Student name:

1 Write down each statement below and its negation. Which of these two statements is true and which is false?

a 12 is divisible by 3.

b 4 is divisible by 12.

c The square of any odd number is even.

d The sum of any three prime numbers is an odd number.

e There is some triangle that has a reflex angle.

2 Write down the contrapositive of each statement.

a If you are frowning, then you are sad.

b If you have run far, then you will be hot.

c If you are in China, then you are in Asia.

d If $x = 2$, then $x^2 = 4$.

e If n^3 is odd, then n is odd.

3 Write down and prove the contrapositive of each statement below:

a If $4x + 3 > 11$, then $x > 2$.

b If $5n + 3$ is even, then n is odd.

c If $n^2 + 2n + 6$ is odd, then n is odd.

d If $5x + 2y > 20$, then $x > 2$ or $y > 5$.

e If $x^2 \neq 9$ then $x \neq 3$.

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Answers to Chapter 6 Skillsheet 6B

1 **a** Statement: 12 is divisible by 3 (true)
Negation: 12 is not divisible by 3 (false)

b Statement: 4 is divisible by 12 (false)
Negation: 4 is not divisible by 12 (true)

c Statement: The square of any odd number is even (false)
Negation: The square of some odd number is odd (true)

d Statement: The sum of any three prime numbers is an odd number (false)
Negation: The sum of some three prime numbers is an odd number (true)

e Statement: There is some triangle that has a reflex angle (false)
Negation: No triangle has a reflex angle (true)

2 Write down the contrapositive of each statement.

a If you are not sad, then you are not frowning.

b If you are not hot, then you have not run far.

c If you are not in Asia, then you are not in China.

d If $x^2 \neq 4$, then $x \neq 2$.

e If n is even, then n^3 is even.

3 Write down and prove the contrapositive of each statement below:

a Contrapositive: If $x \leq 2$, then $4x + 3 \leq 11$.

Proof: If $x \leq 2$, then

$$4x + 3 \leq 4 \times 2 + 3 = 11.$$

b Contrapositive: If n is even, then $5n + 3$ is odd.

Proof: If n is even, then $n = 2k$ for some integer k. Therefore,

$$\begin{aligned} 5n + 3 &= 5(2k) + 3 \\ &= 10k + 3 \\ &= 2(5k + 1) + 1 \end{aligned}$$

is odd.

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c Contrapositive: If n is even, then $n^2 + 2n + 6$ is even.

Proof: If n is even, then $n = 2k$ for some integer k. Therefore,

$$\begin{aligned} n^2 + 2n + 6 &= (2k)^2 + 2(2k) + 6 \\ &= 4k^2 + 4k + 6 \\ &= 2(2k^2 + 2k + 3) \end{aligned}$$

is even.

d Contrapositive: If $x \le 2$ and $y \le 5$, then $5x + 2y \le 20$.

Proof: If $x \le 2$ and $y \le 5$, then

$$5x + 2y \le 5 \times 2 + 2 \times 5 = 20.$$

e Contrapositive: If $x = 3$, then $x^2 = 9$.

Proof: If $x = 3$, then $x^2 = 3^2 = 9$.

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