

Definition: Morphism

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A **morphism** (also called an arrow) is a fundamental concept in **Category** theory representing a structure-preserving map between objects.

Formal Definition

In a **Category** \mathcal{C} , a morphism $f : A \rightarrow B$ consists of: - A **domain** (or source) object A - A **codomain** (or target) object B - An abstract “arrow” f from A to B

Key Properties

For morphisms in a category, we have:

1. **Composition:** If $f : A \rightarrow B$ and $g : B \rightarrow C$, then there exists a composite morphism $g \circ f : A \rightarrow C$
2. **Identity:** For each object A , there exists an identity morphism $\text{id}_A : A \rightarrow A$
3. **Associativity:** $(h \circ g) \circ f = h \circ (g \circ f)$ whenever the compositions are defined
4. **Identity laws:** $f \circ \text{id}_A = f$ and $\text{id}_B \circ f = f$ for $f : A \rightarrow B$

Notation

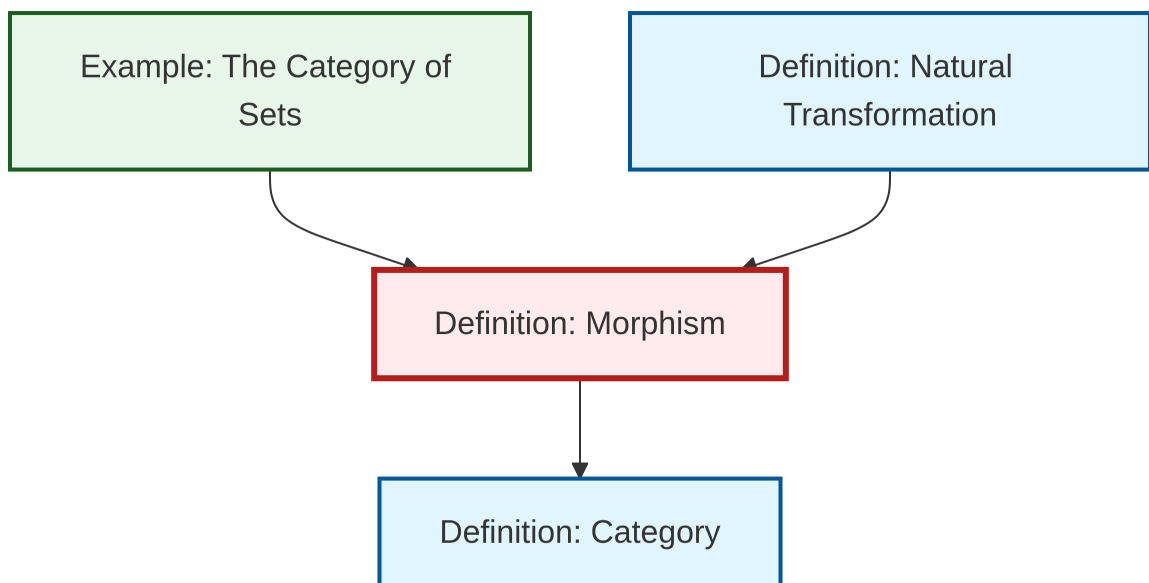
The collection of all morphisms from A to B in category \mathcal{C} is denoted: - $\text{Hom}_{\mathcal{C}}(A, B)$ or - $\mathcal{C}(A, B)$ or - $\text{Mor}_{\mathcal{C}}(A, B)$

Types of Morphisms

Special types of morphisms include: - **Monomorphism:** A categorical generalization of injective functions - **Epimorphism:** A categorical generalization of surjective functions - **Isomorphism:** A morphism with a two-sided inverse



Dependency Graph



Local dependency graph