Sketch of Solutions for Assignment #3. CSCI 338

1. (1.1)

$S \to A \mid B$
$A \to aaS_1b$
$S_1 \to A \mid aS_1 \mid a$
(for $\{a^nb^m \mid n > 2m\}$)

$B \to aS_2b$
$S_2 \to B \mid aaS_2b \mid Bb \mid b \mid \varepsilon$
(for $\{a^nb^m \mid n < 2m\}$)

(1.2)

$S \to S_1 \mid S_2$
$S_1 \to S_1c \mid A \mid \varepsilon$
$A \to aAb \mid \varepsilon$
$S_2 \to aS_2 \mid B \mid \varepsilon$
$B \to bBc \mid \varepsilon$

(1.3)

$S \to aaaSb \mid \varepsilon$

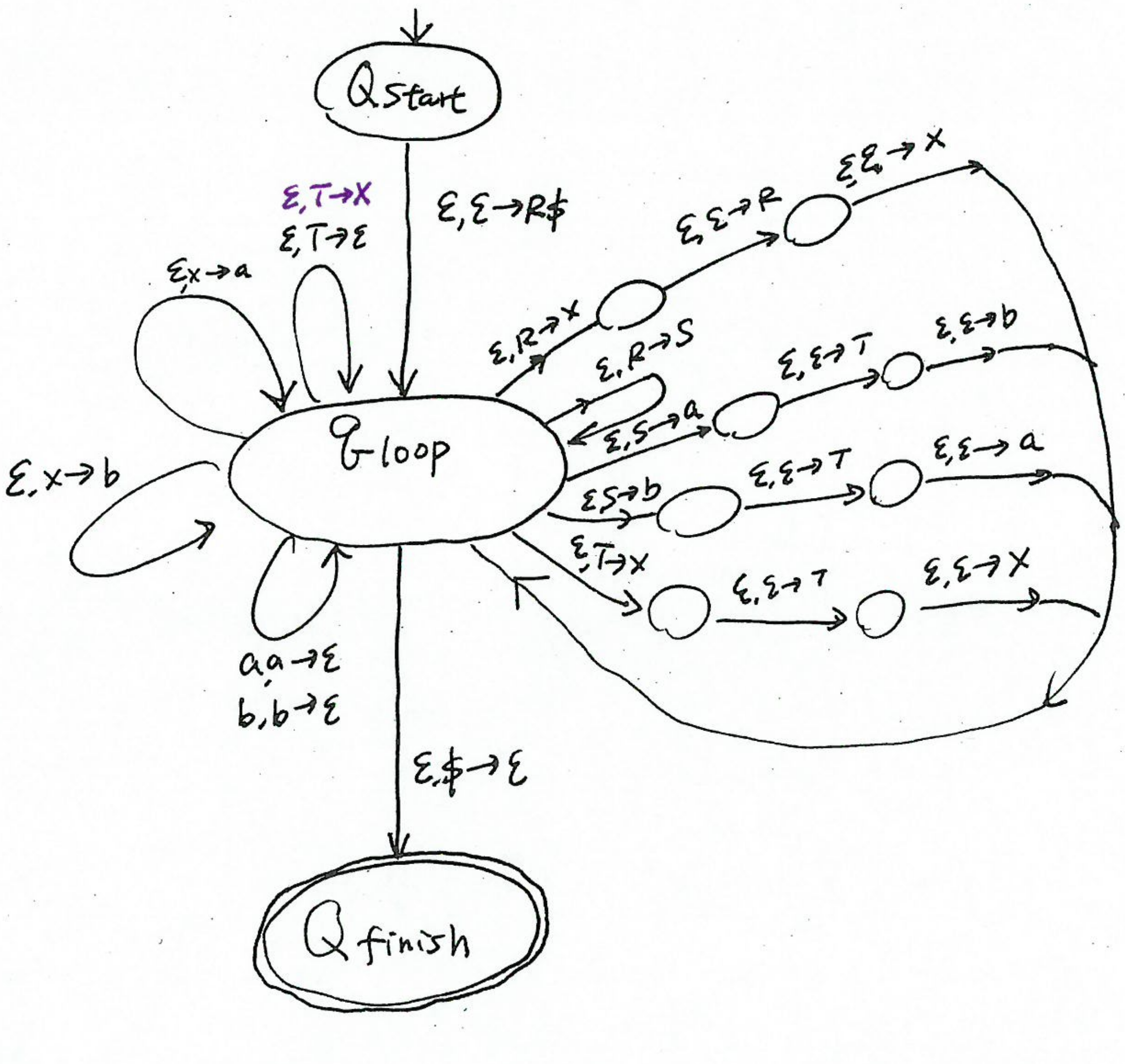
(1.4)

$S \to AB$
$A \to aaa \mid aa \mid a \mid \varepsilon$
$B \to aBb \mid b \mid \varepsilon$

2. It is ambiguous:

$S \Rightarrow aaB \Rightarrow aab$
$S \Rightarrow AB \Rightarrow AaB \Rightarrow aaB \Rightarrow aab$

3.



4. (a) $0^i \# 0^j \# 0^k$ or $0^i \# 0^{2i}$

(b) Select $s = 0^p \# 0^{2p}$

By the pumping lemma for regular languages.

$s = xyz$ s.t.

(1) $xy^iz \in A = L(G)$, for all $i \geq 0$

(2) $|xy| \leq p$

(3) $|y| > 0$

By (2), xy must be before $\#$.

Either pumping up or down (say set $i = 2$)

we have $0^{p+k} \# 0^{2p} \notin A$

Therefore, A is not regular.

5.
$$
\begin{aligned}
S_0 &\rightarrow BA_1 \mid BA \mid AB \mid BB \mid CC \mid \varepsilon \\
A &\rightarrow BA_1 \mid BA \mid AB \mid BB \mid CC \\
A_1 &\rightarrow AB \\
B &\rightarrow CC \\
C &\rightarrow 0
\end{aligned}
$$

(details omitted)

G.(1) $L = \{a^n b^j c^k \mid k = nj\}$

<u>proof</u>. Assume that L is context-free and choose $s = a^p b^{p+1} c^{p(p+1)}$

So by the pumping lemma, $s = uvxyz$ s.t.

① $uv^ixy^iz \in L$, for $i \geq 0$

② $|vxy| \leq p$

③ $|vy| > 0$

Case A. If vxy contains only a's, b's or c's then pumping up you will have uv^ixy^iz having more a's, b's or c's (with the # of other 2 symbols staying the same). Hence $uv^ixy^iz \notin L$.

Case B. If vxy straddle the break points of a's and b's (or b's and c's), pumping down you will have ux^0vy^0z having less a's or b's while the # of c's stays the same. Hence

$$uv^0xy^0z \notin L.$$

Therefore, the assumption that L is context-free is wrong. □

6.2 $L = \{a^n b^j \mid n \geq (j-1)^3\}$

Proof. Assume L is context-free and choose $s = a^{p^3} b^{p+1}$, p being the pumping length.

By the pumping lemma, $s = uvxyz$ s.t.

① $uv^ixy^iz \in L$

② $|vxy| \leq p$

③ $|vy| > 0$.

Case A. If vxy contains only a's, then uv^0xy^0z contains less than p^3 # of a's, so $n < (j-1)^3$.

Case B. If vxy contains only b's, then uv^ixy^iz $(i>0)$ contains more than $p+1$ b's, so $n < (j-1)^3$.

Case C. If v contains a's and y contains b's, then we pump up to have uv^2xy^2z, where # of b's increases by at least 1 and # of a's increases by at most p. Therefore, number of b's is at least $(p+2)$, number of a's is at most p^3+p. Obviously,

$$p^3+p \not\geq [(p+2)-1]^3 = p^3+3p^2+3p+1.$$

Hence, $uv^2xy^2z \notin L$.

Case D. If v or y contains a mixture of a's and b's, then pumping up to uv^ixy^iz $(i>0)$ will change the order of a's and b's.

Therefore, all these cases violate the pumping lemma and L is not context-free. $\square$