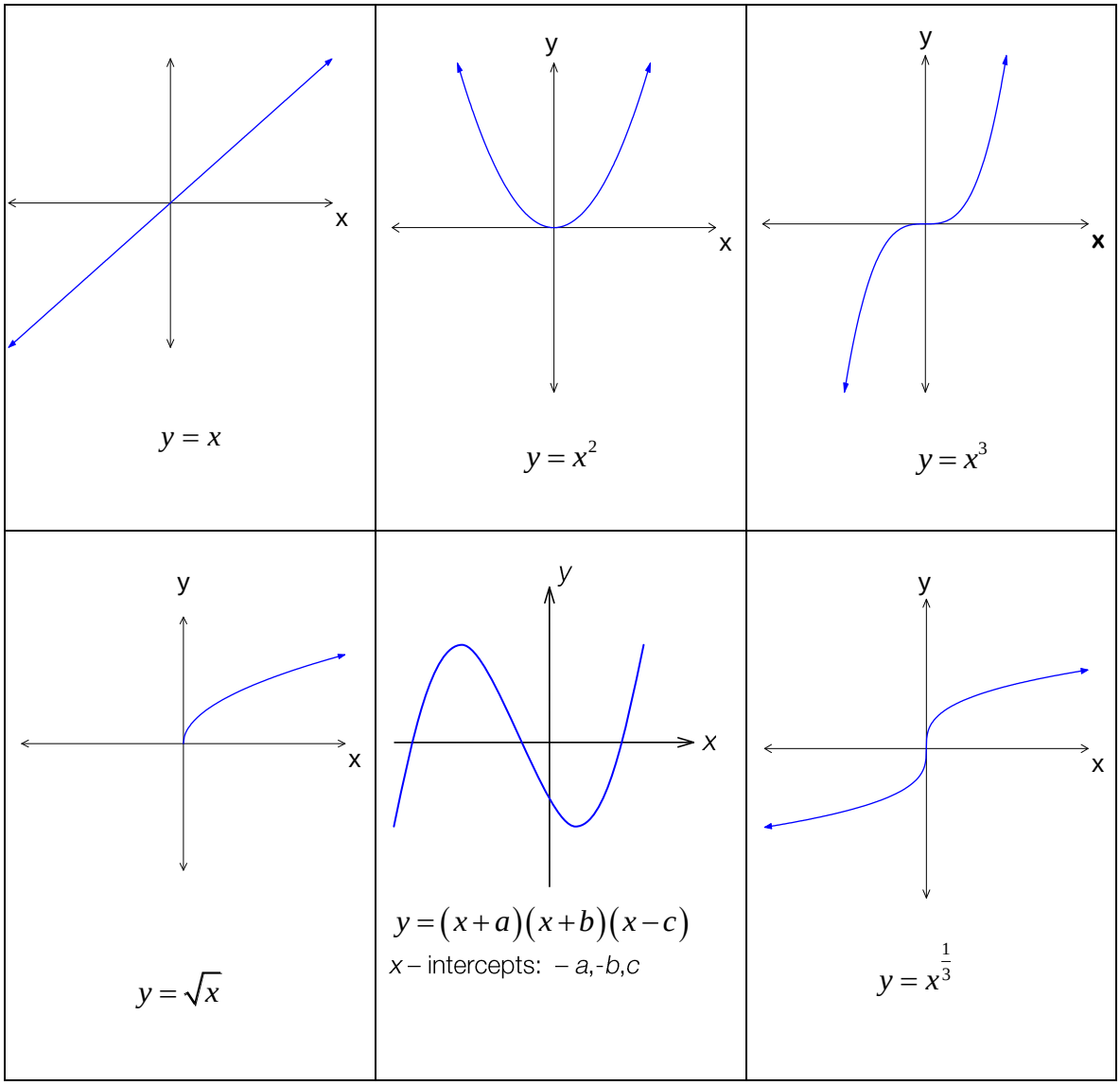
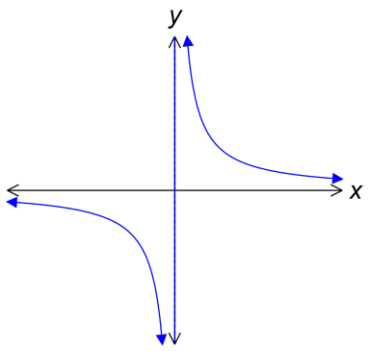
GR1.3: GRAPHS AND TRANSFORMATIONS

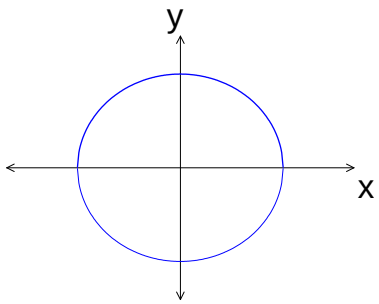
The known graphs of some simple functions and relations can be used to sketch related, but more complicated functions.

Some graphs that would be useful to remember are:



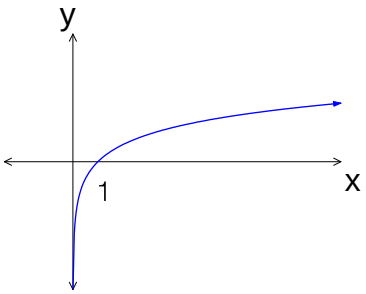


$y = \frac{1}{x}$

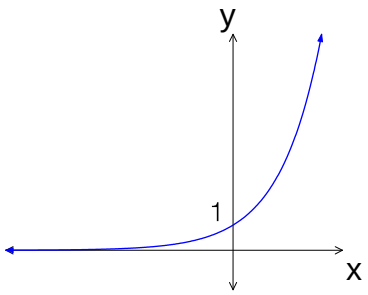


$x^2 + y^2 = r^2$

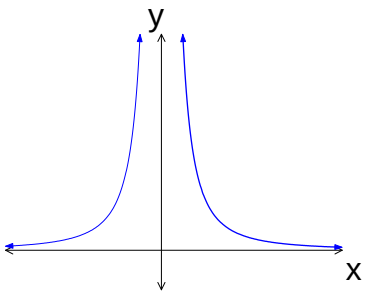
Circle of radius r



$y = \log x$



$y = a^x$



$y = \frac{1}{x^2}$

To sketch a graph look for:

- *x* and *y* intercepts
- turning points
- behaviour as *x* tends to $\pm\infty$
- asymptotes (eg when the denominator of a fraction = 0)

Graphs should be named, axes labelled and any intercepts, turning points or asymptotes marked.

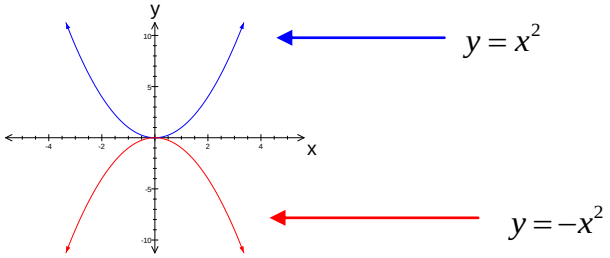
Reflections

If $y = f(x)$ then

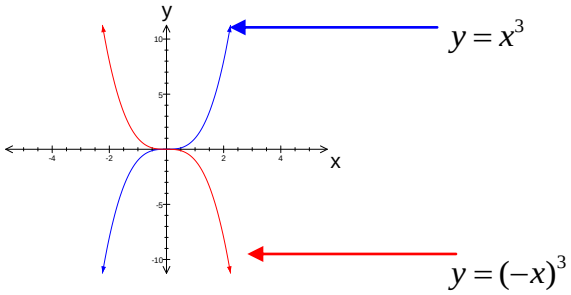
$y = f(-x)$ is the reflection of the graph of about the ***y*-axis**
$y = -f(x)$ is the reflection of the graph of about the ***x*-axis**

Examples

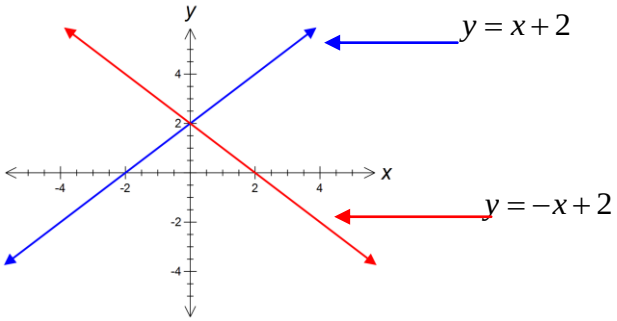
1. If $f(x) = x^2$, $-f(x) = -x^2$. The graph of $y = -x^2$ (red) is the graph of $y = x^2$ reflected about the *x*-axis.



2.If $f(x) = x^3$, $f(-x) = (-x)^3 = -x^3$. The graph of $y = -x^3$ (red) is the graph of $y = x^3$ reflected about the *y*-axis.



3.If $f(x) = x + 2$, $f(-x) = -x + 2$. The graph of $y = -x + 2$ (red) is the graph of $y = x + 2$ reflected about the *y*-axis.



Translations

A translation may be a horizontal shift or a vertical shift.

Horizontal

The graph of $y = f(x-a)$ is a shift of the graph $y = f(x)$ '*a*' units to the **right.**

The graph of $y = f(x+a)$ is a shift of the graph $y = f(x)$ '*a*' units to the **left**

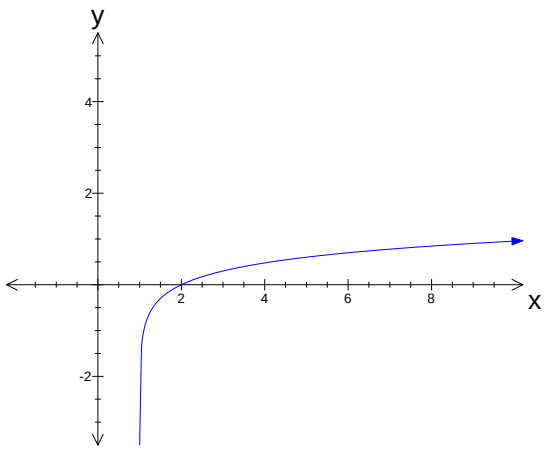
Vertical

The graph of $y = f(x)+b$ is a shift of the graph $y = f(x)$ '*b*' units **up.**

The graph of $y = f(x)-b$ is a shift of the graph $y = f(x)$ '*b*' units **down.**

Examples

1. $y = \log(x-1)$



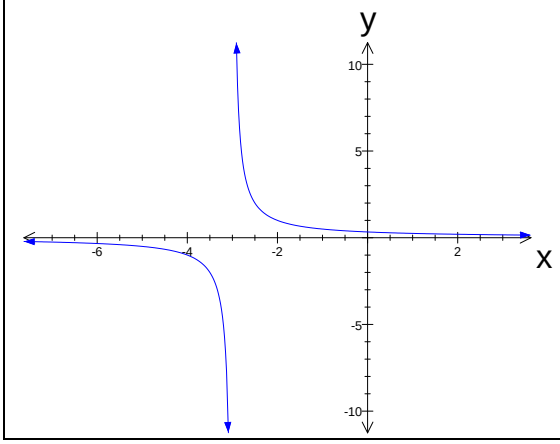
Basic graph is . $y = \log x$

Replacing *x* with (*x* –1) shifts the graph **1 unit to the right**

$y = \log(x-1)$

shifted 1 unit right

2. $y = \dfrac{1}{(x+3)}$



Basic graph is $y = \dfrac{1}{x}$.

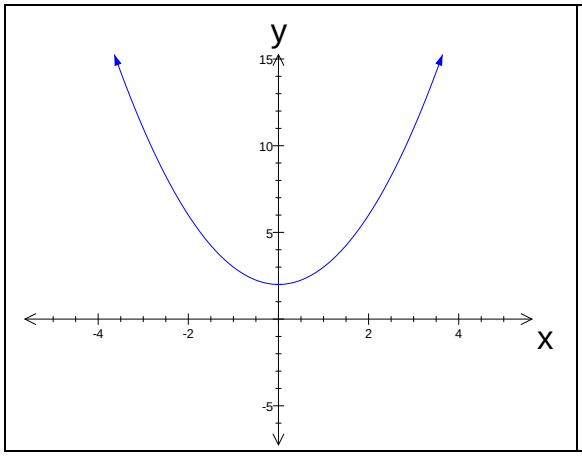
Replacing *x* with (*x* + 3) shifts the graph **3 units to the left.**

$y = \dfrac{1}{(x+3)}$

shifted 3 units left

Asymptote is at $x = -3$, when $x + 3 = 0$

3. $y = x^2 + 2$



Basic graph is $y = x^2$.

Adding 2 shifts the graph **up** 2.

$y = x^2 + 2$

shifted 2 units up

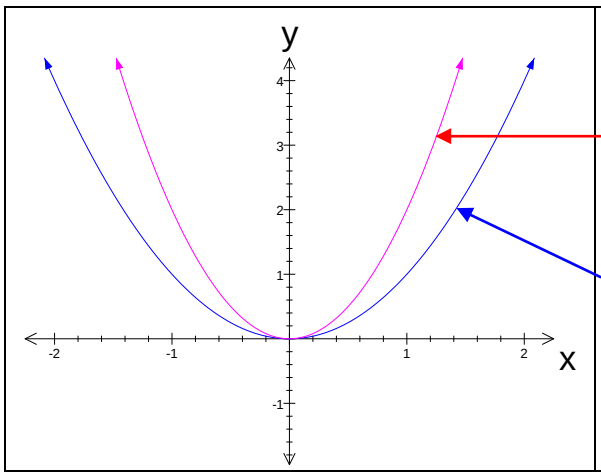
Dilations

A dilation is a stretching or a squashing.

The graph of $y = af(x)$ is a dilation of the graph of $y = f(x)$ by a factor of '*a*' units parallel to the ***y*-axis.**

The graph of $y = f(bx)$ is a dilation of the graph of $y = f(x)$ by a factor of $\frac{1}{b}$ units parallel to the ***x*-axis.**

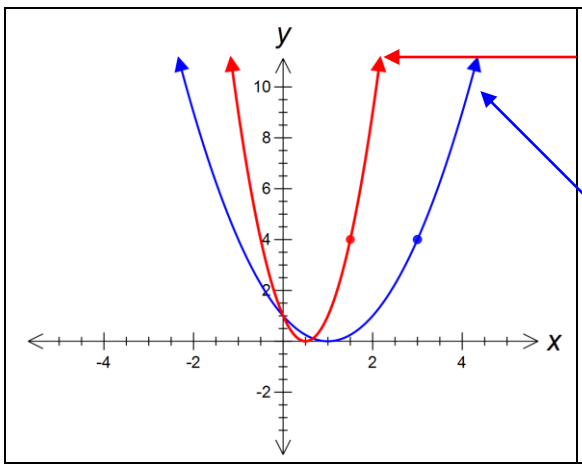
Examples



Stretched by a factor of 2 parallel to the *y* =axis.

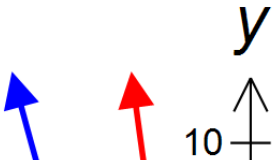
Each value of $y = 2x^2$ is obtained by multiplying the corresponding value of $y = x^2$ by 2.
eg.. $x = 1$ gives $y = 1$ for $y = x^2$ and $y = 2$ for $y = 2x^2$

2. $y = (x-1)^2$ and $y = (2x-1)^2$



Dilated by a factor of 1/2 parallel to the *x* =axis.

In this case *x* values that differ by a factor ½ give the same *y* – values.
e.g The value of $y = 4$ is given by $x = 3$ for $y = (x-1)^2$ and by $x = 3/2$ for $y = (2x-1)^2$

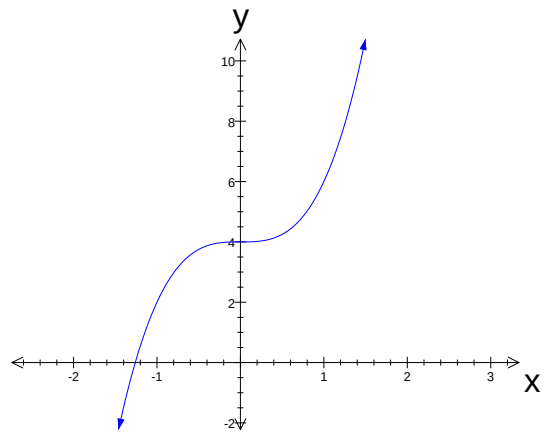


Combinations

More complicated graphs can be sketched by using combinations of dilation, reflection and translation. Dilations must always be considered before reflection and translation.

Examples

1. $y = 2x^3 + 4$



Basic graph is $y = x^3$

Multiplying by 2 stretches the graph by a factor of 2 parallel to the *y* axis

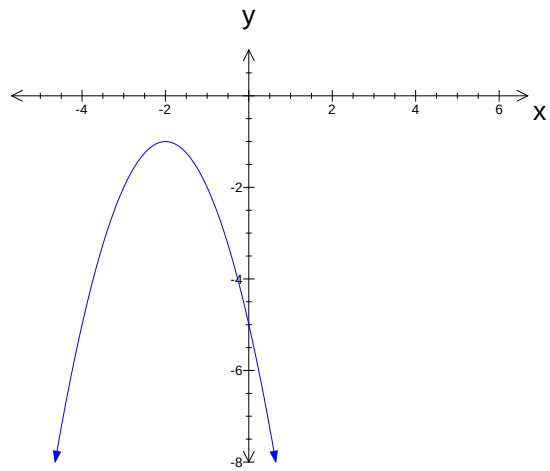
Adding 4 shifts the graph **up 4 units.**

$y = 2x^3 + 4$

Stretched by factor 2 vertically

shifted 4 units up

2. $y = -(x+2)^2 - 1$



Basic graph is $y = x^2$

Replacing *x* with (*x* + 2) shifts the graph **2 units to the left.**

Adding -1 shifts the graph **1 unit down.**

The negative sign before the bracket **reflects the graph about the *x* – axis**

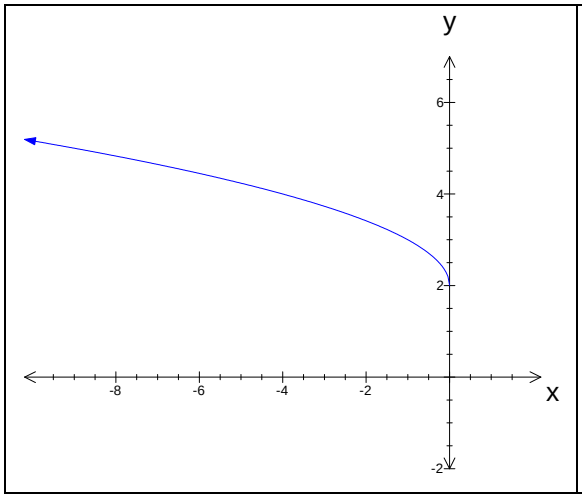
$y = -(x+2)^2 - 1$

reflected about ***x*** – **axis**

Shifted **2** units **left**

shifted **1** unit **down**

3. $y=\sqrt{-x}+2$



Basic graph is $y=\sqrt{x}$

Replacing x with $-x$ **reflects the graph about the $y-$ axis.**

Adding +2 shifts the **graph up 2 units.**

$y=\sqrt{-x}+2$

reflected about $y-$ axis

Shifted 2 units up

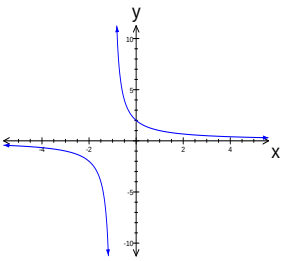
Exercises

Exercise 1

Sketch the following graphs.

1. $y=\dfrac{2}{(x+1)}$
2. $y=\log(x+3)$
3. $y=x^2-25$
4. $(x-2)^2+(y-1)^2=16$
5. $y=2-(x+1)^2$
6. $y=4^x$

Answers



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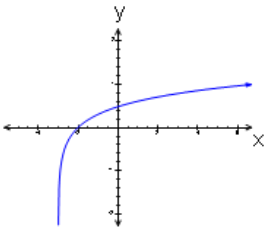

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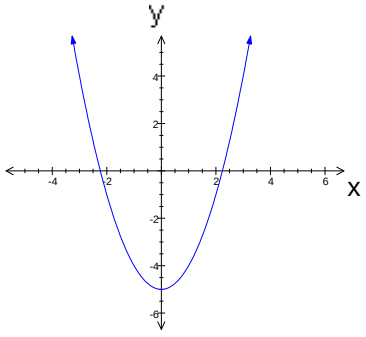

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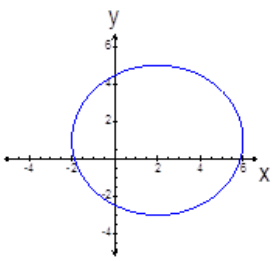

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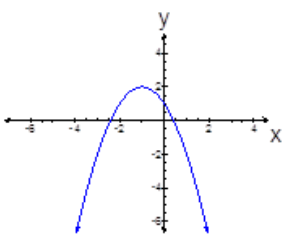

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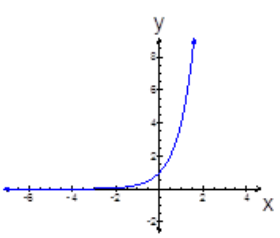

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