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Numerical Computation
Gaussian Elimination

For an n×m matrix, the Gaussian elimination method is a technique used to solve systems of linear equations by transforming the matrix into an upper triangular form. This process involves a series of row operations to simplify the system, making it easier to solve for the unknowns. Specifically, the goal is to eliminate variables from successive rows, eventually resulting in a system that can be solved by back substitution. If the system is consistent and the matrix is of full rank, Gaussian elimination will provide a unique solution. This project demonstrates the application of Gaussian elimination to efficiently solve linear systems.

First Matrix

```
Enter the number of variables (v): 3
Enter the augmented matrix (each row as space-separated values, including the right-hand side):
Enter row 1: 3 1 -4 7
Enter row 2: -2 3 1 -5
Enter row 3: 2 0 5 10
Solution: [3.20987654 0.2345679  0.71604938]
```

Second Matrix

```
Enter the number of variables (v): 3
Enter the augmented matrix (each row as space-separated values, including the right-hand side):
Enter row 1: 1 -2 4 6
Enter row 2: 8 -3 2 2
Enter row 3: -1 10 2 4
Solution: [-0.11320755  0.0754717   1.56603774]
```

The gauss_Elimination function solves a system of linear equations using Gaussian elimination. It first performs forward elimination, transforming the matrix into an upper triangular form by selecting pivot elements (the largest absolute values). If a pivot element is zero, the function returns `None`, indicating no unique solution. Rows are swapped when necessary, and terms below the pivot are eliminated. Afterward, back substitution is used to solve for the variables. The solution vector `x` is calculated by working backward from the last row and using the previously solved variables to find the remaining unknowns. If any pivot elements are zero during elimination, the function returns `None`. Otherwise, it returns the solution vector `x`.

The `main` function allows the user to input a system of linear equations and solve it using Gaussian elimination. It first prompts the user to enter the number of variables \(v \), then initializes an augmented matrix `X` with dimensions \(v \times (v+1) \) to represent the system. The user is asked to input the rows of the augmented matrix, where each row consists of space-separated values, including the right-hand side of the equations. If a solution exists, it is printed to the screen. If the system has no unique solution, the function outputs a message indicating this.

Problem f)

So I tested if the roots for x,y,z are correct. I used a gaussian elimination system of linear equations calculator and checked for each matrix.

For first matrix

Solution set:

$x_1 = 260/81$
$x_2 = 19/81$
$x_3 = 58/81$

(260/81) ~ 3.20987654
(19/81) ~ 0.2345679
(58/81) ~ 0.716049338

For second matrix

Solution set:

$x_1 = -6/53$
$x_2 = 4/53$
$x_3 = 83/53$

(-6/53) ~ -0.11320755
(4/53) ~ 0.07544717
(83/53) ~ 1.56603774

It matches the value that was collected for the results roots (x,y,z).

In conclusion, Gaussian elimination is a powerful and widely-used method for solving systems of linear equations. By systematically transforming a matrix into an upper triangular form through row operations, it simplifies the process of solving for unknown variables. The method not only ensures that solutions are found efficiently but also provides a clear path for determining whether a system has a unique solution. This project highlights the practical implementation of Gaussian elimination and its effectiveness in solving linear systems, demonstrating its importance in numerical analysis and various real-world applications.

[1] GeeksforGeeks, "Gaussian Elimination," GeeksforGeeks, 2020, https://www.geeksforgeeks.org/gaussian-elimination/.