# Definition: Expected Value

# **Definition: Expected Value**

The **expected value** (or **expectation**) of a random variable is a measure of the central tendency of its distribution, representing the average value the random variable takes over many trials.

#### Discrete Case

For a discrete random variable X with probability mass function p(x):

$$E[X] = \sum_x x \cdot p(x)$$

where the sum is taken over all possible values of X.

#### Continuous Case

For a continuous random variable X with probability density function f(x):

$$E[X] = \int_{-\infty}^{\infty} x \cdot f(x) \, dx$$

#### General Definition

For a random variable X on a probability space  $(\Omega, \mathcal{F}, P)$ :

$$E[X] = \int_{\Omega} X(\omega) \, dP(\omega)$$

This is the Lebesgue integral of X with respect to the probability measure P.

#### **Properties**

- 1. Linearity: E[aX + bY] = aE[X] + bE[Y] for constants a, b
- 2. Monotonicity: If  $X \leq Y$  (almost surely), then  $E[X] \leq E[Y]$
- 3. Constant: E[c] = c for any constant c
- 4. Non-negativity: If  $X \ge 0$  (almost surely), then  $E[X] \ge 0$

#### Existence

The expected value exists if and only if  $E[|X|] < \infty$ , i.e., when:

$$\int_{\Omega} |X(\omega)| \, dP(\omega) < \infty$$

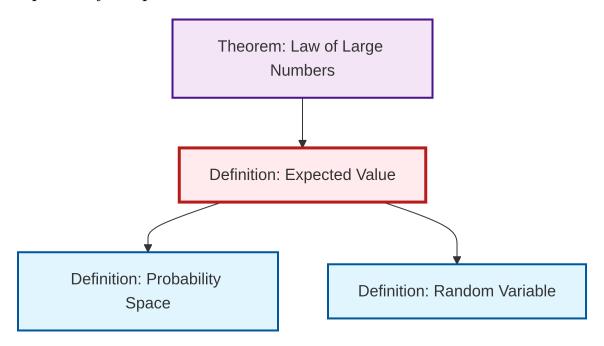
## Examples

- 1. **Bernoulli**: If  $X \sim \text{Bernoulli}(p)$ , then E[X] = p2. **Binomial**: If  $X \sim \text{Binomial}(n, p)$ , then E[X] = np3. **Normal**: If  $X \sim \mathcal{N}(\mu, \sigma^2)$ , then  $E[X] = \mu$ 4. **Exponential**: If  $X \sim \text{Exp}(\lambda)$ , then  $E[X] = 1/\lambda$
- Mermaid Diagram

```
graph TD
   A[Expected Value] --> B[Discrete: Σ x·p(x)]
A --> C[Continuous: x·f(x) dx]
A --> D[General: X dP]
A --> E[Properties]
E --> F[Linearity]
E --> G[Monotonicity]
A --> H[Existence: E[|X|] < ω]

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## Dependency Graph



Local dependency graph