March 1

Ex 2. $C = \{ww \mid w \in \{0,1\}^*\}$ is not context free.

proof. Suppose that C is context-free.

Pick $s = 0^p 1^p 0^p 1^p$, p being the pumping length.

By the pumping lemma, s can be decomposed into $s = uvxyz$, s.t. ① for $i \geq 0$, $uv^ixy^iz \in C$,

② $|vy| > 0$,

③ $|vxy| \leq p$.

a) If vxy does not straddle (cross) the midpoint of S, $uv^2xy^2z \notin C$ as it can't be written as ww.

b) If vxy straddles (crosses) the midpoint of S, set $i=0$, $uv^0xy^0z = uxz$ has the form $0^p 1^i 0^j 1^p$ with either $i<p$ or $j<p$.

$\therefore$ $uxz \notin C$, which violates the pumping lemma.

$\therefore$ C is not context free. $\square$

Ex 3. $L = \{a^n b^j \mid n = j^2\}$ is not context free.

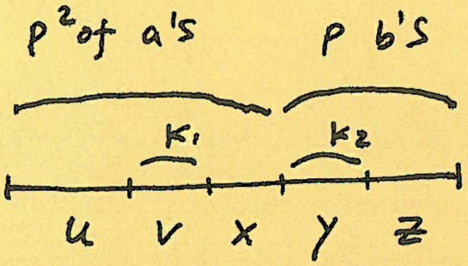
proof. Assume that L is context-free.

We pick $s = a^{p^2} b^p$, p being the pumping length.

By the pumping lemma, s can be decomposed into $s = uvxyz$, s.t. ① $uv^ixy^iz \in L$, for $i \geq 0$,

② $|vy| > 0$,

③ $|vxy| \leq p$.



with the case shown on the left. we have

$p^2 + (i-1)\cdot k_1$ a's, and

$p + (i-1)\cdot k_2$ b's,

if we pump down by setting $i=0$.

__Case 1.__ If $k_1 \neq 0$, and $k_2 \neq 0$, then we pick $i=0$, as

$$\begin{aligned}(p-k_2)^2 &\leq (p-1)^2 \\ &= p^2 - 2p + 1 \\ &< p^2 - k_1 \quad // \quad k_1 \leq p\end{aligned}$$

Therefore, the # of a's and b's in uxz can not be in the form of q^2 vs q. $\therefore uxz \notin L$.

__Case 2.__ If one of k_1 and k_2 is zero, then again we pick $i=0$ and obtain that

$uxz \notin L$.

$\therefore L$ is not context-free. $\square$

Ex4. $E = \{ a^i b^j c^k \mid 0 \leq i \leq j \leq k \}$

take-home exercise. try to work on it,

I will go over it on wednesday.