RMIT UNIVERSITY

IN1.1: INTEGRATION OF BASIC FUNCTIONS

Integrals of the form $\frac{1}{x}$

The rule for integrating x^n cannot be used when $n = -1$. (Why?)

Differentiating $\log_e x$ gives $\frac{1}{x}$, therefore an antidifferential of $\frac{1}{x}$ is $\log_e x$

$$\therefore \boxed{\int \frac{1}{x} dx = \log_e |x| + c}.$$

cannot take the log of a negative number

In general $$\int \frac{1}{(ax+b)} dx = \frac{1}{a} \log_e (ax+b)$$

The denominator **must be a linear function.**

Examples

1. $\int \frac{5}{x} dx = 5 \log_e |x| + c$

2. $\int \frac{1}{(2x-5)} dx = \frac{1}{2} \log_e |2x-5| + c$ a = 2 and b = – 5

3. $\int \frac{1}{(1-3x)} dx = -\frac{1}{3} \log_e |1-3x| + c$ a = – 3 and b = 1

See Exercise 1

Exponential function

If $f(x) = e^x$ then $f'(x) = e^x$, therefore an antidifferential of e^x is e^x

$$\therefore \boxed{\int e^x dx = e^x + c}$$

In general $$\int e^{ax+b} dx = \frac{1}{a} e^{ax+b} + c$$

Examples

1. $\int 2e^x dx = 2e^x + c$
2. $\int e^{5x+1} dx = \frac{1}{5} e^{5x+1} + c$ a = 5 and b =1
3. $\int e^{-6x} dx = -\frac{1}{6} e^{-6x} + c$ a =– 6 and b = 0
4. $\int e^{\frac{x}{3}+4} dx = 3e^{\frac{x}{3}+4} + c$ a = 1/3 and b = 4

See Exercise 2

Trigonometric functions

Using the derivatives of the trigonometric functions the following integrals can be written down:

$$\int \sin x dx = -\cos x + c$$
$$\int \cos x dx = \sin x + c$$
$$\int \sec^2 x dx = \tan x + c$$

These can be generalised to give the following

$$\int \sin(ax+b) = -\frac{1}{a}\cos(ax+b) + c$$
$$\int \cos(ax+b) = \frac{1}{a}\sin(ax+b) + c$$
$$\int \sec^2(ax+b) = \frac{1}{a}\tan(ax+b) + c$$

Examples

$\int 5\cos x dx = 5\sin x + c$

$\int \cos 2x dx = \frac{1}{2}\sin 2x + c$ a = 2 b =0

$\int \sin(5x+2) dx = -\frac{1}{5}\cos(5x+2) + c$ a=5 b = 2

$\int 3\cos(3-2x) dx = \frac{3}{-2}\sin(3-2x) + c$ a = –2 b = 3

$\int \sec^2 \frac{1}{2} x dx = 2\tan\frac{1}{2} x + c$ a = 1/2 b =0

See Exercise 3

Exercises

Exercise 1

Integrate the following:

(a) $\int \frac{1}{x+5}dx$ (b) $\int \frac{1}{3x}dx$ (c) $\int \frac{4}{3x+5}dx$

(d) $\int \frac{1}{2-3x}dx$ (e) $\int \frac{5}{(6-5x)}dx$ (f) $\int \left(\frac{1}{3x+5}-\frac{1}{x-2}\right)dx$

Exercise 2

Integrate the following

(a) $\int e^{3x}dx$ (b) $\int e^{x+4}dx$ (c) $\int 5e^{2x-7}dx$

(d) $\int e^{2-5x}dx$ (e) $\int \left(e^{-3x}+4\right)dx$ (f) $\int \frac{9e^{3x-4}+5}{e^{2x}}dx$

Exercise 3

Find the following integrals.

(a) $\int \cos 8x dx$ (b) $\int \sin(3x+2)\,dx$ (c) $\int 2\cos(1-x)\,dx$

(d) $\int \left(\cos\frac{\theta}{2}+\sin\frac{\theta}{2}\right)d\theta$ (e) $\int \sec^2 4\theta d\theta$ (f) $\int 5\cos\left(\pi+\frac{2\theta}{3}\right)d\theta$

(g) $\int \frac{1}{2}\left(\cos(4x-1)-\sin(4x-1)\right)dx$ (h) $\int -3\sin\left(2-\frac{1}{2}x\right)dx$

(i) $\int 2\sin\left(\frac{5-3x}{4}\right)dx$

Answers

1 (a) $\log_e|x+5|+c$ (b) $\frac{1}{3}\log_e|x|+c$ (or $\frac{1}{3}\log_e|3x|+c$) (c) $\frac{4}{3}\log_e|3x+5|+c$

(d) $\frac{-1}{3}\log_e|2-3x|+c=\frac{1}{3}\log_e\frac{1}{|2-3x|}+c$ (e) $-\log_e|6-5x|+c=\log_e\frac{1}{|6-5x|}+c$

(f) $\frac{1}{3}\log_e|3x+5|-\log_e|x-2|+c=\log_e\left|\frac{\sqrt[3]{3x+5}}{x-2}\right|+c$

2 (a) $\frac{e^{3x}}{3}+c$ (b) $e^{x+4}+c$ (c) $\frac{5e^{2x-7}}{2}+c$ (d) $\frac{e^{2-5x}}{-5}+c$

(e) $\frac{e^{-3x}}{-3}+4x+c$ (f) $9e^{x-4}-\frac{5}{2e^{2x}}+c$

3 (a) $\frac{1}{8}\sin 8x+c$ (b) $-\frac{1}{3}\cos(3x+2)+c$ (c) $-2\sin(1-x)+c$

(d) $2\left(\sin\frac{\theta}{2}-\cos\frac{\theta}{2}\right)+c$ (e) $\frac{1}{4}\tan 4\theta+c$ (f) $\frac{15}{2}\sin\left(\pi+\frac{2\theta}{3}\right)+c$

(g) $\frac{1}{8}\left(\sin(4x-1)+\cos(4x-1)\right)+c$ (h) $-6\cos\left(2-\frac{1}{2}x\right)+c$ (i) $\frac{8}{3}\cos\left(\frac{5-3x}{4}\right)+c$