Definition: Compact Space

Compact Space

A Topological Space is **compact** if every open cover has a finite subcover.

Formal Definition

A topological space (X, τ) is compact if for every collection $\{U_i\}_{i \in I}$ of Open Set such that $X = \bigcup_{i \in I} U_i$, there exists a finite subset $J \subseteq I$ such that $X = \bigcup_{j \in J} U_j$.

Equivalent Characterizations

For a topological space X, the following are equivalent:

1. X is compact
2. Every collection of closed sets with the finite intersection property has non-empty intersection
3. Every net in X has a convergent subnet
4. Every filter on X has a cluster point

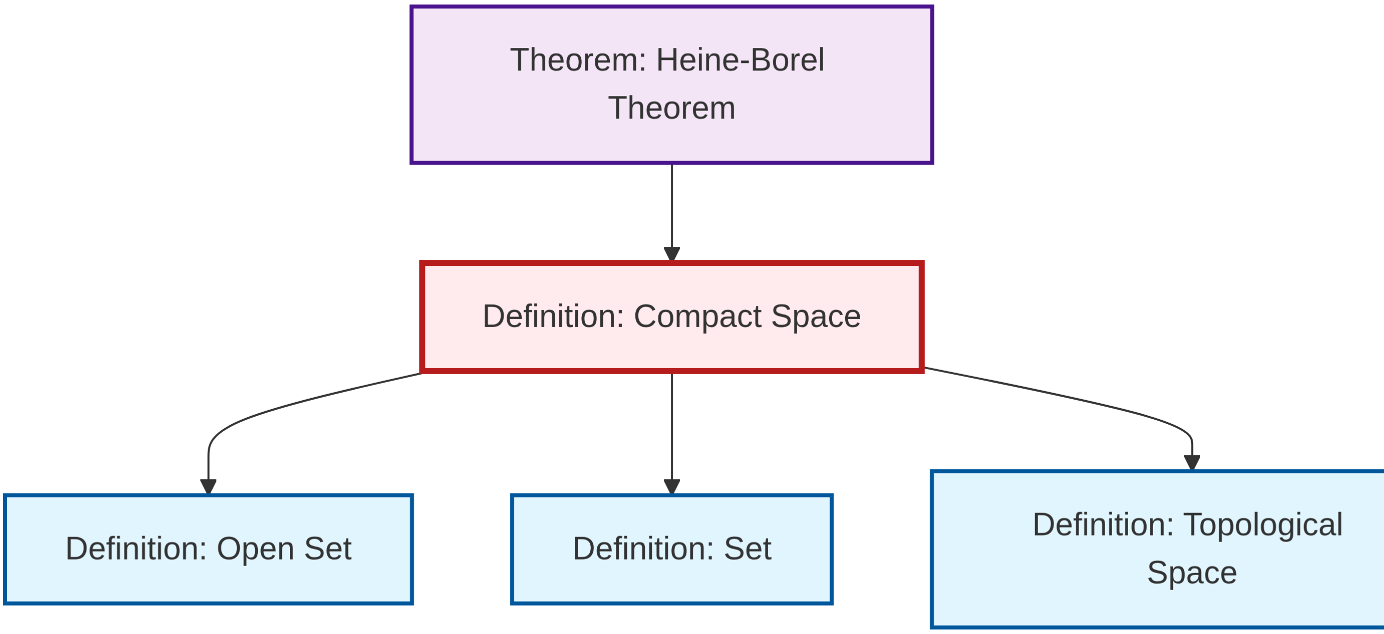
Properties

- Closed subsets of compact spaces are compact
- Compact subsets of Hausdorff spaces are closed
- The continuous image of a compact space is compact
- The product of compact spaces is compact (Tychonoff's theorem)

Examples

- Any finite topological space is compact
- The closed interval $[a, b]$ in $\mathbb{R}$ is compact (Heine-Borel theorem)
- The unit sphere S^n in $\mathbb{R}^{n+1}$ is compact

Dependency Graph



Local dependency graph