Jan 13

Sets: A set is a group of similar objects.

- $\{a, c, g, t\}$.
- $\{1, 3, 5, 7, \cdots\} = \{\text{all positive odd integers}\}$
- objects are called elements.

$a \in \{a, g, c, t\}$

$o \notin \{a, c, g, t\}$

- A is a subset of B, written $A \subseteq B$, if all elements in A are also in B.

$\{a, c\} \subseteq \{a, c, g, t\}$.

Q: If $|A| = n$ (A has n elements), how many subsets does A have?

$\boxed{\text{Answer: } 2^n}$

Ex: $A = \{a, b, c\}$

$2^A = \{\{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, c\}, \{a, b, c\}, \phi\}$, $|2^A| = 2^3 = 8$.

2^n: $n = 10$, $2^n \sim 1000$ $(= 1024)$

$n = 20$, $2^{20} \sim 1{,}000{,}000 = 10^6$

$n = 40$, $2^{40} \sim 10^{12}$

A is a proper subset of B, if $A \subseteq B$, and $A \neq B$,

denoted as $A \subset B$ or $A \subsetneq B$.

Multisets: repeated elements are allowed.

$\{8\}$, $\{8, 8, 8\}$ are diff multisets.

N — set of natural numbers

$N = \{1, 2, 3, 4, \cdots\}$.

Z - set of integers

$Z = \{\cdots, -2, -1, 0, 1, 2, \cdots\}$

Q: How to define concisely a special set?

$S = \{x \mid \text{rule about } x\}$.

<u>Ex</u>. $X = \{x \mid x = m^2 \text{ for } m \in N\}$.

$X = \{1, 4, 9, 16, \cdots\}$

Q: set operations, union ($\cup$), intersection ($\cap$)

complementation ($-$).

<u>Ex</u>. $A = \{1, 3, 5, 6\}$. $B = \{2, 4, 6, 8\}$

$A \cup B = \{1, 2, 3, 4, 5, 6, 8\}$

$A \cap B = \{6\}$

$U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

$\overline{A} = \{2, 4, 7, 8, 9\}$

Sequences and tuples:

A **sequence** is a list of **ordered** objects.

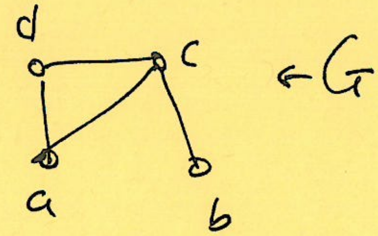
$$(1,3,5,7) \neq (1,5,3,7)$$

- Finite sequences are also called **tuples**

  5-tuple: $\langle 5,4,3,2,1 \rangle$

- 2-tuples are also called **pairs**

Graphs $G = \langle V, E \rangle$



- V is the set of vertices.

  Ex: $V = \{a, b, c, d\}$

- E is the set of edges, each being a pair.

  Ex. $E = \{(a,c), (a,d), (b,c), (c,d)\}$

Theorems and proofs

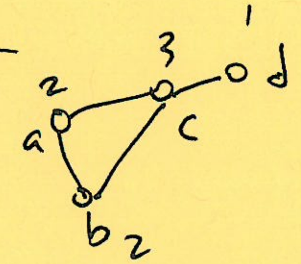
- A theorem is a true mathematical statement.
  - Mergesort takes $\Theta(n\log n)$ steps to sort n elements.
- A proof is a convincing logic argument that a statement is true.
- You can prove a statement by providing a proof.
- You can disprove a statement by a counterexample.
  (by proof).

<u>proof methods</u>:
1. Direct method
2. By contradiction
3. By induction
4. By construction

~~By example~~

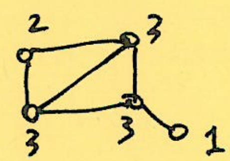
<u>Direct method</u>:

EX 1: Given a graph $G=\langle V,E\rangle$,
let $deg(v)$ be the # of edges incident to $v\in V$.
Then, $\sum_{v\in V} deg(v)$ is even.



$2+2+3+1=8$

<u>proof</u>: When counting the degree,
for edge (u,v), it is counted
twice (once for $deg(u)$, once for $deg(v)$).



$2+3+3+3+1=12$

$\therefore \sum_{v\in V} deg(v) = 2\cdot|E|$, which is always even.

$\square$

<u>Ex 2</u>. $\overline{A \cup B} = \bar{A} \cap \bar{B}$.

proof: "$\Rightarrow$" we show $\overline{A \cup B} \subseteq \bar{A} \cap \bar{B}$.

If $x \in \overline{A \cup B}$, then by definition x is not in $A \cup B$; i.e., x is not in A and x is not in B. Therefore, $x \in \bar{A}$ and $x \in \bar{B}$. Hence, $x \in \bar{A} \cap \bar{B}$.

"$\Leftarrow$" Similar (take home exercise).

$\square$