FA1.5: FACTORISATION: PERFECT SQUARES

Perfect squares

Examples of perfect squares are 5^2, x^2, $a^2 b^2$, $(xy)^2$ and $(a+b)^2$

$(a+b)^2$ can be expanded as follows

$$(a+b)^2 = a^2 + 2ab + b^2$$

similarly $\qquad (a-b)^2 = a^2 - 2ab + b^2$

Note that:

- the expansion gives *three* terms

$(a+b)^2$ *does not equal* $a^2 + b^2$	$(a-b)^2$ *does not equal* $a^2 - b^2$

- the **first** and **last** terms of the expansion must be **positive**
- the **middle term** is **twice** the **product of the first and last terms**
- the middle term may be positive or negative

The rule for expanding perfect squares can be used in reverse for factorising perfect squares.

Examples

Check each of the following expressions and, if it is a perfect square, state the perfect square.

1. $x^2 + 14x + 49$

   $x^2 + 14x + 49 = (x+7)^2$

   first term is the square of x
   last term is positive and is the square of ± 7
   middle term $= 2 \times x \times (+7)$

2. $y^2 - 20y + 25$

   $y^2 - 20y + 25$ *is not a perfect square*

   first term is the square of y
   last term is positive and is the square of ± 5
   middle term **does not equal** $2 \times y \times (\pm 5)$

3. $4a^2 - 12a - 9$

   $4a^2 - 12a - 9$ *is not a perfect square*

   first term is the square of 2a
   last term is negative (9 is a perfect square but – 9 is **not**)

4. $100x^2 - 180x + 81$

   $100x^2 - 180x + 81 = (10x - 9)^2$

   first term is $(10x)^2$
   last term is $(\pm 9)^2$
   middle term is $2 \times 10x \times (-9)$

5. $50x^2 + 80x + 32 = 2(25x^2 + 40x + 16$

$50x^2 + 80x + 32 = 2(5x + 4)^2$

neither the first or last term is a perfect square therefore the expression cannot be a perfect square, but to factorise the expression:

take out a common factor of 2.

factorise $25x^2 + 40x + 16$ which is a perfect square.

Exercise

Check each of the following expressions. If it is a perfect square state the perfect square.

1. $a^2 + 2a + 1$
2. $x^2 - 4x + 4$
3. $25x^2 - 10x + 1$
4. $4y^2 - 6y + 9$
5. $81x^2 + 108x + 36$
6. $9a^2 - 24a - 16$
7. $16x^2 - 40xy + 25y^2$
8. $121z^2 + 88z + 64$
9. $2x^2 + 8x + 8$

Answers

1. $(a + 1)^2$
2. $(x - 2)^2$
3. $(5x - 1)^2$
4. Not a perfect square
5. $(9x + 6)^2$
6. Not a perfect square
7. $(4x - 5y)^2$
8. Not a perfect square
9. Not a perfect square but
$2x^2 + 8x + 8 = 2(x^2 + 4x + 4) = 2(x + 2)^2$