

# Advanced concepts of Reinforcement learning

## a practical guide

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# What's the goal

**Learn how to make good decisions under uncertainty**

# Goals of today

- We don't necessarily develop new algorithms!
- Learn how to:
  - identify and set up RL problems
  - apply RL appropriately
  - deal with common problems
- RL is not easy, don't expect to understand or solve things immediately

# Stop, Think, Apply!

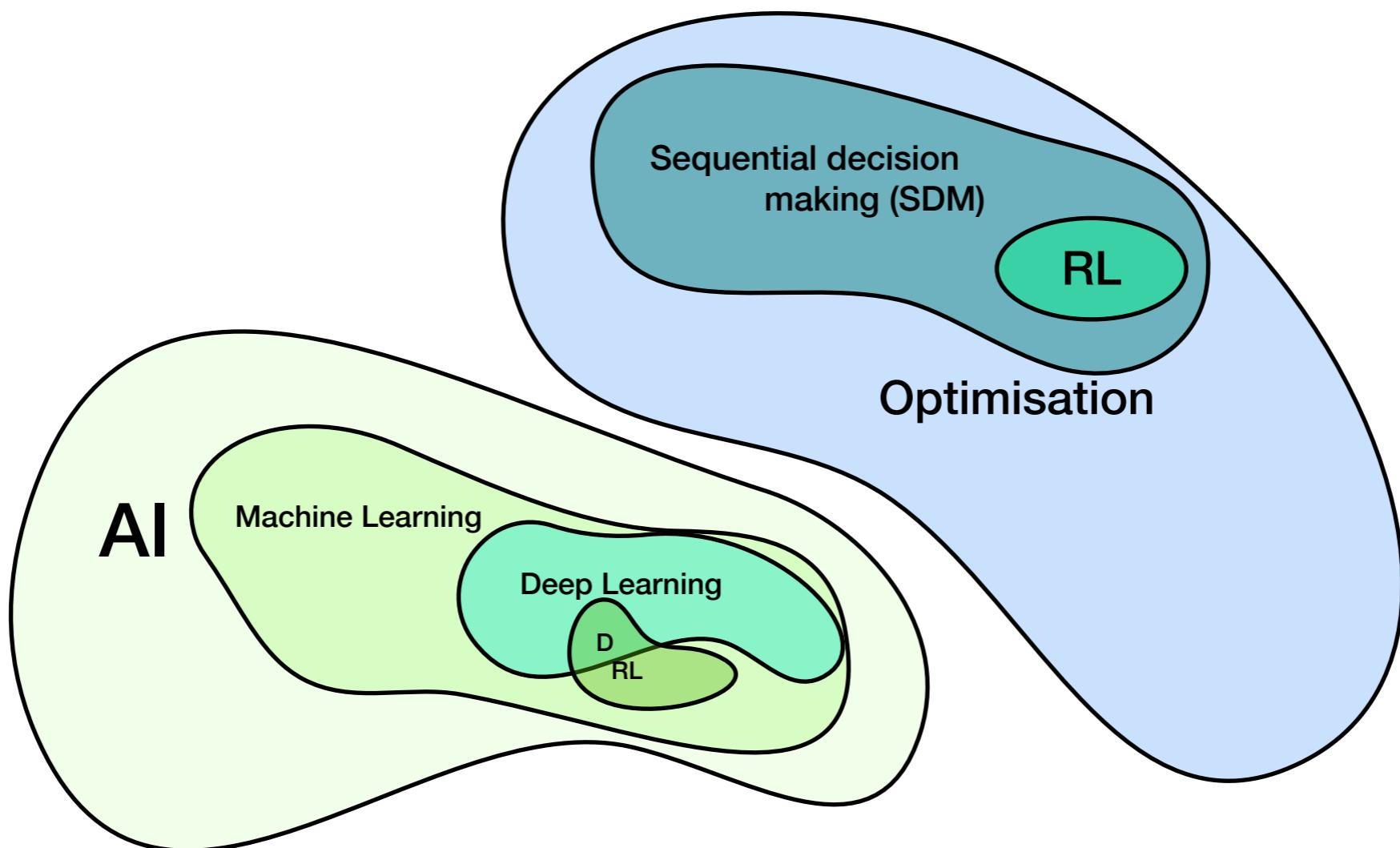
- Pick the right problems!
  - Ask: does this have a chance of solving an important problem? Does my optimization problem have a chance to be solved? Do I solve the right problem?

# RL - what is it today?

- What is addressed by RL: position in AI, community of researchers applying tools, data-driven dynamic programming
- Little knowledge of probabilistic mechanism how data and rewards change over time
- Probe and learn dynamics to find control

B. Recht, 2018

# AI and optimization viewpoint

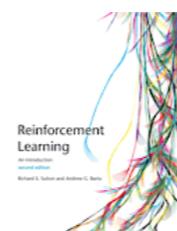


# How to approach RL

- Pick the right problems (important and solvable)!
  - ➔ What is my goal?
  - ➔ What are my observables and my actions?
- Model the problem appropriately
- Training and evaluation of RL
- Are there better alternatives?

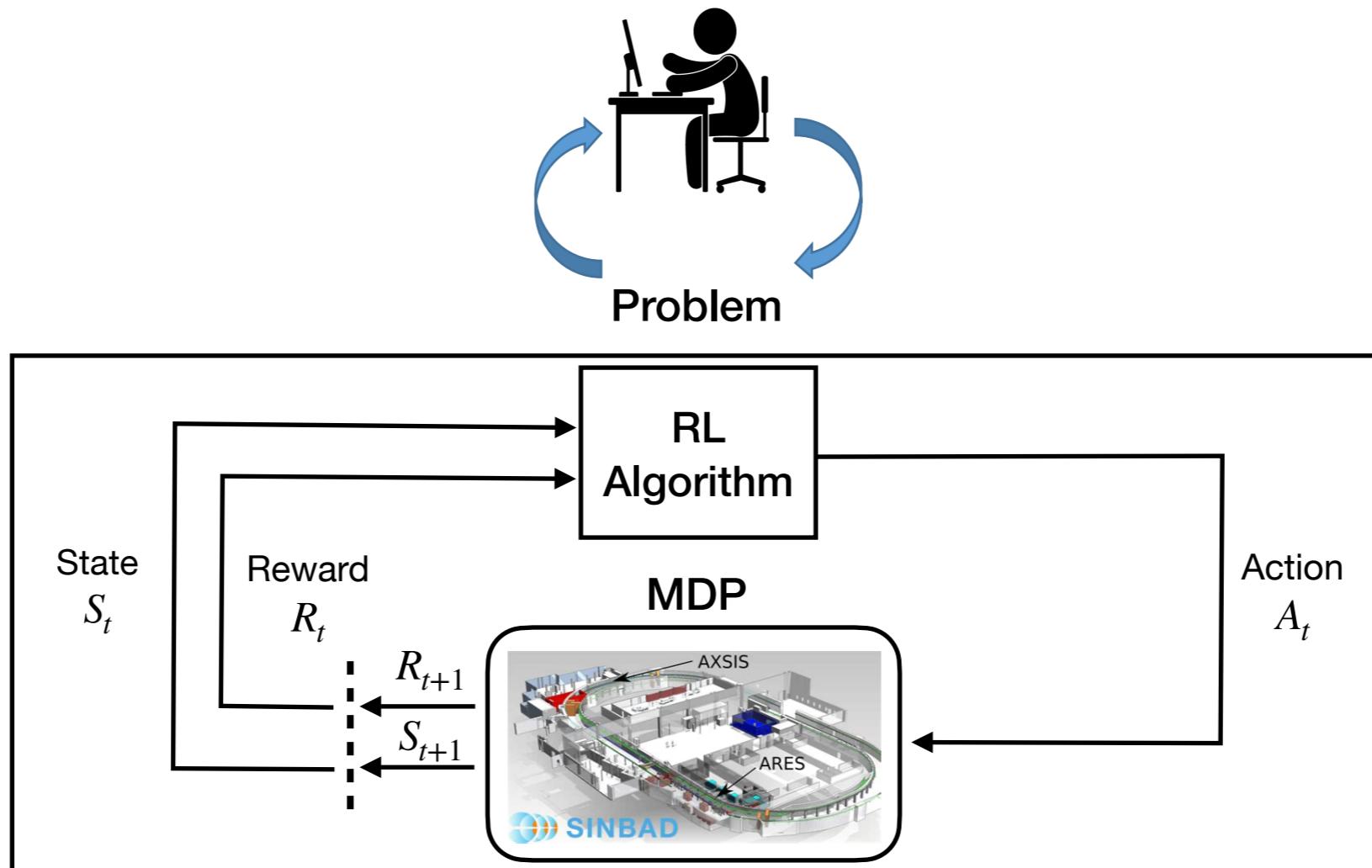
# The entire problem

**SB: 1, 3, 10, 17**



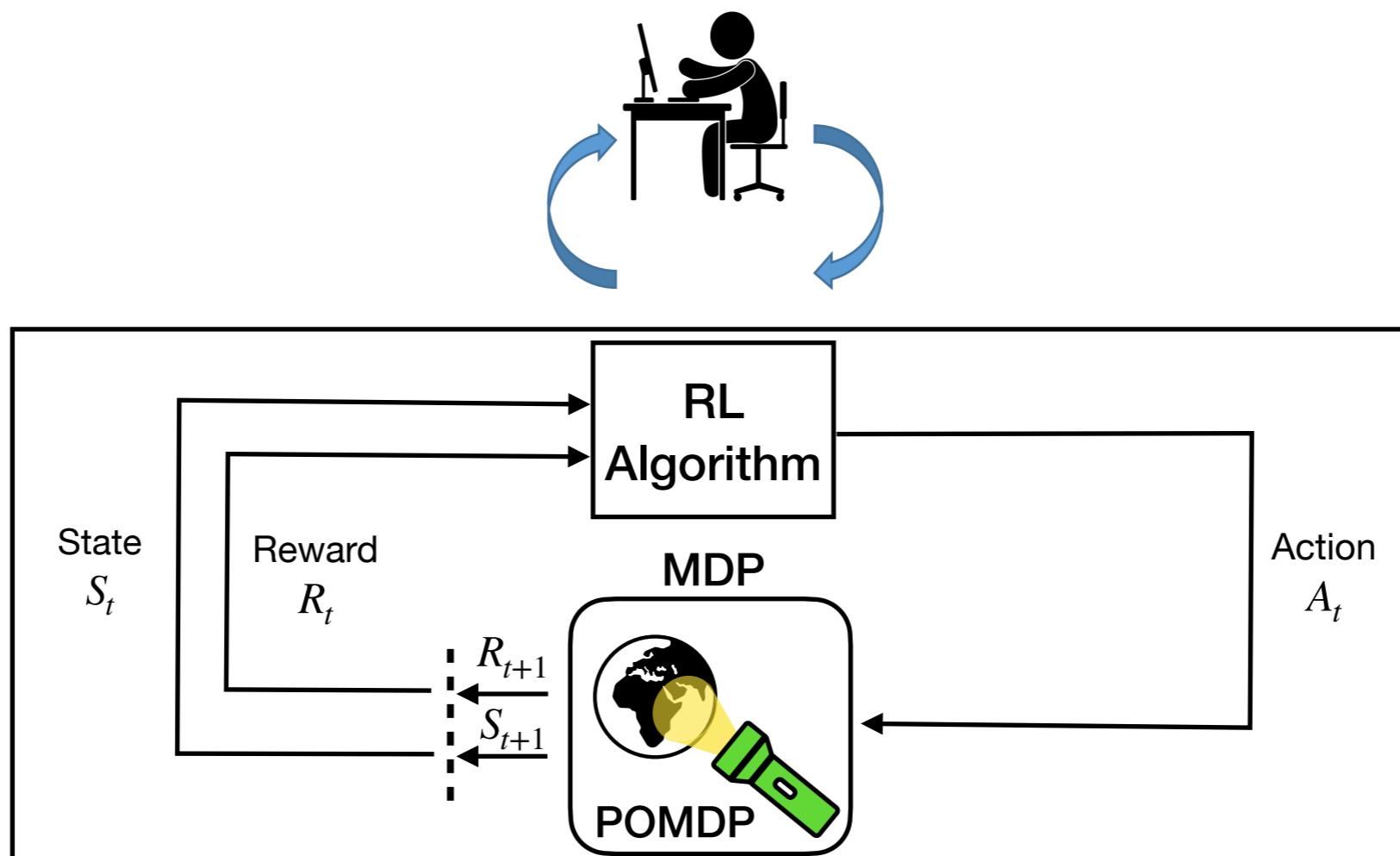
# The entire problem

Markov decision process - MDP



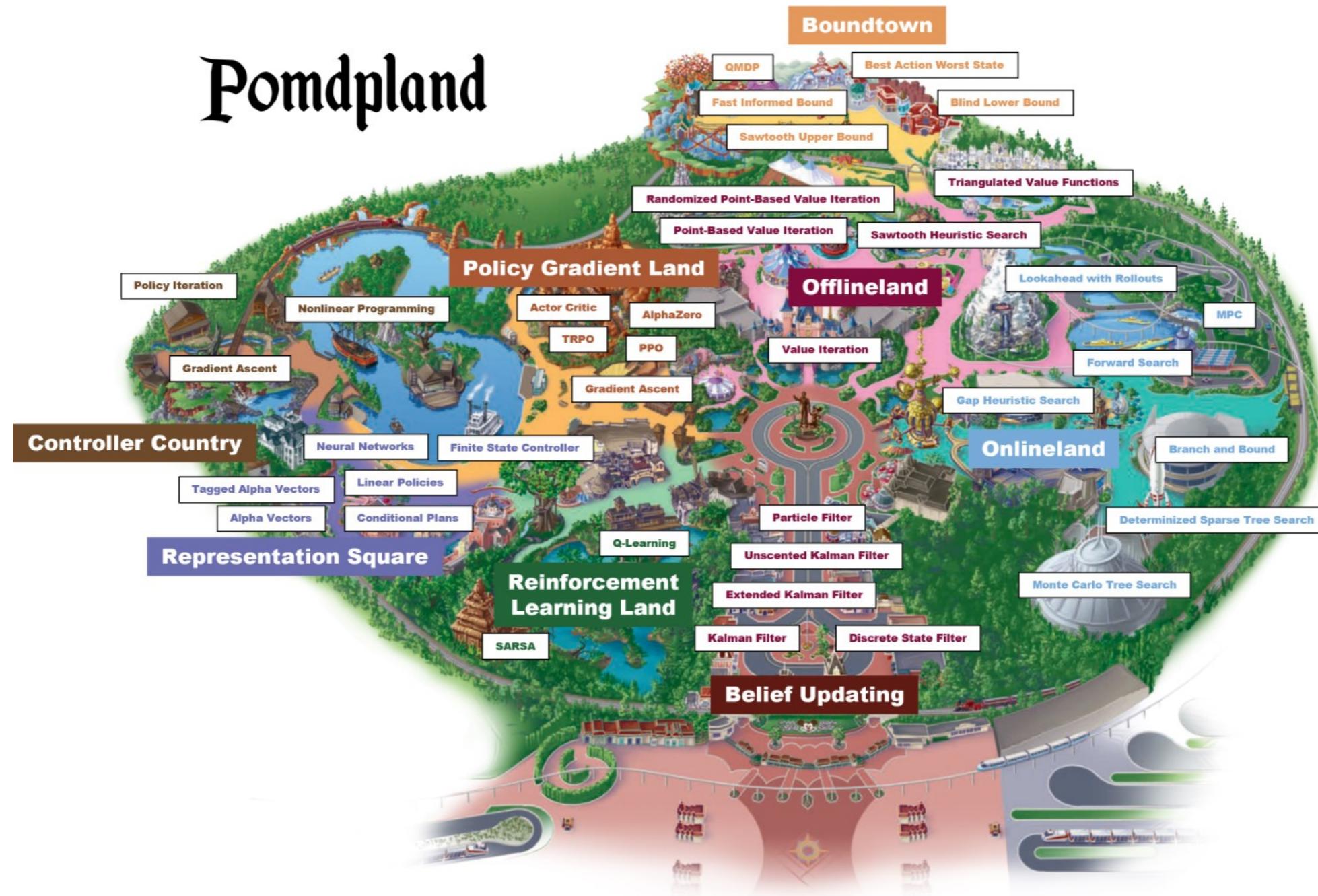
# Common set-up

Partially observable Markov decision process - POMDP



# Wellcome to POMDPs

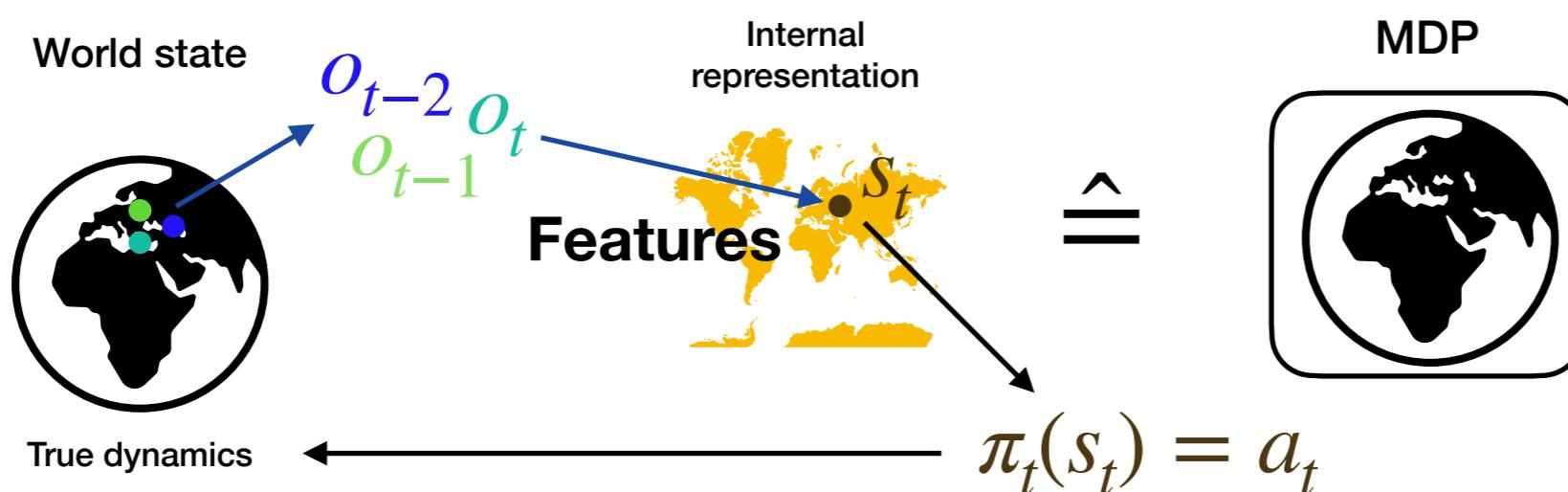
## Pomdpland



From Mykel Kochenderfer

# Problem design - capture the right thing

- We want to solve an SDM problem: Information→Decision→Information→Decision→...
- Such problems are generally be stochastic!
- Consequently we build a feedback system not planning too far in the future:
  - We define a **state**  $s_t = h_t(o_t, a_{t-1}, o_{t-1}, a_{t-2}, o_{t-2} \dots)$ , as a function holding **sufficient statistics** until time step  $t$  for a decision - (example pong)
  - We look for a decision based on  $s_t$  via:  $a_t = \pi_t(s_t)$  - the policy - optimise an expected aggregate of future rewards



- Rarely the observation  $o$  is the state  $s$ , the world state is, but often we assume it is!
- Generally POMDP - today MDPs!

# POMDPs and non stationarity

- POMDP generally P-Space hard (not on average)
- To find a proper state we have to solve the additional prediction problem  $s_t = h_t(o_t, a_{t-1}, o_{t-1}, a_{t-2}, o_{t-2} \dots)$
- In the non-stationary, finite horizon formulation the MDP has the form  $(S, A, \{P\}_h, \{r\}_h, H, \rho_0) \Rightarrow$  Value-functions  $Q_h(s, a)$  get time depended  $\Rightarrow$  similar form of Bellman equations
- We can incorporate time into state e.g.  $\tilde{s} = (s, h) \Rightarrow$  standard MDP
- Generally Bellman equation nice in discounted, stationary formulation  $\Rightarrow$  this is what we usually see and most libraries build on this formulation

# Remarks on MDPs

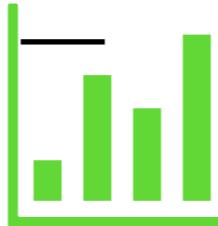
- Mainly we have POMDPs
  - Try to find a state which provides sufficient statistics to solve your problem (internal representation of the agent) - not world state
- Why is MDP so popular?
  - It always possible to make an MDP - by including sufficient information
  - What if not Markov?
    - What happens? Q-learning example
    - Montecarlo - No need for Markov assumption
    - History inclusion - RNNs, LSTMs
- Extreme: Bandits → no states (little knowledge about state)

# The problem modelling

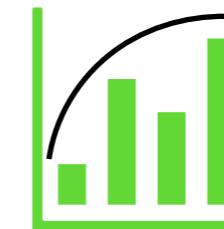
# MDP - the ingredients

- MDP - discounted version:  $(S, A, R, P, \rho_0, \gamma)$ 
  - $P(s', r | s, a) = \mathbb{P}(S_t = s', R_t = r | S_{t-1} = s, A_{t-1} = a)$  - dynamics function
  - Mainly defined by system (and my state and reward definition)
- What do I want to solve? What's the objective function? How to reach the goal? The expectation of reward in the future!

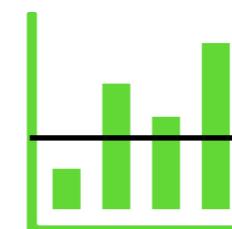
1. Finite horizon:  $\max_{\pi} \mathbb{E}_{\pi} \left[ \sum_{t=0}^T R_t \right]$



2. Infinite horizon:  $\max_{\pi} \mathbb{E}_{\pi} \lim_{T \rightarrow \infty} \left[ \sum_{t=0}^T \gamma^t R_t \right]$



3. Average reward:  $\max_{\pi} \mathbb{E}_{\pi} \lim_{T \rightarrow \infty} \frac{1}{T} \left[ \sum_{t=0}^T R_t \right]$



- Mostly:  $\max_{\pi} \mathbb{E}_{\pi} \lim_{T \rightarrow \infty} \left[ \sum_{t=0}^T \gamma^t R_t \right]$  - solution: stationary policy  $\pi(s)$

# RL and optimization

- All said falls into the domain of optimization:
  - An optimiser tries to find the arguments of a function to maximise the function value (optimization is greedy!)
  - RL algorithms look to find a mapping (the policy) from states to actions maximising the expected cumulative reward rather than just a single optimal function value
    - If parametric function approximation is used, we try to find the values of the parameters of the approximated function (either a value function or the policy directly) to obtain this mapping (this a classical optimisation problem).
- RL is comparable to calculus of variation (its origin is in classical mechanics
  - HJB equation) instead of function optimization

## Optimization

$$\begin{aligned} & \text{maximise}_{\{A_t\}} \sum_{t=0}^T R(S_t, A_t, W_t) \\ & \text{subject to: } S_{t+1} = f(S_t, A_t, W_t) \end{aligned}$$

## RL

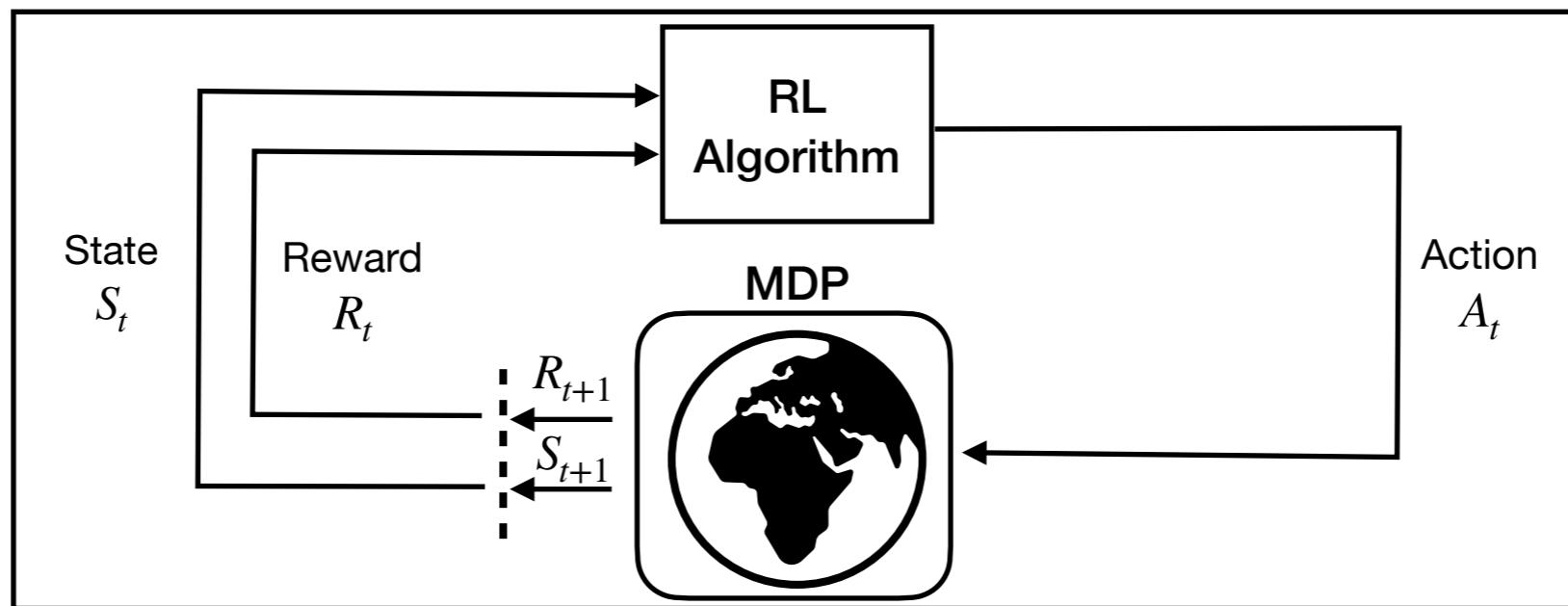
$$\begin{aligned} & \text{maximise}_{\pi_t} \mathbb{E}_{W_t} \left[ \sum_{t=0}^T R_t(S_t, A_t, W_t) \right] \\ & \text{subject to: } S_{t+1} = f(S_t, A_t, W_t) \\ & \qquad \qquad \qquad A_t = \pi_t(S_t, S_{t-1}, \dots) \end{aligned}$$

**Feedback structure takes noise into account**

Often optimization is performed only for one step horizon:

$$\text{maximise}_a R(\cdot, a, W_t)$$

# Episodic training: we probe the system



- The system generates noisy trajectories:  
 $\tau_i = o_{0,i}, a_{0,i}, o_{1,i}, r_{1,i}, a_{1,i}, o_{2,i}, r_{2,i}, \dots$
- From these probes we learn, but how?

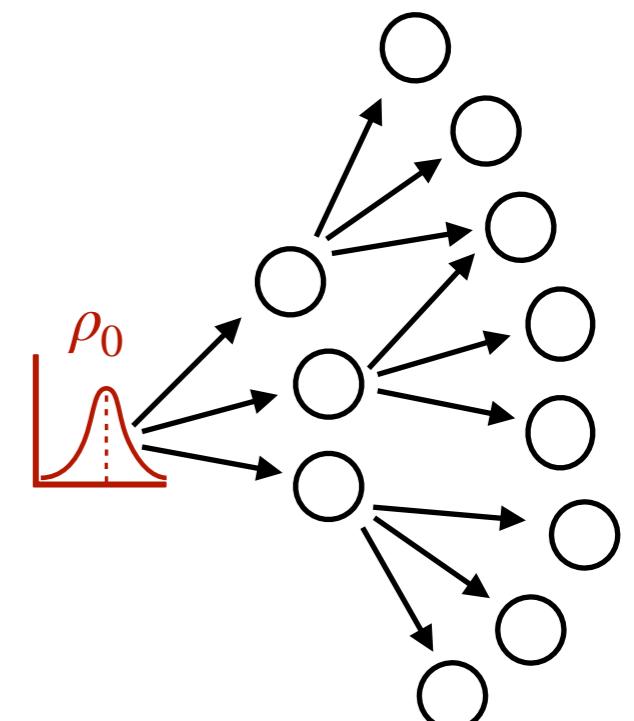
# Learning from episodes

- $(S, A, R, P, \rho_0, \gamma)$

- Rarely we have a full online learning problem
- The problem either is naturally episodic or we train in an episodic manner and are reset (adding absorbing state):

$$\tau_i = o_{0,i} \sim \rho_0, a_{0,i}, o_{1,i}, r_{1,i}, a_{1,i}, o_{2,i}, r_{2,i}, \dots$$

- Design the episodic training:
  - Is it an infinite horizon problem → stability forever?
  - Is it a finite horizon problem with stationary dynamics?
- What is the role of  $\rho_0$ ?
- What is the role of a finite maximum length?
- Exploration (finite time in infinite problem) - not part of the problem



# Reward design and shaping

- $(S, A, R, P, \rho_0, \gamma)$
- The reward includes the goal and how to reach the goal!
- There is an equivalence class of problem formulations leading to the same goal → differences in algorithmic efficiency
  - Example: Negative/Positive/Normalised Reward
- Generally probabilistic:  
$$P(r | s, a, s') = \sum_{s'} P(r, s' | S_t = s, A_t = a)$$
- Can be formulated in dependence of  $s$  and  $a$  or given as a direct feedback signal
- To improve exploration or to solve sparse reward problems - reward shaping during training!

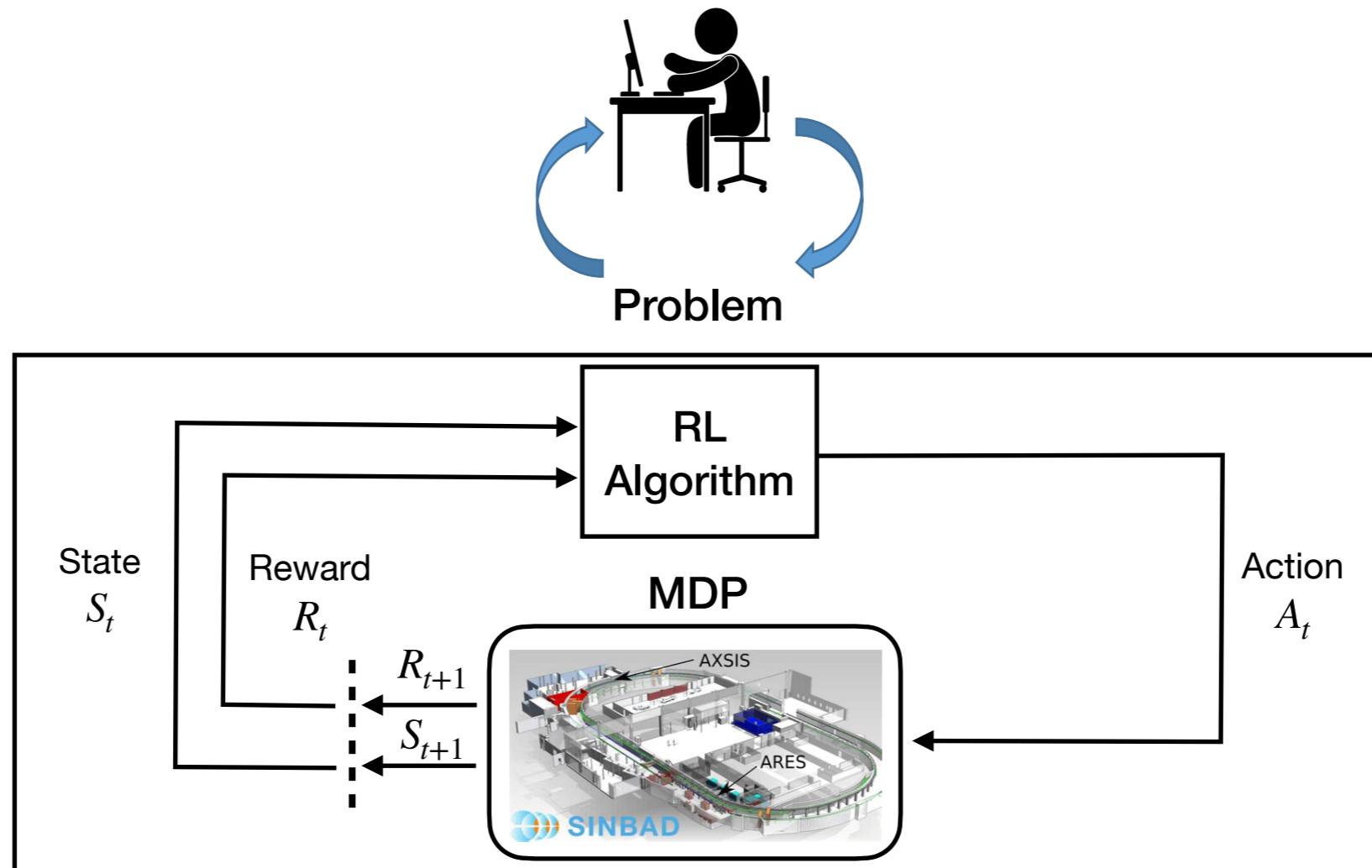
# The discounting $\gamma$ - a hyperparameter?

- $(S, A, R, P, \rho_0, \gamma)$
- Generally we don't need discounting
- Introduced due to mathematical convenience:  
convergence of cumulative sum of rewards as  
alternative to mean reward: 
$$\sum_t^N \gamma^t R_t \leq \frac{1}{1-\gamma}, \text{ when } R_t \text{ is bounded in } [0,1]$$
- Can be used influence training performance
- Not needed in naturally episodic problems!

# Summary

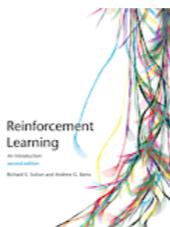
- Capture the right problem
- Formulate the MDP appropriately
- (Problem equivalent) Design has impact on RL algorithm
- Problem I solve = Designed MDP + Reward objective

# The entire problem



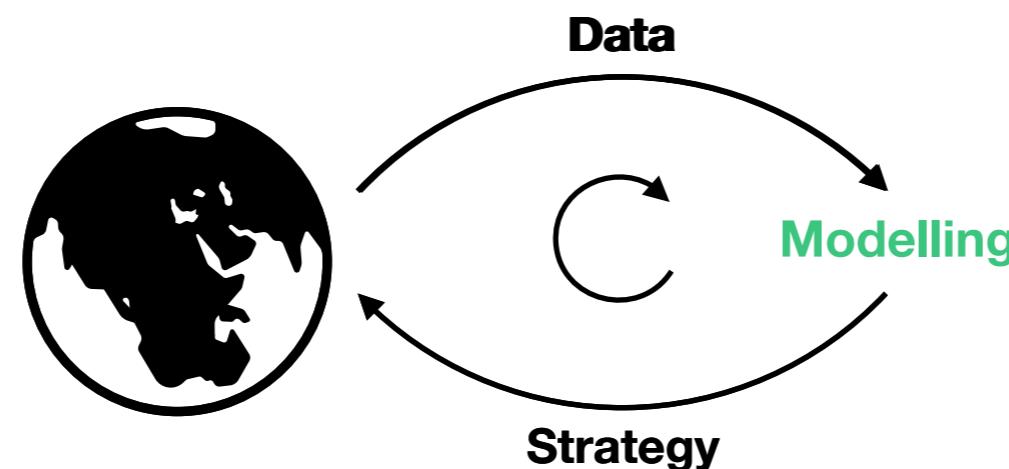
# About Machine Learning

**SB: 9**



# RL and decision theory

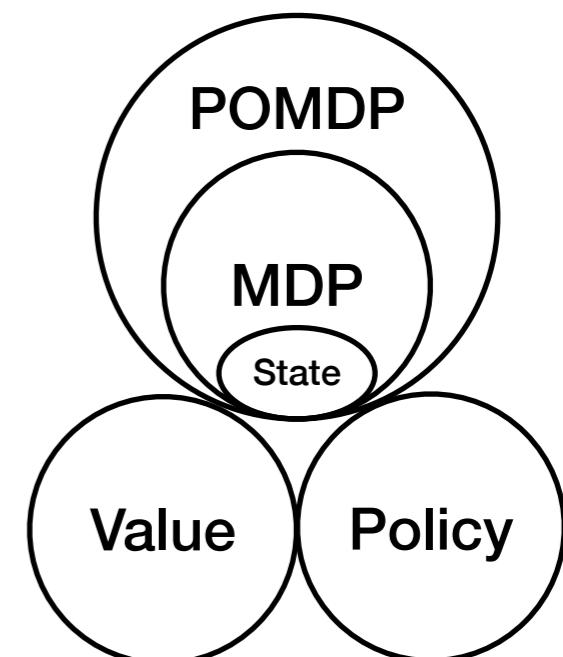
Information → decision → Information → decision → Information → ...

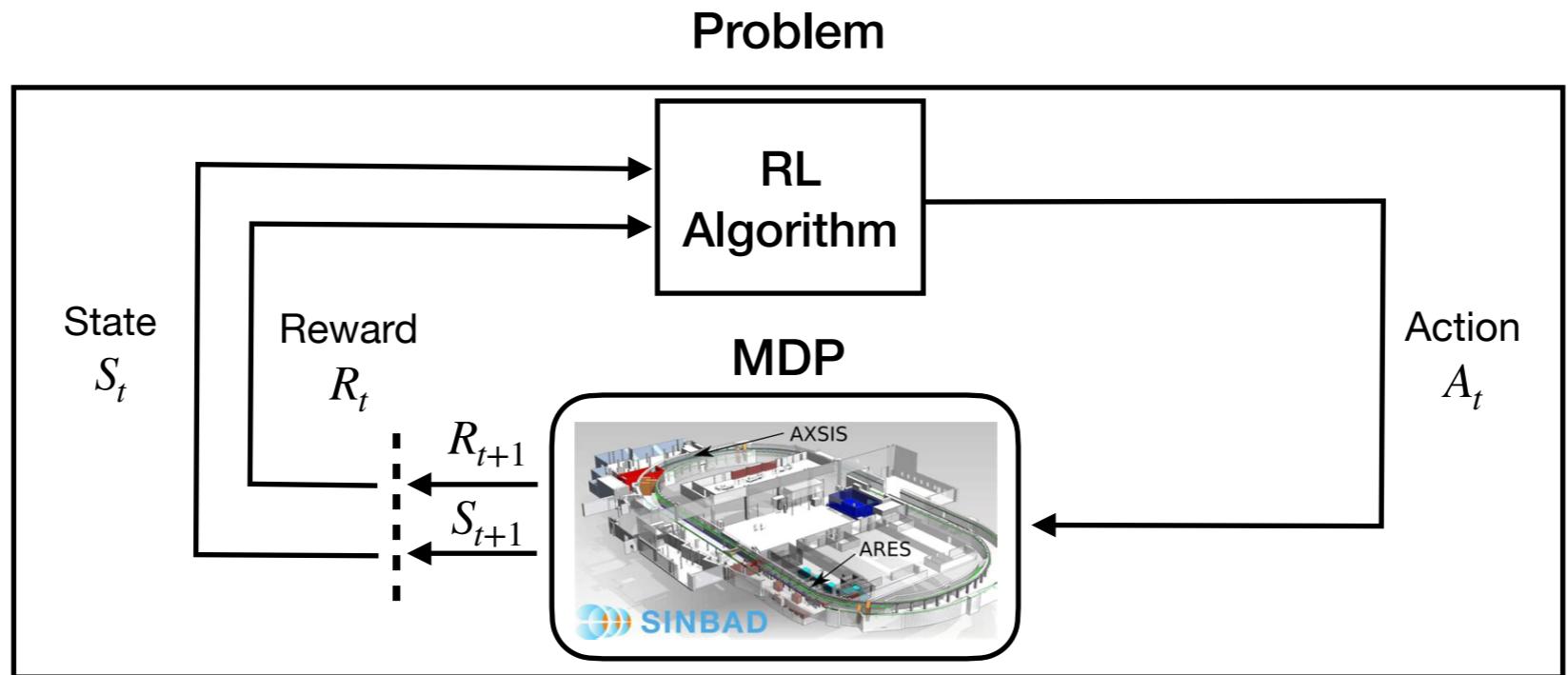


- One step horizon offline RL  $\Rightarrow$  Prediction  $\mathbb{P}(Y_i | X_i)$  - pattern recognition or supervised learning (SL)
- One step horizon RL  $\Rightarrow$  active Learning - e.g. system identification
- RL is a multi step **optimization** problem - we learn about the dynamics of the world

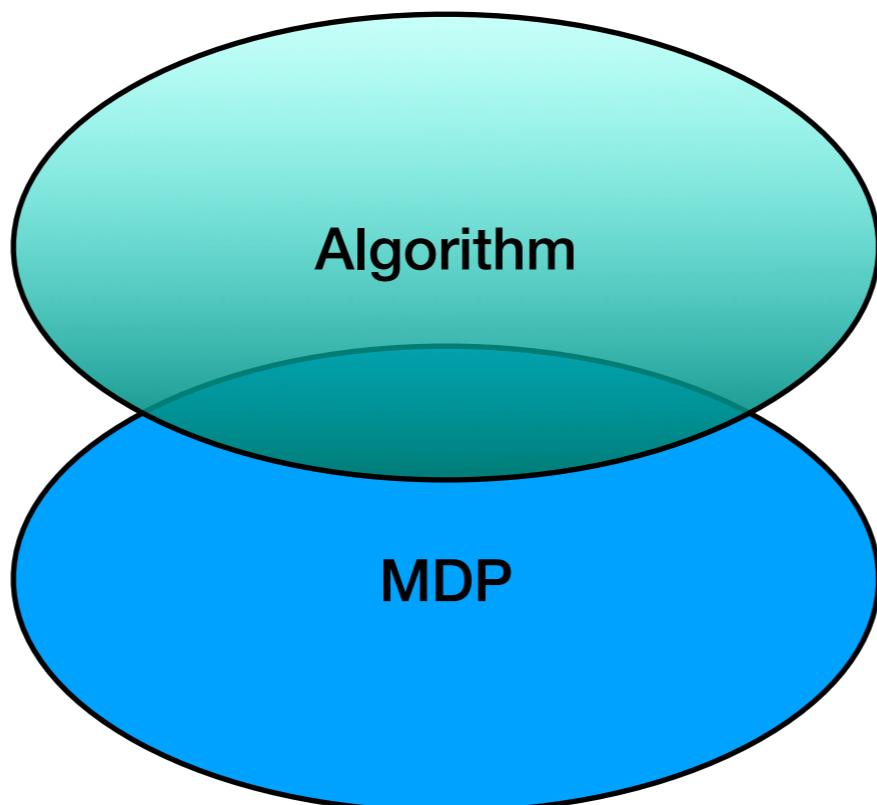
# Prediction

- Function approximation (FA):
  - Parametric - compact approximation of a function using a parametrised representation  $f(x) \approx \hat{f}(x, \bar{\theta})$ , where  $\bar{\theta}$  are parameters to be adapted
    - Fixed representational power
    - Constant computationally complexity - fixed set of parameters
    - Example: Artificial neural networks (ANNs), linear approximations...
  - Non-parametric - memory based:
    - $f(x) \approx \hat{f}(x, \mathcal{D}) = \sum_{x' \in \mathcal{D}} k(x, x') g(x')$ 
      - Data
      - Kernel
      - Weight
    - No fixed representational power
    - Parameters are not learned directly
    - Computationally complexity grows with data
    - Example: Gaussian processes, Kernel-based methods,...

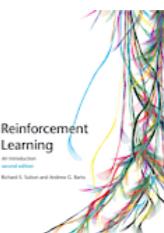




# Solving the problem



**SB: 6, 8, 9, 10, 11, 13**



# Most common issues RL - algorithmic side

- Sample efficiency
- Stability
- Run time
- Hyperparameter tuning
- Exploration

# Remarks on RL

- Trial and error
- Only rewards (labels of visited state, actions)
- Policy decides what we learn - usually censored
- Only valid estimates of things sufficiently learned

# Special cases

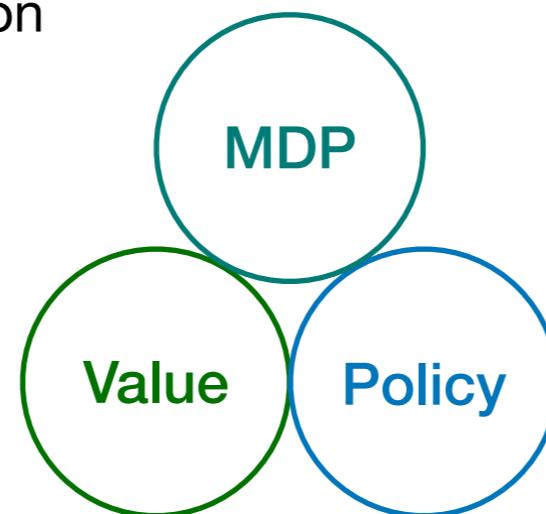
- We know  $P(s', r | s, a)$  and the MDP is finite - frame work of dynamic problem can be used (Bellman update is exact):
  - Good theoretic properties
  - Playground of classical control theory
  - If large, we use sampling: approximative dynamic programming
- If  $P(s', r | s, a)$  is known, the dynamics is linear and we design a quadratic dependence of  $s$  and  $a$  of the reward: analytical solutions (the popular Linear Quadratic Regulator -LQR).  
Stationary dynamics → Bellman → Riccati equation - static state feedback.
- Alternative solution methods?
  - E.g. linear programming

# Hidden Markov Models - POMDPs

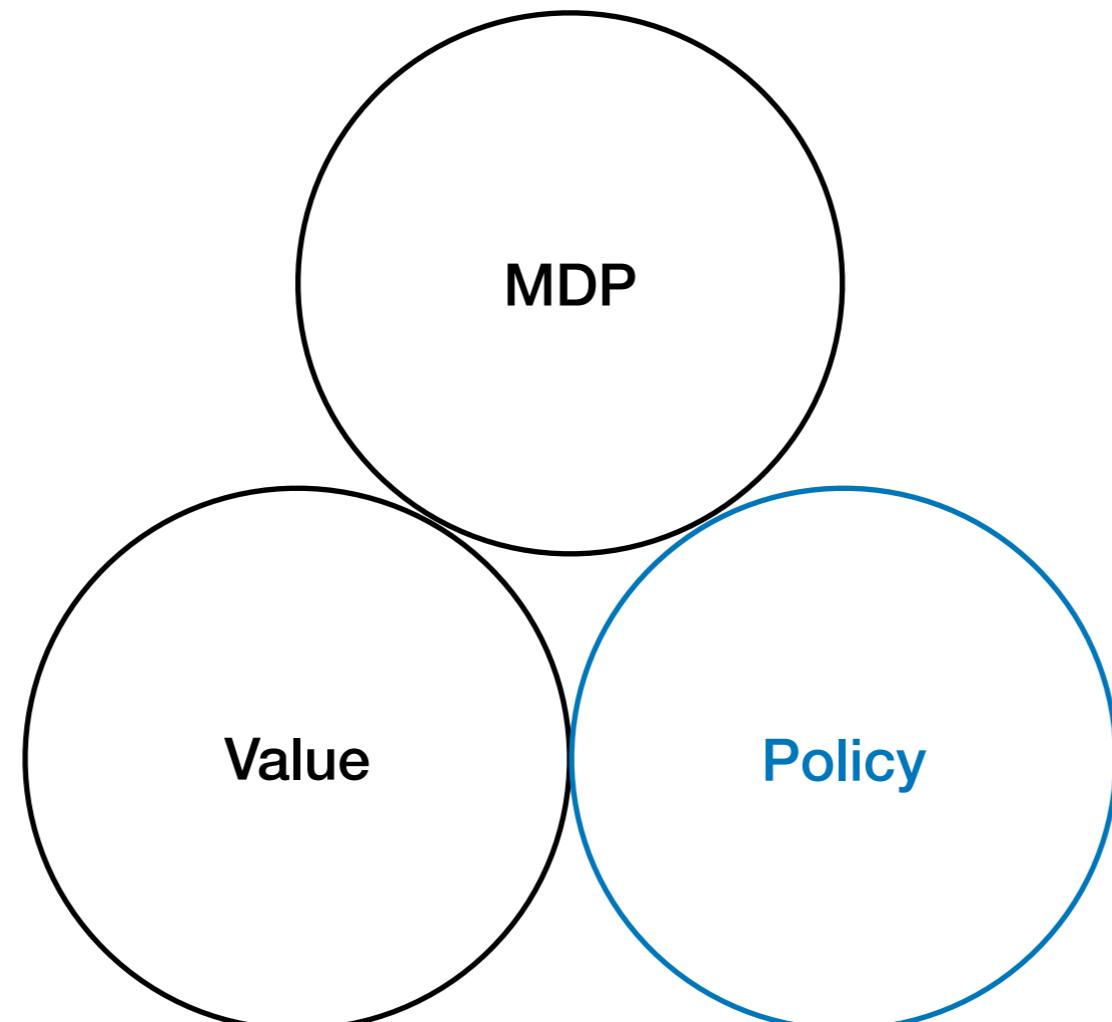
- Linear POMDP: belief state -  $O_t = h_t(S_t, A_t, W_t)$ 
  - Static output feedback is NP hard (linear in O and dynamics)
  - General POMDPs are PSPACE hard
- There are ways out - separation principle:
  - Filtering  $\hat{S}_t = f(\{o_t\})$  - prediction problem
  - Action based on certainty equivalence
  - Optimal filtering - if dynamics are linear and noise is Gaussian - Kalman filtering - general belief propagation - LQG
  - Kalman filtered state - duality between estimation and control
  - Estimate state with prediction  $S_t = h(\tau_t)$ ,  $\tau_t$  are time lags

# General cases

- $P(s', r | s, a)$  unknown → approximative methods
- MDP finite: approximative dynamic programming
- General MDPs: Continuous  $A, S$  spaces
  - Value function approximated with FA
  - Policy approximated with FA
  - Model of the MDP (learn from data or from simulator) use certainty equivalence
  - Trained in a stochastic fashion



# Policy based



# Direct policy search

- RL as derivative free optimization:
  - maximise $_{z \in \mathbb{R}^d} R(z) \Rightarrow \text{maximise}_{p(z)} \mathbb{E}_p[R(z)]$
  - Parametrise a distribution  $p(z; \theta) \Rightarrow \text{maximise}_{p(\theta)} \mathbb{E}_{p(z; \theta)}[R(z)]$
  - Likelihood trick - estimate the derivative:

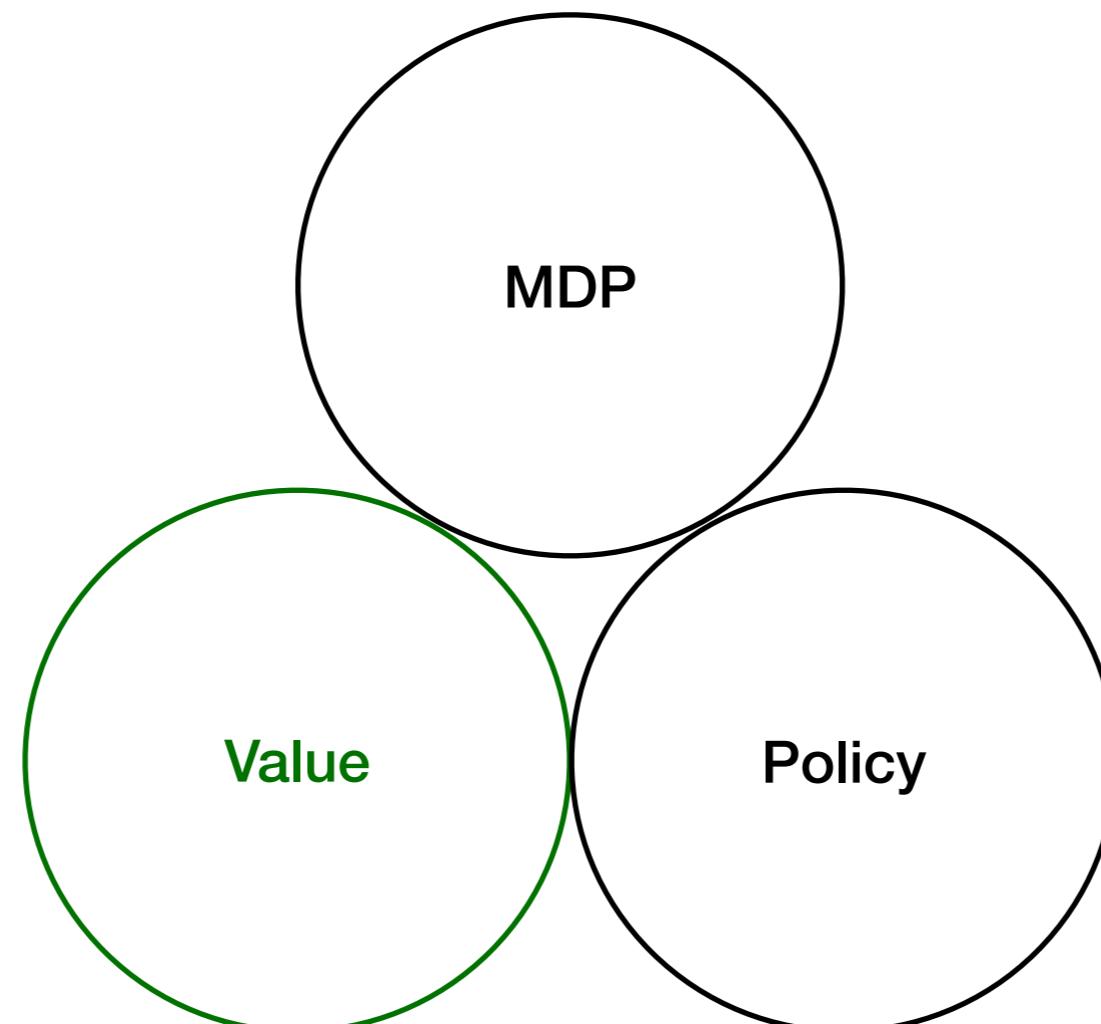
$$\begin{aligned}\nabla_\theta J(\theta) &= \int R(z) \nabla_\theta p(z; \theta) dz = \int R(z) \frac{\nabla_\theta p(z; \theta)}{p(z; \theta)} p(z; \theta) dz \\ &\bullet \quad = \int R(z) \nabla_\theta \log p(z; \theta) p(z; \theta) dz = \mathbb{E}_{p(z; \theta)}[R(z) \boxed{\nabla_\theta \log p(z; \theta)}]\end{aligned}$$

- Unbiased gradient estimate of  $J$ , if sample efficiently from  $p(z; \theta)$  and  $\log p(z; \theta)$
- High variance

# Probabilistic trajectories

- Objective if episodic:  $J(\theta) = V^{\pi_\theta}(s_0) := V(\theta)$ 
  - Stochastic search: pure random search, Simplex, Bayesian optimization
- Using the gradient:
  - $V(\theta) = \sum_{\tau} P(\tau; \theta) R(\tau)$ 
    - Trajectory probability
    - Trajectory reward
  - $\nabla_{\theta} V(\theta) = \sum_{\tau} P(\tau; \theta) R(\tau) \nabla_{\theta} \log P(\tau; \theta) = \mathbb{E}[R(\tau) \nabla_{\theta} \log P(\tau; \theta)]$ 
    - Stochastic gradient
    - Log likelihood trick
  - Sampling of  $A_t \sim p(\cdot | \tau_t; \theta)$ 
    - Handle probabilistic policies (example)
    - High dimensional and continuous action spaces
    - Reinforce algorithm considers temporal structure
  - Finite difference approximation  $\hat{=}$  Reinforce algorithm

# Value based



**The value-function is introduced to compare policies**

# Basis of Q-Learning - Temporal difference (TD) learning

- If one had to identify one idea as central and novel to reinforcement learning, it would undoubtedly be temporal-difference learning. (Sutton and Barto, p.113)
- $V^\pi(s) = \mathbb{E}_\pi[\sum_t \gamma^t R_{t+1} | S_t = s] = \mathbb{E}_\pi[R_{t+1} + \gamma V(s') | S_t = s]$
- Estimated via sampling: TD(0) error
- $\hat{V}(S_t) \leftarrow \hat{V}(S_t) - \alpha [R_{t+1} + \gamma \hat{V}(S_{t+1}) - \hat{V}(S_t)]$   
Learn rate
- Can be used in episodic and non-episodic scenarios
- Immediate update of the estimator
- If probabilities known - exact update

Bootstrapping =  
estimating from estimator

# Q-learning - issues

Bellman equation:

$$Q^*(s, a) = \mathbb{E}[R_t + \max_{a'} Q^*(s', a')]$$

Bellman-operator is a contraction operator ( $L^2$ norm) - converges to a fixed point

Here - stochastic approximation:

$$\hat{Q}(s, a) \approx \hat{Q}(s, a) + \alpha[R_t - \max_{a'} \hat{Q}(s', a')]$$

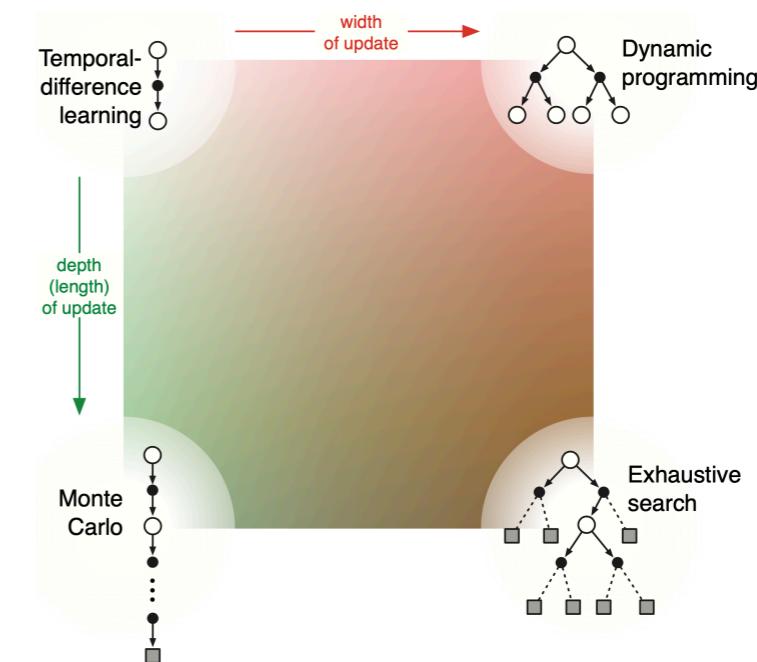
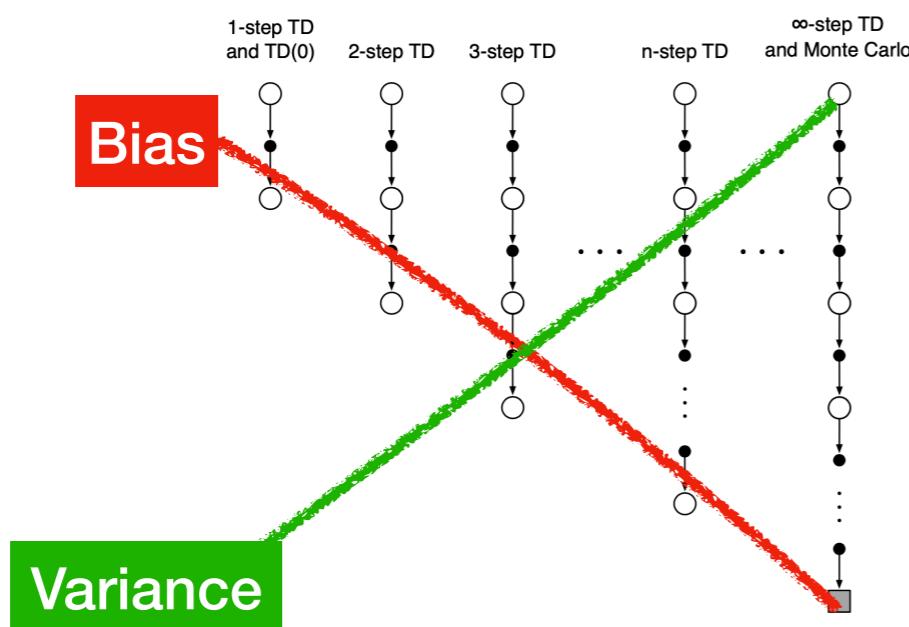
- $\pi(s) = \operatorname{argmax}_a Q(s, a)$
- Two immediate consequences:
  - Maximisation bias (expectation and maximisation don't commute)
  - Bias through bootstrapping
- Inefficient update (compare to e.g. SL)
- If FA is used contraction property might lost
- Might diverge - **deadly triad**:
  - FA
  - Bootstrapping
  - Off-policy

# Deep approximate dynamic programming

- Value dominated
- Tries to mitigate maximisation bias (double networks)
- Stabilises training of networks through tricks (random batches from replay buffer, target network, action noise) - (DDPG, TD3)
- More recent: distributional learning (D4PG), truncating trajectories (TQC)
- ....

# Modern algorithms

- Interplay of Bias and Variance
- Policy dominated: add baseline - the critic!
- DDPG: maximisation operator in Bellman equation is approximated - the actor!

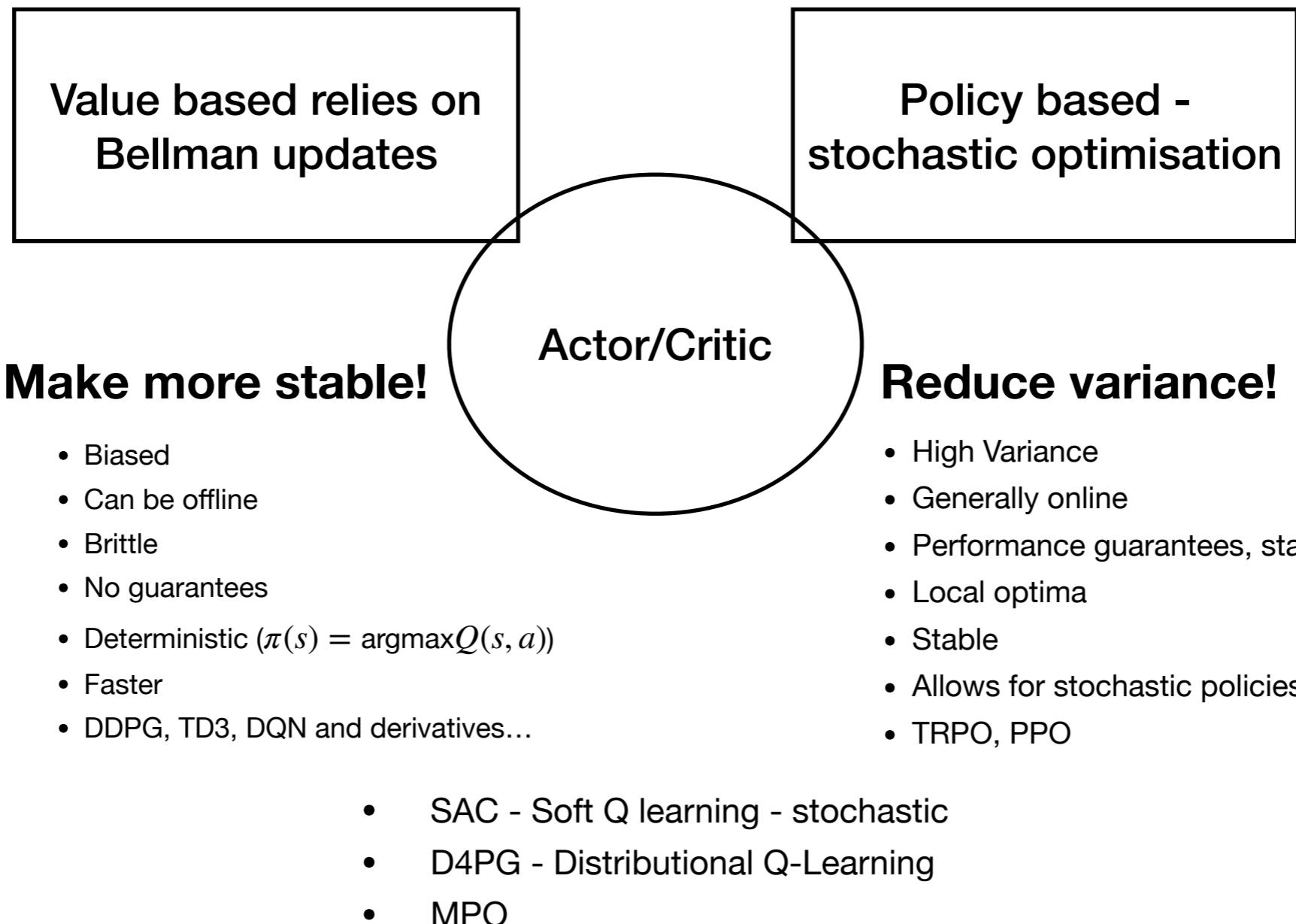


From Sutton and Barto

# Modern landscape

- Bias-Variance trade-off
- Regulated via Policy based and Value based methods
- Policy gradient regulated via update-length of value function

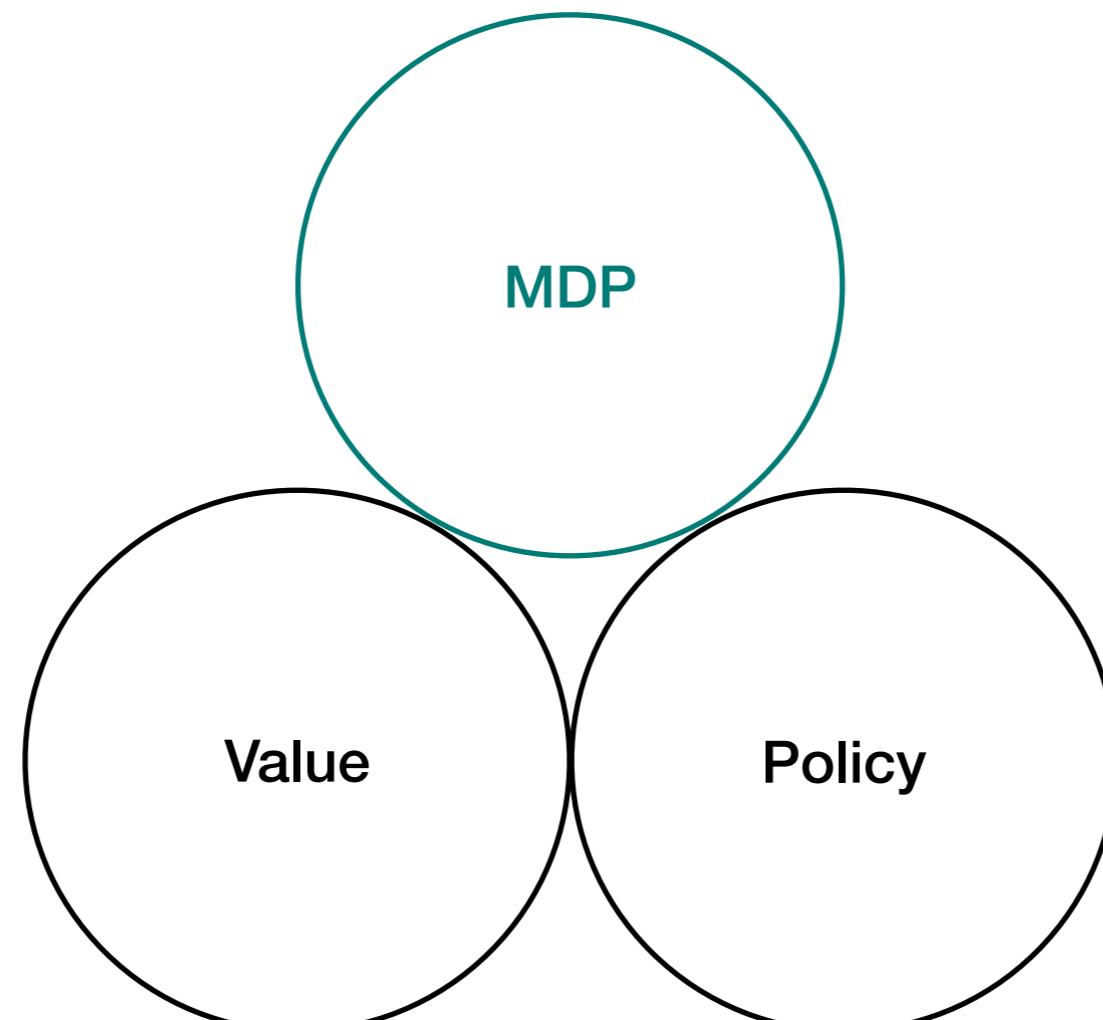
# Value vs policy dominated



# Summary

- Value based: Approximate bellman updates - biased
- Policy based: high variance but stable
- Modern actor critic algorithms: bias-variance trade-off

# Model of the MDP



**Separation heuristics**

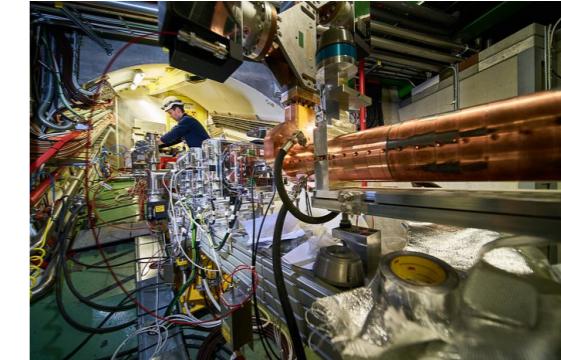
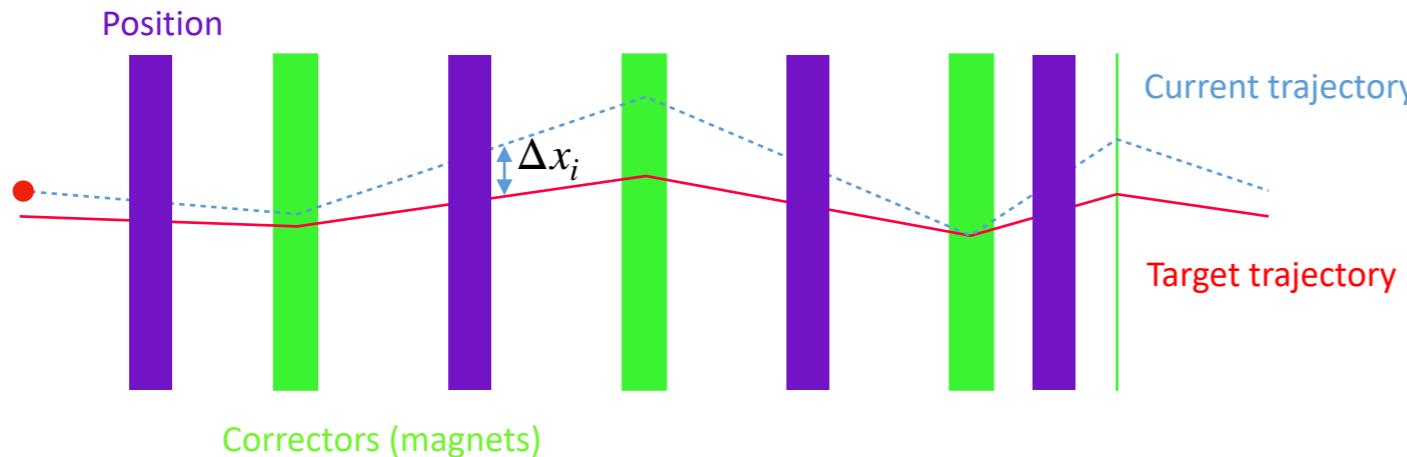
# Modelling the MDP

- We can have a **simulation**  $\hat{P}(r, s' | s, a)$  which models the real (true) MPD  $P(r, s' | s, a)$
- We can learn the MDP from real data using FA
- During training the RL algorithm we use  $\hat{P}(r, s' | s, a)$  as it would be  $P(r, s' | s, a)$  and hope to solve the problem (certainty equivalence)
- We can use a mixture of both
- Deal with consequences, try to learn  $\hat{P}(r, s' | s, a)$  as accurate as possible (system identification)

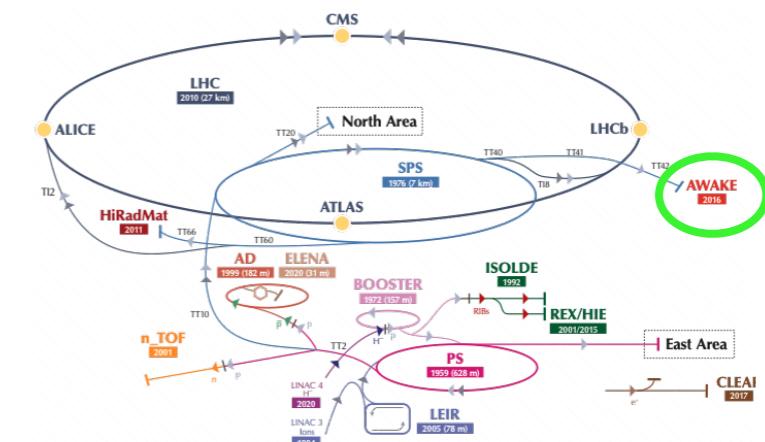
# Train a good MDP

- What degrees of freedom to excite to learn the control problem?
- Model the uncertainties appropriately
  - Learn a model capturing the epistemic and aleatoric uncertainty (the noise)
  - Take the epistemic uncertainty appropriately into account
  - Examples: ensembles of ANNs, Bayesian ANNs, GPs...
- Having a model allows for planning
  - Horizon is critical!
  - Safety constraints can be taken into account

# AWAKE Simulation Benchmark



- $R = - \left( \sum_i \Delta x_i^2 \right)^{\frac{1}{2}}$
- 10 magnets =  $\{k_1, k_2, \dots, k_{10}\}$
- 10 positions =  $\{\Delta x_1, \Delta x_2, \dots, \Delta x_{10}\}$

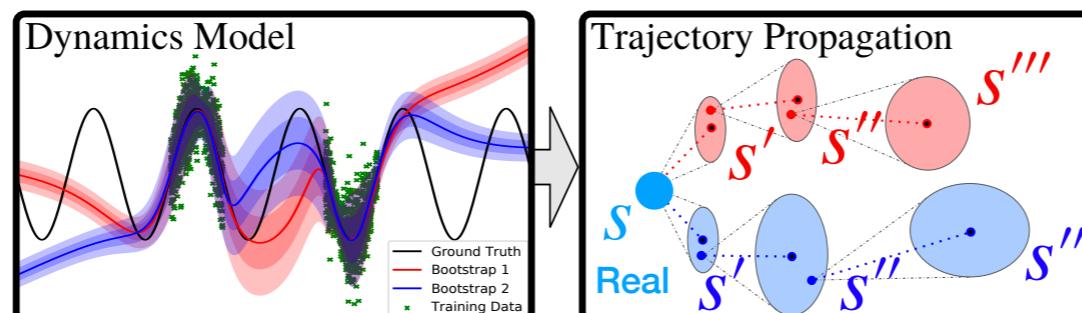
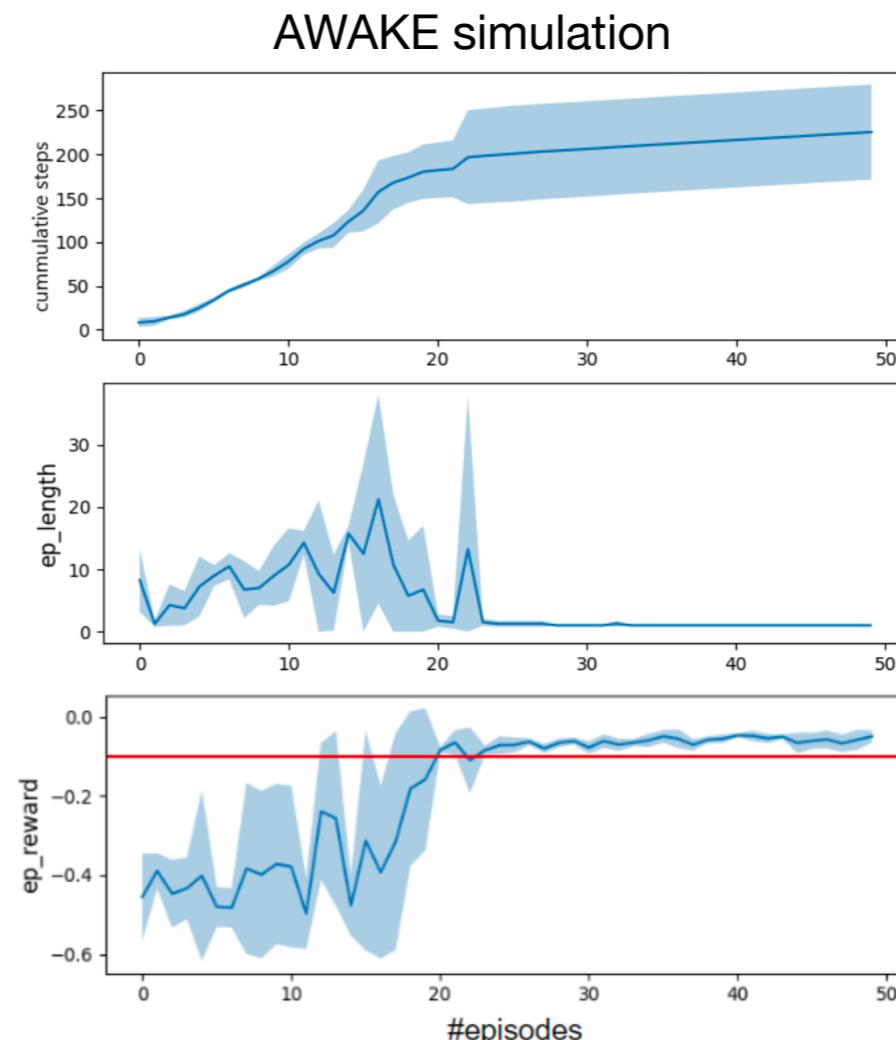


Target: trajectory steering - correct the trajectory in as little steps as possible.

# Example: Model based policy optimization

- Dyna style algorithm
- Model: Bayesian ANN bootstrapped ensemble
- RL algorithm: SAC
- Short roll-outs from real interactions
- Monotonic behaviour
- Many hyper-parameters

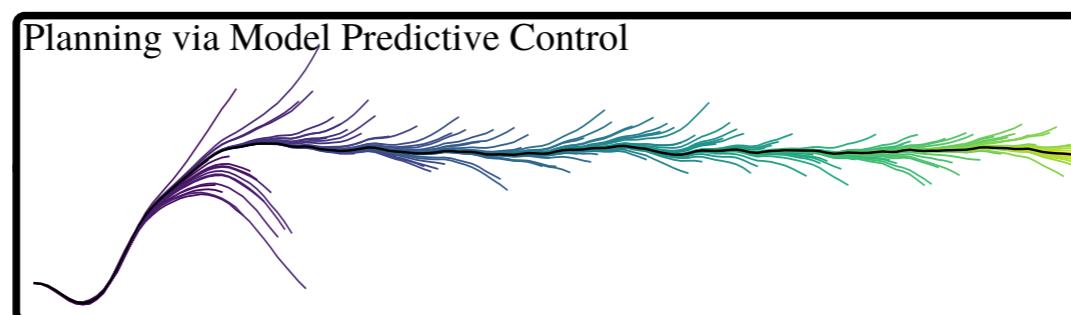
<https://arxiv.org/abs/1906.08253>



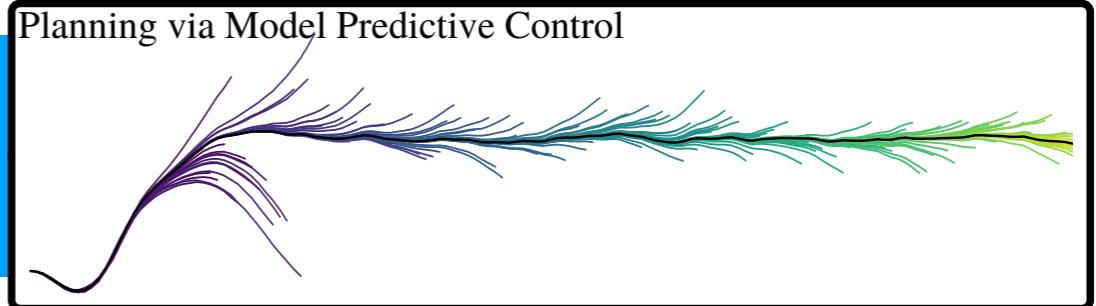
Adapted from <https://arxiv.org/abs/1805.12114>

# Example: Model predictive control (MPC)

- If convergence quickly - no long term planning needed - solve infinite horizon problem with short term planning (open loop)
- Plans action sequence (optimisation) and takes first action only then replan
- Might retrain MDP model each step



# Example: MPC



- Solves the infinite optimization problem ( $W_t$ =random variable):

$$\text{maximise}_{\pi_t} \mathbb{E} \left[ \sum_{t=0}^{\infty} R_t(S_t, A_t, W_t) \right]$$

Optimise finite horizon

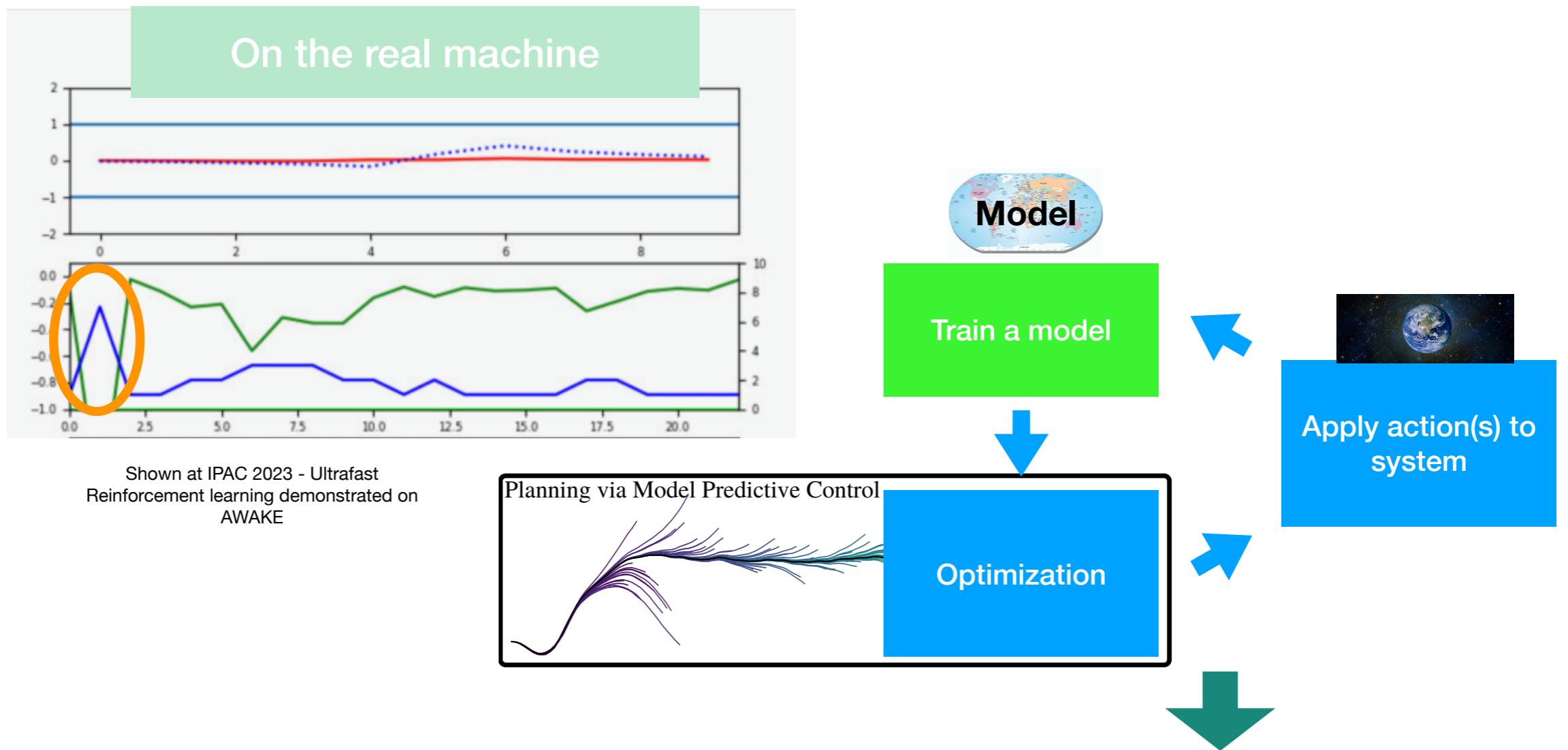
Final cost performance and robustness

$$\approx \text{maximise}_{\pi_t} \mathbb{E}_{W_t} \left[ \sum_{t=0}^T R_t(S_t, A_t, W_t) + V(S_{T+1}) \right]$$

- subject to:  $S_{t+1} = f(S_t, A_t, W_t)$
- $A_t = \pi_t(S_t, S_{t-1}, \dots), \quad S_0 = s$

# MPC with Gaussian Processes

- Data-Efficient Reinforcement Learning with Probabilistic Model Predictive Control
- Few shot RL learning on AWAKE



# Summary

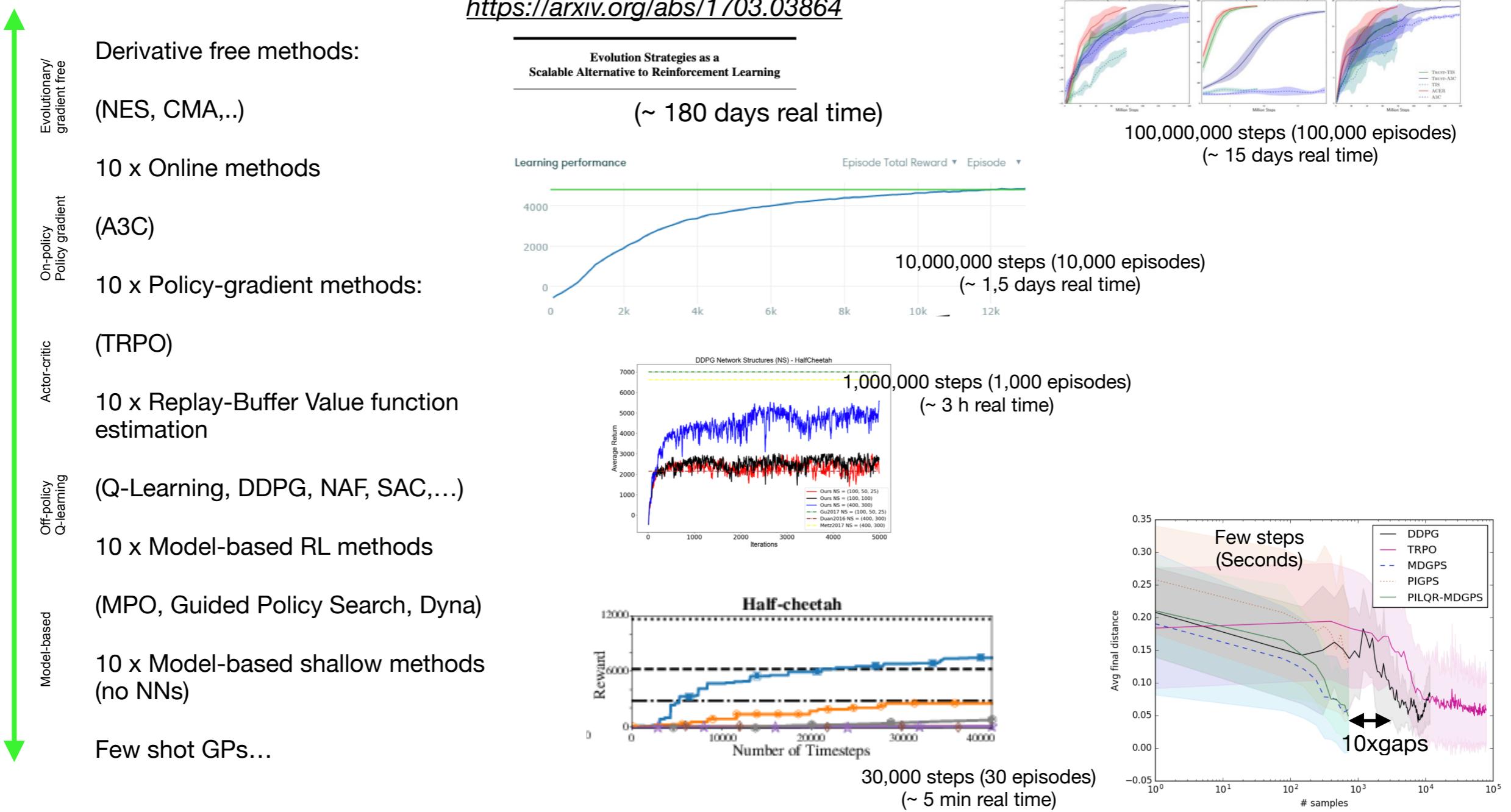
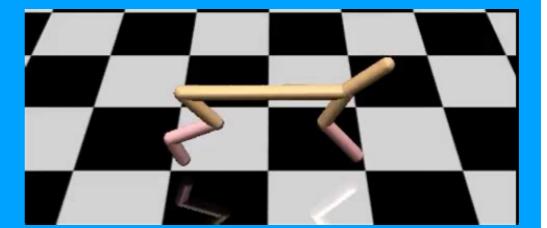
- Modelling the MDP - certainty equivalence
- Modelling epistemic and aleatoric error
- Short horizons and feedbacks avoid model bias
- With a model safety concerns can be considered

# General comments on RL

# Exploration/Exploitation

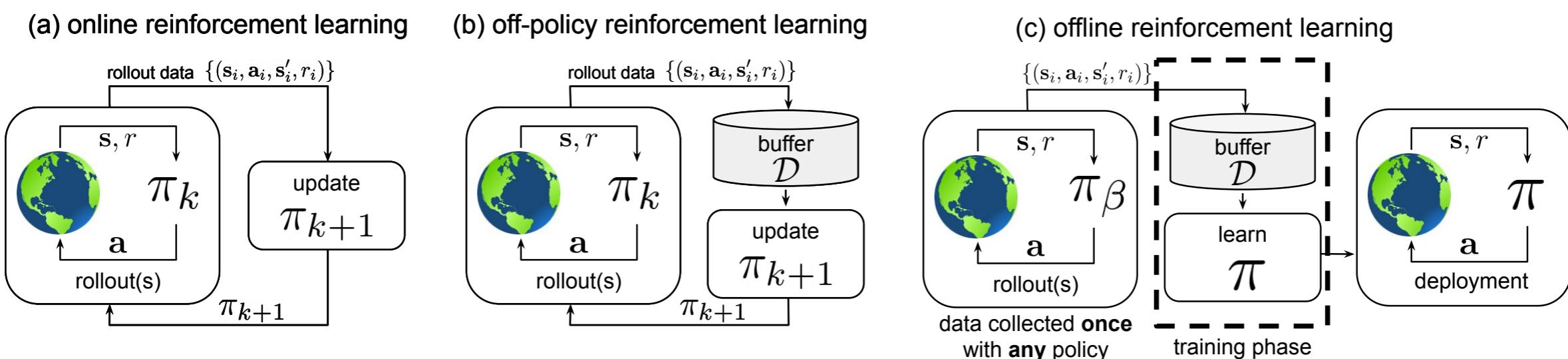
- You only can estimate things you have sufficiently learned!
- Finite MDP:  $\varepsilon$  - greedy, what does this mean?
- Gaussian noise - continuous actions
- Boltzmann exploration
- Theory: Bandit algorithms to study trade-offs (Book)

# Sample-Complexity



# Use existing data?

- On-policy, off-policy, offline training



<https://arxiv.org/format/2005.01643>

# Some training tips

- How to measure the performance of RL algorithms?
  - Take different seeds!
  - Return per episode on validation during training
  - Episode length
- How to set up your algorithmic environment?
  - Only start to implement, if there is no established library
  - Debugging is tricky in Monte-Carlo experiments, start simple and slowly increase complexity
  - More in our tutorial
- The solution is on MDP + objective function

# Limitations

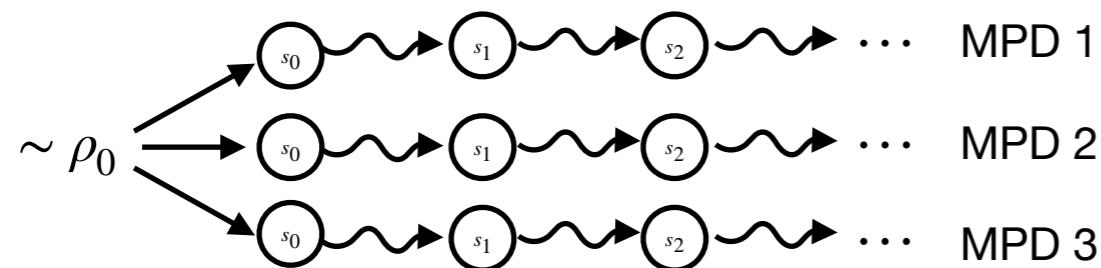
- Stability
- Safety
- Sample-efficiency
- Sufficient observability
- Interpretability
- ...

# Summary

- Exploration/Exploitation
- Sample-efficiency
- On-policy, off-policy, offline RL
- Training
- Limitations

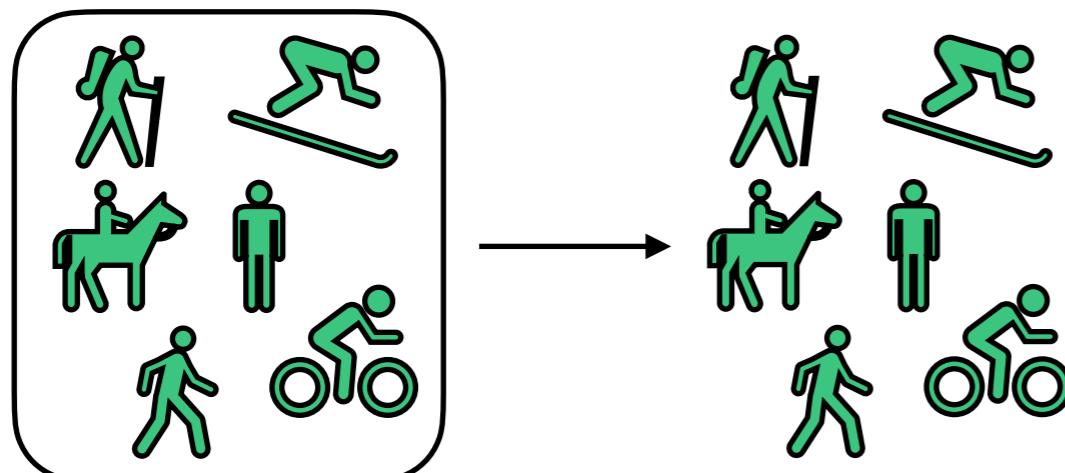
# Advanced topics - outlook

# Beyond the classical MDP



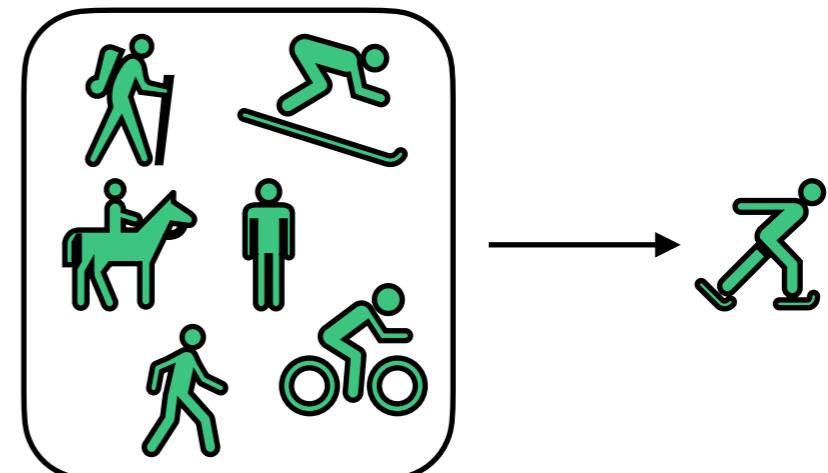
**Contextual MDPs**

**Multi task RL**



Common learning can improve sample efficiency

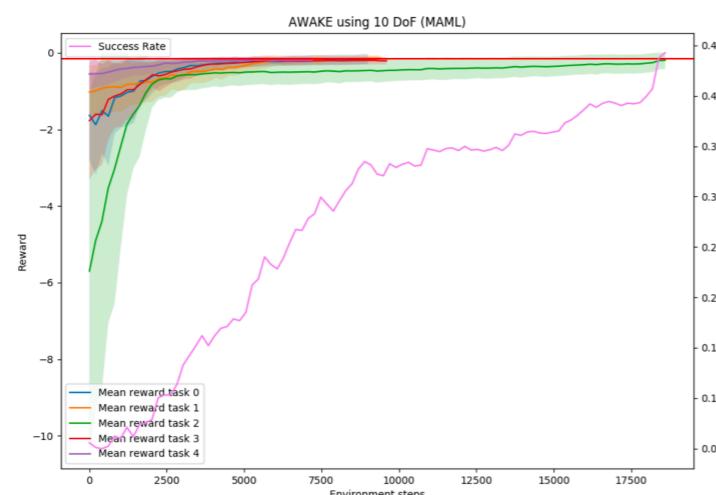
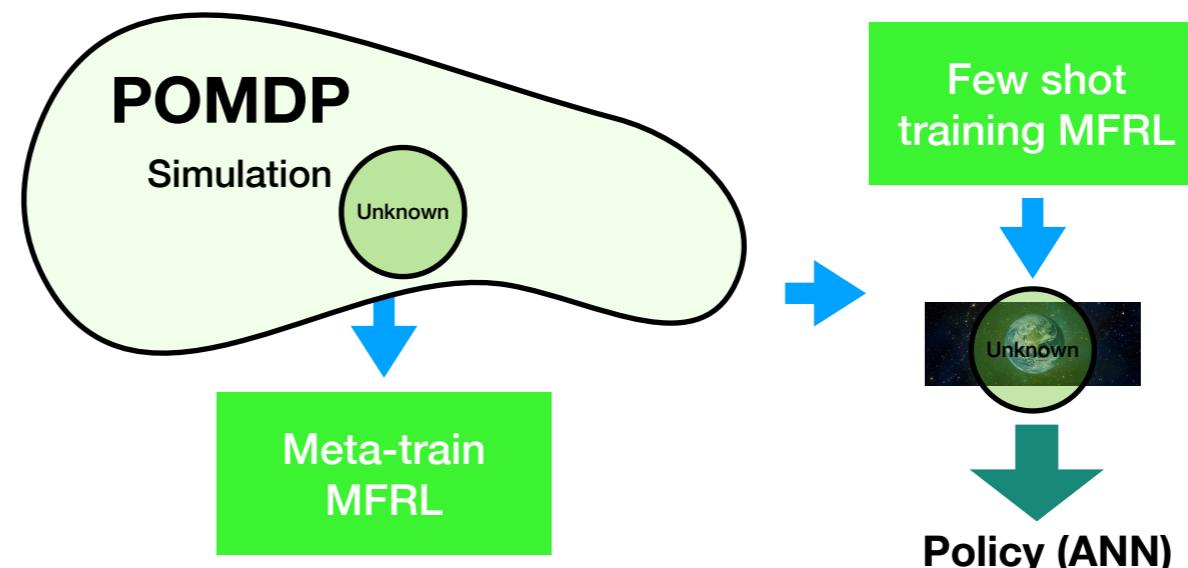
**Meta RL**



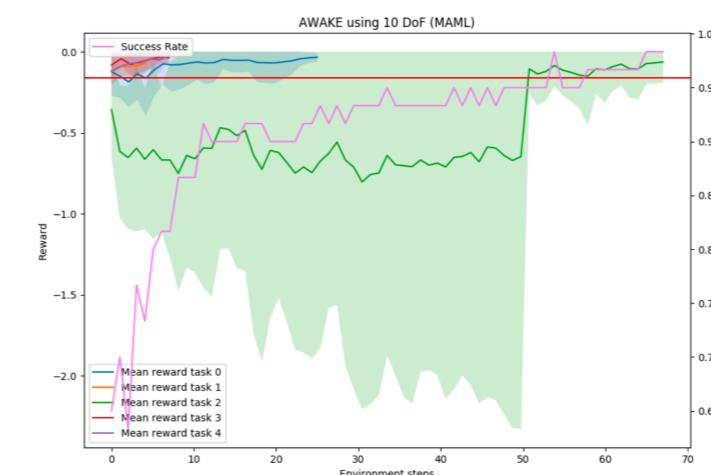
Accelerates learning enormously

# Example: Meta-RL

- Awake simulation, varying quads
- Few shot stable adaption
- TRPO with some guarantees



Untrained ~ 18000 samples 40% success



Meta-trained ~80 samples 100% success ~ **few steps on the machine**



Demonstrated in experiment AWAKE - to be published

# Further advanced topics

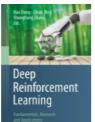
- **Meta RL**
- **Multi task RL**
- Contextual RL
- Multi-agent RL
- Hierarchical RL
- Distributional RL
- Inverse RL/Imitation learning
- ...

# Summary

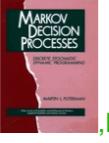
- Overview of designing correct problem
- Function approximation
- Different RL solution approaches: Value, Policy, MDP-model
- RL algorithmic challenges
- Beyond classical RL

# Selected Literature

- General:

-  Reinforcement Learning: An Introduction <http://incompleteideas.net/book/the-book-2nd.html>
-  Deep Reinforcement Learning: Fundamentals, Research and Applications <https://link.springer.com/book/10.1007/978-981-15-4095-0>
-   
Reinforcement Learning:  
Theory and Algorithms  
Akhil Agarwal, Nan Jiang, Sham M. Kakade, Wen Sun  
January 31, 2022  
  
WORKING DRAFT:  
Please email [book@rltheorybook.net](mailto:book@rltheorybook.net) with any typos or errors you find.  
We appreciate it!  
<https://rltheorybook.github.io/>

- POMDPs:

-  Algorithms for Decision Making <https://algorithmsbook.com/>
-  Decision Making Under Uncertainty: Theory and Application: <http://web.stanford.edu/group/sisl/public/dmu.pdf>
-  Markov Decision Processes: Discrete Stochastic Dynamic Programming: <https://onlinelibrary.wiley.com/doi/book/10.1002/9780470316887>
-  Reinforcement Learning and Stochastic Optimization: A Unified Framework for Sequential Decisions: <https://onlinelibrary.wiley.com/doi/book/10.1002/9781119815068>