

## Definition: Binomial Coefficient

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The **binomial coefficient**  $\binom{n}{k}$  (read “n choose k”) counts the number of ways to choose  $k$  objects from  $n$  objects without regard to order.

### Formal Definition

For non-negative integers  $n$  and  $k$  with  $k \leq n$ , the binomial coefficient is defined as:

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

where  $n!$  denotes the factorial of  $n$ .

### Alternative Formulations

1. **In terms of Combination:**  $\binom{n}{k} = C(n, k)$  - the number of  $k$ -combinations from  $n$  elements
2. **In terms of Permutation:**  $\binom{n}{k} = \frac{P(n, k)}{k!}$  where  $P(n, k)$  is the number of  $k$ -permutations
3. **Recursive definition:**

$$\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$$

with base cases  $\binom{n}{0} = \binom{n}{n} = 1$

### Special Values

- $\binom{n}{0} = 1$  (one way to choose nothing)
- $\binom{n}{1} = n$  ( $n$  ways to choose one item)
- $\binom{n}{n} = 1$  (one way to choose everything)
- $\binom{n}{k} = 0$  if  $k > n$  (cannot choose more items than available)

### Properties

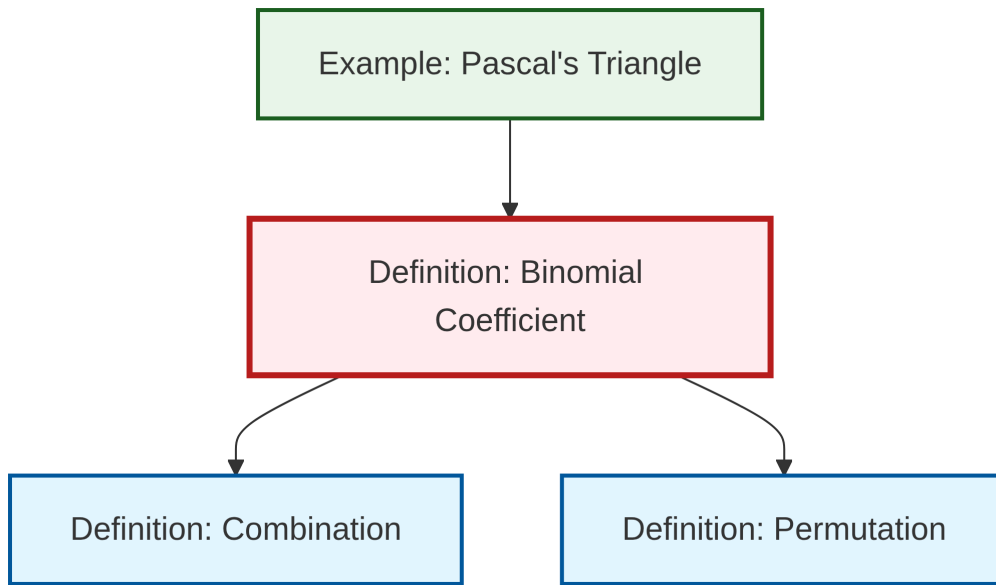
1. **Symmetry:**  $\binom{n}{k} = \binom{n}{n-k}$
2. **Pascal's Identity:**  $\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$
3. **Binomial Theorem:**  $(x + y)^n = \sum_{k=0}^n \binom{n}{k} x^k y^{n-k}$

## Applications

- Counting subsets of a given size
- Expansion of binomial expressions
- Probability calculations in discrete distributions
- Pascal's triangle construction



## Dependency Graph



Local dependency graph