Definition: Combination

Combination

A **combination** is a selection of items from a Set where the order does not matter.

Formal Definition

Let S be a set with n elements. A k-combination of S is a subset of S with exactly k elements, where $0 \leq k \leq n$.

Notation and Formula

The number of k-combinations from a set of n elements is denoted: - $\binom{n}{k}$ (binomial coefficient) - $C(n,k)$ or ${}_nC_k$ - C_n^k

The formula is:

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

Properties

1. **Symmetry**: $\binom{n}{k} = \binom{n}{n-k}$
2. **Pascal's identity**: $\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$
3. **Boundary conditions**: $\binom{n}{0} = \binom{n}{n} = 1$
4. **Sum**: $\sum_{k=0}^{n} \binom{n}{k} = 2^n$

Relationship to Permutations

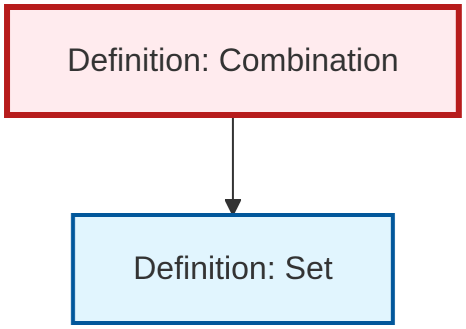
The number of k-permutations divided by $k!$ gives the number of k-combinations:

$$\binom{n}{k} = \frac{P(n,k)}{k!}$$

Applications

- Counting subsets of a given size
- Binomial theorem coefficients
- Probability calculations
- Combinatorial optimization

Dependency Graph



Local dependency graph