

Years spanning model

$$\min_{N_{y,m,\sigma}, P_{m,i}, R_{m,i,\sigma} \in \mathbb{Z}^*} \frac{1}{|S|} \cdot \sum_{\sigma \in S} \left(\sum_{m \in M} \left[\sum_{y \in Y} (N_{y,m,\sigma} \cdot c_{y,m}) + \sum_{i \in I^P} (P_{m,i} \cdot d_P \cdot e_m^H) + \sum_{i \in I^R} \left[\sum_{m'=0}^m (f_{m',i,\sigma} - R_{m',i,\sigma}) \right] \cdot e_m^H \cdot H_m \right] \right) \quad (1)$$

subject to:

$$L_y \cdot N_{y,m,\sigma} + L_{y,m}^{Inst} \geq \sum_{i \in I^P} (P_{m,i} \cdot d_y^P) + \sum_{i \in I^R} (R_{m,i,\sigma} \cdot d_{y,i}^R) \quad \forall \sigma \in S, \forall m \in M, \forall y \in Y \quad (2)$$

$$\sum_{m'=0}^{m-1} f_{m',i,\sigma} \geq \sum_{m'=0}^m R_{m',i,\sigma} \quad \forall \sigma \in S, \forall m \in M, \forall i \in I^R \quad (3)$$

$$P_{m,i} \leq \sum_{m'=0}^{m-G^{MIN}} P_{m',(i-1)} - \sum_{m'=0}^{m-1} P_{m',i} \quad \forall m \in M, \forall i \in I^P \quad (4)$$

$$P_{m,i} \geq \sum_{m'=0}^{m-G^{MAX}} P_{m',(i-1)} - \sum_{m'=0}^{m-1} P_{m',i} \quad \forall m \in M, \forall i \in I^P \quad (5)$$

$$N_{y,m,\sigma} \leq A_{y,m} - N_{y,m}^{Inst} \quad \forall \sigma \in S, \forall m \in M, \forall y \in Y \quad (6)$$

Years model Explanation

- (1) The objective takes an average cost of all scenarios by counting up the cost per month. Each month has a cost for vessels chartered (by type), the energy lost due to planned maintenance, and the energy lost due to unrepaired failures
- (2) For each vessel type the available time (based on charters and spare time from installation) needs to surpass the estimated total time all tasks take
- (3) For any type of reactive task, the amount of tasks performed cannot exceed the amount of failures of that type (with a 1 month delay before a failure can be solved)
- (4) Ensures planned tasks are not scheduled too close together by counting up the turbines that were last visited G^{MIN} or more months ago and subtracting all that were revisited later
- (5) Ensures planned tasks are not scheduled too far apart, in similar manner as the previous constraint
- (6) Limits the maximum amount of vessels chartered based on normal availability and vessels used for installation

Years model notation overview

Sets:

- S : Scenarios
- M : Months
- Y : Vessel types
- I^P : Indices for the planned tasks (first time a turbine is visited is index 1, second time is index 2, etc...)
- I^R : Types of failures reactive tasks

Decision variables:

- $N_{y,m,\sigma}$: The amount of vessels of type y chartered in month m in scenario σ
- $P_{m,i}$: Planned i th tasks in month m
- $R_{m,i,\sigma}$: Reactive tasks of type i in month m and scenario σ

Parameters:

- $c_{y,m}$: The cost of chartering a vessel of type y in month m
- d_P : The duration of a preventive task
- e_m^H : The energy produced by a single turbine per hour in month m
- $f_{m,i,\sigma}$: The amount of failures of type i in month m in scenario σ
- H_m : The number of hours in month m
- L_y : The amount of hours a vessel of type y is available if chartered for a month
- $L_{y,m}^{Inst}$: Leftover hours vessels of type y are available in month m based on the installation schedule
- d_y^P : The duration of a preventive task for vessel type y
- $d_{y,i}^R$: The duration of a reactive task i for vessel type y
- G^{MIN} & G^{MAX} : The minimum and maximum amounts of months between two planned maintenance tasks
- $A_{y,m}$: The number of maximum available vessels of type y in month m
- $N_{y,m}^{Inst}$: Amount of vessels of type y used by the installation schedule in month m

Months spanning model

$$\min_{\substack{s_i \in \mathbb{R}_{\geq 0} \\ a_{v,i,j} \in \{0,1\}}} \sum_{i \in I} c_i \cdot (s_i + \max_{y \in Y} (s_{y,i} + d_{y,i})) \quad (7)$$

subject to:

$$\sum_{i \in I} a_{v,i,j} \leq 1 \quad \forall v \in V, \forall j \in J \quad (8)$$

$$\sum_{i \in I} a_{v,i,j} \leq \sum_{i \in I} a_{v,i,(j-1)} \quad \forall v \in V, \forall j \in J - \{0\} \quad (9)$$

$$\rho_{y,i} \leq \sum_{v \in V_y} \sum_{j \in J} a_{v,i,j} \quad \forall y \in Y, \forall i \in I \quad (10)$$

$$M \cdot (a_{v,i,j} + a_{v,i',(j-1)}) + d_{y,i'} \cdot a_{v,i',(j-1)} - 2M \leq s_i + s_{y,i} - s_{i'} - s_{i',y} \quad \forall y \in Y, \forall v \in V_y, \forall i, i' \in I, \forall j \in J - \{0\} \quad (11)$$

$$s_i + \max_{y \in Y} (s_{y,i} + d_{y,i}) \leq T \quad \forall i \in I \quad (12)$$

$$\sum_{j \in J} a_{v,i,j} = a_{v,i}^{Inst} \quad \forall v \in V, \forall i \in I^{Inst} \quad (13)$$

$$s_i = s_i^{Inst} \quad \forall i \in I^{Inst} \quad (14)$$

Months model Explanation

- (7) Objective is to minimise costs of tasks being uncompleted (the $+max$ bit is optional as it's about constants)
- (8) Ensures every vessel only does one task at a time
- (9) Ensures that if a vessel has an x th task it also has an $x - 1$ th task
- (10) Ensures every task has enough resources assigned to it
- (11) Ensures the starting times of consecutive tasks are separated by at least the duration of the first task (the M factors ensure that if the tasks are not consecutive the starting times don't matter)
- (12) Ensures all tasks are finished on time
- (13) Forces every installation task to be assigned as in the installation schedule
- (14) Forces every installation task to be started as in the installation schedule

Months model notation overview

Sets:

- Y : Vessel types
- V : Individual vessels
- $V_y \subseteq V$: Vessels of type $y \in Y$
- I : Tasks to be completed in this month
- $I^{Inst} \subset I$: Installation tasks completed this month by vessels that are also available for maintenance tasks
- J : Maximum amount of tasks assigned to a single vessel

Decision variables:

- s_i : Start time of task i
- $a_{v,i,j}$: Binary variable which is 1 if vessel v performs task i as its j th task

Parameters:

- c_i : The cost per hour of a task not being completed
- $s_{y,i}$: The start time offset between task i starting and vessel type y being used for it
- $d_{y,i}$: The duration of task i for vessel type y
- $\rho_{y,i}$: The amount of vessels of type y required for task i
- M : A large number
- T : The end time of the month
- s_i^{Inst} : Start time of task $i \in I^{Inst}$, as in the installation schedule
- $a_{v,i}^{Inst}$: Binary assignment variable of task $i \in I^{Inst}$, as in the installation schedule