MATB61 Tutorial 1 Notes

1. In the below questions, set up a linear programming model of the situation described. Then, determine if the model is in standard form. If it is not, state what must be changed to put the model into standard form.
   a. A coffee packer blends Brazilian coffee and Colombian coffee to prepare two products: Super and Deluxe brands. Each kilogram of Super coffee contains 0.5 kg of Brazilian coffee and 0.5 kg of Colombian coffee, whereas each kilogram of Deluxe coffee contains 0.25 kg of Brazilian coffee and 0.75 kg of Colombian coffee. The packer has 120 kg of Brazilian coffee and 160 kg of Colombian coffee on hand. If the profit on each kilogram of Super coffee is 20 cents and the profit on each kilogram of Deluxe coffee is 30 cents, how many kilograms of each type of coffee should be blended to maximize profit?

Soln:

Let x be the num of kg of Super.

Let y be the num of kg of Deluxe.

	Brazilian	Colombian	Profit (in \$)
x	0.5	0.5	0.2
y	0.25	0.75	0.3

Max $p = 0.2x + 0.3y$ subject to:

1. $0.5x + 0.25y \leq 120$
2. $0.5x + 0.75y \leq 160$
3. $x, y \geq 0$

This is already in SLP.

b. Consider an airshed in which there is one major contributor to air pollution, a cement-manufacturing plant whose annual production capacity is 2,500,000 barrels of cement. Figures are not available to determine whether the plant has been operating at capacity. Although the kilns are equipped with mechanical collectors for air pollution control, the plant still emits 2.0 lb of dust per barrel of cement produced. There are two types of electrostatic precipitators that can be installed to control dust emission. The four-field type would reduce emissions by 1.5 lb of dust/barrel and would cost $0.14/barrel to operate. The five-field type would reduce emissions by 1.8 lb of dust/barrel and would cost $0.18/barrel to operate. The EPA requires that particulate emissions be reduced by at least 84%. How many barrels of cement should be produced using each new control process to minimize the cost of controls and still meet the EPA requirements?

Soln:

Let x be the num of barrels produced using the four-field.

Let y be the num of barrels produced using the five-field.

Min $c = 0.14x + 0.18y$ subject to:

1. $x + y \leq 2{,}500{,}000$
2. $1.5x + 1.8y \geq 0.84(2x + 2y)$
3. $x \geq 0,\ y \geq 0$

To turn it into SLP form:

Let $c' = -c$

$= -0.14x - 0.18y$

Max $c' = -0.14x - 0.18y$ subject to:

1. $x + y \leq 2{,}500{,}000$
2. $-1.5x - 1.8y \leq -0.84(2x + 2y)$
3. $x, y \geq 0$

c. The administrator of a \$200,000 trust fund set up by Mr. Smith's will must adhere to certain guidelines. The total amount of \$200,000 need not be fully invested at any one time. The money may be invested in three different types of securities: a utilities stock paying a 9% dividend, an electronics stock paying a 4% dividend, and a bond paying 5% interest. Suppose that the amount invested in the stocks cannot be more than half the total amount invested; the amount invested in the utilities stock cannot exceed \$40,000; and the amount invested in the bond must be at least \$70,000. What investment policy should be pursued to maximize the return?

Soln:

Let x be the amount invested in u. stocks.
Let y be the amount invested in e. stocks.
Let z be the amount invested in bonds.

Max $p = 0.09x + 0.04y + 0.05z$ subject to:

1. $x+y+z \leq 200{,}000$
2. $x+y \leq \frac{1}{2}(x+y+z) \rightarrow \frac{x}{2} + \frac{y}{2} - \frac{z}{2} \leq 0$
3. $x \leq 40{,}000$
4. $z \geq 70{,}000$
5. $x, y, z \geq 0$

To change to SLP form, multiply both sides by -1 for #4.

I.e. $z \geq 40{,}000 \rightarrow -z \leq -40{,}000$

d. A local health food store packages three types of snack foods, Chewy, Crunchy, and Nutty, by mixing sunflower seeds, raisins, and peanuts. The specifications for each mixture are given in the accompanying table.

Mixture	Sunflower seeds	Raisins	Peanuts	Selling price per kilogram ($)
Chewy		At least 60%	At most 20%	2.00
Crunchy	At least 60%			1.60
Nutty	At most 20%		At least 60%	1.20

The suppliers of the ingredients can deliver each week at most 100 kg of sunflower seeds at $1.00/kg, 80 kg of raisins at $1.50/kg, and 60 kg of peanuts at $0.80/kg. Determine a mixing scheme that will maximize the store's profit.

Soln:

Let x_{ij} be the amount of kg of ingredient i in mixture j.

Ingredients	Mixture
S.S	Chewy
Raisins	Crunchy
Peanuts	Nutty

	Chewy	Crunchy	Nutty
S.S	X_{11}	X_{21}	X_{31}
Raisins	X_{12}	X_{22}	X_{32}
Peanuts	X_{13}	X_{23}	X_{33}

$$\text{Max } p = 2\sum_{i=1}^{3} x_{i1} + 1.6\sum_{i=1}^{3} x_{i2} + 1.2\sum_{i=1}^{3} x_{i3}$$

$$- \sum_{j=1}^{3} x_{1j} - 1.5\sum_{j=1}^{3} x_{2j} - 0.8\sum_{j=1}^{3} x_{3j}$$

Subject to:

1. $\sum_{j=1}^{3} x_{1j} \leq 100$

2. $\sum_{j=1}^{3} x_{2j} \leq 80$

3. $\sum_{j=1}^{3} x_{3j} \leq 60$

4. $x_{21} \geq 0.6(x_{11} + x_{21} + x_{31})$

$x_{21} \geq 0.6x_{11} + 0.6x_{21} + 0.6x_{31}$

$0 \geq 0.6x_{11} - 0.4x_{21} + 0.6x_{31}$

$0.6x_{11} - 0.4x_{21} + 0.6x_{31} \leq 0$

5. $x_{31} \leq 0.2(x_{11} + x_{21} + x_{31})$

$\leq 0.2x_{11} + 0.2x_{21} + 0.2x_{31}$

$-0.2x_{11} - 0.2x_{21} + 0.8x_{31} \leq 0$

6. $x_{12} \geq 0.6(x_{12} + x_{22} + x_{32})$

$\geq 0.6x_{12} + 0.6x_{22} + 0.6x_{32}$

$0 \geq -0.4x_{12} + 0.6x_{22} + 0.6x_{32}$

$-0.4x_{12} + 0.6x_{22} + 0.6x_{32} \leq 0$

7. $X_{13} \leq 0.2(X_{13} + X_{23} + X_{33})$

$\leq 0.2X_{13} + 0.2X_{23} + 0.2X_{33}$

$0.8X_{13} - 0.2X_{23} - 0.2X_{33} \leq 0$

8. $X_{33} \geq 0.6(X_{13} + X_{23} + X_{33})$

$\geq 0.6X_{13} + 0.6X_{23} + 0.6X_{33}$

$0 \geq 0.6X_{13} + 0.6X_{23} - 0.4X_{33}$

$0.6X_{13} + 0.6X_{23} - 0.4X_{33} \leq 0$

9. $x_{ij} \geq 0$ for i in 1, 2, 3

for j in 1, 2, 3

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2. Convert the following LP problems into standard form:
   a. Max z = 4a + 2b + c subject to:
      -a + 3b - c >= 1
      5a +3c = 5
      a + b + c <= 1
      -1 <= a, b <= 2
      c >= 3

Soln:

In SLP Form, there must be these 3 things:

1. Max obj func
2. The constraints are all $\leq$.
3. All vars must be ≥ 0.

$-1 \leq a \rightarrow -1-a \leq 0$

$= a+1 \geq 0$

Let a' be a slack variable, s.t. $a' = a+1$.

$a = a'-1$

$b \leq 2 \rightarrow 0 \leq 2-b$

Let b' be a slack var s.t. $b' = 2-b$.

$b = b'-2$

Rewriting everything by replacing a with $a'-1$ and b with $b'-2$, we get:

Max $z = 4(a'-1)+3(b'-2)+c$ subject to

1. $-(a'-1) + 3(b'-2) - c \geq 1$
2. $5(a'-1) + \quad 3c = 5$
3. $(a'-1) + (b'-2) + c \leq 1$

Simplifying everything, we get

Max $z = 4a' + 3b' + c - 10$ subject to

1. $-a' + 3b' - c \geq 6$
2. $5a' + 3c = 10$
3. $a' + b' + c \leq 4$
4. $c \geq 3 \longrightarrow -c \leq -3$

To change 1 into $\leq$, we multiply both sides by -1.

1. $a' - 3b' + c \leq -6$

To change 2 into $\leq$, we split it up into 2 pieces, a $\leq$ and a $\geq$ and then multiply both sides of the $\geq$ to change it to a $\leq$.

2. $5a' + 3c = 10$ can be split into:
   a) $5a' + 3c \leq 10$
   b) $5a' + 3c \geq 10 \rightarrow -5a' - 3c \leq -10$

As 3 is already in $\leq$ form, we don't need to change it.

Putting it all together, we get

Max $z = 4a' + 3b' - c - 10$ subject to

1. $a' - 3b' + c \leq -6$
2. $5a' + 3c \leq 10$
3. $-5a' - 3c \leq -10$
4. $a' + b' + c \leq 4$
5. $-c \leq -3$
6. $a', b', c \geq 0$

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b. Max z = a + 2b + 4c subject to
|4a +3b -7c| <= a + b + c
a, b, c >= 0

Soln:

$|4a+3b-7c| \leq a+b+c$ can be split into:

1. $4a+3b-7c \leq a+b+c$

   $3a+2b-8c \leq 0$

2. $-a-b-c \leq 4a+3b-7c$

   $0 \leq 5a+4b-6c$

   $-5a-4b+6c \leq 0$

Putting it all together, we get

Max $z = a+2b+4c$ subject to

1. $3a+2b-8c \leq 0$
2. $-5a-4b+6c \leq 0$
3. $a, b, c \geq 0$

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3. Convert the following LP problems into canonical form:
   a. Max z = -a -2b -c subject to
      a - b + c >= 2
      |a + b| <= 3
      a, b >= 0

Soln:

In CLP form, there must be these 3 things:

1. Max obj func.
2. Constraints are equal.
3. Vars are ≥ 0.

To make $c \geq 0$, let's introduce 2 vars, c^+ and c^-, s.t. $c^+, c^- \geq 0$.

Let $c = c^+ - c^-$.

Replacing $c^+ - c^-$ for c, we get

Max $z = -a - 2b - c^+ + c^-$ subject to:

1. $a - b + c^+ - c^- \geq 2$
2. $|a+b| \leq 3$

To deal with 1, let's introduce a slack var, s_1, s.t. $s_1 \geq 0$. Then:

$a - b + c^+ - c^- - s_1 = 2$

To deal with 2, let's split the abs value into 2 inequalities and add slack vars for each inequality.

$|a+b| \leq 3 \rightarrow -3 \leq a+b \leq 3$

$-3 \leq a+b$:

Let's introduce another slack var, S_2, s.t. $S_2 \geq 0$.

$a+b-S_2=-3$

$a+b \leq 3$:

Let's introduce another slack var, S_3, s.t. $S_3 \geq 0$.

$a+b+S_3=3$

Putting it all together, we get

Max $z=-a-2b-c^+ + c^-$ subject to

1. $a-b+c^+-c^--S_1=2$
2. $a+b-S_2=-3$
3. $a+b+S_3=3$
4. $a, b, c^+, c^-, S_1, S_2, S_3 \geq 0$

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b. Min z = a - 12b +3c subject to
5a - b -2c = 10
2a + b -10c >= -30
b <= 0
1 <= c <= 4

Soln:

Let a^+ and a^- be 2 vars s.t. $a^+, a^- \geq 0$

and $a = a^+ - a^-$.

Let $b' = -b$.
$b' \geq 0$

Replacing a with $a^+ - a^-$ and b with $-b'$, we get:

Min $z = a^+ - a^- + 12b' + 3c$ subject to

1. $5(a^+ - a^-) + b' - 2c = 10$
2. $2(a^+ - a^-) - b' - 10c \geq -30$
3. $1 \leq c \leq 4$

To change a min obj function to a max obj func, we multiply the obj func by -1 and then max it.

Let $z' = -z$
$= -a^+ + a^- - 12b' - 3c$

We don't need to do too much to 1, just expand and simplify. 13

$5(a^+ - a^-) + b' - 2c = 10 \rightarrow 5a^+ - 5a^- + b' - 2c = 10$

For 2, we need to introduce a slack var.

Let $S_1 \geq 0$.

$2a^+ - 2a^- - b' - 10c - S_1 = 30$

For 3, we can split it into 2 parts and add a slack var for each inequality/part.

$1 \leq c \leq 4 \longrightarrow \begin{array}{l} 1 \leq c \\ c \leq 4 \end{array}$

$1 \leq c$:

Let $S_2 \geq 0$.

$c - S_2 = 1$

$c \leq 4$:

Let $S_3 \geq 0$

$c + S_3 = 4$

Putting it all together, we get

Max $z' = a^+ - a^- + 12b' + 3c$ subject to

1. $5a^+ - 5a^- + b' - 2c = 10$
2. $2a^+ - 2a^- - b' - 10c - S_1 = 30$
3. $c - S_2 = 1$
4. $c + S_3 = 4$
5. $a^+, a^-, b', c, S_1, S_2, S_3 \geq 0$