Definition: Subgroup

Subgroup

A **subgroup** of a Group G is a Set $H \subseteq G$ that is itself a group under the same operation as G.

Formal Definition

Let $(G, \cdot)$ be a group. A non-empty subset $H \subseteq G$ is a subgroup of G if:

1. **Closure**: For all $a, b \in H$, we have $a \cdot b \in H$
2. **Identity**: The identity element e of G is in H
3. **Inverses**: For all $a \in H$, the inverse $a^{-1} \in H$

We denote this as $H \leq G$.

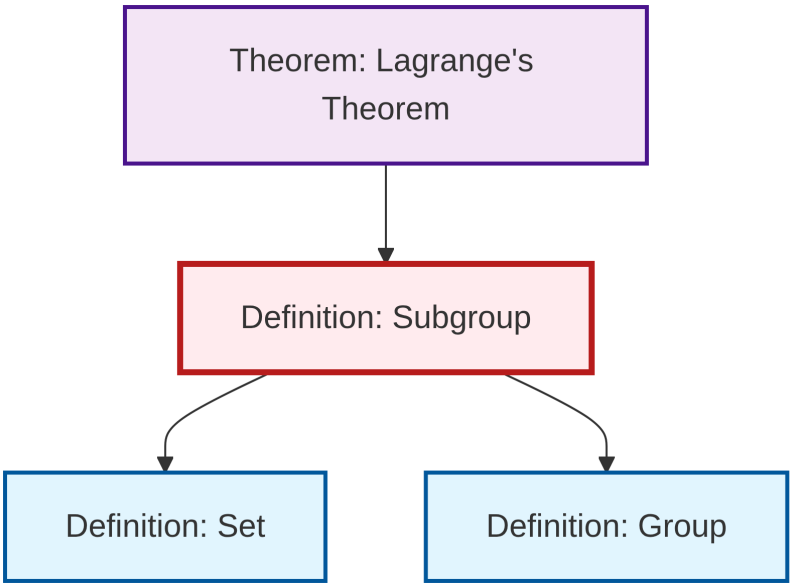
Subgroup Test

A subset H of a group G is a subgroup if and only if: - H is non-empty - For all $a, b \in H$, we have $a \cdot b^{-1} \in H$

Properties

- Every group G has at least two subgroups: the trivial subgroup $\{e\}$ and G itself
- The intersection of any collection of subgroups is a subgroup
- Subgroups inherit associativity from the parent group

Dependency Graph



Local dependency graph