CSCI 338: Quiz 4 (due: Friday, March 26, 8:00pm)

Your Name:

This is an open-book quiz (not an attendance counting), so you should try your best. After you finish, upload a pdf file on D2L under Quiz-4. A solution will be posted on D2L after the deadline.

Problem 1

On March 19, we covered an undecidability proof for $REGULAR_{TM}$, where we construct a TM M_2 with the property that either $L(M_2) = \Sigma^*$ or $L(M_2) = \{0^n 1^n | n \geq 0\}$ — depending on whether M accepts w or not. (This is basically Theorem 5.2 in the textbook.) On the other hand, notice that in this two cases $L(M_2)$ have overlaps (as $\{0^n 1^n\} \subset \Sigma^*$).

In this quiz, you are asked to write a new undecidability proof for $REGULAR_{TM}$ such that in the two cases (i.e., when M accepts w, and when M doesn't accept w), the corresponding two languages $L(M_2)$ do not overlap at all.

IDEA: First identify a language X which is regular and

$X \cap \{0^n 1^n\} = \phi$.

I will choose $\{1^+\}$, which can be generated as $S \rightarrow 1S | 1$, which is certainly regular.

<u>proof</u>: Construct a TM M_2 on input x,

M_2 on X:

If x is not in the form of 1^+ or 0^n1^n, <u>reject</u>

otherwise, run M on w and accept 1^+ if M accepts w;

accept 0^n1^n if M doesn't accept w.

// if M accepts $w \Rightarrow M_2$ accepts $1^+ \Rightarrow L(M_2)$ is regular

if M doesn't accept $w \Rightarrow M_2$ accepts $0^n1^n \Rightarrow L(M_2)$ is not regular

Now, assuming TM R decides $REGULAR_{TM}$, we construct a TM S for A_{TM}.

S for A_{TM}, on $\langle M, w\rangle$:

1. Construct TM M_2 as above.
2. Run R on $\langle M_2\rangle$.
3. If R accepts, <u>accept</u>,
   If R rejects, <u>reject</u>.

$\therefore$ we have a decider S for A_{TM}, a contradiction to the fact that A_{TM} is undecidable

$\therefore REGULAR_{TM}$ is undecidable $\square$