Table 4.2: Common Distributions

Distribution	PMF/PDF and Support	Mean	Variance	MGF
Bernoulli Bern(p), $p \in [0,1]$	$\mathbb{P}(X=k) = p^k(1-p)^{1-k}$ $k \in \{0,1\}$	p	$p(1-p)$	$(1-p)+pe^t$
Binomial Bin(n,p), $p \in [0,1]$	$\mathbb{P}(X=k) = \binom{n}{k}p^k(1-p)^{n-k}$ $k \in \{0,1,2,\ldots n\}$	np	$np(1-p)$	$[(1-p)+pe^t]^n$
Discrete uniform Unif($\{1,2,\ldots,N\}$), $N=1,2,\ldots$	$\mathbb{P}(X=k) = \frac{1}{N}$ $k \in \{1,2,\ldots,N\}$	$\frac{N+1}{2}$	$\frac{(N+1)(N-1)}{12}$	$\frac{1}{N}\sum_{k=1}^{N} e^{kt}$
Geometric Geo(p), $p \in [0,1]$	$\mathbb{P}(X=k) = (1-p)^{k-1}p$ $k \in \{1,2,\ldots\}$	$1/p$	$(1-p)/p^2$	$\frac{pe^t}{1-(1-p)e^t}$, $t < -\log(1-p)$
Hypergeometric HGeom(N,M,n), $0 \le M \le N$, $N=1,2\ldots$	$\mathbb{P}(X=k) = \frac{\binom{M}{k}\binom{N-M}{n-k}}{\binom{N+M}{n}}$ $k \in \{0,1,2,\ldots,n\}$	$\frac{nM}{N}$	$\frac{nM}{N}\frac{(N-M)(N-n)}{N(N-1)}$	messy
Negative Binomial NBin(r,p), $r=1,2,\ldots$	$\mathbb{P}(X=k) = \binom{r+k-1}{k}p^r(1-p)^k$ $k \in \{0,1,2,\ldots\}$	$\frac{r(1-p)}{p}$	$\frac{r(1-p)}{p^2}$	$\left[\frac{p}{1-(1-p)e^t}\right]^r$, $t < -\log(1-p)$
Poisson Poisson(λ), $\lambda \ge 0$	$\mathbb{P}(X=k) = \frac{\lambda^k}{k!}e^{-\lambda}$ $k \in \{0,1,2,\ldots\}$	λ	λ	$e^{\lambda(e^t-1)}$
Beta Beta(α,β), $\alpha,\beta > 0$	$f_X(x) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}x^{\alpha-1}(1-x)^{\beta-1}$ $x \in [0,1]$	$\frac{\alpha}{\alpha+\beta}$	$\frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$	exists but has no analytic expression
Cauchy Cauchy(μ,σ), $\mu \in \mathbb{R}$, $\sigma > 0$	$f_X(x) = \frac{1}{\pi\sigma}\frac{1}{1+\left(\frac{x-\mu}{\sigma}\right)^2}$ $x \in \mathbb{R}$	does not exist	does not exist	does not exist
Chi Square χ^2_n, $n=1,2,\ldots$	$f_X(x) = \frac{1}{2^{n/2}\Gamma(n/2)}x^{n/2-1}e^{-x/2}$ $x \ge 0$	n	$2n$	$(1-2t)^{-n/2}$, $t < 1/2$
Exponential Exp(λ), $\lambda > 0$	$f_X(x) = \lambda e^{-\lambda x}$ $x \ge 0$	$\frac{1}{\lambda}$	$\frac{1}{\lambda^2}$	$\left(1-\frac{t}{\lambda}\right)^{-1}$, $t < \lambda$
Fisher $F_{p,q}$, $p,q=1,2,\ldots$	$f_X(x) = \frac{\Gamma\left(\frac{p+q}{2}\right)}{\Gamma\left(\frac{p}{2}\right)\Gamma\left(\frac{q}{2}\right)}\left(\frac{p}{q}\right)^{\frac{p}{2}}x^{\frac{p}{2}-1}\left(1+\frac{p}{q}x\right)^{-\frac{p+q}{2}}$ $x \ge 0$	$\frac{q}{q-2}$, if $q > 2$	$2\left(\frac{q}{q-2}\right)^2\frac{p+q-2}{p(q-4)}$, if $q > 4$	does not exist
Gamma Gamma(α,λ), $\alpha,\lambda > 0$	$f_X(x) = \frac{\lambda^\alpha}{\Gamma(\alpha)}x^{\alpha-1}e^{-\lambda x}$ $x \ge 0$	$\frac{\alpha}{\lambda}$	$\frac{\alpha}{\lambda^2}$	$\left(1-\frac{t}{\lambda}\right)^{-\alpha}$, $t < \lambda$
Laplace (Double Exponential) Laplace(μ,σ), $\mu \in \mathbb{R}$, $\sigma > 0$	$f_X(x) = \frac{1}{2\sigma}e^{-\lvert x-\mu\rvert/\sigma}$ $x \in \mathbb{R}$	μ	$2\sigma^2$	$\frac{e^{\mu t}}{1-(\sigma t)^2}$, $\lvert t\rvert < \sigma^{-1}$
Logistic Logistic(μ,σ), $\mu \in \mathbb{R}$, $\sigma > 0$	$f_X(x) = \frac{1}{\sigma}\frac{e^{-(x-\mu)/\sigma}}{[1+e^{-(x-\mu)/\sigma}]^2}$ $x \in \mathbb{R}$	μ	$\frac{\pi^2\sigma^2}{3}$	$e^{\mu t}\Gamma(1-\sigma t)\Gamma(1+\sigma t)$, $\lvert t\rvert < \sigma^{-1}$
Log-Normal $\mathcal{LN}(\mu,\sigma^2)$, $\mu \in \mathbb{R}$, $\sigma > 0$	$f_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}}x^{-1}e^{-\frac{(\ln x-\mu)^2}{2\sigma^2}}$ $x > 0$	$e^{\mu+\sigma^2/2}$	$e^{2\mu+\sigma^2}(e^{\sigma^2}-1)$	does not exist
Normal $\mathcal{N}(\mu,\sigma^2)$	$f_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}}e^{-\frac{(x-\mu)^2}{2\sigma^2}}$ $x \in \mathbb{R}$	μ	σ^2	$e^{t\mu+\frac{\sigma^2t^2}{2}}$
Pareto Pareto(α,β), $\alpha,\beta > 0$	$f_X(x) = \frac{\beta\alpha^\beta}{x^{\beta+1}}$ $x > \alpha$	$\frac{\alpha\beta}{\beta-1}$ if $\beta > 1$	$\frac{\alpha^2\beta}{(\beta-1)^2(\beta-2)}$ if $\beta > 2$	does not exist
Student-t t_n, $n=1,2,\ldots$	$f_X(x) = \frac{\Gamma((n+1)/2)}{\sqrt{n\pi}\Gamma(n/2)}(1+x^2/n)^{-(n+1)/2}$ $x \in \mathbb{R}$	0 if $n > 1$	$\frac{n}{n-2}$ if $n > 2$	does not exist
Uniform U(a,b), $a < b$	$f_X(x) = \frac{1}{b-a}$ $x \in [a,b]$	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$	$\frac{e^{bt}-e^{at}}{(b-a)t}$
Weibull Weibull(γ,β), $\gamma,\beta > 0$	$f_X(x) = \frac{\gamma}{\beta}x^{\gamma-1}e^{-x^\gamma/\beta}$ $x \ge 0$	$\beta^{1/\gamma}\Gamma\left(1+\frac{1}{\gamma}\right)$	$\beta^{2/\gamma}\left[\Gamma\left(1+\frac{2}{\gamma}\right)-\Gamma^2\left(1+\frac{2}{\gamma}\right)\right]$	exists for $\gamma > 1$, but is not helpful