# Definition: Power Set

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The **power set** of a set A, denoted  $\mathcal{P}(A)$  or  $2^A$ , is the set of all subsets of A.

#### Formal Definition

$$\mathcal{P}(A) = \{B : B \subseteq A\}$$

In other words,  $B \in \mathcal{P}(A)$  if and only if every element of B is also an element of A.

## **Properties**

- 1. Always contains the empty set:  $\emptyset \in \mathcal{P}(A)$  for any set A
- 2. Always contains the original set:  $A \in \mathcal{P}(A)$
- 3. Cardinality: If |A| = n (finite), then  $|\mathcal{P}(A)| = 2^n$
- 4. Ordering:  $\mathcal{P}(A)$  forms a partially ordered set under inclusion

## Examples

- 1. If  $A = \emptyset$ , then  $\mathcal{P}(A) = \{\emptyset\}$
- 2. If  $A = \{1\}$ , then  $\mathcal{P}(A) = \{\emptyset, \{1\}\}$
- 3. If  $A = \{1, 2\}$ , then  $\mathcal{P}(A) = \{\emptyset, \{1\}, \{2\}, \{1, 2\}\}$
- 4. If  $A = \{a, b, c\}$ , then:

$$\mathcal{P}(A) = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}\}\}$$

# **Set-Theoretic Properties**

- $A \subseteq B$  if and only if  $\mathcal{P}(A) \subseteq \mathcal{P}(B)$
- $\mathcal{P}(A \cap B) = \mathcal{P}(A) \cap \mathcal{P}(B)$
- $\mathcal{P}(A) \cup \mathcal{P}(B) \subseteq \mathcal{P}(A \cup B)$  (but not always equal)

#### Mermaid Diagram

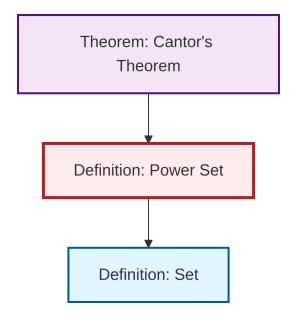
```
graph TD
    A[Power Set P(A)] --> B[Contains ]
    A --> C[Contains A]
    A --> D[All Subsets of A]
    D --> E[Cardinality: 2^|A|]
    D --> F[Partial Order by ]

A --> G[Example: A = {1,2}]
```

```
G --> H[P(A) = { , {1}, {2}, {1,2}}]

style A fill:#f9f,stroke:#333,stroke-width:2px
style D fill:#bbf,stroke:#333,stroke-width:2px
style E fill:#bfb,stroke:#333,stroke-width:2px
```

# Dependency Graph



Local dependency graph