

# Definition: Sample Space

A **sample space** is the [Set](#) of all possible outcomes of a random experiment or probabilistic scenario. It is typically denoted by  $\Omega$  (omega) or  $S$ .

## Formal Definition

A sample space  $\Omega$  is a non-empty set such that: 1. It contains all possible outcomes of the experiment 2. The outcomes are mutually exclusive (only one can occur) 3. The outcomes are collectively exhaustive (one must occur)

## Types of Sample Spaces

### Discrete Sample Spaces

- **Finite:** Contains a finite number of outcomes
  - Example: Rolling a die:  $\Omega = \{1, 2, 3, 4, 5, 6\}$
- **Countably Infinite:** Contains countably many outcomes
  - Example: Number of coin flips until first heads:  $\Omega = \{1, 2, 3, \dots\}$

### Continuous Sample Spaces

- Contains uncountably many outcomes
- Example: Measuring reaction time:  $\Omega = [0, \infty)$

## Examples

1. **Coin Flip:**  $\Omega = \{H, T\}$  (heads, tails)
2. **Two Dice:**  $\Omega = \{(i, j) : i, j \in \{1, 2, 3, 4, 5, 6\}\}$  (36 outcomes)
3. **Random Point in Unit Square:**  $\Omega = [0, 1] \times [0, 1]$
4. **Lifetime of a Lightbulb:**  $\Omega = [0, \infty)$

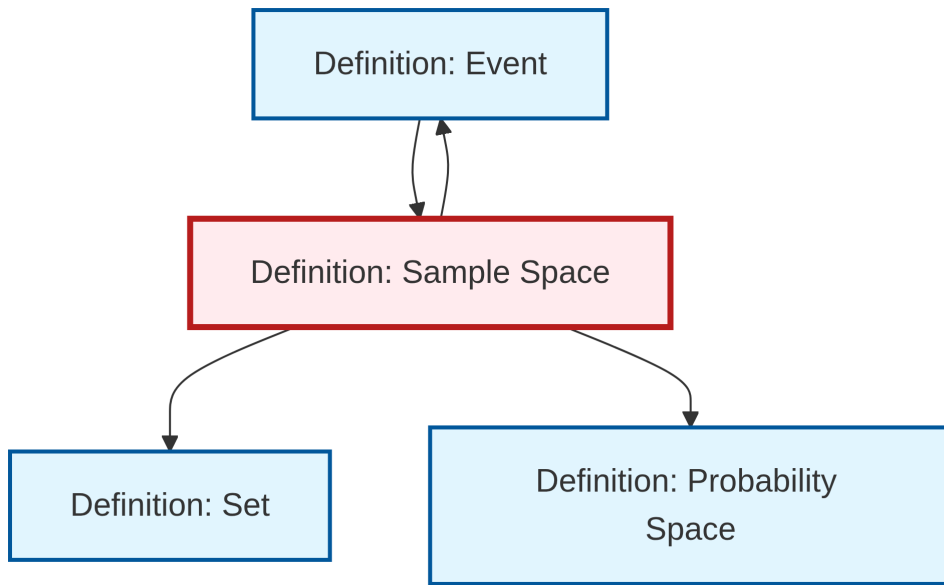
## Relationship to Probability

- Every element  $\omega \in \Omega$  is called an **elementary outcome**
- Subsets of  $\Omega$  are called **events** (see [Event](#))
- A [Probability Space](#) is built upon a sample space
- The entire sample space  $\Omega$  has probability 1:  $P(\Omega) = 1$

## Properties

- The empty set  $\emptyset$  represents an impossible event
- The sample space  $\Omega$  represents the certain event
- Sample spaces must be defined carefully to capture all relevant outcomes
- The choice of sample space affects how probabilities are calculated

## Dependency Graph



Local dependency graph