Integral Notes

1. $\int a\, dx = ax + c$, where a is a constant
2. $\int x^n\, dx = \dfrac{x^{n+1}}{n+1} + c$, where $n \neq -1$
3. $\int \dfrac{1}{x}\, dx = \ln|x| + c$
4. $\int a^x\, dx = \dfrac{a^x}{\ln(a)} + c$
5. $\int e^x\, dx = e^x + c$
6. $\int \ln(x)\, dx = x\ln(x) - x + c$
7. $\int \sin(x)\, dx = -\cos(x) + c$
8. $\int \cos(x)\, dx = \sin(x) + c$
9. $\int \sec^2(x)\, dx = \tan(x) + c$
10. $\int cf(x)\, dx = c\int f(x)\, dx$, where c is a constant
11. $\int (f \pm g)\, dx = \int f\, dx \pm \int g\, dx$

Integration

12. by Parts

- Is often useful when 2 functions are multiplied together.

- $\int uv\,dx = u\int v\,dx - \int u'\left(\int v\,dx\right)dx$

- E.g. 1 $\int x\cos(x)\,dx$

$u = x$

$v = \cos(x)$

$u\int v\,dx - \int u'\left(\int v\,dx\right)dx$

$= x\int \cos(x)\,dx - \int x'\left(\int \cos(x)\,dx\right)dx$

$= x\sin(x) - \int \sin\,dx$

$= x\sin(x) + \cos(x) + c$

- E.g. 2 $\int \ln(x)\,dx$

$u = \ln(x)$

$v = 1$

$u\int v\,dx - \int u'\left(\int v\,dx\right)dx$

$= \ln(x)\int 1\,dx - \int (\ln(x))'\left(\int 1\,dx\right)dx$

$= x\ln(x) - \int \left(\frac{1}{x}\right)(x)\,dx$

$= x\ln(x) - \int 1\,dx$

$= x\ln(x) - x + c$

- E.g. 3 $\int \frac{\ln(x)}{x^2}\,dx$

$u = \ln(x)$
$v = \frac{1}{x^2}$

$$u\int v\,dx - \int u'\left(\int v\,dx\right)dx$$
$$= \ln(x)\int x^{-2}\,dx - \int (\ln(x))'\left(\int x^{-2}\,dx\right)dx$$
$$= \frac{-\ln(x)}{x} - \int \left(\frac{1}{x}\right)\left(\frac{-1}{x}\right)dx$$
$$= \frac{-\ln(x)}{x} + \int \frac{1}{x^2}\,dx$$
$$= \frac{-\ln(x)}{x} - \frac{1}{x} + c$$
$$= -\left(\frac{\ln(x)+1}{x}\right) + c$$

- E.g. 4 $\int e^x x\,dx$

$u = x$
$v = e^x$

$$u\int v\,dx - \int u'\left(\int v\,dx\right)dx$$
$$= x\int e^x\,dx - \int x'\left(\int e^x\,dx\right)dx$$
$$= xe^x - \int e^x\,dx$$
$$= xe^x - e^x + c$$
$$= e^x(x-1) + c$$

Note: If you chose e^x as u and x as v, you would have had a messy equation.

We need to choose u and v carefully. We should choose a u that gets simpler when you differentiate it and a v that doesn't get any more complicated when you integrate it.

As a general rule, choose u based on which of these comes first:

1. Inverse trig functions
2. Log functions
3. Algebraic functions
4. Trig functions
5. Exponential functions

Integration By Substitution

- The first and most vital step is to write the integral in this form:

$$\int f(g(x))\, g'(x)\, dx$$

- When the integration is set up like above, we can do:

$$\int f(\underbrace{g(x)}_{\downarrow})\ \underbrace{g'(x)\,dx}_{\downarrow}$$

$$\int f(u) \qquad du$$

- E.g. 1 $\int \cos(x^2)\cdot 2x\, dx$

let $u = x^2$

$$\frac{du}{dx} = 2x \longrightarrow dx = \frac{du}{2x}$$

$$\int \cos(u)\, du$$

$$= \sin(u)$$

$$= \sin(x^2) + C$$

- E.g. 2 $\int \cos(x^2)\, 6x\, dx$

$= 3\int \cos(x^2)\, 2x\, dx$

$= 3\sin(x^2) + c$

- E.g. 3 $\int \frac{x}{x^2+1}\, dx$

let $u = x^2+1$

$\frac{du}{dx} = 2x$

$dx = \frac{du}{2x}$

$\int \frac{1}{2u}\, du$

$= \frac{1}{2}\int \frac{1}{u}\, du$

$= \frac{1}{2}(\ln|u|)$

$= \frac{1}{2}(\ln(x^2+1)) + c$