Definition: Expected Value

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The **expected value** (or **expectation**) of a random variable is a measure of the central tendency of its distribution, representing the average value the random variable takes over many trials.

Discrete Case

For a discrete random variable X with probability mass function $p(x)$:

$$E[X] = \sum_{x} x \cdot p(x)$$

where the sum is taken over all possible values of X.

Continuous Case

For a continuous random variable X with probability density function $f(x)$:

$$E[X] = \int_{-\infty}^{\infty} x \cdot f(x)\, dx$$

General Definition

For a random variable X on a probability space $(\Omega, \mathcal{F}, P)$:

$$E[X] = \int_{\Omega} X(\omega)\, dP(\omega)$$

This is the Lebesgue integral of X with respect to the probability measure P.

Properties

1. **Linearity**: $E[aX + bY] = aE[X] + bE[Y]$ for constants a, b
2. **Monotonicity**: If $X \leq Y$ (almost surely), then $E[X] \leq E[Y]$
3. **Constant**: $E[c] = c$ for any constant c
4. **Non-negativity**: If $X \geq 0$ (almost surely), then $E[X] \geq 0$

Existence

The expected value exists if and only if $E[|X|] < \infty$, i.e., when:

$$\int_{\Omega} |X(\omega)|\, dP(\omega) < \infty$$

Examples

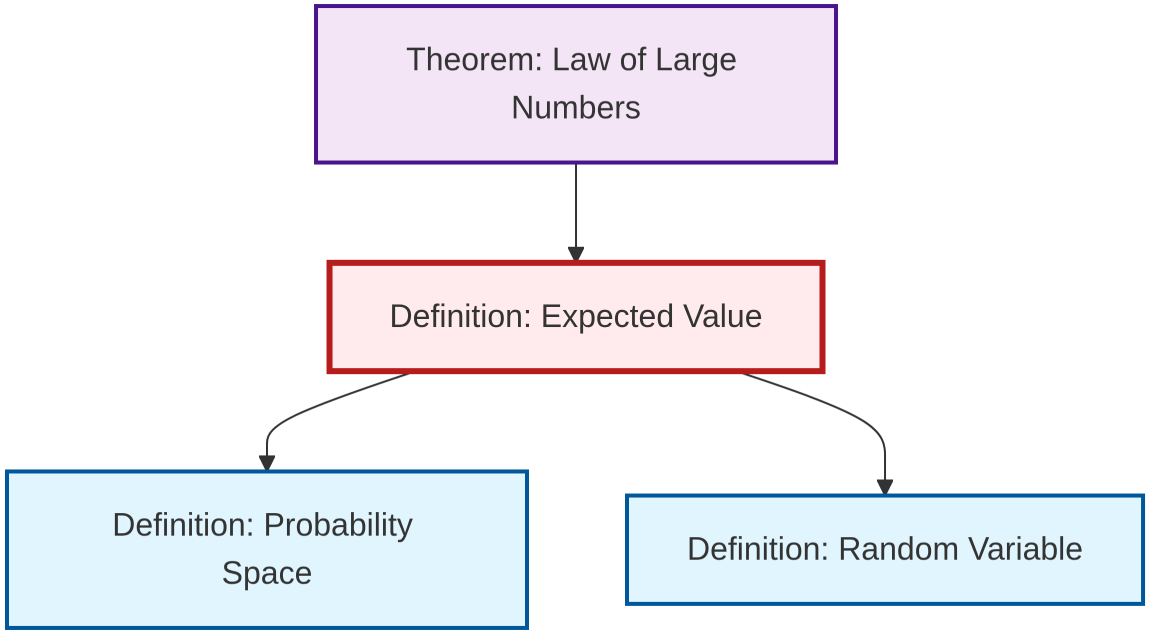
1. **Bernoulli**: If $X \sim \text{Bernoulli}(p)$, then $E[X] = p$
2. **Binomial**: If $X \sim \text{Binomial}(n, p)$, then $E[X] = np$
3. **Normal**: If $X \sim \mathcal{N}(\mu, \sigma^2)$, then $E[X] = \mu$
4. **Exponential**: If $X \sim \text{Exp}(\lambda)$, then $E[X] = 1/\lambda$

Mermaid Diagram

```
graph TD
    A[Expected Value] --> B[Discrete: Σ x·p(x)]
    A --> C[Continuous:  x·f(x) dx]
    A --> D[General:  X dP]
    A --> E[Properties]
    E --> F[Linearity]
    E --> G[Monotonicity]
    A --> H[Existence: E[|X|] < ∞]

    style A fill:#f9f,stroke:#333,stroke-width:2px
    style B fill:#bbf,stroke:#333,stroke-width:2px
    style C fill:#bbf,stroke:#333,stroke-width:2px
    style D fill:#bbf,stroke:#333,stroke-width:2px
```

Dependency Graph



Local dependency graph