Theorem: Uniqueness of Identity

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Let $(G, \star)$ be a Group. Then the identity element is unique.

Statement

If e and e' are both identity elements in G, then $e = e'$.

Proof

Suppose e and e' are both identity elements in the group $(G, \star)$.

Since e is an identity element, for all $a \in G$:

$$a \star e = e \star a = a$$

Since e' is an identity element, for all $a \in G$:

$$a \star e' = e' \star a = a$$

Now consider the element $e \star e'$:

1. Since e' is an identity, we have: $e \star e' = e$
2. Since e is an identity, we have: $e \star e' = e'$

Therefore, $e = e \star e' = e'$.

Thus, the identity element is unique. □

Consequences

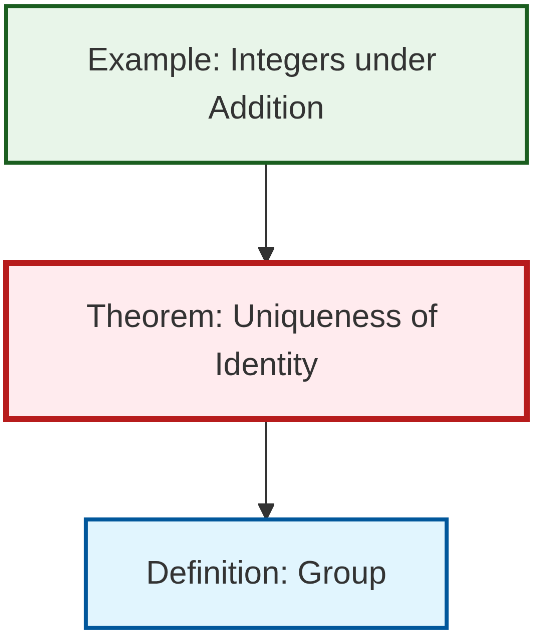
This theorem justifies our use of "the" identity element rather than "an" identity element when discussing groups. We often denote the unique identity element as:

- e or 1 in multiplicative notation
- 0 in additive notation

Related Results

- Theorem: Uniqueness of Inverses (coming soon)
- Group

Dependency Graph



Local dependency graph