

Definition: Combination

Combination

A **combination** is a selection of items from a [Set](#) where the order does not matter.

Formal Definition

Let S be a set with n elements. A k -combination of S is a subset of S with exactly k elements, where $0 \leq k \leq n$.

Notation and Formula

The number of k -combinations from a set of n elements is denoted: - $\binom{n}{k}$ (binomial coefficient)
- $C(n, k)$ or ${}_nC_k$ - C_n^k

The formula is:

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

Properties

1. **Symmetry:** $\binom{n}{k} = \binom{n}{n-k}$
2. **Pascal's identity:** $\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$
3. **Boundary conditions:** $\binom{n}{0} = \binom{n}{n} = 1$
4. **Sum:** $\sum_{k=0}^n \binom{n}{k} = 2^n$

Relationship to Permutations

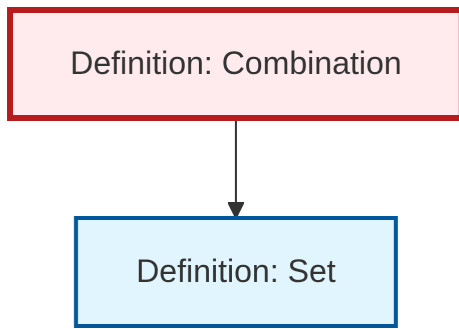
The number of k -permutations divided by $k!$ gives the number of k -combinations:

$$\binom{n}{k} = \frac{P(n, k)}{k!}$$

Applications

- Counting subsets of a given size
- Binomial theorem coefficients
- Probability calculations
- Combinatorial optimization

Dependency Graph



Local dependency graph