Definition: Subset

A Set A is a **subset** of a set B, denoted $A \subseteq B$, if every element of A is also an element of B.

Formal Definition

$$A \subseteq B \iff \forall x (x \in A \implies x \in B)$$

In words: A is a subset of B if and only if for all x, if x is in A, then x is in B.

Notation and Terminology

- $A \subseteq B$: "A is a subset of B" or "A is contained in B"
- $B \supseteq A$: "B is a superset of A" or "B contains A"
- $A \subset B$: "A is a proper subset of B" (when $A \subseteq B$ and $A \neq B$)
- $A \not\subseteq B$: "A is not a subset of B"

Properties

1. **Reflexivity**: Every set is a subset of itself: $A \subseteq A$
2. **Transitivity**: If $A \subseteq B$ and $B \subseteq C$, then $A \subseteq C$
3. **Antisymmetry**: If $A \subseteq B$ and $B \subseteq A$, then $A = B$
4. **Empty set**: The empty set $\emptyset$ is a subset of every set

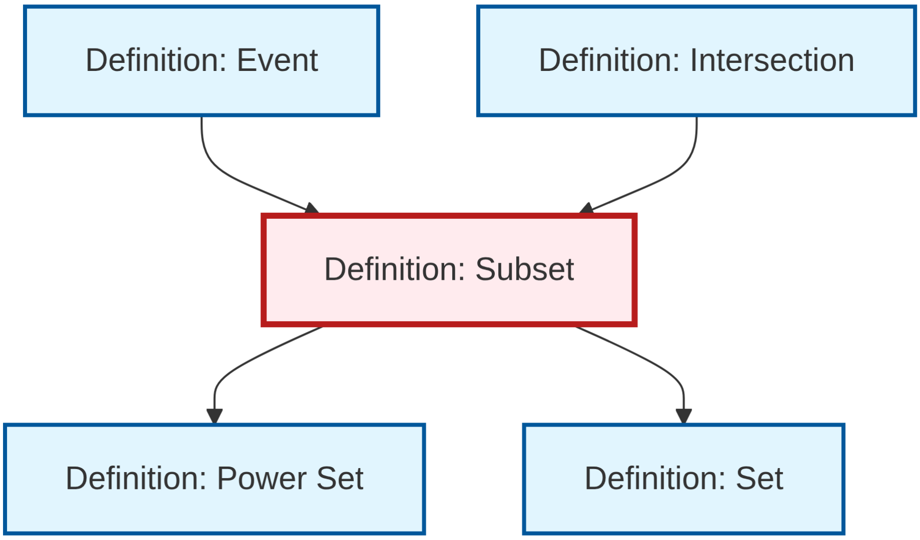
Examples

- $\{1, 2\} \subseteq \{1, 2, 3, 4\}$
- $\mathbb{N} \subseteq \mathbb{Z} \subseteq \mathbb{Q} \subseteq \mathbb{R} \subseteq \mathbb{C}$
- For any set A: $\emptyset \subseteq A \subseteq A$

Related Concepts

- Power Set: The set of all subsets of a given set
- Set equality can be proven by showing mutual subset relations
- The subset relation defines a partial order on the power set

Dependency Graph



Local dependency graph