Feb. 1

Thm 1.39

Every NFA has an equivalent DFA.

NFA $N = (Q, \Sigma, \delta, q_0, F)$

$\Downarrow$

DFA $M = (Q', \Sigma, \delta', q_0', F')$

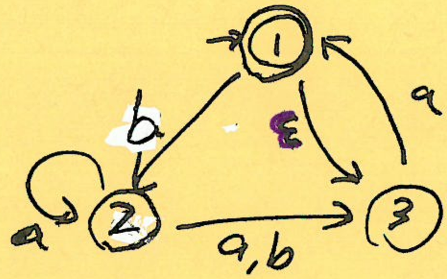
Proof B with ε-transitions.

1. $Q' = P(Q)$

$E(R) = \{q \mid q$ can be reached from R by traveling 0 or more ε-transitions$\}$

2. $\delta'(R, a) = \{q \in Q \mid q \in E(\delta(r, a))$ for some $r \in R\}$.
3. $q_0' = E(\{q_0\})$
4. $F' = \{R \in Q' \mid R$ contains an accept state of $N\}$.

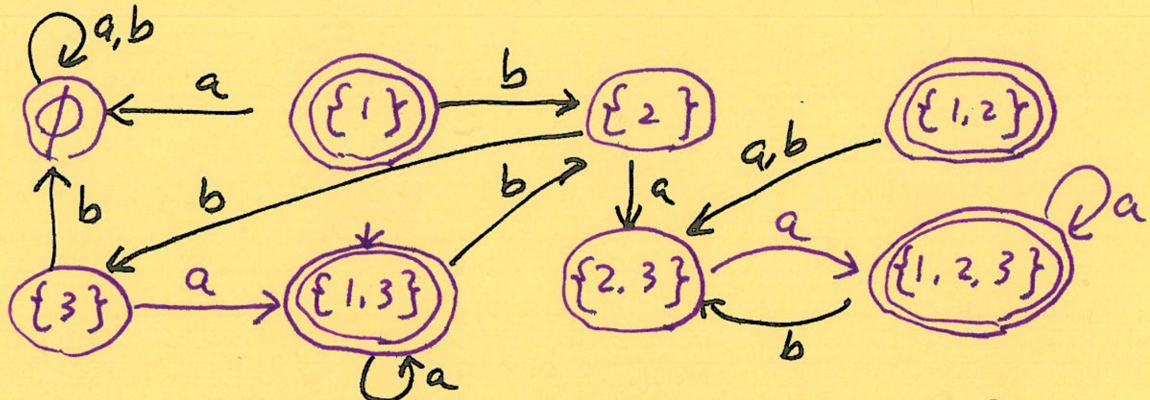
NFA: N



Ex $E(\{1\}) = \{1, 3\}$.

$E(\{2,3\}) = \{2, 3\}$

DFA M:



Ex $\delta'(\{3\}, a) = \{q \in Q \mid q \in E(\delta(r,a)) \text{ for } r = 3\}$

$= \{q \in Q \mid q \in E(\delta(3,a))\}$

$= \{q \in Q \mid q \in E(\{1\})\}$

$= \{q \in Q \mid q \in \{1,3\}\}$

$= \{1, 3\}$

Ex $\delta'(\overset{R}{\{2,3\}}, a) = \{q \in Q \mid q \in E(\delta(2,a)) \text{ or } q \in E(\delta(3,a))\}$

$= \{q \in Q \mid q \in E(\{2,3\})\} \cup \{q \in Q \mid q \in E(\{1\})\}$

$= \{2,3\} \cup \{1,3\}$

$= \{1, 2, 3\}$

Ex $\delta'(\overset{R}{\overbrace{\{1,2,3\}}}, a)$

$= E(\delta(1,a)) \cup E(\delta(2,a)) \cup E(\delta(3,a))$

$= \emptyset \cup E(\{2,3\}) \cup E(\{1\})$

$= \emptyset \cup \{2,3\} \cup \{1,3\}$

$= \{1, 2, 3\}$.

R is a regular expression, if R is

1. a, for some $a \in \Sigma$, // $L(R) = \{a\}$
2. ε, // $L(R) = \{\varepsilon\}$
3. ϕ. // $L(R) = \phi$
4. $R_1 \cup R_2$, where R_1 and R_2 are regular expressions
5. $R_1 \circ R_2$, where — — ——— ——— —— –
6. R_1^*, where R_1 is a regular expression.

Ex. $R \cup \phi = R$, $R \cup \varepsilon \neq R$.

eg. $R = 0$, $L(R) = \{0\}$

$L(R \cup \varepsilon) = \{0, \varepsilon\}$.

Thm 1.54

A language is regular if and only if some regular expression describes it.

"If part"

Lem 1.55. If a language is described by a regular expression, then it is a regular language.

proof: Let R be a regular expression describing some language A. We show how to convert R into an NFA N (recognizing A)

1. $R = a$, for some $a \in \Sigma$, $L(R) = \{a\}$

NFA: