

# Theorem: Lagrange's Theorem

## Lagrange's Theorem

For a finite **Group**  $G$  and a **Subgroup**  $H$  of  $G$ , the order of  $H$  divides the order of  $G$ .

### Statement

If  $G$  is a finite group and  $H \leq G$ , then:

$$|G| = |H| \cdot [G : H]$$

where  $|G|$  denotes the order (number of elements) of  $G$ , and  $[G : H]$  is the index of  $H$  in  $G$  (the number of distinct left cosets of  $H$  in  $G$ ).

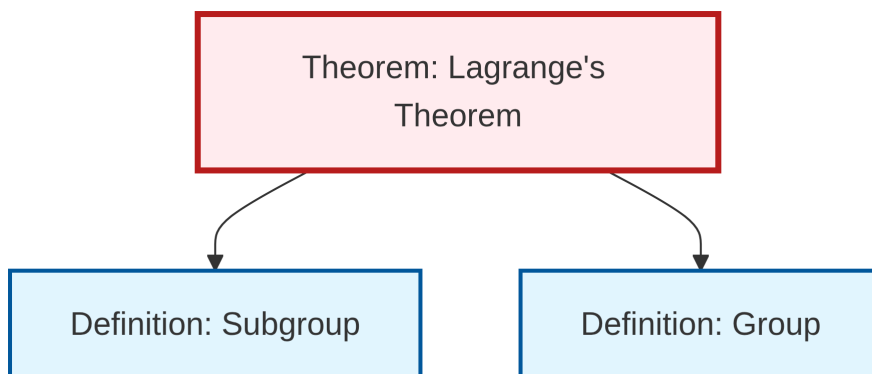
### Corollaries

1. The order of any element  $a \in G$  divides the order of  $G$
2. If  $|G|$  is prime, then  $G$  is cyclic and has no proper non-trivial subgroups
3. Any group of prime order is isomorphic to  $\mathbb{Z}_p$  for some prime  $p$

### Applications

Lagrange's theorem is fundamental in group theory and has numerous applications: - Determining possible subgroup structures - Proving Fermat's Little Theorem - Classifying groups of small order

### Dependency Graph



Local dependency graph