Definition: Metric Space

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A **metric space** is a Set X together with a function $d : X \times X \to \mathbb{R}$ called a **metric** or **distance function** that satisfies the following axioms for all $x, y, z \in X$:

Metric Axioms

1. **Non-negativity**: $d(x, y) \geq 0$
2. **Identity of indiscernibles**: $d(x, y) = 0$ if and only if $x = y$
3. **Symmetry**: $d(x, y) = d(y, x)$
4. **Triangle inequality**: $d(x, z) \leq d(x, y) + d(y, z)$

Notation

A metric space is denoted as the ordered pair (X, d) where: - X is the underlying set - d is the metric on X

Important Concepts

Given a metric space (X, d):

- **Open ball**: For $x \in X$ and $r > 0$, the open ball centered at x with radius r is $B(x, r) = \{y \in X : d(x, y) < r\}$
- **Closed ball**: $\overline{B}(x, r) = \{y \in X : d(x, y) \leq r\}$
- **Bounded set**: A subset $A \subseteq X$ is bounded if there exists $x \in X$ and $r > 0$ such that $A \subseteq B(x, r)$

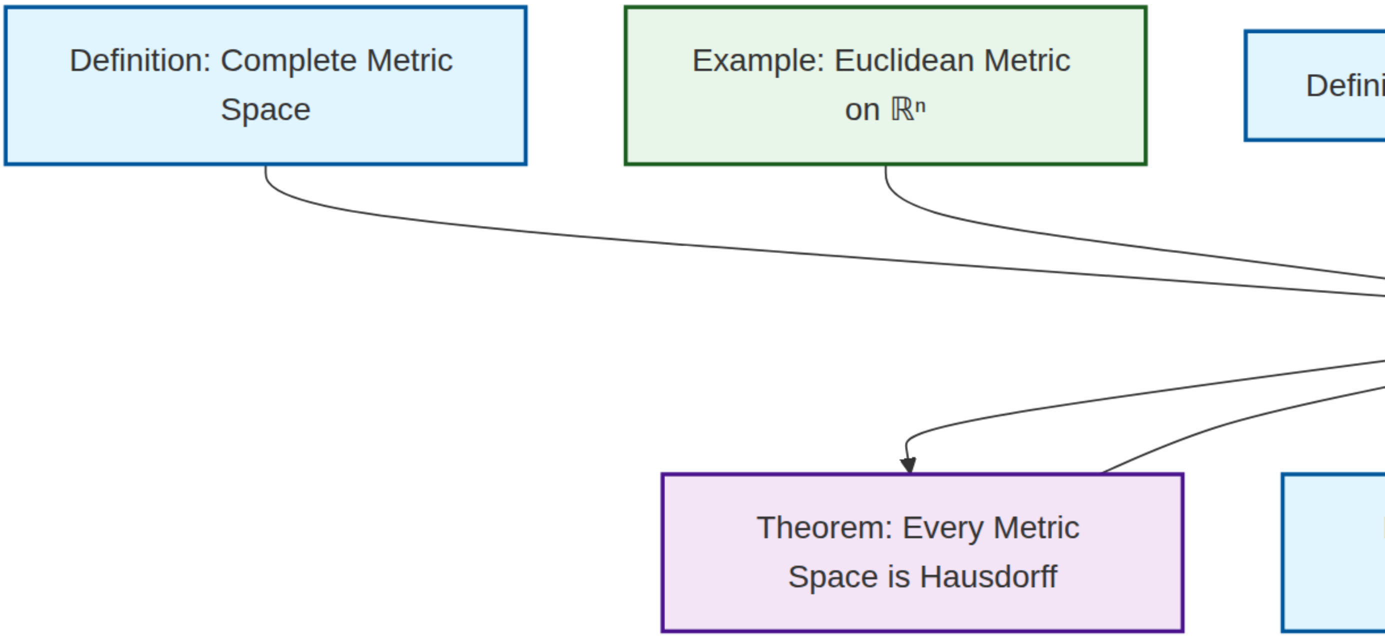
Relationship to Topology

Every metric space (X, d) induces a topology on X where the open sets are those sets that can be expressed as unions of open balls. This makes every metric space a Topological Space.

See Also

- Real Line with Standard Metric - The real line with the standard metric
- Continuity - Continuity can be defined in terms of metrics
- Every Metric Space is Hausdorff - Every metric space is a Hausdorff space

Dependency Graph



Local dependency graph