# CSCI 338: Quiz 5 (due: Friday, April 2, 8:00pm)

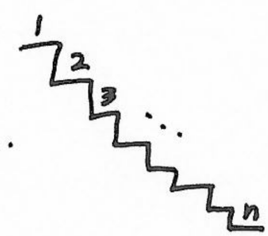
## Your Name:

This is an open-book quiz (not an attendance counting), so you should try your best. After you finish, upload a pdf file on D2L under Quiz-5. A solution will be posted on D2L after the deadline.

### Problem 1

On March 24, we covered the crystal ball problem.

In this quiz, you are asked to solve the second problem, i.e., if Peter has exactly two balls, how could he find the smallest step $i$ (efficiently, in better than $O(n)$ time) such that his ball would break at step $i$ (but not at step $i-1$). Note that once both balls are broken Peter can't use any replacement.




① Use ball-1 to test on steps $1$, $\lceil\sqrt{n}\rceil+1$, $2\lceil\sqrt{n}\rceil+1$, ... until it breaks. With this info, we can compute $j$ such that ball-1 breaks at step $(j+1)\lceil\sqrt{n}\rceil+1$ but doesn't break at step $j\lceil\sqrt{n}\rceil+1$.

② Use ball-2 to test sequentially at $j\lceil\sqrt{n}\rceil+2$, $j\lceil\sqrt{n}\rceil+3$, ... until it breaks at step $i = j\lceil\sqrt{n}\rceil+k$ ($k \leq \lceil\sqrt{n}\rceil$); if not, $i=(j+1)\lceil\sqrt{n}\rceil+1$.

Ball-1 and Ball-2 would both be tested at at most $\lceil\sqrt{n}\rceil$ steps.

$\therefore$ the running time of this algorithm (with 2 balls) is $2\lceil\sqrt{n}\rceil = O(\sqrt{n})$.