CMPG-767 Image Processing and Analysis

STATISTICAL ANALYSIS OF IMAGES

Statistical Characteristics of Images

- Statistical analysis of images is very important for

- global contrast evaluation and its enhancement
- local contrast evaluation and its enhancement
- estimation of noise
- evaluation of the filtering/compression/deblurring results
- evaluation of statistical similarities

Histogram

- The histogram of an NxM digital image with intensity levels {0,1,…,*L*-1} is a discrete function

$$h(r_k) = n_k; k = 0,1,\ldots, L-1$$

where r_k is the *k*th intensity value and n_k is the number of pixels in the image with intensity r_k

- Evidently, $$\sum_{k=0}^{L-1} h(r_k) = MN$$

Normalized Histogram

- The normalized histogram of an NxM image with intensity levels {0,1,…,*L*-1} is its histogram normalized by the product NM (the number of pixels in the image). The normalized histogram is an estimate of the probability of occurrence of each intensity level in the image:

$$p(r_k) = \frac{h_k(r_k)}{NM}; k = 0, 1, \ldots, L-1$$

- Evidently,

$$\sum_{k=0}^{L-1} p(r_k) = 1$$

Mean (Statistically Correct Definition)

- The global mean value of an image is the average intensity over all the pixels in the image
- Let *A* be an $N\text{x}M$ image. Then its global mean

$$m = \sum_{k=0}^{L-1} r_k p(r_k)$$

where r_k is the *k*th intensity value, $p(r_k)$ is the probability of occurrence the intensity r_k

Sampling Mean

- The global sampling mean value of an image is the average intensity of all the pixels in the image
- Let A be an NxM image. Then its global mean

$$m = \frac{1}{NM}\sum_{i=0}^{N-1}\sum_{j=0}^{M-1} A(i,j)$$

- Evidently, the global sampling mean is equal to the global mean (statistical). We will call them both simply mean

Variance–
Statistically Correct Definition

- Variance is the second moment of intensity about its mean

$$\sigma^2 = \mu_2(r) = \sum_{k=0}^{L-1} (r_k - m)^2 \, p(r_k)$$

where m is the mean, r_k is the *k*th intensity value, $p(r_k)$ is the probability of occurrence the intensity r_k

Sampling Variance

- Variance is the second moment of intensity about its mean

$$\sigma^2 = \mu_2(r) = \frac{1}{NM}\sum_{i=0}^{N-1}\sum_{j=0}^{M-1}\left(A(i,j) - m\right)^2$$

  where m is the mean

- Evidently, the sampling variance is equal to the variance (statistical). We will call them both simply variance (dispersion)

Standard Deviation

- Standard Deviation is the square root of the variance

$$\sigma = \sqrt{\sigma^2} = \sqrt{\sum_{k=0}^{L-1} (r_k - m)^2 \, p(r_k)} = \sqrt{\frac{\sum_{i=0}^{N-1} \sum_{j=0}^{M-1} \left(A(i,j) - m\right)^2}{NM}}$$

Importance of Mean, Variance, and Standard Deviation

- Mean is a measure of average intensity
- The closer is a mean to the middle of the dynamic intensity range, the higher contrast should be expected (if a variance is large enough)
- Variance (standard deviation) is a measure of contrast in an image
- The larger is variance (standard deviation), the higher is contrast

Signal-to-Noise Ratio (SNR)

- SNR is a measure , which is used to quantify how much a signal has been corrupted by noise

$$SNR = \frac{P_{image}}{P_{noise}} = \left(\frac{A_{image}}{A_{noise}}\right)^2$$

$$SNR_{DB} = 10\log_{10}\left(\frac{P_{image}}{P_{noise}}\right) = P_{signal,DB} - P_{noise,DB}$$

$$SNR_{DB} = 10\log_{10}\left(\frac{A_{image}}{A_{noise}}\right)^2 = 20\log_{10}\left(\frac{A_{image}}{A_{noise}}\right)$$

- P_{image} is image power, P_{noise} is noise power
- A_{signal} is image amplitude, A_{noise} is noise amplitude

Signal-to-Noise Ratio (SNR)

- Since amplitudes of a clear image and noise cannot be distinguished, to estimate SNR, the following method is used

$$SNR \approx \frac{m}{\sigma}$$

- where m is the mean and σ is the standard deviation of an image

Signal-to-Noise Ratio (SNR)

- The Rose criterion says that if an image has SNR>5, then a level of noise is so small that it shall be considered negligible, and the image shall be considered clean. If SNR<5, then some noise should be expected.
- The lower is SNR, the stronger is noise
- <u>Small-detailed images (for example, satellite images) may have lower SNR because the presence of many small details always lead to the higher standard deviation</u>

Mean Square Error and Standard Deviation Between Two Images

- For two *NxM* digital images *A* and *B* the mean square error (deviation) (MSE) is defined as follows:

$$MSE = \frac{1}{NM} \sum_{i=0}^{N-1} \sum_{j=0}^{M-1} \left(A(i,j) - B(i,j) \right)^2$$

- For two *NxM* digital images *A* and *B* the standard deviation (the root mean square error - RMSE) is defined as follows:

$$RMSE = \sqrt{MSE} = \sqrt{\frac{1}{NM} \sum_{i=0}^{N-1} \sum_{j=0}^{M-1} \left(A(i,j) - B(i,j) \right)^2}$$

Mean Square Error and Standard Deviation Between Two Images

- MSE and RMSE are used to evaluate closeness of images to each other

- This is important, for example to evaluate how close is an image resulted from any kind of filtering or compression to an ideal image

Peak Signal-to-Noise Ratio (PSNR)

- PSNR is the ratio between the maximum possible power of an image and a power of corrupting noise

- To estimate PSNR of an image, it is necessary to compare this image to an “ideal” clear image with the maximum possible power

Peak Signal-to-Noise Ratio (PSNR)

$$PSNR = 10\log_{10}\left(\frac{(L-1)^2}{MSE}\right) = 20\log_{10}\left(\frac{L-1}{RMSE}\right)$$

- where L is the number of maximum possible intensity levels (the minimum intensity level suppose to be 0) in an image, MSE is the mean square error, RMSE is the root mean square error between a tested image and an “ideal” image

Peak Signal-to-Noise Ratio (PSNR)

- PSNR is commonly used to estimate the efficiency of filters, compressors, etc.
- How it works: a clear image should be distorted by noise or compressed, respectively. Then a noisy image, a filtered image or a compressed image shall be compared to the clear ("ideal") one in terms of PSNR
- The larger is PSNR, the more efficient a corresponding filter or compression method is, and the less an analyzed image differs from the "ideal" one

Peak Signal-to-Noise Ratio (PSNR)

- PSNR <30.0 (this corresponds to RMSE>8.6) is commonly considered low. This means the presence of clearly visible noise, smoothed edges and many small details “washed”, that is significantly distorted

- PSNR > 30.0 (this corresponds to RMSE <8.6) is commonly considered acceptable if it resulted from processing of a heavily corrupted image (some noise is still visible and small details are still smoothed)

- PSNR > 33.0 (this corresponds to RMSE <5.6) is commonly considered good if it resulted from processing of a significantly corrupted image (some noise leftovers may still be visible, and some smallest details may still be smoothed)

- PSNR > 35.0 (this corresponds to RMSE <4.5) is commonly considered excellent (it is usually not possible to find any visual distinction from the “ideal” image)

Mean Absolute Error (Difference) between two images (MAE)

- Mean Absolute Error between two $N\text{x}M$ digital images *A* and *B* measures the absolute closeness of these images to each other:

$$MAE = \frac{1}{NM}\sum_{i=0}^{N-1}\sum_{j=0}^{M-1}\left|A(i,j)-B(i,j)\right|$$

Contrast Enhancement

- If an actual dynamic range of an image

$$\{r_0, r_1, ..., r_k\}, k << L; r_k - r_0 << L$$

  is too narrow and (or) its histogram is also too narrow, as a result, this image has a low contrast
- To take care of this problem, the dynamic range of the image intensities should be extended, and its histogram should be equalized, to ensure that more intensity values are presented in the image and there are no sharp spikes in the histogram

Histogram Equalization

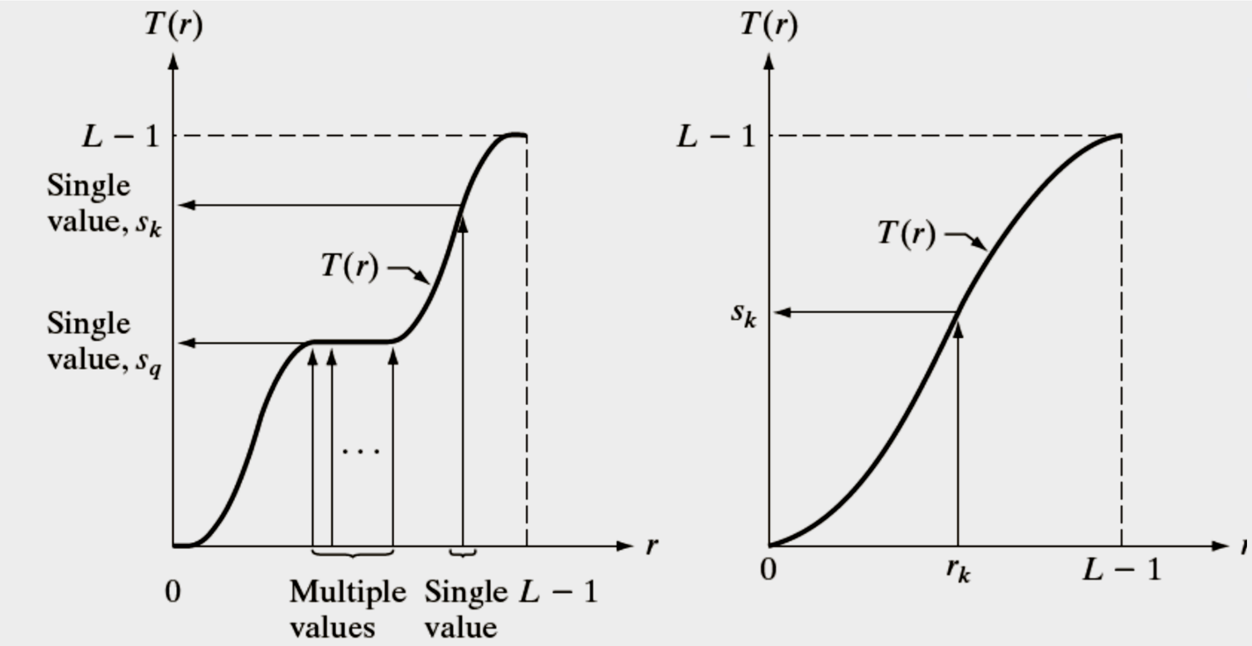
- The histogram of an image can be equalized using the following transformation

$$s = T(r); r_0 \le r \le r_k$$

  where r is the initial intensity value, s is the intensity value after the transformation

- Function T must be monotonically increasing and limited $\{s_0, \dots, s_k\}$ is a new range $s_0 \le T(r) \le s_k$

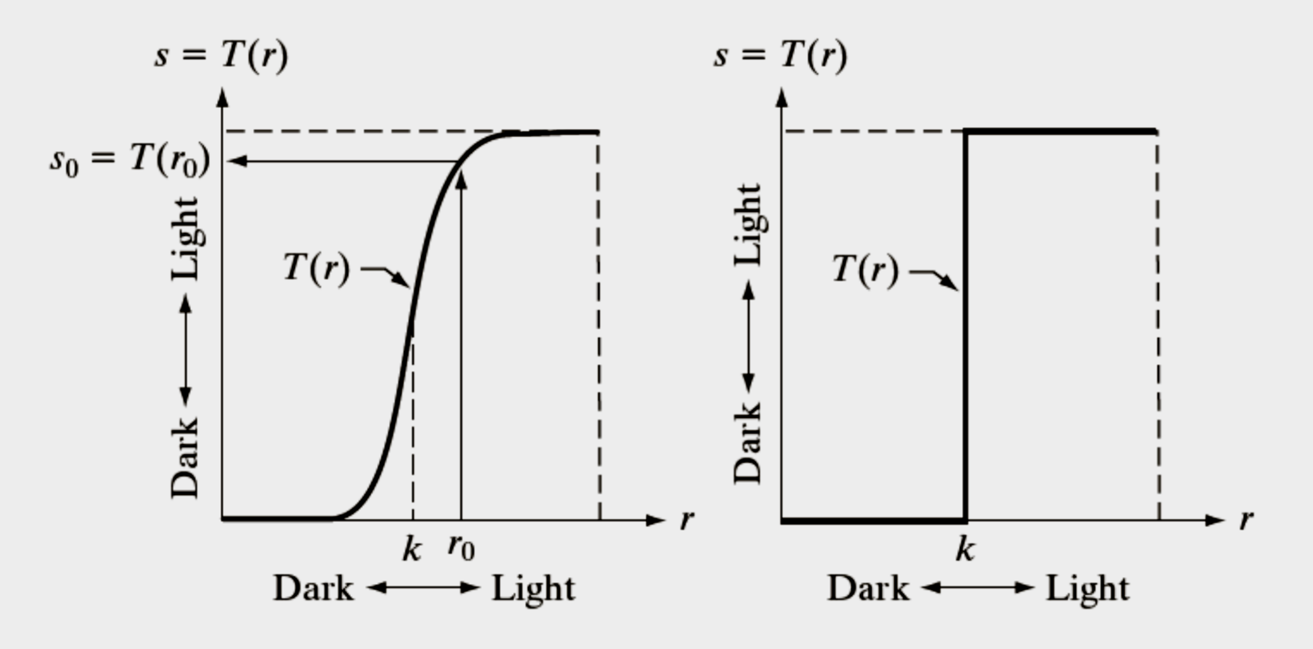
Histogram Equalization



a b

FIGURE 3.17 (a) Monotonically increasing function, showing how multiple values can map to a single value. (b) Strictly monotonically increasing function. This is a one-to-one mapping, both ways.

Example of Contrast Enhancement

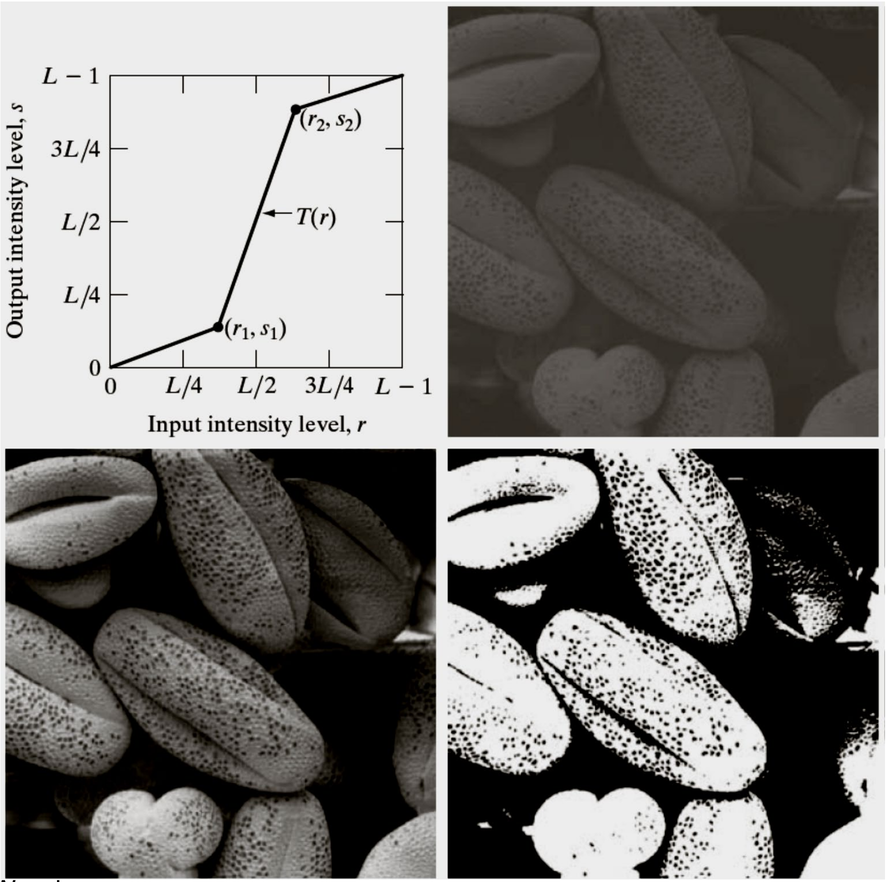


a b

FIGURE 3.2 Intensity transformation functions. (a) Contrast-stretching function. (b) Thresholding function.

Contrast Stretching
(“Piecewise Linear”, “Broken Line” Correction)

- Contrast Stretching is a process that expands the range of intensity levels in an image so that it spans the full intensity range

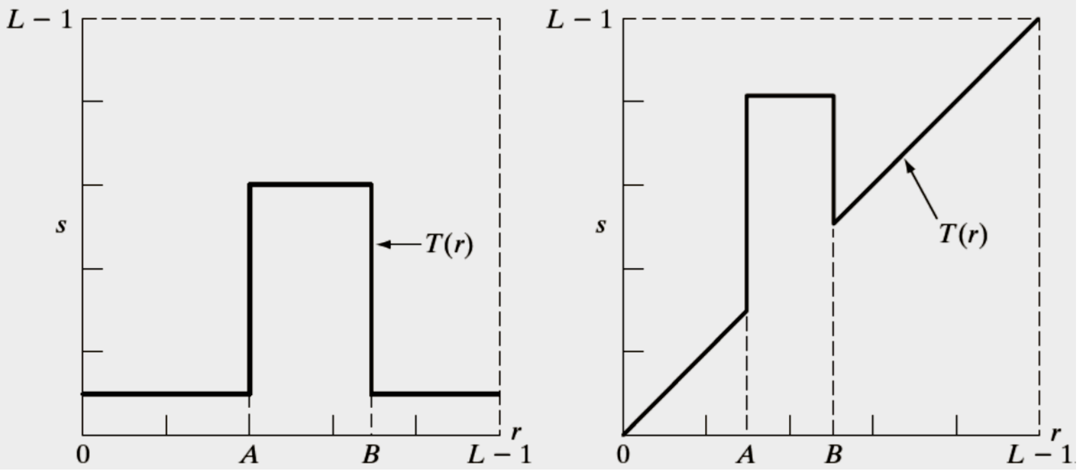


a b
c d

FIGURE 3.10 Contrast stretching. (a) Form of transformation function. (b) A low-contrast image. (c) Result of contrast stretching. (d) Result of thresholding. (Original image courtesy of Dr. Roger Heady, Research School of Biological Sciences, Australian National University, Canberra, Australia.)

Intensity Level Slicing

- Intensity level slicing is a process of highlighting of some intensity subrange with a possible suppression of other subranges.



a b

FIGURE 3.11 (a) This transformation highlights intensity range [A, B] and reduces all other intensities to a lower level. (b) This transformation highlights range [A, B] and preserves all other intensity levels.

Linear Histogram Equalization

- Linear histogram equalization for an image with the intensity range $\{r_0, r_1, \ldots, r_k\}, k \le L-1$ to the intensity range $\{s_0, s_1, \ldots, s_k\}, k \le L-1$ can be done as follows

New min

New max

$$s_i = T(r_i) = (s_k - s_0)\sum_{j=0}^{i} p_r(r_j) + r_0 = \frac{s_k - s_0}{MN}\sum_{j=0}^{i} n_j + s_0$$

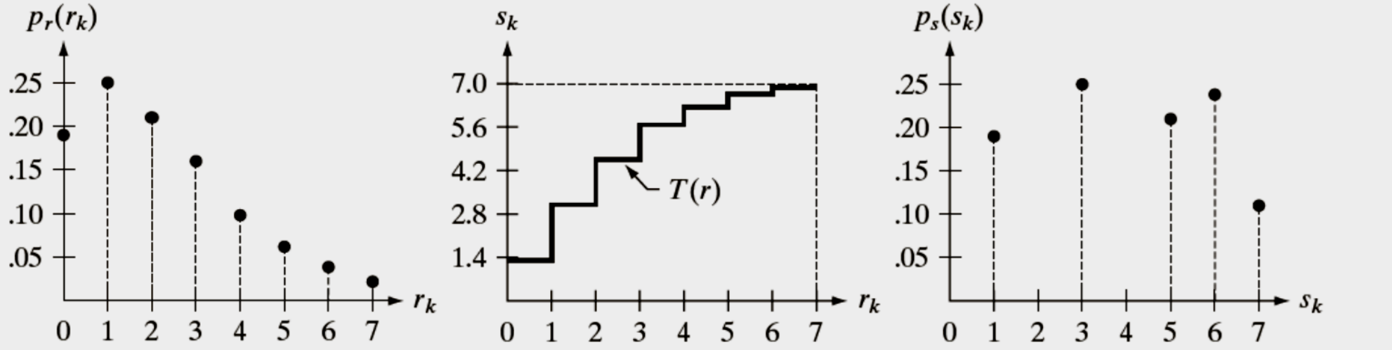
New intensity

- If $s_0 = 0$, then

$$s_i = T(r_i) = s_k \sum_{j=0}^{i} p_r(r_j) = \frac{s_k}{MN}\sum_{j=0}^{i} n_j$$

$i = 0, 1, \ldots, k$ n_j - # of j^{th} intensity appearances in the original image

Linear Histogram Equalization



a b c

FIGURE 3.19 Illustration of histogram equalization of a 3-bit (8 intensity levels) image. (a) Original histogram. (b) Transformation function. (c) Equalized histogram.

See also Fig. 3.20. Let us make the same experiments

Linear Contrast Correction

- Linear contrast correction is a kind of histogram equalization for an image with the intensity range $\{r_0, r_1, \ldots, r_k\}, k \le L-1$, mean m_{old} and standard deviation σ_{old} which produces an output image with the pre-determined mean m_{new} and standard deviation σ_{new} and a new range $\{s_0, s_1, \ldots, s_k\}, k \le L-1$

$$s_i = r_i \frac{\sigma_{new}}{\sigma_{old}} + \left(m_{new} - m_{old} \frac{\sigma_{new}}{\sigma_{old}} \right)$$

New intensity

$$i = 0, 1, \ldots, k$$