RMIT UNIVERSITY

MATHS FOR PRICES & MARKETS:

DERIVATIVE FUNCTIONS

A derivative is a formula derived from the equation of a function which can be used to show whether that function is increasing, decreasing or stationary.

Some symbols that are used to indicate a derivative function are $\frac{dy}{dx}$ or $\frac{\partial TR}{\partial Q}$ or $\frac{\partial}{\partial Q}(TC)$

$\frac{dy}{dx}$ is read as "the derivative of y" and y will be expressed in terms of x

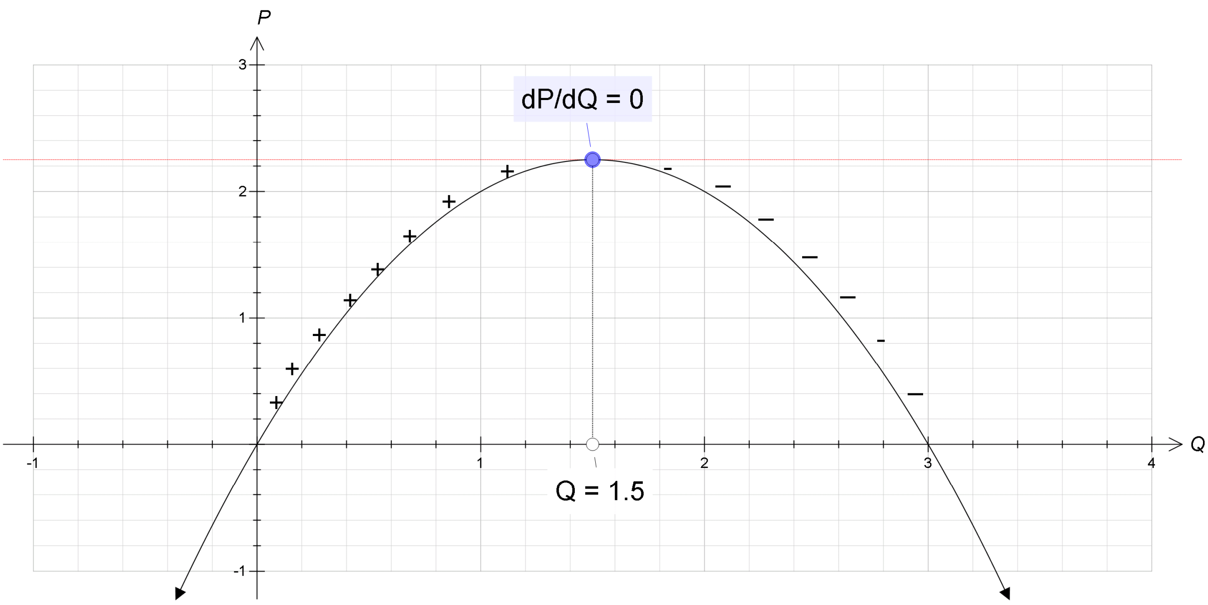
eg. $y = 10x^2 + 2x$ and by the rules below $\frac{dy}{dx} = 20x + 2$

$\frac{\partial TR}{\partial Q}$ is read as " the derivative of total revenue" and revenue will be a function of Q

eg. $TR = 200Q - 15Q^2$ and by the rules below $\frac{\partial TR}{\partial Q} = 200 - 30Q$

$\frac{\partial}{\partial Q}(TC)$ is read as "the derivative of total cost"

Consider the graph of the function with equation $P = 3Q - Q^2$ for values of Q between 0 and 3



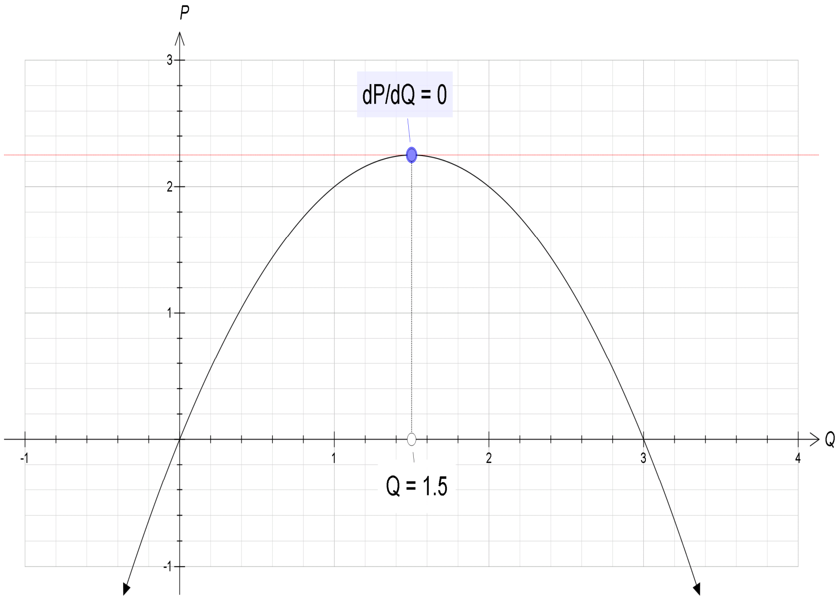
Between Q = 0 and Q = 1.5 the function is increasing and its derivative will be a positive value

Between Q = 1.5 and Q = 3 the function is decreasing and its derivative will be a negative value

At Q = 1.5 the function is neither increasing nor decreasing – it is stationary.

We can use derivatives to find where a function has its minimum or maximum values.

Maximum or minimum values of a function occur where the value of the function's derivative is zero.



The maximum value of this function occurs where Q = 1.5

Rules for finding derivative functions

[n drops down]

1. If $y = x^n$, then $\frac{\partial y}{\partial x} = nx^{n-1}$ ← [power loses 1]

   Eg $y = x^2 \Rightarrow \frac{\partial y}{\partial x} = 2x^{2-1}$ or $\frac{\partial y}{\partial x} = 2x$ [since $x^1 = x$]

2. If y = k, then $\frac{\partial y}{\partial x} = 0$

   Eg $y = 30 \Rightarrow \frac{\partial y}{\partial x} = 0$ [y never changes as the 0 value for the derivative indicates]

3. If $y = kx^n$, then $\frac{\partial y}{\partial x} = knx^{n-1}$ [NB: If y = kx , then $\frac{\partial y}{\partial x} = k$]

   Eg $y = 30x^2 \Rightarrow \frac{\partial y}{\partial x} = 30.2x^{2-1}$ or $\frac{\partial y}{\partial x} = 60x$

4. If functions are added/subtracted their corresponding derivatives are added/subtracted

   Eg $y = x^2 + 10x - 20 \Rightarrow \frac{\partial y}{\partial x} = 2x^{2-1} + 10x^{1-1} - 0$ or $\frac{\partial y}{\partial x} = 2x + 10$

Examples

1. *Find the derivative function for $Z = U^5$*

$$\frac{\partial Z}{\partial U} = 5U^{5-1} = 5U^4$$

2. *Find the derivative function for $P = Q^6 + Q^2$*

$$\frac{\partial P}{\partial Q} = 6Q^{6-1} + 2Q^{2-1} = 6Q^5 + 2Q$$

3. *Find the derivative function for $W = \frac{V^8}{V}$*

$$W = \frac{V^8}{V} = V^7$$ [First simplify the function]

$$\frac{\partial W}{\partial V} = 7V^{7-1} = 7V^6$$

4. *Find the derivative function for $H = 10F^2(F - 2)$*

$$H = 10F^2(F - 2) = 10F^2xF - 10F^2x2 = 10F^3 - 20F^2$$ [First simplify the function]

$$\frac{\partial H}{\partial F} = 10x3F^{3-1} - 20x2F^{2-1}$$

$$= 30F^2 - 40F$$

5. *Find the derivative function for $TC = 0.5Q^3 - 10Q^2 + 120Q + 500$*

$$\frac{\partial TC}{\partial Q} = 0.5x3Q^{3-1} - 10x2Q^{2-1} + 120Q^{1-1} + 0 = 1.5Q^2 - 20Q + 120$$

6. *Find the maximum value of the function $C = 200Q - 10Q^2$*

$$\frac{\partial C}{\partial Q} = 200 - 10x2Q^{2-1} = 200 - 20Q$$

The maximum value will occur when $\frac{\partial C}{\partial Q} = 0$

ie $200 - 20Q = 0$

ie $200 - 200 - 20Q = 0 - 200$

ie $-20Q = -200$

ie $Q = \frac{-200}{-20}$

ie $Q = 10$

To find the maximum value of C we substitute this value into $C = 200Q - 10Q^2$

When $Q = 10$, $C = 200x10 - 10x10^2$

$= 2000 - 1000$

$= 1000$

Exercises

Exercise 1

1) Find the derivative of each of the following functions
   a. $y = x^7$
   b. $Z = Q^4$
   c. $P = 20Q^6$
   d. $y = 100x$
   e. $M = 200 + 5Q^2$
   f. $R = 12.5Q^2 - Q$
   g. $C = 0.8d^2$
   h. $I = 250J^3 - 80J^2 + 32J$
   i. $TR = 8Q - 0.2Q^2$
   j. $P = 60 - 0.5Q$

2) Differentiate each of the following
   a. $Z = \dfrac{X^6}{X^2}$
   b. $M = \dfrac{Q^2 + 10Q}{Q}$
   c. $P = \dfrac{10Q + 5}{10}$
   d. $TC = 4Q^3 - 10Q^2 + 0.5Q + 100$

3) Find $\dfrac{\partial P}{\partial Q}$ if $20P - 40Q - 70 = 0$ [Hint: First make 'P' the subject of the equation]

4) Find $\dfrac{\partial}{\partial Q}(TR)$ if $TR = (200 - 5Q)Q$

5) Find $\dfrac{\partial P}{\partial Q}$ and $\dfrac{\partial Q}{\partial P}$ if $5P + 2Q - 30 = 0$

Exercise 2

Find the minimum or maximum value for each of the following functions

1) $Z = 60X - 2X^2$
2) $TP = 200Q - 4Q^2$
3) $y = x^2 - 6x + 6$
4) $Q = 120P - 2.5P^2$
5) $M = 5 - 0.1P^2 + P$

Answers

Exercise1

1) a. $\dfrac{\partial y}{\partial x} = 7x^6$ b. $\dfrac{\partial Z}{\partial Q} = 4Q^3$ c. $\dfrac{\partial P}{\partial Q} = 120Q^5$ d. $\dfrac{\partial y}{\partial x} = 100$ e. $\dfrac{\partial M}{\partial Q} = 10Q$

f. $\dfrac{\partial R}{\partial Q} = 25Q - 1$ g. $\dfrac{\partial C}{\partial d} = 1.6d$ h. $\dfrac{\partial I}{\partial J} = 750J^2 - 160J + 32$ i. $\dfrac{\partial TR}{\partial Q} = 8 - 0.4Q$ j. $\dfrac{\partial P}{\partial Q} = -0.5$

2) a $\dfrac{\partial Z}{\partial X} = 4X^3$ b. $\dfrac{\partial M}{\partial Q} = 1$ c. $\dfrac{\partial P}{\partial Q} = 1$ d. $\dfrac{\partial TC}{\partial Q} = 12Q^2 - 20Q + 0.5$

3) $P = 3.5 + 2Q$, $\dfrac{\partial P}{\partial Q} = 2$

4) $TR = 200Q - 5Q^2$, $\dfrac{\partial TR}{\partial Q} = 200 - 10Q$

5) $P = 6 - 0.4Q$, $\dfrac{\partial P}{\partial Q} = -0.4$ and $Q = 15 - 2.5P$, $\dfrac{\partial Q}{\partial P} = -2.5$

Exercise 2

1. Max value occurs when X = 15, Z = 450
2. Max value occurs when Q = 25, TP = 2500
3. Min value occurs when x = 3, y = -3
4. Max value occurs when P = 24, Q = 1440
5. Max value occurs when P = 5, M = 7.5