Post's Correspondence Problem (PCP)

– a practical undecidable problem

~ You are given some domino cards, with strings written below and above the mid-line at each card.

~ Stack cards one by one, so that the strings above and below the mid-line read the same

~ You can use a card as many number of times as you want (or, for each card, you have an infinite # of copies).

<u>Ex1</u>.

card 1	card 2	card 3	card 4
b	a	ca	abc
ca	ab	a	c

Try to spend 10 – 15 minutes, note that you don't have to use all cards.

2	1	3	2	4
a	b	ca	a	abc
ab	ca	a	ab	c

// Solution might not be unique

Ex2. $\overset{1}{\left[\dfrac{aaa}{aa}\right]}, \overset{2}{\left[\dfrac{abb}{bba}\right]}, \overset{3}{\left[\dfrac{aa}{aaa}\right]}$

Solutions: $\overset{3}{\left[\dfrac{aa}{aaa}\right]}\overset{1}{\left[\dfrac{aaa}{aa}\right]}$ or $\overset{3}{\left[\dfrac{aa}{aaa}\right]}\overset{2}{\left[\dfrac{abb}{bba}\right]}\overset{1}{\left[\dfrac{aaa}{aa}\right]}$

Ex3. $\left[\dfrac{bb}{b}\right], \left[\dfrac{bba}{aa}\right], \left[\dfrac{abbb}{baa}\right]$ No solution.

Thm PCP is undecidable // we'll not go over the proof

other practical undecidable problems.

1. Test whether a number is random.
   // rand() is pseudo-random.

2. write a shortest program to solve a problem.
   // Kolmogorov complexity.

Reduction via Computation Histories

- Motivation: for some problem like

$$ALL_{CFG} = \{\langle G \rangle \mid G \text{ is a CFG and } L(G) = \Sigma^*\}$$

A_{TM} ? ALL_{CFG}

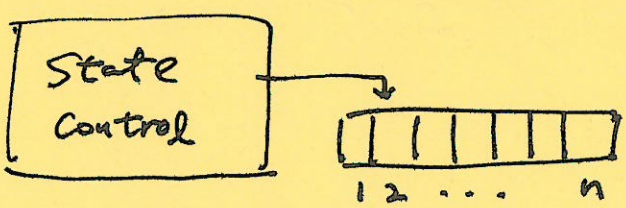
$\langle M, w \rangle$ $\langle G \rangle$

M and G are different models.

- **Def.** Let M be a TM and w an input string.
  An accepting computation history is a sequence $C_1, C_2, \cdots, C_\ell$, where
  - C_1 is the starting configuration,
  - C_ℓ is the accepting configuration for M accepting w,
  - C_i follows from C_{i-1} according to the rules of M.

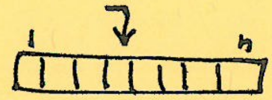
// If C_ℓ is a rejecting configuration, then $C_1, C_2, \cdots, C_\ell$ is a rejecting history.

- **Def.** A linear bounded automaton (LBA) is a TM with a fixed amount of memory.



✓ $A_{LBA} = \{ \langle M, w \rangle \mid M \text{ is an LBA that accepts } w \}$

✗ $E_{LBA} = \{ \langle M \rangle \mid M \text{ is an LBA and } L(M) = \emptyset \}$



Thm 5.9 A_{LBA} is decidable.

proof: First, there are qng^n distinct configurations of M for a tape of length n, where

q — # of states,

g — # of symbols in the tape alphabet.

TM L: On input $\langle M, w \rangle$

1. Simulate M on w for qng^n steps or until it halts.
2. If M halts, accept if M has been accepted
   reject if M has been rejected.

   If M doesn't halt, reject.

// If M exhausts all configurations and still doesn't halt, it is in an infinite loop.

Can we use the same idea on E_{LBA}?