March 08

Difference between decidable and recognizable.

decidable: yes for YES instance,
No for NO instance.

recognizable: yes for YES instance.

Ex. Check whether $P(x) = c_0 x^n + c_1 x^{n-1} + c_2 x^{n-2} + \cdots + c_{n-1} x^1 + c_n$

has an integral root (solution); i.e. check whether there is an x_0 s.t. $x_0 \in I$ and $P(x_0) = 0$.

Algorithm (TM):

Check whether x_0 is a root for

$x_0 = 0, +1, -1, +2, -2, +3, -3, \cdots$

Q: Does the algorithm decide the problem?

NO $\quad P(x) = x^{10} - 5, \quad x_0 = \sqrt[10]{5}$

Q: Does the algorithm recognize the problem/language?

yes.

Chapter 4 Decidability

- Decidable languages / problems

$A_{DFA} = \{\langle B, w\rangle \mid B \text{ is a DFA that accepts } w\}$

$E_{DFA} = \{\langle A\rangle \mid A \text{ is a DFA and } L(A) = \phi\}$

$EQ_{DFA} = \{\langle A, B\rangle \mid A, B \text{ are DFA's and } L(A) = L(B)\}$.

<u>Thm 4.1</u> A_{DFA} is decidable.

<u>proof</u>: we present a TM M that decides A_{DFA}.

M: 1. Simulate the DFA B on input w.

2. If the simulation stops at an accept state, <u>accept</u>. If it stops at a non-accepting state, <u>reject</u>. □

A_{NFA}, A_{REX} // similar.

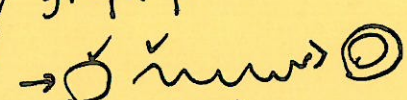
Thm 4.4 E_{DFA} is decidable.

<u>proof</u>: TM T:

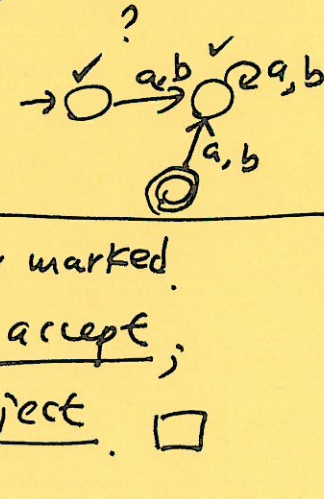
1. Mark the start state of A.
2. Repeat (until no new state gets marked)

   Mark q if $p \to q$ and p is already marked.
3. If no accept state in A is marked, <u>accept</u>; otherwise, <u>reject</u>. □

graph problem



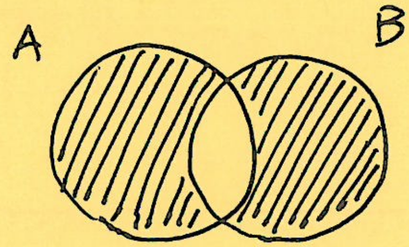
Ex



EQ_{DFA}, how would you proceed?

- Feed all w on A and B,
  if one of them accepts (the other rejects),
  then <u>reject</u>.

* But # of w's is infinite!



shaded part — C

$C = (A \cap \overline{B}) \cup (\overline{A} \cap B)$

More formally, $L(C) = (L(A) \cap \overline{L(B)}) \cup (\overline{L(A)} \cap L(B))$.

- $L(A) = L(B)$ iff $L(C) = \phi$
- Note that, regular languages are closed under
  $\cup$, $\cap$, $-$ operations.