

# Initial models for optimisation

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# Initial model for installation

$$\max_{\substack{O_p, N_{rp} \in \mathbb{Z}^* \\ s_{ait} \in \{0,1\}}} \sum_{p \in P} [DIS^p(O_p \cdot v_p - \sum_{r \in R} N_{rp} \cdot C_{rp})] \quad (1)$$

subject to:

$$s_{ait} \leq s_{ai(t+1)} \quad \forall a \in A, \forall i \in I, \forall t \in T \quad (2)$$

$$s_{ai\sigma_{itN}} \geq 1 \quad \forall a \in A, \forall i \in I \quad (3)$$

$$s_{ajt} \leq s_{ai\sigma_{it}} \quad \forall a \in A, \forall (i,j) \in IP, \forall t \in T \quad (4)$$

$$N_{rp} \geq \sum_{a \in A} \sum_{i \in I} (\rho_{ir} \cdot (s_{ait} - s_{ai\sigma_{it}})) \quad \forall r \in R, \forall p \in P, \forall t \in T_p \quad (5)$$

$$O_p = \sum_{a \in A} s_{aiN\sigma_{iN}t_p} \quad \forall p \in P \quad (6)$$

$$N_{rp} \leq m_{rp} \quad \forall r \in R, \forall p \in P \quad (7)$$

# Installation Model Explanation

- (1) Objective function, sums up profits from energy made, subtracts money used on resources (vessels), and multiplies it all with a discount factor
- (2) Makes every task that is started stay started
- (3) Forces every task to be started and finished by the final timestep
- (4) For every precedence relation  $(i, j)$ , ensures that  $i$  is finished before  $j$  is started
- (5) Counts up the resources needed in a time period by adding the resources needed by all tasks started, and subtracting the resources needed by all tasks finished
- (6) Counts the number of turbines which finished installing by the end of a period
- (7) Sets a limit on the amount of vessels that can be charatered in a given period

# Notation overview

## Sets:

- $P$ : All time periods (large scale)
- $T$ : All time intervals (small scale)  
 $[t_0, \dots, t_N]$
- $T_p \subset T$ : All time intervals (small scale)  
in period  $p$
- $R$ : All resources
- $I$ : All tasks per asset  $[1, \dots, i_N]$
- $IP$ : All precedence pairs  $(i, j)$
- $A$ : All assets

## Decision variables:

- $O_p$ : Number of online turbines after  
period  $p$
- $N_{rp}$ : Number of resources  $r$  used in  
period  $p$
- $s_{ait}$ : Binary variable, 1 if task  $i \in I$  for  
asset  $a$  has started at or before time  $t$

## Parameters:

- $DIS$ : The discount factor per period
- $v_p$ : The value of energy a single turbine  
produces in period  $p$
- $C_{rp}$ : The cost of chartering resource  $r$   
in period  $p$
- $\sigma_{it}$ : Indicates the timestep at which  
task  $i$  should have been started for it to  
be finished by timestep  $t$ , taking into  
account the duration and the weather  
conditions
- $\rho_{ir}$ : The amount of resource  $r$  used by  
task  $i$
- $t_p$ : The final time interval (from  $T$ )  
before period  $p$
- $m_{rp}$ : The maximum amount of  
resources  $r$  that can be chartered in  
period  $p$

# Initial model for maintenance

$$\max_{\substack{O_t, N_{rp} \in \mathbb{Z}^* \\ s_{ait}, b_{at} \in \{0,1\}}} \sum_{p \in P} [DIS^p(\sum_{t \in T_p} (O_t \cdot v_t) - \sum_{r \in R} (N_{rp} \cdot C_{rp}))] \quad (8)$$

subject to:

$$s_{ait} \leq s_{ai(t+1)} \quad \forall a \in A, \forall i \in M, \forall t \in T \quad (9)$$

$$s_{ai\sigma_{itN}} \geq 1 \quad \forall a \in A, \forall i \in M^M \quad (10)$$

$$N_{rp} \geq \sum_{a \in A} \sum_{i \in M} (\rho_{ir} \cdot (s_{ait} - s_{ai\sigma_{it}})) \quad \forall r \in R, \forall p \in P, \forall t \in T_p \quad (11)$$

$$b_{at} > \sum_{i \in M} [s_{ai\sigma_{i(t-\lambda_a)}} - s_{ai\sigma_{it}}] \quad \forall a \in A, \forall t \in T \quad (12)$$

$$O_t = |A| - \sum_{a \in A} b_{at} \quad \forall t \in T \quad (13)$$

$$N_{rp} \leq m_{rp} \quad \forall r \in R, \forall p \in P \quad (14)$$

# Maintenance Model Explanation

- (8) Objective function, sums up profits from energy made, subtracts money used on resources (vessels), and multiplies it all with a discount factor
- (9) Makes every task that is finished stay finished
- (10) Forces every mandatory maintenance task to be done at some point
- (11) Counts up the resources needed in a time period by adding the resources needed by all tasks started, and subtracting the resources needed by all tasks finished
- (12) If no maintenance tasks have finished in the past  $\lambda_a$  timesteps this asset is broken
- (13) The number of active (online) turbines is equal to everything that isn't broken
- (14) Sets a limit on the amount of vessels that can be charatered in a given period

# Notation overview

## Sets:

- $P$ : All time periods (large scale)
- $T$ : All time intervals (small scale)  
 $[t_0, \dots, t_N]$
- $T_p \subset T$ : All time intervals (small scale)  
in period  $p$
- $R$ : All resources
- $A$ : All assets
- $M = M^M \cup M^O$ : All (mandatory and optional) maintenance tasks

## Decision variables:

- $O_t$ : Number of active turbines at timestep  $t$
- $N_{rp}$ : Number of resources  $r$  used in period  $p$
- $s_{ait}$ : Binary variable, 1 if maintenance task  $i$  for asset  $a$  has started at or before time  $t$
- $b_{at}$ : Binary variable, 1 if asset  $a$  is broken at timestep  $t$

## Parameters:

- $DIS$ : The discount factor per time period
- $v_t$ : The value of energy a single turbine produces at timestep  $t$
- $C_{rp}$ : The cost of chartering resource  $r$  in period  $p$
- $\lambda_a$ : The number of timesteps after the last maintenance before asset  $a$  fails
- $\sigma_{it}$ : Indicates the timestep at which task  $i$  should have been started for it to be finished by timestep  $t$ , taking into account the duration and the weather conditions
- $\rho_{ir}$ : The amount of resource  $r$  used per task for maintenance task  $i$
- $m_{rp}$ : The maximum amount of resources  $r$  that can be chartered in period  $p$

# Initial 2-mixed model

$$\max_{\substack{N_{rp} \in \mathbb{Z}^* \\ s_{ait}, o_{at} \in \{0,1\}}} \sum_{p \in P} [DIS^p(\sum_{t \in T_p} \sum_{a \in A} (o_{at} \cdot v_t) - \sum_{r \in R} (N_{rp} \cdot C_{rp}))] \quad (15)$$

subject to:

$$s_{ait} \leq s_{ai(t+1)} \quad \forall a \in A, \forall i \in \mathcal{I}, \forall t \in T \quad (16)$$

$$s_{ai\sigma_{it_N}} \geq 1 \quad \forall a \in A, \forall i \in I \cup M^M \quad (17)$$

$$s_{ajt} \leq s_{ai\sigma_{it}} \quad \forall a \in A, \forall (i,j) \in IP, \forall t \in T \quad (18)$$

$$N_{rp} \geq \sum_{a \in A} \sum_{i \in \mathcal{I}} (\rho_{ir} \cdot (s_{ait} - s_{ai\sigma_{it}})) \quad \forall r \in R, \forall p \in P, \forall t \in T_p \quad (19)$$

$$o_{at} \leq s_{ai_N\sigma_{i_N t}} \quad \forall a \in A, \forall t \in T \quad (20)$$

$$o_{at} \leq \sum_{i \in M \cup \{i_N\}} [s_{ai\sigma_{it}} - s_{ai\sigma_{i(t-\lambda_a)}}] \quad \forall a \in A, \forall t \in T \quad (21)$$

$$N_{rp} \leq m_{rp} \quad \forall r \in R, \forall p \in P \quad (22)$$



## 2-Mixed Model Explanation

- (15) Objective function, sums up profits from energy made, subtracts money used on resources (vessels), and multiplies it all with a discount factor
- (16) Makes every task that is started stay started
- (17) Forces every installation and mandatory maintenance task to be started and finished by the final timestep
- (18) For every precedence relation  $(i, j)$ , ensures that  $i$  is finished before  $j$  is started
- (19) Counts up the resources needed in a time period by adding the resources needed by all tasks started, and subtracting the resources needed by all tasks finished
- (20) An asset can only be online if it finished installation
- (21) An asset can only be online if it had work done on it recently enough
- (22) Sets a limit on the amount of vessels that can be charatered in a given period

# Notation overview

## Sets:

- $P$ : All time periods (large scale)
- $T$ : All time intervals (small scale)  $[t_0, \dots, t_N]$
- $T_p \subset T$ : All time intervals (small scale) in period  $p$
- $R$ : All resources
- $I$ : All installation tasks per asset  $[1, \dots, i_N]$
- $M = M^M \cup M^O$ : all mandatory and optional maintenance tasks
- $\mathcal{I} = I \cup M$ : All tasks
- $IP$ : All precedence pairs  $(i, j)$
- $A$ : All assets

## Decision variables:

- $o_{at}$ : Binary variable, 1 if asset  $a$  is online at timestep  $t$
- $N_{rp}$ : Number of resources  $r$  used in period  $p$
- $s_{ait}$ : Binary variable, 1 if task  $i \in \mathcal{I}$  for asset  $a$  has started at or before time  $t$

## Parameters:

- $DIS$ : The discount factor per time period
- $v_t$ : The value of energy a single turbine produces at timestep  $t$
- $C_{rp}$ : The cost of chartering resource  $r$  in period  $p$
- $\lambda_a$ : The number of timesteps after the last maintenance before asset  $a$  fails
- $\sigma_{it}$ : Indicates the timestep at which task  $i$  should have been started for it to be finished by timestep  $t$ , taking into account the duration and the weather conditions
- $\rho_{ir}$ : The amount of resource  $r$  used for task  $i \in \mathcal{I}$
- $m_{rp}$ : The maximum amount of resources  $r$  that can be chartered in period  $p$

# Initial model for decommission

$$\max_{\substack{N_{rp} \in \mathbb{Z}^* \\ s_{ait} \in \{0,1\}}} \sum_{p \in P} [DIS^p(\sum_{t \in T_p} \sum_{a \in A} ((1 - s_{ai_0 t}) \cdot v_t) - \sum_{r \in R} (N_{rp} \cdot C_{rp}))] \quad (23)$$

subject to:

$$s_{ait} \leq s_{ai(t+1)} \quad \forall a \in A, \forall i \in D, \forall t \in T \quad (24)$$

$$s_{ai\sigma_{it_N}} \geq 1 \quad \forall a \in A, \forall i \in D \quad (25)$$

$$s_{ajt} \leq s_{ai\sigma_{it}} \quad \forall a \in A, \forall (i,j) \in IP, \forall t \in T \quad (26)$$

$$N_{rp} \geq \sum_{a \in A} \sum_{i \in I} (\rho_{ir} \cdot (s_{ait} - s_{ai\sigma_{it}})) \quad \forall r \in R, \forall p \in P, \forall t \in T_p \quad (27)$$

$$N_{rp} \leq m_{rp} \quad \forall r \in R, \forall p \in P \quad (28)$$

# Installation Model Explanation

- (23) Objective function, sums up profits from energy made, subtracts money used on resources (vessels), and multiplies it all with a discount factor
- (24) Makes every task that is started stay started
- (25) Forces every task to be started and finished by the final timestep
- (26) For every precedence relation  $(i, j)$ , ensures that  $i$  is finished before  $j$  is started
- (27) Counts up the resources needed in a time period by adding the resources needed by all tasks started, and subtracting the resources needed by all tasks finished
- (28) Sets a limit on the amount of vessels that can be charatered in a given period

# Notation overview

## Sets:

- $P$ : All time periods (large scale)
- $T$ : All time intervals (small scale)  
 $[t_0, \dots, t_N]$
- $T_p \subset T$ : All time intervals (small scale)  
in period  $p$
- $R$ : All resources
- $D$ : All tasks per asset  $[i_0, \dots, i_N]$
- $IP$ : All precedence pairs  $(i, j)$
- $A$ : All assets

## Decision variables:

- $N_{rp}$ : Number of resources  $r$  used in  
period  $p$
- $s_{ait}$ : Binary variable, 1 if task  $i \in I$  for  
asset  $a$  has started at or before time  $t$

## Parameters:

- $DIS$ : The discount factor per period
- $v_t$ : The value of energy a single turbine  
produces in timestep  $t$
- $C_{rp}$ : The cost of chartering resource  $r$   
in period  $p$
- $\sigma_{it}$ : Indicates the timestep at which  
task  $i$  should have been started for it to  
be finished by timestep  $t$ , taking into  
account the duration and the weather  
conditions
- $\rho_{ir}$ : The amount of resource  $r$  used by  
task  $i$
- $t_p$ : The final time interval (from  $T$ )  
before period  $p$
- $m_{rp}$ : The maximum amount of  
resources  $r$  that can be charatered in  
period  $p$

# Initial 3-mixed model

$$\max_{\substack{N_{rp} \in \mathbb{Z}^* \\ s_{ait}, o_{at} \in \{0,1\}}} \sum_{p \in P} [DIS^p(\sum_{t \in T_p} \sum_{a \in A} (o_{at} \cdot v_t) - \sum_{r \in R} (N_{rp} \cdot C_{rp}))] \quad (29)$$

subject to:

$$s_{ait} \leq s_{ai(t+1)} \quad \forall a \in A, \forall i \in \mathcal{I}, \forall t \in T \quad (30)$$

$$s_{ai\sigma_{itN}} \geq 1 \quad \forall a \in A, \forall i \in \mathcal{I} - M^O \quad (31)$$

$$s_{ai_0^D t} - 1 \leq s_{ai\sigma_{it}} - s_{ait} \quad \forall a \in A, \forall i \in \mathcal{I} - D, \forall t \in T \quad (32)$$

$$s_{ajt} \leq s_{ai\sigma_{it}} \quad \forall a \in A, \forall (i, j) \in IP, \forall t \in T \quad (33)$$

$$m_{rp} \geq N_{rp} \geq \sum_{a \in A} \sum_{i \in \mathcal{I}} (\rho_{ir} \cdot (s_{ait} - s_{ai\sigma_{it}})) \quad \forall r \in R, \forall p \in P, \forall t \in T_p \quad (34)$$

$$o_{at} \leq s_{ai_N^I \sigma_{i_N^I t}} - s_{ai_0^D t} \quad \forall a \in A, \forall t \in T \quad (35)$$

$$o_{at} \leq \sum_{i \in M \cup \{i_N^I\}} (s_{ai\sigma_{it}} - s_{ai\sigma_{i(t-\lambda_a)}}) \quad \forall a \in A, \forall t \in T \quad (36)$$

$$o_{at} \leq 1 + s_{ai\sigma_{it}} - s_{ait} \quad \forall a \in A, \forall i \in M, \forall t \in T \quad (37)$$

### 3-Mixed Model Explanation

- (29) Objective function, sums up profits from energy made, subtracts money used on resources (vessels), and multiplies it all with a discount factor
- (30) Makes every task that is started stay started
- (31) Forces every installation and mandatory maintenance task to be started and finished by the final timestep
- (32) Ensures every non-decommission task is inactive during decomission
- (33) For every precedence relation  $(i,j)$ , ensures that  $i$  is finished before  $j$  is started
- (34) Counts up the resources needed in a time period by adding the resources needed by all tasks started, and subtracting the resources needed by all tasks finished. It also sets a limit on the amount of vessels that can be charatered in a given period
- (35) An asset can only be online if installation is complete and decomission has not started yet
- (36) An asset can only be online if it had maintenance done recently (or only completed installation recently)
- (37) Ensures an asset if offline is maintenance work is going on at this moment

# Notation overview

## Sets:

- $P$ : All time periods (large scale)
- $T$ : All time intervals (small scale)  $[t_0, \dots, t_N]$
- $T_p \subset T$ : All time intervals (small scale) in period  $p$
- $R$ : All resources
- $I$ : All installation tasks per asset  $[i_0^I, \dots, i_N^I]$
- $M = M^M \cup M^O$ : all mandatory and optional maintenance tasks
- $D$ : All decommission tasks per asset  $[i_0^D, \dots, i_N^D]$
- $\mathcal{I} = I \cup M \cup D$ : All tasks
- $IP$ : All precedence pairs  $(i, j)$ . Includes relations stating that the final installation task is finished before maintenance and decommission are started.
- $A$ : All assets

## Decision variables:

- $o_{at}$ : Binary variable, 1 if asset  $a$  is online at timestep  $t$
- $N_{rp}$ : Number of resources  $r$  used in period  $p$
- $s_{ait}$ : Binary variable, 1 if task  $i \in \mathcal{I}$  for asset  $a$  has started at or before time  $t$

## Parameters:

- $DIS$ : The discount factor per time period
- $v_t$ : The value of energy a single turbine produces at timestep  $t$
- $C_{rp}$ : The cost of chartering resource  $r$  in period  $p$
- $\lambda_a$ : The number of timesteps after the last maintenance before asset  $a$  fails
- $\sigma_{it}$ : Indicates the timestep at which task  $i$  should have been started for it to be finished by timestep  $t$ , generated duration and the weather conditions
- $\rho_{ir}$ : The amount of resource  $r$  used for task  $i \in \mathcal{I}$
- $m_{rp}$ : The maximum amount of resources  $r$  that can be chartered in period  $p$



# Initial Corrective model

$$\max_{\substack{N_{rp} \in \mathbb{Z}^* \\ s_{ait}, o_{at} \in \{0,1\}}} \sum_{p \in P} [DIS^p(\sum_{t \in T_p} \sum_{a \in A} (o_{at} \cdot v_t) - \sum_{r \in R} (N_{rp} \cdot C_{rp}))] \quad (38)$$

subject to:

$$s_{ait} \leq s_{ai(t+1)} \quad \forall a \in A, \forall i \in \mathcal{I}, \forall t \in T \quad (39)$$

$$s_{ai\sigma_{itN}} \geq 1 \quad \forall a \in A, \forall i \in \mathcal{I} - M^C \quad (40)$$

$$s_{ai0}^D - 1 \leq s_{ai\sigma_{it}} - s_{ait} \quad \forall a \in A, \forall i \in \mathcal{I} - D, \forall t \in T \quad (41)$$

$$s_{ajt} \leq s_{ai\sigma_{it}} \quad \forall a \in A, \forall (i,j) \in IP, \forall t \in T \quad (42)$$

$$m_{rp} \geq N_{rp} \geq \sum_{a \in A} \sum_{i \in \mathcal{I}} (\rho_{ir} \cdot (s_{ait} - s_{ai\sigma_{it}})) \quad \forall r \in R, \forall p \in P, \forall t \in T_p \quad (43)$$

$$o_{at} \leq s_{aiN}^{iI\sigma_{iI}t} - s_{ai0}^D - f_{at} \quad \forall a \in A, \forall t \in T \quad (44)$$

$$f_{at} \geq s_{aiN}^{iI\sigma_{iI}t} - s_{ai0}^D - \sum_{i \in M \cup \{i_N^I\}} (s_{ai\sigma_{it}} - s_{ai\sigma_{i(t-\lambda_{ai})}}) \quad \forall a \in A, \forall t \in T \quad (45)$$

$$f_{a(t-1)} \geq s_{ait} - s_{ai(t-1)} \quad \forall a \in A, \forall i \in M^C, \forall t \in T \quad (46)$$

$$o_{at} \leq 1 + s_{ai\sigma_{it}} - s_{ait} \quad \forall a \in A, \forall i \in M, \forall t \in T \quad (47)$$

# Corrective Model Explanation

- (38) Objective function, sums up profits from energy made, subtracts money used on resources (vessels), and multiplies it all with a discount factor
- (39) Makes every task that is started stay started
- (40) Forces every installation and preventive maintenance task to be started and finished by the final timestep
- (41) Ensures every non-decommission task is inactive during decomission
- (42) For every precedence relation  $(i, j)$ , ensures that  $i$  is finished before  $j$  is started
- (43) Counts up the resources needed in a time period by adding the resources needed by all tasks started, and subtracting the resources needed by all tasks finished. It also sets a limit on the amount of vessels that can be charatered in a given period
- (44) An asset can only be online if installation is complete, decomission has not started yet, and it isn't in a failure state
- (45) An asset is in a failure state if installation is complete, decomission has not started yet, and it has not gotten maintained recently enough
- (46) A corrective maintenance task can only be started if the asset is in a failure state
- (47) Ensures an asset if offline is maintenance work is going on at this moment

# Notation overview

## Sets:

- $P$ : All time periods (large scale)
- $T$ : All time intervals (small scale)  $[t_0, \dots, t_N]$
- $T_p \subset T$ : All time intervals (small scale) in period  $p$
- $R$ : All resources
- $I$ : All installation tasks per asset  $[i_0^I, \dots, i_N^I]$
- $M = M^P \cup M^C$ : all preventive and corrective maintenance tasks
- $D$ : All decommission tasks per asset  $[i_0^D, \dots, i_N^D]$
- $\mathcal{I} = I \cup M \cup D$ : All tasks
- $IP$ : All precedence pairs  $(i, j)$ . Includes relations stating that the final installation task is finished before maintenance and decommission are started.
- $A$ : All assets

## Decision variables:

- $o_{at}$ : Binary variable, 1 if asset  $a$  is online at timestep  $t$
- $N_{rp}$ : Number of resources  $r$  used in period  $p$
- $s_{ait}$ : Binary variable, 1 if task  $i \in \mathcal{I}$  for asset  $a$  has started at or before time  $t$
- $f_{at}$ : Binary variable, 1 if asset  $a$  has failed at time  $t$

## Deterministic parameters:

- $DIS$ : The discount factor per time period
- $C_{rp}$ : The cost of chartering resource  $r$  in period  $p$
- $\rho_{ir}$ : The amount of resource  $r$  used for task  $i \in \mathcal{I}$
- $m_{rp}$ : The maximum amount of resources  $r$  that can be chartered in period  $p$

## Stochastic parameters:

- $v_t$ : The value of energy a single turbine produces at timestep  $t$
- $\lambda_{ai}$ : The number of timesteps after the last maintenance before asset  $a$  fails
- $\sigma_{it}$ : Indicates the timestep at which task  $i$  should have been started for it to be finished by timestep  $t$ , generated based on duration and the weather conditions (stochastic based on those)