THE SURFACTANT MODEL IN MRST

ABSTRACT. In this document, we describe briefly the surfactant model which is implemented in MRST.

Our starting point is the mass conservation equations for oil, water, and surfactant, which are given by

$$(1) \qquad \frac{\partial}{\partial t}(\rho_\alpha \phi S_\alpha) + \nabla \cdot (\rho_\alpha \vec{v}_\alpha) = 0, \qquad \alpha \in \{w, o\},$$

and

$$(2) \qquad \frac{\partial}{\partial t}(\rho_w \phi S_w c) + \frac{\partial}{\partial t}\left(\rho_{r,\text{ref}}(1 - \phi_{\text{ref}})c^a\right) + \nabla \cdot (c \rho_w \vec{v}_{wp}) = 0.$$

Let us describe the terms involved in (1) and (2). The values ρ_α, $\vec{v}_\alpha$, and S_α denote density, velocity, and saturation of the phase α. The porosity is denoted by ϕ and is assumed to be a function $\phi(p)$ of pressure only, c is the surfactant concentration, and $\vec{v}_{wp}$ the velocity of water containing diluted polymer. The quantity c^a denotes the mass of surfactant which is adsorbed to a unit mass of rock. Sources and sinks may be included in a manner equivalent to boundary conditions, and are left out of the above equations.

Let us describe how these quantities are determined. The fluxes $\vec{v}_\alpha$ are given by

$$\vec{v}_\alpha = \frac{k_\alpha(s_\alpha, c)}{\mu_\alpha(p, c)} \nabla p_\alpha,$$

where the relative permeabilities k_α depends on the saturations and the surfactant concentrations and the viscosities depend on the surfactant concentration. The phase pressures are given by the relation

$$(3) \qquad p_o = p_w + p_c(s_w, c)$$

where the capillary pressure function p_c depends on the water saturation and surfactant concentration. The model for relative permeability and viscosity follow Eclipse implementation, see [1, 2]. First, we introduce the interfacial tension σ. At the pore level, the effect of surfactant is to modify the interfacial tension so that σ is a function of the surfactant concentration, $\sigma(c)$. At a upscaled level, the change in relative permeability will depend on the capillary number N_c which measures the ratio between the viscous and

capillary forces and is defined as

$$(4) \qquad N_c = \frac{|K\nabla p|}{\sigma}.$$

Thus, the relative permeability is given as $k_\alpha(s_\alpha, N_c)$. The values of $k_\alpha(s_\alpha, N_c)$ are computed by interpolating the relative permeability curves for minimal and maximal N_c, in the following way. Let N_c^{surf} and N_c^{nosurf} denote the minimal and maximal values of the capillary numbers for which the relative permeability curves are given as

$$k_\alpha^{\mathrm{surf}}(s_\alpha) \quad \text{and} \quad k_\alpha^{\mathrm{nosurf}}(s_\alpha).$$

We denote the residual saturation values for each phase as $s_{\alpha.res}^{\mathrm{surf}}$ and $s_{\alpha.res}^{\mathrm{surf}}$. The interpolation is logarithmic so that we define the interpolation factor m as

$$(5) \qquad m(N_c) = \frac{\log_{10}(N_c) - \log_{10}(N_c^{\mathrm{surf}})}{\log_{10}(N_c^{\mathrm{nosurf}}) - \log_{10}(N_c^{\mathrm{surf}})},$$

which belongs to $[0, 1]$. The residual saturation values for the relative permeabilities $k_\alpha(s_\alpha, N_c)$ is obtained by interpolation

$$(6) \qquad s_{\alpha,res}^{N_c} = m(N_c)\, s_{\alpha,res}^{\mathrm{surf}} + (1 - m(N_c))\, s_{\alpha,res}^{\mathrm{nosurf}}.$$

The relative permeabilities k_r^{surf} and k_r^{nosurf} are rescaled to match these endpoints. Hence, we define the function $k_{r.\alpha}^{\mathrm{surf}}$ as

$$(7) \qquad k_\alpha^{\mathrm{surf}}(s_\alpha, N_c) = k_\alpha^{\mathrm{surf}}(s_\alpha^{\mathrm{surf}}(N_c))$$

where $s_\alpha^{\mathrm{surf}}(N_c)$ is defined through the relation

$$(8) \qquad \frac{s_\alpha^{\mathrm{surf}}(N_c) - s_{\alpha,res}^{\mathrm{surf}}}{1 - s_{\alpha,res}^{\mathrm{surf}} - s_{\alpha',res}^{\mathrm{surf}}} = \frac{s_\alpha - s_{\alpha,res}^{N_c}}{1 - s_{\alpha,res}^{N_c} - s_{\alpha',res}^{N_c}}.$$

In (8), α' denotes the phase which is not the phase α. Similarly we define $k_\alpha^{\mathrm{nosurf}}$. Finally, we define the relative permeability k_α by interpolation, that is,

$$(9) \qquad k_\alpha(s, N_c) = m(N_c)\, k_\alpha^{\mathrm{surf}}(s_\alpha, N_c) + (1 - m(N_c))\, k_\alpha^{\mathrm{nosurf}}(s_\alpha, N_c).$$

The change in viscosity is only modeled for the water phase with multiplier functions. We have

$$(10) \qquad \mu_w(p, c) = \frac{\mu_w^0(p)}{\mu_w^0(p_{\mathrm{ref}})} \mu_s(c),$$

where μ_w^0 denotes the viscosity without surfactant and $\mu_s(c)$ denotes the viscosity as a function of surfactant concentration at pressure p_{ref}. For the adsorption term, we use an approach based on Langmuir isotherm so that the adsorbed concentration is a function of the surfactant concentration, that is of the from $c^a(s)$. We consider the reversible case and the case without desorption where the term c^a is replaced by $\hat{c}^a$ which denotes the maximum

value that the adsorption concentration has reached. More precisely, if we define

$$\hat{c}^a_{\max}(t, x) = \max_{t' < t}(\hat{c}^a(t', x)), \tag{11}$$

then we have defined as

$$\hat{c}^a(t, x) = \max(c^a(c(t, x)), \hat{c}^a_{\max}(t, x)). \tag{12}$$

Keyword	Description
SURFACT	Model initialization
SURFST	IFT data
SURFVISC	Viscosity modifier
SURFCAPD	Capillary de-saturation
SURFADS	Surfactant adsorption
SURFROCK	Rock properties
SURFNUM	Grid region specification
WSURFACT	Injected concentration

TABLE 1. List of Eclipse keyword that have been implemented.

REFERENCES

[1] Schlumberger, editor. *Eclipse Technical Description*. 2011.
[2] Kristian Jørgensen. "Implementation of a Surfactant Model in MRST With Basis in Schlumberger's Eclipse". MSc thesis. NTNU, 2013.
[3] M. R. Todd and W. J Longstaff. "The Development, Testing, and Application Of a Numerical Simulator for Predicting Miscible Flood Performance". In: *J. Petrol. Tech.* 24.7 (1972), pages 874–882.