Feb 08

Q: what is the use of pumping lemma?

A: Show many problems can't be solved with DFA or NFA.

$A \to B$

To disprove A, you assume A is true and try to obtain some $\neg B$ instance.

Ex1 $B = \{0^n 1^n \mid n \geq 0\}$ is not regular.

proof. (By contradiction + pumping lemma)

Assume that to contrary that B is regular.

Choose $s = 0^p 1^p$, p being the pumping length. // you choose s.

By the pumping lemma, s can be decomposed into $s = xyz$, s.t.

① $xy^iz \in B$ for $i \geq 0$

② $|xy| \leq p$

③ $|y| > 0$.

// $p = 2$

$s = 0011$

ⓐ $s = xyz$

$= 0\underbrace{0}_{y}11$

ⓑ $s = xyz$

$= 00\underline{11}_{y}$

We consider 3 cases:

ⓐ y contains only of 0's. Then xyyz contains more 0's than 1's, violating condition ①. hence can't be in B.

ⓑ y contains only of 1's. symmetric to ⓐ.

ⓒ y contains of 0's and 1's.

$xyyz$ has 0's and 1's out of order (i.e., we have a subsequence 0101), hence $\notin B$.

// ⓒ $s = xyz$
$= 0\underline{01}1$
y

As we have a contradiction in all cases, the assumption that B is regular is incorrect. $\therefore$ B is not regular. $\square$

Ex 2. $A_1 = \{0^n 1^n 2^n \mid n \geq 0\}$ is not regular.

<u>proof</u> (By contradiction).

Assume that A_1 is regular.

Select $s = 0^p 1^p 2^p$, p being the pumping length.

By the pumping Lemma, s can be decomposed into $s = xyz$, s.t.

① $xy^iz \in A_1$, for $i \geq 0$

② $|y| > 0$, and

③ $|xy| \leq p$.

By ③, $|xy| \leq p$, therefore y must be composed of 0's.

Then $xyyz$ would have more 0's than 1's and 2's.

Hence, $xyyz \notin A_1$. A contradiction to the pumping Lemma.

$\therefore$ A_1 is not regular. $\square$

Ex3. $F = \{ww \mid w \in \{0,1\}^*\}$ is not regular.

Proof. Assume that F is regular.

Select $s = 0^p 1 0^p 1$, p being the pumping length.

By the pumping lemma, s can be decomposed into $s = xyz$, s.t.

① $xy^iz \in F$. for $i \geq 0$

② $|y| > 0$, and

③ $|xy| \leq p$.

$p=3$

$\underbrace{00}_{y}010001$

By ③, $|xy| \leq p$. So y must contain only 0's (before the first 1)

Then $xyyz \notin F$ as it contains more 0's before the first 1 (compared with after it).

∴ A contradiction to condition ① in the pumping lemma.

Hence, F is not regular.

Ex 4. $E = \{0^i 1^j \mid i > j\}$ is not regular.