

### Other Facts.

1. Let  $A$  be a  $n \times n$  matrix. Then,  $AA^{-1} = I_{n \times n}$ .

2. If you divide a matrix, you are multiplying by its inverse.  
E.g.  $AB = c$   
 $B = A^{-1}c$

3. Multiplying a matrix by the identity matrix gets you the same matrix.  
E.g. Let  $A$  be a  $n \times n$  matrix.  
 $AI_{n \times n} = A$

4. Let  $A$  be a  $n \times n$  matrix.  
Let  $c$  be a constant.

$$\det(cA) = c^n \cdot \det(A)$$

E.g. Let  $A$  be a  $3 \times 3$  matrix.

$$\text{Let } \det(A) = 2$$

$$\det(3A) = 3^3 \cdot \det(A)$$

$$= 27 \cdot 2$$

$$= 54$$

$$\begin{aligned} 5. \det(A^{-1}) &= \frac{\det(I)}{\det(A)} \\ &= \frac{1}{\det(A)} \end{aligned}$$

E.g. If  $\det(A) = 2$ , find  $\det(A^{-1})$

$$\det(A \cdot A^{-1}) = \det(I) = 1$$

$$1 = \det(A) \cdot \det(A^{-1})$$

$$\frac{1}{\det(A)} = \det(A^{-1})$$

$$\frac{1}{2} = \det(A^{-1})$$



6. A singular matrix is a matrix that is not invertible.  
If  $A$  is a singular matrix, then  $\det(A) = 0$ .
7. To find the inverse image of a linear transformation, find the inverse of the standard matrix rep of that linear transformation.