

## CSCI 338: Assignment 2 (7 points)

River Kelly

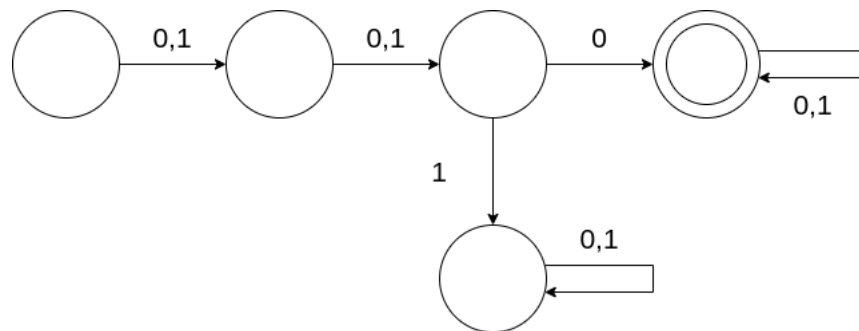
Feb 18, 2021

## Problem 1

**1.1** Give state diagrams of DFAs recognizing the following languages. In all parts, the alphabet is  $\{0, 1\}$ .

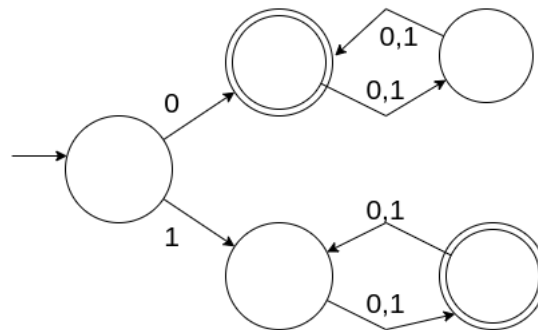
**1.6.d**

$\{w \mid w \text{ has length at least 3 and its third symbol is a } 0\}$



**1.6.e**

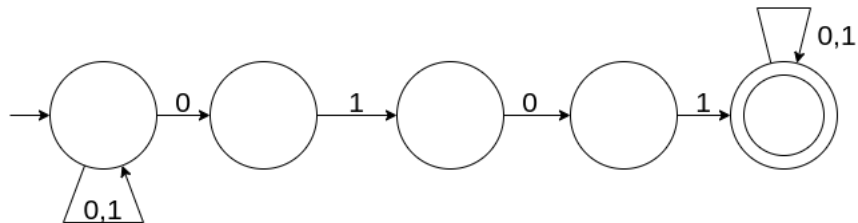
$\{w \mid w \text{ starts with } 0 \text{ and has odd length, or starts with } 1 \text{ and has even length} \}$



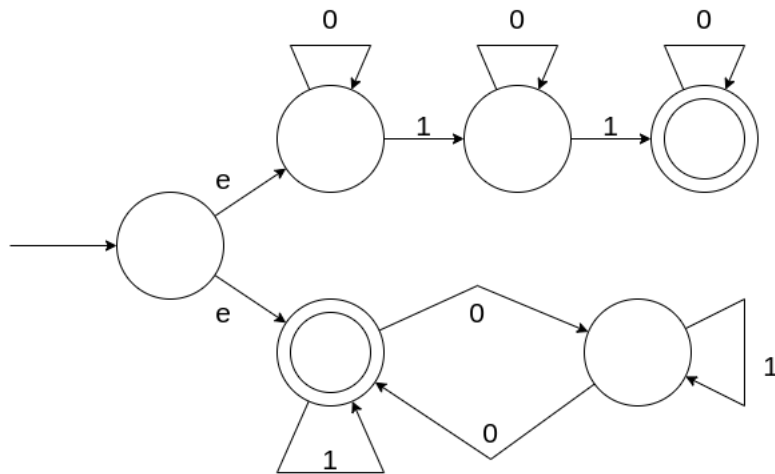
**1.2** Give state diagrams of NFAs with the specified number of states recognizing each of the following languages. In all parts, the alphabet is  $\{0, 1\}$ .

**1.7.b** The language of Exercise 1.6c with five states.

Five states that accept the strings over the alphabet  $\Sigma = \{0, 1\}$  and contains the sub-string 0101.



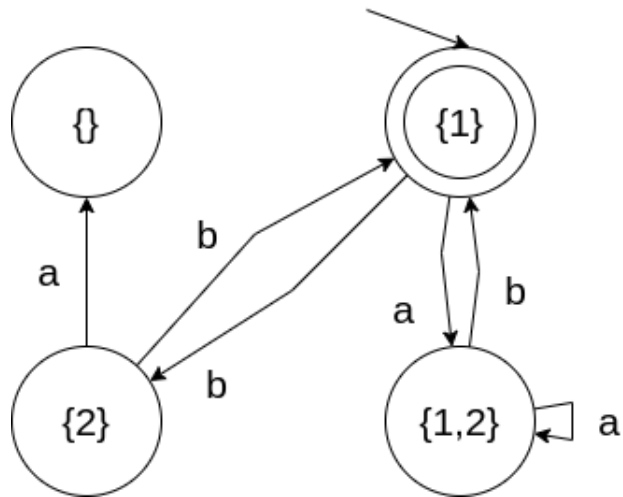
**1.7.c** The language of Exercise 1.6l with six states



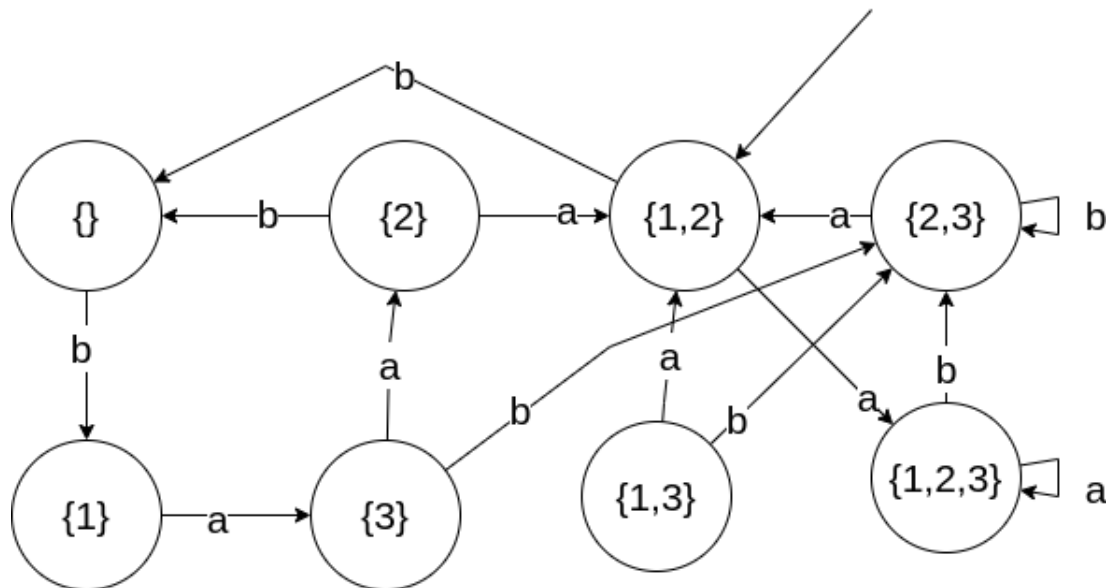
## Problem 2

Problem 1.16.a, Problem 1.16.b (page 86).

**1.16.a**



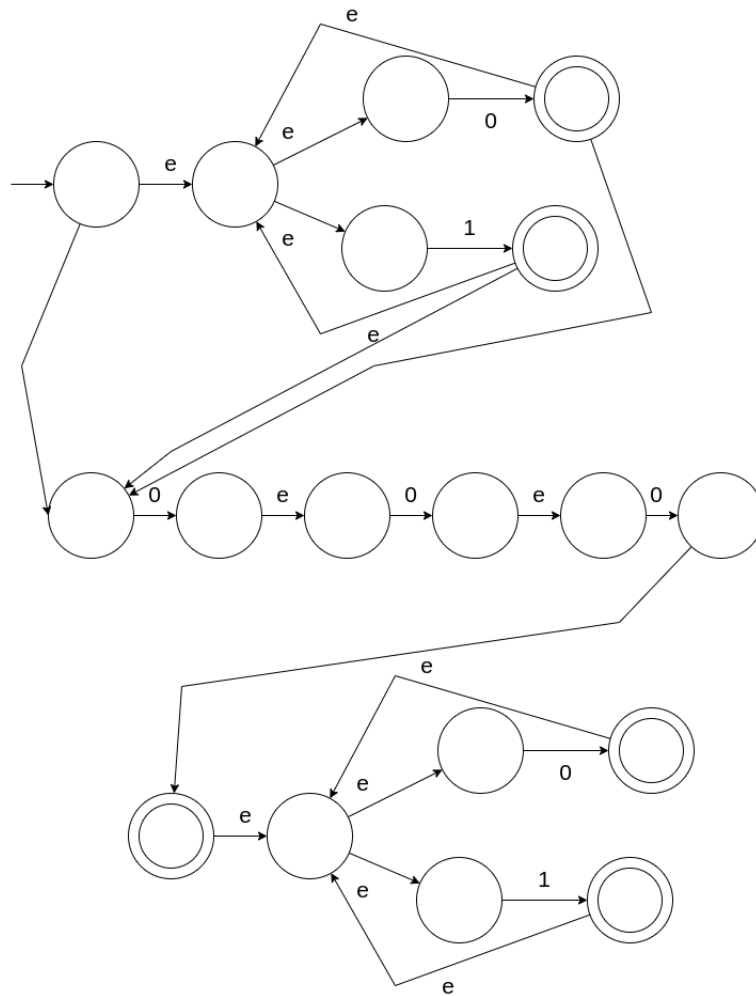
**1.16.b**



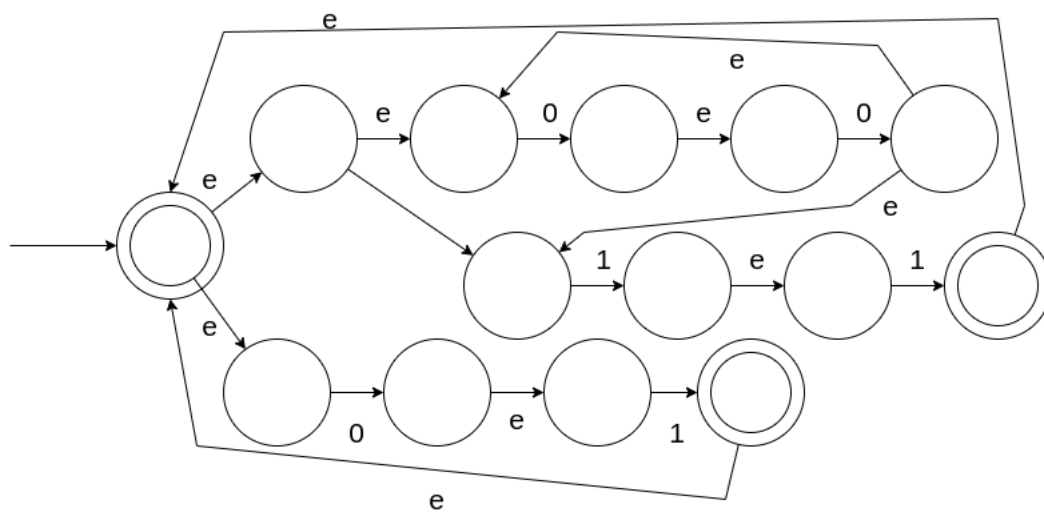
### Problem 3

Use the procedure described in Lemma 1.55 to convert the following regular expressions to nondeterministic finite automata. (page 86).

**1.19.a.**  $(0 \cup 1)^* 000(0 \cup 1)^*$

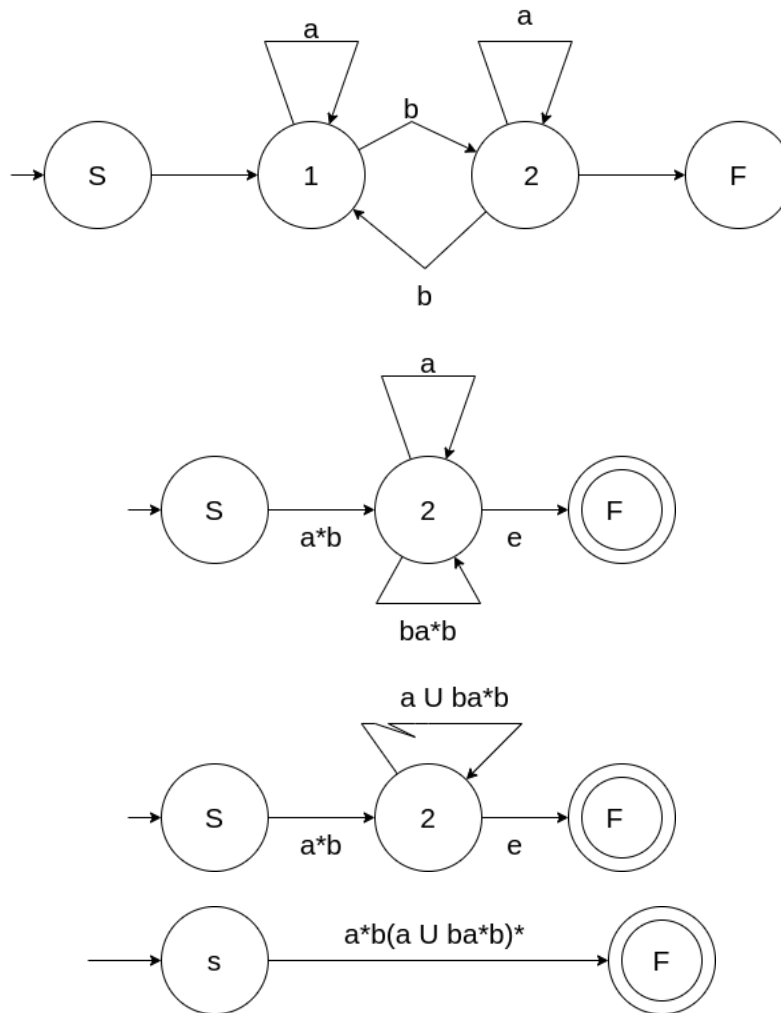


1.19.b.  $((00)^* (11)^* \cup 01)^*$



## Problem 4

Problem 1.21.a (page 86).



The regular expression is:  $a^*b(a \cup ba^*b)^*$

## Problem 5

Prove the following languages are not regular.

1.  $xy^iz \in A$  for  $i \geq 0$
2.  $|y| > 0$
3.  $|xy| \leq p$

(5.1)  $A = \{a^{n^3} | n \geq 0\}$ . Here  $a^x$  means a string of  $x$   $a$ 's.

*Proof.* Assume that  $A$  is regular. Let  $S = a^{p^3}$  where  $p$  is the pumping length. By the pumping lemma,  $S$  decomposes into  $xyz$  s.t.

By 3,  $|y| \leq p$ .

Pumping up,  $|xy^2z| \leq p^3 + p < p^3 + 3p^2 + 3p + 1 = (p+1)^3$ .

By 2,  $|y| > 0$ , hence  $p^3 < |xy^2z| < (p+1)^3$ . Thus  $xy^2z \in A$ , a contradiction of the pumping lemma.

$\therefore A$  is not regular. □

(5.2)  $B = \{0^n 1^m 0^n | m, n \geq 0\}$ .

*Proof.* Assume  $B$  is regular. Let  $S = 0^P 1 0^P$  where  $p$  is the pumping length. Then,  $S$  decomposes in  $xyz$  s.t.

By 3,  $y$  consists of only 0's.

Let  $\delta = |y|$  then by 2,  $\delta > 0$ .

The pumping up,  $xy^2z = 0^{P+\delta} 1 0^P \notin B$  because the same number of 0's is not the same before and after the 1.

A contradiction of the pumping lemma.

$\therefore B$  is not regular. □