

# Definition: Category

## Category

A **category** is a mathematical structure consisting of objects and morphisms (arrows) between them, with a composition operation satisfying certain axioms.

### Formal Definition

A category  $\mathcal{C}$  consists of:

1. A collection of **objects**, denoted  $\text{Ob}(\mathcal{C})$
2. For each pair of objects  $A, B$ , a **Set** of **morphisms** (or arrows) from  $A$  to  $B$ , denoted  $\text{Hom}_{\mathcal{C}}(A, B)$
3. For each triple of objects  $A, B, C$ , a **composition operation**:

$$\circ : \text{Hom}(B, C) \times \text{Hom}(A, B) \rightarrow \text{Hom}(A, C)$$

satisfying:

### Axioms

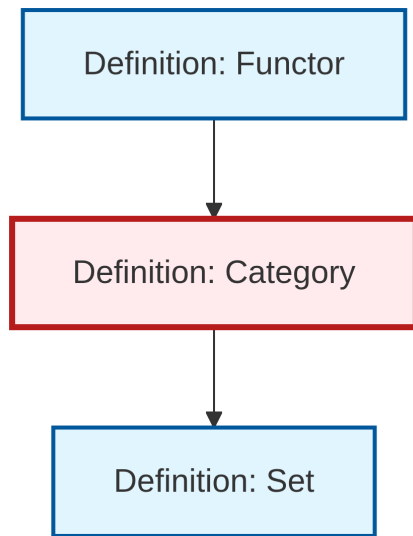
1. **Identity**: For each object  $A$ , there exists an identity morphism  $\text{id}_A \in \text{Hom}(A, A)$  such that:
  - $f \circ \text{id}_A = f$  for all  $f : A \rightarrow B$
  - $\text{id}_A \circ g = g$  for all  $g : B \rightarrow A$
2. **Associativity**: For morphisms  $f : A \rightarrow B, g : B \rightarrow C, h : C \rightarrow D$ :

$$h \circ (g \circ f) = (h \circ g) \circ f$$

### Examples

- **Set**: Objects are sets, morphisms are functions
- **Grp**: Objects are groups, morphisms are group homomorphisms
- **Top**: Objects are topological spaces, morphisms are continuous functions
- **Vect**: Objects are vector spaces, morphisms are linear transformations

## Dependency Graph



Local dependency graph