STUDY AND LEARNING CENTRE

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STUDY TIPS



INTRODUCTORY PROBABILITY

A probability is written as a number between zero and one: $0 \le Pr(A) \le 1$

Pr(A) = 0 means that event A is impossible.

Pr(A) = 1 means that event A is certain.

When considering a set of all possible outcomes an *event* is a particular outcome of interest. For example,

- In tossing a coin the particular event of interest might be 'obtaining a head'
- In considering the weather for Saturday the event of interest might be 'it doesn't rain'
- In planning a two child family he particular event of interest might be 'a boy and a girl'.

The probability of an event E can be found with the formula:

$$Pr(E) = \frac{number of ways E can occur}{total number of possible outcomes}$$

[assuming all outcomes are equally likely]

Examples:

1. If two coins are tossed find the probability of obtaining two heads.

Let E be the event 'two heads'

The possible outcomes are HH HT TH TT

$$\therefore Pr(E) = \frac{\text{number of ways E can occur}}{\text{total number of possible outcomes}}$$
$$= \frac{1}{4}$$

2. If a die is thrown find the probability of obtaining an odd number

Let E be the event 'an odd number'

The possible outcomes are 1 2 3 4 5 6

∴ Pr (E) =
$$\frac{\text{number of ways E can occur}}{\text{total number of possible outcomes}}$$

= $\frac{3}{6}$
= $\frac{1}{2}$

The multiplication principle

Two events, A and B, are independent if the fact that A occurs does not affect the probability of B occurring. Because successive tosses of a coin are independent events, an alternate way of calculating the probability in example one would be to use the multiplication principle.

If A and B are independent events then $Pr(A \text{ and } B) = Pr(A \cap B) = Pr(A) \times Pr(B)$

The probability of a head on the first toss (H_1) and a head on the second toss (H_2)

$$= Pr (H_1 \cap H_2)$$

$$= \frac{1}{2} \times \frac{1}{2}$$

$$= \frac{1}{4}$$

The addition principle

$$Pr(A \text{ or } B) = Pr(A \cup B) = Pr(A) + Pr(B) - Pr(A \cap B)$$

If A and B are mutually exclusive (cannot happen together) then
 $Pr(A \text{ or } B) = Pr(A \cup B) = Pr(A) + Pr(B)$

If we are tossing a single die twice and want to calculate the probability that a 6 occurs, then the 6 could occur on the first toss (S_1) or on the second toss (S_2) :

$$\begin{split} \text{Pr}(S_1 \text{ or } S_2) &= \text{Pr}(S_1 \cup S_2) = \text{ Pr}(S_1) + \text{ Pr}(S_2) \text{ - Pr}(S_1 \cap S_2) \quad \text{[because the events are not mutually exclusive]} \\ &= \frac{1}{6} + \frac{1}{6} - \frac{1}{36} \\ &= \frac{11}{36} \end{split}$$

Complementary events

If E is an event in then (not E) or \overline{E} or E' is called the complement of E. Examples of complementary events:

- 'winning the grand final' and 'not winning the grand final'
- 'passing a test' and 'failing a test'
- 'being left handed' and 'being right handed'

Because
$$P(E) + P(E') = 1$$
 it follows that $P(E') = 1 - P(E)$

In the previous example where a die was tossed twice the probability of not getting a 6 on either the first or second toss = $1 - \frac{11}{36}$

Exercise

- **1.** If 1000 tickets are sold in a raffle and one winning ticket is chosen at random, what is my probability of winning the raffle if I buy 5 tickets?
- **2.** If I roll a die, what is the probability that the number uppermost is greater than 4?
- **3.** A bag contains 6 white marbles and 4 black marbles. A marble is chosen, the colour recorded and then replaced three times. What is the probability that all three marbles are white?
- **4.** The probability that person A is alive in 30 years time is 0.7. The probability that person B is alive in 30 years time is 0.4.

Find the probability that:

- (a) both are alive in 30 years. (b) neither are alive in 30 years
- (c) only one is alive in 30 years time (d) at least one is alive in 30 years time.

Answers

1.
$$\frac{1}{200}$$
 2. $\frac{1}{3}$ 3. 0.216 4. (a) 0.28 (b) 0.18 (c) 0.54 (d) 0.82