FU1.2: ABSOLUTE VALUE

The absolute value of a number $|x|$ gives a measure of its size or magnitude regardless of whether it is positive or negative. If a number is plotted on a number line then its absolute value can be considered to be the *distance* from zero.

Examples

Eg (i) $|2| = 2$

(ii) $|-2| = 2$

(iii) $|-4+3| = |-1| = 1$

(iv) $|-8| + |-1| = 8 + 1 = 9$

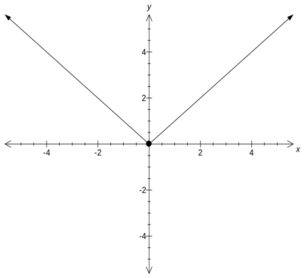
(v) $|x| = 7 \Rightarrow x = 7$ or $x = -7$

The absolute value function and its graph

The absolute value function is a hybrid function defined as follows:

$$f{:}R \to R \quad \text{where} \quad f(x) = |x| = \begin{cases} -x, & x < 0 \\ x, & x \geq 0 \end{cases}$$

with graph

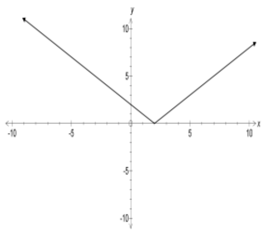


NB: The domain of $f(x) = |x|$ is R

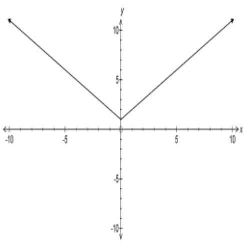
The range is $R^+ \cup 0$

The graph of $y = |x|$ may be translated in the same way as the graphs of other functions.

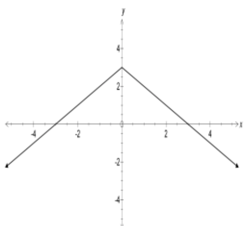
Compare the graphs of the following functions with that of $y = |x|$



1. $y = |x-2|$



2. $y = |x| + 1$



3. $y = 3 - |x| = -|x| + 3$

$y = |x|$ translated horizontally 3 units to the right

$y = |x|$ translated vertically one unit up

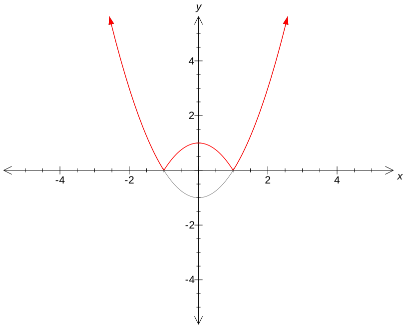
$y = |x|$ reflected in the x-axis followed by a vertical shift of 3 units

To sketch the graph of $y = |f(x)|$ we need to sketch the graph of y = f(x) first and then reflect in the x-axis the portion of the graph which is below the x-axis.

Example

Sketch $\{ (x,y): \ y = |x^2 - 1| \}$

The graph of this function is the graph of $y = x^2 - 1$ with the portion below the x-axis reflected across the x-axis.



Equations & inequalities involving $|f(x)|$

Because $y = |f(x)|$ is a hybrid function two cases must be considered when solving equations and inequalties

Examples

1. *Solve* $|x - 2| = 3$
2.

If $|x - 2| = 3$

then $x - 2 = 3$ or $-(x - 2) = 3$

ie $x = 5$ or $x = -1$

With an absolute value expression on each side of the equation it is easier to square both sides:

2. *Solve* $|2x + 1| = |x - 5|$

If $|2x + 1| = |x - 5|$

then $(2x+1)^2 = (x - 5)^2$

ie $4x^2 + 4x + 1 = x^2 - 10x + 25$

ie $3x^2 + 14x - 24 = 0$

ie $(3x - 4)(x + 6) = 0$

ie $x = \frac{4}{3}$ or $x = -6$

NB: Care must be taken when multiplying or dividing by a negative to reverse an inequality

3. *Solve* $\left|\frac{2 - x}{3}\right| < 4$

$$\left|\frac{2-x}{3}\right| < 4 \quad \Rightarrow \quad |2 - x| < 12$$

$$\Rightarrow \; -12 < 2 - x < 12$$

$$\Rightarrow \; -14 < -x < 10$$

$$\Rightarrow \; 14 > x > -10 \quad \text{or} \quad -10 < x < 14$$

Exercises

Exercise 1

Evaluate

1. $|-11|$
2. $|-9+4|$
3. $-|4|-|-5|$
4. $|-12|-|3|$
5. $|-30| \div |5|$

Exercise 2

Sketch the graph of

1. y = $|x+4|$
2. y = $|x-1|-3$
3. y = $|3-x^2|$

Exercise 3

Find for x $\in$ **R**

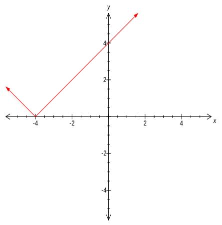
1. {x: $|x|$ = 6}
2. {x: $|x - 1| < 3$}
3. {x: $\left|\frac{x-3}{2}\right| \geq 1$}
4. $\{\left|\frac{x}{2}\right| = |x+2|\}$

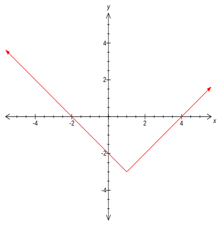
Answers

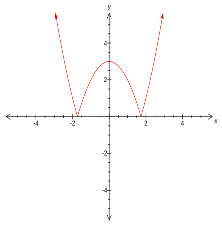
Exercise 1

1) 11 2) 5 3) -9 4) 9 5) 6

Exercise 2

1. 

2. 

3. 

Exercise 3

1. {-6,6}
2. {x: -2<x<4}
3. {x: x $\leq$ 1} $\cup$ {x: x $\geq$ 5}
4. $\{-4, -\frac{4}{3}\}$