

## Definition: Connected Space

### Connected Space

A **Topological Space** is **connected** if it cannot be represented as the union of two disjoint non-empty **Open Set** sets.

#### Formal Definition

A topological space  $(X, \tau)$  is connected if the only subsets of  $X$  that are both open and **Closed Set** are  $\emptyset$  and  $X$  itself.

Equivalently,  $X$  is connected if and only if: -  $X$  cannot be written as  $X = U \cup V$  where  $U, V$  are non-empty, disjoint open sets - There does not exist a continuous surjection  $f : X \rightarrow \{0, 1\}$  (with discrete topology)

#### Disconnected Spaces

A space is disconnected if it can be written as  $X = U \cup V$  where: -  $U, V$  are non-empty -  $U \cap V = \emptyset$  -  $U, V$  are both open (or equivalently, both closed)

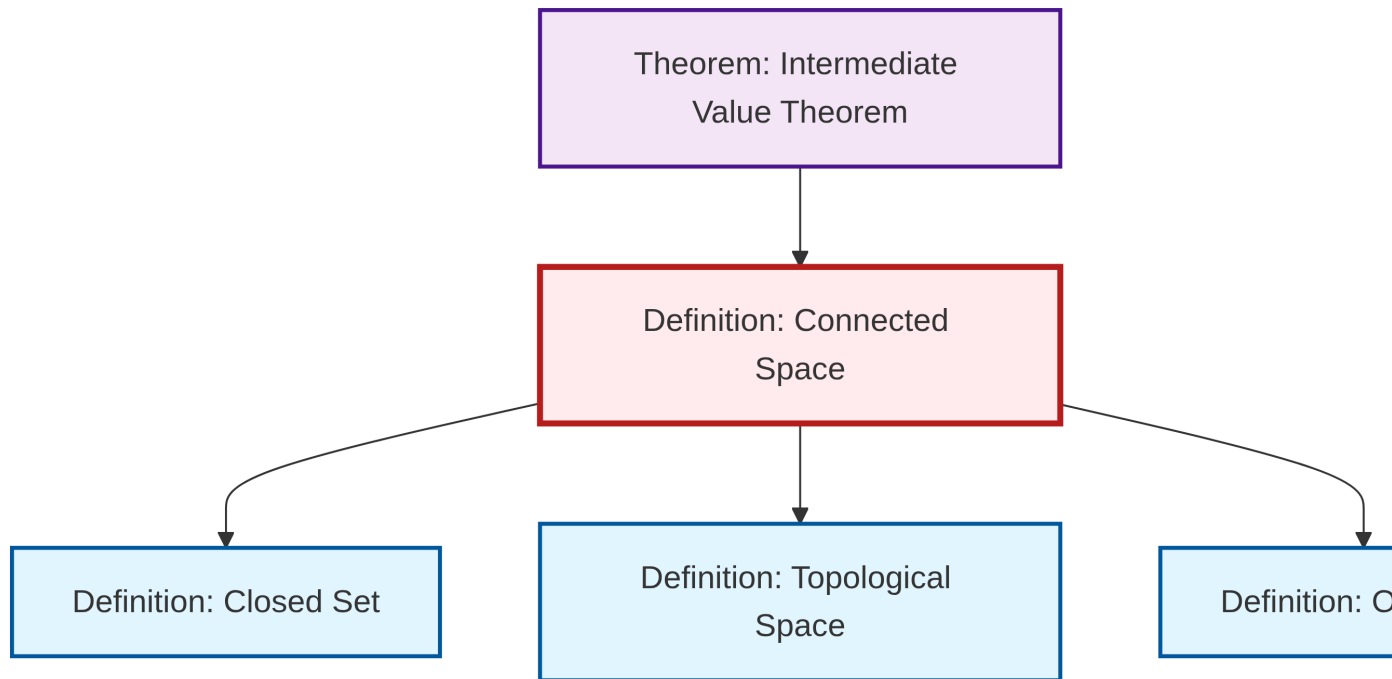
#### Properties

- The continuous image of a connected space is connected
- The closure of a connected set is connected
- If  $\{A_i\}$  is a collection of connected sets with  $\bigcap A_i \neq \emptyset$ , then  $\bigcup A_i$  is connected
- A space is connected if and only if every continuous function to  $\{0, 1\}$  is constant

#### Examples

- The real line  $\mathbb{R}$  is connected
- Any interval  $[a, b]$ ,  $(a, b)$ ,  $[a, b)$  in  $\mathbb{R}$  is connected
- The space  $\mathbb{R} \setminus \{0\}$  is disconnected

## Dependency Graph



Local dependency graph