

Definition: Euclidean Space

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An n -dimensional **Euclidean space**, denoted \mathbb{E}^n , is the **vector space** \mathbb{R}^n equipped with the standard inner product and the induced **metric**.

Formal Definition

The n -dimensional Euclidean space consists of:

1. **The underlying set:** $\mathbb{R}^n = \{(x_1, x_2, \dots, x_n) : x_i \in \mathbb{R}\}$
2. **The standard inner product:** For $\mathbf{x} = (x_1, \dots, x_n)$ and $\mathbf{y} = (y_1, \dots, y_n)$,

$$\langle \mathbf{x}, \mathbf{y} \rangle = \sum_{i=1}^n x_i y_i$$

3. **The Euclidean norm:**

$$\|\mathbf{x}\| = \sqrt{\langle \mathbf{x}, \mathbf{x} \rangle} = \sqrt{\sum_{i=1}^n x_i^2}$$

4. **The Euclidean metric:**

$$d(\mathbf{x}, \mathbf{y}) = \|\mathbf{x} - \mathbf{y}\| = \sqrt{\sum_{i=1}^n (x_i - y_i)^2}$$

Properties

- **Complete metric space:** Every Cauchy sequence converges
- **Finite-dimensional:** Has dimension n as a vector space
- **Locally compact:** Every point has a compact neighborhood
- **Simply connected:** For $n \geq 2$
- **Homogeneous:** Looks the same at every point

Special Cases

- \mathbb{E}^1 : The real line
- \mathbb{E}^2 : The Euclidean plane
- \mathbb{E}^3 : Three-dimensional Euclidean space

Mermaid Diagram

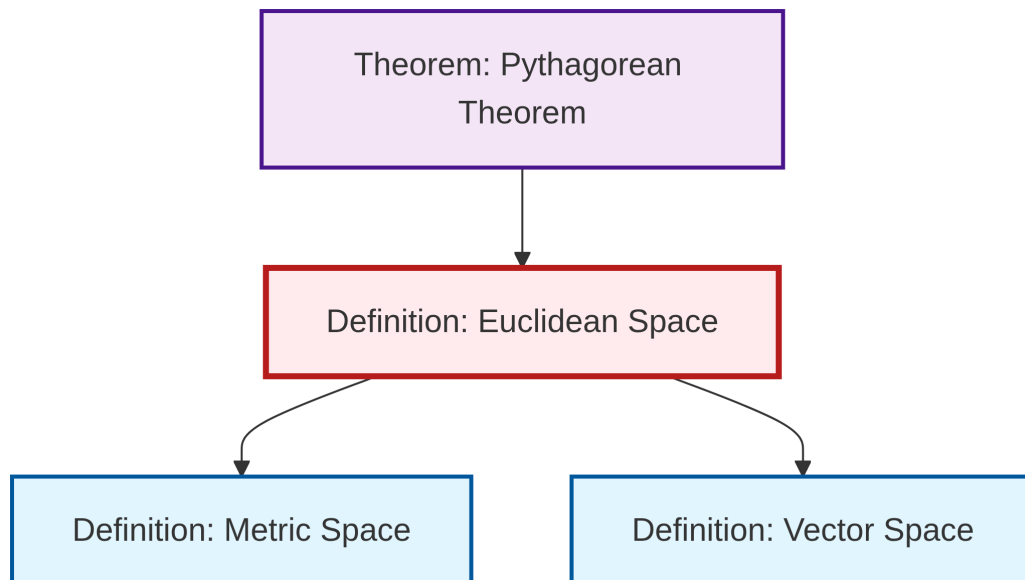
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graph TD
  A[Euclidean Space] --> B[Vector Space]
  A --> C[Inner Product]
  A --> D[Euclidean Metric]
  C --> E[Norm]
  E --> D
  B --> F[Linear Structure]
  D --> G[Metric Structure]

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  style B fill:#bbf,stroke:#333,stroke-width:2px
  style C fill:#bbf,stroke:#333,stroke-width:2px
  style D fill:#bbf,stroke:#333,stroke-width:2px

```

Dependency Graph



Local dependency graph