

## Example: Standard Basis of

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The standard basis of  $\mathbb{R}^n$  is the most fundamental example of a [Basis](#) for a [Vector Space](#).

#### Definition

The standard basis of  $\mathbb{R}^n$  consists of the vectors: -  $e_1 = (1, 0, 0, \dots, 0)$  -  $e_2 = (0, 1, 0, \dots, 0)$  -  $e_3 = (0, 0, 1, \dots, 0)$  -  $\vdots$  -  $e_n = (0, 0, 0, \dots, 1)$

where  $e_i$  has a 1 in the  $i$ -th position and 0s elsewhere.

#### Verification as a Basis

To show that  $\{e_1, e_2, \dots, e_n\}$  is a basis, we verify:

##### 1. Linear Independence

The vectors are [Linear Independence](#). If  $c_1 e_1 + c_2 e_2 + \dots + c_n e_n = 0$ , then:

$$(c_1, c_2, \dots, c_n) = (0, 0, \dots, 0)$$

This implies  $c_1 = c_2 = \dots = c_n = 0$ .

##### 2. Spanning

The vectors [Span](#) all of  $\mathbb{R}^n$ . Any vector  $(x_1, x_2, \dots, x_n) \in \mathbb{R}^n$  can be written as:

$$(x_1, x_2, \dots, x_n) = x_1 e_1 + x_2 e_2 + \dots + x_n e_n$$

#### Special Cases

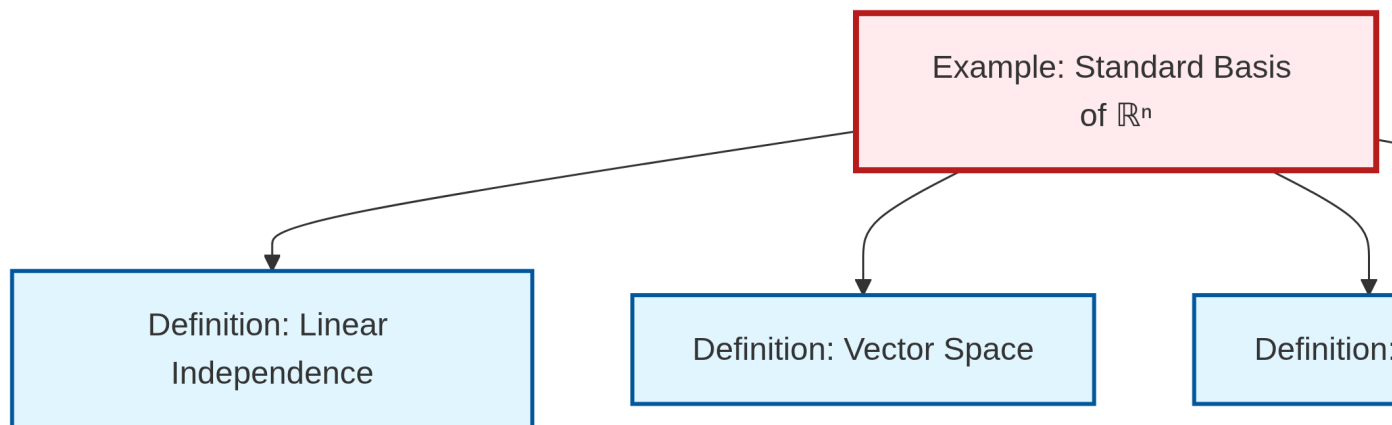
- $n = 2$ : The standard basis of  $\mathbb{R}^2$  is  $\{(1, 0), (0, 1)\}$  - the familiar unit vectors along the x and y axes
- $n = 3$ : The standard basis of  $\mathbb{R}^3$  is  $\{(1, 0, 0), (0, 1, 0), (0, 0, 1)\}$  - often denoted  $\{\mathbf{i}, \mathbf{j}, \mathbf{k}\}$  in physics

#### Properties

- The standard basis is **orthonormal** with respect to the standard inner product
- Any vector's coordinates in the standard basis are simply its components
- The matrix of any linear transformation in the standard basis is particularly simple to compute



## Dependency Graph



Local dependency graph