Definition: Binary Operation

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Let S be a Set. A **binary operation** on S is a function $\star : S \times S \to S$.

Explanation

A binary operation takes two elements from a set and produces another element from the same set. We often write $a \star b$ instead of $\star(a, b)$.

Properties of Binary Operations

A binary operation $\star$ on a set S may satisfy various properties:

1. **Closure**: By definition, $a \star b \in S$ for all $a, b \in S$
2. **Associativity**: $(a \star b) \star c = a \star (b \star c)$ for all $a, b, c \in S$
3. **Commutativity**: $a \star b = b \star a$ for all $a, b \in S$
4. **Identity element**: There exists $e \in S$ such that $a \star e = e \star a = a$ for all $a \in S$
5. **Inverse elements**: For each $a \in S$, there exists $b \in S$ such that $a \star b = b \star a = e$ (where e is the identity)

Examples

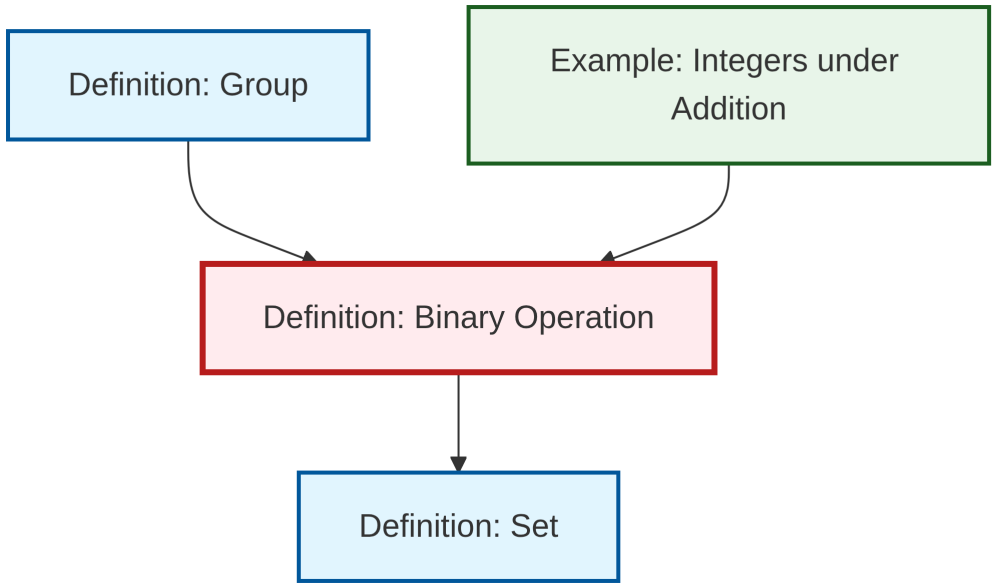
Common binary operations include:

- Addition ($+$) on the integers $\mathbb{Z}$
- Multiplication ($\cdot$) on the real numbers $\mathbb{R}$
- Matrix multiplication on $n \times n$ matrices
- Composition of functions

See Also

- Definition: Group (uses this concept)
- Example: Integers under Addition (illustrates this concept)

Dependency Graph



Local dependency graph