# Initial models for optimisation

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March 24, 2020

# Initial model for installation

$$\text{maximize} \sum_{p \in P} [DIS^p (A_p \cdot v_p - \sum_{r \in R} N_{rp} \cdot C_{rp})] \qquad (1)$$

subject to:

$$1 = \sum_{t \in T} s_{it} = \sum_{t \in T} f_{it} \qquad \forall i \in I \qquad (2)$$

$$1 \leq \sum_{t=\dot{t}}^{\hat{t}} [\sum_{t'=\dot{t}}^{t} f_{jt'} \cdot \sum_{t'=t}^{\hat{t}} s_{it'}] \qquad \forall (i,j) \in IP \qquad (3)$$

$$d_i \geq (f_{it''} + s_{it'} - 1) \cdot \sum_{t=t'}^{t''} \omega_{it} \qquad \forall i \in I, \forall t'' \geq t' + d_i, t', t'' \in T \qquad (4)$$

$$N_{rp} \geq \sum_{i \in I} \sum_{t'=\dot{t}}^{t} \sum_{t''=t}^{\hat{t}} s_{it'} \cdot f_{it''} \cdot \rho_{ir} \qquad \forall r \in R, \forall p \in P, \forall t \in T_p \qquad (5)$$

$$A_p = \sum_{t=\dot{t}}^{\tau_p} \sum_{i \in F} f_{it} \qquad \forall p \in P \qquad (6)$$

# Notation overview

Sets:

- $P$: All time periods (large scale)
- $T$: All time intervals $[\dot{t}, \dots, \hat{t}]$
- $T_p \in T$: All time intervals (small scale) in period $p$
- $R$: All resources
- $I$: All tasks
- $F \subset I$: All final tasks that complete a turbine
- $IP$: All precedency pairs $(i, j)$

Decision variables:

- $A_p$: Number of active turbines after period $p$
- $N_{rp}$: Number of resources $r$ used in period $p$
- $s_{it}$: Binary variable, 1 if task $i$ starts at time $t$
- $f_{it}$: Binary variable, 1 if task $i$ ends at time $t$

Parameters:

- $DIS$: The discount factor per period
- $v_p$: The value of energy a single turbine produces in period $p$
- $C_{rp}$: The cost of chartering resource $r$ in period $p$
- $d_i$: The duration of task $i$
- $\omega_{it}$: Binary parameter representing weather, 1 if task $i$ can be completed at time $t$, 0 otherwise
- $\rho_{ir}$: The amount of resource $r$ used by task $i$
- $\tau_p$: The final time interval (from $T$) in period $p$

# Initial model for maintenance

$$\text{maximize} \sum_{p \in P} [DIS^p (A_p \cdot v_p - \sum_{r \in R} N_{rp} \cdot C_{rp})] \qquad (7)$$

subject to:

$$s_{ic} = \max(\gamma \cdot c, \max_{j \in IP_i}(f_{jc})) \qquad \forall i \in I \quad \forall c \in C \qquad (8)$$

$$d_{ic} = \sum_{t=s_{ic}}^{f_{ic}} \omega_{it} \qquad \forall i \in I \quad \forall c \in C \qquad (9)$$

$$N_{rp} \geq \sum_{i \mid s_{ic} \leq t \leq f_{ic}, \forall t \in T_p} \rho_{ir} \qquad \forall r \in R \quad \forall p \in P \quad \forall c \in C \qquad (10)$$

$$A_p = A_{p-1} \cdot \lambda + \text{number fully repaired} \qquad \forall p \in P \quad \forall i \in F \qquad (11)$$

# Initial mixed model

Sets:

- $P$: All time periods (large scale)
- $T_p$: All time intervals (small scale) in period $p$
- $C$: All cycles
- $R$: All resources
- $I$: All tasks
- $F \subset I$: All final tasks that complete a turbine
- $IP_i$: All prerequisite tasks of task $i$

Decision variables:

- $\gamma$: The lenth of a cycle
- $A_p$: Number of active turbines after period $p$
- $N_{rp}$: Number of resources $r$ used in period $p$
- $s_{ic}$: Starting time of task $i$ in cycle $c$
- $f_{ic}$: Finishing time of task $i$ in cycle $c$

Parameters:

- $DIS$: The discount factor per period
- $v_p$: The value of energy a single turbine produces in period $p$
- $C_{rp}$: The cost of chartering resource $r$ in period $p$
- $d_i$: The duration of task $i$
- $\omega_{it}$: Binary parameter representing weather, 1 if task $i$ can be completed at time $t$, 0 otherwise
- $\rho_{ir}$: The amount of resource $r$ used by task $i$
- $\tau_p$: The final time interval (from $T$) in period $p$