Student No.	Name

CSCI 338 Computer Science Theory

Test 1 — 55 minutes (10 points)

Note: If you don't have a printer, you should write the answers on separate blank papers. If you don't have a scanner to scan a .pdf file to D2L (under Assignments/Test 1), you can email the .jpg files (taken as photos) to `bhz@montana.edu` at the end of the test. *Note also that this is an open book test, while all physical resources are allowed, resorting for external human help constitutes a* **plagiarism.**

Question 1

Circle each of the following 4 statements as either True (T) or False (F).

Let A be a language over $\Sigma = \{a, b\}$, define $\overline{A} = \{x | x \notin A\}$. We say that $\overline{A}$ is obtained from A by the *complementation* operation.

T F Regular languages are not closed under the concatenation ($\circ$) operation.

T F Regular languages are not closed under the star ($\star$) operation.

T F Regular languages is a proper subset of regular expressions.

T F Regular languages are closed under the complementation operation.

Question 2

Let a forest F be composed of r trees with a total of n vertices. Prove that F has $n - r$ edges.

Proof. (direct argument)

Let the r trees in F be $T_1, T_2, \cdots, T_r$.

By the definition of a tree, we have

$$V(T_i) - 1 = E(T_i),$$

where $V(T_i)$ and $E(T_i)$ are the vertices and edges of T_i.

Then, $\sum_{i=1}^{r} (V(T_i) - 1) = \sum_{i=1}^{r} E(T_i) = E(F)$.

$$\therefore \ E(F) = \sum_{i=1}^{r} \{V(T_i) - 1\} = \sum_{i=1}^{r} V(T_i) - r$$

$$= n - r. \quad \square$$

Note 1: We only use the fact that a tree with k vertices has $k-1$ edges.

Note 2: You can't use Euler's formula directly on F as F is a forest, which is <u>not</u> connected.

Question 2

Let a forest F be composed of r trees with a total of n vertices. Prove that F has $n - r$ edges.

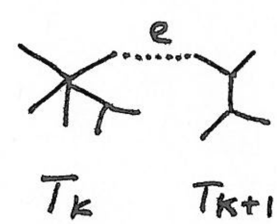
Proof: (by induction on r)

Basis. when $r=1$, F is a tree with n vertices, so F has $n-1 = n-r$ edges.

IH. Assume that any forest with k trees and n total vertices has $n-k$ edges.

IS. Let F be a forest with $k+1$ trees and n vertices in total.

We pick 2 trees in F, say T_k, T_{k+1}.



Connecting T_k and T_{k+1} with an edge e, we have a new forest F' with k trees. By IH, F' has

$$n-k$$

edges.

$\therefore$ F has $(n-k)-1$ // this "1" corresponds to edge e.

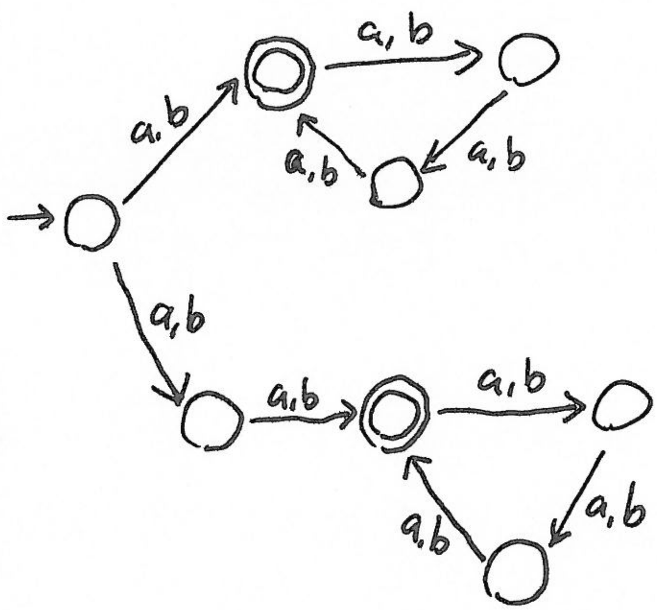
$$= n-(k+1)$$

edges, with $k+1$ trees in F. $\square$

Question 3

Let $\Sigma = \{a, b\}$. Construct a DFA or NFA for the following language:

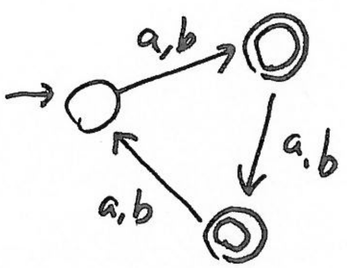
$A = \{w|\ \textit{the length of } w \textit{ is not equal to } 3x, x \textit{ is an integer and } x \geq 0\}$.



// 3x+1

// 3x+2

or,



pump up

Question 4

Let $\Sigma = \{a, b\}$. Use the pumping lemma to show that the following language is not regular:
$B = \{a^{n!} | n \geq 0\}$. Here a^y means a string of y a's and $n! = 1 \times 2 \times \cdots \times (n-1) \times n$.

Proof: Assume that B is regular, we select $s = a^{P!}$, P being the pumping length. By the pumping lemma, s can be decomposed into $s = xyz$, s.t.,

① $xy^iz \in B$, for $i \geqslant 0$,

② $|y| > 0$,

③ $|xy| \leq P$.

By ② and ③, $1 \leq |y| \leq P$. We pump up by setting $i=2$.

Then,

$$P! < |xy^2z| = |xyz| + |y| \leq P! + P.$$

$$P! + P < P! + (P+1) < P! \cdot (P+1)$$
$$= (P+1)!, \text{ when } P \geqslant 2.$$

// $a+b < a \cdot b$ when $a, b \geqslant 2$ and one of them > 2.

$$\therefore \quad P! < |xy^2z| < (P+1)!, \text{ when } P \geqslant 2.$$

Hence, when $P \geqslant 2$, $xy^2z \notin B$, which is a contradiction with the pumping lemma.

$\therefore$ B is not regular. $\square$

pump down

Question 4

Let $\Sigma = \{a, b\}$. Use the pumping lemma to show that the following language is not regular:
$B = \{a^{n!} | n \geq 0\}$. Here a^y means a string of y a's and $n! = 1 \times 2 \times \cdots \times (n-1) \times n$.

Proof: Assume that B is regular, we select $s = a^{P!}$, P being the pumping length. By the pumping lemma, s can be decomposed into $s = xyz$, s.t.,

① $xy^iz \in B$. for $i \geqslant 0$.

② $|y| > 0$,

③ $|xy| \leqslant P$.

Let $|y| = k$. We have $1 \leq k \leq P$. We pump down by setting $i=0$.

Then, $P! - P \leq |xy^0z| \leq P! - 1$; and moreover,

$(P-1)! < |xy^0z| < P!$ when $P \geqslant 3$. // $P! - P = (P-1)![P - \frac{P}{(P-1)!}] > (P-1)!$, when $P \geqslant 3$.

Hence, when $P \geqslant 3$, $xy^0z \notin B$, which is a contradiction with the pumping lemma.

$\therefore$ B is not regular. $\square$