

# Definition: Ring

## Ring

A **ring** is a [Set](#) equipped with two [Binary Operation](#) operations that generalize the arithmetic of integers.

### Formal Definition

A ring  $(R, +, \cdot)$  consists of a set  $R$  together with two binary operations, addition  $(+)$  and multiplication  $(\cdot)$ , such that:

#### Addition Axioms

1.  $(R, +)$  is an abelian [Group](#):
  - **Associativity:**  $(a + b) + c = a + (b + c)$
  - **Commutativity:**  $a + b = b + a$
  - **Identity:** There exists  $0 \in R$  such that  $a + 0 = a$
  - **Inverses:** For each  $a \in R$ , there exists  $-a \in R$  such that  $a + (-a) = 0$

#### Multiplication Axioms

2. Multiplication is associative:  $(a \cdot b) \cdot c = a \cdot (b \cdot c)$

#### Distributive Laws

3. Left distributivity:  $a \cdot (b + c) = a \cdot b + a \cdot c$
4. Right distributivity:  $(a + b) \cdot c = a \cdot c + b \cdot c$

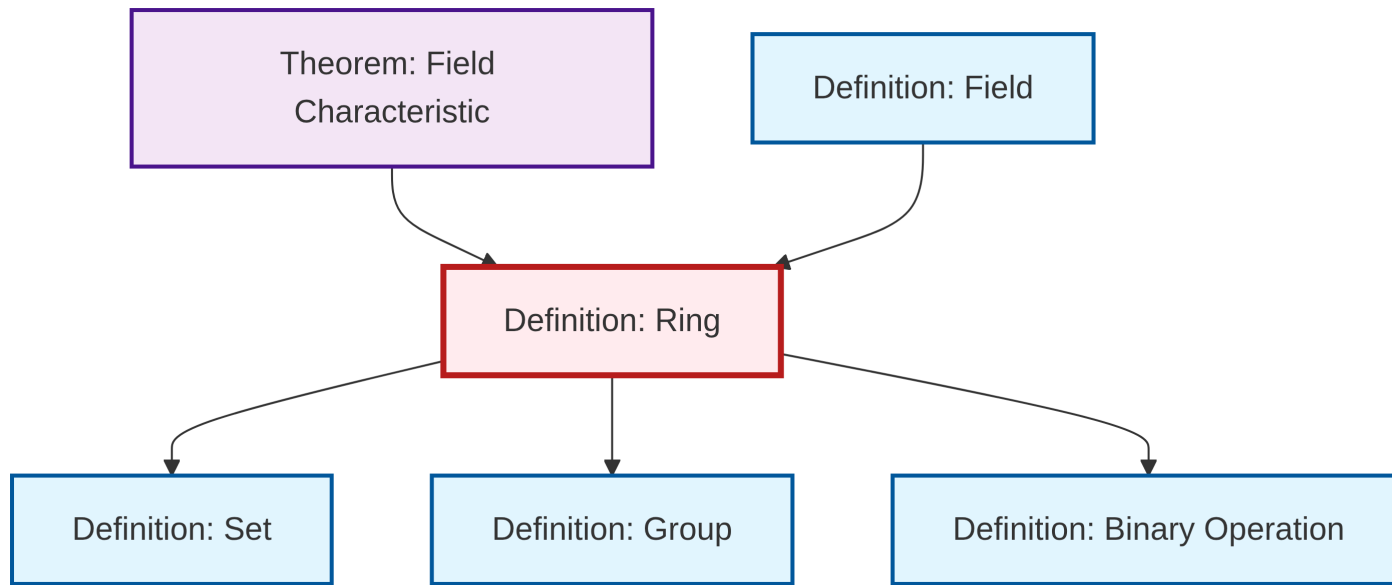
## Types of Rings

- **Commutative ring:**  $a \cdot b = b \cdot a$  for all  $a, b$
- **Ring with unity:** There exists  $1 \in R$  such that  $1 \cdot a = a \cdot 1 = a$
- **Integral domain:** Commutative ring with unity and no zero divisors
- **Field:** Commutative ring where every non-zero element has a multiplicative inverse

## Examples

- $(\mathbb{Z}, +, \cdot)$  - integers under addition and multiplication
- $M_n(\mathbb{R})$  -  $n \times n$  matrices with real entries
- $\mathbb{Z}[x]$  - polynomials with integer coefficients

## Dependency Graph



Local dependency graph