MATHS FOR PRICES & MARKETS:

ALGEBRA SKILLS

Variables are letters that we use to abbreviate or represent some commonly used expressions. Sometimes we may know what the variable(s) stands for:

Q – quantity
P – price
C - cost
$\prod$ - profit
MR – marginal revenue
TC – total cost
AVC – average variable cost
$\sum x$ - the sum (addition) of all the individual values of x

NB: Different textbooks and courses may use the same variable to mean different things. In Prices and Markets Q is used to represent 'quantity' but in Business Statistics Q is used to mean 'quartile'. Make sure you are very clear what any variables and subscripts are referring to in each of your courses.

We may use subscripts or superscripts to be more precise and to identify special cases of the usual variables:

Q_s – quantity supplied
P_o – the initial price
P_1, P_2 - the first price, the second price
TR_{2011} – the total revenue for 2011
$\prod^{PC}$ - profit under perfect competition
$\prod^{\max}$ - maximum profit

Operations

When working with variables the usual rules of algebra apply:

Addition and Subtraction:

$10Q - 30Q + 70Q = (10 - 30 + 70)Q = 40Q$ [add and subtract the numbers in front to find the total amount of Q]

$10Q + 30Q = 40Q$ BUT $10Q + 30Q + 25P = 40Q + 25P$ [only like terms can be combined by addition or subtraction]
$= 25P + 40Q$ [order is unimportant]

Multiplication and powers:

$10Q = 10 * Q = 10 \times Q = 10 \bullet Q$ [many different symbols, or none at all, are used for multiplication]

$10 * Q = Q * 10 = 10Q$ [by convention we write numbers in front of variables]

$Q^2 = Q * Q$

$10Q * 20Q = 200Q^2$ [multiply the numbers and then use a power to show how many factors of Q]

Brackets

$Q(200 - 45Q) = Q * (200 - 45Q)$
$= Q * 200 - Q * 45Q$
$= 200Q - 45Q^2$

Fractions:

$$\frac{10Q^2}{5Q} = \frac{1\cancel{0}^{2}\cancel{Q}Q}{\cancel{5}^{1}\cancel{Q}} = 2Q$$

$$\frac{10Q^2 + 20Q}{5Q} = \frac{10Q^2}{5Q} + \frac{20Q}{5Q}$$ [NB: denominators must be the same to do this]

$$= 2Q + 4$$ [after simplifying]

Substitution and evaluation

The process of replacing a *variable* (or pronumeral or letter) in an expression or formula with a specific value is called ***substitution***.
Within a particular problem once a value has been allocated to a variable, eg Q = 30, then whenever Q appears in that problem we give it the value 30. For subsequent problems the value of the variable may change (or vary!).

When variables have been allocated values it is then possible to ***evaluate*** the algebraic expression.

Examples

1. *If TR = P * Q find the value of TR when P = 150 and Q = 20*

   TR = P * Q [write the formula]
   = 150 * 20 [replace the variables with their values]
   = 3000 [calculate]

2. *If* $\Pi = 4000 - Q^2 - 20Q$ *evaluate* Π *when Q = 25*

   $\Pi = 4000 - Q^2 - 20Q$ [write the formula]
   $= 4000 - 25^2 - 20 * 25$ [replace the variables with their values]
   $= 2875$ [calculate]

Exercises

Exercise 1

Simplify

1. a) $10Q + 15Q$ b) $25P_D - 12P_D$ c) $10P + 30Q - 15Q + 20P$

2. a) $5 \times 12P$ b) $10Q \times 5Q$ c) $10Q + 5Q^2 + 25Q$

3. a) $10(4G - 5)$ b) $2P(30 - 3P)$ c) $10Q(Q + 25)$

4 a) $\frac{5Q^2}{Q}$ b) $\frac{5Q^2 + 8Q}{2Q}$ c) $\frac{5Q^2 + 25Q}{10Q}$

Exercise 2

1. If $Q_D = 50 - 0.6P$ find Q_D when (i) $P = 30$ (ii) $P = 80$

2. The formula in $ for an average cost function for laser printers is given as

   $AC = 500 + \frac{325}{Q}$ where Q is the quantity of printers produced.

   Evaluate AC to the nearest cent if Q is equal to (i) 60 (ii) 100 (iii) 500

3. In statistics the mean of a set of values is calculated using the formula

   $\bar{x} = \frac{\sum x}{n}$. Find the mean $\bar{x}$ if (i) $\sum x = 360$ and $n = 24$.

   (ii) $\sum x = 45$ and $n = 10$.

4. Use the formula $z = \frac{x - u}{\sigma}$ to evaluate z when (i) $x = 20$, $u = 15$ and $\sigma = 2$

   (ii) $x = 120$, $u = 100$ and $\sigma = 10$

   (iii) $x = 5.5$, $u = 6$ and $\sigma = 1.2$

5. If $\Pi = 1500 - Q^2 - 30Q$ find the value of Π when (i) $Q = 25$
   (ii) $Q = 10$
   (iii) $Q = 4$

6. Evaluate T when X = 20 if $T = 5X(30 - 0.8X)$

Answers

Exercise 1

1 a) $25Q$ b) $13P_D$ c) $30P + 15Q$
2 a) $60P$ b) $50Q^2$ c) $5Q^2 + 35Q$
3 a) $40G - 50$ b) $60P - 6P^2$ c) $10Q^2 + 250Q$
4 a) $5Q$ b) $\frac{5Q}{2} + 4$ c) $\frac{Q}{2} + \frac{5}{2}$

Exercise 2

1. (i) 32 (ii) 2
2. (i) 505.42 (ii) 503.25 (iii) 500.65
3. (i) 15 (ii) 4.5
4. (i) 2.5 (ii) 2 iii) -0.42 rounded to 2 dec places
5. (i) 125 (ii) 1100 (iii) 1364
6. 1400