Example: Exponential Homomorphism

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The exponential function provides a classic example of a Group Homomorphism between Group structures.

The Homomorphism

Consider the function:

$$\exp : (\mathbb{R}, +) \to (\mathbb{R}^+, \cdot)$$
$$\exp(x) = e^x$$

where $(\mathbb{R}, +)$ is the additive group of real numbers and $(\mathbb{R}^+, \cdot)$ is the multiplicative group of positive real numbers.

Verification

To verify this is a homomorphism, we check:

$$\exp(x + y) = e^{x+y} = e^x \cdot e^y = \exp(x) \cdot \exp(y)$$

Properties

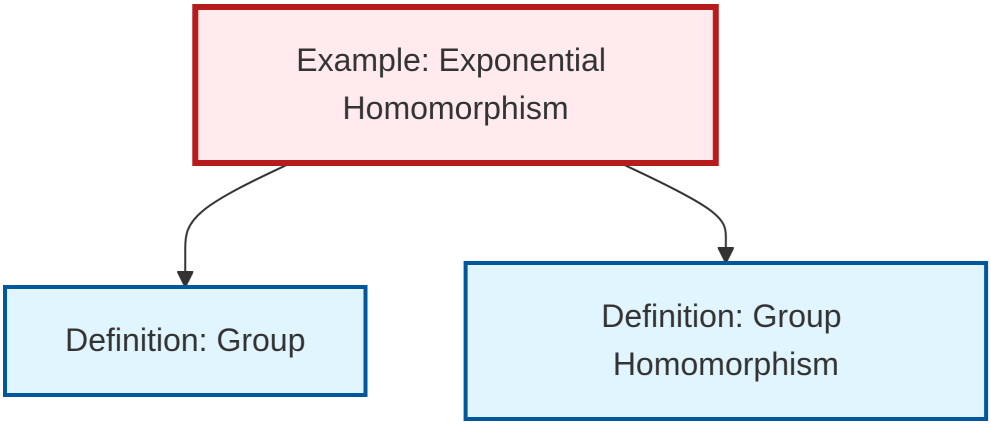
- **Injective**: Yes, since $e^x = e^y$ implies $x = y$
- **Surjective**: Yes, for any $a \in \mathbb{R}^+$, we have $\exp(\ln a) = a$
- **Bijective**: Therefore, exp is an isomorphism

Inverse

The natural logarithm $\ln : (\mathbb{R}^+, \cdot) \to (\mathbb{R}, +)$ is the inverse isomorphism:

$$\ln(xy) = \ln(x) + \ln(y)$$

Dependency Graph



Local dependency graph