Example: Standard Basis of

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The standard basis of $\mathbb{R}^n$ is the most fundamental example of a Basis for a Vector Space.

Definition

The standard basis of $\mathbb{R}^n$ consists of the vectors: - $e_1 = (1, 0, 0, ..., 0)$ - $e_2 = (0, 1, 0, ..., 0)$ - $e_3 = (0, 0, 1, ..., 0)$ - $\vdots$ - $e_n = (0, 0, 0, ..., 1)$

where e_i has a 1 in the i-th position and 0s elsewhere.

Verification as a Basis

To show that $\{e_1, e_2, ..., e_n\}$ is a basis, we verify:

1. Linear Independence

The vectors are Linear Independence. If $c_1e_1 + c_2e_2 + ... + c_ne_n = 0$, then:

$$(c_1, c_2, ..., c_n) = (0, 0, ..., 0)$$

This implies $c_1 = c_2 = ... = c_n = 0$.

2. Spanning

The vectors Span all of $\mathbb{R}^n$. Any vector $(x_1, x_2, ..., x_n) \in \mathbb{R}^n$ can be written as:

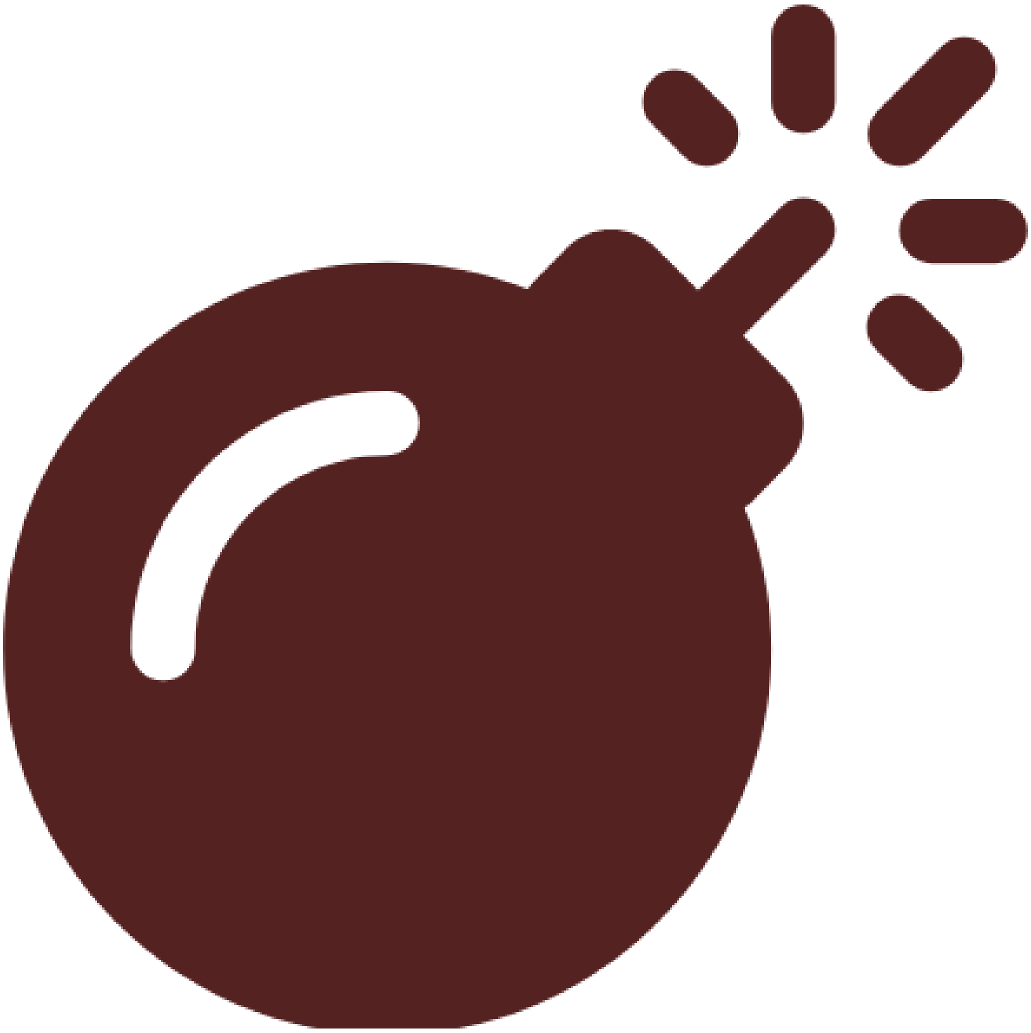
$$(x_1, x_2, ..., x_n) = x_1e_1 + x_2e_2 + ... + x_ne_n$$

Special Cases

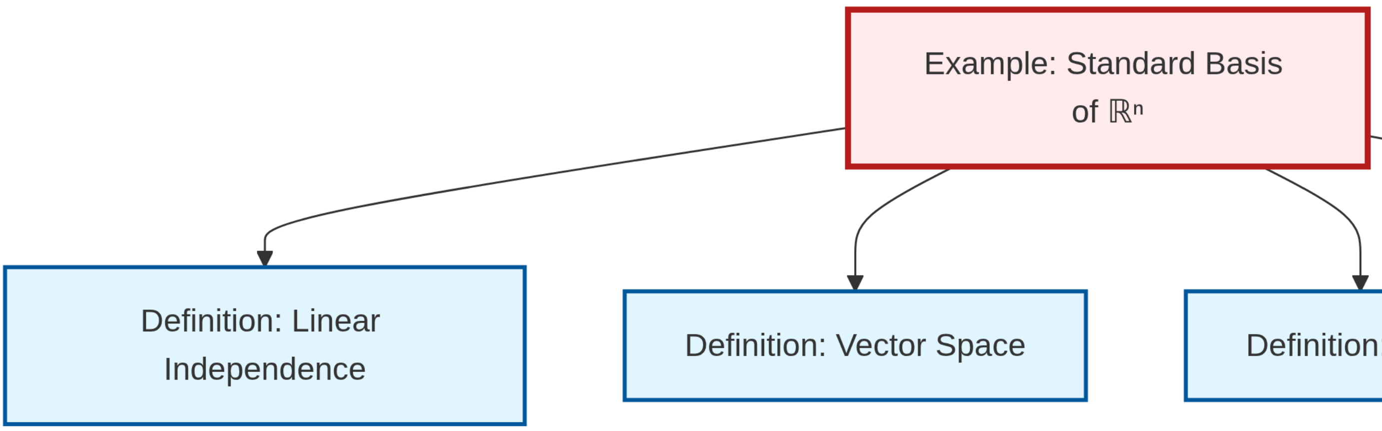
- $n = 2$: The standard basis of $\mathbb{R}^2$ is $\{(1, 0), (0, 1)\}$ - the familiar unit vectors along the x and y axes
- $n = 3$: The standard basis of $\mathbb{R}^3$ is $\{(1, 0, 0), (0, 1, 0), (0, 0, 1)\}$ - often denoted $\{\mathbf{i}, \mathbf{j}, \mathbf{k}\}$ in physics

Properties

- The standard basis is **orthonormal** with respect to the standard inner product
- Any vector's coordinates in the standard basis are simply its components
- The matrix of any linear transformation in the standard basis is particularly simple to compute



Dependency Graph



Local dependency graph