In the Name of God

Iran University of Science and Technology

Reinforcement Learning in Control

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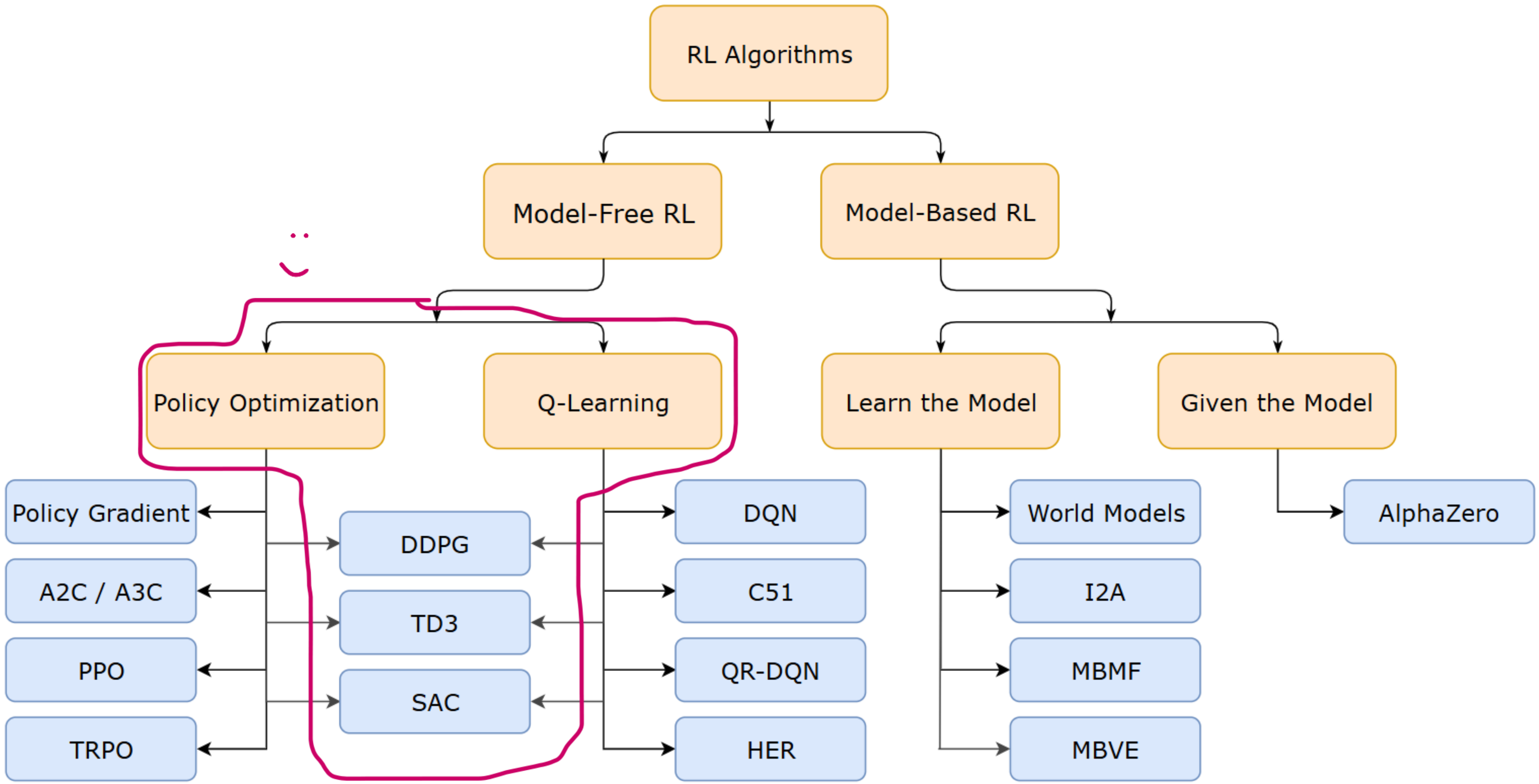
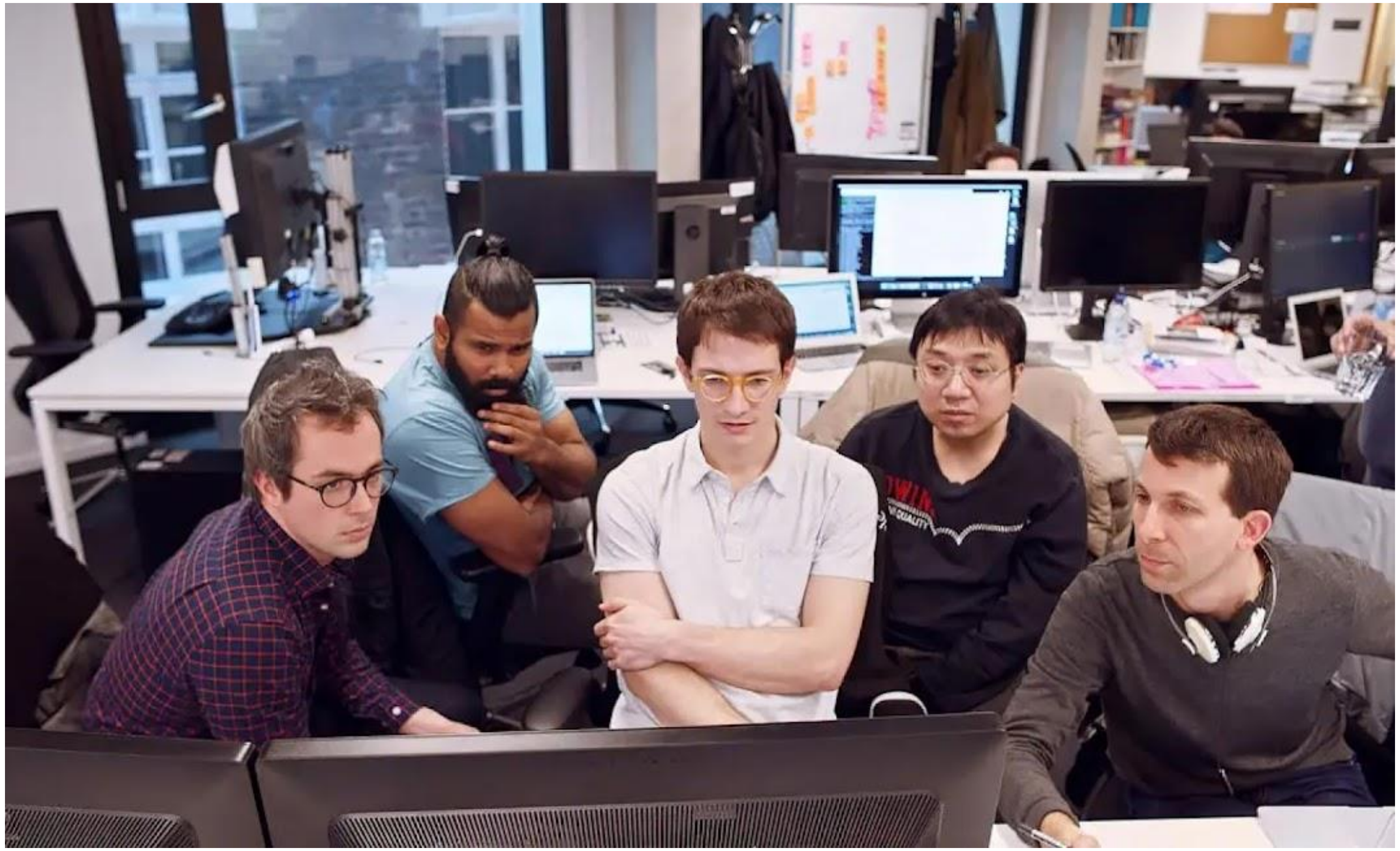
Fall 2025 | 4041

(Deep) Deterministic Policy Gradient

RL Algorithms



Once Again, Google DeepMind!



Deterministic Policy Gradient

Deterministic Policy Gradient Algorithms

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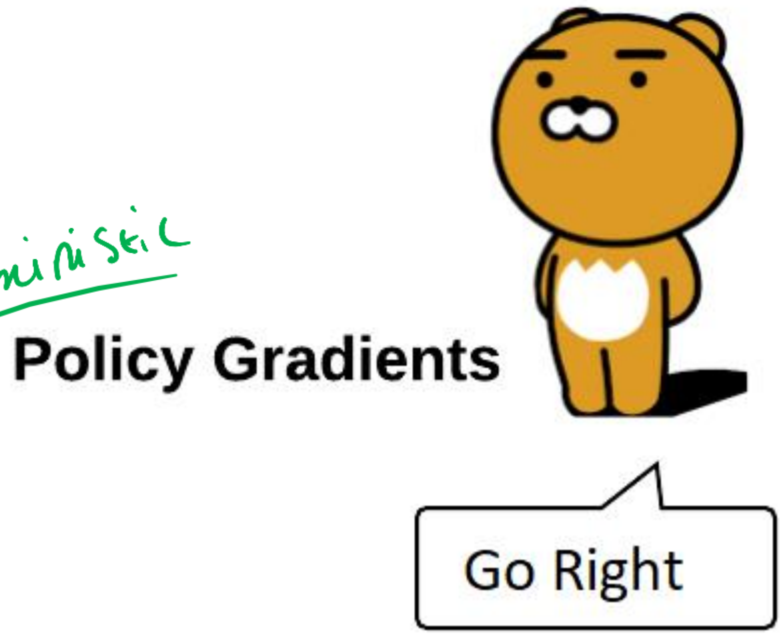
Deterministic Policy Gradient Algorithms (Paper)

Deterministic Policy Gradient

Deterministic



exploration?

- ✓ Continuous Action Space
- ✓ Off-Policy
- ✓ Stochastic Behaviour Policy

Deterministic Policy Gradient

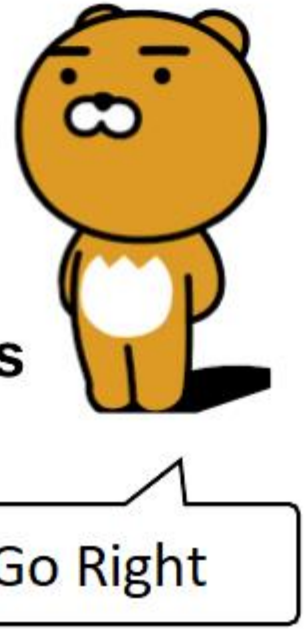
Reminder Box

$$\pi_\theta(a|s) = \mathbb{P}\left[a|s;\theta\right]$$

Stochastic Policy

Adjusting policy parameters using gradient ascent

Policy Gradients



Deterministic Policy Gradient

"In this paper we instead consider *deterministic* policies"

$$a = \mu_\theta(s)$$

Q: Does a deterministic policy gradient actually exist?

"yes! Deterministic policy gradient is the limiting case, as policy variance tends to zero, of the stochastic policy gradient."

Stochastic PG vs Deterministic PG

Stochastic PG integrates over states + actions $\rightsquigarrow$ Needs more samples!

Deterministic PG integrates only over states $\rightsquigarrow$ More sample efficient!

Q: Does this mean stochastic policies are no longer useful?
What happens to exploration?

Off-Policy DPG!

Quick Review of RL

A Markov Decision Process with: An action Space $\mathcal{A}$ and a state space $\mathcal{S}$

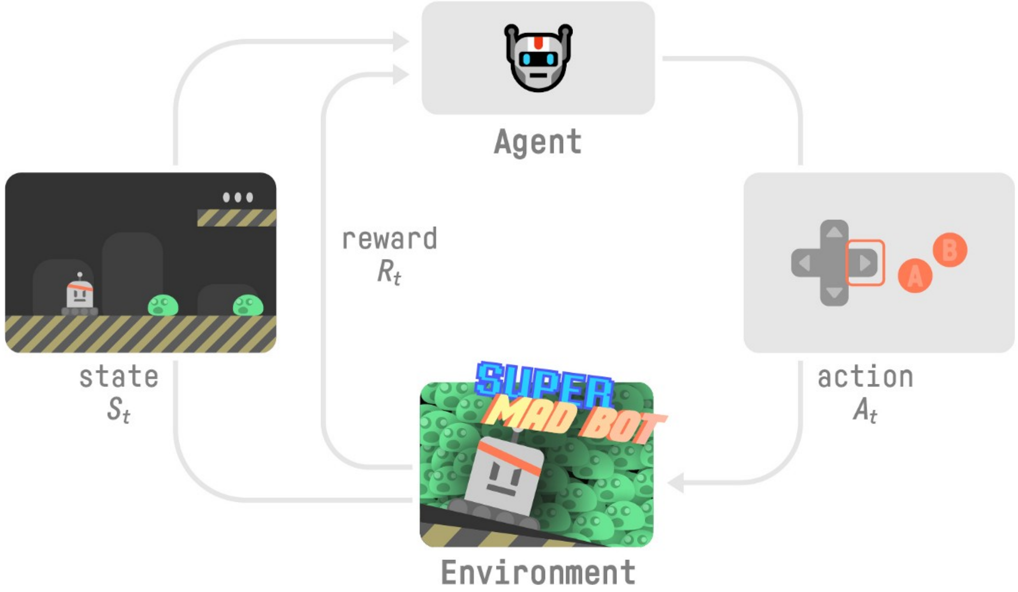
$$p(s_{t+1}|s_1, a_1, \ldots, s_t, a_t) = p(s_{t+1}|s_t, a_t)$$

A reward function $r: \mathcal{S} \times \mathcal{A} \longrightarrow \mathbb{R}$

A policy

$$\pi_\theta : \mathcal{S} \to \mathcal{P}(\mathcal{A})$$

is the set of probability measures on $\mathcal{A}$

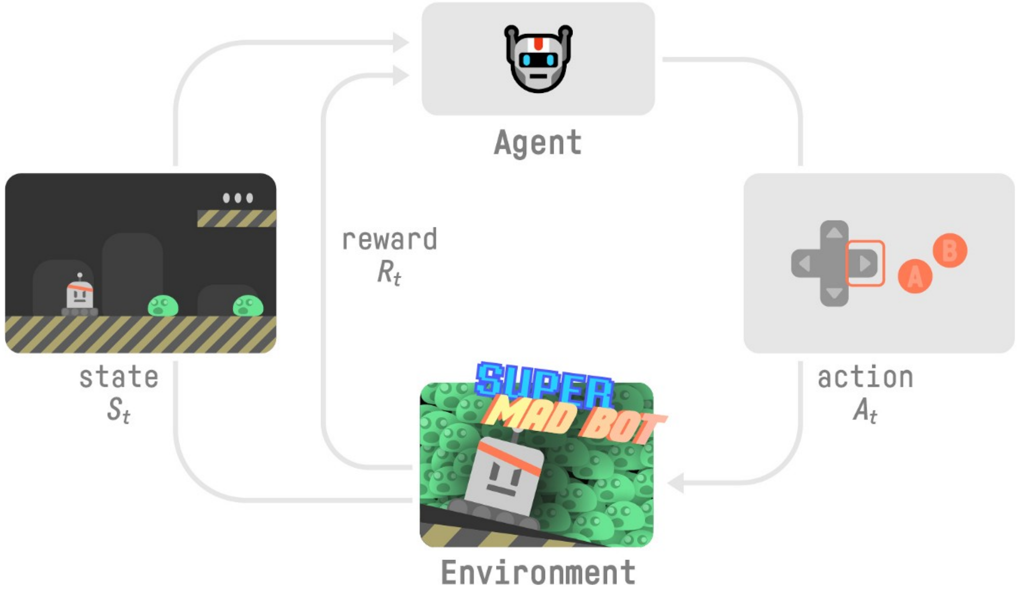


Quick Review of RL

$$h_{1:T} = s_1, a_1, r_1 \ldots, s_T, a_T, r_T \text{ over } \mathcal{S} \times \mathcal{A} \times \mathbb{R}$$ $\rightsquigarrow$ Trajectory

$$r_t^\gamma = \sum_{k=t}^{\infty} \gamma^{k-t} r(s_k, a_k)$$ return

$$\underbrace{\begin{aligned} V^\pi(s) &= \mathbb{E}\left[r_1^\gamma | S_1 = s; \pi\right] \\ Q^\pi(s,a) &= \mathbb{E}\left[r_1^\gamma | S_1 = s, A_1 = a; \pi\right] \end{aligned}}_{\text{Value functions}}$$



Quick Review of RL

Performance Objective:

$$J(\pi) = \mathbb{E}\left[r_1^{\gamma} | \pi\right]$$

(Improper) Discounted state distribution:

$$\rho^{\pi}(s') := \int_{\mathcal{S}} \sum_{t=1}^{\infty} \gamma^{t-1} p_1(s) p(s \rightarrow s', t, \pi) \mathrm{d}s$$

Initial state distribution

(the density at state *s'* after transitioning for *t* time steps from state *s*)

Then we can write:

$$\begin{aligned} J(\pi_\theta) &= \int_{\mathcal{S}} \rho^{\pi}(s) \int_{\mathcal{A}} \pi_\theta(s,a) r(s,a) \mathrm{d}a\mathrm{d}s \\ &= \mathbb{E}_{s \sim \rho^{\pi}, a \sim \pi_\theta}\left[r(s,a)\right] \end{aligned}$$

Quick Review of RL

Stochastic Policy Gradient:

$$\begin{aligned}\nabla_\theta J(\pi_\theta) &= \int_{\mathcal{S}} \rho^\pi(s) \int_{\mathcal{A}} \nabla_\theta \pi_\theta(a|s) Q^\pi(s,a) \mathrm{d}a\mathrm{d}s \\ &= \mathbb{E}_{s\sim\rho^\pi, a\sim\pi_\theta}\left[\nabla_\theta \log \pi_\theta(a|s) Q^\pi(s,a)\right]\end{aligned}$$

Stochastic Actor-Critic:

$$\nabla_\theta J(\pi_\theta) = \mathbb{E}_{s\sim\rho^\pi, a\sim\pi_\theta}\left[\nabla_\theta \log \pi_\theta(a|s) Q^w(s,a)\right]$$

Off-Policy Actor-Critic

It is often useful to estimate the policy gradient *off-policy* from trajectories sampled from a distinct behaviour policy $\beta(a|s) \neq \pi_\theta(a|s)$

$$\begin{aligned} J_\beta(\pi_\theta) &= \int_{\mathcal{S}} \rho^\beta(s) V^\pi(s) \mathrm{d}s \\ &= \int_{\mathcal{S}} \int_{\mathcal{A}} \rho^\beta(s) \pi_\theta(a|s) Q^\pi(s,a) \mathrm{d}a\mathrm{d}s \end{aligned}$$

$$\begin{aligned} \nabla_\theta J_\beta(\pi_\theta) &\approx \int_{\mathcal{S}} \int_{\mathcal{A}} \rho^\beta(s) \nabla_\theta \pi_\theta(a|s) Q^\pi(s,a) \mathrm{d}a\mathrm{d}s \\ &= \mathbb{E}_{s\sim\rho^\beta, a\sim\beta}\left[\frac{\pi_\theta(a|s)}{\beta_\theta(a|s)} \nabla_\theta \log \pi_\theta(a|s) Q^\pi(s,a) \right] \end{aligned}$$

Off-Policy Actor-Critic

It is often useful to estimate the policy gradient *off-policy* from trajectories sampled from a distinct behaviour policy $\beta(a|s) \neq \pi_\theta(a|s)$

$$\begin{aligned} J_\beta(\pi_\theta) &= \int_{\mathcal{S}} \rho^\beta(s) V^\pi(s) \mathrm{d}s \\ &= \int_{\mathcal{S}} \int_{\mathcal{A}} \rho^\beta(s) \pi_\theta(a|s) Q^\pi(s,a) \mathrm{d}a \mathrm{d}s \end{aligned}$$

$$\begin{aligned} \nabla_\theta J_\beta(\pi_\theta) &\approx \int_{\mathcal{S}} \int_{\mathcal{A}} \rho^\beta(s) \nabla_\theta \pi_\theta(a|s) Q^\pi(s,a) \mathrm{d}a \mathrm{d}s \\ &= \mathbb{E}_{s\sim\rho^\beta, a\sim\beta}\left[\frac{\pi_\theta(a|s)}{\beta_\theta(a|s)} \nabla_\theta \log \pi_\theta(a|s) Q^\pi(s,a)\right] \end{aligned}$$

Off-Policy Actor-Critic

$$\nabla_\theta J_\beta(\pi_\theta) \approx \int_{\mathcal{S}}\int_{\mathcal{A}} \rho^\beta(s)\nabla_\theta\pi_\theta(a|s)Q^\pi(s,a)\mathrm{d}a\mathrm{d}s$$

$$= E\left[\int_{\mathcal{A}} \nabla_\theta\pi_\theta(a|s)Q^\pi(s,a)\middle| s\sim\rho^\beta\right]$$

$$= E\left[\underbrace{\int_{\mathcal{A}} \beta_\theta(a|s)}_{E[\dots|a\sim\beta]} \frac{\pi_\theta(a|s)}{\beta_\theta(a|s)} \underbrace{\frac{\nabla_\theta\pi_\theta(a|s)}{\pi_\theta(a|s)}}_{\nabla_\theta log\pi_\theta(a|s)} Q^\pi(s,a)\middle| s\sim\rho^\beta\right]$$

$$= E\left[\frac{\pi_\theta(a|s)}{\beta_\theta(a|s)}\nabla_\theta log\pi_\theta(a|s)Q^\pi(s,a)\middle| s\sim\rho^\beta, a\sim\beta\right]$$

$$= \mathbb{E}_{s\sim\rho^\beta, a\sim\beta}\left[\frac{\pi_\theta(a|s)}{\beta_\theta(a|s)}\nabla_\theta \log\pi_\theta(a|s)Q^\pi(s,a)\right]$$

Gradients of Deterministic Policies: Action-Value Gradients

Reminder Box

Policy Improvement:

$$\mu^{k+1}(s) = \underset{a}{\operatorname{argmax}}\ Q^{\mu^k}(s, a)$$

Q: What about continuous action space?

"Requiring a global maximisation at every step
Instead, move the policy in the direction of the gradient of Q"

Gradients of Deterministic Policies: Action-Value Gradients

For each visited state s, the policy parameters θ^{k+1}, are updated in proportion to the gradient $\nabla_\theta Q^{\mu^k}(s, \mu_\theta(s))$.

Each state suggests a different direction of policy improvement; these may be averaged together by taking an expectation with respect to the state distribution $\rho^\mu(s)$.

$$\theta^{k+1} = \theta^k + \alpha \mathbb{E}_{s\sim\rho^{\mu^k}} \left[\nabla_\theta Q^{\mu^k}(s, \mu_\theta(s)) \right]$$

chain rule

$$\theta^{k+1} = \theta^k + \alpha \mathbb{E}_{s\sim\rho^{\mu^k}} \left[\nabla_\theta \mu_\theta(s) \left. \nabla_a Q^{\mu^k}(s, a) \right|_{a=\mu_\theta(s)} \right]$$

Deterministic Policy Gradient Theorem

A deterministic policy $\mu_\theta : S \to A,\ \theta \in R^n$

Performance Objective $J(\mu_\theta) = E[r_1^\gamma | \mu]$

Probability Distribution $\rho(s \to s', t, \mu)$

Discounted state distribution $\rho^\mu(s)$

Then:

$$J(\mu_\theta) = \int_S \rho^\mu(s) r(s, \mu_\theta(s)) \mathrm{d}s$$

$$= \mathbb{E}_{s \sim \rho^\mu} \left[r(s, \mu_\theta(s)) \right]$$

Deterministic Policy Gradient Theorem

Theorem 1 (Deterministic Policy Gradient Theorem)

Suppose that the MDP satisfies condition below:

$p(s'|s,a),\ \nabla_a p(s'|s,a),\ \mu_\theta(s),\ \nabla_\theta \mu_\theta(s),\ r(s,a),\ \nabla_a r(s,a),\ p_1(s)$

are continuous in all parameters and variables s, a, s', x.

These imply that $\nabla_\theta \mu_\theta(s)$ and $\nabla_a Q^\mu(s,a)$ exist *and that the deterministic policy gradient exists. Then*

$$\begin{aligned}\nabla_\theta J(\mu_\theta) &= \int_{\mathcal{S}} \rho^\mu(s) \nabla_\theta \mu_\theta(s)\ \nabla_a Q^\mu(s,a)\big|_{a=\mu_\theta(s)} \,\mathrm{d}s \\ &= \mathbb{E}_{s\sim\rho^\mu}\left[\nabla_\theta \mu_\theta(s)\ \nabla_a Q^\mu(s,a)\big|_{a=\mu_\theta(s)}\right]\end{aligned}$$

Limit of the Stochastic Policy Gradient

For a wide class of stochastic policies, including many bump functions, the deterministic policy gradient is indeed a special (limiting) case of the stochastic policy gradient.

Theorem 2

Consider a stochastic policy $\pi_{\mu_\theta,\sigma}$ where μ_θ is a deterministic policy and σ is a parameter controlling the variance. Then,

$$\lim_{\sigma\downarrow 0} \nabla_\theta J(\pi_{\mu_\theta,\sigma}) = \nabla_\theta J(\mu_\theta)$$

$$\boxed{\begin{array}{c}\sigma \rightarrow 0 \\ \pi_{\mu_\theta,0} \equiv \mu_\theta\end{array}}$$

Where on the l.h.s. the gradient is the standard stochastic policy gradient and on the r.h.s. the gradient is the deterministic policy gradient.

Deterministic Actor-Critic: On-Policy

Q: Optimality?

Algorithm: On-Policy Actor-Critic

In this *deterministic actor-critic* algorithm, the critic uses Sarsa updates to estimate the action-value function.

$$\delta_t = r_t + \gamma Q^w(s_{t+1}, a_{t+1}) - Q^w(s_t, a_t)$$

$$w_{t+1} = w_t + \alpha_w \delta_t \nabla_w Q^w(s_t, a_t) \qquad \text{Critic}$$

$$\theta_{t+1} = \theta_t + \alpha_\theta \nabla_\theta \mu_\theta(s_t) \left. \nabla_a Q^w(s_t, a_t) \right|_{a=\mu_\theta(s)} \qquad \text{Actor}$$

Deterministic Actor-Critic: Off-Policy

Learn a deterministic target policy $\mu_\theta(s)$, from trajectories generated by an arbitrary stochastic behaviour policy $\pi(a|s) = \beta(a|s)$.

$$
\begin{aligned}
J_\beta(\mu_\theta) &= \int_{\mathcal{S}} \rho^\beta(s) V^\mu(s) \mathrm{d}s \\
&= \int_{\mathcal{S}} \rho^\beta(s) Q^\mu(s, \mu_\theta(s)) \mathrm{d}s
\end{aligned}
$$

$$
\begin{aligned}
\nabla_\theta J_\beta(\mu_\theta) &\approx \int_{\mathcal{S}} \rho^\beta(s) \nabla_\theta \mu_\theta(a|s) Q^\mu(s, a) \mathrm{d}s \\
&= \mathbb{E}_{s \sim \rho^\beta} \left[\nabla_\theta \mu_\theta(s) \; \nabla_a Q^\mu(s, a)|_{a=\mu_\theta(s)} \right]
\end{aligned}
$$

Deterministic Actor-Critic: Off-Policy

Q: Importance Sampling?

"We now develop an actor-critic algorithm that updates the policy in the direction of the off-policy deterministic policy gradient."

Algorithm: On-Policy Actor-Critic

Substitute a differentiable action-value function $Q^w(s,a)$ in place of the true action-value function $Q^\mu(s,a)$. A critic estimates the action-value function $Q^w(s,a) \approx Q^\mu(s,a)$ off-policy from trajectories generated by $\beta(a|s)$, and uses Q-learning updates to estimate the action-value function.

$$\delta_t = r_t + \gamma Q^w(s_{t+1}, \mu_\theta(s_{t+1})) - Q^w(s_t, a_t)$$

$$w_{t+1} = w_t + \alpha_w \delta_t \nabla_w Q^w(s_t, a_t) \qquad \text{Critic}$$

$$\theta_{t+1} = \theta_t + \alpha_\theta \nabla_\theta \mu_\theta(s_t) \left. \nabla_a Q^w(s_t, a_t) \right|_{a=\mu_\theta(s)} \qquad \text{Actor}$$

For further details on the algorithms, including compatible function approximation and related methods, please refer to the original paper. We will not go into more depth here.

Deep Deterministic Policy Gradient

Published as a conference paper at ICLR 2016

CONTINUOUS CONTROL WITH DEEP REINFORCEMENT LEARNING

Timothy P. Lillicrap[*], Jonathan J. Hunt[*], Alexander Pritzel, Nicolas Heess,
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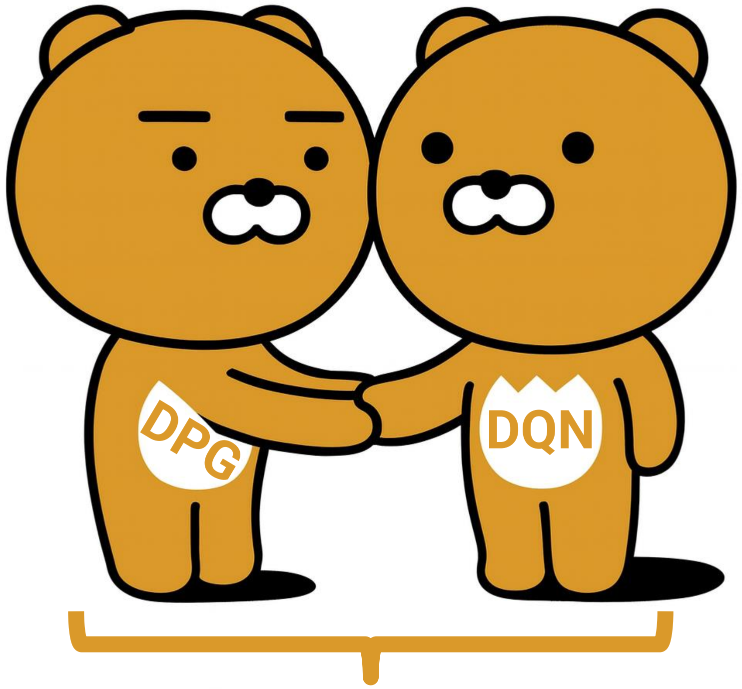
Continuous Control with Deep Reinforcement Learning (Paper)

Deep Deterministic Policy Gradient

learn policies "end-to-end": directly from raw pixel inputs.



$\approx$ ***DDPG*** →

- Actor-Critic
- Model-Free
- Off-Policy
- Based on DPG
- For Continuous Action Space

DQN: Discussion Questions

Q: What is the main limitation of standard DQN? A: Discrete/Low Dim Action Space!

Q: Why can't DQN be directly applied to continuous action spaces? A: Maximization!

well, discretize the action space!

Q: What problems arise when discretizing continuous action spaces, and why is it inefficient? A: Curse of dimentionality exploration, ...

Example: 7 DOF × 3 values each: $3^7 = 2187$actions.

DQN: Discussion Questions

Q: Is DQN only limited and problematic, or does it also have important advantages?

A: Replay buffer
Target network
} Stability in training 😎

Reminder Box

$$R_t = \sum_{i=t}^{T} \gamma^{(i-t)} r(s_i, a_i)$$
$$J = \mathbb{E}_{r_i, s_i \sim E, a_i \sim \pi} [R_1]$$

DDPG

In the DPG paper, we saw

$$\nabla_\theta J_\beta(\mu_\theta) \approx \int_{\mathcal{S}} \rho^\beta(s) \nabla_\theta \mu_\theta(a|s) Q^\mu(s,a) \mathrm{d}s$$
$$= \mathbb{E}_{s\sim\rho^\beta}\left[\nabla_\theta \mu_\theta(s)\ \nabla_a Q^\mu(s,a)|_{a=\mu_\theta(s)}\right]$$

In the DDPG paper, the notation is slightly different:

$$\nabla_{\theta^\mu} J \approx \mathbb{E}_{s_t\sim\rho^\beta}\left[\nabla_{\theta^\mu} Q(s,a|\theta^Q)|_{s=s_t, a=\mu(s_t|\theta^\mu)}\right]$$
$$= \mathbb{E}_{s_t\sim\rho^\beta}\left[\nabla_a Q(s,a|\theta^Q)|_{s=s_t, a=\mu(s_t)} \nabla_{\theta_\mu} \mu(s|\theta^\mu)|_{s=s_t}\right]$$

DDPG → 4 NNs + Batch Norm technique (?)

Similar to DQN, we also use:

- separate target networks, **Q:** Why?
- and a replay buffer here

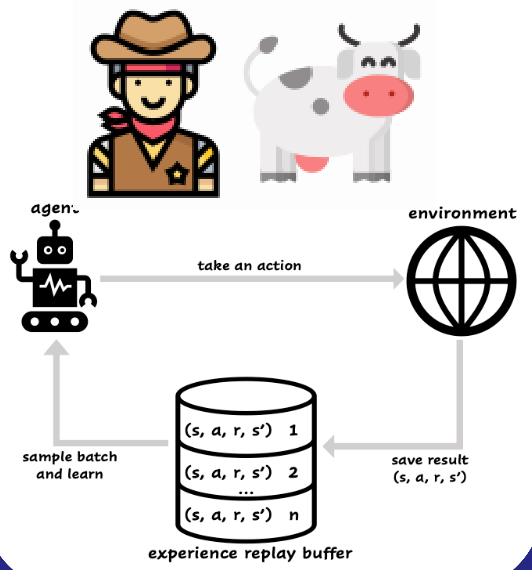
s, a → **Critic network** → $Q(s,a|\theta^{Q})$

s → **Actor network** → $\mu(s|\theta^{\mu})$

s, a → **Target critic network** → $Q'(s,a|\theta^{Q'})$

s → **Target actor network** → $\mu'(s|\theta^{\mu'})$

Reminder Box



The weights of these target networks are then updated by having them slowly track the learned networks: $\theta' \leftarrow \tau\theta + (1-\tau)\theta', \qquad \tau \ll 1$

Exploration Challenge

"A major challenge of learning in continuous action spaces is exploration. An advantage of off-policies algorithms such as DDPG is that we can treat the problem of exploration independently from the learning algorithm. We constructed an exploration policy μ' by adding noise sampled from a noise process $\mathcal{N}$ to our actor policy

$$\mu'(s_t) = \mu(s_t|\theta_t^\mu) + \mathcal{N}$$

$\mathcal{N}$ can be chosen to suit the environment."

DDPG: Algorithm

Algorithm 1 DDPG algorithm

Randomly initialize critic network $Q(s, a|\theta^Q)$ and actor $\mu(s|\theta^\mu)$ with weights θ^Q and θ^μ.
Initialize target network Q' and μ' with weights $\theta^{Q'} \leftarrow \theta^Q$, $\theta^{\mu'} \leftarrow \theta^\mu$
Initialize replay buffer R
for episode = 1, M **do**
  Initialize a random process $\mathcal{N}$ for action exploration
  Receive initial observation state s_1
  for t = 1, T **do**
    Select action $a_t = \mu(s_t|\theta^\mu) + \mathcal{N}_t$ according to the current policy and exploration noise
    Execute action a_t and observe reward r_t and observe new state s_{t+1}
    Store transition (s_t, a_t, r_t, s_{t+1}) in R
    Sample a random minibatch of N transitions (s_i, a_i, r_i, s_{i+1}) from R
    Set $y_i = r_i + \gamma Q'(s_{i+1}, \mu'(s_{i+1}|\theta^{\mu'})|\theta^{Q'})$
    Update critic by minimizing the loss: $L = \frac{1}{N}\sum_i (y_i - Q(s_i, a_i|\theta^Q))^2$
    Update the actor policy using the sampled policy gradient:

$$\nabla_{\theta^\mu} J \approx \frac{1}{N} \sum_i \nabla_a Q(s, a|\theta^Q)|_{s=s_i, a=\mu(s_i)} \nabla_{\theta^\mu} \mu(s|\theta^\mu)|_{s_i}$$

    Update the target networks:

$$\theta^{Q'} \leftarrow \tau\theta^Q + (1-\tau)\theta^{Q'}$$

$$\theta^{\mu'} \leftarrow \tau\theta^\mu + (1-\tau)\theta^{\mu'}$$

  end for
end for

End of Part I