Initial models for optimisation

R. Kuipers

June 15, 2020

Initial model for installation

$$\max_{\substack{O_p, N_{rp} \in \mathbb{Z}^* \\ s_{ait} \in \{0,1\}}} \sum_{p \in P} [DIS^p (O_p \cdot v_p - \sum_{r \in R} N_{rp} \cdot C_{rp})] \tag{1}$$

subject to:

$$s_{ait} \leq s_{ai(t+1)} \qquad \forall a \in A, \forall i \in I, \forall t \in T \tag{2}$$

$$s_{ai\sigma_{it_N}} \geq 1 \qquad \forall a \in A, \forall i \in I \tag{3}$$

$$s_{ajt} \leq s_{ai\sigma_{it}} \qquad \forall a \in A, \forall (i,j) \in IP, \forall t \in T \tag{4}$$

$$N_{rp} \geq \sum_{a \in A} \sum_{i \in I} (\rho_{ir} \cdot (s_{ait} - s_{ai\sigma_{it}})) \qquad \forall r \in R, \forall p \in P, \forall t \in T_p \tag{5}$$

$$O_p = \sum_{a \in A} s_{ai_N \sigma_{it_p}} \qquad \forall p \in P \tag{6}$$

$$N_{rp} \leq m_{rp} \qquad \forall r \in R, \forall p \in P \tag{7}$$

Installation Model Explanation

(1) Objective function, sums up profits from energy made, subtracts money used on resources (vessels), and multiplies it all with a discount factor
(2) Makes every task that is finished stay finished
(3) Forces every task to be starded and finished by the final timestep
(4) For every precedence relation (i, j), ensures that i is finished before j is started
(5) Counts up the resources needed in a time period by adding the resources needed by all tasks started, and subtracting the resources needed by all tasks finished
(6) Counts the number of turbines which finished installing by the end of a period
(7) Sets a limit on the amount of vessels that can be charatered in a given period

Notation overview

Sets:

- P: All time periods (large scale)
- T: All time intervals (small scale) $[t_0, \dots, t_N]$
- $T_p \in T$: All time intervals (small scale) in period p
- R: All resources
- I: All tasks per asset $[1, \dots, i_N]$
- IP: All precedency pairs (i, j)
- A: All assets

Decision variables:

- O_p: Number of online turbines after period p
- N_{rp}: Number of resources r used in period p
- s_{ait}: Binary variable, 1 if task $i \in I$ for asset a has started at time t

Parameters:

- DIS: The discount factor per period
- v_p: The value of energy a single turbine produces in period p
- C_{rp}: The cost of chartering resource r in period p
- σ_{it}: Indicates the timestep at which task i should have been started for it to be finished by timestep t, taking into account the duration and the weather conditions
- ρ_{ir}: The amount of resource r used by task i
- t_p: The final time interval (from T) before period p
- m_{rp}: The maximum amount of resources r that can be charatered in period p

Initial model for maintenance

$$\max_{\substack{O_t, N_{rp} \in \mathbb{Z}^* \\ s_{act}, f_{act}, b_{at} \in \{0,1\}}} \sum_{p \in P} [DIS^p (\sum_{t \in T_p} (O_t \cdot v_t) - \sum_{r \in R} (N_{rp} \cdot C_{rp}))] \tag{8}$$

subject to (1):

$$1 = \sum_{t \in T} s_{act} = \sum_{t \in T} f_{act} \qquad \forall a \in A, \forall c \in C^M \tag{9}$$

$$1 \geq \sum_{t \in T} s_{act} \qquad \forall a \in A, \forall c \in C^O \tag{10}$$

$$\sum_{t \in T} s_{act} = \sum_{t \in T} f_{act} \qquad \forall a \in A, \forall c \in C^O \tag{11}$$

Initial model for maintenance

$$\max_{\substack{O_t, N_{rp} \in \mathbb{Z}^* \\ s_{act}, f_{act}, b_{at} \in \{0,1\}}} \sum_{p \in P} [DIS^p (\sum_{t \in T_p} (O_t \cdot v_t) - \sum_{r \in R} (N_{rp} \cdot C_{rp}))] \tag{8}$$

subject to (2):

$$(s_{act_1} + f_{act_2} - 1) \cdot d_c \leq \frac{s_{act_1} + f_{act_2}}{2} \cdot \sum_{t_3 = t_1}^{t_2} \omega_{ct_3} \qquad \forall a \in A, \forall c \in C, \forall t_1, t_2 \in T \tag{12}$$

$$N_{rp} \geq \sum_{a \in A} \sum_{c \in C} \sum_{t_1 = t_0}^{t} (\rho_{ir} \cdot (s_{act_1} - f_{ac(t_1 - 1)})) \qquad \forall r \in R, \forall p \in P, \forall t \in T_p \tag{13}$$

$$b_{at} > \sum_{c \in C} \sum_{t_1 = t - \lambda_a}^{t} - f_{act_1} \qquad \forall a \in A, \forall t \in T \tag{14}$$

$$O_t = |A| - \sum_{a \in A} b_{at} \qquad \forall t \in T \tag{15}$$

$$N_{rp} \leq m_{rp} \qquad \forall r \in R, \forall p \in P \tag{16}$$

Maintenance Model Explanation

(8) Objective function, sums up profits from energy made, subtracts money used on resources (vessels), and multiplies it all with a discount factor
(9) Forces every mandatory maintenance cycle to be done at some point
(10) Ensures each optional maintenance cycle to be started at most once
(11) Ensures that every maintenance cycle for a particular asset that is started is also finished
(12) Ensures that between the starting and finish times of each cycle are enough timesteps with acceptable weather
(13) Counts up the resources needed in a time period by adding the resources needed by all tasks started, and subtracting the resources needed by all tasks finished
(14) If no maintenance tasks have finished in the past λ_a timesteps this asset is broken
(15) The number of active (online) turbines is equal to everything that isn't broken
(16) Sets a limit on the amount of vessels that can be charatered in a given period

Notation overview

Sets:

- P: All time periods (large scale)
- T: All time intervals (small scale) $[t_0, \ldots, t_N]$
- $T_p \in T$: All time intervals (small scale) in period p
- R: All resources
- A: All assets
- $C = C^M \cup C^O$: All (mandatory and optional) maintenance cycles

Decision variables:

- O_t: Number of active turbines at timestep t
- N_{rp}: Number of resources r used in period p
- s_{act}: Binary variable, 1 if maintenance cycle c for asset a starts at time t
- f_{act}: Binary variable, 1 if maintenance cycle c for asset a finishes at time t
- b_{at}: Binary variable, 1 if asset a is broken at timestep t

Parameters:

- DIS: The discount factor per time period
- v_t: The value of energy a single turbine produces at timestep t
- C_{rp}: The cost of chartering resource r in period p
- d_c: The duration per task during maintenance cycle c
- λ_a: The number of timesteps after the last maintenance before asset a fails
- ω_{ct}: Binary parameter representing weather, 1 if maintenance cycle c can be completed at time t, 0 otherwise
- ρ_{cr}: The amount of resource r used per task for maintenance cycle c
- m_{rp}: The maximum amount of resources r that can be charatered in period p

Initial mixed model

$$\max_{\substack{O_t, N_{rp} \in \mathbb{Z}^* \\ s_{ait}, f_{ait}, o_{at} \in \{0,1\}}} \sum_{p \in P} [DIS^p(\sum_{t \in T_p} (O_t \cdot v_t) - \sum_{r \in R} (N_{rp} \cdot C_{rp}))] \tag{17}$$

subject to:

$$1 = \sum_{t \in T} s_{ait} = \sum_{t \in T} f_{ait} \qquad \forall i \in I \cup M^M, \forall a \in A \tag{18}$$

$$1 \geq s_{ajt_1} + f_{ait_2} \qquad \begin{array}{r} \forall (i,j) \in IP, \forall a \in A, \\ \forall t_1, t_2 \in T | t_1 \leq t_2 \end{array} \tag{19}$$

$$1 \geq \sum_{t \in T} s_{ait} \qquad \forall a \in A, \forall i \in M^O \tag{20}$$

$$\sum_{t \in T} s_{ait} = \sum_{t \in T} f_{ait} \qquad \forall a \in A, \forall i \in M^O \tag{21}$$

$$0 = \sum_{t_1 = t_0}^{t} s_{ajt_1} + \sum_{t_2 = t}^{t_N} f_{ait_2} \qquad \begin{array}{r} \forall a \in A, \forall t \in T, \\ \forall i \in I, \forall j \in M \end{array} \tag{22}$$

Initial mixed model

$$\max_{\substack{O_t, N_{rp} \in \mathbb{Z}^* \\ s_{ait}, f_{ait}, o_{at} \in \{0,1\}}} \sum_{p \in P} [DIS^p (\sum_{t \in T_p} (O_t \cdot v_t) - \sum_{r \in R} (N_{rp} \cdot C_{rp}))] \quad (17)$$

subject to (2):

$$(s_{ait_1} + f_{ait_2} - 1) \cdot d_i \leq \frac{s_{ait_1} + f_{ait_2}}{2} \cdot \sum_{t_3 = t_1}^{t_2} \omega_{it_3} \qquad \forall i \in \mathcal{I}, \forall a \in A, \forall t_1, t_2 \in T \quad (23)$$

$$N_{rp} \geq \sum_{a \in A} \sum_{i \in \mathcal{I}} \sum_{t_1 = t_0}^{t} (\rho_{ir} \cdot (s_{ait_1} - f_{ai(t_1 - 1)})) \qquad \forall r \in R, \forall p \in P, \forall t \in T_p \quad (24)$$

$$o_{at} \leq \frac{1}{2} \cdot (\sum_{t_1 = t_0}^{t} f_{ai_N t_1} + \sum_{i \in M \cup \{i_N\}} \sum_{t_2 = t - \lambda_a}^{t} f_{ait_2}) \qquad \forall a \in A, \forall t \in T \quad (25)$$

$$O_t = \sum_{a \in A} o_{at} \qquad \forall t \in T \quad (26)$$

$$N_{rp} \leq m_{rp} \qquad \forall r \in R, \forall p \in P \quad (27)$$

Mixed Model Explanation

(17) Objective function, sums up profits from energy made, subtracts money used on resources (vessels), and multiplies it all with a discount factor
(18) Forces every mandatory task to be starded and finished at some point
(19) For every precedence relation (i, j) it ensures that if task i finishes at time t_2 there is no $t_1 \leq t_2$ at which task j starts
(20) Ensures each optional maintenance task to be started at most once
(21) Ensures that every maintenance task for a particular asset that is started is also finished
(22) Ensures an asset is fully installed before maintenance starts
(23) Ensures that between the starting and finish times of each task are enough timesteps with acceptable weather
(24) Counts up the resources needed in a time period by adding the resources needed by all tasks started, and subtracting the resources needed by all tasks finished
(25) Sets an asset to be online if it installed and had work done on it recently
(26) Counts how many assets are online
(27) Sets a limit on the amount of vessels that can be charatered in a given period

Notation overview

Sets:

- P: All time periods (large scale)
- T: All time intervals (small scale) $[t_0, \dots, t_N]$
- $T_p \in T$: All time intervals (small scale) in period p
- R: All resources
- I: All installation tasks per asset $[1, \dots, i_N]$
- $M = M^M \cup M^O$: all mandatory and optional maintenance tasks
- $\mathcal{I} = I \cup M$: All tasks
- IP: All precedency pairs (i, j)
- A: All assets

Decision variables:

- O_t: Number of online turbines at timestep t
- o_{at}: Binary variable, 1 if asset a is online at timestep t
- N_{rp}: Number of resources r used in period p
- s_{ait}: Binary variable, 1 if task $i \in \mathcal{I}$ for asset a starts at time t
- f_{ait}: Binary variable, 1 if task $i \in \mathcal{I}$ for asset a finishes at time t

Parameters:

- DIS: The discount factor per time period
- v_t: The value of energy a single turbine produces at timestep t
- C_{rp}: The cost of chartering resource r in period p
- d_i: The duration of task $i \in \mathcal{I}$
- λ_a: The number of timesteps after the last maintenance before asset a fails
- ω_{it}: Binary parameter representing weather, 1 if task $i \in \mathcal{I}$ can be completed at time t, 0 otherwise
- ρ_{ir}: The amount of resource r used for task $i \in \mathcal{I}$
- m_{rp}: The maximum amount of resources r that can be charatered in period p