Definition: Intersection

The **intersection** of two Sets A and B, denoted $A \cap B$, is the set containing all elements that belong to both A and B.

Formal Definition

$$A \cap B = \{x : x \in A \text{ and } x \in B\}$$

Equivalently, using logical notation:

$$x \in A \cap B \iff (x \in A) \land (x \in B)$$

Properties

1. **Commutativity**: $A \cap B = B \cap A$
2. **Associativity**: $(A \cap B) \cap C = A \cap (B \cap C)$
3. **Identity**: $A \cap U = A$ (where U is the universal set)
4. **Annihilator**: $A \cap \emptyset = \emptyset$
5. **Idempotence**: $A \cap A = A$
6. **Absorption**: If $A \subseteq B$, then $A \cap B = A$

Generalized Intersection

For a non-empty collection of sets $\{A_i : i \in I\}$:

$$\bigcap_{i \in I} A_i = \{x : \forall i \in I, x \in A_i\}$$

Special cases: - Finite intersection: $\bigcap_{i=1}^{n} A_i = A_1 \cap A_2 \cap \cdots \cap A_n$ - Infinite intersection: $\bigcap_{i=1}^{\infty} A_i$

Disjoint Sets

Two sets A and B are **disjoint** if their intersection is empty:

$$A \cap B = \emptyset$$

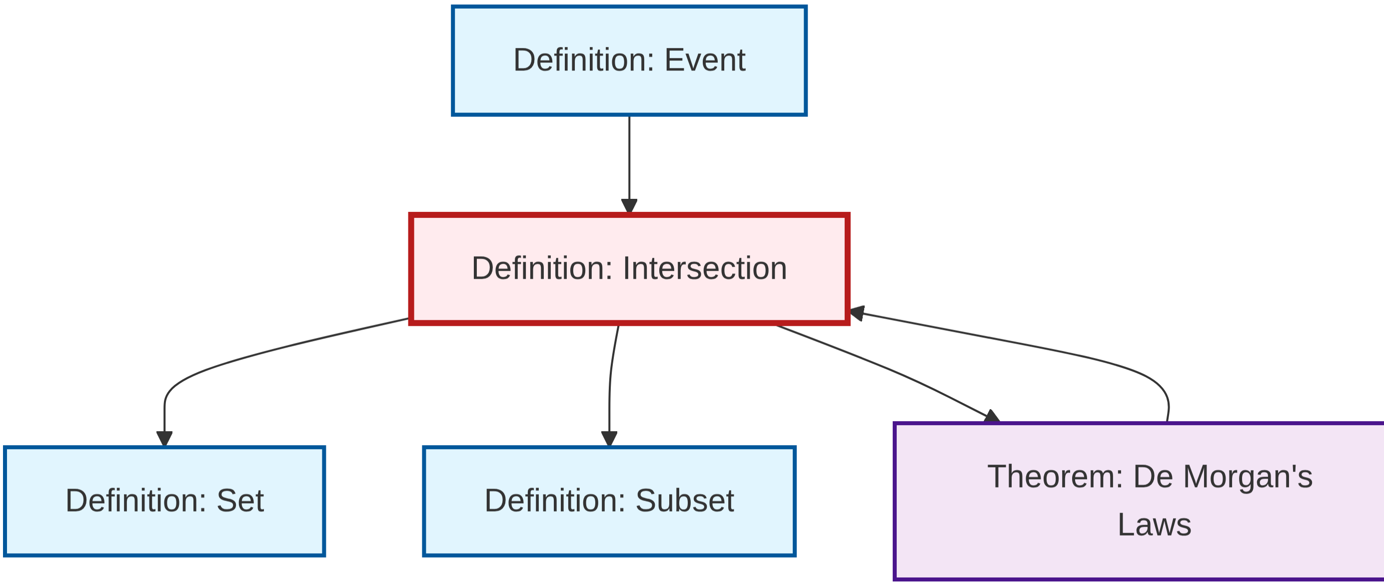
Examples

- $\{1, 2, 3\} \cap \{2, 3, 4\} = \{2, 3\}$
- $\mathbb{Z} \cap \mathbb{N} = \mathbb{N}$ (integers intersect naturals equals naturals)
- For intervals: $[0, 2] \cap [1, 3] = [1, 2]$
- Even and odd integers are disjoint: $2\mathbb{Z} \cap (2\mathbb{Z} + 1) = \emptyset$

Relationship with Other Operations

- De Morgan's Laws: $(A \cap B)^c = A^c \cup B^c$ (see De Morgan's Laws)
- Distributivity: $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
- With Subset: $A \cap B = A \iff A \subseteq B$

Dependency Graph



Local dependency graph