

## Definition: Subgroup

### Subgroup

A **subgroup** of a **Group**  $G$  is a **Set**  $H \subseteq G$  that is itself a group under the same operation as  $G$ .

### Formal Definition

Let  $(G, \cdot)$  be a group. A non-empty subset  $H \subseteq G$  is a subgroup of  $G$  if:

1. **Closure:** For all  $a, b \in H$ , we have  $a \cdot b \in H$
2. **Identity:** The identity element  $e$  of  $G$  is in  $H$
3. **Inverses:** For all  $a \in H$ , the inverse  $a^{-1} \in H$

We denote this as  $H \leq G$ .

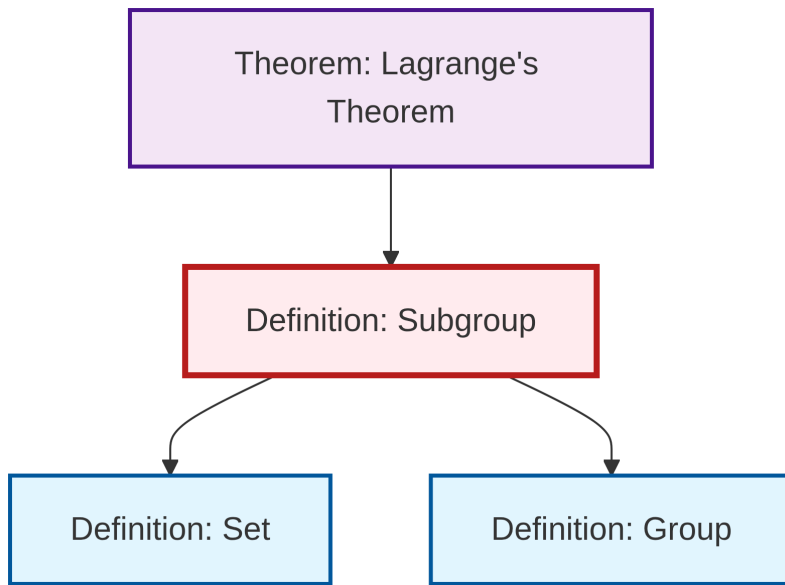
### Subgroup Test

A subset  $H$  of a group  $G$  is a subgroup if and only if: -  $H$  is non-empty - For all  $a, b \in H$ , we have  $a \cdot b^{-1} \in H$

### Properties

- Every group  $G$  has at least two subgroups: the trivial subgroup  $\{e\}$  and  $G$  itself
- The intersection of any collection of subgroups is a subgroup
- Subgroups inherit associativity from the parent group

## Dependency Graph



Local dependency graph