Buying Derivatives - A Monte Carlo Approach

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Chapter 1

Introduction

Over the course of the last year, I have developed an algorithm that calculates the expected outcome for security[1] derivatives[2]. In this paper, I am specifically referring to put options[3] and call options[4] on stocks when I reference security

[1] In finance, a security refers to a tradable financial asset that has monetary value and can be bought or sold in a financial market. Securities include a wide range of assets, such as stocks, bonds, options, futures, and exchange-traded funds (ETFs), among others.

[2] A derivative is a financial instrument whose value is derived from the value of an underlying security, such as a stock, bond, commodity, or currency. In other words, the value of a derivative depends on the value of the underlying asset. There are many types of derivatives, including options, futures, forwards, and swaps.For example, an option is a type of derivative that gives the holder the right, but not the obligation, to buy or sell an underlying asset at a specific price within a specified time period.

[3] A put option is a type of financial contract that gives the holder the right, but not the obligation, to *sell* an underlying security at a predetermined price (known as the strike price) within a specific period of time (known as the strike date).In other words, a put option gives the holder the ability to sell an asset at a certain price, even if the market price of the asset falls below the strike price. This can be useful for investors who believe that the price of an asset is likely to decline, as it allows them to profit from the decline without actually owning the asset. For example, if an investor buys a put option for a certain stock with a strike price of $50 and an expiration date of one month, and the stock's market price falls to $40 during that time, the investor can exercise the put option to sell the stock at the higher strike price of $50, making a profit of $10 per share. However, if the stock's market price remains above the strike price of $50, the investor may choose not to exercise the put option and will lose the premium paid to purchase the option.

[4] A call option is a type of financial contract that gives the holder the right, but not the obligation, to *buy* an underlying asset, such as a stock, bond, or commodity, at a predetermined price (known as the strike price) within a specific period of time.In other words, a call option gives the holder the ability to purchase an asset at a certain price, even if the market price of the asset rises above the strike price. This can be useful for investors who believe that the price of an asset is likely to increase, as it allows them to profit from the increase without actually owning the asset.For example, if an investor buys a call option for a certain stock with a strike price of $50 and an expiration date of one month, and the stock's market price rises to $60 during that time, the investor can exercise the call option to buy the stock at the lower strike price of $50, making a profit of $10 per share. However, if the stock's market price remains below the strike price of $50, the investor may choose not to exercise the call

derivatives. To do this, I calculate several probabilities[5] and find the expected gain or cost, then combine them to calculate the expected value[6] and understand if it is worthwhile to purchase.

These probabilities are bootstrapped[7] through the use of a Monte Carlo simulation[8].

I am first simulating the distribution of the price of the security on the strike date using the Monte Carlo simulation. I am then able, with this information, to obtain the probability the security will be above (below) a certain strike and the expected value of the security if it does go above (below) the strike for a call (put). Combining this with the cost to purchase that derivative, I am able to create an expected value for a range of options.

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[5]Probability is a measure of the likelihood or chance that a specific event will occur, expressed as a number between 0 and 1. A probability of 0 means that the event is impossible, while a probability of 1 means that the event is certain to occur. A probability of 0.5 (or 50%) means that the event has an equal chance of occurring or not occurring.

[6]Expected value is a concept used in statistics to calculate the long-term average value of a random variable, taking into account the probability of each possible outcome.The expected value is calculated by multiplying each possible outcome by its probability and adding up all the products. In other words, it is the weighted average of all possible outcomes.

[7]A bootstrap sample is a sample of data that is created by randomly selecting observations with replacement from a larger data set. The term "bootstrap" comes from the idea of pulling oneself up by one's bootstraps, as the bootstrap method allows us to estimate the properties of a population by repeatedly resampling from a smaller sample. To create a bootstrap sample, we start by randomly selecting an observation from the original data set and adding it to the new sample. We then put the observation back in the original data set and repeat this process, selecting a new observation with replacement each time, until we have a sample of the desired size. By selecting with replacement, some observations may be selected more than once, while others may not be selected at all. Bootstrap sampling is commonly used in statistics to estimate the sampling distribution of a statistic, such as the mean or variance, when the underlying population distribution is unknown or difficult to model. By repeatedly resampling from the available data, we can generate many different bootstrap samples and use them to estimate the variability and uncertainty of the statistic of interest.

[8]A Monte Carlo simulation is a computational method used to estimate the probability distribution of an outcome by generating a large number of random samples or scenarios and analyzing their aggregate behavior.

Chapter 2

Calculating an Option's Expected Value

The expected value for each option can be calculated as follows:

$$E_p(k) = (P_{ae} * C) + (P_{be} * G) \tag{2.1}$$

$$E_c(k) = (P_{be} * C) + (P_{ae} * G) \tag{2.2}$$

where

- $E_p(k)$ is the expected value of a put at strike price k
- $E_c(k)$ is the expected value of a call at strike price k
- P_{ae} is the probability a security price on the strike date is above the *effective price*[1], k_e
  - This is calculated with the Monte Carlo simulation
  - $P_{ae} = \frac{r_{ae}}{r_t}$ where r_{ae} is the number of runs above the expected price and r_t are the total runs
- P_{be} = 1 - P_{ae} is the probability a security price on the strike date is below the *effective price*, k_e
- $C = 100c$ is the cost to buy the option
  - c is the cost of a single option (options are typically bought/sold in lots of 100)
- $G = 100(E_{p_s}(p_s|p_s > k_e) - c)$ is the expected gain if the derivative is in the money

[1] The effective price is the strike price plus the option cost (for calls) - you cannot make money on the option until you cross this threshold

- $E_{p_s}(p_s | p_s > k_e)$ is the expected value of the of the price of the stock, p_s, given that the price of the stock has exceeded the effective price, k_e (for calls).