Definition: Binomial Coefficient

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The **binomial coefficient** $\binom{n}{k}$ (read "n choose k") counts the number of ways to choose k objects from n objects without regard to order.

Formal Definition

For non-negative integers n and k with $k \leq n$, the binomial coefficient is defined as:

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

where $n!$ denotes the factorial of n.

Alternative Formulations

1. **In terms of Combination**: $\binom{n}{k} = C(n,k)$ - the number of k-combinations from n elements
2. **In terms of Permutation**: $\binom{n}{k} = \frac{P(n,k)}{k!}$ where $P(n,k)$ is the number of k-permutations
3. **Recursive definition**:
$$\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$$
with base cases $\binom{n}{0} = \binom{n}{n} = 1$

Special Values

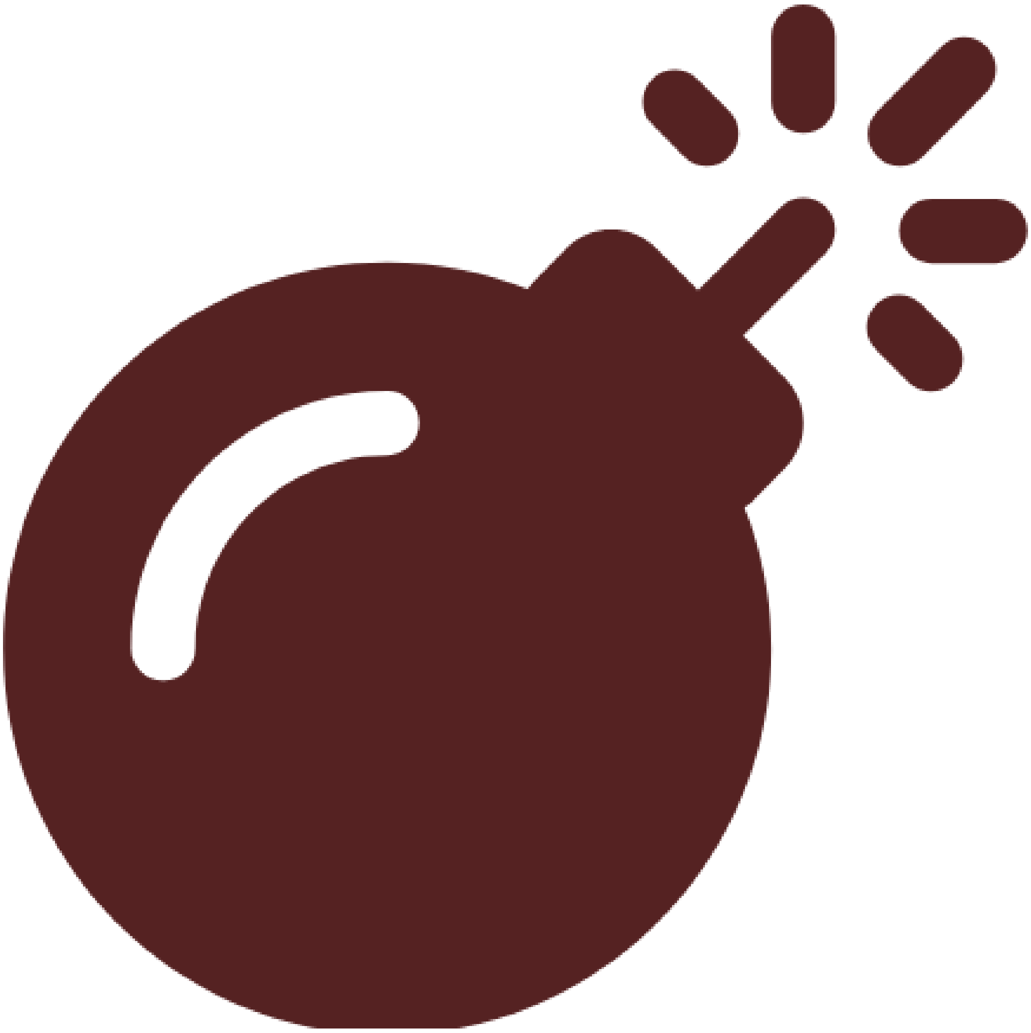
- $\binom{n}{0} = 1$ (one way to choose nothing)
- $\binom{n}{1} = n$ (n ways to choose one item)
- $\binom{n}{n} = 1$ (one way to choose everything)
- $\binom{n}{k} = 0$ if $k > n$ (cannot choose more items than available)

Properties

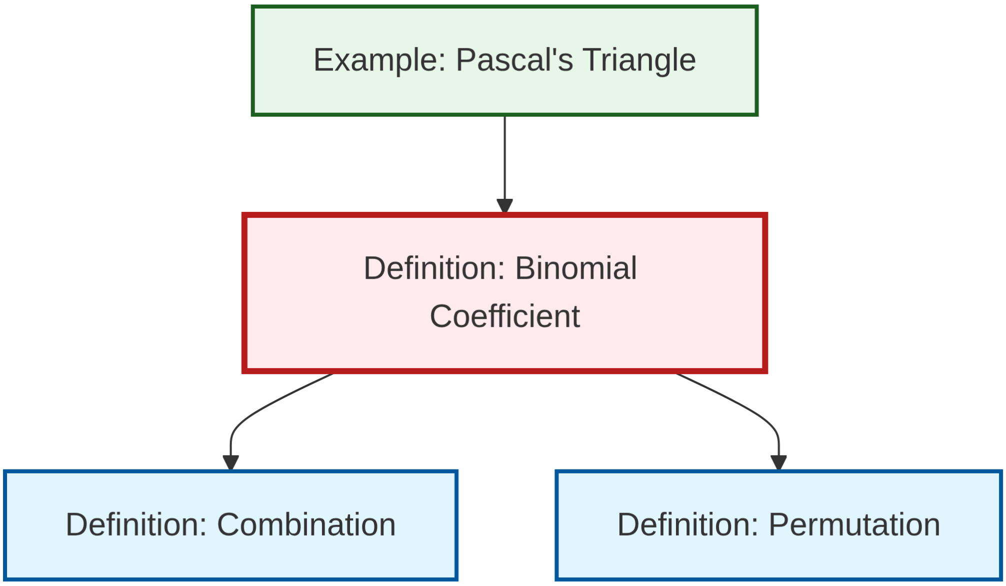
1. **Symmetry**: $\binom{n}{k} = \binom{n}{n-k}$
2. **Pascal's Identity**: $\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$
3. **Binomial Theorem**: $(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^k y^{n-k}$

Applications

- Counting subsets of a given size
- Expansion of binomial expressions
- Probability calculations in discrete distributions
- Pascal's triangle construction



Dependency Graph



Local dependency graph