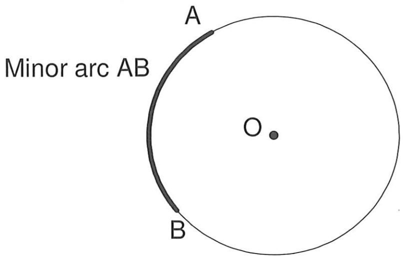
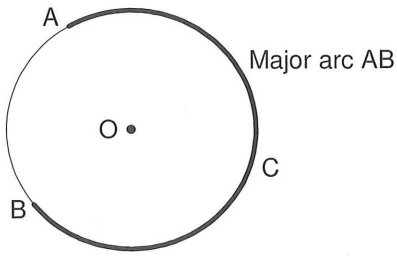
Chapter 13 Circular Measure

REVIEW OF ARCS, SECTORS AND SEGMENTS

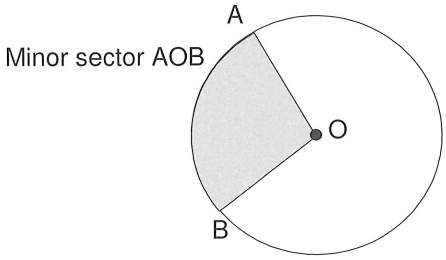
Arc: an arc is a portion of the circumference of a circle.

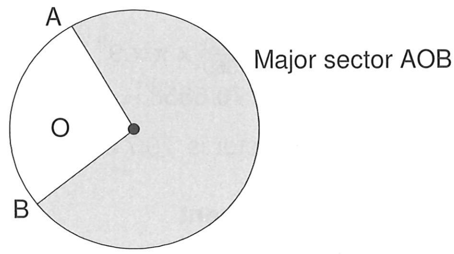




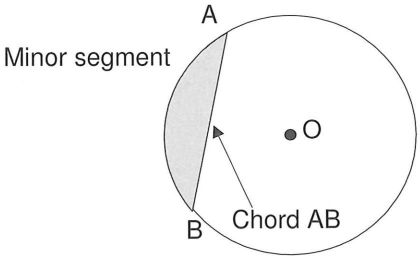
Naming arcs only by their endpoints may be ambiguous. Using the terms minor and major avoids ambiguity in naming arcs. Alternatively using the convention that the endpoints only are used for minor arcs and for major arcs the endpoints and an additional point between the endpoints is included in the naming of the arc. For the above, the arc on the left is simply arc AB and the major arc right is named as arc ACB.

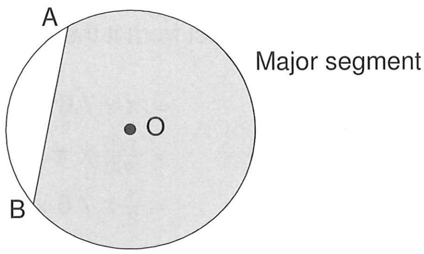
Sector: a sector is part of a circle enclosed by two radii and an arc of the circle.





Segment: a segment is part of a circle enclosed by an arc and the line segment connecting the endpoints of the arc. The line segment joining the endpoints of an arc is called a chord.





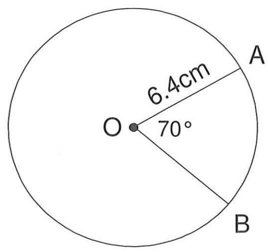
Finding the length of an arc

Example 1

Arc AB subtends an angle of 70° at the centre of circle centre O and radius 6.4 cm. Find the length of arc AB.

Solution

To find the length of the arc AB, we first need to find the circumference of the circle and then that portion of the circumference that subtends an angle of 70° at the centre of the circle.



Circumference of circle $= 2 \times \pi \times 6.4$ cm

$= 12.8\pi$ cm

Length of arc AB $= \frac{70}{360} \times 12.8\pi$ cm

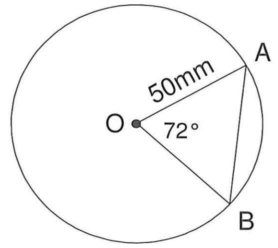
$= 7.82$ cm (rounded to 2 decimal places.

Hence arc AB is of length 7.82 cm.

Alternatively the answer may be given exactly as $\frac{112\pi}{45}$ cm.

Finding the length of a chord
Example 2
Chord AB subtends an angle of 72° at the centre of circle centre O and radius 50 mm. Find the length of chord AB. Give your answer to the nearest millimetre.

Solution
To find the length of chord AB we need to consider ΔAOB and apply the cosine rule to find the length of the side AB.



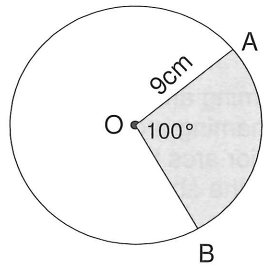
Applying the cosine rule: $AB^2 = 50^2 + 50^2 - 2(50)(50)\cos 72°$

$AB^2 = 3454.9150\ldots$

Hence $AB = 58.8$ (rounded to 1 decimal place)

The length of the chord AB is 59 mm

Finding the area of a sector
Example 3
Find the area of the sector shown shaded in the diagram on the right. Round your answer to 1 decimal place.



Solution
To find the area of sector AOB we need to find that proportion of the circle area that the arc AB subtends. In this case it is $\frac{100}{360}^{\text{ths}}$ of the area of the circle.

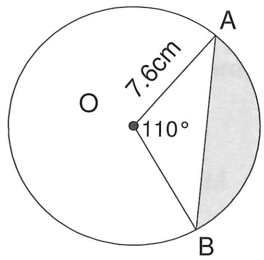
Area of circle $= \pi \times 9^2$

Area of sector $= \frac{100}{360} \times \pi \times 9^2$

$= 70.6858\ldots$

Hence area of shaded sector is 70.7 cm^2 rounded to 1 decimal place.

Finding the area of a segment
Example 4
Find the area of the segment shown shaded in the diagram on the right.



Solution
To find the area of the shaded segment we need to firstly find the area of the minor sector AOB and subtract from it the area of triangle AOB.

Area of circle $= \pi \times 7.6^2$

Area of minor sector AOB $= \frac{110}{360} \times \pi \times 7.6^2$

Area of triangle AOB $= \frac{1}{2} \times 7.6 \times 7.6 \times \sin 110°$

Area of segment = Area of minor sector AOB – Area of triangle AOB

$= \frac{110}{360} \times \pi \times 7.6^2 - \frac{1}{2} \times 7.6 \times 7.6 \times \sin 110°$

$= 28.3072\ldots$

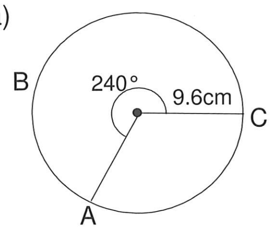
Hence area of the segment is 28.3 cm^2 rounded to 1 decimal place.

EXERCISE 13A

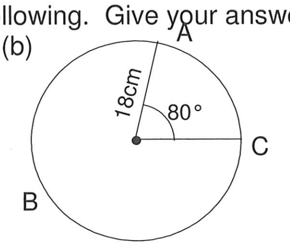
1. Find the lengths of the arcs with the following radii and angles subtended at the centre of the circle. \
Give your answers rounded to 1 decimal place.
(a) 7 cm, 80° (b) 9.5 cm, 250° (c) 75 mm, 290°

2. Find the length of the arc ABC in each of the following. Give your answer as an exact value.

(a)



(b)

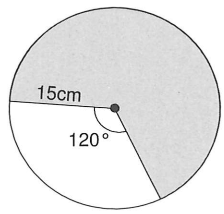


3. Find the length of the radius of the circle if an arc of length 8 cm subtends an angle of 36°.

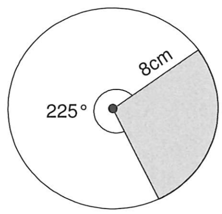
4. Find the angle subtended at the centre of a circle of radius length 10 cm and arc length 25 cm. Give your answer to the nearest minute.

5. Find the exact area of the shaded sectors below.

(a)

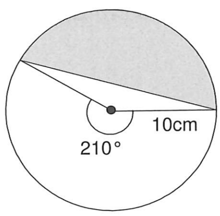


(b)

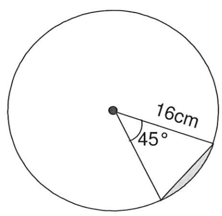


6. Find the exact area of the shaded segments below.

(a)



(b)



7. A chord subtends an angle of $120°$ at the centre of a circle of radius 9 cm. Determine the difference in length of the chord and the minor arc cut off by the chord. Round your answer to 2 decimal places.

8. A chord of length 10 cm is drawn in a circle of radius 7 cm. Find the area of the minor segment cut off by this chord. Give your answer in square centimetres correct to two decimal places.

9. Two circles have radii 5 cm and 12 cm respectively and their centres are 13 cm apart.
   (a) Find the length of the arc cut off from the larger circle.
   (b) Find the length of the common chord.

10. Two circles have radii 12 cm and 15 cm respectively and their centres are 20 cm apart. Calculate the area common to both circles.

CIRCULAR MEASURE

Circular measure is the measurement of an angle in units called **radians** instead of degrees.
When working with circles, graphs of trigonometric functions and calculus it is much easier to use radians than degrees.
For example, when finding the length of an arc using degrees, where θ is the angle measured in degrees and r the radius of the circle we need to apply the following formula:

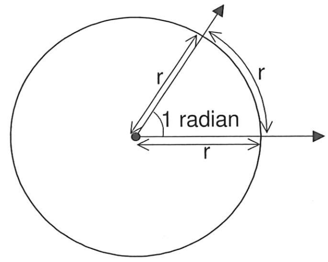
$$\text{Length of arc} = \frac{\theta}{360} \times 2\pi r$$

The equivalent formula where θ is the angle measured in radians and r the radius of the circle is as follows:

$$\text{Length of arc} = r\theta$$

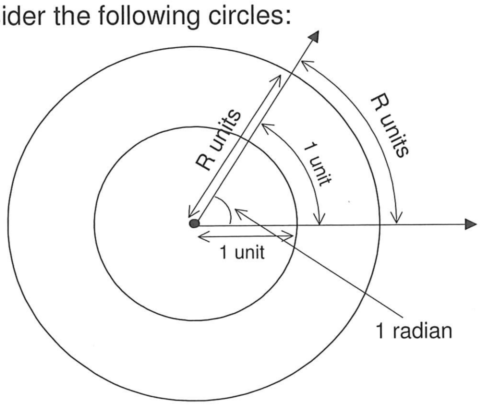
Radian

For any circle, an angle of one radian subtends an arc equal in length to the radius of the circle.



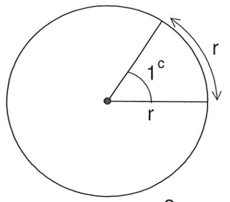
The radian may be represented by the symbol rad or ^{c} for circular measure.
A value of say 1.6 radians may be written as 1.6 rad, 1.6 r, 1.6^{rad}, 1.6^{R}, 1.6^{c} or simply just 1.6.

Consider the following circles:

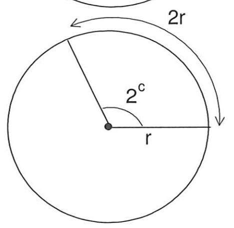


The diagram on the left shows an angle of **one radian** in the unit circle and in the circle of radius R units.

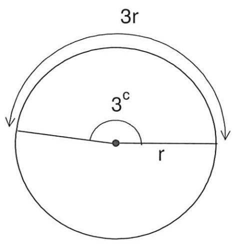
Consider the following circles each with a radius of r units.



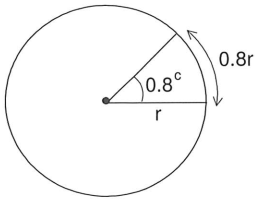
The length of the arc is r units and hence forms an angle of 1 radian at the centre.



The length of the arc is 2r units and hence forms an angle of 2 radians at the centre.



The length of the arc is 3r units and hence forms an angle of 3 radian at the centre.
Note that 3 radians is approximately equal to 180°.

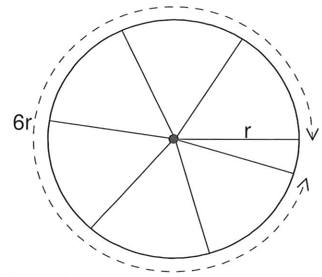


The length of the arc is 0.8r units and hence forms an angle of 0.8 radians at the centre.

Note that radians measure angles by the length of the arc that subtends an angle at the centre of a circle.

Relationship between degrees and radians
Consider the following circle with radius r units.

The diagram on the right shows that approximately 6.28 radians is the same as 360°.
On further investigation it has been found that the radius r fits around the circumference of a circle **exactly** 2π times.



Now one revolution is 360°, hence 360° = 2π radians
Therefore 180° = π radians

$$\boxed{180° = \pi \text{ radians}}$$

Converting degrees to radians
Example 5
Convert (a) 60° (b) 100° and (c) 225° to radians.

Solution

(a) $180° = \pi$ radians

$1° = \frac{\pi}{180}$ radians

$\therefore 60° = \frac{\pi}{180} \times 60$ radians

$= \frac{\pi}{3}$ radians

(b) $180° = \pi$ radians

$1° = \frac{\pi}{180}$ radians

$\therefore 100° = \frac{\pi}{180} \times 100$ radians

$= \frac{5\pi}{9}$ radians

(c) $180° = \pi$ radians

$1° = \frac{\pi}{180}$ radians

$\therefore 225° = \frac{\pi}{180} \times 225$ radians

$= \frac{5\pi}{4}$ radians

Converting radians to degrees
Example 6
Convert (a) $\frac{\pi}{6}$ radians (b) $\frac{5\pi}{12}$ radians and (c) 2.5 radians to degrees.

Solution

(a) π radians = 180°

$\frac{\pi}{6}$ radians $= \frac{180°}{6}$

$= 30°$

(b) π radians = 180°

$\frac{5\pi}{12}$ radians $= \frac{5 \times 180°}{12}$

$= 75°$

(c) π radians = 180°

1 radians $= \frac{180°}{\pi}$

2.5 radians $= \frac{2.5 \times 180°}{\pi}$

$= (\frac{450}{\pi})°$

Note for example 6 the exact answers have been given. If the conversion for part (c) was to be given rounded to two decimal place then it would be written as 143.24°.

Trigonometric ratios and radians
If angles of triangles are given in radians instead of degrees there is no need to change the radian measure of the angle into degree measure before applying the trigonometric ratio. When using a calculator to evaluate a trigonometric ratio in radians the calculator needs to be changed from working in degree mode to working in radian mode.

Example 7
Evaluate each of the following to two decimal places:

(a) $\sin\frac{\pi}{4}$ (b) $\cos\frac{4\pi}{5}$ (c) $\tan 2.3$ (d) $2\sin 4 \cos 3$

Solution
Make sure that your calculator is set to operate in radian mode.

(a) $\sin\frac{\pi}{4} = 0.71$ (b) $\cos\frac{4\pi}{5} = -0.81$ (c) $\tan 2.3 = -1.12$ (d) $2\sin 4\cos 3 = 1.50$

EXERCISE 13B

1. Express each of the following in radians.

(a) 180° (b) 360° (c) 90° (d) 720°

(e) 270° (f) 45° (g) −135° (h) 18°

(i) $-120°$ (j) $240°$ (k) $225°$ (l) $-150°$

(m) $5°$ (n) $-630°$ (o) $-108°$ (p) $-180°$

(q) $405°$ (r) $100°$ (s) $54°$ (t) $99°$

2. Express each of the following in degrees.
(a) 2π (b) $\frac{\pi}{2}$ (c) $-\frac{\pi}{4}$ (d) -3π

(e) $\frac{3\pi}{2}$ (f) $\frac{\pi}{12}$ (g) $\frac{3\pi}{4}$ (h) $\frac{5\pi}{2}$

(i) $\frac{4\pi}{3}$ (j) $\frac{\pi}{5}$ (k) $\frac{11\pi}{6}$ (l) $-\frac{7\pi}{4}$

(m) $\frac{3\pi}{8}$ (n) $-\frac{3\pi}{5}$ (o) $\frac{3\pi}{7}$ (p) $\frac{11\pi}{20}$

3. Convert each of the following to radians correct to two decimal places.
(a) $66°$ (b) $194°$ (c) $34°24'$ (d) $275°54'$

4. Convert each of the following to degrees correct to two decimal places.
(a) 1 rad (b) 3.2 rads (c) 0.8 rads (d) $0.\dot{6}$ rads

5. Evaluate each of the following to two decimal places:
(a) cos 1 (b) tan 2 (c) cos 3 (d) tan 1.5

6. (a) Given $\sin\theta = 0.5$ determine the size of angle θ if $0° < \theta < 90°$.

(b) Convert your answer to (a) into radians in terms of π.

7. (a) Given $\cos\theta = 0.5$ determine the size of angle θ if $0° < \theta < 90°$.

(b) Convert your answer to (a) into radians in terms of π.

8. (a) Given $\tan\theta = 1$ determine the size of angle θ if $0° < \theta < 90°$.

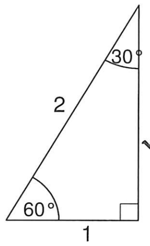
(b) Convert your answer to (a) into radians in terms of π.

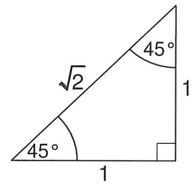
9. Given $\sin\theta = \frac{\sqrt{3}}{2}$ determine the size of angle θ in terms of π if $0 < \theta < \frac{\pi}{2}$.

10. Given $\tan\theta = \sqrt{3}$ determine the size of angle θ in terms of π if $0 < \theta < \frac{\pi}{2}$.

EXACT TRIGONOMETRIC VALUES

In chapter 11 we investigated exact values of sine, cosine and tangent at integer values of 30°, 60° and 45°. The results are summarised below.



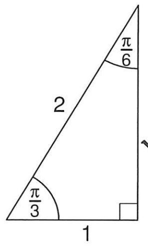


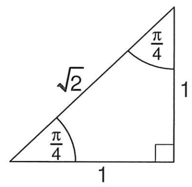
$\cos 30° = \frac{\text{adj}}{\text{hyp}} = \frac{\sqrt{3}}{2}$ $\quad \cos 60° = \frac{\text{adj}}{\text{hyp}} = \frac{1}{2}$ $\quad \cos 45° = \frac{\text{opp}}{\text{adj}} = \frac{1}{\sqrt{2}}$

$\sin 30° = \frac{\text{opp}}{\text{hyp}} = \frac{1}{2}$ $\quad \sin 60° = \frac{\text{opp}}{\text{hyp}} = \frac{\sqrt{3}}{2}$ $\quad \sin 45° = \frac{\text{opp}}{\text{hyp}} = \frac{1}{\sqrt{2}}$

$\tan 30° = \frac{\text{opp}}{\text{adj}} = \frac{1}{\sqrt{3}}$ $\quad \tan 60° = \frac{\text{opp}}{\text{adj}} = \sqrt{3}$ $\quad \tan 45° = \frac{\text{opp}}{\text{adj}} = 1$

Exact trigonometric ratios can be found using radians instead of degrees as shown below.





$\cos\frac{\pi}{6} = \frac{\text{adj}}{\text{hyp}} = \frac{\sqrt{3}}{2}$ $\quad \cos\frac{\pi}{3} = \frac{\text{adj}}{\text{hyp}} = \frac{1}{2}$ $\quad \cos\frac{\pi}{4} = \frac{\text{opp}}{\text{adj}} = \frac{1}{\sqrt{2}}$

$\sin\frac{\pi}{6} = \frac{\text{opp}}{\text{hyp}} = \frac{1}{2}$ $\quad \sin\frac{\pi}{3} = \frac{\text{opp}}{\text{hyp}} = \frac{\sqrt{3}}{2}$ $\quad \sin\frac{\pi}{4} = \frac{\text{opp}}{\text{hyp}} = \frac{1}{\sqrt{2}}$

$\tan\frac{\pi}{6} = \frac{\text{opp}}{\text{adj}} = \frac{1}{\sqrt{3}}$ $\quad \tan\frac{\pi}{3} = \frac{\text{opp}}{\text{adj}} = \sqrt{3}$ $\quad \tan\frac{\pi}{4} = \frac{\text{opp}}{\text{adj}} = 1$

Example 8

Find the exact value of each of the following ratios (a) $\sin\frac{\pi}{3}$ (b) $\cos\frac{\pi}{3}\sin\frac{\pi}{3}\tan\frac{\pi}{6}$ (c) $\tan\frac{\pi}{3} + 2\tan^2\frac{\pi}{4}$

Solution

(a) $\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$

(b) $\cos\frac{\pi}{3}\sin\frac{\pi}{3}\tan\frac{\pi}{6} = (\frac{1}{2})(\frac{\sqrt{3}}{2})(\frac{1}{\sqrt{3}})$
$= \frac{1}{4}$

(c) $\tan^2\frac{\pi}{3} + 2\tan^2\frac{\pi}{4} = (\sqrt{3})^2 + 2(1)^2$
$= 5$

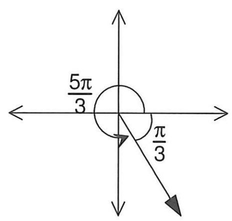
Example 9

Find the exact value of $\cos\frac{5\pi}{3}$.

Solution

An angle of $\frac{5\pi}{3}$ lies in the 4th quadrant where cosine is positive and makes an angle of $\frac{\pi}{3}$ with the x axis.

Hence $\cos\frac{5\pi}{3} = \cos\frac{\pi}{3} = \frac{1}{2}$



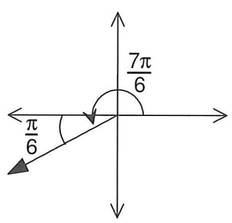
Example 10

Find the exact value of $\sin\frac{7\pi}{6}$

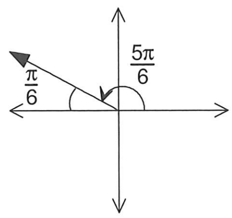
Solution

An angle of $\frac{7\pi}{6}$ lies in the 3rd quadrant where sine is negative and makes an angle of $\frac{\pi}{6}$ with the x axis.

Hence $\sin\frac{7\pi}{6} = -\sin\frac{\pi}{6} = -\frac{1}{2}$



Example 11
Find the exact value of $\tan\frac{5\pi}{6}$

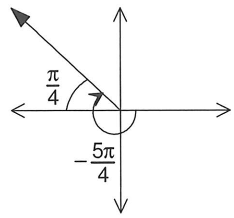


Solution
An angle of $\frac{5\pi}{6}$ lies in the 2nd quadrant where tangent is negative

and makes an angle of $\frac{\pi}{6}$ with the x axis.

Hence $\tan\frac{5\pi}{6} = -\tan\frac{\pi}{6} = -\frac{1}{\sqrt{3}}$

Example 12
Find the exact value of $\cos(-\frac{5\pi}{4})$



Solution
An angle of $-\frac{5\pi}{4}$ lies in the 2nd quadrant where cosine is negative

and makes an angle of $\frac{\pi}{4}$ with the x axis.

Hence $\cos(-\frac{5\pi}{4}) = -\cos\frac{\pi}{4} = -\frac{1}{\sqrt{2}}$

EXERCISE 13C

1. Evaluate each of the following without the aid of a calculator.

(a) $\cos\frac{4\pi}{3}$ (b) $\cos\frac{5\pi}{6}$ (c) $\cos\frac{3\pi}{2}$ (d) $\cos(-\frac{\pi}{4})$

(e) $\cos\frac{7\pi}{6}$ (f) $\cos\frac{5\pi}{3}$ (g) $\cos\frac{8\pi}{3}$ (h) $\cos(-\frac{5\pi}{4})$

2. Evaluate each of the following without the aid of a calculator.

(a) $\sin\frac{7\pi}{4}$ (b) $\sin\frac{5\pi}{6}$ (c) $\sin\frac{3\pi}{2}$ (d) $\sin(-\frac{7\pi}{4})$

(e) $\sin\frac{13\pi}{6}$ (f) $\sin\frac{5\pi}{2}$ (g) $\sin(-\frac{8\pi}{3})$ (h) $\sin(-\frac{23\pi}{6})$

3. Evaluate each of the following without the aid of a calculator.

(a) $\tan\frac{7\pi}{6}$ (b) $\tan(-\frac{5\pi}{6})$ (c) $\tan\frac{3\pi}{4}$ (d) $\tan(-\frac{7\pi}{4})$

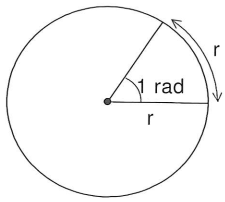
(e) $\tan\frac{11\pi}{6}$ (f) $\tan\frac{3\pi}{2}$ (g) $\tan(-\frac{5\pi}{3})$ (h) $\tan(-\frac{13\pi}{6})$

4. Evaluate each of the following without the aid of a calculator.

(a) $\cos\frac{\pi}{6}\sin(-\frac{5\pi}{6})\tan\frac{5\pi}{3}$ (b) $\tan^2\frac{\pi}{6} + 2\sin\frac{\pi}{3} + \tan\frac{5\pi}{4} - \tan\frac{\pi}{3} + \cos^2\frac{5\pi}{6}$

Length of arc using circular measure

We know that if an arc of length equal to the radius of a circle subtends and angle at the centre of the circle then the size of the angle is 1 radian.

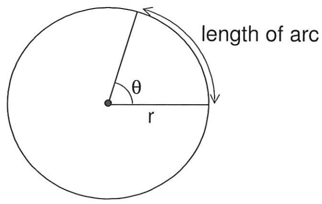


Therefore, the size of the central angle, in radians, subtended by an arc is given by the ratio $\dfrac{\text{length of arc}}{\text{radius of circle}}$.

Thus if θ radians is the size of the central angle and r the radius of the circle,

then $\theta = \dfrac{\text{length of arc}}{r}$.

Rearranging the above equation we obtain Length of arc = $r\theta$



$$\boxed{\text{Length of arc} = r\theta}$$

Example 13

Arc AB subtends an angle of 1.4 radians at the centre of circle centre O and radius 6.5 cm. Find the length of arc AB.

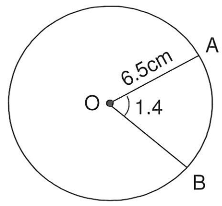
Solution

Length of arc AB $= r\theta$

$= 6.5 \times 1.4$ cm

$= 9.1$ cm

Hence arc AB is of length 9.1 cm.



Length of chord using circular measure

Consider chord AB shown below in figure 1.

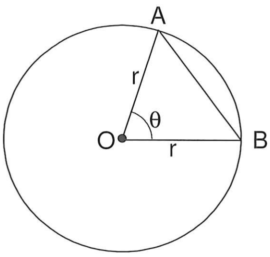


Figure 1

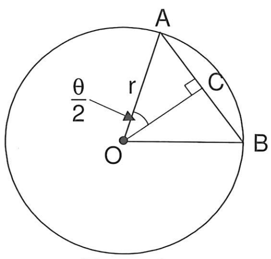


Figure 2

Figure 2 is obtained by drawing in line segment OC, the bisector of angle AOB and hence the perpendicular bisector or the chord AB. That is AC = BC.

Now in ΔAOC $\quad \sin\frac{\theta}{2} = \dfrac{AC}{AO} = \dfrac{AC}{r}$

Hence $\quad AC = r\sin\frac{\theta}{2}$

Therefore $\quad AB = 2r\sin\frac{\theta}{2}$ as $AC = \frac{1}{2}AB$

$$\boxed{\text{Length of chord} = 2r\sin\tfrac{\theta}{2}}$$

Example 14

Chord AB subtends an angle of 1.5 radians at the centre of circle centre O and radius 50 mm. Find the length of chord AB. Give your answer to the nearest millimetre.

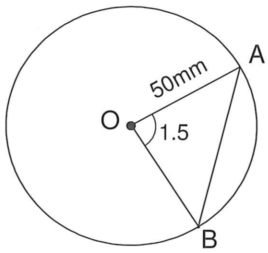
Solution

Length of chord $= 2r\sin\frac{\theta}{2}$

$= 2 \times 50 \times \sin\frac{1.5}{2}$ mm

$= 68.1638\ldots$ mm

Hence the length of the chord AB is 68 mm.



Area of sector using circular measure

Consider sector AOB shown right with radius r and central angle θ.

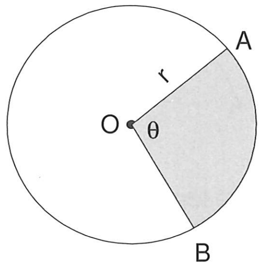
The area of sector may be found by comparing the ratios of an area of a sector and the angle it makes at the centre of the circle.

Hence we obtain

$$\frac{\text{Area of sector}}{\text{Area of circle}} = \frac{\text{angle of sector in radians}}{\text{angle of circle in radians}}$$

$$\frac{\text{Area of sector}}{\pi r^2} = \frac{\theta}{2\pi}$$

$$\text{Area of sector} = \frac{\theta\ \pi r^2}{2\pi} = \frac{\theta r^2}{2} = \tfrac{1}{2}r^2\theta$$



$$\boxed{\text{Area of sector} = \tfrac{1}{2}r^2\theta}$$

NOTE: Area of sector $= \frac{1}{2}r^2\theta = \frac{1}{2}r \times r\theta = \frac{1}{2}r \times \ell = \frac{1}{2}r\ell$ where ℓ is the length of the arc of the sector.

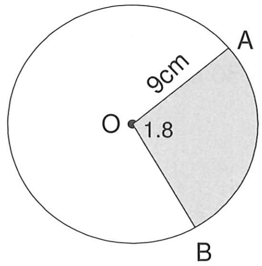
Example 15

Find the area of the sector shown shaded in the diagram on the right.

Solution

$$\begin{aligned}\text{Area of sector} &= \tfrac{1}{2}r^2\theta \\ &= \tfrac{1}{2} \times 9^2 \times 1.8\,\text{cm}^2 \\ &= 72.9\,\text{cm}^2\end{aligned}$$

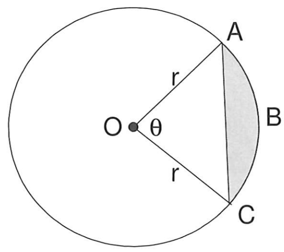
Hence area of shaded sector is $72.9\,\text{cm}^2$.



Area of segment using circular measure

Consider the segment ABC.

The area of segment ABC can be found by subtracting the area of ΔAOC from the area of minor sector AOC.



$$\begin{aligned}\text{Area of segment} &= \text{Area of minor sector AOC} - \text{Area of triangle AOC} \\ &= \tfrac{1}{2}r^2\theta - \tfrac{1}{2}(r)(r)\sin\theta \\ &= \tfrac{1}{2}r^2\theta - \tfrac{1}{2}r^2\sin\theta \\ &= \tfrac{1}{2}r^2(\theta - \sin\theta)\end{aligned}$$

$$\boxed{\text{Area of segment} = \tfrac{1}{2}r^2(\theta - \sin\theta)}$$

Example 16

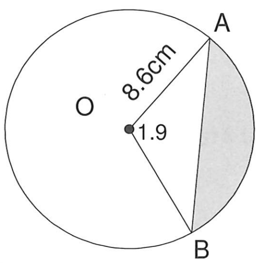
Find the area of the segment shown shaded in the diagram on the right.

Round your answer to one decimal place.

Solution

$$\begin{aligned}\text{Area of segment} &= \tfrac{1}{2}r^2(\theta - \sin\theta) \\ &= \tfrac{1}{2} \times 8.6^2 \times (1.9 - \sin 1.9) \\ &= 35.2678\ldots\end{aligned}$$

Hence area of the segment is 35.3 cm^2 rounded to 1 decimal place.



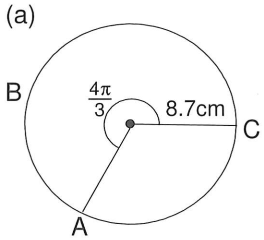
EXERCISE 13D

1. Find the lengths of the arcs with the following radii and angles subtended at the centre of the circle.

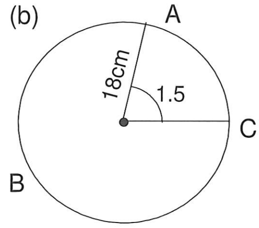
   (a) 12 cm, 2.4 (b) 8.7 cm, $\frac{2\pi}{3}$ (c) 45 mm, 3.6

2. Find the length of the arc ABC in each of the following. Give your answer as an exact value.

(a)



(b)



3. Find the length of the radii of the circular arcs with the following lengths and angles subtended at the centre.

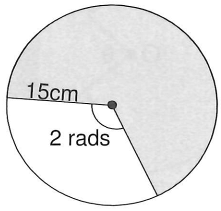
(a) 36 cm; 1.2 (b) $\frac{7\pi}{2}$ cm; $\frac{2\pi}{5}$ (c) 12m; $\frac{3\pi}{4}$

4. Find the angles subtended at the centre of circular arcs with the following lengths and radii.

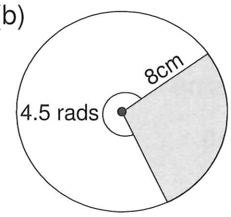
(a) 150 mm; 120 mm (b) $\frac{4\pi}{3}$ cm; 8 cm (c) 42 cm; 30 cm

5. Find the area of the shaded sectors below. Give answers rounded to one decimal place.

(a)

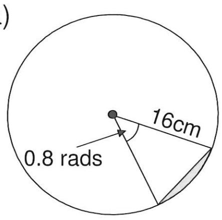


(b)

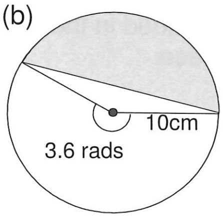


6. Find the area of the shaded segments below. Give answers rounded to one decimal place.

(a)



(b)



7. The minor arc of a circle with radius 20 cm subtends an angle of 1.2 radians at its centre.

(a) Calculate the area of the minor sector. (b) Calculate the area of the major sector.

8. A chord subtends an angle of $\frac{2\pi}{3}$ radians at the centre of a circle of radius 12 cm. Determine the difference in length of the chord and the minor arc cut off by the chord. Round your answer to two decimal places.

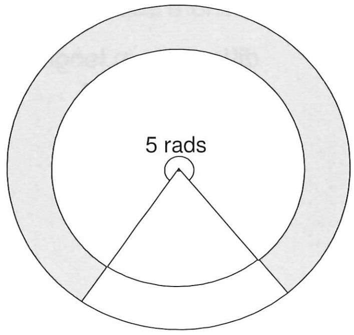
9. A chord of length 10 cm is drawn in a circle of radius 8 cm. Find the area of the minor segment cut off by this chord. Give your answer in square centimetres correct to two decimal places.

10. The area of the minor sector of a circle radius 8 cm is $48\,\text{cm}^2$.
(a) Determine the length of the minor arc.
(b) Determine the area of the minor segment cut off by the chord of the minor arc.

11. The length of the minor arc PQ of circle centre O and radius 12 cm is 30 cm.
(a) Find the area of the minor sector POQ.
(b) Find the area of the major segment cut off by chord PQ.

12 The area of the minor segment cut off by a chord in a circle of radius 10 cm is $30\,\text{cm}^2$. Calculate the area of the minor sector of this circle. Give your answer rounded to 2 decimal places.

13. The diagram on the right is made up of concentric circles with radii 36 cm and 26 cm. Calculate the area of the shaded region.



14. A chord of length 9 cm is drawn in a circle of radius 15 cm.
(a) Find the length of the minor arc cut off by this chord.
(b) Find the area of the minor segment.

15. Two circles of radii 8 cm and 15 cm have their centres 17 cm apart. Find the length of the minor arc of each circle. Give your answer rounded to two decimal places.

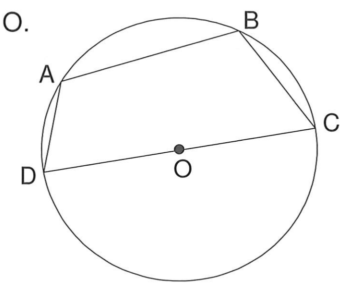
16. The radii of two circles are 7 cm and 4 cm respectively. The centres of these circles are 8 cm apart. Calculate the length of the arc of the smaller circle which is cut off by the larger circle. Give your answer rounded to two decimal places.

17. Two circles have radii 7 cm and 8 cm respectively and their centres are 12 cm apart. Calculate the area common to both circles. Give your answer rounded to two decimal places.

18. The centres of two circles are 10 cm apart. If the radii of the circles are 6 cm and 8 cm, calculate the area which is common to both circles. Give your answer rounded to two decimal places.

19. A go-kart racing track is made up of two circular arcs connected by two straights. The circular arcs are determined by circles of radii 32 metres and 60 metres and their centres are 68 metres apart. Calculate the length of the racing track to the nearest tenth of metre.

20. ABCD is a cyclic quadrilateral with DC the diameter of the circle centre O. DC is 24 cm, AD is 8 cm and minor arc BC is 3π cm.

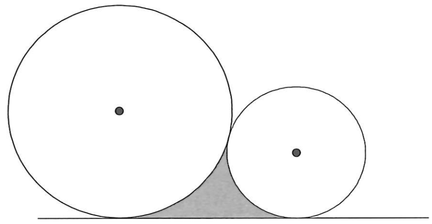


(a) Determine the exact area of sector BOC.

(b) Determine the area of the minor segment bound by chord AD.

(c) Determine the area of quadrilateral ABCD.

21. Two circles with radii 25 cm and 9 cm have a common tangent and touch each other as shown right.
(a) Determine the perimeter of the shaded region.

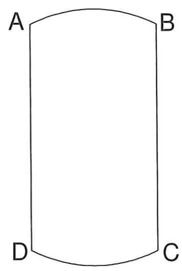


(b) Determine the area of the shaded region.

22. A goat is tethered to a post which is 5 metres from a fence. If the rope is 13 long, over what area can the goat graze? Give your answer to the nearest tenth of a square metre.

23. The diagram right shows the design of a window. The window comprises of a rectangle with circular segments attached to the top and bottom as shown. The segments are parts of a circle having its centre at the intersection of the diagonals of the rectangle.

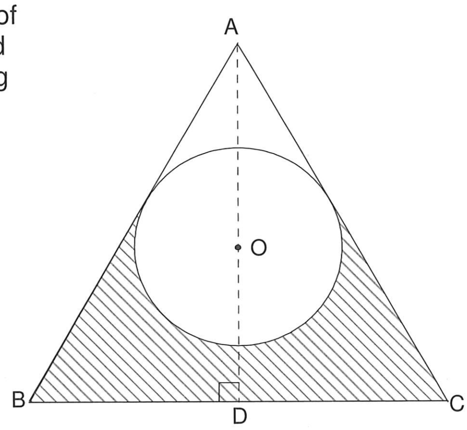
Determine the perimeter (to the nearest cm) and area (to the nearest square cm) of this window if AB = 1 m metre and BC = 2.4 metres.



24. If the human eye can detect a movement of 2.54 mm per second, how long must the minute hand of a clock be for the movement of its tip to be detected?

25. A belt fits tightly around two circular pulleys whose radii are 8 cm and 15 cm. If the centres of the pulleys are 25 cm apart find, correct to 2 decimal places, the length of the belt.

26. Consider the diagram shown right. The circle has a radius of 6 cm. AD the perpendicular from A to BC is 19 cm long and AO is 10 cm long. Find the area of the shaded region giving your answer to the nearest square centimetre.



27. A goat is tethered at a point P on the circumference of a circular garden of radius 24 metres. If the rope is 14 metres long, determine the area inside the garden over which the goat can graze.

CHAPTER THIRTEEN CHECKLIST

You now should be able to:

- calculate length of arcs and chords using degree and radian measure
- calculate areas of sectors and segments using degree and radian measure
- define and use radian measure and understand its relationship with degree measure
- recognise and use the exact values of $\cos\theta$, $\sin\theta$ and $\tan\theta$ at integer multiples of $\frac{\pi}{6}$ and $\frac{\pi}{4}$

CHAPTER THIRTEEN REVIEW EXERCISE

Questions numbered 1 to 6 should be completed without the aid of a calculator.

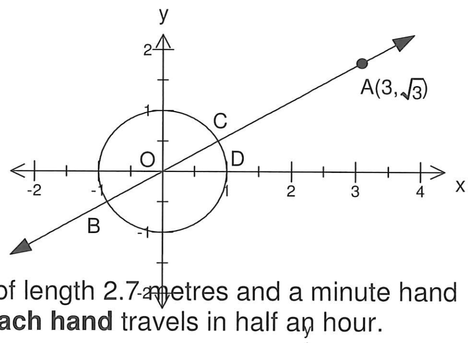
1. (a) Convert $\frac{8\pi}{3}$ into degrees. (b) Convert $285°$ into radians in terms of π.

2. A, B and C are acute angles such that $\sin A = 0.8$, $\cos B = \frac{12}{13}$ and $\tan C = \frac{1}{3}$.
   Determine the exact values of
   (a) $\cos A$ (b) $\sin B$ (c) $\sin C$

3. Determine the exact values of each of the following:
   (a) $\cos\frac{5\pi}{6}$ (b) $\sin\frac{5\pi}{3}$ (c) $\tan(-\frac{\pi}{3})$ (d) $\cos(-\frac{\pi}{4})$
   (e) $\sin 405°$ (f) $\tan\frac{7\pi}{6}$ (g) $\cos(-\frac{17\pi}{6})$ (h) $\tan(-\frac{3\pi}{2})$

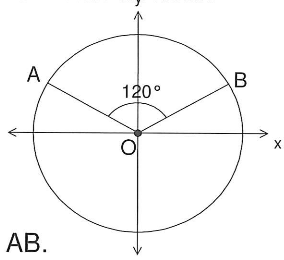
4. Determine the exact value of each of the following:
   (a) $\sin\frac{\pi}{4}\cos\frac{\pi}{4} + \tan\frac{5\pi}{6}\cos\frac{4\pi}{3}\sin(-\frac{13\pi}{3})$ (b) $\sin(-\frac{7\pi}{6})\cos\frac{2\pi}{3} - \sin(-\frac{3\pi}{2})\cos(-3\pi)\tan^2(\frac{10\pi}{3})$

5. Consider the diagram right.
   (a) State the equation of the circle and line AB.
   (b) Determine the co-ordinates of B, the point of intersection of the line AB and the circle.
   (c) Write down the value of the sine of angle BOD.
   (d) Hence, determine the length of minor arc BD.



6. The Great Clock of Westminster (Big Ben) has an hour hand of length 2.7 metres and a minute hand of length 4.3 metres. Calculate the exact distance the tip of **each hand** travels in half an hour.

7. The diagram on the right shows a circle with centre the origin of the axes of the Cartesian Coordinate Plane.
   The minor arc AB has a length of 5π cm and subtends an angle of $120°$ at the centre of the circle.
   (a) Find the length of the radius of the circle shown.
   (b) Find the exact area of the major sector AOB.
   (c) Find the area of the minor segment bounded by minor arc AB and chord AB.

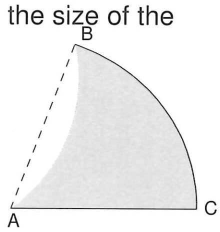


8. The perimeter and area of a sector are 28 cm and $48\,\text{cm}^2$ respectively. Determine the radius and central angle of such a sector.

9. Two circles overlap and have a common chord of length 30 cm. If the radii of the circles are 15 cm and 30 cm respectively, calculate the area common to both circles. Give your answer to the nearest square centimetre.

10. Two circles with radii 25 cm and 20 cm have their centres 30 cm apart. Determine the size of the common area to both circle correct to 0.1 square centimetres.

11. Consider the diagram shown right. The size of angle BAC is $70°$ and the circular arc BC has centre A and radius 9 cm. The circular arc AB has a radius of 8 cm.
    Calculate the perimeter and area of the shaded region.



12. A belt fits tightly around two circular pulleys whose radii are 20 cm and 11 cm. If the centres of the pulleys are 41 cm apart find, correct to 2 decimal places, the length of the belt.

13. Three circular garden beds are arranged as shown such that each circular flower bed touched the other two.
    The radii of circles centred A, B and C are 5 metres, 4 metres and 3 metres respectively.
    Find the area of the shaded region.