Definition: Prime Number

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A natural number $p > 1$ is called a **prime number** (or simply a **prime**) if its only positive divisors are 1 and p itself.

Formal Definition

A natural number $p > 1$ is prime if and only if:

$$\forall a \in \mathbb{N}, \quad a \mid p \implies a = 1 \text{ or } a = p$$

where $a \mid p$ means "a divides p" (i.e., there exists $k \in \mathbb{N}$ such that $p = ak$).

Equivalent Characterizations

1. p has exactly two positive divisors
2. p cannot be written as a product of two natural numbers both greater than 1
3. If $p \mid ab$ for integers a, b, then $p \mid a$ or $p \mid b$ (prime property)

Examples and Non-Examples

Prime numbers: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, …

Composite numbers (non-prime numbers > 1): 4, 6, 8, 9, 10, 12, 14, 15, …

Note: - 2 is the only even prime number - 1 is not considered prime (by modern convention)

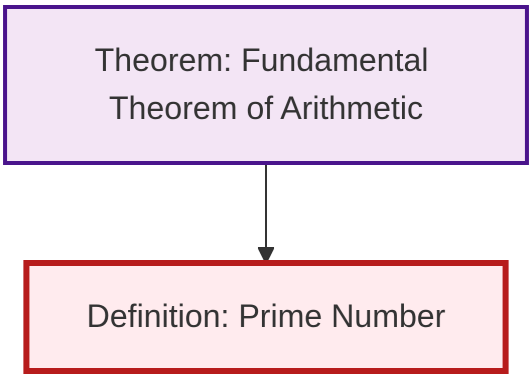
Importance

Prime numbers are the "atoms" of number theory: - Every natural number > 1 can be uniquely factored into primes (Fundamental Theorem of Arithmetic) - They appear in numerous areas of mathematics and applications (cryptography, coding theory) - Their distribution remains a central mystery (Riemann Hypothesis)

Related Concepts

- **Composite number**: A natural number > 1 that is not prime
- **Coprime**: Two integers are coprime if their greatest common divisor is 1
- **Prime factorization**: Expression of a number as a product of prime powers

Dependency Graph



Local dependency graph