Example: Euclidean Metric on

Example: Euclidean Metric on

The n-dimensional Euclidean space $\mathbb{R}^n$ with the Euclidean distance forms a Metric Space.

Definition of the Euclidean Metric

For points $x = (x_1, x_2, ..., x_n)$ and $y = (y_1, y_2, ..., y_n)$ in $\mathbb{R}^n$, the Euclidean distance is:

$$d(x, y) = \sqrt{\sum_{i=1}^{n} (x_i - y_i)^2}$$

Verification of Metric Space Axioms

We verify that d satisfies all properties of a metric:

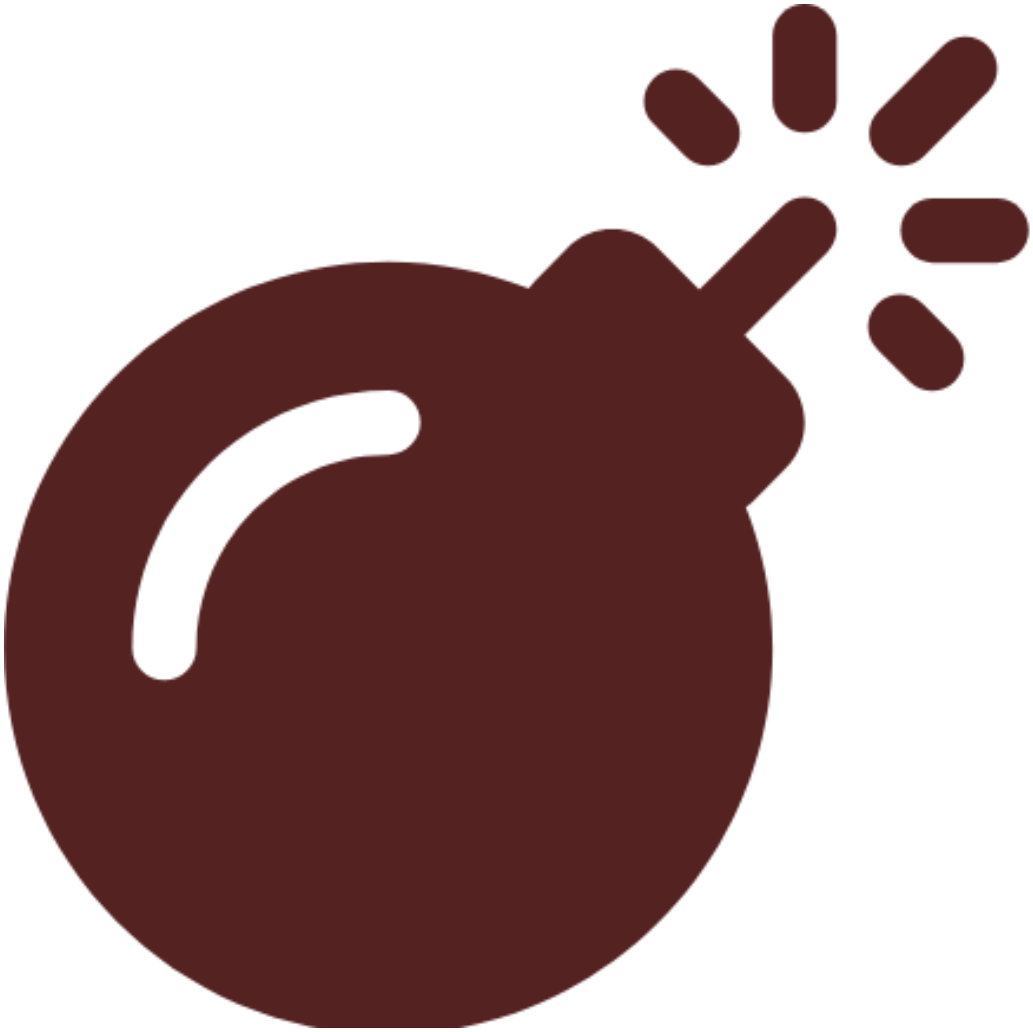
1. **Non-negativity**: $d(x, y) \geq 0$ since it's a square root of a sum of squares.
2. **Identity of indiscernibles**:
   - If $x = y$, then $x_i = y_i$ for all i, so $d(x, y) = 0$.
   - If $d(x, y) = 0$, then $\sum_{i=1}^{n} (x_i - y_i)^2 = 0$, which implies $x_i = y_i$ for all i, so $x = y$.
3. **Symmetry**: $d(x, y) = d(y, x)$ since $(x_i - y_i)^2 = (y_i - x_i)^2$.
4. **Triangle inequality**: For any $x, y, z \in \mathbb{R}^n$:

   $$d(x, z) \leq d(x, y) + d(y, z)$$

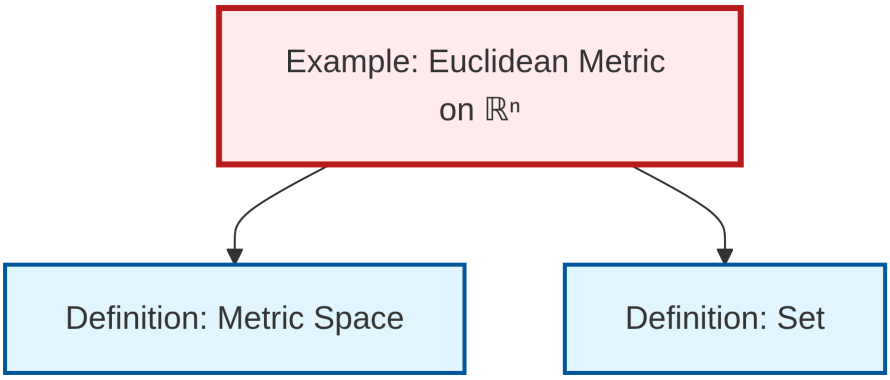
   This follows from the Cauchy-Schwarz inequality.

Special Cases

- $n = 1$: The usual absolute value metric on $\mathbb{R}$: $d(x, y) = |x - y|$
- $n = 2$: The familiar distance in the plane: $d((x_1, y_1), (x_2, y_2)) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
- $n = 3$: The distance in 3D space used in physics and engineering



Dependency Graph



Local dependency graph