

Initial models for optimisation

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April 17, 2020

Initial model for installation

$$\text{maximize } \sum_{p \in P} [DIS^p(O_p \cdot v_p - \sum_{r \in R} N_{rp} \cdot C_{rp})] \quad (1)$$

subject to:

$$1 = \sum_{t \in T} s_{it} = \sum_{t \in T} f_{it} \quad \forall i \in I \quad (2)$$

$$1 \leq \sum_{t=\hat{t}}^{\hat{t}} \left[\sum_{t'=\hat{t}}^t f_{it'} \cdot \sum_{t''=\hat{t}}^{\hat{t}} s_{jt''} \right] \quad \forall (i,j) \in IP \quad (3)$$

$$d_i \geq (f_{it''} + s_{it'} - 1) \cdot \sum_{t=t'}^{t''} \omega_{it} \quad \forall i \in I, \forall t'' \geq t' + d_i, t', t'' \in T \quad (4)$$

$$N_{rp} \geq \sum_{i \in I} \sum_{t'=\hat{t}}^t \sum_{t''=\hat{t}}^{\hat{t}} s_{it'} \cdot f_{it''} \cdot \rho_{ir} \quad \forall r \in R, \forall p \in P, \forall t \in T_p \quad (5)$$

$$O_p = \sum_{t=\hat{t}}^{\tau_p} \sum_{i \in F} f_{it} \quad \forall p \in P \quad (6)$$

Installation Model Explanation

- (1) Objective function, sums up profits from energy made, subtracts money used on resources (vessels), and multiplies it all with a discount factor
- (2) Forces every task to be started and finished at some point
- (3) For every precedence relation (i, j) it ensures there is a t such that i has a finish time before t , and j a starting time after t
- (4) Ensures that between the starting and finish times of each task are enough timesteps with acceptable weather
- (5) Counts up the resources needed in a time period summing up over all active tasks (an s before and f after t)
- (6) Counts the number of turbines which finished installing by the end of a period

Notation overview

Sets:

- P : All time periods (large scale)
- T : All time intervals $[\hat{t}, \dots, \hat{t}]$
- $T_p \in T$: All time intervals (small scale) in period p
- R : All resources
- I : All tasks
- $F \subset I$: All final tasks that complete a turbine
- IP : All precedence pairs (i, j)

Decision variables:

- O_p : Number of online turbines after period p
- N_{rp} : Number of resources r used in period p
- s_{it} : Binary variable, 1 if task i starts at time t
- f_{it} : Binary variable, 1 if task i ends at time t

Parameters:

- DIS : The discount factor per period
- v_p : The value of energy a single turbine produces in period p
- C_{rp} : The cost of chartering resource r in period p
- d_i : The duration of task i
- ω_{it} : Binary parameter representing weather, 1 if task i can be completed at time t , 0 otherwise
- ρ_{ir} : The amount of resource r used by task i
- τ_p : The final time interval (from T) in period p

Initial model for maintenance

$$\text{maximize } \sum_{p \in P} [DIS^p(\sum_{t \in T_p} (O_t \cdot v_t) - \sum_{r \in R} (N_{rp} \cdot C_{rp}))] \quad (7)$$

subject to (1):

$$1 = \sum_{t \in T} s_{act} = \sum_{t \in T} f_{act} \quad \forall a \in A, \forall c \in C^M \quad (8)$$

$$1 \geq \sum_{t \in T} s_{act} \quad \forall a \in A, \forall c \in C^O \quad (9)$$

$$\sum_{t \in T} s_{act} = \sum_{t \in T} f_{act} \quad \forall a \in A, \forall c \in C^O \quad (10)$$

Initial model for maintenance

$$\text{maximize } \sum_{p \in P} [DIS^p(\sum_{t \in T_p} (O_t \cdot v_t) - \sum_{r \in R} (N_{rp} \cdot C_{rp}))] \quad (7)$$

subject to (2):

$$d_a \geq (f_{act''} + s_{act'} - 1) \cdot \sum_{t=t'}^{t''} \omega_{it} \quad \forall a \in A, \forall c \in C, \quad (11)$$

$$\forall t'' \geq t' + d_a, t', t'' \in T$$

$$N_{rp} \geq \sum_{a \in A} \sum_{c \in C} \sum_{t'=i}^t \sum_{t''=t}^{\hat{t}} s_{act'} \cdot f_{act''} \cdot \rho_{ar} \quad \forall r \in R, \forall p \in P, \forall t \in T_p \quad (12)$$

$$b_{at} > \sum_{c \in C} \sum_{t'=t-\lambda_a}^t -f_{act'} \quad \forall a \in A, \forall t \in T \quad (13)$$

$$O_t = |A| - \sum_{a \in A} b_{at} \quad \forall t \in T \quad (14)$$

Maintenance Model Explanation

- (7) Objective function, sums up profits from energy made, subtracts money used on resources (vessels), and multiplies it all with a discount factor
- (8) Forces every mandatory maintenance cycle to be done at some point
- (9) Ensures each optional maintenance cycle to be started at most once
- (10) Ensures that every maintenance cycle for a particular asset that is started is also finished
- (11) Ensures that between the starting and finish times of each cycle are enough timesteps with acceptable weather
- (12) Counts up the resources needed in a time period summing up over all active tasks (an s before and f after t)
- (13) If no maintenance tasks have finished in the past λ_a timesteps this asset is broken
- (14) The number of active (online) turbines is equal to everything that isn't broken

Notation overview

Sets:

- P : All time periods (large scale)
- T : All time intervals (small scale) $[\hat{t}, \dots, \hat{t}]$
- $T_p \in T$: All time intervals (small scale) in period p
- R : All resources
- A : All assets
- $C = C^M \cup C^O$: All (mandatory and optional) maintenance cycles

Decision variables:

- O_t : Number of active turbines at timestep t
- N_{rp} : Number of resources r used in period p
- s_{act} : Binary variable, 1 if maintenance cycle c for asset a starts at time t
- f_{act} : Binary variable, 1 if maintenance cycle c for asset a finishes at time t
- b_{at} : Binary variable, 1 if asset a is broken at timestep t

Parameters:

- DIS : The discount factor per time period
- v_t : The value of energy a single turbine produces at timestep t
- C_{rp} : The cost of chartering resource r in period p
- d_a : The duration of a maintenance cycle for asset a
- λ_a : The number of timesteps after the last maintenance before asset a fails
- ω_{it} : Binary parameter representing weather, 1 if task i can be completed at time t , 0 otherwise
- ρ_{ar} : The amount of resource r used for maintenance of asset a

Initial mixed model

$$\text{maximize } \sum_{p \in P} [DIS^p(\sum_{t \in T_p} (O_t \cdot v_t) - \sum_{r \in R} (N_{rp} \cdot C_{rp}))] \quad (15)$$

subject to:

$$1 = \sum_{t \in T} s_{ait}^I = \sum_{t \in T} f_{ait}^I \quad \forall i \in I, \forall a \in A \quad (16)$$

$$1 \leq \sum_{t=i}^{\hat{t}} [\sum_{t'=i}^t f_{ait'}^I \cdot \sum_{t'=t}^{\hat{t}} s_{ajt'}^I] \quad \forall (i,j) \in IP, \forall a \in A \quad (17)$$

$$1 = \sum_{t \in T} s_{act}^M = \sum_{t \in T} f_{act}^M \quad \forall a \in A, \forall c \in \{1, \dots, c_M\} \quad (18)$$

$$1 \geq \sum_{t \in T} s_{act}^M \quad \forall a \in A, \forall c \in \{c_M + 1, \dots, c_N\} \quad (19)$$

$$\sum_{t \in T} s_{act}^M = \sum_{t \in T} f_{act}^M \quad \forall a \in A, \forall c \in \{c_M + 1, \dots, c_N\} \quad (20)$$

$$0 = \sum_{t=i}^{t'} s_{act}^M \cdot \sum_{t=t'}^{\hat{t}} f_{ait}^I \quad \begin{matrix} \forall a \in A, \forall t' \in T, \\ \forall i \in I, \forall c \in C \end{matrix} \quad (21)$$

Initial mixed model

$$\text{maximize } \sum_{p \in P} [DIS^p(\sum_{t \in T_p} (O_t \cdot v_t) - \sum_{r \in R} (N_{rp} \cdot C_{rp}))] \quad (15)$$

subject to (2):

$$d_i \geq (f_{ait''}^I + s_{ait'}^I - 1) \cdot \sum_{t=t'}^{t''} \omega_{ait}^I \quad \forall i \in I, \forall a \in A, \quad (22)$$

$$\forall t'' \geq t' + d_i, t', t'' \in T$$

$$d_a \geq (f_{act''}^M + s_{act'}^M - 1) \cdot \sum_{t=t'}^{t''} \omega_{act}^M \quad \forall a \in A, \forall c \in C, \quad (23)$$

$$\forall t'' \geq t' + d_a, t', t'' \in T$$

$$N_{rp} \geq \sum_{a \in A} \sum_{t'=t}^t \sum_{t''=t}^{\hat{t}} [\sum_{i \in I} (s_{ait'}^I \cdot f_{ait''}^I \cdot \rho_{air}^I) + \sum_{c \in C} (s_{act'}^M \cdot f_{act''}^M \cdot \rho_{ar}^M)] \quad \forall r \in R, \forall p \in P, \forall t \in T_p \quad (24)$$

$$o_{at} \leq \sum_{t'=t}^t f_{ana't'}^I \cdot \sum_{c \in C} \sum_{t'=t-\lambda_a}^t (f_{act'}^M + f_{ana't'}^I) \quad \forall a \in A, \forall t \in T \quad (25)$$

$$O_t = \sum_{a \in A} o_{at} \quad \forall t \in T \quad (26)$$

Mixed Model Explanation

- (15) Objective function, sums up profits from energy made, subtracts money used on resources (vessels), and multiplies it all with a discount factor
- (16) (Installation) Forces every task to be started and finished at some point
- (17) (Installation) For every precedence relation (i, j) it ensures there is a t such that i has a finish time before t , and j a starting time after t
- (18) (Maintenance) Forces every mandatory maintenance cycle to be started and finished at some point
- (19) (Maintenance) Ensures each optional maintenance cycle to be started at most once
- (20) (Maintenance) Ensures that every maintenance cycle for a particular asset that is started is also finished
- (21) (Mixed) Ensures an asset is fully installed before maintenance starts
- (22) (Installation) Ensures that between the starting and finish times of each task are enough timesteps with acceptable weather
- (23) (Maintenance) Ensures that between the starting and finish times of each cycle are enough timesteps with acceptable weather
- (24) (Mixed) Counts up the resources needed in a time period summing up over all active tasks (an s before and f after t)
- (25) (Mixed) Sets an asset to be online if it installed and had work done on it recently
- (26) (Mixed) Counts how many assets are online

Notation overview

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- P : All time periods (large scale)
- T : All time intervals (small scale) $[\hat{t}, \dots, \hat{t}]$
- $T_p \in T$: All time intervals (small scale) in period p
- R : All resources
- I : All installation tasks per asset $[1, \dots, n_a]$
- A : All assets
- C : All (c_M mandatory and ($c_N - c_M$) optional) maintenance cycles $[1, \dots, c_M, \dots, c_N]$

Decision variables:

- O_t : Number of online turbines at timestep t
- o_{at} : Binary variable, 1 if asset a is online at timestep t
- N_{rp} : Number of resources r used in period p
- s_{ait}^I : Binary variable, 1 if installation task i for asset a starts at time t
- f_{ait}^I : Binary variable, 1 if installation task i for asset a finishes at time t
- s_{act}^M : Binary variable, 1 if maintenance cycle c for asset a starts at time t
- f_{act}^M : Binary variable, 1 if maintenance cycle c for asset a finishes at time t

Parameters:

- DIS : The discount factor per time period
- v_t : The value of energy a single turbine produces at timestep t
- C_{rp} : The cost of chartering resource r in period p
- d_i : The duration of installation task i
- d_a : The duration of a maintenance cycle for asset a
- λ_a : The number of timesteps after the last maintenance before asset a fails
- ω_{ait}^I : Binary parameter representing weather, 1 if task i on asset a can be completed at time t , 0 otherwise
- ω_{act}^M : Binary parameter representing weather, 1 if maintenance cycle c on asset a can be completed at time t , 0 otherwise
- ρ_{air}^I : The amount of resource r used for installation task i of asset a
- ρ_{ar}^M : The amount of resource r used for maintenance of asset a