

## Example: Integers Form a Ring

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The set of integers  $\mathbb{Z}$  with usual addition and multiplication forms a [Ring](#), denoted  $(\mathbb{Z}, +, \cdot)$ .

#### Ring Axioms Verification

##### Additive Group Structure

The integers form an abelian [Group](#) under addition: - **Associativity**:  $(a + b) + c = a + (b + c)$  for all  $a, b, c \in \mathbb{Z}$  - **Identity**: 0 is the additive identity - **Inverses**: For each  $a \in \mathbb{Z}$ , the inverse is  $-a$  - **Commutativity**:  $a + b = b + a$  for all  $a, b \in \mathbb{Z}$

##### Multiplication Structure

Multiplication satisfies: - **Associativity**:  $(a \cdot b) \cdot c = a \cdot (b \cdot c)$  for all  $a, b, c \in \mathbb{Z}$  - **Identity**: 1 is the multiplicative identity

##### Distributive Laws

For all  $a, b, c \in \mathbb{Z}$ : - **Left distributivity**:  $a \cdot (b + c) = a \cdot b + a \cdot c$  - **Right distributivity**:  $(a + b) \cdot c = a \cdot c + b \cdot c$

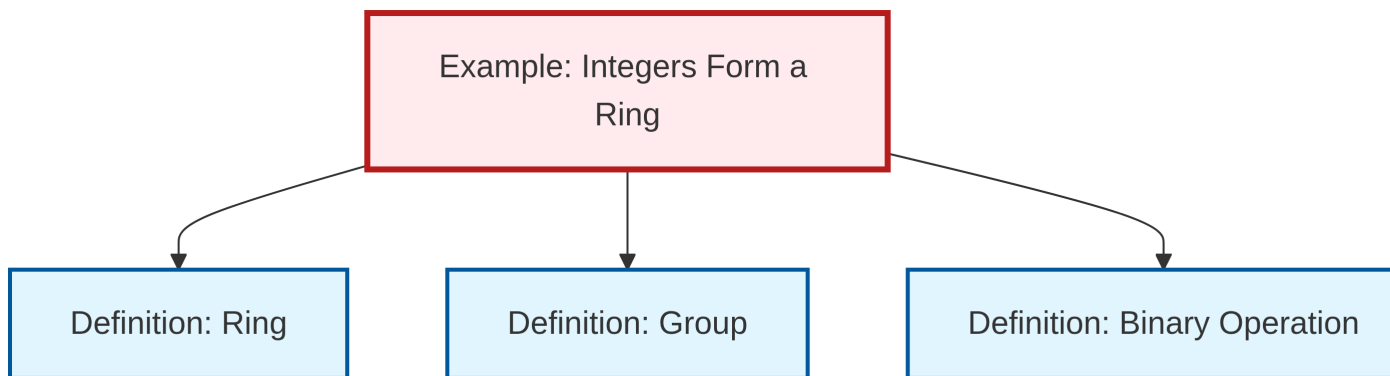
##### Special Properties

$\mathbb{Z}$  is actually a **commutative ring with unity**: - **Commutativity**:  $a \cdot b = b \cdot a$  for all  $a, b \in \mathbb{Z}$  - **Unity**: The multiplicative identity 1 exists

Furthermore,  $\mathbb{Z}$  is an **integral domain** since it has no zero divisors: if  $a \cdot b = 0$ , then either  $a = 0$  or  $b = 0$ .



## Dependency Graph



## Local dependency graph