Definition: Closed Set

Closed Set

In a Topological Space, a **closed set** is a Set whose complement is an Open Set.

Formal Definition

Let (X, τ) be a topological space. A subset $C \subseteq X$ is closed if and only if its complement $X \setminus C$ is open (i.e., $X \setminus C \in \tau$).

Properties of Closed Sets

The collection of closed sets in a topological space (X, τ) satisfies:

1. **Empty set and whole space**: $\emptyset$ and X are closed
2. **Arbitrary intersections**: The intersection of any collection of closed sets is closed
3. **Finite unions**: The union of finitely many closed sets is closed

Relationship with Open Sets

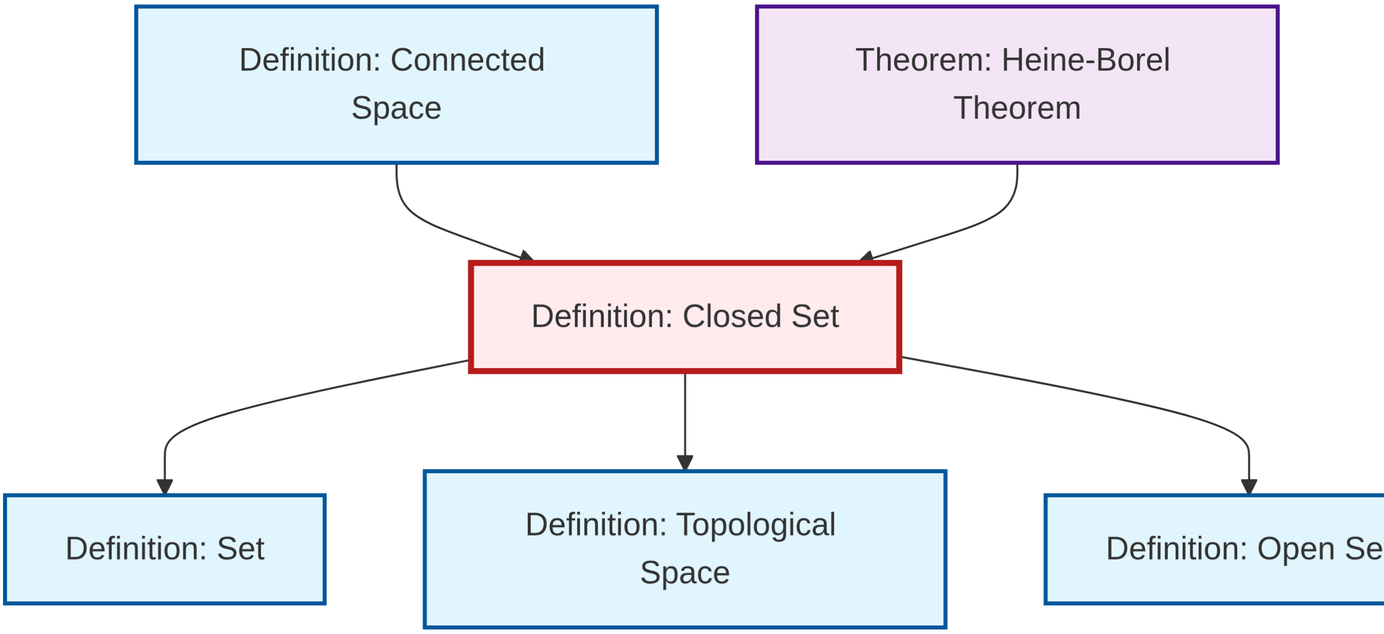
- A set can be both open and closed (clopen)
- A set can be neither open nor closed
- The closed sets form the dual notion to open sets

Closure Operator

For any subset $A \subseteq X$, the closure $\overline{A}$ is the smallest closed set containing A:

$$\overline{A} = \bigcap \{C : A \subseteq C \text{ and } C \text{ is closed}\}$$

Dependency Graph



Local dependency graph