Example: Finite Field

Example: The Finite Field $\mathbb{F}_5$

The integers modulo 5 form a Field, denoted $\mathbb{F}_5$ or $\mathbb{Z}/5\mathbb{Z}$.

Elements and Operations

The field $\mathbb{F}_5 = \{0, 1, 2, 3, 4\}$ with operations:

Addition Table (mod 5)

```
+   | 0  1  2  3  4
---|---------------
0   | 0  1  2  3  4
1   | 1  2  3  4  0
2   | 2  3  4  0  1
3   | 3  4  0  1  2
4   | 4  0  1  2  3
```

Multiplication Table (mod 5)

```
×   | 0  1  2  3  4
---|---------------
0   | 0  0  0  0  0
1   | 0  1  2  3  4
2   | 0  2  4  1  3
3   | 0  3  1  4  2
4   | 0  4  3  2  1
```

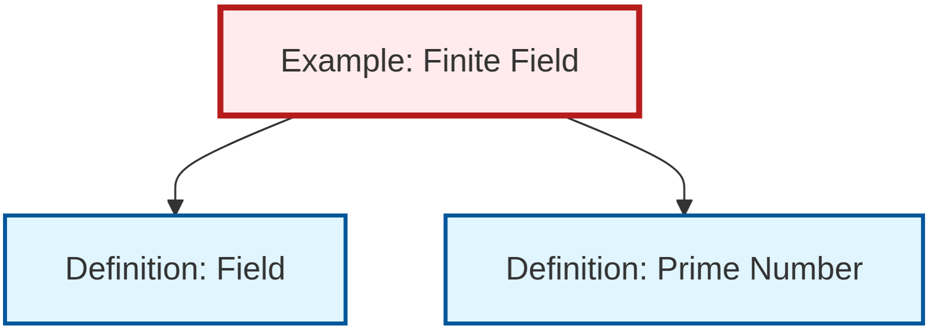
Verification

- **Additive identity**: 0
- **Multiplicative identity**: 1
- **Additive inverses**: $-1 = 4$, $-2 = 3$, $-3 = 2$, $-4 = 1$
- **Multiplicative inverses**: $1^{-1} = 1$, $2^{-1} = 3$, $3^{-1} = 2$, $4^{-1} = 4$

General Result

For any Prime Number p, the set $\mathbb{F}_p = \{0, 1, ..., p - 1\}$ with arithmetic modulo p forms a field. This works because: - Every non-zero element has a multiplicative inverse (by Fermat's Little Theorem) - There are no zero divisors when p is prime

Dependency Graph



Local dependency graph