

## Definition: Parallel Lines

Two **Lines** are **parallel** if they lie in the same plane and do not intersect, no matter how far they are extended.

### Formal Definition

In Euclidean geometry, lines  $\ell$  and  $m$  are parallel (denoted  $\ell \parallel m$ ) if: 1. They are coplanar (lie in the same plane) 2. They have no **Points** in common:  $\ell \cap m = \emptyset$

### Characterizations

#### In Coordinate Geometry

Two lines in  $\mathbb{R}^2$  are parallel if and only if they have the same slope: - Line 1:  $y = m_1x + b_1$  - Line 2:  $y = m_2x + b_2$  - Parallel iff  $m_1 = m_2$  and  $b_1 \neq b_2$

#### Using Direction Vectors

Lines with direction vectors  $\vec{v}_1$  and  $\vec{v}_2$  are parallel iff:

$$\vec{v}_1 = k\vec{v}_2 \text{ for some } k \neq 0$$

### Euclid's Parallel Postulate

The famous fifth postulate states: > Through a point not on a given line, there exists exactly one line parallel to the given line.

This postulate: - Cannot be proven from other axioms - Distinguishes Euclidean from non-Euclidean geometries

### Properties

1. **Reflexivity**: Every line is parallel to itself (by convention)
2. **Symmetry**: If  $\ell \parallel m$ , then  $m \parallel \ell$
3. **Transitivity**: If  $\ell \parallel m$  and  $m \parallel n$ , then  $\ell \parallel n$
4. **Equal distance**: Parallel lines maintain constant perpendicular distance

### Parallel Lines and Transversals

When a transversal crosses two parallel lines: - **Corresponding angles** are equal - **Alternate interior angles** are equal - **Co-interior angles** are supplementary (sum to  $180^\circ$ )

### Examples

1. **Railroad tracks**: Classic real-world example of parallel lines
2. **Coordinate axes translations**:  $y = 2$  and  $y = 5$  are parallel
3. **Opposite sides of a rectangle**: Always parallel by definition

## Non-Euclidean Variations

### Hyperbolic Geometry

Through a point not on a line, there exist infinitely many lines parallel to the given line.

### Spherical Geometry

No parallel lines exist - all great circles intersect.

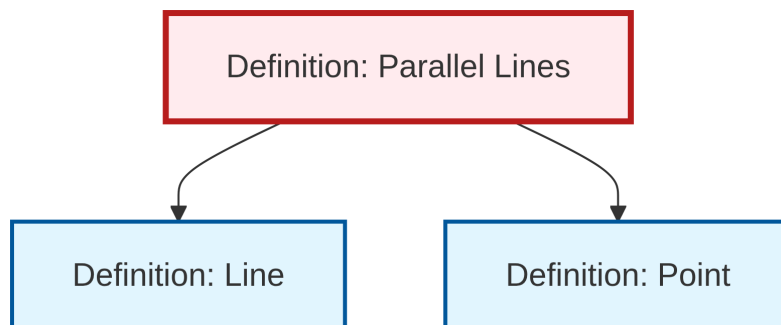
### Related Concepts

- **Perpendicular lines:** Intersect at  $90^\circ$
- **Skew lines:** Non-parallel lines in 3D that don't intersect
- **Parallel planes:** Extension to higher dimensions
- **Parallelogram:** Quadrilateral with opposite sides parallel

### Applications

- **Architecture:** Ensuring structural stability
- **Computer graphics:** Ray tracing and collision detection
- **Navigation:** Latitude lines are parallel
- **Linear algebra:** Parallel vectors and subspaces

### Dependency Graph



Local dependency graph