频率域滤波

频率域滤波器锐化图像 High pass filter

$$H(u,v)=\begin{cases}0 & D(u,v)\le D_0\\ 1 & D(u,v)>D_0\end{cases}$$

理想高通滤波器

$$H(u,v)=\frac{1}{1+[D_0/D(u,v)]^{2n}}$$

布特沃斯高通滤波器 Butterworth HPF

$$H(u,v)=1-e^{-D^2(u,v)/2D_0^2}$$

高斯高通滤波器 Gaussian HPF

频率域滤波器平滑图像 Low pass filter

$$H(u,v)=\begin{cases}1 & D(u,v)\le D_0\\ 0 & D(u,v)>D_0\end{cases}$$

其中，D(u,v)是频率域中点(u,v)与频率矩形中心的距离

理想低通滤波器Idea low pass filter

D0 -> Non-negative value / D(u,v): Distance point(u,v)

$$H(u,v)=\frac{1}{1+[D(u,v)/D_0]^{2n}}$$

n 阶布特沃斯低通滤波器 Butterworth LPF

$$H(u,v)=e^{-D^2(u,v)/2D_0^2}$$

高斯低通滤波器 Gaussian LPF

频率域滤波基础

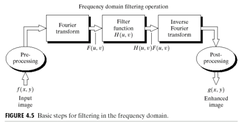


FIGURE 4.5 Basic steps for filtering in the frequency domain.

Basic steps

1、 $(-1)^{x+y}\times$ 原图像
2、 $F(u,v)$
3、 $H(u,v)\times F(u,v)$
4、反DEF
5、实部
6、用 $(-1)^{x+y}\times(5)$结果。

被滤波图像 $=\wp^{-1}\left[G(u,v)\right]$

$$G(u,v)=H(u,v)F(u,v)$$

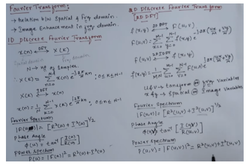
In Text

$$H(u,v)=\begin{cases}0 & (u,v)=(\frac{M}{2},\frac{N}{2})\\ 1 & 其它\end{cases}$$

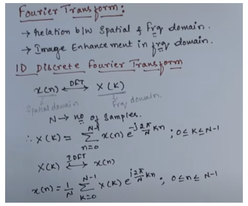
陷波滤波器：→平均值

Image Enhancement in Frequency domain

- Process
  - From spacial domain -> frequency domain -> process
  - Invest transform is applied to bring back into special domain
- Frequency domain: Filter
  - -> Smoothing/Sharpening image -> removing high/low frequency components
  - change -> whole image(unlike special domain-> which takes pixels by pixels)
- Types of filter
  - Low pass filter (remove high frequency components image) -> smoothing
  - High pass filter (remove low frequency components image) -> sharpening

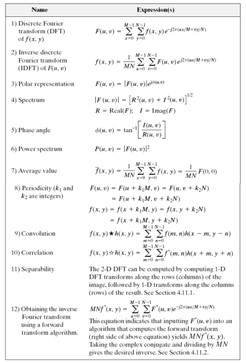


Some draft



离散傅立叶变换(DFT)

- Terms
  - 傅里叶级数（FS）
    - 任何周期函数都可以表示为不同频率的正弦或余弦之和的形式每个正弦项和余弦项都乘以不同的系数
  - 傅里叶变换（FT）
    - 任何非周期函数（其曲线下的面积是有限的）可以表示为正弦和/或余弦乘以加权函数的积分来表示
- Fomula conclusion



Name	Expression(s)
1) Discrete Fourier transform (DFT) of $f(x,y)$	$F(u,v)=\sum_{x=0}^{M-1}\sum_{y=0}^{N-1}f(x,y)e^{-2\pi i(ux/M+vy/N)}$
2) Inverse discrete Fourier transform (IDFT) of $F(u,v)$	$f(x,y)=\frac{1}{MN}\sum_{u=0}^{M-1}\sum_{v=0}^{N-1}F(u,v)e^{2\pi i(ux/M+vy/N)}$
3) Polar representation	$F(u,v)=\lvert F(u,v)\rvert e^{j\phi(u,v)}$
4) Spectrum	$\lvert F(u,v)\rvert=\left[R^2(u,v)+I^2(u,v)\right]^{1/2}$ $R=\mathrm{Real}(F);\quad I=\mathrm{Imag}(F)$
5) Phase angle	$\phi(u,v)=\tan^{-1}\left[\frac{I(u,v)}{R(u,v)}\right]$
6) Power spectrum	$P(u,v)=\lvert F(u,v)\rvert^2$
7) Average value	$\bar{f}(x,y)=\frac{1}{MN}\sum_{x=0}^{M-1}\sum_{y=0}^{N-1}f(x,y)=\frac{1}{MN}F(0,0)$
8) Periodicity (k_1 and k_2 are integers)	$F(u,v)=F(u+k_1M,v)=F(u,v+k_2N)$ $=F(u+k_1M,v+k_2N)$ $f(x,y)=f(x+k_1M,y)=f(x,y+k_2N)$ $=f(x+k_1M,y+k_2N)$
9) Convolution	$f(x,y)\star h(x,y)=\sum_{m=0}^{M-1}\sum_{n=0}^{N-1}f(m,n)h(x-m,y-n)$
10) Correlation	$f(x,y)\circ h(x,y)=\sum_{m=0}^{M-1}\sum_{n=0}^{N-1}f^*(m,n)h(x+m,y+n)$
11) Separablity	The 2-D DFT can be computed by computing 1-D DFT transforms along the rows (columns) of the image, followed by 1-D transforms along the columns (rows) of the result. See Section 4.11.1.
12) Obtaining the inverse Fourier transform using a forward transform algorithm.	$MNf^*(x,y)=\sum_{u=0}^{M-1}\sum_{v=0}^{N-1}F^*(u,v)e^{-2\pi i(ux/M+vy/N)}$ This equation indicates that inputting $F^*(u,v)$ into an algorithm that computes the forward transform (right side of above equation) yields $MNf^*(x,y)$. Taking the complex conjugate and dividing by MN gives the desired inverse; See Section 4.11.2.