# Definition: Subgroup

## Subgroup

A **subgroup** of a Group $G$ is a Set $H \subseteq G$ that is itself a group under the same operation as $G$.

## Formal Definition

Let $(G, \cdot)$ be a group. A non-empty subset $H \subseteq G$ is a subgroup of $G$ if:

1. **Closure**: For all $a, b \in H$, we have $a \cdot b \in H$
2. **Identity**: The identity element $e$ of $G$ is in $H$
3. **Inverses**: For all $a \in H$, the inverse $a^{-1} \in H$
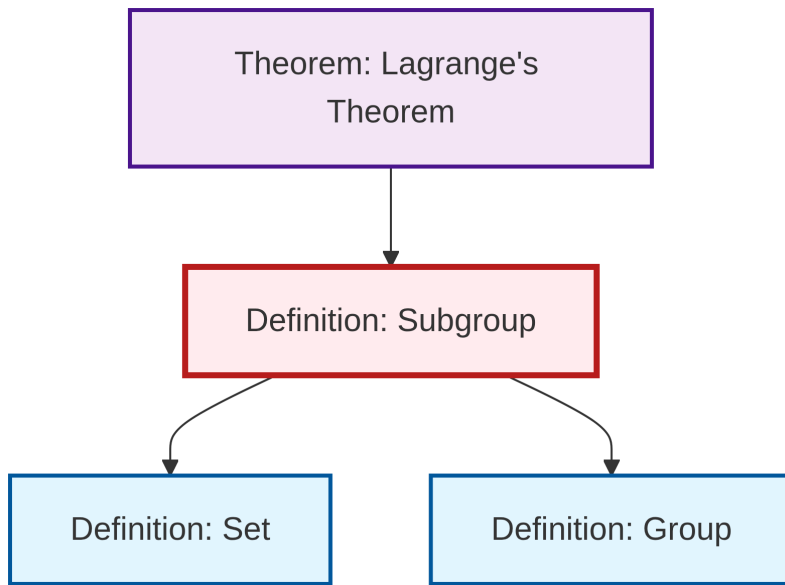
We denote this as $H \leq G$.

## Subgroup Test

A subset $H$ of a group $G$ is a subgroup if and only if: - $H$ is non-empty - For all $a, b \in H$, we have $a \cdot b^{-1} \in H$

## Properties

- Every group $G$ has at least two subgroups: the trivial subgroup $\{e\}$ and $G$ itself
- The intersection of any collection of subgroups is a subgroup
- Subgroups inherit associativity from the parent group

**Dependency Graph**

Theorem: Lagrange's Theorem

Definition: Subgroup

Definition: Set

Definition: Group

Local dependency graph