Definition: Line

A **line** is a one-dimensional geometric object that extends infinitely in both directions. It is completely determined by any two distinct Points that lie on it.

Formal Definition

In different mathematical contexts:

1. **Euclidean Geometry**: A line is an undefined primitive satisfying Euclid's axioms, particularly:
   - Through any two distinct points, there exists exactly one line
   - A line contains infinitely many points
2. **Coordinate Geometry**: In $\mathbb{R}^2$, a line is the set of all points (x, y) satisfying:

   $$ax + by + c = 0$$

   where a, b, $c \in \mathbb{R}$ and $(a, b) \neq (0, 0)$
3. **Vector Spaces**: A line through point P in direction $\vec{v}$ is:

   $$\ell = \{P + t\vec{v} : t \in \mathbb{R}\}$$

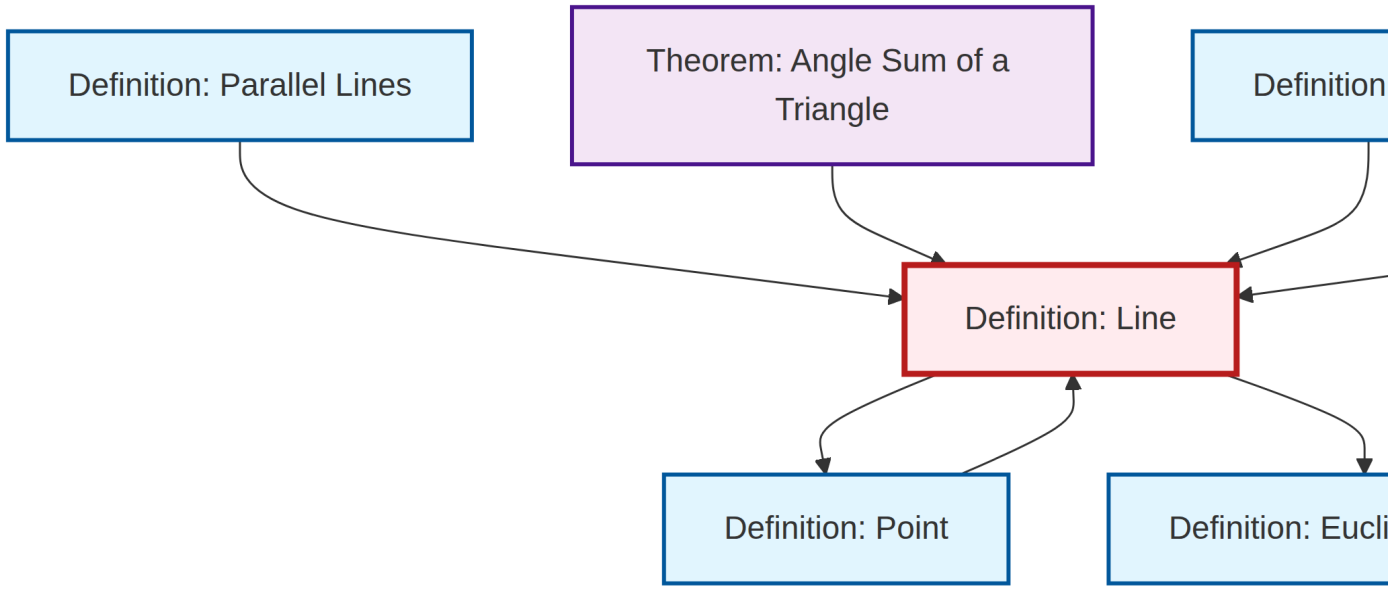
Properties

- Lines have no thickness or width
- Any two distinct lines in a plane either intersect at exactly one point or are parallel
- In Euclidean Space, the shortest path between two points lies along the line connecting them

Related Concepts

- **Line segment**: The portion of a line between two points
- **Ray**: A half-line starting at a point and extending infinitely in one direction
- **Parallel lines**: Lines in the same plane that never intersect
- **Perpendicular lines**: Lines that intersect at a 90° angle

Dependency Graph



Local dependency graph