Definition: Open Set

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Let (X, τ) be a Topological Space. A subset $U \subseteq X$ is called an **open set** if and only if $U \in \tau$.

Characterization

By the definition of a topology, open sets have the following properties:

1. The empty set $\emptyset$ and the entire space X are open
2. Any union of open sets is open
3. Any finite intersection of open sets is open

Examples in Common Spaces

In $\mathbb{R}$ with the standard topology:

- Open intervals $(a, b) = \{x \in \mathbb{R} : a < x < b\}$ are open sets
- Closed intervals $[a, b]$ are not open sets
- Half-open intervals $[a, b)$ and $(a, b]$ are not open sets
- The entire real line $\mathbb{R}$ is open
- Any union of open intervals is open

In a discrete space:

- Every subset is an open set

In an indiscrete space:

- Only $\emptyset$ and X are open sets

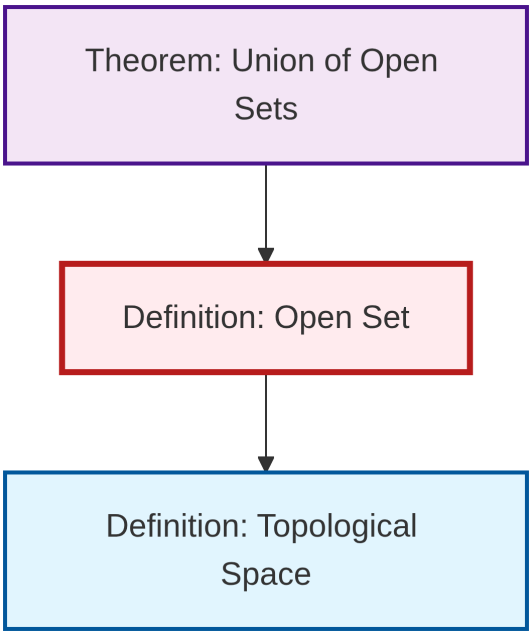
Intuition

Open sets can be thought of as sets where every point has some "wiggle room" around it that stays within the set. This intuition is formalized in metric spaces, where a set is open if every point has an open ball around it contained in the set.

Related Concepts

- Definition: Closed Set (complement of an open set)
- Theorem: Union of Open Sets (proves a fundamental property)
- Definition: Interior of a Set (coming soon)
- Definition: Neighborhood (coming soon)

Dependency Graph



Local dependency graph