CSCI 338: Assignment 2 (7 points)

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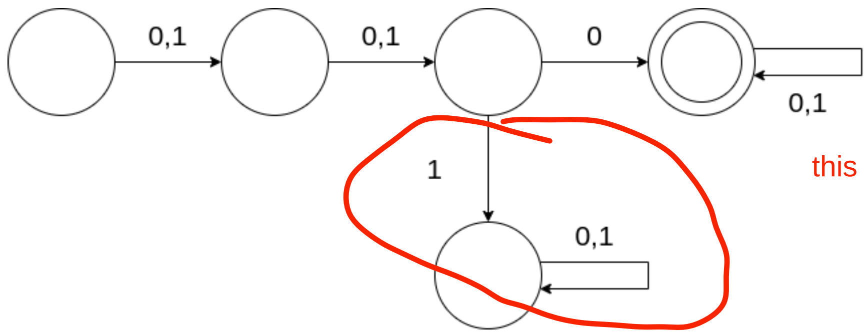
Problem 1

1.1 Give state diagrams of DFAs recognizing the following languages. In all parts, the alphabet is $\{0, 1\}$.

1.6.d

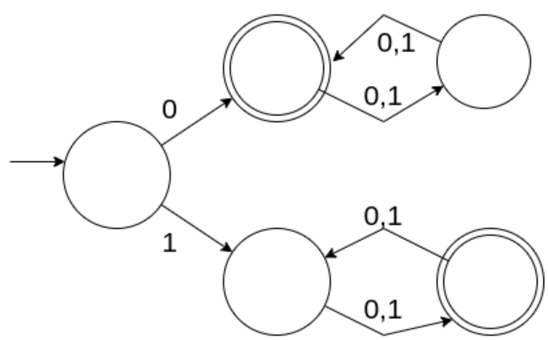
$\{w \mid w$ has length at least 3 and its third symbol is a $0\}$



this part can be removed.

1.6.e

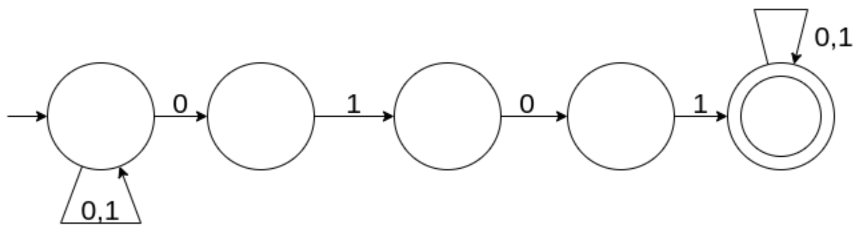
$\{w \mid w$ starts with 0 and has odd length, or starts with 1 and has even length $\}$



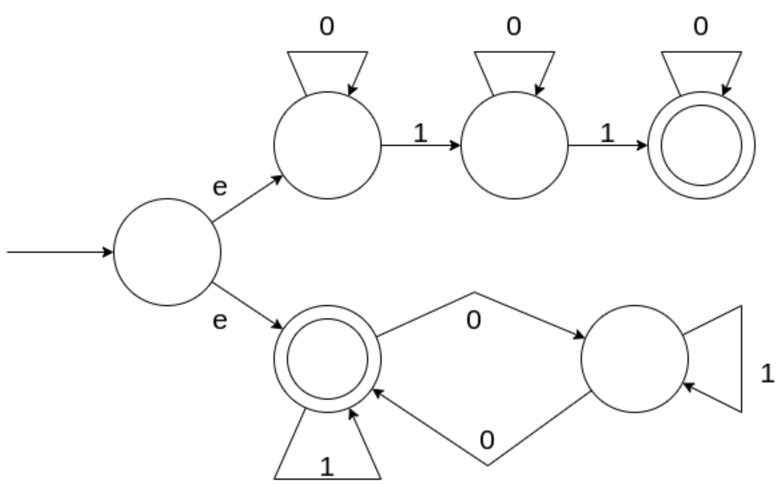
1.2 Give state diagrams of NFAs with the specified number of states recognizing each of the following languages. In all parts, the alphabet is $\{0, 1\}$.

1.7.b The language of Exercise 1.6c with five states.

Five states that accept the strings over the alphabet $\Sigma = \{0, 1\}$ and contains the sub-string 0101.



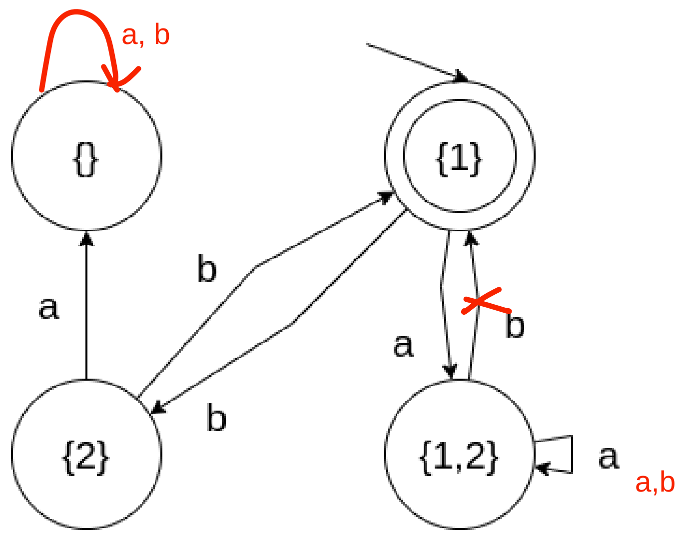
1.7.c The language of Exercise 1.6l with six states



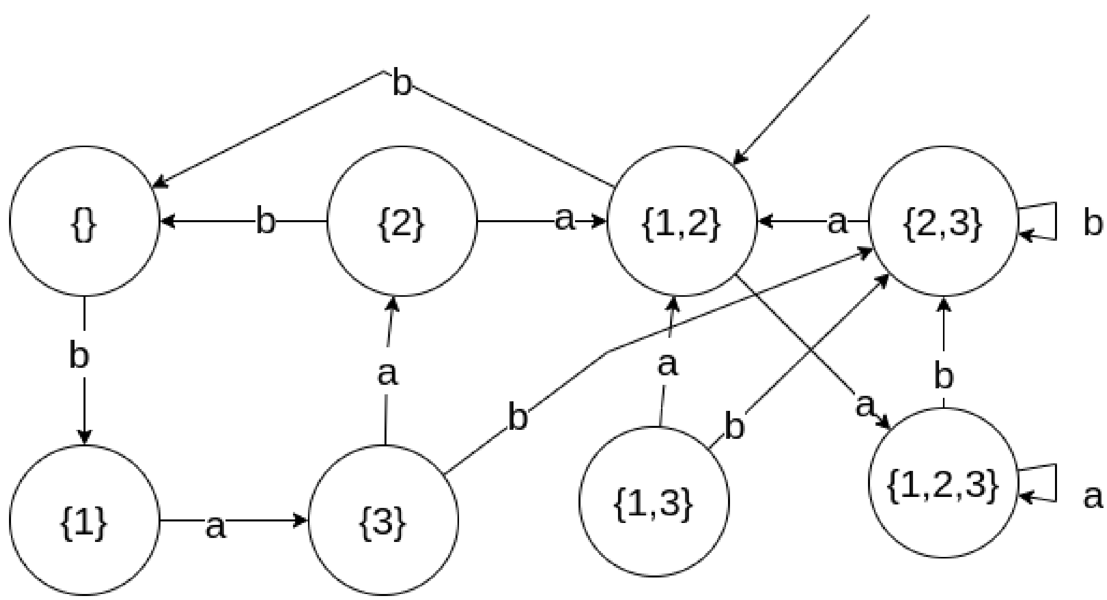
Problem 2

Problem 1.16.a, Problem 1.16.b (page 86).

1.16.a



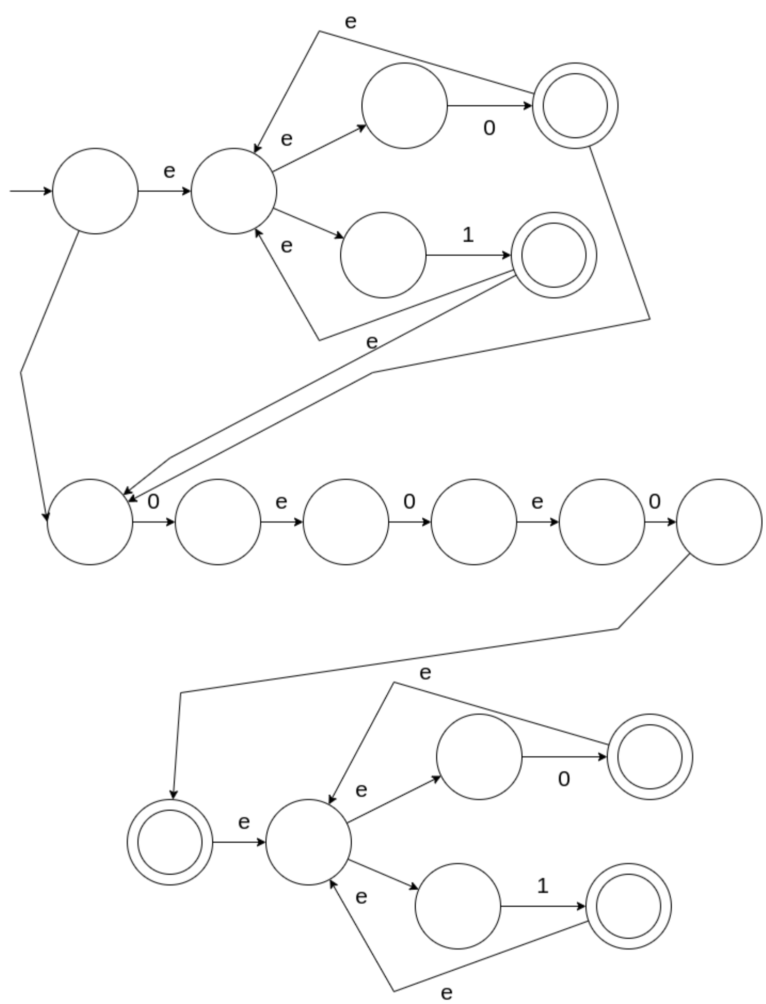
1.16.b



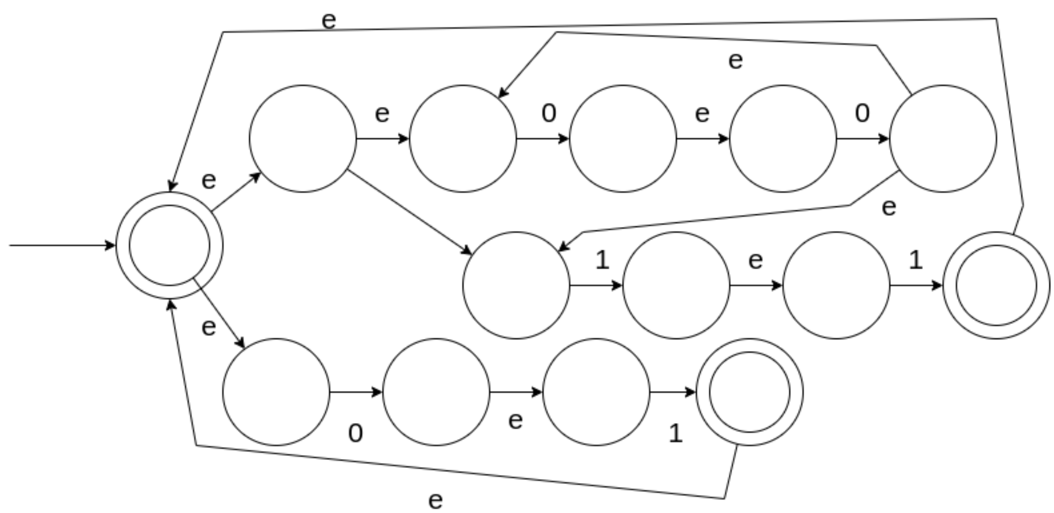
Problem 3

Use the procedure described in Lemma 1.55 to convert the following regular expressions to nondeterministic finite automata. (page 86).

1.19.a. $(0 \cup 1) * 000(0 \cup 1*)$

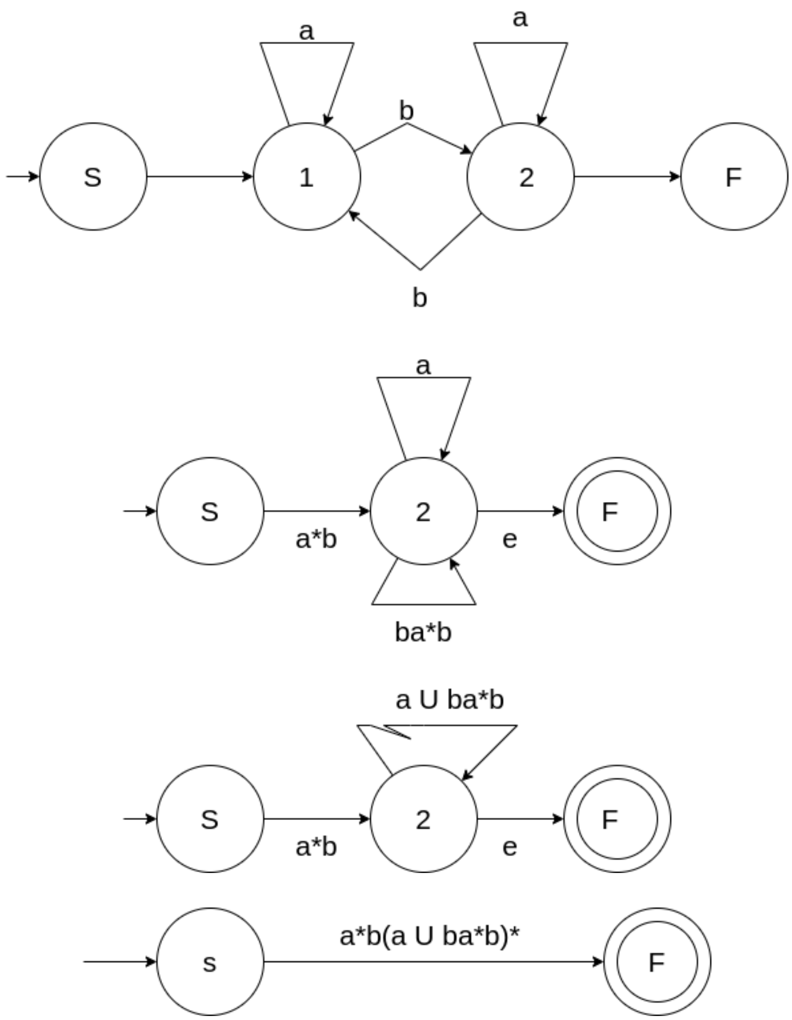


1.19.b. $(((00)*(11)) \cup 01)*$



Problem 4

Problem 1.21.a (page 86).



The regular expression is: $a * b(a \cup ba * b)*$

Problem 5

Prove the following languages are not regular.

1. $xy^iz \in A$ for $i \geq 0$
2. $\mid y \mid > 0$
3. $\mid xy \mid \leq p$

(5.1) $A = \{a^{n^3} | n \geq 0\}$. Here a^x means a string of x a's.

Proof. Assume that A is regular. Let $S = a^{p^3}$ where p is the pumping length. By the pumping lemma, S decomposes into xyz s.t.

By 3, $\mid y \mid \leq p$.
Pumping up, $\mid xy^2z \mid \leq p^3 + p < p^3 + 3p^2 + 3p + 1 = (p+1)^3$.
By 2, $\mid y \mid > 0$, hence $p^3 < \mid xy^2z \mid < (p+1)^3$. Thus $xy^2z \in A$, a contradiction of the pumping lemma.
$\therefore A$ is not regular. $\square$

(5.2) $B = \{0^n 1^m 0^n | m, n \geq 0\}$.

Proof. Assume B is regular. Let $S = 0^P 1 0^P$ where p is the pumping length. Then, S decomposes in xyz s.t.

By 3, y consists of only 0's.
Let $\delta = \mid y \mid$ then by 2, $\delta > 0$.
The pumping up, $xy^2z = 0^{P+\delta} 1 0^P \notin B$ because the same number of 0's is not the same before and after the 1.
A contradiction of the pumping lemma.
$\therefore B$ is not regular. $\square$