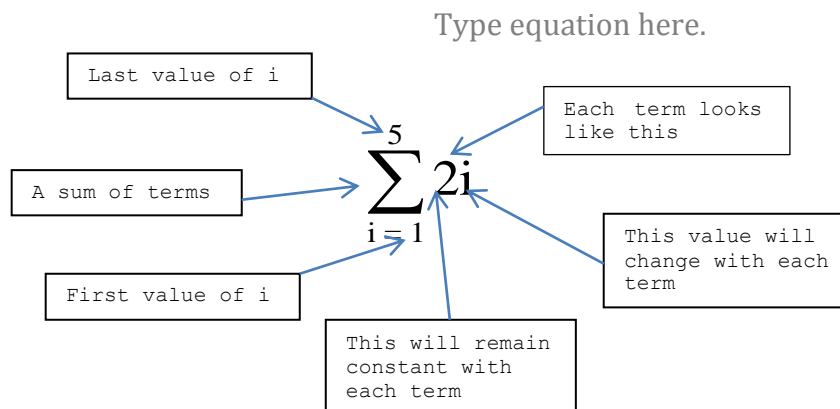# SUMMATION NOTATION

Summation notation or sigma notation is a shorthand method of writing the sum or addition of a string of similar terms. A typical element of the sequence which is being summed appears to the right of the summation sign.

Type equation here.

| Last value of i |
| A sum of terms |
| First value of i |

$$\sum_{i=1}^{5} 2i$$

| Each term looks like this |
| This value will change with each term |
| This will remain constant with each term |

To expand we replace i by its starting value (below the sigma symbol) and obtain each successive term by adding 1 to the previous value until the final value of i (above the sigma symbol)

For the above sequence:

$$\sum_{i=1}^{5} 2i = 2 \times 1 + 2 \times 2 + 2 \times 3 + 2 \times 4 + 2 \times 5 = 30$$

Examples:

1. *Expand and evaluate* $\displaystyle\sum_{i=0}^{3}(i^2 - 3)$

$$\sum_{i=0}^{3}(i^2 - 3) = (0^2 - 3) + (1^2 - 3) + (2^2 - 3) + (3^2 - 3)$$
$$= (-3) + (-2) + 1 + 6$$
$$= 2$$

2. Given the set of data $x_1 = 1$, $x_2 = 2$, $x_3 = 4$, $x_4 = 5$ evaluate

(a) $\displaystyle \bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$

(b) $\displaystyle s^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}$

(a) $\bar{x} = \dfrac{\sum\limits_{i=1}^{n} x_i}{n}$

$= \dfrac{x_1 + x_2 + x_3 + x_4}{n}$

$= \dfrac{1 + 2 + 4 + 5}{4}$

$= 3$

(b) $s^2 = \dfrac{\sum\limits_{i=1}^{n}\left(x_i - \bar{x}\right)^2}{n-1}$

$= \dfrac{\left(x_1 - \bar{x}\right)^2 + \left(x_2 - \bar{x}\right)^2 + \left(x_3 - \bar{x}\right)^2 + \left(x_4 - \bar{x}\right)^2}{4-1}$

$= \dfrac{\left(1-3\right)^2 + \left(2-3\right)^2 + \left(4-3\right)^2 + \left(5-3\right)^2}{4-1}$

$= \dfrac{4+1+1+4}{3}$

$= \dfrac{10}{3}$

NB: If n is not specified then it is assumed to be the number of scores or values.
$\sum x$ means the sum of all the scores.

Exercise

1. Find (a) $\displaystyle\sum_{i=1}^{3}(5i - 2)$ (b) $\displaystyle\sum_{i=1}^{3}(5i) - 2$

2. Given $x_1 = -2$, $x_2 = 0$, $x_3 = 1$, $x_4 = 3$, $x_5 = 3$

   find (a) $\displaystyle\sum_{i=1}^{5} 10x_i$ (b) $10\displaystyle\sum_{i=1}^{5} x_i$ (c) $\displaystyle\sum_{i=1}^{5}(x_i)^2$ (d) $\left(\displaystyle\sum_{i=1}^{5} x_i\right)^2$

   (e) $\displaystyle\sum_{i=1}^{5} i(x_i)$ (f) $\bar{x} = \dfrac{\sum\limits_{i=1}^{n} x_i}{n}$

Answers: 1. (a) 24   (b) 28
2. (a) 50   (b) 50   (c) 23   (d) 25   (e) 28   (f) 1