Definition: Power Set

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The **power set** of a set A, denoted $\mathcal{P}(A)$ or 2^A, is the set of all subsets of A.

Formal Definition

$$\mathcal{P}(A) = \{B : B \subseteq A\}$$

In other words, $B \in \mathcal{P}(A)$ if and only if every element of B is also an element of A.

Properties

1. **Always contains the empty set**: $\emptyset \in \mathcal{P}(A)$ for any set A
2. **Always contains the original set**: $A \in \mathcal{P}(A)$
3. **Cardinality**: If $|A| = n$ (finite), then $|\mathcal{P}(A)| = 2^n$
4. **Ordering**: $\mathcal{P}(A)$ forms a partially ordered set under inclusion

Examples

1. If $A = \emptyset$, then $\mathcal{P}(A) = \{\emptyset\}$
2. If $A = \{1\}$, then $\mathcal{P}(A) = \{\emptyset, \{1\}\}$
3. If $A = \{1, 2\}$, then $\mathcal{P}(A) = \{\emptyset, \{1\}, \{2\}, \{1, 2\}\}$
4. If $A = \{a, b, c\}$, then:

$$\mathcal{P}(A) = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}\}$$

Set-Theoretic Properties

- $A \subseteq B$ if and only if $\mathcal{P}(A) \subseteq \mathcal{P}(B)$
- $\mathcal{P}(A \cap B) = \mathcal{P}(A) \cap \mathcal{P}(B)$
- $\mathcal{P}(A) \cup \mathcal{P}(B) \subseteq \mathcal{P}(A \cup B)$ (but not always equal)

Mermaid Diagram

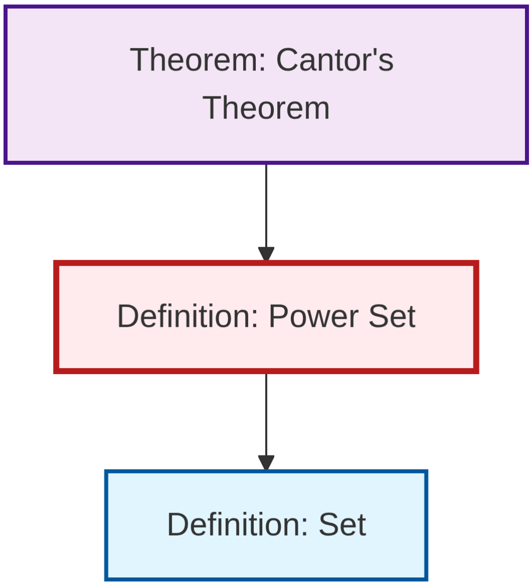
```
graph TD
    A[Power Set P(A)] --> B[Contains  ]
    A --> C[Contains A]
    A --> D[All Subsets of A]
    D --> E[Cardinality: 2^|A|]
    D --> F[Partial Order by  ]

    A --> G[Example: A = {1,2}]
```

```
    G --> H[P(A) = { , {1}, {2}, {1,2}}]

    style A fill:#f9f,stroke:#333,stroke-width:2px
    style D fill:#bbf,stroke:#333,stroke-width:2px
    style E fill:#bfb,stroke:#333,stroke-width:2px
```

Dependency Graph



Local dependency graph