Definition: Matrix

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A **matrix** over a field F is a rectangular array of elements from F arranged in rows and columns.

Notation

An $m \times n$ matrix A has m rows and n columns, and is written as:

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}$$

where $a_{ij} \in F$ is the entry in the i-th row and j-th column.

Alternative Notations

- Compact form: $A = (a_{ij})_{m \times n}$ or $A = [a_{ij}]$
- The set of all $m \times n$ matrices over F is denoted $M_{m \times n}(F)$ or $F^{m \times n}$

Special Types of Matrices

- **Square matrix**: When $m = n$
- **Row vector**: A $1 \times n$ matrix
- **Column vector**: An $m \times 1$ matrix
- **Zero matrix**: All entries are 0, denoted O or $0_{m \times n}$
- **Identity matrix**: Square matrix with 1's on the diagonal and 0's elsewhere, denoted I_n

Matrix Operations

1. **Addition**: $(A + B)_{ij} = a_{ij} + b_{ij}$ (for matrices of the same size)
2. **Scalar multiplication**: $(cA)_{ij} = c \cdot a_{ij}$ for $c \in F$
3. **Matrix multiplication**: For $A \in M_{m \times n}(F)$ and $B \in M_{n \times p}(F)$:

$$(AB)_{ij} = \sum_{k=1}^{n} a_{ik} b_{kj}$$

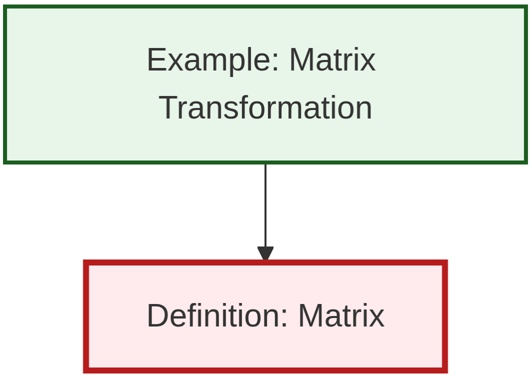
Transpose

The **transpose** of an $m \times n$ matrix A is the $n \times m$ matrix A^T where:

$$(A^T)_{ij} = a_{ji}$$

Matrices are fundamental objects in linear algebra, representing linear transformations, systems of equations, and bilinear forms.

Dependency Graph



Local dependency graph