CSCI 338: Exercise 01

I will give non-regular exercises for you to try (remember, you must work on these besides following my lectures or read book sections!). These will not be graded. Solutions will be posted on D2L a bit later.

Problem 1

Prove that $\sqrt{6}$ is irrational.

Proof: - Suppose $\sqrt{6}$ is rational, then by definition, $\sqrt{6}$ can be expressed as $\sqrt{6} = \frac{m}{n}$, with $m, n \in N$ and $\gcd(m, n) = 1$.

- Take squares at both sides of $\sqrt{6} = \frac{m}{n}$, we have $6 = \frac{m^2}{n^2}$, or $6n^2 = m^2$. Then m must be even, or $m = 2k$. Consequently, $6n^2 = (2k)^2$, or $3n^2 = 2k^2$. This implies that $3n^2$ is even, hence n^2 is even. Thus n must also be even.

- Since m, n are both even, $\gcd(m,n) \geq 2$. This is a contradiction with the assumption.

$\therefore \sqrt{6}$ is irrational. $\square$

Problem 2

Prove that $\sqrt{7}$ is irrational.

Proof: Suppose that $\sqrt{7}$ is rational, by definition.

$\sqrt{7} = \frac{m}{n}$, with $m, n \in N$ and $\gcd(m,n) = 1$.

Take squares at both sides of $\sqrt{7} = \frac{m}{n}$, we have

$$7 = \frac{m^2}{n^2}, \text{ or } 7n^2 = m^2.$$

Then m must be a multiple of 7, or $m = 7k$.

plug $m = 7k$ back to $7n^2 = m^2$, we have $7n^2 = (7k)^2$, or $n^2 = 7k^2$. Thus n is also a multiple of 7.

Since m, n are both multiples of 7, $\gcd(m,n) \geq 7$.

This contradicts with the assumption that $\gcd(m,n) = 1$.

$\therefore \sqrt{7}$ is irrational. $\square$