

## POLARIZATION AND PHASE SHIFTS IN ${}^6\text{Li}(p, p){}^6\text{Li}$ FROM 0.5 TO 5.6 MeV

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Received 7 January 1969

**Abstract:** Angular distributions of polarization in  ${}^6\text{Li}(p, p){}^6\text{Li}$  have been measured to an accuracy of about 0.01 at 11 energies from 1.2 to 3.2 MeV with a target of 20 keV thickness at 2 MeV. These data and cross section measurements of others are reproduced well by a set of phase shifts, which vary reasonably with energy and which are complex because a reaction channel is open. The  ${}^2\text{S}_-$ ,  ${}^4\text{S}_-$  and  ${}^4\text{P}_-$  phases suffice for the analysis. Neither  ${}^2\text{P}$ -phases nor mixing parameters are found to be important. The  ${}^4\text{P}_{\frac{3}{2}}$  phase resonates at 1.8 MeV as expected. The broad resonance at 5 MeV can be reproduced equally well with either the  ${}^4\text{P}_{\frac{3}{2}}$  or the  ${}^4\text{P}_{\frac{1}{2}}$  phase resonating, the former agreeing with a previous spin assignment. Measurements of cross section, proton polarization and  ${}^6\text{Li}$  vector polarization cannot distinguish between the two sets, but measurements of  ${}^6\text{Li}$  tensor polarization could. The uniqueness of the phases is discussed and a polarization contour map is presented.

E

NUCLEAR REACTION  ${}^6\text{Li}$  (polarized p, p),  $E = 1.2$  to 3.2 MeV; measured proton polarization; deduced phase shifts for  $E = 0.5$  to 5.6 MeV. Enriched target.

### 1. Introduction

Recently two of us reported an experimental study of the  ${}^6\text{Li}(p, {}^3\text{He}){}^4\text{He}$  reaction induced with polarized protons<sup>1</sup>). In that experiment data on the elastic scattering of polarized protons on  ${}^6\text{Li}$  were recorded simultaneously, but the data were neither analysed nor published. The equations for describing the elastic scattering of spin  $-\frac{1}{2}$  particles by spin  $-1$  particles were set forth some years ago by various authors<sup>2-4</sup>); Seyler<sup>5</sup>) corrected and extended this early work, and his equations were the basis of a phase shift analysis of proton deuteron scattering<sup>6</sup>). At about the same time and independently Arvieux<sup>7</sup>) derived the equations for the restricted but physically useful case that has no mixing between the quadruplet and doublet states. Seyler's equations, when simplified to the same assumption about mixing, agree with those of Arvieux.

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The experimental basis for our present knowledge of the structure of  ${}^7\text{Be}$  in the energy range up to 5 MeV proton lab energy is given in the paper <sup>1)</sup> on the  ${}^6\text{Li}(p, {}^3\text{He}){}^4\text{He}$  reaction. Bashkin and Richards <sup>8)</sup> first measured the elastic scattering cross section for  ${}^6\text{Li}(p, p){}^6\text{Li}$  from 1.1 to 3.1 MeV but only at lab angle  $164^\circ$ . More than a decade later experimental techniques were improved to the extent that the cross section could be measured over a wide range of scattering angle <sup>9-11)</sup>.

McCray <sup>9)</sup> showed in a simplified phase shift analysis that the resonance at 1.8 MeV, corresponding to the 7.2 MeV level of  ${}^7\text{Be}$ , is a  ${}^4P_{\frac{3}{2}}$  state. Measurements of elastic scattering cross section are unable to distinguish between  $\frac{3}{2}^-$  and  $\frac{1}{2}^-$  as the spin and parity of the 5 MeV resonance, but Harrison <sup>12)</sup> demonstrated with inelastic scattering that it is  $\frac{3}{2}^-$ . We attempted a general phase shift analysis, in spite of the large number of parameters possible, because we had an extensive amount of polarization data.

## 2. Calculations

We have modified to include inelastic channels a computer program based on the equations of Seyler <sup>5)</sup>, which was used for  $D(p, p)D$  analysis <sup>6)</sup>. It calculates various

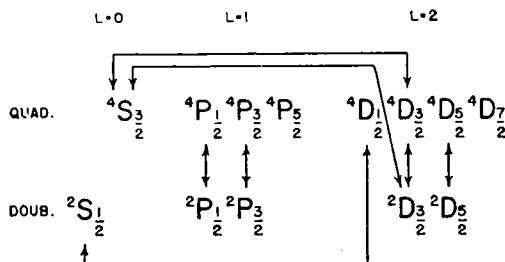


Fig. 1. Parameters describing the states for the elastic scattering of protons by  ${}^6\text{Li}$ . For orbital angular momentum  $l_{\text{max}} = 2$  there are 13 phase shifts associated with the 13 states. Since inelastic channels are open the phase shifts are complex. The seven arrows represent mixing parameters coupling states of the same  $J\pi$ . If mixing parameters are real, a total of 33 parameters are needed to describe  ${}^6\text{Li}(p, p){}^6\text{Li}$  for S-, P- and D-waves.

measurable quantities from phase shifts and mixing parameters: differential elastic cross section, proton polarization, total reaction cross section, Wolfenstein parameters and  ${}^6\text{Li}$  vector and tensor polarization. At present the program has no provision for calculating partial waves of orbital angular momentum greater than  $l = 2$ . Fig. 1 gives the states for which there are phase shifts and indicates with arrows the possible mixing between these states. There are 13 phase shifts, which are complex, and 7 mixing parameters, making a total of 33 parameters, if the mixing parameters are real. This large number of parameters is the great problem of the analysis, and we shall devote considerable space to a recount of our efforts in determining the importance, more correctly the unimportance, of many of these parameters.

We have elected to express the absorption of a partial wave by a parameter  $X$ , defined by Spiger and Tombrello<sup>13)</sup> as

$$\cos^2 X = \exp(-2\delta_{\text{im}}),$$

where  $\delta_{\text{im}}$  is the imaginary component of the complex phase shift. Thus  $X$  has a range of values between 0 and  $90^\circ$ , corresponding to no absorption and complete absorption respectively. We shall present evidence later that mixing can be neglected. With that assumption the total cross section for all inelastic processes is

$$\sigma_{\text{t}} = -\frac{\pi}{6k^2} \sum_{jls} (2j+1)(1 - \cos^4 X_{ls}^j),$$

where the superscript of  $X$  is the total angular momentum quantum number  $j$ , and the subscripts are the orbital angular momentum quantum number  $l$  and the channel spin  $s$ .

### 3. Data

Fig. 2 shows the measured datum points for the polarization analysing efficiency in  ${}^6\text{Li}(\text{p}, \text{p}){}^6\text{Li}$  and curves fitted to them by this analysis. The Basel sign convention applies. A table of the numerical values of the data has been deposited with the National Auxiliary Publications Service<sup>†</sup>. The uncertainty of these data is about 0.01, a value larger than the statistical accuracy of the experiment. Experimental uncertainties arise from geometrical effects resulting from the finite size of beam and target. These are estimated to be 0.0045 from measurements of polarization in proton-proton scattering, which were made with a similar experimental arrangement<sup>14)</sup> and which are in fact measurements of the geometrical errors, since the proton-proton polarization is very small at 3 MeV. The tabulated experimental error is the quadratic sum of the statistical and geometrical uncertainties.

The measurements of differential cross section were made by McCray<sup>9)</sup>, Fasoli *et al.*<sup>10)</sup> and Harrison and Whitehead<sup>11)</sup>. The total reaction cross section is needed for a fit of the absorption parameters. In the energy range of interest here only  ${}^6\text{Li}(\text{p}, {}^3\text{He}){}^4\text{He}$  need be considered; we use the measurements of four groups<sup>15-18)</sup>. It is a difficult experimental problem to determine the thickness of a lithium target, and for this reason the absolute values of all cross-section measurements have large errors even though the relative errors are small. Spiger and Tombrello<sup>18)</sup> determined an absolute normalization factor for cross section by observing the inverse reaction  ${}^4\text{He}({}^3\text{He}, \text{p}){}^6\text{Li}$ , which gets around the lithium difficulty because the target is gaseous. Harrison<sup>12)</sup> prepared a table for normalizing all cross-section data. We were unable to improve significantly the quality of fit by altering Spiger and Tombrello's normalization constant.

<sup>†</sup> For a copy of this table order Document NAPS-00325 from the National Auxiliary Publications Service of the American Society for Information Sciences, c/o CCM Information Sciences, Inc., 22 West 34th Street, New York, N.Y. 10001, remitting \$ 1.00 for microfiche or \$ 3.00 for photocopies.

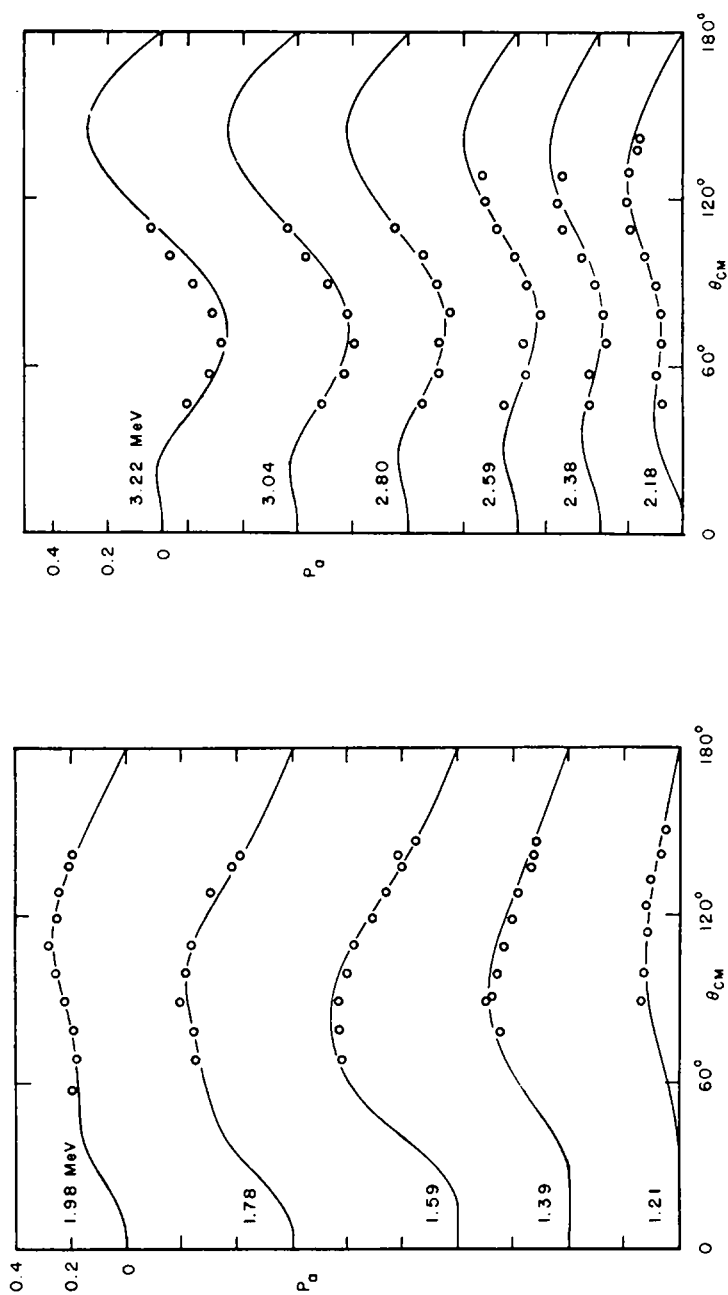


Fig. 2. Angular distributions of the polarization efficiency of  ${}^6\text{Li}(p, p){}^6\text{Li}$  plotted as functions of c.m. scattering angle. The vertical scale is the same for all drawings and can be ascertained at the upper left margin of each drawing. The radius of the circles is approximately the experimental error. The curves are calculated from the absorption parameters of fig. 3 and the smooth curves drawn through the points of the phase shifts in fig. 4.

#### 4. Determination of the phase shifts

The large number of parameters that describe elastic scattering here requires us to describe in detail how we arrive at the final set of phase shifts and especially how we are able to ignore so many. The method for determining the phase shifts is the gradient search, which locates minima of the function

$$\varepsilon^2 = \frac{1}{n+m} \left[ \sum_{i=1}^n \left( \frac{P_{\text{cal}}(\theta_i) - P_{\text{exp}}(\theta_i)}{\Delta P} \right)^2 + \sum_{j=1}^m \left( \frac{\sigma_{\text{cal}}(\theta_j) - \sigma_{\text{exp}}(\theta_j)}{\Delta \sigma} \right)^2 \right],$$

where  $P(\theta)$  and  $\sigma(\theta)$  are the polarization and differential cross section,  $\Delta P$  and  $\Delta \sigma$  their respective experimental uncertainties, and the subscripts cal and exp designate values calculated from the phases being tried and values from the experimental data. We also made searches in which the total reaction cross section was included in  $\varepsilon^2$ . We found, however, that the differential cross section and the polarization are not very sensitive to the absorption parameters, so we elected to set their values by hand calculation of the total cross section rather than allow the computer to search for a minimum with them. Their inclusion shifts the values of the phases slightly and improves the quality of fit.

##### 4.1. THREE PHASE FIT

The dominant structure of  ${}^7\text{Be}$  in this energy range is the resonance at 1.8 MeV, which has been identified previously as  $\frac{5}{2}^-$ . Thus for starting values at 1.8 MeV we set the  ${}^4P_{\frac{3}{2}}$  phase equal to  $90^\circ$  and the two S-phases equal to zero. Even though no other parameters were freed the search program quickly produced a good value of  $\varepsilon^2$ . We were unable to fit the data with any other single P-phase resonating. The three phase fit could be extended up to 2 MeV and down to 0.5 MeV with smooth energy variation and good quality of fit; below 1.2 MeV only cross-section data are available. Searches were made near the resonance from 30 sets of small starting phases; two sets were found with good quality of fit for 1.0 and 1.2 MeV but they could be extended to higher energies only by abrupt changes, and therefore, were discounted.

##### 4.2. FIVE PHASE FIT

The three phase fit is unsatisfactory above 2 MeV. Releasing the two remaining quadruplet P-phases, the  ${}^4P_{\frac{1}{2}}$  and the  ${}^4P_{\frac{3}{2}}$  phase, suffices to give good fits up to 3.2 MeV, where the polarization data end. The  ${}^4P_{\frac{1}{2}}$  and the  ${}^4P_{\frac{3}{2}}$  phases are indistinguishable, in that one phase can be increased a few degrees without worsening the quality of fit so long as the other phase is correspondingly decreased; such variations have little effect on the other three phases. We tried unsuccessfully to find a set of phases for which the absolute value of the  ${}^4S_{\frac{1}{2}}$  phase is greater than the  ${}^2S_{\frac{1}{2}}$ .

##### 4.3. FITS WITH ALL S- AND P-PHASES AND MIXING PARAMETERS FREE

Releasing the doublet P-phases, starting either at zero or at small positive or negative values, had very little effect. The same was true if we released the parameters

that mix quadruplet and doublet P-states, the only mixing possible, if D- and higher partial waves are excluded. In order to ease somewhat our worries about other sets of phases that could satisfy the data, we undertook 40 searches at 1.8 MeV with random starting values for all seven variables; since we are reasonably certain that the  ${}^4P_{\frac{1}{2}}$  phase resonates at that energy, we took  $90^\circ$  as its starting value for half of the searches. The 40 searches produced 24 sets of parameters for which  $\varepsilon^2$  was 5 or less. These 24 sets were then used as starting values for energies just above and below the resonance. This procedure eliminated all but four sets. The four remaining sets all show more than one P-phase large at low energies, which is scarcely possible unless more than one state is located there.

#### 4.4. INCLUSION OF ABSORPTION

At the beginning of our analysis we had noted that the absorption parameters are not sensitive to the differential cross section and polarization data, hence they were taken up rather late in the study. At that point we decided that there was sufficient evidence for us to set the doublet P-phases and the mixing parameters to zero. We also found it more satisfactory to calculate the total reaction cross section by hand in terms of the  $X$ -parameters. Beaumevielle *et al.* <sup>16)</sup> divided the total reaction cross section into three parts, attributable to absorption of the  ${}^4S_{\frac{1}{2}}$ ,  ${}^4P_{\frac{1}{2}}$  and  ${}^4P_{\frac{3}{2}}$  waves. Since the five phase fit shows the  ${}^2S_{\frac{1}{2}}$  wave to be important and since there seems to be little evidence for a  $\frac{3}{2}^+$  state at 0.8 MeV, which figured in the work of Beaumevielle, we set the absorption parameters for the two S-waves equal, accepted Beaumevielle's value for the  $\frac{3}{2}^-$  resonance, using the more recent normalization factor, of course, and assumed the  ${}^4P_{\frac{3}{2}}$  wave to be the important one near 4 and 5 MeV. This procedure is arbitrary, but there is little in the data to guide us. Inclusion of these fixed values for the  $X$ -parameters in searches with five phases improved the quality of fit but caused only minor variation in the phase shifts. A few attempts were made using a combination of  $X$ -parameters different from that just described but which gives the measured reaction cross section; this had little effect on the phase shifts. The absorption parameters that we used are shown in fig. 3.

#### 4.5. EXTENSION OF ANALYSIS TO ENERGIES WHERE NO POLARIZATION DATA EXIST

The evidence that five phase shifts can explain the data from 1.2 to 3.2 MeV encouraged us to extend the analysis to energies where only differential cross-section measurements exist. Harrison <sup>12)</sup> had identified the state at 5 MeV to be  $\frac{3}{2}^-$ , so we influenced the starting values of that phase shift accordingly. The results of these searches are plotted in fig. 4. The points are for individual fits and are included with the polarization data in the document stored with the Natl. Aux. Pub. Ser. The smooth curves are drawn by eye. No attempt to find other sets of phases that satisfy cross-section data alone was made. Without doubt there are many. Of particular note: the interrelated behavior of the  ${}^4P_{\frac{1}{2}}$  and the  ${}^4P_{\frac{3}{2}}$  phases, already noted in sub-

sect. 4.2, persists over the entire energy range. The two separate curves shown in fig. 4 give the best fits, but their minima are not sharp. If the plotted values of the  ${}^4P_{\frac{1}{2}}$  and  ${}^4P_{\frac{3}{2}}$  phases are interchanged, i.e., if we change the designation of the plots, and leave the other phases unaltered, the differential cross section and proton polarization are so nearly the same that experiments of present day accuracy could scarcely distinguish between them. We designate as Set I the phase shifts plotted in fig. 4. Set II is obtained from Set I by interchanging the  ${}^4P_{\frac{1}{2}}$  and  ${}^4P_{\frac{3}{2}}$  phases.

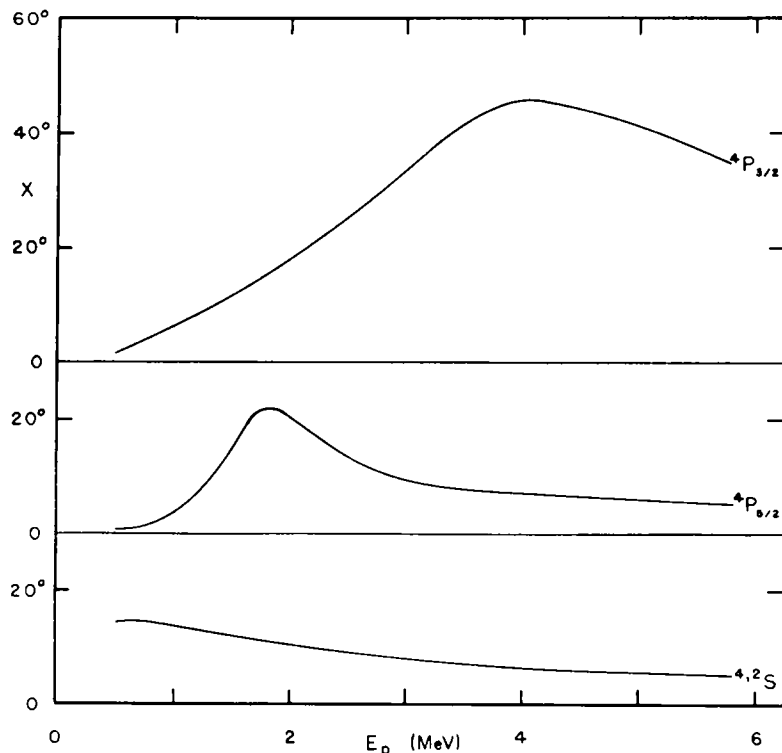


Fig. 3. Absorption parameters as functions of energy. The parameters are not very sensitive to the differential cross section or the polarization data. They were set arbitrarily: the parameters of the two S-waves are equal, and only the parameters for the two resonating P-waves are included. The absorption parameters plotted here give the measured total reaction cross section. Altering their relative strengths has little effect on the phase shifts.

#### 4.6. D-PHASES

The D-phases contribute little to the quality of fit below 4 MeV. When released in a search they scatter about zero. Searches were made with them either locked together or independent of one another with little effect on the result. Above 4 MeV their inclusion does improve the fit. They are negative by a few degrees. They are too inaccurate for inclusion in the plot.

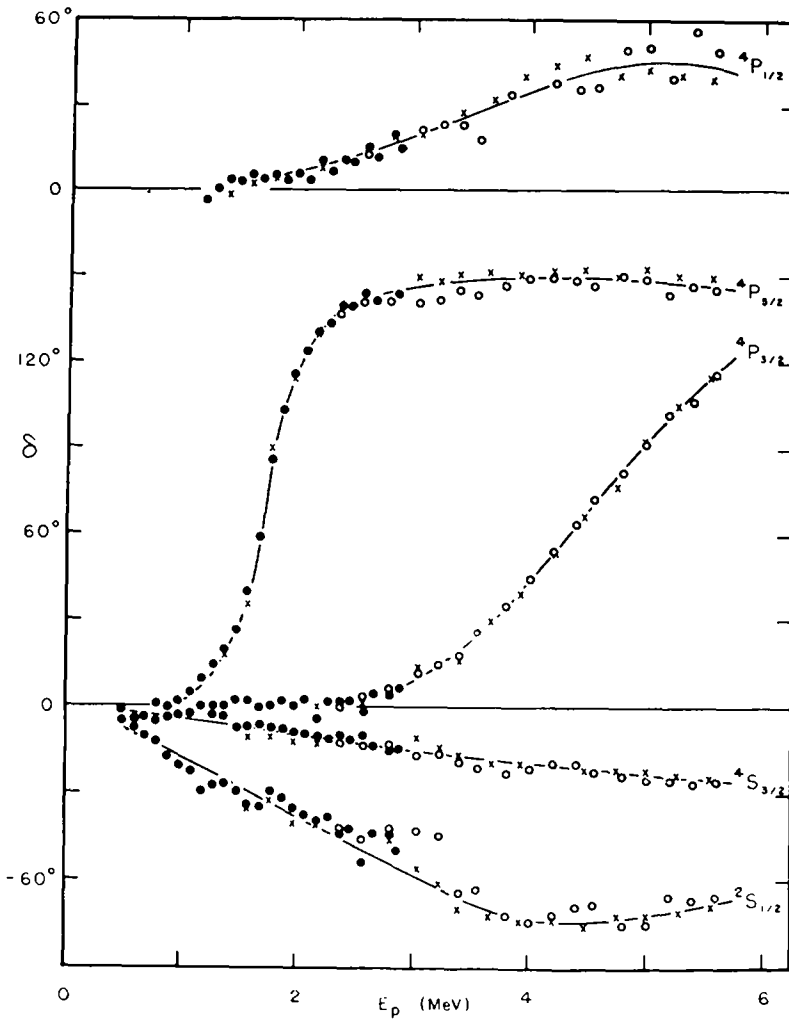


Fig. 4. Phase shifts as functions of energy. The points show the phase shifts that were obtained as the best fit to the experimental data. Angular distributions of polarization efficiency were included with the differential elastic cross section at the energies given in fig. 2. The source of the cross-section data is denoted by the shape of the points: closed circles are ref. <sup>9)</sup>, crosses are ref. <sup>10)</sup> and open circles are ref. <sup>11)</sup>. The smooth curves were drawn by eye. A second, equally good set of phase shifts results, if one interchanges the  ${}^4P_{3/2}$  and the  ${}^4P_{1/2}$  phases. The second set is eliminated by a previous identification <sup>12)</sup> of the state near 5 MeV as  $\frac{3}{2}^-$ .

### 5. Tensor moments and Wolfenstein parameters

Our efforts to locate other sets of phases and determine whether or not they are physically significant make us aware of the difficulty of ascertaining the uniqueness of the results. We offer only two energy dependent sets, one of which is excluded by Harrison's inelastic scattering experiments. Nevertheless, we wish to note here how



other measurements of elastic scattering can distinguish Set I from Set II and probably from any other set, as yet unknown. Measurements of Wolfenstein parameters in  ${}^6\text{Li}(p, p){}^6\text{Li}$  appears at the present to be a very difficult experiment indeed. Furthermore, the two sets of phases do not give dramatically different results for them. On the other hand sources of polarized lithium exist <sup>19,20</sup>, and such a source allows the examination of  $\text{H}({}^6\text{Li}, {}^6\text{Li})\text{H}$ . Our calculations show that the two sets would

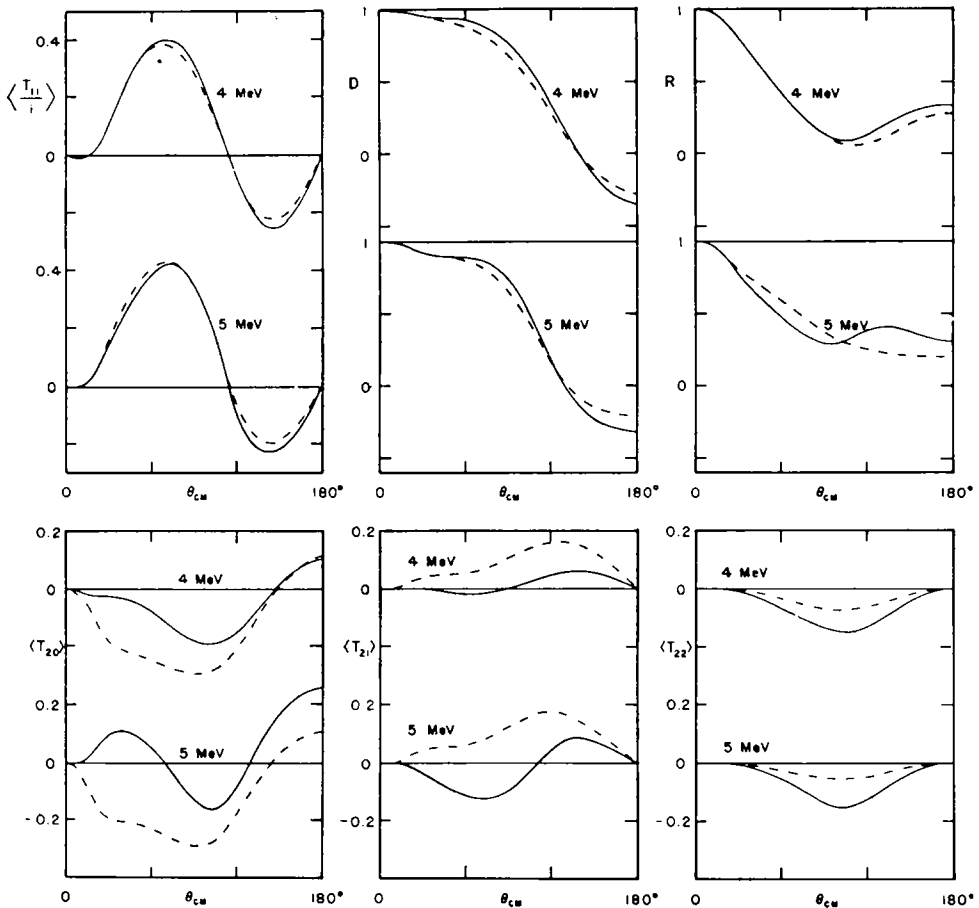


Fig. 5. Suggestions for future experiment. The large number of parameters that describe the scattering causes doubts about the uniqueness of the phases reported here. We show here six measurable quantities calculated with Sets I and II as examples of possible profitable experiments. Vector polarization of  ${}^6\text{Li}$ ,  $\langle T_{11}/i \rangle$ , Wolfenstein parameters  $D$  and  $R$  and the tensor moments of  ${}^6\text{Li}$   $\langle T_{20} \rangle$ ,  $\langle T_{21} \rangle$  and  $\langle T_{22} \rangle$  are each shown for 4 and 5 MeV as calculated with Set I (full curve) and Set II (dashed curve). Measurements with tensor polarized  ${}^6\text{Li}$  could easily distinguish Set I from Set II and, one suspects, would give strong confirmation or denial to the phases presented here.

be indistinguishable if one measures the vector polarization efficiency of  ${}^6\text{Li}$  scattered by hydrogen, but would be easily distinguishable if one measured the tensor moments

$\langle T_{20} \rangle$ ,  $\langle T_{21} \rangle$  and  $\langle T_{22} \rangle$ . Samples of these calculations are shown in fig. 5. Such measurements might give more conclusive evidence about doublet phases and mixing, which are not sensitive to cross section and polarization.

## 6. Conclusions

We obtained fits to cross section data from 0.5 to 5 MeV and to proton polarization data from 1.2 to 3.2 with only five, physically reasonable phase shifts. The  ${}^4P_{\frac{3}{2}}$  phase

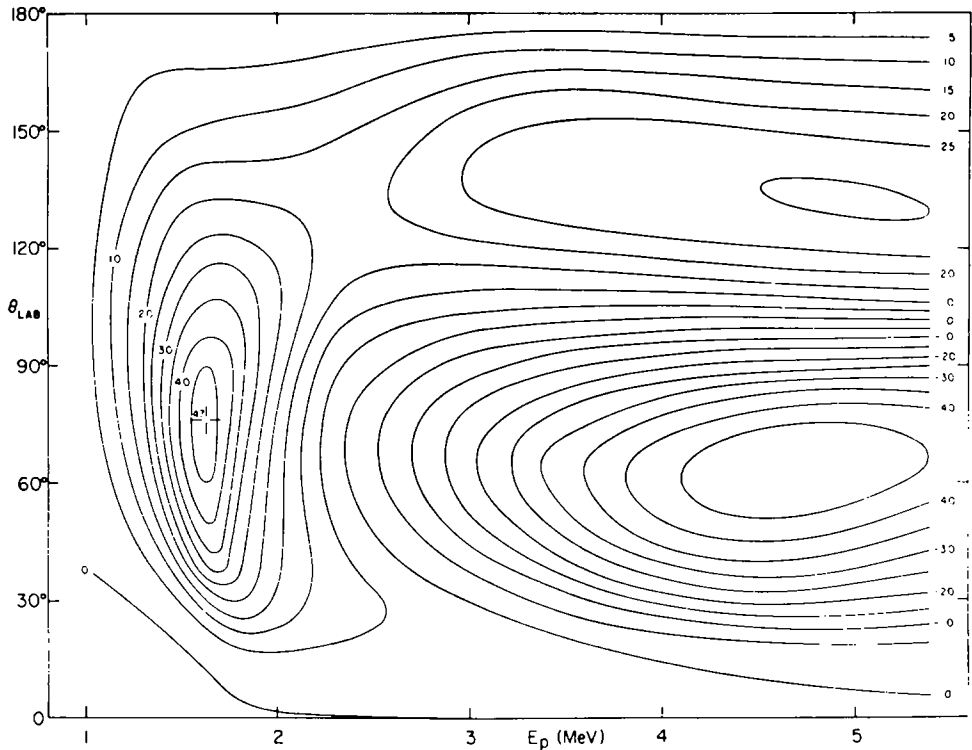


Fig. 6. Polarization contour map of  ${}^6\text{Li}(p, p){}^6\text{Li}$ . Analysing efficiency contours of 0.05 spacing, which are calculated from the smooth curves of the phase shifts in fig. 4 together with the absorption parameters of fig. 3, are shown plotted on a coordinate system of lab scattering angle against lab proton energy. Polarization data were used in the phase shift analysis only up to 3.2 MeV, so this map may be unreliable above that energy.

resonates at 1.8 MeV, and we could not find any other single phase that does. The doublet P-phases have little effect but may not be zero, as we have set them. There is no evidence that mixing between the quadruplet and doublet states is important, but it cannot be if the doublet P-phases are small. We did not investigate the effect of mixing between S- and D-states, since the D-waves contribute so little, particularly at lower energies where polarization data exist. Inclusion of the absorption parameters

improves the quality of fit; their values were set rather arbitrarily with the requirements that they give the value of the total reaction cross section and have all P-wave absorption in the two partial waves that resonate. They have little influence on the differential cross section or the polarization. The qualities of fit for polarization and cross section data were generally in the range  $\varepsilon^2 = 1$  to 2. Fasoli's cross section at 54.7° from 2.8 to 3.2 MeV lies far from our calculated values, which agree, however, with the data of Harrison and Whitehead.

We find only two sets of phase shifts that are smoothly varying with energy and that have reasonable values at low energies. The matter of uniqueness is not settled, as we have not proved there are only Sets I and II, and we recommend experiments with polarized lithium, which could even distinguish between Sets I and II. Measurements of proton polarization above 3.2 MeV would improve the accuracy of the phases and help establish their uniqueness.

Fig. 6 is a contour map of proton polarization that has been calculated from the smooth curves drawn through the points in fig. 4. It is essentially the same for either set of phases. No polarization data were used in the analysis above 3.2 MeV, so the map is of uncertain accuracy at the high energy end.

We are grateful to the Ohio State University Computer Center for the generous amount of computer time provided for this research.

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