

Example: Euclidean Metric on

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The n -dimensional Euclidean space \mathbb{R}^n with the Euclidean distance forms a [Metric Space](#).

Definition of the Euclidean Metric

For points $x = (x_1, x_2, \dots, x_n)$ and $y = (y_1, y_2, \dots, y_n)$ in \mathbb{R}^n , the Euclidean distance is:

$$d(x, y) = \sqrt{\sum_{i=1}^n (x_i - y_i)^2}$$

Verification of Metric Space Axioms

We verify that d satisfies all properties of a metric:

1. **Non-negativity:** $d(x, y) \geq 0$ since it's a square root of a sum of squares.
2. **Identity of indiscernibles:**
 - If $x = y$, then $x_i = y_i$ for all i , so $d(x, y) = 0$.
 - If $d(x, y) = 0$, then $\sum_{i=1}^n (x_i - y_i)^2 = 0$, which implies $x_i = y_i$ for all i , so $x = y$.
3. **Symmetry:** $d(x, y) = d(y, x)$ since $(x_i - y_i)^2 = (y_i - x_i)^2$.
4. **Triangle inequality:** For any $x, y, z \in \mathbb{R}^n$:

$$d(x, z) \leq d(x, y) + d(y, z)$$

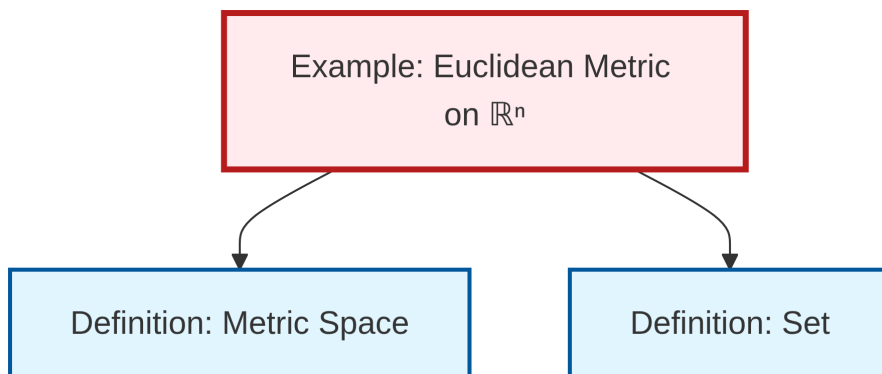
This follows from the Cauchy-Schwarz inequality.

Special Cases

- $n = 1$: The usual absolute value metric on \mathbb{R} : $d(x, y) = |x - y|$
- $n = 2$: The familiar distance in the plane: $d((x_1, y_1), (x_2, y_2)) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
- $n = 3$: The distance in 3D space used in physics and engineering



Dependency Graph



Local dependency graph