Years spanning model

$$\min_{\substack{N_{y,m,\sigma},P_{m,ip}\in\mathbb{Z}^*\\ R_{m,ir,\sigma},U_{m,ir,\sigma}\in\mathbb{Z}^*}} \frac{1}{|S|}\cdot\sum_{\sigma\in S}(\sum_{m\in M}[\sum_{y\in Y}(N_{y,m,\sigma}\cdot c_{y,m}) + e^H_m\cdot(\sum_{ir\in I^R}[U_{m,ir,\sigma}\cdot H_m + R_{m,ir,\sigma}\cdot(d^R_{ir}+d^D_{ir})] \sum_{ip\in I^P}[P_{m,ip}\cdot d^P])] + \sum_{ir\in I^R}[F^U_{\hat{m},ir,\sigma}\cdot\lambda_{ir}]) \tag{1}$$

subject to:

$$L_y\cdot N_{y,m,\sigma} + L^{Inst}_{y,m} \geq \sum_{ip\in I^P}(P_{m,ip}\cdot d^P_y\cdot\rho^P_y) + \sum_{ir\in I^R}(R_{m,ir,\sigma}\cdot d^R_{y,ir}\cdot\rho^R_{y,ir}) \qquad \forall\sigma\in S,\forall m\in M,\forall y \tag{2}$$

$$(N_{y,m,\sigma}+N^{Inst}_{y,m})/\rho^P_y \geq M^P_{y,m,\sigma} \geq \sum_{ip\in I^P}P_{m,ip}\cdot d^P_y/L_y \qquad \forall\sigma\in S,\forall m\in M,\forall y \tag{3}$$

$$(N_{y,m,\sigma}+N^{Inst}_{y,m})/\rho^R_{y,ir} \geq M^R_{y,m,\sigma,ir} \geq R_{m,ir,\sigma}\cdot d^R_{y,ir}/L_y \qquad \forall\sigma\in S,\forall m\in M,\forall y\in Y,\forall ir \tag{4}$$

$$N_{y,m,\sigma} \leq A_{y,m} \qquad \forall\sigma\in S,\forall m\in M,\forall y \tag{5}$$

$$F^T_{m,ir,\sigma} + U_{m-1,ir,\sigma} = R_{m,ir,\sigma} + U_{m,ir,\sigma} \qquad \forall\sigma\in S,\forall m\in M,\forall ir \tag{6}$$

$$\sum_{m'=0}^{m-G^U}P_{m',(ip-1)} - \sum_{m'=0}^{m-1}P_{m',ip} \leq P_{m,ip} \leq \sum_{m'=0}^{m-G^L}P_{m',(ip-1)} - \sum_{m'=0}^{m-1}P_{m',ip} \qquad \forall m\in M,\forall ip \tag{7}$$

Years model Explanation

(1) The objective takes an average cost of all scenarios by counting up the cost per month. Each month has a cost for vessels chartered (by type), the cost of failures (broken up in unhandled and repaired failures), andthe energy lost due to planned maintenance. Finally there is a cost for unhandled failures in the final month.
(2) For each vessel type the available time (based on charters and spare time from installation) minus the planned time the vessels are used (for planned maintenance) needs to be greater than the time spend fixing new failures.
(3) Ensures enough vessels are assigned to a given month (based on feedback from that month), and the amount of vessels chartered doesn't surpass the available vessels
(4) Every failure (new and leftover from previous months) needs to be repaired or unhandled
(5) Ensures planned tasks are not scheduled too close together or too far apart by counting up the turbines that were last visited G^U and G^L or more months ago respectively, and subtracting all that were revisited more recently

Years model notation overview

Sets:

- S: Scenarios
- $M = \{0, \dots, \hat{m}\}$: Months
- Y: Vessel types
- I^P: Indices for the planned tasks (first time a turbine is visited is index 1, second time is index 2, etc...)
- I^R: Types of failures and reactive tasks

Decision variables:

- $N_{y,m,\sigma}$: The amount of vessels of type y chartered in month m in scenario σ
- $P_{m,ip}$: Planned tasks in iteration $ip \in I^P$ in month m
- $R_{m,ir,\sigma}$: Repair tasks of type $ir \in I^R$ in month m and scenario σ
- $U_{m,ir,\sigma}$: Unhandled failures of type $ir \in I^R$ in month m and scenario σ

Parameters:

- $c_{y,m}$: The cost of chartering a vessel of type y in month m
- λ_{ir}: The cost penalty for leftover failures of type ir in the final month
- e^H_m: The energy produced by a single turbine per hour in month m
- H_m: The number of hours in month m

Parameters (cont):

- $F^T_{m,ir,\sigma}$: The total amount of failures of type $ir \in I^R$ in month m in scenario σ
- $\epsilon_{\sigma,m,y}$: Feedback parameter that enforces leighway in the amount of vessel time available to month m
- $\rho_{\sigma,m,y}$: Feedback parameter that enforces a minimum amount of a certain vessel assigned to month m
- L_y: The amount of hours a vessel of type y is available if chartered for a month
- $L^{Inst}_{y,m}$: Leftover hours vessels of type y are available in month m based on the installation schedule
- $N^{Inst}_{y,m}$: Amount of vessels of type y used by the installation schedule in month m that can also partially help out with maintenance operations
- $d^P \& d^P_y$: The duration of a preventive task (specified for vessel type y)
- $d^R_{ir} \& d^R_{y,ir}$: The duration of reactive task ir (specified for vessel type y)
- d^D_{ir}: The delay before a failure of type ir is detected and can be worked on
- G^U & G^L: The Upper and Lower bounds months between two planned maintenance tasks
- $A_{y,m}$: The number of maximum available vessels of type y in month m

Months spanning model

$$\min_{\substack{f_i, s_{v,j} \in \mathbb{R}_{\geq 0} \\ a_{v,i,j} \in \{0,1\}}} \sum_{i \in I} c_i \cdot (f_i - r_i) \tag{8}$$

subject to:

$$\sum_{i \in I} a_{v,i,j} \leq \sum_{i \in I} a_{v,i,(j-1)} \leq 1 \qquad \forall v \in V, \forall j \in J - \{0\} \tag{9}$$

$$\sum_{j \in J} a_{v,i,j} \leq A_i \qquad \forall v \in V, \forall i \in I^{Maint} \tag{10}$$

$$\sum_{v \in V_y} \sum_{j \in J} a_{v,i,j} = A_i \cdot \rho_{y,i} \qquad \forall y \in Y, \forall i \in I^{Maint} \tag{11}$$

$$s_{v,j} - s_{v,(j-1)} \geq \sum_{i \in I} (a_{v,i,(j-1)} \cdot d_{y,i}) \qquad \forall y \in Y, \forall v \in V_y, \forall j \in J - \{0\} \tag{12}$$

$$s_{v,j} \geq r_i \cdot a_{v,i,j} \qquad \forall v \in V, \forall I \in I, \forall j \in J \tag{13}$$

$$s_{v,j} + d_{y,i} \leq f_i + M \cdot (1 - a_{v,i,j}) \qquad \forall y \in Y, \forall v \in V_y, \forall i \in I, \forall j \in J \tag{14}$$

$$f_i \leq T \qquad \forall i \in I \tag{15}$$

$$f_i - a_{v,i',j} \cdot M \leq s_{v,j} \qquad \forall (i, i') \in PR, \forall v \in V, \forall j \in J \tag{16}$$

$$\sum_{j \in J} a_{v,i,j} = a^{Inst}_{v,i} \qquad \forall v \in V, \forall i \in I^{Inst} \tag{17}$$

$$s^{Inst}_i \cdot a_{v_i,i,j} \leq s_{v_i,j} \leq T - a_{v_i,i,j} \cdot (T - s^{Inst}_i) \qquad \forall i \in I^{Inst}, \forall j \in J \tag{18}$$

Months model Explanation

(6) Objective is to minimise the waiting time of tasks, weighed by their cost over time

(7) Ensures any vessels is assigned to at most 1 task at a time, and only has a jth task if it has a $(j-1)$th task

(8) Ensures no task is visited more than that task should be visited

(9) Ensures every task has enough resourses assigned to it (for every iteration)

(10) Ensures the starting times of consecutive (for a given vessel) tasks are far enough apart

(11) Ensures all tasks are started after their release time

(12) Links the start and finish time of a task together with its duration

(13) Ensures tasks are finished on time

(14) Enforces precedency relations between tasks

(15) Forces every installation task to be assigned as in the installation schedule

(16) Forces installation tasks to start at the time dictated by the installation schedule

Months model notation overview

Sets:

- Y: Vessel types
- V: Individual vessels
- $V_y \subseteq V$: Vessels of type $y \in Y$
- I^{Inst}: Installation tasks completed this month by vessels that are also available for maintenance tasks
- I^{Maint}: Maintenance tasks to be scheduled this month
- $I = I^{Inst} \cup I^{Maint}$: Tasks to be completed in this month
- PR: Pairs of tasks with a precedence relationship (where the first task needs to be finished before the second task can be started)

Decision variables:

- $s_{v,j}$: Start time of the jth assignment to vessel v
- f_i: Finish time of the ith task
- $a_{v,i,j}$: Binary variable which is 1 if vessel v performs task i as its jth task

Parameters:

- c_i: The cost per hour of a task not being completed
- r_i: The release time of task i; it cannot be started before this time
- A_i: The amount of times task i needs to be completed (tasks with the same characteristics are grouped this way)
- $d_{y,i}$: The duration of task i for vessel type y
- $\rho_{y,i}$: The amount of vessels of type y required for task i
- M: A large number
- T: The end time of the month
- s_i^{Inst}: Start time of task $i \in I^{Inst}$, as in the installation schedule
- $a_{v,i}^{Inst}$: Binary assignment variable of task $i \in I^{Inst}$, as in the installation schedule
- v_i: For $i \in I^{Inst}$ this represents the vessel assigned to installation task i