CSCI 338: Assignment 5 (7 points)

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Problem 1

We are given 5 matrices $M_1, ..., M_5$, their dimensions (i.e., rows by columns) are as follows: M_1 is 15 $\times$ 20, M_2 is 20 $\times$ 30, M_3 is 30 $\times$ 10, M_4 is 10 $\times$ 50, and M_5 is 50 $\times$ 8.

(1.1) Run the dynamic programming algorithm for *matrix chain multiplication* that we covered in class to produce the table $m[-,-]$.

	1	2	3	4	5
1	0	9000	9000	165000	13600
2	-	0	6000	16000	11200
3	-	-	0	15000	6400
4	-	-	-	0	4000
5	-	-	-	-	0

(1.2) What is the optimal solution value? Where do you find it?

The optimal solution value is found at at $m\,[1,5]$, and is 13600.

Problem 2

We are given a context-free grammar G as follows:

G: $S \to AS|SB$

$A \to AD|DA|a$

$B \to BB|BD|b$

$D \to DD|d.$

We are also given a string $w = bdbdd$.

(2.1) Run the dynamic programming algorithm for A_{CFG} that we covered in class to produce the table $table[-,-]$.

	1	2	3	4	5
1	B	B	B	B	B
2	-	D	$\emptyset$	$\emptyset$	$\emptyset$
3	-	-	B	B	B
4	-	-	-	D	D
5	-	-	-	-	D

(2.2) How do we know whether G generates w from the table?

The following rule determines whether or not G generates w:

$$w \in L(G) \Longleftrightarrow S \in table\,[1,5]$$

Since $S \notin table\,[1,5]$, we know that G does not generate w.

Problem 3

Show that $ALL_{DFA} \in$ P.

Proof. First, recall that ALL_{DFA} accepts Σ^* if, and only if, all reachable states from the start state are accepting. We will construct a Turning machine to decide for ALL_{DFA}.

S = "On input $\langle D \rangle$, where D is some DFA:
1. Preform Breadth First Search on D starting at the start state.
2. If at any point a non-accepting state is visited, *reject*.
3. If only accepting states are found, *accept*."

Breadth First Search runs in polynomial time, therefor we have created a decider that runs in $O(n^k)$ time. Sine S decides ALL_{DFA} as S visits all possible states, and runs in polynomial time, by definition, $ALL_{DFA} \in P$.

$\therefore ALL_{DFA} \in$ P. $\square$

Problem 4

Show that Independent Set $\in$ NP.

Proof. We will create a non-deterministic Turning Machine that decides in polynomial time to decide an Independent Set.

S = "On input $\langle G, k \rangle$ where G is a graph ($G = (V, E)$), and k is an integer such that $k \leq |V|$
1. Non-deterministically choose a subset c of k vertices from G
2. Test whether G contains any of the $\binom{k}{2}$ edges connecting nodes in c
3. If test is true, the an edge $u, v \in c$ in G and *reject*, Otherwise *accept*."

S runs in $O(\binom{k}{2}) = O(n^2)$ time. So we have found a non-deterministic decider that runs in polynomial time. So Independent Set $\in NP$ by Theorem 7.20

$\square$