Definition: Group

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A **group** is a set G together with a binary operation $\star : G \times G \to G$ that satisfies the following axioms:

Group Axioms

1. **Associativity**: For all $a, b, c \in G$,

$$(a \star b) \star c = a \star (b \star c)$$

2. **Identity element**: There exists an element $e \in G$ such that for all $a \in G$,

$$a \star e = e \star a = a$$

3. **Inverse elements**: For each $a \in G$, there exists an element $b \in G$ such that

$$a \star b = b \star a = e$$

   where e is the identity element. We denote this inverse as a^{-1}.

Notation

- A group is often denoted as $(G, \star)$ to explicitly show both the set and the operation
- When the operation is clear from context, we may simply write G
- The operation symbol is often omitted: ab instead of $a \star b$

Important Properties

From these axioms, we can prove:

- The identity element is unique
- Each element has a unique inverse
- Cancellation laws hold: if $ab = ac$ then $b = c$

Examples

- $(\mathbb{Z}, +)$: The integers under addition
- $(\mathbb{Q} \setminus \{0\}, \cdot)$: The non-zero rationals under multiplication
- $(GL_n(\mathbb{R}), \cdot)$: The invertible $n \times n$ matrices under matrix multiplication

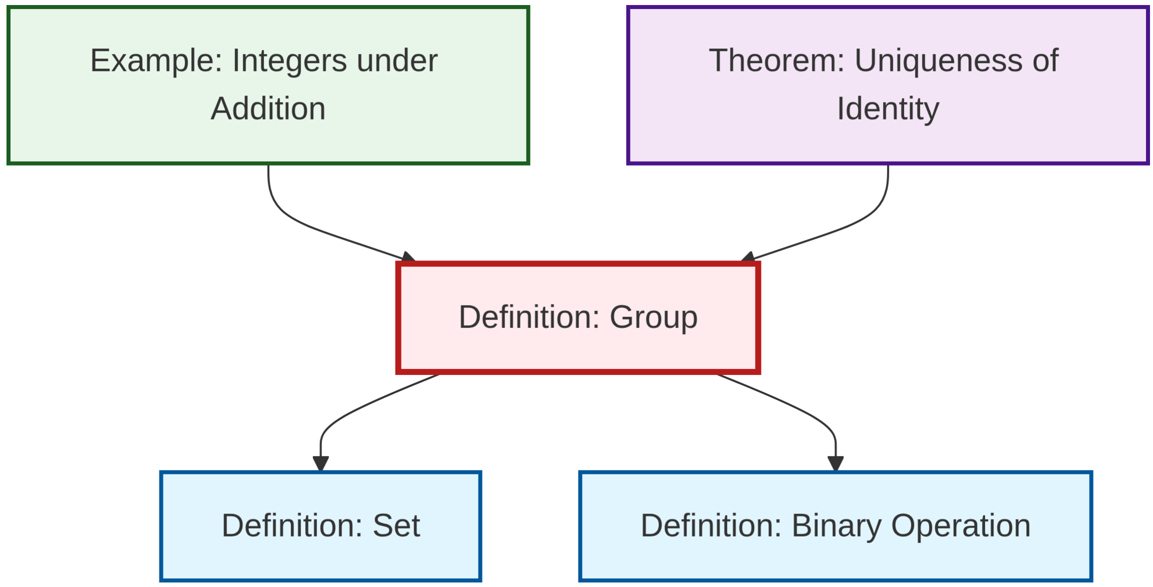
Special Types of Groups

- **Abelian group**: A group where the operation is commutative
- **Finite group**: A group with finitely many elements
- **Cyclic group**: A group generated by a single element

See Also

- Theorem: Uniqueness of Identity (proves a property of groups)
- Definition: Subgroup (coming soon)
- Definition: Group Homomorphism (coming soon)

Dependency Graph



Local dependency graph

Interactive Visualization

Explore the local knowledge graph neighborhood interactively:

You can: - **Drag** nodes to rearrange the layout - **Zoom** in/out using your mouse wheel - **Hover** over nodes to see their details - View the full interactive version in a separate window