Theorem: Lagrange's Theorem

Lagrange's Theorem

For a finite Group G and a Subgroup H of G, the order of H divides the order of G.

Statement

If G is a finite group and $H \leq G$, then:

$$|G| = |H| \cdot [G : H]$$

where $|G|$ denotes the order (number of elements) of G, and $[G : H]$ is the index of H in G (the number of distinct left cosets of H in G).

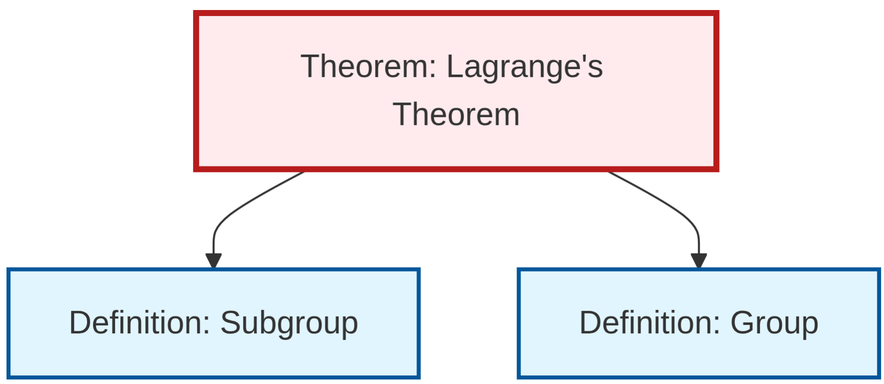
Corollaries

1. The order of any element $a \in G$ divides the order of G
2. If $|G|$ is prime, then G is cyclic and has no proper non-trivial subgroups
3. Any group of prime order is isomorphic to $\mathbb{Z}_p$ for some prime p

Applications

Lagrange's theorem is fundamental in group theory and has numerous applications: - Determining possible subgroup structures - Proving Fermat's Little Theorem - Classifying groups of small order

Dependency Graph



Local dependency graph