Definition: Span

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Let V be a Vector Space over a field F, and let  $S = \{\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_k\} \subseteq V$  be a set of vectors. The **span** of S, denoted span(S), is the set of all linear combinations of vectors in S:

$$\operatorname{span}(S) = \left\{ \sum_{i=1}^{k} a_i \mathbf{v}_i : a_i \in F \right\}$$

### **Alternative Notation**

The span is also commonly denoted as: -  $\langle S \rangle$  or  $\langle \mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_k \rangle$  - L(S) (for "linear span") - Lin(S)

## **Properties**

- 1. **Subspace**: span(S) is always a subspace of V
- 2. Smallest containing subspace: span(S) is the smallest subspace of V that contains S
- 3. Closure under linear combinations: If  $\mathbf{u}, \mathbf{w} \in \operatorname{span}(S)$  and  $a, b \in F$ , then  $a\mathbf{u} + b\mathbf{w} \in \operatorname{span}(S)$

## **Special Cases**

- $\operatorname{span}(\emptyset) = \{\mathbf{0}\}$  (the zero subspace)
- $\operatorname{span}(\{\mathbf{v}\}) = \{a\mathbf{v} : a \in F\}$  (the line through  $\mathbf{v}$ )
- If S spans V, we say S is a spanning set or generating set for V

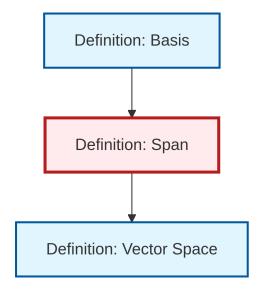
#### Infinite Sets

For an infinite set  $S \subseteq V$ , the span consists of all *finite* linear combinations:

$$\mathrm{span}(S) = \left\{ \sum_{i=1}^n a_i \mathbf{v}_i : n \in \mathbb{N}, \mathbf{v}_i \in S, a_i \in F \right\}$$

The concept of span connects individual vectors to the subspaces they generate.

# Dependency Graph



Local dependency graph