

## Definition: Group

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A **group** is a **set**  $G$  together with a **binary operation**  $\star : G \times G \rightarrow G$  that satisfies the following axioms:

#### Group Axioms

1. **Associativity:** For all  $a, b, c \in G$ ,

$$(a \star b) \star c = a \star (b \star c)$$

2. **Identity element:** There exists an element  $e \in G$  such that for all  $a \in G$ ,

$$a \star e = e \star a = a$$

3. **Inverse elements:** For each  $a \in G$ , there exists an element  $b \in G$  such that

$$a \star b = b \star a = e$$

where  $e$  is the identity element. We denote this inverse as  $a^{-1}$ .

#### Notation

- A group is often denoted as  $(G, \star)$  to explicitly show both the set and the operation
- When the operation is clear from context, we may simply write  $G$
- The operation symbol is often omitted:  $ab$  instead of  $a \star b$

#### Important Properties

From these axioms, we can prove:

- The identity element is unique
- Each element has a unique inverse
- Cancellation laws hold: if  $ab = ac$  then  $b = c$

#### Examples

- $(\mathbb{Z}, +)$ : The integers under addition
- $(\mathbb{Q} \setminus \{0\}, \cdot)$ : The non-zero rationals under multiplication
- $(GL_n(\mathbb{R}), \cdot)$ : The invertible  $n \times n$  matrices under matrix multiplication

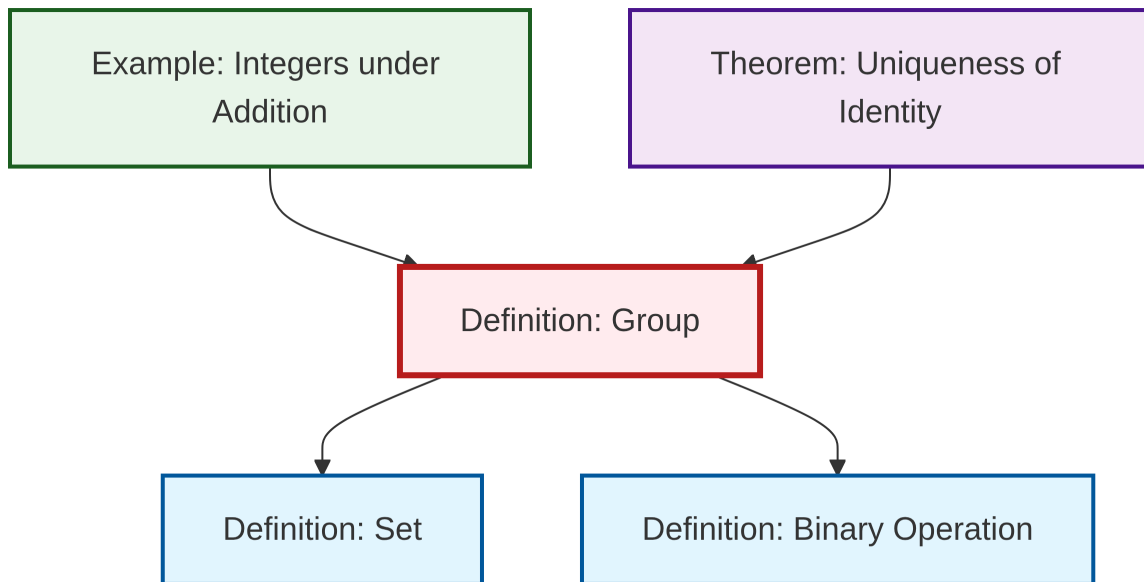
#### Special Types of Groups

- **Abelian group:** A group where the operation is commutative
- **Finite group:** A group with finitely many elements
- **Cyclic group:** A group generated by a single element

## See Also

- Theorem: Uniqueness of Identity (proves a property of groups)
- Definition: Subgroup (coming soon)
- Definition: Group Homomorphism (coming soon)

## Dependency Graph



Local dependency graph

## Interactive Visualization

Explore the local knowledge graph neighborhood interactively:

You can: - **Drag** nodes to rearrange the layout - **Zoom** in/out using your mouse wheel - **Hover** over nodes to see their details - View the full interactive version in a [separate window](#)