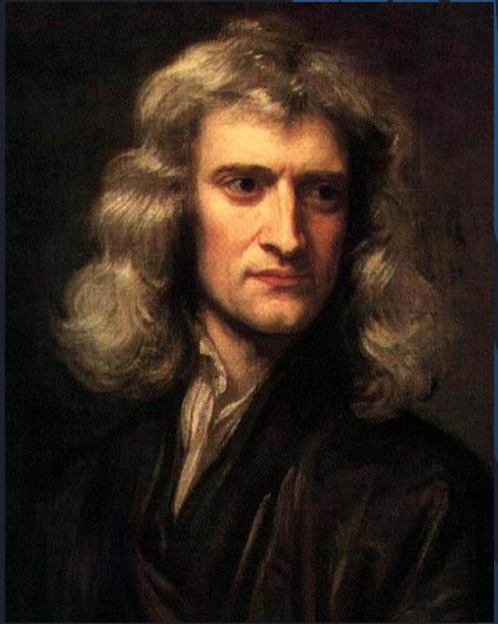
Linear Force and Momentum



Programming – Physics for Games

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Contents

- Rigid Body Dynamics
- Particles vs Rigid Bodies
- Properties of Particles
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Rigid Body Dynamics

- Physics is a huge field
- In games, we're only interested in a small subset of physics called *dynamics*
- Even more specifically, *rigid body dynamics*

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Rigid Body Dynamics

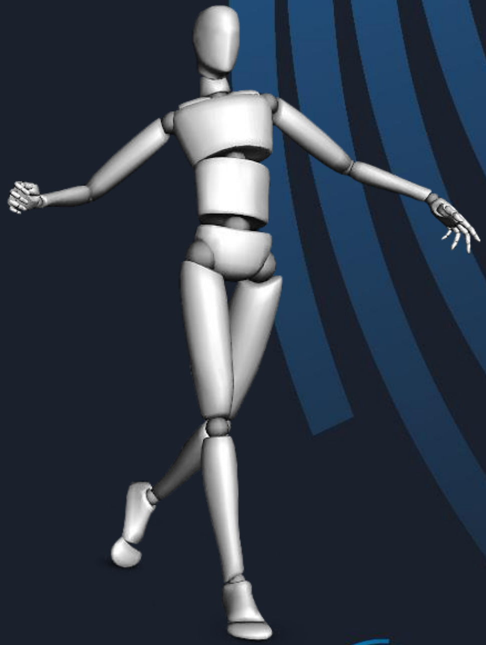
- *Kinematics* is the study of movement over time
  - It doesn't concern itself with what causes the object to start moving in the first place, it just deals with the actual movement itself
- *Dynamics* is the study of forces and masses that cause the kinematic qualities to change over time
  - The "*rigid body*" part refers to constraints we place on the objects we're simulating
  - A rigid body's shape does not change during the simulation

Rigid Body Dynamics

- *Kinematics*
  - How far a baseball travels in a straight line after 10 seconds at 50 km/hr is a kinematics problem
- *Dynamics*
  - How far a baseball travels in earth's gravity if I smack it with a bat is a dynamics problem

Rigid Body Dynamics

- We can create articulated characters by building each section of the figure (like forearms, feet, etc.) out of rigid bodies and putting joints between them
- This still doesn’t account for the flexing of bones or ‘squishy’ flesh
- But it does let us simplify our equations while giving us interesting dynamic behaviour



Particles vs Rigid Bodies

- In the real world, objects are composed of millions of tiny particles
  - How could we possibly simulate all these complicated quantum-electrical forces?
- We can treat all these particles as a single object provided the body stays rigid
  - Meaning that all pairs of points within the object maintain a fixed distance from each other
  - Any internal parts are not accelerating relative to each other
  - This means the internal forces are perfectly balanced (and can be ignored)
- Even if you have two or more objects joined together, provided they stay rigid, you can treat them as a single object

Particles vs Rigid Bodies

- If we ignore rotations on rigid bodies, something interesting happens

- We can treat all forces acting on a rigid body as if their vector sum is acting on a point at the centre of mass
  - This is because all particles within the body maintain a fixed distance from each other
  - This point, called the *centre of mass*, has the mass of the entire body

- We can treat the whole rigid body as a single point, or *particle*

Properties of Particles

- Position
  - A location in space, in either 2 or 3 dimensions
  - Stored as a vector
  - We say that position is the integral of velocity
- Velocity
  - A vector describing how the position changes over time
  - We say that the velocity is the derivative of position, and the integral of acceleration
- Acceleration
  - A vector describing how the velocity changes over time
  - We say that the acceleration is the derivative of acceleration

Properties of Particles

- Mass
  - The amount of matter in the object
  - It is the measure of an object’s resistance to acceleration
  - Not the same as weight (although often determined by measuring the object’s weight)
    - An object on the moon would weigh less than on earth, but would still have the same mass
- Momentum
  - The mass of the particle multiplied by its velocity
- Force
  - Measured as the change in momentum over time

Newtonian Physics

- We can use Newton's three laws to see how forces affect acceleration
  - These laws of motion describe how objects move in our world
- This is referred to as *classical mechanics*
  - When we use the term *Physics* in games, we mean *classical mechanics*
  - Principals of relativistic mechanics or quantum mechanics are beyond what we need to create realistic physical simulations in games

Newton’s First Law

- “Every object persists in its state of rest or uniform motion in a straight line unless it is compelled to change that state by forces impressed on it”
  - If no forces are applied to an object, it’s velocity will not change
    - If it was moving, it will keep moving in a straight line
    - If it was at rest, it will stay in the same position

Newton's Second Law

F = force
m = mass
a = acceleration

- "Force is equal to the change in momentum per change in time. For a constant mass, force equals mass times acceleration"
  - This gives us the formula:

$$F = ma \qquad \text{or} \qquad a = \frac{F}{m}$$

  - If we give an object a push, it begins to move
  - The harder we push, the faster it accelerates
  - The heavier the object, the harder we have to push

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Newton's Second Law

- In the real world, we can't directly change the position of objects
- We can't even directly change the velocity of objects
- All we can ever do is apply forces
- Applying a force will accelerate an object
  - Which changes its velocity,
  - Which in turn changes its position

Momentum

- Momentum is the mass of an object multiplied by its velocity:

$$\mathbf{P} = mv$$

  - You can think of *momentum* as the "total amount of pushing" required to stop a moving object

- The law of *conservation of momentum* tells us that the total momentum of a system remains unchanged
  - The total system momentum is found by adding the momentum of all objects in the system

P = momentum

m = mass

v = velocity

(Don't confuse the symbol for momentum with position – which is traditionally indicated with a lower-case ***p***)

Newton’s Third Law

- “For every action, there is an equal and opposite re-action”
  - Force is a change in momentum over time
  - The total momentum of a system does not change
  - When applying a force to an object, we’re changing its momentum
  - In order to keep the total momentum constant, an equal an opposite force must be applied to another object
    - For example, the other object involved in a collision

Applying Newton's Laws

- We treat rigid bodies as single point masses (particles)
- For a given force, we find the acceleration of the point by dividing the force by the mass
- This gives us the acceleration to use in our integration, and we can calculate the object's movement

Summary

- The three laws of motion succinctly describe what many of us intuitively already know
- They provide a realistic model for us to implement in our game engines
- By treating rigid body objects as particles, we simplify the maths

References

- Chris Hecker. 1997. *Physics, Part1: The Next Frontier*. [ONLINE] Available at: http://chrishecker.com/images/d/df/Gdmphys1.pdf. [Accessed 6 June 2017].
- Dunn, F, 2011. *3D Math Primer for Graphics and Game Development, 2nd Edition*. 2. A K Peters/CRC Press.

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