Definition: Union

The **union** of two Sets A and B, denoted $A \cup B$, is the set containing all elements that belong to either A or B (or both).

Formal Definition

$$A \cup B = \{x : x \in A \text{ or } x \in B\}$$

Equivalently, using logical notation:

$$x \in A \cup B \iff (x \in A) \vee (x \in B)$$

Properties

1. **Commutativity**: $A \cup B = B \cup A$
2. **Associativity**: $(A \cup B) \cup C = A \cup (B \cup C)$
3. **Identity**: $A \cup \emptyset = A$
4. **Idempotence**: $A \cup A = A$
5. **Absorption**: If $A \subseteq B$, then $A \cup B = B$

Generalized Union

For a collection of sets $\{A_i : i \in I\}$:

$$\bigcup_{i \in I} A_i = \{x : \exists i \in I, x \in A_i\}$$

Special cases: - Finite union: $\bigcup_{i=1}^{n} A_i = A_1 \cup A_2 \cup \cdots \cup A_n$ - Infinite union: $\bigcup_{i=1}^{\infty} A_i$

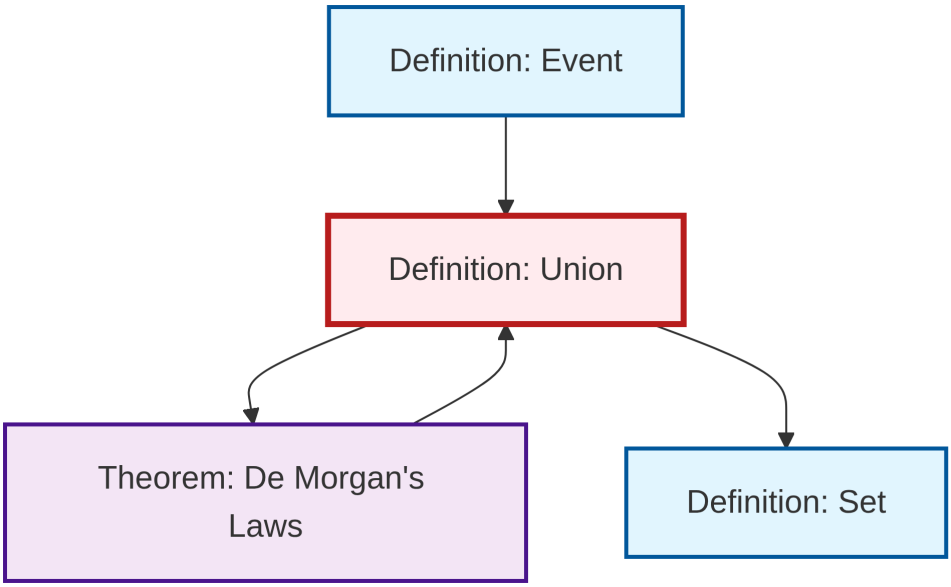
Examples

- $\{1, 2, 3\} \cup \{3, 4, 5\} = \{1, 2, 3, 4, 5\}$
- $\mathbb{Q} \cup \mathbb{I} = \mathbb{R}$ (rationals union irrationals equals reals)
- For intervals: $[0, 2] \cup [1, 3] = [0, 3]$

Relationship with Other Operations

- De Morgan's Laws: $(A \cup B)^c = A^c \cap B^c$ (see De Morgan's Laws)
- Distributivity: $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
- Cardinality: $|A \cup B| = |A| + |B| - |A \cap B|$ (inclusion-exclusion)

Dependency Graph



Local dependency graph