RMIT UNIVERSITY

IN2 INTEGRATION OF POLYNOMIALS

$\int f(x)\,dx$ is read as "the integral of $f(x)$" and indicates that we wish to find the antiderivative of $f(x)$. The process of finding the antiderivative is called *integration*.

When 'c' is unspecified the result of integration is an *indefinite integral*.

Operational rules

$\int f(x) \pm g(x)\,dx = \int f(x)\,dx \pm \int g(x)\,dx$

$\int kf(x)\,dx = k\int f(x)\,dx$

Integrating powers of x

Using the rule for finding the antiderivative of x^n:

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + \text{c}, \text{ (providing n} \neq \text{-1)}$$

Examples

1. $\int (x^3 + 4)dx = \int x^3\,dx + \int 4\,dx$ [Each term can be integrated separately]

$= \frac{x^4}{4} + 4\text{x} + \text{c}$

[Only one constant of integration is needed]

2. $\int \frac{1}{3\sqrt{x}}\,dx = \frac{1}{3}\int x^{-\frac{1}{2}}\,dx$ [Using $\int kf(x)\,dx = k\int f(x)\,dx$]

$= \frac{1}{3}\frac{x^{\frac{1}{2}}}{\frac{1}{2}} + \text{c}$

$= \frac{2}{3}x^{\frac{1}{2}} + \text{c}$

$= \frac{2\sqrt{x}}{3} + \text{c}$ [it is conventional to give the answer in the same form as the question]

3. $\int \frac{3s^4 - s^3 + 7}{s^2}\,ds = \int 3s^2 - s + 7s^{-2}ds$ [Divide each term by s^2]

$= \frac{3s^3}{3} - \frac{s^2}{2} - \frac{7}{s} + c$

Integrating powers of linear functions of x

Functions of the form $f(x) = (ax + b)^n$ can be integrated using the following rule:

$$\int (ax+b)^n \, dx = \frac{(ax+b)^{n+1}}{a(n+1)} + c, \text{ (providing n} \neq -1)$$

Examples

1. $\int (8x-5)^9 \, dx = \frac{(8x-5)^{9+1}}{8(9+1)} + c$

$= \frac{(8x-5)^{10}}{80} + c$

2. $\int (7-2x)^{-3} \, dx = \frac{(7-2x)^{-3+1}}{-2(-3+1)} + c$

$= \frac{(7-2x)^{-2}}{6} + c$

$= \frac{1}{6(7-2x)^2} + c$

Exercises

1. Find the following integrals.

(a) $\int 3x^2 \, dx$ (b) $\int 4x^5 - 2x^3 + 9 \, dx$ (c) $\int x^2 + \frac{1}{x^2} dx$

(d) $\int \frac{t^4 - 2t^3 + 1}{2t^2} \, dt$ (e) $\int s + \frac{2}{3\sqrt{s^3}} \, ds$

2. (a) $\int (5x+1)^4 \, dx$ (b) $\int \sqrt{x-9} \, dx$ (c) $\int \frac{2}{(x-3)^3} \, dx$

Answers

1. (a) $x^3 + c$ (b) $\frac{2x^6}{3} - \frac{x^4}{2} + 9x + c$ (c) $\frac{x^3}{3} - \frac{1}{x} + c$

(d) $\frac{t^3}{6} - \frac{t^2}{2} - \frac{1}{2t} + c$ (e) $\frac{s^2}{2} - \frac{4}{3\sqrt{s}} + c$

2. (a) $\frac{(5x+1)^5}{25} + c$ (b) $\frac{2(x-9)^{\frac{3}{2}}}{3} + c$ (c) $-\frac{1}{(x-3)^2} + c$