Initial models for optimisation

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Initial model for installation

$$\max_{\substack{O_p, N_{rp} \in \mathbb{Z}^* \\ s_{ait} \in \{0,1\}}} \sum_{p \in P} [DIS^p (O_p \cdot v_p - \sum_{r \in R} N_{rp} \cdot C_{rp})] \tag{1}$$

subject to:

$$s_{ait} \leq s_{ai(t+1)} \qquad \forall a \in A, \forall i \in I, \forall t \in T \tag{2}$$

$$s_{ai\sigma_{it_N}} \geq 1 \qquad \forall a \in A, \forall i \in I \tag{3}$$

$$s_{ajt} \leq s_{ai\sigma_{it}} \qquad \forall a \in A, \forall (i,j) \in IP, \forall t \in T \tag{4}$$

$$N_{rp} \geq \sum_{a \in A} \sum_{i \in I} (\rho_{ir} \cdot (s_{ait} - s_{ai\sigma_{it}})) \qquad \forall r \in R, \forall p \in P, \forall t \in T_p \tag{5}$$

$$O_p = \sum_{a \in A} s_{ai_N \sigma_{i_N} t_p} \qquad \forall p \in P \tag{6}$$

$$N_{rp} \leq m_{rp} \qquad \forall r \in R, \forall p \in P \tag{7}$$

Installation Model Explanation

(1) Objective function, sums up profits from energy made, subtracts money used on resources (vessels), and multiplies it all with a discount factor
(2) Makes every task that is started stay started
(3) Forces every task to be starded and finished by the final timestep
(4) For every precedence relation (i, j), ensures that i is finished before j is started
(5) Counts up the resources needed in a time period by adding the resources needed by all tasks started, and subtracting the resources needed by all tasks finished
(6) Counts the number of turbines which finished installing by the end of a period
(7) Sets a limit on the amount of vessels that can be charatered in a given period

Notation overview

Sets:

- P: All time periods (large scale)
- T: All time intervals (small scale) $[t_0, \dots, t_N]$
- $T_p \subset T$: All time intervals (small scale) in period p
- R: All resources
- I: All tasks per asset $[1, \dots, i_N]$
- IP: All precedency pairs (i, j)
- A: All assets

Decision variables:

- O_p: Number of online turbines after period p
- N_{rp}: Number of resources r used in period p
- s_{ait}: Binary variable, 1 if task $i \in I$ for asset a has started at or before time t

Parameters:

- DIS: The discount factor per period
- v_p: The value of energy a single turbine produces in period p
- C_{rp}: The cost of chartering resource r in period p
- σ_{it}: Indicates the timestep at which task i should have been started for it to be finished by timestep t, taking into account the duration and the weather conditions
- ρ_{ir}: The amount of resource r used by task i
- t_p: The final time interval (from T) before period p
- m_{rp}: The maximum amount of resources r that can be charatered in period p

Initial model for maintenance

$$\max_{\substack{O_t, N_{rp} \in \mathbb{Z}^* \\ s_{ait}, b_{at} \in \{0,1\}}} \sum_{p \in P} [DIS^p (\sum_{t \in T_p} (O_t \cdot v_t) - \sum_{r \in R} (N_{rp} \cdot C_{rp}))] \tag{8}$$

subject to:

$$s_{ait} \leq s_{ai(t+1)} \qquad \forall a \in A, \forall i \in M, \forall t \in T \tag{9}$$

$$s_{ai\sigma_{it_N}} \geq 1 \qquad \forall a \in A, \forall i \in M^M \tag{10}$$

$$N_{rp} \geq \sum_{a \in A} \sum_{i \in M} (\rho_{ir} \cdot (s_{ait} - s_{ai\sigma it})) \qquad \forall r \in R, \forall p \in P, \forall t \in T_p \tag{11}$$

$$b_{at} > \sum_{i \in M} [s_{ai\sigma_{i(t-\lambda_a)}} - s_{ai\sigma_{it}}] \qquad \forall a \in A, \forall t \in T \tag{12}$$

$$O_t = |A| - \sum_{a \in A} b_{at} \qquad \forall t \in T \tag{13}$$

$$N_{rp} \leq m_{rp} \qquad \forall r \in R, \forall p \in P \tag{14}$$

Maintenance Model Explanation

(8) Objective function, sums up profits from energy made, subtracts money used on resources (vessels), and multiplies it all with a discount factor
(9) Makes every task that is finished stay finished
(10) Forces every mandatory maintenance task to be done at some point
(11) Counts up the resources needed in a time period by adding the resources needed by all tasks started, and subtracting the resources needed by all tasks finished
(12) If no maintenance tasks have finished in the past λ_a timesteps this asset is broken
(13) The number of active (online) turbines is equal to everything that isn't broken
(14) Sets a limit on the amount of vessels that can be charatered in a given period

Notation overview

Sets:

- P: All time periods (large scale)
- T: All time intervals (small scale) $[t_0, \ldots, t_N]$
- $T_p \subset T$: All time intervals (small scale) in period p
- R: All resources
- A: All assets
- $M = M^M \cup M^O$: All (mandatory and optional) maintenance tasks

Decision variables:

- O_t: Number of active turbines at timestep t
- N_{rp}: Number of resources r used in period p
- s_{ait}: Binary variable, 1 if maintenance task i for asset a has started at or before time t
- b_{at}: Binary variable, 1 if asset a is broken at timestep t

Parameters:

- DIS: The discount factor per time period
- v_t: The value of energy a single turbine produces at timestep t
- C_{rp}: The cost of chartering resource r in period p
- λ_a: The number of timesteps after the last maintenance before asset a fails
- σ_{it}: Indicates the timestep at which task i should have been started for it to be finished by timestep t, taking into account the duration and the weather conditions
- ρ_{ir}: The amount of resource r used per task for maintenance task i
- m_{rp}: The maximum amount of resources r that can be charatered in period p

Initial 2-mixed model

$$\max_{\substack{N_{rp}\in\mathbb{Z}^* \\ s_{ait},o_{at}\in\{0,1\}}} \sum_{p\in P}[DIS^p(\sum_{t\in T_p}\sum_{a\in A}(o_{at}\cdot v_t) - \sum_{r\in R}(N_{rp}\cdot C_{rp}))] \tag{15}$$

subject to:

$$\begin{aligned}
s_{ait} &\leq s_{ai(t+1)} && \forall a\in A, \forall i\in\mathcal{I}, \forall t\in T && (16)\\
s_{ai\sigma_{it_N}} &\geq 1 && \forall a\in A, \forall i\in I\cup M^M && (17)\\
s_{ajt} &\leq s_{ai\sigma_{it}} && \forall a\in A, \forall(i,j)\in IP, \forall t\in T && (18)\\
N_{rp} &\geq \sum_{a\in A}\sum_{i\in\mathcal{I}}(\rho_{ir}\cdot(s_{ait}-s_{ai\sigma_{it}})) && \forall r\in R, \forall p\in P, \forall t\in T_p && (19)\\
o_{at} &\leq s_{ai_N\sigma_{i_N t}} && \forall a\in A, \forall t\in T && (20)\\
o_{at} &\leq \sum_{i\in M\cup\{i_N\}}[s_{ai\sigma_{it}} - s_{ai\sigma_{i(t-\lambda_a)}}] && \forall a\in A, \forall t\in T && (21)\\
N_{rp} &\leq m_{rp} && \forall r\in R, \forall p\in P && (22)
\end{aligned}$$

2-Mixed Model Explanation

(15) Objective function, sums up profits from energy made, subtracts money used on resources (vessels), and multiplies it all with a discount factor
(16) Makes every task that is started stay started
(17) Forces every installation and mandatory maintenance task to be starded and finished by the final timestep
(18) For every precedence relation (i, j), ensures that i is finished before j is started
(19) Counts up the resources needed in a time period by adding the resources needed by all tasks started, and subtracting the resources needed by all tasks finished
(20) An asset can only be online if it finished installation
(21) An asset can only be online if it had work done on it recently enough
(22) Sets a limit on the amount of vessels that can be charatered in a given period

Notation overview

Sets:

- P: All time periods (large scale)
- T: All time intervals (small scale) $[t_0, \dots, t_N]$
- $T_p \subset T$: All time intervals (small scale) in period p
- R: All resources
- I: All installation tasks per asset $[1, \dots, i_N]$
- $M = M^M \cup M^O$: all mandatory and optional maintenance tasks
- $\mathcal{I} = I \cup M$: All tasks
- IP: All precedency pairs (i, j)
- A: All assets

Decision variables:

- o_{at}: Binary variable, 1 if asset a is online at timestep t
- N_{rp}: Number of resources r used in period p
- s_{ait}: Binary variable, 1 if task $i \in \mathcal{I}$ for asset a has started at or before time t

Parameters:

- DIS: The discount factor per time period
- v_t: The value of energy a single turbine produces at timestep t
- C_{rp}: The cost of chartering resource r in period p
- λ_a: The number of timesteps after the last maintenance before asset a fails
- σ_{it}: Indicates the timestep at which task i should have been started for it to be finished by timestep t, taking into account the duration and the weather conditions
- ρ_{ir}: The amount of resource r used for task $i \in \mathcal{I}$
- m_{rp}: The maximum amount of resources r that can be charatered in period p

Initial model for decommission

$$\max_{\substack{N_{rp} \in \mathbb{Z}^* \\ s_{ait} \in \{0,1\}}} \sum_{p \in P} [DIS^p (\sum_{t \in T_p} \sum_{a \in A} ((1 - s_{ai_0 t}) \cdot v_t) - \sum_{r \in R} (N_{rp} \cdot C_{rp}))] \tag{23}$$

subject to:

$$s_{ait} \leq s_{ai(t+1)} \qquad \forall a \in A, \forall i \in D, \forall t \in T \tag{24}$$

$$s_{ai\sigma_{it_N}} \geq 1 \qquad \forall a \in A, \forall i \in D \tag{25}$$

$$s_{ajt} \leq s_{ai\sigma_{it}} \qquad \forall a \in A, \forall (i,j) \in IP, \forall t \in T \tag{26}$$

$$N_{rp} \geq \sum_{a \in A} \sum_{i \in I} (\rho_{ir} \cdot (s_{ait} - s_{ai\sigma_{it}})) \qquad \forall r \in R, \forall p \in P, \forall t \in T_p \tag{27}$$

$$N_{rp} \leq m_{rp} \qquad \forall r \in R, \forall p \in P \tag{28}$$

Installation Model Explanation

(23) Objective function, sums up profits from energy made, subtracts money used on resources (vessels), and multiplies it all with a discount factor
(24) Makes every task that is started stay started
(25) Forces every task to be starded and finished by the final timestep
(26) For every precedence relation (i, j), ensures that i is finished before j is started
(27) Counts up the resources needed in a time period by adding the resources needed by all tasks started, and subtracting the resources needed by all tasks finished
(28) Sets a limit on the amount of vessels that can be charatered in a given period

Notation overview

Sets:

- P: All time periods (large scale)
- T: All time intervals (small scale) $[t_0, \ldots, t_N]$
- $T_p \subset T$: All time intervals (small scale) in period p
- R: All resources
- D: All tasks per asset $[i_0, \ldots, i_N]$
- IP: All precedency pairs (i, j)
- A: All assets

Decision variables:

- N_{rp}: Number of resources r used in period p
- s_{ait}: Binary variable, 1 if task $i \in I$ for asset a has started at or before time t

Parameters:

- DIS: The discount factor per period
- v_t: The value of energy a single turbine produces in timestep t
- C_{rp}: The cost of chartering resource r in period p
- σ_{it}: Indicates the timestep at which task i should have been started for it to be finished by timestep t, taking into account the duration and the weather conditions
- ρ_{ir}: The amount of resource r used by task i
- t_p: The final time interval (from T) before period p
- m_{rp}: The maximum amount of resources r that can be charatered in period p

Initial 3-mixed model

$$\max_{\substack{N_{rp}\in\mathbb{Z}^* \\ s_{ait},o_{at}\in\{0,1\}}} \sum_{p\in P}[DIS^p(\sum_{t\in T_p}\sum_{a\in A}(o_{at}\cdot v_t) - \sum_{r\in R}(N_{rp}\cdot C_{rp}))] \tag{29}$$

subject to:

$$\begin{aligned}
s_{ait} &\leq s_{ai(t+1)} && \forall a\in A, \forall i\in\mathcal{I}, \forall t\in T && (30)\\
s_{ai\sigma_{itN}} &\geq 1 && \forall a\in A, \forall i\in\mathcal{I}-M^O && (31)\\
s_{ai_0^D t} - 1 &\leq s_{ai\sigma_{it}} - s_{ait} && \forall a\in A, \forall i\in\mathcal{I}-D, \forall t\in T && (32)\\
s_{ajt} &\leq s_{ai\sigma_{it}} && \forall a\in A, \forall (i,j)\in IP, \forall t\in T && (33)\\
m_{rp} \geq N_{rp} &\geq \sum_{a\in A}\sum_{i\in\mathcal{I}}(\rho_{ir}\cdot(s_{ait}-s_{ai\sigma_{it}})) && \forall r\in R, \forall p\in P, \forall t\in T_p && (34)\\
o_{at} &\leq s_{ai_N^I\sigma_{i_N^I t}} - s_{ai_0^D t} && \forall a\in A, \forall t\in T && (35)\\
o_{at} &\leq \sum_{i\in M\cup\{i_N^I\}}(s_{ai\sigma_{it}} - s_{ai\sigma_{i(t-\lambda_a)}}) && \forall a\in A, \forall t\in T && (36)\\
o_{at} &\leq 1 + s_{ai\sigma_{it}} - s_{ait} && \forall a\in A, \forall i\in M, \forall t\in T && (37)
\end{aligned}$$

3-Mixed Model Explanation

(29) Objective function, sums up profits from energy made, subtracts money used on resources (vessels), and multiplies it all with a discount factor
(30) Makes every task that is started stay started
(31) Forces every installation and mandatory maintenance task to be started and finished by the final timestep
(32) Ensures every non-decommission task is inactive during decomission
(33) For every precedence relation (i, j), ensures that i is finished before j is started
(34) Counts up the resources needed in a time period by adding the resources needed by all tasks started, and subtracting the resources needed by all tasks finished. It also sets a limit on the amount of vessels that can be charatered in a given period
(35) An asset can only be online if installation is complete and decomission has not started yet
(36) An asset can only be online if it had maintenance done recently (or only completed installation recently)
(37) Ensures an asset if offline is maintenance work is going on at this moment

Notation overview

Sets:

- P: All time periods (large scale)
- T: All time intervals (small scale) $[t_0, \ldots, t_N]$
- $T_p \subset T$: All time intervals (small scale) in period p
- R: All resources
- I: All installation tasks per asset $[i_0^I, \ldots, i_N^I]$
- $M = M^M \cup M^O$: all mandatory and optional maintenance tasks
- D: All decomission tasks per asset $[i_0^D, \ldots, i_N^D]$
- $\mathcal{I} = I \cup M \cup D$: All tasks
- IP: All precedency pairs (i, j). Includes relations stating that the final installation task is finished before maintenance and decomission are started.
- A: All assets

Decision variables:

- o_{at}: Binary variable, 1 if asset a is online at timestep t
- N_{rp}: Number of resources r used in period p
- s_{ait}: Binary variable, 1 if task $i \in \mathcal{I}$ for asset a has started at or before time t

Parameters:

- DIS: The discount factor per time period
- v_t: The value of energy a single turbine produces at timestep t
- C_{rp}: The cost of chartering resource r in period p
- λ_a: The number of timesteps after the last maintenance before asset a fails
- σ_{it}: Indicates the timestep at which task i should have been started for it to be finished by timestep t, generated duration and the weather conditions
- ρ_{ir}: The amount of resource r used for task $i \in \mathcal{I}$
- m_{rp}: The maximum amount of resources r that can be charatered in period p

Initial Corrective model

$$\max_{\substack{N_{rp}\in\mathbb{Z}^* \\ s_{ait},o_{at}\in\{0,1\}}} \sum_{p\in P}[DIS^p(\sum_{t\in T_p}\sum_{a\in A}(o_{at}\cdot v_t) - \sum_{r\in R}(N_{rp}\cdot C_{rp}))] \tag{38}$$

subject to:

$$s_{ait} \leq s_{ai(t+1)} \qquad \forall a\in A, \forall i\in\mathcal{I}, \forall t\in T \tag{39}$$

$$s_{ai\sigma_{itN}} \geq 1 \qquad \forall a\in A, \forall i\in\mathcal{I}-M^C \tag{40}$$

$$s_{ai_0^D t} - 1 \leq s_{ai\sigma_{it}} - s_{ait} \qquad \forall a\in A, \forall i\in\mathcal{I}-D, \forall t\in T \tag{41}$$

$$s_{ajt} \leq s_{ai\sigma_{it}} \qquad \forall a\in A, \forall (i,j)\in IP, \forall t\in T \tag{42}$$

$$m_{rp} \geq N_{rp} \geq \sum_{a\in A}\sum_{i\in\mathcal{I}}(\rho_{ir}\cdot(s_{ait} - s_{ai\sigma_{it}})) \qquad \forall r\in R, \forall p\in P, \forall t\in T_p \tag{43}$$

$$o_{at} \leq s_{ai_N^I \sigma_{i_N^I t}} - s_{ai_0^D t} \qquad \forall a\in A, \forall t\in T \tag{44}$$

$$o_{at} \leq \sum_{i\in M\cup\{i_N^I\}}(s_{ai\sigma_{it}} - s_{ai\sigma_{i(t-\lambda_{ai})}}) \qquad \forall a\in A, \forall t\in T \tag{45}$$

$$s_{ait} - s_{ai(t-1)} \leq 1 - \sum_{j\in M\cup\{i_N^I\}} \frac{(s_{aj\sigma_{jt}} - s_{aj\sigma_{j(t-\lambda_{aj})}})}{L} \qquad \forall a\in A, \forall i\in M^C, \forall t\in T \tag{46}$$

$$o_{at} \leq 1 + s_{ai\sigma_{it}} - s_{ait} \qquad \forall a\in A, \forall i\in M, \forall t\in T \tag{47}$$

Corrective Model Explanation

(38) Objective function, sums up profits from energy made, subtracts money used on resources (vessels), and multiplies it all with a discount factor
(39) Makes every task that is started stay started
(40) Forces every installation and preventive maintenance task to be started and finished by the final timestep
(41) Ensures every non-decommission task is inactive during decomission
(42) For every precedence relation (i, j), ensures that i is finished before j is started
(43) Counts up the resources needed in a time period by adding the resources needed by all tasks started, and subtracting the resources needed by all tasks finished. It also sets a limit on the amount of vessels that can be charatered in a given period
(44) An asset can only be online if installation is complete, and decomission has not started yet
(45) An asset is offline if it has not gotten maintained recently enough
(46) A corrective maintenance task can only be started if the asset is in a failure state (division by L means any number of tasks can have been recent enough for their time to matter)
(47) Ensures an asset if offline is maintenance work is going on at this moment

Notation overview

Sets:

- P: All time periods (large scale)
- T: All time intervals (small scale) $[t_0, \ldots, t_N]$
- $T_p \subset T$: All time intervals (small scale) in period p
- R: All resources
- I: All installation tasks per asset $[i_0^I, \ldots, i_N^I]$
- $M = M^P \cup M^C$: all preventive and corrective maintenance tasks
- D: All decomission tasks per asset $[i_0^D, \ldots, i_N^D]$
- $\mathcal{I} = I \cup M \cup D$: All tasks
- IP: All precedency pairs (i, j). Includes relations stating that the final installation task is finished before maintenance and decomission are started.
- A: All assets

Decision variables:

- o_{at}: Binary variable, 1 if asset a is online at timestep t
- N_{rp}: Number of resources r used in period p
- s_{ait}: Binary variable, 1 if task $i \in \mathcal{I}$ for asset a has started at or before time t

Deterministic parameters:

- DIS: The discount factor per time period
- C_{rp}: The cost of chartering resource r in period p
- ρ_{ir}: The amount of resource r used for task $i \in \mathcal{I}$
- m_{rp}: The maximum amount of resources r that can be charatered in period p
- L: A large number (at least $|M| + 1$)

Stochastic parameters:

- v_t: The value of energy a single turbine produces at timestep t
- λ_{ai}: The number of timesteps after the last maintenance before asset a fails
- σ_{it}: Indicates the timestep at which task i should have been started for it to be finished by timestep t, generated based on duration and the weather conditions (stochastic based on those)

Initial Stochastic model

$$\max_{\substack{N_{rp} \in \mathbb{Z}^* \\ s_{ait}, o_{at\sigma} \in \{0,1\}}} \sum_{\sigma \in S} \sum_{p \in P} [DIS^p(\sum_{t \in T_p} \sum_{a \in A} (o_{at\sigma} \cdot v_{t\sigma}) - \sum_{r \in R} (N_{rp} \cdot C_{rp}))] \tag{48}$$

subject to:

$$s_{ait} \leq s_{ai(t+1)} \qquad \forall a \in A, \forall i \in M, \forall t \in T \tag{49}$$

$$s_{ai\omega_{it_N}} \geq 1 \qquad \forall a \in A, \forall i \in M^P \tag{50}$$

$$m_{rp} \geq N_{rp} \geq \sum_{a \in A} \sum_{i \in \mathcal{I}} (\rho_{ir} \cdot (s_{ait} - s_{ai\omega_{it}})) \qquad \forall r \in R, \forall p \in P, \forall t \in T_p \tag{51}$$

$$o_{at\sigma} \leq \sum_{i \in M} (s_{ai\omega_{it}} - s_{ai\omega_{i(t-\lambda_{ai\sigma})}}) \qquad \forall a \in A, \forall t \in T, \forall \sigma \in S \tag{52}$$

$$s_{ait} - s_{ai(t-1)} \leq 1 - \sum_{j \in M} \frac{(s_{aj\omega_{jt}} - s_{aj\omega_{j(t-\lambda_{aj\sigma})}})}{L} \qquad \forall a \in A, \forall i \in M^C, \forall t \in T, \forall \sigma \in S \tag{53}$$

$$o_{at\sigma} \leq 1 + s_{ai\omega_{it}} - s_{ait} \qquad \forall a \in A, \forall i \in M, \forall t \in T, \forall \sigma \in S \tag{54}$$

Stochastic Model Explanation

(48) Objective function, sums up profits from energy made, subtracts money used on resources (vessels), and multiplies it all with a discount factor
(49) Makes every task that is started stay started
(50) Forces every installation and preventive maintenance task to be started and finished by the final timestep
(51) Counts up the resources needed in a time period by adding the resources needed by all tasks started, and subtracting the resources needed by all tasks finished. It also sets a limit on the amount of vessels that can be charatered in a given period
(52) An asset is offline if it has not gotten maintained recently enough
(53) A corrective maintenance task can only be started if the asset is in a failure state (division by L means any number of tasks can have been recent enough for their time to matter)
(54) Ensures an asset if offline is maintenance work is going on at this moment

Notation overview

Sets:

- P: All time periods (large scale)
- T: All time intervals (small scale) $[t_0, \ldots, t_N]$
- $T_p \subset T$: All time intervals (small scale) in period p
- R: All resources
- $M = M^P \cup M^C$: all preventive and corrective maintenance tasks
- A: All assets
- S: All scenarios

Decision variables:

- $o_{at\sigma}$: Binary variable, 1 if asset $a \in A$ is online at timestep $t \in T$ in scenario $\sigma \in S$
- N_{rp}: Number of resources $r \in R$ used in period $p \in P$
- s_{ait}: Binary variable, 1 if task $i \in M$ for asset $a \in A$ has started at or before time $t \in T$

Deterministic parameters:

- DIS: The discount factor per time period
- C_{rp}: The cost of chartering resource $r \in R$ in period $p \in P$
- ρ_{ir}: The amount of resource $r \in R$ used for task $i \in M$
- m_{rp}: The maximum amount of resources $r \in R$ that can be charatered in period $p \in P$
- L: A large number (at least $|M| + 1$)

Stochastic parameters:

- $v_{t\sigma}$: The value of energy a single turbine produces at timestep $t \in T$ in scenario $\sigma \in S$
- $\lambda_{ai\sigma}$: The number of timesteps after the completion of $i \in M$ until asset $a \in A$ fails in scenario $\sigma \in S$
- ω_{it}: Indicates the timestep at which task $i \in M$ should have been started for it to be finished by timestep $t \in T$, generated based on duration and the weather conditions (stochastic based on those)

Initial Multilevel model - 1

$$\min_{\substack{N_{ym\sigma}, P_m, R_{m\sigma} \in \mathbb{Z}^* \\ \gamma_\sigma \in \mathbb{R}_{\geq 0}}} \begin{array}{c} \frac{1}{|S|} \cdot \sum_{\sigma \in S} (\sum_{m \in M} [\sum_{y \in Y} (N_{ym\sigma} \cdot c_{ym}) + P_m \cdot d_P \cdot e_m^H + \\ \sum_{m'=0}^{m} (f_{m'\sigma} - R_{m'\sigma}) \cdot e_m^M] - u \cdot \gamma_\sigma) \end{array} \tag{55}$$

subject to:

$$l_y \cdot N_{ym\sigma} \geq P_m \cdot d_y^P + R_{m\sigma} \cdot d_y^R + \gamma_\sigma \qquad \forall \sigma \in S, \forall m \in M, \forall y \in Y \tag{56}$$

$$\sum_{m'=0}^{m-1} f_{m'\sigma} \geq \sum_{m'=0}^{m} R_{m'\sigma} \qquad \forall \sigma \in S, \forall m \in M \tag{57}$$

$$\sum_{m \in M} P_m \geq A \tag{58}$$

Initial Multilevel mode - 2

$$\min_{\substack{s_i \in \mathbb{R}_{\geq 0} \\ a_{vij} \in \{0,1\}}} \sum_{i \in I} c_i \cdot (s_i + \max_{y \in Y}(s_{yi} + d_{yi})) \tag{59}$$

subject to:

$$\sum_{i \in I} a_{vij} \leq 1 \qquad \forall v \in V, \forall j \in J \tag{60}$$

$$\sum_{i \in I} a_{vij} \leq \sum_{i \in I} a_{vi(j-1)} \qquad \forall v \in V, \forall j \in J - \{0\} \tag{61}$$

$$\rho_{yi} \leq \sum_{v \in V_y} \sum_{j \in J} a_{vij} \qquad \forall y \in Y, \forall i \in I \tag{62}$$

$$\begin{matrix} M \cdot (a_{vij} + a_{vi'(j-1)}) + \\ d_{yi'} \cdot a_{vi'(j-1)} - 2M \end{matrix} \leq s_i + s_{yi} - s_{i'} - s_{i'y} \qquad \begin{matrix} \forall y \in Y, \forall v \in V_y, \\ \forall i, i' \in I, \forall j \in J - \{0\} \end{matrix} \tag{63}$$

Multilevel Model Explanation

(55) Objective is to minimise costs over all scenarios, summing up resource, downtime and failure costs, and subtracting the value of the mimimum buffer
(56) Ensures the durations of the planned tasks do not exceed the capacity of the assigned vessels
(57) Ensures repairs are only done after failures have occured
(58) Ensures all assets have preventive maintenance done at least once within the timespan

(59) Objective is to minimise costs of tasks being uncompleted (the +max bit is optional as it's about constants)
(60) Ensures every vessel only does one task at a time
(61) Ensures that if a vessel has an xth task it also has an $x-1$th task
(62) Ensures every task has enough resourses assigned to it
(63) Ensures the starting times of consequtive tasks are separated by at least the duration of the first task (the M factors ensure that if the tasks are not consecutive the starting times don't matter)

Notation overview