Definition: Group Homomorphism

Group Homomorphism

A **group homomorphism** is a function between two Group structures that preserves the group Binary Operation.

Formal Definition

Let $(G, \cdot)$ and $(H, *)$ be groups. A function $f : G \to H$ is a group homomorphism if:

$$f(a \cdot b) = f(a) * f(b)$$

for all $a, b \in G$.

Properties

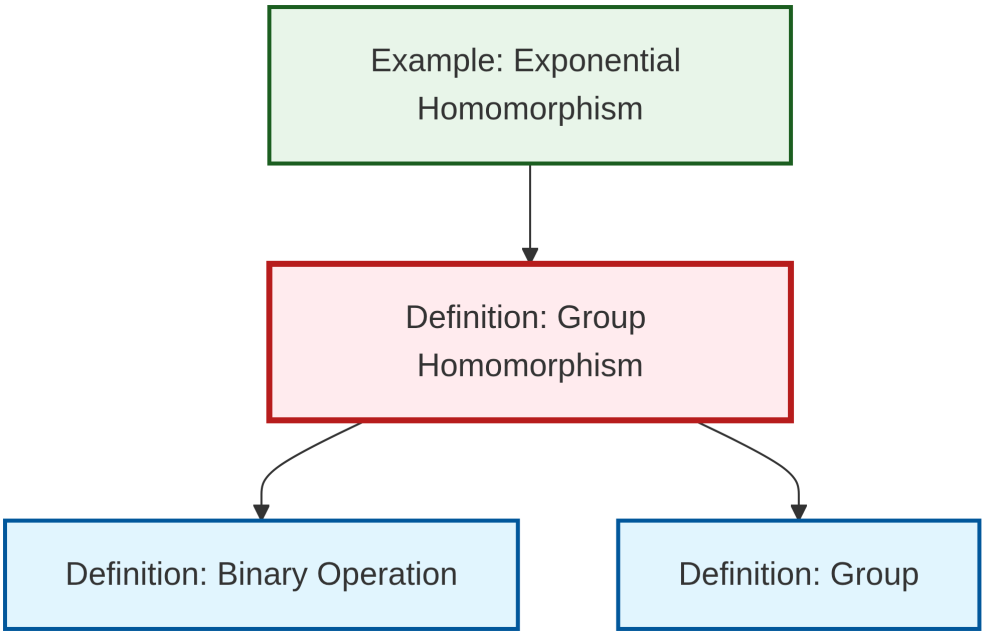
For any group homomorphism $f : G \to H$:

1. **Identity preservation**: $f(e_G) = e_H$ where e_G and e_H are the identity elements
2. **Inverse preservation**: $f(a^{-1}) = f(a)^{-1}$ for all $a \in G$
3. **Subgroup preservation**: If $K \leq G$, then $f(K) \leq H$

Special Types

- **Monomorphism**: An injective homomorphism
- **Epimorphism**: A surjective homomorphism
- **Isomorphism**: A bijective homomorphism
- **Endomorphism**: A homomorphism from a group to itself
- **Automorphism**: A bijective endomorphism

Dependency Graph



Local dependency graph