MATA22 Booklet 2 Notes

Definitions:

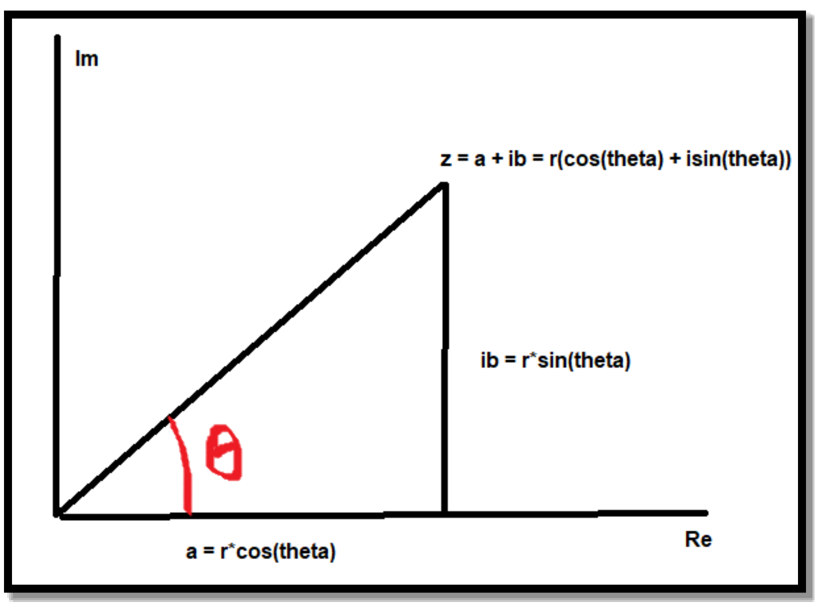
1. The set of all complex numbers, denoted by C, is the set of all numbers in the form of a + ib, where a and b are real numbers.
   a is the real part, denoted by Re.
   b is the imaginary part, denoted by Im.

   Let z = a + ib

2. The modulus or magnitude of z is denoted by |z| or r.
   $|z| = \sqrt{a^2 + b^2}$

3. The polar form of z is r(cos(Θ) + isin(Θ)).

4. The angle Θ is called the argument of z, denoted by Arg(z).
   If $-\pi \leq \Theta \leq \pi$, then Θ is the principal argument of z.
   $\Theta = \tan^{-1}(\frac{b}{a})$



5. The conjugate of z, denoted by $\bar{z}$ is a – ib.

   If x = a – ib, then $\bar{x}$ = a + ib.

Complex Number Arithmetic:

Let x = a + ib.
Let y = c + id.
Let r be a scalar.

1. **Addition:**
   x + y = (a + ib) + (c + id)
   = (a + c) + i(b + d)

2. **Multiplication:**
   xy = (a + ib)(c + id)
   = ac + iad + ibc + i^2bd
   = (ac – bd) + i(ad + bc)

3. **Scalar Multiplication:**
   rx = r(a + ib)
   = ra + irb

Theorem:

Let $z_1 = a + ib = r_1(\cos(\Theta_1) + i\sin(\Theta_1))$
Let $z_2 = c + id = r_2(\cos(\Theta_2) + i\sin(\Theta_2))$

1. $(z_1)(z_2) = (r_1)(r_2)[\cos(\Theta_1 + \Theta_2) + i\sin(\Theta_1 + \Theta_2)]$
2. $\frac{z_1}{z_2} = \frac{r_1}{r_2}[\cos(\Theta_1 - \Theta_2) + i\sin(\Theta_1 - \Theta_2)]$
3. $\overline{\overline{z_1}} = Z_1$

4. $|z_1|^2 = (z_1)\boxed{\overline{z_1}}$

5. $\boxed{\overline{z_1 + z_2} = \overline{z_1} + \overline{z_2}}$

6. $\boxed{z_1^{-1} = \dfrac{\overline{z_1}}{\|z_1\|^2}}$

7. $\boxed{\overline{z_1 / z_2} = \overline{z_1} / \overline{z_2}}$

8. $|z_1 + z_2| \le |z_1| + |z_2|$ (Triangle Inequality)

Examples:

1. Express $(1 + i)^8$ in the form of a + ib.

   Step 1: Find r
   r = $\sqrt{a^2 + b^2}$
   = $\sqrt{1^2 + 1^2}$
   = $\sqrt{2}$

   Step 2: Find Θ
   Θ = $\tan^{-1}(\frac{b}{a})$
   = $\tan^{-1}(\frac{1}{1})$
   = $\frac{\pi}{4}$

   Step 3: Write it in polar form
   z = r(cos(Θ) + isin(Θ))
   = $\sqrt{2}(\cos(\frac{\pi}{4}) + i\sin(\frac{\pi}{4}))$

   Step 4: Plug it into the equation
   $z^n = r^n(\cos(n\Theta) + i\sin(n\Theta))$
   $z^8 = (\sqrt{2})^8(\cos(\frac{8\pi}{4}) + i\sin(\frac{8\pi}{4}))$
   = 16

2. Find the four fourth roots of -16.

3. Step 1: Find r
   r = $\sqrt{(-16)^2}$
   = 16

   Step 2: Find Θ
   Θ = $\tan^{-1}(\frac{0}{-16})$
   = 0

   Step 3: Plug it into the equation
   $z^{1/n} = r^{1/n}(\cos(\frac{\Theta}{n} + \frac{2k\pi}{n}) + i\sin(\frac{\Theta}{n} + \frac{2k\pi}{n}))$
   $z^{1/4} = 2(\cos(\frac{k\pi}{2}) + i\sin(\frac{k\pi}{2}))$

$k = 0, 1, 2, \ldots n - 1$
Plug in the different values for k and solve.