CP Violation in Kaon Decays

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1. Abstract

This is a theoretical project to understand the CP violation in the K-meson and B-meson decays. The project aims at developing a mathematical intuition of the subject based on the literature review. The CPT (charge conjugation, parity transformation, and time inversion) symmetry is one of the most fundamental symmetries observed in nature. This symmetry is observed in the most basic building blocks of the universe from the subatomic realms. It is expected that all the fundamental interactions in nature should be symmetric under C, P, and T operations. This symmetry facilitates our understanding of particles and their antiparticles. How these subatomic particles behave under the symmetry transformations reveals a lot about the observational evidence of the universe that we have long gathered. However, there are interactions that blow apart this symmetry. The expected outcome of this project is an understanding of such interactions which are asymmetric under the CPT transformations. In this segment, I present my current understanding of the subject from a mathematical perspective.

2. Introduction

T.D. Lee and C.N.Yang realized in 1956 the possibility of Partiy violation. This soon was confirmed by experiments conducted by C.S. Wu and her collaborators in 1957[1]. Such an astounding result was confronted with great confusion and excitement from the scientific community. However, later it was believed for a while though the violations are observed individually under discrete symmetries, the CP symmetry should be perfectly conserved. But soon the same belief was broken by the experiment conducted by Christenson, Cronin, Fitch, and Turlay at Alternating Gradient Synchrotron (AGS). They observed that the long-lived state of neutral Kaons (K_L^0) having assigned an odd parity, rarely decayed into the two π^0 that had an even parity. This experiment showed that K-meson decays exhibited a CP symmetry violation. It rather took a while for the scientific community to accept this bitter truth of nature but only until

[1] Wu and collaborators aligned Co^{60} and counted decay electrons in the direction of nuclear spin and opposite to the spin. They observed more electrons in direction opposite to spin. This confirmed the Parity violation

the implications of such a symmetry violation were explored. This violation implied that the particles and antiparticles were not fundamentally the same in nature. They had an intrinsic difference that has produced the universe as observed today. In 1967 A.D. Sakharov explored the nature of this symmetry violation and proposed three conditions, known as Sakharov conditions, that were necessary to observe the dominance of matter over anti-matter[2]. Soon the CP violation was observed in B-meson decays also. This matter-antimatter discrepancy is what makes our existence possible. Therefore understanding such symmetry violation holds importance in particle physics.

3. CP violation in the K-meson system

3.1 Kaons and their existence in different states

Kaons exist in four states. These states are characterized as below[3]:

Particle name	Particle symbol	Antiparticle symbol	Particle Quark content	Rest mass (MeV/c^2)	S	C	B
Neutral Kaon	K^0	$K^{0\ bar}$	$d\bar{s}/ds$	497.611±0.013	1	0	0
Charged Kaon	K^+	K^-	us/us	493.677±0.016	1	0	0

The neutral kaon states exhibit a phenomenon of strangeness oscillations and hence are practically indistinguishable from each other.

3.2 Discrete symmetries of C, P, T and their action on strangeness eigenstates[4,5]

Given below are the set of discrete transformations of C, P, and T symmetries in the non-relativistic fashion:

$$P|K^0> \;=\; -\,|K^0>, \qquad P|K^{0\,bar}> \;=\; -\,|K^{0bar}>, \tag{1.1}$$

$$C|K^0> \;= e^{i\Omega}|K^{0\,bar}>, \qquad C|K^{0\,bar}> \;= e^{-i\Omega}|K^0\;>, \tag{1.2}$$

$$T|K^0> \;=\; e^{i(\Gamma-\Omega)}|K^0>, \quad T|K^{0\,bar}> \;=\; e^{i(\Gamma+\Omega)}|K^{0\,bar}>, \tag{1.3}$$

Where Γ, Ω are arbitrary phases.

This shows us that the $|K^0>$ and $|K^{0\,bar}>$ states are pseudoscalars since they acquire a negative sign under the parity transformation (P). The C (charge conjugation) operator can switch these two states into each other with an introduction of some arbitrary phase. With this it is easy to see that:

$$\Theta|K^0> \;=\; -\,e^{i\xi(\Omega,\Gamma)}|K^{0\,bar}>, \quad \Theta|K^{0\,bar}> \;=\; -\,e^{i\xi(-\Omega,\Gamma)}|K^0\;> \tag{1.4}$$

Where $\Theta|\psi> \;=\; CPT|\psi>$ and $\xi(\Omega,\Gamma), \xi(-\;\Omega,\Gamma)$ are functions of arbitrary phases Ω, Γ. Similarly,

$$\Theta^2|K^0> \;=\; e^{i\xi'(\Omega,\Gamma)}|K^0\;>,\; \Theta^2|K^{0bar}\;> \;=\; e^{i\xi'(-\Omega,\Gamma)}|K^{0\,bar}\;> \tag{1.5}$$

Where $\xi'(\Omega,\Gamma), \xi'(-\;\Omega,\Gamma)$ are different functions of arbitrary phases. So we understand that the $|K^0>$ and $|K^{0\,bar}>$ are not eigenstates of Θ but are eigenstates of Θ^2. This suggests that the Θ^2 eigenstates are the mass eigenstates when Θ^2 is perfectly conserved.

$$[H, \Theta^2] \;=\; 0; \tag{1.6}$$

Let $H|K^0> \;=\; m|K^0>$ and $H|K^{0\,bar}> \;=\; m^{bar}|K^{0\,bar}>$

$$< K^0|H|K^0> \;=\; < K^0|CPT^{-1}\,CPT^{-1}\,CPT\;\;CPT\;\;H\,CPT^{-1}\,CPT^{-1}\,CPT\;\;CPT\;\;|K^0>$$
$$= \;< K^0|CPT^{-1}H\,CPT\;\;|K^0>$$
$$= \;< K^{0\,bar}|\,H\,|\,K^{0\,bar}>$$
$$= \;m^{bar}$$

Thus we see that $m \;=\; m^{bar} = m_0$ (1.6.1)

Therefore we have arrived at a formalism where we now can identify if the CP is conserved in the kaon system.

3.3 Which states are CP eigenstates and which ones are not?

As discussed in the previous section, $|K^0>$ and $|K^{0\,bar}>$ are not eigenstates of Θ and hence are not eigenstates of CP. Hence under an approximation that CP is conserved, one has to construct such states. The simplest way to do so is by using the superposition of fundamental Kaon states.

$$|K_1> \;=\; (|K^0>+\;|K^{0\,bar}>)/\sqrt{2} \qquad (1.7)$$
$$|K_2> \;=\; (|K^0>-\;|K^{0\,bar}>)/\sqrt{2} \qquad (1.8)$$

$|K_1>$ and $|K_2>$ are the eigenstates of CP operator with eigenvalues -1 and +1 respectively. This is readily seen by performing the following operations:

$$CP\,|K_1> \;=\; (-\;C|K^0>-\;C|K^{0\,bar}>)/\sqrt{2} \;=\; -\;|K_1> \qquad (1.9)$$
$$CP\,|K_2> \;=\; (-\;C|K^0>+\;C|K^{0\,bar}>)/\sqrt{2} \;=\; |K_2> \qquad (2.0)$$

These eigenstates under the conserved CP approximation, are the mass eigenstates also. Hence these are the observable states and it is possible to experimentally study their decays.

3.4 Kaon decays[6]

Kaons decay through the following three processes:

a) Purely Leptonic decays: $K \rightarrow lv, ll\ etc.$
b) Semi Leptonic decays $K \rightarrow \pi lv, 2\pi lv\ etc.$
c) Purely Hadronic decays: $K \rightarrow 2\pi, 3\pi\ etc.$

These three processes are $\Delta s = \pm 1$ processes where the strangeness quantum number sees a change of +1 or -1. For the case of CP eigenstates we have created, we are interested in the purely hadronic decay regime.

It is possible to characterize $|K_1>$ and $|K_2>$ states through their decay product. Since we are working under an assumption that CP is conserved, one can expect the decay products to conserve the CP eigenvalues. Eigenvalues for the decay products can be calculated in the following way.

For 2π decay:

$$CP|\pi^0\pi^0> = (-1)^L(-1)(-1)|\pi^0\pi^0> = |\pi^0\pi^0> \tag{2.1}$$

Same procedure works for the $|\pi^+\pi^->$ decay product.

For 3π decay:

$$CP|\pi^0\pi^0\pi^0> = (-1)^L(-1)(-1)(-1)|\pi^0\pi^0\pi^0> = -|\pi^0\pi^0\pi^0> \tag{2.2}$$

Same procedure works for $|\pi^+\pi^-\pi^0>$ the decay product.

Thus one of the decay products i.e the two pion decay has a CP eigenvalue of +1 while the three pion decay product has CP eigenvalue of -1. As mentioned above, to conserve CP:

$$\text{CP eigenvalue of Reactant} = \text{CP eigenvalue of the Product} \tag{2.3}$$

Using this we can characterize $|K_1>$ and $|K_2>$ states as:

$|K_1> \;\rightarrow\; |\pi^0\pi^0\pi^0> \;/\; |\pi^+\pi^-\pi^0>$

$|K_2> \;\rightarrow\; |\pi^0\pi^0> \;/\; |\pi^+\pi^->$

These different decay modes have a profound impact on $|K_1>$ and $|K_2>$. Because of such differing decay modes, the partial width for these two decays are different and so are the lifetimes of $|K_1>$ and $|K_2>$ states. Based on these differing lifetimes, these states can be classified as, $|K_1> = |K_S>$ and $|K_2> = |K_L>$. As mentioned before, decays of $|K_S>$ and $|K_L>$ can be characterized by differing lifespans of decays and a slight mass difference between these two states. The mass splitting between these states is $\Delta m \sim 3.5 \times 10^{-15}\, GeV$ which is very less. But there is a significant difference in the partial widths of 2π and 3π decay.

$\Gamma(K_S \rightarrow \pi\ \pi\ ) = 7.4 \times 10^{-15}\, GeV$, $\quad \Gamma(K_L \rightarrow \pi\ \pi\ \pi\ ) = 4.3 \times 10^{-18}\, GeV$

This is because of the fact that since the mass of three pions is close to the mass of kaon, the phase space available for two pion decay is more and subsequently the lifetime of $|K_S>$ is considerably lesser than the lifetime of $|K_L>$.

$\tau(K_S) = 8.9 \times 10^{-11}\, s$, $\;\tau(K_L) = 5.2 \times 10^{-8}\, s$

Experimentally it is possible to detect these decay rates for two pion and three pion decays. The experimental setup used by Cronin et al. aimed at doing the same. But the results of the experiment turned out to be different than they expected.

3.5 The experiment that detected CP violation

The experiment, performed by J. H. Christenson, J. W. Cronin, V. L. Fitch, and R. Turlay aimed at identifying and quantifying decay products and rates from the Kaon decays.

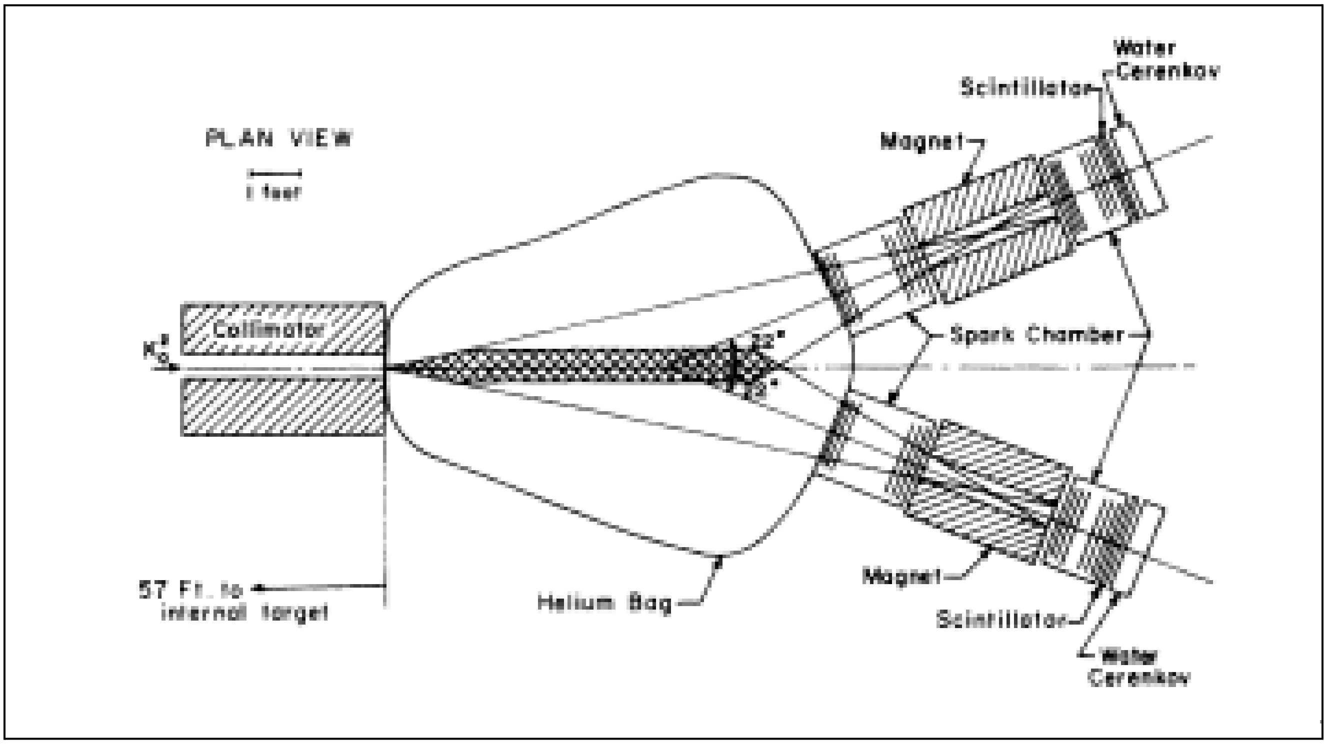

Figure 1: Figure shows the experimental setup employed by Cronin et al.[7]

The K^0 beam was produced at the Brookhaven AGS in an internal Be target by bombarding it with 30-BeV protons. Once the decay events were produced, they were detected using two spectrometers each composed of two spark chambers. These decays were observed from a helium chamber (bag) to minimize the interaction. The analysis program was employed to calculate the vector momenta of decay products. This included determining two-body and three-body decays and respective angles between the vectors. It was observed that out of a total of 22700 decays, 45$\pm$9 two pion decays were observed for the $|K_L>$ state.

Based on this data, the branching ratio for two pion decays can be calculated to be:

$$\zeta = \Gamma(K_L \to \pi\pi)/\Gamma(K_L \to \pi\pi\pi) = = (2.0 \pm 0.4) \times 10^{-3}$$

This proves that there in fact is CP violation in kaon decays. There are two contending explanations of ‘Direct CP violation’ and ‘Indirect CP violation’. To explain this CP violation we have to accept that the CP eigenstates that we created are not pure eigenstates but do involve a mixing parameter. Thus CP violation has profound impacts on particle physics.

4 The way ahead

Ahead of this I am working on understanding CP violation in the B-meson system. This system is particularly important since a considerable amount of CP violation can be observed in this system. The kaon system does show the CP violation but the mixing parameter is of order 10^{-3} which is fairly small. With the understanding of CP violation in B-meson, one can dive into the realm of the CKM matrix which tries to explain the CP violation through imaginary phase factors that show up in the interaction matrix. Thus my plan ahead is to get an understanding of the Cabbibo matrix and then shift to the CKM matrix to better understand the physical description of CP violation.

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[1] Appendix 1: The discovery of parity non-conservation, Stanford Encyclopedia of Philosophy, Allan Franklin & Slobodan Perovic (2019)

[2]A. D. Sakharov JETP Lett.-USSR 5,24 (1967)

[3]Wikipedia Article, ‘Kaon’

[4],[5]Giancarlo D'Ambrosio & Gino Isidori LNF-96/036 (P), INFNNA-IV-96-29,(1996)

[6] Nuclear and Particle Physics: Neutral Kaon decays and oscillations (lecture 16)

[7]J.H.Christonson et al. Phys. Review Lett. 13,4 (1964)