به نام خدا

Iran University
of Science and
Technology

یادگیری تقویتی در کنترل

دکتر سعید شمقدری

دانشکده مهندسی برق
گروه کنترل

نیمسال اول ۱۴۰۵–۱۴۰۴

Temporal Difference Learning

Temporal Difference Learning

Q Box

مشابهت با DP؟

مشابهت با MC؟

Reminder Box

تخمین تابع Value در DP و MC

$$\underbrace{V(S_t)}_{\text{New value of state t}} \leftarrow \underbrace{V(S_t)}_{\substack{\text{Former estimation of value of state t}\\ \text{(= Expected return starting at that state)}}} + \underbrace{\alpha}_{\text{Learning Rate}}[\underbrace{G_t}_{\text{Return at timestep t}} - \underbrace{V(S_t)}_{\substack{\text{Former estimation of value of state t}\\ \text{(= Expected return starting at that state)}}}]$$

روش Constant alpha در MC (نیاز به اتمام اپیزود)

استفاده از Return به عنوان تخمین امید ریاضی (Sampling)

Temporal Difference Prediction

تخمین تابع Value در TD

$$\underbrace{V(S_t)}_{\text{New value of state t}} \leftarrow \underbrace{V(S_t)}_{\text{Former estimation of value of state t}} + \underbrace{\alpha}_{\text{Learning Rate}}[\underbrace{R_{t+1}}_{\text{Reward}} + \underbrace{\gamma V(S_{t+1})}_{\text{Discounted value of next state}} - \underbrace{V(S_t)}]$$

Q Box:
Target in MC and TD?

Temporal Difference Prediction

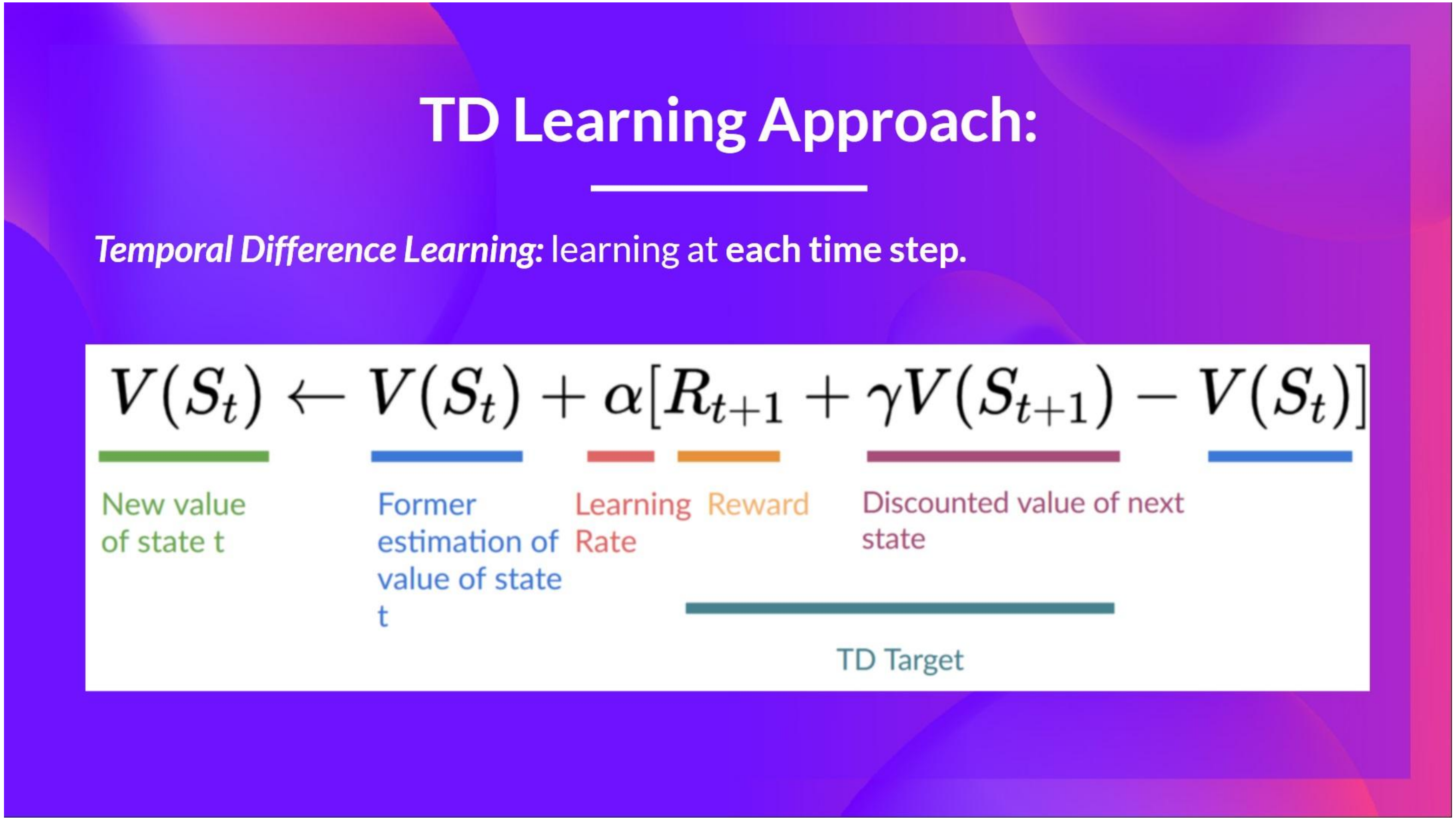
تخمین تابع Value در TD

$$\underbrace{V(S_t)}_{\text{New value of state t}} \leftarrow \underbrace{V(S_t)}_{\text{Former estimation of value of state t}} + \underbrace{\alpha}_{\text{Learning Rate}}[\underbrace{\underbrace{R_{t+1}}_{\text{Reward}} + \underbrace{\gamma V(S_{t+1})}_{\text{Discounted value of next state}}}_{\text{TD Target}} - V(S_t)]$$

TD(0): one step TD

Q Box:
Target in MC and TD?

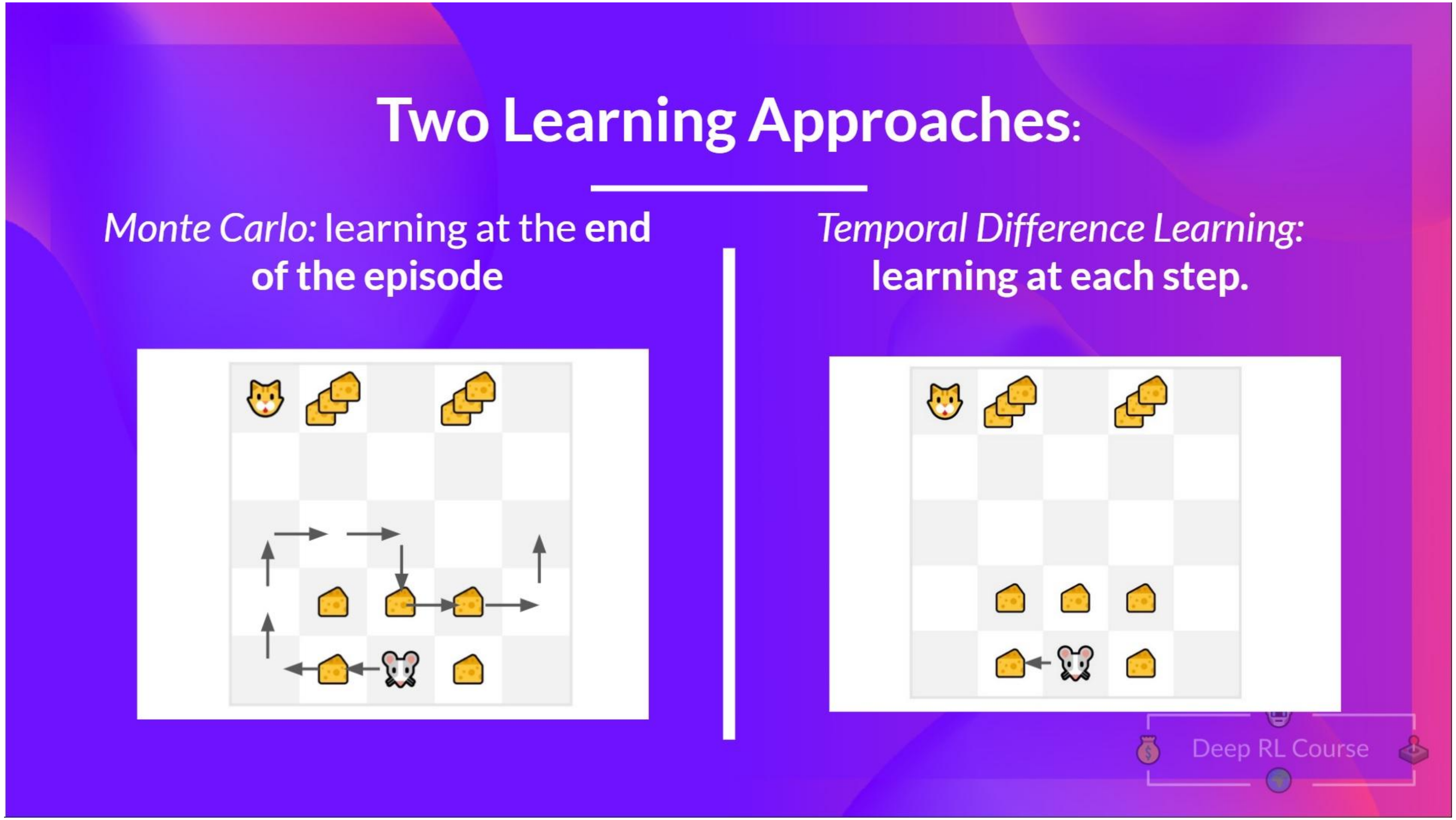
جمع‌بندی ...


TD Learning Approach:
Temporal Difference Learning: learning at each time step.
$V(S_t) \leftarrow V(S_t) + \alpha[R_{t+1} + \gamma V(S_{t+1}) - V(S_t)]$
New value of state t
Former estimation of value of state t
Learning Rate
Reward
Discounted value of next state
TD Target

جمع‌بندی ...


TD Approach:
At the end of one step (State, Action, Reward, Next State):
- We have Rt+1 and St+1
- We update V(St):
- We estimate Gt by adding Rt+1 and the discounted value of next state.
TD target : Rt+1 + gamma * V(St+1)
$V(S_t) \leftarrow V(S_t) + \alpha[R_{t+1} + \gamma V(S_{t+1}) - V(S_t)]$
Now we continue to interact with this environment with our updated value function.
By running more and more steps, the agent will learn to play better and better.

جمع‌بندی ...


Two Learning Approaches:
Monte Carlo: learning at the end of the episode
Temporal Difference Learning: learning at each step.
Deep RL Course

Temporal Difference Prediction

تخمین تابع Value در TD

$$
\begin{aligned}
v_\pi(s) &\doteq \mathbb{E}_\pi[G_t \mid S_t = s] \\
&= \mathbb{E}_\pi[R_{t+1} + \gamma G_{t+1} \mid S_t = s] \\
&= \mathbb{E}_\pi[R_{t+1} + \gamma v_\pi(S_{t+1}) \mid S_t = s]
\end{aligned}
$$

- DP: محاسبه امید ریاضی با مدل، استفاده از تخمین قبلی تابع Value در تخمین جدید (bootstrapping)
- TD: Sampling، استفاده از تخمین قبلی تابع Value در تخمین جدید (bootstrapping)

الگوریتم

Tabular TD(0) for estimating v_π

Input: the policy π to be evaluated
Algorithm parameter: step size $\alpha \in (0, 1]$
Initialize $V(s)$, for all $s \in \mathcal{S}^+$, arbitrarily except that $V(\textit{terminal}) = 0$

Loop for each episode:
    Initialize S
    Loop for each step of episode:
        $A \leftarrow$ action given by π for S
        Take action A, observe R, S'
        $V(S) \leftarrow V(S) + \alpha\big[R + \gamma V(S') - V(S)\big]$
        $S \leftarrow S'$
    until S is terminal

TD Error

$$\delta_t \doteq R_{t+1} + \gamma V(S_{t+1}) - V(S_t)$$

Monte Carlo Error

$$\begin{aligned}
G_t - V(S_t) &= R_{t+1} + \gamma G_{t+1} - V(S_t) + \gamma V(S_{t+1}) - \gamma V(S_{t+1}) \\
&= \delta_t + \gamma\big(G_{t+1} - V(S_{t+1})\big) \\
&= \delta_t + \gamma\delta_{t+1} + \gamma^2\big(G_{t+2} - V(S_{t+2})\big) \\
&= \delta_t + \gamma\delta_{t+1} + \gamma^2\delta_{t+2} + \cdots + \gamma^{T-t-1}\delta_{T-1} + \gamma^{T-t}\big(G_T - V(S_T)\big) \\
&= \delta_t + \gamma\delta_{t+1} + \gamma^2\delta_{t+2} + \cdots + \gamma^{T-t-1}\delta_{T-1} + \gamma^{T-t}(0 - 0) \\
&= \sum_{k=t}^{T-1} \gamma^{k-t}\delta_k.
\end{aligned}$$

Driving Home

مثال

Value function approximation TD

State	*Elapsed Time (minutes)*	*Predicted Time to Go*	*Predicted Total Time*
leaving office, friday at 6	0	30	30
reach car, raining	5	35	40
exiting highway	20	15	35
2ndary road, behind truck	30	10	40
entering home street	40	3	43
arrive home	43	0	43

Q Box

Reward?
Return? T_Go
Value? E(T_Go)

Driving Home

مثال

Value function approximation TD

State	*Elapsed Time (minutes)*	*Predicted Time to Go*	*Predicted Total Time*
leaving office, friday at 6	0	30	30
reach car, raining	5	35	40
exiting highway	20	15	35
2ndary road, behind truck	30	10	40
entering home street	40	3	43
arrive home	43	0	43

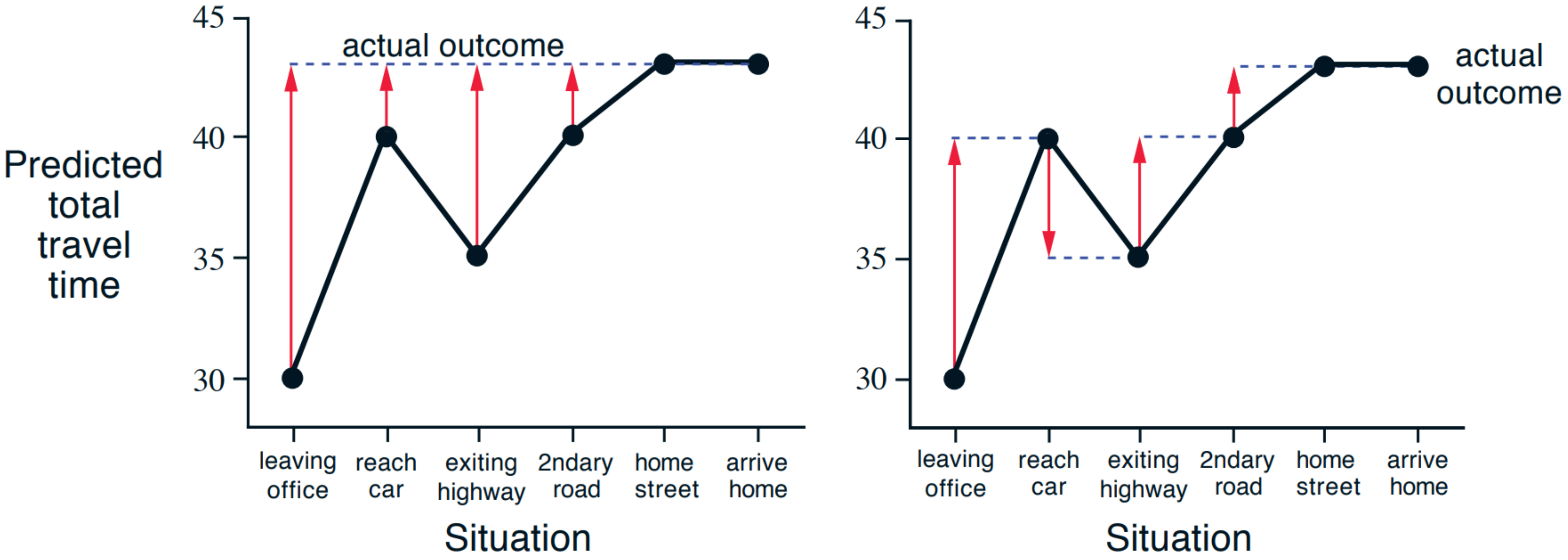
Q Box

خطای تخمین در خروجی بزرگراه (بر اساس MC)؟

اصلاح تخمین در خروجی بزرگراه (بر اساس MC) به ازای alpha=0.5؟

$$\alpha\big(G_t - V(s_t)\big)?$$

TD: یادگیری سریعتر



Temporal Difference Learning

همگرایی TD

- مقدار α بسیار کوچک
- α با شرایط کاهشی

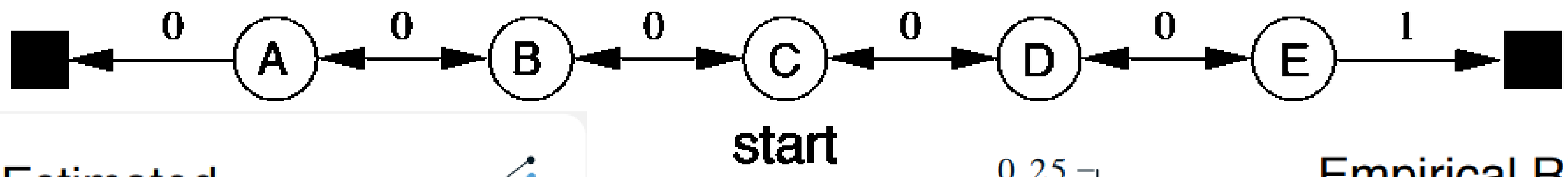
$$\sum_{n=1}^{\infty} \alpha_n(a) = \infty \qquad \text{and} \qquad \sum_{n=1}^{\infty} \alpha_n^2(a) < \infty$$

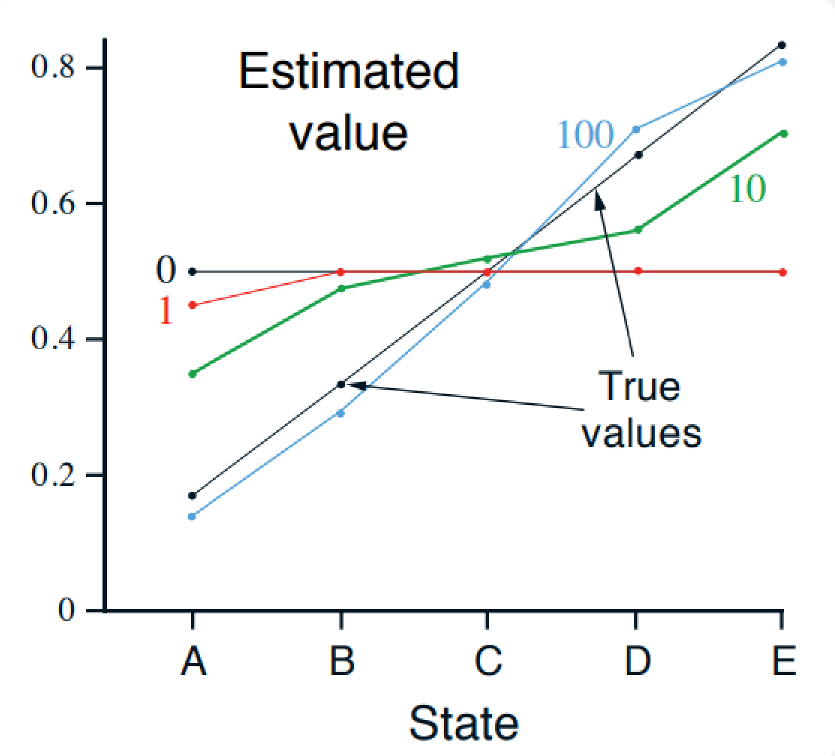
سرعت همگرایی MC و TD

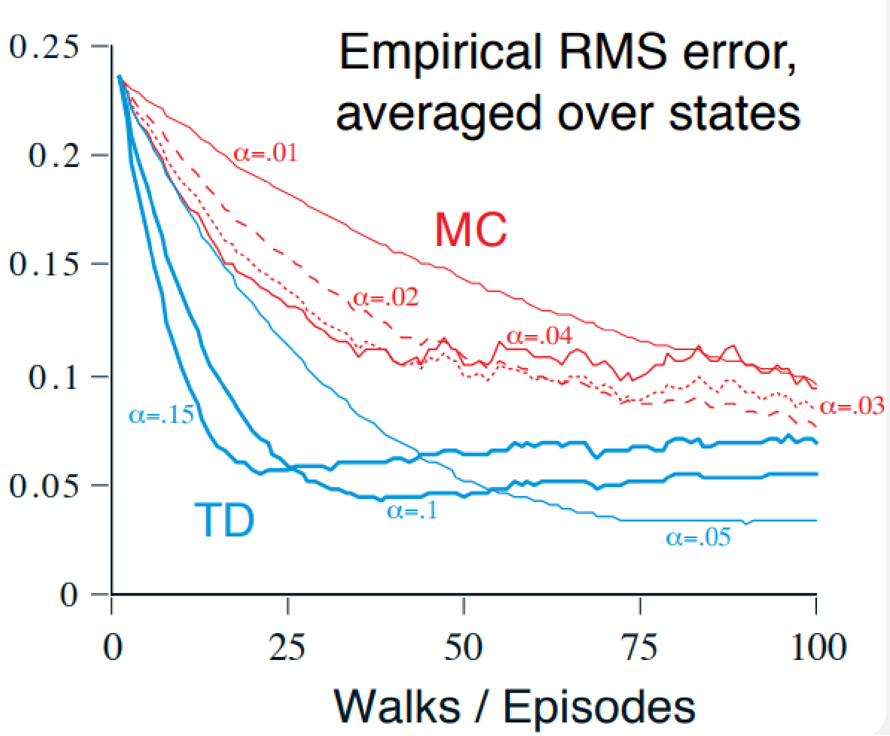
Markov Reward Process: Random Walk

مثال

In this example we empirically compare the prediction abilities of TD(0) and constant-α MC when applied to the following Markov reward process:







Random Walk Under Batch Updating

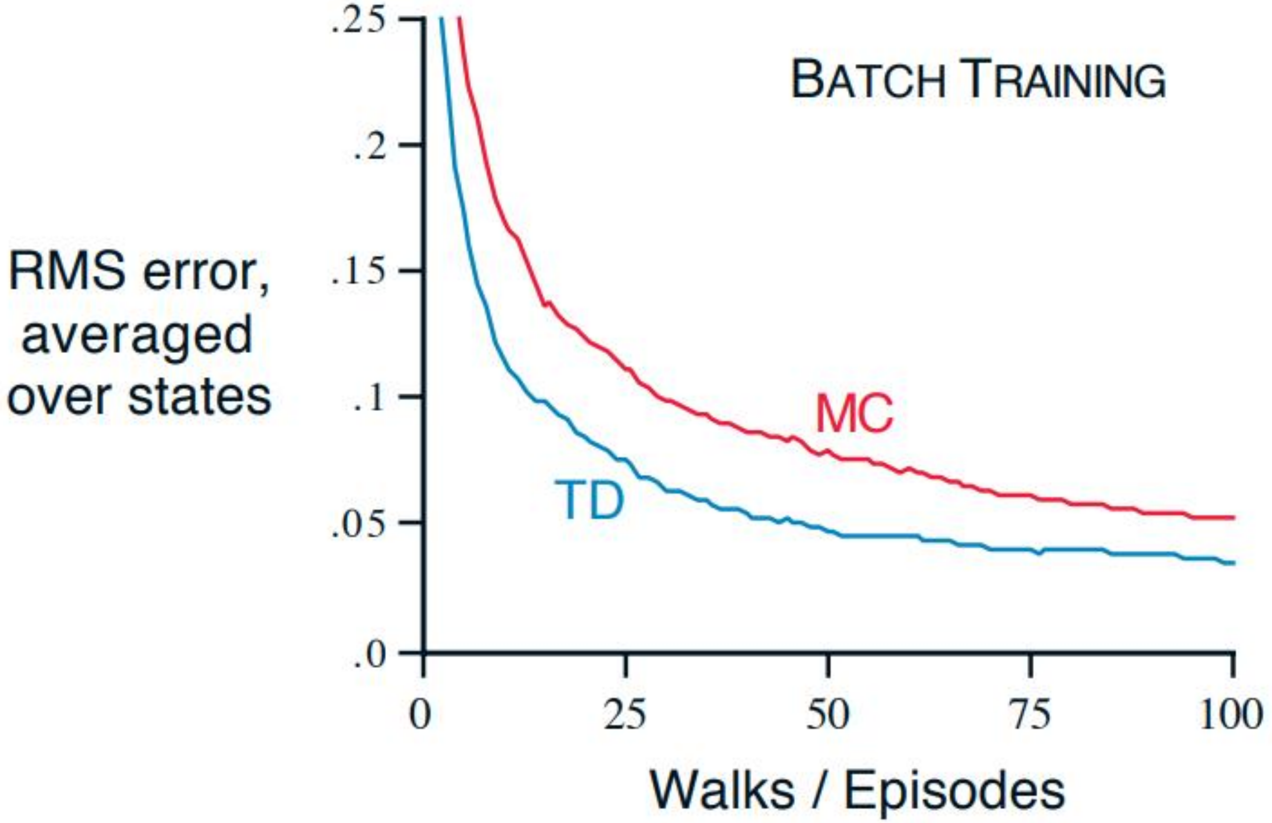
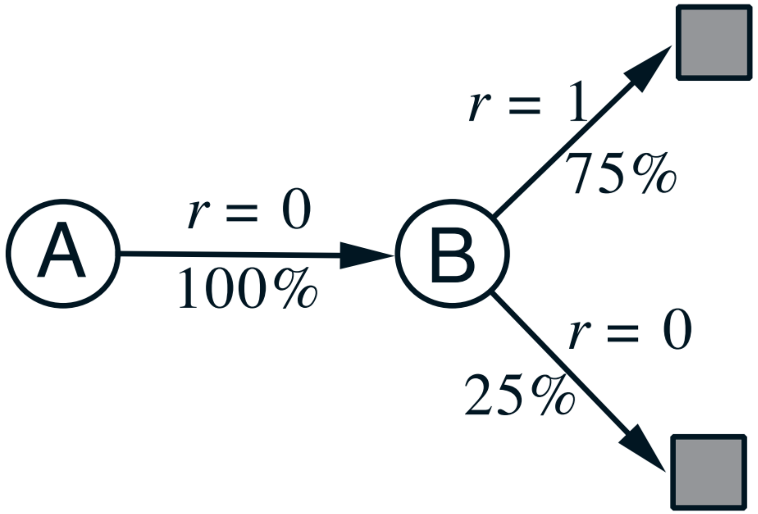


Figure 6.2: Performance of TD(0) and constant-α MC under batch training on the random walk task.

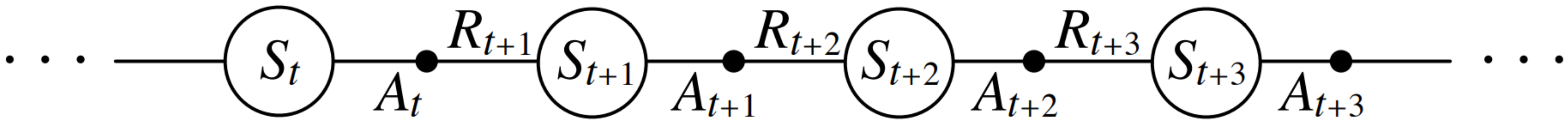
You are the Predictor

مثال

$A, 0, B, 0$
$B, 1$
$B, 1$
$B, 1$
$B, 1$
$B, 1$
$B, 1$
$B, 0$

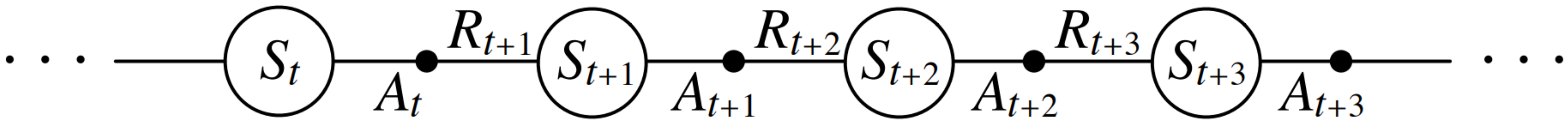


SARSA: On-Policy TD Control



$$Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha \Big[R_{t+1} + \gamma Q(S_{t+1}, A_{t+1}) - Q(S_t, A_t) \Big]$$

SARSA: On-Policy TD Control



$$\underbrace{Q(S_t, A_t)}_{\text{Updated Q Value}} \leftarrow \underbrace{Q(S_t, A_t)}_{\text{Current Q Value}} + \alpha \left[\overbrace{R_{t+1} + \underbrace{\gamma Q(S_{t+1}, A_{t+1})}_{\text{Target Policy is always as same as Behaviour Policy}}}^{\text{Target Q Value}} - \underbrace{Q(S_t, A_t)}_{\text{Current Q Value}} \right]$$

الگوریتم

Sarsa (on-policy TD control) for estimating $Q \approx q_*$

Algorithm parameters: step size $\alpha \in (0, 1]$, small $\varepsilon > 0$
Initialize $Q(s, a)$, for all $s \in \mathcal{S}^+, a \in \mathcal{A}(s)$, arbitrarily except that $Q(terminal, \cdot) = 0$

Loop for each episode:
    Initialize S
    Choose A from S using policy derived from Q (e.g., ε-greedy)
    Loop for each step of episode:
        Take action A, observe R, S'
        Choose A' from S' using policy derived from Q (e.g., ε-greedy)
        $Q(S, A) \leftarrow Q(S, A) + \alpha\big[R + \gamma Q(S', A') - Q(S, A)\big]$
        $S \leftarrow S'$; $A \leftarrow A'$;
    until S is terminal

درک بهتر!

Select Action

Agent chooses action based on policy

Observe State

Agent perceives current environment

Update Q-value

Agent adjusts Q-values based on experience

Execute Action!

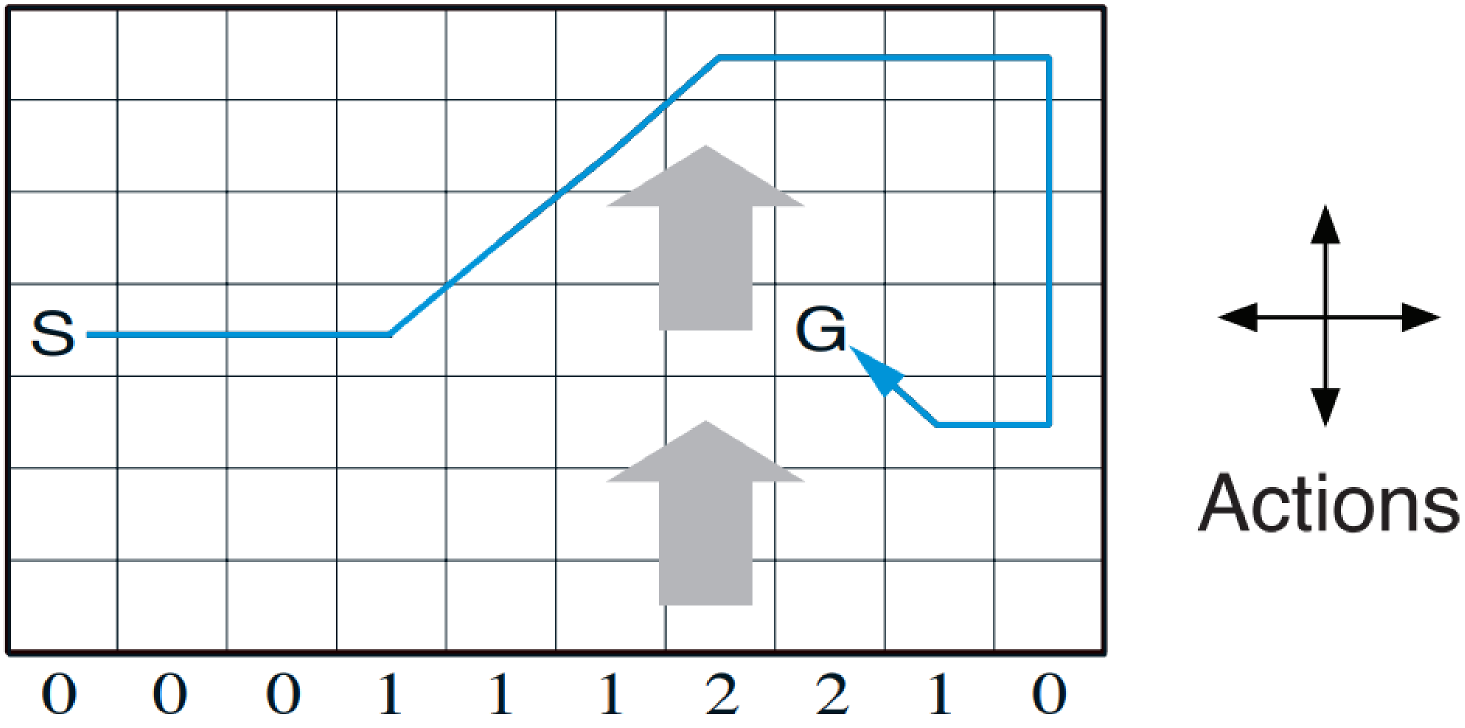
Agent performs selected action

Observe Reward

Agent receives feedback from environment

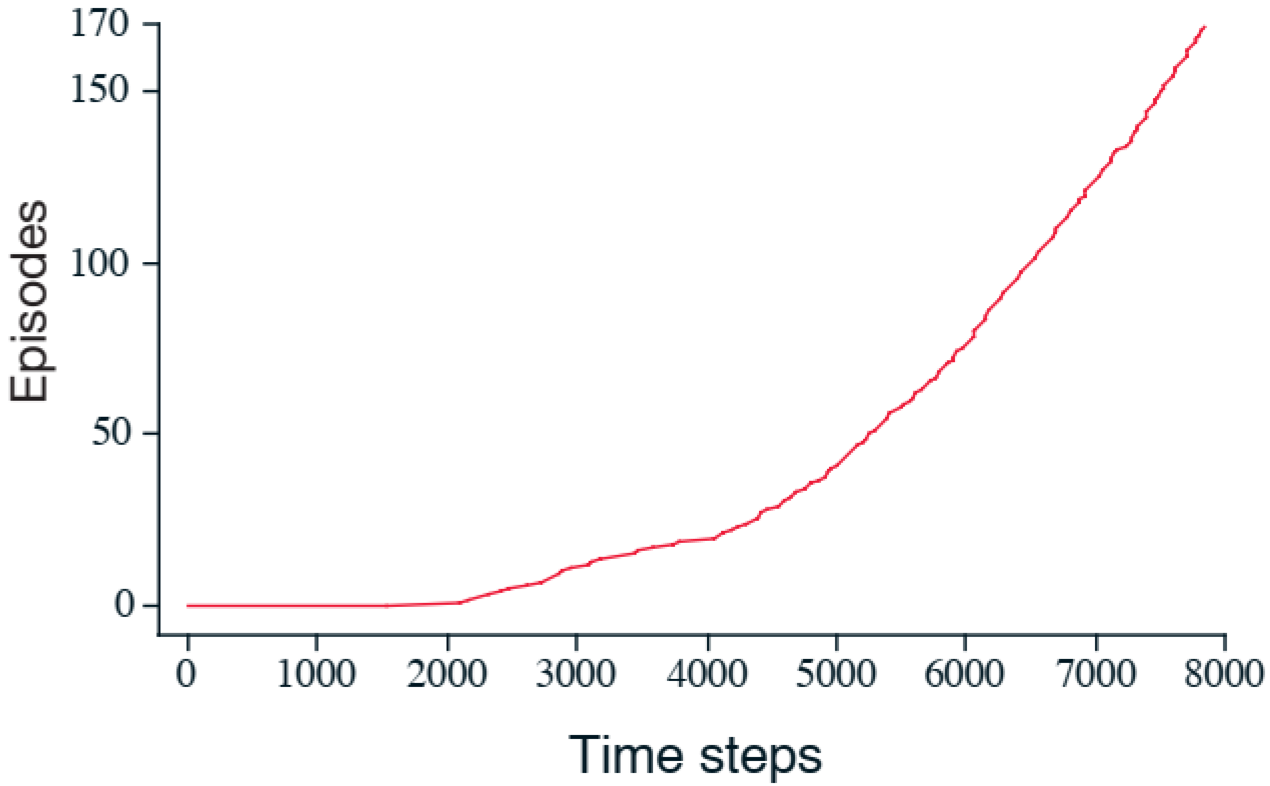
Windy Grid

مثال



Windy Grid

مثال



Q-Learning: Off-Policy TD Control

$$\underbrace{Q(S_t, A_t)}_{\text{New Q-value estimation}} \leftarrow \underbrace{Q(S_t, A_t)}_{\text{Former Q-value estimation}} + \underbrace{\alpha}_{\text{Learning Rate}}[\underbrace{\underbrace{\underbrace{R_{t+1}}_{\text{Immediate Reward}} + \underbrace{\gamma max_a Q(S_{t+1}, a)}_{\text{Discounted Estimate optimal Q-value of next state}}}_{\text{TD Target}} - \underbrace{Q(S_t, A_t)}_{\text{Former Q-value estimation}}}_{\text{TD Error}}]$$

Example

- You always start at the **same starting point.**
- The goal: eat the **big pile of cheese** (at the bottom right-hand corner) and **avoid the poison.**
- The episode ends if we eat the poison, eat the big pile of cheese or if we spent more than 5 steps.
- Learning rate = 0.1
- Gamma = 0.99

Example

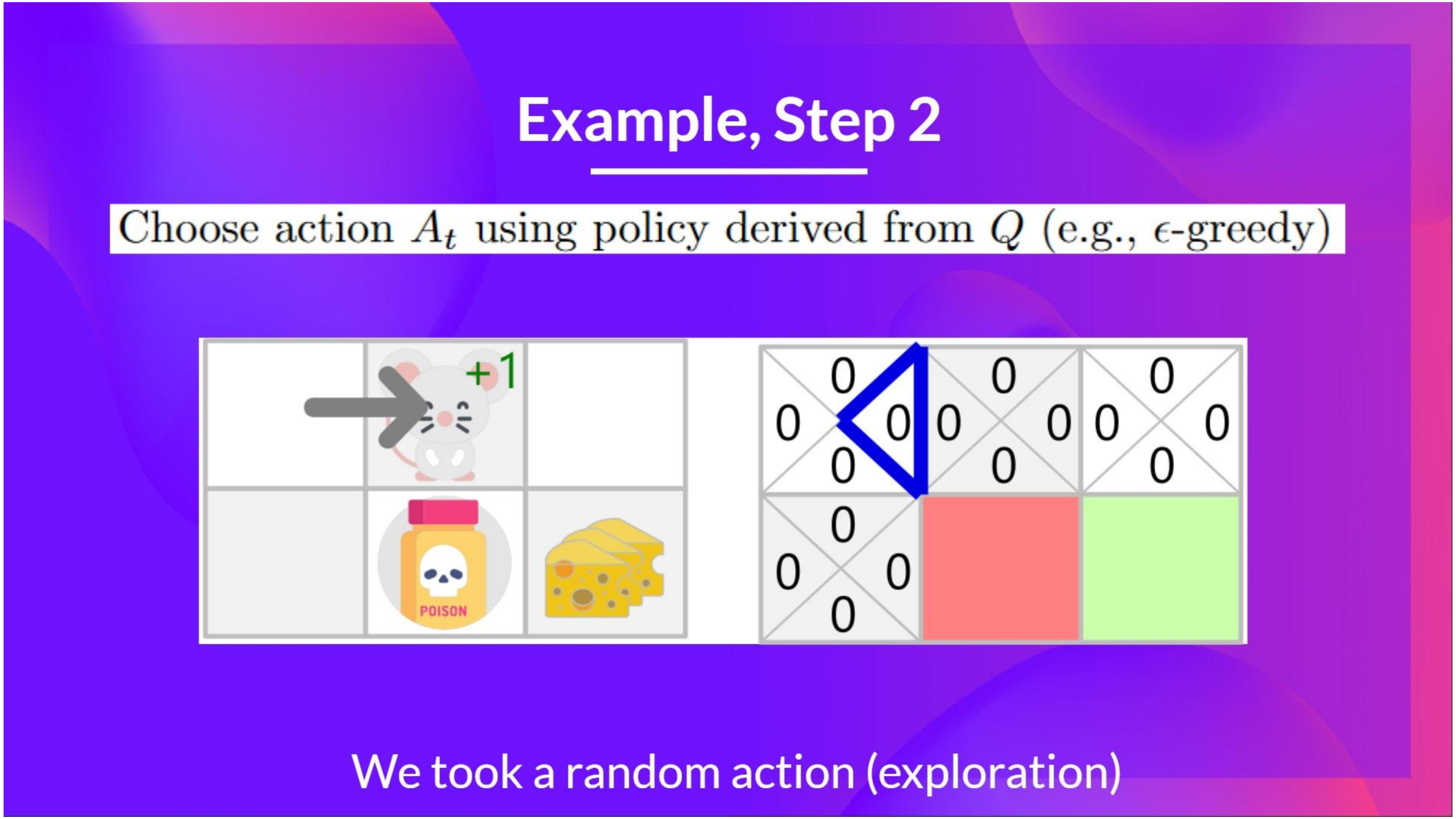
- The reward function:
  - 0: Going to a state **with no cheese in it.**
  - +1: Going to a state with a **small cheese in it.**
  - +10: Going to the state with **the big pile of cheese.**
  - -10: Going to the state **with the poison and thus die.**

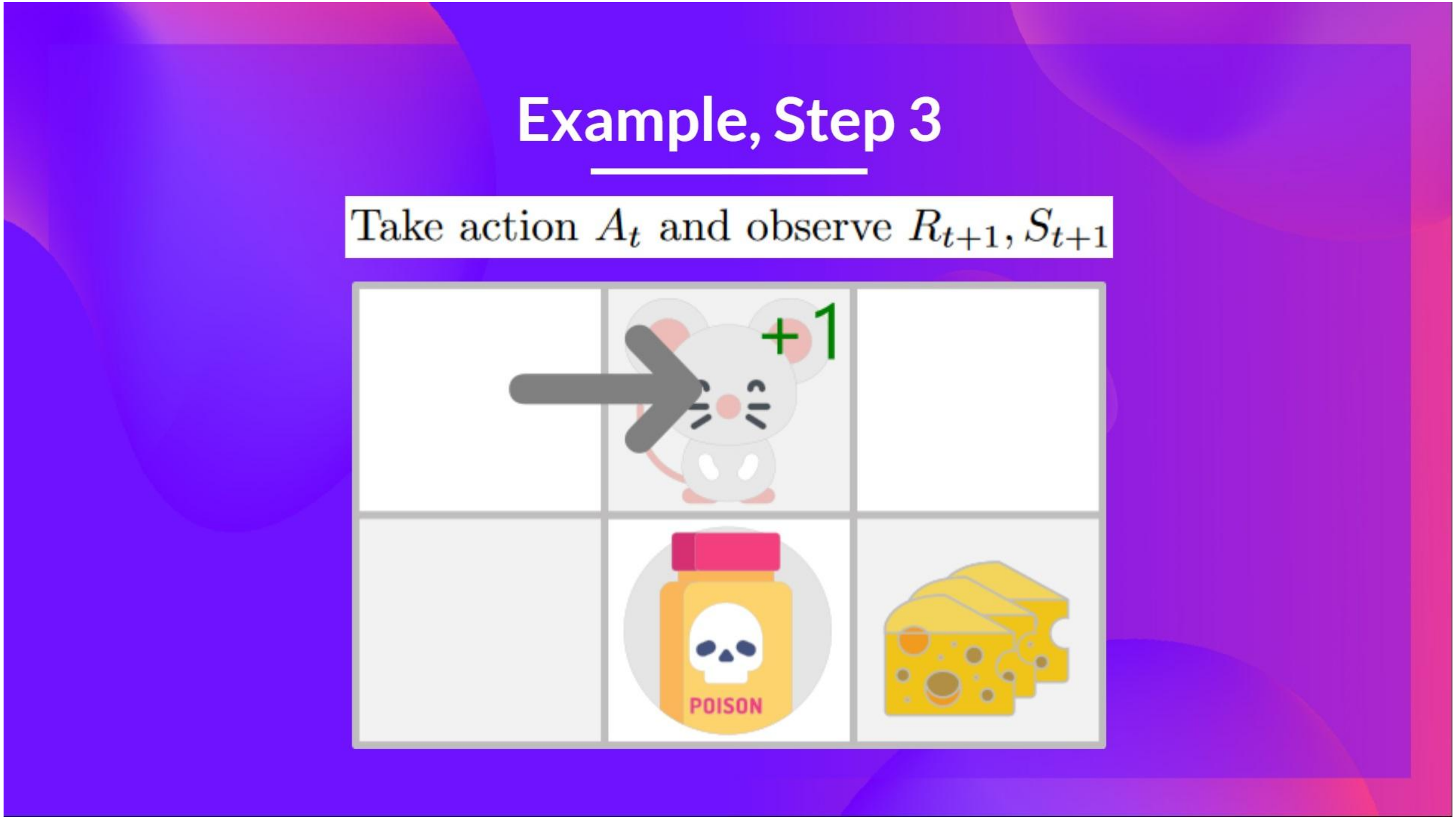
Example, Step 1

Initialize Q arbitrarily (e.g., $Q(s, a) = 0$ for all $s \in \mathcal{S}$ and $a \in \mathcal{A}(s)$, and $Q(\textit{terminal-state}, \cdot) = 0$)

	←	→	↑	↓
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0

We initialize the Q-Table


Example, Step 2
Choose action A_t using policy derived from Q (e.g., ϵ-greedy)
+1
POISON
We took a random action (exploration)


Example, Step 3
Take action A_t and observe R_{t+1}, S_{t+1}
+1
POISON

Example, Step 4

$$\underbrace{Q(S_t, A_t)}_{\text{New Q-value estimation}} \leftarrow \underbrace{Q(S_t, A_t)}_{\text{Former Q-value estimation}} + \underbrace{\alpha}_{\text{Learning Rate}}[\underbrace{R_{t+1}}_{\text{Immediate Reward}} + \underbrace{\gamma max_a Q(S_{t+1}, a)}_{\text{Discounted Estimate optimal Q-value of next state}} - \underbrace{Q(S_t, A_t)}_{\text{Former Q-value estimation}}]$$

TD Target: $R_{t+1} + \gamma max_a Q(S_{t+1}, a)$

TD Error: $R_{t+1} + \gamma max_a Q(S_{t+1}, a) - Q(S_t, A_t)$

Update our Q-value estimation

Example, Step 4

$$Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha(R_{t+1} + \gamma \max_a Q(S_{t+1}, a) - Q(S_t, A_t))$$

Q(Initial state, Right) = 0 + 0.1 * [1 + 0.99 * 0 - 0]
Q(Initial state, Right) = 0.1

	←	→	↑	↓
	0	0.1	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0

Q-Learning Recap

الگوریتم

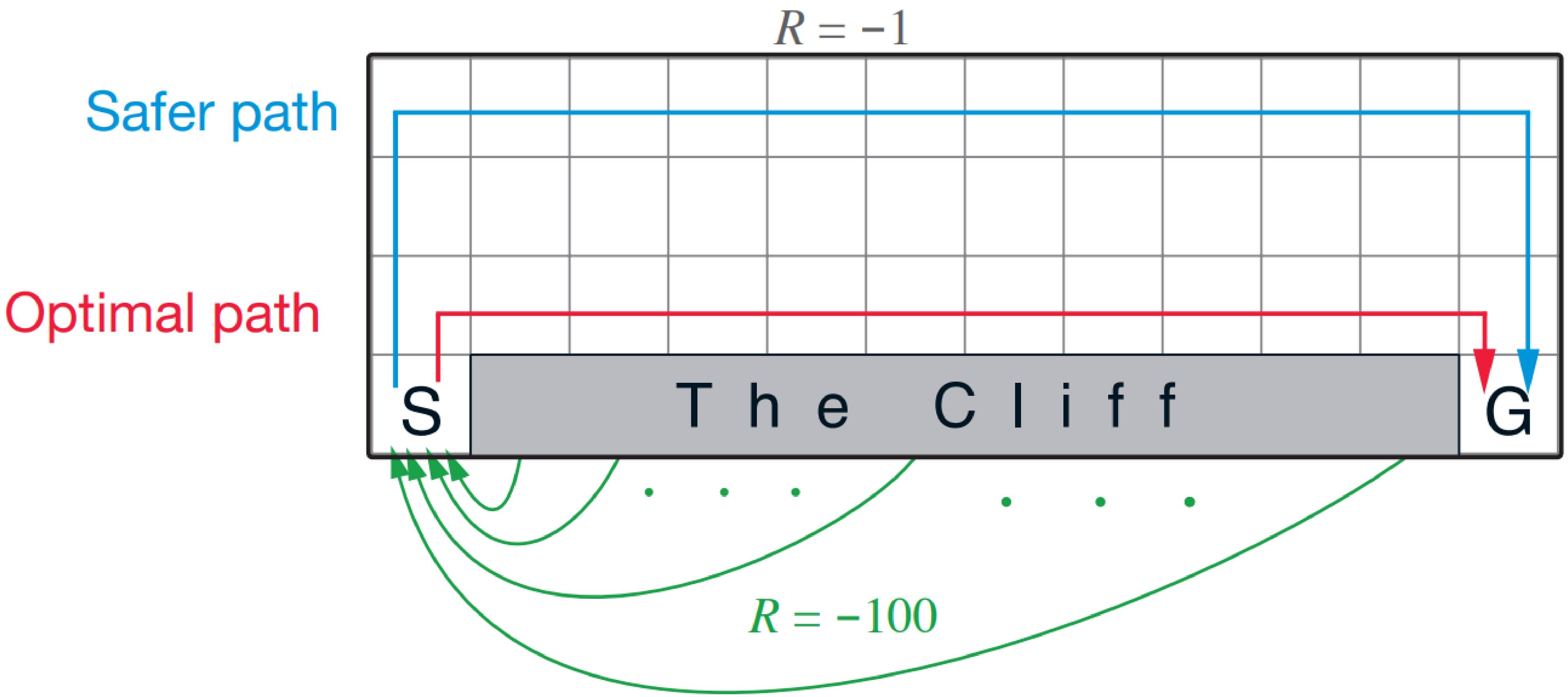
Q-learning (off-policy TD control) for estimating $\pi \approx \pi_*$

Algorithm parameters: step size $\alpha \in (0, 1]$, small $\varepsilon > 0$
Initialize $Q(s, a)$, for all $s \in \mathcal{S}^+, a \in \mathcal{A}(s)$, arbitrarily except that $Q(terminal, \cdot) = 0$

Loop for each episode:
  Initialize S
  Loop for each step of episode:
    Choose A from S using policy derived from Q (e.g., ε-greedy)
    Take action A, observe R, S'
    $Q(S, A) \leftarrow Q(S, A) + \alpha\big[R + \gamma \max_a Q(S', a) - Q(S, A)\big]$
    $S \leftarrow S'$
  until S is terminal

Cliff Walking

مثال



Cliff Walking

مثال