Feb 10

Ex4 $E = \{0^i 1^j \mid i > j\}$ is not regular.

proof: Assume that E is regular.

Select $s = 0^{p+1} 1^p$, p being the pumping length.

By the pumping lemma, s can be decomposed into $s = xyz$, s.t.,

① $xy^iz \in E$, for $i \geq 0$.

② $|y| > 0$.

③ $|xy| \leq p$.

By ③, y contains of some of the first p 0's.

By setting $i = 0$, $s = xy^0z = xz \notin E$, as at least one 0's would be deleted. (Removing y decreases the number of 0's.)

∴ A contradiction to the pumping lemma.

∴ E is not regular $\square$

// pump down example.

Ex5. $D = \{1^{n^2} \mid n \geq 0\}$ is not regular

1^x — x copies of 1's.

proof. Assume that D is regular.

Select $s = 1^{p^2}$, p being the pumping length.

By the pumping lemma, s can be decomposed into

$s = xyz$, s.t.,

① $xy^iz \in D$, for $i \geq 0$.

② $|y| > 0$.

③ $|xy| \leq p$.

By ③, $|xy| \leq p$, so $|y| \leq p$.

We have $|s| = |xyz| = p^2$. so,

$|xy^2z| \leq p^2 + p$.

But $p^2 + p < p^2 + 2p + 1 = (p+1)^2$.

So $p^2 < |xy^2z| < (p+1)^2$, as by ②, $|y| \geq 1$.

Hence, $xy^2z \notin D$. A contradiction to the pumping lemma.

$\therefore$ D is not regular. $\square$

End of regular languages.

Chapter 2. Context free languages

Ex. $0^n 1^n$ is not regular.

Ex1. $A \to 0A1$

$A \to B$

$B \to \#(\varepsilon, \text{or } \lambda)$

$$A \Rightarrow 0A1 \Rightarrow 00A11 \Rightarrow 000A111 \Rightarrow 000B111 \Rightarrow 000\#111$$

Ex2 $S \to AS_1 \mid S_1B$

$S_1 \to aS_1b \mid \lambda$

$A \to aA \mid a$

$B \to bB \mid b$

$\{a^i b^j \mid i \neq j\}$