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SCATTERING OF ^{3}He AND ^{4}He FROM POLARIZED ^{3}He BETWEEN 4 AND 10 MeV

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Abstract: The polarization analyzing power of ^{3}He-^{3}He and ^{3}He-^{4}He elastic scattering is reported for bombarding energies from 4.33 to 9.83 MeV. A phase-shift analysis of existing ^{3}He-^{4}He elastic scattering data is presented with attention to the errors assigned to the phase shifts. In contrast to earlier work at low energies, the p-wave phase shifts may be parameterized by single-level formulas to give reasonable values of the nuclear reaction radius and of the ratio of the reduced width to the Wigner limit. The analyzing power of ^{3}He-^{3}He elastic scattering is consistent with zero at $\theta_{c.m.} = 60°$.

E | NUCLEAR REACTIONS $^3\vec{\mathrm{He}}(\alpha, \alpha)^3$He, E = 4.40-9.15 MeV; measured polarization analyzing power; deduced phase shifts. ^{7}Be deduced resonance parameters. $^3\vec{\mathrm{He}}(\tau, \tau)^3$He, E = 4.33–9.83 MeV; measured polarization analyzing power.

1. Introduction

In recent years a number of experiments involving ^{3}He-^{4}He and ^{3}He-^{3}He scattering below 18 MeV have been conducted [1-6]). Of these only three [1,4,6]) report polarization data in ^{3}He-^{4}He scattering. In view of the small amount of polarization data available, especially at low energies, the present experiment was planned to determine more precisely the ^{3}He-^{4}He scattering phase shifts and to look for any polarization effects in ^{3}He-^{3}He scattering, using, in both cases, a ^{3}He target polarized by the method of optical pumping.

Scattered particles were detected at the c.m. angles 71.6°, 87.0° and 120.0° for the ^{3}He-^{4}He elastic scattering, and 60.0° for the ^{3}He-^{3}He elastic scattering. Since most studies have been made using ^{3}He beams, the energy referred to in this paper concerning ^{3}He-^{4}He elastic scattering will be, unless otherwise stated, the equivalent ^{3}He lab energy E_3, defined as the ^{4}He lab bombarding energy on target multiplied by the ratio of the mass of ^{3}He to that of ^{4}He. The equivalent ^{3}He lab energy range was 3.30 to 6.86 MeV, which slightly overlaps that of ref. [5]). The ^{3}He bombarding energy range for the ^{3}He-^{3}He elastic scattering was 4.33 to 9.83 MeV.

Earlier phase-shift analyses [2,3]) of ^{3}He-^{4}He elastic scattering data agree reasonably well with the present analysis, and the splitting in the P-wave phase shifts is the same,

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namely $\delta_{\frac{1}{2}}^{-} > \delta_{\frac{3}{2}}^{-}$. In contrast with earlier work [2])† single-level parameterizations of the P-wave phase shifts using R-matrix theory [7]) have been performed and give reasonable values for the nuclear reaction radius and the ratio of the reduced width to the Wigner limit [8]).

2. Apparatus

The experimental apparatus was similar to that described by Baker *et al.* [9]) and Hardy *et al.* [10]). Scattering asymmetries were observed using scattering chambers built to observe left and right scattering at lab angles 30.0° and 45.0° with rms angular resolutions of 1.1° and 1.7°, respectively, calculated by taking into account multiple scattering in the entrance foil, beam collimation and scattered-particle collimation. The optically measured target polarization, which ranged from 0.09 to 0.15, was calculated as in ref. [9]). The parameter f, which appears in the optical determination of the ^{3}He polarization and which is defined in ref. [9]), was taken as 0.7. The analyzing power A_y was also computed using the equations in ref. [9]). No correction for background was made in the elastic scattering data because the background events always represented a negligibly small fraction of the elastic scattering events.

3. The ^{3}He-^{4}He scattering

3.1. ANALYZING POWER A_y

Table 1 gives the polarization analyzing power A_y in ^{3}He-^{4}He elastic scattering at corresponding values of E_3. The error includes counting statistics and estimates of

TABLE 1

Analyzing power A_y and errors (standard deviation) for ^{3}He-^{4}He elastic scattering

E_3 (MeV)	c.m. angle (deg.) 71.6 [a])	87.0 [b])	120.0 [a])
3.30	-0.04 ± 0.06	0.04 ± 0.03	0.15 ± 0.14
3.51	0.00 ± 0.02	0.04 ± 0.03	0.11 ± 0.05
3.88	-0.03 ± 0.02	0.04 ± 0.13	0.12 ± 0.05
4.37	0.09 ± 0.02	0.16 ± 0.06	0.03 ± 0.05
4.46	-0.06 ± 0.03	0.14 ± 0.05	0.04 ± 0.07
4.64	-0.22 ± 0.03	-0.26 ± 0.06	0.09 ± 0.06
4.79	0.25 ± 0.03	0.40 ± 0.06	0.08 ± 0.05
4.95	-0.48 ± 0.03	-0.58 ± 0.10	0.01 ± 0.04
5.09	-0.60 ± 0.04	-0.49 ± 0.02	0.08 ± 0.03
5.21	-0.58 ± 0.02	0.06 ± 0.05	0.02 ± 0.01
6.04	0.35 ± 0.04	0.90 ± 0.15	0.34 ± 0.12
6.45	0.38 ± 0.03	0.91 ± 0.08	-0.02 ± 0.10
6.86	0.22 ± 0.02	0.88 ± 0.12	0.01 ± 0.06

The equivalent ^{3}He lab energies E_3 are accurate to ± 20 keV. The errors in A_y do not include any contribution from possible systematic errors discussed in the text (subsect. 3.1).

[a]) Data at 71.6° and 120.0° are from ref. [18]).

[b]) Data at 87.0° are interpolated values based on the data of ref. [14]).

† To obtain numerical tables of the data in this paper, order document NAPS 01580 from ASIS National Auxiliary Publication Service, c/o CCM Information Corporation, 866 3rd Ave., New York, New York 10022; remitting \$ 2.00 for each microfiche or \$ 5.00 for each photocopy.

the precision with which the target polarization is measured. It does not include any contribution due to systematic errors in the measurement of the target polarization [9]. As a result all the experimental asymmetries given in table 1 may be multiplied by a single factor between 0.85 and 1.15.

3.2. PHASE-SHIFT ANALYSIS

To perform the phase-shift analysis a computer program was employed using the formulas of ref. [11]. The program was required to fit the data by minimizing χ^2:

$$\chi^2 = \sum_i [(D(\theta_i)_{\mathrm{exp}} - D(\theta_i)_{\mathrm{calc}})/\varepsilon(\theta_i)]^2,$$

where $D(\theta_i)_{\mathrm{exp}}$ is the measured value of the cross section or polarization at the c.m. angle θ_i, $D(\theta_i)_{\mathrm{calc}}$ is the calculated value at the same angle, and $\varepsilon(\theta_i)$ is the experimental error associated with $D(\theta_i)_{\mathrm{exp}}$.

The data used in the phase-shift analysis consisted of the cross-section data of Barnard *et al.* [2] ($E_3 \leqq 5.21$ MeV), the cross-section data of Spiger and Tombrello [3] ($E_3 > 5.21$ MeV), the polarization data at $\theta_{\mathrm{c.m.}} = 71.6°$, $87.0°$ and $120.0°$ from the present experiment, and the polarization data of Hardy *et al.* [5] at $\theta_{\mathrm{c.m.}} = 114.0°$. Linear interpolations of the data of refs. [2,3,5] were necessary to obtain data at the values of E_3 shown in table 1. Any error introduced by this process should be small compared to the quoted experimental errors.

The phase shifts for $l \geqq 4$ were set to zero because even at higher energies there is no indication that these depart significantly from zero [3]. The D-waves were also fixed at zero because they too are close to zero at higher energies [3,5] and there are no known D-states in the ^{7}Be system [12]. (We discuss the D-waves further later.) The energy range of the experiment was below the p-^{6}Li threshold, and thus the five parameters in the search were the real S-, D- and F-wave phase shifts.

The method used in determining the phase shifts was to hold one phase shift constant and allow the program to vary the other four. When the minimum value of χ^2 had been found for this set of values of the one fixed and four variable phase shifts, the fixed phase shift was incremented by 0.1° and the procedure repeated. From the set of χ^2 values and corresponding phase shifts generated in this way the most probable value δ_0 of the phase shift corresponding to the smallest value of χ^2, namely χ_0^2, was found. An illustration of this procedure is provided by the solid curve in fig. 1.

The phase-shift error was assigned in the following manner. The relative likelihood L of a given fit to the experimental data can be estimated by [13] $L = \exp(-\frac{1}{2}\chi^2)$, and for the best fit is $L_0 = \exp(-\frac{1}{2}\chi_0^2)$. The error $\Delta\delta$ in the phase shift is taken to be the average of the upward and downward displacements from δ_0 which reduce the likelihood function to a value $L_1 = L_0 \exp(-\frac{1}{2})$, corresponding to a change in δ of one standard deviation if the likelihood function is Gaussian. Thus $L_1 = \exp(-\frac{1}{2}\chi_1^2)$, where $\chi_1^2 = \chi_0^2 + 1$. The phase-shift error is then obtained from

$$\Delta\delta = \tfrac{1}{2}|\delta_{\mathrm{U}} - \delta_{\mathrm{L}}|,$$

where δ_U and δ_L are the upper and lower values, respectively, of the phase shift corresponding to χ_1^2. Plots such as fig. 1 were then used to determine $\Delta\delta$.

It should be noted that the phase-shift error determined by the foregoing method does not include the effects of possible systematic errors in the data. Such effects are discussed later in this section.

The starting values of the phase shifts for the search procedure were taken from refs. [2,3]). This choice of initial phase-shift values may tend to bias somewhat the final results because these phase shifts already fit the cross-section data and are therefore probably at or near a minimum in χ^2. In an attempt to locate other minima in χ^2 the range of each of the fixed phase shifts was extended to at least the values $\delta_0 \pm 15°$. In the majority of the cases χ^2 continued to increase as the phase shift in question was moved away from δ_0. In the few cases where this was not so, there was usually only one other minimum, and in no case were there more than two others. The value of L at these other minima was ordinarily much smaller than L_0; moreover, these extra minima occurred only at isolated values of E_3 and did not represent a trend with energy.

TABLE 2
Phase shifts (deg.) in ^{3}He-^{4}He elastic scattering

E_3 (MeV)	$\delta_{\frac{1}{2}}^{+}$	$\delta_{\frac{1}{2}}^{-}$	$\delta_{\frac{3}{2}}^{-}$	$\delta_{\frac{5}{2}}^{-}$	$\delta_{\frac{7}{2}}^{-}$
3.30	-23 ± 2	162 ± 3	165 ± 1	0 ± 1	2 ± 1
3.51	-24 ± 1	161 ± 1	164 ± 1	2 ± 1	2 ± 1
3.88	-27 ± 1	159 ± 2	162 ± 1	2 ± 1	3 ± 1
4.37	-30 ± 1	156 ± 2	158 ± 1	3 ± 1	7 ± 1
4.46	-31 ± 1	156 ± 2	158 ± 1	5 ± 1	7 ± 1
4.64	33 ± 2	153 ± 2	158 ± 1	5 ± 1	10 ± 1
4.79	34 ± 1	151 ± 2	157 ± 2	5 ± 1	15 ± 1
4.95	-40 ± 6	150 ± 3	154 ± 2	3 ± 2	23 ± 4
5.09	-49 ± 7	146 ± 5	153 ± 3	4 ± 1	40 ± 4
5.21	-38 ± 5	149 ± 3	153 ± 2	-3 ± 3	78 ± 4
6.04	-50 ± 7	137 ± 5	142 ± 2	11 ± 4	165 ± 3
6.46	-49 ± 5	139 ± 3	140 ± 2	15 ± 3	169 ± 2
6.86	-48 ± 9	139 ± 4	140 ± 4	17 ± 4	174 ± 3

The equivalent ^{3}He lab energy E_3 is accurate to ± 20 keV. The errors do not include any contribution from systematic errors discussed in the text (subsect. 3.2).

The results of the phase-shift searches are given in table 2. The P-wave phase shifts are plotted in fig. 2. The sign of the splitting is the same as that reported in earlier studies [2,3]). Various starting values were used for the P-wave searches, including both sets in which $\delta_{\frac{1}{2}}^-$ was larger and smaller than $\delta_{\frac{3}{2}}^-$. In all cases the search procedure resulted in the values reported. This is an improvement over the results of Barnard *et al.* [2]), who reported finding two sets of phase shifts, one with $\delta_{\frac{1}{2}}^- < \delta_{\frac{3}{2}}^-$ and one with $\delta_{\frac{1}{2}}^- > \delta_{\frac{3}{2}}^-$, both of which fit the cross-section data equally well except at the resonance around $E_3 = 5.2$ MeV.

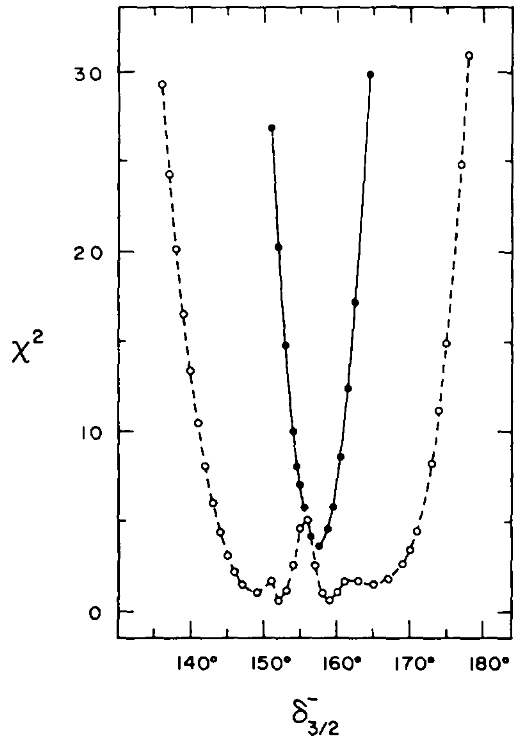

Fig. 1. The value of χ^2 *versus* $\delta_{\frac{3}{2}}^-$ at $E_3 = 4.64$ MeV. All other phase shifts were varied to obtain the lowest value of χ^2. The values represented by solid circles were generated using both cross-section and polarization data, and the open circles by cross-section data only. The curves serve only to connect the points. It is seen that in this case the polarization data eliminate the multiple minima that occur with cross-section data alone.

When the search program was allowed to vary the D-wave phase shifts the values obtained at the various energies were scattered with no discernible trend in a band between $-4°$ and $+4°$. It was felt that this reason, along with the previous discussion concerning the D-waves, gave sufficient justification for holding the D-waves constant at zero in all other searches. The phase shifts for $l \geqq 4$ were assumed to be zero. The phase-shift errors in table 3 do not include any effects due to possible systematic errors in either the polarization data A_y or the cross-section data. However, as discussed next, the effects of such errors on the phase shifts would be relatively minor.

The polarization data could systematically be in error by as much as 15 %, as discussed in subsect. 3.1. Multiplying A_y by 0.85 (corresponding to $f = 0.5$) and 1.1 (corresponding to $f = 0.9$) and repeating the phase-shift searches for both cases had a negligible effect on the phase shifts, usually a few tenths of a degree, and the errors were not affected.

The cross-section data reported in ref. [2]) are about 10 % smaller than those reported in ref. [3]). Multiplying the cross-section data of ref. [2]) by 1.1 and repeating the phase-shift search procedure in the energy range where those data were used resulted in an average correction of $-3°$ in the S- and P-wave phase shifts and an average correction of $+1°$ in the F-wave phase shifts. The phase-shift errors were not affected.

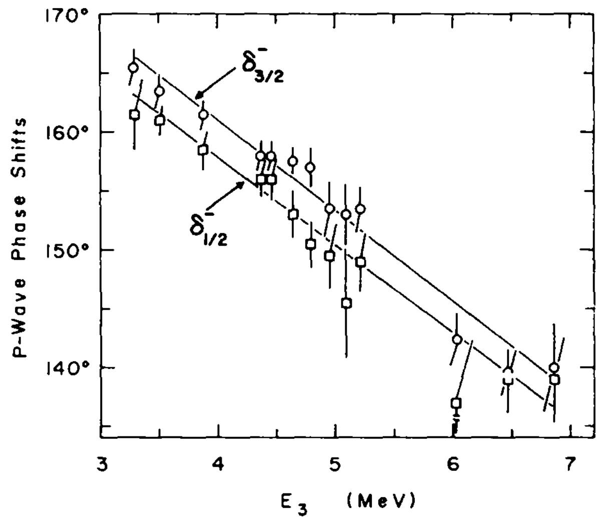

Fig. 2. P-wave phase shifts *versus* the equivalent ^{3}He lab energy E_3 for ^{3}He-^{4}He elastic scattering. Squares represent values of $\delta_{\frac{1}{2}}^-$ and circles represent values of $\delta_{\frac{3}{2}}^-$. The upper and lower lines indicate single-level fits to $\delta_{\frac{3}{2}}^-$ and $\delta_{\frac{1}{2}}^-$, respectively, over the entire energy range shown.

TABLE 3

Single-level parameters for the $\frac{1}{2}^-$ and $\frac{3}{2}^-$ states in ^{7}Be

State	J^π	E_R (MeV)	r (fm)	γ^2 (MeV)	γ^2/γ_0^2
ground	$\frac{3}{2}^-$	-1.587	$3.1^{+0.4}_{-0.3}$	$1.18^{-0.52}_{+0.43}$	$0.31^{-0.09}_{+0.04}$
first excited	$\frac{1}{2}^-$	-1.155	$3.5^{+0.5}_{-0.4}$	$0.87^{-0.57}_{+0.46}$	$0.29^{-0.16}_{+0.07}$

The values of the resonance energy E_R are from ref. [11]). The errors for the radius r, the reduced width γ^2 and the ratio of the reduced width to the Wigner limit γ^2/γ_0^2 do not include any contribution from systematic errors. See subsect. 3.3 for further discussion.

Because other authors have employed only cross-section data in their phase-shift analyses it seemed instructive to determine the effect of the use of the polarization data on the phase-shift searches. Fig. 1 is a plot of the values of minimum χ^2 *versus* the P-wave phase shift $\delta_{\frac{1}{2}}^-$ for $E_3 = 4.64$ MeV. The solid curve is the result of using both cross-section and polarization data while the dashed curve is the result of using cross-section data alone. The solid curve, whose shape was used to determine $\Delta\delta_{\frac{1}{2}}^-$, has a single minimum. In contrast to this the dashed curve has several minima. This is presumably the reason that Barnard *et al.* [2]) found two sets of P-wave phase shifts which fit their cross section data. In a few of the phase-shift searches the plot of χ^2 had a single minimum when only cross-section data were used. However, in such cases the phase-shift error was larger by a factor of from 1.2 to 4 compared with the error determined when polarization data were included in the analysis.

From a set of smoothed phase shifts (subsect. 3.3) which have been extrapolated to lower energies, the polarization analyzing power of ^{3}He-^{4}He elastic scattering has been calculated. The polarization is plotted in fig. 3 as a function of c.m. angle and equivalent ^{3}He lab energy. Other such maps are given in refs. [2-4]).

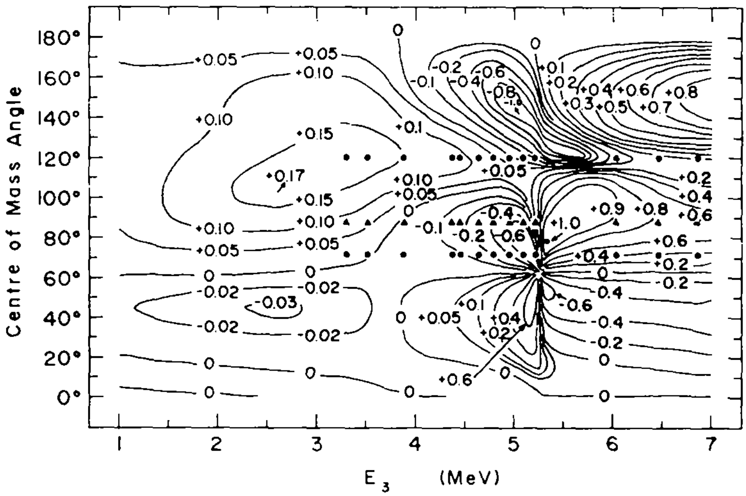

Fig. 3. Polarization analyzing power in ^{3}He-^{4}He elastic scattering *versus* equivalent ^{3}He lab energy E_3 and the c.m. scattering angle. The method for obtaining the phase shifts from which this map was obtained is described in the text. The numbers not labeling a contour are accompanied by arrows to indicate the location of the point at which that value is found. Solid circles and triangles show the locations of the data in table 1.

3.3. PARAMETERIZATION OF THE PHASE SHIFTS

A single-level parameterization † of the S- and P-wave phase shifts in table 2 was made using the R-matrix formalism of Lane and Thomas [15]) for the phase shifts and the formulas of Jackson and Blatt [16]) and Buck *et al.* [17]) for the Coulomb functions. A least-squares technique was used to fit the phase shifts.

The S-waves are adequately fit by assuming scattering by a hard sphere, and the best fit was obtained using a radius of 2.8 fm. The S-wave phase shifts used to construct the polarizations map (fig. 3) were assumed to be hard-sphere phase shifts for a radius of 2.8 fm.

† A computer program developed by Dr. E. V. Hungerford of Rice University was used in this analysis.

The P-wave phase shifts in the range $3.30 \leqq E_3 \leqq 4.95$ were fit by varying the level parameters r (reaction radius) and γ^2 (reduced width). The values of the resonance energies E_R were taken to be those of the $\frac{1}{2}^-$ and $\frac{3}{2}^-$ levels in ^{7}Be below the ^{3}He-^{4}He threshold [12]). The procedure for determining the values of r and γ^2 and their errors was much the same as that previously described in subsect. 3.2 for obtaining the phase shifts from the scattering data. Table 3 lists the values obtained for r and γ^2, as well as the ratio of γ^2 to the Wigner limit [8]) $\gamma_0^2 = 3\hbar^2/2\mu r^2$. It is seen that the ^{3}He-^{4}He strengths in these first two states in ^{7}Be are substantial and are approximately equal. The errors quoted in table 3 indicate the values of the parameters (r and γ^2) which give values of the likelihood function which are down by a factor $\exp(-\frac{1}{2})$ from the highest values (obtained at the best fit). They do not include effects due to any systematic errors in the phase shifts. The values of the parameters r and γ^2 are correlated, and in table 3 the positive deviation in r corresponds to the negative deviation in γ^2, and vice versa.

As noted above, the level parameters given in table 3 were computed using the phase shifts corresponding to equivalent ^{3}He lab energies between 3.30 and 4.95 MeV only. The fit to the $\frac{3}{2}^-$ phase shifts was somewhat worse if the phase shifts over the range 3.30 to 6.86 MeV were used, although the values of the level parameters were not changed by more than the errors assigned to the parameters. A possible explanation for this could be that the $\frac{3}{2}^-$ states in ^{7}Be at 9.9 and 10.8 MeV are contributing significantly to the phase shifts between 5 and 7 MeV so that the single-level formula becomes inadequate. The P-wave phase shifts used to construct the polarization map in fig. 3 were obtained from the single-level fitting procedure described above, using the best fit in each case to the data at all energies. These fits are indicated by the lines in fig. 2.

TABLE 4

Polarization analyzing power in ^{3}He-^{3}He elastic scattering at $\theta_{c.m.} = 60°$

E_3 (MeV)	A_y
4.33	-0.009 ± 0.015
4.89	-0.005 ± 0.012
5.48	-0.006 ± 0.012
6.01	-0.017 ± 0.009
6.59	-0.028 ± 0.021
7.11	-0.005 ± 0.010
7.67	0.015 ± 0.022
8.21	-0.002 ± 0.010
8.77	-0.072 ± 0.025
9.25	-0.026 ± 0.014
9.79	0.024 ± 0.011
9.83	-0.077 ± 0.025

Analyzing power A_y and errors (standard deviation) for ^{3}He-^{3}He elastic scattering are presented. The ^{3}He bombarding energies E_3 are accurate to ± 20 keV. As in table 1, the errors in A_y do not include any contribution from the systematic errors discussed in the text (see subsect. 3.1).

It was not possible to obtain a satisfactory fit to the low-energy F-wave phase shifts using a single-level formula; a reasonable fit to the $\frac{5}{2}^-$ resonance at $E_3 = 5.2$ MeV results in phase shifts which are too small below the resonance. For purposes of constructing the polarization map, the F-wave phase shifts were represented by a combination of polynomial and spline fits.

4. The ^{3}He-^{3}He scattering

The polarization analyzing power A_y in ^{3}He-^{3}He elastic scattering was measured at $\theta_{\mathrm{c.m.}} = 60.0°$ for ^{3}He bombarding energies from 4.33 to 9.83 MeV. The results are given in table 4 and are consistent with zero, as are those reported in ref. [5]) for higher energies (9.3–17.5 MeV). Such data are consistent with a description of the scattering which leaves unsplit the P-wave and F-wave phase shifts.

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