

## Definition: Intersection

The **intersection** of two [Sets](#)  $A$  and  $B$ , denoted  $A \cap B$ , is the set containing all elements that belong to both  $A$  and  $B$ .

### Formal Definition

$$A \cap B = \{x : x \in A \text{ and } x \in B\}$$

Equivalently, using logical notation:

$$x \in A \cap B \iff (x \in A) \wedge (x \in B)$$

### Properties

1. **Commutativity:**  $A \cap B = B \cap A$
2. **Associativity:**  $(A \cap B) \cap C = A \cap (B \cap C)$
3. **Identity:**  $A \cap U = A$  (where  $U$  is the universal set)
4. **Annihilator:**  $A \cap \emptyset = \emptyset$
5. **Idempotence:**  $A \cap A = A$
6. **Absorption:** If  $A \subseteq B$ , then  $A \cap B = A$

### Generalized Intersection

For a non-empty collection of sets  $\{A_i : i \in I\}$ :

$$\bigcap_{i \in I} A_i = \{x : \forall i \in I, x \in A_i\}$$

Special cases: - Finite intersection:  $\bigcap_{i=1}^n A_i = A_1 \cap A_2 \cap \dots \cap A_n$  - Infinite intersection:  $\bigcap_{i=1}^{\infty} A_i$

### Disjoint Sets

Two sets  $A$  and  $B$  are **disjoint** if their intersection is empty:

$$A \cap B = \emptyset$$

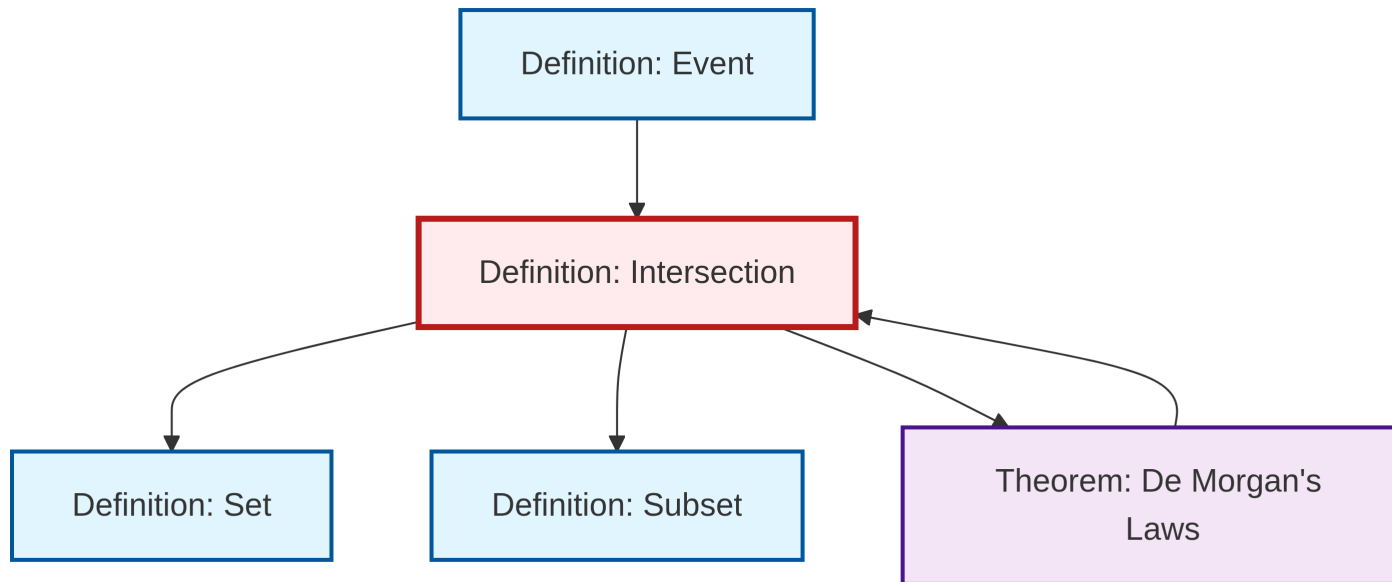
### Examples

- $\{1, 2, 3\} \cap \{2, 3, 4\} = \{2, 3\}$
- $\mathbb{Z} \cap \mathbb{N} = \mathbb{N}$  (integers intersect naturals equals naturals)
- For intervals:  $[0, 2] \cap [1, 3] = [1, 2]$
- Even and odd integers are disjoint:  $2\mathbb{Z} \cap (2\mathbb{Z} + 1) = \emptyset$

## Relationship with Other Operations

- De Morgan's Laws:  $(A \cap B)^c = A^c \cup B^c$  (see [De Morgan's Laws](#))
- Distributivity:  $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
- With [Subset](#):  $A \cap B = A \iff A \subseteq B$

## Dependency Graph



Local dependency graph