

Definition: Parallel Lines

Two **Lines** are **parallel** if they lie in the same plane and do not intersect, no matter how far they are extended.

Formal Definition

In Euclidean geometry, lines ℓ and m are parallel (denoted $\ell \parallel m$) if: 1. They are coplanar (lie in the same plane) 2. They have no **Points** in common: $\ell \cap m = \emptyset$

Characterizations

In Coordinate Geometry

Two lines in \mathbb{R}^2 are parallel if and only if they have the same slope: - Line 1: $y = m_1x + b_1$ - Line 2: $y = m_2x + b_2$ - Parallel iff $m_1 = m_2$ and $b_1 \neq b_2$

Using Direction Vectors

Lines with direction vectors \vec{v}_1 and \vec{v}_2 are parallel iff:

$$\vec{v}_1 = k\vec{v}_2 \text{ for some } k \neq 0$$

Euclid's Parallel Postulate

The famous fifth postulate states: > Through a point not on a given line, there exists exactly one line parallel to the given line.

This postulate: - Cannot be proven from other axioms - Distinguishes Euclidean from non-Euclidean geometries

Properties

1. **Reflexivity**: Every line is parallel to itself (by convention)
2. **Symmetry**: If $\ell \parallel m$, then $m \parallel \ell$
3. **Transitivity**: If $\ell \parallel m$ and $m \parallel n$, then $\ell \parallel n$
4. **Equal distance**: Parallel lines maintain constant perpendicular distance

Parallel Lines and Transversals

When a transversal crosses two parallel lines: - **Corresponding angles** are equal - **Alternate interior angles** are equal - **Co-interior angles** are supplementary (sum to 180°)

Examples

1. **Railroad tracks**: Classic real-world example of parallel lines
2. **Coordinate axes translations**: $y = 2$ and $y = 5$ are parallel
3. **Opposite sides of a rectangle**: Always parallel by definition

Non-Euclidean Variations

Hyperbolic Geometry

Through a point not on a line, there exist infinitely many lines parallel to the given line.

Spherical Geometry

No parallel lines exist - all great circles intersect.

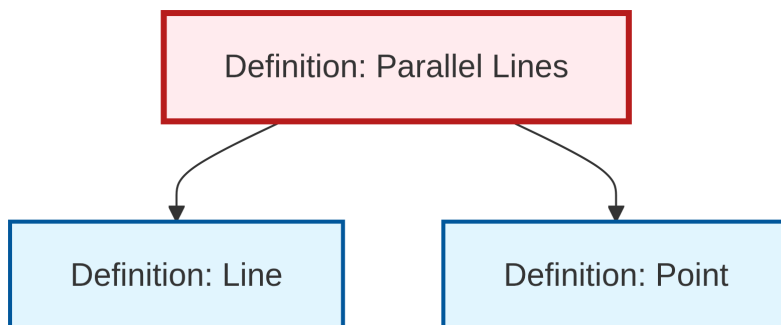
Related Concepts

- **Perpendicular lines:** Intersect at 90°
- **Skew lines:** Non-parallel lines in 3D that don't intersect
- **Parallel planes:** Extension to higher dimensions
- **Parallelogram:** Quadrilateral with opposite sides parallel

Applications

- **Architecture:** Ensuring structural stability
- **Computer graphics:** Ray tracing and collision detection
- **Navigation:** Latitude lines are parallel
- **Linear algebra:** Parallel vectors and subspaces

Dependency Graph



Local dependency graph