CSCI 338: Exercise 04

I will give non-regular exercises for you to try (remember, you must work on these besides following my lectures or read book sections!). These will not be graded. Solutions will be posted on D2L a bit later.

Problem 1

Let $\Sigma = \{a\}$. Use the pumping lemma to show that the following language is not regular:
$A = \{a^{2^n} | n \geq 0\}$. Here a^{2^n} means a string of 2^n a's.

Proof: Assume A is regular, we select $s = a^{2^p}$, where p is the pumping length. By the pumping lemma, s can be decomposed into $s = xyz$, s.t.,

① $xy^iz \in A$, for $i \geq 0$,

② $|y| > 0$, and

③ $|xy| \leq p$.

By ③, $|xy| \leq p$. Since $p < 2^p$, we have $|y| < 2^p$.

$\therefore |xy^2z| = |xyz| + |y| < 2^p + 2^p = 2^{p+1}$.

As $|y| \geq 1$ (by ②), $|xy^2z| = |xyz| + |y| > |xyz| = 2^p$.

$\therefore 2^p < |xy^2z| < 2^{p+1}$.

Hence, when $i = 2$, $xy^2z \notin A$ as its length is not a power of 2.

A contradiction to the pumping lemma.

$\therefore$ A is not regular. $\square$