

# SAMPLE SPACES

A list or diagram showing all possible outcomes in a probability experiment is called a **sample space**.

$$\text{Then } \Pr(E) = \frac{\text{number of ways E can occur}}{\text{number of outcomes in the sample space}} = \frac{n(E)}{n(S)}$$

- For tossing a single die the sample space is 1, 2, 3, 4, 5, 6 and  $\Pr(1) = \Pr(2) = \Pr(3) = \Pr(4) = \Pr(5) = \Pr(6) = \frac{1}{6}$



- For this spinner, which has 4 equal sectors, the sample space is Red, Green, Yellow, Blue  
And  $\Pr(R) = \Pr(G) = \Pr(Y) = \Pr(B) = \frac{1}{4}$

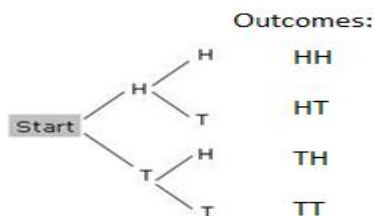


NB: The sum of the probabilities of the distinct outcomes within a sample space is 1.

## Tree diagrams

A tree diagram can be used to find the sample space.

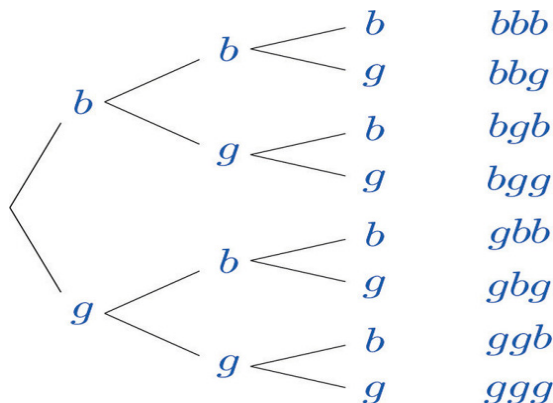
For example, if two coins are tossed there are four possible outcomes:



The sample space for tossing two coins is HH HT TH TT

If E is the event 'at least one head' then  $\Pr(E) = \Pr(HH \text{ or } HT \text{ or } TH) = \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{3}{4}$

The sample space for a three child family is shown below:



If E is the event 'first child a girl' then  $\Pr(E) = \frac{4}{8} = \frac{1}{2}$

## Other sample spaces and diagrams

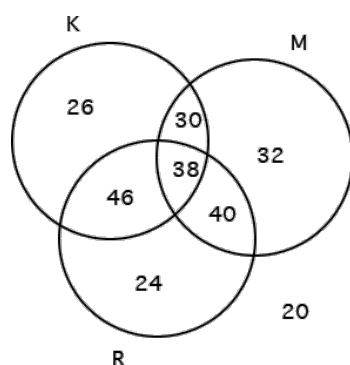
### Sample Space for Choosing a Card from a Deck

Ace	2	3	4	5	6	7	8	9	10	Jack	Queen	King
♥	♥	♥	♥	♥	♥	♥	♥	♥	♥	♥	♥	♥
Ace	2	3	4	5	6	7	8	9	10	Jack	Queen	King
♦	♦	♦	♦	♦	♦	♦	♦	♦	♦	♦	♦	♦
Ace	2	3	4	5	6	7	8	9	10	Jack	Queen	King
♠	♠	♠	♠	♠	♠	♠	♠	♠	♠	♠	♠	♠
Ace	2	3	4	5	6	7	8	9	10	Jack	Queen	King
♣	♣	♣	♣	♣	♣	♣	♣	♣	♣	♣	♣	♣

If a single card is drawn from the deck and

- (a) D is the event 'the card is a diamond' then  $\Pr(D) = \frac{13}{52} = \frac{1}{4}$
- (b) E is the event 'the card is a diamond (D) or an ace (A)'  
then  $\Pr(E) = \Pr(D \text{ or } A)$   
 $= \Pr(D \cup A)$   
 $= \Pr(D) + \Pr(A) - \Pr(D \cap A)$   
 $= \frac{13}{52} + \frac{4}{52} - \frac{1}{52}$   
 $= \frac{16}{52}$   
 $= \frac{4}{13}$

Tables and Venn diagrams can also be used to organise information that makes finding probabilities easier



The diagram shows the number of people in a survey of 256 who regularly ate Kit Kats, Mars Bars or Rocky Road

From the diagram we can see

$$\Pr(K) = \frac{26+46+30+38}{256} = \frac{140}{256} = \frac{35}{64}$$

$$\Pr(M \cap R) = \frac{78}{256} = \frac{39}{128}$$

$$\Pr(\text{KitKat and MarsBar but not Rocky Road}) = \frac{30}{256} = \frac{15}{128}$$

$$\Pr(\text{at least one of these things}) = 1 - \frac{20}{256} = \frac{236}{256} = \frac{59}{64}$$

[using complementary events]

	Lung Cancer yes	Lung Cancer no	Total
Smokers yes	30	70	100
Non-smokers no	10	90	100
	40	160	200

The table shows the results of a study that looked at the association between smoking (S) and lung cancer (C).

From the table we can see

$$\Pr(S) = \frac{100}{200} = \frac{1}{2}$$

$$\Pr(C') = \frac{160}{200} = \frac{4}{5}$$

$$\Pr(S \cap C) = \frac{30}{200} = \frac{3}{20}$$

### Exercise

- Use a tree diagram to find the sample space for a two child family. Hence find
  - The probability that both children are girls
  - The probability that the oldest child is a girl
  - The probability that at least one child is a girl
- The diagram shows the sample space for tossing a single die twice.

		Second throw					
		1	2	3	4	5	6
First throw	1	(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
	2	(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
	3	(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
	4	(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
	5	(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
	6	(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

Find the probability that

- the first toss is a 4
  - the sum of the two tosses is 5
  - at least one toss is a 6
  - neither toss is a 6
- In a classroom of 20 Yr 12 VCE students 10 study Maths Methods, 7 study Specialist maths and 5 study both. Organise the information in a Venn diagram and find the probability that a student chosen at random
    - Studies neither of these maths subjects
    - Studies Maths Methods but not Specialist Maths
  - Find the probability that a card drawn at random from a pack is
    - A red card
    - Lower than a 5 (ace low)

### Answers

- $\frac{1}{4}$
  - $\frac{1}{2}$
  - $\frac{3}{4}$
- $\frac{1}{6}$
  - $\frac{1}{9}$
  - $\frac{11}{36}$
  - $\frac{25}{36}$
- $\frac{2}{5}$
  - $\frac{1}{4}$
- $\frac{1}{2}$
  - $\frac{4}{13}$