Definition: Angle

An **angle** is the figure formed by two rays (half-lines) that share a common endpoint. The common endpoint is called the **vertex** of the angle, and the two rays are called the **sides** of the angle.

Formal Definition

An angle can be defined as:

1. **Geometric Definition**: Given three Points A, B, C where $A \neq B$ and $C \neq B$, the angle $\angle ABC$ is the union of rays $\overrightarrow{BA}$ and $\overrightarrow{BC}$.
2. **Measure Definition**: An angle is associated with a real number called its **measure**, typically expressed in:
   - **Degrees**: A full rotation is $360°$
   - **Radians**: A full rotation is 2π radians
   - **Gradians**: A full rotation is 400 gradians
3. **Vector Definition**: The angle θ between two non-zero vectors $\vec{u}$ and $\vec{v}$ is defined by:

$$\cos\theta = \frac{\vec{u} \cdot \vec{v}}{|\vec{u}||\vec{v}|}$$

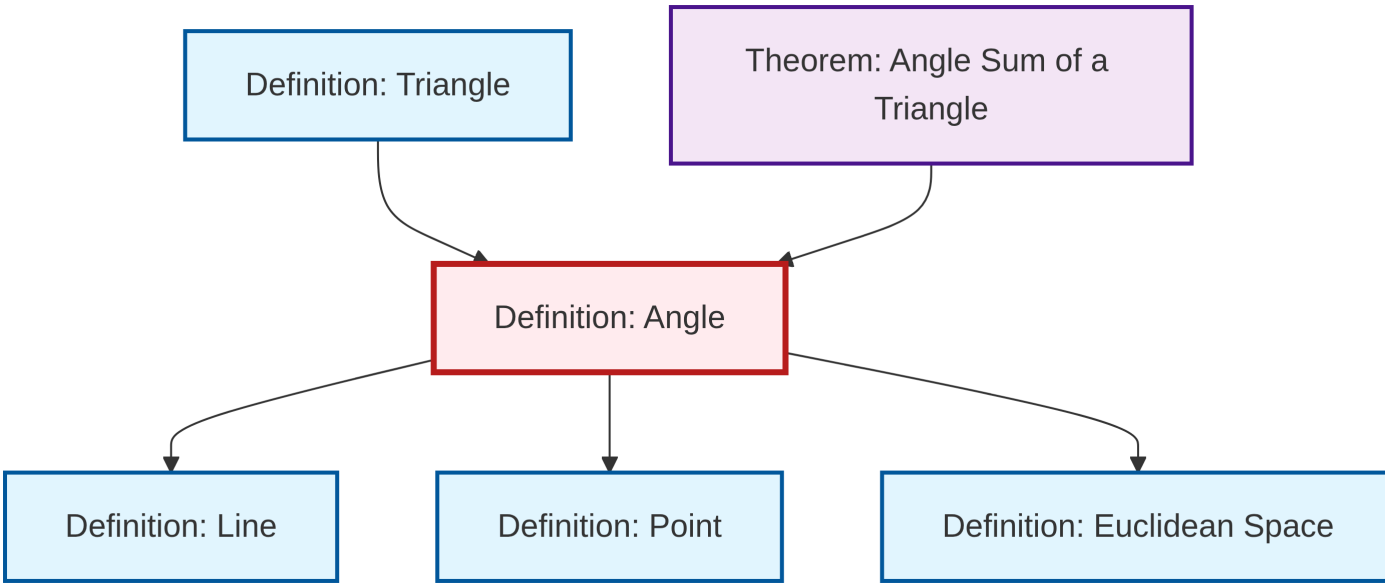
Classification

Angles are classified by their measure: - **Acute angle**: $0° < \theta < 90°$ - **Right angle**: $\theta = 90°$ - **Obtuse angle**: $90° < \theta < 180°$ - **Straight angle**: $\theta = 180°$ - **Reflex angle**: $180° < \theta < 360°$

Properties

- The angle measure is invariant under rotation and translation
- In Euclidean Space, angles satisfy the angle addition postulate
- Two angles are **congruent** if they have the same measure
- **Complementary angles** sum to $90°$
- **Supplementary angles** sum to $180°$

Dependency Graph



Local dependency graph