

Name: Makena WilliamsonDate: 10/20/2022Pledge: I pledge my honor that I have abided by the Stevens Honor System.

Use the Master Theorem to find the complexity of each recurrence relation listed below.

1. $T(n) = T\left(\frac{n}{2}\right) + n^2$
Complexity: $\Theta(n^2)$
 $d=2$ $a=1$ $b=2$ $a < b^d$ $1 < 2^2 \checkmark \rightarrow \Theta(n^b)$
2. $T(n) = 4T\left(\frac{n}{2}\right) + n^2$
Complexity: $\Theta(n^2 \lg n)$
 $d=2$ $a=4$ $b=2$ $a = b^d$ $4 = 2^2 \checkmark \rightarrow \Theta(n^d \log_b n)$
3. $T(n) = 3T\left(\frac{n}{3}\right) + \sqrt{n}$
Complexity: $\Theta(n)$
 $d=\frac{1}{2}$ $a=3$ $b=3$ $a > b^d$ $3 > \sqrt{3} \checkmark \rightarrow \Theta(n^{\log_b a})$
 $n^{\log_3(3)} = n^1 = n$

For each function below, write the recurrence relation for its running time and then use the Master Theorem to find its complexity.

4. `int f(int arr[], int n) {
 if (n == 0) {
 return 0;
 }
 int sum = 0;
 for (int j = 0; j < n; ++j) { // n
 sum += arr[j];
 }
 return f(arr, n / 2) + sum + f(arr, n / 2);
}`

$$a=2 \quad a=b^d$$

$$b=2 \quad 2=2^1 \checkmark$$

$$d=1$$

Recurrence: $T(n) = 2T\left(\frac{n}{2}\right) + n$
 Complexity: $\Theta(n \lg n)$

5. `void g(int n, int arrA[], int arrB[]) {
 if (n == 0) {
 return;
 }
 for (int i = 0; i < n; ++i) { // n
 for (int j = 0; j < n; ++j) { // n
 arrB[j] += arrA[i];
 }
 }
 g(n / 2, arrA, arrB);
}`

$$a=1 \quad d < b^d$$

$$b=2 \quad 1 < 2^2 \checkmark$$

$$d=2$$

Recurrence: $T(n) = T\left(\frac{n}{2}\right) + n^2$
 Complexity: $\Theta(n^2)$