

# UNIT NO.5

# REASONING UNDER UNCERTAINTY



# UNCERTAINTY IN AI

- Uncertainty in AI refers to the **lack of complete certainty in decision-making due to incomplete, ambiguous, or noisy data.**
- AI models handle uncertainty by using probabilistic methods, fuzzy logic, and Bayesian inference.
- Proper uncertainty representation enables AI systems to make **informed predictions and improve reliability in real-world applications.**



# WHAT IS UNCERTAINTY IN AI?

- Uncertainty in Artificial Intelligence (AI) refers to **the inability of models to make fully confident predictions due to incomplete, ambiguous, or noisy data**.
- AI systems must account for uncertainty to make accurate and reliable decisions, especially in dynamic environments where information is inconsistent or evolving.
- **Examples of Uncertainty in Real-World AI Applications:**
- **Autonomous Vehicles:** AI-powered self-driving cars must navigate unpredictable road conditions, such as sudden pedestrian movement or bad weather.
- **Healthcare Diagnostics:** AI models analyzing medical images face uncertainty due to variability in symptoms, leading to multiple possible diagnoses.
- **Natural Language Processing (NLP):** AI chatbots and language models encounter contextual ambiguity, making it difficult to infer user intent accurately.
- **Fraud Detection:** In finance, AI models must distinguish between genuine and fraudulent transactions, often dealing with uncertain patterns in data.



# WHAT IS UNCERTAINTY IN AI?

- Uncertainty affects AI reliability, risk assessment, and trustworthiness.
- **A model that fails to handle uncertainty may provide misleading predictions, leading to incorrect actions in critical fields like medicine, finance, and cybersecurity.**
- Managing uncertainty through probabilistic reasoning, fuzzy logic, and Bayesian inference allows AI models to adapt to variability and improve decision-making accuracy.



# SOURCES OF UNCERTAINTY IN AI

## ■ Data Uncertainty:

- Data uncertainty occurs when **AI models rely on incomplete, noisy, or inconsistent data.**
- This can result from measurement errors, missing values, or biased datasets.
- For example, in medical diagnosis, an AI system **analyzing low-quality X-ray images may struggle to detect anomalies accurately.**
- Similarly, AI-powered speech recognition faces **uncertainty when dealing with background noise or unclear speech patterns.**

## ■ Model Uncertainty:

- Model uncertainty stems from limitations in **AI algorithms and training processes.**
- If an AI model lacks sufficient training data or has an inadequate architecture, it may struggle to generalize beyond its training set.
- For example, an **AI-powered recommendation system trained on limited user preferences may fail to suggest relevant content to new users.**



# SOURCES OF UNCERTAINTY IN AI

- **Computational Uncertainty:**
  - Many AI models use approximation techniques to make predictions, leading to computational uncertainty.
  - This occurs in deep learning models, where numerical computations involve rounding errors and algorithmic approximations.
  - In fields like autonomous robotics, computational uncertainty affects AI's ability to process real-time sensor data efficiently.
- **Environmental Uncertainty:**
  - AI models operate in dynamic environments, where external factors introduce uncertainty.
  - In self-driving cars, unexpected weather conditions or roadblocks can impact AI decision-making.
  - Similarly, in financial forecasting, sudden economic changes can disrupt predictive models.



# **TYPES OF UNCERTAINTY IN AI**

- Uncertainty in AI can be categorized into different types **based on its nature and cause**.
- Understanding these types helps in designing robust AI models that can manage unpredictability more effectively.

## **1. Aleatoric Uncertainty**

- Aleatoric uncertainty, also known as statistical uncertainty, **arises due to randomness or inherent noise in data**.
- It cannot be reduced by collecting more data because it is intrinsic to the system.
- **Examples:**
- In self-driving cars, **aleatoric uncertainty occurs due to weather conditions (fog, rain) or sensor noise, affecting the AI's perception of objects**.
- In medical AI, variability in patient test results due to biological differences leads to aleatoric uncertainty in diagnosis.



# TYPES OF UNCERTAINTY IN AI

## 2. Epistemic Uncertainty

- Epistemic uncertainty **arises from a lack of knowledge or insufficient training data.**
- Unlike aleatoric uncertainty, it can be reduced by improving data quality and model architecture.
- **Examples:**
  - An AI chatbot trained on a limited dataset may struggle with understanding new dialects or slang.
  - In financial forecasting, AI models trained on historical data may perform poorly when predicting unprecedented market crashes.

## 3. Computational Uncertainty

- This type of **uncertainty occurs due to rounding errors, numerical approximations, and hardware limitations in AI computations.**
- **Example:**
  - In deep learning, slight variations in weight initialization can lead to different model outcomes, affecting consistency.



# **TYPES OF UNCERTAINTY IN AI**

## **4. Perceptual Uncertainty**

- AI systems **relying on sensor-based perception face uncertainty when sensor limitations affect data collection.**
- **Example:**
- Autonomous drones may misinterpret objects due to low-resolution cameras or motion blur, leading to incorrect navigation decisions.



# TECHNIQUES FOR ADDRESSING UNCERTAINTY IN AI

## ▪ Probabilistic Logic Programming

- Probabilistic logic programming integrates probability theory with logic-based reasoning, allowing AI models to deal with uncertainty through probability distributions.
- This technique is commonly used in fields like medical diagnosis, where AI must evaluate multiple possible conditions based on uncertain symptom data.
- **By assigning probability values to different outcomes, probabilistic logic helps AI make flexible decisions rather than relying on rigid rule-based logic.**

## ▪ Fuzzy Logic Programming

- Fuzzy logic enables AI to handle imprecise or vague data, making it useful in real-world applications where absolute truth values (true/false) are insufficient.
- Unlike traditional binary logic, fuzzy logic represents values in degrees, such as low, medium, or high.
- This method is widely used in self-driving cars, where AI must interpret traffic conditions, pedestrian movement, and weather variations to make driving decisions.
- By considering multiple factors with partial truths, fuzzy logic improves AI's ability to function in complex, unpredictable environments.



# TECHNIQUES FOR ADDRESSING UNCERTAINTY IN AI

## ■ Nonmonotonic Logic Programming

- Nonmonotonic logic is designed for AI models **that must revise their conclusions when new data becomes available.**
- Unlike classical logic systems that assume all knowledge remains static, nonmonotonic logic allows AI to update its reasoning dynamically.
- This is particularly **useful in cybersecurity applications, where AI must adapt to emerging threats, revise attack predictions, and correct previous assumptions when new evidence arises.**

## ■ Paraconsistent Logic Programming

- Paraconsistent logic is useful for AI systems that encounter contradictory or conflicting information.
- **Instead of rejecting inconsistent data outright, this approach enables AI to analyze contradictions and extract meaningful insights.**
- In financial analysis, for example, AI-powered trading models often deal with conflicting market signals, and paraconsistent logic allows them to make informed decisions without discarding critical data points.



# TECHNIQUES FOR ADDRESSING UNCERTAINTY IN AI

- **Hybrid Logic Programming**
- Hybrid logic programming combines multiple logic techniques to enhance AI's reasoning and adaptability in uncertain conditions.
- By integrating elements of probabilistic, fuzzy, and nonmonotonic logic, hybrid models achieve greater flexibility in handling ambiguous inputs and incomplete datasets.
- AI-powered customer service chatbots often rely on hybrid logic to manage varied user queries, interpret intent more accurately, and refine their responses over time.



# WAYS TO SOLVE PROBLEMS WITH UNCERTAIN KNOWLEDGE

## 1. Bayes' Rule

- Bayes' Rule is a fundamental principle in probability theory that allows AI systems to **update their beliefs based on new evidence.**
- **It calculates the probability of an event occurring given prior knowledge and new data.**
- This makes it particularly useful in diagnostic AI, fraud detection, and medical decision-making, where new information continuously refines the probability of different outcomes.
- For example, in AI-driven spam detection, Bayes' Rule helps classify emails as spam or legitimate by assessing the probability of specific words appearing in spam messages.
- As the AI model processes more data, it updates its probability estimates, improving its classification accuracy.



# WAYS TO SOLVE PROBLEMS WITH UNCERTAIN KNOWLEDGE

## 2. Bayesian Statistics

- Bayesian statistics extends the idea of Bayes' Rule by **incorporating prior probabilities into AI models**. Unlike traditional frequentist statistics, which rely only on observed data, Bayesian models combine historical data with current observations to improve predictions. This method is widely used in predictive analytics, finance, and autonomous systems.
- In self-driving cars, Bayesian inference helps predict pedestrian movements by factoring in both previous traffic patterns and real-time sensor data. This allows the AI system to adjust its predictions dynamically, making smarter navigation decisions.
- By leveraging Bayes' Rule and Bayesian statistics, AI models can manage uncertainty more effectively, leading to more reliable and adaptable decision-making in unpredictable environments.



# PRIOR PROBABILITY

- The **prior probability** (or simply prior) represents what we know (or assume) about an event **before** observing any new data or evidence.
- It expresses **our initial belief** about an outcome.
- $P(H)$
- Where:
  - $H$ = hypothesis or event of interest
  - $P(H)$  =prior probability of  $H$  being true
- **Example:**
  - Suppose 1% of people have a certain disease.  
Before any medical test, the **prior probability** that a randomly chosen person has the disease is:
  - $P(\text{Disease}) = 0.01$
  - This is your **belief before seeing test results.**



# PRIOR PROBABILITY

- *Prior probability is defined as the **initial assessment or the likelihood of the event or an outcome before any new data is considered**.*
- *In simple words, it tells us about what we know based on previous knowledge or experience.*



# PRIOR PROBABILITY

- priors are classified based on the amount of information content in it. Below discussed are some of the types of priors:
- Informative Priors
- Weakly Informative Priors
- Non-informative Priors
- Improper Priors



# PRIOR PROBABILITY

## ▪ **Informative Priors**

- These kind of priors have detailed knowledge or are **decided from the expert opinions**.
- These priors are **chosen based on the past or historical data, or under expert guidance**.
- These priors have a **significant impact on the posterior distribution**.
- These kind of priors are **useful only when we have strong information that can drive the analysis**.

## ▪ **Weakly Informative Priors**

- These kind of priors are **in-between informative and non-informative priors**.
- They have some prior knowledge but cannot eventually influence the posterior distribution.
- These priors **provide some regularization and also prevents from noise fitting but it still allows the data to influence the posterior distribution**.
- Normal prior with the high variance can be considered as weakly informative priors.



# PRIOR PROBABILITY

- **Non-informative Priors**

- Non informative priors are also known as **uninformative priors**.
- These kind of priors have **very little or practically no prior knowledge about the parameter**.
- They have a **minimum influence to posterior distribution**, allowing the data to primarily drive the inference.
- Uniform prior is an example of non-informative prior that assigns equal probabilities to all the possible outcomes, reflecting the lack of prior knowledge.

- **Improper Priors**

- These are **non-informative priors but it does not integrate over one parameter space, i.e. they do not have a valid probability distribution**.
- These kind of priors are still used in Bayesian statistics as long as the resulting posterior distribution remains proper which integrates over one parameter space.
- These priors are generally used for parameters with unbounded ranges such as it uses  $1/\theta$  over a parameter  $\theta$ .



# APPLICATIONS OF PRIOR PROBABILITY

▪ Various application of Prior Probability includes:

- **Medical Diagnosis:** Prior probabilities are used in medical diagnosis in order to determine the likelihood of the disease before testing.
- **Spam Filtering:** Prior probabilities are used in email filtering to classify an email as a spam or not spam based on the previous historical data.
- **Financial Forecasting:** Investors consider the prior probability in order to assess the risk of the investments before considering market trends and data.
- **Machine Learning:** In the field of Machine Learning, prior probabilities are integrated with various algorithms in order to improve model performance and its accuracy



# POSTERIOR PROBABILITY

- The **posterior probability** (or *posterior*) represents the **updated probability** of an event **after observing new evidence**.
- In Bayesian statistics, posterior probability is the revised or updated probability of an event after taking into account new information.
- The posterior probability is calculated by updating the prior probability using the Bayes Theorem



# POSTERIOR PROBABILITY

- Formula:

$$P(A|B) = \frac{P(B|A) P(A)}{P(B)}$$



# POSTERIOR PROBABILITY

- where:
- $P(A|B)$  is Posterior Probability of event A occurring given that event B is observed
  - Posterior probability (updated belief after evidence)
- $P(B|A)$  is Probability of observing event B given that event A has occurred
  - Likelihood (the chance of something happening)
- $P(A)$  is Prior Probability of event A occurring before observing event B
  - Prior probability (initial belief)
- $P(B)$  is Probability of Observing Event B
  - Marginal likelihood (probability of observing the evidence)



# INFERENCE USING FULL JOINT DISTRIBUTION

## ■ What is a Full Joint Distribution?

- A full joint distribution is a complete tabulation of probabilities for every possible combination of values across all random variables in the domain.
- Each row represents a distinct scenario and its associated probability, ensuring the entire table sums to 1.0.
- This comprehensive “knowledge base” allows you to derive any marginal or conditional probability.
- The **Full Joint Probability Distribution** represents the probability of **every possible combination** of values for **all random variables** in a system.
- If we have random variables  $X_1, X_2, \dots, X_n$ , the **full joint distribution** is:
- $P(X_1, X_2, \dots, X_n)$
- It gives a complete description of the probabilistic relationships among all variables.



# STEP-BY-STEP GUIDE TO INFERENCE WITH A FULL JOINT DISTRIBUTION

- **Step 1: Identify the Query**
- Decide what probability need to compute.
- For example:
- $P(X | E = e)$
- Where
- $X$ = **query variable** (the one you want to find the probability of)
- $E = e$ = **evidence variable(s)** with observed value(s)



## **Step 2: Write the Definition of Conditional Probability**

- Use the basic rule of conditional probability:
- $P(X | E = e) = \frac{P(X, E = e)}{P(E=e)}$
- This expresses the desired probability in terms of the **joint probabilities**.

## **Step 3: Expand the Joint Probability**

- The joint probability  $P(X, E = e)$  may include **hidden (unobserved)** variables, say  $Y$ .
- To handle that, **marginalize (sum out)** the hidden variables:
- $P(X, E = e) = \sum_Y P(X, E, Y)$
- You use the **Full Joint Distribution**  $P(X_1, X_2, \dots, X_n)$  to find these values.



## **Step 4: Compute the Denominator $P(E = e)$**

- To normalize the probability, compute the **total probability** of the evidence:
- This ensures all conditional probabilities add up to 1.

## **Step 5: Substitute and Normalize**

- Plug the values of  $P(X, E = e)$  and  $P(E = e)$  into:
- $$P(X | E = e) = \frac{P(X, E = e)}{P(E = e)}$$
- This gives you the **posterior (updated)** probability.

## **Step 6: Interpret the Result**

- Your result  $P(X | E = e)$  represents the **new belief** about  $X$  after observing evidence  $e$ .



# EXAMPLE

- **Example: Inference Using Full Joint Distribution**
- We have:
  - $R$ : Whether it is raining (True or False)
  - $U$ : Whether someone carries an umbrella (True or False)
- We want to perform **inference**, e.g.:
  - $P(R = \text{True} \mid U = \text{True})$  “→ Given that someone has an umbrella, what is the probability that it is raining?”



# EXAMPLE

- **Step 1: Full Joint Distribution Table**
- Assume we know this joint probability distribution  $P(R, U)$ :

Rain (R)	Umbrella (U)	( $P(R, U)$ )
True	True	0.30
True	False	0.10
False	True	0.20
False	False	0.40
<b>Total</b>		<b>1.00</b>



- **Step 2: Write the Query**

- We want:
- $P(R = T \mid U = T)$
- By definition of conditional probability:

- $$P(R \mid U) = \frac{P(R, U)}{P(U)}$$

- **Step 3: Compute Denominator (Evidence Probability)**

- We must find  $P(U = T)$ :
- $$P(U = T) = P(R' = T, U = T) + P(R' = F, U = T)$$
- Substitute values:
- $$P(U = T) = 0.30 + 0.20 = 0.50$$



- **Step 4: Compute Numerator**

- $P(R = T | U = T) = 0.30$

- **Step 5: Apply the Formula**

- $$P(R = T | U = T) = \frac{P(R = T \cap U = T)}{P(U = T)} = \frac{0.30}{0.50} = 0.6$$

- **Result:**

- $$P(\text{Rain=True} | \text{Umbrella=True}) = 0.6$$



# EXAMPLE 2

- We have three Boolean variables:

Variable	Meaning
B	Burglary occurs
E	Earthquake occurs
A	Alarm goes off



- **Full Joint Distribution**

- Let's assume we know the full joint probability distribution  $P(B, E, A)$ :

B	E	A	$P(B, E, A)$
T	T	T	0.0001
T	T	F	0.0000
T	F	T	0.0040
T	F	F	0.0010
F	T	T	0.0020
F	T	F	0.0080
F	F	T	0.09
F	F	F	0.8949



- **Query:**  $P(B \mid A)$
- **If the alarm goes off, what's the probability that there was a burglary?**



# BAYES' THEOREM IN AI

- Bayes' Theorem in AI, also known as Bayes' rule or Bayes' law, is a fundamental concept in probability theory and statistics.
- **It provides a way to update our beliefs or the probability of an event occurring based on new evidence or information.**
- It is named after the 18th-century statistician and philosopher Thomas Bayes.



# BAYES' THEOREM IN AI

- Bayes' Theorem comprises four key components:
- **1. Prior Probability ( $P(A)$ ):**
  - This is the initial probability or belief in an event A before considering any new evidence.
  - It represents what we know or assume about A based on prior knowledge.
- **2. Likelihood ( $P(B|A)$ ):**
  - The likelihood represents the probability of observing evidence B given that the event A is true.
  - It quantifies how well the evidence supports the event.



# BAYES' THEOREM IN AI

## ■ 3. Evidence ( $P(B)$ ):

- Evidence, also known as the marginal likelihood, is the probability of observing evidence B, regardless of the truth of A.
- It serves as a normalizing constant, ensuring that the posterior probability is a valid probability distribution.

## ■ 4. Posterior Probability ( $P(A|B)$ ):

- The posterior probability represents the updated belief in event A after considering the new evidence B.
- It answers the question, "What is the probability of A being true given the observed evidence B?"



# BAYES' THEOREM IN AI

- Here's how it works:
- It starts with a prior probability  $P(A)$ , which represents initial belief.
- The likelihood  $P(B|A)$  quantifies how likely the observed evidence is if prior belief is true.
- calculate the evidence  $P(B)$ , which is the probability of observing the evidence regardless of prior belief.
- Using these components, Bayes' Theorem computes the posterior probability  $P(A|B)$ , which is updated belief in A after taking the new evidence into account.



# MATHEMATICAL DERIVATION OF BAYES' RULE

- Bayes' Rule is derived from the definition of conditional probability. Let's start with the definition:

- $P(A | B) = \frac{P(A \cap B)}{P(B)}$

This equation states that the probability of event  $A$  given event  $B$  is equal to the probability of both events happening (the intersection of  $A$  and  $B$ ) divided by the probability of event  $B$ .

- Similarly, we can write the conditional probability of event  $B$  given event  $A$ :

- $P(B | A) = \frac{P(A \cap B)}{P(A)}$



# MATHEMATICAL DERIVATION OF BAYES' RULE

■ By rearranging this equation, we get:

- $P(A \cap B) = P(B | A) \cdot P(A)$

- Now, we have two expressions for  $P(A \cap B)$ , since both expressions are equal to  $P(A \cap B)$ , we can set them equal to each other:

- $P(A|B) \cdot P(B) = P(B|A) \cdot P(A)$

- To get  $P(A | B)$ , we divide both sides by  $P(B)$ :

- $$P(A | B) = \frac{P(B)}{P(B|A) \cdot P(A)}$$



# BAYES' THEOREM IN AI

$$P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B)}$$

Probability **B** Will Happen Given Evidence **A** Has Already Happened      Probability **A** Will Happen

Probability **A** Will Happen Given Evidence **B** Has Already Happened

Probability **B** Will Happen

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# BAYES' THEOREM IN AI

- **Problem Statement**
- Suppose a patient is tested for a rare disease that affects **1% of the population**.
- The test is **95% accurate**, meaning:
  - If a person has the disease, the test correctly identifies it **95%** of the time (True Positive Rate).
  - If a person does not have the disease, the test correctly returns negative **95%** of the time (True Negative Rate).
- **Now, if a patient's test result comes back positive, what is the probability that the patient actually has the disease?**



# BAYES' THEOREM IN AI

## Given Data

- $P(\text{Disease}) = 0.01$  (Prior probability of disease)
- $P(\text{No Disease}) = 0.99$
- $P(\text{Positive}|\text{Disease}) = 0.95$  (Test sensitivity)
- $P(\text{Positive}|\text{No Disease}) = 0.05$  (False positive rate)

We need to find:  $P(\text{Disease}|\text{Positive})$  (Posterior probability).



# Step-by-Step Calculation Using Bayes' Rule

Bayes' Theorem formula:

$$P(\text{Disease}|\text{Positive}) = \frac{P(\text{Positive}|\text{Disease}) \times P(\text{Disease})}{P(\text{Positive})}$$

First, calculate  $P(\text{Positive})$  (total probability of a positive test):

$$P(\text{Positive}) = P(\text{Positive}|\text{Disease}) \times P(\text{Disease}) + P(\text{Positive}|\text{No Disease}) \times P(\text{No Disease})$$

$$P(\text{Positive}) = (0.95 \times 0.01) + (0.05 \times 0.99) = 0.0095 + 0.0495 = 0.059$$

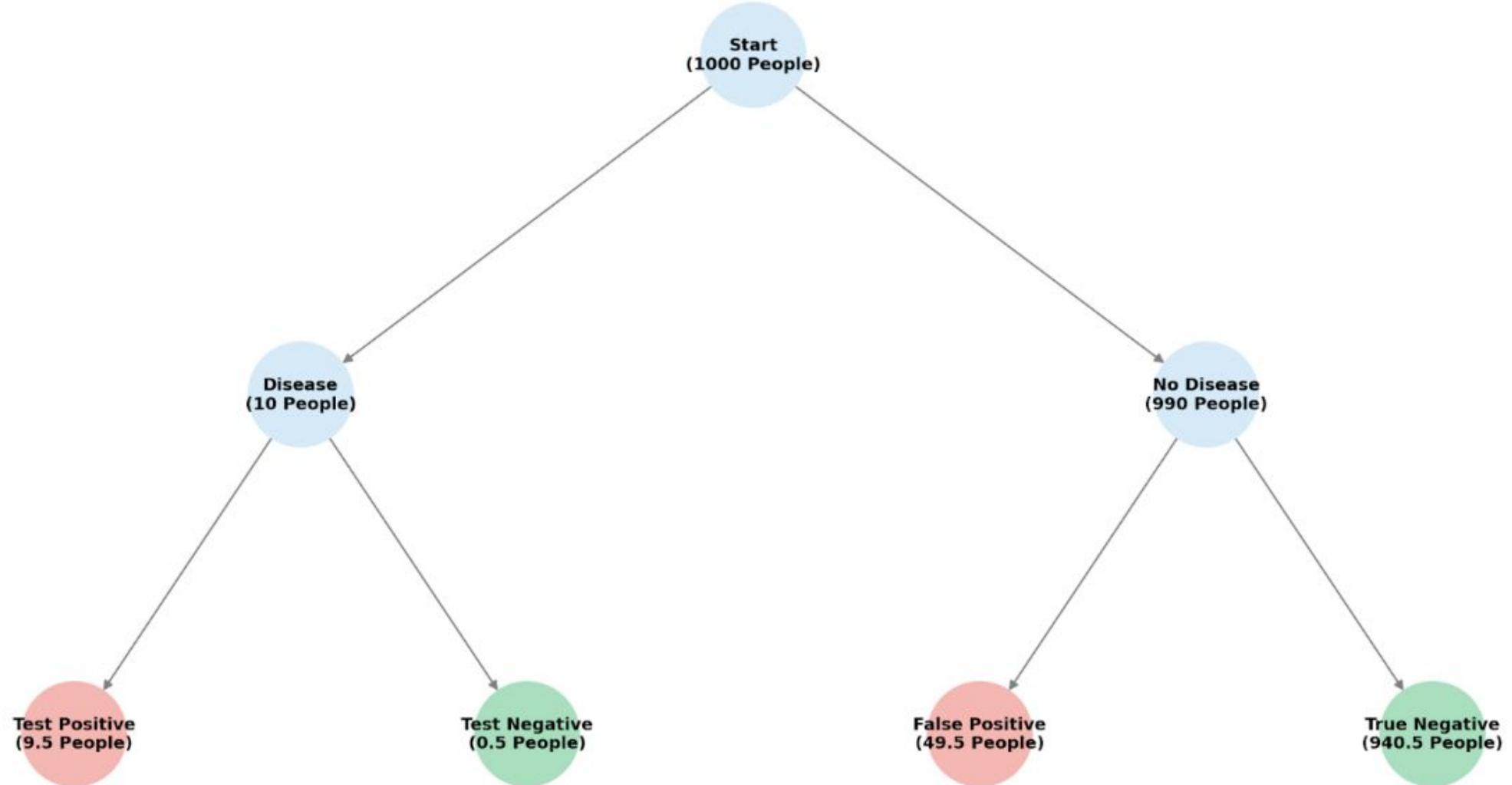
Now, applying Bayes' Theorem:

$$P(\text{Disease}|\text{Positive}) = \frac{0.95 \times 0.01}{0.059} = \frac{0.0095}{0.059} \approx 0.161$$

Thus, even after a positive test, the probability that the patient actually has the disease is only about 16.1%.

# Tree Diagram

Enhanced Visual Tree Diagram for Disease Diagnosis (Bayes' Theorem)



# EXAMPLE 2

- You have two coins in a bag:
- **Coin F (fair):**  $P(H) = 0.5$ .
- **Coin B (biased):**  $P(H) = 0.75$ .
- You pick one coin at random (each with prior probability 0.5) and flip it 3 times, observing **HHH** (three heads).  
Question: **what is the probability you picked the biased coin given the data — i.e.  $P(B | HHH)$ ?**



# BAYES' THEOREM IN AI

- **Application of Bayes Theorem in AI:**
- **1. Spam Email Classification:** Bayes' Theorem is employed in spam email classification. By analyzing the likelihood of certain words or phrases occurring in spam or legitimate emails, a spam filter can calculate the probability that an email is spam. This probability is then used to classify emails as either spam or not.
- **2. Medical Diagnosis:** In medical diagnosis, Bayes' Theorem is used to update the probability of a patient having a disease based on diagnostic test results and the prevalence of the disease. It aids healthcare professionals in making more informed decisions about patient care and treatment options.
- **3. Natural Language Processing (NLP):** Bayes' Theorem is applied in various NLP tasks, including language modeling, part-of-speech tagging, sentiment analysis, and information retrieval. For example, in text classification, Bayes' Theorem can help determine the probability that a given document belongs to a particular category, such as topic classification or sentiment classification



# BAYESIAN BELIEF NETWORKS

- **Bayesian Belief Network (BBN)** is a **graphical model that represents the probabilistic relationships among variables**.
- It is used to handle uncertainty and make predictions or decisions based on probabilities.
- **Graphical Representation:** Variables are represented as nodes in a **directed acyclic graph (DAG)**, and their dependencies are shown as edges.
- **Conditional Probabilities:** Each node's probability depends on its parent nodes, expressed as  $P(\text{Variable} \mid \text{Parent})$ .
- **Probabilistic Model:** Built from probability distributions, BBNs apply probability theory for tasks like prediction and anomaly detection.
- Bayesian Belief Networks are valuable tools for understanding and solving problems involving uncertain events. They are also known as **Bayes networks, belief networks, decision networks, or Bayesian models**.



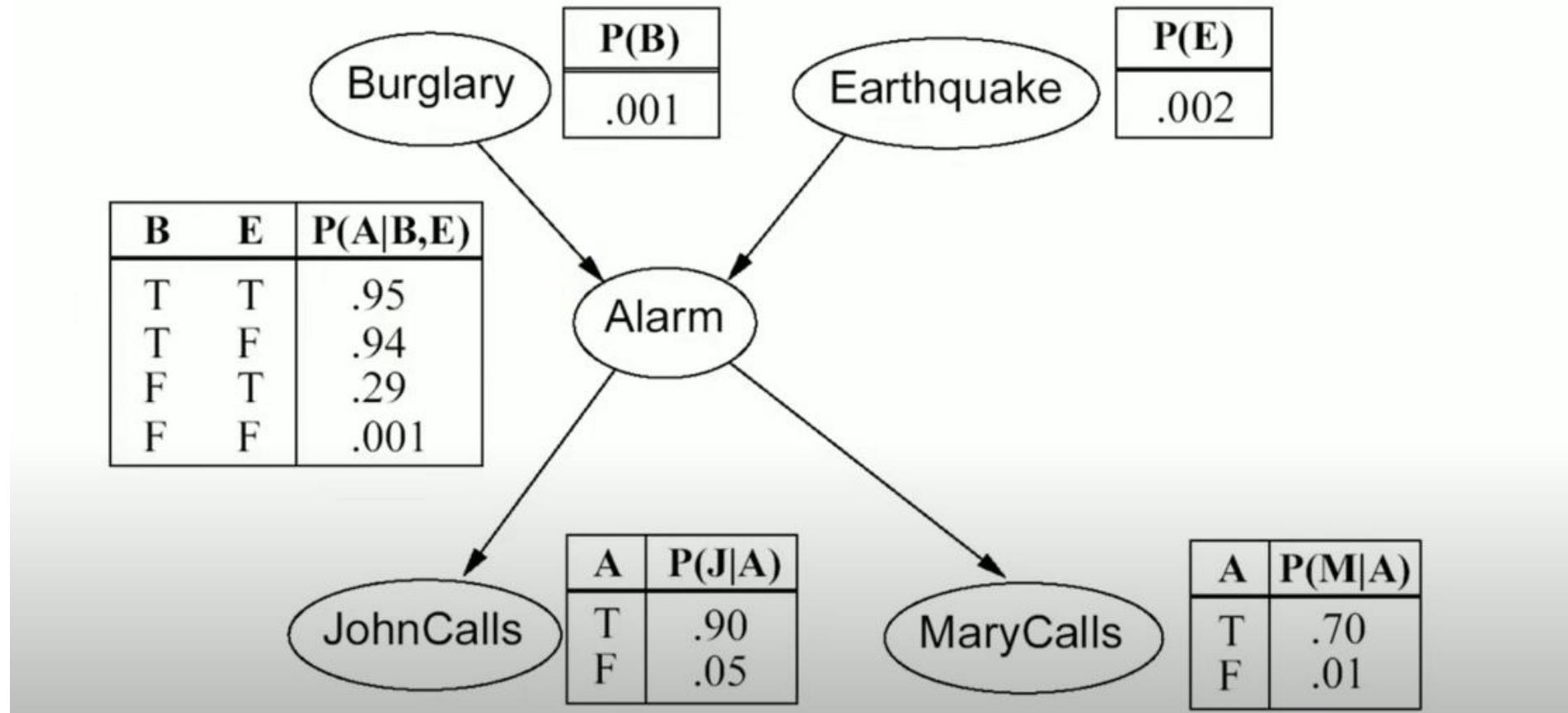
# BAYESIAN BELIEF NETWORKS

## EXAMPLE 1

- You have a new burglar alarm installed at home.
- It is fairly reliable at detecting burglary, but also sometimes responds to minor earthquakes.
- You have two neighbors, John and Merry , who promised to call you at work when they hear the alarm.
- John always calls when he hears the alarm, but sometimes confuses telephone ringing with the alarm and calls too.
- Merry likes loud music and sometimes misses the alarm.
- Given the evidence of who has or has not called, we would like to estimate the probability of a burglary.

# BAYESIAN BELIEF NETWORKS

## BAYESIAN BELIEF NETWORKS – EXAMPLE – 1



1. What is the probability that the alarm has sounded but neither a burglary nor an earthquake has occurred, and both John and Merry call?



## 2. What is the probability that John call?

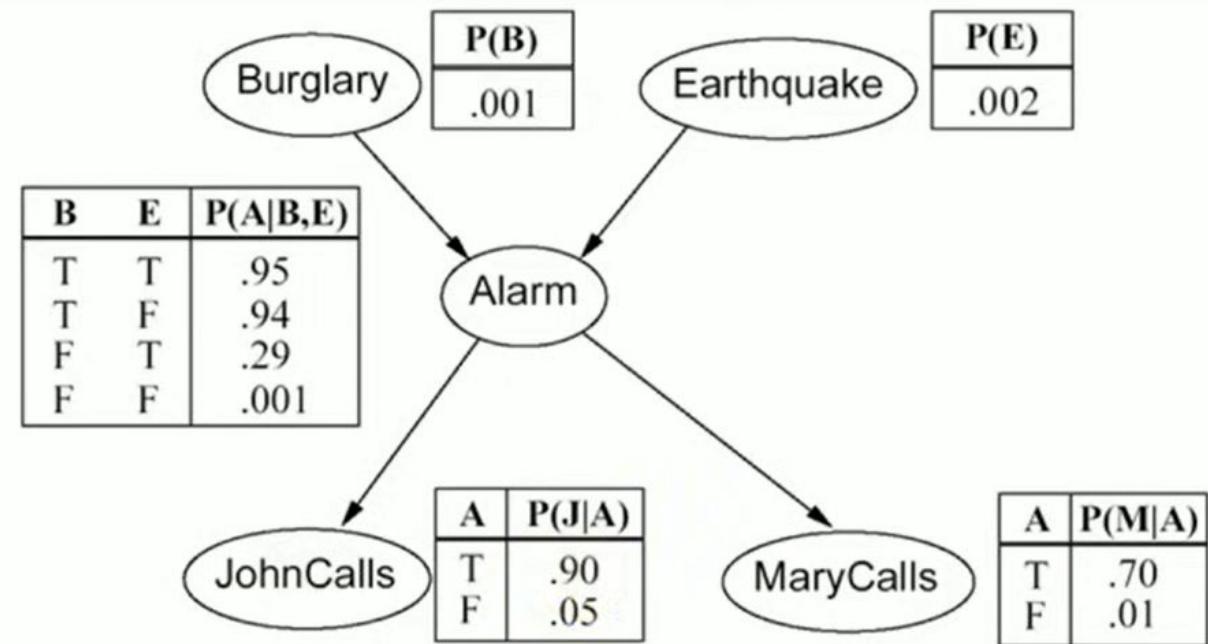
Solution:

$$P(j) = P(j | a) P(a) + P(j | \neg a) P(\neg a)$$

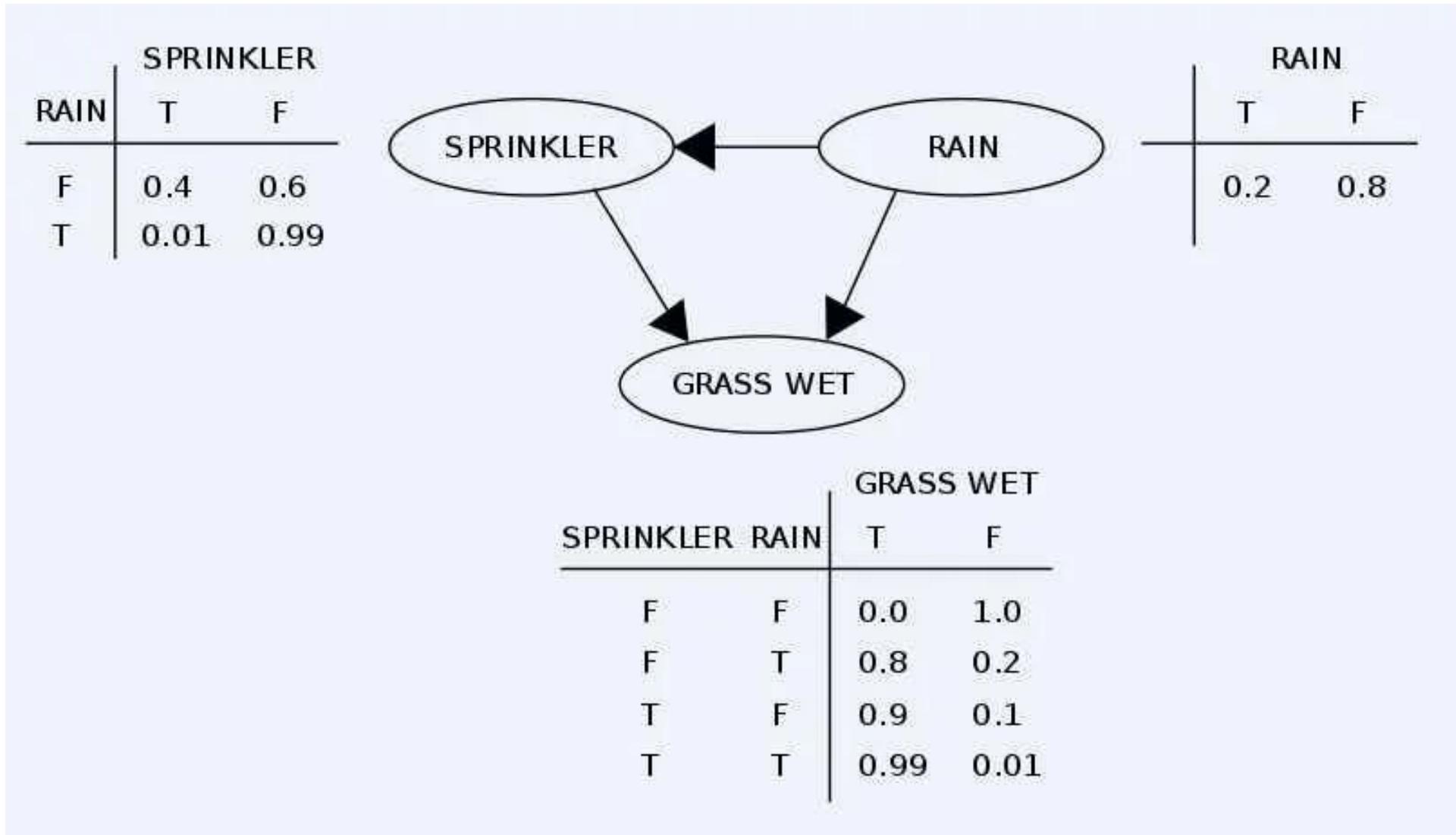
$$= P(j | a) \{P(a | b, e) * P(b, e) + P(a | \neg b, e) * P(\neg b, e) + P(a | b, \neg e) * P(b, \neg e) + P(a | \neg b, \neg e) * P(\neg b, \neg e)\}$$

$$+ P(j | \neg a) \{P(\neg a | b, e) * P(b, e) + P(\neg a | \neg b, e) * P(\neg b, e) + P(\neg a | b, \neg e) * P(b, \neg e) + P(\neg a | \neg b, \neg e) * P(\neg b, \neg e)\}$$

$$= 0.90 * 0.00252 + 0.05 * 0.9974 = 0.0521$$



# EXAMPLE 2



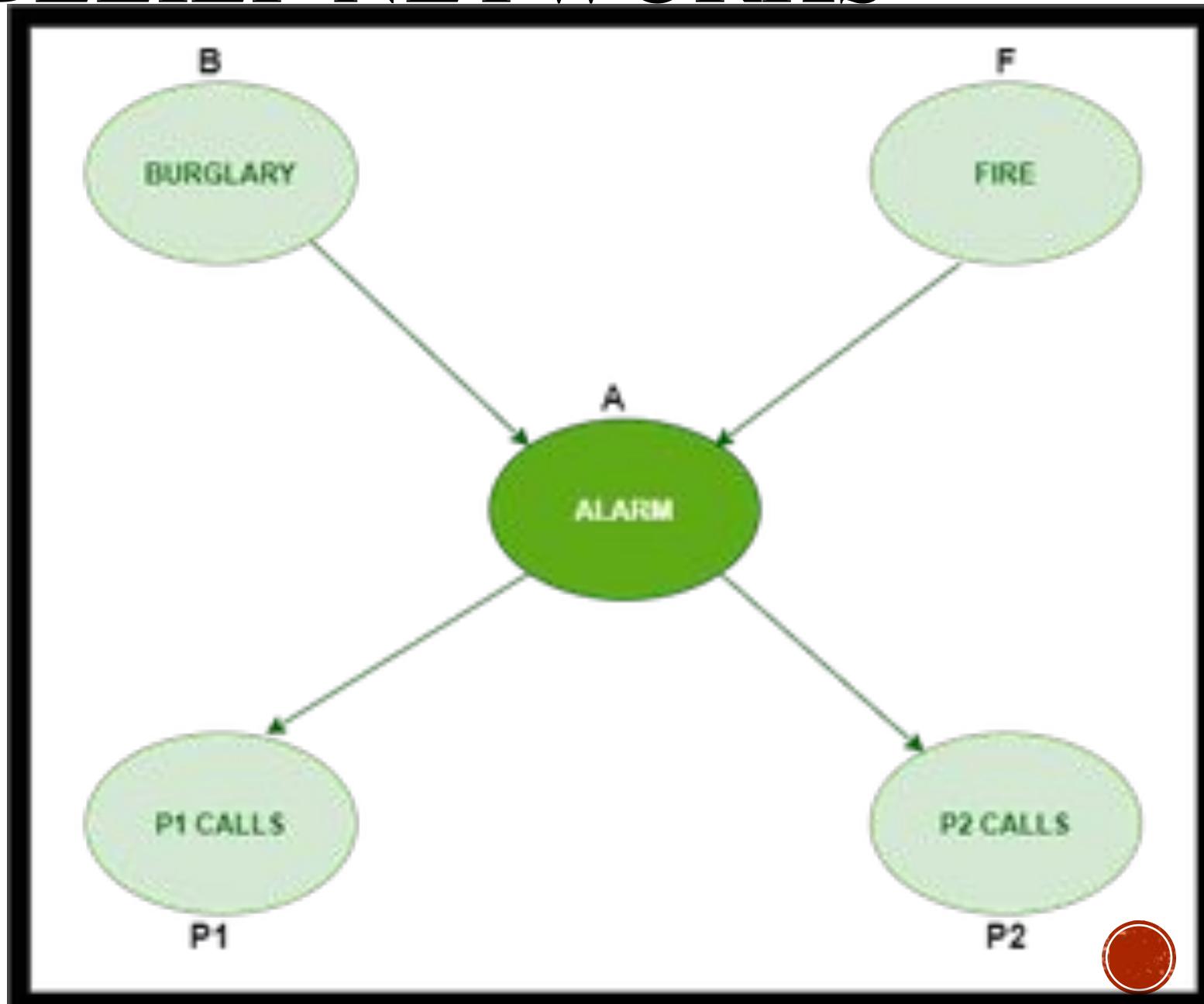
# EXAMPLE 2

- Find Probability of wet grass when there is a sprinkler and rain?
- $P(G,S,R) = ?$



# BAYESIAN BELIEF NETWORKS

- Consider this example



- In the above figure, we have an alarm 'A' - a node, say installed in a house of a person 'gfg',
- which rings upon two probabilities i.e burglary 'B' and fire 'F', which are - parent nodes of the alarm node.
- The alarm is the parent node of two probabilities P1 calls 'P1' & P2 calls 'P2' person nodes.
- Upon the instance of burglary and fire, 'P1' and 'P2' call person 'gfg', respectively.
- But, there are few drawbacks in this case, as sometimes 'P1' may forget to call the person 'gfg', even after hearing the alarm, as he has a tendency to forget things, quick.
- Similarly, 'P2', sometimes fails to call the person 'gfg', as he is only able to hear the alarm, from a certain distance.



# BAYESIAN BELIEF NETWORKS

- Calculating Conditional Probability of Events in a Bayesian Network
- Find the probability that
  - 'P1' is true (P1 has called 'gfg'),
  - 'P2' is true (P2 has called 'gfg')
  - when the alarm 'A' rang, but no burglary 'B' and fire 'F' has occurred.
  - =>  $P(P1, P2, A, \neg B, \neg F)$  [ where- P1, P2 & A are 'true' events and ' $\neg B$ ' & ' $\neg F$ ' are 'false' events]



# BAYESIAN BELIEF NETWORKS

▪ *Burglary 'B'* -

- $P(B=T) = 0.001$  ('B' is true i.e burglary has occurred)
- $P(B=F) = 0.999$  ('B' is false i.e burglary has not occurred)

▪ *Fire 'F'* -

- $P(F=T) = 0.002$  ('F' is true i.e fire has occurred)
  - $P(F=F) = 0.998$  ('F' is false i.e fire has not occurred)
- *Alarm 'A'* -

B	F	$P(A=T)$	$P(A=F)$
T	T	0.95	0.05
T	F	0.94	0.06
F	T	0.29	0.71
F	F	0.001	<u>0.999</u>

# BAYESIAN BELIEF NETWORKS

- The alarm 'A' node can be 'true' or 'false' ( i.e may have rung or may not have rung). It has two parent nodes burglary 'B' and fire 'F' which can be 'true' or 'false' (i.e may have occurred or may not have occurred) depending upon different conditions.
- *Person 'P1'* –

A	P (P1=T)	P (P1=F)
T	<u>0.95</u>	0.05
F	0.05	0.95

The person 'P1' node can be 'true' or 'false' (i.e may have called the person 'gfg' or not) . It has a parent node, the alarm 'A', which can be 'true' or 'false' (i.e may have rung or may not have rung ,upon burglary 'B' or fire 'F').



# BAYESIAN BELIEF NETWORKS

▪ *Person 'P2'* –

A	P (P2=T)	P (P2=F)
T	<u>0.80</u>	0.20
F	0.01	0.99

The person 'P2' node can be 'true' or false' (i.e may have called the person 'gfg' or not). It has a parent node, the alarm 'A', which can be 'true' or 'false' (i.e may have rung or may not have rung, upon burglary 'B' or fire 'F').



# BAYESIAN BELIEF NETWORKS

- **Solution:** Considering the observed probabilistic scan -

- With respect to the question —  $P(P1, P2, A, \sim B, \sim F)$  , we need to get the probability of 'P1'. We find it with regard to its parent node - alarm 'A'. To get the probability of 'P2', we find it with regard to its parent node — alarm 'A'.
- We find the probability of alarm 'A' node with regard to ' $\sim B$ ' & ' $\sim F$ ' since burglary 'B' and fire 'F' are parent nodes of alarm 'A'.
- From the observed probabilistic scan, we can deduce -
  - $P(P1, P2, A, \sim B, \sim F)$
  - $= P(P1/A) * P(P2/A) * P(A/\sim B \sim F) * P(\sim B) * P(\sim F)$
  - $= 0.95 * 0.80 * 0.001 * 0.999 * 0.998$
  - $= 0.00075$

