Initial models for optimisation

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Initial model for installation

$$\text{maximize} \sum_{p \in P} [DIS^p(A_p \cdot v_p - \sum_{r \in R} N_{rp} \cdot C_{rp})] \tag{1}$$

subject to:

$$1 = \sum_{t \in T} s_{it} = \sum_{t \in T} f_{it} \qquad \forall i \in I \tag{2}$$

$$1 \leq \sum_{t=\dot{t}}^{\hat{t}} [\sum_{t'=\dot{t}}^{t} f_{jt'} \cdot \sum_{t'=t}^{\hat{t}} s_{it'}] \qquad \forall (i,j) \in IP \tag{3}$$

$$d_i \geq (f_{it''} + s_{it'} - 1) \cdot \sum_{t=t'}^{t''} \omega_{it} \qquad \forall i \in I, \forall t'' \geq t' + d_i, t', t'' \in T \tag{4}$$

$$N_{rp} \geq \sum_{i \in I} \sum_{t'=\dot{t}}^{t} \sum_{t''=t}^{\hat{t}} s_{it'} \cdot f_{it''} \cdot \rho_{ir} \qquad \forall r \in R, \forall p \in P, \forall t \in T_p \tag{5}$$

$$A_p = \sum_{t=\dot{t}}^{\tau_p} \sum_{i \in F} f_{it} \qquad \forall p \in P \tag{6}$$

Notation overview

Sets:

- P: All time periods (large scale)
- T: All time intervals $[\dot{t}, \dots, \hat{t}]$
- $T_p \in T$: All time intervals (small scale) in period p
- R: All resources
- I: All tasks
- $F \subset I$: All final tasks that complete a turbine
- IP: All precedency pairs (i, j)

Decision variables:

- A_p: Number of active turbines after period p
- N_{rp}: Number of resources r used in period p
- s_{it}: Binary variable, 1 if task i starts at time t
- f_{it}: Binary variable, 1 if task i ends at time t

Parameters:

- DIS: The discount factor per period
- v_p: The value of energy a single turbine produces in period p
- C_{rp}: The cost of chartering resource r in period p
- d_i: The duration of task i
- ω_{it}: Binary parameter representing weather, 1 if task i can be completed at time t, 0 otherwise
- ρ_{ir}: The amount of resource r used by task i
- τ_p: The final time interval (from T) in period p

Initial model for maintenance

$$\text{maximize} \sum_{p \in P} [DIS^p (A_p \cdot v_p - \sum_{r \in R} N_{rp} \cdot C_{rp})] \tag{7}$$

subject to (1):

$$1 = \sum_{t \in T} s_{act} = \sum_{t \in T} f_{act} \qquad \forall a \in A, \forall c \in C^M \tag{8}$$

$$1 \geq \sum_{t \in T} s_{act} \qquad \forall a \in A, \forall c \in C^O \tag{9}$$

$$\sum_{t \in T} s_{act} = \sum_{t \in T} f_{act} \qquad \forall a \in A, \forall c \in C^O \tag{10}$$

$$\sum_{t \in T} s_{act} \geq \sum_{t \in T} s_{ac't} \qquad \forall a \in A, \forall c + 1 = c', c, c' \in C^O \tag{11}$$

Initial model for maintenance

$$\text{maximize} \sum_{p \in P} [DIS^p (A_p \cdot v_p - \sum_{r \in R} N_{rp} \cdot C_{rp})] \qquad (7)$$

subject to (2):

$$d_a \geq (f_{act''} + s_{act'} - 1) \cdot \sum_{t=t'}^{t''} \omega_{it} \qquad \forall a \in A, \forall c \in C, \forall t'' \geq t' + d_i, t', t'' \in T \qquad (12)$$

$$N_{rp} \geq \sum_{a \in A} \sum_{c \in C} \sum_{t'=\dot{t}}^{t} \sum_{t''=t}^{\hat{t}} s_{act'} \cdot f_{act''} \cdot \rho_{ar} \qquad \forall r \in R, \forall p \in P, \forall t \in T_p \qquad (13)$$

$$A_p = \sum_{t=\dot{t}}^{\tau_p} \sum_{i \in F} f_{it} \qquad \forall p \in P \qquad (14)$$

Notation overview

Sets:

- P: All time periods (large scale)
- T: All time intervals $[\dot{t}, \ldots, \hat{t}]$
- $T_p \in T$: All time intervals (small scale) in period p
- R: All resources
- I: All tasks
- $F \subset I$: All final tasks that complete a turbine
- IP: All precedency pairs (i, j)

Decision variables:

- A_p: Number of active turbines after period p
- N_{rp}: Number of resources r used in period p
- s_{it}: Binary variable, 1 if task i starts at time t
- f_{it}: Binary variable, 1 if task i ends at time t

Parameters:

- DIS: The discount factor per period
- v_p: The value of energy a single turbine produces in period p
- C_{rp}: The cost of chartering resource r in period p
- d_i: The duration of task i
- ω_{it}: Binary parameter representing weather, 1 if task i can be completed at time t, 0 otherwise
- ρ_{ir}: The amount of resource r used by task i
- τ_p: The final time interval (from T) in period p

Initial model for maintenance

$$\text{maximize} \sum_{p\in P}[DIS^p(A_p \cdot v_p - \sum_{r\in R} N_{rp} \cdot C_{rp})] \tag{15}$$

subject to:

$$s_{ic} = \max(\gamma \cdot c, \max_{j\in IP_i}(f_{jc})) \qquad \forall i \in I \quad \forall c \in C \tag{16}$$

$$d_{ic} = \sum_{t=s_{ic}}^{f_{ic}} \omega_{it} \qquad \forall i \in I \quad \forall c \in C \tag{17}$$

$$N_{rp} \geq \sum_{i|s_{ic}\leq t\leq f_{ic},\forall t\in T_p} \rho_{ir} \qquad \forall r \in R \quad \forall p \in P \quad \forall c \in C \tag{18}$$

$$A_p = A_{p-1} \cdot \lambda + \text{number fully repaired} \qquad \forall p \in P \quad \forall i \in F \tag{19}$$

Initial mixed model

Sets:

- P: All time periods (large scale)
- T_p: All time intervals (small scale) in period p
- C: All cycles
- R: All resources
- I: All tasks
- $F \subset I$: All final tasks that complete a turbine
- IP_i: All prerequisite tasks of task i

Decision variables:

- γ: The lenth of a cycle
- A_p: Number of active turbines after period p
- N_{rp}: Number of resources r used in period p
- s_{ic}: Starting time of task i in cycle c
- f_{ic}: Finishing time of task i in cycle c

Parameters:

- DIS: The discount factor per period
- v_p: The value of energy a single turbine produces in period p
- C_{rp}: The cost of chartering resource r in period p
- d_i: The duration of task i
- ω_{it}: Binary parameter representing weather, 1 if task i can be completed at time t, 0 otherwise
- ρ_{ir}: The amount of resource r used by task i
- τ_p: The final time interval (from T) in period p