

Example: Exponential Homomorphism

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The exponential function provides a classic example of a [Group Homomorphism](#) between [Group](#) structures.

The Homomorphism

Consider the function:

$$\begin{aligned}\exp : (\mathbb{R}, +) &\rightarrow (\mathbb{R}^+, \cdot) \\ \exp(x) &= e^x\end{aligned}$$

where $(\mathbb{R}, +)$ is the additive group of real numbers and (\mathbb{R}^+, \cdot) is the multiplicative group of positive real numbers.

Verification

To verify this is a homomorphism, we check:

$$\exp(x + y) = e^{x+y} = e^x \cdot e^y = \exp(x) \cdot \exp(y)$$

Properties

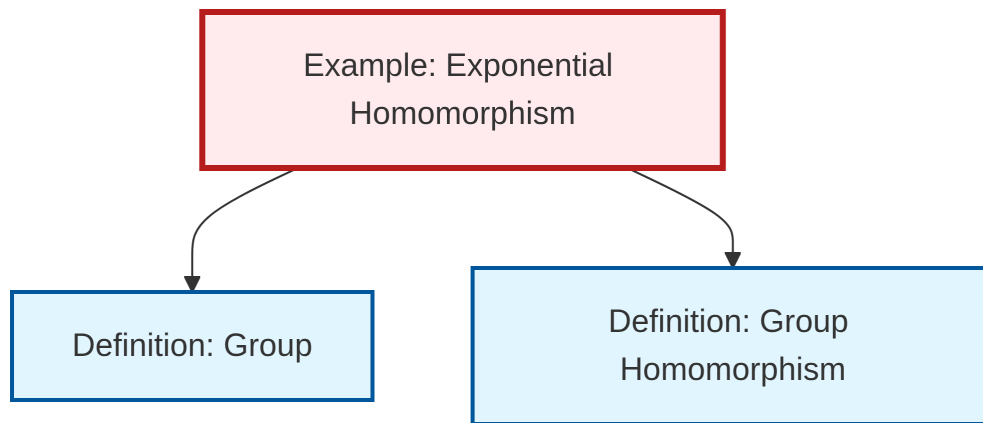
- **Injective:** Yes, since $e^x = e^y$ implies $x = y$
- **Surjective:** Yes, for any $a \in \mathbb{R}^+$, we have $\exp(\ln a) = a$
- **Bijjective:** Therefore, \exp is an isomorphism

Inverse

The natural logarithm $\ln : (\mathbb{R}^+, \cdot) \rightarrow (\mathbb{R}, +)$ is the inverse isomorphism:

$$\ln(xy) = \ln(x) + \ln(y)$$

Dependency Graph



Local dependency graph