AS1.1: ALGEBRAIC OPERATIONS

Like Terms

Like terms contain exactly the same *pronumerals* (letters, variables)
.

Like Terms	Unlike terms
$3x$, $5x$	$3x$, $4y$
$2a$, $-3a$	$3a$, 3
$3m^2$, m^2	$3m^2$, $3m$
$2ab$, $3ab$	$2a^2b$, $3ab^2$
$3ef$, $5fe$	$3ef$, $7fg$

NB: Order is unimportant but alphabetical order of pronumerals is conventional

See Exercise 1

Addition and Subtraction

Only like terms may be added or subtracted.

1. $7e + 10e = (7 + 10)e$ [eventually it is possible to just THINK the second step and go straight to the answer]
$\qquad\qquad = 17e$

2. $3x^2 - x^2 - 4x^2 = (3 - 1 - 4)x^2$
$\qquad\qquad\qquad = -2x^2$

3. $3m - 4n + 6m + n = (3 + 6)m + (-4 + 1)n$
$\qquad\qquad\qquad\quad = 9m - 3n$

4. $3a - b - 5a + 4ab - 3b + ab = (3 - 5)a + (-1-3)b + (4 + 1)ab$
$\qquad\qquad\qquad\qquad\qquad = -2a - 4b + 5ab$

5. $3x - x^2$ cannot be simplified

6. $p + 2p - 3 = 3p - 3$

7. $8uv + 3u - 10vu = -2uv + 3u$

8. $6r^2s - 2rs^2$ cannot be simplified

See Exercise 2

Multiplication

Consider the sign (positive or negative) of the answers to the following simple multiplication problems

$\qquad 2 \times 3 = 6$ $\qquad\qquad$ *positive x positive* $\quad\rightarrow\quad$ ***positive***

$\qquad -2 \times 3 = -6$ $\qquad\qquad$ *negative x positive* $\rightarrow$ *negative*

$\qquad 2 \times -3 = -6$ $\qquad\qquad$ *positive x negative* $\rightarrow$ *negative*

$\qquad -2 \times -3 = 6$ $\qquad\qquad$ *negative x negative* $\quad\rightarrow\quad$ ***positive***

Terms can be multiplied whether they are like or unlike terms.
When multiplying two or more terms consider
- the sign of the answer
- the product of the numbers

and use an index to show
- how many factors of each pronumeral

1. $(-4) \times (-3b) = 12b$

2. $-2 \times 6y = -12y$

3. $2e \times (-5e^2) = -10e^3$

4. $(-2u^2v) \times (-4v) = 8u^2v^2$

5. $-3pq \times (-2q) \times p = 6p^2q^2$

In algebra a fraction line means to divide   eg. $\frac{1}{2} = 1 \div 2$, $\frac{3}{4} = 3 \div 4$

When dividing algebraic expressions
- ● rewrite as a fraction if necessary
- ● expand any powers
- ● establish the sign of the answer
- ● cancel factors
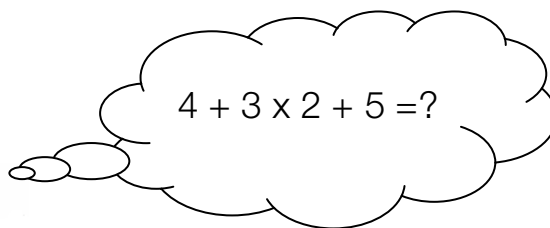- ● write the answer in simplified form

1. $-12wyz \div 3yz = \dfrac{-12wyz}{3yz} = -4w$

2. $-2m^2n \div 6mn^2 = \dfrac{-2m^2n}{6mn^2} = \dfrac{-2mmn}{6mnn} = -\dfrac{m}{3n}$

3. $6a^2 \times 4b \div 12ab = \dfrac{6aax4b}{12ab} = 2a$

NB: $\dfrac{m+2}{4m}$ CANNOT be simplified!!!

See Exercise 3

$4 + 3 \times 2 + 5 = ?$

Order of Operations

Consider   4 + 3 × 2 + 5.   *What is the answer?*

   4 + 3 × 2 + 5 → 7 × 7 = 49?

   4 + 3 × 2 + 5 → 4 + 3 × 7 = 25?

   4 + 3 × 2 + 5 → 7 × 2 + 5 = 19?

To avoid such confusion we perform operations in the following order

1.   **B**rackets   B

2.   **I**ndice*s*   I

3*.*   **M**ultiplication and **D**ivision from left to right   M

                                                                     D

4*.*   **A**ddition and **S**ubtraction from left to right   A

                                                                     S

To make it easier to remember the rule for order of operations is abbreviated to   B I M D A S

Using this rule   4 + 3 × 2 + 5 = 4 + 6 + 5 = 15!!

Examples:

1.   3 × 2 + 4 = 6 + 4 = 10
2.   3 + 2 × 4 = 3 + 8 = 11
3.   (3 + 2) × 4 = 5 × 4 = 20
4.   3 - 2^2 = 3 − 4 = -1
5. $(3 − 2)^2 = 1^2 = 1$
6   3^2 - 2^2 = 3 × 3 − 2 × 2 = 9 − 4 = 5
7   3st – 3s × 4t = 3st – 12st = -9st

See Exercise 4

Exercises

Exercise 1
Which of the five terms on the right is a like term with the term on the left?

1) *3x*	3	2x	$3x^2$	2xy	$4x^2a$
2) *2ab*	2a	2b	$3x^2$	6abc	12ab
3) *$2x^2$*	x	2x	$5x^2$	4x	$4x^3$
4) *$3xy^2$*	3	3xy	$3x^2$	xy^2	$3y^2$
5) *$2m^2n$*	n	mn^2	8mn	$2m^2$	$4nm^2$
6) *$4ab^2c$*	4	$2ab^2c$	4abc	$8b^2c$	4cba

Exercise 2

Simplify each of the following.

1. a) $5x + 3x$ b) $12x - 7x$
   c) $15xy + 5xy$ d) $11mn - 5mn$
   e) $10abc - 3bca$ f) $10m - 22m$
   g) $6x + 3x + 4x$ h) $4ab + 5ab - 2ab$
   i) $m + 2m - 9m$ j) $14xy - 4xy + 2xy$

2. a) $13x + 4 - 3x - 1$ b) $10mn + 5m + 12mn + 6m$
   c) $3xy^2 + 2xy + 5xy^2 + 3xy$ d) $5xy + 6m - 2xy - 2m$
   e) $x + y + 2x - y$ f) $3a + 5b - a - 6b$
   g) $x + 4y - x - 2y$ h) $7x - 4m - 5x - 3m$
   i) $4x - 5x - 3y + 5x$ j) $9mn - 3m - n + 4m^2$

Exercise 3

Simplify the following algebraic expressions

1. a) $5 \times 2k$ b) $4a \times 3ab$
   c) $y \times 3y$ d) $4m \times (-3mn)$
   e) $m \times 3p \times 5$ f) $2ab \times 3bc \times (-4)$
   g) $2ab^2 \times 3ac$ h) $4m \times (-5kmp)$

2. a) $18ef \div 6f$ b) $-100uvw \div 100w$
   c) $24gh^2 \div 8gh$ d) $3m^2n \div 12mn^2$
   e) $rs \times 2st \div 2s$ f) $3jk \times 12km \div 9jkm$
   g) $10p \times 3pq \div 16pq$ h) $4yz \times 5w^2z \div 10wy$

Exercise 4

Simplify

1) $18 + (3 \times -5)$ 2) $3 \times -4 + (8 \times 2)$
3) $10 - 5^2 + 3$ 4) $(10 - 5)^2 + 3$
5) $10^2 - 5^2$ 6) $(10 - 5)^2$
7) $3m + 2 \times 3m$ 8) $6ab - 3a \times 4b$
9) $16gh - 4gh \times 4$ 10) $9b - 3b \times 2k + 2k \times b$

Answers

Exercise 1

1. $2x$ 2. $12ab$ 3. $5x^2$ 4. xy^2 5. $4nm^2$ 6. $2ab^2c$

Exercise 2

1. a. $8x$ b. $5x$ c. $20xy$ d. $6mn$ e. $7abc$
   f. $-12m$ g. $13x$ h. $7ab$ i. $-6m$ j. $12xy$
2. a. $10x + 3$ b. $22mn + 11m$
   c. $8xy^2 + 5xy$ d. $3xy + 4m$
   e. $3x$ f. $2a - b$
   g. $2y$ h. $2x - 7m$
   i. $4x - 3y$ j. $9mn - 3m - n + 4m^2$

Exercise 3

1. a. $10k$ b. $12a^2b$ c. $3y^2$ d. $-12m^2n$ e. $15mp$ f. $-24ab^2c$ g. $6a^2b^2c$ h. $-20km^2p$

2. a. $3e$ b. $-uv$ c. $3h$ d. $\dfrac{m}{4n}$ e. rst f. $4k$ g. $\dfrac{15p}{8}$ h. $2wz^2$

Exercise 4

1. 3 2. 4 3. -12 4. 28 5. 75
6. 25 7. $9m$ 8. $-6ab$ 9. 0 10. $9b - 4bk$