

# Definition: Morphism

## Definition: Morphism

A **morphism** (also called an arrow) is a fundamental concept in **Category** theory representing a structure-preserving map between objects.

### Formal Definition

In a **Category**  $\mathcal{C}$ , a morphism  $f : A \rightarrow B$  consists of: - A **domain** (or source) object  $A$  - A **codomain** (or target) object  $B$  - An abstract “arrow”  $f$  from  $A$  to  $B$

### Key Properties

For morphisms in a category, we have:

1. **Composition:** If  $f : A \rightarrow B$  and  $g : B \rightarrow C$ , then there exists a composite morphism  $g \circ f : A \rightarrow C$
2. **Identity:** For each object  $A$ , there exists an identity morphism  $\text{id}_A : A \rightarrow A$
3. **Associativity:**  $(h \circ g) \circ f = h \circ (g \circ f)$  whenever the compositions are defined
4. **Identity laws:**  $f \circ \text{id}_A = f$  and  $\text{id}_B \circ f = f$  for  $f : A \rightarrow B$

### Notation

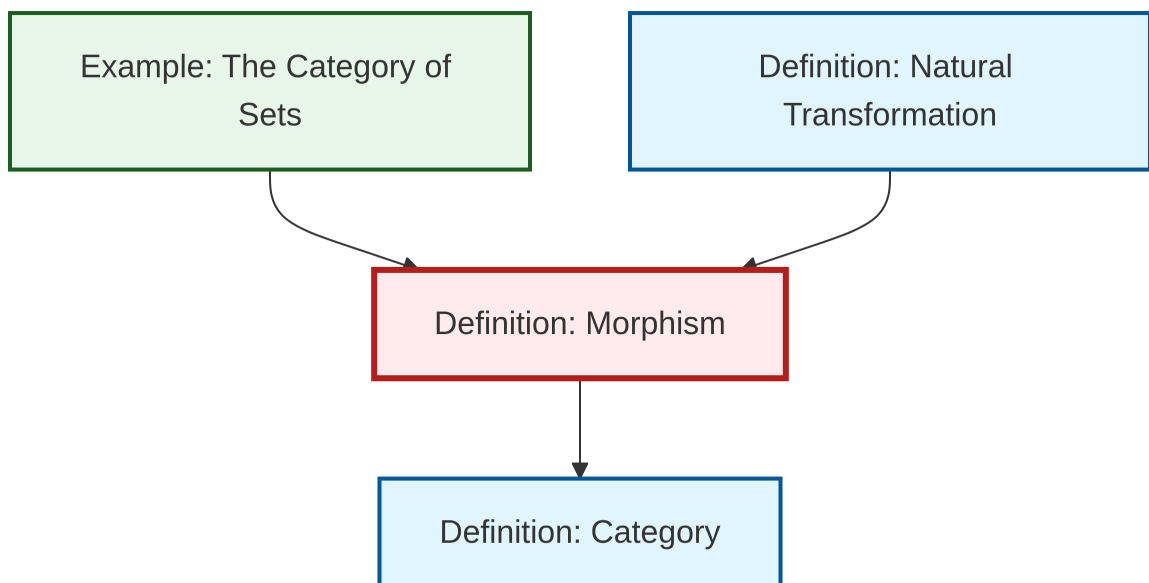
The collection of all morphisms from  $A$  to  $B$  in category  $\mathcal{C}$  is denoted: -  $\text{Hom}_{\mathcal{C}}(A, B)$  or -  $\mathcal{C}(A, B)$  or -  $\text{Mor}_{\mathcal{C}}(A, B)$

### Types of Morphisms

Special types of morphisms include: - **Monomorphism:** A categorical generalization of injective functions - **Epimorphism:** A categorical generalization of surjective functions - **Isomorphism:** A morphism with a two-sided inverse



### Dependency Graph



Local dependency graph