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CSCI-347
Homework 04
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Show your work. Include any code snippets you used to generate an answer, using comments in the code to clearly indicate which problem corresponds to which code

Code Setup

[16]:
```
# import libraries
import numpy as np
import matplotlib.pyplot as plt
```

Part 1 (2 points) Matrix Vector

Consider matrix A and vector v. Compute the matrix-vector product Av.

$$A = \begin{pmatrix} 2 & 1 \\ 1 & 3 \end{pmatrix}, v = \begin{pmatrix} -1 & 1 \end{pmatrix}$$

[17]:
```
# build matrix 'A'
A = np.ndarray(shape=(2,2), dtype=int)
A[0] = np.array([2, 1])
A[1] = np.array([1, 3])
# build vector 'v'
V = np.array([-1, 1], dtype=int)
dot_product = np.array([0] * A.shape[1])
# loop through row is matrix A
for i in range(A.shape[1]):
    # get the current row
    a_row = A[i, :]
    # loop over the values in the current row
    # and compute the dot product
    for index, row_value in enumerate(a_row):
        # compute the dot product for i in row
        dot_product[i] += row_value * V[index]
# show the result dot pot_product
dot_product
```

[17]:
```
array([-1,  2])
```

Part 2

Consider matrix A and data set D:

$$A = \begin{pmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix}, D = \begin{pmatrix} 1 & 1.5 \\ 1 & 2 \\ 3 & 4 \\ -1 & -1 \\ -1 & 1 \\ 1 & -2 \\ 2 & 2 \\ 2 & 3 \end{pmatrix}$$

Helper Code

[18]:
```
A = np.matrix([
    [((3**(1/2))/2), -(1/2)],
    [(1/2), ((3**(1/2))/2)]
])
# show matrix 'A'
A
```

[18]:
```
matrix([[ 0.8660254, -0.5      ],
        [ 0.5      ,  0.8660254]])
```

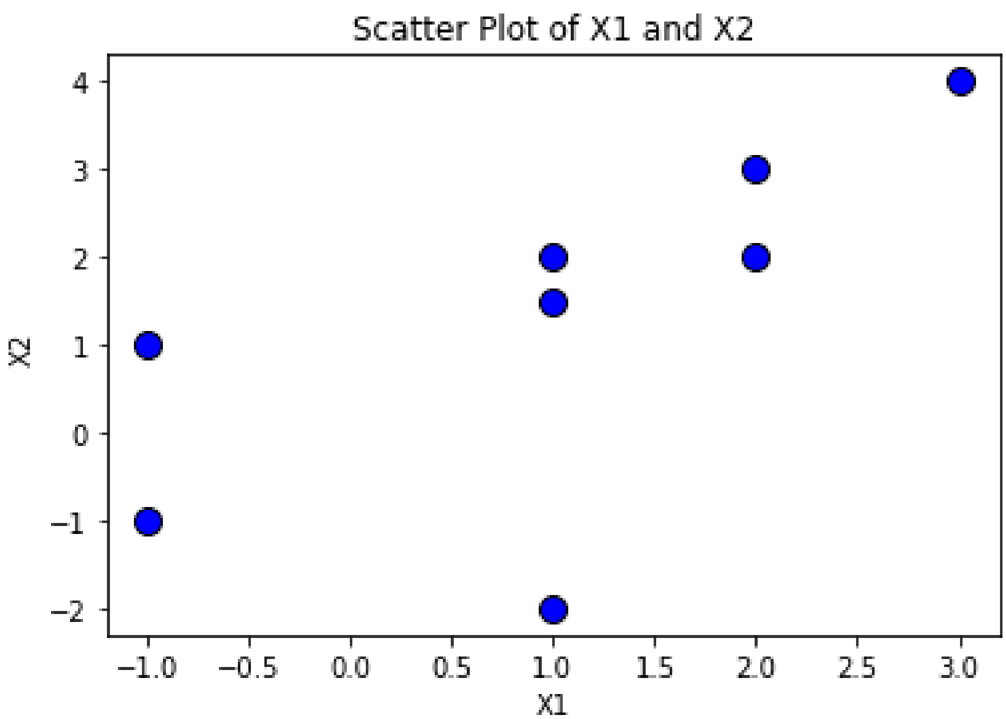
[19]:
```
# build data set 'D'
D = np.ndarray(shape=(8, 2))
D[0] = [1, 1.5]
D[1] = [1, 2]
D[2] = [3, 4]
D[3] = [-1,-1]
D[4] = [-1, 1]
D[5] = [1, -2]
D[6] = [2, 2]
D[7] = [2, 3]
# show data set 'D'
D
```

[19]:
```
array([[ 1. ,  1.5],
       [ 1. ,  2. ],
       [ 3. ,  4. ],
       [-1. , -1. ],
       [-1. ,  1. ],
       [ 1. , -2. ],
       [ 2. ,  2. ],
       [ 2. ,  3. ]])
```

2.a Scatter Plot (2 points)

Let $X1$ and $X2$ be the first and second attributes of the data, respectively. Use Python to create a scatter plot of the data, where the x-axis is $X1$ and the y-axis is $X2$.

[20]:
```python
X1 = D[:,0]
X2 = D[:,1]
plt.scatter(x=X1, y=X2, color='blue', marker='o', s=100, edgecolors='black')
plt.xlabel('X1')
plt.ylabel('X2')
plt.title('Scatter Plot of X1 and X2')
plt.show()
```



2.b Linear Transformation (4 points)

Treating each row as a 2-dimensional vector, apply the linear transformation A to each row. In other words, let x_i be the i-th row of D. For each x_i, find the matrix-vector product Ax_i.

For example,

$$x2 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

[21]:
```
linearTransformationData = np.ndarray(shape=D.shape)
for row_index, row in enumerate(D):
    dot = np.dot(A, row)
    linearTransformationData[row_index] = [dot[0,0], dot[0,1]]
```

[22]:
```
# print out the transformed data
for index, row in enumerate(linearTransformationData):
    print(f"x{index+1}: ", row)
```

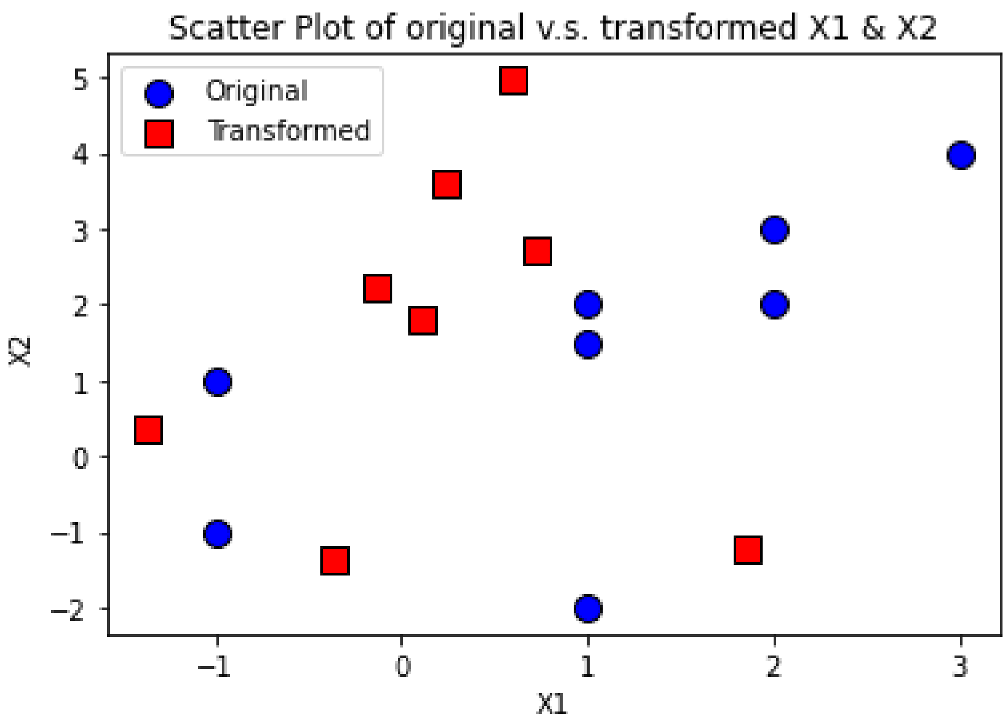
```
x1:  [0.1160254  1.79903811]
x2:  [-0.1339746   2.23205081]
x3:  [0.59807621 4.96410162]
x4:  [-0.3660254 -1.3660254]
x5:  [-1.3660254  0.3660254]
x6:  [ 1.8660254  -1.23205081]
x7:  [0.73205081 2.73205081]
x8:  [0.23205081 3.59807621]
```

2.c Transformed Data (3 points)

UsePythontocreateaplotshowingboththeoriginaldataandthetransformed data, with the x-axis still corresponding to $X1$ and the y-axis corresponding to $X2$. Use different colors and markers to differentiate between the original and transformed data. That is, each transformed data point in the plot should be one matrix-vector product Ax_i, which is a 2-dimensional vector. Each original point in the plot should have the same coordinates as it did in part 2.1.

[23]:

```
X1_transformend = linearTransformationData[:,0]
X2_transformend = linearTransformationData[:,1]
plt.scatter(x=X1, y=X2, color='blue', marker='o', s=100, edgecolors='black')
plt.scatter(x=X1_transformend, y=X2_transformend, color='red', marker='s',␣
 ↪s=100, edgecolors='black')
plt.xlabel('X1')
plt.ylabel('X2')
plt.title('Scatter Plot of original v.s. transformed X1 & X2')
plt.legend(['Original', 'Transformed'])
plt.show()
```



2.d Multi-Dimensional Mean (1 point)

Write down the multi-dimensional mean of the data. (Remember that this should be a 2-dimensional vector)

[24]:
```
def multiDimensionalMean(m):
    # output array (i.e. mean array)
    mean = [0] * m.shape[1]
    # iterate over columns
    for col_index in range(m.shape[1]):
        # get column array
        col_arr = m[:,col_index]
        # column mean
        col_mean = col_arr.mean()
        # set column mean to mean (output) array
        mean[col_index] = col_mean
    # return multi-dimensional mean
    return mean
multiDimMean = multiDimensionalMean(D)
multiDimMean
```

[24]:
```
[1.0, 1.3125]
```

2.e Mean-Centered Data (2 points)

Mean-center the data. Write down the mean-centered data matrix.

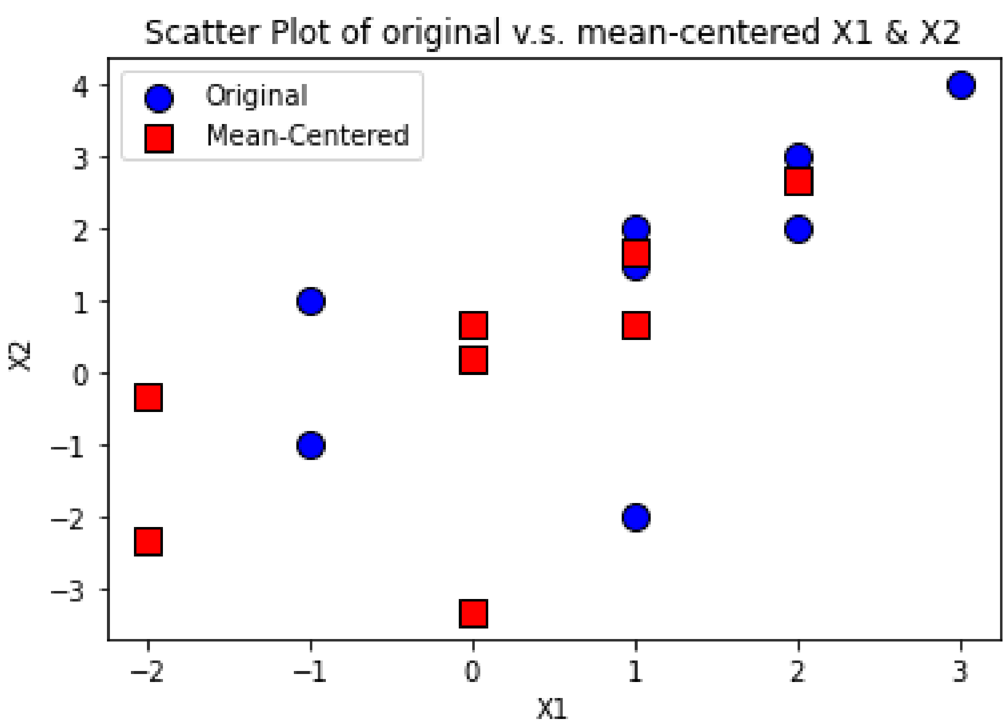
[25]:
```
meanCenteredData = np.ndarray(shape=D.shape)
for row_index, row in enumerate(D):
    for row_col_index, value in enumerate(row):
        meanCenteredData[row_index][row_col_index] = value -␣
 ↪multiDimMean[row_col_index]
meanCenteredData
```

[25]:
```
array([[ 0.    ,  0.1875],
       [ 0.    ,  0.6875],
       [ 2.    ,  2.6875],
       [-2.    , -2.3125],
       [-2.    , -0.3125],
       [ 0.    , -3.3125],
       [ 1.    ,  0.6875],
       [ 1.    ,  1.6875]])
```

2.f Plot of Original v.s. Mean-Centered Data (2 points)

Use Python to create a scatter plot showing both the original data and the mean-centered data, where the x-axis is $X1$ and the y-axis is $X2$. Use different colors and markers to differentiate between the original and mean-centered data.

[26]:
```
X1_mean_centered = meanCenteredData[:,0]
X2_mean_centered = meanCenteredData[:,1]
plt.scatter(x=X1, y=X2, color='blue', marker='o', s=100, edgecolors='black')
plt.scatter(x=X1_mean_centered, y=X2_mean_centered, color='red', marker='s',␣
 ↪s=100, edgecolors='black')
plt.xlabel('X1')
plt.ylabel('X2')
plt.title('Scatter Plot of original v.s. mean-centered X1 & X2')
plt.legend(['Original', 'Mean-Centered'])
plt.show()
```



2.g Covariance Matrix (2 points)

Write down the covariance matrix of the data matrix D. Use estimated covariance.

[27]:
```
def covariance(v1, v2 = None):
    if v2 is None: v2 = v1
    # vector 1 mean
    v1_mean = v1.mean()
    # vector 2 mean
    v2_mean = v2.mean()
    # co_var (the covariance between v1 and v2)
    co_var = 0
    # loop through v1 and v2 values
    for i in range(v1.shape[0]):
        co_var += (v1[i] - v1_mean) * (v2[i] - v2_mean)
    # calculate and return the co-variance between v1 and v2
    return (co_var / (v1.shape[0] - 1))

def covarianceMatrix(m):
    # co-variance matrix
    covar_m = np.ndarray((m.shape[1], m.shape[1]))
    # loop through input matrix rows
    for i in range(m.shape[1]):
        # loop through input matrix rows (again)
        for j in range(m.shape[1]):
            # set x_ij covariance value
            covar_m[i, j] = covariance(m[:,i], m[:,j])
    # return the covariance matrix
    return covar_m
covarianceMatrix(D)
```

[27]:
```
array([[2.        , 1.85714286],
       [1.85714286, 3.92410714]])
```

2.h Covariance Matrix of Z (Centered Mean) (3 points)

Write down the covariance matrix of the centered data matrix Z. Use estimated covariance.

[28]:
```
covarianceMatrix(meanCenteredData)
```

[28]:
```
array([[2.        , 1.85714286],
       [1.85714286, 3.92410714]])
```

2.i Covariance Matric of Standard Normalization (3 points)

Write down the covariance matrix of the data after applying standard normalization.

[29]:
```
def zScoreNormalize(m):
    # create normlized matrix based on shape of input matrix
    z_score = np.ndarray(m.shape)
    # loop through input matrix rows
    for row_index in range(m.shape[0]):
        # loop through input matrix columns
        for col_index in range(m.shape[1]):
            # get current column array
            col_arr = m[:,col_index]
            # calculate the standard devieation for the current column
            col_std_div = (covariance(col_arr)) ** (1/2)
            # calculate the column's mean
            col_mean = col_arr.mean()
            # get the x_ij value from the imput matix
            x_ij = m[row_index, col_index]
            # calculate the x_ji z-score
            x_ij_zscore = (x_ij - col_mean) / col_std_div
            # set x_ij normalized value in normalized matrix
            z_score[row_index, col_index] = x_ij_zscore
    # return the normalized array
    return z_score
covarianceMatrix(zScoreNormalize(D))
```

[29]:
```
array([[1.        , 0.66291811],
       [0.66291811, 1.        ]])
```