Definition: Random Variable

Random Variable

A **random variable** is a measurable function from a Probability Space to a measurable space.

Formal Definition

Given a probability space $(\Omega, \mathcal{F}, P)$, a random variable is a function $X : \Omega \to \mathbb{R}$ such that for every Borel set $B \subseteq \mathbb{R}$:

$$X^{-1}(B) = \{\omega \in \Omega : X(\omega) \in B\} \in \mathcal{F}$$

This condition ensures that we can assign probabilities to events of the form $\{X \in B\}$.

Types of Random Variables

1. **Discrete Random Variable**: Takes countably many values
   - Characterized by probability mass function (PMF): $p_X(x) = P(X = x)$
2. **Continuous Random Variable**: Takes uncountably many values
   - Characterized by probability density function (PDF): $f_X(x)$ where $P(a \leq X \leq b) = \int_a^b f_X(x)dx$

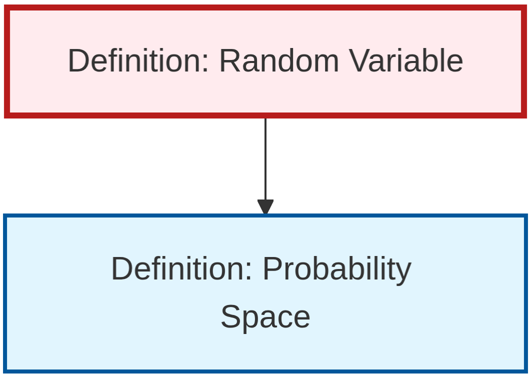
Properties

- **Cumulative Distribution Function (CDF)**: $F_X(x) = P(X \leq x)$
- **Expected Value**: $E[X] = \int_\Omega X(\omega)dP(\omega)$
- **Variance**: $\text{Var}(X) = E[(X - E[X])^2]$

Examples

- Bernoulli: $X \in \{0, 1\}$ with $P(X = 1) = p$
- Normal: $X \sim N(\mu, \sigma^2)$
- Poisson: $P(X = k) = \frac{\lambda^k e^{-\lambda}}{k!}$

Dependency Graph



Local dependency graph