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CSCI 338 Computer Science Theory  
Test 1 — 55 minutes (10 points)

### Question 1

- **F**
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### Question 2

By definition, a forest  $F$  is an undirected graph in which any two vertices are connected by at most one path. We will prove that a forest  $F$  is composed of  $r$  trees with a total of  $n$  vertices such that  $F$  has  $n - r$  edges. This proof will be done by induction. Let us assume that the total number of vertices  $n$  is equal to 1.

Base case:  $n = 1$

In the base case, there is only one such tree,  $r = 1$ , and one isolated vertex,  $n = 1$ . Thus,  $1 - 1 = 0$ , hence a single isolated vertex does not have any edges. Therefore, the base case is proven to be true.

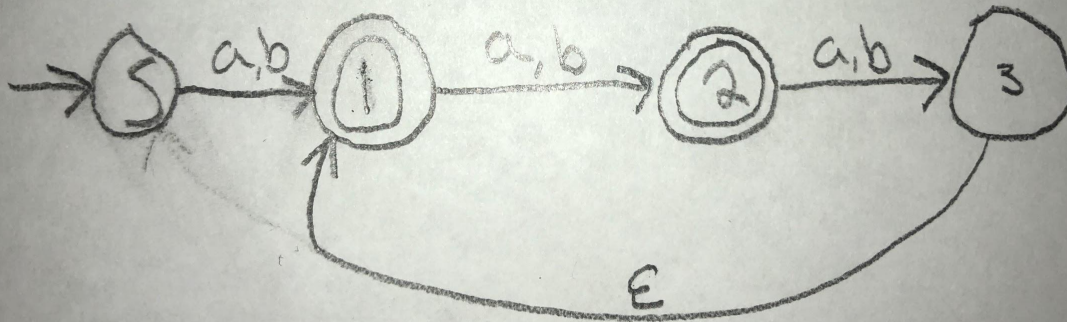
Inductive Step: Assume that the number of edges in the forest  $F$  is also true for vertices  $n$ , such that  $n \geq 2$  and that the base,  $n = 1$  is also true.

- Let  $v$  be any leaf in the forest  $F$
- By removing leaf  $v$  from forest  $F$ , and thereby removing its unique edge which connects it to the tree, we now have a new forest  $H$ .
- $H$  must be connected such that no cycles exist
- With the removal of a leaf, we also lose an edge
- Then  $G$  has  $(n-2) + 1 = n - 1$  edges

Therefore, any forest  $F$  has  $n - r$  edges.

Question 3

Question 3.



## Question 4

Let's assume that  $B$  is a regular language. If  $B$  is Regular, then by definition, there must exist some pumping length  $p$  for a substring  $s$  in  $B$  that can be pumped such that  $|s| \geq p$ .

All possible string from language  $B$  are some multiple of a factorial of  $a$ . Thus, by decomposing  $s$  into  $x y^i z$ , By 3  $|y| \leq p$ . But if  $y$  substring begins at index 0 and  $x$  is an empty set, this is a contradiction to rule 3.