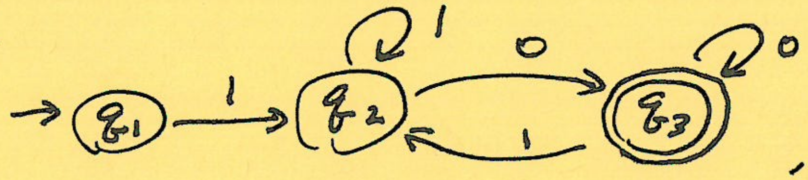
Jan 25

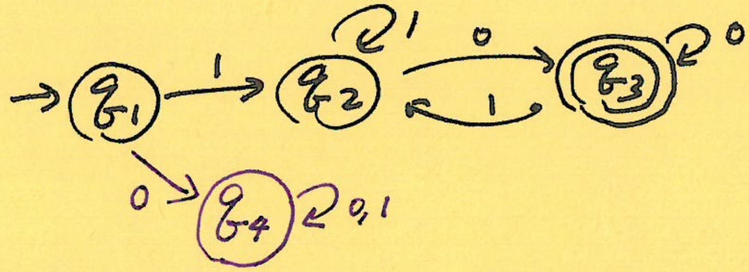
Question asked in the previous lecture:

Q: In EX1



,

do we have to include a state q_4, s.t.



?

<u>Answer</u>: <u>Formally, Yes.</u> Remember that $\delta: Q \times \Gamma \to Q$. So at each state you must have a transition for any letter in Σ.

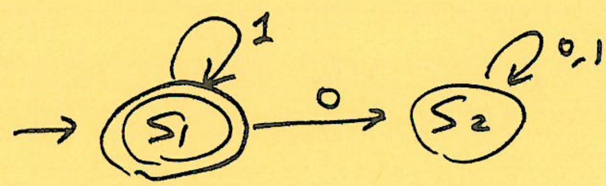
On the other hand, q_4 is not an accept state, so leaving out q_4 (i.e., to use a <u>short form</u> as I did in class) is fine.

- Identify the languages accepted by the following DFA (FSM)

FSM → finite state machine

DFA → deterministic finite automaton

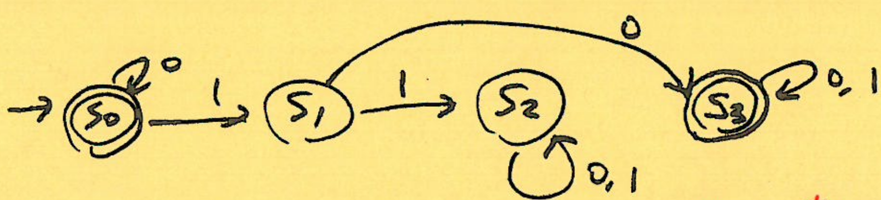
<u>Ex1</u>



$\{1^*\}$ or

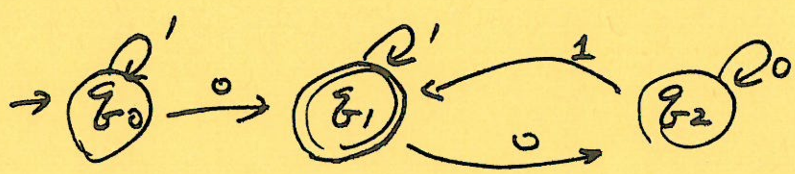
$\{1^n \mid n = 0, 1, 2, \cdots\}$

<u>Ex2</u>



$\{0^*, 0^*10(0|1)^*\}$

<u>Ex3</u>



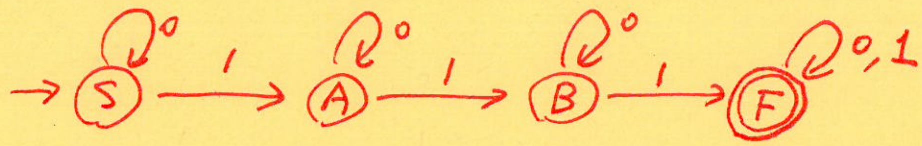
$\{1^*01^*, 1^*01^*0^+1^+\}$

$+$ — at least 1

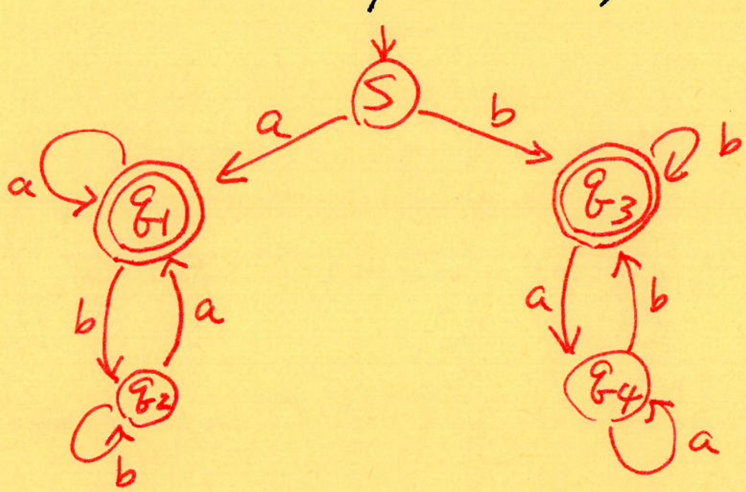
$*$ — at least 0

- A language is called <u>regular</u> if some finite state automaton recognizes (accepts) it.
- Q: How to design finite automaton?

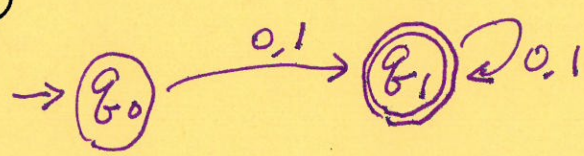
Ex1: $\{w \mid w \text{ contains at least three 1's}\}$, $\Sigma = \{0,1\}$.



Ex2: $\{w \mid w \text{ starts and ends with the same symbol}\}$, $\Sigma = \{a,b\}$



Ex3: Everything but the empty string. // take-home exercise



– <u>Regular operations</u>

Let A, B be languages.

- $A \cup B = \{x \mid x \in A \text{ or } x \in B\}$ // union
- $A \circ B = \{xy \mid x \in A \text{ and } y \in B\}$ // concatenation
- $A^* = \{x_1 x_2 \cdots x_k \mid k \geq 0, \text{ and } x_i \in A\}$ // star or repetition operation.

- Thm 1.25

If A_1 and A_2 are regular languages, so is $A_1 \cup A_2$. // Regular languages are closed under the union operation.

Ex. for $x \in N$, $y \in N$, $x+y \in N$.

// Natural numbers are closed under the addition operation.

Ex. for $7 \in N$, $22 \in N$, $7/22 \notin N$.

// Natural numbers are not closed under the division operation.

Thm 1.25

If A_1 and A_2 are regular languages, so is $A_1 \cup A_2$.

<u>proof</u>. (by construction).

By def, let M_1 recognize A_1, $M_1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$

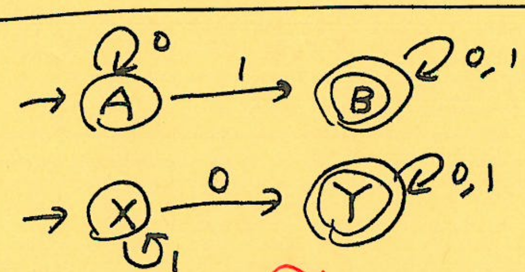
let M_2 recognize A_2, $M_2 = (Q_2, \Sigma, \delta_2, q_2, F_2)$.

We construct $M = (Q, \Sigma, \delta, q_0, F)$ as follows.

1. $Q = \{(r_1, r_2) \mid r_1 \in Q_1, r_2 \in Q_2\}$. // $|Q| = |Q_1| * |Q_2|$
2. Σ. // $\Sigma = \Sigma_1 \cup \Sigma_2$ if M_1, M_2 are on diff Σ's.
3. $\delta((r_1, r_2), a) = (\delta_1(r_1, a), \delta_2(r_2, a))$
4. $q_0 = (q_1, q_2)$
5. $F = \{(r_1, r_2) \mid r_1 \in F_1$ OR $r_2 \in F_2\}$ □

<u>Ex</u>.

Q_1 :

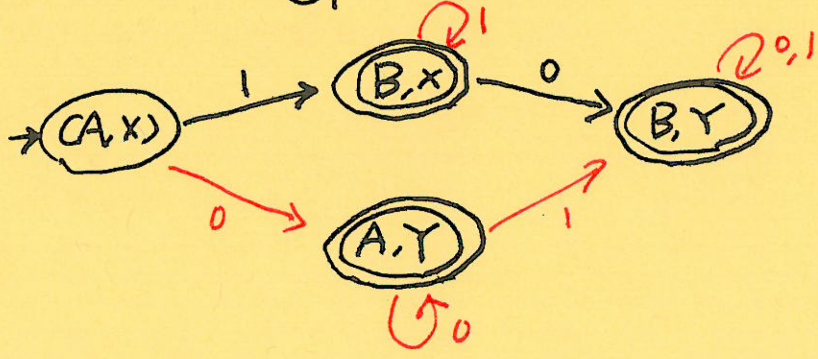


Q_2 :

$0^*1(0|1)^*$

$1^*0(0|1)^*$

M:



$\delta((A,X), 1)$
$= (\delta_1(A,1), \delta_2(X,1))$
$= (B, X)$

$\delta((B,X), 0)$
$= (\delta_1(B,0), \delta_2(X,0))$
$= (B, Y)$