Definition: Euclidean Space

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An **n-dimensional Euclidean space**, denoted $\mathbb{E}^n$, is the vector space $\mathbb{R}^n$ equipped with the standard inner product and the induced metric.

Formal Definition

The n-dimensional Euclidean space consists of:

1. **The underlying set**: $\mathbb{R}^n = \{(x_1, x_2, \ldots, x_n) : x_i \in \mathbb{R}\}$
2. **The standard inner product**: For $\mathbf{x} = (x_1, \ldots, x_n)$ and $\mathbf{y} = (y_1, \ldots, y_n)$,
$$\langle \mathbf{x}, \mathbf{y} \rangle = \sum_{i=1}^{n} x_i y_i$$
3. **The Euclidean norm**:
$$\|\mathbf{x}\| = \sqrt{\langle \mathbf{x}, \mathbf{x} \rangle} = \sqrt{\sum_{i=1}^{n} x_i^2}$$
4. **The Euclidean metric**:
$$d(\mathbf{x}, \mathbf{y}) = \|\mathbf{x} - \mathbf{y}\| = \sqrt{\sum_{i=1}^{n} (x_i - y_i)^2}$$

Properties

- **Complete metric space**: Every Cauchy sequence converges
- **Finite-dimensional**: Has dimension n as a vector space
- **Locally compact**: Every point has a compact neighborhood
- **Simply connected**: For $n \geq 2$
- **Homogeneous**: Looks the same at every point

Special Cases

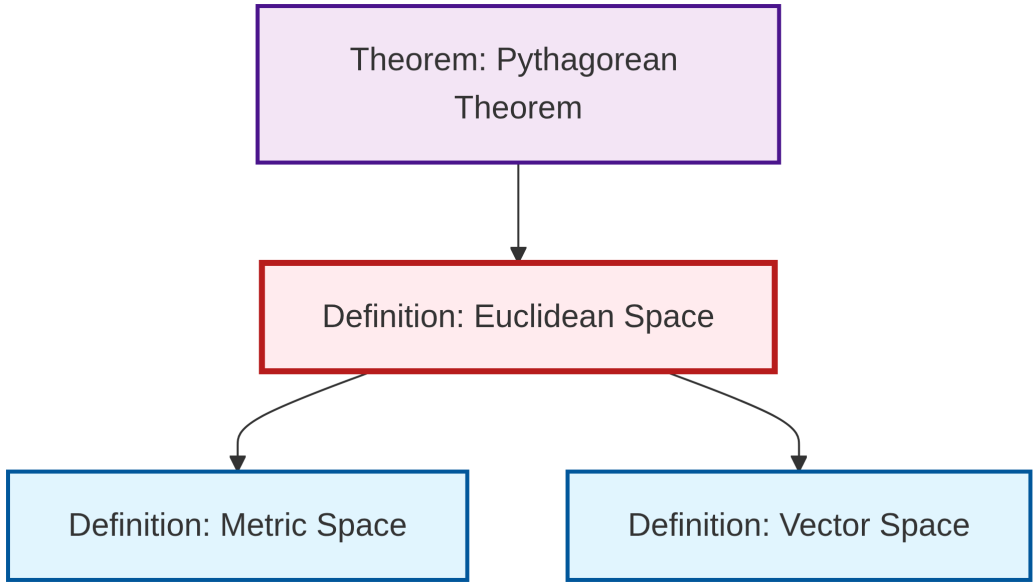
- $\mathbb{E}^1$: The real line
- $\mathbb{E}^2$: The Euclidean plane
- $\mathbb{E}^3$: Three-dimensional Euclidean space

Mermaid Diagram

```
graph TD
    A[Euclidean Space] --> B[Vector Space  ]
    A --> C[Inner Product]
    A --> D[Euclidean Metric]
    C --> E[Norm]
    E --> D
    B --> F[Linear Structure]
    D --> G[Metric Structure]

    style A fill:#f9f,stroke:#333,stroke-width:2px
    style B fill:#bbf,stroke:#333,stroke-width:2px
    style C fill:#bbf,stroke:#333,stroke-width:2px
    style D fill:#bbf,stroke:#333,stroke-width:2px
```

Dependency Graph



Local dependency graph