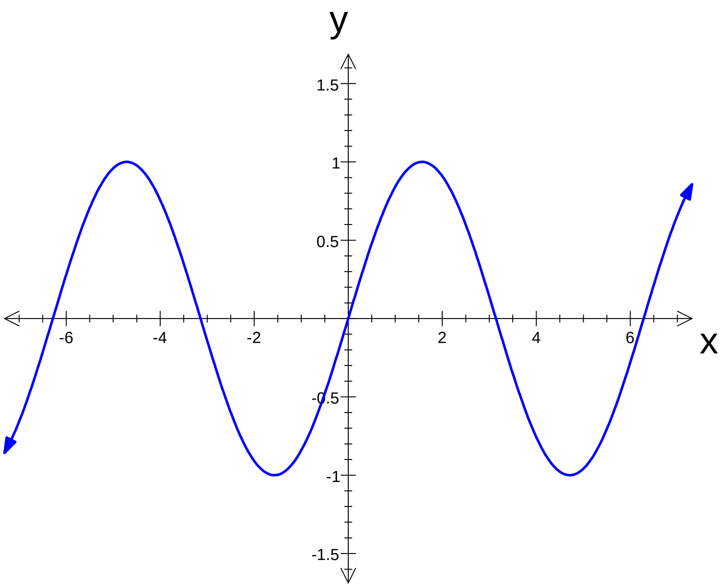
TR1.7: GRAPHS OF SINE AND COSINE FUNCTIONS

Both the functions $y = \sin x$ and $y = \cos x$ have a domain of R and a range of $[-1,1]$ The graphs of both functions have an **amplitude of** 1 and **a period of** 2π **radians**. (repeats every 2π units).

(Remember $\pi \approx 3.142$ so $2\pi \approx 6.284$)

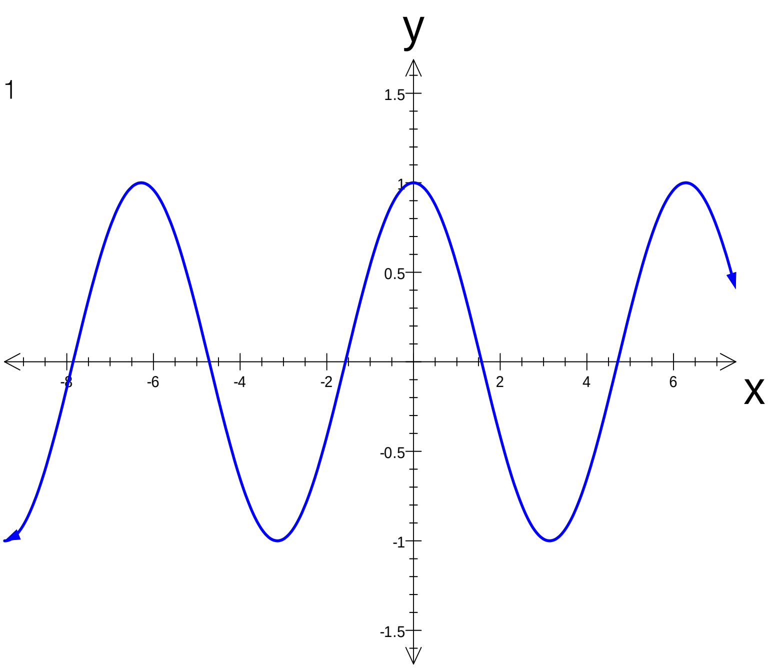
Sine function y = sin(x)

$y = \sin x$
amplitude = 1
period = 2π



Cosine function y = cos(x)

$y = \cos x$
amplitude = 1
period = 2π



$y = a\sin(nx)$, $y = a\cos(nx)$

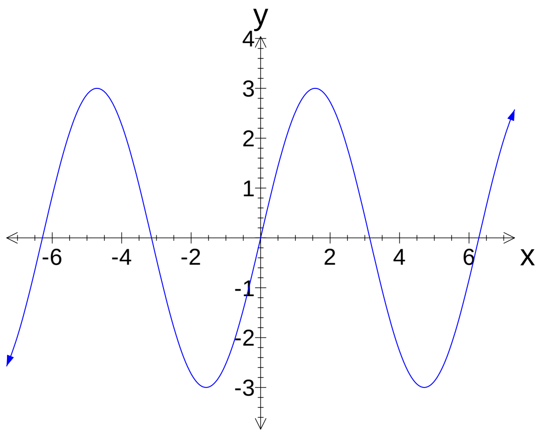
The graphs of both $y = a\sin nx$ and $y = a\cos nx$ have:

amplitude = 'a'

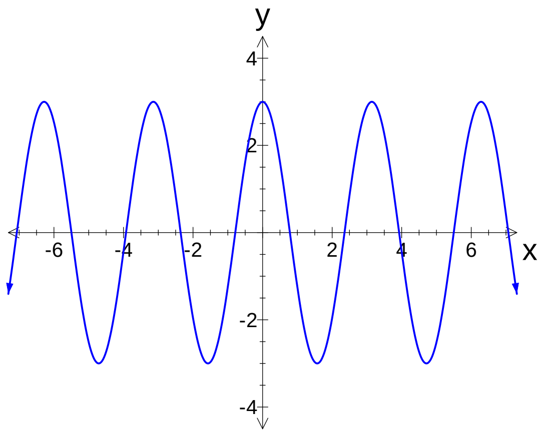
period = $\dfrac{2\pi}{n}$

Examples

$y = 3\sin x$



$y = 3\cos 2x$



$y = 3\sin x$ has:
an amplitude of 3 (a = 3)
a period of 2π , (n = 1)

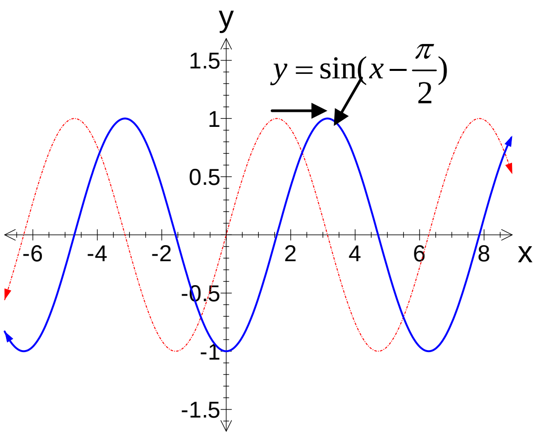
The graph of $y = 3\cos 2x$ has:
an amplitude of 3 (a = 3)
a period of $\dfrac{2\pi}{2} = \pi$, (n = 2)

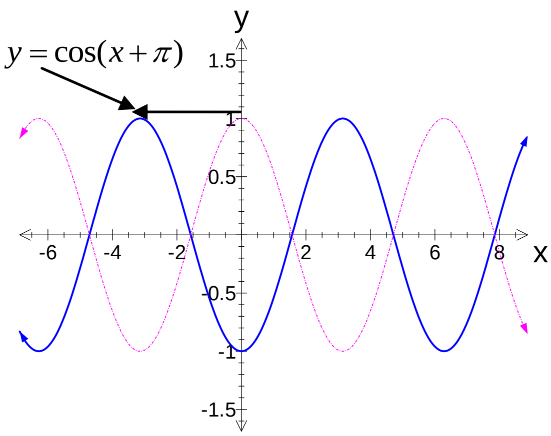
$y = a\sin(x - \phi)$, $y = a\cos(x - \phi)$

Replacing x with $(x - \phi)$ shifts the graphs of $y = \sin x$ and $y = \cos x$ horizontally ϕ units right.

$y = a\sin(x + \phi)$, $y = a\cos(x + \phi)$

Replacing x with $(x + \phi)$ shifts the graphs of $y = \sin x$ and $y = \cos x$ horizontally ϕ units left.





$y = \sin(x - \dfrac{\pi}{2})$ is the graph of $y = \sin x$ (dotted red), shifted $\dfrac{\pi}{2}$ ($\approx$ 1.571) **right.**

Amplitude = 1 Period = 2π

$y = \cos(x + \pi)$ is the graph of $y = \cos x$ (dotted red), shifted π ($\approx$ 3.142) **left.**

Amplitude = 1 Period = 2π

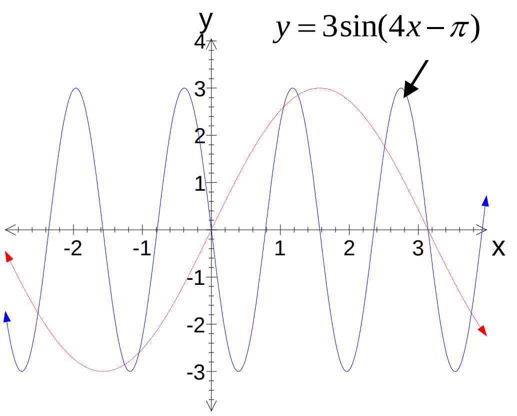
Examples

$y = a\sin(nx - \phi)$, $y = a\cos(nx - \phi)$

To graph $y = a\sin(nx - \phi)$ or $y = a\cos(nx - \phi)$ first rearrange to the form $y = a\sin n(x - \frac{\phi}{n})$ or $y = a\cos n(x - \frac{\phi}{n})$ respectively by taking a factor of '*n*' from (*nx* – *ϕ*).

The graph of $y = a\sin n(x - \frac{\phi}{n})$ has amplitude '*a*', period $\frac{2\pi}{n}$ and shifted horizontally $\frac{\phi}{n}$ units right from the basic sine graph with amplitude *a* and period $\frac{2\pi}{n}$	The graph of $y = a\cos n(x - \frac{\phi}{n})$ has amplitude '*a*', period $\frac{2\pi}{n}$ and shifted horizontally $\frac{\phi}{n}$ units right from the basic cosine graph with amplitude *a* and period $\frac{2\pi}{n}$

Example



For example to sketch the graph of $y = 3\sin(4x - \pi)$ rearrange to:

$y = 3\sin 4(x - \frac{\pi}{4})$.

The period is $\frac{2\pi}{4} = \frac{\pi}{2}$.

The amplitude is 3.

There is a horizontal shift right of $\frac{\pi}{4}$.

Dotted red graph is $y = 3\sin 4(x)$

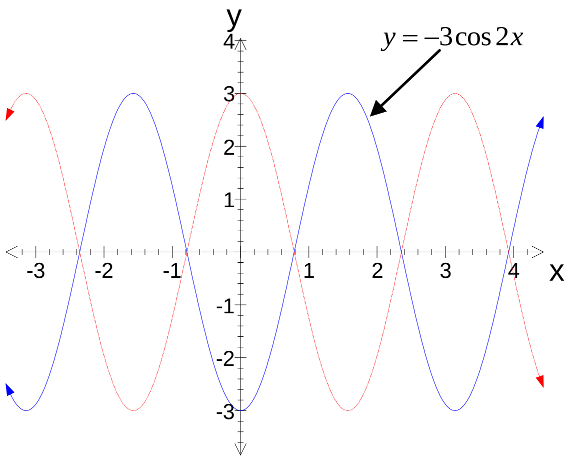
Similarly to sketch the graphs of $y = a\sin(nx + \phi)$ and $y = a\cos(nx + \phi)$, first rearrange to $y = a\sin n(x + \frac{\phi}{n})$ and $y = a\cos n(x + \frac{\phi}{n})$.

Reflection

If the coefficient of the function is negative the graph will be reflected about the *x* – axis.
The amplitude remains positive.

Example

$y = -3\cos 2x$



The graph of $y = -3\cos 2x$ has an amplitude of 3 and a period of π.

It is a reflection of the graph of $y = 3\cos 2x$, (dotted red), about the x – axis.

Exercises

1.Sketch the graph the following for one complete cycle stating the amplitude and period:

(a) $y = 2\cos x$ (b) $y = 2\sin 3x$ (c) $y = \frac{1}{2}\sin 2x$ (d) $y = 3\cos\frac{x}{2}$ (e) $y = -2\sin 3x$

2. Sketch the graph the following for one complete cycle stating the amplitude and period.

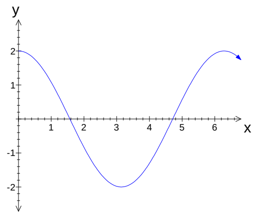
(a) $y = 2\sin(x-\pi)$ (b) $y = \cos(x+\frac{\pi}{2})$

3. Sketch the graph the following for one complete cycle stating the amplitude and period.

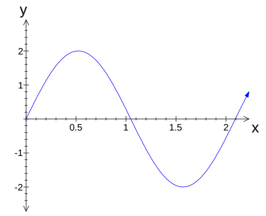
(a) $y = 2\sin(3x-\pi)$ (b) $y = 3\cos(4x-2\pi)$ (c) $y = 2\sin(2x+\frac{\pi}{3})$

Answers

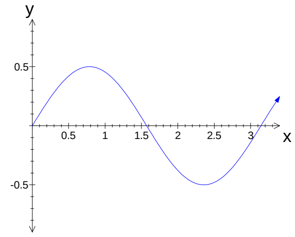
1(a)
Amplitude = 2. Period = 2π



1(b)
Amplitude = 2. Period = $2\pi/3$

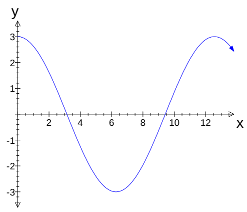


1(c)
Amplitude = 0.5. Period = π



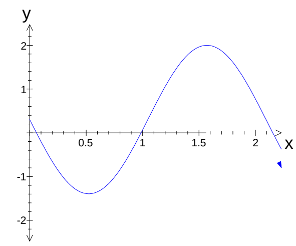
1(d)

Amplitude = 3. Period = 4π

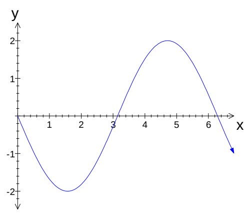


1(e)

Amplitude = 2. Period = $2\pi/3$

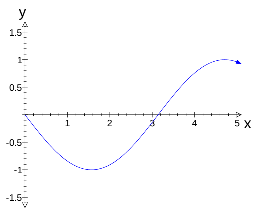


2(a)
Amplitude = 2. Period = 2π

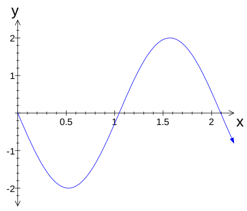


2(b)

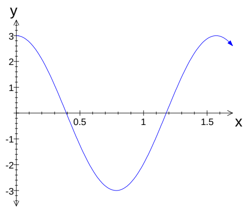
Amplitude = 1. Period = 2π



3(a)
Amplitude = 2. Period = $2\pi/3$



3(b)
Amplitude = 3. Period = $\pi/2$



3(c)
Amplitude = 2. Period = π