Definition: Derivative

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The **derivative** of a function at a point measures the instantaneous rate of change of the function at that point. It is defined as a limit of difference quotients.

Definition

Let $f : (a, b) \to \mathbb{R}$ and let $c \in (a, b)$. The derivative of f at c, denoted $f'(c)$, is defined as:

$$f'(c) = \lim_{h \to 0} \frac{f(c + h) - f(c)}{h}$$

provided this limit exists.

Alternative Formulation

Equivalently, using a different variable:

$$f'(c) = \lim_{x \to c} \frac{f(x) - f(c)}{x - c}$$

Differentiability

A function f is **differentiable at** c if $f'(c)$ exists. A function is **differentiable on an interval** if it is differentiable at every point in the interval.

Notation

Various notations for the derivative include: - $f'(x)$ (Lagrange notation) - $\frac{df}{dx}$ or $\frac{d}{dx}f(x)$ (Leibniz notation) - $Df(x)$ or $D_x f$ (Operator notation) - $\dot{f}$ (Newton notation, typically for time derivatives)

Geometric Interpretation

The derivative $f'(c)$ represents: 1. The slope of the tangent line to the graph of f at the point $(c, f(c))$ 2. The instantaneous rate of change of f at $x = c$

Properties

If f is differentiable at c, then: 1. f is continuous at c 2. The tangent line at $(c, f(c))$ has equation: $y - f(c) = f'(c)(x - c)$

Examples

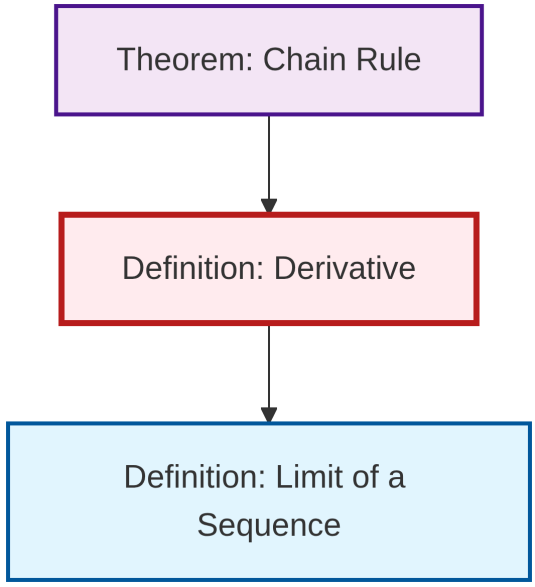
1. **Constant function**: If $f(x) = k$, then $f'(x) = 0$
2. **Power function**: If $f(x) = x^n$ (n), then $f'(x) = nx^{n-1}$
3. **Exponential**: If $f(x) = e^x$, then $f'(x) = e^x$
4. **Sine**: If $f(x) = \sin(x)$, then $f'(x) = \cos(x)$

Mermaid Diagram

```
graph TD
    A[Derivative f'(c)] --> B[Limit of Difference Quotient]
    B --> C[lim (f(c+h) - f(c))/h]
    A --> D[Geometric Meaning]
    D --> E[Slope of Tangent Line]
    D --> F[Rate of Change]
    A --> G[Properties]
    G --> H[Differentiable   Continuous]

    style A fill:#f9f,stroke:#333,stroke-width:2px
    style B fill:#bbf,stroke:#333,stroke-width:2px
    style C fill:#bfb,stroke:#333,stroke-width:2px
    style D fill:#bbf,stroke:#333,stroke-width:2px
```

Dependency Graph



Local dependency graph