CSCI 338: Assignment 1 (7 points)

This assignment is due on **Thursday, Jan 28, 8:00pm**. You will need to use Latex to generate a single pdf file and upload it under *Assignment 1* on D2L. There will be a penalty for not using Latex (to finish the assignment). This is **not** a group-assignment, so you must finish the assignment by yourself.

Problem 1

Prove that $1^4 + 2^4 + 3^4 + \cdots + n^4 = \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30}$.

Problem 2

Given a planar graph $P = (V, E)$, we have Euler's formula: $|V| + |F| - |E| = 2$, where F is the set of faces of P and E is the set of edges of P. Let $|V| = n$, where V is the set of vertices of P. Prove that $|F|$ is at most $2n$.

Problem 3

Prove that in any simple graph there is a path from any vertex of odd degree to some other vertex of odd degree.

Problem 4

A fully binary tree T is a tree such that all internal nodes have two children. Prove that a fully binary tree with n internal nodes in total has $n + 1$ leaves.

Problem 5

Given an undirected graph $G = (V, E)$, the breadth-first-search starting at $v \in V$ ($bfs(v)$ for short) is to generate a shortest path tree starting at vertex $v \in V$. The diameter of G is the longest of all shortest paths $\delta(u, v), u, v \in V$.

When G is a tree, the following algorithm is proposed to compute the diameter of G.

1. Run $bfs(w), w \in V$, and compute the vertex $x \in V$ furthest from w.
2. Run $bfs(x)$ and compute the vertex $y \in V$ furthest from x.
3. Return $\delta(x, y)$ as the diameter of G.

Prove that this algorithm is correct; i.e., $\delta(x, y)$ is in fact the longest among all the shortest paths between $u, v \in V$.