

## Definition: Hausdorff Space

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A [Topological Space](#)  $(X, \tau)$  is called a **Hausdorff space** (or  $T_2$  **space**) if for any two distinct points  $x, y \in X$  with  $x \neq y$ , there exist [open sets](#)  $U, V \in \tau$  such that:

1.  $x \in U$
2.  $y \in V$
3.  $U \cap V = \emptyset$

In other words, any two distinct points can be “separated” by disjoint open neighborhoods.

### Intuition

The Hausdorff property ensures that points in the space are “distinguishable” from each other using the topology. This is one of the most important separation axioms in topology.

### Properties

- **Uniqueness of limits:** In a Hausdorff space, sequences and nets have at most one limit
- **Closed points:** Every singleton set  $\{x\}$  is closed in a Hausdorff space
- **Diagonal property:** A space  $X$  is Hausdorff if and only if the diagonal  $\Delta = \{(x, x) : x \in X\}$  is closed in  $X \times X$  with the product topology

### Examples and Non-Examples

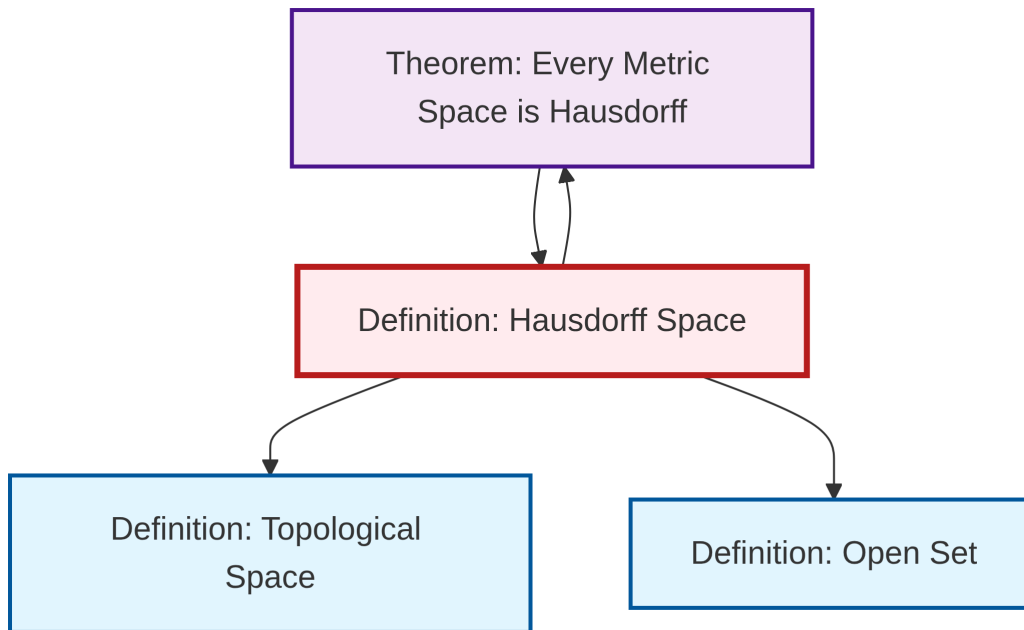
**Examples of Hausdorff spaces:** - Any metric space (see [Every Metric Space is Hausdorff](#)) - The real line  $\mathbb{R}$  with the standard topology - Any discrete space

**Non-examples:** - The cofinite topology on an infinite set - The Zariski topology on algebraic varieties (in general)

### See Also

- [Topological Space](#) - The basic structure on which this property is defined
- [Every Metric Space is Hausdorff](#) - Every metric space is Hausdorff
- [Compact Hausdorff space](#) - Compact Hausdorff spaces have especially nice properties

## Dependency Graph



Local dependency graph