

## Definition: Group Homomorphism

### Group Homomorphism

A **group homomorphism** is a function between two [Group](#) structures that preserves the group [Binary Operation](#).

#### Formal Definition

Let  $(G, \cdot)$  and  $(H, *)$  be groups. A function  $f : G \rightarrow H$  is a group homomorphism if:

$$f(a \cdot b) = f(a) * f(b)$$

for all  $a, b \in G$ .

#### Properties

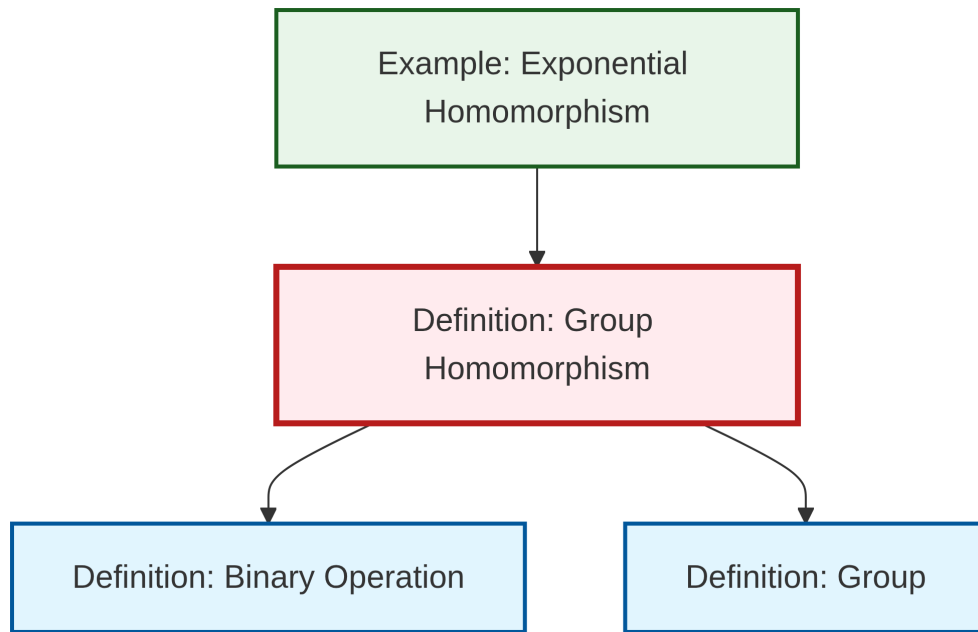
For any group homomorphism  $f : G \rightarrow H$ :

1. **Identity preservation:**  $f(e_G) = e_H$  where  $e_G$  and  $e_H$  are the identity elements
2. **Inverse preservation:**  $f(a^{-1}) = f(a)^{-1}$  for all  $a \in G$
3. **Subgroup preservation:** If  $K \leq G$ , then  $f(K) \leq H$

#### Special Types

- **Monomorphism:** An injective homomorphism
- **Epimorphism:** A surjective homomorphism
- **Isomorphism:** A bijective homomorphism
- **Endomorphism:** A homomorphism from a group to itself
- **Automorphism:** A bijective endomorphism

## Dependency Graph



Local dependency graph