Introduction of fourth generation lepton (ℓ_4) and the study of $\mu, \mu \to \ell_4, \ell_4$ kinematics at Muon collider with $\sqrt{s} = 14\ \mathrm{TeV}$

Ramkrishna Joshi

Indian Institute Of Technology Hyderabad

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Contents Overview

Extending the Standard Model: Introducing a Fourth-Generation Lepton

- No fundamental principle limits the number of fermion generations.
- Could address open questions in neutrino masses, dark matter, and baryon asymmetry.
- Constraints from LEP and LHC imply the fourth-generation neutrino (ν_4) must be heavy and possibly sterile.

$$\text{Lepton Doublets:} \quad \begin{pmatrix} \nu_e \\ e \end{pmatrix}, \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}, \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}, \begin{pmatrix} \nu_4 \\ \ell_4 \end{pmatrix}$$

$$\text{Quark Doublets:} \quad \begin{pmatrix} u \\ d \end{pmatrix}, \begin{pmatrix} c \\ s \end{pmatrix}, \begin{pmatrix} t \\ b \end{pmatrix}, \begin{pmatrix} t' \\ b' \end{pmatrix}$$

The mass of the fourth generation lepton is set to be 113.6 GeV in the paper titled "Joint analysis of Higgs decays and electroweak precision observables in the Standard Model with a sequential fourth generation",Eberhardt et al. (2012). We consider this as our reference.

ℓ_4 Model Creation in SARAH

```
(* Matter Fields *)

FermionFields[[1]] = {q, 3, {uL, dL},      1/6, 2,
FermionFields[[2]] = {l, 4, {vL, eL},     -1/2, 2,
FermionFields[[3]] = {d, 3, conj[dR],      1/3, 1,
FermionFields[[4]] = {u, 3, conj[uR],     -2/3, 1,
FermionFields[[5]] = {e, 4, conj[eR],        1, 1,

ScalarFields[[1]] =  {H, 1, {Hp, H0},      1/2, 2,
```

(a)

```
{Fe,  {Description -> "Leptons",
 LaTeX -> "e",
 OutputName -> "Fe",
 PDG -> {11, 13, 15, 17},   (* Added PDG for e4 *)
 Mass -> Automatic,
 ElectricCharge -> -1,
 LHPC -> True }},
{Fv, {Description -> "Neutrinos",
LaTeX -> "\\nu",
OutputName -> "Fv",
PDG -> {12, 14, 16, 18},   (* Added PDG for ν4 *)
ElectricCharge -> 0,
LHPC -> True }}
```

(b)

Figure: Steps in implementing the ℓ_4 model in SARAH: (a) Particle content and quantum numbers, (b) Gauge group definitions

Model Initialization in CalcHEP

Parameters

Clr Del Size Read ErrMes

Name	Value	> Comment
Mh	125	higgs mass
MFe1	0.000511	electron mas
MFe2	0.1057	muon mass
MFe3	1.777	taon mass
MFe4	113	foron mass
WZ	2.4952	width
MZ	91.1876	
WWp	2.141	width
Wd1	0	width
Md1	0.0035	
Wd2	0	width
Md2	0.104	
Wd3	0	width
Md3	4.2	
Wu1	0	width
Mu1	0.0015	
Wu2	0	width
Mu2	1.27	
Wu3	1.51	width
Mu3	171.2	
WFe1	0	width

F1 F2 Xgoto Ygoto Find Write

Definition of model parameters in vars.mdl including physical constants.

Full name	A	A+
hh	h	h
VG	g	g
VP	A	A
VZ	Z	Z
VWp	Wp	Wm
Fv[{1}]	Fv1	fv1
Fv[{2}]	Fv2	fv2
Fv[{3}]	Fv3	fv3
Fv[{4}]	Fv4	fv4
Fd[{1}]	d1	D1
Fd[{2}]	d2	D2
Fd[{3}]	d3	D3
Fu[{1}]	u1	U1
Fu[{2}]	u2	U2
Fu[{3}]	u3	U3
Fe[{1}]	Fe1	fe1
Fe[{2}]	Fe2	fe2
Fe[{3}]	Fe3	fe3
Fe[{4}]	Fe4	fe4

Particle content setup in prtcls.mdl.

Particle Information for ℓ_4 and ν_4 in CalcHEP

```
Particle information
Patricle Fe4(fe4),  PDG = 17,  Mass= 1.130E+02  Width=2.27E-01
Quantum numbers:  spin=1/2,  charge(el.)=-1  color=1
 Branchings & Decay channels:
 1.00E+00      Fe4 -> Wm,Fv4
```

(a)

```
Particle information
Patricle Fv4(fv4),  PDG = 18,  Mass= Zero
Quantum numbers:  spin=1/2,  charge(el.)=0  color=1
```

(b)

Figure: Definition of (a) fourth-generation charged lepton ℓ_4 and (b) neutrino ν_4 in `prtcls.mdl` on CalcHEP

.

Initializing $\mu^+\mu^- \to \ell_4^+\ell_4^-$

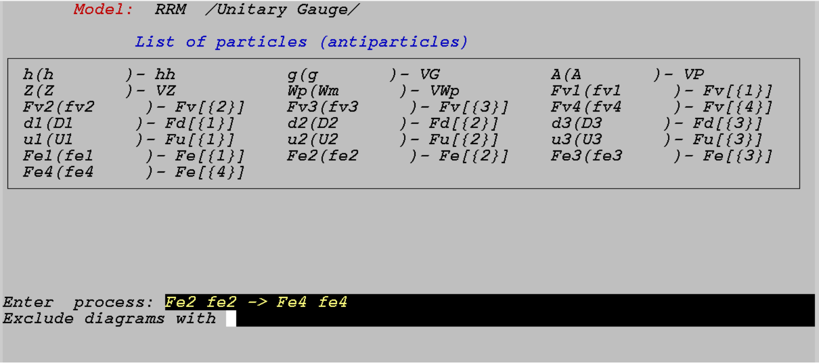
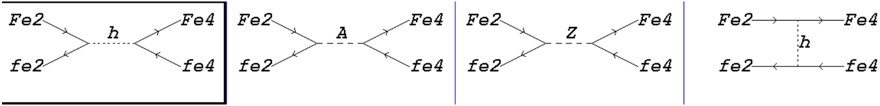
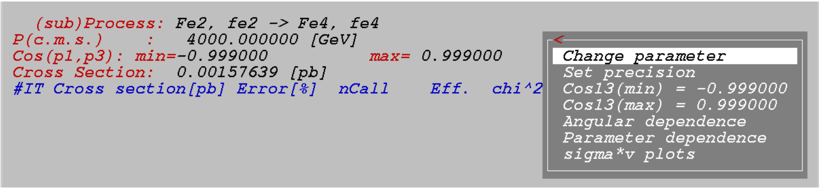


Figure: CalcHEP (anti)particle listing and defining scattering process

Cross section calculations in CalcHEP



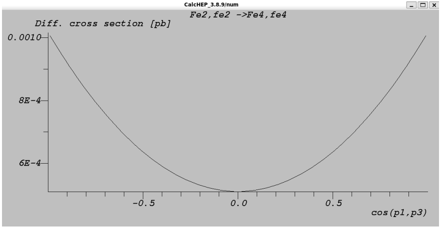
(a)



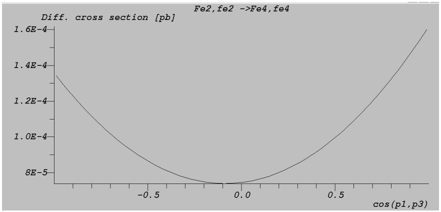
(b)

Figure: (a) Process listing and (b) Total cross section from 1D integration results at 8000 GeV centre of mass energy from CalcHEP.

Angular dependence for Photon mediated and Z-Boson mediated processes



(a)



(b)

Figure: Angular dependence for the processes mediated by (a) Photon (A) (b)Z boson.

Generating events in ClacHEP

```
rkjoshi123@DESKTOP-M7HQSN7:~/CPP/calchep/calchep_3.8.9/work/results$ ../bin/subproc_cycle 5000
5000 events are requested
Use second parameter to specify flux in [1/fb]
#Subprocess 1 ( Fe2, fe2 -> Fe4, fe4 ) Cross section = 5.1555E-04 pb (1.80E-06%) , 5000 events
5.156E-04  -total cross section[pb]
5000    -maximum number of events
 5000 events are generated
Events in LHE format: events_1_1.lhe
Total Cross Section 5.156E-04 [pb] (1.800E-06%)
See details in directory 1_1
```

Figure: Generating 5000 events of the scattering process

- Due to the small cross section ($\sim 5.16 \times 10^{-4}$ pb), it's recommended to override the default flux normalization to achieve sufficient statistics. The number of events is computed using:

$$N = \sigma \times \mathcal{L}$$

Where N = number of events, sigma= cross section in pb ,$\mathcal{L}$ = luminosity in pb^{-1}.If the default luminosity is $\mathcal{L} = 1\,\mathrm{fb}^{-1} = 10^3\,\mathrm{pb}^{-1}$, and $\sigma = 1.57 \times 10^{-3}\,$pb, then $N = 5.156 \times 10^{-3} \times 10^3 = 5$ events. Therefore to generate sufficient events, I override the flux.

Event Simulation and Kinematics at Muon collider card in Delphes.

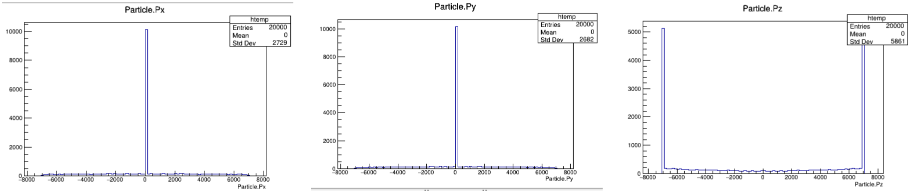


Figure: Momentum components of particles from the process $\mu^+\mu^- \to \ell_4^+\ell_4^-$ simulated in Delphes. The detector is symmetric; particles are boosted along the z-axis (beam axis), while p_x and p_y show transverse dynamics.

K_T Jet kinematics at Delphes Muon collider card

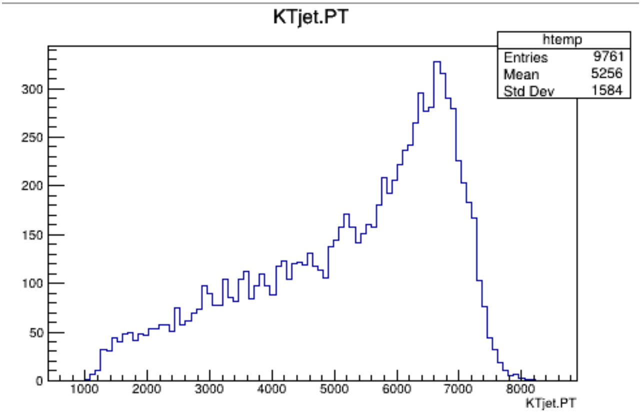
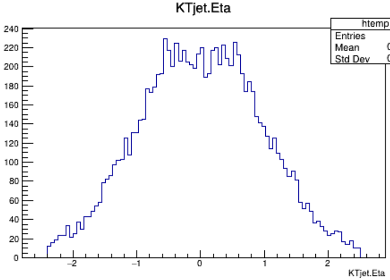
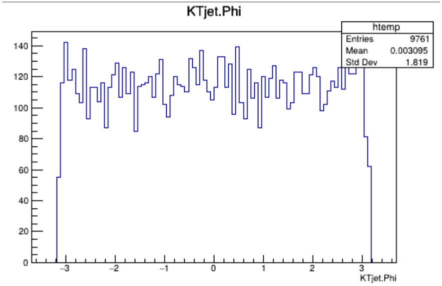


Figure: Plot represents transverse momentum for K_T Jets.The jets are very energetic peaking around 6.5–7 TeV, which strongly implies they're coming from the decays of very heavy particles like fourth-generation leptons.



(a)



(b)

Angular distributions of k_T jets from the simulated $\mu^+\mu^- \to \ell_4^+\ell_4^-$ process. (a) The pseudo-rapidity (η) and (b) azimuthal angle (ϕ) reflect the detector coverage and event topology.

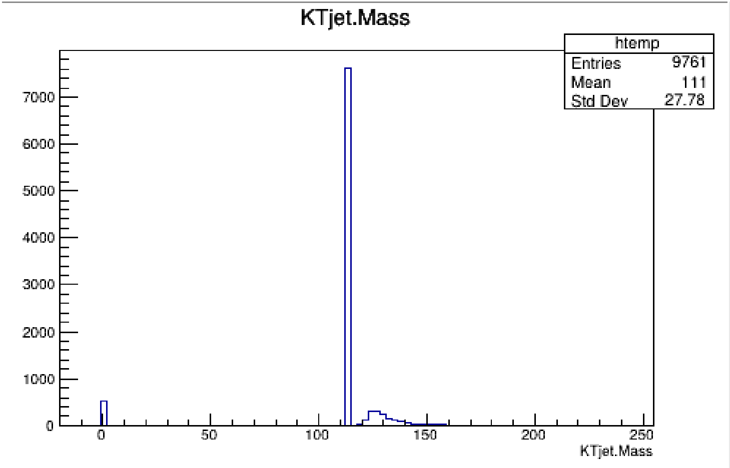


Figure: Figure shows the mass of K_T Jets. The model incorporates the fourth generation lepton with a mass of 113.6 GeV. The sharp peak in the plot corresponds to high-quality, high-pT jets retaining the mass of the decaying ℓ_4. Other smaller peaks may represent noise, soft jets, beam remnants.

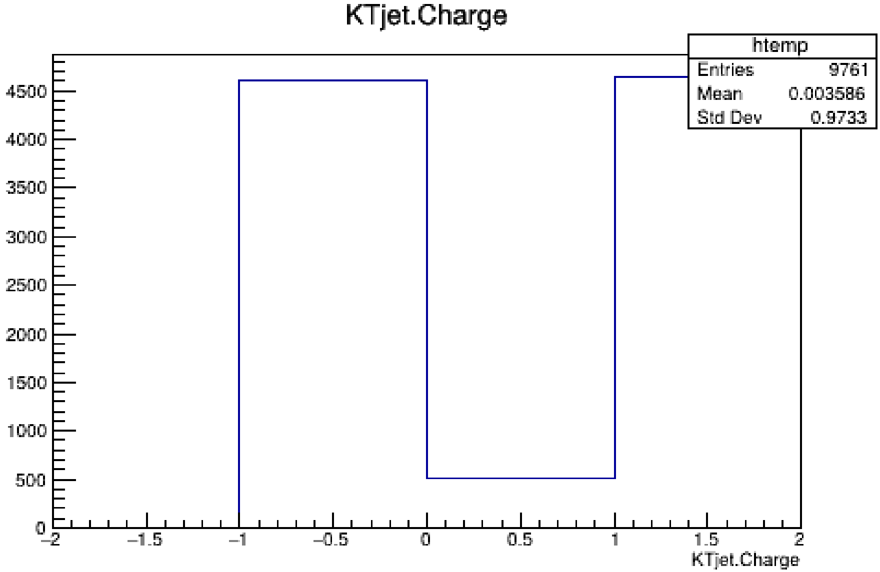


Figure: Charge distribution (-1,0,+1) of K_T Jets from the simulated $\mu^+\mu^- \to \ell_4^+\ell_4^-$ process.

The distribution of jet charges shows approximately equal numbers of jets with charges $+1$ and -1, with a smaller fraction being neutral. This is consistent with the expected hadronic decays of W bosons from fourth-generation lepton pair production,

$$\mu^+\mu^- \to \ell_4^+\ell_4^- \to W^+\nu\, W^-\bar{\nu}.$$

The W bosons can decay hadronically via

$$W^+ \to u\bar{d},\ c\bar{s} \quad \text{and} \quad W^- \to \bar{u}d,\ \bar{c}s,$$

leading to jets with net positive or negative charge. The small neutral component likely arises from artifacs related to jet reconstruction algorithm of Delphes or likely gluon initiated jets.

*VLCJETR*07*N*2 Jet kinematics at Delphes Muon collider card

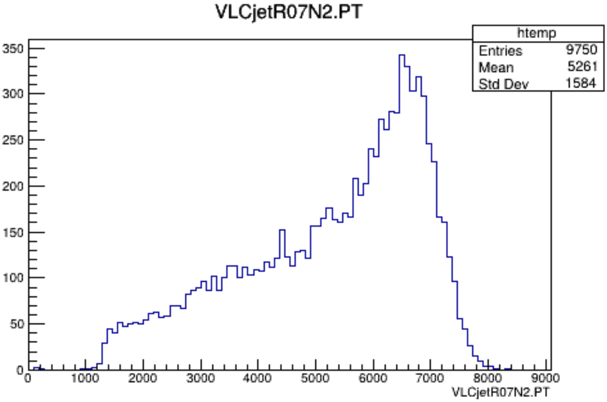
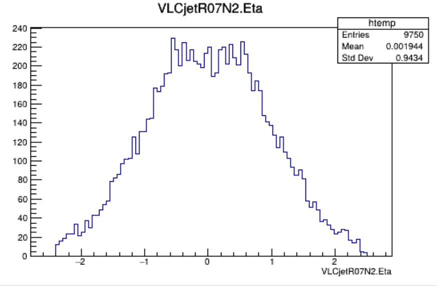
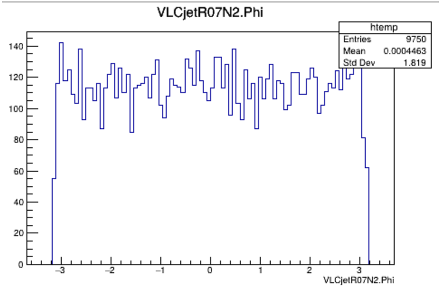


Figure: Plot represents transverse momentum for *VLC* Jets with R=0.7 and N=2.The jets are very energetic peaking around 6.5–7 TeV, which strongly implies they're coming from the decays of very heavy particles like ℓ_4.



(a)



(b)

Angular distributions of $VLCR07N2$ jets from the simulated $\mu^+\mu^- \rightarrow \ell_4^+\ell_4^-$ process. (a) The pseudo-rapidity (η) and (b) azimuthal angle (ϕ) reflect the detector coverage and event topology.

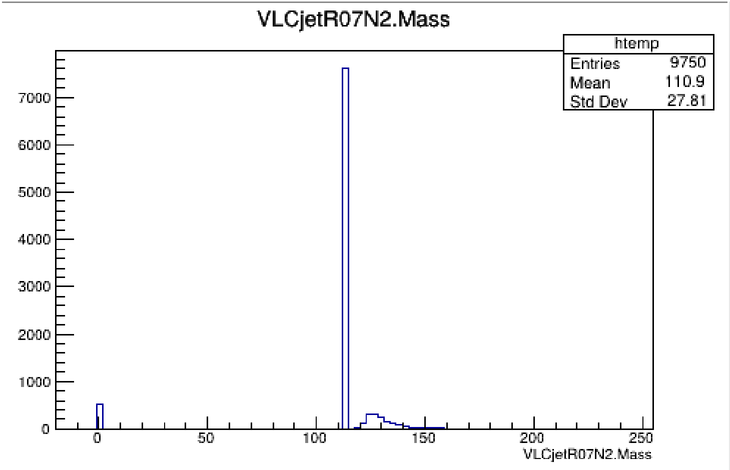
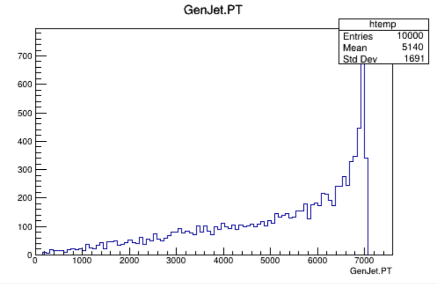
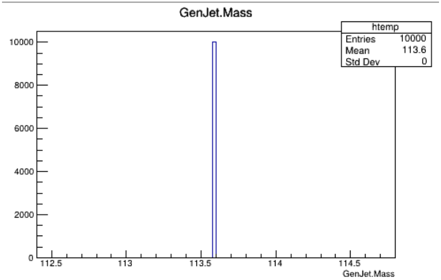


Figure: Figure shows the mass of $VLCR07N2$ Jets. The model incorporates the fourth generation lepton with a mass of 113.6 GeV. The sharp peak in the plot corresponds to high-quality, high-pT jets retaining the mass of the decaying ℓ_4. Other smaller peaks may represent noise, soft jets, beam remnants.

Comparing with GenJet kinematics.

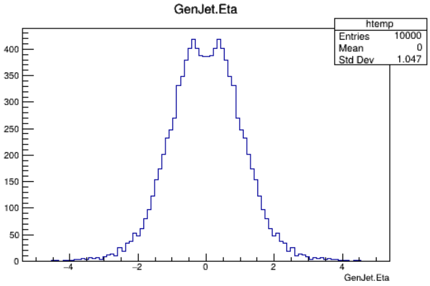


(a)

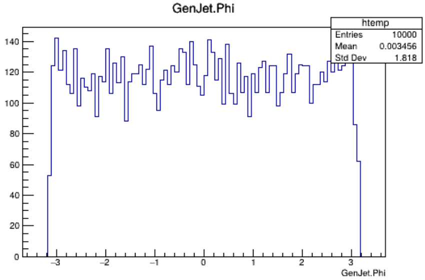


(b)

Figure: Distributions of (a) transverse momentum (p_T) and (b) mass of GenJets generated at the muon collider in Delphes. Since detector effects are not included, the distributions exhibit significantly less smearing and broadening compared to reconstructed jets from k_T and VLC algorithms.



(a)



(b)

Angular distributions of Genjets from the simulated $\mu^+\mu^- \to \ell_4^+\ell_4^-$ process. (a) The pseudo-rapidity (η) and (b) azimuthal angle (ϕ) reflect the detector coverage and event topology.

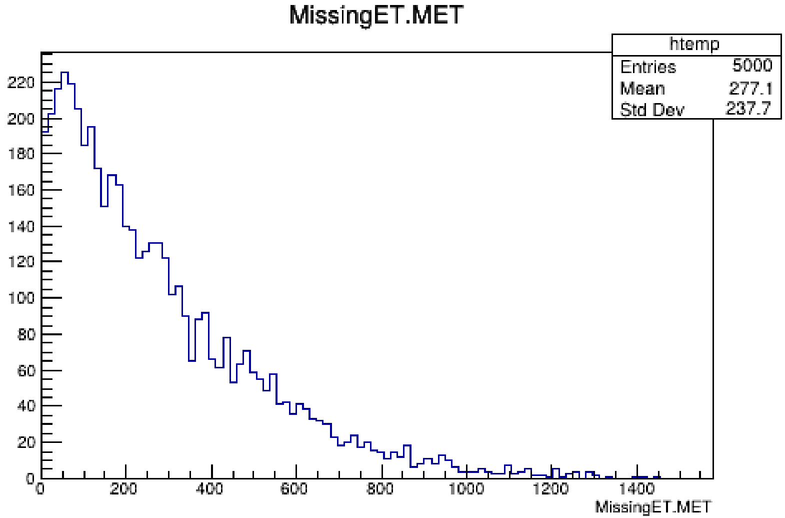
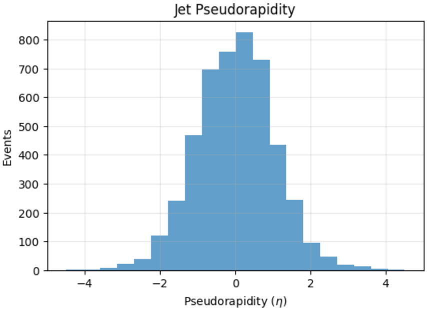
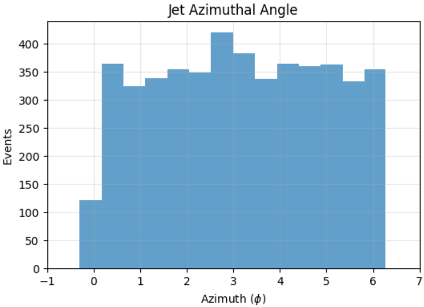


Figure: Plot shows the missing transverse energy carried off by invisible neutrinos in the simulated events at the muon collider.

Pythia analysis and kinematics for jets



(a)



(b)

Angular distributions of jets reconstructed in PYTHIA from the simulated $\mu^+\mu^- \to \ell_4^+\ell_4^-$ process. Plots represents (a) The pseudo-rapidity (η) and (b) azimuthal angle (ϕ)

Jet Multiplicity

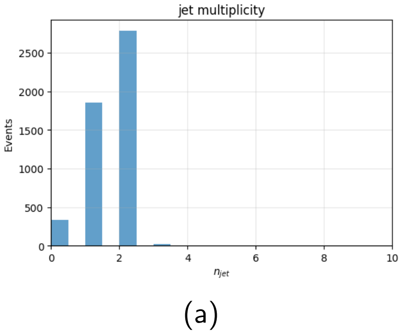


Figure: Jet multiplicity comparison for $\mu^+\mu^- \to \ell_4^+\ell_4^-$ process with (a) ISR+FSR on and (b) ISR+FSR off.

Interpretation of Jet Multiplicity for $\mu^+\mu^- \to \ell_4\bar{\ell}_4$

- The process involves production of heavy fourth-generation leptons ℓ_4, which decay via:

$$\ell_4 \to W\nu_4 \quad \text{with} \quad W \to jj$$

- This decay chain can lead to up to 4 jets per event, assuming both W bosons decay hadronically.
- However, jet multiplicity may be lower due to:
  - Some W bosons decaying leptonically,
  - Detector acceptance
  - Overlapping or soft jets not being reconstructed.
- Both plots show a clear peak at $n_{\text{jet}} = 2$, indicating that in most events, only two jets are reconstructed.

Dijet mass histogram

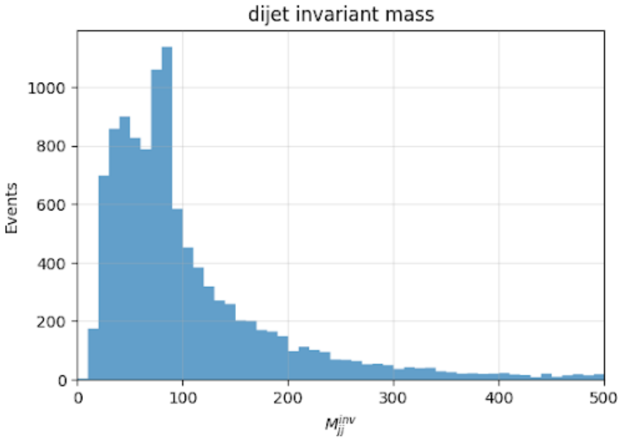
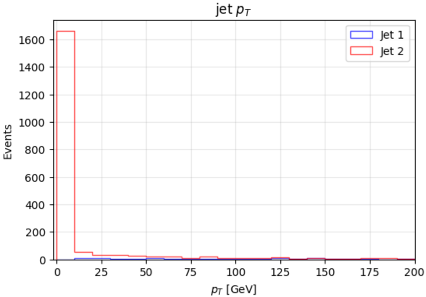
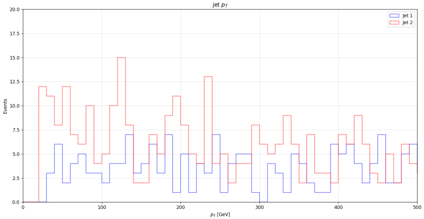


Figure: Plot shows histogram of Dijet invariant mass. There are two peaks visible. The peak in the range 80-90 GeV corresponds to W events since the fourth generation lepton decays to a W boson and fourth generation neutrino.

Jet p_T histogram



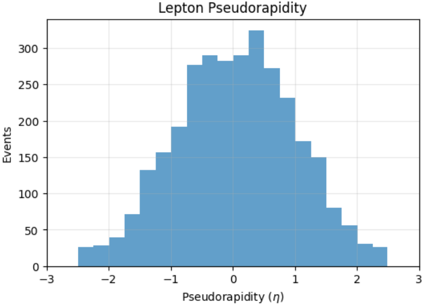
(a)



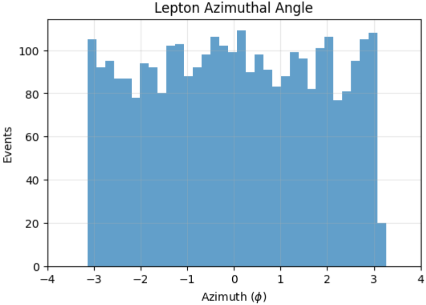
(b)

Histogram of jet p_T. Putting a threshold on minimum p_T gets rid of all the soft jets and likely misconstructed jets.

Pythia analysis and kinematics for leptons



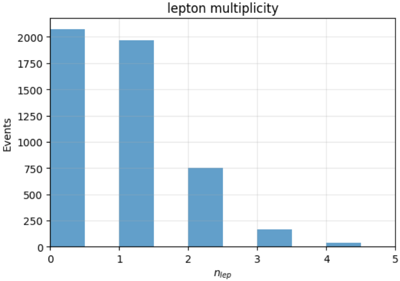
(a)



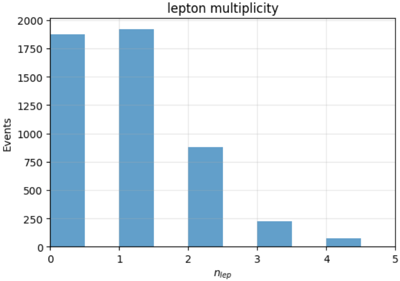
(b)

Angular distributions of leptons reconstructed in PYTHIA from the simulated $\mu^+\mu^- \to \ell_4^+\ell_4^-$ process. Plots represents (a) The pseudo-rapidity (η) and (b) azimuthal angle (ϕ)

Lepton Multiplicity



(a)



(b)

Figure: Lepton multiplicity comparison for $\mu^+\mu^- \to \ell_4^+\ell_4^-$ process with (a) ISR+FSR on and (b) ISR+FSR off.

Interpretation: Lepton Multiplicity for $\mu^+\mu^- \to \ell_4^+\ell_4^-$

- The fourth-generation charged lepton decays as:

$$\ell_4^{\pm} \to W^{\pm}\nu_4, \quad \text{with } W^{\pm} \to \ell\nu \text{ or } q\bar{q}'$$

  - 0–1 leptons: hadronic W decay(s) or leptons outside detector acceptance.
  - 2 leptons: both W bosons decay leptonically.
  - ≥ 3 leptons: Maybe from ISR/FSR-induced radiation of additional leptons.

Lepton p_T

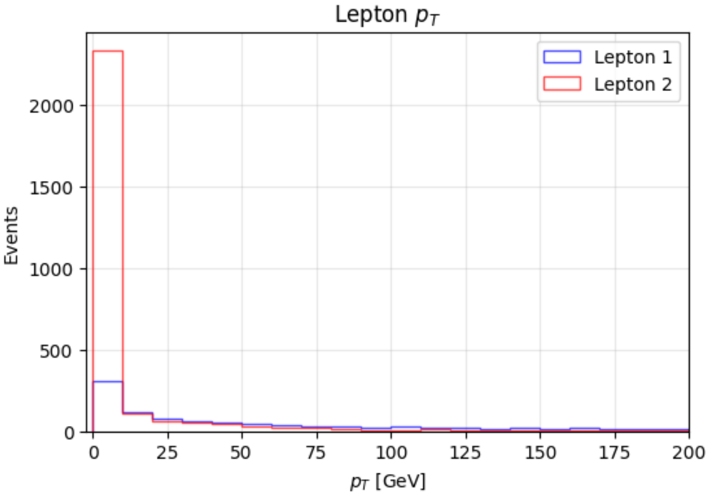


Figure: Histogram of lepton p_T

Summary and future work

- In this project, we successfully incorporate a heavy fourth generation lepton and neutrino using SARAH and study various scattering processes.
- I have demonstrated the $\mu\mu$ scattering and simulated the events at a Delphes muon collider card and demonstrated the kinematics of the same.
- The dijet amss confirms the W decay channel as a result of the fourth generation lepton decay.
- There seems to be a discrepancy with the jet p_T which may be due to clustering problem, misidentification or generation of a lot of soft jets.
- Further constraints on the mass can be studied by incorporating a dirac neutrino.