Example: Integers Form a Ring

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The set of integers $\mathbb{Z}$ with usual addition and multiplication forms a Ring, denoted $(\mathbb{Z}, +, \cdot)$.

Ring Axioms Verification

Additive Group Structure

The integers form an abelian Group under addition: - **Associativity**: $(a+b)+c = a+(b+c)$ for all $a, b, c \in \mathbb{Z}$ - **Identity**: 0 is the additive identity - **Inverses**: For each $a \in \mathbb{Z}$, the inverse is $-a$ - **Commutativity**: $a+b = b+a$ for all $a, b \in \mathbb{Z}$

Multiplication Structure

Multiplication satisfies: - **Associativity**: $(a \cdot b) \cdot c = a \cdot (b \cdot c)$ for all $a, b, c \in \mathbb{Z}$ - **Identity**: 1 is the multiplicative identity

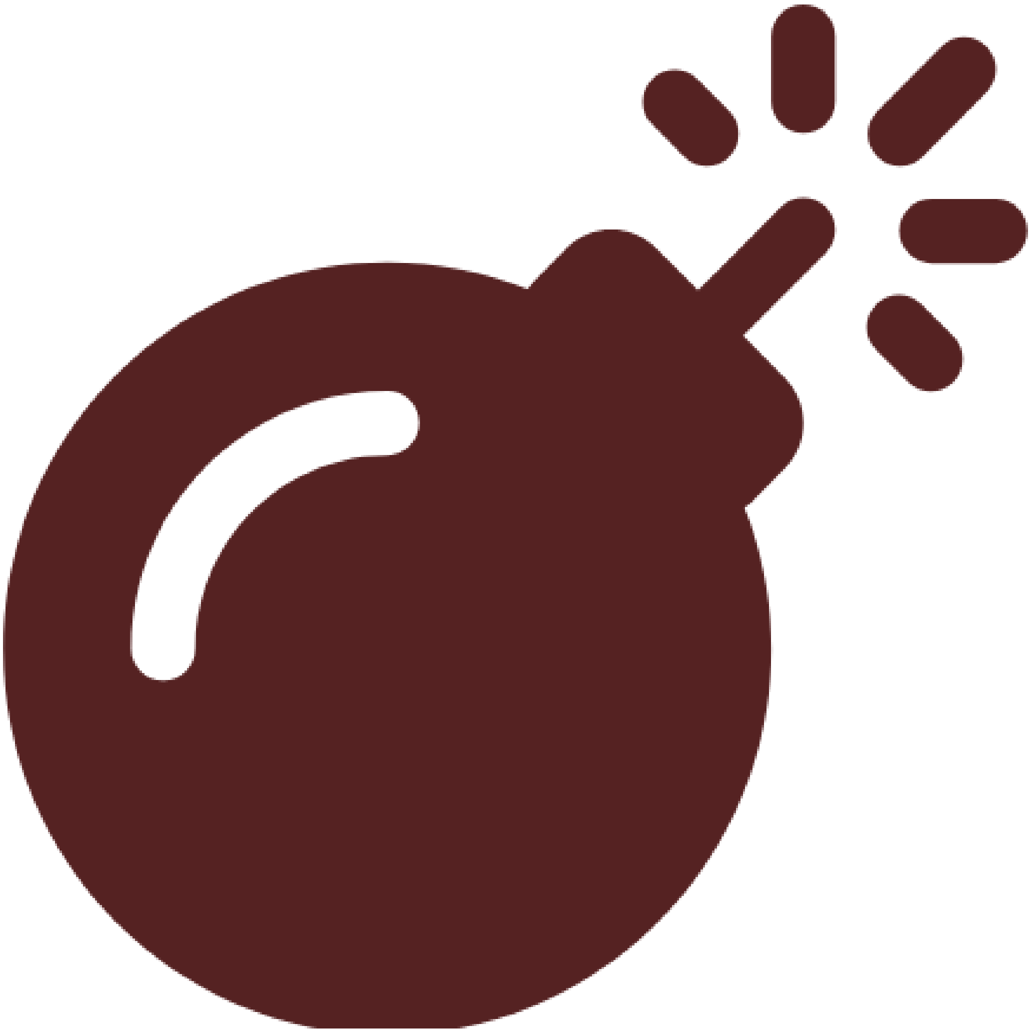
Distributive Laws

For all $a, b, c \in \mathbb{Z}$: - **Left distributivity**: $a \cdot (b+c) = a \cdot b + a \cdot c$ - **Right distributivity**: $(a+b) \cdot c = a \cdot c + b \cdot c$

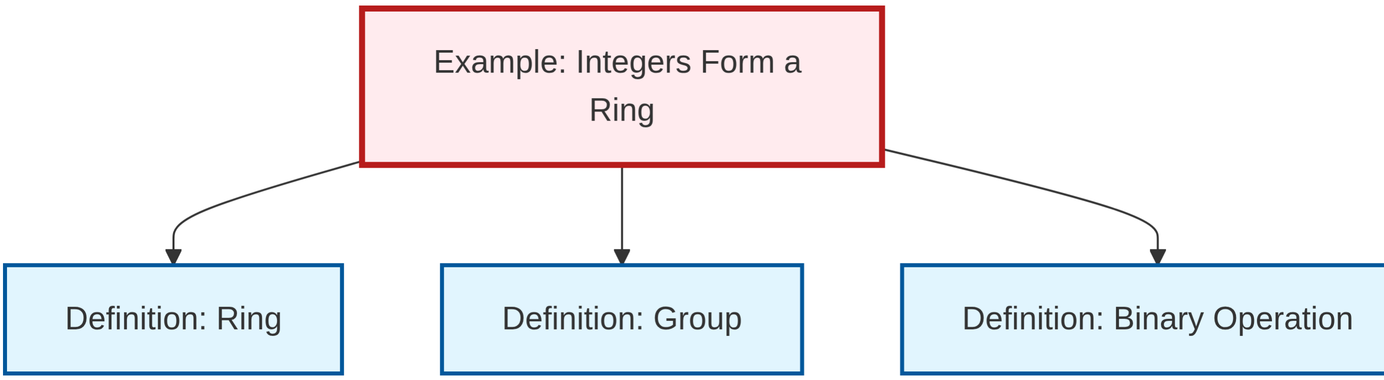
Special Properties

$\mathbb{Z}$ is actually a **commutative ring with unity**: - **Commutativity**: $a \cdot b = b \cdot a$ for all $a, b \in \mathbb{Z}$ - **Unity**: The multiplicative identity 1 exists

Furthermore, $\mathbb{Z}$ is an **integral domain** since it has no zero divisors: if $a \cdot b = 0$, then either $a = 0$ or $b = 0$.



Dependency Graph



Local dependency graph