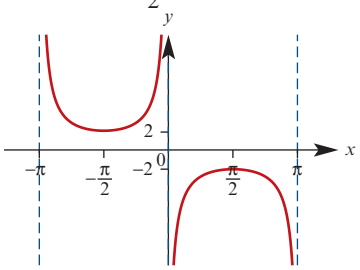
e



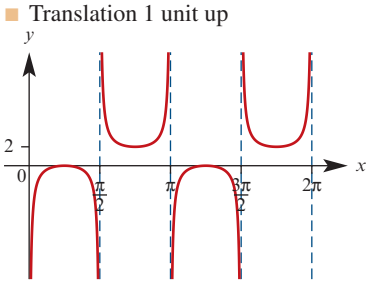
f



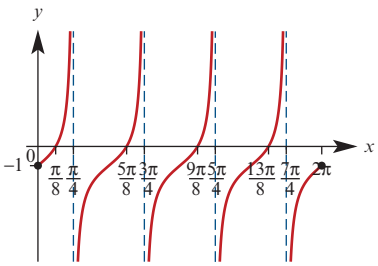
3 ■ Reflection in the x-axis
■ Dilation of factor 2 from the x-axis
■ Translation $\frac{\pi}{2}$ units to the right



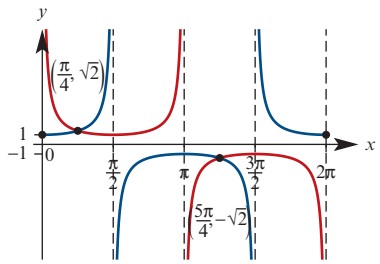
4 ■ Reflection in the y-axis
■ Dilation of factor $\frac{1}{2}$ from the y-axis
■ Translation 1 unit up



5 ■ Reflection in the x-axis
■ Dilation of factor $\frac{1}{2}$ from the y-axis
■ Translation $\frac{\pi}{4}$ units to the right and 1 unit down



6



Exercise 9C

1 a $\frac{\sqrt{2}+\sqrt{6}}{4}$ **b** $\frac{1-\sqrt{3}}{2\sqrt{2}} = \frac{\sqrt{2}-\sqrt{6}}{4}$

2 a $\frac{\sqrt{6}-\sqrt{2}}{4}$ **b** $\frac{\sqrt{3}+1}{\sqrt{3}-1} = 2+\sqrt{3}$

3 a $\frac{\sqrt{3}-1}{2\sqrt{2}} = \frac{\sqrt{6}-\sqrt{2}}{4}$

b $\frac{\sqrt{3}-1}{2\sqrt{2}} = \frac{\sqrt{6}-\sqrt{2}}{4}$

c $\frac{1-\sqrt{3}}{1+\sqrt{3}} = -2+\sqrt{3}$

4 For $u, v \in \left(0, \frac{\pi}{2}\right)$, $\sin(u+v) = \frac{63}{65}$;

For $u, v \in \left(\frac{\pi}{2}, \pi\right)$, $\sin(u+v) = -\frac{63}{65}$;

For $u \in \left(0, \frac{\pi}{2}\right)$, $v \in \left(\frac{\pi}{2}, \pi\right)$, $\sin(u+v) = -\frac{33}{65}$;

For $u \in \left(\frac{\pi}{2}, \pi\right)$, $v \in \left(0, \frac{\pi}{2}\right)$, $\sin(u+v) = \frac{33}{65}$

5 a $\frac{\sqrt{3}}{2}\sin\theta + \frac{1}{2}\cos\theta$ **b** $\frac{1}{\sqrt{2}}(\cos\varphi + \sin\varphi)$

c $\frac{\tan\theta + \sqrt{3}}{1-\sqrt{3}\tan\theta}$ **d** $\frac{1}{\sqrt{2}}(\sin\theta - \cos\theta)$

6 a $\sin u$ **b** $\cos u$

7 a $-\frac{119}{169}$ **b** $\frac{24}{25}$ **c** $\frac{24}{7}$ **d** $-\frac{169}{119}$

e $-\frac{33}{65}$ **f** $-\frac{16}{65}$ **g** $-\frac{65}{33}$ **h** $\frac{7}{24}$

8 a $\frac{63}{16}$ **b** $-\frac{24}{7}$ **c** $\frac{56}{65}$ **d** $\frac{24}{25}$

9 a $\frac{7}{25}$ **b** $\frac{3}{5}$ **c** $\frac{117}{44}$ **d** $\frac{-336}{625}$

10 a $-\frac{\sqrt{3}}{2}$ for $\theta = \frac{5\pi}{3}$ **b** $-\frac{1}{2}$

11 a $1 - \sin(2\theta)$ **b** $\cos(2\theta)$

Exercise 9D

1 a 5, −5 **b** 2, −2 **c** $\sqrt{2}, -\sqrt{2}$

d $\sqrt{2}, -\sqrt{2}$ **e** $2\sqrt{3}, -2\sqrt{3}$

f 2, −2 **g** 4, 0 **h** $5+\sqrt{13}, 5-\sqrt{13}$

Cambridge Senior Maths for Western Australia
Mathematics Specialist 1&2
ISBN 978-1-009-25961-3

2 a $\frac{\pi}{2}, \pi$ **b** $0, \frac{2\pi}{3}, 2\pi$

c $\frac{\pi}{6}, \frac{3\pi}{2}$ **d** $0, \frac{5\pi}{3}, 2\pi$

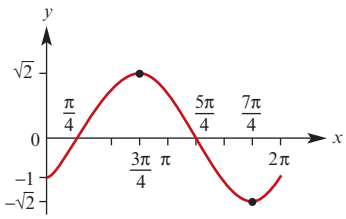
e 53.13° **f** 95.26°, 155.26°

3 $2\cos\left(2x + \frac{\pi}{6}\right)$

4 $\sqrt{2}\sin\left(3x - \frac{5\pi}{4}\right)$

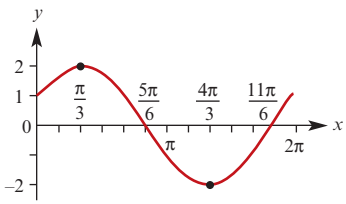
5 a $f(x) = \sin x - \cos x = \sqrt{2}\cos\left(x - \frac{3\pi}{4}\right)$

$= \sqrt{2}\sin\left(x + \frac{7\pi}{4}\right) = \sqrt{2}\sin\left(x - \frac{\pi}{4}\right)$



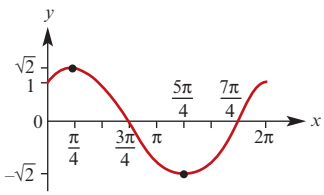
b $f(x) = \sqrt{3}\sin x + \cos x$

$= 2\cos\left(x - \frac{\pi}{3}\right) = 2\sin\left(x + \frac{\pi}{6}\right)$



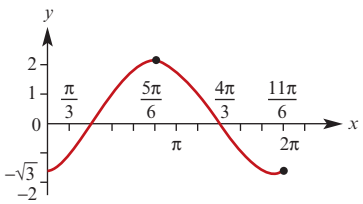
c $f(x) = \sin x + \cos x$

$= \sqrt{2}\cos\left(x - \frac{\pi}{4}\right) = \sqrt{2}\sin\left(x + \frac{\pi}{4}\right)$



d $f(x) = \sin x - \sqrt{3}\cos x = 2\cos\left(x - \frac{5\pi}{6}\right)$

$= 2\sin\left(x + \frac{5\pi}{3}\right) = 2\sin\left(x - \frac{\pi}{3}\right)$



Exercise 9E

1 a $\sin(5\pi t) + \sin(\pi t)$ **b** $\frac{1}{2}(\sin 50° - \sin 10°)$

c $\sin(\pi x) + \sin\left(\frac{\pi x}{2}\right)$ **d** $\sin(A) + \sin(B + C)$

2 $\cos(\theta) - \cos(5\theta)$

3 $\sin A - \sin B$

5 a $2\sin 39° \cos 17°$ **b** $2\cos 39° \cos 17°$

c $2\cos 39° \sin 17°$ **d** $-2\sin 39° \sin 17°$

6 a $2\sin(4A)\cos(2A)$ **b** $2\cos\left(\frac{3x}{2}\right)\cos\left(\frac{x}{2}\right)$

c $2\sin\left(\frac{x}{2}\right)\cos\left(\frac{7x}{2}\right)$ **d** $-2\sin(2A)\sin(A)$

11 a $-\frac{5\pi}{6}, -\frac{3\pi}{4}, -\frac{\pi}{2}, -\frac{\pi}{4}, \frac{\pi}{6}, \frac{\pi}{2}, \frac{3\pi}{4}, \frac{5\pi}{6}$

b $-\pi, -\frac{2\pi}{3}, -\frac{\pi}{2}, -\frac{\pi}{3}, 0, \frac{\pi}{3}, \frac{\pi}{2}, \frac{2\pi}{3}, \pi$

c $-\pi, -\frac{3\pi}{4}, -\frac{2\pi}{3}, -\frac{\pi}{3}, -\frac{\pi}{4}, 0, \frac{\pi}{4}, \frac{\pi}{3}, \frac{2\pi}{3}, \frac{3\pi}{4}, \pi$

d $-\pi, -\frac{5\pi}{6}, -\frac{\pi}{2}, -\frac{\pi}{6}, 0, \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \pi$

12 a $\frac{\pi}{6}, \frac{5\pi}{6}$ **b** $0, \frac{\pi}{6}, \frac{\pi}{3}, \frac{2\pi}{3}, \frac{5\pi}{6}, \pi$

c $0, \frac{\pi}{12}, \frac{\pi}{3}, \frac{5\pi}{12}, \frac{7\pi}{12}, \frac{2\pi}{3}, \frac{11\pi}{12}, \pi$

d $\frac{\pi}{10}, \frac{\pi}{6}, \frac{3\pi}{10}, \frac{\pi}{2}, \frac{7\pi}{10}, \frac{5\pi}{6}, \frac{9\pi}{10}$

17 $\frac{1 - \cos(100x)}{2\sin(x)}$

Chapter 9 review

Short-answer questions

1 a 5, 1 **b** 4, −2 **c** 4, −4 **d** 2, 0 **e** $1, \frac{1}{3}$

2 a $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$ **b** $\frac{3\pi}{8}, \frac{7\pi}{8}, \frac{11\pi}{8}, \frac{15\pi}{8}$

c $\frac{\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}$ **d** $\frac{\pi}{8}, \frac{7\pi}{8}, \frac{9\pi}{8}, \frac{15\pi}{8}$

4 a $\frac{140}{221}$ **b** $\frac{-21}{221}$ **c** $\frac{171}{140}$

5 a $\frac{1}{2}$ **b** 1

6 a 1 **b** 0

8 a $\frac{-1}{9}$ **b** $\frac{-4\sqrt{5}}{9}$ **c** $\frac{8\sqrt{5}}{81}$

10 $2 - \sqrt{3}$

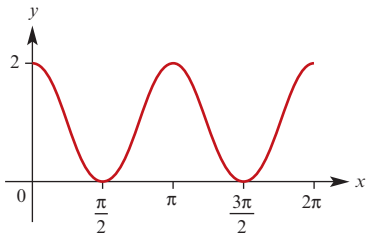
11 a $0, \frac{\pi}{2}, 2\pi$ **b** $\frac{7\pi}{6}, \frac{11\pi}{6}$ **c** $0, \pi, 2\pi$

d $\frac{\pi}{2}, \frac{3\pi}{2}$ **e** $\frac{\pi}{6}, \frac{\pi}{3}, \frac{7\pi}{6}, \frac{4\pi}{3}$

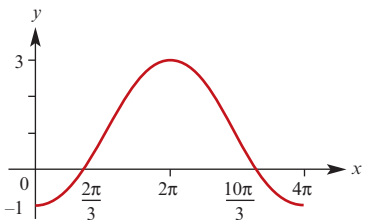
f $\frac{7\pi}{12}, \frac{3\pi}{4}, \frac{19\pi}{12}, \frac{7\pi}{4}$

ISBN 978-1-009-25961-3

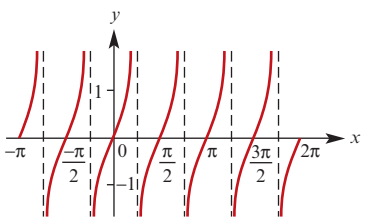
12 a $y = 2\cos^2 x$



b $y = 1 - 2\sin\left(\frac{\pi}{2} - \frac{x}{2}\right)$



c $f(x) = \tan(2x)$



13 $\frac{2}{9}$

14 a $\sqrt{85}\cos(\theta - \alpha)$ where $\alpha = \cos^{-1}\left(\frac{2}{\sqrt{85}}\right)$

b i $\sqrt{85}$ **ii** $\frac{2}{\sqrt{85}}$

iii $\theta = \cos^{-1}\left(\frac{2}{\sqrt{85}}\right) + \cos^{-1}\left(\frac{1}{\sqrt{85}}\right)$

15 a $0, \frac{\pi}{3}, \frac{\pi}{2}, \frac{2\pi}{3}, \pi$ **b** $0, \frac{\pi}{3}, \pi$

Extended-response questions

1 b $P = 10\sqrt{5}\cos(\theta - \alpha)$ where $\alpha = \cos^{-1}\left(\frac{2}{\sqrt{5}}\right)$; $\theta = 70.88°$

c $k = 25$ **d** $\theta = 45°$

2 a $AD = \cos\theta + 2\sin\theta$

b $AD = \sqrt{5}\cos(\theta - \alpha)$ where $\alpha = \cos^{-1}\left(\frac{1}{\sqrt{5}}\right) \approx 63°$

c Max length of AD is $\sqrt{5}$ m when $\theta = 63°$

d $\theta = 79.38°$

3 b ii $a = 1,\ b = 1$

c $\frac{1 + \sqrt{2} - \sqrt{3}}{1 + \sqrt{3} + \sqrt{6}} = \frac{2\sqrt{2} - \sqrt{3} - 1}{\sqrt{3} - 1}$

$= \sqrt{6} + \sqrt{2} - \sqrt{3} - 2$

4 a ii $2\cos\left(\frac{\pi}{5}\right)$

b iii $4\cos^2\left(\frac{\pi}{5}\right) - 2\cos\left(\frac{\pi}{5}\right) - 1 = 0$

iv $\frac{1 + \sqrt{5}}{4}$

5 b $-\frac{2}{3}$ or $\frac{1}{2}$

Chapter 10

Short-answer questions

3 a If n is odd, then $5n + 3$ is even.

c If n is even, then $5n + 3$ is odd.

7 a 90° **b** 54° **c** 80° **d** 220°

e $x = 96°,\ y = 70°$ **f** 46°

8 a 4 **b** $\frac{3\sqrt{10}}{2}$ **c** 12

9 a $\frac{\pi}{2}, \frac{5\pi}{2}, \frac{13\pi}{2}, \frac{17\pi}{2}$

b $\frac{-17\pi}{24}, \frac{-11\pi}{24}, \frac{7\pi}{24}, \frac{13\pi}{24}$

10 $x = 2n\pi \pm \frac{\pi}{6},\ n \in \mathbb{Z}$

11 a $\frac{5}{4}$ **b** $\frac{4}{3}$ **c** $-\frac{\sqrt{3}}{3}$ **d** $\frac{2\sqrt{3}}{3}$

12 $\pm\frac{\sqrt{6}}{3}$

14 a $\frac{1}{2}\sin(4x) - \frac{1}{2}\sin(2x)$

b $\theta = \frac{(2n+1)\pi}{2}$ or $\theta = \frac{(2n+1)\pi}{4},\ n \in \mathbb{Z}$

Extended-response questions

2 b $\angle BCA = x°,\ \angle BOA = 2x°,\ \angle TAB = x°,\ \angle TBA = x°$

4 a $x = 4,\ y = 9$

b i 4 **ii** 2

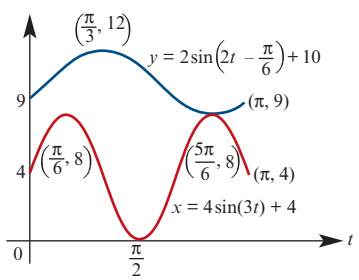
c i 8, 0 **ii** 12, 8

d i $\frac{2\pi}{3}$ **ii** π

e $\frac{\pi}{6}$ s **f** $\frac{5\pi}{6}, \frac{3\pi}{2}, \frac{13\pi}{6}, \frac{17\pi}{6}$

g $\frac{5\pi}{6}, \frac{11\pi}{6}, \frac{17\pi}{6}, \frac{23\pi}{6}$

h

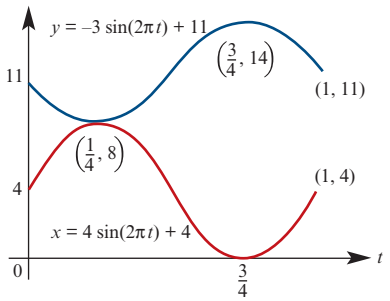


i $\frac{5\pi}{6}$ s **j** 2π s

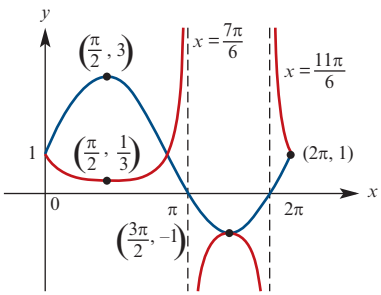
Cambridge Senior Maths for Western Australia Mathematics Specialist 1&2
ISBN 978-1-009-25961-3

5 a One possible set of values is $a = 4$, $b = 4$, $n = 2\pi$ and $c = -3$, $d = 11$, $m = 2\pi$

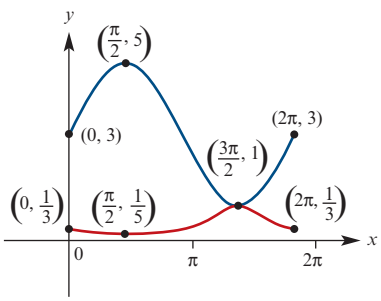
b



6 a i

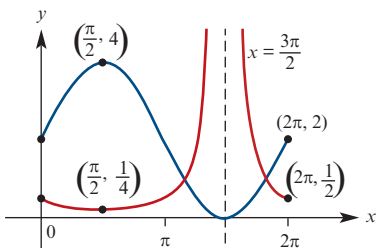


ii



b $k = 2$

c



Chapter 11

Exercise 11A

1 a 2×2 **b** 2×3 **c** 1×4 **d** 4×1

2 a $\begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \end{bmatrix}$ **b** $\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$

3 a $\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$ **b** $\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix}$

c $\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$

4 $\begin{bmatrix} 200 & 180 & 135 & 110 & 56 & 28 \\ 110 & 117 & 98 & 89 & 53 & 33 \end{bmatrix}$

5 a $\begin{bmatrix} 0 & x \end{bmatrix} = \begin{bmatrix} 0 & 4 \end{bmatrix}$ if $x = 4$

b $\begin{bmatrix} 4 & 7 \\ 1 & -2 \end{bmatrix} = \begin{bmatrix} x & 7 \\ 1 & -2 \end{bmatrix}$ if $x = 4$

c $\begin{bmatrix} 2 & x & 4 \\ -1 & 10 & 3 \end{bmatrix} = \begin{bmatrix} y & 0 & 4 \\ -1 & 10 & 3 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 4 \\ -1 & 10 & 3 \end{bmatrix}$ if $x = 0$, $y = 2$

6 a $x = 2$, $y = 3$ **b** $x = 3$, $y = 2$

c $x = 4$, $y = -3$ **d** $x = 3$, $y = -2$

7 $\begin{bmatrix} 21 & 5 & 5 \\ 8 & 2 & 3 \\ 4 & 1 & 1 \\ 14 & 8 & 60 \\ 0 & 1 & 2 \end{bmatrix}$

Exercise 11B

1 $\mathbf{X} + \mathbf{Y} = \begin{bmatrix} 4 \\ -2 \end{bmatrix}$ $2\mathbf{X} = \begin{bmatrix} 2 \\ -4 \end{bmatrix}$ $4\mathbf{Y} + \mathbf{X} = \begin{bmatrix} 13 \\ -2 \end{bmatrix}$

$\mathbf{X} - \mathbf{Y} = \begin{bmatrix} -2 \\ -2 \end{bmatrix}$ $-3\mathbf{A} = \begin{bmatrix} -3 & 3 \\ -6 & -9 \end{bmatrix}$

$-3\mathbf{A} + \mathbf{B} = \begin{bmatrix} 1 & 3 \\ -7 & -7 \end{bmatrix}$

2 $2\mathbf{A} = \begin{bmatrix} 2 & -2 \\ 0 & 4 \end{bmatrix}$ $-3\mathbf{A} = \begin{bmatrix} -3 & 3 \\ 0 & -6 \end{bmatrix}$

$-6\mathbf{A} = \begin{bmatrix} -6 & 6 \\ 0 & -12 \end{bmatrix}$

3 a Yes **b** Yes

4 a $\begin{bmatrix} 6 & 4 \\ -4 & -4 \end{bmatrix}$ **b** $\begin{bmatrix} 0 & -9 \\ 12 & 3 \end{bmatrix}$

c $\begin{bmatrix} 6 & -5 \\ 8 & -1 \end{bmatrix}$ **d** $\begin{bmatrix} -6 & -13 \\ 16 & 7 \end{bmatrix}$

5 a $\begin{bmatrix} 0 & 1 \\ 2 & 3 \end{bmatrix}$ **b** $\begin{bmatrix} -2 & 3 \\ 6 & 3 \end{bmatrix}$

c $\begin{bmatrix} 3 & 3 \\ -1 & 7 \end{bmatrix}$

Cambridge Senior Maths for Western Australia Mathematics Specialist 1&2 ISBN 978-1-009-25961-3

6 $\mathbf{X} = \begin{bmatrix} 2 & 4 \\ 0 & -3 \end{bmatrix}$, $\mathbf{Y} = \begin{bmatrix} \frac{-9}{2} & \frac{-23}{2} \\ \frac{-1}{2} & 11 \end{bmatrix}$

7 $\mathbf{X} + \mathbf{Y} = \begin{bmatrix} 310 & 180 & 220 & 90 \\ 200 & 0 & 125 & 0 \end{bmatrix}$ represents the total production at two factories in two successive weeks

Exercise 11C

1 $\mathbf{AX} = \begin{bmatrix} 4 \\ -5 \end{bmatrix}$ $\mathbf{BX} = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$ $\mathbf{AY} = \begin{bmatrix} -5 \\ 8 \end{bmatrix}$

$\mathbf{IX} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$ $\mathbf{AC} = \begin{bmatrix} 0 & -1 \\ 1 & 2 \end{bmatrix}$

$\mathbf{CA} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$ $(\mathbf{AC})\mathbf{X} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$\mathbf{C}(\mathbf{BX}) = \begin{bmatrix} 9 \\ 5 \end{bmatrix}$ $\mathbf{AI} = \begin{bmatrix} 1 & -2 \\ -1 & 3 \end{bmatrix}$

$\mathbf{IB} = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$ $\mathbf{AB} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\mathbf{BA} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $\mathbf{A}^2 = \begin{bmatrix} 3 & -8 \\ -4 & 11 \end{bmatrix}$

$\mathbf{B}^2 = \begin{bmatrix} 11 & 8 \\ 4 & 3 \end{bmatrix}$ $\mathbf{A}(\mathbf{CA}) = \begin{bmatrix} 1 & -3 \\ -1 & 4 \end{bmatrix}$

$\mathbf{A}^2\mathbf{C} = \begin{bmatrix} -2 & -5 \\ 3 & 7 \end{bmatrix}$

2 Defined: **AY**, **CI**;
Not defined: **YA**, **XY**, $\mathbf{X}^2$, **XI**

3 $\mathbf{AB} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

4 No

5 One possible answer is $\mathbf{A} = \begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix}$

6 $\mathbf{LX} = \begin{bmatrix} 7 \end{bmatrix}$, $\mathbf{XL} = \begin{bmatrix} 4 & -2 \\ -6 & 3 \end{bmatrix}$

7 **AB** and **BA** are not defined unless $m = n$

8 b $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

9 One possible answer is
$\mathbf{A} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, $\mathbf{B} = \begin{bmatrix} -2 & 1 \\ 1.5 & -0.5 \end{bmatrix}$

10 One possible answer is
$\mathbf{A} = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$, $\mathbf{B} = \begin{bmatrix} 0 & 1 \\ 2 & 3 \end{bmatrix}$, $\mathbf{C} = \begin{bmatrix} -1 & 2 \\ -2 & 1 \end{bmatrix}$,
$\mathbf{A}(\mathbf{B} + \mathbf{C}) = \begin{bmatrix} -1 & 11 \\ -4 & 24 \end{bmatrix}$, $\mathbf{AB} + \mathbf{AC} = \begin{bmatrix} -1 & 11 \\ -4 & 24 \end{bmatrix}$,
$(\mathbf{B} + \mathbf{C})\mathbf{A} = \begin{bmatrix} 11 & 7 \\ 16 & 12 \end{bmatrix}$

11 For example: $\mathbf{A} = \begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix}$ and $\mathbf{B} = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$

12 a $\begin{bmatrix} 29 \\ 8.50 \end{bmatrix}$, John took 29 minutes to eat food costing \$8.50

b $\begin{bmatrix} 29 & 22 & 12 \\ 8.50 & 8.00 & 3.00 \end{bmatrix}$,
John's friends took 22 and 12 minutes to eat food costing \$8.00 and \$3.00 respectively

13 $\mathbf{A}^2 = \begin{bmatrix} -3 & 4 \\ -4 & -3 \end{bmatrix}$, $\mathbf{A}^4 = \begin{bmatrix} -7 & -24 \\ 24 & -7 \end{bmatrix}$,
$\mathbf{A}^8 = \begin{bmatrix} -527 & 336 \\ -336 & -527 \end{bmatrix}$

14 $\mathbf{A}^2 = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$, $\mathbf{A}^3 = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$, $\mathbf{A}^4 = \begin{bmatrix} 1 & 4 \\ 0 & 1 \end{bmatrix}$,
$\mathbf{A}^n = \begin{bmatrix} 1 & n \\ 0 & 1 \end{bmatrix}$

Exercise 11D

1 a 1 **b** $\begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix}$ **c** 2 **d** $\frac{1}{2}\begin{bmatrix} 2 & 2 \\ -3 & -2 \end{bmatrix}$

2 a $\begin{bmatrix} -1 & 1 \\ -4 & 3 \end{bmatrix}$ **b** $\begin{bmatrix} \frac{2}{7} & \frac{-1}{14} \\ \frac{1}{7} & \frac{3}{14} \end{bmatrix}$

c $\begin{bmatrix} 1 & 0 \\ 0 & \frac{1}{k} \end{bmatrix}$ **d** $\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$

4 a $\mathbf{A}^{-1} = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ 0 & -1 \end{bmatrix}$, $\mathbf{B}^{-1} = \begin{bmatrix} 1 & 0 \\ -3 & 1 \end{bmatrix}$

b $\mathbf{AB} = \begin{bmatrix} 5 & 1 \\ -3 & -1 \end{bmatrix}$, $(\mathbf{AB})^{-1} = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ -\frac{3}{2} & -\frac{5}{2} \end{bmatrix}$

c $\mathbf{A}^{-1}\mathbf{B}^{-1} = \begin{bmatrix} -1 & \frac{1}{2} \\ 3 & -1 \end{bmatrix}$,
$\mathbf{B}^{-1}\mathbf{A}^{-1} = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ -\frac{3}{2} & -\frac{5}{2} \end{bmatrix}$, $(\mathbf{AB})^{-1} = \mathbf{B}^{-1}\mathbf{A}^{-1}$

5 a $\begin{bmatrix} -\frac{1}{2} & \frac{3}{2} \\ 1 & -2 \end{bmatrix}$ **b** $\begin{bmatrix} 0 & 7 \\ 1 & -8 \end{bmatrix}$

c $\begin{bmatrix} \frac{5}{2} & -\frac{7}{2} \\ \frac{11}{2} & -\frac{21}{2} \end{bmatrix}$

6 a $\begin{bmatrix} \frac{-3}{8} & \frac{11}{8} \\ \frac{1}{16} & \frac{7}{16} \end{bmatrix}$ **b** $\begin{bmatrix} \frac{-11}{16} & \frac{17}{16} \\ \frac{-1}{4} & \frac{3}{4} \end{bmatrix}$

7 $\begin{bmatrix} \frac{1}{a_{11}} & 0 \\ 0 & \frac{1}{a_{22}} \end{bmatrix}$

Cambridge Senior Maths for Western Australia
Mathematics Specialist 1&2
ISBN 978-1-009-25961-3

9 $\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}, \begin{bmatrix}-1 & 0\\0 & -1\end{bmatrix}, \begin{bmatrix}1 & 0\\k & -1\end{bmatrix}, \begin{bmatrix}-1 & 0\\k & 1\end{bmatrix}, \begin{bmatrix}1 & k\\0 & -1\end{bmatrix}, \begin{bmatrix}-1 & k\\0 & 1\end{bmatrix}, k \in \mathbb{R}, \begin{bmatrix}a & b\\\frac{1-a^2}{b} & -a\end{bmatrix}, b \neq 0$

10 $a = \pm\sqrt{2}$

Exercise 11E

1 a $\begin{bmatrix}3\\10\end{bmatrix}$ **b** $\begin{bmatrix}5\\17\end{bmatrix}$

2 a $x = -\frac{1}{7},\ y = \frac{10}{7}$ **b** $x = 4,\ y = 1.5$

3 $(2, -1)$

4 Book \$12, CD \$18

5 a $\begin{bmatrix}2 & -3\\4 & -6\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}3\\6\end{bmatrix}$

b $\begin{bmatrix}2 & -3\\4 & -6\end{bmatrix}$ is non-invertible

c System has solutions (not a unique solution)

d Solution set contains infinitely many pairs

6 a $\mathbf{A}^{-1}\mathbf{C}$ **b** $\mathbf{B}^{-1}\mathbf{A}^{-1}\mathbf{C}$ **c** $\mathbf{A}^{-1}\mathbf{C}\mathbf{B}^{-1}$

d $\mathbf{A}^{-1}\mathbf{C} - \mathbf{B}$ **e** $\mathbf{A}^{-1}(\mathbf{C} - \mathbf{B})$

f $(\mathbf{A} - \mathbf{B})\mathbf{A}^{-1} = \mathbf{I} - \mathbf{B}\mathbf{A}^{-1}$

Chapter 11 review

Short-answer questions

1 a $\begin{bmatrix}0 & 0\\12 & 8\end{bmatrix}$ **b** $\begin{bmatrix}0 & 0\\8 & 8\end{bmatrix}$

2 $\begin{bmatrix}a\\2 - \frac{3}{4}a\end{bmatrix},\ a \in \mathbb{R}$

3 a Exist: **AC**, **CD**, **BE**; Does not exist: **AB**

b $\mathbf{DA} = \begin{bmatrix}14 & 0\end{bmatrix},\quad \mathbf{A}^{-1} = \frac{1}{7}\begin{bmatrix}1 & 2\\3 & -1\end{bmatrix}$

4 $\mathbf{AB} = \begin{bmatrix}2 & 0\\2 & -2\end{bmatrix},\quad \mathbf{C}^{-1} = \begin{bmatrix}-2 & 1\\\frac{3}{2} & -\frac{1}{2}\end{bmatrix}$

5 $\begin{bmatrix}-1 & 2\\-3 & 5\end{bmatrix}$

6 $\mathbf{A}^2 = \begin{bmatrix}4 & 0 & 0\\0 & 4 & 0\\0 & 0 & 4\end{bmatrix},\quad \mathbf{A}^{-1} = \begin{bmatrix}\frac{1}{2} & 0 & 0\\0 & 0 & \frac{1}{2}\\0 & \frac{1}{2} & 0\end{bmatrix}$

7 8

8 a i $\begin{bmatrix}3 & -5\\5 & 8\end{bmatrix}$ **ii** $\begin{bmatrix}1 & -18\\18 & 19\end{bmatrix}$ **iii** $\frac{1}{7}\begin{bmatrix}3 & 1\\-1 & 2\end{bmatrix}$

b $x = 2,\ y = 1$

Extended-response questions

1 a i $\begin{bmatrix}2 & -3\\4 & 1\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}3\\5\end{bmatrix}$

ii $\det(\mathbf{A}) = 14,\ \mathbf{A}^{-1} = \frac{1}{14}\begin{bmatrix}1 & 3\\-4 & 2\end{bmatrix}$

iii $\frac{1}{7}\begin{bmatrix}9\\-1\end{bmatrix}$

iv $\left(\frac{9}{7}, -\frac{1}{7}\right)$ is the point of intersection of the two lines

b i $\begin{bmatrix}2 & 1\\4 & 2\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}3\\8\end{bmatrix}$

ii $\det(\mathbf{A}) = 0$, so $\mathbf{A}$ is non-invertible

c Equations of two parallel lines

2 a $\begin{bmatrix}79 & 78 & 80\\80 & 78 & 82\end{bmatrix}$ **b** $\begin{bmatrix}0.2\\0.3\\0.5\end{bmatrix}$

c Semester 1: 79.2; Semester 2: 80.4

d Semester 1: 83.8; Semester 2: 75.2

e No, total score is 318.6

f 3 marks

3 a $\begin{bmatrix}10 & 2\\8 & 4\\8 & 8\\6 & 10\end{bmatrix}$ **b** $\begin{bmatrix}70\\60\end{bmatrix}$

c Term 1: \$820; Term 2: \$800; Term 3: \$1040; Term 4: \$1020

d $\begin{bmatrix}2 & 2 & 1\\2 & 2 & 1\\3 & 4 & 2\\3 & 4 & 2\end{bmatrix}$ **e** $\begin{bmatrix}60\\55\\40\end{bmatrix}$

f Term 1: \$270; Term 2: \$270; Term 3: \$480; Term 4: \$480

g Term 1: \$1090; Term 2: \$1070; Term 3: \$1520; Term 4: \$1500

Chapter 12

Exercise 12A

1 a $(-2, 6)$ **b** $(-8, 22)$

c $(26, 2)$ **d** $(-4, -2)$

2 a $(3, 2)$ **b** $(-4, 9)$

c $(8, 3)$ **d** $(7, 11)$

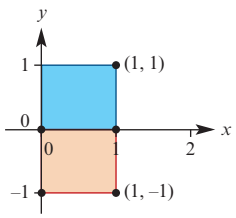
3 a $\begin{bmatrix}2 & 3\\4 & 5\end{bmatrix}$ **b** $\begin{bmatrix}11 & -3\\3 & -8\end{bmatrix}$

c $\begin{bmatrix}2 & 0\\1 & -3\end{bmatrix}$ **d** $\begin{bmatrix}0 & 1\\-1 & 0\end{bmatrix}$

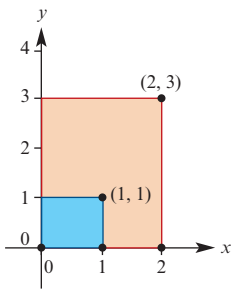
Cambridge Senior Maths for Western Australia Mathematics Specialist 1&2 ISBN 978-1-009-25961-3

4 Unit square is blue; image is red

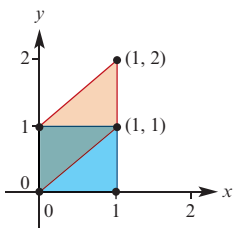
a



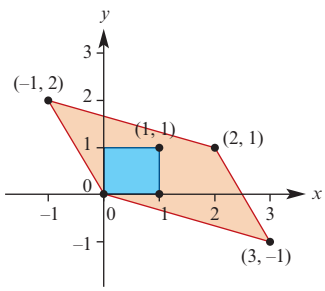
b



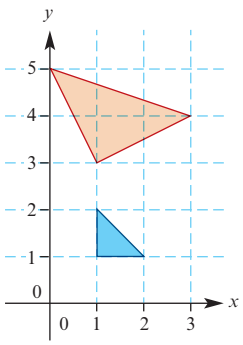
c



d



5



6 $\begin{bmatrix} 3 & 5 \\ 4 & 6 \end{bmatrix}\begin{bmatrix} -2 \\ 4 \end{bmatrix} = \begin{bmatrix} 14 \\ 16 \end{bmatrix}$

7 $\begin{bmatrix} -3 & 1 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} -3 \\ 1 \end{bmatrix}$

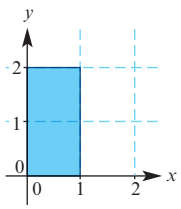
8 a $\begin{bmatrix} 1 & 1 \\ -1 & 2 \end{bmatrix}$ or $\begin{bmatrix} 1 & 1 \\ 2 & -1 \end{bmatrix}$

b $\begin{bmatrix} 1 & -2 \\ -1 & 1 \end{bmatrix}$ or $\begin{bmatrix} -2 & 1 \\ 1 & -1 \end{bmatrix}$

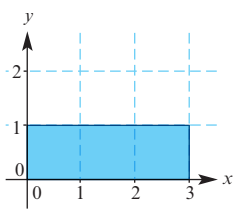
c $\begin{bmatrix} 1 & -2 \\ -1 & -3 \end{bmatrix}$ or $\begin{bmatrix} -2 & 1 \\ -3 & -1 \end{bmatrix}$

Exercise 12B

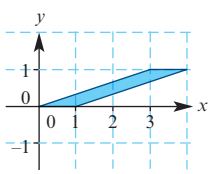
1 a i $\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$ **ii**



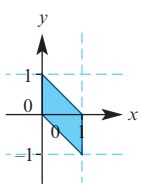
b i $\begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix}$ **ii**



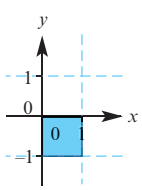
c i $\begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$ **ii**



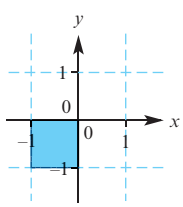
d i $\begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}$ **ii**



e i $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ **ii**



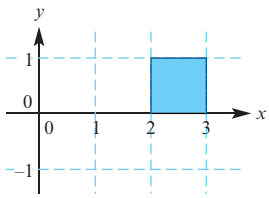
f i $\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$ **ii**



Cambridge Senior Maths for Western Australia
Mathematics Specialist 1&2
ISBN 978-1-009-25961-3

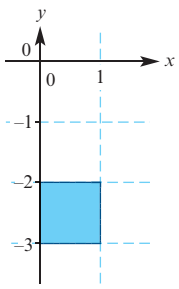
2 a i $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 2 \\ 0 \end{bmatrix} = \begin{bmatrix} x+2 \\ y \end{bmatrix}$

ii



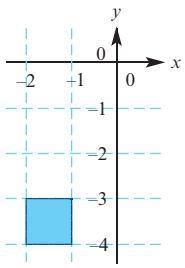
b i $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 0 \\ -3 \end{bmatrix} = \begin{bmatrix} x \\ y-3 \end{bmatrix}$

ii



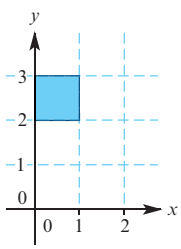
c i $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} -2 \\ -4 \end{bmatrix} = \begin{bmatrix} x-2 \\ y-4 \end{bmatrix}$

ii



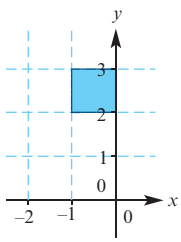
d i $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 0 \\ 2 \end{bmatrix} = \begin{bmatrix} x \\ y+2 \end{bmatrix}$

ii



e i $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} x-1 \\ y+2 \end{bmatrix}$

ii



Exercise 12C

1 a $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ **b** $\begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$

c $\begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$ **d** $\begin{bmatrix} -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix}$

2 a $(-3, 2)$ **b** $\left(\frac{5\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

3 a $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ **b** $\begin{bmatrix} -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$

c $\begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & -\frac{1}{2} \end{bmatrix}$ **d** $\begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix}$

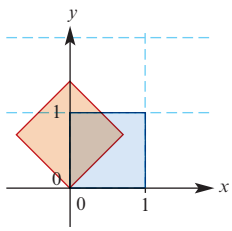
4 a $\begin{bmatrix} -\frac{4}{5} & \frac{3}{5} \\ \frac{3}{5} & \frac{4}{5} \end{bmatrix}$ **b** $\begin{bmatrix} -\frac{12}{13} & \frac{5}{13} \\ \frac{5}{13} & \frac{12}{13} \end{bmatrix}$

c $\begin{bmatrix} \frac{5}{13} & \frac{12}{13} \\ \frac{12}{13} & -\frac{5}{13} \end{bmatrix}$ **d** $\begin{bmatrix} -\frac{4}{5} & -\frac{3}{5} \\ -\frac{3}{5} & \frac{4}{5} \end{bmatrix}$

5 a $\begin{bmatrix} \frac{1-m^2}{m^2+1} & \frac{2m}{m^2+1} \\ \frac{2m}{m^2+1} & \frac{m^2-1}{m^2+1} \end{bmatrix}$ **b** $\left(\frac{-23}{37}, \frac{47}{37}\right)$

6 a $\begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix}$

b



c $\sqrt{2} - 1$

7 a $C\left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right), B\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

b Equilateral

c $y = -\sqrt{3}x,\ y = 0,\ y = \sqrt{3}x$

Exercise 12D

1 $\begin{bmatrix} -1 & 0 \\ 0 & 3 \end{bmatrix}$ **2** $\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$

Cambridge Senior Maths for Western Australia Mathematics Specialist 1&2 ISBN 978-1-009-25961-3

3 a $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

b $\begin{bmatrix} \cos 180° & -\sin 180° \\ \sin 180° & \cos 180° \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

4 a $\begin{bmatrix} 2 & 0 \\ 0 & -1 \end{bmatrix}$ **b** $\begin{bmatrix} 2 & 0 \\ 0 & -1 \end{bmatrix}$ **c** No

5 a $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$ **b** $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ **c** Yes

6 a $(x, y) \to (-x - 3, y + 5)$

b $(x, y) \to (-x + 3, y + 5)$ **c** Yes

7 a $\begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ **b** $\begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

c $\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ **d** $\begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$

8 a $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

b $\begin{bmatrix} \cos 90° & -\sin 90° \\ \sin 90° & \cos 90° \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

9 $\theta = 180°k$, where $k \in \mathbb{Z}$

10 a 2θ

b $\begin{bmatrix} \cos^2\theta - \sin^2\theta & -2\sin\theta\cos\theta \\ 2\sin\theta\cos\theta & \cos^2\theta - \sin^2\theta \end{bmatrix}$

c $\cos(2\theta) = \cos^2\theta - \sin^2\theta$

$\sin(2\theta) = 2\sin\theta\cos\theta$

11 a $x' = y + 1$ **b** $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$

$y' = x + 2$

12 a $\begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$ **b** $\begin{bmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix}$

c $\begin{bmatrix} \frac{\sqrt{2}+\sqrt{6}}{4} & \frac{\sqrt{2}-\sqrt{6}}{4} \\ \frac{\sqrt{6}-\sqrt{2}}{4} & \frac{\sqrt{6}+\sqrt{2}}{2} \end{bmatrix}$

d $\cos 15° = \frac{\sqrt{2}+\sqrt{6}}{4}$, $\sin 15° = \frac{\sqrt{6}-\sqrt{2}}{4}$

13 $\begin{bmatrix} \cos(2\theta - 2\varphi) & -\sin(2\theta - 2\varphi) \\ \sin(2\theta - 2\varphi) & \cos(2\theta - 2\varphi) \end{bmatrix}$, rotation matrix for angle $2\theta - 2\varphi$

Exercise 12E

1 a $\begin{bmatrix} 1 & -1 \\ -3 & 4 \end{bmatrix}$ **b** $\begin{bmatrix} \frac{2}{7} & \frac{1}{14} \\ \frac{1}{7} & -\frac{3}{14} \end{bmatrix}$

c $\begin{bmatrix} \frac{2}{3} & -\frac{1}{2} \\ \frac{1}{3} & 0 \end{bmatrix}$ **d** $\begin{bmatrix} \frac{5}{7} & -\frac{3}{7} \\ \frac{4}{7} & -\frac{1}{7} \end{bmatrix}$

2 a $(x, y) \to (x - 2y, 2x - 5y)$

b $(x, y) \to (y, -x + y)$

3 a $(-1, 1)$ **b** $\left(-\frac{1}{2}, 1\right)$

4 $\begin{bmatrix} -4 & 3 \\ -1 & 1 \end{bmatrix}$

5 $(0, 0), (-1, -2), (1, 1), (0, -1)$

6 a $\mathbf{A} = \begin{bmatrix} k & 0 \\ 0 & 1 \end{bmatrix}$ **b** $\mathbf{A}^{-1} = \begin{bmatrix} \frac{1}{k} & 0 \\ 0 & 1 \end{bmatrix}$

7 a $\mathbf{A} = \begin{bmatrix} 1 & k \\ 0 & 1 \end{bmatrix}$ **b** $\mathbf{A}^{-1} = \begin{bmatrix} 1 & -k \\ 0 & 1 \end{bmatrix}$

8 a $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

b Reflecting twice in the same axis will return any point (x, y) to its original position

9 a $\begin{bmatrix} \cos(2\theta) & \sin(2\theta) \\ \sin(2\theta) & -\cos(2\theta) \end{bmatrix}$

b Reflecting twice in the same line will return any point (x, y) to its original position

Exercise 12F

1 a $y = -3x - 1$ **b** $y = \frac{x}{2} + 1$ **c** $y = \frac{9x}{2} + 3$

d $y = 3x - 1$ **e** $y = -9x + 3$ **f** $y = \frac{-x - 1}{3}$

g $y = \frac{x - 1}{3}$

2 a $y = 6 - \frac{9x}{2}$ **b** $y = \frac{x + 2}{3}$

c $y = \frac{2 - 3x}{7}$ **d** $y = \frac{5x - 2}{12}$

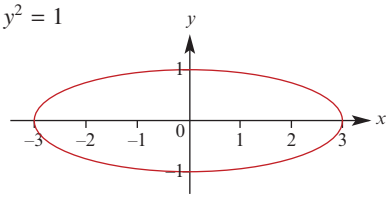
3 $\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$

4 $\begin{bmatrix} -3 & 0 \\ 0 & 6 \end{bmatrix}$

5 $y = -(x + 1)^2 - 1$

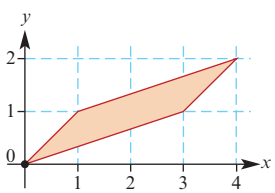
6 $y = (x - 1)^2 - 3$

7 $\frac{x^2}{3^2} + y^2 = 1$



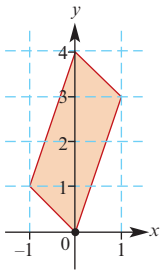
Exercise 12G

1 a Area = 2

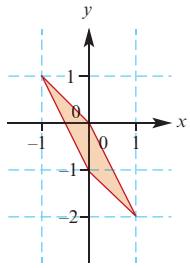


Cambridge Senior Maths for Western Australia
Mathematics Specialist 1&2
ISBN 978-1-009-25961-3

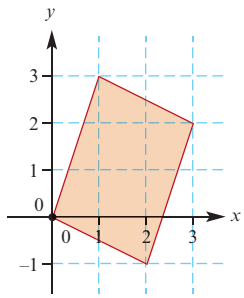
b Area = 4



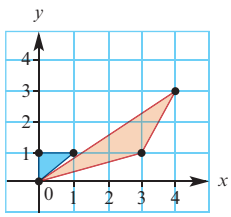
c Area = 1



d Area = 7

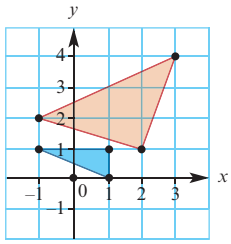


2 a



b Original area = $\frac{1}{2}$; image area = $\frac{5}{2}$

3 a



b Original area = 1; image area = 5

4 $m = \pm 2$

5 $m = -1, 2$

6 a **i** $\det\begin{bmatrix} 1 & k \\ 0 & 1 \end{bmatrix} = 1$

ii $\det\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} = 1$

iii $\det\begin{bmatrix} \cos(2\theta) & \sin(2\theta) \\ \sin(2\theta) & -\cos(2\theta) \end{bmatrix} = -1$

b **i** Dilation of factor k from the y-axis and dilation of factor $\frac{1}{k}$ from the x-axis

ii Determinant of matrix is 1

7 b $x = -1$

8 $m > 2$ or $m < 1$

9 $\begin{bmatrix} 1 & \pm\frac{\sqrt{3}}{2} \\ 0 & \pm\frac{1}{2} \end{bmatrix}$

10 a $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$

Exercise 12H

1 $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} y \\ -x+4 \end{bmatrix}$ **2** $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} -x-2 \\ -y+2 \end{bmatrix}$

3 a $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} y+1 \\ x-1 \end{bmatrix}$ **b** $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} -y-1 \\ -x-1 \end{bmatrix}$

c $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} x \\ -y+2 \end{bmatrix}$ **d** $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} -x-4 \\ y \end{bmatrix}$

4 a $\mathbf{A} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ **b** $\mathbf{B} = \begin{bmatrix} 1 & 0 \\ 0 & k \end{bmatrix}$

c $\mathbf{C} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$

d $\mathbf{CBA} = \begin{bmatrix} \cos^2\theta + k\sin^2\theta & \cos\theta\sin\theta - k\sin\theta\cos\theta \\ \cos\theta\sin\theta - k\sin\theta\cos\theta & \sin^2\theta + k\cos^2\theta \end{bmatrix}$

5 $\begin{bmatrix} \cos^2\theta & \cos\theta\sin\theta \\ \cos\theta\sin\theta & \sin^2\theta \end{bmatrix}$

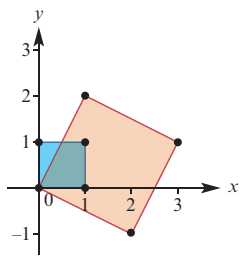
6 $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} x+1 \\ y-1 \end{bmatrix}$

Chapter 12 review

Short-answer questions

1 a $(7, 4)$ **b** $\begin{bmatrix} 2 & 1 \\ -1 & 2 \end{bmatrix}$

c Area = 5



d $(x, y) \to \left(\frac{2}{5}x - \frac{1}{5}y, \frac{1}{5}x + \frac{2}{5}y\right)$

2 a $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$ **b** $\begin{bmatrix} 1 & 0 \\ 0 & 5 \end{bmatrix}$ **c** $\begin{bmatrix} 1 & -3 \\ 0 & 1 \end{bmatrix}$

d $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ **e** $\begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$ **f** $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

3 a $\begin{bmatrix} -\frac{4}{5} & \frac{3}{5} \\ \frac{3}{5} & \frac{4}{5} \end{bmatrix}$ **b** $\left(\frac{4}{5}, \frac{22}{5}\right)$

Cambridge Senior Maths for Western Australia Mathematics Specialist 1&2 ISBN 978-1-009-25961-3

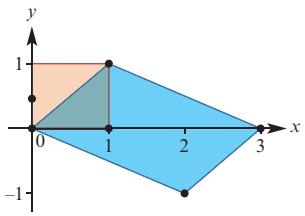
4 a $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ **b** $\begin{bmatrix} 0 & -1 \\ 2 & 0 \end{bmatrix}$ **c** $\begin{bmatrix} 0 & 1 \\ 1 & 2 \end{bmatrix}$

5 a $(x, y) \to (x - 3, -y + 4)$

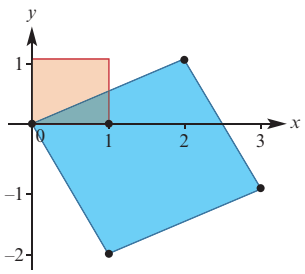
b $(x, y) \to (x - 3, -y - 4)$

6 a $\mathbf{A} = \begin{bmatrix} 1 & 0 \\ k & 1 \end{bmatrix}$ **b** $\mathbf{A}^{-1} = \begin{bmatrix} 1 & 0 \\ -k & 1 \end{bmatrix}$

7 a Area of image = 3 square units

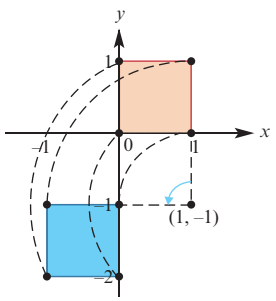


b Area of image = 5 square units



8 a $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} -y \\ x - 2 \end{bmatrix}$ **b** $(1, 0)$

c



Extended-response questions

1 a $\begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$ **b** $\begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$

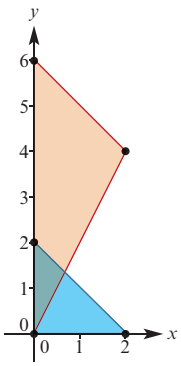
c Product of these two matrices:

$$\begin{bmatrix} \frac{-1+\sqrt{3}}{2\sqrt{2}} & -\frac{1+\sqrt{3}}{2\sqrt{2}} \\ \frac{1+\sqrt{3}}{2\sqrt{2}} & \frac{-1+\sqrt{3}}{2\sqrt{2}} \end{bmatrix}$$

d $\cos 75° = \frac{-1+\sqrt{3}}{2\sqrt{2}} = \frac{-\sqrt{2}+\sqrt{6}}{4}$

$\sin 75° = \frac{1+\sqrt{3}}{2\sqrt{2}} = \frac{\sqrt{2}+\sqrt{6}}{4}$

2 a



b Original area = 2 square units;
Image area = 6 square units

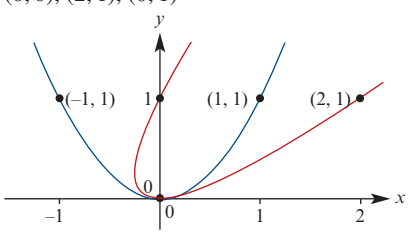
c 8π cubic units

3 a $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

b Shear of factor 1 parallel to the x-axis

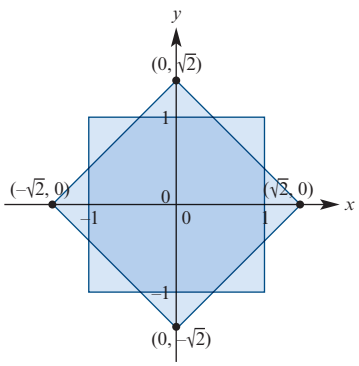
c $(0, 0), (2, 1), (0, 1)$

d



4 a $(0, \sqrt{2}), (\sqrt{2}, 0), (0, -\sqrt{2}), (-\sqrt{2}, 0)$

b



c $13 - 8\sqrt{2}$ square units

5 b i The composition of two rotations is a rotation

ii The composition of two reflections is a rotation

iii The composition of a reflection followed by a rotation is a reflection

iv The composition of a rotation followed by a reflection is a reflection

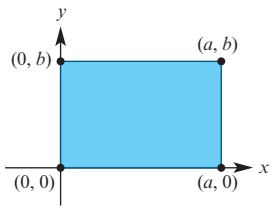
c $\begin{bmatrix} -\frac{1}{2} & -\frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & -\frac{1}{2} \end{bmatrix}$

Cambridge Senior Maths for Western Australia
Mathematics Specialist 1&2
ISBN 978-1-009-25961-3

6 a $\begin{bmatrix} \frac{3}{5} & \frac{4}{5} \\ \frac{4}{5} & -\frac{3}{5} \end{bmatrix}$ **b** $A'(-1, -3)$ **c** $2\sqrt{10}$

d Isosceles **f** $2\sqrt{10}$

7 a



b $O(0, 0)$, $A(a\cos\theta, a\sin\theta)$,
$B(-b\sin\theta, b\cos\theta)$,
$C(a\cos\theta - b\sin\theta, a\sin\theta + b\cos\theta)$

8 a $y = \frac{1}{m} - \frac{x}{m}$; $(1, 0)$, $\left(\frac{1-m^2}{1+m^2}, \frac{2m}{1+m^2}\right)$

b $y = 1 - \frac{x}{m}$; $(0, 1)$, $\left(\frac{2m}{1+m^2}, \frac{m^2-1}{1+m^2}\right)$

c $\begin{bmatrix} \frac{1-m^2}{1+m^2} & \frac{2m}{1+m^2} \\ \frac{2m}{1+m^2} & \frac{m^2-1}{1+m^2} \end{bmatrix}$

Chapter 13

See solutions supplement

Chapter 14

Exercise 14A

1

	Re(z)	Im(z)
a	2	3
c	$\frac{1}{2}$	$-\frac{3}{2}$
e	0	3

	Re(z)	Im(z)
b	4	5
d	−4	0
f	$\sqrt{2}$	$-2\sqrt{2}$

2 a $a = 2,\ b = -2$

b $a = 3,\ b = 2$ or $a = 2,\ b = 3$

c $a = 5,\ b = 0$ **d** $a = \frac{2}{3},\ b = -\frac{1}{3}$

3 a $6 - 8i$ **b** $6 - i$ **c** $-6 - 2i$

d $7 - 3\sqrt{2}i$ **e** $-2 - 3i$ **f** $4 + 2i$

g $6 - 4i$ **h** $-4 + 6i$ **i** $-1 + 11i$

j -1

4 a $4i$ **b** $6i$ **c** $\sqrt{2}i$

d $-i$ **e** -1 **f** 1

g -2 **h** -12 **i** -4

5 a $1 + 2i$ **b** $-3 + 4i$

c $-\sqrt{2} - 2i$ **d** $-\sqrt{6} - 3i$

Exercise 14B

1 a $15 + 8i$ **b** $-8i$ **c** $-2 + 16i$

d $2i$ **e** 5 **f** $-4 + 19i$

2 a $2 + 5i$ **b** $-1 - 3i$

c $\sqrt{5} + 2i$ **d** $5i$

3 a $2 + i$ **b** $-3 - 2i$ **c** $-4 + 7i$

d $-4 - 7i$ **e** $-4 - 7i$ **f** $-1 + i$

g $-1 - i$ **h** $-1 - i$

4 a $2 + 4i$ **b** 20 **c** 4

d $8 - 16i$ **e** $-8i$ **f** 8

g $\frac{1}{10}(1 + 2i)$ **h** $-4 - 2i$

5 $a = \frac{1}{29},\ b = -\frac{17}{29}$

6 a $\frac{7}{17} - \frac{6}{17}i$ **b** i **c** $\frac{7}{2} - \frac{1}{2}i$

d $-\frac{1}{2} - \frac{1}{2}i$ **e** $\frac{2}{13} + \frac{3}{13}i$ **f** $\frac{3}{20} + \frac{1}{20}i$

7 $a = \frac{5}{2},\ b = -\frac{3}{2}$

8 a $-\frac{42}{5}(1 - 2i)$ **b** $-\frac{1}{2}(1 - i)$

c $\frac{1}{17}(4 + i)$ **d** $\frac{1}{130}(6 + 43i)$

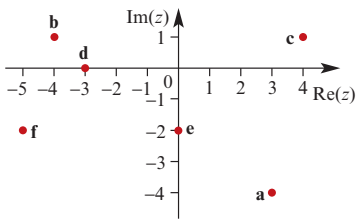
e $2 - 2i$

Exercise 14C

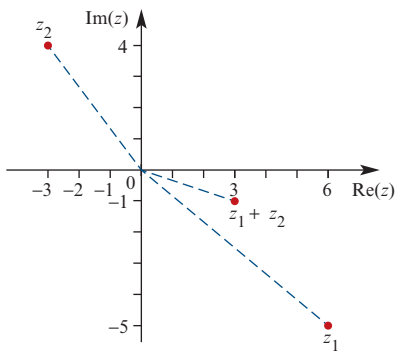
1 $A = 3 + i$, $B = 2i$, $C = -3 - 4i$

$D = 2 - 2i$, $E = -3$, $F = -1 - i$

2

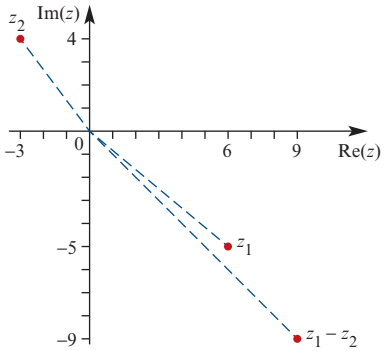


3 a $z_1 + z_2 = 3 - i$

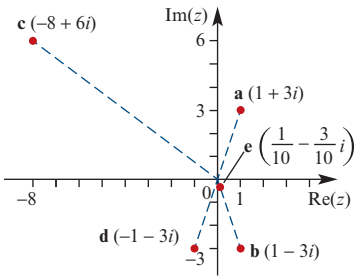


Cambridge Senior Maths for Western Australia Mathematics Specialist 1&2 ISBN 978-1-009-25961-3

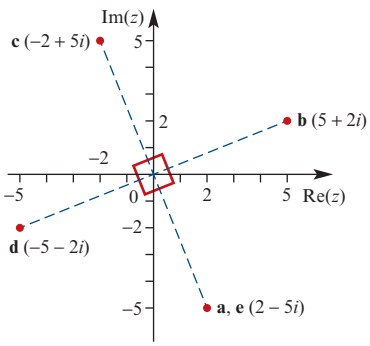
b $z_1 - z_2 = 9 - 9i$



4



5



Exercise 14D

1 **a** $\pm 2i$ **b** $\pm 3i$ **c** $\pm\sqrt{5}i$ **d** $2 \pm 4i$
e $-1 \pm 7i$ **f** $1 \pm \sqrt{2}i$ **g** $\frac{1}{2}(-3 \pm \sqrt{3}i)$
h $\frac{1}{4}(-5 \pm \sqrt{7}i)$ **i** $\frac{1}{6}(1 \pm \sqrt{23}i)$
j $1 \pm 2i$ **k** $\frac{1}{2}(3 \pm \sqrt{11}i)$ **l** $3 \pm \sqrt{5}i$

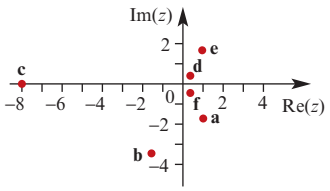
2 **a** $(z + 3i)(z - 3i)$ **b** $(z + \sqrt{3}i)(z - \sqrt{3}i)$
c $3(z + 2i)(z - 2i)$
d $(z + 1 + 2i)(z + 1 - 2i)$
e $\left(z - \frac{3}{2} + \frac{\sqrt{15}}{2}i\right)\left(z - \frac{3}{2} - \frac{\sqrt{15}}{2}i\right)$
f $2\left(z + \frac{1}{2} + \frac{1}{2}i\right)\left(z + \frac{1}{2} - \frac{1}{2}i\right)$

Chapter 14 review

Short-answer questions

1 **a** $(2m + 3p) + (2n + 3q)i$ **b** $p - qi$
c $(mp + nq) + (np - mq)i$
d $\frac{(mp + nq) + (np - mq)i}{p^2 + q^2}$ **e** $2m$
f $(m^2 - n^2 - p^2 + q^2) + (2mn - 2pq)i$
g $\frac{m - ni}{m^2 + n^2}$
h $\frac{(mp + nq) + (mq - np)i}{m^2 + n^2}$
i $\frac{3((mp + nq) + (np - mq)i)}{p^2 + q^2}$

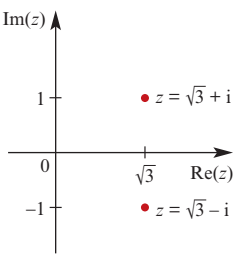
2



a $1 - \sqrt{3}i$ **b** $-2 - 2\sqrt{3}i$ **c** -8
d $\frac{1}{4}(1 + \sqrt{3}i)$ **e** $1 + \sqrt{3}i$ **f** $\frac{1}{4}(1 - \sqrt{3}i)$

Extended-response questions

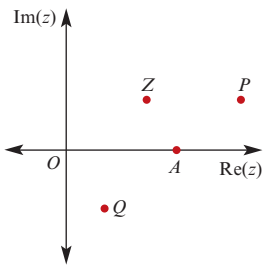
1 **a** $z = \sqrt{3} + i$ or $z = \sqrt{3} - i$
b **i**



ii $x^2 + y^2 = 4$ **iii** $a = 2$

2 **a** **i** $6\sqrt{2}$ **ii** 6

3 **a**



b $\sqrt{2} + 1$

6 **a** $|z + 1| = \sqrt{2 + 2a}$
b $|z - 1| = \sqrt{2 - 2a}$
c $\left|\frac{z - 1}{z + 1}\right| = \sqrt{\frac{2 - a}{2 + a}}$

Cambridge Senior Maths for Western Australia
Mathematics Specialist 1&2
ISBN 978-1-009-25961-3

7 a $\Delta = b^2 - 4ac$

b $b^2 < 4ac$

c i $-\frac{b}{a}, \frac{\sqrt{4ac}}{2a}$ **ii** $\frac{b^2}{2ac} - 1$

8 a $z_1 = \frac{1}{2}(-1 + \sqrt{3}i),\ z_2 = \frac{1}{2}(-1 - \sqrt{3}i)$

c $\frac{\sqrt{3}}{4}$

Chapter 15

Short-answer questions

1 a All defined except **AB**

b $\mathbf{DA} = \begin{bmatrix} 6 & -12 \end{bmatrix}$, $\mathbf{A}^{-1} = \begin{bmatrix} \frac{1}{9} & \frac{4}{9} \\ \frac{2}{9} & -\frac{1}{9} \end{bmatrix}$

2 a $\begin{bmatrix} -2 & 4 \\ 18 & -24 \end{bmatrix}$ **b** $\begin{bmatrix} -10 & -19 \\ 7 & -16 \end{bmatrix}$

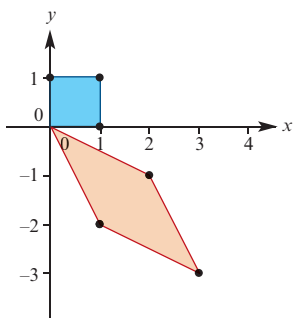
3 8

4 $\mathbf{A} = \begin{bmatrix} t \\ 3t - 5 \end{bmatrix}, t \in \mathbb{R}$

5 $\mathbf{AB} = \begin{bmatrix} -9 & -8 \\ -15 & 10 \end{bmatrix}$, $\mathbf{C}^{-1} = \begin{bmatrix} 2 & 1 \\ \frac{3}{2} & \frac{1}{2} \end{bmatrix}$

6 a $(7, -8)$ **b** $\begin{bmatrix} 2 & 1 \\ -1 & -2 \end{bmatrix}$

c Area = 3



d $(x, y) \to \left(\frac{2}{3}x + \frac{1}{3}y, -\frac{1}{3}x - \frac{2}{3}y\right)$

7 a $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ **b** $\begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix}$ **c** $\begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$

d $\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$ **e** $\begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix}$

f $\begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$ **g** $\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$ **h** $\begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{bmatrix}$

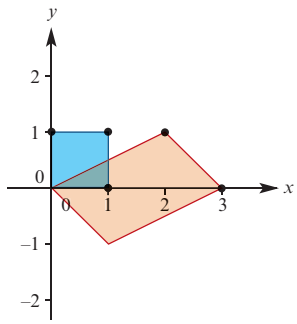
8 a $\begin{bmatrix} -\frac{15}{17} & \frac{8}{17} \\ \frac{8}{17} & \frac{15}{17} \end{bmatrix}$ **b** $\left(\frac{2}{17}, \frac{76}{17}\right)$

9 a $\begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix}$ **b** $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ **c** $\begin{bmatrix} -2 & -1 \\ -1 & 0 \end{bmatrix}$

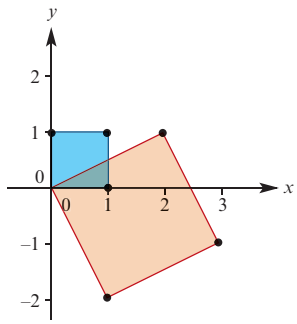
10 a $(x, y) \to (-x + 2, y - 1)$

b $(x, y) \to (-x - 2, y - 1)$

11 a Area = 3

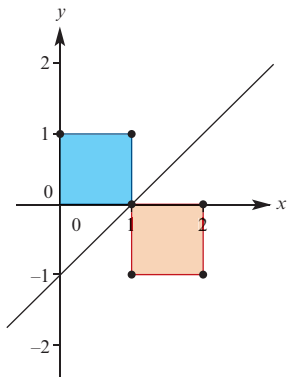


b Area = 5



12 a $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} y + 1 \\ x - 1 \end{bmatrix}$ **b** $(0, 0) \to (1, -1)$

c



16 a 6 **b** $4i$ **c** 13 **d** 10

e 36 **f** -16 **g** $24i$ **h** $24i$

17 a $3 - 5i$ **b** $-1 + i$ **c** $-4 - 7i$ **d** $\frac{8 - i}{13}$

e $2 + i$ **f** $\frac{-2 + i}{5}$ **g** $-2 - i$ **h** $\frac{8 + i}{5}$

i $\frac{13 - i}{34}$ **j** $3 - i$ **k** $\frac{-1 - 3i}{2}$ **l** $-3 - 4i$

Cambridge Senior Maths for Western Australia
Mathematics Specialist 1&2
ISBN 978-1-009-25961-3

18 a $(z-7i)(z+7i)$

b $(z-1-3i)(z-1+3i)$

c $9\left(z-\frac{1}{3}-\frac{2}{3}i\right)\left(z-\frac{1}{3}+\frac{2}{3}i\right)$

d $4\left(z+\frac{3}{2}-i\right)\left(z+\frac{3}{2}+i\right)$

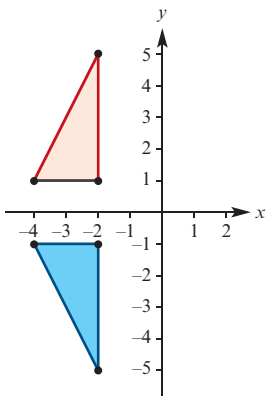
19 a $2+i,\ -2-i$ **b** $z=-1-i$ or $z=-i$

Extended-response questions

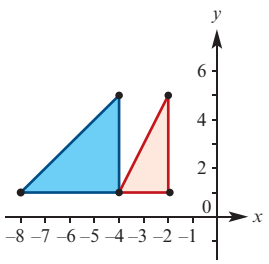
1 a i $\begin{bmatrix} a^2+bc & ab+bd \\ ac+dc & d^2+bc \end{bmatrix}$ **ii** $\begin{bmatrix} 3a & 3b \\ 3c & 3d \end{bmatrix}$

2 a $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} x+6 \\ y+3 \end{bmatrix}$

b



c



d $y=2(x+3)^2+2$ **e** $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} x+3 \\ -2y+4 \end{bmatrix}$

3 a $(4,1)$

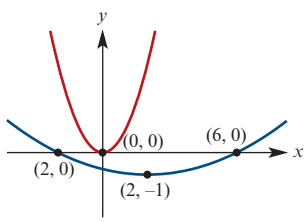
b i Rectangle with vertices $A'(0,0)$, $B'(0,1)$, $C'(4,1)$, $E'(4,0)$

ii 1 **iii** 4 **iv** k

c $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 4x \\ y \end{bmatrix}$

d i $y=\frac{1}{16}x^2$ **ii** $y=\frac{1}{16}(x-2)^2-1$

iii



e $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} x+2 \\ \frac{1}{5}(y+3) \end{bmatrix}$

4 b i $x^2+(y-1)^2=1$

ii $\left(x-\frac{4}{5}\right)^2+\left(y-\frac{3}{5}\right)^2=1$

c $(0,0),\left(\frac{4}{5},\frac{8}{5}\right)$

5 a $(-3,11)$ **b** $\frac{1}{10}\begin{bmatrix} 3 & -1 \\ -2 & 4 \end{bmatrix}$

c $a=2,\ b=3$ **d** $(5a,5a)$

e $\lambda=2,\ b=-2a;\ \lambda=5,\ b=a$

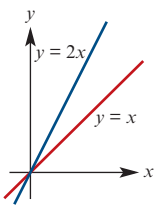
6 a $\begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix}$ **b** $\begin{bmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix}$

c $a=\sqrt{2},\ b=0$ **d** $c=\frac{3\sqrt{2}}{2},\ d=\frac{\sqrt{2}}{2}$

e i $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{2}}{2}x'+\frac{\sqrt{2}}{2}y' \\ -\frac{\sqrt{2}}{2}x'+\frac{\sqrt{2}}{2}y' \end{bmatrix}$

ii $\sqrt{2}(y-x)=(x+y)^2$

7 a

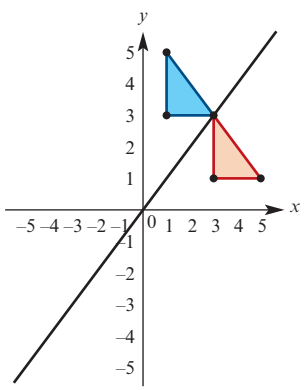


b $a=2,\ b=\frac{\pi}{4}$

c $\begin{bmatrix} \frac{3}{\sqrt{10}} & -\frac{1}{\sqrt{10}} \\ \frac{1}{\sqrt{10}} & \frac{3}{\sqrt{10}} \end{bmatrix}$

8 a i $(3,1)$ **ii** $A'(3,1),\ B'(5,1),\ C'(3,3)$

iii



b ii $(-1,-1),(2,2)$

iv $(-1,-1),(2,2),$

$\left(\frac{1}{2}(-1+\sqrt{5}),\frac{1}{2}(-1-\sqrt{5})\right),$

$\left(\frac{1}{2}(-1-\sqrt{5}),\frac{1}{2}(-1+\sqrt{5})\right)$

Cambridge Senior Maths for Western Australia
Mathematics Specialist 1&2
ISBN 978-1-009-25961-3