Example: Even Integers Form a Subgroup

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The set of even integers $2\mathbb{Z} = \{..., -4, -2, 0, 2, 4, ...\}$ forms a Subgroup of the Integers under Addition.

Verification

To show that $2\mathbb{Z}$ is a subgroup of $(\mathbb{Z}, +)$, we need to verify:

1. **Non-empty**: $0 \in 2\mathbb{Z}$ since $0 = 2 \cdot 0$.
2. **Closure**: If $a, b \in 2\mathbb{Z}$, then $a = 2m$ and $b = 2n$ for some $m, n \in \mathbb{Z}$. Thus:

$$a + b = 2m + 2n = 2(m + n) \in 2\mathbb{Z}$$

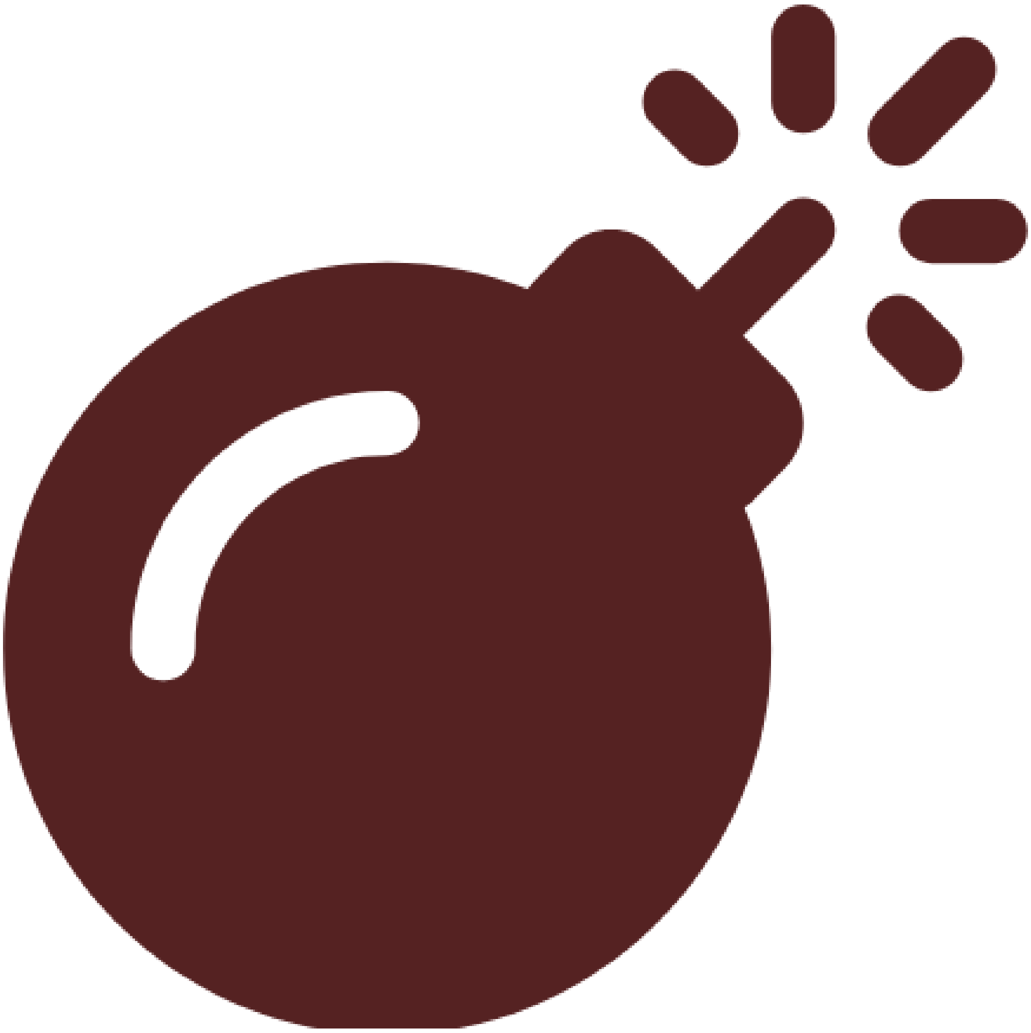
3. **Inverses**: If $a \in 2\mathbb{Z}$, then $a = 2m$ for some $m \in \mathbb{Z}$. The inverse is:

$$-a = -(2m) = 2(-m) \in 2\mathbb{Z}$$

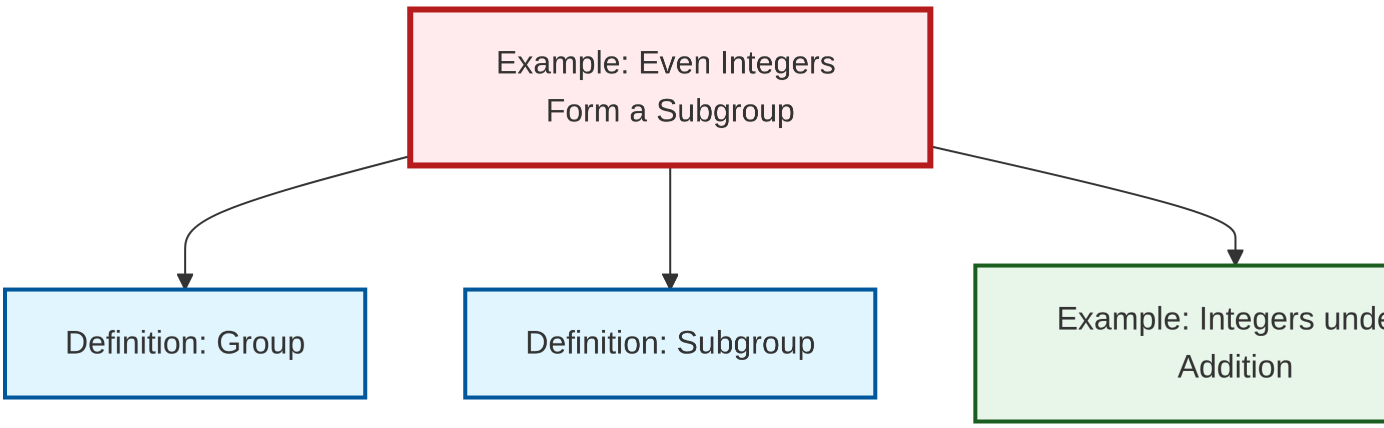
Therefore, $2\mathbb{Z}$ is a subgroup of $\mathbb{Z}$ under addition.

Properties

- This is a proper subgroup since $1 \in \mathbb{Z}$ but $1 \notin 2\mathbb{Z}$.
- This subgroup has index 2 in $\mathbb{Z}$, meaning there are exactly 2 cosets.
- The cosets are $2\mathbb{Z}$ (even integers) and $1 + 2\mathbb{Z}$ (odd integers).



Dependency Graph



Local dependency graph