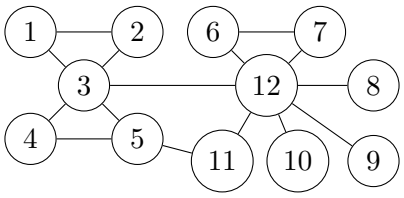
River Kelly
CSCI 347
Homework 03

Show your work. Include any code snippets you used to generate an answer, using comments in the code to clearly indicate which problem corresponds to which code

Consider the following graph:



1. (3 points) Without using networkx or other graph analysis packages (though you may use them to check your answer), find the closeness centrality of vertices 3 and 12.

Notes

Closeness Centrality of x_i:

$$cc(x_i) = \frac{1}{\sum_{j=1}^{n} d(x_i, x_j)}$$

Where $d(x_i, x_j)$ is the shortest path between x_i and x_j.

The closeness centrality of vertices 3:

$$\begin{aligned} cc(x_3) &= \frac{1}{\sum_{j=1}^{n} d(x_3, x_j)} \\ &= \frac{1}{1+1+0+1+1+2+2+2+2+2+2+1} \\ &= \frac{1}{17} \\ &= 0.05882.... \end{aligned}$$

The closeness centrality of vertices 12:

$$\begin{aligned} cc(x_{12}) &= \frac{1}{\sum_{j=1}^{n} d(x_{12}, x_j)} \\ &= \frac{1}{2+2+1+2+2+1+1+1+1+1+1+0} \\ &= \frac{1}{15} \\ &= 0.06666.... \end{aligned}$$

Problem 1 Answer - Closeness Centrality

Note: This solution was done by hand.

The closeness centrality of vertex 3: $\frac{1}{17}$ or 0.059
The closeness centrality of vertex 12: $\frac{1}{15}$ or 0.067

2. (3 points) Without using networkx or other graph analysis packages (though you may use them to check your answer), find the eccentricity of vertices 3, 12, and 11.

Notes

Eccentricity of x_i is:

$$e(x_i) = \max_i \{d(x_i, x_j)\}$$

$$e(x_3) = 2$$
$$e(x_{12}) = 2$$
$$e(x_{11}) = 3$$

Problem 2 Answer - Eccentricity

Note: This solution was done by hand.

The eccentricity of vertex 3: 2
The eccentricity of vertex 12: 2
The eccentricity of vertex 11: 3

3. (3 points) Without using networkx or other graph analysis packages (though you may use them to check your answer), find the clustering coefficient of vertex 3.

Notes

Let $G_i = (V_i, E_i)$ be the subgraph included by the neighbors of node x_i.
Let $n_i =\mid V_i \mid$ and $m_i =\mid VE_i \mid$,
Clustering Coefficient of x_i:

$$\frac{m_i}{\binom{n_i}{2}} = \frac{\#\text{ of edges amoung neighbors of } x_i}{\#\text{ of possible edges amoung neighbors of } x_i}$$

$$\begin{aligned}\frac{m_3}{\binom{n_3}{2}} &= \frac{2}{10} \\ &= \frac{1}{5} \\ &= 0.2\end{aligned}$$

Problem 3 Answer - Clustering Coefficient of x_3

Note: This solution was done by hand.

The clustering coefficient of vertex 3 is $\frac{1}{5}$ or 0.2

4. (3 points) Without using networkx or other graph analysis packages (though you may use them to check your answer), find the clustering coefficient of the graph.

x_i	$\frac{m_i}{\binom{n_i}{2}}$
x_1	1
x_2	1
x_3	$\frac{1}{5}$
x_4	1
x_5	$\frac{1}{3}$
x_6	1
x_7	1
x_8	0
x_9	0
x_{10}	0
x_{11}	0
x_{12}	$\frac{1}{21}$

Clustering Coefficient of the graph:

$$\begin{aligned} C(G) &= \frac{1}{n}\sum_{i=1}^{n} \text{clust coeff } x_i \\ &= \frac{1}{12}(1+1+\frac{1}{5}+1+\frac{1}{3}+1+1+0+0+0+0+\frac{1}{21}) \\ &= \frac{1}{12}\times\frac{586}{105} \\ &= \frac{293}{630} \\ &= 0.46507\ldots \end{aligned}$$

Problem 4 Answer - Clustering Coefficient of graph G

The clustering coefficient of graph G 3 is $\frac{293}{630}$ or 0.46508

5. (3 points) Find the betweenness centrality of vertices 3 and 12. You may use networkx or other graph analysis packages, but include the code used to generate your answer in your submission.

```
# libraries
import networkx as nx
import matplotlib.pyplot as plt

# Init Graph
G = nx.Graph()

# Add edges to graph
G.add_edge(1, 2)
G.add_edge(1, 3)
G.add_edge(2, 3)
G.add_edge(3, 4)
G.add_edge(3, 5)
G.add_edge(3, 12)
G.add_edge(4, 5)
G.add_edge(5, 11)
G.add_edge(6, 7)
G.add_edge(6, 12)
G.add_edge(7, 12)
G.add_edge(8, 12)
G.add_edge(9, 12)
G.add_edge(10, 12)
G.add_edge(11, 12)
```

Betweenness Centrality:

```
# using definition in the slides
nx.betweenness_centrality(G, normalized=False)
```

Output:

```
{
    1: 0.0,
    2: 0.0,
    3: 27.0,
    4: 0.0,
    5: 2.5,
    12: 40.5,
    11: 3.0,
    6: 0.0,
    7: 0.0,
    8: 0.0,
    9: 0.0,
    10: 0.0
}
```

Problem 5 Answer - Betweenness

The betweenness centrality of vertices 3 and 12 is 27 and 40.5, respectively.

6. (3 points) Using networkx, find the prestige centrality of vertices 3 and 12. Include the code used to generate the answer. (Note that networkx calls the prestige centrality "eigenvector centrality")

```
# prestige centrality
nx.eigenvector_centrality(G)
```

Output:

```
{
    1: 0.21086454283237194,
    2: 0.21086454283237194,
    3: 0.46528704789070097,
    4: 0.23879994540475255,
    5: 0.3004407253859518,
    12: 0.5310014523926223,
    11: 0.25929496322149226,
    6: 0.24064930403878465,
    7: 0.24064930403878465,
    8: 0.1655995303513709,
    9: 0.1655995303513709,
    10: 0.1655995303513709
}
```

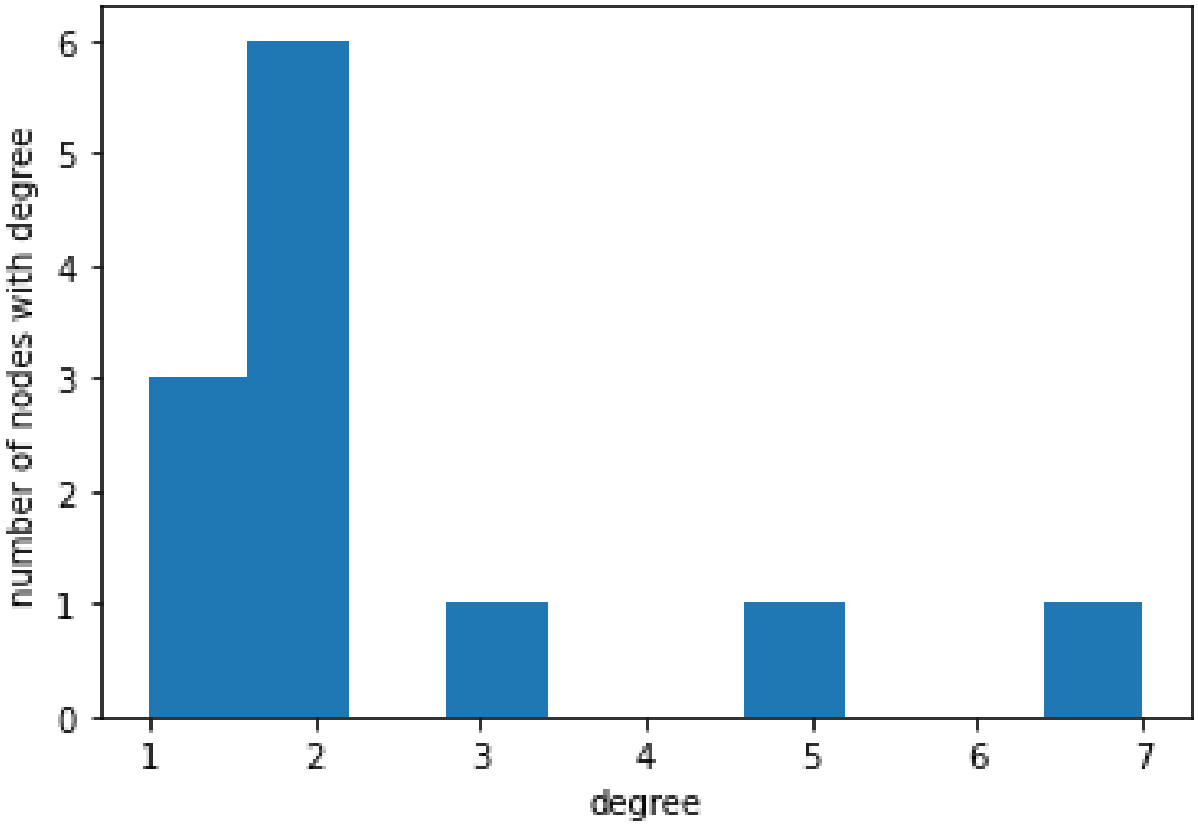
Problem 6 Answer - Eigenvector Centrality

The prestige centrality of vertices 3 and 12 is 0.4653 and 0.5310, respectively.

7. (3 points) Use Python to create a plot for the degree distribution of this graph. Include the code used to generate the plot as well as the plot in your submission.

```
deg_view = nx.degree(G)
deg_vals = dict(deg_view).values()

plt.hist(deg_vals)
plt.xlabel('degree')
plt.ylabel('number of nodes with degree')
```



Acknowledgements: Homework problems adapted from assignments of Veronika Strnadova-Neeley.