March 3

Ex 4. $E = \{a^i b^j c^k \mid 0 \le i \le j \le k\}$ is not CF.

Proof: Assume that E is CF, select $s = a^p b^p c^p$, where p is the pumping length. By the pumping lemma, s can be decomposed into $s = uvxyz$, s.t.

① $uv^ixy^iz \in E$ for $i \ge 0$,

② $|vy| > 0$,

③ $|vxy| \le p$.

a) When both v and y contain only one type of symbols (e.g., a's and b's),

a.1) a's do not appear in v or y.
$uv^0xy^0z = uxz \notin E$ // # of a's would be more than b's or c's

a.2) b's do not appear
If a's appear in v or y, $uv^2xy^2z \notin E$ // # of a's ↑
If c's appear in v or y, $uv^0xy^0z \notin E$ // as in a.1).

a.3) c's do not appear
we pump up, $uv^2xy^2z \notin E$ // # of c's < # of a's or b's.

b) Either v or y contains more than one type of symbols, $uv^2xy^2z \notin E$ as we could have either a-b-a-b or b-c-b-c as subsequences (i.e, out of order)

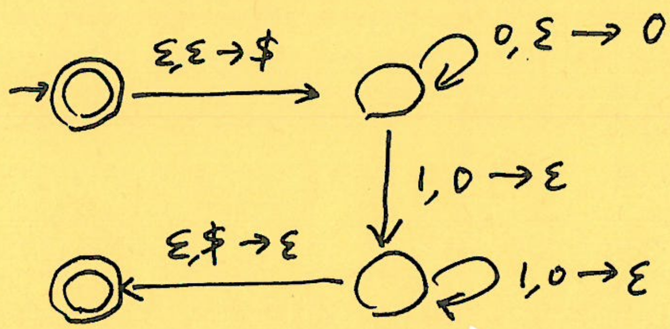
∴ A contradiction to the pumping lemma!

∴ E is not CF. □

In compilers, etc, we use deterministic CFL, or deterministic PDA:

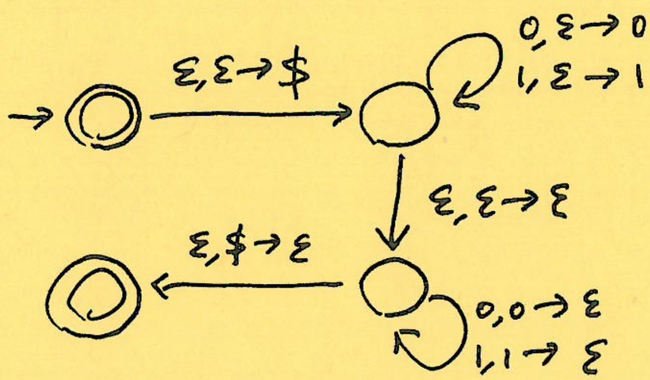
$$\text{Exactly one of } \left.\begin{array}{l} \delta(q, a, x) \\ \delta(q, a, \varepsilon) \\ \delta(q, \varepsilon, x) \\ \delta(q, \varepsilon, \varepsilon) \end{array}\right\} \text{ is not } \phi$$

Ex.



deterministic

Ex



non-deterministic

The diagonalization method (Cantor, 1873)

- How do we measure the size of a set?

  Finite set —— just count!

  Infinite set —— not trivial
  - $N = \{1, 2, 3, 4 \cdots\}$
  - R — real numbers

- Def: $f: A \to B$

  f is <u>one to one</u> if $f(a) \neq f(b)$ whenever $a \neq b$.

  $$abs(x) = \begin{cases} x, & \text{if } x \geq 0 \\ -x, & \text{if } x < 0 \end{cases}$$

  $abs(-2) = 2$

  $abs(2) = 2$

  $$\begin{matrix} -2 \\ 2 \end{matrix} \rightarrow 2$$

  f is <u>onto</u> if it hits every element of B.

  $square(x) = x^2$

  — 3 can't be hit when the domain is N.

  f is a <u>correspondence</u> if it is <u>one-to-one</u> and it is also <u>onto</u>.

  A and B are of the same size if there is a correspondence between them.

- Def. A set A is <u>countable</u> if it is <u>finite</u> or it has the same size as N (natural numbers or $\{1, 2, 3 \cdots\}$).

Ex 1. $f(n) = 2n$, $n \in N$ (set of positive even integers).

Ex 2. $Q = \{ m/n \mid m, n \in N \}$ is countable.

m \ n	1	2	3	4	$\cdots$
1	$\frac{1}{1}$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	$\cdots$
2	$2/1$	$2/2$	$2/3$	$2/4$	$\cdots$
3	$3/1$	$3/2$	$3/3$	$3/4$	$\cdots$
4	$\cdots$				

Def. If an infinite set has <u>no</u> correspondence with N, then it is called <u>uncountable</u>.

Def. R – set of real numbers.

Thm 4.17 R is uncountable.