## Change of Basis

## 1. Coordinate Vector Relative to Basis:

Cut B=(bi, bz,...bn) be an ordered boois for a vector space V. If v is a vector in V, then V= ribi t rzbzt ... rnbn. The coordinate vector of v relative to B, denoted by VB, is VB= [Yi, Yz,...Yn].

2. Change of Basis Matrix.

For this section, we need VB to be a column vector.

I.e. VB = Y1 instead of [Y1, Y2, ... Yn].

Cet MB be the matrix having the vectors in the ordered basis B as column vectors. This is the basis matrix for B. B is an ordered basis for Rn.

Mg= | b1 b2 1... bn

Note: MBVB=V, where v is a vector in Rn

Furthermore, a vector space can have more than I basis.

So, if B' is another ordered basis for R", then MB' VB' = V.

This shows that MBVB=MB'VB' and VB' = M-1'B' MBVB.

Therefore, for any 2 ordered bases B and B', there exists an invertible matrix C, s.t., C= M-'B'MB. C is called the change of basis matrix.

Furthermore, for all v in Rn, VB' = CVB

Important Notations:

CB,B' means we are changing from wordinates relative to B to coordinates relative to B'.

I.e. you read the subscripts from left to right.

Inverse of Change of Coordinate Matrix:

Suppose we have VB' = CB,B'VB. Then,

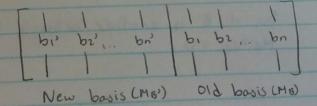
VB = C'B,B' VB', where C'B,B' is the inverse

of CB,B'. Note: CB',B = C'B,B'.

Steps For Finding the Change of Coordinate Matrix:

We know C= M'B'MB. However, if H'B' is not already available, we need to form the augmented matrix [MB'IMB] and then RREF it to [IIC].

Another way of thinking about this is



~ [I|CB,B']

Fig. 1 (it B= ([1,1,0], [2,0,1], [1,-1,0]) and let

E= (e1, e2, e3) be the Standard Ordered basis

Of R3. Find the Change of coordinate matrix

CE,B.

## Solution

1	1	2	. 1	,	0	0	-
1	1	0	-1	0	1	0	-
	0	1	b	0	0	1	J

New Basis (MB) Old Basis (ME)