Definition: Probability Space

Probability Space

A **probability space** is a mathematical construct that provides a formal model for random phenomena.

Formal Definition

A probability space is a triple $(\Omega, \mathcal{F}, P)$ where:

1. **Sample space** Ω: A Set of all possible outcomes
2. **-algebra** $\mathcal{F}$: A collection of subsets of Ω (events) satisfying:
   - $\Omega \in \mathcal{F}$
   - If $A \in \mathcal{F}$, then $A^c \in \mathcal{F}$ (closed under complements)
   - If $A_1, A_2, ... \in \mathcal{F}$, then $\bigcup_{i=1}^{\infty} A_i \in \mathcal{F}$ (closed under countable unions)
3. **Probability measure** $P : \mathcal{F} \to [0, 1]$ satisfying:
   - $P(\Omega) = 1$ (normalization)
   - $P(A) \geq 0$ for all $A \in \mathcal{F}$ (non-negativity)
   - For disjoint events $A_1, A_2, ...$: $P(\bigcup_{i=1}^{\infty} A_i) = \sum_{i=1}^{\infty} P(A_i)$ (countable additivity)

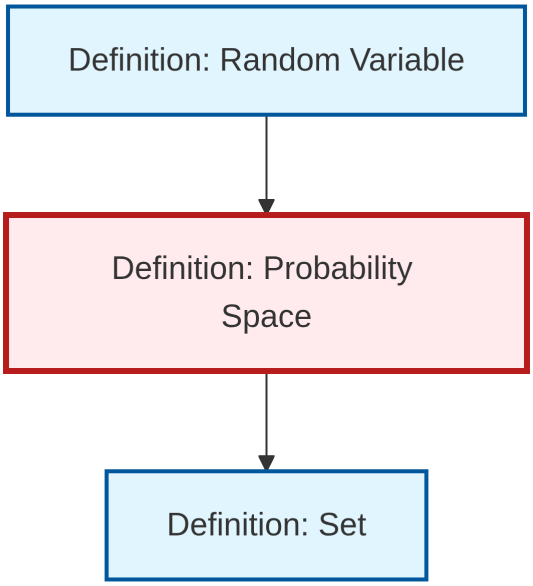
Properties

- $P(\emptyset) = 0$
- $P(A^c) = 1 - P(A)$
- If $A \subseteq B$, then $P(A) \leq P(B)$
- $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Examples

- Coin flip: $\Omega = \{H, T\}$, $\mathcal{F} = 2^{\Omega}$, $P(H) = P(T) = 1/2$
- Die roll: $\Omega = \{1, 2, 3, 4, 5, 6\}$, with uniform probability
- Continuous: $\Omega = [0, 1]$ with Lebesgue measure

Dependency Graph



Local dependency graph