Definition: Sequence

A **sequence** is an ordered list of elements, typically numbers, indexed by the natural numbers. Formally, a sequence is a function from the natural numbers to a set.

Formal Definition

A sequence in a set S is a function:
$$a : \mathbb{N} \to S$$

We denote: - The sequence as $(a_n)_{n=1}^{\infty}$ or simply (a_n) - The n-th term as $a_n = a(n)$

Notation

Common notations for sequences: - $(a_n)_{n=1}^{\infty} = (a_1, a_2, a_3, ...)$ - $(a_n)_{n \in \mathbb{N}}$ - $\{a_n\}_{n=1}^{\infty}$ (though this can be confused with set notation)

Types of Sequences

By Domain

- **Infinite sequences**: Domain is all of $\mathbb{N}$
- **Finite sequences**: Domain is $\{1, 2, ..., N\}$ for some N

By Codomain

- **Real sequences**: $a_n \in \mathbb{R}$
- **Complex sequences**: $a_n \in \mathbb{C}$
- **Vector sequences**: $a_n \in \mathbb{R}^d$ or other vector spaces
- **Function sequences**: a_n are functions

Examples

1. **Arithmetic sequence**: $a_n = a_1 + (n-1)d$
    - Example: $(2, 5, 8, 11, ...)$ with $a_1 = 2$, $d = 3$
2. **Geometric sequence**: $a_n = a_1 \cdot r^{n-1}$
    - Example: $(3, 6, 12, 24, ...)$ with $a_1 = 3$, $r = 2$
3. **Harmonic sequence**: $a_n = \frac{1}{n}$
    - $(1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, ...)$
4. **Fibonacci sequence**: $a_1 = 1, a_2 = 1, a_n = a_{n-1} + a_{n-2}$
    - $(1, 1, 2, 3, 5, 8, 13, ...)$

Properties

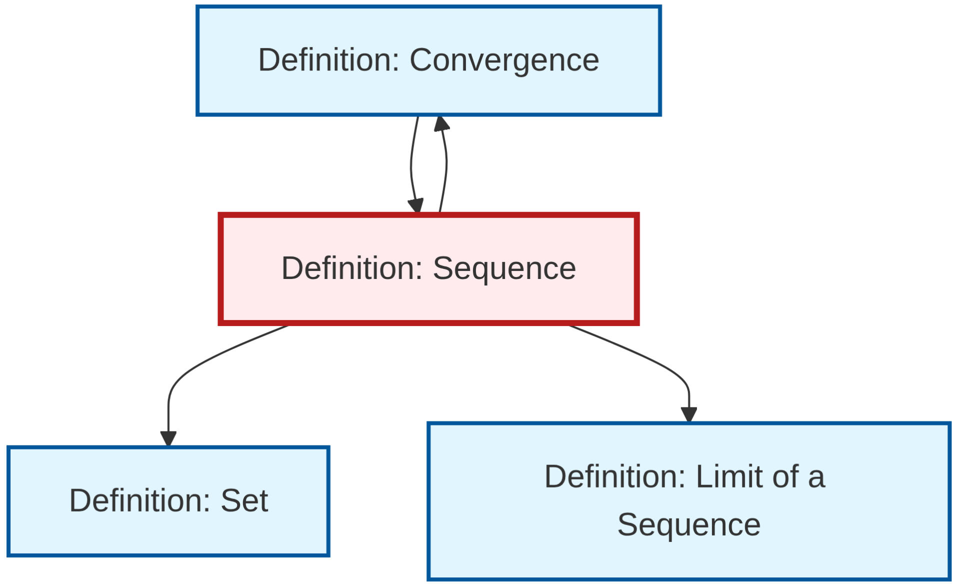
- **Bounded**: $\exists M > 0$ such that $|a_n| \leq M$ for all n
- **Monotonic**: Either increasing $(a_n \leq a_{n+1})$ or decreasing

- **Periodic**: $\exists p$ such that $a_{n+p} = a_n$ for all n
- **Cauchy**: $\forall \varepsilon > 0, \exists N$ such that $|a_m - a_n| < \varepsilon$ for all $m, n > N$

Related Concepts

- Convergence: When sequences approach a limit
- Limit of a Sequence: The value a convergent sequence approaches
- Series: Sum of sequence terms
- Subsequences: Sequences extracted from a sequence

Dependency Graph



Local dependency graph