Definition: Ring

Ring

A **ring** is a Set equipped with two Binary Operation operations that generalize the arithmetic of integers.

Formal Definition

A ring $(R, +, \cdot)$ consists of a set R together with two binary operations, addition $(+)$ and multiplication $(\cdot)$, such that:

Addition Axioms

1. $(R, +)$ is an abelian Group:
   - **Associativity**: $(a + b) + c = a + (b + c)$
   - **Commutativity**: $a + b = b + a$
   - **Identity**: There exists $0 \in R$ such that $a + 0 = a$
   - **Inverses**: For each $a \in R$, there exists $-a \in R$ such that $a + (-a) = 0$

Multiplication Axioms

2. Multiplication is associative: $(a \cdot b) \cdot c = a \cdot (b \cdot c)$

Distributive Laws

3. Left distributivity: $a \cdot (b + c) = a \cdot b + a \cdot c$
4. Right distributivity: $(a + b) \cdot c = a \cdot c + b \cdot c$

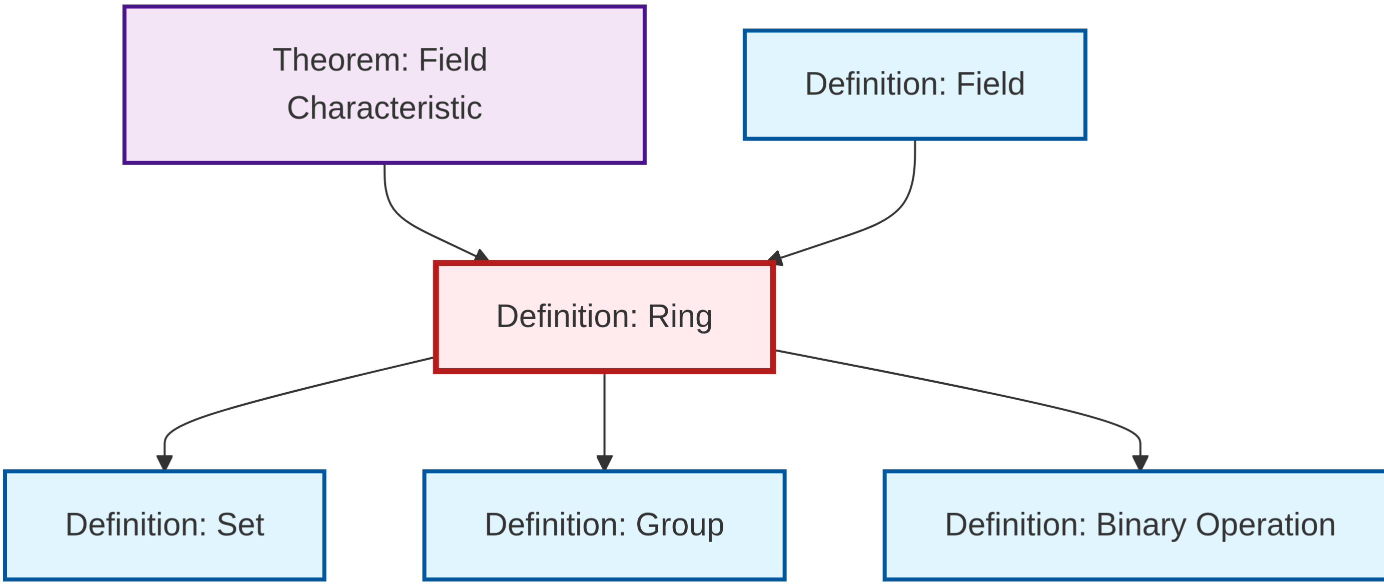
Types of Rings

- **Commutative ring**: $a \cdot b = b \cdot a$ for all a, b
- **Ring with unity**: There exists $1 \in R$ such that $1 \cdot a = a \cdot 1 = a$
- **Integral domain**: Commutative ring with unity and no zero divisors
- **Field**: Commutative ring where every non-zero element has a multiplicative inverse

Examples

- $(\mathbb{Z}, +, \cdot)$ - integers under addition and multiplication
- $M_n(\mathbb{R})$ - $n \times n$ matrices with real entries
- $\mathbb{Z}[x]$ - polynomials with integer coefficients

Dependency Graph



Local dependency graph