Definition: Basis

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Let V be a Vector Space over a field F. A subset $B \subseteq V$ is called a **basis** for V if:

1. B is linearly independent
2. B spans V (i.e., $\text{span}(B) = V$)

Equivalent Characterizations

The following are equivalent for a subset B of a vector space V:

1. B is a basis for V
2. B is a maximal linearly independent set in V
3. B is a minimal spanning set for V
4. Every vector in V can be expressed uniquely as a linear combination of vectors in B

Types of Bases

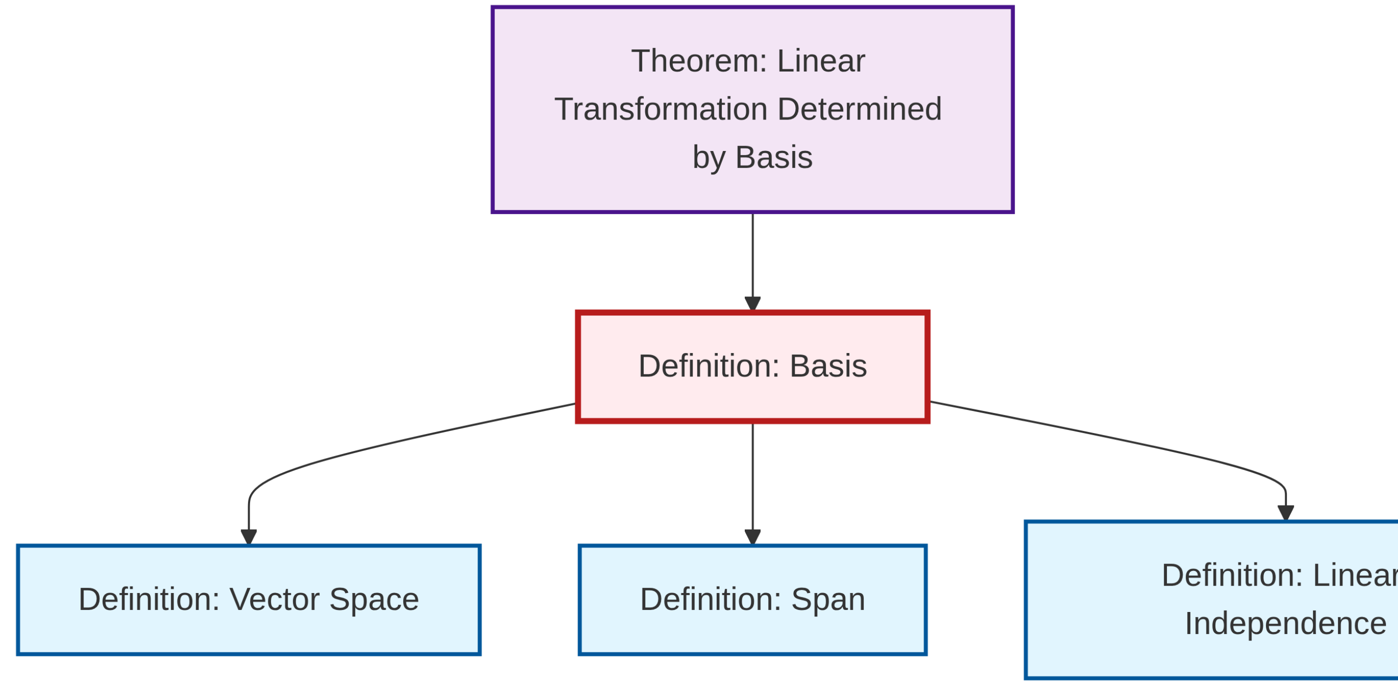
- **Standard basis**: For $\mathbb{R}^n$, the standard basis is $\{\mathbf{e}_1, \mathbf{e}_2, \ldots, \mathbf{e}_n\}$ where $\mathbf{e}_i$ has 1 in the i-th position and 0 elsewhere
- **Ordered basis**: A basis with a specified ordering of its elements
- **Orthonormal basis**: In an inner product space, a basis where all vectors have unit length and are mutually orthogonal

Dimension

If V has a finite basis with n elements, then: - Every basis of V has exactly n elements - We say V has **dimension** n, written $\dim(V) = n$ - If no finite basis exists, V is **infinite-dimensional**

The concept of basis is fundamental to linear algebra, providing a coordinate system for vector spaces.

Dependency Graph



Local dependency graph