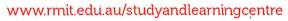
STUDY AND LEARNING CENTRE



STUDY TIPS

WORKED SOLUTIONS



LT2.1 LAPLACE TRANSFORMS: ELECTRICAL CIRCUITS

Question

For an LRÇ circuit given by

$$L\frac{di}{dt} + Ri + \frac{1}{C}\int idt = E_0 \qquad \frac{dq}{dt}(0) = q(0) = 0 \qquad \cdots \qquad \boxed{0}$$

and L = 1, R = 3, C = 0.5, $E_0 = 10$ find

- (a) the charge q(t) on the capacitor
- (b) the resulting current i(t) in the circuit at time t using Laplace transforms.

Solution

$$L \frac{d^2q}{dt^2} + R \frac{dq}{dt} + \frac{q}{c} = E_0 \dots 2$$

Note: Sidt = Sdq. dt = Sdq = q

Taking Laplace Transforms of (2) and substituting for L,R,C & E₀: $(5^2 Q(5) - 5 Q(0) - 9(0)) + 3 (5 Q(5) - 9(0)) + 2 Q(5) = 10/5$

Substituting initial values
$$dg_t(0) = q(0) = 0$$
 gives

$$5^{2}Q(5) + 35Q(5) + 2Q(5) = \frac{1}{5}$$

$$\Rightarrow$$
 Q(s) $[5^2 + 35 + 2] = \frac{10}{5} \Rightarrow Q(s) = \frac{10}{5(5+1)(5+2)}$

Resolving into partial fractions gives

$$10 = \frac{A}{S} + \frac{B}{(S+1)} + \frac{C}{(S+2)}$$
 ... (3)

Setting S=0: $10 = A(1)(2) = 2A \Rightarrow A = 5$

Substituting back into 3 gives

$$Q(s) = \frac{5}{s} - \frac{10}{(s+1)} + \frac{5}{s+2}$$

Taking inverse Laplace transforms:

$$q(t) = L^{-1} \left[\frac{5}{5} - \frac{10}{(5+1)} + \frac{5}{(5+2)} \right]$$

$$q(t) = 5 - 10e^{-t} + 5e^{-2t}$$

(b) The resulting current i(t) at time t is

$$i(t) = \frac{dq}{dt} = \frac{d}{dt}(5 - 10e^{-t} + 5e^{-2t}) = 10e^{-10}e^{-2t}$$

Note: We could have found the current by taking Laplace transforms of (1), however the mathematics is more complicated

This question could have been solved more casily using a 2nd order DE had the question not asked specifically for its solution using Laplace transforms.