

Initial models for optimisation

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Initial model for installation

$$\max_{\substack{O_p, N_{rp} \in \mathbb{Z}^* \\ s_{it}, f_{it} \in \{0,1\}}} \sum_{p \in P} [DIS^p(O_p \cdot v_p - \sum_{r \in R} N_{rp} \cdot C_{rp})] \quad (1)$$

subject to:

$$1 = \sum_{t \in T} s_{it} = \sum_{t \in T} f_{it} \quad \forall i \in I \quad (2)$$

$$1 \geq s_{jt_1} + f_{it_2} \quad \forall (i,j) \in IP, \quad (3)$$
$$\forall t_1, t_2 \in T | t_1 < t_2$$

$$d_i \geq (f_{it_2} + s_{it_1} - 1) \cdot \sum_{t_3=t_1}^{t_2} \omega_{it_3} \quad \forall i \in I, \forall t_1, t_2 \in T | t_2 \geq t_1 + d_i \quad (4)$$

$$N_{rp} \geq \sum_{i \in I} \sum_{t_1=t_0}^t (\rho_{ir} \cdot (s_{it_1} - f_{it_1})) \quad \forall r \in R, \forall p \in P, \forall t \in T_p \quad (5)$$

$$O_p = \sum_{t=t_0}^{t_p} \sum_{i \in F} f_{it} \quad \forall p \in P \quad (6)$$

Installation Model Explanation

- (1) Objective function, sums up profits from energy made, subtracts money used on resources (vessels), and multiplies it all with a discount factor
- (2) Forces every task to be started and finished at some point
- (3) For every precedence relation (i, j) it ensures that if task i finishes at time t_2 there is no $t_1 < t_2$ at which task j starts
- (4) Ensures that between the starting and finish times of each task are enough timesteps with acceptable weather
- (5) Counts up the resources needed in a time period by adding the resources needed by all tasks started, and subtracting the resources needed by all tasks finished
- (6) Counts the number of turbines which finished installing by the end of a period

Notation overview

Sets:

- P : All time periods (large scale)
- T : All time intervals $[t_0, \dots, t_N]$
- $T_p \in T$: All time intervals (small scale) in period p
- R : All resources
- I : All tasks
- $F \subset I$: All final tasks that complete a turbine
- IP : All precedence pairs (i, j)

Decision variables:

- O_p : Number of online turbines after period p
- N_{rp} : Number of resources r used in period p
- s_{it} : Binary variable, 1 if task i starts at time t
- f_{it} : Binary variable, 1 if task i ends at time t

Parameters:

- DIS : The discount factor per period
- v_p : The value of energy a single turbine produces in period p
- C_{rp} : The cost of chartering resource r in period p
- d_i : The duration of task i
- ω_{it} : Binary parameter representing weather, 1 if task i can be completed at time t , 0 otherwise
- ρ_{ir} : The amount of resource r used by task i
- t_p : The final time interval (from T) in period p

Initial model for maintenance

$$\max_{\substack{O_t, N_{rp} \in \mathbb{Z}^* \\ s_{act}, f_{act}, b_{at} \in \{0,1\}}} \sum_{p \in P} [DIS^p(\sum_{t \in T_p} (O_t \cdot v_t) - \sum_{r \in R} (N_{rp} \cdot C_{rp}))] \quad (7)$$

subject to (1):

$$1 = \sum_{t \in T} s_{act} = \sum_{t \in T} f_{act} \quad \forall a \in A, \forall c \in C^M \quad (8)$$

$$1 \geq \sum_{t \in T} s_{act} \quad \forall a \in A, \forall c \in C^O \quad (9)$$

$$\sum_{t \in T} s_{act} = \sum_{t \in T} f_{act} \quad \forall a \in A, \forall c \in C^O \quad (10)$$

Initial model for maintenance

$$\max_{\substack{O_t, N_{rp} \in \mathbb{Z}^* \\ s_{act}, f_{act}, b_{at} \in \{0,1\}}} \sum_{p \in P} [DIS^p(\sum_{t \in T_p} (O_t \cdot v_t) - \sum_{r \in R} (N_{rp} \cdot C_{rp}))] \quad (7)$$

subject to (2):

$$d_c \geq (f_{act_2} + s_{act_1} - 1) \cdot \sum_{t_3=t_1}^{t_2} \omega_{ct_3} \quad \forall a \in A, \forall c \in C, \quad (11)$$

$$\forall t_1, t_2 \in T | t_2 \geq t_1 + d_c$$

$$N_{rp} \geq \sum_{a \in A} \sum_{c \in C} \sum_{t_1=t_0}^t (\rho_{ir} \cdot (s_{act_1} - f_{act_1})) \quad \forall r \in R, \forall p \in P, \forall t \in T_p \quad (12)$$

$$b_{at} > \sum_{c \in C} \sum_{t_1=t-\lambda_a}^t -f_{act_1} \quad \forall a \in A, \forall t \in T \quad (13)$$

$$O_t = |A| - \sum_{a \in A} b_{at} \quad \forall t \in T \quad (14)$$

Maintenance Model Explanation

- (7) Objective function, sums up profits from energy made, subtracts money used on resources (vessels), and multiplies it all with a discount factor
- (8) Forces every mandatory maintenance cycle to be done at some point
- (9) Ensures each optional maintenance cycle to be started at most once
- (10) Ensures that every maintenance cycle for a particular asset that is started is also finished
- (11) Ensures that between the starting and finish times of each cycle are enough timesteps with acceptable weather
- (12) Counts up the resources needed in a time period by adding the resources needed by all tasks started, and subtracting the resources needed by all tasks finished
- (13) If no maintenance tasks have finished in the past λ_a timesteps this asset is broken
- (14) The number of active (online) turbines is equal to everything that isn't broken

Notation overview

Sets:

- P : All time periods (large scale)
- T : All time intervals (small scale) $[t_0, \dots, t_N]$
- $T_p \in T$: All time intervals (small scale) in period p
- R : All resources
- A : All assets
- $C = C^M \cup C^O$: All (mandatory and optional) maintenance cycles

Decision variables:

- O_t : Number of active turbines at timestep t
- N_{rp} : Number of resources r used in period p
- s_{act} : Binary variable, 1 if maintenance cycle c for asset a starts at time t
- f_{act} : Binary variable, 1 if maintenance cycle c for asset a finishes at time t
- b_{at} : Binary variable, 1 if asset a is broken at timestep t

Parameters:

- DIS : The discount factor per time period
- v_t : The value of energy a single turbine produces at timestep t
- C_{rp} : The cost of chartering resource r in period p
- d_c : The duration per task during maintenance cycle c
- λ_a : The number of timesteps after the last maintenance before asset a fails
- ω_{ct} : Binary parameter representing weather, 1 if maintenance cycle c can be completed at time t , 0 otherwise
- ρ_{cr} : The amount of resource r used per task for maintenance cycle c

Initial mixed model

$$\max_{\substack{O_t, N_{rp} \in \mathbb{Z}^* \\ s_{ait}, f_{ait}, o_{at} \in \{0,1\}}} \sum_{p \in P} [DIS^p(\sum_{t \in T_p} (O_t \cdot v_t) - \sum_{r \in R} (N_{rp} \cdot C_{rp}))] \quad (15)$$

subject to:

$$1 = \sum_{t \in T} s_{ait} = \sum_{t \in T} f_{ait} \quad \forall i \in I \cup M^M, \forall a \in A \quad (16)$$

$$1 \geq s_{ajt_1} + f_{ait_2} \quad \forall (i, j) \in IP, \forall a \in A, \quad (17)$$

$$1 \geq \sum_{t \in T} s_{ait} \quad \forall a \in A, \forall i \in M^O \quad (18)$$

$$\sum_{t \in T} s_{ait} = \sum_{t \in T} f_{ait} \quad \forall a \in A, \forall i \in M^O \quad (19)$$

$$0 = \sum_{t_1=t_0}^t s_{ajt_1} + \sum_{t_2=t}^{t_N} f_{ait_2} \quad \forall a \in A, \forall t \in T, \quad (20)$$

Initial mixed model

$$\max_{\substack{O_t, N_{rp} \in \mathbb{Z}^* \\ s_{ait}, f_{ait}, o_{at} \in \{0,1\}}} \sum_{p \in P} [DIS^p(\sum_{t \in T_p} (O_t \cdot v_t) - \sum_{r \in R} (N_{rp} \cdot C_{rp}))] \quad (15)$$

subject to (2):

$$d_i \geq (f_{ait_2} + s_{ait_1} - 1) \cdot \sum_{t_3=t_1}^{t_2} \omega_{it_3} \quad \forall i \in \mathcal{I}, \forall a \in A, \quad (21)$$

$$\forall t_1, t_2 \in T \mid t_2 \geq t_1 + d_i$$

$$N_{rp} \geq \sum_{a \in A} \sum_{i \in \mathcal{I}} \sum_{t_1=t_0}^t (\rho_{ir} \cdot (s_{ait_1} - f_{ait_1})) \quad \forall r \in R, \forall p \in P, \forall t \in T_p \quad (22)$$

$$o_{at} \leq \frac{1}{2} \cdot \left(\sum_{t_1=t_0}^t f_{ai_N t_1} + \sum_{i \in M \cup \{i_N\}} \sum_{t_2=t-\lambda_a}^t f_{ait_2} \right) \quad \forall a \in A, \forall t \in T \quad (23)$$

$$O_t = \sum_{a \in A} o_{at} \quad \forall t \in T \quad (24)$$

Mixed Model Explanation

- (15) Objective function, sums up profits from energy made, subtracts money used on resources (vessels), and multiplies it all with a discount factor
- (16) Forces every mandatory task to be started and finished at some point
- (17) For every precedence relation (i, j) it ensures that if task i finishes at time t_2 there is no $t_1 < t_2$ at which task j starts
- (18) Ensures each optional maintenance task to be started at most once
- (19) Ensures that every maintenance task for a particular asset that is started is also finished
- (20) Ensures an asset is fully installed before maintenance starts
- (21) Ensures that between the starting and finish times of each task are enough timesteps with acceptable weather
- (22) Counts up the resources needed in a time period by adding the resources needed by all tasks started, and subtracting the resources needed by all tasks finished
- (23) Sets an asset to be online if it installed and had work done on it recently
- (24) Counts how many assets are online

Notation overview

Sets:

- P : All time periods (large scale)
- T : All time intervals (small scale) $[t_0, \dots, t_N]$
- $T_p \in T$: All time intervals (small scale) in period p
- R : All resources
- I : All installation tasks per asset $[1, \dots, i_N]$
- $M = M^M \cup M^O$: all mandatory and optional maintenance tasks
- $\mathcal{I} = I \cup M$: All tasks
- IP : All precedence pairs (i, j)
- A : All assets

Decision variables:

- O_t : Number of online turbines at timestep t
- o_{at} : Binary variable, 1 if asset a is online at timestep t
- N_{rp} : Number of resources r used in period p
- s_{ait} : Binary variable, 1 if task $i \in \mathcal{I}$ for asset a starts at time t
- f_{ait} : Binary variable, 1 if task $i \in \mathcal{I}$ for asset a finishes at time t

Parameters:

- DIS : The discount factor per time period
- v_t : The value of energy a single turbine produces at timestep t
- C_{rp} : The cost of chartering resource r in period p
- d_j : The duration of task $i \in \mathcal{I}$
- λ_a : The number of timesteps after the last maintenance before asset a fails
- ω_{it} : Binary parameter representing weather, 1 if task $i \in \mathcal{I}$ can be completed at time t , 0 otherwise
- ρ_{ir} : The amount of resource r used for task $i \in \mathcal{I}$