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CSCI 338 Computer Science Theory

Test 2 — 70 minutes (10 points)

Note: If you don't have a printer, you should write the answers on white papers. After you finish, scan a .pdf file to D2L (under Assignments/Test 2). *Note also that this is an open book test, while all physical resources are allowed, resorting for external human help constitutes a **plagiarism**.*

Question 1

Let R_1 be the set of all the positive real numbers less than 1, i.e.,

$$R_1 = \{x | 0 < x < 1\}.$$

Prove that R_1 is uncountable.

Question 2

Determine whether the following grammar is ambiguous. Explain your reason.

$$S \rightarrow aSbS \mid bSaS \mid \epsilon.$$

Question 3

Is the language $L = \{a^i b^j c^k \mid i < j < k\}$ context-free? You must give enough details to justify your answer.

Question 4

In class we mentioned that the general Post Correspondence Problem (PCP), i.e., when $|\Sigma| \geq 2$, is undecidable. Show that if $\Sigma = \{a\}$ then the restricted problem PCP-1 is in fact decidable.

Question 5

Define $ALL_{TM} = \{ \langle M \rangle \mid M \text{ is a Turing machine with } L(M) = \Sigma^* \}$. Prove that ALL_{TM} is undecidable.