Definition: Morphism

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A **morphism** (also called an arrow) is a fundamental concept in Category theory representing a structure-preserving map between objects.

Formal Definition

In a Category $\mathcal{C}$, a morphism $f : A \to B$ consists of: - A **domain** (or source) object A - A **codomain** (or target) object B - An abstract "arrow" f from A to B

Key Properties

For morphisms in a category, we have:

1. **Composition**: If $f : A \to B$ and $g : B \to C$, then there exists a composite morphism $g \circ f : A \to C$
2. **Identity**: For each object A, there exists an identity morphism $\mathrm{id}_A : A \to A$
3. **Associativity**: $(h \circ g) \circ f = h \circ (g \circ f)$ whenever the compositions are defined
4. **Identity laws**: $f \circ \mathrm{id}_A = f$ and $\mathrm{id}_B \circ f = f$ for $f : A \to B$

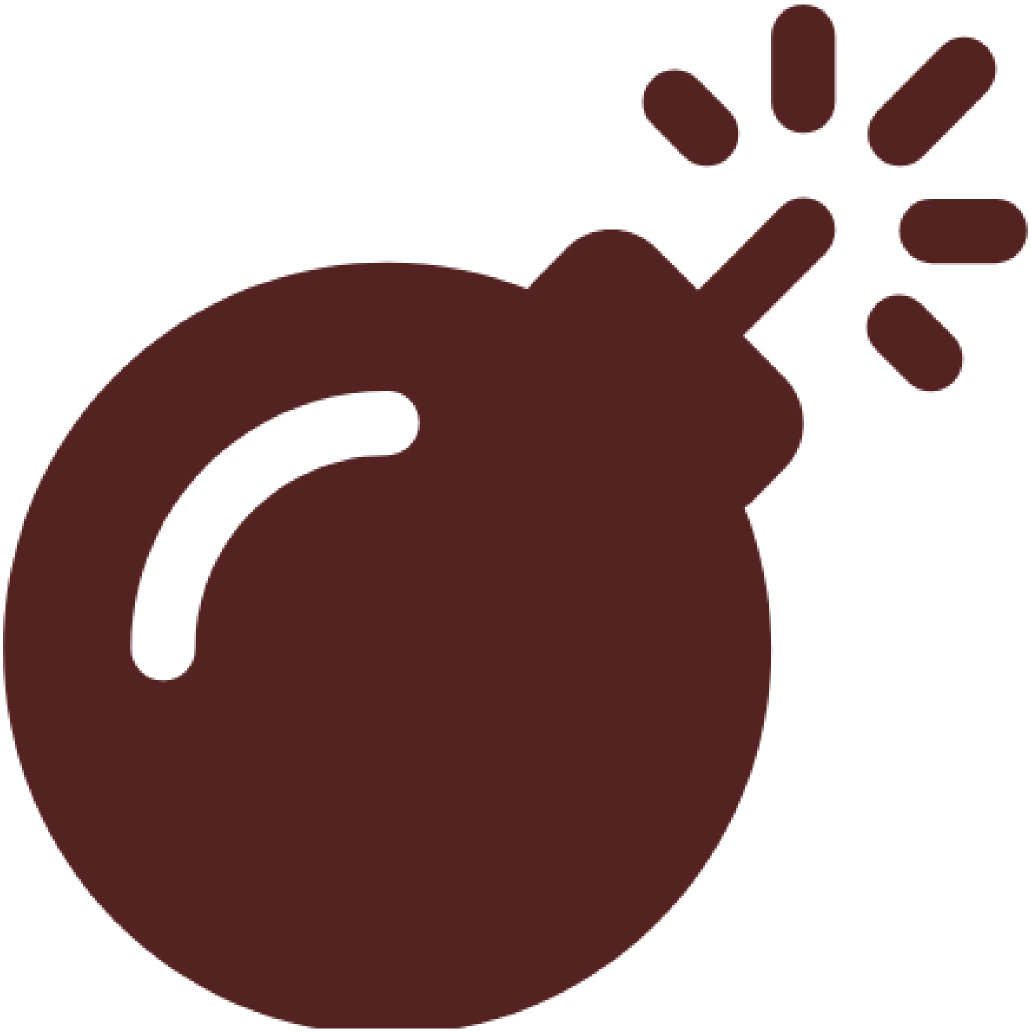
Notation

The collection of all morphisms from A to B in category $\mathcal{C}$ is denoted: - $\mathrm{Hom}_{\mathcal{C}}(A, B)$ or - $\mathcal{C}(A, B)$ or - $\mathrm{Mor}_{\mathcal{C}}(A, B)$

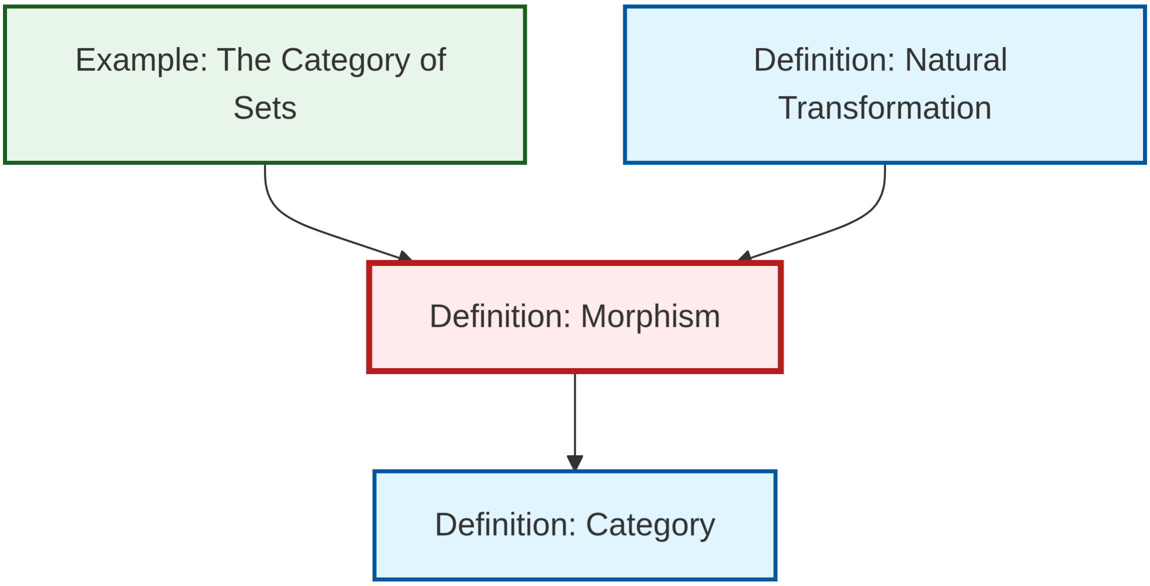
Types of Morphisms

Special types of morphisms include: - **Monomorphism**: A categorical generalization of injective functions - **Epimorphism**: A categorical generalization of surjective functions - **Isomorphism**: A morphism with a two-sided inverse

Local dependency graph



Dependency Graph



Local dependency graph