CSCI 338: Exercise 02

I will give non-regular exercises for you to try (remember, you must work on these besides following my lectures or read book sections!). These will not be graded. Solutions will be posted on D2L a bit later.

Problem 1

Given a (connected) planar graph $P = (V, E)$, prove Euler's formula: $|V| + |F| - |E| = 2$, where F (resp. E) is the set of faces (resp. edges) of P and $|F|$ (resp. $|E|$) is the size of F (resp. E).
Hint: Use induction on $|V|$, or $|E|$, or $|F|$.

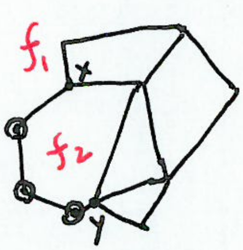
Proof: We use induction on $|F|$.

Basis. when $|F| = 1$, P must be a tree with $|E| = |V| - 1$.

$\therefore\ |V| + |F| - |E| = |V| + 1 - (|V| - 1) = 2.$

IH. suppose $|V| + |F| - |E| = 2$ for all planar graphs with $|F| \le k$ faces.

IS. Take the unbounded face f_1 of P and any bounded face f_2 of P, suppose they share a path $\langle x, y \rangle$. If we remove all internal vertices of $\langle x, y \rangle$, say z of them, we end up with a new planar graph P' (where f_1 and f_2 becomes the new unbounded face in P'). By IH, for P' we have



$$(|V| - z) + (|F| - 1) - \{|E| - (z+1)\} = 2.$$

This clearly implies that

$$|V| + |F| - |E| = 2.$$

$\square$