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ORIGINAL ARTICLE

The astrophysical S-Factor of ^{4}He(^{3}He,γ)^{7}Be reaction at very low-energies

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Abstract The astrophysical S-factor for ^{4}He-^{3}He radiative capture is calculated at very low-energies. We construct conserved two- and three-body electromagnetic currents, using minimal substitution in the explicit momentum dependence of the two- and three-cluster interactions. The realistic Argonne v_{18} two-nucleon and Urbana IX or Tucson-Melbourne three-cluster interactions are considered for calculation. The zero energy S-factor is found to be $S(0) = 0.563$ (0.581) keV b, with (without) three-body interactions, in satisfactory agreement with other theoretical results and experiment data.

Keywords The astrophysical S-factor · ^{4}He-^{3}He system · Radiative capture · Few-body systems

1 Introduction

The ^{4}He(^{3}He,γ)^{7}Be reaction plays a major role in the solar proton-proton chain in Hydrogen burning from which the ^{7}Be and ^{8}B neutrinos are generated. It is also a fundamental reaction in Big-Bang Nucleosynthesis (BBN) and significantly important in the verification of the Standard Solar Model. The astrophysical S-factors, determining the threshold behavior of cross sections, are very important parameters in models of big-bang nuclear synthesis, stellar hydrogen burning, solution of the Sun-neutrino puzzle etc. This capture process occurs through the formation of a ^{7}Be nucleus with the emission of γ-radiation coming from the direct capture into the ground state and the first excited state of ^{7}Be. Solar neutrino fluxes depend on nuclear physics inputs, namely on the cross sections of the reactions responsible for neutrino production inside the Solar core.

The experimental situation regarding the capture cross section was not clear, for a long time, due to conflicting experimental results (Adelberger et al. 1998; Alexander et al. 1984; Nara Singh et al. 2004; Bemmerer et al. 2007; Confortola et al. 2007; Brown et al. 2007; Di Leva et al. 2009). Recently, a new series of measurements has begun, starting with an activation measurement. These new studies tried to measure the reaction with high precision and therefore to investigate the possible discrepancy between two different experimental approaches that could be given underestimated systematic errors and possible non radiative transitions. The aim of these experiments were therefore to provide high precision data obtained simultaneously using different methods (Nara Singh et al. 2004; Brown et al. 2007; Confortola et al. 2007; Bemmerer et al. 2007). Cross section of the ^{4}He(^{3}He,γ)^{7}Be reaction has been remeasured by Weizmann (Nara Singh et al. 2004), the LUNA Collaboration (Bemmerer et al. 2007; Confortola et al. 2007), the Seattle group (Brown et al. 2007), and the ERNA Collaboration (Di Leva et al. 2009) now providing consistent high precision data. Furthermore, it is still not possible to reach the low-energies relevant in nuclear burning.

In theoretical frameworks, some groups are studying the radiative capture cross section for ^{4}He(^{3}He,γ)^{7}Be reaction. For such reactions, solving the many-body problem with realistic interactions is hard and consistent *ab initio* reaction calculations have been possible only for single nucleon projectiles (Nollett et al. 2007; Quaglioni and Navratil 2009; Navratil et al. 2007; Neff 2011). Up to now, none of calculations is successful in describing the energy dependence of the capture cross section data. Neff has investigated consistently the bound state properties, and the scattering phase

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shifts as well as the normalization and energy dependence of this capture cross section (Neff 2011). Their results deviates from the correlation between the ground state quadrupole moment and zero-energy S-factor found in cluster models using phenomenological interactions.

It is the purpose of the present paper to present the calculation of the ^{4}He(^{3}He,γ)^{7}Be cross section and S-factor at very low-energies. We describe consistently bound and scattering states starting from the realistic modern nucleon-nucleon Argonne v18(AV18) two-body and Urbana IX three-body interactions, that reproduces the scattering cross section. The emphasis is on constructing three-body currents corresponding to two- and three-cluster interactions and comparison of the our result with those of other theoretical methods and experimental data.

In Sect. 2, we briefly overview theoretical framework including interactions, two- and three-body currents, multipole transition, the Faddeev integral equation for calculating of ^{4}He-^{3}He radiative capture at thermal energies. Next, we present the obtained results in comparison with the corresponding experimental and theoretical data, in Sect. 3. In particular, we compare the calculated the astrophysical S-factor to experimental data and, further, calculated cross sections in very low-energy range. Summary and conclusions are given in Sect. 4.

2 Formulation

2.1 Two- and three-body currents and multipole transition of ^{4}He-^{3}He radiative capture

We can write in a direct way the matrix element for any threshold process, in a general form, using only the symmetry properties of the short-range and the electromagnetic interactions. Let us recall the most important symmetry principles of fundamental interactions which allow us to establish the spin structure of the matrix element:

- Isotropy of space (conservation of total angular momentum).
- Invariance of relative inversion of the space coordinates, P-transformation and invariance.
- The gauge invariance (in case of photonuclear or radiative capture reactions).
- The Pauli principle for identical fermions.
- Isotopic invariance of the short-range interaction and generalized Pauli principle (for non-identical fermions).
- Invariance under charge conjugation (C-invariance).

These are a crucial role in calculation of the cross section and observables measured in the astrophysical radiative captures.

In this section, we present the nuclear electromagnetic current operator, in addition to one-body convection and spin-magnetization terms, two- and three-body terms (for more details see Marcucci et al. 2005), with respect to the above symmetry principles of fundamental interactions.

The nuclear electromagnetic charge and current operators is given by

$$\rho(\mathbf{q}) = \sum_i \rho_i(\mathbf{q}) + \sum_{i<j} \rho_{ij}(\mathbf{q}) + \sum_{i<j<k} \rho_{ijk}(\mathbf{q}) \dots, \qquad (1)$$

$$\mathbf{J}(\mathbf{q}) = \sum_i \mathbf{J}_i(\mathbf{q}) + \sum_{i<j} \mathbf{J}_{ij}(\mathbf{q}) + \sum_{i<j<k} \mathbf{J}_{ijk}(\mathbf{q}) \dots. \qquad (2)$$

where $\rho_i(\mathbf{q})$ and $\mathbf{J}_i(\mathbf{q})$ are one-body operators that derived from the non-relativistic reduction of the covariant single-nucleon current. $\rho_{ij}(\mathbf{q})(\rho_{ijk}(\mathbf{q}))$ and $\mathbf{J}_{ij}(\mathbf{q})(\rho_{ijk}(\mathbf{q}))$ are also two-body (three-body) terms that operate on the cluster degrees of freedom, the one-body charge operator in configuration space is given by

$$\rho_i(\mathbf{q}) = \rho_{i,\mathrm{NR}}(\mathbf{q}) + \rho_{i,\mathrm{RC}}(\mathbf{q}), \quad \rho_{i,\mathrm{NR}}(\mathbf{q}) = \epsilon_i e^{\mathrm{i}\mathbf{q}\cdot\mathbf{r}_i}, \qquad (3)$$

where $\epsilon_i = \frac{1}{2}[G_E^S(q_\mu^2) + G_E^V(q_\mu^2)\tau_{i,z}]$ $G_E^S(q_\mu^2)$ and $G_E^V(q_\mu^2)$ are the isoscalar and isovector combinations of the nucleon electric Sachs form factors, respectively, evaluated at the four-momentum transfer q_μ^2 (Marcucci et al. 2005). The relativistic correction term, $\rho_{i,\mathrm{RC}}(\mathbf{q})$, is proportional to $1/m^2$. The isospin-conserving momentum-independent part of the two-body interaction v_{ij}^0, can be written as

$$v_{ij}^0 = v_{1,ij} + v_{2,ij}\boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j, \qquad (4)$$

where $\boldsymbol{\tau}_i$, $\boldsymbol{\tau}_j$, v_1 and v_2 the isospin Pauli matrices and functions of the positions and spin operators of the two nucleons (Marcucci et al. 2005).

Consider again the isospin-conserving momentum-independent part of the potential given in Eq. (4). The isospin operator $\boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j$ can be expressed in terms of P_{ij}, using the formula

$$\boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j = -1 - (1 + \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j) P_{ij}, \qquad (5)$$

where $P_{ij} f(\mathbf{r}_i, \mathbf{r}_j) \equiv e^{\mathbf{r}_{ji}\cdot\nabla_i + \mathbf{r}_{ij}\cdot\nabla_j} f(\mathbf{r}_i, \mathbf{r}_j) = f(\mathbf{r}_j, \mathbf{r}_i)$. In the presence of an electromagnetic interactions, the space-exchange operator, after performing minimal substitution, becomes

$$P_{ij} \to P_{ij}^{\mathbf{A}} = e^{\mathbf{r}_{ji}\cdot[\nabla_i - \mathrm{i}\epsilon_i \mathbf{A}(\mathbf{r}_i)] + \mathbf{r}_{ij}\cdot[\nabla_j - \mathrm{i}\epsilon_j \mathbf{A}(\mathbf{r}_j)]}$$
$$\equiv e^{\mathbf{r}_{ji}\cdot\nabla_i + g_i(\mathbf{r}_i)} e^{\mathbf{r}_{ij}\cdot\nabla_j + g_j(\mathbf{r}_j)}, \qquad (6)$$

where $\mathbf{A}(\mathbf{r})$ and the functions $g_{i(j)}$ are the vector potential and $g_{i(j)}(\mathbf{r}_{i(j)}) \equiv -\mathrm{i}\epsilon_{i(j)}\mathbf{r}_{ji} \cdot \mathbf{A}(\mathbf{r}_{i(j)})$.

A conserved current can then be derived by considering an infinitesimal gauge transformation. When electro-nuclear observables are typically carried out in first-order perturbation theory in these fields, the current in the limit of weak electromagnetic fields, after gauge transformation and retain only linear terms in the vector potential, we find

$$\begin{aligned} v_{ij} &\to v_{1,ij} + v_{2,ij}\big[-1-(1+\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j)P_{ij}^{\mathbf{A}}\big] \\ &\simeq v_{ij}^0 + v_{2,ij}\bigg[-\mathrm{i}\epsilon_i\int d\mathbf{s}\cdot\mathbf{A}(\mathbf{s}) - \mathrm{i}\epsilon_j\int d\mathbf{s}'\cdot\mathbf{A}(\mathbf{s}')\bigg] \\ &\quad\times(1+\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j) \\ &\equiv v_{ij}^0 - \int \mathbf{J}_{ij}(\mathbf{x})\cdot\mathbf{A}(\mathbf{x})d\mathbf{x}, \end{aligned} \tag{7}$$

then, Fourier transform of the current density operator is given by

$$\mathbf{J}_{ij}(\mathbf{q}) = \mathrm{i}v_{2,ij}\bigg(\epsilon_i\int_{\gamma_{ij}} d\mathbf{s}\mathrm{e}^{\mathrm{i}\mathbf{q}\cdot\mathbf{s}} + \epsilon_j\int_{\gamma'_{ji}} d\mathbf{s}'\mathrm{e}^{\mathrm{i}\mathbf{q}\cdot\mathbf{s}'}\bigg)(1+\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j). \tag{8}$$

It is important to note that, when $q\to 0$, the current operator becomes unique, and is given by (Marcucci et al. 2005)

$$\mathbf{J}_{ij}(\mathbf{q}=0) = -v_{2,ij}G_E^V\big(q_\mu^2\big)(\boldsymbol{\tau}_i\times\boldsymbol{\tau}_j)_z\mathbf{r}_{ij}. \tag{9}$$

The current operators associated with the momentum-dependent operators of the two-nucleon interaction, in the same way. For spin-orbit interactions,

$$\begin{aligned} v_{1,ij}^p &= v_b(r)\mathbf{L}\cdot\mathbf{S}, \\ v_{2,ij}^p &= v_{b\tau}(r)\mathbf{L}\cdot\mathbf{S}, \end{aligned} \tag{10}$$

where v_1^p and v_2^p of Eq. (4) are is used, and $\mathbf{L}=\mathbf{r}_{ij}\times(\mathbf{p}_i-\mathbf{p}_j)/2$, $\mathbf{p}_i$ and $\mathbf{p}_j$ being the particles' momentum operators. By performing minimal substitution and keeping only terms linear in the vector potential $\mathbf{A}$, the associated current is found to be (Marcucci et al. 2005)

$$\begin{aligned} \mathbf{J}_{ij}(\mathbf{q};b\tau) &= \frac{1}{4}v_{b\tau}(r)\mathbf{S}\times\mathbf{r}_{ij}\big(\eta_j\mathrm{e}^{\mathrm{i}\mathbf{q}\cdot\mathbf{r}_i} - \eta_i\mathrm{e}^{\mathrm{i}\mathbf{q}\cdot\mathbf{r}_j}\big) \\ &\quad+\frac{1}{2}v_{b\tau}(r)G_E^V\big(q_\mu^2\big)(\boldsymbol{\tau}_i\times\boldsymbol{\tau}_j)_z \\ &\quad\times\bigg(\mathbf{L}\cdot\mathbf{S}\int_{\gamma_{ij}} d\mathbf{s}\mathrm{e}^{\mathrm{i}\mathbf{q}\cdot\mathbf{s}} + \int_{\gamma_{ij}} d\mathbf{s}\mathrm{e}^{\mathrm{i}\mathbf{q}\cdot\mathbf{s}}\mathbf{L}\cdot\mathbf{S}\bigg), \end{aligned} \tag{11}$$

where $\eta_i = G_E^S(q_\mu^2)\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j + G_E^V(q_\mu^2)\tau_{i,z}$.

Three-body currents involving the Δ-isobar resonance were derived recently in the study of explicit Δ components in the three-nucleon wave functions (Deltuva 2009). The Urbana-type of the three-body interaction is written as sum of a short-range spin- and isospin-independent term and a term involving the excitation of an intermediate Δ. The central term is ignored and the Δ-excitation term is given by (Marcucci et al. 2005)

$$\begin{aligned} V_{ijk} = \sum_{\text{cyclic } ijk} A_{2\pi}\bigg(&\{X_{ij},X_{jk}\}\{\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j,\boldsymbol{\tau}_j\cdot\boldsymbol{\tau}_k\} \\ &+\frac{1}{4}[X_{ij},X_{jk}][\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j,\boldsymbol{\tau}_j\cdot\boldsymbol{\tau}_k]\bigg), \end{aligned} \tag{12}$$

$\{\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j,\boldsymbol{\tau}_j\cdot\boldsymbol{\tau}_k\} = 2\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_k$ and $X_{ij} = v_{\sigma\tau}^{II}(r)\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j + v_{t\tau}^{II}(r)S_{ij}$, where $v_{\sigma\tau}^{II}(r)$ and $v_{t\tau}^{II}(r)$ are the standard spin-isospin and tensor-isospin functions occurring in the one-pion-exchange interaction. The parameter $A_{2\pi}$ is determined by reproducing the triton binding energy in an approximate hypernetted-chain variational calculation (Marcucci et al. 2005). Consider the isospin dependence of the three-Body interaction. The associated current $\mathbf{J}_{j;ki}^A(\mathbf{q})$ is derived with the same methods discussed for two-body and is given by (see Eq. (8))

$$\begin{aligned} \mathbf{J}_{j;ki}^A(\mathbf{q}) &= 2\mathrm{i}A_{2\pi}\{X_{ij},X_{jk}\}\bigg(\epsilon_i\int d\mathbf{s}\mathrm{e}^{\mathrm{i}\mathbf{q}\cdot\mathbf{s}} + \epsilon_k\int d\mathbf{s}'\mathrm{e}^{\mathrm{i}\mathbf{q}\cdot\mathbf{s}'}\bigg) \\ &\quad\times(1+\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_k). \end{aligned} \tag{13}$$

Gauging the gradient operators and retaining only linear terms in the vector potential (in the same way of two-body interaction) lead to the following current $\mathbf{J}_{j;ki}^C(\mathbf{q})$ from the commutator term of the three-body interaction (Marcucci et al. 2005):

$$\begin{aligned} \mathbf{J}_{j;ki}^C(\mathbf{q}) = \frac{\mathrm{i}}{4}A_{2\pi}[X_{ij},X_{jk}] \\ &\times\bigg[\bigg(\epsilon_i\int d\mathbf{s}\mathrm{e}^{\mathrm{i}\mathbf{q}\cdot\mathbf{s}} + \epsilon_j\int d\mathbf{s}\mathrm{e}^{\mathrm{i}\mathbf{q}\cdot\mathbf{s}} + \epsilon_k\int d\mathbf{s}\mathrm{e}^{\mathrm{i}\mathbf{q}\cdot\mathbf{s}}\bigg) \\ &\times(1+\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j)(1+\boldsymbol{\tau}_j\cdot\boldsymbol{\tau}_k) \\ &-\bigg(\epsilon_i\int d\mathbf{s}\mathrm{e}^{\mathrm{i}\mathbf{q}\cdot\mathbf{s}} + \epsilon_j\int d\mathbf{s}\mathrm{e}^{\mathrm{i}\mathbf{q}\cdot\mathbf{s}} + \epsilon_k\int d\mathbf{s}\mathrm{e}^{\mathrm{i}\mathbf{q}\cdot\mathbf{s}}\bigg) \\ &\times(1+\boldsymbol{\tau}_j\cdot\boldsymbol{\tau}_k)(1+\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j)\bigg]. \end{aligned} \tag{14}$$

We then obtain (Marcucci et al. 2005):

$$\begin{aligned} \mathbf{J}_{j;ki}(\mathbf{q}) &= \mathbf{J}_{j;ki}^A(\mathbf{q}) + \mathbf{J}_{j;ki}^C(\mathbf{q}) \\ &= 2A_{2\pi}G_E^V\big(q_\mu^2\big)\{X_{ij},X_{jk}\}(\boldsymbol{\tau}_i\times\boldsymbol{\tau}_k)_z\int d\mathbf{s}\mathrm{e}^{\mathrm{i}\mathbf{q}\cdot\mathbf{s}} \\ &\quad+\frac{\mathrm{i}}{4}A_{2\pi}G_E^V\big(q_\mu^2\big)[X_{ij},X_{jk}] \\ &\quad\times\bigg[(\tau_{i,z}\boldsymbol{\tau}_j\cdot\boldsymbol{\tau}_k - \tau_{j,z}\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_k)\int d\mathbf{s}\mathrm{e}^{\mathrm{i}\mathbf{q}\cdot\mathbf{s}} \\ &\quad+(\tau_{k,z}\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j - \tau_{j,z}\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_k)\int d\mathbf{s}\mathrm{e}^{\mathrm{i}\mathbf{q}\cdot\mathbf{s}}\bigg]. \end{aligned} \tag{15}$$

As a case of two-body currents, the limit $\mathbf{q}=0$ is

$$\mathbf{J}_{j;ki}(\mathbf{q}=0)=\mathrm{i}\left[V_{j;ki},\int d\mathbf{x}\mathbf{x}\left[\rho_{i,\mathrm{NR}}(\mathbf{x})+\rho_{j,\mathrm{NR}}(\mathbf{x})+\rho_{k,\mathrm{NR}}(\mathbf{x})\right]\right]. \tag{16}$$

The polarization of real photons can be characterized by the three-vector $\vec{e}$, which satisfies the Lorentz condition ($\vec{e}\cdot\hat{\vec{k}}=0$, where $\hat{\vec{k}}$ is the unit vector along the photon three-momentum). The sum over the photon polarizations is given by

$$\sum_{i=1,2} e_a^{(i)*}e_b^{(i)}=\delta_{ab}-\hat{k}_a\hat{k}_b,$$

where the upper index i numerates the two possible independent polarization vectors of the photon.

The first step in the determination of the spin structure of the matrix element is the analysis of the possible multipole (for processes involving photons) or partial transitions, which are allowed for the considered process by the above mentioned symmetry properties. In photonuclear processes $\gamma+a\to b+c$ or radiative capture $a+b\to c+\gamma$, in threshold, the photon can be characterized by the smallest value of j. Thus, the basic formulas for the construction of the matrix elements for electromagnetic processes from the corresponding combinations of polarization $\vec{e}$ and unit vector $\hat{\vec{k}}$ are given by

$$\vec{e}\to E1 \quad \text{(electric dipole)},$$

$$\vec{e}\times\hat{\vec{k}}\to M1 \quad \text{(magnetic dipole)},$$

$$e_a\hat{k}_b+e_b\hat{k}_a\equiv E_{ab}\to E2 \quad \text{(electric quadrupole)},$$

$$(\vec{e}\times\vec{k})_a\hat{k}_b+(\vec{e}\times\vec{k})_b\hat{k}_a\equiv M_{ab}\to M2 \quad \text{(magnetic quadrupole)}.$$

The degree of this three-vector $\hat{\vec{k}}$ is directly related to the value of the orbital angular momentum of colliding particles or the multiplicity of the photon. The zero degree in $\hat{\vec{k}}$ describes the interaction of the initial particles in S-state, the first degree in P-state and the second degree in D-state.

The radiative capture cross section, in corresponding to each multipole transitions, can then be expressed in terms of those amplitudes as

$$\sigma^{E1,M1}=\frac{16\pi}{9}\left(\frac{k_\gamma}{k}\right)^3\left(\frac{\mu}{\hbar c}\right)\left(\frac{e^2}{\hbar c}\right)\frac{1}{2s+1}\times\frac{1}{2I_t+1}\sum_{LSJ}\left|T^{E_1{}^{LSJ},M_1{}^{LSJ}}\right|^2 \tag{17}$$

$$\sigma^{E2}=\frac{4\pi}{75}\left(\frac{k_\gamma^5}{k^3}\right)\left(\frac{\mu}{\hbar c}\right)\left(\frac{e^2}{\hbar c}\right)\frac{1}{2s+1}\times\frac{1}{2I_t+1}\sum_{LSJ}\left|T^{E_2{}^{LSJ}}\right|^2 \tag{18}$$

where I_t and s denote the spin of target nucleus and incoming particle, respectively. μ stands for the reduced mass of the system.

2.2 The Faddeev integral equation for the ^{4}He-proton-deuteron system

The nuclear three-cluster system that may be considered realistic in the sense that the interactions are given by high precision potentials valid over a broad energy range. Nevertheless, in the same way one considers the nucleon as a single particle by neglecting its inner quark structure, in a further approximation one can consider a cluster of nucleons (composite nucleus) to be a single particle that interacts with other nucleons or nuclei via effective potentials whose parameters are determined from the two-body data. A classical example is the particle, a tightly bound four-nucleon cluster. The description of the (Helium-4, deuteron and nucleon) three-cluster system, with the Argonne V18 two-body and UIX/Tucson-Melbourne three-body forces, is quite successful at low-energies. The methods based on Faddeev equation are applied. We describe the scattering process in a system of three-clusters interacting via with short-range, strong interactions v_α, $\alpha=1,2,3$; where, v_1 is the potential between clusters αp, pd and αd (Glöckle 1983; Miyagawa and Glöckle 1993). In the framework of non relativistic quantum mechanics the center-of-mass (c.m.) and the internal motion can be separated by introducing Jacobi momenta

$$\vec{p}_\alpha=\frac{m_\gamma\vec{k}_\beta-m_\beta\vec{k}_\gamma}{m_\beta+m_\gamma}, \tag{19}$$

$$\vec{q}_\alpha=\frac{m_\alpha(\vec{k}_\beta+\vec{k}_\gamma)-(m_\beta+m_\gamma)\vec{k}_\alpha}{m_\alpha+m_\beta+m_\gamma}, \tag{20}$$

where $(\alpha\beta\gamma)$, $\vec{k}_\alpha$ and m_α are the cyclic permutations of (123), the individual cluster momenta and the masses, respectively. We now consider the particle or cluster β scattering from the pair β that is bound with energy ϵ_β. The initial channel state $|b_\beta\vec{q}_\beta\rangle$ is the product of the bound state wave function $|b_\beta\rangle$ for the pair β and a plane wave with the relative particle-pair β momentum $\mathbf{q}_\beta$; the dependence on the discrete quantum numbers is suppressed in our notation.

We start from the triad of Lippmann-Schwinger equations (Miyagawa and Glöckle 1993) acting on a three-cluster initial state given by

$$\Phi_\beta^{(c)}=|\mathbf{p}_\beta\rangle^{(c)}|\mathbf{q}_\beta\rangle, \tag{21}$$

where $|\mathbf{p}_\beta\rangle^{(c)}$ is a two-cluster state, and the index $\beta = 1, 2, 3$ indicates the three choices of pairs characterized by the third particle or cluster. The final channel state is the particle-pair state in the same or different configuration $|b_\alpha \vec{q}_\alpha\rangle$ in the case of elastic and rearrangement scattering or, in the case of breakup, it is the state of three free particles $|\vec{p}_\gamma \vec{q}_\gamma\rangle$ with the same energy $E = p_\gamma^2/2\mu_\gamma + q_\gamma^2/2M_\gamma$ and pair γ reduced mass $\mu_\gamma = m_\alpha m_\gamma/(m_\alpha + m_\gamma)$; any set of Jacobi momenta can be used equally well for the breakup state. Furthermore, $U^\beta = \sum_{\gamma\neq\beta} U_\gamma$, where U_γ ($\gamma = 1, 2, 3$) are the pair forces. Three-body forces can be also incorporated in a straightforward fashion. The LS equations are given by:

$$\Psi_0^{(c)} = \Phi_\beta^{(c)} + G_\beta U^\beta \Psi_0^{(c)}, \tag{22}$$

where $G_\beta^{-1} = (E + i\varepsilon - H_0 - U_\beta)^{-1}$ is Green's function. By using standard Jacobi momenta $\mathbf{p}_\alpha$ and $\mathbf{q}_\alpha$, Eq. (19) and suitable multiplication of the three equations in the triad from the left by V_γ one obtains the transition operators $V_{\beta 0} \equiv (U_\gamma + U_\theta)\Psi_0^{(c)}$, with $\gamma \neq \beta, \beta \neq \theta$, which fulfill the set of equations

$$V_{\beta 0} = \sum_{\gamma\neq\beta} t_\gamma \Phi_0 + \sum_{\gamma\neq\beta} t_\gamma G_0 U_{\gamma 0}, \tag{23}$$

where $\Phi_0 = |\mathbf{p}\rangle|\mathbf{q}\rangle$ is the three-cluster state. We consider the system of three-clusters with charges z_β of equal sign interacting via pairwise strong short-range and screened Coulomb potentials $V_\beta + W_{\beta R}$ with β being 1, 2, or 3. The corresponding two-particle transition matrices are calculated with the full channel interaction, see (Deltuva 2009)

$$T_\beta^{(R)} = (V_\beta + W_{\beta R}) + (V_\beta + W_{\beta R}) G_0 T_\beta^{(R)}. \tag{24}$$

For the three-body break-up operator, one can generate the multiple scattering series directly by decomposing V_{00} as

$$V_{00} \equiv \sum_\theta V_\theta, \tag{25}$$

where V_θ is the coupled set of Faddeev equations

$$V_\theta = T_\theta + T_\theta G_0 \sum_{\alpha\neq\theta} V_\beta. \tag{26}$$

By iterating Eq. (26) and inserting the result into Eq. (25), it leads exactly to the multiple scattering series. So, we have a set of three coupled equations (Khalidi et al. 2010)

$$\begin{aligned} V_1 &= T_1 + T_1 G_0 (V_2 + V_3) \\ V_2 &= T_2 + T_2 G_0 (V_3 + V_1) \\ V_3 &= T_3 + T_3 G_0 (V_1 + V_2). \end{aligned} \tag{27}$$

For calculations, the ^{4}He-nnn four-body cluster is quite challenging, because of: (i) three species of particles (α, proton and neutron) and (ii) three different kinds of interactions (^{4}He-nucleon, ^{4}He-deuteron, and ^{4}He-^{3}He) involved.

Fixing arbitrarily clusters, like the neutron as spectator and label it as "1" and the two others (^{4}He-d, nn two-body and nnn three-body clusters). By applying the driving terms to the free state, we obtain (Khalidi et al. 2010):

$$\begin{aligned} V_1 \Phi_{0,a} &= T_1 \Phi_{0,a} + T_1 G_0 (1 - P_{23}) V_2 \Phi_{0,a} \\ V_2 \Phi_{0,a} &= T_2 \Phi_{0,a} + T_2 G_0 (-P_{23} V_2 \Phi_{0,a} + V_1 \Phi_{0,a}). \end{aligned} \tag{28}$$

where, $\Phi_{0,a} \equiv (1 - P_{23})|\mathbf{p}_1 \mathbf{q}_1\rangle|0 m_2 m_3\rangle|0\frac{1}{2}\frac{1}{2}\rangle$, which is antisymmetric under exchange of the two clusters.

The matrix element for the $^4\text{He} + {}^3\text{He} \to {}^7\text{Be} + \gamma$ process is simply related to the time reversed photo disintegration process of ^{7}Be into three free clusters. It is necessary that one can formulate photodisintegration of ^{7}Be based on an four-particle picture. Let O be the photon absorption operator and $|\Psi_{^7Be}\rangle$ the ^{7}Be ground state. The break-up amplitude into $\text{n} + \text{nn} + {}^4\text{He}$ can then be written as an infinite series of processes

$$\begin{aligned} \langle \Phi_{0,a}|V_0|\Psi_{^7Be}\rangle &= \langle \Phi_{0,a}|O|\Psi_{^7Be}\rangle \\ &\quad + \sum_i \langle \Phi_{0,a}|U_i G_0 O|\Psi_{^7Be}\rangle \\ &\quad + \sum_{ij} \langle \Phi_{0,a}|U_i G_0 U_j G_0 O|\Psi_{^7Be}\rangle + \cdots . \end{aligned} \tag{29}$$

Here U_i are the pair forces among the ^{4}He-nucleon, ^{4}He-deuteron, and ^{4}He-^{3}He particles. This infinite series in terms of pair forces represents the final state interactions. The first term is the direct break-up process generated by O. By defining

$$\langle \Phi_{0,a}|V_0|\Psi_{^7Be}\rangle = \langle \Phi_{0,a}|O|\Psi_{^7Be}\rangle\rangle + \sum_i \langle \Phi_{0,a}|V_{0i}|\Psi_{^7Be}\rangle, \tag{30}$$

one can obtain the T-matrices T_i which leads to three coupled Faddeev equations ($i = 1, 2, 3$) (Khalidi et al. 2010),

$$V_{0i}|\Psi_{^7Be}\rangle = T_i G_0 O|\Psi_{^7Be}\rangle + T_i G_0 \sum_{j\neq i} V_{0j}|\Psi_{^7Be}\rangle \tag{31}$$

and the complete break-up amplitude

$$\begin{aligned} \langle \Phi_{0,a}|U_0|\Psi_{^7Be}\rangle &= \langle \Phi_{0,a}|O|\Psi_{^7Be}\rangle + \langle \Phi_{0,a}|U_{01}|\Psi_{^7Be}\rangle \\ &\quad + \langle \Phi_{0,a}|(1 - P_{23}) U_{02}|\Psi_{^7Be}\rangle. \end{aligned} \tag{32}$$

We employ the two- and three-body subsystem interactions and single particle currents, two-body currents, these coupled equations can be solved by standard techniques (Golak et al. 2005).

3 Results and discussion

We derived two coupled Faddeev equations for the three-cluster scattering amplitudes. We derived the results of three-body Faddeev-type calculations for systems of three-particles interacting through short-range nuclear plus the long-range Coulomb potentials and two- and three-body currents. Realistic applications of three-body theory to three-cluster nuclear reactions only became possible to address in recent years when a reliable and practical momentum-space treatment of the Coulomb interaction has been developed (Deltuva 2009). In the present proton-deuteron-^{4}He, three-body system, it is absolutely necessary that any sub-cluster systems composed of the one, two, or three constituent particles are reasonably described by taking the two- and three-body interactions among these systems, as discussed in Sect. 2. We provided Faddeev equations for the $^3\mathrm{He}+{}^4\mathrm{He}$ radiative capture process to the ^{7}Be ground state. We numerically solved the Faddeev integral equation.

The astrophysical S-factor of the ^{4}He(^{3}He,γ)^{7}Be radiative capture reaction at thermal energies is given by

$$S(E) = E\sigma_t(E)\exp^{2\pi\eta}, \tag{33}$$

where $2\pi\eta = 164.12/E^{1/2}$, E (in keV) is center of mass kinetic energy and $\sigma_t(E)$ is the total cross section. Calculation of $S(E)$ with a center of mass energy of zero, cannot be measured directly, but instead must be extrapolated from values taken at higher energies.

The results for the astrophysical S-factor of this reaction is presented in Tables 1, 2 and 3, along with the experimental data (Costantini et al. 2008; Nara Singh et al. 2004; Carmona-Gallardo 2012), at solar energies 0.1–0.2 MeV (Costantini et al. 2008), 0.4–1 MeV (Nara Singh et al. 2004) and at medium energies 1–2 MeV (Carmona-Gallardo 2012), respectively. Table 4 compares the S-factor of the ^{4}He(^{3}He,γ)^{7}Be reaction at zero energy for various experimental and theoretical works.

Here, no significant difference has been seen between the results obtained with the present model based on the modern nucleon-nucleon potential model, for the nuclear current operator, and experimental results. The agreement between the theoretical predictions and the experimental data, especially for the very recent LUNA data (Confortola et al. 2007), is good. However, the addition of the three-body currents, which give a rather sizable contribution as can be seen from the column labeled "two- and three-body" in Tables 1, 2, 3, 4.

As will be shown in the following, this observable is also influenced by three-body current contributions. The obtained S-factor is shown in Fig. 1 together with the experimental data. Our results with two- and three-body interactions are in good agreement with the recent measurements regarding both the absolute normalization and the energy dependence.

The astrophysical S-factor for the ^{4}He(^{3}He,γ)^{7}Be reaction at 0–2 MeV, calculated with the AV18 and AV18/UIX are compared with the experimental data of Nara Singh et al. (2004), Carmona-Gallardo (2012), Krawinkel (1982), Bemmerer et al. (2007), Confortola et al. (2007), in Fig. 1. The short-dashed line are calculated by adopting purely the AV18 two-body force alone that obtained using the corresponding wave functions and including, in addition to the one- and two-body current operator, $J_{ij}(q)$. The solid line are obtained with the AV18/UIX Hamiltonian and include the corresponding set of one-, two-, and three-body currents in the new meson exchange scheme (Marcucci et al. 2005). The Fermionic Molecular Dynamics results are also given by the long-dashed lines. Recent experimental data (Nara Singh et al. 2004; Carmona-Gallardo 2012; Krawinkel 1982) are shown as dark colored symbols and older data (Krawinkel 1982) as light symbols. The cross and plus signs are also the results of the LUNA Collaboration (Bemmerer et al. 2007; Confortola et al. 2007).

In order to compare $S(0)$-factor value for the activation method, following Table 4 it is instructive to list the extrapolated $S(0)$-factor values for the different theoretical and experimental studies together with their quoted uncertainty at different years. The obtained data are shown the better agreement in comparison with the experimental based on activation studies. The extrapolated S-factor at astrophysical energies obtained in this work, $S(0) = 0.581$ and 0.563 keV b, from the two-body and the three-body interactions, respectively, is in good agreement with the one obtained in the calculation based on Fermionic Molecular Dynamics, $S(0) = 0.593$ keV b (Neff 2011), or the value obtained by of Cyburt et al., $S(0) = 0.580$ keV b (Cyburt and Davids 2008).

Three-body currents give small but significant contributions to the astrophysical S-factor in the ^{4}He(^{3}He,γ)^{7}Be reaction at low-energies. Carlson et al. find a small astrophysical S-factor, because of sensitive cancellations in the wave function and destructive interference between the one- and many-body current contributions (Carlson et al. 1990). These studies showed that inferring the S-factor from the measured cross section can be misleading, because of the large contributions associated with the many-body components of the electro weak current operator, and their destructive interference with the one-body current contributions.

In this respect, this study of radiative capture process is a particularly important reaction for two reasons: it is most importantly, due to the one-, two-, and three-body current contributions are comparable in magnitude and different sign (Carlson et al. 1991); secondly, many-body axial currents, specifically conserved current can then be derived by

Table 1 The results for the astrophysical S-factor of ^{4}He(^{3}He,γ)^{7}Be reaction and estimated error in percentage in comparison with the experimental data (Costantini et al. 2008), at solar energies 0.1–0.2 MeV

Energy (keV)	Exp. (keV b)	Exp. error (keV b)	S-Factor (two-body) (keV b)	Error (%)	S-Factor (two & three-body) (keV b)	Error (%)
92.9	0.53	0.02	0.54(5)	1.8	0.52(7)	1.8
105.7	0.49	0.01	0.54(2)	10	0.52(4)	6
126.5	0.51	0.04	0.53(0)	4	0.51(2)	0.1
147.7	0.50	0.07	0.51(8)	3.6	0.50(4)	0.1
168.9	0.48	0.19	0.51(4)	6	0.49(8)	3.5
169.5	0.51	0.01	0.51(2)	0.3	0.49(7)	2.5

Table 2 The results for the astrophysical S-factor of ^{4}He(^{3}He,γ)^{7}Be reaction and estimated error in percentage in comparison with the experimental data (Nara Singh et al. 2004), 0.4–1 MeV

Energy (keV)	Exp. (keV b)	Exp. error (keV b)	S-Factor (two-body) (keV b)	Error (%)	S-Factor (two & three-body) (keV b)	Error (%)
420	0.42	0.01	0.43(6)	3.8	0.41(8)	0.4
506	0.38	0.01	0.41(7)	10	0.39(8)	2
605	0.37	0.01	0.39(3)	5.4	0.37(9)	1
614.5	0.36	0.01	0.39(0)	8.3	0.37(4)	3.8
624	0.35	0.01	0.38(6)	10	0.36(9)	5
950	0.32	0.01	0.36(5)	12	0.32(7)	2

Table 3 The results for the astrophysical S-factor of ^{4}He(^{3}He,γ)^{7}Be reaction and estimated error in percentage in comparison with the experimental data (Carmona-Gallardo 2012), at medium energies 1–2 MeV

Energy (MeV)	Exp. (keV b)	Exp. error (keV b)	S-Factor (two-body) (keV b)	Error (%)	S-Factor (two & three-body) (keV b)	Error (%)
1.5	0.31	0.02	0.34(8)	12	0.33(0)	6
2.0	0.30	0.02	0.38(1)	25	0.34(0)	13

Table 4 Comparison between different theoretical and experimental results for the $S(0)$-factor values of ^{4}He(^{3}He,γ)^{7}Be reaction, obtained from this work in comparison with a weighted average of the $S(0)$-factor values from the activation measurements (Brown et al. 2007; Bemmerer et al. 2007; Confortola et al. 2007; Nara Singh et al. 2004) and the prompt measurements (Alexander et al. 1984; Brown et al. 2007; Confortola et al. 2007). The two last rows show our two- and three-body interactions results

	Year	$S(0)$-Factor (keV b)	Method
Experiment			
Alexander et al.	1984	0.47 ± 0.04	Prompt γ-rays
Brown et al.	2007	0.59 ± 0.02	Prompt γ-rays
Confortola et al.	2007	0.56 ± 0.02	Prompt γ-rays
Brown et al.	2007	0.59 ± 0.01	^{7}Be activity
Bemmerer et al.	2007	0.63 ± 0.04	^{7}Be activity
Nara Singh et al.	2004	0.53 ± 0.02	^{7}Be activity
Theory			
Cyburt et al.	2008	0.58(0)	Analyzing of modern data
Neff	2011	0.59(3)	Fermionic Molecular Dynamics
This work (two-body)	2013	0.58(1)	Realistic two & three-cluster interactions
This work (two & three-body)	2013	0.56(3)	Realistic two & three-cluster interactions

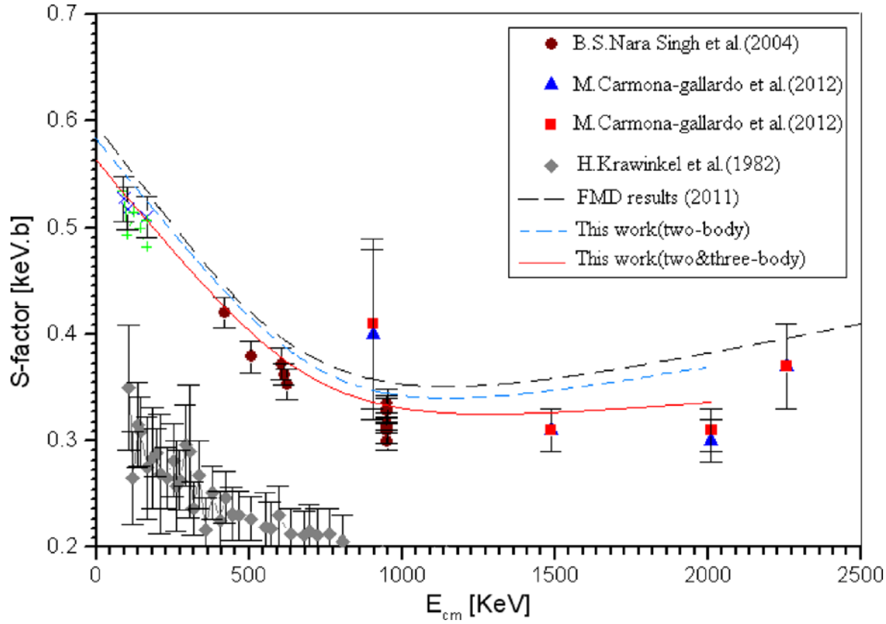

Fig. 1 The astrophysical S-factor for the ^{4}He(^{3}He,γ)^{7}Be reaction. The AV18 and AV18/UIX and the fermionic molecular dynamics results are given by the *short-dashed*, *solid* and *long-dashed lines*, respectively. Recent experimental data (Nara Singh et al. 2004; Carmona-Gallardo 2012; Krawinkel 1982) are shown as *dark colored symbols* and older data (Krawinkel 1982) as *light symbols*. The cross and plus signs are also the results of the LUNA Collaboration (Bemmerer et al. 2007; Confortola et al. 2007)

considering an infinitesimal gauge transformation as Marcucci et al. (2005), procedure in the two- and three-body current operator in the minimal-substitution scheme.

In this study, the destructive interference between one- and many-body currents occurs with the difference that there the leading components of the many-body currents (are model independent), and give a much larger contribution than that associated with the one-body current. The cancellation between the one and many-body matrix elements of transition amplitudes has the effect of enhancing the importance of P-wave capture channels. The contribution due to P-waves, the latter only considered the long-wavelength form of the multipole transition amplitudes, namely, their $q = 0$ limit. In P capture, only the multipole transition amplitudes survives in this limit and the corresponding $S(0)$-factor is calculated including two- and three-body contributions. The suppressed one-body contribution also comes mostly from transitions involving the D-state components of the ^{3}He and ^{4}He wave functions, while the many-body contributions are predominantly due to transitions connecting the S-state in ^{3}He to the D-state in ^{4}He, or vice versa.

4 Summary and conclusion

We have presented the results of three-body Faddeev-type calculations for systems of three-particles, two of which are charged, interacting through short-range nuclear plus the long-range Coulomb potentials. Realistic applications of two- and three-body currents and interactions to three-cluster nuclear reactions only became possible to address in recent years when a reliable and practical momentum-space treatment of the Coulomb interaction has been developed.

We focus on photonuclear process, and study a reaction, namely the ^{4}He(^{3}He,γ)^{7}Be reaction, that is the object of much interest as modern topics of nuclear astrophysics research. We have considered bound and scattering states starting from the realistic modern nucleon-nucleon Argonne v18(AV18) two- and Urbana IX three-body interactions, that reproduces the astrophysical S-factor.

The range of applications of this theoretical framework developed is then expanded to include nuclear radiative capture processes. The analysis of these processes in this framework requires the introduction of photonuclear processes involving two- and three-body currents and interactions, a development that has important application in the Nuclear Astrophysics field since many nuclear reactions important in Astrophysics involve photonuclear processes. We use the ^{4}He(^{3}He,γ)^{7}Be radiative capture process as a case study. In the present calculation, we employ the interactions so that those severe constraints are also successfully met in our two-, three-body subsystems. The zero energy S-factor is found to be $S(0) = 0.563$ (0.581) keV b, with (without) three-body interactions, in good agreement with available theoretical results and experiment data. This study allow the correct description of the available experimental data and different values for the astrophysical S-factor at very low-energies.

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