

Definition: Permutation

Permutation

A **permutation** of a **Set** is a bijective function from the set to itself.

Formal Definition

Let S be a set. A permutation of S is a bijection $\sigma : S \rightarrow S$.

For a finite set $S = \{1, 2, \dots, n\}$, a permutation can be represented as a rearrangement of the elements.

Notation

Permutations can be written in several ways:

1. **Two-line notation:**

$$\sigma = \begin{pmatrix} 1 & 2 & 3 & \dots & n \\ \sigma(1) & \sigma(2) & \sigma(3) & \dots & \sigma(n) \end{pmatrix}$$

2. **One-line notation:** $\sigma = [\sigma(1), \sigma(2), \dots, \sigma(n)]$

3. **Cycle notation:** $(a_1 \ a_2 \ \dots \ a_k)$ means $a_1 \mapsto a_2 \mapsto \dots \mapsto a_k \mapsto a_1$

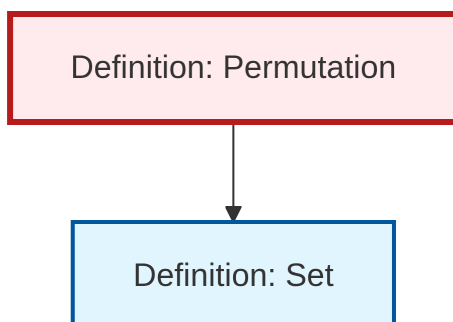
Properties

- The set of all permutations of n elements forms the symmetric group S_n
- The number of permutations of n distinct elements is $n!$
- Every permutation can be decomposed into disjoint cycles
- Every permutation can be expressed as a product of transpositions

Examples

For $S = \{1, 2, 3\}$: - Identity: $(1)(2)(3)$ - Transposition: $(1\ 2)(3)$ - 3-cycle: $(1\ 2\ 3)$

Dependency Graph



Local dependency graph