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A windowed Gaussian notch filter for quasi-periodic noise removal

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Abstract

This paper presents an efficient method of quasi-periodic noise detection and filtering. Taking into account that periodic noise leaves peaks in the amplitude spectrum, the proposed approach focuses on their detection and elimination. The detection is performed semi-automatically using a local median whereupon the localized peaks are eliminated by a modified Gaussian notch filter. The proposed approach demonstrates high efficiency for images corrupted by both pure-periodic and quasi-periodic noise.

Keywords: Nonlinear filtering; Spectrum filtering; Fourier transform; Periodic noise

1. Introduction

Periodic noise is usually caused by electrical or electromechanical interference during image acquisition. Having an unpredictable appearance in spatial domain, periodic noise has a very specific spectral counterpart, and is revealed in the Fourier amplitude spectrum as spike-like components at specific frequencies [1–4]. As a consequence, periodic and quasi-periodic noise can be efficiently removed by correcting the amplitude spectrum components altered by the noise. An example of quasi-periodic noise and its influence on the amplitude spectrum can be seen in Fig. 1, where four star-like peaks around the DC (zero-frequency) component are clearly visible (there are also at least eight very small peaks in the high-frequency domain marked manually for better visibility).

Usually the peaks caused by purely periodic noise are extremely narrow and of very high magnitude, which makes them clearly distinguishable from the rest of spectral coefficients. They are easy to localize using thresholding techniques. Being localized they can be corrected by the band-reject or notch filters [1–4]. However, in most cases periodic noise is not pure, and its spectral representation, usually, does not contain easily detectable sharp peaks. All the coefficients in a close neighborhood around the peak are affected by the noise. As noted by Gonzalez and Woods [3]: "Star-like components in Fourier spectrum indicate more than one sinusoidal pattern". In the following, the term "peaks" will refer to these star-like components consisting of the highest point – apex – in the center and its closest neighborhood affected by the noise. These peaks are sometimes visually indistinguishable, especially in the high-frequency area, nevertheless they have to be found and eliminated.

There are several solutions proposed in the literature, like Wiener filter, notch filters [3,4] or masking filters [1,2,5]. The Wiener filter requires an accurate noise model per case. Approaches, based on peak detection using thresholding of the amplitude spectrum, are suitable only for pure periodic noise, characterized by narrow and strong peaks. They can localize only a limited number of sharp and high peaks. Notch filters eliminate peaks in predetermined regions of amplitude spectrum by correcting the corresponding frequencies. That has a limited applicability.

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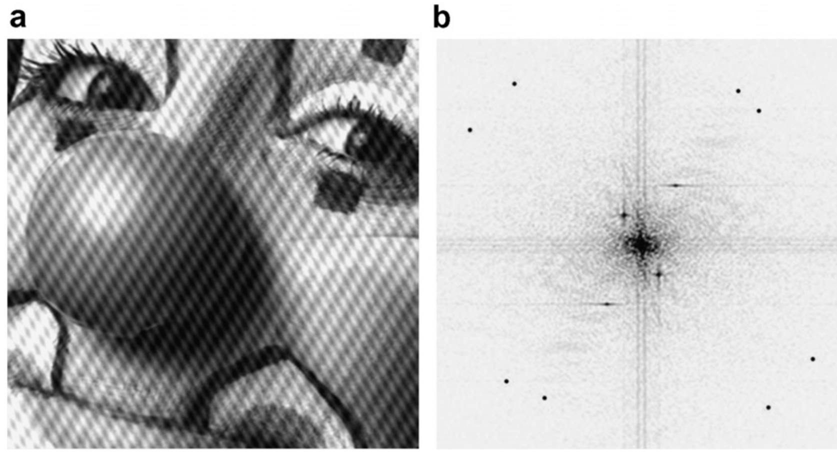

Fig. 1. An example of quasi-periodic noise (a) and the corresponding Fourier amplitude spectrum (b), where the quasi-periodic structures are represented by a number of star-like components. The noticeable peaks in the high-frequency domain have been manually marked.

Recently, we introduced a spectral peak detector [6,7], where each spectral coefficient was compared to the local median in the spectrum to identify peaks. The apices of the detected peaks were replaced by the local median. This approach has shown good performance in eliminating peaks corresponding to pure periodic noise. However correcting only the apices is not sufficient for suppressing quasi-periodic noise. To this end, Al Hudhud and Turner [8] have proposed to apply the median filter to the neighborhood of peaks, considering that they are already localized, and demonstrated the results comparable to those obtained by notch filters.

The approach that we propose here is a successor of our previous work [6,7]. We propose a detection and filtering framework that corrects spectral peaks along with their closest neighborhood. The new approach can efficiently detect peaks in both the low- and high-frequency domain and correct them using a modification of the Gaussian notch reject filter.

2. Windowed Gaussian notch filter

It is well known that periodic and quasi-periodic patterns in spatial domain are represented by peaks in the corresponding Fourier amplitude spectrum (Fig. 1). While periodic noise may have an unpredictable appearance in the spatial domain, contaminating the whole image, the spectrum is affected only locally with peaks having approximately fixed shape and varying mainly in their magnitude and location. Thus, it is much faster and easier to filter periodic noise in the spectral domain by removing the corresponding peaks. However, special care must be taken, as modifying a spectrum one can easily introduce other kinds of noise.

As demonstrated in [6] and [7], local median can serve well to detect spectral peaks, but the filters proposed in the above references correct only the apices of detected peaks, which proved to be insufficient to eliminate quasi-

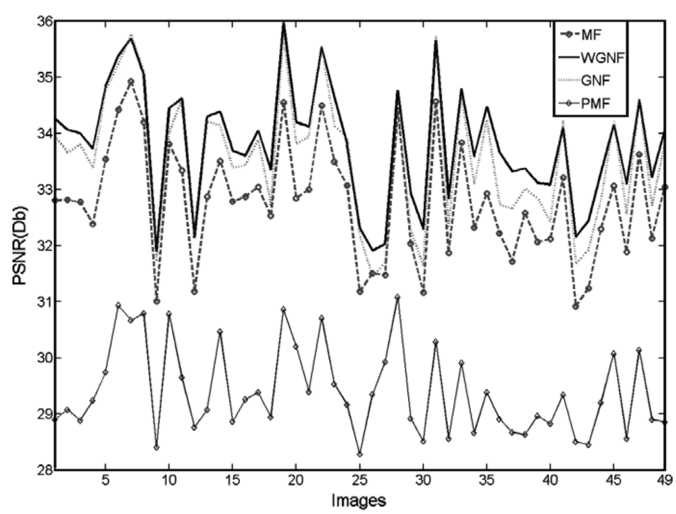

Fig. 3. Comparison of the PSNR for the different filtering techniques on a set of 49 images.

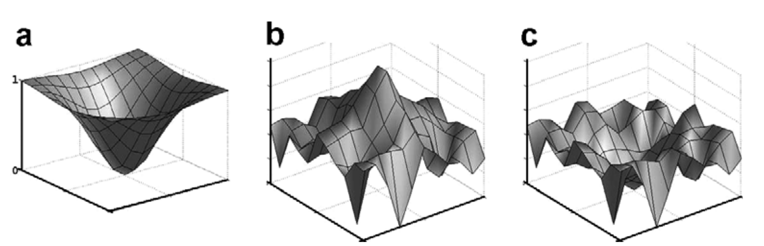

Fig. 2. Peak filtering illustration: (a) 11×11 Gaussian surface (4), $A = 1$, $B = 0.1$; (b) a sample 11×11 peak from a spectrum; (c) filtered peak (the Hadamard product of peaks in (a) and (b)).

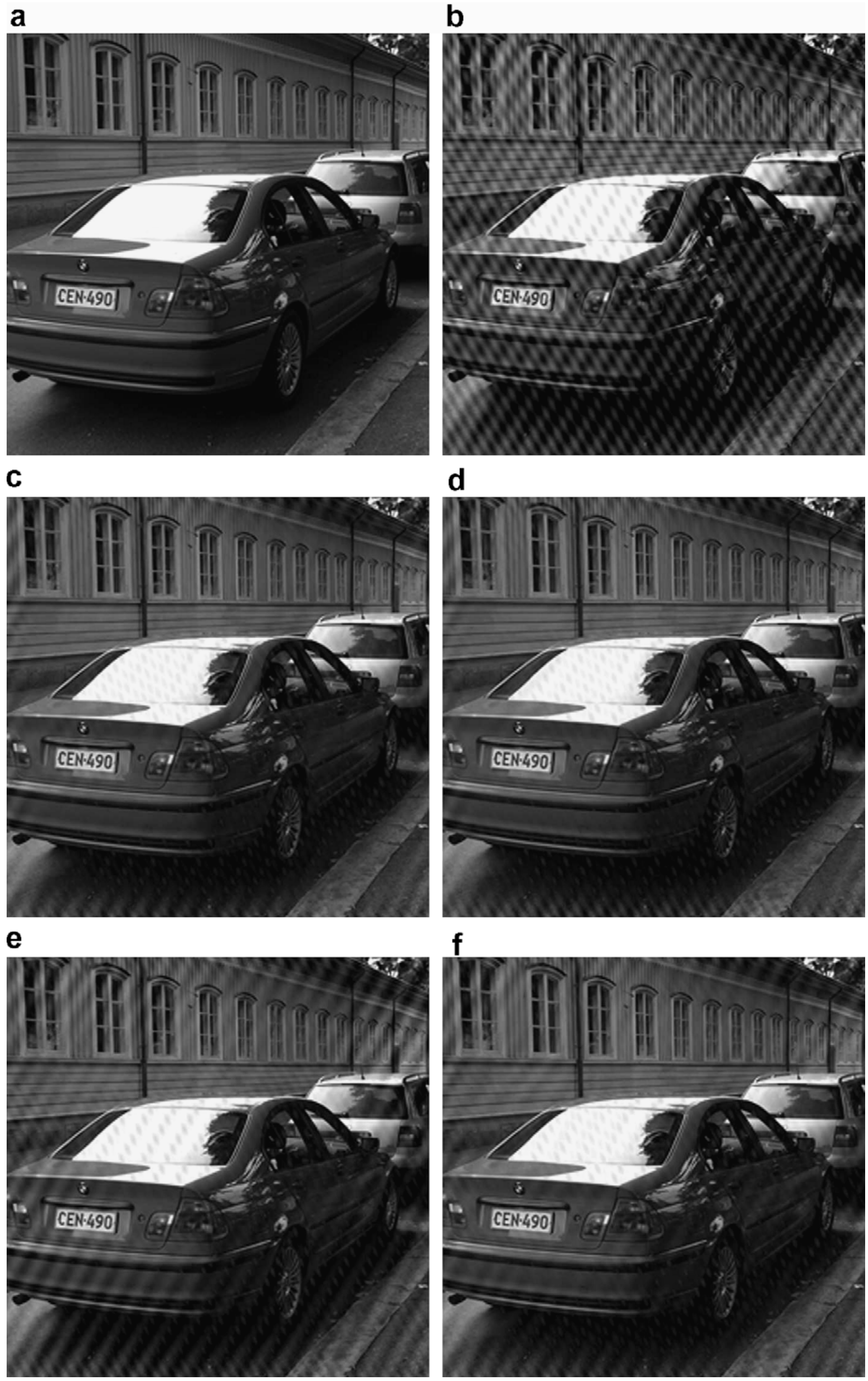

Fig. 4. Filtering a sample image from the test set: (a) clean image "Car"; (b) "Car" corrupted by quasi-periodic noise; (c) "Car" filtered by WGNF, PSNR = 33.3; (d) "Car" filtered by GNF, PSNR = 32.8; (e) "Car" filtered by PMF, PSNR = 29.2; (f) "Car" filtered by MF, PSNR = 32.3.

periodic noise completely. A convenient general solution would be to detect peaks using the local median, and then filter them out using the Gaussian Notch Reject Filter (GNF) [3] defined by:

$$H(u,v) = 1 - \mathrm{Exp}\left[-\frac{D_1(u,v)D_2(u,v)}{2D_0^2}\right] \tag{1}$$

where D_1 and D_2 are the Euclidean distances from the point (u,v) to the peak and its symmetric counterpart, and D_0 is a predefined radius of the neighborhood of each of the two peaks. It is easy to see that, when the peak is far away from the DC component, the surface H will be quite large. Multiplying a spectrum by such a surface is computationally intensive and, in the presence of many

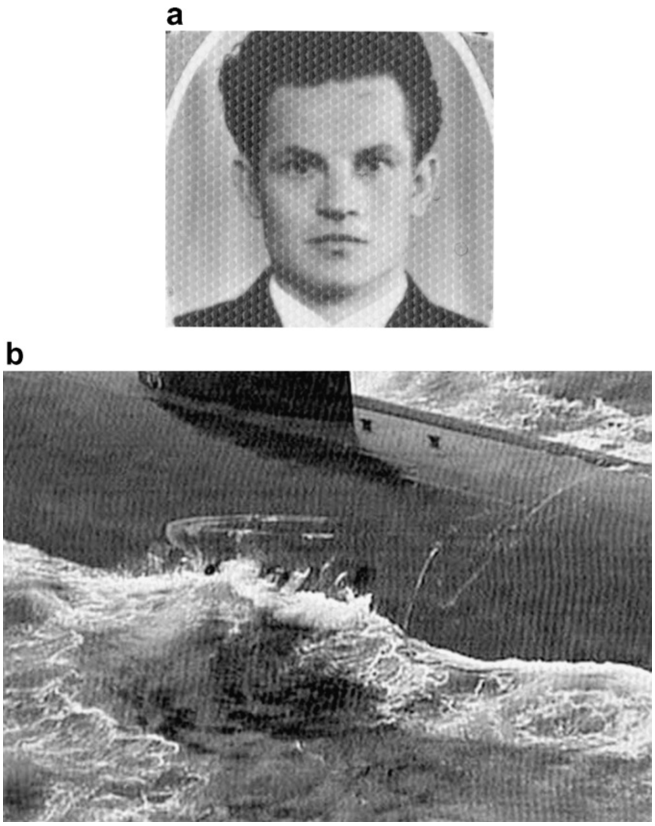

Fig. 5. Two non-synthetic noisy images.

peaks, would become prohibitively long. However, the area occupied by one peak is very small compared to the size of the generated surface H, therefore most of the spectral coefficients are unaffected by the operation. On the other hand, although the width of the generated peaks is defined by D_0, the surface H, being an exponential, approaches the plane $H(u,v) = 1$ asymptotically. Thus it differs from 1 in many points beyond the radius D_0. Under such conditions it is difficult to know which coefficients are corrected by the multiplication and which not.

To solve the above outlined problems, we modify the GNF and efficiently combine it with the semi-automatic peak detection framework developed in [6,7]. Let R_{LF} be the area of low-frequency spectral coefficients whose values must not be altered. These coefficients make a significant contribution to the signal energy, and their change may lead to an undesirable distortion of an image. Let C_{ij} be the (i,j)th coefficient of a Fourier amplitude spectrum. The spectrum is considered to have the DC coefficient C_{00} at its center. Normally R_{LF} should be defined as a circle around C_{00}, but this area is to be small, otherwise there is a risk of including noisy coefficients. R_{LF} therefore can be a square or rectangle to simplify its computer implementation.

The peak detector is formulated as follows. We say that a peak is centered at the spectral coefficient $C_{ij} \notin R_{\mathrm{LF}}$, if for a predefined Θ:

$$\frac{C_{ij}}{\underset{m\times n}{\mathrm{MED}}(C_{ij})} \geqslant \Theta \tag{2}$$

where $\underset{m\times n}{\mathrm{MED}}(C_{ij})$ is the local median in the $m \times n$ (m and n are odd) filtering window around C_{ij}.

Once the peaks are detected, each C_{ij} , corresponding to a peak, together with its closest surroundings D^{ij}_{mn} are corrected by the windowed Gaussian notch filter (WGNF) defined as follows:

$$\widetilde{D}^{ij}_{mn} = \begin{cases} D^{ij}_{mn} \circ G_{mn}, & \text{if (2) holds} \\ D^{ij}_{mn}, & \text{otherwise} \end{cases} \tag{3}$$

where D^{ij}_{mn} is the $m \times n$ area around C_{ij}; $\widetilde{D}^{ij}_{mn}$ is the same area after the filtering. "$\circ$" denotes the Hadamard (element-wise) matrix multiplication. G_{mn} is an $m \times n$ matrix, whose (x,y)th element is defined as:

$$\begin{gathered} G_{mn}(x,y) = 1 - A\,\mathrm{e}^{-B(x^2+y^2)}, \\ x = -\left[\frac{n}{2}\right],\ldots,\left[\frac{n}{2}\right]; \quad y = -\left[\frac{m}{2}\right],\ldots,\left[\frac{m}{2}\right] \end{gathered} \tag{4}$$

with $[\cdot]$ denoting the integer part of a real number, $0 < A \leqslant 1$ being the magnitude of the generated peak and $B > 0$ being a scaling coefficient along the **X** and **Y** axes.

One can see that (4) is essentially the same surface as that of the GNF (1) but with a single peak in the center of the surface and the surface is of the same size as the filtering window used in (2). The benefit of using (4), as compared to (1), is that it is applied locally, affecting only coefficients within the filtering window. It can also be easily precomputed as opposed to (1), which requires recomputation for every peak. Another good point about (4) is that it is possible to define the width of the generated peak (controlled by B), its magnitude (controlled by A) and the corrected area (m and n) independently, allowing a wide peak in a narrow window to attenuate strong spectral distortions. The latter is impossible to achieve by (1) as the Gaussian surface with a wide base has long non-zero tails.

An example of filtering of one peak is presented in Fig. 2. Fig. 2a demonstrates a typical surface G defined by (4) with $A = 1$ and $B = 0.1$. Fig. 2b shows one of the peaks (within 11×11 window) extracted from the amplitude spectrum in Fig. 1b. Fig. 2c shows how this peak is eliminated by the Hadamard product of the surfaces in Figs. 2a and b according to (3). As it can be observed the peak has been eliminated and the whole 11×11 surrounding area has been attenuated.

3. Parameter selection

The Fourier amplitude spectrum of a typical image is known to decrease with increasing of frequency. Some studies demonstrate that its decay can be modeled by a power law [8]. Therefore, noting that, given a monotonous signal, the median in the neighborhood of a point is equal to the value of this point, it is expected that, on average, $C_{ij}/\underset{m\times n}{\mathrm{MED}}(C_{ij})$ should be close to unity. Since the peaks

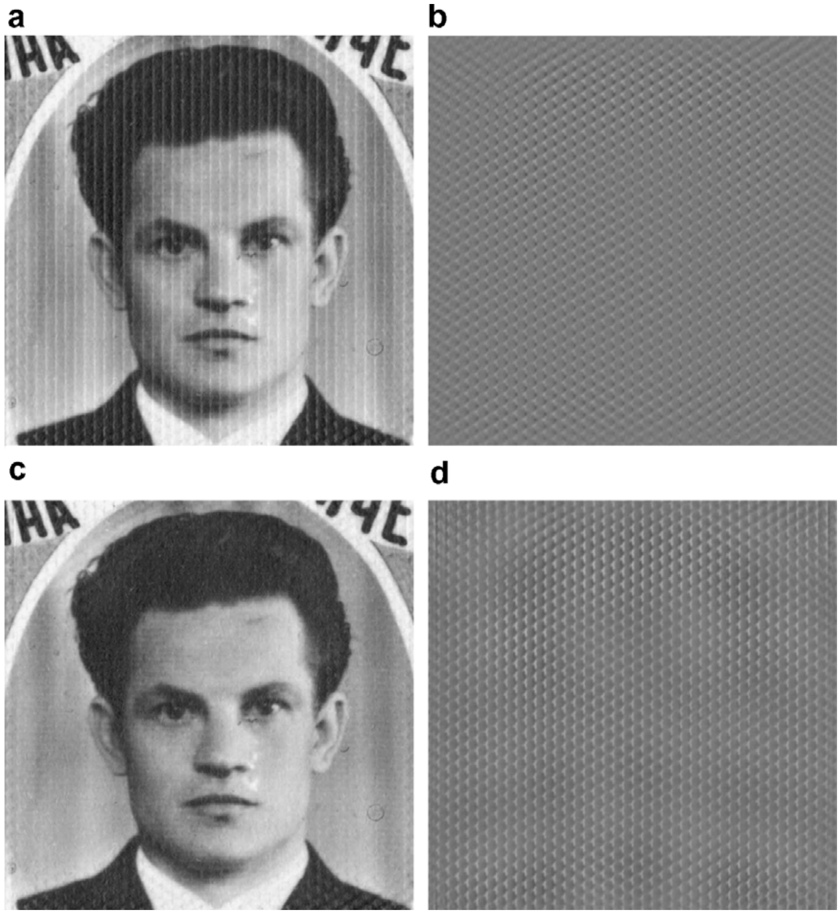

Fig. 6. Filtering of Fig. 5a: (a,b) filtering by GNF and the pixelwise difference with the original noisy image, and (c,d) filtering by WGNF and the pixelwise difference with the original noisy image.

are higher than their surroundings, a lower bound on Θ can be established: $\Theta > 1$. The analysis of spectra of a number of clean images has confirmed that the ratio, corresponding to the filtering windows ranging from 5×5 to 17×17, is equal to 1.05–1.09 with the standard deviation of about 0.6–0.8. It must be noted that we work with the amplitude spectra normalized by the number of pixels, so that the amplitude of the DC component equals the mean intensity of the image. The choice of a proper Θ might require 2–3 additional steps. Our experience shows that taking Θ equal to 7 or 8 gives a good starting point. If the result still contains noise as it occasionally happens, then Θ must be reduced. If the result shows distortion then Θ must be increased. Our experiments suggest that the detector is relatively insensitive to the accurate choice of the threshold. A proper value can be found by increments of 0.5 or 1. It is also possible to use dichotomy for its estimation.

The rest of the parameters can be fixed. Empirically it was found that 7×7 square window is a suitable choice for R_{LF}. Usually this area covers all the important low frequencies keeping the periodic noise components outside. An $m \times n$ filtering window of detector (2) can be fixed to 11×11. The value of A determines the strength of peak attenuation and setting A to 1 allows to eliminate the apices of peaks completely. As for B, its value[1] can be set equal to $12.5/\min(m,n)^2$. This value corresponds to a peak $G_{mn}(x,y)$, which has approximately 99% of its area within the filtering $m \times n$ window (as in Fig. 2a), with boundary values close to unity, so that multiplication of spectrum fragment by this surface will not introduce sharp discontinuities in the spectrum.

4. Simulation results

Here, we compare the proposed WGNF with the GNF defined by (1), with the filter considered in [6,7], called here the Point Fourier-domain Median Filter (PMF), which was applied only to the apices of detected peaks, and with the Median Filter (MF) applied to the whole filtering window in the spectrum as proposed in [8]. To evaluate the performance, we have used 49 different images artificially corrupted by the same quasi-periodic noise added in the

[1] This value is easily derived by noting that the surface (4) is very similar to PDF of normal distribution with $\sigma^2 = 1/(2B)$. The latter is known to have approximately 99% of its area within $\pm 2.5\sigma$ [9]. The value in the text is an approximation to a more precise value $2.5^2/(2\min([m/2],[n/2])^2)$ obtained as a ratio of the area of the generated surface on the interval $\pm\min([m/2],[n/2])$ to its area on the infinite interval.

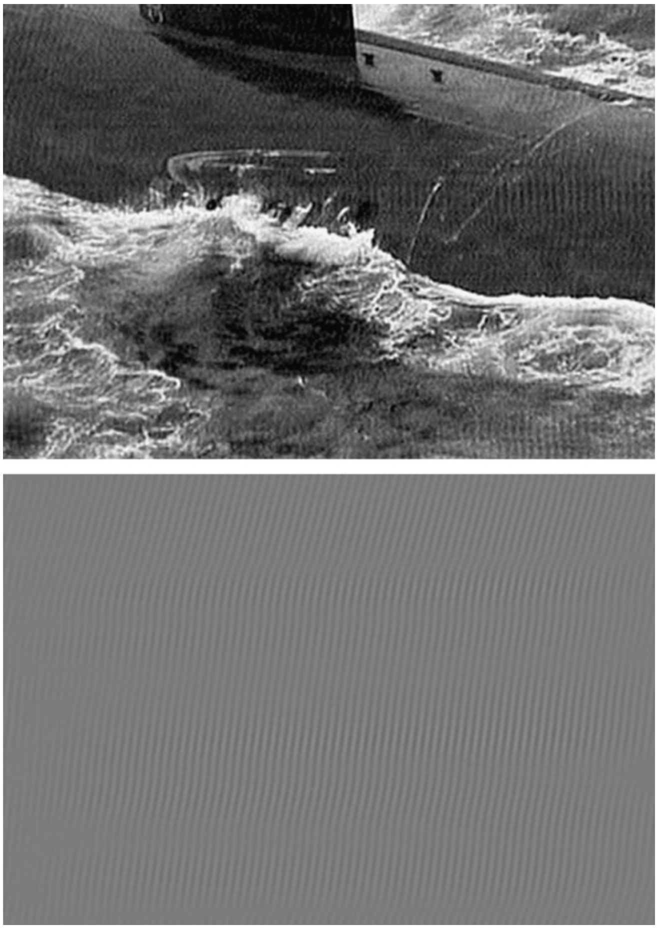

Fig. 7. Filtering of Fig. 5b by GNF and the pixelwise difference with the original noisy image.

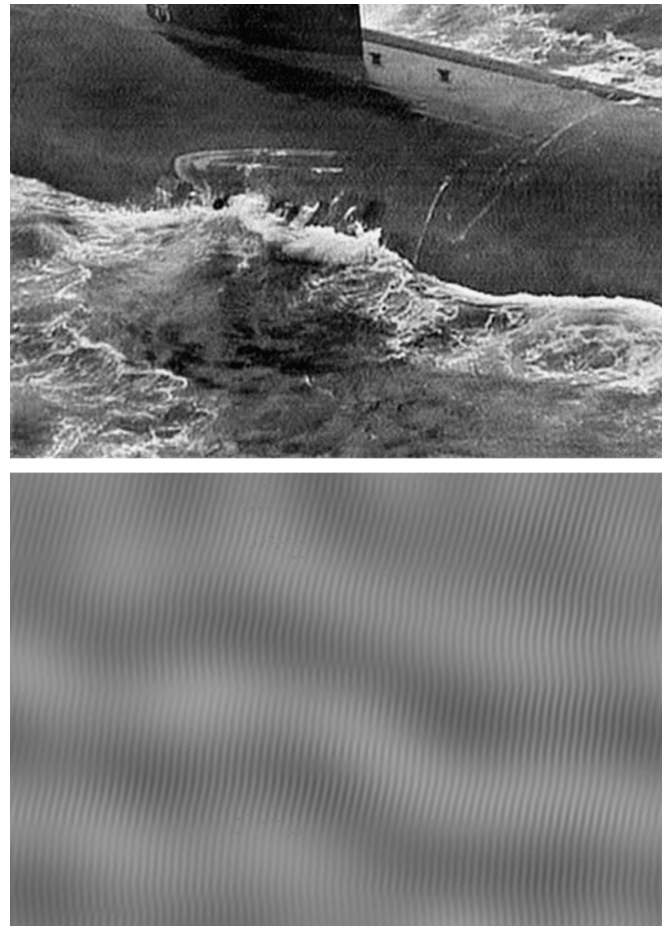

Fig. 8. Filtering of Fig. 5b by WGNF and the pixelwise difference with the original noisy image.

spatial domain (see Fig. 4a for an example). The Fourier amplitude spectrum was computed for each image, the peaks were localized using the detector (2) and eliminated by each of the four methods under comparison. The parameter settings are as follows: 11×11 filtering and 7×7 R_{LF} windows, $\Theta = 7$. Additionally $A = 1, B = 0.1$ for WGNF and $D_0 = 7$ for GNF, respectively. The Peak Signal-to-Noise Ratio (PSNR) for the resulting images was computed and plotted in Fig. 3. One can observe that the WGNF, GNF and MF perform comparably well, although the WGNF is slightly better than the others. A sample image from the testing set is shown in Fig. 4 whereon a test image (Fig. 4a) was artificially corrupted by quasi-periodic noise (Fig. 4b) and filtered by the WGNF, GNF, PMF and MF. While there is no noticeable difference between the WGNF and GNF filtering results, it should be noted that Fig. 4f, corresponding to the MF, still contains periodic noise on the dark parts of a car, particularly visible on the rear bumper. The result for the PMF in Fig. 4e still exhibits some periodic noise, which demonstrates that correcting only the apices is insufficient for quasi-periodic noise suppression. Filtering of several images from real life can be seen in Figs. 5–8.

5. Discussion

Although the proposed filter demonstrates good performance on a whole image, in some cases it is better to filter the image by overlapping blocks. Such procedure can suppress non-uniform quasi-periodic noise (like non-uniform moiré). This kind of noise can entail either very weakly marked peaks in the amplitude spectrum of the entire image or no distinguishable peaks at all. Nevertheless the spectra of regions with approximately uniform periodic pattern will contain peaks and those of clean regions will not. In this case, the spectrum of each block is filtered by the WGNF separately. Making the blocks overlap serves two purposes. On the one hand it prevents appearance of visible boundaries between the blocks after the filtering. On the other hand, as experiments show, there are always noise leftovers on the block boundaries after the filtering, which can be simply discarded due to the overlapping of the blocks. An example of such filtering is presented in Fig. 9. In comparison with the regular approach, block-wise filtering produces much clearer image. That is especially visible at the image boundaries.

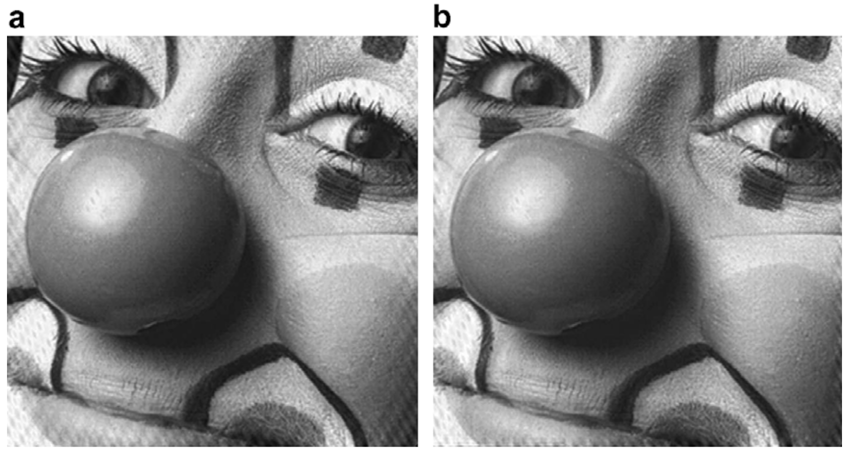

Fig. 9. Comparison of the regular and block-wise filtering of 330×330 image "Clown" (Fig. 1a): (a) filtering by the regular WGNF with $\Theta = 6$, $A = 1$, $B = 0.1$ and 11×11 filtering and R_{LF} windows; (b) filtering by nine 128×128 overlapping blocks using the same parameters for each block.

6. Conclusions

A new windowed Gaussian notch filter for quasi-periodic and periodic noise filtering has been designed in this paper. The method shows better results than those obtained by the widely used Gaussian notch reject filter and the spectral median filter, providing at the same time a semi-automatic way to detect spectral components corresponding to the noise. Finally, block-wise filtering was considered as a way to improve filtering results as well as to filter out non-uniform quasi-periodic noise. The filtering parameters are selected manually and an automated method of their estimation is the subject of our future research.

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