# Initial models for optimisation

R. Kuipers

May 14, 2020

# Initial model for installation

$$\max_{\substack{O_p, N_{rp} \in \mathbb{Z}^* \\ s_{it}, f_{it} \in \{0,1\}}} \sum_{p \in P} [DIS^p (O_p \cdot v_p - \sum_{r \in R} N_{rp} \cdot C_{rp})] \tag{1}$$

subject to:

$$1 = \sum_{t \in T} s_{it} = \sum_{t \in T} f_{it} \qquad \forall i \in I \tag{2}$$

$$1 \geq s_{jt_1} + f_{it_2} \qquad \forall (i,j) \in IP, \ \forall t_1, t_2 \in T | t_1 \leq t_2 \tag{3}$$

$$(s_{it_1} + f_{it_2} - 1) \cdot d_i \leq \frac{s_{it_1} + f_{it_2}}{2} \cdot \sum_{t_3 = t_1}^{t_2} \omega_{it_3} \qquad \forall i \in I, \forall t_1, t_2 \in T \tag{4}$$

$$N_{rp} \geq \sum_{i \in I} \sum_{t_1 = t_0}^{t} (\rho_{ir} \cdot (s_{it_1} - f_{it_1})) \qquad \forall r \in R, \forall p \in P, \forall t \in T_p \tag{5}$$

$$O_p = \sum_{t = t_0}^{t_p} \sum_{i \in F} f_{it} \qquad \forall p \in P \tag{6}$$

# Installation Model Explanation

(1) Objective function, sums up profits from energy made, subtracts money used on resources (vessels), and multiplies it all with a discount factor
(2) Forces every task to be starded and finished at some point
(3) For every precedence relation $(i, j)$ it ensures that if task $i$ finishes at time $t_2$ there is no $t_1 < t_2$ at which task $j$ starts
(4) Ensures that between the starting and finish times of each task are enough timesteps with acceptable weather
(5) Counts up the resources needed in a time period by adding the resources needed by all tasks started, and subtracting the resources needed by all tasks finished
(6) Counts the number of turbines which finished installing by the end of a period

# Notation overview

Sets:

- $P$: All time periods (large scale)
- $T$: All time intervals $[t_0, \ldots, t_N]$
- $T_p \in T$: All time intervals (small scale) in period $p$
- $R$: All resources
- $I$: All tasks
- $F \subset I$: All final tasks that complete a turbine
- $IP$: All precedency pairs $(i, j)$

Decision variables:

- $O_p$: Number of online turbines after period $p$
- $N_{rp}$: Number of resources $r$ used in period $p$
- $s_{it}$: Binary variable, 1 if task $i$ starts at time $t$
- $f_{it}$: Binary variable, 1 if task $i$ ends at time $t$

Parameters:

- $DIS$: The discount factor per period
- $v_p$: The value of energy a single turbine produces in period $p$
- $C_{rp}$: The cost of chartering resource $r$ in period $p$
- $d_i$: The duration of task $i$
- $\omega_{it}$: Binary parameter representing weather, 1 if task $i$ can be completed at time $t$, 0 otherwise
- $\rho_{ir}$: The amount of resource $r$ used by task $i$
- $t_p$: The final time interval (from $T$) in period $p$

# Initial model for maintenance

$$\max_{\substack{O_t, N_{rp} \in \mathbb{Z}^* \\ s_{act}, f_{act}, b_{at} \in \{0,1\}}} \sum_{p \in P} [DIS^p (\sum_{t \in T_p} (O_t \cdot v_t) - \sum_{r \in R} (N_{rp} \cdot C_{rp}))] \tag{7}$$

subject to (1):

$$1 = \sum_{t \in T} s_{act} = \sum_{t \in T} f_{act} \qquad \forall a \in A, \forall c \in C^M \tag{8}$$

$$1 \geq \sum_{t \in T} s_{act} \qquad \forall a \in A, \forall c \in C^O \tag{9}$$

$$\sum_{t \in T} s_{act} = \sum_{t \in T} f_{act} \qquad \forall a \in A, \forall c \in C^O \tag{10}$$

# Initial model for maintenance

$$\max_{\substack{O_t, N_{rp} \in \mathbb{Z}^* \\ s_{act}, f_{act}, b_{at} \in \{0,1\}}} \sum_{p \in P} [DIS^p (\sum_{t \in T_p} (O_t \cdot v_t) - \sum_{r \in R} (N_{rp} \cdot C_{rp}))] \tag{7}$$

subject to (2):

$$(s_{act_1} + f_{act_2} - 1) \cdot d_c \leq \frac{s_{act_1} + f_{act_2}}{2} \cdot \sum_{t_3 = t_1}^{t_2} \omega_{ct_3} \qquad \forall a \in A, \forall c \in C, \forall t_1, t_2 \in T \tag{11}$$

$$N_{rp} \geq \sum_{a \in A} \sum_{c \in C} \sum_{t_1 = t_0}^{t} (\rho_{ir} \cdot (s_{act_1} - f_{act_1})) \qquad \forall r \in R, \forall p \in P, \forall t \in T_p \tag{12}$$

$$b_{at} > \sum_{c \in C} \sum_{t_1 = t - \lambda_a}^{t} - f_{act_1} \qquad \forall a \in A, \forall t \in T \tag{13}$$

$$O_t = |A| - \sum_{a \in A} b_{at} \qquad \forall t \in T \tag{14}$$

# Maintenance Model Explanation

(7) Objective function, sums up profits from energy made, subtracts money used on resources (vessels), and multiplies it all with a discount factor
(8) Forces every mandatory maintenance cycle to be done at some point
(9) Ensures each optional maintenance cycle to be started at most once
(10) Ensures that every maintenance cycle for a particular asset that is started is also finished
(11) Ensures that between the starting and finish times of each cycle are enough timesteps with acceptable weather
(12) Counts up the resources needed in a time period by adding the resources needed by all tasks started, and subtracting the resources needed by all tasks finished
(13) If no maintenance tasks have finished in the past $\lambda_a$ timesteps this asset is broken
(14) The number of active (online) turbines is equal to everything that isn't broken

# Notation overview

Sets:

- $P$: All time periods (large scale)
- $T$: All time intervals (small scale) $[t_0, \ldots, t_N]$
- $T_p \in T$: All time intervals (small scale) in period $p$
- $R$: All resources
- $A$: All assets
- $C = C^M \cup C^O$: All (mandatory and optional) maintenance cycles

Decision variables:

- $O_t$: Number of active turbines at timestep $t$
- $N_{rp}$: Number of resources $r$ used in period $p$
- $s_{act}$: Binary variable, 1 if maintenance cycle $c$ for asset $a$ starts at time $t$
- $f_{act}$: Binary variable, 1 if maintenance cycle $c$ for asset $a$ finishes at time $t$
- $b_{at}$: Binary variable, 1 if asset $a$ is broken at timestep $t$

Parameters:

- $DIS$: The discount factor per time period
- $v_t$: The value of energy a single turbine produces at timestep $t$
- $C_{rp}$: The cost of chartering resource $r$ in period $p$
- $d_c$: The duration per task during maintenance cycle $c$
- $\lambda_a$: The number of timesteps after the last maintenance before asset $a$ fails
- $\omega_{ct}$: Binary parameter representing weather, 1 if maintenance cycle $c$ can be completed at time $t$, 0 otherwise
- $\rho_{cr}$: The amount of resource $r$ used per task for maintenance cycle $c$

# Initial mixed model

$$\max_{\substack{O_t, N_{rp} \in \mathbb{Z}^* \\ s_{ait}, f_{ait}, o_{at} \in \{0,1\}}} \sum_{p \in P} [DIS^p ( \sum_{t \in T_p} (O_t \cdot v_t) - \sum_{r \in R} (N_{rp} \cdot C_{rp}))] \tag{15}$$

subject to:

$$1 = \sum_{t \in T} s_{ait} = \sum_{t \in T} f_{ait} \qquad \forall i \in I \cup M^M, \forall a \in A \tag{16}$$

$$1 \geq s_{ajt_1} + f_{ait_2} \qquad \forall (i,j) \in IP, \forall a \in A, \forall t_1, t_2 \in T | t_1 \leq t_2 \tag{17}$$

$$1 \geq \sum_{t \in T} s_{ait} \qquad \forall a \in A, \forall i \in M^O \tag{18}$$

$$\sum_{t \in T} s_{ait} = \sum_{t \in T} f_{ait} \qquad \forall a \in A, \forall i \in M^O \tag{19}$$

$$0 = \sum_{t_1 = t_0}^{t} s_{ajt_1} + \sum_{t_2 = t}^{t_N} f_{ait_2} \qquad \forall a \in A, \forall t \in T, \forall i \in I, \forall j \in M \tag{20}$$

# Initial mixed model

$$\max_{\substack{O_t, N_{rp} \in \mathbb{Z}^* \\ s_{ait}, f_{ait}, o_{at} \in \{0,1\}}} \sum_{p \in P} [DIS^p(\sum_{t \in T_p} (O_t \cdot v_t) - \sum_{r \in R} (N_{rp} \cdot C_{rp}))] \tag{15}$$

subject to (2):

$$(s_{ait_1} + f_{ait_2} - 1) \cdot d_i \leq \frac{s_{ait_1} + f_{ait_2}}{2} \cdot \sum_{t_3 = t_1}^{t_2} \omega_{it_3} \qquad \forall i \in \mathcal{I}, \forall a \in A, \forall t_1, t_2 \in T \tag{21}$$

$$N_{rp} \geq \sum_{a \in A} \sum_{i \in \mathcal{I}} \sum_{t_1 = t_0}^{t} (\rho_{ir} \cdot (s_{ait_1} - f_{ait_1})) \qquad \forall r \in R, \forall p \in P, \forall t \in T_p \tag{22}$$

$$o_{at} \leq \frac{1}{2} \cdot (\sum_{t_1 = t_0}^{t} f_{ai_N t_1} + \sum_{i \in M \cup \{i_N\}} \sum_{t_2 = t - \lambda_a}^{t} f_{ait_2}) \qquad \forall a \in A, \forall t \in T \tag{23}$$

$$O_t = \sum_{a \in A} o_{at} \qquad \forall t \in T \tag{24}$$

# Mixed Model Explanation

(15) Objective function, sums up profits from energy made, subtracts money used on resources (vessels), and multiplies it all with a discount factor
(16) Forces every mandatory task to be starded and finished at some point
(17) For every precedence relation $(i, j)$ it ensures that if task $i$ finishes at time $t_2$ there is no $t_1 < t_2$ at which task $j$ starts
(18) Ensures each optional maintenance task to be started at most once
(19) Ensures that every maintenance task for a particular asset that is started is also finished
(20) Ensures an asset is fully installed before maintenance starts
(21) Ensures that between the starting and finish times of each task are enough timesteps with acceptable weather
(22) Counts up the resources needed in a time period by adding the resources needed by all tasks started, and subtracting the resources needed by all tasks finished
(23) Sets an asset to be online if it installed and had work done on it recently
(24) Counts how many assets are online

# Notation overview

Sets:

- $P$: All time periods (large scale)
- $T$: All time intervals (small scale) $[t_0, \ldots, t_N]$
- $T_p \in T$: All time intervals (small scale) in period $p$
- $R$: All resources
- $I$: All installation tasks per asset $[1, \ldots, i_N]$
- $M = M^M \cup M^O$: all mandatory and optional maintenance tasks
- $\mathcal{I} = I \cup M$: All tasks
- $IP$: All precedency pairs $(i, j)$
- $A$: All assets

Decision variables:

- $O_t$: Number of online turbines at timestep $t$
- $o_{at}$: Binary variable, 1 if asset $a$ is online at timestep $t$
- $N_{rp}$: Number of resources $r$ used in period $p$
- $s_{ait}$: Binary variable, 1 if task $i \in \mathcal{I}$ for asset $a$ starts at time $t$
- $f_{ait}$: Binary variable, 1 if task $i \in \mathcal{I}$ for asset $a$ finishes at time $t$

Parameters:

- $DIS$: The discount factor per time period
- $v_t$: The value of energy a single turbine produces at timestep $t$
- $C_{rp}$: The cost of chartering resource $r$ in period $p$
- $d_i$: The duration of task $i \in \mathcal{I}$
- $\lambda_a$: The number of timesteps after the last maintenance before asset $a$ fails
- $\omega_{it}$: Binary parameter representing weather, 1 if task $i \in \mathcal{I}$ can be completed at time $t$, 0 otherwise
- $\rho_{ir}$: The amount of resource $r$ used for task $i \in \mathcal{I}$