Max Likelihood Examples

Steps:
1. Take the product of the pdf or pmf.
2. Take the log of that to turn it into a sum.
3. Take the derivative and set it to 0.

Examples:
1. The PMF of Y is $f(y) = \theta(1-\theta)^{y-1}$, $y = 1, 2, 3, ...$.
   The observed values for y are $2, 2, 1, 1, 5, 1, 1, 1, 2, 1$.
   Find the MLE of θ.

Soln:
$N = 10$

$$L(\theta) = \prod_{i=1}^{10} f(y)$$

$$= \prod_{i=1}^{10} \theta(1-\theta)^{(y-1)}$$

$$\ln(L) = \sum_{i=1}^{10} \ln(\theta(1-\theta)^{(y-1)})$$

$$= \sum_{i=1}^{10} \ln(\theta) + \sum_{i=1}^{10} \ln(1-\theta)^{y-1}$$

$$= \sum_{i=1}^{10} \ln(\theta) + \sum_{i=1}^{10} \ln(1-\theta)(y^{(i)}-1)$$

$$= 10\ln(\theta) + \sum_{i=1}^{10} y^{(i)} \ln(1-\theta) - \sum_{i=1}^{10} \ln(1-\theta)$$

$$= 10\ln(\theta) + 17\ln(1-\theta) - 10\ln(1-\theta)$$
$$= 10\ln(\theta) + 7\ln(1-\theta)$$

$$\frac{\partial L}{\partial \theta} = \frac{10}{\theta} - \frac{7}{1-\theta} = 0$$

$$0 = 10(1-\theta) - 7\theta$$
$$= 10 - 10\theta - 7\theta$$
$$= 10 - 17\theta$$
$$\frac{10}{17} = \theta$$