MATA22 Booklet 1 Notes

Definitions:

1. If n is a positive integer, then the Euclidean n – space, denoted by R^n, is the collection of all n – tuples of real numbers.

   There are 2 kinds of n – tuples in R^n:
   1. $(a_1, a_2, \ldots a_n)$ denotes a point in R^n.
   2. $[a_1, a_2, \ldots a_n]$ denotes a vector in R^n.

2. The zero vector in R^n is denoted by **0** and is a vector with all of its entries consisting of 0s only.

3. A scalar only has magnitude. It does not have direction.

4. A vector has both magnitude and direction.

5. 2 vectors are parallel if they are both non – zero and they can be written as a non – zero multiple of each other.
   E.g. [1, 2, 3] and [2, 4, 6] are parallel because [1, 2, 3] = $\left(\frac{1}{2}\right)$[2, 4, 6].

6. Given the vectors $\mathbf{v_1}, \mathbf{v_2}, \ldots \mathbf{v_n}$ in R^n and the scalars $s_1, s_2, \ldots s_n$ in R, the linear combination of those vectors with those scalars is
   $s_1\mathbf{v_1} + s_2\mathbf{v_2} + \ldots s_n\mathbf{v_n}$.

7. The set of all linear combinations of $\mathbf{v_1}, \mathbf{v_2}, \ldots \mathbf{v_n}$ is called the span of $\mathbf{v_1}, \mathbf{v_2}, \ldots \mathbf{v_n}$ and is denoted as $sp(\mathbf{v_1}, \mathbf{v_2}, \ldots \mathbf{v_n})$.

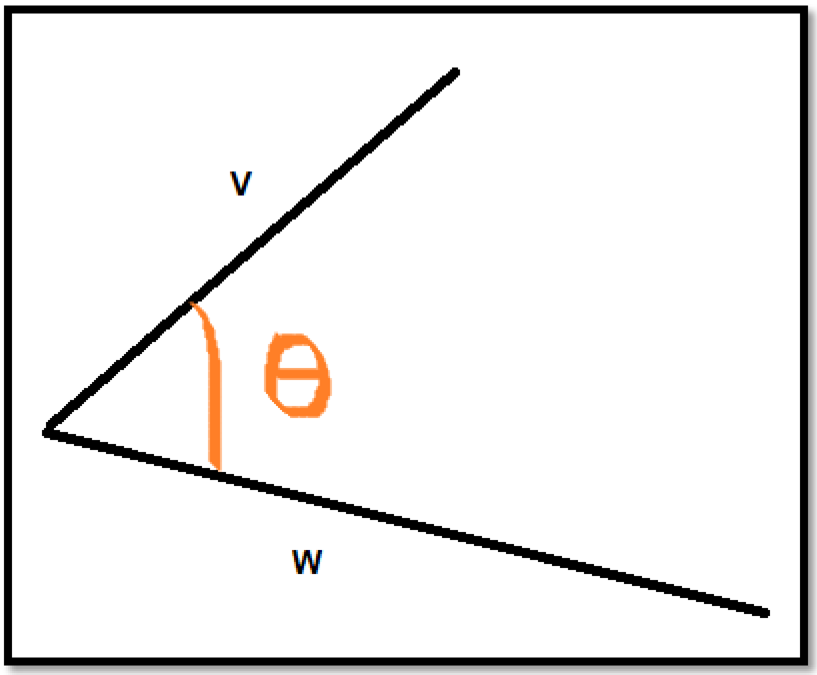
8. A vector is in standard position if it starts at the origin. If a vector does not start at the origin, then that vector has been translated (moved).

9. Two vectors are equal iff all of their entries are equal.
   I.e. $\mathbf{v} = \mathbf{w}$ iff $v_i = w_i$ for all i = 1, 2, … n
   E.g. [1, 2, 3] = [1, 2, 3] because all of their entries are equal.

10. The magnitude or norm of a vector, **v**, is denoted by $||\mathbf{v}||$.
    $||\mathbf{v}|| = \sqrt{(V_1)^2 + \ldots + (V_n)^2}$
    E.g. Let **v** = [1, 2, 3] $||\mathbf{v}|| = \sqrt{1^2 + 2^2 + 3^2} = \sqrt{14}$

11. A unit vector is a vector with a magnitude of 1. It is equal to $\frac{\mathbf{v}}{||\mathbf{v}||}$.

12. The dot product of 2 vectors, **v** and **w** is defined by **v** * **w** = (||**v**||)(||**w**||)(cos(Θ)) where Θ is the angle between the vectors.



Let **v** = [v_1, v_2, …, v_n].
Let **w** = [w_1, w_2, …, w_n].
v * w = (v_1)(w_1) + (v_2)(w_2) + … + (v_n)(w_n)

E.g.
Let **v** = [1, 2, 3].
Let **w** = [2, 3, 4].

v * w = (1)(2) + (2)(3) + (3)(4)
= 20

13. $\Theta = \cos^{-1}\left(\frac{\mathbf{v}*\mathbf{w}}{(||\mathbf{v}||)(||\mathbf{w}||)}\right)$

14. Two vectors are perpendicular if their dot product equals to 0.

15. **v** * **v** = $(||\mathbf{v}||)^2$

16. The orthogonal projection of **b** onto **a** is denoted by $\text{Proj}_{\mathbf{a}}{}^{\mathbf{b}}$.

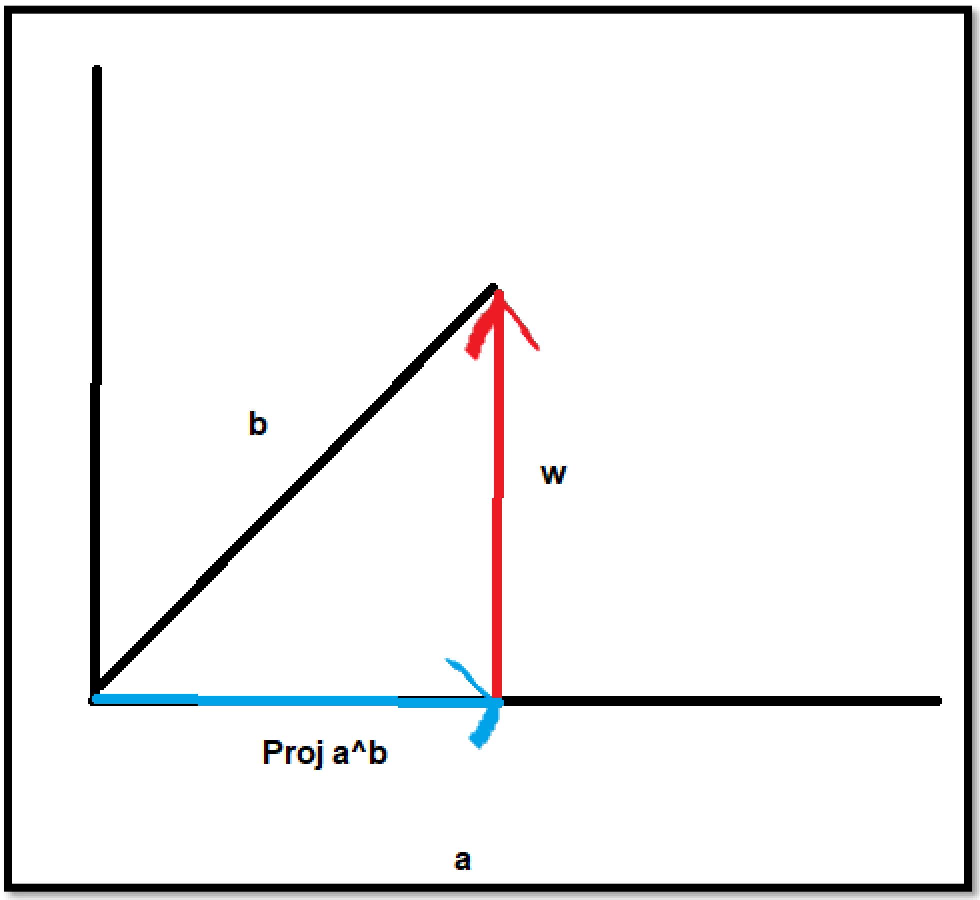
$$\text{Proj}_{\mathbf{a}}{}^{\mathbf{b}} = \left(\frac{\mathbf{a} * \mathbf{b}}{||\mathbf{a}||^2}\right)\mathbf{a}$$

E.g.

Find the orthogonal projection of [1, 2, 4] onto [3, 5, 2].

$$\text{Proj}_{\mathbf{v}}{}^{\mathbf{w}} = \left(\frac{[1,2,4] * [3,5,2]}{||[\mathbf{3,5,2}]||^2}\right)[3,5,2]$$

$$= \left(\frac{21}{38}\right)[3,5,2]$$



17. **w** is the vector component of **b** onto **a.**

$$\mathbf{w} = \mathbf{b} - \text{Proj}_{\mathbf{a}}{}^{\mathbf{b}}$$

Vector Math:

Let $\mathbf{v} = [v_1, v_2, \ldots, v_n]$.
Let $\mathbf{w} = [w_1, w_2, \ldots, w_n]$.
Let r be a scalar.

1. Vector Addition:

$\mathbf{v} + \mathbf{w} = [v_1, v_2, \ldots, v_n] + [w_1, w_2, \ldots, w_n]$
$= [v_1 + w_1, v_2 + w_2, \ldots, v_n + w_n]$

2. Vector Subtraction:

$\mathbf{v} - \mathbf{w} = [v_1, v_2, \ldots, v_n] - [w_1, w_2, \ldots, w_n]$
$= [v_1 - w_1, v_2 - w_2, \ldots, v_n - w_n]$

3. Scalar Multiplication:

$r(\mathbf{v}) = r([v_1, v_2, \ldots, v_n])$
$= [rv_1, rv_2, \ldots, rv_n])$

Theorem:

Let $\mathbf{v} = [v_1, v_2, \ldots, v_n]$.
Let $\mathbf{w} = [w_1, w_2, \ldots, w_n]$.
Let $\mathbf{u} = [u_1, u_2, \ldots, u_n]$.
Let r and s be scalars.

1. $\mathbf{v} + (\mathbf{u} + \mathbf{w}) = (\mathbf{v} + \mathbf{u}) + \mathbf{w}$
2. $\mathbf{v} + \mathbf{u} = \mathbf{u} + \mathbf{v}$
3. $\mathbf{0} + \mathbf{v} = \mathbf{v}$
4. $r(\mathbf{v} + \mathbf{u}) = r\mathbf{v} + r\mathbf{u}$
5. $(r+s)\mathbf{v} = r\mathbf{v} + s\mathbf{v}$
6. $r(s\mathbf{v}) = (rs)\mathbf{v}$
7. $\mathbf{1v} = \mathbf{v}$
8. $||\mathbf{v}|| \geq 0$ and $||\mathbf{v}|| = 0$ iff $\mathbf{v} = \mathbf{0}$
9. $||r\mathbf{v}|| = (|r|)(||\mathbf{v}||)$
10. $||\mathbf{v} + \mathbf{w}|| \leq ||\mathbf{v}|| + ||\mathbf{w}||$ (Triangle Inequality)
11. $\mathbf{v*w} = \mathbf{w*v}$
12. $\mathbf{u}*(\mathbf{v} + \mathbf{w}) = \mathbf{u*w} + \mathbf{u*v}$
13. $r(\mathbf{u*w}) = r\mathbf{u} + r\mathbf{w}$
14. $|\mathbf{u*w}| \leq (||\mathbf{u}||)(||\mathbf{w}||)$ (Cauchy – Schwartz Inequality)