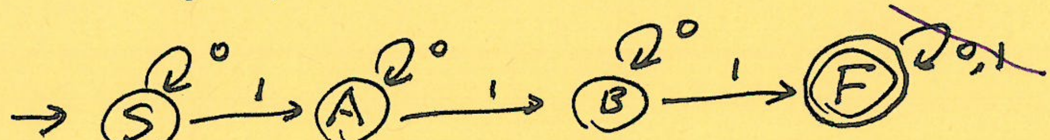
Jan 27

Questions from the previous lecture:

① Ex1: $\{w \mid w \text{ contains at least three 1's}\}$. $\Sigma = \{0,1\}$.



What if I delete the transition at F $\cancel{0,1}$?

<u>Answer</u>: You would miss some string like

$111\underline{0000}$, $010100\underline{111111}$, ...

So, logically it would be an incomplete solution.

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② $A \circ B = \{xy \mid x \in A, y \in B\}$.

How important is the order xy ?

<u>Answer</u>: We just use an example.

Let $A = \{0^*\}$, $B = \{1^+\}$.

$00011, 011111 \in A \circ B$.

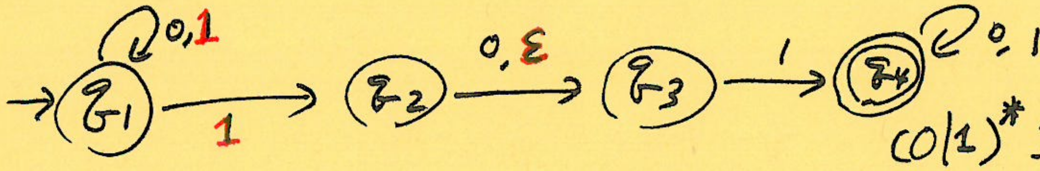
$11111 \in A \circ B$ // $x = \varepsilon = 0^0$.

But $111100 \notin A \circ B$.

So, in essence, y can't be preceeding x !

Deterministic — Given the current state and input, we know where to go next exactly.

Nondeterminism —

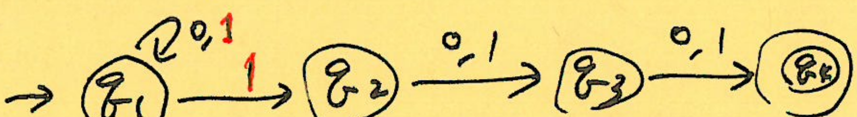


$(0|1)^* 11 (0|1)^*$

$(0|1)^* 101 (0|1)^*$

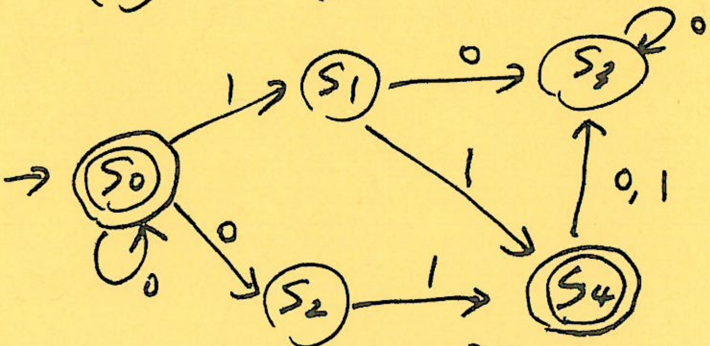
How does an NFA compute?

- If any one of the accept state can be reached by reading the input x in any way, we say the NFA accepts x.

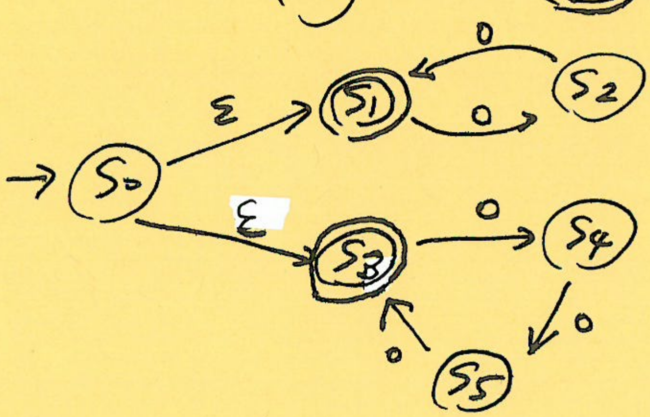
Ex1: 

$(0|1)^* 1 (1|0)(1|0)$

$x^* 1 x x,\ x \in \{0,1\}$

Ex2. 

$\{0^n, 0^n 11, 0^n 0 1\}, n \geq 0$

Ex3 

$\{0^k \mid k$ is a multiple of 2 or 3$\}$

- A nondeterministic finite automaton (NFA) is a 5-tuple $(Q, \Sigma, \delta, q_0, F)$, where

1. Q is the set of states
2. Σ is a finite alphabet
3. $\delta: Q \times \Sigma_\varepsilon \to P(Q)$ is the transition function

   $\Sigma_\varepsilon = \Sigma \cup \{\varepsilon\}$. $P(Q)$ — power set of Q.
4. $q_0 \in Q$ is the start state
5. $F \subseteq Q$ is the set of accept states.

Ex 3 $\delta(s_0, \varepsilon) = \{s_1, s_3\}$

Ex 1 $\delta(q_1, 1) = \{q_1, q_2\}$.

- Let $N = (Q, \Sigma, \delta, q_0, F)$ be an NFA and $w = w_1 \dots w_m$

  $w_i \in \Sigma_\varepsilon$

  N accepts w if

  $\exists\, r_0, r_1, \cdots, r_m \in Q$, s.t.

  ① $r_0 = q_0$

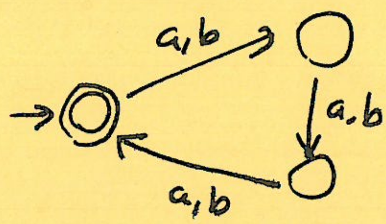
  ② $r_{i+1} \in \delta(r_i, w_{i+1})$, $i = 0, \cdots, m-1$

  ③ $r_m \in F$.
- Two machines are <u>equivalent</u> if they accept the same language.

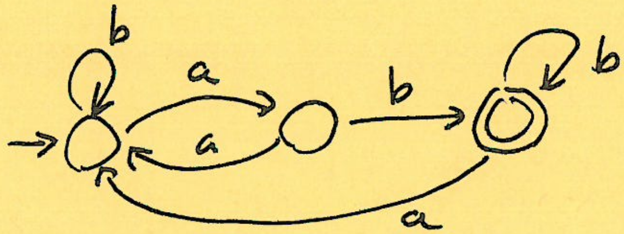
Exercises on DFA:

Let $\Sigma = \{a, b\}$, construct DFA's for the following languages.

① $A = \{w \mid$ the length of w is a multiple of $3\}$

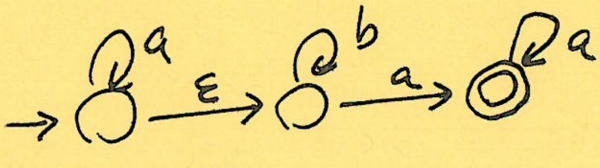


② $C = \{w \mid w$ has an odd number of a's and ends with $b\}$



Exercises on NFA:

① $A = \{a^* b^* a^+\}$



② $B = \{w \mid w$ doesn't contain aab$\}$