

## Definition: Closed Set

### Closed Set

In a [Topological Space](#), a **closed set** is a [Set](#) whose complement is an [Open Set](#).

### Formal Definition

Let  $(X, \tau)$  be a topological space. A subset  $C \subseteq X$  is closed if and only if its complement  $X \setminus C$  is open (i.e.,  $X \setminus C \in \tau$ ).

### Properties of Closed Sets

The collection of closed sets in a topological space  $(X, \tau)$  satisfies:

1. **Empty set and whole space:**  $\emptyset$  and  $X$  are closed
2. **Arbitrary intersections:** The intersection of any collection of closed sets is closed
3. **Finite unions:** The union of finitely many closed sets is closed

### Relationship with Open Sets

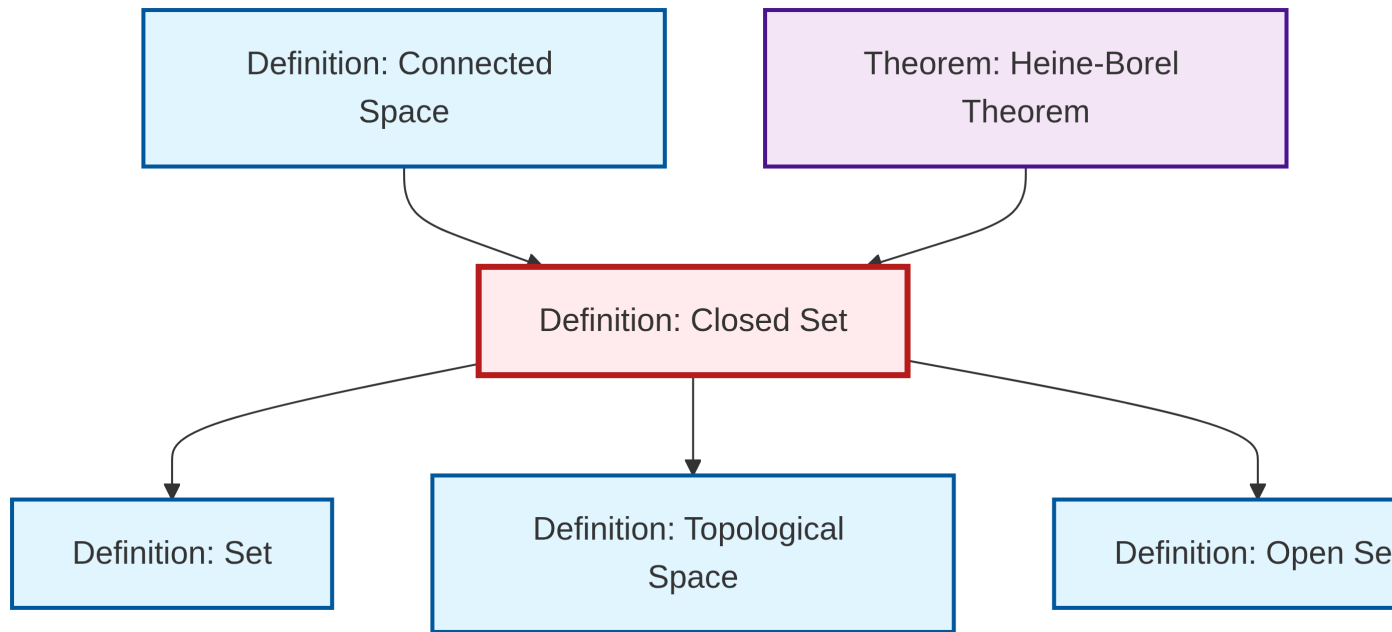
- A set can be both open and closed (clopen)
- A set can be neither open nor closed
- The closed sets form the dual notion to open sets

### Closure Operator

For any subset  $A \subseteq X$ , the closure  $\overline{A}$  is the smallest closed set containing  $A$ :

$$\overline{A} = \bigcap \{C : A \subseteq C \text{ and } C \text{ is closed}\}$$

## Dependency Graph



## Local dependency graph