

## Definition: Linear Transformation

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Let  $V$  and  $W$  be  $\mathbf{s}$  over the same field  $F$ . A function  $T : V \rightarrow W$  is called a **linear transformation** (or **linear map**) if it satisfies the following two properties:

#### Linearity Conditions

For all vectors  $\mathbf{u}, \mathbf{v} \in V$  and all scalars  $a, b \in F$ :

1. **Additivity:**  $T(\mathbf{u} + \mathbf{v}) = T(\mathbf{u}) + T(\mathbf{v})$
2. **Homogeneity:**  $T(a\mathbf{u}) = aT(\mathbf{u})$

These two conditions can be combined into a single condition:

$$T(a\mathbf{u} + b\mathbf{v}) = aT(\mathbf{u}) + bT(\mathbf{v})$$

#### Important Properties

If  $T : V \rightarrow W$  is a linear transformation, then:

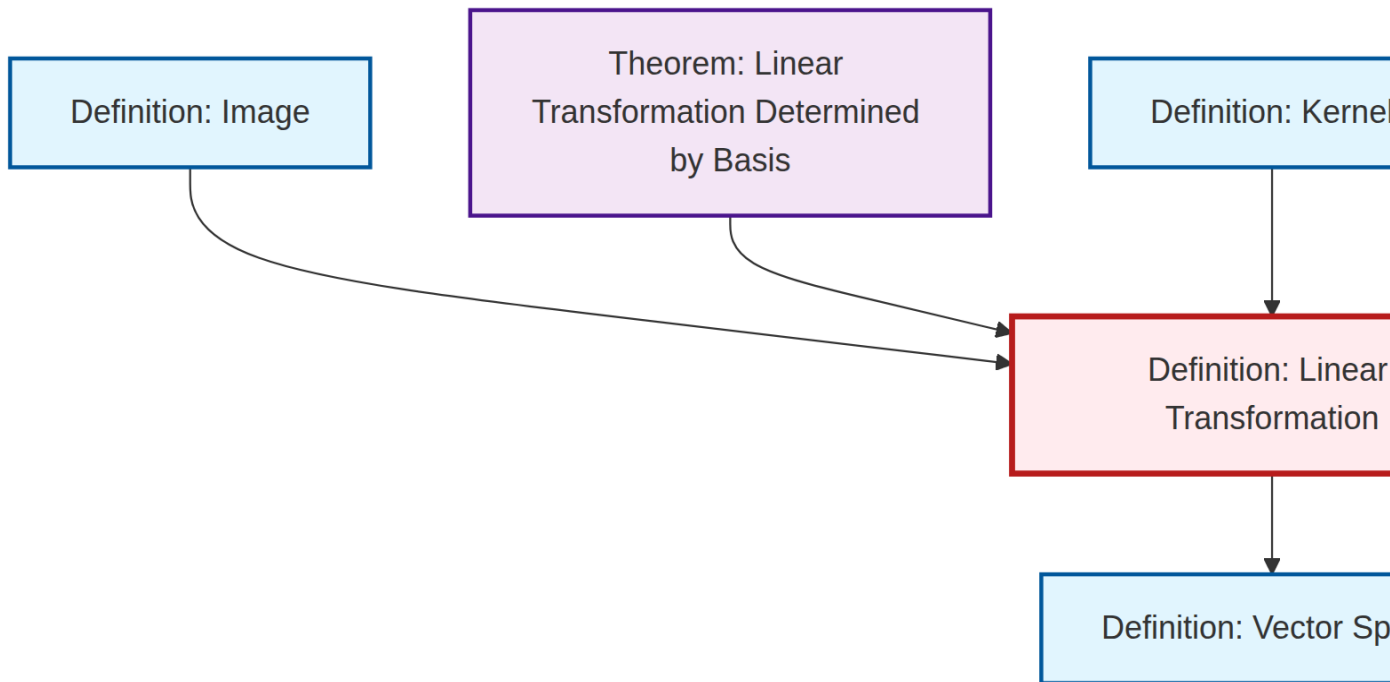
- $T(\mathbf{0}_V) = \mathbf{0}_W$  (maps zero to zero)
- $T(-\mathbf{v}) = -T(\mathbf{v})$  for all  $\mathbf{v} \in V$
- $T(\sum_{i=1}^n a_i \mathbf{v}_i) = \sum_{i=1}^n a_i T(\mathbf{v}_i)$  for any linear combination

#### Special Types

- **Linear functional:** When  $W = F$  (the field of scalars)
- **Linear operator:** When  $V = W$  (transformation from a space to itself)
- **Isomorphism:** When  $T$  is bijective (one-to-one and onto)

Linear transformations preserve the vector space structure and are the morphisms in the category of vector spaces.

## Dependency Graph



## Local dependency graph