

# Theorem: Angle Sum of a Triangle

The sum of the interior **Angles** of any **Triangle** equals  $180^\circ$  (or  $\pi$  radians).

## Statement

For any triangle  $\triangle ABC$  with interior angles  $\alpha$ ,  $\beta$ , and  $\gamma$ :

$$\alpha + \beta + \gamma = 180^\circ$$

## Proof (Euclidean Geometry)

Given triangle  $\triangle ABC$ :

1. Through vertex  $A$ , construct a **Line** parallel to side  $\overline{BC}$
2. This creates two additional angles adjacent to  $\angle BAC$
3. By the parallel postulate and properties of transversals:
  - One angle equals  $\angle ABC$  (alternate interior angles)
  - The other equals  $\angle BCA$  (alternate interior angles)
4. The three angles at vertex  $A$  form a straight angle, summing to  $180^\circ$
5. Therefore:  $\angle BAC + \angle ABC + \angle BCA = 180^\circ$

## Consequences

This theorem has several important implications:

1. **No triangle can have two right angles:** If two angles were  $90^\circ$  each, the third would be  $0^\circ$
2. **Exterior angle theorem:** An exterior angle equals the sum of the two non-adjacent interior angles
3. **Classification constraint:** In any triangle, at most one angle can be obtuse

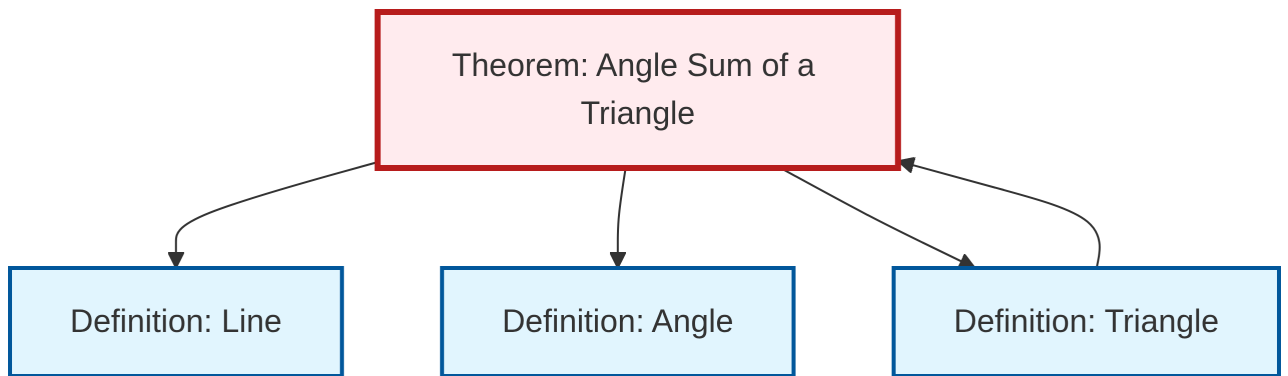
## Generalizations

- In hyperbolic geometry, the angle sum is less than  $180^\circ$
- In spherical geometry, the angle sum is greater than  $180^\circ$
- The difference from  $180^\circ$  relates to the curvature of the space

## Applications

- Navigation and surveying
- Determining if three angles can form a triangle
- Solving for unknown angles in geometric problems
- Foundation for trigonometry

## Dependency Graph



Local dependency graph