Example: Euclidean Space

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The n**-dimensional Euclidean space** $\mathbb{R}^n$ is a fundamental example of a Vector Space over the field $\mathbb{R}$ of real numbers.

Construction

$\mathbb{R}^n$ consists of all ordered n-tuples of real numbers:

$$\mathbb{R}^n = \{(x_1, x_2, \dots, x_n) : x_i \in \mathbb{R} \text{ for all } i = 1, 2, \dots, n\}$$

Operations

For vectors $\mathbf{u} = (u_1, u_2, \dots, u_n)$ and $\mathbf{v} = (v_1, v_2, \dots, v_n)$ in $\mathbb{R}^n$, and scalar $a \in \mathbb{R}$:

1. **Vector addition**:
$$\mathbf{u} + \mathbf{v} = (u_1 + v_1, u_2 + v_2, \dots, u_n + v_n)$$

2. **Scalar multiplication**:
$$a\mathbf{u} = (au_1, au_2, \dots, au_n)$$

Special Cases

- $\mathbb{R}^1 = \mathbb{R}$: The real line
- $\mathbb{R}^2$: The Euclidean plane
- $\mathbb{R}^3$: Three-dimensional Euclidean space

Verification

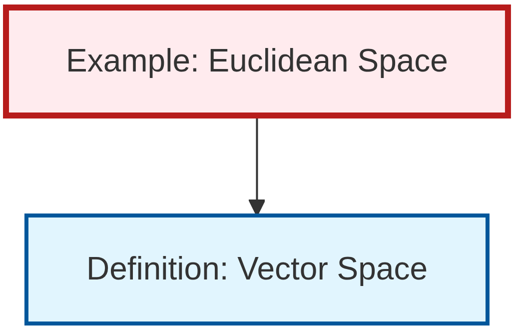
The zero vector is $\mathbf{0} = (0, 0, \dots, 0)$, and the additive inverse of $\mathbf{u}$ is $-\mathbf{u} = (-u_1, -u_2, \dots, -u_n)$.

All vector space axioms can be verified component-wise using the field properties of $\mathbb{R}$.

Geometric Interpretation

$\mathbb{R}^n$ represents the familiar geometric spaces: - Vectors can be visualized as arrows from the origin - Addition corresponds to the parallelogram rule - Scalar multiplication stretches or shrinks vectors

Dependency Graph



Local dependency graph