Mar 10

Decidable Languages (Problems) Concerning Context-free languages.

$$A_{CFG} = \{\langle G, w\rangle \mid G \text{ is a CFG that generates } w\}$$

$$E_{CFG} = \{\langle G\rangle \mid G \text{ is a CFG and } L(G) = \phi\}$$

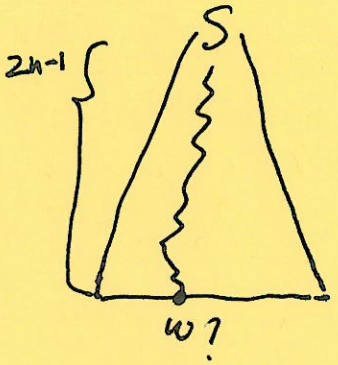
$$EQ_{CFG} = \{\langle G, H\rangle \mid G, H \text{ are CFGs and } L(G) = L(H)\}$$

Thm. 4.7 A_{CFG} is decidable.

- If G were in Chomsky Normal Form, any derivation of w, $|w| = n$, has $\underline{2n-1}$ steps. Why?
- $S \overset{*}{\Rightarrow} A_1 A_2 \cdots A_n$ ($n-1$ steps)
- $A_i \to a_i$ (n steps)

proof: Take $\langle G, w\rangle$, where G is a CFG, construct TM S for A_{CFG}:

1. Convert G into CNF.
2. List all derivations with $2n-1$ steps, $n = |w|$. (For $n=0$, list all derivations with one step.)
3. If any of these derivations generates w, accept; otherwise, reject. □



cost: $|G|^{2n-1}$

We now study $E_{CFG} = \{\langle G \rangle \mid G \text{ is a CFG and } L(G) = \phi\}$.

Thm 4.8 E_{CFG} is decidable.

Proof: Construct TM R for E_{CFG}:

1. Mark all terminal symbols of G
2. Repeat until no new variables get marked
   - Mark A, where $A \to U_1 U_2 \cdots U_k$ is a rule in G and U_i's are marked.
3. If the start variable is not marked, accept; otherwise, reject.

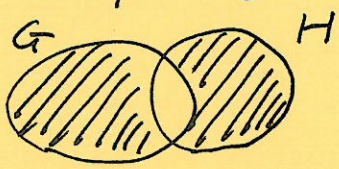
Example: G: $S \to ABX \mid BAX$
$A \to AX$
$B \to BX$
$X \to a$

a, $X \to a$, $A \to AX$, $B \to BX$, $S \to ABX$, $S \to BAX$

∴ S is not marked, and $L(G) = \phi$.

$EQ_{CFG} = \{\langle G, H\rangle \mid G$ and H are CFG's and $L(G) = L(H)\}$.

We can certainly try the trick used for EQ_{DFA}:



C — shaded region.

$C = (L(G) \cap \overline{L(H)}) \cup (\overline{L(G)} \cap L(H))$

$L(G) = L(H)$ iff $L(C) = \phi$

- But this idea only works if CFL's are closed under Union, intersection and complementation operations.

Exercise 1: CFL's are closed under the union operations.

Exercise 2: CFL's are NOT closed under the intersection operations.

Exercise 3: CFL's are NOT closed under the complementation operations.

Try to spend 30 minutes, solutions will be shown next.