

## Example: Integers under Addition

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The integers  $\mathbb{Z} = \{\dots, -2, -1, 0, 1, 2, \dots\}$  form a [Group](#) under the [Binary Operation](#) of addition.

#### Verification of Group Axioms

We denote this group as  $(\mathbb{Z}, +)$  and verify each group axiom:

##### 1. Closure

For any  $a, b \in \mathbb{Z}$ , the sum  $a + b$  is also an integer. This follows from the definition of integer addition.

##### 2. Associativity

For all  $a, b, c \in \mathbb{Z}$ :

$$(a + b) + c = a + (b + c)$$

This is a fundamental property of integer addition.

##### 3. Identity Element

The integer 0 serves as the identity element: - For any  $a \in \mathbb{Z}$ :  $a + 0 = 0 + a = a$

By [Uniqueness of Identity](#), this identity element is unique.

##### 4. Inverse Elements

For each  $a \in \mathbb{Z}$ , the integer  $-a$  is its inverse: -  $a + (-a) = (-a) + a = 0$

#### Additional Properties

The group  $(\mathbb{Z}, +)$  has several important additional properties:

1. **Commutativity:**  $a + b = b + a$  for all  $a, b \in \mathbb{Z}$ 
  - This makes  $(\mathbb{Z}, +)$  an **abelian group**
2. **Infinite order:** The group has infinitely many elements
3. **Cyclic:** The group is generated by the single element 1:
  - Every integer can be written as  $n \cdot 1$  for some  $n \in \mathbb{Z}$

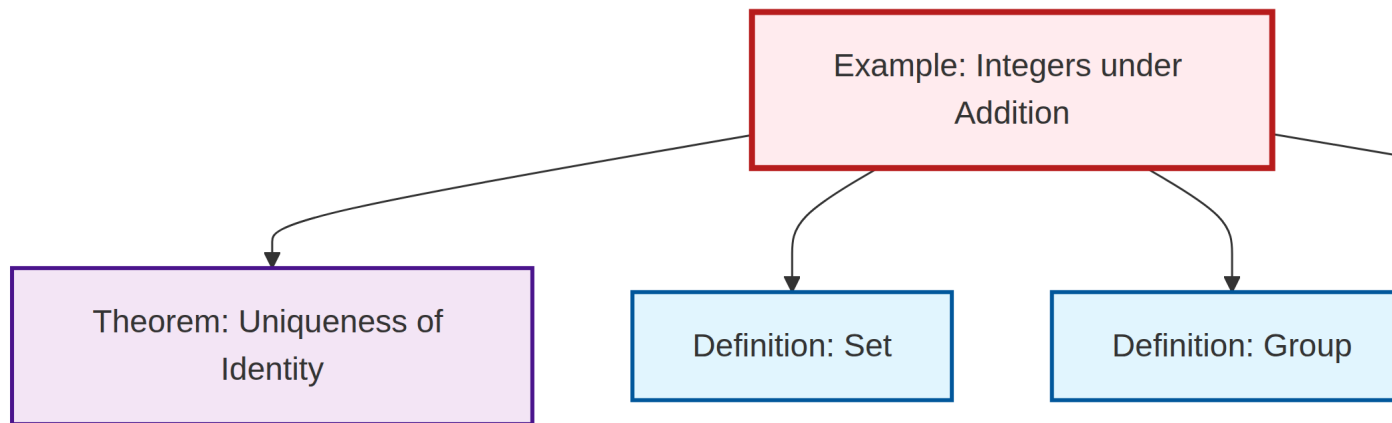
#### Subgroups

Notable subgroups of  $(\mathbb{Z}, +)$  include: -  $n\mathbb{Z} = \{nk : k \in \mathbb{Z}\}$  for any  $n \in \mathbb{N}$  - For example,  $2\mathbb{Z}$  is the subgroup of even integers

## Related Examples

- The rationals  $(\mathbb{Q}, +)$  form a group containing  $(\mathbb{Z}, +)$  as a subgroup
- The integers modulo  $n$ , denoted  $\mathbb{Z}/n\mathbb{Z}$ , form finite groups

## Dependency Graph



Local dependency graph