River Kelly
CSCI-347
Homework 04

Show your work. Include any code snippets you used to generate an answer, using comments in the code to clearly indicate which problem corresponds to which code

Code Setup

[16]:
```
# import libraries
import numpy as np
import matplotlib.pyplot as plt
```

Part 1 - (2 points) Matrix Vector

Consider matrix A and vector v. Compute the matrix-vector product Av.

$$A = \begin{pmatrix} 2 & 1 \\ 1 & 3 \end{pmatrix}, v = \begin{pmatrix} -1 & 1 \end{pmatrix}$$

[17]:
```
# build matrix 'A'
A = np.ndarray(shape=(2,2), dtype=int)
A[0] = np.array([2, 1])
A[1] = np.array([1, 3])
# build vector 'v'
V = np.array([-1, 1], dtype=int)
dot_product = np.array([0] * A.shape[1])
# loop through row is matrix A
for i in range(A.shape[1]):
    # get the current row
    a_row = A[i, :]
    # loop over the values in the current row
    # and compute the dot product
    for index, row_value in enumerate(a_row):
        # compute the dot product for i in row
        dot_product[i] += row_value * V[index]
# show the result dot pot_product
dot_product
```

[17]:
```
array([-1,  2])
```

Part 2

Consider matrix A and data set D:

$$A = \begin{pmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix}, D = \begin{pmatrix} 1 & 1.5 \\ 1 & 2 \\ 3 & 4 \\ -1 & -1 \\ -1 & 1 \\ 1 & -2 \\ 2 & 2 \\ 2 & 3 \end{pmatrix}$$

Helper Code

[18]:
```
A = np.matrix([
    [((3**(1/2))/2), -(1/2)],
    [(1/2), ((3**(1/2))/2)]
])
# show matrix 'A'
A
```

[18]:
```
matrix([[ 0.8660254, -0.5      ],
        [ 0.5      ,  0.8660254]])
```

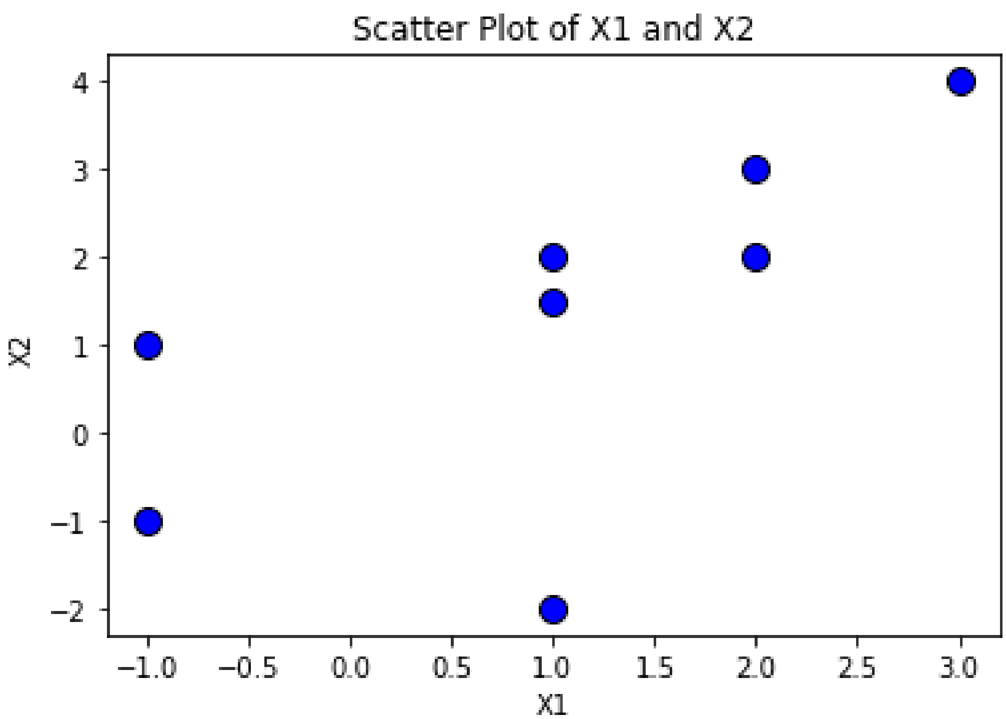
[19]:
```
# build data set 'D'
D = np.ndarray(shape=(8, 2))
D[0] = [1, 1.5]
D[1] = [1, 2]
D[2] = [3, 4]
D[3] = [-1,-1]
D[4] = [-1, 1]
D[5] = [1, -2]
D[6] = [2, 2]
D[7] = [2, 3]
# show data set 'D'
D
```

[19]:
```
array([[ 1. ,  1.5],
       [ 1. ,  2. ],
       [ 3. ,  4. ],
       [-1. , -1. ],
       [-1. ,  1. ],
       [ 1. , -2. ],
       [ 2. ,  2. ],
       [ 2. ,  3. ]])
```

Scatter Plot

Let $X1$ and $X2$ be the first and second attributes of the data, respectively. Use Python to create a scatter plot of the data, where the x-axis is $X1$ and the y-axis is $X2$.

[20]:
```
X1 = D[:,0]
X2 = D[:,1]
plt.scatter(x=X1, y=X2, color='blue', marker='o', s=100, edgecolors='black')
plt.xlabel('X1')
plt.ylabel('X2')
plt.title('Scatter Plot of X1 and X2')
plt.show()
```



Linear Transformation

Treating each row as a 2-dimensional vector, apply the linear transformation A to each row. In other words, let x_i be the i-th row of D. For each x_i, find the matrix-vector product Ax_i.

For example,

$$x2 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

[21]:
```
linearTransformationData = np.ndarray(shape=D.shape)
for row_index, row in enumerate(D):
    dot = np.dot(A, row)
    linearTransformationData[row_index] = [dot[0,0], dot[0,1]]
```

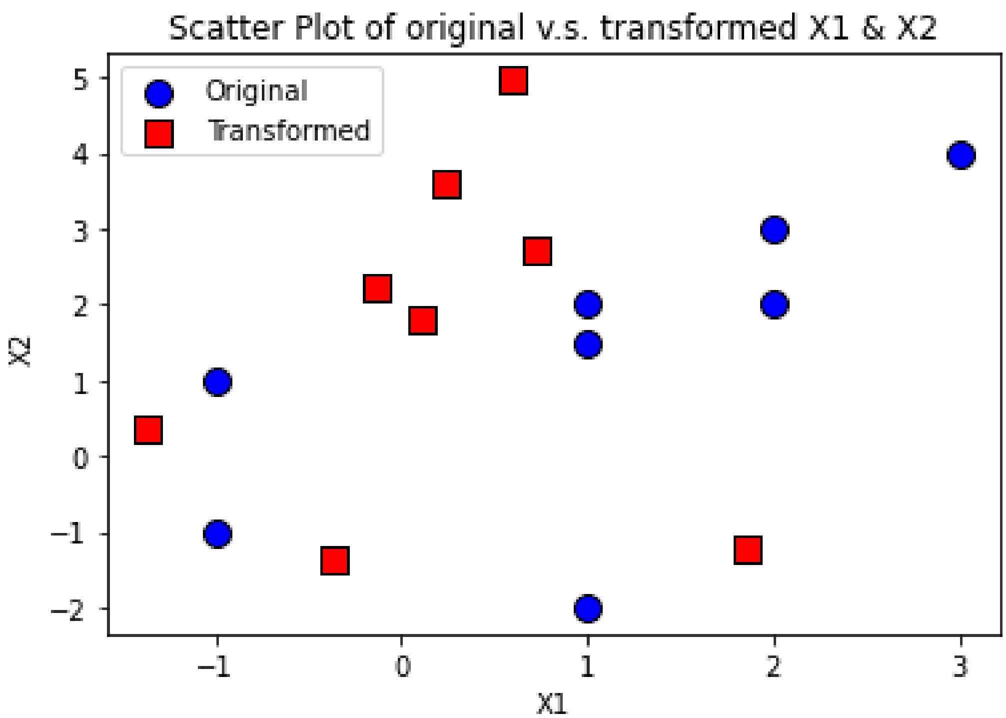
[22]:
```
# print out the transformed data
for index, row in enumerate(linearTransformationData):
    print(f"x{index+1}: ", row)
```

```
x1:  [0.1160254  1.79903811]
x2:  [-0.1339746   2.23205081]
x3:  [0.59807621 4.96410162]
x4:  [-0.3660254 -1.3660254]
x5:  [-1.3660254  0.3660254]
x6:  [ 1.8660254  -1.23205081]
x7:  [0.73205081 2.73205081]
x8:  [0.23205081 3.59807621]
```

Transformed Data

UsePythontocreateaplotshowingboththeoriginaldataandthetransformed data, with the x-axis still corresponding to $X1$ and the y-axis corresponding to $X2$. Use different colors and markers to differentiate between the original and transformed data. That is, each transformed data point in the plot should be one matrix-vector product Ax_i, which is a 2-dimensional vector. Each original point in the plot should have the same coordinates as it did in part 2.1.

[23]:
```
X1_transformend = linearTransformationData[:,0]
X2_transformend = linearTransformationData[:,1]
plt.scatter(x=X1, y=X2, color='blue', marker='o', s=100, edgecolors='black')
plt.scatter(x=X1_transformend, y=X2_transformend, color='red', marker='s',␣
↪s=100, edgecolors='black')
plt.xlabel('X1')
plt.ylabel('X2')
plt.title('Scatter Plot of original v.s. transformed X1 & X2')
plt.legend(['Original', 'Transformed'])
plt.show()
```



Multi-Dimensional Mean

Write down the multi-dimensional mean of the data. (Remember that this should be a 2-dimensional vector)

[24]:
```
def multiDimensionalMean(m):
    # output array (i.e. mean array)
    mean = [0] * m.shape[1]
    # iterate over columns
    for col_index in range(m.shape[1]):
        # get column array
        col_arr = m[:,col_index]
        # column mean
        col_mean = col_arr.mean()
        # set column mean to mean (output) array
        mean[col_index] = col_mean
    # return multi-dimensional mean
    return mean
multiDimMean = multiDimensionalMean(D)
multiDimMean
```

[24]:
```
[1.0, 1.3125]
```

Mean-Centered Data

Mean-center the data. Write down the mean-centered data matrix.

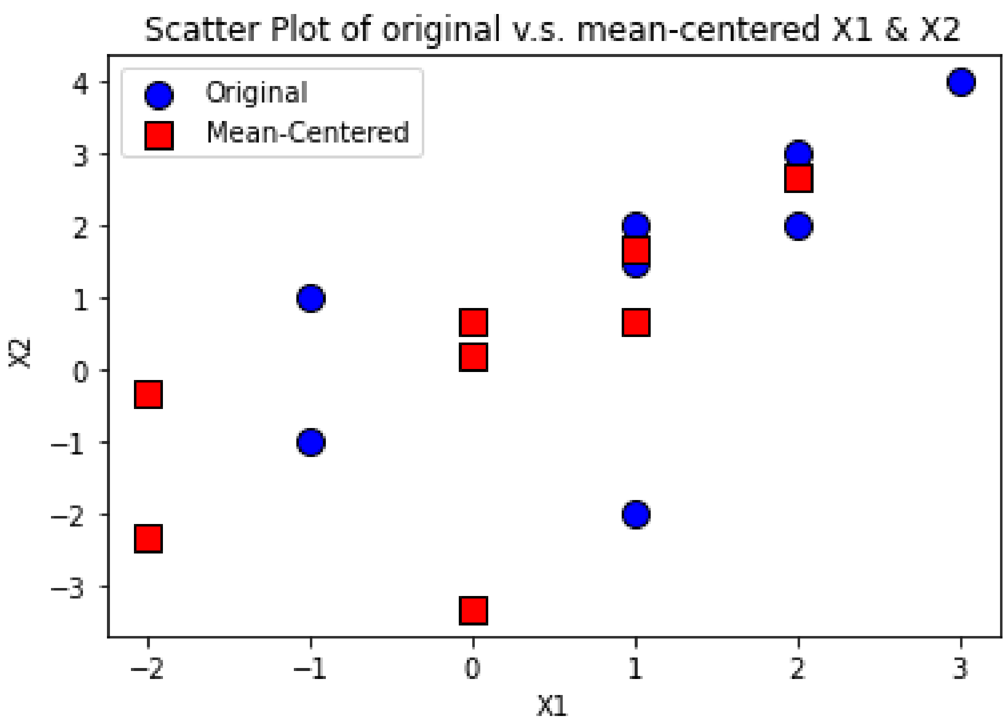
[25]:
```
meanCenteredData = np.ndarray(shape=D.shape)
for row_index, row in enumerate(D):
    for row_col_index, value in enumerate(row):
        meanCenteredData[row_index][row_col_index] = value - multiDimMean[row_col_index]
meanCenteredData
```

[25]:
```
array([[ 0.    ,  0.1875],
       [ 0.    ,  0.6875],
       [ 2.    ,  2.6875],
       [-2.    , -2.3125],
       [-2.    , -0.3125],
       [ 0.    , -3.3125],
       [ 1.    ,  0.6875],
       [ 1.    ,  1.6875]])
```

Plot of Original v.s. Mean-Centered Data

Use Python to create a scatter plot showing both the original data and the mean-centered data, where the x-axis is $X1$ and the y-axis is $X2$. Use different colors and markers to differentiate between the original and mean-centered data.

[26]:
```
X1_mean_centered = meanCenteredData[:,0]
X2_mean_centered = meanCenteredData[:,1]
plt.scatter(x=X1, y=X2, color='blue', marker='o', s=100, edgecolors='black')
plt.scatter(x=X1_mean_centered, y=X2_mean_centered, color='red', marker='s',␣
 ↪s=100, edgecolors='black')
plt.xlabel('X1')
plt.ylabel('X2')
plt.title('Scatter Plot of original v.s. mean-centered X1 & X2')
plt.legend(['Original', 'Mean-Centered'])
plt.show()
```



Covariance

Write down the covariance matrix of the data matrix D. Use estimated covariance.

[27]:
```
def covariance(v1, v2 = None):
    if v2 is None: v2 = v1
    # vector 1 mean
    v1_mean = v1.mean()
    # vector 2 mean
    v2_mean = v2.mean()
    # co_var (the covariance between v1 and v2)
    co_var = 0
    # loop through v1 and v2 values
    for i in range(v1.shape[0]):
        co_var += (v1[i] - v1_mean) * (v2[i] - v2_mean)
    # calculate and return the co-variance between v1 and v2
    return (co_var / (v1.shape[0] - 1))

def covarianceMatrix(m):
    # co-variance matrix
    covar_m = np.ndarray((m.shape[1], m.shape[1]))
    # loop through input matrix rows
    for i in range(m.shape[1]):
        # loop through input matrix rows (again)
        for j in range(m.shape[1]):
            # set x_ij covariance value
            covar_m[i, j] = covariance(m[:,i], m[:,j])
    # return the covariance matrix
    return covar_m
covarianceMatrix(D)
```

[27]:
```
array([[2.        , 1.85714286],
       [1.85714286, 3.92410714]])
```

Covariance Matrix of Z (Centered Mean)

Write down the covariance matrix of the centered data matrix Z. Use estimated covariance.

[28]:
```
covarianceMatrix(meanCenteredData)
```

[28]:
```
array([[2.        , 1.85714286],
       [1.85714286, 3.92410714]])
```

Covariance Matric of Standard Normalization

Write down the covariance matrix of the data after applying standard normalization.

[29]:
```
def zScoreNormalize(m):
    # create normlized matrix based on shape of input matrix
    z_score = np.ndarray(m.shape)
    # loop through input matrix rows
    for row_index in range(m.shape[0]):
        # loop through input matrix columns
        for col_index in range(m.shape[1]):
            # get current column array
            col_arr = m[:,col_index]
            # calculate the standard devieation for the current column
            col_std_div = (covariance(col_arr)) ** (1/2)
            # calculate the column's mean
            col_mean = col_arr.mean()
            # get the x_ij value from the imput matix
            x_ij = m[row_index, col_index]
            # calculate the x_ji z-score
            x_ij_zscore = (x_ij - col_mean) / col_std_div
            # set x_ij normalized value in normalized matrix
            z_score[row_index, col_index] = x_ij_zscore
    # return the normalized array
    return z_score
covarianceMatrix(zScoreNormalize(D))
```

[29]:
```
array([[1.        , 0.66291811],
       [0.66291811, 1.        ]])
```