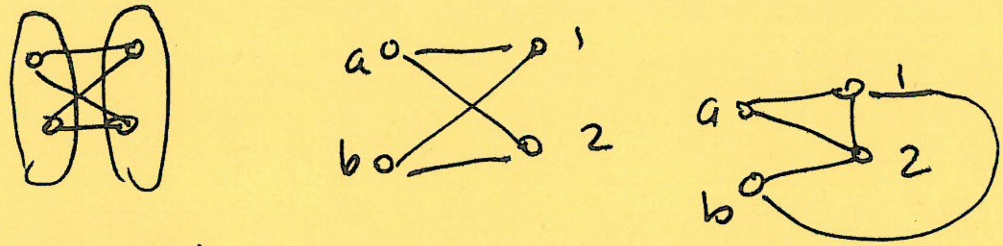
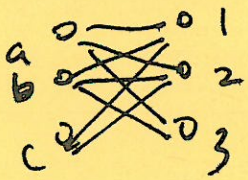
Jan 22

Planar graph: a graph whose vertices and edges can be drawn on the plane with no edge crossing

$K_{2,2}$



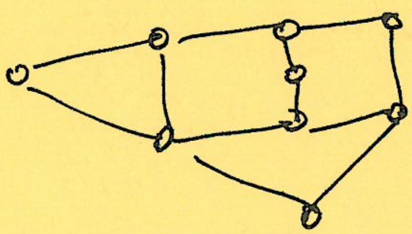
$K_{3,3}$



Can't be drawn that way

- Euler's formula.

$$|V| + |F| - |E| = 2$$ // prove by induction on $|V|$.



$|V| = 9$
$|F| = 5$
$|E| = 12$

$9 + 5 - 12 = 2$

Logic and Boolean logic

$\vee$, $\wedge$, $\neg$, XOR ($\oplus$), $\rightarrow$ "implies", $\leftrightarrow$ "equivalent to"

$P \vee Q \Leftrightarrow \neg(\neg P \wedge \neg Q)$

$P \rightarrow Q \Leftrightarrow \neg P \vee Q$

$P \leftrightarrow Q \Leftrightarrow (P \rightarrow Q) \wedge (Q \rightarrow P)$.

$\neg(P \wedge Q)$

I don't want P&Q to be selected.

$= \neg P \vee \neg Q$.

- distributive law

$P \wedge (Q \vee R) \Leftrightarrow (P \wedge Q) \vee (P \wedge R)$

$P \vee (Q \wedge R) \Leftrightarrow (P \vee Q) \wedge (P \vee R)$.

How to disprove $P \rightarrow Q$?

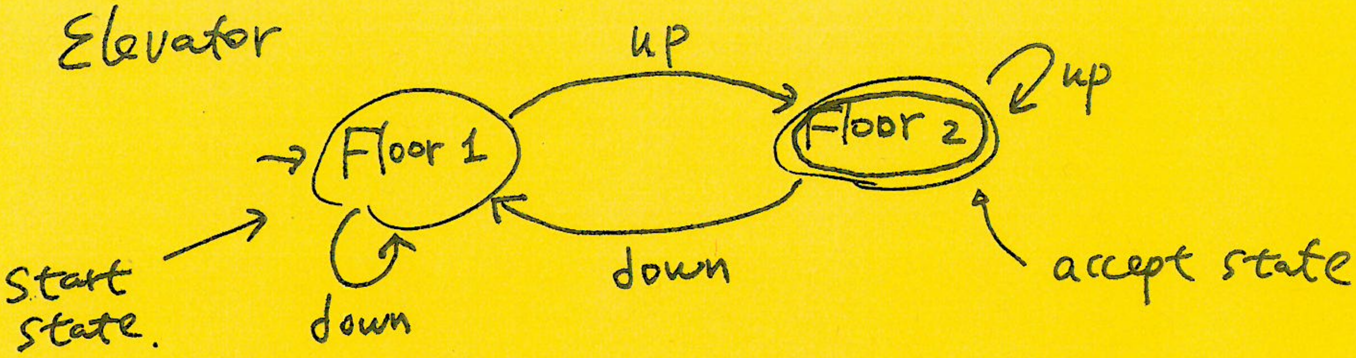
I am happy

it snows.

show: <u>P sometimes implies $\neg Q$.</u>

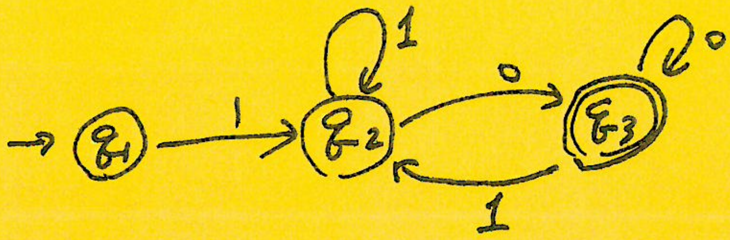
Chapter 1 Regular languages.

Elevator



Pattern: the last button pressed is "up".

Ex 1:

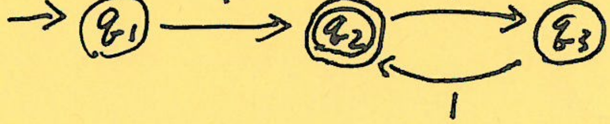


10
10100
11110

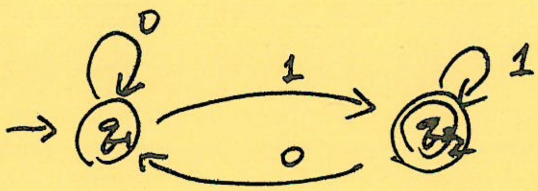
binary sequence starting with 1, and ending with 0.

Ex 2: Design a finite state machine which accepts $A = \{ w \mid \text{the length of } w \text{ is odd} \}$.

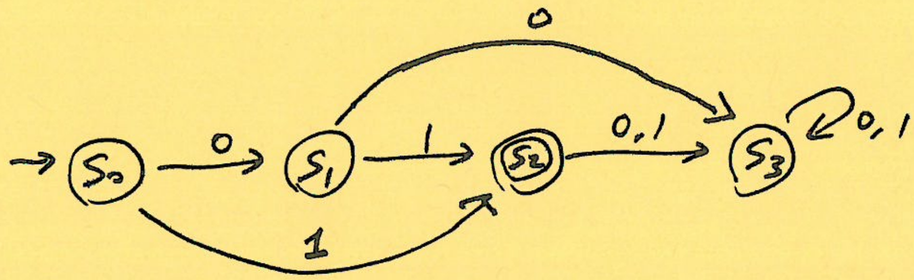
$\Sigma = \{1\}$



Elevator:

M_1: 

$L(M_1) = \{(0|1)^*1\}$

M_2: 

$L(M_2) = \{1, 01\}$

- A finite state automaton is a 5-tuple $(Q, \Sigma, \delta, q_0, F)$, where
  1. Q is the set of states. // $Q(M_1) = \{q_1, q_2\}$
  2. Σ is a finite set called alphabet. // $\Sigma(M_1) = \{0, 1\}$
  3. $\delta: Q \times \Sigma \to Q$ // $\delta(q_1, 1) = q_2$
  4. $q_0 \in Q$ is the start state // $q_0(M_1) = q_1$
  5. $F \subseteq Q$ is the set of accept states // $F(M_1) = \{q_2\}$.
- If A is the set of all strings that the machine M accepts, then $L(M) = A$.
- Formally, given $M = (Q, \Sigma, \delta, q_0, F)$ and $w = w_1 w_2 \cdots w_n$

  M accepts w if ① $\exists\, r_0, r_1, \cdots, r_n$ s.t.,

  $r_0 = q_0$, $\delta(r_i, w_{i+1}) = r_{i+1}$ and $r_n \in F$.