CMPG-767 Digital Image Processing

RESTORATION OF BLURRED IMAGES

Image Degradation and Restoration

FIGURE 5.1
A model of the image degradation/restoration process.

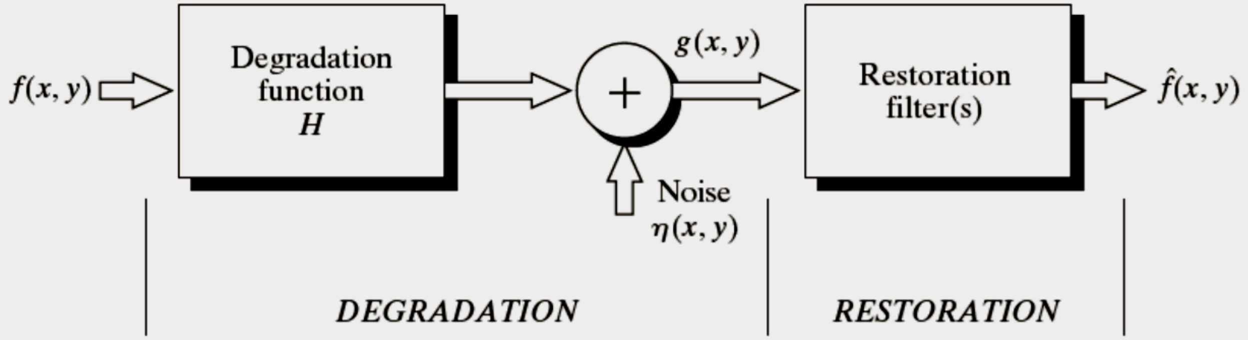


Image Acquisition: Mathematical Model

- Mathematically a variety of image capturing principles can be described by the Fredholm integral of the first kind

$$g(x,y) = \int h(x,y) f(x,y) dt + \eta(x,y)$$

- where $h(x,y)$ is a point-spread function (PSF) of an optical system, $f(x,y)$ is an "ideal" image of a real object, $\eta(x,y)$ is an additive noise, and $g(x,y)$ is an actually observed signal.

Image Acquisition: Mathematical Model

- If the PSF *h* is a linear and shift-invariant function, then the degraded image is expressed in the spatial domain by its convolution with the spatial representation of the PSF *h*

$$g(x, y) = h(x, y) * f(x, y) + \eta(x, y)$$

Frequency Domain Representation

- An equivalent frequency domain representation is as follows

$$G(u,v) = H(u,v) \circ F(u,v) + N(u,v)$$

Where $G(u,v)$ is the Fourier transform of $g(x,y)$,
$H(u,v)$ is the Fourier transform of $h(x,y)$,
$F(u,v)$ is the Fourier transform of $f(x,y)$,
$\circ$ is a component-wise multiplication,
and $N(u,v)$ is the Fourier transform of $\eta(x,y)$

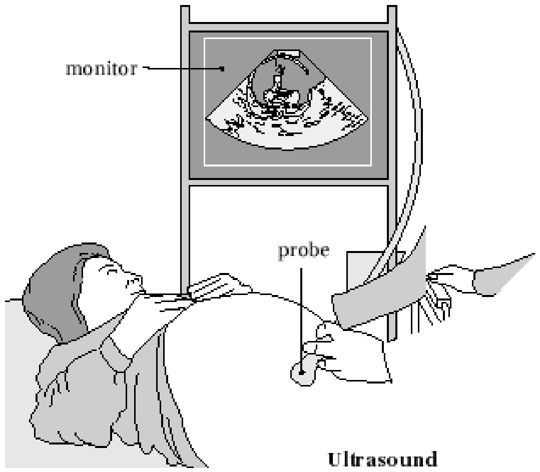
Image acquisition

- Mathematical model of image acquisition

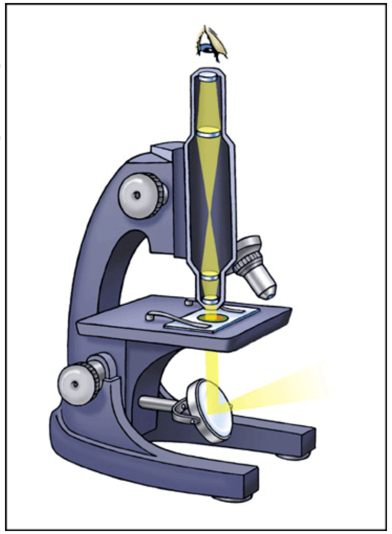
$$g(x,y) = h(x,y) * f(x,y) + \eta(x,y)$$

- where $h(x,y)$ is a point-spread function (PSF) of a system, $f(x,y)$ is a function of a real object and $g(x,y)$ is an observed signal.

- Photo

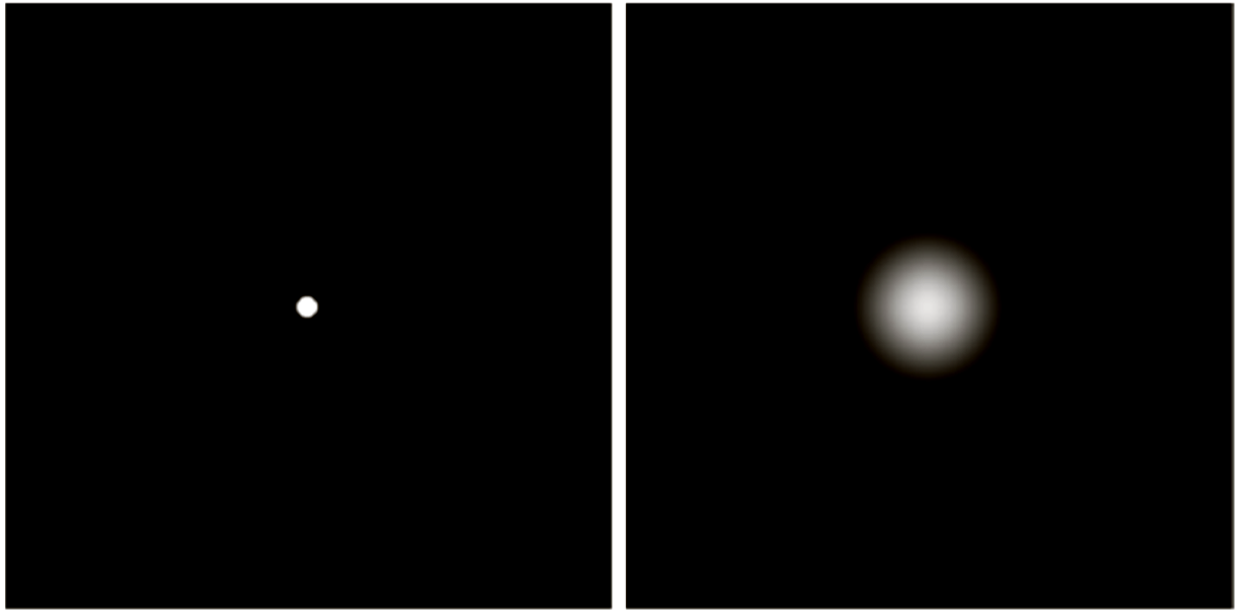


- Micro



- Tomography

Point-Spread Function (PSF)



a b

FIGURE 5.24 Degradation estimation by impulse characterization. (a) An impulse of light (shown magnified). (b) Imaged (degraded) impulse.

What causes image degradation?

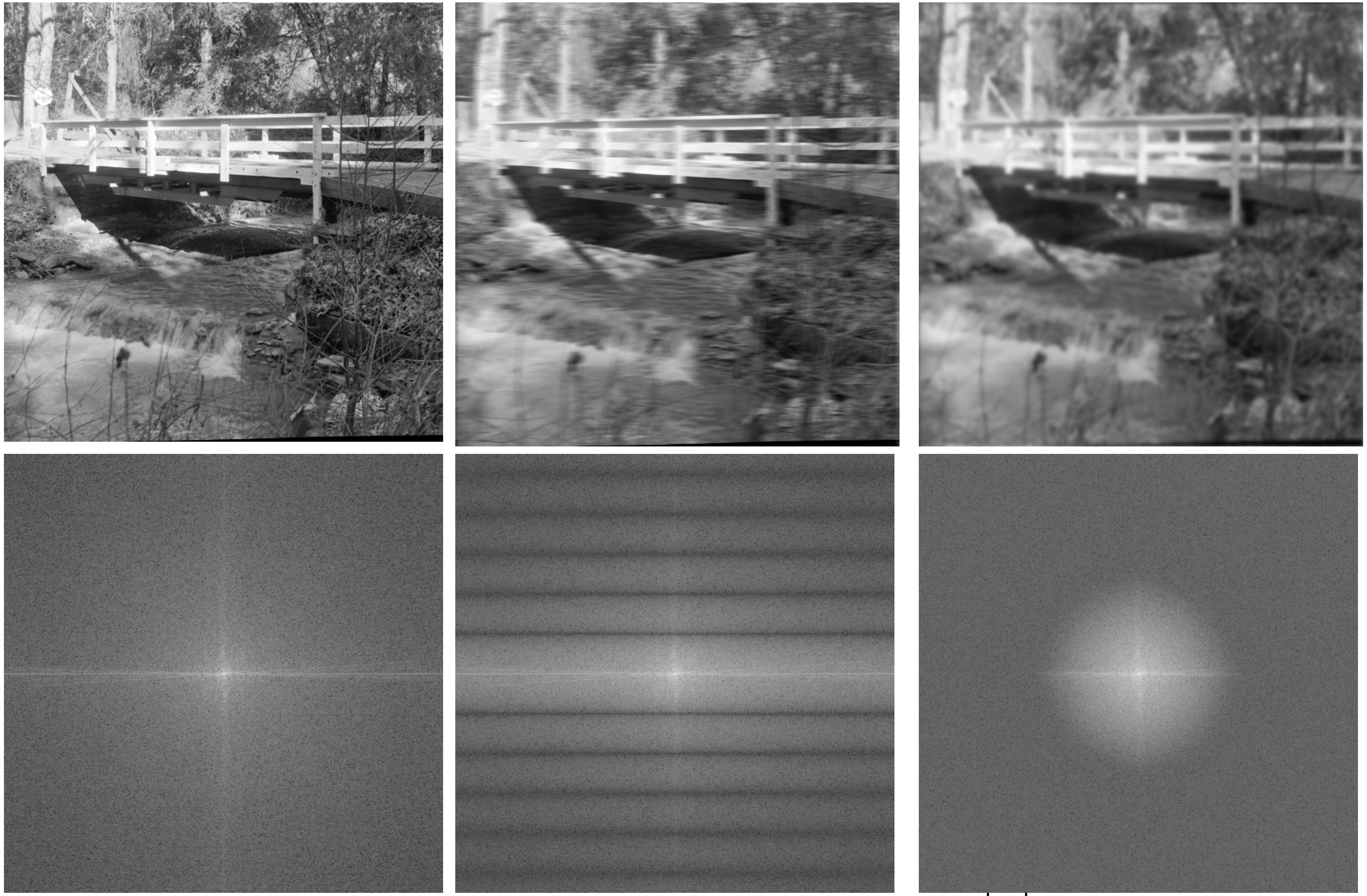
- Image degradation (blurring) is caused by a point-spread function (PSF) of a system used to acquire an image

- The same mathematical model of convolution with PSF is used to describe blur caused by object/camera motion, luck of luminosity, turbulence of atmosphere, etc.

Degradation in Spatial and Frequency Domain

- In the spatial domain, image degradation (blurring) is observed as fuzziness when small image details and edges look unsharpened and many of them become even indistinguishable
- In the frequency domain, image degradation (blurring) significantly affects a power spectrum (as a result of convolution with PSF whose Fourier transform contains many zeros and small values, some frequencies “disappear” from the Fourier transform of a blurred image)
- At the same time, symmetric PSFs almost do not affect a phase spectrum (particularly its low and medium frequency areas are mostly not affected)

Degradation in Spatial and Frequency Domain



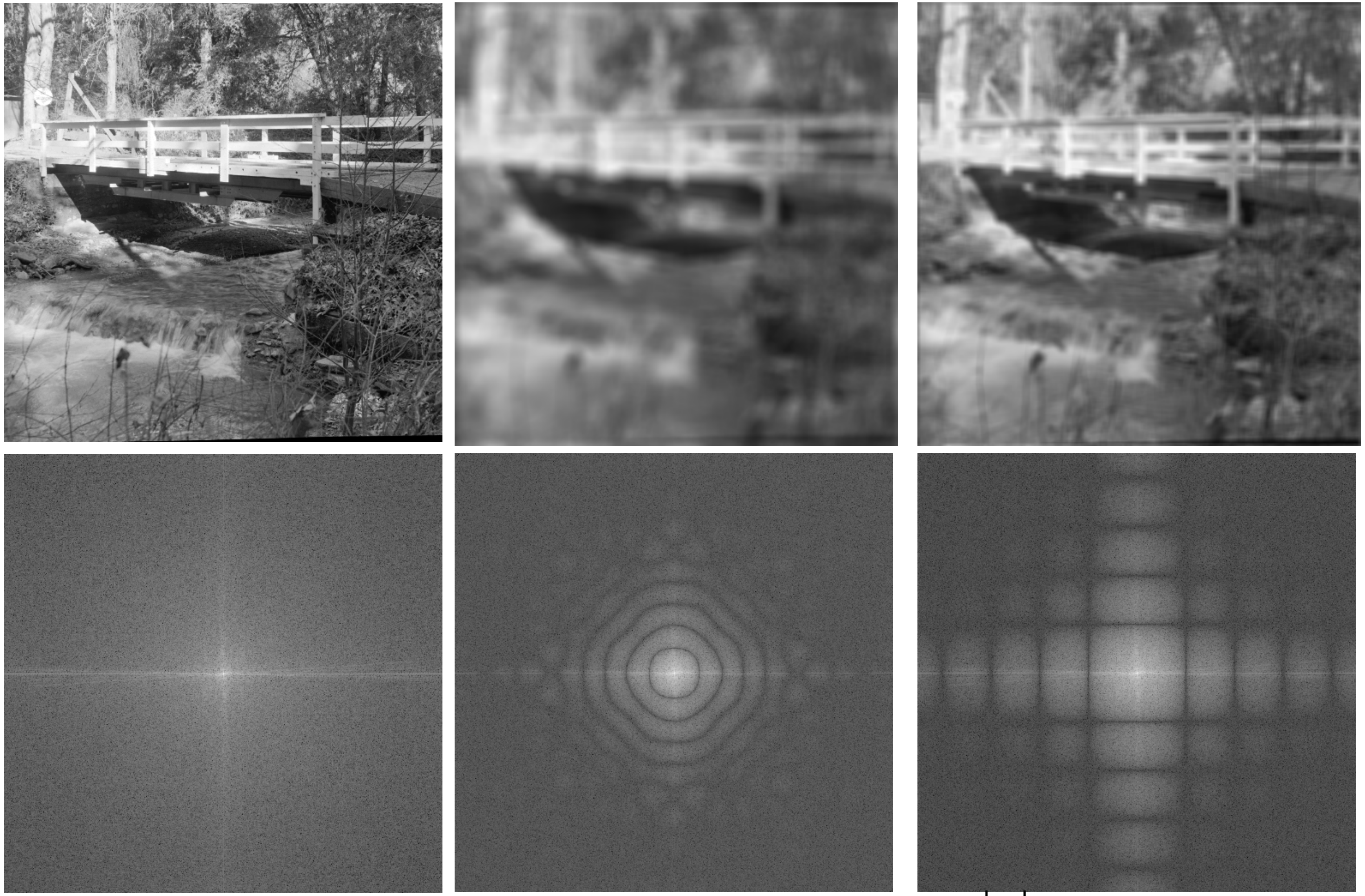
Images and log of their Power Spectra $\log|Z|$

True Image

Horizontal Motion blur

Gaussian blur

Degradation in Spatial and Frequency Domain



Images and log of their Power Spectra $\log|Z|$

True Image

Optical
Defocus blur

Rectangular
(2D motion) blur

Examples of Different PSFs

The **Gaussian** PSF:

σ^2 is a parameter (variance)

$$h(x,y)=\frac{1}{2\pi\sigma^2}\exp\left(-\frac{x^2+y^2}{\sigma^2}\right)$$

The **uniform linear motion**:

t is a parameter (the length of motion)

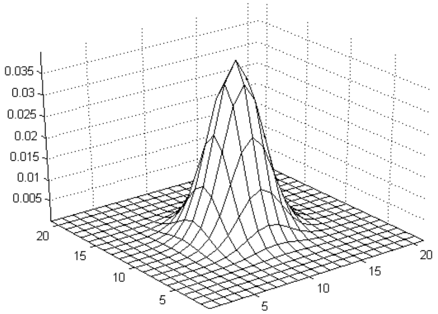
$$h(x,y)=\begin{cases}\frac{1}{t}, & \sqrt{x^2+y^2}<t/2,\ x\cos\phi=y\sin\phi,\\ 0, & \text{otherwise,}\end{cases}$$

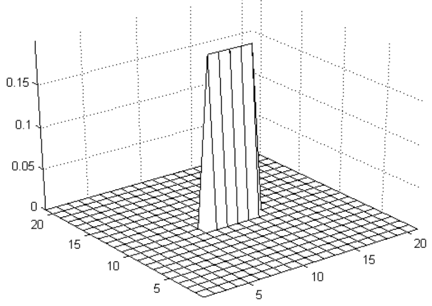
The **uniform optical defocus**:
t is a parameter
(the size of smoothing area)

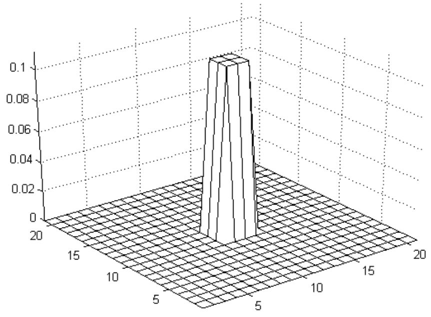
$$h(x,y)=\begin{cases}\frac{1}{t^2}, & |x|<\frac{t}{2},\ |y|<\frac{t}{2},\\ 0, & \text{otherwise,}\end{cases}$$

Examples of Different PSFs

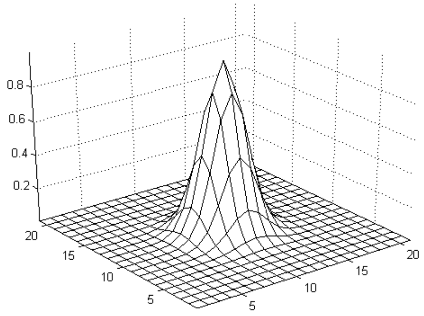
PSF in spatial domain

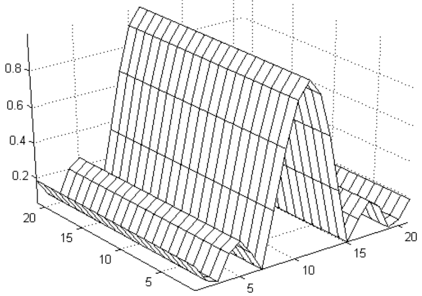


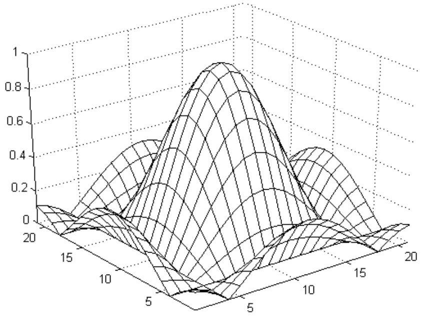




PSF in frequency domain

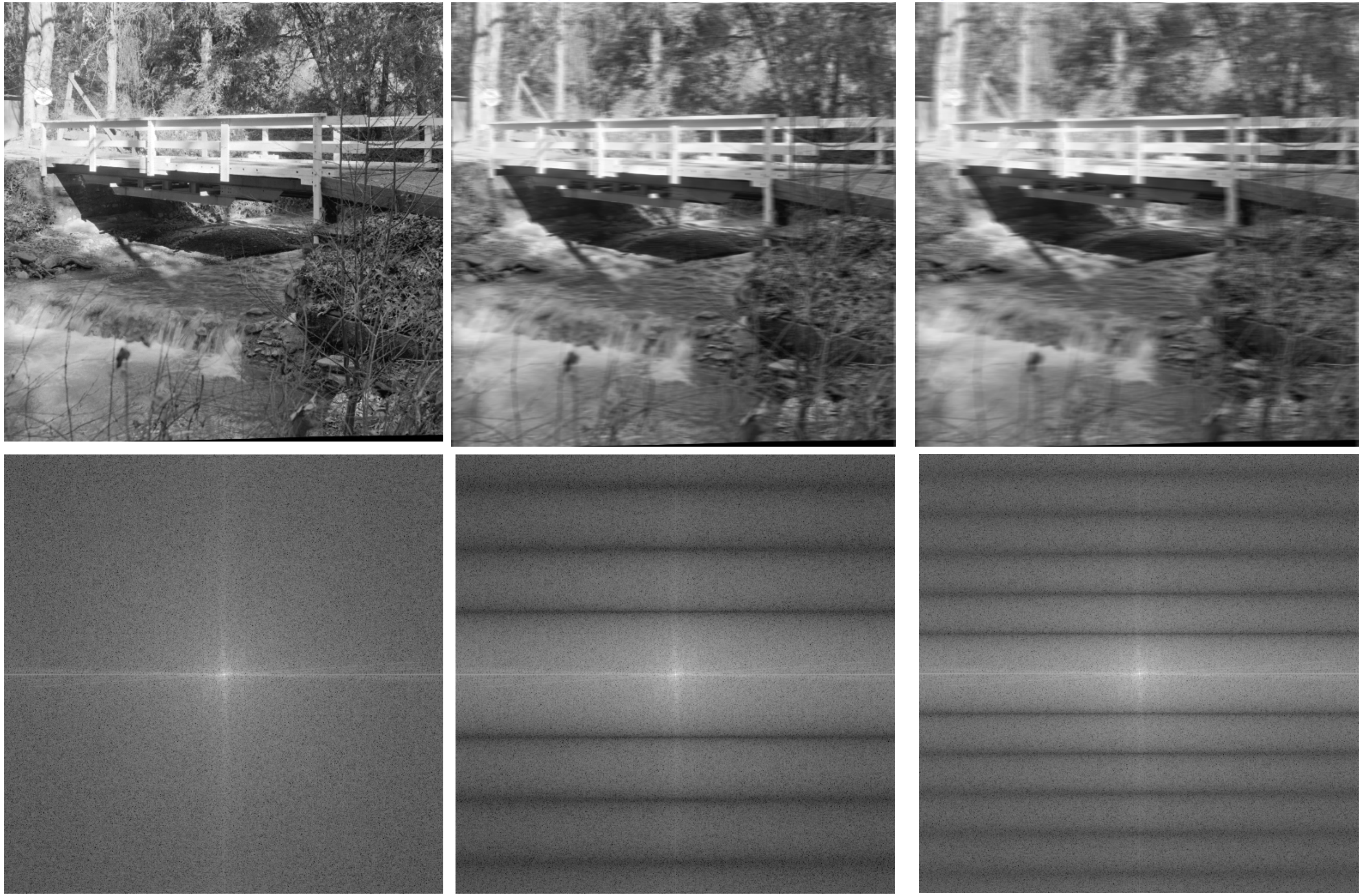






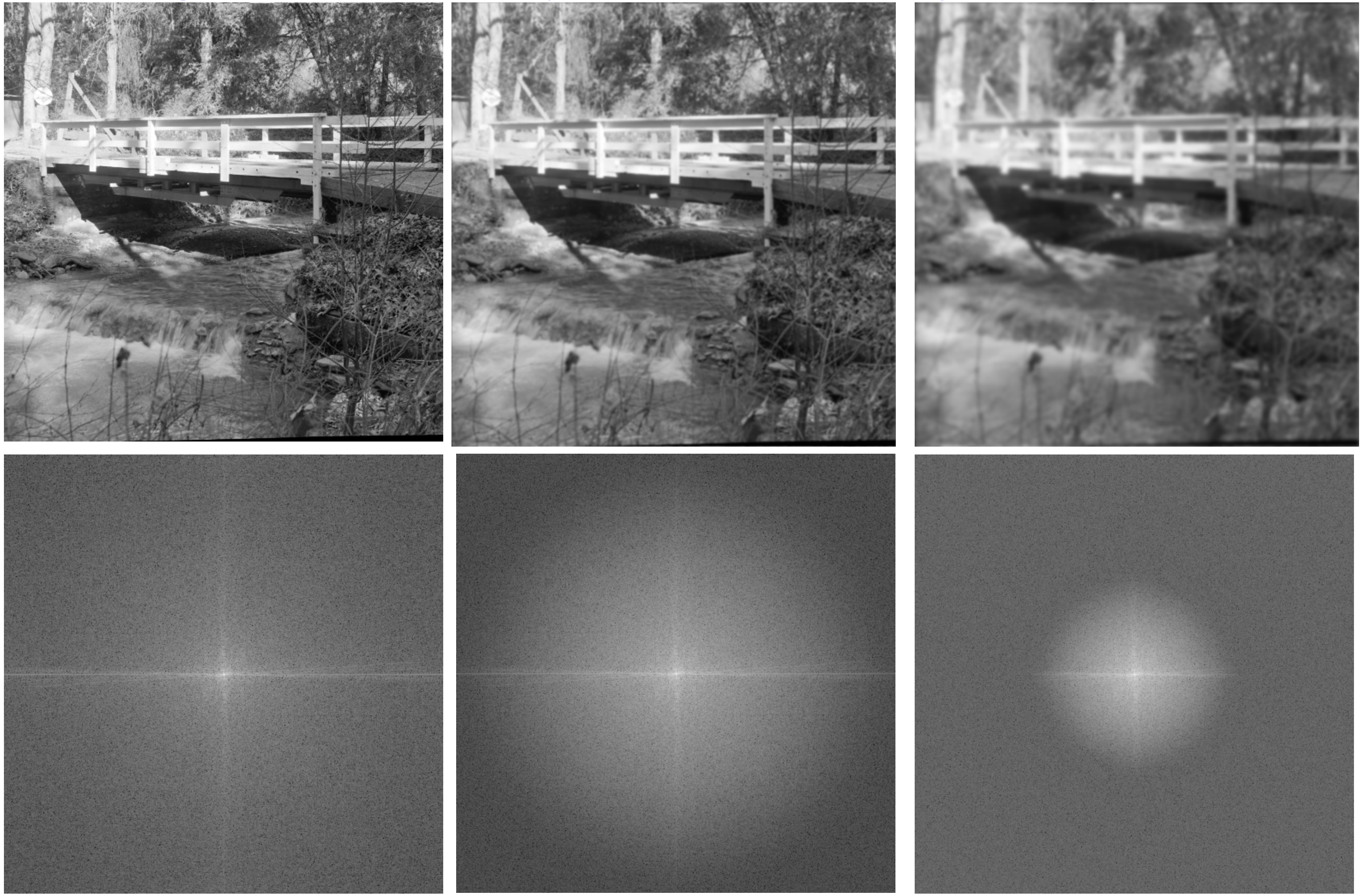
Gaussian | Motion | Defocus

Degradation in Spatial and Frequency Domain



True Image | Motion blur, 6 pixels | Motion blur, 10 pixels

Degradation in Spatial and Frequency Domain

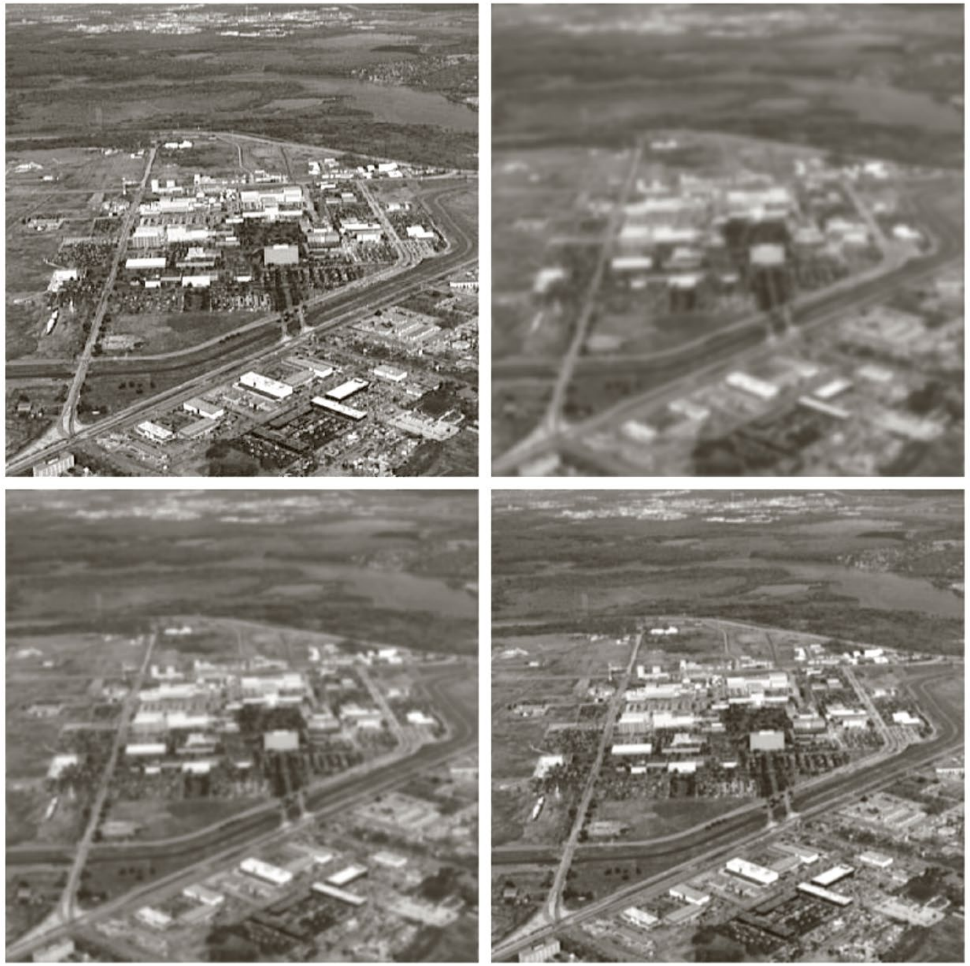


True Image

Gaussian blur, σ=2

Gaussian blur, σ =5

Atmosphere Turbulence Blur Simulated by Gaussian Model



a b
c d

FIGURE 5.25 Illustration of the atmospheric turbulence model. (a) Negligible turbulence. (b) Severe turbulence, $k = 0.0025$. (c) Mild turbulence, $k = 0.001$. (d) Low turbulence, $k = 0.00025$. (Original image courtesy of NASA.)

"Ideal" Model in Spatial Domain

- Still assuming that the effect of noise is negligible, in spatial domain we obtain

$$g(x,y) = h(x,y) * f(x,y)$$

$$G(u,v) = H(u,v)F(u,v)$$

- Respectively,

$$f(x,y) = g(x,y) * \left(h(x,y)\right)^{*1}$$

- This is equation is called "deconvolution". *1 means a backward convolutional kernel.
- Neither in spatial, nor in frequency domain it can be solved directly.
- In the spatial domain, - due to the difficulty in finding a backward convolutional kernel. In the frequency domain, - due to zeros and very small numbers contained in H.

“Ideal” Frequency Domain Model

- Assuming that the effect of noise is negligible (which, in fact, is almost never possible for real-world images), we may use a simpler frequency domain model

$$G(u,v) = H(u,v) \circ F(u,v)$$

- Respectively, to restore our image, we have to solve this equation for $F(u,v)$

$$\hat{F}(u,v) = \circ\frac{G(u,v)}{H(u,v)}$$

$A \circ B$ is a component-wise matrix multiplication of matrices A and B

$\circ\frac{A}{B}$ is a component-wise division of matrices A and B

“Ideal” Frequency Domain Model

- **Disadvantages**:
  1) This model does a terrible job since it divides in the frequency domain by numbers that are very small or even equal to zero
  2) Even assuming that there are no zeros there, but there are only small numbers, we face a situation when any light noise corrupting an image will be significantly enhanced

Methods of Solving Deconvolution Problem

- A necessary condition to restore a blurred image is to know the exact PSF model – to know function $h(x,y)$ and its Fourier transform $H(u,v)$.
- The alternative is to apply a blind deconvolution technique, which is resulted in the long iterative process whose results can be good or not good. So it is "50/50". Thus, it is preferable to identify PSF

Methods of Solving Deconvolution Problem – Inverse Filtering

- Inverse filtering – deconvolution equation is going to be solved in the frequency domain. Those frequencies where $H(u,v)=0$, are not involved in the operation

$$\frac{1}{H(u,v)} = \left(H(u,v)\right)^{-1} = \frac{\bar{H}(u,v)}{\left|H(u,v)\right|^2}$$

$$\hat{F}(u,v) = \circ\frac{G(u,v)}{H(u,v)} = \frac{\bar{H}(u,v)}{\left|H(u,v)\right|^2} \circ G(u,v)$$

$A \circ B$ is a component-wise matrix multiplication of matrices A and B

$\circ\frac{A}{B}$ is a component-wise division of matrices A and B

Methods of Solving Deconvolution Problem – Inverse Filtering with **regularization**

- Inverse filtering with regularization – deconvolution equation is going to be solved in the frequency domain. To avoid division by 0 where $H(u,v)=0$, a regularization parameter α should be used

$$\frac{1}{H(u,v)} = \left(H(u,v)\right)^{-1} = \frac{\bar{H}(u,v)}{\left|H(u,v)\right|^2}$$

$$\hat{F}(u,v) = \circ\frac{G(u,v)}{H(u,v)+\alpha} = \left(\alpha + \frac{\bar{H}(u,v)}{\left|H(u,v)\right|^2}\right) \circ G(u,v)$$

- α should be equal to $\begin{cases} 0.0001 \le \alpha \le 0.01^{-2}, & \text{if } \left|H(u,v)\right| \le 10^{-4} \\ 0, & \text{if } \left|H(u,v)\right| > 10^{-4} \end{cases}$

$A \circ B$ is a component-wise matrix multiplication of matrices A and B

$\circ\frac{A}{B}$ is a component-wise division of matrices A and B

Methods for PSF identification

- To identify PSF, if an image is even slightly corrupted by noise, it is necessary to use machine learning tools (a neural network, a support vector machine, etc.) and to analyze a power spectrum
- A problem is that when an image is noisy, there are no actual "zeros" (gaps) in its power spectrum, because the Fourier transform of an image contains the one of noise as an additive component

Methods of Solving Deconvolution Problem – Wiener Filtering

- Wiener filtering is a minimum Mean Square Error Filtering. The error is

$$e^2 = E\left\{(f - \hat{f})^2\right\}$$

- Thus, Wiener filtering is based on the minimization of the mean square error between an ideal image and its estimation obtained as a result of filtering

Methods of Solving Deconvolution Problem – Wiener Filtering

- Wiener filtering is a minimum Mean Square Error Filtering. The error is $e^2 = E\left\{(f - \hat{f})^2\right\}$
- The filter in frequency domain is determined as follows
- $$\hat{F}(u,v) = \left[\circ \frac{\bar{H}(u,v)}{|H(u,v)|^2 + \frac{|N(u,v)|^2}{|F(u,v)|^2}} \right] \circ G(u,v)$$
- $N(u,v)$ is a Fourier transform of noise, $F(u,v)$ is a Fourier transform of an ideal image, $H(u,v)$ is a Fourier transform of PSF, $G(u,v)$ is a Fourier transform of an image to be restored, $\bar{H}$ is complex-conjugated H

$A \circ B$ is a component-wise matrix multiplication of matrices A and B

$\circ \frac{A}{B}$ is a component-wise division of matrices A and B

Wiener Filtering for White Noise Case

$$\hat{F}(u,v) = \left[\circ \frac{\bar{H}(u,v)}{|H(u,v)|^2 + K} \right] \circ G(u,v)$$

- K (a ratio of power spectrum of noise to power spectrum of an image) is a constant for white noise and can be estimated as follows:

1) Apply some simple low pass filter to an image
2) Subtract the result from a noisy image
3) This difference is an estimation of noise. Hence, we can measure its variance σ^2_{Noise}. For a non-normalized one it is $NM\sigma^2_{Noise}$ for an N x M image. Hence $\frac{|N(u,v)|^2}{|F(u,v)|^2} = K \approx \frac{\sigma^2_{Noise}}{\sigma^2_{image}}$ or $= \frac{(NM)\sigma^2_{Noise}}{\sigma^2_{image}}$

Wiener Filtering for White Noise Case

- In this estimation of K

$$\frac{|N(u,v)|^2}{|F(u,v)|^2} = K \approx \frac{\sigma^2_{Noise}}{\sigma^2_{image}} \text{ or } \approx \frac{(NM)\sigma^2_{Noise}}{\sigma^2_{image}}$$

- σ^2_{image} is a variance either of input (blurred) image $\mathbf{g}$ or a filtered image obtained from $\mathbf{g}$ when we estimate noise performing low pass filtering

Wiener Filtering and Regularization

- Thus, Wiener filtering

$$\hat{F}(u,v) = \left[\circ \frac{\bar{H}(u,v)}{|H(u,v)|^2 + K} \right] \circ G(u,v)$$

- is a way for regularization of solving a problem of deconvolution. K regularizes the issue of zeros and very small numbers in

$$\boxed{\frac{1}{H(u,v)} = \left(H(u,v)\right)^{-1} = \frac{\bar{H}(u,v)}{|H(u,v)|^2}}$$

Wiener Filtering and Regularization

- Wiener filtering with extra Regularization employes a regularization parameter similarly to inverse filter with regularization

$$\hat{F}(u,v)=\left[\circ\frac{\bar{H}(u,v)}{\left|H(u,v)\right|^2+\alpha K}\right]\circ G(u,v)$$

where α is a regularization parameter. Small α makes it possible to enhance high frequencies significantly (enhancing noise too). It should usually be in the range of $0.01 \le \alpha \le 0.0001$

Restoration: Examples

a b c

FIGURE 5.28 Comparison of inverse and Wiener filtering. (a) Result of full inverse filtering of Fig. 5.25(b). (b) Radially limited inverse filter result. (c) Wiener filter result.

Restoration of an image after deconvolution is done

- After restoring an image Fourier transform $\hat{F}(u,v)$ by solving a deconvolution problem, an image can be restored by finding its inverse Fourier transform

$$\hat{f}\left(x,y\right)=\Phi^{-1}\left(\hat{F}(u,v)\right)$$

Evaluation of the quality of a restored image

- The quality of restoration can be evaluated as usually in terms of PSNR
- In image deblurring there are also two other measures, which are preferable:

➢ the blurred signal-to-noise ratio (BSNR) and

➢ the improved signal-to-noise-ratio (ISNR) – it can be used to test a restoration method when an ideal image is known (like PSNR is used for filter design)

Criteria: BSNR and ISNR

- Blurred Signal to Noise Ratio (BSNR)

$$BSNR = 10 \cdot \log_{10} \frac{\sum \left(g - \hat{f}\right)^2}{NM\sigma_{Noise}^2}$$

where g is a blurred image without noise, $\hat{f}$ is a restored image, NM is the number of pixels, and σ_{Noise}^2 is the noise variance

- Improved Signal to Noise Ratio (ISNR)

$$ISNR = 10 \cdot \log_{10} \frac{\sum (f - g)^2}{\sum \left(f - \hat{f}\right)^2}$$

where f is an ideal image, g is a degraded image, and $\hat{f}$ is a restored image