

Theorem: Angle Sum of a Triangle

The sum of the interior **Angles** of any **Triangle** equals 180° (or π radians).

Statement

For any triangle $\triangle ABC$ with interior angles α , β , and γ :

$$\alpha + \beta + \gamma = 180^\circ$$

Proof (Euclidean Geometry)

Given triangle $\triangle ABC$:

1. Through vertex A , construct a **Line** parallel to side \overline{BC}
2. This creates two additional angles adjacent to $\angle BAC$
3. By the parallel postulate and properties of transversals:
 - One angle equals $\angle ABC$ (alternate interior angles)
 - The other equals $\angle BCA$ (alternate interior angles)
4. The three angles at vertex A form a straight angle, summing to 180°
5. Therefore: $\angle BAC + \angle ABC + \angle BCA = 180^\circ$

Consequences

This theorem has several important implications:

1. **No triangle can have two right angles:** If two angles were 90° each, the third would be 0°
2. **Exterior angle theorem:** An exterior angle equals the sum of the two non-adjacent interior angles
3. **Classification constraint:** In any triangle, at most one angle can be obtuse

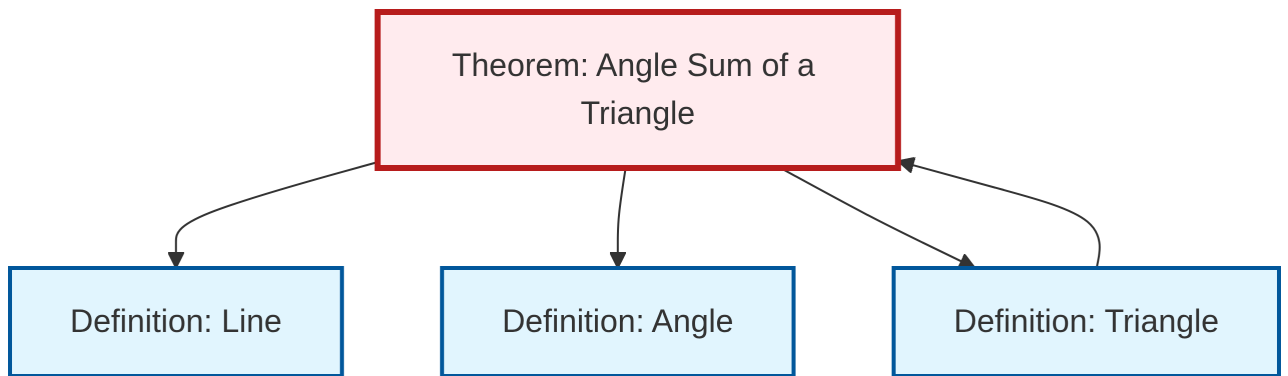
Generalizations

- In hyperbolic geometry, the angle sum is less than 180°
- In spherical geometry, the angle sum is greater than 180°
- The difference from 180° relates to the curvature of the space

Applications

- Navigation and surveying
- Determining if three angles can form a triangle
- Solving for unknown angles in geometric problems
- Foundation for trigonometry

Dependency Graph



Local dependency graph