

# SEE 2021 Entry

Rishabh Jain

My Entry is [2, 2, 59, 16, 18, 1, 1, 1, 0, 0]

## 1 Overview

The most important thing that I immediately noticed is that there is no optimal solution or a Nash equilibrium for this problem. Given any arrangement, my opponent can simply construct a new one with only a few soldiers moved to beat me. For this reason, while there are certain strategies that I will discuss, my entry is mostly based on what I expect my opponents to submit.

There are two more interesting factors about this particular competition. First, the W/L ratio doesn't matter at all, as we are being ranked on average score. This means that I want to win by the biggest possible margin, as well as still score points when I lose. Next, winning three castles automatically wins all of the remaining castles. This innocent provision actually warps all strategy around itself, because it is the easiest way to win the most possible points with the least investment into different castles.

## 2 Further Analysis

I approached this puzzle in two ways. I thought about what a smart person with some knowledge of game theory would enter. I also used code to generate thousands of different entries and see what tended to win, and test my own entry. The total number of different arrangements is  $\binom{109}{100} \approx 4.2$  trillion, which is way too many to compare all of them. However, I realized that a large majority of these are terrible distributions that no one would ever actually use. So what I did was test entries against a sample of entries that I thought other people would pick, and I added in some random ones just to see how mine fared. This is the `smartComparison()` method.

One simple strategy is to try to blitz unsuspecting opponents by winning the first three castles, something like [33, 33, 34, 0, ..., 0]. When this strategy works, it usually wins 55-0, which is the best possible margin and leads to extremely high scores. Unfortunately, because of its strength, I think that most people will have thought of it, and it is pretty simple to counter by perhaps shifting to something like [0, 0, 40, 30, 30, 0, ..., 0]. Someone might then counter that with something like [1, 1, 41, 29, 28, 0, ..., 0]. This could continue endlessly, and it's a matter of figuring out how far to go. What I realized is that a lot of strategies

will try to win either the first three castles, or castles 4, 5, and 6. For this reason, castles 7 through 10 are not very useful, because the vast majority of games will have been won or lost already through the “three in a row” provision.

So if most people are going to try to win 3 in a row, why not completely prevent that while picking up as many castles as possible? An entry for this looked something like [1, 1, 45, 1, 2, 45, 2, 1, 1, 1]. The problem with this method was that its average score wasn’t that high, maybe in the high 20s or low 30s depending on the competition. It was fairly consistent, because it was difficult to win a triple castle versus it, but because of all the 1s, it tended to only win the last couple of castles. In the end I think the best way is to try to win a triple castle, while accounting for the possibilities where that doesn’t work out.

As a reminder, my entry is [2, 2, 59, 16, 18, 1, 1, 1, 0, 0]. Because of the focus on early castles, I decided to concentrate my troops on castle 3. This covers a possible blitz attack on castles 1 to 3, 2 to 4, and 3 to 5. In any of those scenarios, I will lose two out of the three “blitzed” castles, but I will eventually assemble a triple of my own and win. If my opponent commits to winning the first three castles, it is almost impossible for them to win unless they specifically try to counter my strategy by placing a lot of soldiers on castle 3. In my testing, my entry usually got an average score of between 35 and 45, depending on what factors I used. Obviously, it’s not perfect. Someone who thinks of the same thing I did and then decides to go one step further with something like [3, 3, 60, 18, 15, 1, 0, 0, 0, 0] would beat me every time. However, I feel like my solution is far enough removed from the chain of counters that there won’t be a ton of people who have something specifically designed to counter my strategy. Overall, I should (hopefully) obtain a decent average.