Example: The Category of Sets

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The category **Set** is one of the most fundamental examples in Category theory, with sets as objects and functions as morphisms.

Definition

The category **Set** consists of: - **Objects**: All Set s - **Morphisms**: Functions between sets - **Composition**: Function composition - **Identity**: The identity function on each set

Verification of Category Axioms

We verify that **Set** satisfies the axioms of a category:

1. Composition is Associative

For functions $f : A \to B$, $g : B \to C$, and $h : C \to D$:

$$(h \circ g) \circ f = h \circ (g \circ f)$$

This holds because function composition is associative: for any $x \in A$,

$$((h \circ g) \circ f)(x) = h(g(f(x))) = (h \circ (g \circ f))(x)$$

2. Identity Laws

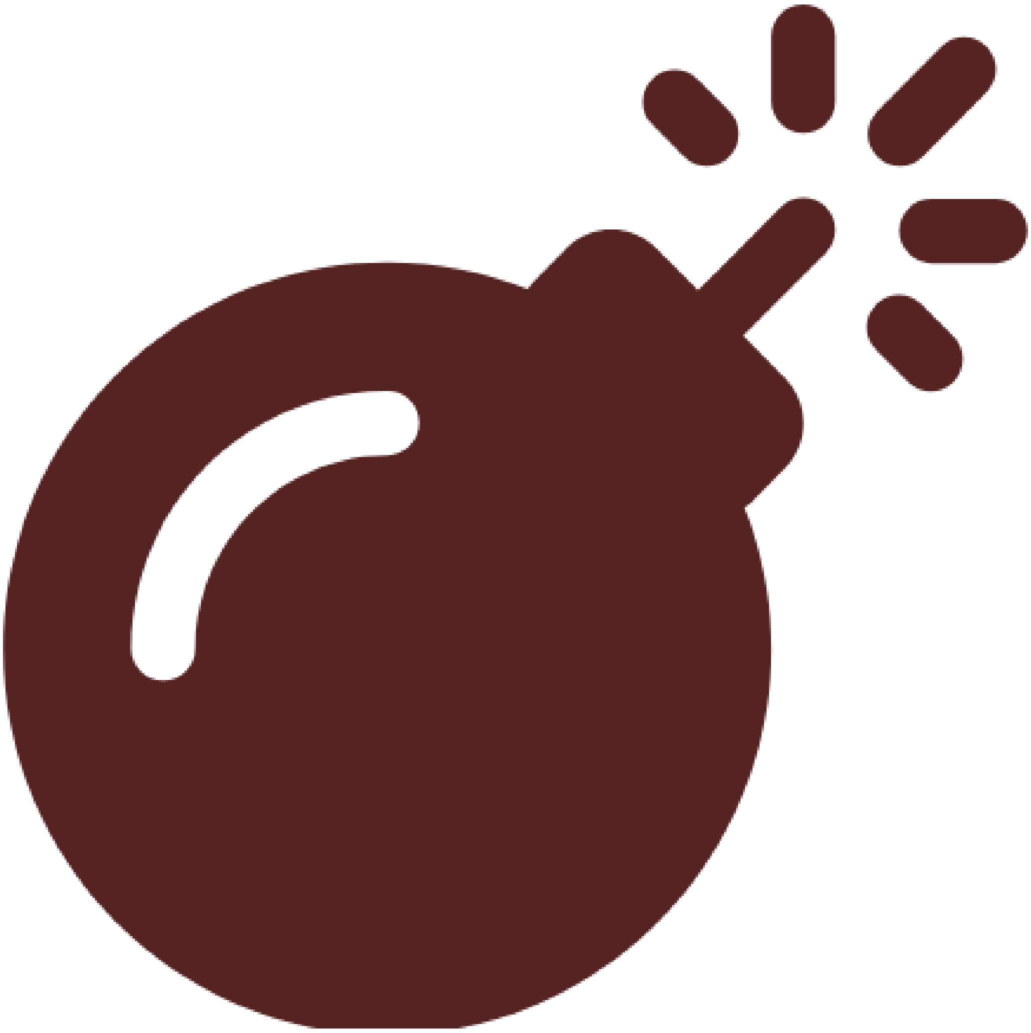
For each set A, the identity function $\mathrm{id}_A : A \to A$ defined by $\mathrm{id}_A(x) = x$ satisfies: - For any $f : A \to B$: $f \circ \mathrm{id}_A = f$ - For any $g : B \to A$: $\mathrm{id}_A \circ g = g$

Properties of Set

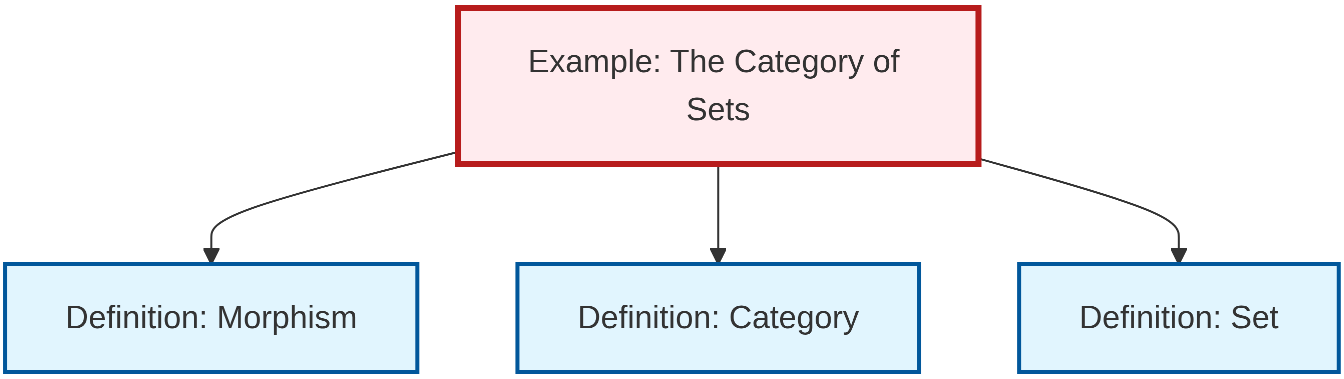
1. **Size**: Set is a **large category** - its collection of objects forms a proper class, not a set
2. **Special Morphisms**:
   - Monomorphisms in Set are exactly the injective functions
   - Epimorphisms in Set are exactly the surjective functions
   - Isomorphisms in Set are exactly the bijective functions
3. **Initial and Terminal Objects**:
   - The empty set $\emptyset$ is the initial object (unique function from $\emptyset$ to any set)
   - Any singleton set $\{*\}$ is a terminal object (unique function from any set to $\{*\}$)

Related Categories

- **FinSet**: The category of finite sets
- **Set**$*$ The category of pointed sets (sets with a distinguished element)



Dependency Graph



Local dependency graph