Definition: Sample Space

A **sample space** is the Set of all possible outcomes of a random experiment or probabilistic scenario. It is typically denoted by Ω (omega) or S.

Formal Definition

A sample space Ω is a non-empty set such that: 1. It contains all possible outcomes of the experiment 2. The outcomes are mutually exclusive (only one can occur) 3. The outcomes are collectively exhaustive (one must occur)

Types of Sample Spaces

Discrete Sample Spaces

- **Finite**: Contains a finite number of outcomes
  - Example: Rolling a die: $\Omega = \{1, 2, 3, 4, 5, 6\}$
- **Countably Infinite**: Contains countably many outcomes
  - Example: Number of coin flips until first heads: $\Omega = \{1, 2, 3, ...\}$

Continuous Sample Spaces

- Contains uncountably many outcomes
- Example: Measuring reaction time: $\Omega = [0, \infty)$

Examples

1. **Coin Flip**: $\Omega = \{H, T\}$ (heads, tails)
2. **Two Dice**: $\Omega = \{(i, j) : i, j \in \{1, 2, 3, 4, 5, 6\}\}$ (36 outcomes)
3. **Random Point in Unit Square**: $\Omega = [0, 1] \times [0, 1]$
4. **Lifetime of a Lightbulb**: $\Omega = [0, \infty)$

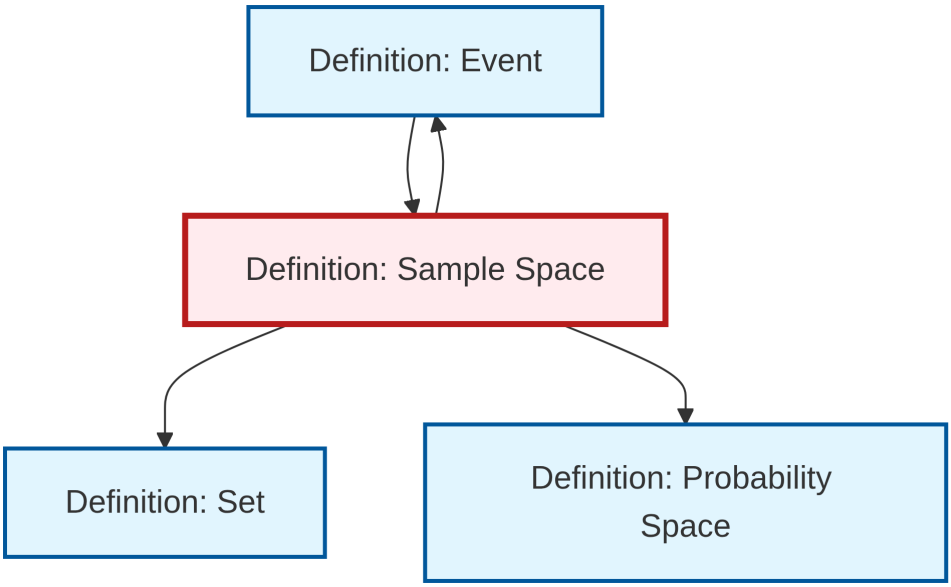
Relationship to Probability

- Every element $\omega \in \Omega$ is called an **elementary outcome**
- Subsets of Ω are called **events** (see Event)
- A Probability Space is built upon a sample space
- The entire sample space Ω has probability 1: $P(\Omega) = 1$

Properties

- The empty set $\emptyset$ represents an impossible event
- The sample space Ω represents the certain event
- Sample spaces must be defined carefully to capture all relevant outcomes
- The choice of sample space affects how probabilities are calculated

Dependency Graph



Local dependency graph