FA1.1: FACTORISATION: COMMON FACTORS

Expansion of brackets (or *removing brackets*) in an algebraic expression is done by multiplying *all* the terms inside the brackets by the term(s) outside the brackets.

e.g

$$5a(3y - 2x) = 15ay - 10ax$$

Each term inside the brackets has been multiplied by $5a$.

Factorisation is the reverse of expansion. To *factorise* a number or algebraic expression means to write the number or expression as a *product* (multiplication).

Examples

1. $-2xyz$ has factors $-2,$ $x, y,$ and z

2. $7(a+b)$ has factors 7 and $(a+b)$

3. $3(x-y)$ has factors 3 and $(x-y)$

4. $x(3a+2b)$ has factors x and $(3a+2b)$

5. $2 p(2r+1)(s+4)$ has factors 2, p, $(2r+1)$ and $(s+4)$
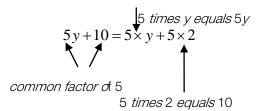
> *Expansion* means *removing brackets*
>
> *Factorisation* means *inserting brackets*

Factorisation by removing a common factor

- Search each term in the expression for a common factor (**every term** must have this factor)
- There may be several common factors. Search until you have found all of them
- If there is more than one common factor multiply them to give highest common factor. (HCF)
- The HCF is placed before the bracket
- The terms inside the bracket are found by dividing each term by the HCF.

Examples

1. $5y + 10$

$$5y + 10 = 5 \times y + 5 \times 2$$

5 times y equals 5y

factorise, common factor 5

common factor of 5

5 times 2 equals 10

$$5y + 10 = 5(y + 2)$$

5 before brackets, (y+2) inside brackets

2. $3x + 3y$

$$3x + 3y = 3 \times x + 3 \times y$$
$$= 3(x + y)$$

common factor 3

3. $p^2 + p$

$$p^2 + p = p \times p + p \times 1$$
$$= p(p + 1)$$

common factor p

4. $7y^2 + 7y$

$$7y^2 + 7y = 7y \times y + 7y \times 1$$
$$= 7y(y + 1)$$

common factors 7 and y HCF=7y

5. $2ab - 12a$

$$2ab - 12a = 2a \times b - 2a \times 6$$
$$= 2a(b - 6)$$

common factor 2, a, and c. HCF = 2ac

See Exercise 1

A common factor may be negative.

Examples

$$-2a - 2b$$
$$-2a - 2b = (-2) \times a + (-2) \times b$$
$$= -2(a + b)$$

common factor - 2

$$-3x + 6xy$$
$$-3x + 6xy = (-3x) \times 1 - (-3x) \times 2y$$
$$= -3x(1 - 2y)$$

HCF = - 3x

See Exercise 2

Exercises

Exercise 1.

Factorise the following (if possible)

a) $3x + 3y$

b) $2a - 2b$

c) $8a - 8b + 8c$

d) $xy - 5x$

e) $x^2 - x$

f) $7x + 21y$

g) $5m - 2n$

h) $c^2 - 2bc - 3c$

i) $5mn - 10n$

j) $3m^2 - 3mnp$

k) $7x + 21x^2$

l) $12m^2 - 18mn$

m) $5xy - 10xz$

n) $5pq - pq^2 - 3pqr$

o) $2ab^2c + 6abc^2$

p) $rst + 5rst^2 - 2rs$

q) $5mn + 10m - pqr$

r) $5xyz - x^2yz^2 + 10x$

Exercise 2

Factorise the following by removing a negative factor.

a) $-3x - 6y$

b) $-15xy + 25xz$

c) $-2xy + 4xyz$

d) $14xyz - 7xy$

e) $-6xyz - 15yz - 3xy^2z$

Answers

Exercise 1

a) $3(x + y)$

b) $2(a - b)$

c) $8(a - b + c)$

d) $x(y - 5)$

e) $x(x - 1)$

f) $7(x + 3y)$

g) no factors

h) $c(c - 2b - 3)$

i) $5n(m - 2)$

j) $3m(m - np)$

k) $7x(1 + 3x)$

l) $6m(2m - 3n)$

m) $5x(y - 2z)$

n) $pq(5 - q - 3r)$

o) $2abc(b + 3c)$

p) $rs(t + 5t^2 - 2)$

q) no factors

r) $x(5yz - xyz^2 + 10)$

Exercise 2

a) $-3(x + 2y)$

b) $-5x(3x - 5z)$

c) $-2xy(1 - 2z)$

d) $-7xy(-2z + 1)$

e) $-3y(2xz + 5z + xyz)$