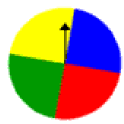
SAMPLE SPACES

A list or diagram showing all possible outcomes in a probability experiment is called a **sample space.**

$$\text{Then } \Pr(E) = \frac{\text{number of ways E can occur}}{\text{number of outcomes in the sample space}} = \frac{n(E)}{n(S)}$$

- For tossing a single die the sample space is 1, 2, 3, 4, 5, 6 and $\Pr(1) = \Pr(2) = \Pr(3) = \Pr(4) = \Pr(5) = \Pr(6) = \frac{1}{6}$

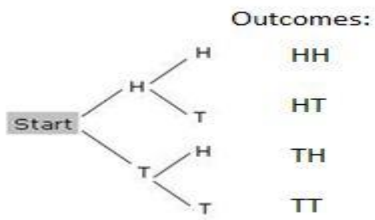


- For this spinner, which has 4 equal sectors, the sample space is Red, Green, Yellow, Blue And $\Pr(R) = \Pr(G) = \Pr(Y) = \Pr(B) = \frac{1}{4}$

NB: The sum of the probabilities of the distinct outcomes within a sample space is 1.

Tree diagrams

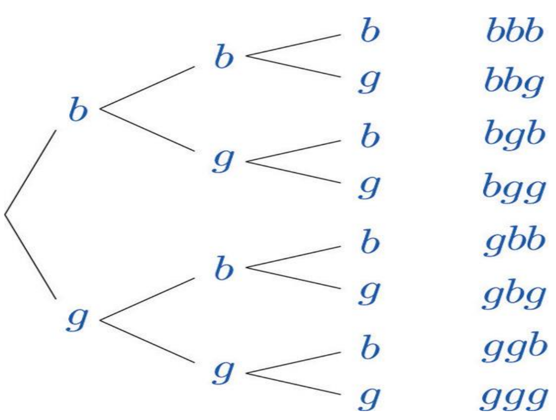
A tree diagram can be used to find the sample space.
For example, if two coins are tossed there are four possible outcomes:



The sample space for tossing two coins is HH HT TH TT

If E is the event 'at least one head' then $\Pr(E) = \Pr(HH \text{ or } HT \text{ or } TH) = \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{3}{4}$

The sample space for a three child family is shown below:



If E is the event 'first child a girl' then $\Pr(E) = \frac{4}{8} = \frac{1}{2}$

Other sample spaces and diagrams

Sample Space for Choosing a Card from a Deck

Ace ♥	2 ♥	3 ♥	4 ♥	5 ♥	6 ♥	7 ♥	8 ♥	9 ♥	10 ♥	Jack ♥	Queen ♥	King ♥
Ace ♦	2 ♦	3 ♦	4 ♦	5 ♦	6 ♦	7 ♦	8 ♦	9 ♦	10 ♦	Jack ♦	Queen ♦	King ♦
Ace ♠	2 ♠	3 ♠	4 ♠	5 ♠	6 ♠	7 ♠	8 ♠	9 ♠	10 ♠	Jack ♠	Queen ♠	King ♠
Ace ♣	2 ♣	3 ♣	4 ♣	5 ♣	6 ♣	7 ♣	8 ♣	9 ♣	10 ♣	Jack ♣	Queen ♣	King ♣

If a single card is drawn from the deck and

(a) D is the event 'the card is a diamond' then $\Pr(D) = \frac{13}{52} = \frac{1}{4}$

(b) E is the event 'the card is a diamond (D) or an ace (A)'

then $\Pr(E)$ $= \Pr\text{ (D or A)}$

$= \Pr(D \cup A)$

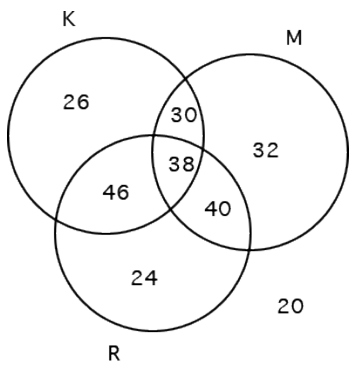
$= \Pr(D) + \Pr(A) - \Pr(D \cap A)$

$= \frac{13}{52} + \frac{4}{52} - \frac{1}{52}$

$= \frac{16}{52}$

$= \frac{4}{13}$

Tables and Venn diagrams can also be used to organise information that makes finding probabilities easier



The diagram shows the number of people in a survey of 256 who regularly ate Kit Kats, Mars Bars or Rocky Road

From the diagram we can see

$\Pr(K) = \frac{26+46+30+38}{256} = \frac{140}{256} = \frac{35}{64}$

$\Pr(M \cap R) = \frac{78}{256} = \frac{39}{128}$

$\Pr(\text{KitKat and MarsBar but not Rocky Road}) = \frac{30}{256} = \frac{15}{128}$

$\Pr(\text{at least one of these things}) = 1 - \frac{20}{256} = \frac{236}{256} = \frac{59}{64}$ [using complementary events]

	Lung Cancer yes	Lung Cancer no	Total
Smokers yes	30	70	100
Non-smokers no	10	90	100
	40	160	200

The table shows the results of a study that looked at the association between smoking (S) and lung cancer (C).

From the table we can see

$\Pr(S) = \frac{100}{200} = \frac{1}{2}$

$\Pr(C') = \frac{160}{200} = \frac{4}{5}$

$\Pr(S \cap C) = \frac{30}{200} = \frac{3}{20}$

Exercise

1. Use a tree diagram to find the sample space for a two child family. Hence find
   (a) The probability that both children are girls
   (b) The probability that the oldest child is a girl
   (c) The probability that at least one child is a girl

2. The diagram shows the sample space for tossing a single die twice.

		Second throw					
		1	2	3	4	5	6
First throw	1	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
	2	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
	3	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
	4	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
	5	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
	6	(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

   Find the probability that
   (a) the first toss is a 4
   (b) the sum of the two tosses is 5
   (c) at least one toss is a 6
   (d) neither toss is a 6

3. In a classroom of 20 Yr 12 VCE students 10 study Maths Methods, 7 study Specialist maths and 5 study both. Organise the information in a Venn diagram and find the probability that a student chosen at random
   (a) Studies neither of these maths subjects
   (b) Studies Maths Methods but not Specialist Maths

4. Find the probability that a card drawn at random from a pack is
   (a) A red card
   (b) Lower than a 5 (ace low)

Answers

1. (a) $\frac{1}{4}$ (b) $\frac{1}{2}$ (c) $\frac{3}{4}$
2. (a) $\frac{1}{6}$ (b) $\frac{1}{9}$ (c) $\frac{11}{36}$ (d) $\frac{25}{36}$
3. (a) $\frac{2}{5}$ (b) $\frac{1}{4}$
4. (a) $\frac{1}{2}$ (b) $\frac{4}{13}$