

# Initial models for optimisation

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# Initial model for installation

$$\max_{\substack{O_p, N_{rp} \in \mathbb{Z}^* \\ s_{ait}, f_{ait} \in \{0,1\}}} \sum_{p \in P} [DIS^p(O_p \cdot v_p - \sum_{r \in R} N_{rp} \cdot C_{rp})] \quad (1)$$

subject to:

$$1 = \sum_{t \in T} s_{ait} = \sum_{t \in T} f_{ait} \quad \forall a \in A, \forall i \in I \quad (2)$$

$$1 \geq s_{ajt_1} + f_{ait_2} \quad \forall a \in A, \forall (i,j) \in IP, \quad (3)$$

$$(s_{ait_1} + f_{ait_2} - 1) \cdot d_i \leq \frac{s_{ait_1} + f_{ait_2}}{2} \cdot \sum_{t_3=t_1}^{t_2} \omega_{it_3} \quad \forall a \in A, \forall i \in I, \quad (4)$$

$$n_{rt} = n_{r(t-1)} + \sum_{a \in A} \sum_{i \in I} (\rho_{ir} \cdot (s_{ait} - f_{ai(t-1)})) \quad \forall r \in R, \forall t \in T \quad (5)$$

$$O_p = \sum_{t=t_0}^{t_p} \sum_{a \in A} f_{ai_N t} \quad \forall p \in P \quad (6)$$

$$n_{rt} \leq N_{rp} \leq m_{rp} \quad \forall r \in R, \forall p \in P, \forall t \in T_p \quad (7)$$

# Installation Model Explanation

- (1) Objective function, sums up profits from energy made, subtracts money used on resources (vessels), and multiplies it all with a discount factor
- (2) Forces every task to be started and finished at some point
- (3) For every precedence relation  $(i, j)$  it ensures that if task  $i$  finishes at time  $t_2$  there is no  $t_1 \leq t_2$  at which task  $j$  starts
- (4) Ensures that between the starting and finish times of each task are enough timesteps with acceptable weather
- (5) Counts up the resources needed in a time period by adding the resources needed by all tasks started, and subtracting the resources needed by all tasks finished
- (6) Counts the number of turbines which finished installing by the end of a period
- (7) Sets a limit on the amount of vessels that can be charatered in a given period

# Notation overview

## Sets:

- $P$ : All time periods (large scale)
- $T$ : All time intervals (small scale)  
 $[t_0, \dots, t_N]$
- $T_p \in T$ : All time intervals (small scale)  
in period  $p$
- $R$ : All resources
- $I$ : All tasks per asset  $[1, \dots, i_N]$
- $IP$ : All precedence pairs  $(i, j)$
- $A$ : All assets

## Decision variables:

- $O_p$ : Number of online turbines after  
period  $p$
- $N_{rp}$ : Number of resources  $r$  used in  
period  $p$
- $s_{ait}$ : Binary variable, 1 if task  $i \in I$  for  
asset  $a$  starts at time  $t$
- $f_{ait}$ : Binary variable, 1 if task  $i \in I$  for  
asset  $a$  finishes at time  $t$

## Parameters:

- $DIS$ : The discount factor per period
- $v_p$ : The value of energy a single turbine  
produces in period  $p$
- $C_{rp}$ : The cost of chartering resource  $r$   
in period  $p$
- $d_i$ : The duration of task  $i$
- $\omega_{it}$ : Binary parameter representing  
weather, 1 if task  $i$  can be completed at  
time  $t$ , 0 otherwise
- $\rho_{ir}$ : The amount of resource  $r$  used by  
task  $i$
- $t_p$ : The final time interval (from  $T$ )  
before period  $p$
- $m_{rp}$ : The maximum amount of  
resources  $r$  that can be charatered in  
period  $p$

# Initial model for maintenance

$$\max_{\substack{O_t, N_{rp} \in \mathbb{Z}^* \\ s_{act}, f_{act}, b_{at} \in \{0,1\}}} \sum_{p \in P} [DIS^p(\sum_{t \in T_p} (O_t \cdot v_t) - \sum_{r \in R} (N_{rp} \cdot C_{rp}))] \quad (8)$$

subject to (1):

$$1 = \sum_{t \in T} s_{act} = \sum_{t \in T} f_{act} \quad \forall a \in A, \forall c \in C^M \quad (9)$$

$$1 \geq \sum_{t \in T} s_{act} \quad \forall a \in A, \forall c \in C^O \quad (10)$$

$$\sum_{t \in T} s_{act} = \sum_{t \in T} f_{act} \quad \forall a \in A, \forall c \in C^O \quad (11)$$

# Initial model for maintenance

$$\max_{\substack{O_t, N_{rp} \in \mathbb{Z}^* \\ s_{act}, f_{act}, b_{at} \in \{0,1\}}} \sum_{p \in P} [DIS^p(\sum_{t \in T_p} (O_t \cdot v_t) - \sum_{r \in R} (N_{rp} \cdot C_{rp}))] \quad (8)$$

subject to (2):

$$(s_{act_1} + f_{act_2} - 1) \cdot d_c \leq \frac{s_{act_1} + f_{act_2}}{2} \cdot \sum_{t_3=t_1}^{t_2} \omega_{ct_3} \quad \forall a \in A, \forall c \in C, \quad (12)$$

$$\forall t_1, t_2 \in T$$

$$N_{rp} \geq \sum_{a \in A} \sum_{c \in C} \sum_{t_1=t_0}^t (\rho_{ir} \cdot (s_{act_1} - f_{ac(t_1-1)})) \quad \forall r \in R, \forall p \in P, \quad (13)$$

$$\forall t \in T_p$$

$$b_{at} > \sum_{c \in C} \sum_{t_1=t-\lambda_a}^t -f_{act_1} \quad \forall a \in A, \forall t \in T \quad (14)$$

$$O_t = |A| - \sum_{a \in A} b_{at} \quad \forall t \in T \quad (15)$$

$$N_{rp} \leq m_{rp} \quad \forall r \in R, \forall p \in P \quad (16)$$

# Maintenance Model Explanation

- (8) Objective function, sums up profits from energy made, subtracts money used on resources (vessels), and multiplies it all with a discount factor
- (9) Forces every mandatory maintenance cycle to be done at some point
- (10) Ensures each optional maintenance cycle to be started at most once
- (11) Ensures that every maintenance cycle for a particular asset that is started is also finished
- (12) Ensures that between the starting and finish times of each cycle are enough timesteps with acceptable weather
- (13) Counts up the resources needed in a time period by adding the resources needed by all tasks started, and subtracting the resources needed by all tasks finished
- (14) If no maintenance tasks have finished in the past  $\lambda_a$  timesteps this asset is broken
- (15) The number of active (online) turbines is equal to everything that isn't broken
- (16) Sets a limit on the amount of vessels that can be charatered in a given period

# Notation overview

## Sets:

- $P$ : All time periods (large scale)
- $T$ : All time intervals (small scale)  $[t_0, \dots, t_N]$
- $T_p \in T$ : All time intervals (small scale) in period  $p$
- $R$ : All resources
- $A$ : All assets
- $C = C^M \cup C^O$ : All (mandatory and optional) maintenance cycles

## Decision variables:

- $O_t$ : Number of active turbines at timestep  $t$
- $N_{rp}$ : Number of resources  $r$  used in period  $p$
- $s_{act}$ : Binary variable, 1 if maintenance cycle  $c$  for asset  $a$  starts at time  $t$
- $f_{act}$ : Binary variable, 1 if maintenance cycle  $c$  for asset  $a$  finishes at time  $t$
- $b_{at}$ : Binary variable, 1 if asset  $a$  is broken at timestep  $t$

## Parameters:

- $DIS$ : The discount factor per time period
- $v_t$ : The value of energy a single turbine produces at timestep  $t$
- $C_{rp}$ : The cost of chartering resource  $r$  in period  $p$
- $d_c$ : The duration per task during maintenance cycle  $c$
- $\lambda_a$ : The number of timesteps after the last maintenance before asset  $a$  fails
- $\omega_{ct}$ : Binary parameter representing weather, 1 if maintenance cycle  $c$  can be completed at time  $t$ , 0 otherwise
- $\rho_{cr}$ : The amount of resource  $r$  used per task for maintenance cycle  $c$
- $m_{rp}$ : The maximum amount of resources  $r$  that can be chartered in period  $p$



# Initial mixed model

$$\max_{\substack{O_t, N_{rp} \in \mathbb{Z}^* \\ s_{ait}, f_{ait}, o_{at} \in \{0,1\}}} \sum_{p \in P} [DIS^p(\sum_{t \in T_p} (O_t \cdot v_t) - \sum_{r \in R} (N_{rp} \cdot C_{rp}))] \quad (17)$$

subject to:

$$1 = \sum_{t \in T} s_{ait} = \sum_{t \in T} f_{ait} \quad \forall i \in I \cup M^M, \forall a \in A \quad (18)$$

$$1 \geq s_{ajt_1} + f_{ait_2} \quad \forall (i, j) \in IP, \forall a \in A, \quad (19)$$

$$1 \geq \sum_{t \in T} s_{ait} \quad \forall a \in A, \forall i \in M^O \quad (20)$$

$$\sum_{t \in T} s_{ait} = \sum_{t \in T} f_{ait} \quad \forall a \in A, \forall i \in M^O \quad (21)$$

$$0 = \sum_{t_1=t_0}^t s_{ajt_1} + \sum_{t_2=t}^{t_N} f_{ait_2} \quad \forall a \in A, \forall t \in T, \quad (22)$$

# Initial mixed model

$$\max_{\substack{O_t, N_{rp} \in \mathbb{Z}^* \\ s_{ait}, f_{ait}, o_{at} \in \{0,1\}}} \sum_{p \in P} [DIS^p(\sum_{t \in T_p} (O_t \cdot v_t) - \sum_{r \in R} (N_{rp} \cdot C_{rp}))] \quad (17)$$

subject to (2):

$$(s_{ait_1} + f_{ait_2} - 1) \cdot d_i \leq \frac{s_{ait_1} + f_{ait_2}}{2} \cdot \sum_{t_3=t_1}^{t_2} \omega_{it_3} \quad \forall i \in \mathcal{I}, \forall a \in A, \forall t_1, t_2 \in T \quad (23)$$

$$N_{rp} \geq \sum_{a \in A} \sum_{i \in \mathcal{I}} \sum_{t_1=t_0}^t (\rho_{ir} \cdot (s_{ait_1} - f_{ai(t_1-1)})) \quad \forall r \in R, \forall p \in P, \forall t \in T_p \quad (24)$$

$$o_{at} \leq \frac{1}{2} \cdot \left( \sum_{t_1=t_0}^t f_{ai_N t_1} + \sum_{i \in M \cup \{i_N\}} \sum_{t_2=t-\lambda_a}^t f_{ait_2} \right) \quad \forall a \in A, \forall t \in T \quad (25)$$

$$O_t = \sum_{a \in A} o_{at} \quad \forall t \in T \quad (26)$$

$$N_{rp} \leq m_{rp} \quad \forall r \in R, \forall p \in P \quad (27)$$

# Mixed Model Explanation

- (17) Objective function, sums up profits from energy made, subtracts money used on resources (vessels), and multiplies it all with a discount factor
- (18) Forces every mandatory task to be started and finished at some point
- (19) For every precedence relation  $(i, j)$  it ensures that if task  $i$  finishes at time  $t_2$  there is no  $t_1 \leq t_2$  at which task  $j$  starts
- (20) Ensures each optional maintenance task to be started at most once
- (21) Ensures that every maintenance task for a particular asset that is started is also finished
- (22) Ensures an asset is fully installed before maintenance starts
- (23) Ensures that between the starting and finish times of each task are enough timesteps with acceptable weather
- (24) Counts up the resources needed in a time period by adding the resources needed by all tasks started, and subtracting the resources needed by all tasks finished
- (25) Sets an asset to be online if it installed and had work done on it recently
- (26) Counts how many assets are online
- (27) Sets a limit on the amount of vessels that can be chartered in a given period

# Notation overview

## Sets:

- $P$ : All time periods (large scale)
- $T$ : All time intervals (small scale)  $[t_0, \dots, t_N]$
- $T_p \in T$ : All time intervals (small scale) in period  $p$
- $R$ : All resources
- $I$ : All installation tasks per asset  $[1, \dots, i_N]$
- $M = M^M \cup M^O$ : all mandatory and optional maintenance tasks
- $\mathcal{I} = I \cup M$ : All tasks
- $IP$ : All precedence pairs  $(i, j)$
- $A$ : All assets

## Decision variables:

- $O_t$ : Number of online turbines at timestep  $t$
- $o_{at}$ : Binary variable, 1 if asset  $a$  is online at timestep  $t$
- $N_{rp}$ : Number of resources  $r$  used in period  $p$
- $s_{ait}$ : Binary variable, 1 if task  $i \in \mathcal{I}$  for asset  $a$  starts at time  $t$
- $f_{ait}$ : Binary variable, 1 if task  $i \in \mathcal{I}$  for asset  $a$  finishes at time  $t$

## Parameters:

- $DIS$ : The discount factor per time period
- $v_t$ : The value of energy a single turbine produces at timestep  $t$
- $C_{rp}$ : The cost of chartering resource  $r$  in period  $p$
- $d_i$ : The duration of task  $i \in \mathcal{I}$
- $\lambda_a$ : The number of timesteps after the last maintenance before asset  $a$  fails
- $\omega_{it}$ : Binary parameter representing weather, 1 if task  $i \in \mathcal{I}$  can be completed at time  $t$ , 0 otherwise
- $\rho_{ir}$ : The amount of resource  $r$  used for task  $i \in \mathcal{I}$
- $m_{rp}$ : The maximum amount of resources  $r$  that can be chartered in period  $p$