

Insert School Logo

Semester One Examination 2019
Question/Answer Booklet

MATHEMATICS SPECIALIST
UNIT 1

Section One:
Calculator-free

Student Name: _____

Teacher's Name: _____

Time allowed for this section

Reading time before commencing work: five minutes

Working time for paper: fifty minutes

Material required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet

Formula Sheet

To be provided by the candidate

Standard items: pens(blue/black preferred), pencils(including coloured), sharpener,
correction tape/fluid, erasers, ruler, highlighters

Special Items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

	Number of questions available	Number of questions to be attempted	Suggested working time (minutes)	Marks available
Section One Calculator—free	6	6	50 minutes	53
Section Two Calculator—assumed	11	11	100 minutes	97
				150

Instructions to candidates

1. The rules for the conduct of Western Australian external examinations are detailed in the *Year 12 Information Handbook 2019*. Sitting this examination implies that you agree to abide by these rules.
2. Answer the questions according to the following instructions.

Section One: Write answers in this Question/Answer Booklet. Answer **all** questions.

Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

It is recommended that you **do not use pencil**, except in diagrams.

3. You must be careful to confine your responses to the specific questions asked and to follow any instructions that are specific to a particular question.
4. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
 - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
 - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.
5. The Formula Sheet is **not** handed in with your Question/Answer Booklet.

Section One: Calculator-free**53 marks**

This section has **six (6)** questions. Attempt **all** questions. Write your answers in the spaces provided.

Working time: 50 minutes

Question 1 (6 marks)

The diagram below shows the vectors \mathbf{u} and \mathbf{v} .

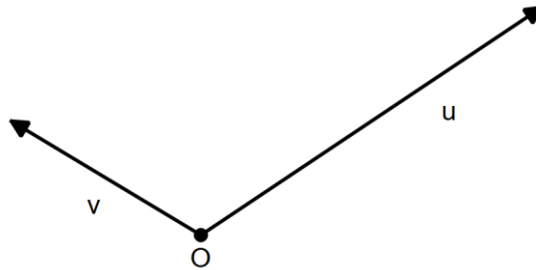
Using a ruler draw an accurate diagram of each of the following on the sketch below.

All your vector answers must start from O.

(a) $\mathbf{u} + 2\mathbf{v}$ (2 marks)

(b) $\mathbf{v} - \mathbf{u}$ (2 marks)

(c) $-\|\mathbf{u}\| \times \frac{\mathbf{v}}{\|\mathbf{v}\|}$ (2 marks)



Question 2 (14 marks)

- (a) Points P and Q have position vectors $\begin{pmatrix} 3 \\ -5 \end{pmatrix}$ and $\begin{pmatrix} 4 \\ \alpha \end{pmatrix}$ respectively, with $\alpha \in \mathbb{R}$.
Justifying your answers, determine the value(s) of α so that:

(i) OP is parallel to OQ. (2 marks)

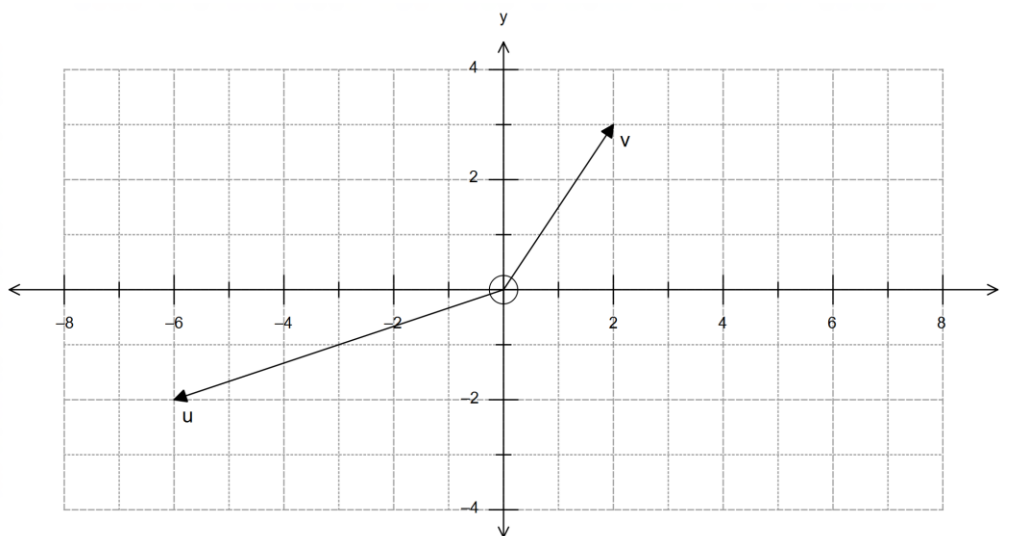
(ii) OQ is perpendicular to $\mathbf{i} + \mathbf{j}$. (2 marks)

(iii) PQ is a unit vector. (2 marks)

(iv) $\triangle OPQ$ is isosceles with PQ as its base. (2 marks)

(Question 2 – Continued)

- (b) The unit vectors \mathbf{i} and \mathbf{j} are called “reference vectors” because they can be combined to obtain any other vector in 2D space, however any other two non-parallel vectors can be used. The vectors \mathbf{u} and \mathbf{v} shown below are chosen as the new reference vectors in 2D.



- (i) Obtain an expression for both \mathbf{i} and \mathbf{j} in terms of \mathbf{u} and \mathbf{v} . Show your working clearly. (4 marks)
- (ii) Hence, or otherwise, write vector $\mathbf{r} = 14\mathbf{i} + 7\mathbf{j}$ in terms of \mathbf{u} and \mathbf{v} . (2 marks)

Question 3 (6 marks)

(a) If $18! = k$, then express each of the following in term of k .

(i) $20! - 18!$ (2 marks)

(ii) $\frac{{}^{20}P_{17}}{{}^{21}C_3}$ (2 marks)

(b) Show that ${}^nC_r = {}^nC_{n-r}$ is true for all integers n, r with $n > r$. (2 marks)

Question 4 (8 marks)

Consider the following statements.

A: If $m > m^2$, with $m \in \mathbb{R}$, then $m < 1$.

B: If a parallelogram has congruent diagonals, then the parallelogram is a rectangle.

C: $\forall p \in \mathbb{Q}, \exists a, b \in \mathbb{Z}: p = \frac{a}{b}$

- (a) State the converse of statement A. Is the converse always true? If yes, then explain why it is always true; if not provide a counter example and adjust the domain of m so that the converse is always true. (4 marks)

- (b) Write down the contrapositive of statement B. Is the contrapositive always true? Explain. (2 marks)

- (c) Rewrite statement C in words. (2 marks)

Question 5 (13 marks)

Consider the portion of Pascal's triangle shown below.

					1		2		1				
					1		3		3		1		
				1		4		6		4		1	
			1		5		10		10		5		1
		1		6		15		20		15		6	
	1		7		21		35		35		21		7
1		8		28		56		70		56		28	
													1

- (a) Determine each of the following powers using the elements of Pascal's triangle. Show your working clearly.

(i) 2^6 (1 mark)

(ii) 11^5 [Hint: $11 = 10 + 1$] (2 marks)

- (b) State the value(s) of x in each case below.

(i) ${}^6C_x = 20$ (1 mark)

(ii) ${}^xC_5 = 21$ (1 mark)

(iii) ${}^xC_2 = {}^xC_6$ (1 mark)

(Question 5 – Continued)

(c) Use the elements of Pascal's triangle to expand and simplify $(2x - y)^5$. (3 marks)

(d) An academic team of 5 members is to be composed from 3 chemists, 3 biologists and 2 environmentalists. How many different teams of 5 academics can be assembled if:

(i) there are no other restrictions? (1 mark)

(ii) both environmentalists must be chosen? (1 mark)

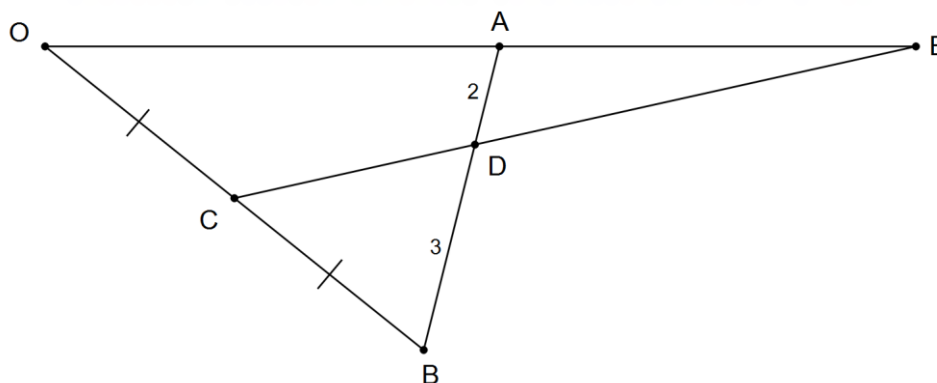
(iii) it must contain at least two chemists? (2 marks)

Question 6 (6 marks)

The figure below shows triangle OAB.

Point C is the midpoint of OB, and CE crosses AB at the point D such that $AD:DB = 2:3$.

Let $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$.



- (a) Determine \overrightarrow{AD} and \overrightarrow{CD} in terms of \mathbf{a} and \mathbf{b} .

(3 marks)

(Question 6 – Continued)

- (b) If $\overrightarrow{OE} = \alpha \overrightarrow{OA}$ and $\overrightarrow{CE} = \beta \overrightarrow{CD}$, use the fact that $\overrightarrow{OC} + \overrightarrow{CE} = \overrightarrow{OE}$ to determine α and β . (3 marks)

End of Section One

Additional working space

Question number(s):

Additional working space

Question number(s):

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Question number(s):

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Question number(s):