

## Definition: Expected Value

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The **expected value** (or **expectation**) of a **random variable** is a measure of the central tendency of its distribution, representing the average value the random variable takes over many trials.

#### Discrete Case

For a discrete random variable  $X$  with probability mass function  $p(x)$ :

$$E[X] = \sum_x x \cdot p(x)$$

where the sum is taken over all possible values of  $X$ .

#### Continuous Case

For a continuous random variable  $X$  with probability density function  $f(x)$ :

$$E[X] = \int_{-\infty}^{\infty} x \cdot f(x) dx$$

#### General Definition

For a random variable  $X$  on a **probability space**  $(\Omega, \mathcal{F}, P)$ :

$$E[X] = \int_{\Omega} X(\omega) dP(\omega)$$

This is the Lebesgue integral of  $X$  with respect to the probability measure  $P$ .

#### Properties

1. **Linearity:**  $E[aX + bY] = aE[X] + bE[Y]$  for constants  $a, b$
2. **Monotonicity:** If  $X \leq Y$  (almost surely), then  $E[X] \leq E[Y]$
3. **Constant:**  $E[c] = c$  for any constant  $c$
4. **Non-negativity:** If  $X \geq 0$  (almost surely), then  $E[X] \geq 0$

#### Existence

The expected value exists if and only if  $E[|X|] < \infty$ , i.e., when:

$$\int_{\Omega} |X(\omega)| dP(\omega) < \infty$$

## Examples

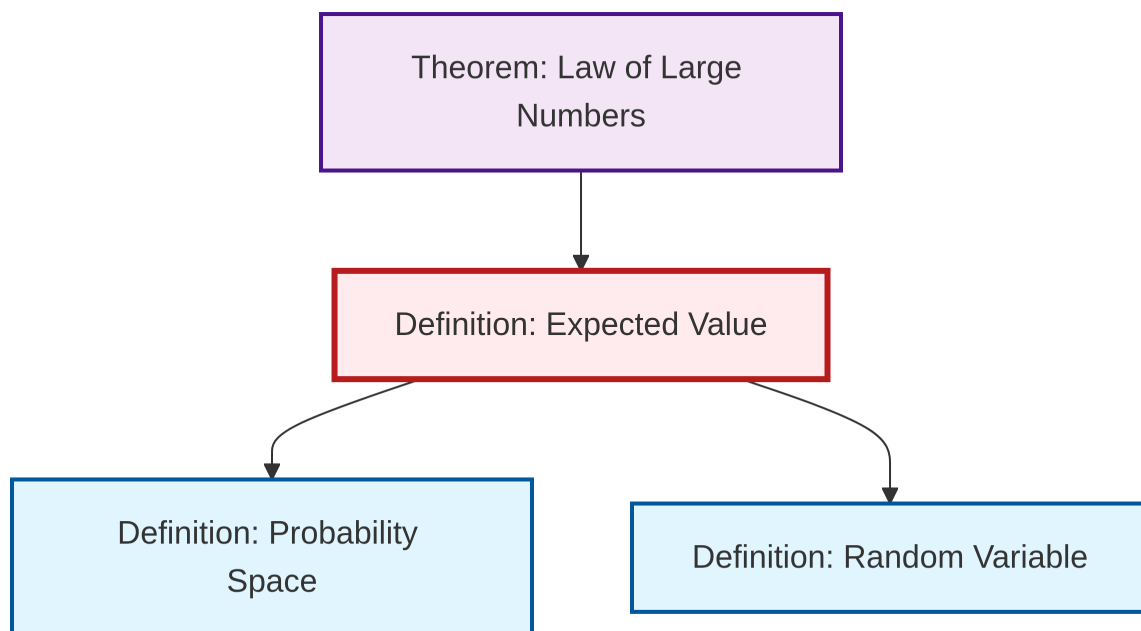
1. **Bernoulli:** If  $X \sim \text{Bernoulli}(p)$ , then  $E[X] = p$
2. **Binomial:** If  $X \sim \text{Binomial}(n, p)$ , then  $E[X] = np$
3. **Normal:** If  $X \sim \mathcal{N}(\mu, \sigma^2)$ , then  $E[X] = \mu$
4. **Exponential:** If  $X \sim \text{Exp}(\lambda)$ , then  $E[X] = 1/\lambda$

## Mermaid Diagram

```
graph TD
    A[Expected Value] --> B[Discrete:  $\sum x \cdot p(x)$ ]
    A --> C[Continuous:  $\int x \cdot f(x) dx$ ]
    A --> D[General:  $\int x dP$ ]
    A --> E[Properties]
    E --> F[Linearity]
    E --> G[Monotonicity]
    A --> H[Existence:  $E[|X|] < \infty$ ]

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    style B fill:#bbf,stroke:#333,stroke-width:2px
    style C fill:#bbf,stroke:#333,stroke-width:2px
    style D fill:#bbf,stroke:#333,stroke-width:2px
```

## Dependency Graph



Local dependency graph