11 Specialist Investigation 1 2023

Answers to questions in take-home part

Some of the answers below include the calculation used to obtain the answer; others do not. (But in the validation test, question parts worth more than 2 marks will – as usual – require working for full marks.)

1. What is the total number of different 7-permutators?

$$\begin{aligned}\text{Total number} &= 7! \\ &= 5040\end{aligned}$$

2. How many different 7-permutators swap the letters within just one pair (and leave all the other letters in the same place)?

$$\begin{aligned}\text{Total number} &= \binom{7}{2} \\ &= 21\end{aligned}$$

3. How many different 7-permutators swap letters within exactly 2 pairs?
   How many swap letters within 3 pairs (always leaving the other letters in the same place)?

$$\text{Number that swap letters within 2 pairs} = 105$$

$$\text{Number that swap letters within 3 pairs} = 105$$

4. How many different 7-permutators move *exactly* 3 letters (leaving the other letters where they are)?

$$\begin{aligned}\text{Number that move exactly 3 letters} &= \binom{7}{3} \times 2 \\ &= 70\end{aligned}$$

5. Can you find examples of 7-permutators which have loop lengths of 3, 4, 5 and 6?

   Here is an example of one with loop length 3:

$$\begin{pmatrix}1 & 2 & 3 & 4 & 5 & 6 & 7\\ 2 & 3 & 1 & 4 & 5 & 6 & 7\end{pmatrix}$$

6. Are there 'different ways' of obtaining 7-permutators with a given loop length (such as 6)?

   Yes. Here are two examples of 7-permutators with loop length 6:

$$\begin{pmatrix}1 & 2 & 3 & 4 & 5 & 6 & 7\\ 2 & 3 & 4 & 5 & 6 & 1 & 7\end{pmatrix}$$

$$\begin{pmatrix}1 & 2 & 3 & 4 & 5 & 6 & 7\\ 2 & 3 & 1 & 5 & 4 & 6 & 7\end{pmatrix}$$

7. What is the total number of 7-permutators with a loop length of 7?

$$\begin{aligned}\text{Total number} &= (7-1)! \\ &= 6! \\ &= 720\end{aligned}$$

8. What is the total number of 7-permutators with a loop length of 5?

$$\text{Total number} = 504$$

9. What is the maximum possible loop length of a 7-permutator?

$$\text{Maximum possible loop length} = 12$$