# Codes for Semiparametric Mixture Regression for Asynchronous Longitudinal Data Using Multivariate Functional Principal Component Analysis

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## Package 'EMERALD'

**Title** Semiparametric Mixture Regression for Asynchronous Longitudinal Data **Description** Estimation of asynchronous longitudinal data using multivariate functional principal component analysis; Estimation of the mixed models including latent class; Mixed models for multivariate longitudinal outcomes using a maximum likelihood estimation method

### 1 SWAN folder

- basis1.csv file provides the nonorthogonal basis for  $X_1$ , which is glucose.
- basis2.csv file provides the nonorthogonal basis for  $X_2$ , which is triglyceride.
- basis 3.csv file provides the nonorthogonal basis for  $X_2$ , which is systolic blood pressure.
- mul.csv file provides the initial value for the mean function of glucose.
- mu2.csv file provides the initial value for the mean function of triglyceride.
- mu3.csv file provides the initial value for the mean function of systolic blood pressure.
- psi1.csv file provides the initial value for the eigenfunction of glucose;
- psi2.csv file provides the initial value for the eigenfunction of triglycerides.
- psi3.csv file provides the initial value for the eigenfunction of systolic blood pressure.
- probability1.csv, probability2.csv, probability3.csv, probability4.csv, and probability5.csv files provide the initial value of classification probability ( $\pi_{ic}$ ) according to different number of clusters.
- new1class.py, new2class.py, new3class.py, new4class.py, and new5class.py files consist of functions **Emerald** need and return a text result file.
- W1.csv, W2.csv, W3.csv, and Ymeasure.csv are data files we would like to analyze.
- bootstrap for SWAN.py calculate the standard errors for each parameter using bootstrap data.
- result\_1cluster\_hpcc.txt, result\_2cluster\_hpcc.txt, result\_3cluster\_hpcc.txt, result\_4cluster\_hpcc.txt, and result\_5cluster\_hpcc.txt files are the results from **Emerald**.
- After determining the number of subgroups, return the results as fit\_beta.csv, fit\_gamma.csv, fit\_mu.csv, fit\_psi.csv, and fit\_score.csv.
- parametric bootstap.R generates the bootstrap sample for SWAN data.

### 2 Simulation study folder

### 2.1 2 subgroup folder

- Corrected Simulation Data for 2 classes.ipynb simulates data for 2-subgroup.
- basis1.csv, basis2.csv provide the initial value of nonorthogonal basis for  $X_1$ ,  $X_2$ , respectively.
- mu1.csv, mu2.csv provide the initial value of the mean function for  $X_1, X_2$ , respectively.
- psi1.csv, psi2.csv provide the initial value of the mean function for  $X_1, X_2$ , respectively.
- probability2.csv provides the initial value of classification probability, which is  $\pi_{ic}$ .
- result\_2cluster\_hpcc.txt is the result file.

### 2.2 3 subgroup

- Corrected Simulation Data for 3 classes.ipynb simulates data for 3-subgroup.
- basis1.csv, basis2.csv provide the initial value of nonorthogonal basis for  $X_1$ ,  $X_2$ , respectively.
- mu1.csv, mu2.csv provide the initial value of the mean function for  $X_1, X_2$ , respectively.
- psi1.csv, psi2.csv provide the initial value of the mean function for  $X_1, X_2$ , respectively.
- probability3.csv provides the initial value of classification probability, which is  $\pi_{ic}$ .
- result\_3cluster\_hpcc.txt is the result file.
- X1measure, X2measure, and Ymeasure folder contain datafile for 200 simulation runs.

### 3 Detail function in *Python* file

- fun\_df2dict (data): transfer each column into a nested dictionary.
- fun\_diag\_blocks (data): transfer a  $p \times 1$  vector into a  $p \times p$  squared matrix.
- fun\_b\_t\_N\_orth\_b\_t (n\_spline): construct a orthogonal matrix. The number of knots is required.
- fun\_theta\_mu (data\_basis, data\_original\_theta\_mu): calculate the matrix of observed mean function for variable v, which is  $\boldsymbol{B}_{iv}\boldsymbol{\theta}_{\mu v}$ .
- fun\_theta\_psi (data\_orth\_basis, data\_original\_theta\_psi, data\_sigma, data\_pick\_n\_pc): after determining the number of principal components, calculate the matrix of observed eigenfunction for each variable v, which is  $\boldsymbol{B}_{iv}\boldsymbol{\Theta}_{\psi v}$ .
- fun\_matrix\_B\_iv (n, dx, data\_id, data\_W, data\_orth\_b\_t): calculate basis matrices for the observed time in all  $\boldsymbol{W}$  variables. Then return to the orthogonal basis.
- fun\_matrix\_unorth\_B\_iv (n, dx, data\_id, data\_W, data\_orth\_b\_t): calculate basis matrices for the observed time in all  $\boldsymbol{W}$  variables. Then return to the nonorthogonal basis.
- fun\_matrix\_tilde\_mu\_iv (n, dx, data\_theta\_mu, data\_B\_unorth\_iv): given the nonorthogonal spline basis matrix, calculate the mean matrix for each subject within each variable of  $\boldsymbol{W}$ .
- fun\_matrix\_tilde\_psi\_iv(n, dx, data\_theta\_psi, data\_B\_iv): given the orthogonal spline basis matrix, calculate the eigenfunction matrix for each subject within each variable of  $\boldsymbol{W}$ .
- fun\_matrix\_B\_star\_iv (n, dx, data\_id, data\_Y, data\_orth\_b\_t): calculate basis matrices for the observed time in **Y** variable. Then return to the orthogonal basis.
- fun\_matrix\_tilde\_mu\_star\_iv(n, dx, data\_theta\_mu, data\_B\_unorth\_star\_iv): given the nonorthogonal spline basis matrix, calculate the mean matrix for each subject within Y.
- fun\_matrix\_tilde\_mu\_star\_ic\_v(n, C, dx, data\_beta, data\_tilde\_mu\_star\_iv): multiple group-specific effect  $\beta_c$  with mean matrix for each subject for all variables in  $\boldsymbol{W}$ .
- fun\_matrix\_tilde\_psi\_star\_iv (n, dx, data\_theta\_psi, data\_B\_star\_iv): given the orthogonal spline basis matrix, calculate the eigenfunction matrix for each subject within **Y**.
- fun\_matrix\_tilde\_psi\_star\_ic\_v (n, C, dx, data\_beta, data\_tilde\_psi\_star\_iv): multiple group-specific effects  $\beta_c$  with a discrete matrix of eigenfunction for each subject for all variables in W.

- fun\_matrix\_tilde\_W\_iv (n, dx, data\_id, data\_W, data\_tilde\_mu\_iv): calculate the value of  $\widetilde{\boldsymbol{W}}_{iv}$ , which is  $\boldsymbol{W}_{iv} \boldsymbol{B}_{iv}^* \boldsymbol{\theta}_{\mu v}$ .
- fun\_matrix\_tilde\_Y\_ic (n, C, dz, data\_id, data\_Z, data\_Y, data\_beta, data\_tilde\_mu\_star\_ic\_v): calculate the value of  $\widetilde{\boldsymbol{Y}}_{ic}$ , which is  $\boldsymbol{Y}_i \beta_{0,c} \boldsymbol{1}_{m_{y,i}} \sum_{v=1}^{d_x} \beta_{x,cv} \boldsymbol{B}_{iv} \boldsymbol{\theta}_{\mu v} \boldsymbol{Z}_i \boldsymbol{\beta}_{z,c}$ .
- fun\_matrix\_G\_y\_ic (n, C, data\_id, data\_Y, data\_var\_Y): construct variance-covariance matrix for each subject in the *c*th subgroup.
- fun\_list\_invert\_lambda\_i (n, dx, data\_id, data\_W, data\_var\_W): construct the inverse matrix of variance in  $\boldsymbol{W}$  for each subject.