## Definition: Prime Number

#### **Definition:** Prime Number

A natural number p > 1 is called a **prime number** (or simply a **prime**) if its only positive divisors are 1 and p itself.

#### Formal Definition

A natural number p > 1 is prime if and only if:

$$\forall a \in \mathbb{N}, \quad a \mid p \implies a = 1 \text{ or } a = p$$

where  $a \mid p$  means "a divides p" (i.e., there exists  $k \in \mathbb{N}$  such that p = ak).

### **Equivalent Characterizations**

- 1. p has exactly two positive divisors
- 2. p cannot be written as a product of two natural numbers both greater than 1
- 3. If  $p \mid ab$  for integers a, b, then  $p \mid a$  or  $p \mid b$  (prime property)

#### **Examples and Non-Examples**

Prime numbers: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, ...

Composite numbers (non-prime numbers > 1): 4, 6, 8, 9, 10, 12, 14, 15, ...

Note: - 2 is the only even prime number - 1 is not considered prime (by modern convention)

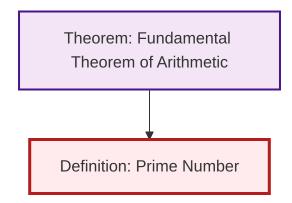
#### **Importance**

Prime numbers are the "atoms" of number theory: - Every natural number > 1 can be uniquely factored into primes (Fundamental Theorem of Arithmetic) - They appear in numerous areas of mathematics and applications (cryptography, coding theory) - Their distribution remains a central mystery (Riemann Hypothesis)

#### Related Concepts

- Composite number: A natural number > 1 that is not prime
- Coprime: Two integers are coprime if their greatest common divisor is 1
- Prime factorization: Expression of a number as a product of prime powers

# Dependency Graph



Local dependency graph