Years spanning model

$$\min_{N_{y,m,\sigma},P_{m,i},R_{m,i,\sigma}\in\mathbb{Z}^*} \quad \frac{1}{|S|}\cdot\sum_{\sigma\in S}(\sum_{m\in M}[\sum_{y\in Y}(N_{y,m,\sigma}\cdot c_{y,m})+\sum_{i\in I^P}(P_{m,i}\cdot d^P\cdot e^H_m)+ \sum_{i\in I^R}[\sum_{m'=0}^{m}(f_{m',i,\sigma}-R_{m',i,\sigma})]\cdot e^H_m\cdot H_m]) \tag{1}$$

subject to:

$$L_y\cdot N_{y,m,\sigma}+L^{Inst}_{y,m}\geq\sum_{i\in I^P}(P_{m,i}\cdot d^P_y)+\sum_{i\in I^R}(R_{m,i,\sigma}\cdot d^R_{y,i}) \qquad \forall\sigma\in S,\forall m\in M,\forall y\in Y \tag{2}$$

$$\sum_{m'=0}^{m-1}f_{m',i,\sigma}\geq\sum_{m'=0}^{m}R_{m',i,\sigma} \qquad \forall\sigma\in S,\forall m\in M,\forall i\in I^R \tag{3}$$

$$P_{m,i}\leq\sum_{m'=0}^{m-G^{MIN}}P_{m',(i-1)}-\sum_{m'=0}^{m-1}P_{m',i} \qquad \forall m\in M,\forall i\in I^P \tag{4}$$

$$P_{m,i}\geq\sum_{m'=0}^{m-G^{MAX}}P_{m',(i-1)}-\sum_{m'=0}^{m-1}P_{m',i} \qquad \forall m\in M,\forall i\in I^P \tag{5}$$

$$N_{y,m,\sigma}\leq A_{y,m}-N^{Inst}_{y,m} \qquad \forall\sigma\in S,\forall m\in M,\forall y\in Y \tag{6}$$

Years model Explanation

(1) The objective takes an average cost of all scenarios by counting up the cost per month. Each month has a cost for vessels chartered (by type), the energy lost due to planned maintenance, and the energy lost due to unrepaired failures
(2) For each vessel type the available time (based on charters and spare time from installation) needs to surpass the estimated total time all tasks take
(3) For any type of reactive task, the amount of tasks performed cannot exceed the amount of failures of that type (with a 1 month delay before a failure can be solved)
(4) Ensures planned tasks are not scheduled too close together by counting up the turbines that were last visited G^{MIN} or more months ago and subtracting all that were revisited later
(5) Ensures planned tasks are not scheduled too far apart, in similar manner as the previous constraint
(6) Limits the maximum amount of vessels chartered based on normal availability and vessels used for installation

Years model notation overview

Sets:

- S: Scenarios
- M: Months
- Y: Vessel types
- I^P: Indices for the planned tasks (first time a turbine is visited is index 1, second time is index 2, etc...)
- I^R: Types of failures reactive tasks

Decision variables:

- $N_{y,m,\sigma}$: The amount of vessels of type y chartered in month m in scenario σ
- $P_{m,i}$: Planned ith tasks in month m
- $R_{m,i,\sigma}$: Reactive tasks of type i in month m and scenario σ

Parameters:

- $c_{y,m}$: The cost of chartering a vessel of type y in month m
- d^P: The duration of a preventive task
- e^H_m: The energy produced by a single turbine per hour in month m
- $f_{m,i,\sigma}$: The amount of failures of type i in month m in scenario σ
- H_m: The number of hours in month m
- L_y: The amount of hours a vessel of type y is available if chartered for a month
- $L^{Inst}_{y,m}$: Leftover hours vessels of type y are available in month m based on the installation schedule
- d^P_y: The duration of a preventive task for vessel type y
- $d^R_{y,i}$: The duration of a reactive task i for vessel type y
- G^{MIN} & G^{MAX}: The minimum and maximum amounts of months between two planned maintenance tasks
- $A_{y,m}$: The number of maximum available vessels of type y in month m
- $N^{Inst}_{y,m}$: Amount of vessels of type y used by the installation schedule in month m

Months spanning model

$$\min_{\substack{s_i \in \mathbb{R}_{\geq 0} \\ a_{v,i,j} \in \{0,1\}}} \sum_{i \in I} c_i \cdot (s_i + \max_{y \in Y}(s_{y,i} + d_{y,i})) \tag{7}$$

subject to:

$$\sum_{i \in I} a_{v,i,j} \leq 1 \qquad \forall v \in V, \forall j \in J \tag{8}$$

$$\sum_{i \in I} a_{v,i,j} \leq \sum_{i \in I} a_{v,i,(j-1)} \qquad \forall v \in V, \forall j \in J - \{0\} \tag{9}$$

$$\rho_{y,i} \leq \sum_{v \in V_y} \sum_{j \in J} a_{v,i,j} \qquad \forall y \in Y, \forall i \in I \tag{10}$$

$$\begin{aligned} M \cdot (a_{v,i,j} + a_{v,i',(j-1)}) + \\ d_{y,i'} \cdot a_{v,i',(j-1)} - 2M \end{aligned} \leq s_i + s_{y,i} - s_{i'} - s_{i',y} \qquad \begin{aligned} \forall y \in Y, \forall v \in V_y, \\ \forall i, i' \in I, \forall j \in J - \{0\} \end{aligned} \tag{11}$$

$$s_i + \max_{y \in Y}(s_{y,i} + d_{y,i}) \leq T \qquad \forall i \in I \tag{12}$$

$$\sum_{j \in J} a_{v,i,j} = a^{Inst}_{v,i} \qquad \forall v \in V, \forall I \in I^{Inst} \tag{13}$$

$$s_i = s^{Inst}_i \qquad \forall I \in I^{Inst} \tag{14}$$

Months model Explanation

(7) Objective is to minimise costs of tasks being uncompleted (the +max bit is optional as it's about constants)
(8) Ensures every vessel only does one task at a time
(9) Ensures that if a vessel has an xth task it also has an $x - 1$th task
(10) Ensures every task has enough resourses assigned to it
(11) Ensures the starting times of consecutive tasks are separated by at least the duration of the first task (the M factors ensure that if the tasks are not consecutive the starting times don't matter)
(12) Ensures all tasks are finished on time
(13) Forces every installation task to be assigned as in the installation schedule
(14) Forces every installation task to be started as in the installation schedule

Months model notation overview

Sets:

- Y: Vessel types
- V: Individual vessels
- $V_y \subseteq V$: Vessels of type $y \in Y$
- I: Tasks to be completed in this month
- $I^{Inst} \subset I$: Installation tasks completed this month by vessels that are also available for maintenance tasks
- J: Maximum amount of tasks assigned to a single vessel

Decision variables:

- s_i: Start time of task i
- $a_{v,i,j}$: Binary variable which is 1 if vessel v performs task i as its jth task

Parameters:

- c_i: The cost per hour of a task not being completed
- $s_{y,i}$: The start time offset between task i starting and vessel type y being used for it
- $d_{y,i}$: The duration of task i for vessel type y
- $\rho_{y,i}$: The amount of vessels of type y required for task i
- M: A large number
- T: The end time of the month
- s_i^{Inst}: Start time of task $i \in I^{Inst}$, as in the installation schedule
- $a_{v,i}^{Inst}$: Binary assignment variable of task $i \in I^{Inst}$, as in the installation schedule