CSCI 338 : Computer Science Theory

Sample Question 2 (30-35 minutes)

Given graph $G = (V, E)$, an *Independent Set* of size K is a set $V' \subseteq V$, $|V'| = K$, such that for every $u, v \in V'$ we have $(u, v) \notin E$. We learn in class that it is NP-complete. This question is about another problem called Set Packing.

INSTANCE: Collection C of finite sets, positive integer $K \leq |C|$.
QUESTION: Does C contain K disjoint sets?

Example. $C = \{\{1, 3, 5\}, \{2, 3, 4\}, \{2, 4, 6\}, \{3, 6\}\}$. {1,3,5} and {2,4,6} are the 2 disjoint sets C contains.

(2.1) Prove that Set Packing is in NP.

- Nondeterministically select K sets from C, e.g., $D_1, D_2, \cdots, D_K$, where $D_i \in C$.
- Check $D_i \cap D_j = \phi$ for all $1 \leq i \neq j \leq K$.
- If all the $\binom{K}{2}$ tests pass, accept; otherwise, reject.

//Checking $D_i \cap D_j = \phi$ can be done by sorting in $O(n \log n)$ time.

The total time is $\binom{K}{2} \cdot O(n \log n) = O(n^3 \log n)$ time.

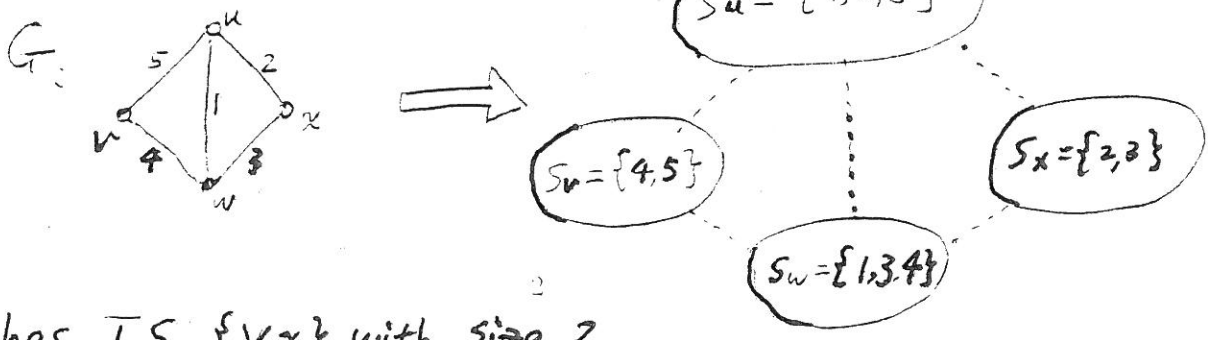
Therefore, Set Packing $\in NP$.

(2.2) Design a polynomial time reduction function which reduces (the decision version of) Independent Set to (the decision version of) Set Packing.

Take an input graph $G=(V,E)$ for IS, construct a collection of $|V|$ finite sets (corresponding to the vertices in V) as follows:

- If there is an edge i between u, v then add i into the sets associated with u, v, S_u and S_v. (Initially S_u is empty, for all $u \in V$.)
- G has an IS of size K iff C has a collection of K disjoint sets.
- The reduction takes $O(|V|+|E|) \in O(n^2)$ time.

Example



- G has IS $\{v, x\}$ with size 2.
- C has 2 disjoint sets $\{4, 5\}$ $\{2, 3\}$