March 05

The diagonalization method.

Thm 4.17 R is uncountable

proof. Contradiction + diagonalization

Assume R is countable, i.e., all numbers can be listed as f(1), f(2), f(3), ...

n	f(n)
1	3. 1 4 1 5 9 ...
2	0. 0 0 0 0 0 ...
3	2. 7 1 8 ......
4	2. 5 0 0 0 0 ...
⋮	⋮

$x = 0.2541\cdots$

Construct a number x, s.t., the ith digit (after .) is different from the ith digit of $f(i)$.

x is a real number, but $x \neq f(i)$ for any i (because they differ at lease at digit i)

∴ f can not be a correspondence between N and R.

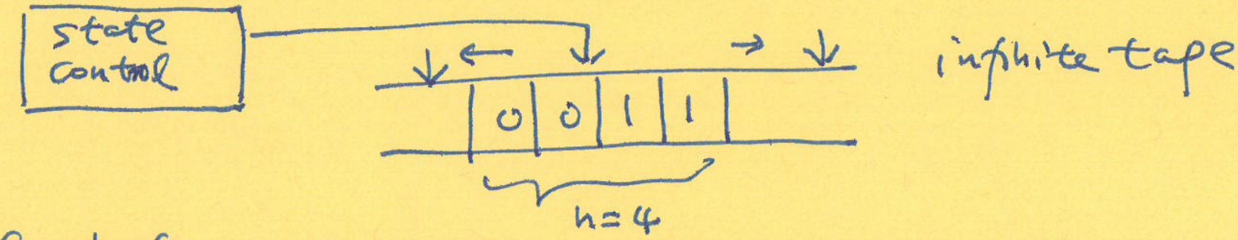
∴ R is uncountable. □

Implication: All the machines can be enumerated. the data to be processed are not countable. ∴ There is sth any machine can't do.

Chapter 3 Church-Turing Thesis

What is a general computer model?

– Turing machine



<u>Formal def</u>:

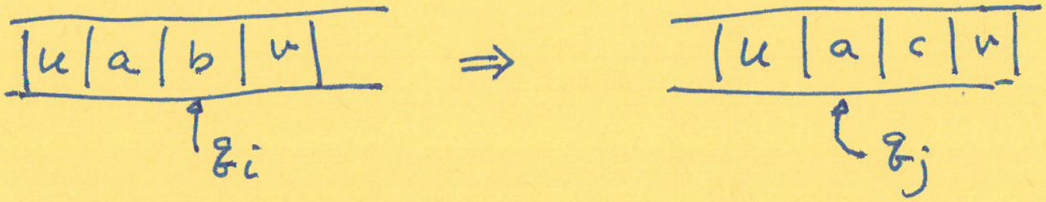
A Turing machine is a 7-tuple

$$(Q, \Sigma, \Gamma, \delta, q_0, q_{accept}, q_{reject})$$

1. Q — set of states
2. Σ — input alphabet not containing "$\sqcup$" — blank symbol. Ex. $\Sigma = \{0,1\}, \{a,b\}$
3. Γ — tape alphabet, where $\sqcup \in \Gamma$ and $\Sigma \subseteq \Gamma$.
4. $\delta: Q \times \Gamma \rightarrow Q \times \Gamma \times \{L, R\}$ is the transition function.
5. $q_0 \in Q$ is the start state
6. $q_{accept} \in Q$ is the accept state
7. $q_{reject} \in Q$ is the reject state, $q_{reject} \neq q_{accept}$.

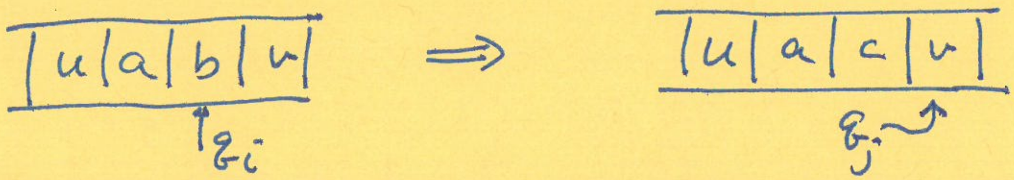
- $\delta(q_i, b) = (q_j, c, L)$

$\Leftrightarrow$ $u a q_i b v$ yields $u q_j a c v$

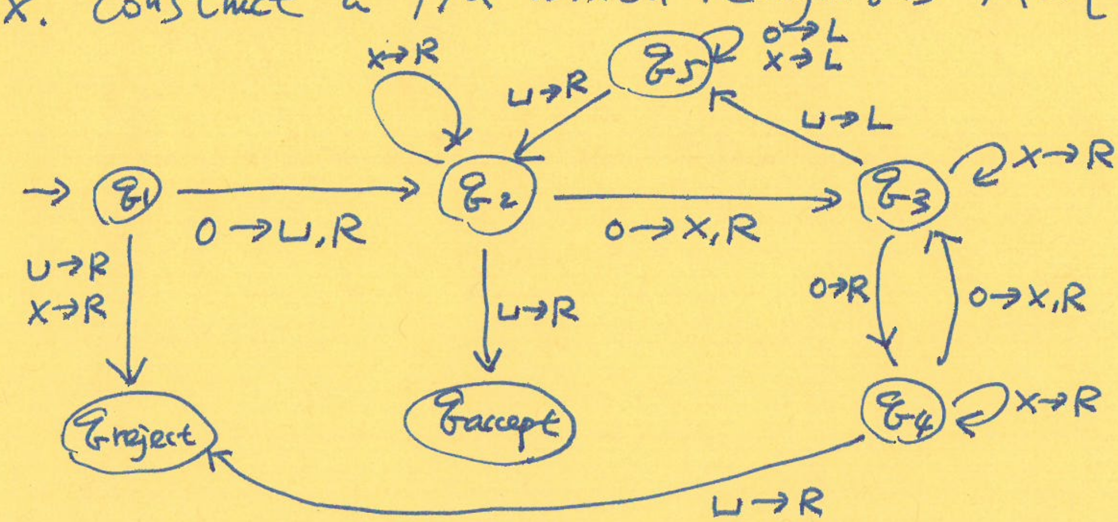


- $\delta(q_i, b) = (q_j, c, R)$

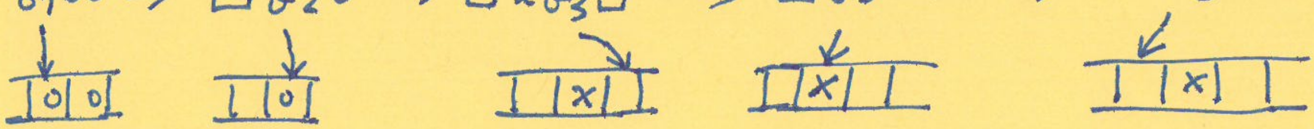
$\Leftrightarrow$ $u a q_i b v$ yields $u a c q_j v$



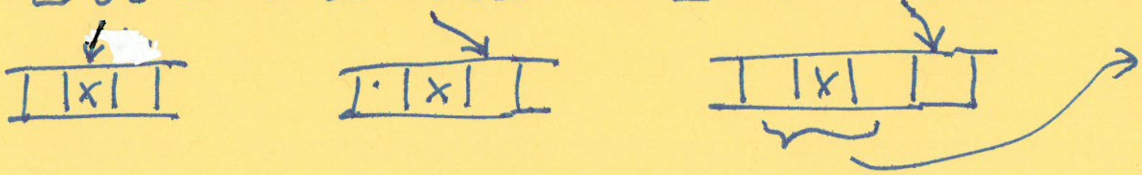
- Ex. Constmct a TM which recognizes $A = \{ 0^{2^n} \mid n \geq 0 \}$



$q_1 00 \Rightarrow \sqcup q_2 0 \Rightarrow \sqcup x q_3 \sqcup \Rightarrow \sqcup q_5 x \sqcup \Rightarrow q_5 \sqcup x \sqcup$



$\Rightarrow \sqcup q_2 x \sqcup \Rightarrow \sqcup x q_2 \sqcup \Rightarrow \sqcup x \sqcup q_{accept} \sqcup$



garbage.

Church-Turing Thesis :

Intuitive algorithms $\Longleftrightarrow$ TM algorithms
(pseudo-codes)