FU1.1: FUNCTIONS AND RELATIONS

Relations

A relation is a set of *ordered pairs*.

For example (1, 2), (2, 6), (3, 4), (x, y) are ordered pairs.

The ***domain*** of a relation is the set of first elements or the ***x-values*** of the ordered pairs.
For the above ordered pairs the domain, $\mathbf{d_m} = \{1, 2, 3, x\}$.

The ***range*** of a relation is the set of second elements or the ***y-values*** of the ordered pairs.
For the above ordered pairs the range, $\mathbf{r_g} = \{2, 6, 4, y\}$.

There is often a *rule* that links the domain and range.

$$S = \{(x, y): \; y > x, \; x \in R\}$$

The set of

ordered pairs

the rule joining the domain and range.

any restrictions placed on the domain or range.

This relation, called S, says that all ordered pairs that have y greater than x are included in the relation.

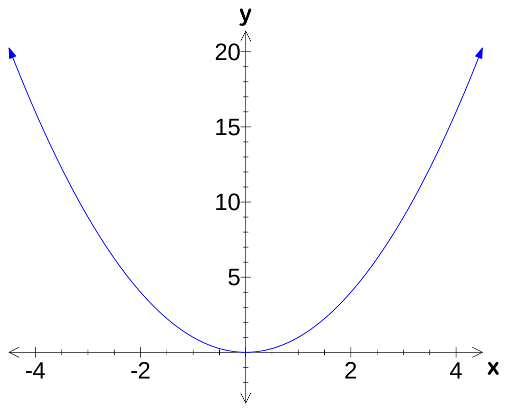
The domain of this relation is, $d_m = R$	$x \in R$ means x belongs to R
The range of this relation is, $r_g = R$.	R is the set of real numbers

Examples

1. Sketch the graph of the following relation and state the domain and range.

$$\{(x, y): y = x^2\}$$

In this example the rule joining the set of ordered pairs (x,*y)* is $y = x^2$



x can be any real number

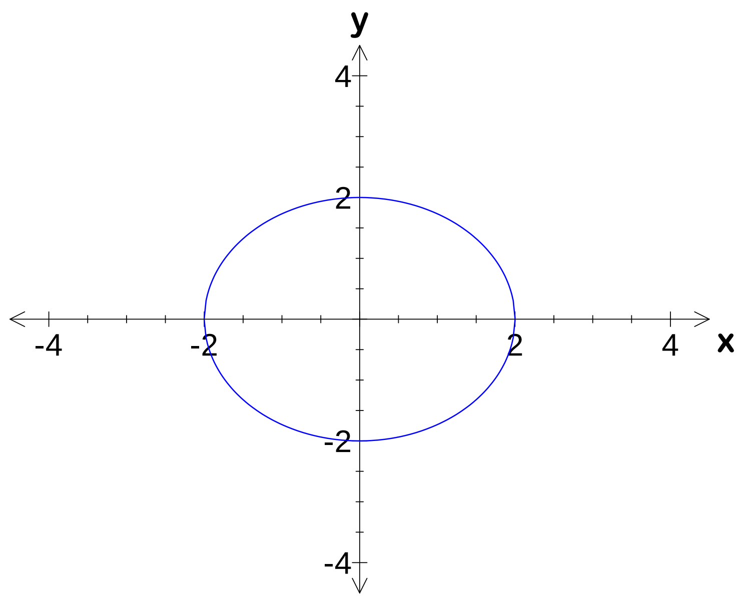
Domain = R

y must be $\geq 0.$

Range is $\{y : y \geq 0\}$

2. Sketch the graph of $x^2 + y^2 = 4$
   State the domain and range of this relation.

In this example the rule joining the set of ordered pairs (x,*y)* is $x^2 + y^2 = 4$



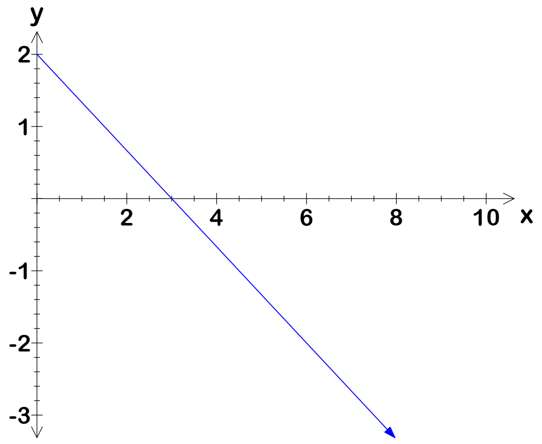
From the graph it can be seen that:

The domain is $\{x : -2 \leq x \leq 2\}$

The range is $\{y : -2 \leq y \leq 2\}$

3. Sketch the graph of $\{(x,\ y): 2x+3y=6,\ x \ge 0\}$
State the domain and range of this relation.

In this example the rule joining the set of ordered pairs (x,*y)* is $2x+3y=6$
The restriction $x \ge 0$ is placed on the domain.



The restriction $x \ge 0$ is specified in the statement of the relation.
If $x = 0$, $y = 2$ (from the relation $2x+3y=6$).

Therefore
The domain is $\{x : x \ge 0\}$ The range is $\{y : y \le 2\}$

The rule of a relation may be thought of as:

Domain → ***RULE*** → **Range**

Values taken from the domain ***produce*** values for the range, after passing through the rule that defines the relation.

See Exercise 1

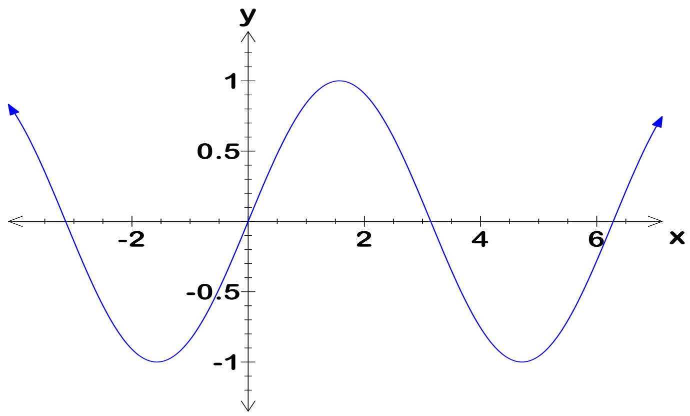
Functions

From some of the previous examples it can be seen that some values in the domain (x-values) may have many, or even an infinite number of corresponding values in the range (y-values).

A ***function*** is a special type of relation.
Each point in the domain of a function has a unique value in the range.
Every value of x may have only one value of y.

Examples

1. $F = \{(x, y) : y = \sin x, x \in R\}$



If we choose any possible value of x, there exists only one corresponding value of y.
$\therefore$ The relation F is a function.

Another way of writing this function is with *mapping notation.*

$$\mathrm{F} : X \to Y, \quad \text{where } \mathrm{F}(x) = \sin x.$$

the domain

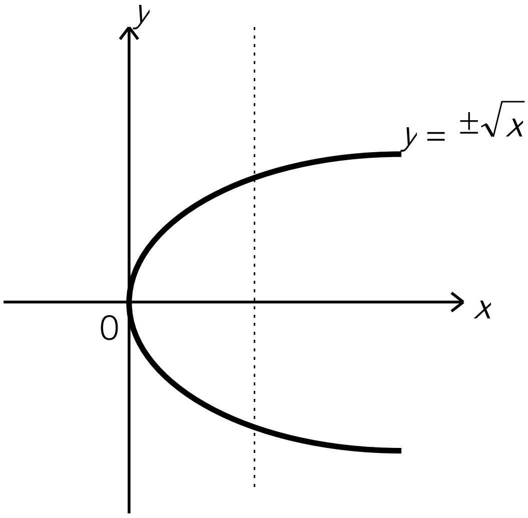
mapped onto the range Y.

rule

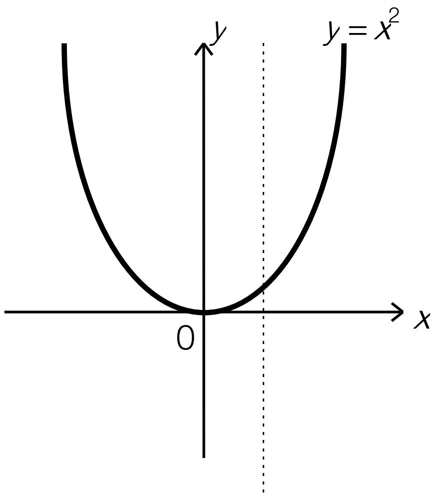
If only the rule is given then we assume that the domain is R.

When relations are represented graphically a *vertical line* test may be applied to decide if they are functions

2.



not a function – x value has two corresponding y values.



a function –x value has only one corresponding y value.

3. The relation $(-1,2),(-1,4),(1,6),(2,8),(3,10)$ is **not a function** because the value $x=-1$ has two corresponding values for *y*. (2 and 4)

4. The relation $(-1,1),(0,2),(1,3),(2,5),(3,7)$ **is a function** because each *x* value has only one corresponding *y* value.

See Exercise 2

Implied Domain

If only the rule of a function is given then we assume that the domain is *R* unless otherwise defined implicitly by the function.

Examples

1. If a function involves a square root the domain, in the real number system, is restricted to those values of *x* that result in a positive number or zero under the square root sign..

The domain of the function $y=+\sqrt{x-4}$ is restricted such that $x-4\geq 0$

The domain is $\{x: x\geq 4\}$

The domain of the function $y=+\sqrt{9-x^2}$ is restricted such that $9-x^2\geq 0$

The domain is $\{x: -3\leq x\leq 3\}$

2. If the function involves a fraction the value the in the denominator must be greater than zero.

The domain of the function $y=\dfrac{3}{x+5}$ is restricted such that $x+5\neq 0$

The domain is $\{x: x$ is a real number and $x\neq -2\}$

The domain of the function $y=\dfrac{3}{2x-8}$ is restricted such that $2x-8\neq 0$

The domain is $\{x: x$ is a real number and $x\neq 4\}$

See Exercise 3

Exercises

Exercise 1.

State the domain and range of the following relations.

(a) $(-2,1),(0,2),(2,5),(2,7),(3,9)$ (b) $(4,1),(5,2),(6,3)$

(c) $x^2+y^2=25$ (d) $\{(x,\ y):2y=6-5x,\ x\geq 2\}$

Exercise 2

Which of the following relations are functions?

(a) $\{(x,\ y):y=2x+4,\ \}$ (b) $\{(x,\ y):y=4-x^2\}$

(c) $\{(x,\ y):x^2+y^2=36\}$ (d) $\{(x,\ y):y=7\}$

(e) $\{(x,\ y):x=-2\}$ (g) $\{(x,\ y):y=-\sqrt{4-x^2}\}$

Exercise 3

State the domain of the following functions.

(a) $\{(x,\ y):y=x+2\}$ (b) $\{(x,\ y):y=4-x^2\}$

(c) $\{(x,\ y):y=+\sqrt{4-x}\}$ (d) $\left\{(x,\ y):y=\dfrac{3}{x+2}\right\}$

(e) $\left\{(x,\ y):y=\dfrac{5}{\sqrt{x\ -7}}\right\}$ (f) $\left\{(x,\ y):y=\dfrac{1}{x+2}-\dfrac{3}{x-4}\right\}$

Answers

Exercise 1

(a) dm = {- 2, 0 2, 3}. rg = {1, 2, 5, 7, 9}. (b) dm = {4, 5, 6}. rg = {1, 2, 3}.

(c) dm $=\{x:-5\leq x\leq 5\ \}$ rg $\{y:-5\leq y\leq 5\ \}$

(d) dm = $\{x\geq 2\}$ rg = $\{y:y\leq -2\}$

Exercise 2

(a), (b), (d), (g)

Exercise 3

(a) $\{R\}$ (b) $\{R\}$ (c) $\{x\leq 4\}$ (d) $\{x:x$ is a real number and $x\neq -2\}$

(e) $\{x:\ x>7\}$ (f) $\{x:x$ is a real number and $x\neq -2$ and $x\neq -4\}$