Definition: Hausdorff Space

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A Topological Space (X, τ) is called a **Hausdorff space** (or T_2 **space**) if for any two distinct points $x, y \in X$ with $x \neq y$, there exist open sets $U, V \in \tau$ such that:

1. $x \in U$
2. $y \in V$
3. $U \cap V = \emptyset$

In other words, any two distinct points can be "separated" by disjoint open neighborhoods.

Intuition

The Hausdorff property ensures that points in the space are "distinguishable" from each other using the topology. This is one of the most important separation axioms in topology.

Properties

- **Uniqueness of limits**: In a Hausdorff space, sequences and nets have at most one limit
- **Closed points**: Every singleton set $\{x\}$ is closed in a Hausdorff space
- **Diagonal property**: A space X is Hausdorff if and only if the diagonal $\Delta = \{(x, x) : x \in X\}$ is closed in $X \times X$ with the product topology

Examples and Non-Examples

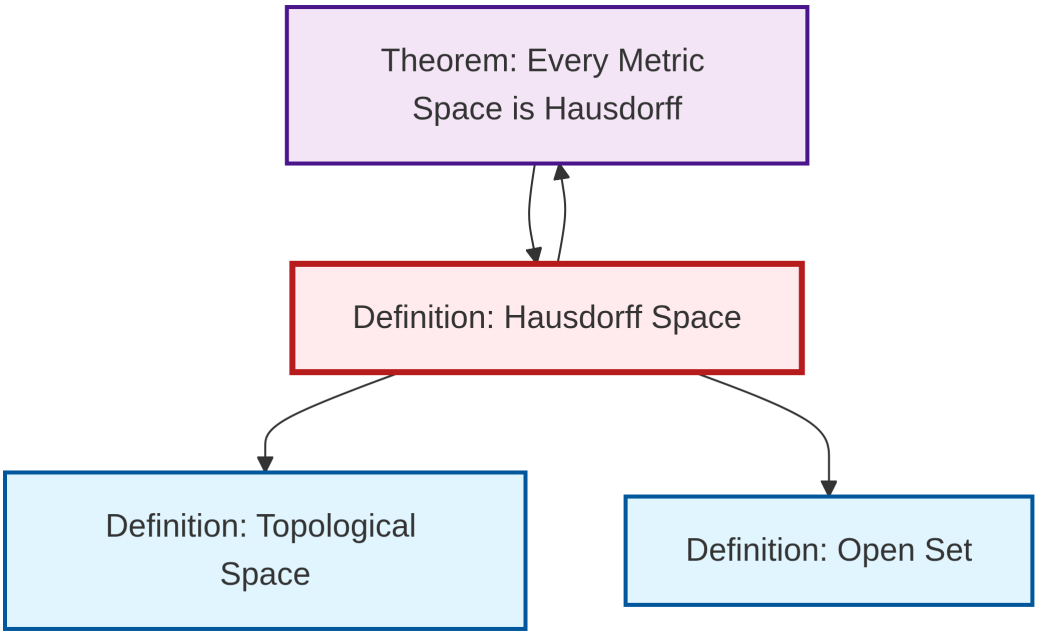
Examples of Hausdorff spaces: - Any metric space (see Every Metric Space is Hausdorff) - The real line $\mathbb{R}$ with the standard topology - Any discrete space

Non-examples: - The cofinite topology on an infinite set - The Zariski topology on algebraic varieties (in general)

See Also

- Topological Space - The basic structure on which this property is defined
- Every Metric Space is Hausdorff - Every metric space is Hausdorff
- **?@def-compact-space** - Compact Hausdorff spaces have especially nice properties

Dependency Graph



Local dependency graph