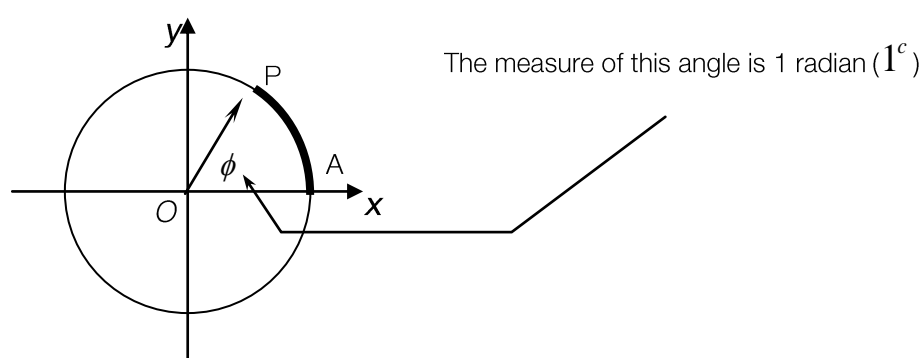


TR1.5: ANGULAR MEASUREMENT

Definition of a radian

So far all angles have been given in degrees. Another common unit of angular measurement is the **radian**.

If we move around the edge of a circle of radius = 1 (unit circle), a distance of 1 unit from A to P, in an anti-clockwise direction, then the angle AOP is formed.



Converting between radians and degrees

The circumference of a circle is: $2 \times \pi \times \text{radius}$ units

Circumference of the unit (radius=1) circle is: $2 \times \pi \times 1 = 2\pi$ units.

Moving around the edge of the unit circle a distance of 2π units from A forms a complete circle.

The angle, in **radians**, formed by one full revolution of a circle is $2\pi^c$.

The angle, in **degrees**, of a circle is 360° .

$$\therefore 2\pi^c = 360^\circ$$

$$\pi^c = 180^\circ$$

Rearranging the above statements gives:

$$1^c = \frac{180^\circ}{\pi} \quad \text{and} \quad 1^\circ = \frac{\pi^c}{180}$$

These two equations are used to convert between radians and degrees.

Examples

1. Convert 60° to radians.

$$1^\circ = \frac{\pi^c}{180}$$

$$60^\circ = \frac{\pi}{180} \times \cancel{60}^c$$

$$60^\circ = \frac{\pi^c}{3}$$

Using $\pi \approx 3.142$, $60^\circ \approx 1.05^c$

2. Convert 250° to radians.

$$1^\circ = \frac{\pi^c}{180}$$

$$250^\circ = \frac{\pi}{180} \times \cancel{250}_{25}^c$$

$$250^\circ = \frac{25\pi^c}{18}$$

Using $\pi \approx 3.142$, $250^\circ \approx 4.36^c$

See Exercise 1

3. Convert $\frac{\pi^c}{4}$ to degrees

$$1^c = \frac{180^\circ}{\pi}$$

$$\frac{\pi^c}{4} = \frac{\cancel{180}^{45}}{\cancel{\pi}} \times \frac{\pi^o}{\cancel{4}}$$

$$\frac{\pi^c}{4} = 45^\circ$$

4. Convert 6.5° to degrees

$$1^c = \frac{180^\circ}{\pi}$$

$$6.5^c = \frac{180}{\pi} \times 6.5$$

$$6.5^c = \frac{1170^\circ}{\pi}$$

Using $\pi \approx 3.142$, $6.5^c \approx 372.4^\circ$

See Exercise 2

Note: The symbol for radian, c , is often omitted.

Exercise

1. Convert the following degrees to radians

1. 30°

4. 450°

2. 270°

5. 135°

3. 20°

6. 57.3°

2. Convert the following radians to degrees

1. $\frac{\pi}{2}$

4. 3.5π

2. $\frac{5\pi}{4}$

5. π

3. $\frac{11\pi}{6}$

6. 1

Answers

1. Degrees to radians

1. $\frac{\pi}{6}$

4. $\frac{5\pi}{2}$

2. $\frac{3\pi}{2}$

5. $\frac{3\pi}{4}$

3. $\frac{\pi}{9}$

6. 1

2. Radians to degrees

1. 90°

4. 630°

2. 225°

5. 180°

3. 330°

6. 57.3°