Definition: Parallel Lines

Two Lines are **parallel** if they lie in the same plane and do not intersect, no matter how far they are extended.

Formal Definition

In Euclidean geometry, lines ℓ and m are parallel (denoted $\ell \parallel m$) if: 1. They are coplanar (lie in the same plane) 2. They have no Points in common: $\ell \cap m = \emptyset$

Characterizations

In Coordinate Geometry

Two lines in \mathbb{R}^2 are parallel if and only if they have the same slope: - Line 1: $y=m_1x+b_1$ - Line 2: $y=m_2x+b_2$ - Parallel iff $m_1=m_2$ and $b_1\neq b_2$

Using Direction Vectors

Lines with direction vectors \vec{v}_1 and \vec{v}_2 are parallel iff:

$$\vec{v}_1 = k\vec{v}_2$$
 for some $k \neq 0$

Euclid's Parallel Postulate

The famous fifth postulate states: > Through a point not on a given line, there exists exactly one line parallel to the given line.

This postulate: - Cannot be proven from other axioms - Distinguishes Euclidean from non-Euclidean geometries

Properties

- 1. **Reflexivity**: Every line is parallel to itself (by convention)
- 2. Symmetry: If $\ell \parallel m$, then $m \parallel \ell$
- 3. Transitivity: If $\ell \parallel m$ and $m \parallel n$, then $\ell \parallel n$
- 4. Equal distance: Parallel lines maintain constant perpendicular distance

Parallel Lines and Transversals

When a transversal crosses two parallel lines: - Corresponding angles are equal - Alternate interior angles are equal - Co-interior angles are supplementary (sum to 180°)

Examples

- 1. Railroad tracks: Classic real-world example of parallel lines
- 2. Coordinate axes translations: y = 2 and y = 5 are parallel
- 3. Opposite sides of a rectangle: Always parallel by definition

Non-Euclidean Variations

Hyperbolic Geometry

Through a point not on a line, there exist infinitely many lines parallel to the given line.

Spherical Geometry

No parallel lines exist - all great circles intersect.

Related Concepts

• Perpendicular lines: Intersect at 90°

• Skew lines: Non-parallel lines in 3D that don't intersect

• Parallel planes: Extension to higher dimensions

• Parallelogram: Quadrilateral with opposite sides parallel

Applications

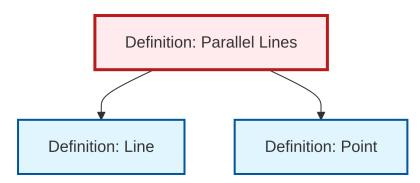
• Architecture: Ensuring structural stability

• Computer graphics: Ray tracing and collision detection

• Navigation: Latitude lines are parallel

• Linear algebra: Parallel vectors and subspaces

Dependency Graph



Local dependency graph