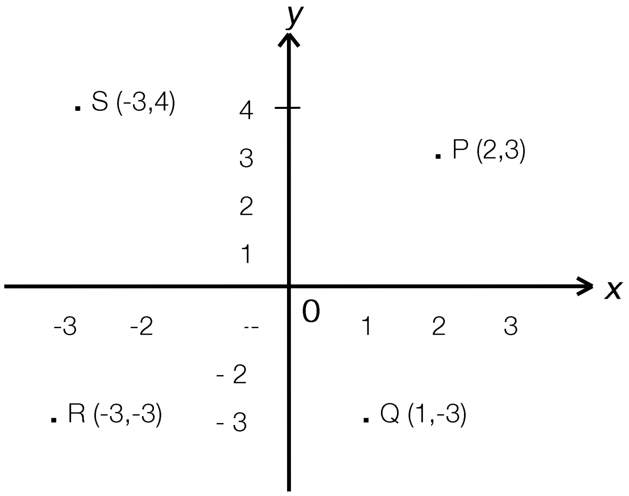
GR1.1: LINEAR GRAPHS

Cartesian Plane



The Cartesian plane is defined by a pair of mutually perpendicular coordinate axes. The **horizontal axis** is the ***x*-axis** and the **vertical axis** the ***y* – axis**.
Their point of intersection is called the origin *O.*

Points on the plane are referred to by their **horizontal distance (*x* – coordinate)** and vertical **distance (*y* – coordinate)** from the origin. Distances to the right of, and up from the origin are positive. Distances to the left of, and down from the origin are negative.

The *x* – coordinate is always given first.

The point P (2, 3) has coordinates $x = 2$ and $y = 3$.

The point Q (1,–3) has coordinates $x = 1$ and $y = –3$.

The point R (–3, –3) has coordinates $x = –3$and $y = –3$.

The point S (–3, 4) has coordinates $x = –3$ and $y = 4$.

Sketch Graphs

When sketching a graph it is not necessary to plot large numbers of points.
Only the basic shape of the graph is required, although some important points need to be clearly labelled.

Intercepts

These are the points where the line crosses the axes.

x-intercept --- where the graph cuts the x – axis ($y = 0$)
y-intercept --- where the graph cuts the y – axis ($x = 0$)

Linear Functions

Linear Graphs

The graph of a linear relationship is a straight line. Only **two** points are needed to sketch the graph of a straight line.

Example

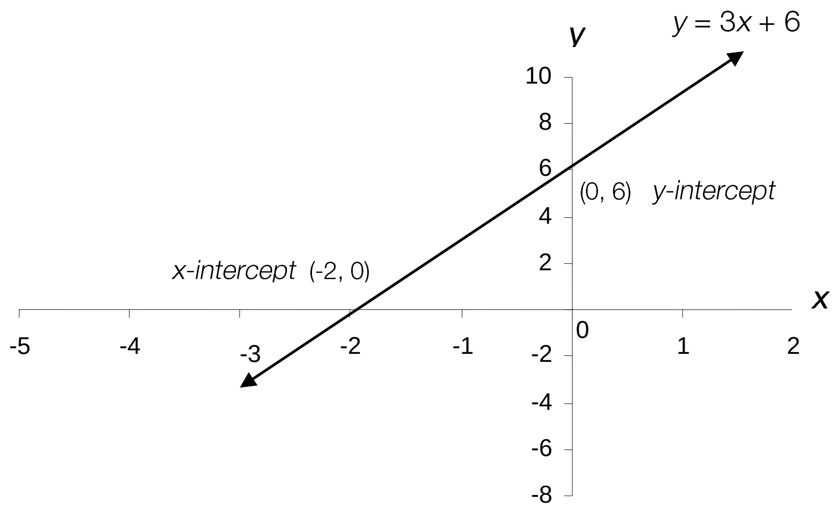
Sketch the function $y = 3x + 6$.

Determine two points on the line. The x- and y-intercepts are usually the easiest.

y-intercept ($x = 0$)
When $x = 0$
$y = 0 + 6$
$y = 6$ The y-intercept is the point (0, 6).

x-intercept ($y = 0$)
When $y = 0$.
$0 = 3x + 6$
$3x = -6$
$x = -2$ The x-intercept is the point (– 2, 0).

Connect the two points with a straight line.



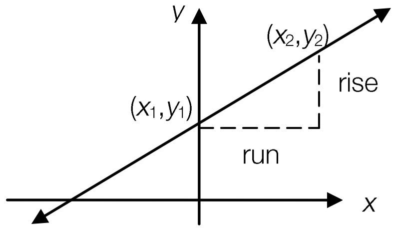
The **x- and y- intercepts** should be marked on the graph.

See Exercise 1.

Gradient

The slope of the a straight line is called the **gradient** . This is the ratio of the vertical change (rise) to the horizontal change (run) between two points (x_1,y_1) and (x_2,y_2) on the line.

$$\text{Gradient} = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$$



Examples

1.Find the gradient of the line $y = 3x + 6$ (see previous example)
The points (0,6), $[(x_1,y_1)]$ and (–2, 0), $[(x_2,y_2)]$ are on the line $y = 3x + 6$.

$$\text{Gradient} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0-6}{-2-0} = \frac{-6}{-2} = 3$$

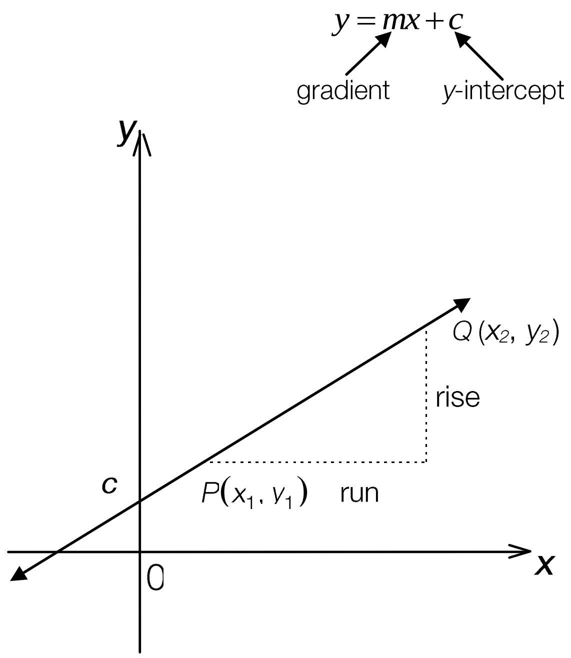
2.Find the gradient of the line joining the two points (1,3) and (4,5).

$$\text{Gradient} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5-3}{4-1} = \frac{2}{3}$$

See Exercise 2

Equation of a straight line

The general equation of a straight line is $\boldsymbol{y = mx + c}$,
where m is the gradient, and c is the value of the intercept on the y-axis.



$$\text{Gradient} = \frac{\text{rise}}{\text{run}}$$

$$= \frac{y_2 - y_1}{x_2 - x_1}$$

= m

y-intercept = c

The equation to the line is:

$$\boldsymbol{y = mx + c}$$

Some properties of linear graphs

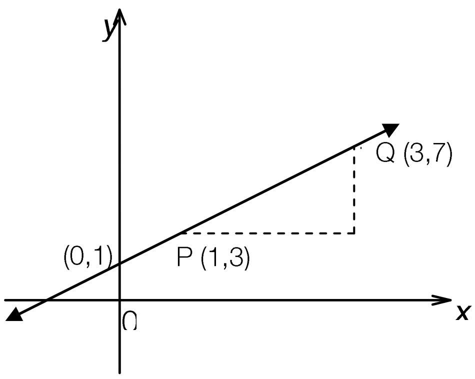
- A line that slopes **up** and to the **right** has a **positive gradient**
- A line that slopes **up** and to the **left** has a **negative gradient**.
- **Parallel** lines have the **same gradient**.
- The gradients of two **mutually perpendicular lines multiply to – 1**
- A line parallel to the *x* – axis has a gradient of 0.
- A line parallel to the *y* – axis, --- the gradient is undefined.

The equation of a straight line can also be written $y - y_1 = m(x - x_1)$ where ***m* is the gradient** and **(x_1, y_1) is a point on the line.** This is called point-slope form.

Examples

1.Find the gradient and the equation of the straight line below.

Positive gradient



gradient = m = $\dfrac{\text{rise}}{\text{run}}$

= $\dfrac{7-3}{3-1}$

= 2

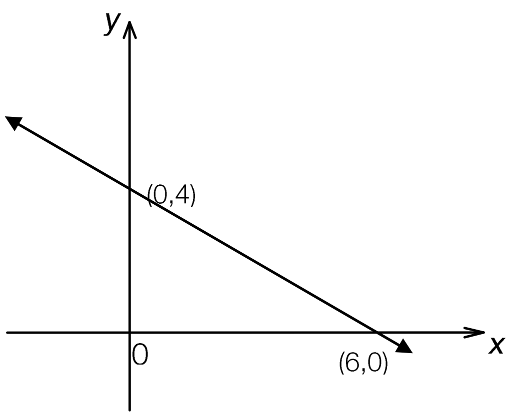
y-intercept = 1

The equation to the line is:

$y = 2x + 1$

2.Find the gradient and the equation of the straight line below.

Negative gradient



Gradient = m = $\dfrac{\text{rise}}{\text{run}}$

= $\dfrac{0-4}{6-0}$

= −2/3

y-intercept = 4

The equation to the line is:

$y = -2/3x + 4$

or

$3y = -2x + 12$

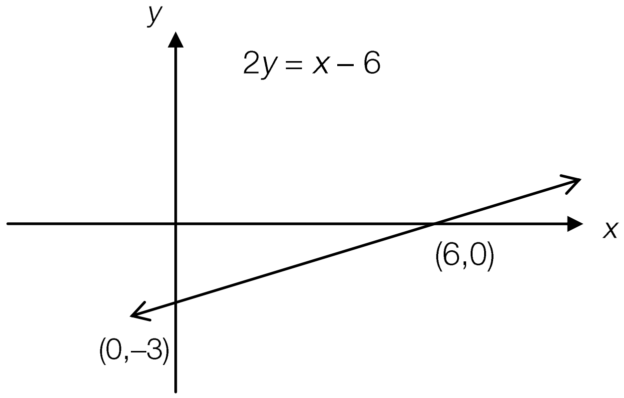
3. Sketch the function $2y = x - 6$ and state the gradient of the straight line.

Determine **x- and y- intercepts.**

$x = 0 \Rightarrow \quad 2y = -6 \quad \therefore \; y = -3$ **y- intercept** (0, – 3)

$y = 0 \Rightarrow \quad 0 = x - 6 \quad \therefore \; x = 6$ **x- intercept** (6, 0)

Draw a straight line through the two points.



To find the gradient, rearrange the equation to the form of $y = mx + c$

$2y = x - 6.$ *Divide both sides by 2*

$y = ½\, x - 3.$

The gradient of the line is ½.

4. Sketch the graph of $3y + 2x = 0$, and state the gradient of the straight line.

Intercepts

$x = 0$ $\Rightarrow \quad 3y = 0 \quad \therefore \; y = 0$ **y- intercept** (0, 0).

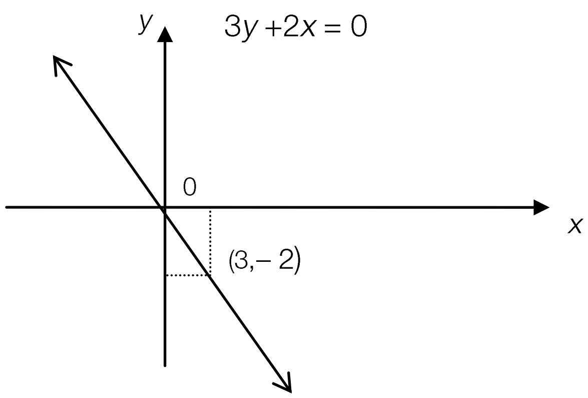
$y = 0$ $\Rightarrow \quad 0 = 2x \quad \therefore \; x = 0$ **x- intercept** (0, 0).

These are the same point. The line passes through the origin.

To find a second point substitute another value for x (or y).

$x = 3$ $\Rightarrow \quad 3y + 6 = 0 \quad \therefore \; y = -2.$

A second point is (3, –2)



Gradient:

$3y + 2x = 0$

$y = -2/3\, x$ *Rearrange to the form of* $y = mx + c$

The gradient of the line is – 2/3

5. Find the equation of the line that passes through the points (–1,3) and (2,0).

Gradient = $m = \dfrac{y_1 - y_2}{x_1 - x_2} = \dfrac{0 - 3}{2 - (-1)} = -1$

$m = -1$

The equation to a straight line is $y = mx + c$

$\therefore\ y = -x + c$

To find c substitute either of the points into the equation for the line.

Using the point (–1,3)

$3 = -(-1) + c$

$c = 2$

$\therefore$ Equation to the line is: $y = -x + 2$

6. Find the equation of the line that passes through the point (3,4) and parallel to the line $y = -2x + 7$. Give your answer in point slope form.

The point-slope form of the equation to a straight line is $y - y_1 = m(x - x_1)$

The gradient of the line $y = -2x + 7$ is -2, so $m = -2$.

The point is (3,4) so $x_1 = 3$ and $y_1 = 4$.

The equation to the line is $y - 4 = -2(x - 3)$

See Exercise 3

Exercise 1

Sketch the graphs of the following equations.

(a) $y = 4x + 3$. (b) $2y = -x + 6$ (c) $3y + 2x = 9$

(d) $y = 2x$ (e) $y = 7$ (f) $2y + x = 0$

Exercise 2

Find the gradient of the following straight lines.

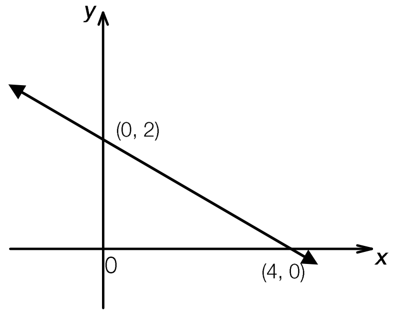
(a) $y = -5x$ (b) $2y = 6x + 9$ (c) $2x - 3y = 0$

(d) $3y + 2x - 2 = 0$ (e) $5y = 8$ (f) $x = -2$

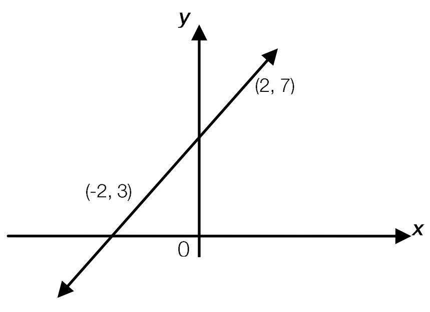
Exercise 3

Find the equation of the straight lines below.

(a)



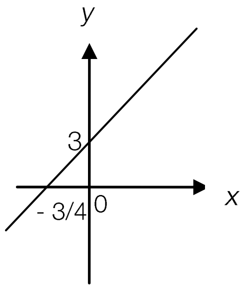
(b)



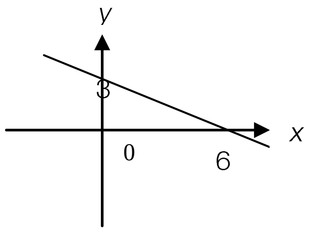
(c) Find the equation, in point-slope form, of the line with a gradient of 3, passing through the point (0, –5).

Answers

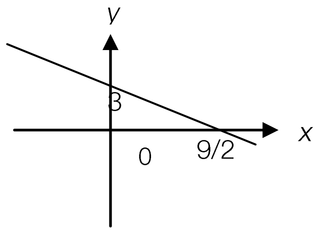
1(a) $y = 4x + 3$



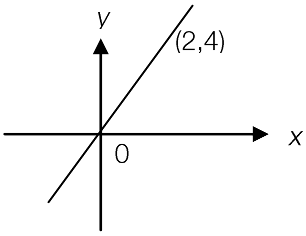
(b) $2y = -x + 6$



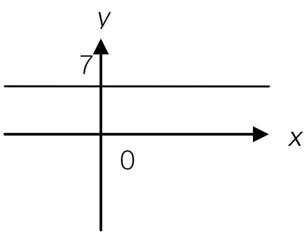
(c) $3y + 2x = 9$



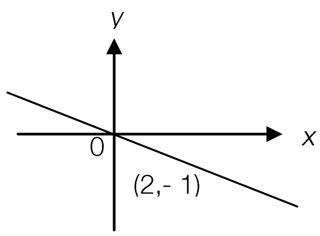
(d) $y = 2x$



(e) $y = 7$



(f) $2y + x = 0$



2. (a) -5 (b) 3 (c) 2/3 (d) - 2/3 (e) 0 (f) undefined

3. (a) $2y = -x + 4$ (b) $y = x + 5$ (c) $y + 5 = 3x$