Jan 29

- Two machines are <u>equivalent</u> if they accept the same language.

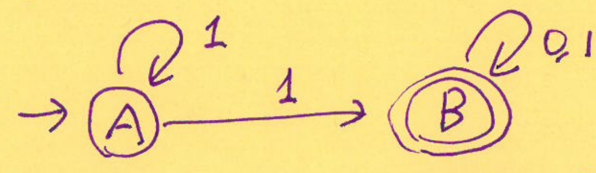
<u>Thm 1.39</u>. Every NFA has an equivalent DFA.

<u>proof</u>: Let $N = (Q, \Sigma, \delta, q_0, F)$ be the NFA accepting/recognizing A.
We'll construct DFA $M = (Q', \Sigma, \delta', q_0', F')$.

ⓐ <u>No ε-transitions (arrows) in N.</u>

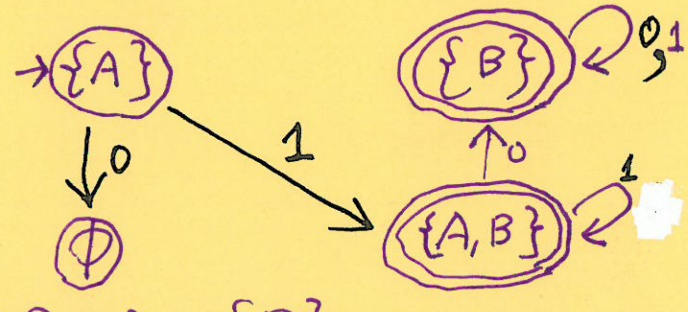
1. $Q' = P(Q)$
2. $\delta'(R, a) = \bigcup_{r \in R} \delta(r, a)$, $R \in Q'$, $a \in \Sigma$.
3. $q_0' = \{q_0\}$
4. $F' = \{R \in Q' \mid R$ contains an accept state of $N\}$

<u>Ex</u>. NFA N:



$1^+(0/1)^*$

DFA M:



$\delta'(\{B\}, 1) = \delta(B, 1) = \{B\}$

$\delta'(\{A,B\}, 0) = \delta(A, 0) \cup \delta(B, 0) = \phi \cup \{B\} = \{B\}$

$\delta'(\{A,B\}, 1) = \delta(A, 1) \cup \delta(B, 1) = \{A,B\} \cup \{B\} = \{A,B\}$

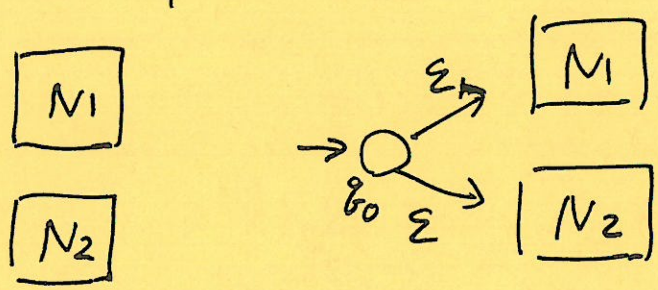
Corollary 1.40.

A language is <u>regular</u> if and only if some NFA accepts it.

<u>Thm 1.45</u>.

The class of regular languages is closed under the union operation.

<u>proof</u>:



Let $N_1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$ recognize A_1,
Let $N_2 = (Q_2, \Sigma, \delta_2, q_2, F_2)$ recognize A_2.

Construct $N = (Q, \Sigma, \delta, q_0, F)$.

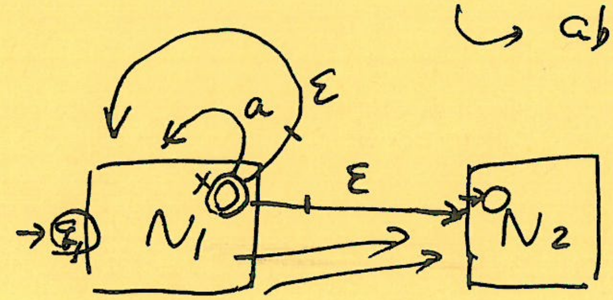
1. $Q = \{q_0\} \cup Q_1 \cup Q_2$
2. q_0 is the (new) start state of N.
3. $F = F_1 \cup F_2$
4. $$\delta(q, a) = \begin{cases} \delta_1(q, a), & q \in Q_1 \\ \delta_2(q, a), & q \in Q_2 \\ \{q_1, q_2\}, & q = q_0, \text{ and } a = \varepsilon \\ \phi, & q = q_0, \text{ and } a \neq \varepsilon \text{ // optional.} \end{cases}$$

Thm 1.47

The class of regular languages is closed under the concatenation operation.

↳ $ab \circ xy = abxy$

proof:



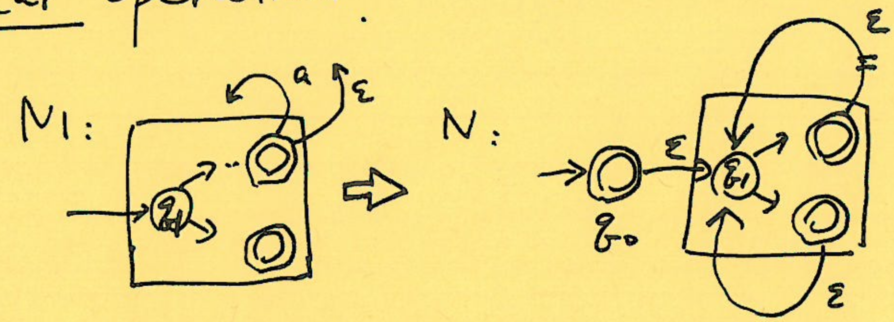
Let $N_1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$ recognize A_1,
Let $N_2 = (Q_2, \Sigma, \delta_2, q_2, F_2)$ recognize A_2.
Construct $N = (Q, \Sigma, \delta, q_1, F)$ to recognize $A_1 \circ A_2$.

1. $Q = Q_1 \cup Q_2$
2. $q_1 = q_1$ (or $q_0 = q_1$)
3. $F = F_2$
4. $$\delta(q,a) = \begin{cases} \delta_1(q,a), & \text{if } q \in Q_1 \text{ and } q \notin F_1 \\ \delta_1(q,a), & \text{if } q \in F_1 \text{ and } a \neq \varepsilon \\ \delta_1(q,a) \cup \{q_2\}, & \text{if } q \in F_1 \text{ and } a = \varepsilon \\ \delta_2(q,a), & \text{if } q \in Q_2 \end{cases}$$

Thm 1.49

The class of regular languages is closed under the <u>star</u> operation.

<u>proof</u>.



Let $N_1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$ recognize A_1.

We construct $N = (Q, \Sigma, \delta, q_0, F)$ to recognize A_1^*.

1. $Q = \{q_0\} \cup Q_1$
2. q_0 is the <u>new</u> start state.
3. $F = \{q_0\} \cup F_1$
4. $$\delta(q,a) = \begin{cases} \delta_1(q,a), & q \in Q_1 \text{ and } q \notin F_1 \\ \delta_1(q,a) & q \in F_1 \text{ and } a \neq \varepsilon \\ \delta_1(q,a) \cup \{q_1\}, & q \in F_1 \text{ and } a = \varepsilon \\ \{q_1\} & , q = q_0 \text{ and } a = \varepsilon \\ \phi & , q = q_0 \text{ and } a \neq \varepsilon \text{ //optional.} \end{cases}$$