

## Example: Matrix Transformation

### Example: Matrix Transformation

Every [Matrix](#) defines a [Linear Transformation](#), providing the most concrete and computational example of linear maps.

#### Construction

Let  $A$  be an  $m \times n$  matrix with entries in a field  $F$ . Define  $T_A : F^n \rightarrow F^m$  by:

$$T_A(\mathbf{x}) = A\mathbf{x}$$

where  $\mathbf{x}$  is viewed as a column vector and  $A\mathbf{x}$  denotes matrix-vector multiplication.

#### Verification of Linearity

For vectors  $\mathbf{x}, \mathbf{y} \in F^n$  and scalars  $a, b \in F$ :

$$T_A(a\mathbf{x} + b\mathbf{y}) = A(a\mathbf{x} + b\mathbf{y}) = aA\mathbf{x} + bA\mathbf{y} = aT_A(\mathbf{x}) + bT_A(\mathbf{y})$$

This follows from the distributive and scalar multiplication properties of matrices.

#### Concrete Example

Consider the  $2 \times 3$  matrix:

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}$$

The linear transformation  $T_A : \mathbb{R}^3 \rightarrow \mathbb{R}^2$  maps:

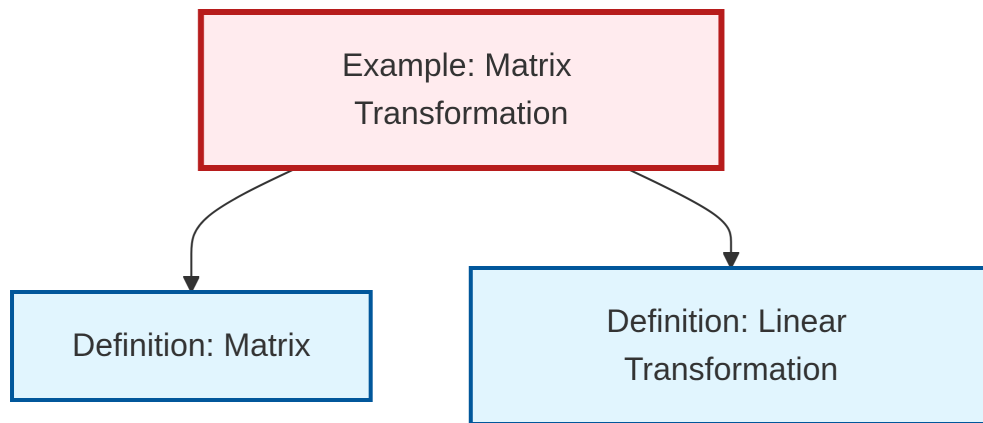
$$T_A \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x + 2y + 3z \\ 4x + 5y + 6z \end{pmatrix}$$

#### Important Facts

- Every linear transformation between finite-dimensional vector spaces can be represented by a matrix
- The columns of  $A$  are the images of the standard basis vectors:  $T_A(\mathbf{e}_i) = \mathbf{a}_i$  (the  $i$ -th column of  $A$ )
- Composition of linear transformations corresponds to matrix multiplication
- The rank of  $A$  equals the dimension of the image of  $T_A$

Matrix transformations provide the computational foundation for linear algebra.

## Dependency Graph



Local dependency graph