From Policy Gradient to Actor-Critic methods

The policy gradient derivation (3/3)

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Policy Gradient with constant baseline

- Reminder:

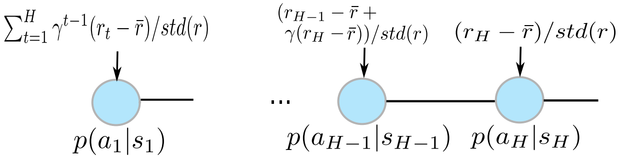
$$\nabla_{\boldsymbol{\theta}} J(\boldsymbol{\theta}) = \frac{1}{m} \sum_{i=1}^{m} \sum_{t=1}^{H} \nabla_{\boldsymbol{\theta}} \log \pi_{\boldsymbol{\theta}}(\mathbf{a}_t^{(i)} | \mathbf{s}_t^{(i)}) [\sum_{k=t}^{H} \gamma^k r(\mathbf{s}_k^{(i)}, \mathbf{a}_k^{(i)})] \tag{1}$$

- If all rewards are positive, the gradient increases all probabilities
- But with renormalization, only the largest increases emerge
- We can substract a "baseline" to (1) without changing its mean:

$$\nabla_{\boldsymbol{\theta}} J(\boldsymbol{\theta}) = \frac{1}{m} \sum_{i=1}^{m} \sum_{t=1}^{H} \nabla_{\boldsymbol{\theta}} \log \pi_{\boldsymbol{\theta}}(\mathbf{a}_t^{(i)} | \mathbf{s}_t^{(i)}) [\sum_{k=t}^{H} \gamma^k r(\mathbf{s}_k^{(i)}, \mathbf{a}_k^{(i)}) - b]$$

- A first baseline is the average return $\bar{r}$ over all states of the batch
- Intuition: returns greater than average get positive, smaller get negative
- Use $(r_t^{(i)} - \bar{r})$ and divide by std $\rightarrow$ get a mean $= 0$ and a std $= 1$
- This improves variance (does the job of renormalization)
- Suggested in https://www.youtube.com/watch?v=tqrcjHuNdmQ

Algorithm 4: adding a constant baseline



- Estimate $\bar{r}$ and $std(r)$ from all rollouts
- Same as Algorithm 2, using $(r_t^{(i)} - \bar{r})/std(r)$
- Suffers from even less variance
- Does not work if all rewards r are identical (e.g. CartPole)

Policy Gradient with state-dependent baseline

- No impact on the gradient as long as the baseline does not depend on action
- A better baseline is $b(\mathbf{s}_t) = V^\pi(\mathbf{s}_t) = \mathbb{E}_\tau[r_t + \gamma r_{t+1} + \gamma^2 r_{t+2} + ... + \gamma^{H-t} r_H]$
- The expectation can be approximated from the batch of trajectories
- Thus we get

$$\nabla_{\boldsymbol{\theta}} J(\boldsymbol{\theta}) = \frac{1}{m} \sum_{i=1}^{m} \sum_{t=1}^{H} \nabla_{\boldsymbol{\theta}} \log \pi_{\boldsymbol{\theta}}(\mathbf{a}_t^{(i)} | \mathbf{s}_t^{(i)}) [Q^{\pi_{\boldsymbol{\theta}}}(\mathbf{s}_t^{(i)} | \mathbf{a}_t^{(i)}) - V^{\pi_{\boldsymbol{\theta}}}(\mathbf{s}_t^{(i)})]$$

- $A^\pi(\mathbf{s}_t, \mathbf{a}_t) = Q^\pi(\mathbf{s}_t | \mathbf{a}_t) - V^\pi(\mathbf{s}_t)$ is the advantage function
- And we get

$$\nabla_{\boldsymbol{\theta}} J(\boldsymbol{\theta}) = \frac{1}{m} \sum_{i=1}^{m} \sum_{t=1}^{H} \nabla_{\boldsymbol{\theta}} \log \pi_{\boldsymbol{\theta}}(\mathbf{a}_t^{(i)} | \mathbf{s}_t^{(i)}) A^{\pi_{\boldsymbol{\theta}}}(\mathbf{s}_t^{(i)}, \mathbf{a}_t^{(i)})$$

https://www.youtube.com/watch?v=S_gwYj1Q-44 (27')

Williams, R. J. (1992) Simple statistical gradient-following algorithms for connectionist reinforcement learning. *Machine Learning*, 8(3-4):229–256

Estimating $V^{\pi}(s)$

- As for estimating $Q^{\pi}(s,a)$, but simpler
- Two approaches:
  - Monte Carlo estimate: Regression against empirical return

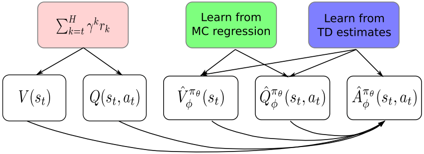
$$\phi_{j+1} \rightarrow arg \min_{\phi_j} \frac{1}{m} \sum_{i=1}^{m} \sum_{t=1}^{H} (\hat{V}^{\pi}_{\phi_j}(\mathbf{s}_t^{(i)}) - (\sum_{k=t}^{H} r(\mathbf{s}_t^{(i)}, \mathbf{a}_t^{(i)})))^2$$

  - Temporal Difference estimate: init $\hat{V}^{\pi}_{\phi_0}$ and fit using (s,a,r,s') data

$$\phi_{j+1} \rightarrow \min_{\phi_j} \sum_{(s,a,r,s')} ||r + \gamma \hat{V}^{\pi}_{\phi_j}(s') - \hat{V}^{\pi}_{\phi_j}(s)||^2$$

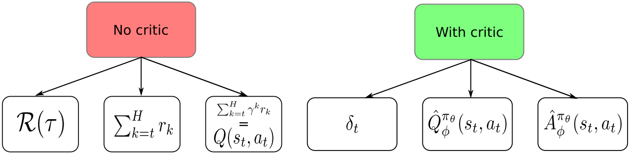
  - May need some regularization to prevent large steps in ϕ

Algorithm 5: adding a state-dependent baseline



- Learn $\hat{V}_\phi^\pi$ from TD, from MC rollouts, or compute $V^{\pi_{\boldsymbol{\theta}}}(\mathbf{s}_t^{(i)})$ from MC
- Learn $\hat{Q}_\phi^\pi$, from TD, from MC rollouts, or compute $Q^{\pi_{\boldsymbol{\theta}}}(\mathbf{s}_t^{(i)}, \mathbf{a}_t^{(i)})$ from MC
- Compute $\hat{A}^\pi(\mathbf{s}_t^{(i)} | \mathbf{a}_t^{(i)}) = \hat{Q}_\phi^\pi(\mathbf{s}_t^{(i)}, \mathbf{a}_t^{(i)}) - \hat{V}_\phi^\pi(\mathbf{s}_t^{(i)})$
- Or even learn $\hat{A}_\phi^\pi$ directly from TD updates using $A^\pi(\mathbf{s}_t, \mathbf{a}_t) = \mathbb{E}[\delta_t]$
- Same as Algorithm 3 using $A^{\pi_{\boldsymbol{\theta}}}(\mathbf{s}_t^{(i)} | \mathbf{a}_t^{(i)})$ instead of $Q^{\pi_{\boldsymbol{\theta}}}(\mathbf{s}_t^{(i)} | \mathbf{a}_t^{(i)})$
- Suffers from even less variance

Synthesis



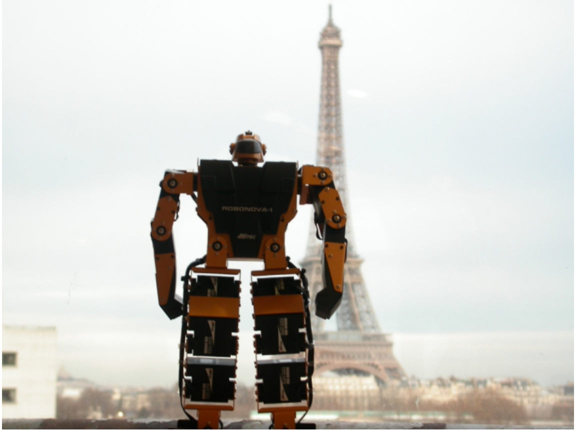
$$\nabla_{\boldsymbol{\theta}} J(\boldsymbol{\theta}) = \mathbb{E}_{\mathbf{s}_t, \mathbf{a}_t \sim \pi_{\boldsymbol{\theta}}(.)}[\nabla_{\boldsymbol{\theta}} \log \pi_{\boldsymbol{\theta}}(\mathbf{a}_t^{(i)} | \mathbf{s}_t^{(i)})] \psi_t \text{ where } \psi_t \text{ can be:}$$

1. $\sum_{t=0}^{H} \gamma^t r_t$: total (discounted) reward of trajectory
2. $\sum_{k=t}^{H} \gamma^{k-t} r_k$: sum of rewards after $\mathbf{a}_t$
3. $\sum_{k=t}^{H} \gamma^{k-t} r_k - b(\mathbf{s}_t)$: sum of rewards after $\mathbf{a}_t$ with baseline
4. $\delta_t = r_t + \gamma V^{\pi}(\mathbf{s}_{t+1}) - V^{\pi}(\mathbf{s}_t)$: TD error, with $V^{\pi}(\mathbf{s}_t) = \mathbb{E}_{\mathbf{a}_t}[\sum_{k=0}^{H} \gamma^k r_{t+l}]$
5. $\hat{Q}_{\phi}^{\pi_{\boldsymbol{\theta}}}(\mathbf{s}_t, \mathbf{a}_t) = \mathbb{E}_{a_{t+1}}[\sum_{k=0}^{H} \gamma^k r_{t+l}]$: action-value function
6. $\hat{A}_{\phi}^{\pi_{\boldsymbol{\theta}}}(\mathbf{s}_t, \mathbf{a}_t) = \hat{Q}_{\phi}^{\pi_{\boldsymbol{\theta}}}(\mathbf{s}_t, \mathbf{a}_t) - \hat{V}_{\phi}^{\pi_{\boldsymbol{\theta}}}(\mathbf{s}_t) = \mathbb{E}[\delta_t]$, advantage function

- Next lesson: Difference to Actor-Critic

John Schulman, Philipp Moritz, Sergey Levine, Michael I. Jordan, and Pieter Abbeel. High-dimensional continuous control using generalized advantage estimation. *arXiv preprint arXiv:1506.02438*, 2015

Any question?



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