Feb 5

Q: When will be the first test? Will there be any question on proof methods?

A: Most likely it will be on Feb 22.

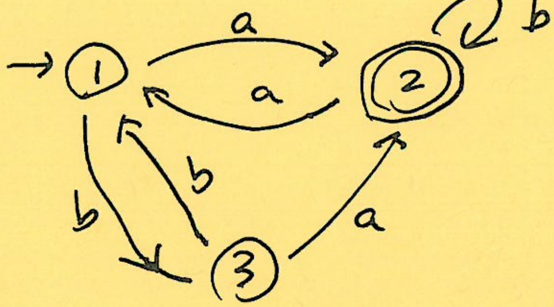
Since we covered (reviewed) proof methods for almost 4 lectures, there will be some questions on those topics.

— But sth like Q5 in Assignment 1 would be too much for a timed test.

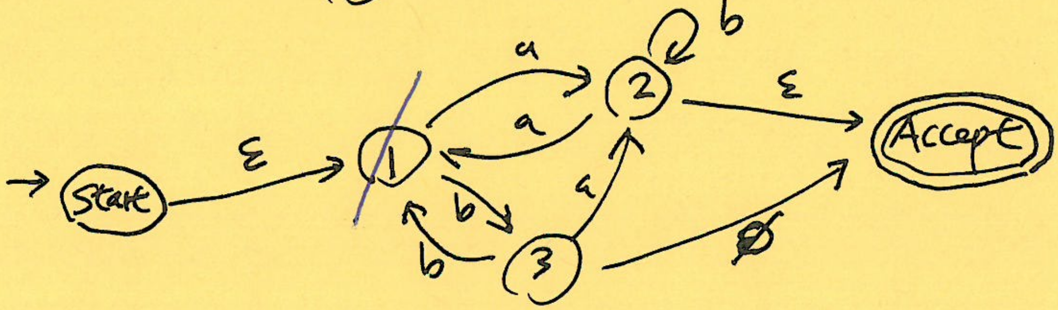
— The test will be done on-line (for most students, I guess). But I will provide the chance for those who want to do it physically in Leon Johnson 339.

Ex

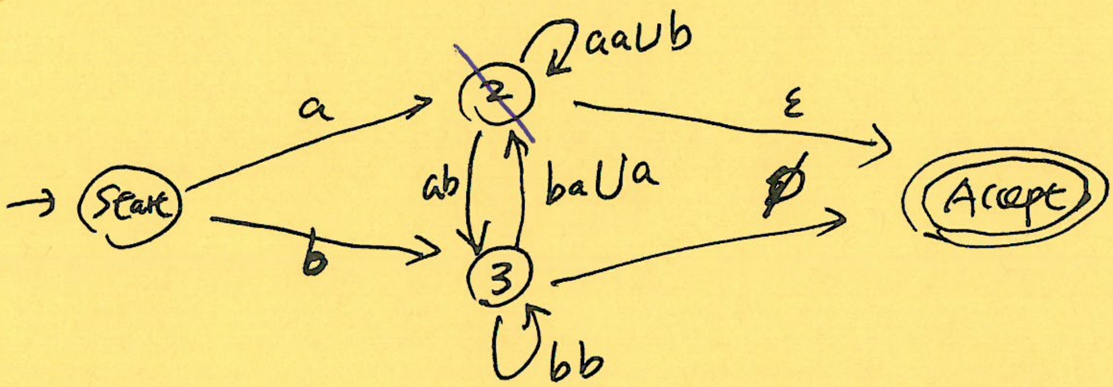
DFA:



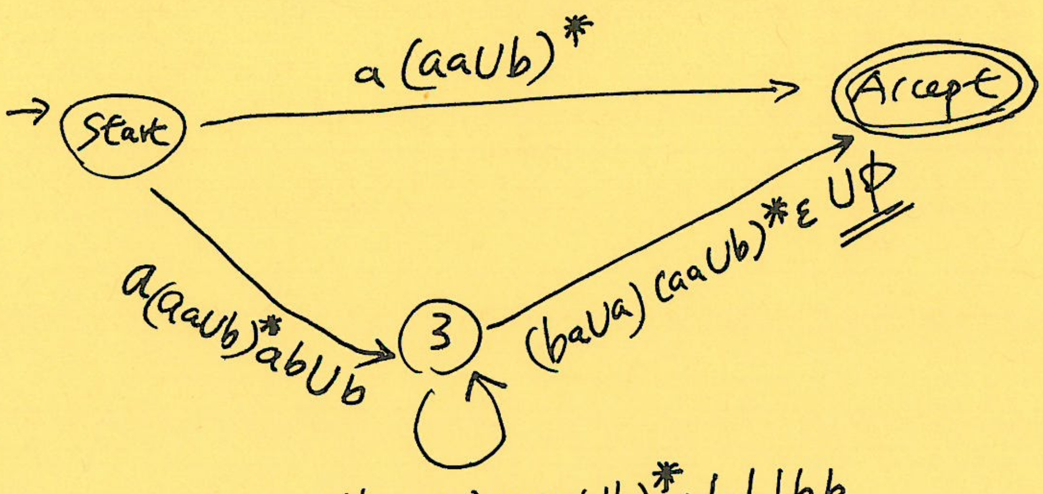
a
a
b
b
b
a
1
2
3



Start
ε
1
a
a
b
b
a
2
b
ε
3
∅
Accept



Start
a
b
2
aa∪b
ab
ba∪a
3
bb
ε
∅
Accept



Start
a(aa∪b)*
Accept
a(aa∪b)*ab∪b
3
(ba∪a)(aa∪b)*ε∪∅
(ba∪a)(aa∪b)*ab∪bb

1.4 Nonregular languages

× $A = \{w \mid w \text{ has more 0's than 1's}\}$

× $B = \{0^n 1^n \mid n \geq 0\}$

× $C = \{w \mid w \text{ has an equal number of 0's and 1's}\}$

✓ $D = \{w \mid w \text{ has an equal number of 01 and 10 as substrings}\}$

Thm 1.70 (Pumping Lemma)

If A is a regular language, then there exists a number p (pumping length) s.t. if $s \in A$ and $|s| \geq p$, the s can be decomposed into $s = xyz$, s.t.

① for $i \geq 0$, $xy^iz \in A$

② $|y| > 0$, and

③ $|xy| \leq p$.

IDEA: p — # of states in DFA $M = (Q, \Sigma, \delta, q_1, F)$ which accepts A.

If s has at least p letters, there must be a repeated state! (pigeonhole principle)

proof (Condensed one)

- Let $M = (Q, \Sigma, \delta, q_1, F)$ be a DFA accepting A and let p be the number of states of M.

  Let $s = s_1 s_2 \cdots s_n \in A$, $n \geq p$.

  Let $r_1, r_2, \ldots, r_{n+1}$ be the sequence of states that M enters while processing s, i.e., $r_{i+1} = \delta(r_i, s_i)$. Note that $n+1 \geq p+1$.

- By the pigeonhole principle, among the first $p+1$ states, two must be the same.

  Let the first be r_j and the second be r_l ($l \leq p+1$).

- Let $x = s_1 s_2 \cdots s_{j-1}$,
  $y = s_j \cdots s_{l-1}$,
  $z = s_l \cdots s_n$

- Clearly, x takes M from r_1 to r_j.
  y takes M from r_j to r_j
  z takes M from r_j to r_{n+1}.

- M must accept xy^iz for $i \geq 0$

  As $j \neq l$, so $|y| > 0$.

  As $l \leq p+1$, $|xy| \leq p$. $\square$