Definition: Connected Space

Connected Space

A Topological Space is **connected** if it cannot be represented as the union of two disjoint non-empty Open Set sets.

Formal Definition

A topological space (X, τ) is connected if the only subsets of X that are both open and Closed Set are $\emptyset$ and X itself.

Equivalently, X is connected if and only if: - X cannot be written as $X = U \cup V$ where U, V are non-empty, disjoint open sets - There does not exist a continuous surjection $f : X \to \{0, 1\}$ (with discrete topology)

Disconnected Spaces

A space is disconnected if it can be written as $X = U \cup V$ where: - U, V are non-empty - $U \cap V = \emptyset$ - U, V are both open (or equivalently, both closed)

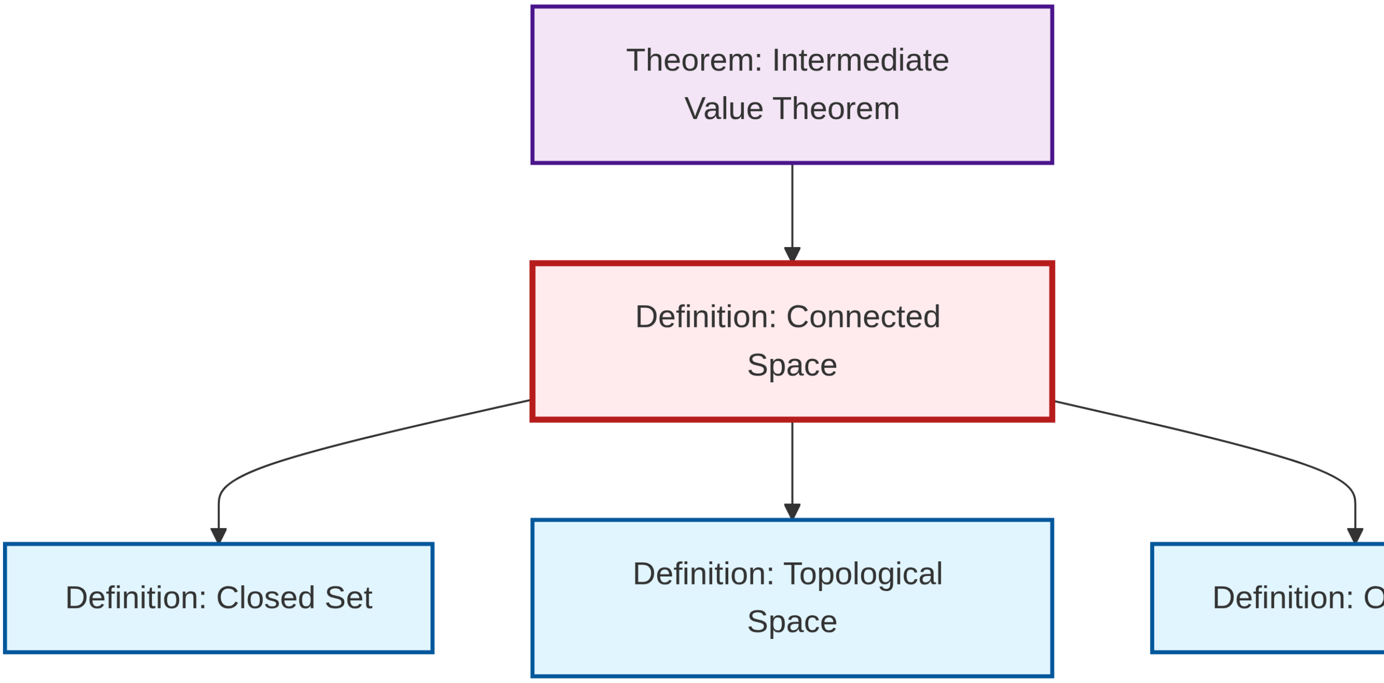
Properties

- The continuous image of a connected space is connected
- The closure of a connected set is connected
- If $\{A_i\}$ is a collection of connected sets with $\bigcap A_i \neq \emptyset$, then $\bigcup A_i$ is connected
- A space is connected if and only if every continuous function to $\{0, 1\}$ is constant

Examples

- The real line $\mathbb{R}$ is connected
- Any interval $[a, b]$, (a, b), $[a, b)$ in $\mathbb{R}$ is connected
- The space $\mathbb{R} \setminus \{0\}$ is disconnected

Dependency Graph



Local dependency graph