SEMESTER ONE

MATHEMATICS METHODS UNITS 1

2018

SOLUTIONS

Calculator–free Solutions

1. (a) (i) $98 = a(16) + 44 + 6$
$a = 3$ ✓
$g(-2) = -4$ ✓
(ii) $x = -\frac{11}{6}$ ✓
(iii) Minimum ✓

(b) $b^2 - 4ac < 0$
$121 - 4a(6) < 0$ ✓
$-24a < -121$
$a > \frac{121}{24}$ ✓ [6]

2. (a) (i) $y = \tan\left(\frac{x}{2}\right) + 1$ ✓✓
(ii) $x = -y^2 + 2$ ✓✓

(b) $y = \tan\left(\frac{x}{2}\right) + 1$ is a function. For each value of x there is only one corresponding y value. (If a vertical line is drawn anywhere on this function it will only intersect the graph a maximum of once.) ✓✓ [6]

3. (a) Line A: $y = \frac{1}{3}x$

Line B: $y = -3x$ ✓

$y = -3(2)$
$= -6$ ✓

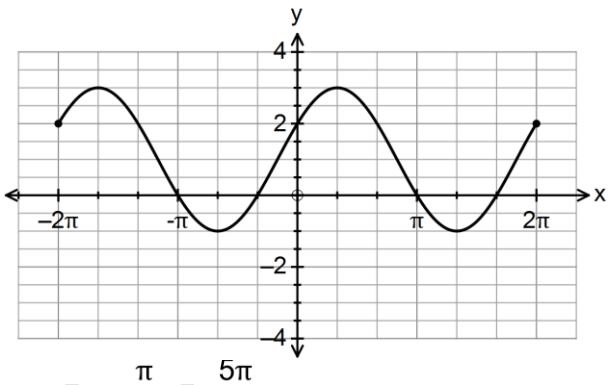
(b) Line CD: $-6 = \frac{1}{3}(2) + c$
$c = -\frac{20}{3}$ $\therefore y = \frac{1}{3}x - \frac{20}{3}$ ✓
Point $C(-1, -7)$ ✓
$\frac{-1 + x}{2} = 2$ $\frac{-7 + y}{2} = -6$ ✓
$x = 5$ and $y = -5$
$D(5, -5)$ ✓ [6]

4. (a) $\frac{4}{5}$ ✓

(b) $\tan\theta = -1$ or $\sin\theta = 0$ or $\sin\theta = 1$ ✓
$\theta = \frac{3\pi}{4}, \frac{7\pi}{4}, -\frac{\pi}{4}, 0, \pi, 2\pi, -\pi, \frac{\pi}{2}$ ✓✓ [4]

5. $x^2 - \frac{7}{5}x + \frac{1}{5} = 0$
$\left(x - \frac{7}{10}\right)^2 - \frac{49}{100} + \frac{1}{5} = 0$ ✓
$x - \frac{7}{10} = \pm\frac{\sqrt{29}}{10}$ ✓
$x = \frac{7 + \sqrt{29}}{10}$ or $\frac{7 - \sqrt{29}}{10}$ ✓ [3]

6. (a)



✓✓✓

(b) (i) $-\pi,\ -\frac{\pi}{3},\ \pi,\ \frac{5\pi}{3}$ ✓✓

(ii) $-\frac{4\pi}{3},\ -\frac{2\pi}{3},\ \frac{2\pi}{3},\ \frac{4\pi}{3}$ ✓✓ [7]

7. (a) $\frac{x^3 + 2x^2 - 5x - 6}{x + 1} = x^2 + x - 6$ ✓

$p(x) = (x + 1)(x + 3)(x - 2)$ ✓✓

(b) (i) $y = \frac{1}{2}(x + 1)(x - 2)^2$ ✓✓

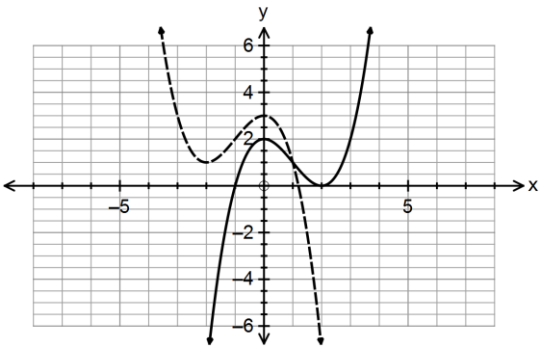
(ii) Vertical dilation, factor $\frac{1}{2}$. ✓

Horizontal translation one unit to the right ✓

Horizontal dilation, factor $\frac{1}{2}$. ✓

(ii) $\left(\frac{1}{2}, 1\right)$ ✓

(iii)



✓✓ [11]

8. (a) (i)

5	1	5	10	10	5	1	
6	1	6	15	20	15	6	1

✓✓

(ii) $p^6 - 6p^5q + 15p^4q^2 - 20p^3q^3 + 15p^2q^4 - 6pq^5 + q^6$ ✓✓

(b) $a = 10 \quad b = 15 \quad c = 10$ ✓✓ [6]

9. $\cos\left(\frac{5\pi}{12}\right) = \cos\left(\frac{\pi}{4} + \frac{\pi}{6}\right) = \cos\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{6}\right) - \sin\left(\frac{\pi}{4}\right)\sin\left(\frac{\pi}{6}\right)$ ✓

$= \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)$ ✓

$= \frac{\sqrt{6} - \sqrt{2}}{4}$ ✓ [3]

Calculator–assumed Solutions

10. (a) $k = 6$ ✓

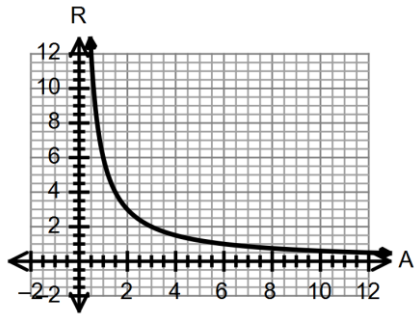
(b) $R = \frac{6}{5} = 1.2$ ohms ✓

(c) $R = \frac{6}{A}$

$\left(\frac{1}{1.35}\right)R = \frac{6}{1.35A}$ ✓

R will decrease by 74% ✓

(d)



✓✓✓ [7]

11. (a)

		Number of toilets			
		1	**2**	**3 or more**	**Total**
Number of bedrooms	**1**	35	**0**	**0**	35
	2	**50**	5	0	55
	3	5	65	**15**	**85**
	4 or more	0	0	**25**	25
Total		**90**	**70**	**40**	**200**

✓✓

(b) (i) $\frac{85}{200}$ ✓

(ii) $\frac{110}{200}$ ✓

(iii) $\frac{65}{200} + \frac{15}{200} = \frac{80}{200}$ ✓✓

(iv) $\frac{5}{70}$ ✓✓

(c) $\frac{{}^{5}\mathbf{C}_1 \times {}^{65}\mathbf{C}_1 \times {}^{15}\mathbf{C}_1}{{}^{85}\mathbf{C}_3} = \frac{4875}{98770} = 0.04936$ ✓✓ [10]

12. (a) $\overset{\frown}{PQ} = r\theta$

$= 12\pi\ cm$ ✓✓

(b) $\frac{\pi}{3}$ ✓

(c) Area of sector $= \frac{1}{2}r^2\theta$

$= \left(\frac{1}{2}\right)(18)^2\left(\frac{2\pi}{3}\right)$

$= 108\pi\ cm^2$ ✓

$\tan\frac{\pi}{3} = \frac{PR}{18} \rightarrow PR = 18\sqrt{3}$ ✓

Area of kite $= 2\left(\frac{1}{2} \times 18 \times 18\sqrt{3}\right) = 324\sqrt{3}\ cm^2$ ✓

Area of shaded area $= 324\sqrt{3} - 108\pi\ cm^2$ ✓ [7]

13. (a) $(x-5)^2 + (y-8)^2 = 25$ ✓

$x^2 - 10x + 25 + y^2 - 16y + 64 = 25$ ✓

$x^2 + y^2 - 10x - 16y + 64 = 0$

(b) (i) M(2, 4) N(8, 4) ✓

MN = 6 units ✓

(ii) $6^2 = 5^2 + 5^2 - 50\cos\theta$

$\theta = 1.2870$ ✓

$A = \frac{1}{2}r^2(\theta - \sin\theta)$

$A = \frac{1}{2}(5^2)(1.287 - \sin 1.287)$ ✓

$A = 4.0875$ units^2 ✓ [7]

14. (a) (i) $\frac{1}{2}$ ✓

(ii) $\frac{1}{12}$ ✓

(iii) $\frac{2}{12}$ ✓

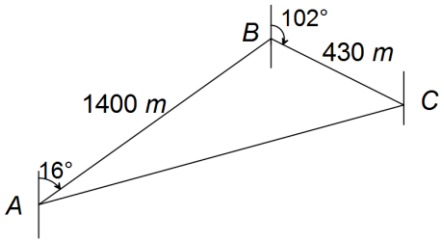
(b) $P(A \cap B) = \frac{1}{12} \neq 0$ ✓

(c) (i) $\frac{2}{12}$ ✓

(ii) $\frac{1}{4}$ ✓

(iii) $\frac{9}{12}$ ✓ [7]

15. (a)



$\angle ABC = 94°$ ✓

$AC^2 = 1400^2 + 430^2 - 2(1400)(430)\cos 94°$ ✓

$AC = 1492.95\ m$ ✓

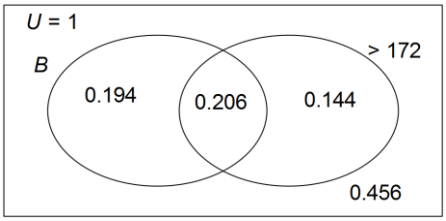
(b) $\frac{\sin C}{1400} = \frac{\sin 94}{1492.95}$ ✓

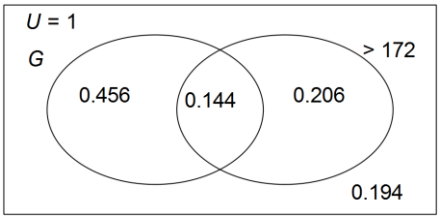
$\angle C = 69.3°$ ✓

$270° - (69.3 - 12) = 212.7°$ or $180 + (102 - 69.3) = 212.7°$

Bearing is 212.7°T ✓ [6]

16. (a)





✓✓✓

(b) (i) 0.144 ✓

(ii) 0.194 ✓

(iii) $\frac{0.144}{0.6} = 0.24$ ✓✓

(c) Girls over 172 cm = 38 ✓

Boys over 172 cm = 55 ✓

38:55 = 1:1.45 ✓ [10]

17. (a) $\sin\left(\frac{\pi}{6}\right) = \frac{PR}{r} \quad \therefore PR = \frac{r}{2}$ ✓

$\cos\left(\frac{\pi}{6}\right) = \frac{OR}{r} \quad \therefore OR = \frac{\sqrt{3}r}{2}$ ✓

$A = \frac{1}{2}\left(\frac{r}{2}\right)\left(\frac{\sqrt{3}r}{2}\right) = \frac{r^2\sqrt{3}}{8}$ ✓

(b) $A = \frac{1}{2}r^2\left(\frac{\pi}{6}\right) = \frac{\pi r^2}{12}$ ✓

$\frac{\pi r^2}{12} - \frac{r^2\sqrt{3}}{8} = \frac{2\pi - 3\sqrt{3}}{6}$ ✓

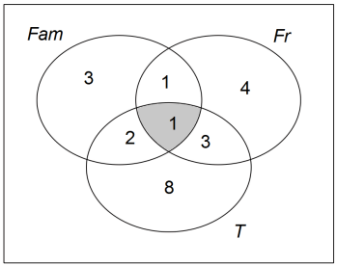
$r = 2\ cm$ ✓ [6]

18. (a) (i) 15! = 1 307 674 368 000 ✓

(ii) ${}^3\mathbf{C}_2 \times {}^4\mathbf{C}_2 \times {}^8\mathbf{C}_2 = 504$ ✓✓

(iii) $\frac{{}^3\mathbf{C}_2 \times {}^4\mathbf{C}_2 \times {}^8\mathbf{C}_2}{{}^{15}\mathbf{C}_6} = \frac{72}{715} = 0.100699$ ✓✓

(b) (i)



✓

This image has family and friends and travel in it. ✓

(ii) $\frac{4}{14}$ ✓✓ [9]

19. (a) D H ✓
(b) C G ✓
(c) G ✓
(d) G ✓
(e) H ✓
(f) A ✓
(g) E ✓
(h) A B ✓ [8]

20. (a) $(0.2 \times 0.75) + 0.8x = 0.39$ ✓
$x = 0.3$ ✓
(b) $\frac{0.15}{0.39} = \frac{5}{13}$ ✓✓
(c) The events are not independent. ✓
$P(S|B) \neq P(S)$
$\frac{15}{39} \neq 0.2$ ✓ [6]

21. (a) (i) $V = \frac{4}{3}\pi(15)^3 = 4500\pi\ cm^3$ ✓
(ii) $\frac{4500\pi}{15^3} = k$ ✓
$k = \frac{4}{3}\pi$
(iii) $\tan\theta = \frac{4\pi}{3}$ ✓
$\theta = 76.57°$ ✓
(b) Let x be the length, then $\frac{20}{x}$ is the width
$(x + 3)\left(\frac{20}{x} + 1\right) = 40$ ✓
$x = 5$ or 12 ✓
Original dimensions are $4m$ x $5m$ or $1\frac{2}{3}m$ x $12m$ ✓
(c) Domain: $\{x : x \neq 0, x \in \mathbb{R}\}$ ✓
Range: $\{y : y \neq 1, y \in \mathbb{R}\}$ ✓ [9]

22. A: $y = \frac{1}{2}\sin x$ B: $y = 3\sin x$ ✓✓
The amplitude for a quieter sound is $\frac{1}{2}$.
The amplitude increases to 3 to produce a louder sound. ✓
C: $y = \sin\frac{x}{2}$ D: $y = \sin 4x$ ✓✓
The period for a deeper pitch is 4π
The period for a higher pitch is $\frac{\pi}{2}$. ✓ [6]