Definition: Permutation

Permutation

A **permutation** of a Set is a bijective function from the set to itself.

Formal Definition

Let S be a set. A permutation of S is a bijection $\sigma : S \to S$.

For a finite set $S = \{1, 2, ..., n\}$, a permutation can be represented as a rearrangement of the elements.

Notation

Permutations can be written in several ways:

1. **Two-line notation**:
$$\sigma = \begin{pmatrix} 1 & 2 & 3 & ... & n \\ \sigma(1) & \sigma(2) & \sigma(3) & ... & \sigma(n) \end{pmatrix}$$
2. **One-line notation**: $\sigma = [\sigma(1), \sigma(2), ..., \sigma(n)]$
3. **Cycle notation**: $(a_1\ a_2\ ...\ a_k)$ means $a_1 \mapsto a_2 \mapsto ... \mapsto a_k \mapsto a_1$

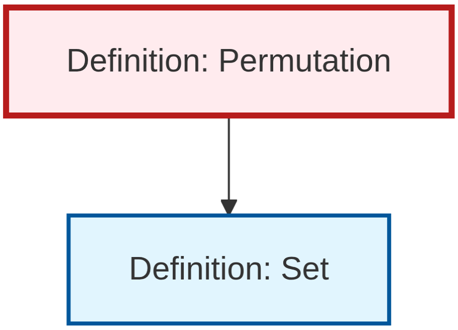
Properties

- The set of all permutations of n elements forms the symmetric group S_n
- The number of permutations of n distinct elements is $n!$
- Every permutation can be decomposed into disjoint cycles
- Every permutation can be expressed as a product of transpositions

Examples

For $S = \{1, 2, 3\}$: - Identity: (1)(2)(3) - Transposition: (1 2)(3) - 3-cycle: (1 2 3)

Dependency Graph



Local dependency graph