RMIT UNIVERSITY

IN1.5: DEFINITE INTEGRALS

$\int_a^b f(x)dx$ is called the definite integral from *a* to *b*.

a is the lower limit of integration
b is the upper limit of integration.

$$\int_a^b f(x)\,dx = \left[F(x)\right]_a^b = F(b) - F(a) \text{ where } F(x) = \int f(x)\,dx$$

This can be calculated only if $f(x)$ is defined for all *x* in the interval $a \le x \le b$

Examples

1.Calculate $\int_2^4 (x+1)\,dx$

$$\int_2^4 (x+1)\,dx = \left[\frac{x^2}{2} + x\right]_2^4$$

Integrate (x+1).

Write the answer in square brackets with the upper(4) and lower(2) limits placed as shown.

$$= \left(\frac{4^2}{2} + 4\right)$$

Evaluate the integral at the upper limit(4) and lower limit(2).

$$= 8$$

Subtract the value at the lower limit from the value at the upper limit.

2.Calculate $\int_0^\pi \left(\cos x + e^{-2x}\right)\,dx$

$$\int_0^\pi \left(\cos x + e^{-2x}\right)\,dx = \left[\sin x - \frac{e^{-2x}}{2}\right]_0^\pi = \left(\sin \pi - \frac{e^{-2\pi}}{2}\right) - \left(\sin 0 - \frac{e^0}{2}\right)$$

$$= \left(0 - \frac{e^{-2\pi}}{2}\right) - \left(0 - \frac{1}{2}\right)$$

$$= \frac{1 - e^{-2\pi}}{2}$$

Applications

Examples

1. The work done moving an object from point *a* to point *b* is given by

W = $\int_a^b (3x^2+2)dx$ joule.

Find the work done in moving the object from the point x = 0 to the point x = 3.

W = $\int_0^3 (3x^2+2)dx = \left[\frac{3x^3}{3}+2x\right]_0^3 = (3^3+6)-(0) = 33$ Joule.

2. The decrease in a koala population is given by $N'(t) = -500e^{-0.2t}$ per year. If the initial population of koalas was 3000 (when t=0), find the approximate koala population after 10 years.

$$N'(t) = -500e^{-0.2t}$$

$$\int_0^{10} \frac{dN}{dt}dt = \int_0^{10} -500e^{-0.2t}dt$$

Integrate both sides with respect to *x*, from t=0 to t=10.

$$\int_0^{10} dN = \int_0^{10} -500e^{-0.2t}dt$$

$$[N]_0^{10} = \left[\frac{-500}{-0.2}e^{-0.2t}\right]_0^{10}$$

$$N_{t=10} - N_{t=0} = \frac{500e^{-2}}{0.2} - \frac{500e^{0}}{0.2}$$

$N_{t=10}$ = number of koalas after 10 years

$N_{t=0}$ = number of koalas when t=0

$$N_{t=10} - 3000 = 2500e^{-2} - 2500$$

$$N_{t=10} = 838.34$$

Number of koalas after 10 years is 838.

Exercises

1.Evaluate:

(a) $\int_1^4 (2x-3)\,dx$ (b) $\int_0^2 (3x^2+x+1)\,dx$ (c) $\int_1^2 \frac{1}{x^2}dx$

(d) $\int_0^{\frac{\pi}{2}} (2\cos x - \sin x)\,dx$ (e) $\int_0^{\pi} (\cos x + \sin 2x)\,dx$ (f) $\int_{-2}^4 2e^{-3x}dx$

(g) $\int_1^4 \frac{1}{3x}dx$

1. 2.The acceleration of a particle is given by $a(t) = 2t^2 + 3e^{-t}$ m/s^2. If its initial velocity, v(0), is 2 m/s find the velocity when t = 3. (acceleration $= \frac{dv}{dt}$)

Answers

1(a) 6 (b) 12 (c) ½ (d) 1 (e) 0 (f) 2/3(e^6-e^{12}) (g) 1/3ln4.
2. 22.88m/s