

Years spanning model

$$\min_{N_{y,m,\sigma}, P_{m,ip}, R_{m,ir}, \sigma \in \mathbb{Z}^*} \frac{1}{|S|} \cdot \sum_{\sigma \in S} \left(\sum_{m \in M} \left[\sum_{y \in Y} (N_{y,m,\sigma} \cdot c_{y,m}) + \sum_{ip \in I^P} (P_{m,ip} \cdot d^P \cdot e_m^H) + \sum_{ir \in I^R} \left[\sum_{m'=0}^m (f_{m',ir,\sigma} - R_{m',ir,\sigma}) \right] \cdot e_m^H \cdot H_m \right] \right) \quad (1)$$

subject to:

$$L_y \cdot N_{y,m,\sigma} + L_{y,m}^{Inst} \geq \sum_{ip \in I^P} (P_{m,ip} \cdot d_y^P) + \sum_{ir \in I^R} (R_{m,ir,\sigma} \cdot d_{y,ir}^R) + \epsilon_{\sigma,m,y} \quad \forall \sigma \in S, \forall m \in M, \forall y \in Y \quad (2)$$

$$N_{y,m,\sigma} + N_{y,m}^{Inst} \geq \rho_{\sigma,m,y} \quad \forall \sigma \in S, \forall m \in M, \forall y \in Y \quad (3)$$

$$\sum_{m'=0}^{m-1} f_{m',ir,\sigma} \geq \sum_{m'=0}^m R_{m',ir,\sigma} \quad \forall \sigma \in S, \forall m \in M, \forall ir \in I^R \quad (4)$$

$$P_{m,ip} \leq \sum_{m'=0}^{m-GMIN} P_{m',(ip-1)} - \sum_{m'=0}^{m-1} P_{m',ip} \quad \forall m \in M, \forall ip \in I^P \quad (5)$$

$$P_{m,ip} \geq \sum_{m'=0}^{m-GMAX} P_{m',(ip-1)} - \sum_{m'=0}^{m-1} P_{m',ip} \quad \forall m \in M, \forall ip \in I^P \quad (6)$$

$$N_{y,m,\sigma} \leq A_{y,m} \quad \forall \sigma \in S, \forall m \in M, \forall y \in Y \quad (7)$$

Years spanning model

$$\min_{\substack{N_{y,m,\sigma}, P_{m,ip} \in \mathbb{Z}^* \\ R_{m,ir,\sigma}, F_{m,ir,\sigma}^U \in \mathbb{Z}^*}} \frac{1}{|S|} \cdot \sum_{\sigma \in S} \sum_{m \in M} \sum_{y \in Y} [\sum (N_{y,m,\sigma} \cdot c_{y,m}) + e_m^H \cdot (\sum_{ip \in I^P} [P_{m,ip} \cdot d^P \cdot e_m^H] + \sum_{ir \in I^R} [F_{m,ir,\sigma}^U \cdot \frac{H_m}{2} + (F_{m,ir,\sigma}^T - F_{m,ir,\sigma}^U) \cdot (d_{ir}^R + d_{ir}^D) + R_{m,ir,\sigma} \cdot d_{ir}^R])] \quad (8)$$

subject to:

$$L_y \cdot N_{y,m,\sigma} + L_{y,m}^{Inst} \geq \epsilon_{\sigma,m,y} + \sum_{ip \in I^P} (P_{m,ip} \cdot d_y^P) + \sum_{ir \in I^R} (R_{m,ir,\sigma} \cdot d_{y,ir}^R) \quad \forall \sigma \in S, \forall m \in M, \forall y \in Y \quad (9)$$

$$\rho_{\sigma,m,y} - N_{y,m}^{Inst} \leq N_{y,m,\sigma} \leq A_{y,m} \quad \forall \sigma \in S, \forall m \in M, \forall y \in Y \quad (10)$$

$$R_{m,ir,\sigma} \leq F_{m-1,ir,\sigma}^U \quad \forall \sigma \in S, \forall m \in M, \forall ir \in I^R \quad (11)$$

$$P_{m,ip} \leq \sum_{m'=0}^{m-G^{MIN}} P_{m',(ip-1)} - \sum_{m'=0}^{m-1} P_{m',ip} \quad \forall m \in M, \forall ip \in I^P \quad (12)$$

$$P_{m,ip} \geq \sum_{m'=0}^{m-G^{MAX}} P_{m',(ip-1)} - \sum_{m'=0}^{m-1} P_{m',ip} \quad \forall m \in M, \forall ip \in I^P \quad (13)$$

$$F_{m,ir,\sigma}^U - F_{m,ir,\sigma}^T + R_{m,ir,\sigma} \geq \frac{1}{d_{y,ir}^R} \cdot (L_y \cdot N_{y,m,\sigma} + L_{y,m}^{Inst} - \epsilon_{\sigma,m,y} - \sum_{ip \in I^P} (P_{m,ip} \cdot d_y^P) - \sum_{ir \in I^R} (R_{m,ir,\sigma} \cdot d_{y,ir}^R)) \quad \forall \sigma \in S, \forall m \in M, \forall y \in Y \quad (14)$$

Years model Explanation

- (1) The objective takes an average cost of all scenarios by counting up the cost per month. Each month has a cost for vessels chartered (by type), the energy lost due to planned maintenance, and the energy lost due to unrepaired failures
- (2) For each vessel type the available time (based on charters and spare time from installation) needs to surpass the estimated total time all tasks take
- (3) Ensures enough vessels are assigned to a given month (based on feedback from that month)
- (4) For any type of reactive task, the amount of tasks performed cannot exceed the amount of failures of that type (with a 1 month delay before a failure can be solved)
- (5) Ensures planned tasks are not scheduled too close together by counting up the turbines that were last visited G^{MIN} or more months ago and subtracting all that were revisited later
- (6) Ensures planned tasks are not scheduled too far apart, in similar manner as the previous constraint
- (7) Limits the maximum amount of vessels chartered based on normal availability and vessels used for installation

Years model notation overview

Sets:

- S : Scenarios
- M : Months
- Y : Vessel types
- I^P : Indices for the planned tasks (first time a turbine is visited is index 1, second time is index 2, etc...)
- I^R : Types of failures reactive tasks

Decision variables:

- $N_{y,m,\sigma}$: The amount of vessels of type y chartered in month m in scenario σ
- $P_{m,ip}$: Planned tasks in iteration ip in month m
- $R_{m,ir,\sigma}$: Reactive tasks of type ir in month m and scenario σ

Parameters:

- $c_{y,m}$: The cost of chartering a vessel of type y in month m
- d^P : The duration of a preventive task
- e_m^H : The energy produced by a single turbine per hour in month m
- $f_{m,ir,\sigma}$: The amount of failures of type ir in month m in scenario σ
- H_m : The number of hours in month m

Parameters (cont):

- $\epsilon_{\sigma,m,y}$: Feedback parameter that enforces leighway in the amount of vessel time available to month m
- $\rho_{\sigma,m,y}$: Feedback parameter that enforces a minimum amount of a certain vessel assigned to month m
- L_y : The amount of hours a vessel of type y is available if chartered for a month
- $L_{y,m}^{Inst}$: Leftover hours vessels of type y are available in month m based on the installation schedule
- $N_{y,m}^{Inst}$: Amount of vessels of type y used by the installation schedule in month m that can also partially help out with maintenance operations
- d_y^P : The duration of a preventive task for vessel type y
- $d_{y,ir}^R$: The duration of a reactive task ir for vessel type y
- G^{MIN} & G^{MAX} : The minimum and maximum amounts of months between two planned maintenance tasks
- $A_{y,m}$: The number of maximum available vessels of type y in month m

Months spanning model

$$\min_{\substack{s_i \in \mathbb{R}_{\geq 0} \\ a_{v,i}^F, a_{v,i}^L, a_{v,i,i'} \in \{0,1\}}} \sum_{i \in I} c_i \cdot (s_i + \max_{y \in Y} (s_{y,i} + d_{y,i})) \quad (15)$$

subject to:

$$\sum_{i \in I} a_{v,i}^F \leq 1 \quad \forall v \in V \quad (16)$$

$$a_{v,i}^F + \sum_{i' \in I - \{i\}} a_{v,i',i} = a_{v,i}^L + \sum_{i' \in I - \{i\}} a_{v,i,i'} \quad \forall v \in V, \forall i \in I \quad (17)$$

$$\sum_{v \in V_y} (a_{v,i}^F + \sum_{i' \in I - \{i\}} a_{v,i',i}) \geq \rho_{y,i} \quad \forall y \in Y, \forall i \in I^{Maint} \quad (18)$$

$$s_{i'} + s_{y,i'} - s_i - s_{i,y} \geq (M + d_{y,i}) \cdot a_{v,i,i'} - M \quad \forall y \in Y, \forall v \in V_y, \forall i, i' \in I \mid i \neq i' \quad (19)$$

$$s_i + \max_{y \in Y} (s_{y,i} + d_{y,i}) \leq T \quad \forall i \in I^{Maint} \quad (20)$$

$$a_{v,i}^F + \sum_{i' \in I - \{i\}} a_{v,i',i} = a_{v,i}^{Inst} \quad \forall v \in V, \forall i \in I^{Inst} \quad (21)$$

$$s_i = s_i^{Inst} \quad \forall i \in I^{Inst} \quad (22)$$

Months model Explanation

- (8) Objective is to minimise costs of tasks being uncompleted (the $+\max$ bit is optional as it's about constants)
- (9) Ensures every vessel has at most one first task
- (10) Ensures that a vessel follows a path without branches
- (11) Ensures every task has enough resources assigned to it
- (12) Ensures the starting times of consecutive tasks are separated by at least the duration of the first task
- (13) Ensures all tasks are finished on time
- (14) Forces every installation task to be assigned as in the installation schedule
- (15) Forces every installation task to be started as in the installation schedule

Months model notation overview

Sets:

- Y : Vessel types
- V : Individual vessels
- $V_y \subseteq V$: Vessels of type $y \in Y$
- I^{Inst} : Installation tasks completed this month by vessels that are also available for maintenance tasks
- I^{Maint} : Maintenance tasks to be scheduled this month
- $I = I^{Inst} \cup I^{Maint}$: Tasks to be completed in this month

Decision variables:

- s_i : Start time of task i
- $a_{v,i}^F$ & $a_{v,i}^L$: Binary variables, 1 for the first and last task i of vessel v respectively
- $a_{v,i,i'}$: Binary variable which is 1 if vessel v performs task i' after task i

Parameters:

- c_i : The cost per hour of a task not being completed
- $s_{y,i}$: The start time offset between task i starting and vessel type y being used for it
- $d_{y,i}$: The duration of task i for vessel type y
- $\rho_{y,i}$: The amount of vessels of type y required for task i
- M : A large number
- T : The end time of the month
- s_i^{Inst} : Start time of task $i \in I^{Inst}$, as in the installation schedule
- $a_{v,i}^{Inst}$: Binary assignment variable of task $i \in I^{Inst}$, as in the installation schedule