

## Example: Real Line with Standard Metric

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The real numbers  $\mathbb{R}$  with the standard metric form one of the most fundamental examples of a [Metric Space](#).

#### Definition

The **standard metric** on  $\mathbb{R}$  is defined by:

$$d(x, y) = |x - y|$$

for all  $x, y \in \mathbb{R}$ , where  $|x - y|$  denotes the absolute value of  $x - y$ .

#### Verification of Metric Axioms

We verify that  $d$  satisfies all four metric axioms:

1. **Non-negativity:** Since absolute value is always non-negative,  $d(x, y) = |x - y| \geq 0$  for all  $x, y \in \mathbb{R}$ .
2. **Identity of indiscernibles:**
  - If  $x = y$ , then  $d(x, y) = |x - y| = |0| = 0$
  - If  $d(x, y) = 0$ , then  $|x - y| = 0$ , which implies  $x - y = 0$ , so  $x = y$
3. **Symmetry:**  $d(x, y) = |x - y| = |-(y - x)| = |y - x| = d(y, x)$
4. **Triangle inequality:** For any  $x, y, z \in \mathbb{R}$ :

$$d(x, z) = |x - z| = |(x - y) + (y - z)| \leq |x - y| + |y - z| = d(x, y) + d(y, z)$$

#### Open Balls

In this metric space, the open ball  $B(a, r)$  centered at  $a$  with radius  $r > 0$  is:

$$B(a, r) = \{x \in \mathbb{R} : |x - a| < r\} = (a - r, a + r)$$

This is simply the open interval of length  $2r$  centered at  $a$ .

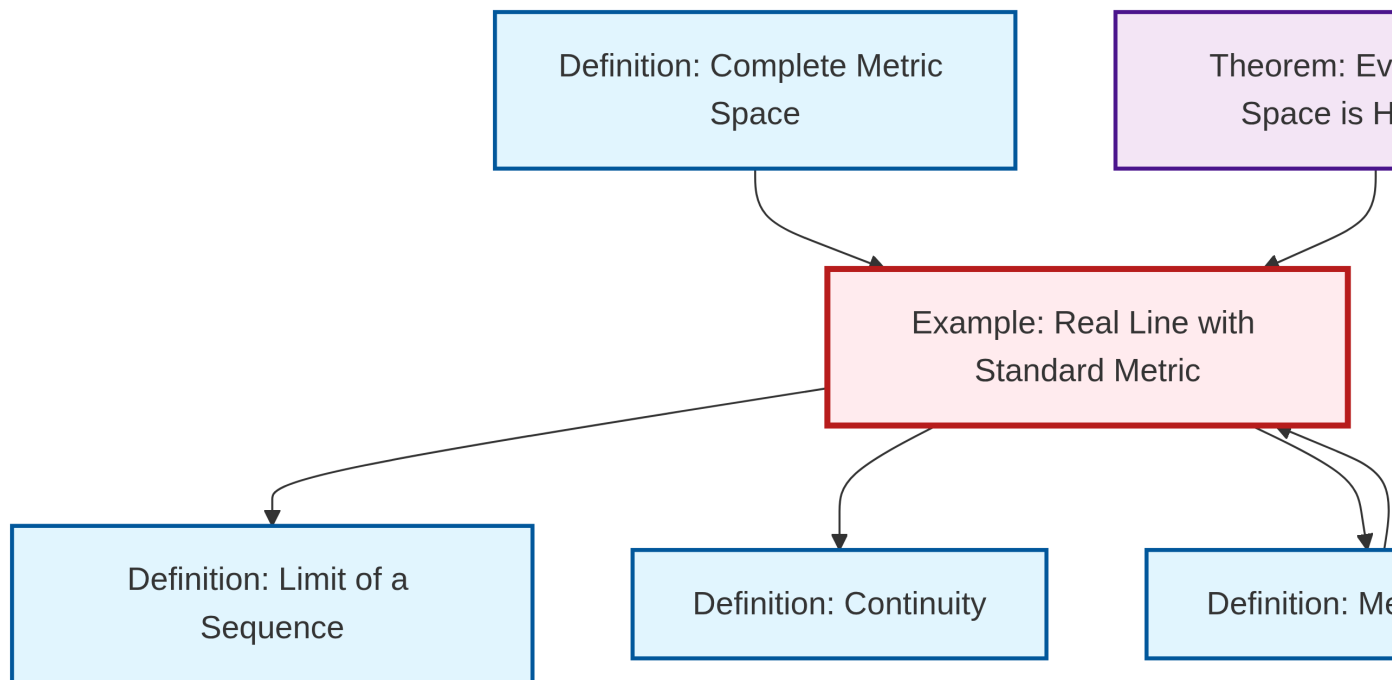
#### Properties

- The metric topology induced by  $d$  is the standard topology on  $\mathbb{R}$
- $(\mathbb{R}, d)$  is a **complete metric space**: every Cauchy sequence converges
- The space is **separable**: the rationals  $\mathbb{Q}$  form a countable dense subset
- It is **connected** but not **compact**

## See Also

- [Limit of a Sequence](#) - Limits in calculus are defined using this metric
- [Continuity](#) - Continuous functions preserve this metric structure

## Dependency Graph



Local dependency graph