Definition: Limit of a Sequence

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Let $(a_n)_{n=1}^{\infty}$ be a sequence of real numbers. We say that the sequence **converges** to a limit $L \in \mathbb{R}$ if:

For every $\varepsilon > 0$, there exists $N \in \mathbb{N}$ such that for all $n > N$:

$$|a_n - L| < \varepsilon$$

Notation

We write this as: - $\lim_{n\to\infty} a_n = L$ - $a_n \to L$ as $n \to \infty$ - $(a_n) \to L$

Formal Definition

Using logical symbols:

$$\lim_{n\to\infty} a_n = L \iff \forall\varepsilon > 0, \exists N \in \mathbb{N}, \forall n > N : |a_n - L| < \varepsilon$$

Geometric Interpretation

The definition means that eventually all terms of the sequence lie within any given distance ε from L. No matter how small we make ε, we can find a point in the sequence after which all terms are within this distance from L.

Uniqueness

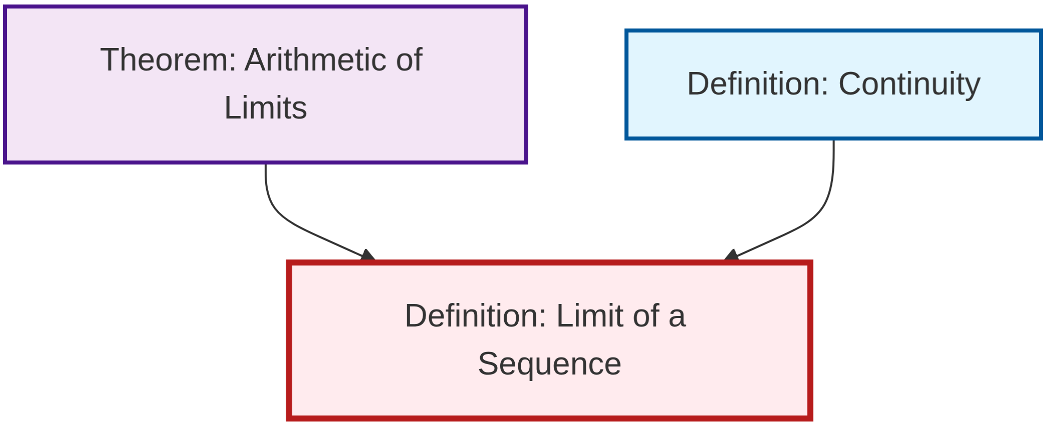
If a sequence converges, its limit is unique. If $a_n \to L_1$ and $a_n \to L_2$, then $L_1 = L_2$.

Divergence

A sequence that does not converge to any limit is said to **diverge**. This includes: - Sequences that oscillate (e.g., $(-1)^n$) - Sequences that grow without bound (e.g., n)

The concept of limit is fundamental to analysis and forms the foundation for continuity, derivatives, and integrals.

Dependency Graph



Local dependency graph