

Initial models for optimisation

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Initial model for installation

$$\max_{\substack{O_p, N_{rp} \in \mathbb{Z}^* \\ s_{ait} \in \{0,1\}}} \sum_{p \in P} [DIS^p(O_p \cdot v_p - \sum_{r \in R} N_{rp} \cdot C_{rp})] \quad (1)$$

subject to:

$$s_{ait} \leq s_{ai(t+1)} \quad \forall a \in A, \forall i \in I, \forall t \in T \quad (2)$$

$$s_{ai\sigma_{it_N}} \geq 1 \quad \forall a \in A, \forall i \in I \quad (3)$$

$$s_{ajt} \leq s_{ai\sigma_{it}} \quad \forall a \in A, \forall (i,j) \in IP, \forall t \in T \quad (4)$$

$$N_{rp} \geq \sum_{a \in A} \sum_{i \in I} (\rho_{ir} \cdot (s_{ait} - s_{ai\sigma_{it}})) \quad \forall r \in R, \forall p \in P, \forall t \in T_p \quad (5)$$

$$O_p = \sum_{a \in A} s_{ai_N \sigma_{i_N t_p}} \quad \forall p \in P \quad (6)$$

$$N_{rp} \leq m_{rp} \quad \forall r \in R, \forall p \in P \quad (7)$$

Installation Model Explanation

- (1) Objective function, sums up profits from energy made, subtracts money used on resources (vessels), and multiplies it all with a discount factor
- (2) Makes every task that is started stay started
- (3) Forces every task to be started and finished by the final timestep
- (4) For every precedence relation (i, j) , ensures that i is finished before j is started
- (5) Counts up the resources needed in a time period by adding the resources needed by all tasks started, and subtracting the resources needed by all tasks finished
- (6) Counts the number of turbines which finished installing by the end of a period
- (7) Sets a limit on the amount of vessels that can be charatered in a given period

Notation overview

Sets:

- P : All time periods (large scale)
- T : All time intervals (small scale)
 $[t_0, \dots, t_N]$
- $T_p \subset T$: All time intervals (small scale)
in period p
- R : All resources
- I : All tasks per asset $[1, \dots, i_N]$
- IP : All precedence pairs (i, j)
- A : All assets

Decision variables:

- O_p : Number of online turbines after
period p
- N_{rp} : Number of resources r used in
period p
- s_{ait} : Binary variable, 1 if task $i \in I$ for
asset a has started at or before time t

Parameters:

- DIS : The discount factor per period
- v_p : The value of energy a single turbine
produces in period p
- C_{rp} : The cost of chartering resource r
in period p
- σ_{it} : Indicates the timestep at which
task i should have been started for it to
be finished by timestep t , taking into
account the duration and the weather
conditions
- ρ_{ir} : The amount of resource r used by
task i
- t_p : The final time interval (from T)
before period p
- m_{rp} : The maximum amount of
resources r that can be chartered in
period p

Initial model for maintenance

$$\max_{\substack{O_t, N_{rp} \in \mathbb{Z}^* \\ s_{ait}, b_{at} \in \{0,1\}}} \sum_{p \in P} [DIS^p(\sum_{t \in T_p} (O_t \cdot v_t) - \sum_{r \in R} (N_{rp} \cdot C_{rp}))] \quad (8)$$

subject to:

$$s_{ait} \leq s_{ai(t+1)} \quad \forall a \in A, \forall i \in M, \forall t \in T \quad (9)$$

$$s_{ai\sigma_{itN}} \geq 1 \quad \forall a \in A, \forall i \in M^M \quad (10)$$

$$N_{rp} \geq \sum_{a \in A} \sum_{i \in M} (\rho_{ir} \cdot (s_{ait} - s_{ai\sigma_{it}})) \quad \forall r \in R, \forall p \in P, \forall t \in T_p \quad (11)$$

$$b_{at} > \sum_{i \in M} [s_{ai\sigma_{i(t-\lambda_a)}} - s_{ai\sigma_{it}}] \quad \forall a \in A, \forall t \in T \quad (12)$$

$$O_t = |A| - \sum_{a \in A} b_{at} \quad \forall t \in T \quad (13)$$

$$N_{rp} \leq m_{rp} \quad \forall r \in R, \forall p \in P \quad (14)$$

Maintenance Model Explanation

- (8) Objective function, sums up profits from energy made, subtracts money used on resources (vessels), and multiplies it all with a discount factor
- (9) Makes every task that is finished stay finished
- (10) Forces every mandatory maintenance task to be done at some point
- (11) Counts up the resources needed in a time period by adding the resources needed by all tasks started, and subtracting the resources needed by all tasks finished
- (12) If no maintenance tasks have finished in the past λ_a timesteps this asset is broken
- (13) The number of active (online) turbines is equal to everything that isn't broken
- (14) Sets a limit on the amount of vessels that can be charatered in a given period

Notation overview

Sets:

- P : All time periods (large scale)
- T : All time intervals (small scale)
 $[t_0, \dots, t_N]$
- $T_p \subset T$: All time intervals (small scale)
in period p
- R : All resources
- A : All assets
- $M = M^M \cup M^O$: All (mandatory and optional) maintenance tasks

Decision variables:

- O_t : Number of active turbines at timestep t
- N_{rp} : Number of resources r used in period p
- s_{ait} : Binary variable, 1 if maintenance task i for asset a has started at or before time t
- b_{at} : Binary variable, 1 if asset a is broken at timestep t

Parameters:

- DIS : The discount factor per time period
- v_t : The value of energy a single turbine produces at timestep t
- C_{rp} : The cost of chartering resource r in period p
- λ_a : The number of timesteps after the last maintenance before asset a fails
- σ_{it} : Indicates the timestep at which task i should have been started for it to be finished by timestep t , taking into account the duration and the weather conditions
- ρ_{ir} : The amount of resource r used per task for maintenance task i
- m_{rp} : The maximum amount of resources r that can be chartered in period p

Initial 2-mixed model

$$\max_{\substack{N_{rp} \in \mathbb{Z}^* \\ s_{ait}, o_{at} \in \{0,1\}}} \sum_{p \in P} [DIS^p(\sum_{t \in T_p} \sum_{a \in A} (o_{at} \cdot v_t) - \sum_{r \in R} (N_{rp} \cdot C_{rp}))] \quad (15)$$

subject to:

$$s_{ait} \leq s_{ai(t+1)} \quad \forall a \in A, \forall i \in \mathcal{I}, \forall t \in T \quad (16)$$

$$s_{ai\sigma_{it_N}} \geq 1 \quad \forall a \in A, \forall i \in I \cup M^M \quad (17)$$

$$s_{ajt} \leq s_{ai\sigma_{it}} \quad \forall a \in A, \forall (i,j) \in IP, \forall t \in T \quad (18)$$

$$N_{rp} \geq \sum_{a \in A} \sum_{i \in \mathcal{I}} (\rho_{ir} \cdot (s_{ait} - s_{ai\sigma_{it}})) \quad \forall r \in R, \forall p \in P, \forall t \in T_p \quad (19)$$

$$o_{at} \leq s_{ai_N \sigma_{i_N t}} \quad \forall a \in A, \forall t \in T \quad (20)$$

$$o_{at} \leq \sum_{i \in M \cup \{i_N\}} [s_{ai\sigma_{it}} - s_{ai\sigma_{i(t-\lambda_a)}}] \quad \forall a \in A, \forall t \in T \quad (21)$$

$$N_{rp} \leq m_{rp} \quad \forall r \in R, \forall p \in P \quad (22)$$

2-Mixed Model Explanation

- (15) Objective function, sums up profits from energy made, subtracts money used on resources (vessels), and multiplies it all with a discount factor
- (16) Makes every task that is started stay started
- (17) Forces every installation and mandatory maintenance task to be started and finished by the final timestep
- (18) For every precedence relation (i, j) , ensures that i is finished before j is started
- (19) Counts up the resources needed in a time period by adding the resources needed by all tasks started, and subtracting the resources needed by all tasks finished
- (20) An asset can only be online if it finished installation
- (21) An asset can only be online if it had work done on it recently enough
- (22) Sets a limit on the amount of vessels that can be charatered in a given period

Notation overview

Sets:

- P : All time periods (large scale)
- T : All time intervals (small scale) $[t_0, \dots, t_N]$
- $T_p \subset T$: All time intervals (small scale) in period p
- R : All resources
- I : All installation tasks per asset $[1, \dots, i_N]$
- $M = M^M \cup M^O$: all mandatory and optional maintenance tasks
- $\mathcal{I} = I \cup M$: All tasks
- IP : All precedence pairs (i, j)
- A : All assets

Decision variables:

- o_{at} : Binary variable, 1 if asset a is online at timestep t
- N_{rp} : Number of resources r used in period p
- s_{ait} : Binary variable, 1 if task $i \in \mathcal{I}$ for asset a has started at or before time t

Parameters:

- DIS : The discount factor per time period
- v_t : The value of energy a single turbine produces at timestep t
- C_{rp} : The cost of chartering resource r in period p
- λ_a : The number of timesteps after the last maintenance before asset a fails
- σ_{it} : Indicates the timestep at which task i should have been started for it to be finished by timestep t , taking into account the duration and the weather conditions
- ρ_{ir} : The amount of resource r used for task $i \in \mathcal{I}$
- m_{rp} : The maximum amount of resources r that can be chartered in period p

Initial model for decommission

$$\max_{\substack{N_{rp} \in \mathbb{Z}^* \\ s_{ait} \in \{0,1\}}} \sum_{p \in P} [DIS^p(\sum_{t \in T_p} \sum_{a \in A} ((1 - s_{ai_0 t}) \cdot v_t) - \sum_{r \in R} (N_{rp} \cdot C_{rp}))] \quad (23)$$

subject to:

$$s_{ait} \leq s_{ai(t+1)} \quad \forall a \in A, \forall i \in D, \forall t \in T \quad (24)$$

$$s_{ai\sigma_{it_N}} \geq 1 \quad \forall a \in A, \forall i \in D \quad (25)$$

$$s_{ajt} \leq s_{ai\sigma_{it}} \quad \forall a \in A, \forall (i,j) \in IP, \forall t \in T \quad (26)$$

$$N_{rp} \geq \sum_{a \in A} \sum_{i \in I} (\rho_{ir} \cdot (s_{ait} - s_{ai\sigma_{it}})) \quad \forall r \in R, \forall p \in P, \forall t \in T_p \quad (27)$$

$$N_{rp} \leq m_{rp} \quad \forall r \in R, \forall p \in P \quad (28)$$

Installation Model Explanation

- (23) Objective function, sums up profits from energy made, subtracts money used on resources (vessels), and multiplies it all with a discount factor
- (24) Makes every task that is started stay started
- (25) Forces every task to be started and finished by the final timestep
- (26) For every precedence relation (i, j) , ensures that i is finished before j is started
- (27) Counts up the resources needed in a time period by adding the resources needed by all tasks started, and subtracting the resources needed by all tasks finished
- (28) Sets a limit on the amount of vessels that can be charatered in a given period

Notation overview

Sets:

- P : All time periods (large scale)
- T : All time intervals (small scale)
 $[t_0, \dots, t_N]$
- $T_p \subset T$: All time intervals (small scale)
in period p
- R : All resources
- D : All tasks per asset $[i_0, \dots, i_N]$
- IP : All precedence pairs (i, j)
- A : All assets

Decision variables:

- N_{rp} : Number of resources r used in
period p
- s_{ait} : Binary variable, 1 if task $i \in I$ for
asset a has started at or before time t

Parameters:

- DIS : The discount factor per period
- v_t : The value of energy a single turbine
produces in timestep t
- C_{rp} : The cost of chartering resource r
in period p
- σ_{it} : Indicates the timestep at which
task i should have been started for it to
be finished by timestep t , taking into
account the duration and the weather
conditions
- ρ_{ir} : The amount of resource r used by
task i
- t_p : The final time interval (from T)
before period p
- m_{rp} : The maximum amount of
resources r that can be charatered in
period p

Initial 3-mixed model

$$\max_{\substack{N_{rp} \in \mathbb{Z}^* \\ s_{ait}, o_{at} \in \{0,1\}}} \sum_{p \in P} [DIS^p(\sum_{t \in T_p} \sum_{a \in A} (o_{at} \cdot v_t) - \sum_{r \in R} (N_{rp} \cdot C_{rp}))] \quad (29)$$

subject to:

$$s_{ait} \leq s_{ai(t+1)} \quad \forall a \in A, \forall i \in \mathcal{I}, \forall t \in T \quad (30)$$

$$s_{ai\sigma_{itN}} \geq 1 \quad \forall a \in A, \forall i \in \mathcal{I} - M^O \quad (31)$$

$$s_{ai_0^D t} - 1 \leq s_{ai\sigma_{it}} - s_{ait} \quad \forall a \in A, \forall i \in \mathcal{I} - D, \forall t \in T \quad (32)$$

$$s_{ajt} \leq s_{ai\sigma_{it}} \quad \forall a \in A, \forall (i, j) \in IP, \forall t \in T \quad (33)$$

$$m_{rp} \geq N_{rp} \geq \sum_{a \in A} \sum_{i \in \mathcal{I}} (\rho_{ir} \cdot (s_{ait} - s_{ai\sigma_{it}})) \quad \forall r \in R, \forall p \in P, \forall t \in T_p \quad (34)$$

$$o_{at} \leq s_{ai_N^I \sigma_{i_N^I t}} - s_{ai_0^D t} \quad \forall a \in A, \forall t \in T \quad (35)$$

$$o_{at} \leq \sum_{i \in M \cup \{i_N^I\}} (s_{ai\sigma_{it}} - s_{ai\sigma_{i(t-\lambda_a)}}) \quad \forall a \in A, \forall t \in T \quad (36)$$

$$o_{at} \leq 1 + s_{ai\sigma_{it}} - s_{ait} \quad \forall a \in A, \forall i \in M, \forall t \in T \quad (37)$$

3-Mixed Model Explanation

- (29) Objective function, sums up profits from energy made, subtracts money used on resources (vessels), and multiplies it all with a discount factor
- (30) Makes every task that is started stay started
- (31) Forces every installation and mandatory maintenance task to be started and finished by the final timestep
- (32) Ensures every non-decommission task is inactive during decomission
- (33) For every precedence relation (i,j) , ensures that i is finished before j is started
- (34) Counts up the resources needed in a time period by adding the resources needed by all tasks started, and subtracting the resources needed by all tasks finished. It also sets a limit on the amount of vessels that can be charatered in a given period
- (35) An asset can only be online if installation is complete and decomission has not started yet
- (36) An asset can only be online if it had maintenance done recently (or only completed installation recently)
- (37) Ensures an asset if offline is maintenance work is going on at this moment

Notation overview

Sets:

- P : All time periods (large scale)
- T : All time intervals (small scale) $[t_0, \dots, t_N]$
- $T_p \subset T$: All time intervals (small scale) in period p
- R : All resources
- I : All installation tasks per asset $[i_0^I, \dots, i_N^I]$
- $M = M^M \cup M^O$: all mandatory and optional maintenance tasks
- D : All decommission tasks per asset $[i_0^D, \dots, i_N^D]$
- $\mathcal{I} = I \cup M \cup D$: All tasks
- IP : All precedence pairs (i, j) . Includes relations stating that the final installation task is finished before maintenance and decommission are started.
- A : All assets

Decision variables:

- o_{at} : Binary variable, 1 if asset a is online at timestep t
- N_{rp} : Number of resources r used in period p
- s_{ait} : Binary variable, 1 if task $i \in \mathcal{I}$ for asset a has started at or before time t

Parameters:

- DIS : The discount factor per time period
- v_t : The value of energy a single turbine produces at timestep t
- C_{rp} : The cost of chartering resource r in period p
- λ_a : The number of timesteps after the last maintenance before asset a fails
- σ_{it} : Indicates the timestep at which task i should have been started for it to be finished by timestep t , generated duration and the weather conditions
- ρ_{ir} : The amount of resource r used for task $i \in \mathcal{I}$
- m_{rp} : The maximum amount of resources r that can be chartered in period p

Initial Corrective model

$$\max_{\substack{N_{rp} \in \mathbb{Z}^* \\ s_{ait}, o_{at} \in \{0,1\}}} \sum_{p \in P} [DIS^p(\sum_{t \in T_p} \sum_{a \in A} (o_{at} \cdot v_t) - \sum_{r \in R} (N_{rp} \cdot C_{rp}))] \quad (38)$$

subject to:

$$s_{ait} \leq s_{ai(t+1)} \quad \forall a \in A, \forall i \in \mathcal{I}, \forall t \in T \quad (39)$$

$$s_{ai\sigma_{itN}} \geq 1 \quad \forall a \in A, \forall i \in \mathcal{I} - M^C \quad (40)$$

$$s_{ai_0^D t} - 1 \leq s_{ai\sigma_{it}} - s_{ait} \quad \forall a \in A, \forall i \in \mathcal{I} - D, \forall t \in T \quad (41)$$

$$s_{ajt} \leq s_{ai\sigma_{it}} \quad \forall a \in A, \forall (i, j) \in IP, \forall t \in T \quad (42)$$

$$m_{rp} \geq N_{rp} \geq \sum_{a \in A} \sum_{i \in \mathcal{I}} (\rho_{ir} \cdot (s_{ait} - s_{ai\sigma_{it}})) \quad \forall r \in R, \forall p \in P, \forall t \in T_p \quad (43)$$

$$o_{at} \leq s_{ai_N^I \sigma_{i_N^I t}} - s_{ai_0^D t} \quad \forall a \in A, \forall t \in T \quad (44)$$

$$o_{at} \leq \sum_{i \in M \cup \{i_N^I\}} (s_{ai\sigma_{it}} - s_{ai\sigma_{i(t-\lambda_{ai})}}) \quad \forall a \in A, \forall t \in T \quad (45)$$

$$s_{ait} - s_{ai(t-1)} \leq 1 - \sum_{j \in M \cup \{i_N^I\}} \frac{(s_{aj\sigma_{jt}} - s_{aj\sigma_{j(t-\lambda_{aj})}})}{L} \quad \forall a \in A, \forall i \in M^C, \forall t \in T \quad (46)$$

$$o_{at} \leq 1 + s_{ai\sigma_{it}} - s_{ait} \quad \forall a \in A, \forall i \in M, \forall t \in T \quad (47)$$

Corrective Model Explanation

- (38) Objective function, sums up profits from energy made, subtracts money used on resources (vessels), and multiplies it all with a discount factor
- (39) Makes every task that is started stay started
- (40) Forces every installation and preventive maintenance task to be started and finished by the final timestep
- (41) Ensures every non-decommission task is inactive during decommission
- (42) For every precedence relation (i, j) , ensures that i is finished before j is started
- (43) Counts up the resources needed in a time period by adding the resources needed by all tasks started, and subtracting the resources needed by all tasks finished. It also sets a limit on the amount of vessels that can be charatered in a given period
- (44) An asset can only be online if installation is complete, and decomission has not started yet
- (45) An asset is offline if it has not gotten maintained recently enough
- (46) A corrective maintenance task can only be started if the asset is in a failure state (division by L means any number of tasks can have been recent enough for their time to matter)
- (47) Ensures an asset if offline is maintenance work is going on at this moment

Notation overview

Sets:

- P : All time periods (large scale)
- T : All time intervals (small scale) $[t_0, \dots, t_N]$
- $T_p \subset T$: All time intervals (small scale) in period p
- R : All resources
- I : All installation tasks per asset $[i_0^I, \dots, i_N^I]$
- $M = M^P \cup M^C$: all preventive and corrective maintenance tasks
- D : All decommission tasks per asset $[i_0^D, \dots, i_N^D]$
- $\mathcal{I} = I \cup M \cup D$: All tasks
- IP : All precedence pairs (i, j) . Includes relations stating that the final installation task is finished before maintenance and decommission are started.
- A : All assets

Decision variables:

- o_{at} : Binary variable, 1 if asset a is online at timestep t
- N_{rp} : Number of resources r used in period p
- s_{ait} : Binary variable, 1 if task $i \in \mathcal{I}$ for asset a has started at or before time t

Deterministic parameters:

- DIS : The discount factor per time period
- C_{rp} : The cost of chartering resource r in period p
- ρ_{ir} : The amount of resource r used for task $i \in \mathcal{I}$
- m_{rp} : The maximum amount of resources r that can be chartered in period p
- L : A large number (at least $|M| + 1$)

Stochastic parameters:

- v_t : The value of energy a single turbine produces at timestep t
- λ_{ai} : The number of timesteps after the last maintenance before asset a fails
- σ_{it} : Indicates the timestep at which task i should have been started for it to be finished by timestep t , generated based on duration and the weather conditions (stochastic based on those)

Initial Stochastic model

$$\max_{\substack{N_{rp} \in \mathbb{Z}^* \\ s_{ait}, o_{at} \in \{0,1\}}} \sum_{p \in P} [DIS^p(\sum_{t \in T_p} \sum_{a \in A} (o_{at} \cdot v_t) - \sum_{r \in R} (N_{rp} \cdot C_{rp}))] \quad (48)$$

subject to:

$$s_{ait} \leq s_{ai}(t+1) \quad \forall a \in A, \forall i \in M, \forall t \in T \quad (49)$$

$$s_{ai\sigma_{it_N}} \geq 1 \quad \forall a \in A, \forall i \in M^P \quad (50)$$

$$m_{rp} \geq N_{rp} \geq \sum_{a \in A} \sum_{i \in \mathcal{I}} (\rho_{ir} \cdot (s_{ait} - s_{ai\sigma_{it}})) \quad \forall r \in R, \forall p \in P, \forall t \in T_p \quad (51)$$

$$o_{at} \leq \sum_{i \in MU\{i_N^l\}} (s_{ai\sigma_{it}} - s_{ai\sigma_{i(t-\lambda_{ai})}}) \quad \forall a \in A, \forall t \in T \quad (52)$$

$$s_{ait} - s_{ai}(t-1) \leq 1 - \sum_{j \in MU\{i_N^l\}} \frac{(s_{aj\sigma_{jt}} - s_{aj\sigma_{j(t-\lambda_{aj})}})}{L} \quad \forall a \in A, \forall i \in M^C, \forall t \in T \quad (53)$$

$$o_{at} \leq 1 + s_{ai\sigma_{it}} - s_{ait} \quad \forall a \in A, \forall i \in M, \forall t \in T \quad (54)$$

Stochastic Model Explanation

- (48) Objective function, sums up profits from energy made, subtracts money used on resources (vessels), and multiplies it all with a discount factor
- (49) Makes every task that is started stay started
- (50) Forces every installation and preventive maintenance task to be started and finished by the final timestep
- (51) Counts up the resources needed in a time period by adding the resources needed by all tasks started, and subtracting the resources needed by all tasks finished. It also sets a limit on the amount of vessels that can be charatered in a given period
- (52) An asset is offline if it has not gotten maintained recently enough
- (53) A corrective maintenance task can only be started if the asset is in a failure state (division by L means any number of tasks can have been recent enough for their time to matter)
- (54) Ensures an asset if offline is maintenance work is going on at this moment

Notation overview

Sets:

- P : All time periods (large scale)
- T : All time intervals (small scale) $[t_0, \dots, t_N]$
- $T_p \subset T$: All time intervals (small scale) in period p
- R : All resources
- $M = M^P \cup M^C$: all preventive and corrective maintenance tasks
- A : All assets

Decision variables:

- o_{at} : Binary variable, 1 if asset a is online at timestep t
- N_{rp} : Number of resources r used in period p
- s_{ait} : Binary variable, 1 if task $i \in \mathcal{I}$ for asset a has started at or before time t

Deterministic parameters:

- DIS : The discount factor per time period
- C_{rp} : The cost of chartering resource r in period p
- ρ_{ir} : The amount of resource r used for task i
- m_{rp} : The maximum amount of resources r that can be chartered in period p
- L : A large number (at least $|M| + 1$)

Stochastic parameters:

- v_t : The value of energy a single turbine produces at timestep t
- λ_{ai} : The number of timesteps after the last maintenance before asset a fails
- σ_{it} : Indicates the timestep at which task i should have been started for it to be finished by timestep t , generated based on duration and the weather conditions (stochastic based on those)