SEMESTER TWO

MATHEMATICS METHODS UNIT 1

2019

SOLUTIONS

Calculator–free Solutions

1. (a) Since $x = 1$ then $y = \dfrac{a+b}{4}$ and $\dfrac{1}{a} + \dfrac{y}{2a} = 1$ ✓

$\therefore \quad \dfrac{1}{a} + \dfrac{a+b}{8a} = 1$ ✓

$\therefore \quad 8 + a + b = 8a \rightarrow b = 7a - 8$ ✓

(b) $y = \dfrac{ax}{4} + \dfrac{b}{4} \rightarrow m_1 = \dfrac{a}{4}$ ✓

$\dfrac{x}{a} + \dfrac{y}{2a} = 1 \rightarrow y = -2x + 2a \rightarrow m_2 = -2$ ✓

Since $\perp$ $\dfrac{a}{4} = \dfrac{1}{2} \rightarrow a = 2$ ✓

and $b = 14 - 8 = 6$ ✓ [7]

2. (a) $-\dfrac{2}{2a} = -2$ ✓

$\therefore \quad -4a = -2 \rightarrow a = \dfrac{1}{2}$ ✓

(b) $f_2(x) = -\dfrac{1}{2}(x^2 + 4x) + b = -\dfrac{1}{2}\left[(x+2)^2 - 4\right] + b$ ✓

$= -\dfrac{1}{2}(x+2)^2 + 2 + b$ ✓

(c) Translation of c up $f_2(x) + c \rightarrow y = -\dfrac{1}{2}x^2 - 2x + (b + c)$ ✓

Since tangential, $b^2 - 4ac = 0$

$\therefore \quad 4 - 4\left(-\dfrac{1}{2}\right)(b + c) = 0$ ✓

$\therefore \quad 4 + 2(b + c) = 0$ ✓

$\therefore \quad c + b = -2 \rightarrow c = -2 - b$ ✓ [8]

3. (a) $y = a(x+1)(x-2)^2$ ✓

$\therefore \quad (0, 2) \rightarrow 2 = 1(4)a \rightarrow a = \dfrac{1}{2}$

$\therefore \quad y = \dfrac{1}{2}(x+1)(x-2)^2$ ✓

(b) $y = a\sqrt{x+4}$ ✓

$\therefore \quad (0, -4) \rightarrow -4 = a\sqrt{4} \rightarrow a = -2$ ✓

$\therefore \quad y = -2\sqrt{x+4}$ [4]

4. (a) $D_x = \{x \in R: -3 \le x \le 3\}$ ✓

$R_y = \{y \in R: -3 \le y \le 3\}$ ✓

(b) $D_x = \{x \in R: x \le 9\}$ ✓

$R_y = \{y \in R\}$ ✓

(c) There is a one to many mapping or vertical line test fails ✓ [5]

5. (a) (i) $140° = 140 \times \dfrac{\pi}{180} = \dfrac{7\pi}{9}$ ✓

(ii) $-\dfrac{360°}{\pi} = -\left(\dfrac{360}{\pi}\right) \times \dfrac{\pi}{180} = -2^R$ ✓

(b) (i) $\dfrac{2\pi}{5} = \dfrac{2\pi}{5} \times \dfrac{180}{\pi} = 72°$ ✓

(ii) $-1{\cdot}5^R = -1{\cdot}5 \times \dfrac{180}{\pi} = -\dfrac{270^R}{\pi}$ ✓

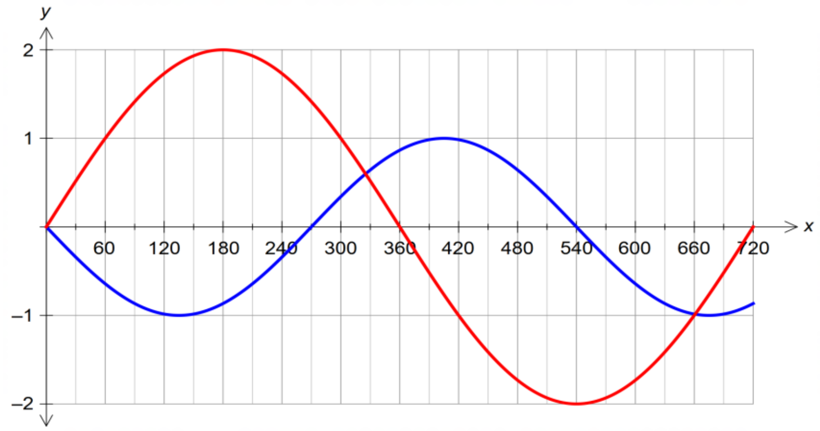
(c) $\sin 2^R \approx \sin 115° > \sin 2°$ ✓
since it has a higher y value in the Unit circle. ✓ [6]

6. (a) $\tan\dfrac{11\pi}{4} = \tan\left(2\pi + \dfrac{3\pi}{4}\right) = \tan\dfrac{3\pi}{4}$ ✓
$= -1$ ✓

(b) $\sin^2 30° - \cos^2 60° = \left(\dfrac{1}{2}\right)^2 - \left(\dfrac{1}{2}\right)^2$ ✓
$= 0$ ✓

(c) $\dfrac{\tan^2\dfrac{\pi}{3} + \cos\pi}{\sin\dfrac{\pi}{2} - \tan\dfrac{3\pi}{4}} = \dfrac{(\sqrt{3})^2 - 1}{1 - (-1)}$ ✓✓
$= \dfrac{2}{2} = 1$ ✓ [7]

7. (a) $a = -1$ ✓
$\dfrac{360}{b} = 540$ ✓
$\therefore\ b = \dfrac{2}{3}$ ✓

(b)


✓✓

(c) 0°, 330°, 660° ✓✓✓ [8]

8. (a) $f(x) = -2x(2x^2 - 9x + 4)$ ✓

$\therefore \quad f(x) = -2x(2x-1)(x-4)$ ✓

(b) (i) $18x^2 - 4x^3 - 8x = 0 \rightarrow -2x(2x-1)(x-4) = 0$

$\therefore \quad x = 0 \text{ or } \dfrac{1}{2} \text{ or } 4$ ✓

(ii) $36y^4 - 16y^3 - 8y^5 = 0 \rightarrow 2y^2(18y^2 - 4y^3 - 8y) = 0$

$\therefore \quad y = 0 \text{ or } \dfrac{1}{2} \text{ or } 4$ ✓✓ [5]

Calculator–Assumed Solutions

9. (a) $n = 5$ ✓

(b) $(x + by)^5 \rightarrow T_4 = \binom{5}{2} b^3 x^2 y^3 = 80x^2y^3$ ✓

$\therefore \quad 10b^3 = 80 \rightarrow b = 2$ ✓

(c) $(x + 2y)^5 \rightarrow T_5 = \binom{5}{4} 2^4 xy^4$ ✓

$\therefore \quad 80xy^4$ ✓ [5]

10. (a) (i) $T = \dfrac{k}{t^2}$ ✓

$12 = \dfrac{k}{4} \rightarrow k = 48$ ✓

(ii) $T = \dfrac{48}{t^2} \rightarrow T = \dfrac{48}{(4{\cdot}75)^2} = 2{\cdot}127$ ✓

$\therefore$ Temperature is 2°C ✓

(b) D ✓ [5]

11. (a) $\sin 43^\circ = \dfrac{x}{6} \rightarrow x = 6\sin 43^\circ = 4{\cdot}09$ ✓

$\therefore \quad \tan a^\circ = \dfrac{5}{4{\cdot}09} \rightarrow a = 50{\cdot}70^\circ$ ✓

(b) $r\,\theta = 24 \rightarrow r = \dfrac{24}{\theta}$ ✓

$54 = \dfrac{1}{2} r^2(\theta - \sin\theta)$ ✓

$\therefore \quad 54 = \dfrac{1}{2}\left(\left(\dfrac{24}{\theta}\right)^2\right)(\theta - \sin\theta)$ ✓

$\therefore \quad \theta = 1.211$ ✓

$\therefore \quad r = 19.8$ cm ✓ [7]

12. (a) $T = 720° = \frac{360}{n} \rightarrow n = \frac{1}{2}$ ✓

$\therefore \quad y = 2\sin\frac{x}{2}$ ✓

(b) $T = 360° \rightarrow n = 1$ ✓

$\therefore \quad y = \cos(x + c)$ ✓

$(45°, 1) \rightarrow 1 = \cos(45° + c) \rightarrow c = -45°$ ✓

$\therefore \quad y = \cos(x - 45°)$

(c) $T = \frac{\pi}{3} \rightarrow \frac{\pi}{3} = \frac{\pi}{n} \rightarrow n = 3$ ✓

$\therefore \quad y = -\tan 3x + 1$ ✓✓ [8]

13. (a) $\frac{1}{2}x + 5 = 2x - 4$ ✓

$\therefore \quad x + 10 = 4x - 8 \rightarrow x = 6$ ✓

$\therefore \quad$ A(6, 8) ✓

(b) $\left(\frac{6 + x}{2}, \frac{8 + y}{2}\right) = (2, 6)$ ✓

$\therefore \quad$ B(–2, 4) ✓

(c) $\overline{\text{CB}}$ has equation $y = -\frac{1}{2}x + c$ (Angle in Semi-circle Theorem)

$\therefore \quad (-2, 4) \rightarrow c = 3 \rightarrow \quad y = -\frac{x}{2} + 3$ ✓

Hence $-\frac{x}{2} + 3 = 2x - 4 \rightarrow x = 2{\cdot}8$ ✓

$\therefore \quad$ C(2.8, 1.6) ✓ [8]

14. (a) (i) $\sqrt{2}\sin(x + 45°) = \sqrt{2}\sin x \cos 45° + \sqrt{2}\sin 45° \cos x$ ✓

$= \sin x + \cos x$ ✓

(ii) $\sqrt{2}\sin 75° = \sqrt{2}\sin(45° + 30°) = \sin 30° + \cos 30°$ ✓

$= \frac{1}{2} + \frac{\sqrt{3}}{2}$ or $\frac{1}{2}(1 + \sqrt{3})$ ✓

(b) $\dfrac{\cos\left(\frac{\pi}{2} + \theta\right)}{\sin\left(\frac{\pi}{2} - \theta\right)} = \dfrac{-\sin\theta}{\cos\theta}$ ✓

$= -\tan\theta$ ✓ [6]

15. (a) It starts in section B. ✓

In section B, $h(0) = 0$ which is the initial height. ✓

(b) Section A = Section B when

$0{\cdot}01t^2 - 2t + 5 = -0{\cdot}001t^2 - t$ ✓

$\therefore \quad t = 5.3$ seconds ✓

(c) Occurs when Section A has its minimum ✓

$\therefore \quad h = 95$ m ✓

(d) Coaster back to height of zero when Section A = 0

$0{\cdot}01t^2 - 2t + 5 = 0$ ✓

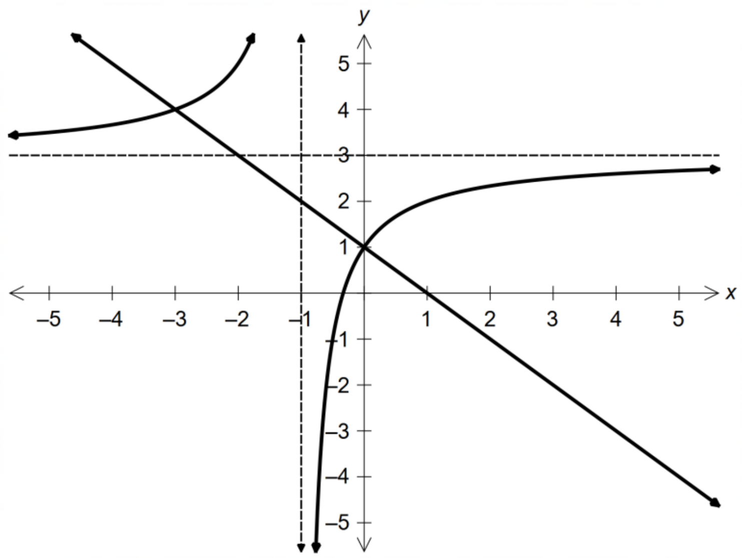
$\therefore \quad t = 197$ seconds ✓ [8]

16. (a) $y = \dfrac{a}{x+1} + 3$ ✓✓

$\therefore \quad (0, 1) \rightarrow 1 = \dfrac{a}{1} + 3$ ✓

$\therefore \quad a = -2$ ✓

(b)



✓✓ [6]

17. (a) (i) 8.6 m ✓

(ii) $t = 18 \rightarrow$ 6 pm ✓

(b) $-2\sin\left(\dfrac{\pi t}{12}\right) + 10 = 9$ ✓

$\therefore \quad t = 2$ and $t = 10$ ✓

Hence between 2 am and 10 am. ✓ [5]

18. (a) $b = 6$

$f = 10$ ✓

(b) $b + c = f$ ✓

(c) 13 ✓

(d) $\begin{pmatrix} 8 \\ 4 \end{pmatrix}$ ✓

(e) $p = \dfrac{1}{2}$ ✓✓ [6]

19. (a) (i) $\binom{5}{3}\binom{5}{2} = 10 \times 10 = 100$ ✓✓

(ii) $\binom{2}{2}\binom{3}{1}\binom{5}{2} + \binom{2}{1}\binom{3}{0}\binom{5}{4} + \binom{2}{2}\binom{3}{0}\binom{5}{3}$ ✓

$= 30 + 10 + 10 = 50$ ✓

(b) How many different selections containing four blue pens and a black pen are possible? ✓✓

(c) (i) $P(\text{at least one}) = 1 - P(0) = 1 - \dfrac{\binom{3}{0}\binom{7}{5}}{\binom{10}{5}}$ ✓

$= 1 - \dfrac{21}{252} = \dfrac{231}{252}$ ✓

(ii) $P(\text{at most two red} \mid \text{at least one red})$

$= \dfrac{\binom{3}{1}\binom{7}{4} + \binom{3}{2}\binom{7}{3}}{231}$ ✓

$= \dfrac{105 + 105}{231} = \dfrac{210}{231}$ ✓ [10]

20. (a) (i) $\dfrac{DB}{\sin 69°} = \dfrac{14}{\sin 23°}$ ✓

$\therefore$ DB = 33 m ✓

(ii) $20^2 = 15^2 + DB^2 - 2(15)(DB)\cos(ABD)$ ✓

$\therefore$ $\angle ABD = 19.84°$ ✓

(b) $\text{Area}(\Delta ABD) = \dfrac{1}{2} \times 15 \times DB \times \sin(ABD)$ ✓

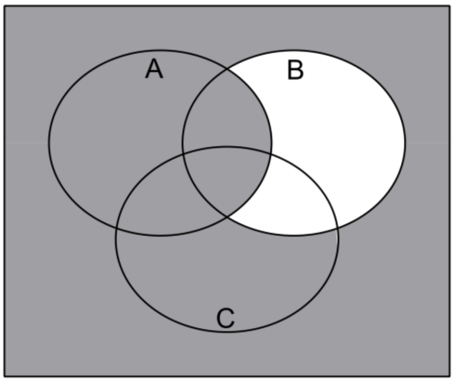
$= 85.16 \text{ m}^2$ ✓

$\text{Area}(\Delta BCD) = \dfrac{1}{2} \times 14 \times DB \times \sin 88°$ ✓

$= 234.01 \text{ m}^2$ ✓

$\therefore$ Total area $= 319.17 \text{ m}^2$ ✓ [7]

21. (a) (i)



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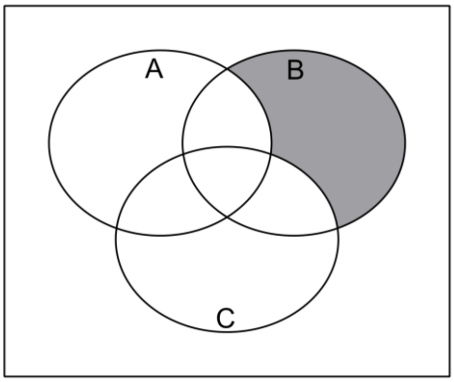
(ii)



✓✓

(b) $E' \cap (D \cup F)$ ✓✓ [6]

22. (a) $P(L1) = 0.7$, $P(L2) = 0.35$, $P(L2|L1) = 0.35$ and $P(L3) = 0.4$

Since $P(L2) = P(L2|L1) = 0.35$ ✓

then independent events. ✓

(b) (i) $P(\text{all three}) = P(L1) \times P(L2) \times P(L3)$

$= 0.7 \times 0.35 \times 0.4 = 0.098$ ✓✓

(ii) $P(L1 \cup L2) = P(L1) + P(L2) - P(L1 \cap L2)$

$= 0.7 + 0.35 - 0.7 \times 0.35 = 0.805$ ✓✓

(iii) $P((L1 \cup L2 \cup L3)' | L1')$

$= P(L2' \cup L3')$ ✓

$= 0.65 + 0.6 - 0.65 \times 0.6 = 0.86$ ✓ [8]

23. (a) (i) $\dfrac{185}{200}$ ✓

(ii) $\dfrac{85}{125}$ ✓✓

(b) $P(Y) = \dfrac{125}{200}$ and $P(7\&8) = \dfrac{100}{200}$

$P(Y \cap 7\&8) = \dfrac{85}{200}$ ✓

Since $P(Y) \times P(7\&8) \neq P(Y \cap 7\&8)$ then not independent. ✓ [5]