Initial models for optimisation

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July 22, 2020

Initial model for installation

$$\max_{\substack{O_p, N_{rp} \in \mathbb{Z}^* \\ s_{ait} \in \{0,1\}}} \sum_{p \in P} [DIS^p (O_p \cdot v_p - \sum_{r \in R} N_{rp} \cdot C_{rp})] \tag{1}$$

subject to:

$$s_{ait} \leq s_{ai(t+1)} \qquad \forall a \in A, \forall i \in I, \forall t \in T \tag{2}$$

$$s_{ai\sigma_{it_N}} \geq 1 \qquad \forall a \in A, \forall i \in I \tag{3}$$

$$s_{ajt} \leq s_{ai\sigma_{it}} \qquad \forall a \in A, \forall (i,j) \in IP, \forall t \in T \tag{4}$$

$$N_{rp} \geq \sum_{a \in A} \sum_{i \in I} (\rho_{ir} \cdot (s_{ait} - s_{ai\sigma_{it}})) \qquad \forall r \in R, \forall p \in P, \forall t \in T_p \tag{5}$$

$$O_p = \sum_{a \in A} s_{ai_N \sigma_{i_N} t_p} \qquad \forall p \in P \tag{6}$$

$$N_{rp} \leq m_{rp} \qquad \forall r \in R, \forall p \in P \tag{7}$$

Installation Model Explanation

(1) Objective function, sums up profits from energy made, subtracts money used on resources (vessels), and multiplies it all with a discount factor
(2) Makes every task that is started stay started
(3) Forces every task to be starded and finished by the final timestep
(4) For every precedence relation (i, j), ensures that i is finished before j is started
(5) Counts up the resources needed in a time period by adding the resources needed by all tasks started, and subtracting the resources needed by all tasks finished
(6) Counts the number of turbines which finished installing by the end of a period
(7) Sets a limit on the amount of vessels that can be charatered in a given period

Notation overview

Sets:

- P: All time periods (large scale)
- T: All time intervals (small scale) $[t_0, \ldots, t_N]$
- $T_p \in T$: All time intervals (small scale) in period p
- R: All resources
- I: All tasks per asset $[1, \ldots, i_N]$
- IP: All precedency pairs (i, j)
- A: All assets

Decision variables:

- O_p: Number of online turbines after period p
- N_{rp}: Number of resources r used in period p
- s_{ait}: Binary variable, 1 if task $i \in I$ for asset a has started at or before time t

Parameters:

- DIS: The discount factor per period
- v_p: The value of energy a single turbine produces in period p
- C_{rp}: The cost of chartering resource r in period p
- σ_{it}: Indicates the timestep at which task i should have been started for it to be finished by timestep t, taking into account the duration and the weather conditions
- ρ_{ir}: The amount of resource r used by task i
- t_p: The final time interval (from T) before period p
- m_{rp}: The maximum amount of resources r that can be charatered in period p

Initial model for maintenance

$$\max_{\substack{O_t, N_{rp} \in \mathbb{Z}^* \\ s_{ait}, b_{at} \in \{0,1\}}} \sum_{p \in P} [DIS^p(\sum_{t \in T_p} (O_t \cdot v_t) - \sum_{r \in R} (N_{rp} \cdot C_{rp}))] \tag{8}$$

subject to:

$$
\begin{aligned}
s_{ait} &\leq s_{ai(t+1)} && \forall a \in A, \forall i \in M, \forall t \in T && (9)\\
s_{ai\sigma_{it_N}} &\geq 1 && \forall a \in A, \forall i \in M^M && (10)\\
N_{rp} &\geq \sum_{a \in A} \sum_{i \in M} (\rho_{ir} \cdot (s_{ait} - s_{ai\sigma it})) && \forall r \in R, \forall p \in P, \forall t \in T_p && (11)\\
b_{at} &> \sum_{i \in M} [s_{ai\sigma_{i(t-\lambda_a)}} - s_{ai\sigma_{it}}] && \forall a \in A, \forall t \in T && (12)\\
O_t &= |A| - \sum_{a \in A} b_{at} && \forall t \in T && (13)\\
N_{rp} &\leq m_{rp} && \forall r \in R, \forall p \in P && (14)
\end{aligned}
$$

Maintenance Model Explanation

(8) Objective function, sums up profits from energy made, subtracts money used on resources (vessels), and multiplies it all with a discount factor
(9) Makes every task that is finished stay finished
(10) Forces every mandatory maintenance task to be done at some point
(11) Counts up the resources needed in a time period by adding the resources needed by all tasks started, and subtracting the resources needed by all tasks finished
(12) If no maintenance tasks have finished in the past λ_a timesteps this asset is broken
(13) The number of active (online) turbines is equal to everything that isn't broken
(14) Sets a limit on the amount of vessels that can be charatered in a given period

Notation overview

Sets:

- P: All time periods (large scale)
- T: All time intervals (small scale) $[t_0, \ldots, t_N]$
- $T_p \in T$: All time intervals (small scale) in period p
- R: All resources
- A: All assets
- $M = M^M \cup M^O$: All (mandatory and optional) maintenance tasks

Decision variables:

- O_t: Number of active turbines at timestep t
- N_{rp}: Number of resources r used in period p
- s_{ait}: Binary variable, 1 if maintenance task i for asset a has started at or before time t
- b_{at}: Binary variable, 1 if asset a is broken at timestep t

Parameters:

- DIS: The discount factor per time period
- v_t: The value of energy a single turbine produces at timestep t
- C_{rp}: The cost of chartering resource r in period p
- λ_a: The number of timesteps after the last maintenance before asset a fails
- σ_{it}: Indicates the timestep at which task i should have been started for it to be finished by timestep t, taking into account the duration and the weather conditions
- ρ_{ir}: The amount of resource r used per task for maintenance task i
- m_{rp}: The maximum amount of resources r that can be charatered in period p

Initial 2-mixed model

$$\max_{\substack{O_t, N_{rp} \in \mathbb{Z}^* \\ s_{ait}, o_{at} \in \{0,1\}}} \sum_{p \in P} [DIS^p(\sum_{t \in T_p} (O_t \cdot v_t) - \sum_{r \in R} (N_{rp} \cdot C_{rp}))] \tag{15}$$

subject to:

$$\begin{aligned}
s_{ait} \leq& s_{ai(t+1)} && \forall a \in A, \forall i \in \mathcal{I}, \forall t \in T && (16)\\
s_{ai\sigma_{it_N}} \geq& 1 && \forall a \in A, \forall i \in I \cup M^M && (17)\\
s_{ajt} \leq& s_{ai\sigma_{it}} && \forall a \in A, \forall (i,j) \in IP, \forall t \in T && (18)\\
N_{rp} \geq& \sum_{a \in A} \sum_{i \in \mathcal{I}} (\rho_{ir} \cdot (s_{ait} - s_{ai\sigma_{it}})) && \forall r \in R, \forall p \in P, \forall t \in T_p && (19)\\
o_{at} \leq& \frac{1}{2} \cdot (s_{ai_N \sigma_{i_N t}} + \sum_{i \in M \cup \{i_N\}} [s_{ai\sigma_{it}} - s_{ai\sigma_{i(t-\lambda_a)}}]) && \forall a \in A, \forall t \in T && (20)\\
O_t =& \sum_{a \in A} o_{at} && \forall t \in T && (21)\\
N_{rp} \leq& m_{rp} && \forall r \in R, \forall p \in P && (22)
\end{aligned}$$

2-Mixed Model Explanation

(15) Objective function, sums up profits from energy made, subtracts money used on resources (vessels), and multiplies it all with a discount factor
(16) Makes every task that is started stay started
(17) Forces every installation and mandatory maintenance task to be starded and finished by the final timestep
(18) For every precedence relation (i, j), ensures that i is finished before j is started
(19) Counts up the resources needed in a time period by adding the resources needed by all tasks started, and subtracting the resources needed by all tasks finished
(20) Sets an asset to be online if it installed and had work done on it recently
(21) Counts how many assets are online
(22) Sets a limit on the amount of vessels that can be charatered in a given period

Notation overview

Sets:

- P: All time periods (large scale)
- T: All time intervals (small scale) $[t_0, \ldots, t_N]$
- $T_p \in T$: All time intervals (small scale) in period p
- R: All resources
- I: All installation tasks per asset $[1, \ldots, i_N]$
- $M = M^M \cup M^O$: all mandatory and optional maintenance tasks
- $\mathcal{I} = I \cup M$: All tasks
- IP: All precedency pairs (i, j)
- A: All assets

Decision variables:

- O_t: Number of online turbines at timestep t
- o_{at}: Binary variable, 1 if asset a is online at timestep t
- N_{rp}: Number of resources r used in period p
- s_{ait}: Binary variable, 1 if task $i \in \mathcal{I}$ for asset a has started at or before time t

Parameters:

- DIS: The discount factor per time period
- v_t: The value of energy a single turbine produces at timestep t
- C_{rp}: The cost of chartering resource r in period p
- λ_a: The number of timesteps after the last maintenance before asset a fails
- σ_{it}: Indicates the timestep at which task i should have been started for it to be finished by timestep t, taking into account the duration and the weather conditions
- ρ_{ir}: The amount of resource r used for task $i \in \mathcal{I}$
- m_{rp}: The maximum amount of resources r that can be charatered in period p

Initial model for decommission

$$\max_{\substack{O_t, N_{rp} \in \mathbb{Z}^* \\ s_{ait}, o_{at} \in \{0,1\}}} \sum_{p \in P} [DIS^p (\sum_{t \in T_p} (O_t \cdot v_t) - \sum_{r \in R} (N_{rp} \cdot C_{rp}))] \tag{23}$$

subject to:

$$
\begin{aligned}
s_{ait} &\leq s_{ai(t+1)} && \forall a \in A, \forall i \in D, \forall t \in T && (24)\\
s_{ai\sigma_{it_N}} &\geq 1 && \forall a \in A, \forall i \in D && (25)\\
s_{ajt} &\leq s_{ai\sigma_{it}} && \forall a \in A, \forall (i,j) \in IP, \forall t \in T && (26)\\
N_{rp} &\geq \sum_{a \in A} \sum_{i \in I} (\rho_{ir} \cdot (s_{ait} - s_{ai\sigma_{it}})) && \forall r \in R, \forall p \in P, \forall t \in T_p && (27)\\
O_t &= \sum_{a \in A} (1 - s_{ai_0 t}) && \forall t \in T && (28)\\
N_{rp} &\leq m_{rp} && \forall r \in R, \forall p \in P && (29)
\end{aligned}
$$

Installation Model Explanation

(23) Objective function, sums up profits from energy made, subtracts money used on resources (vessels), and multiplies it all with a discount factor

(24) Makes every task that is started stay started

(25) Forces every task to be starded and finished by the final timestep

(26) For every precedence relation (i, j), ensures that i is finished before j is started

(27) Counts up the resources needed in a time period by adding the resources needed by all tasks started, and subtracting the resources needed by all tasks finished

(28) Counts the number of turbines which have not started decomissioning by a given timestep

(29) Sets a limit on the amount of vessels that can be charatered in a given period

Notation overview

Sets:

- P: All time periods (large scale)
- T: All time intervals (small scale) $[t_0, \dots, t_N]$
- $T_p \in T$: All time intervals (small scale) in period p
- R: All resources
- D: All tasks per asset $[i_0, \dots, i_N]$
- IP: All precedency pairs (i, j)
- A: All assets

Decision variables:

- O_t: Number of online turbines at timestep t
- N_{rp}: Number of resources r used in period p
- s_{ait}: Binary variable, 1 if task $i \in I$ for asset a has started at or before time t

Parameters:

- DIS: The discount factor per period
- v_t: The value of energy a single turbine produces in timestep t
- C_{rp}: The cost of chartering resource r in period p
- σ_{it}: Indicates the timestep at which task i should have been started for it to be finished by timestep t, taking into account the duration and the weather conditions
- ρ_{ir}: The amount of resource r used by task i
- t_p: The final time interval (from T) before period p
- m_{rp}: The maximum amount of resources r that can be charatered in period p

Initial 3-mixed model

$$\max_{\substack{O_t, N_{rp} \in \mathbb{Z}^* \\ s_{ait}, o_{at} \in \{0,1\}}} \sum_{p \in P} [DIS^p(\sum_{t \in T_p} (O_t \cdot v_t) - \sum_{r \in R} (N_{rp} \cdot C_{rp}))] \tag{30}$$

subject to:

$$\begin{aligned}
s_{ait} &\leq s_{ai(t+1)} && \forall a \in A, \forall i \in \mathcal{I}, \forall t \in T && (31)\\
s_{ai\sigma_{it_N}} &\geq 1 && \forall a \in A, \forall i \in \mathcal{I} - M^O && (32)\\
s_{ajt} &\leq s_{ai\sigma_{it}} && \forall a \in A, \forall (i,j) \in IP, \forall t \in T && (33)\\
N_{rp} &\geq \sum_{a \in A} \sum_{i \in \mathcal{I}} (\rho_{ir} \cdot (s_{ait} - s_{ai\sigma_{it}})) && \forall r \in R, \forall p \in P, \forall t \in T_p && (34)\\
o_{at} &\leq s_{ai^I_N \sigma_{i^I_N t}} - s_{ai^D_0 t} && \forall a \in A, \forall t \in T && (35)\\
o_{at} &\leq \sum_{i \in M \cup \{i^I_N\}} (s_{ai\sigma_{it}} - s_{ai\sigma_{i(t-\lambda_a)}}) && \forall a \in A, \forall t \in T && (36)\\
O_t &= \sum_{a \in A} o_{at} && \forall t \in T && (37)\\
N_{rp} &\leq m_{rp} && \forall r \in R, \forall p \in P && (38)
\end{aligned}$$

3-Mixed Model Explanation

(30) Objective function, sums up profits from energy made, subtracts money used on resources (vessels), and multiplies it all with a discount factor
(31) Makes every task that is started stay started
(32) Forces every installation and mandatory maintenance task to be starded and finished by the final timestep
(33) For every precedence relation (i, j), ensures that i is finished before j is started
(34) Counts up the resources needed in a time period by adding the resources needed by all tasks started, and subtracting the resources needed by all tasks finished
(35) An asset can only be online if installation is complete and decomission has not started yet
(36) An asset can only be online if it had maintenance done recently (or only completed installation recently)
(37) Counts how many assets are online
(38) Sets a limit on the amount of vessels that can be charatered in a given period

Notation overview

Sets:

- P: All time periods (large scale)
- T: All time intervals (small scale) $[t_0, \ldots, t_N]$
- $T_p \in T$: All time intervals (small scale) in period p
- R: All resources
- I: All installation tasks per asset $[i_0^I, \ldots, i_N^I]$
- $M = M^M \cup M^O$: all mandatory and optional maintenance tasks
- D: All decomission tasks per asset $[i_0^D, \ldots, i_N^D]$
- $\mathcal{I} = I \cup M \cup D$: All tasks
- IP: All precedency pairs (i, j)
- A: All assets

Decision variables:

- O_t: Number of online turbines at timestep t
- o_{at}: Binary variable, 1 if asset a is online at timestep t
- N_{rp}: Number of resources r used in period p
- s_{ait}: Binary variable, 1 if task $i \in \mathcal{I}$ for asset a has started at or before time t

Parameters:

- DIS: The discount factor per time period
- v_t: The value of energy a single turbine produces at timestep t
- C_{rp}: The cost of chartering resource r in period p
- λ_a: The number of timesteps after the last maintenance before asset a fails
- σ_{it}: Indicates the timestep at which task i should have been started for it to be finished by timestep t, taking into account the duration and the weather conditions
- ρ_{ir}: The amount of resource r used for task $i \in \mathcal{I}$
- m_{rp}: The maximum amount of resources r that can be charatered in period p