FU1.5: INVERSE FUNCTIONS

Definition of an inverse function

If $f^{-1}(x)$ is the inverse function of a one-to-one function $f(x)$ then $f^{-1}(x)$ is the set of ordered pairs obtained by interchanging the first and second elements in each ordered pair.

So if (a,b) $\in$ f then (b,a) $\in$ f^{-1} and if $f(a)=b$ then $f^{-1}(b)=a$

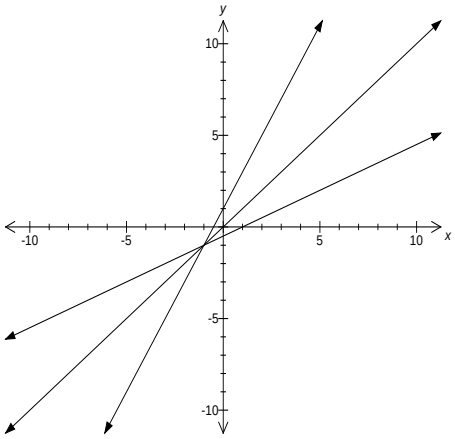
The domain of f is the range of f^{-1} and the range of f is the domain of f^{-1}.

For example the function $f: R \rightarrow R$ defined by y = f(x) = $\frac{x-1}{2}$ has an inverse function with rule $f^{-1}(x) = 2x + 1$.

So (3,1) belongs to f and (1,3) belongs to f^{-1}, and (-7,-4) belongs to f and (-4,-7) belongs to f^{-1}.

Graph of an inverse function

The graphs of any one-to-one function f and its inverse f^{-1} are symmetric about the line $y = x$.



Finding an inverse function for y = f(x)

To obtain the rule for an inverse function swap the x and y coordinates in f and rearrange to express y in terms of x:

Example

Find the inverse function of f where f(x)= 2 – 3x

y = 2 – 3x

x = 2 – 3y [swap x and y]

x – 2 = -3y [rearrange to make 'y' the subject]

-x + 2 = 3y

$\frac{-x+2}{3}$ = y

$\therefore$ $f^{-1}(x) = \frac{-x+2}{3}$

Exercise

Find the inverse of each of the following one-to-one functions:

1) $y = x + 5$

2) $y = 4x$

3) $y = \frac{2x+1}{3}$

4) $y = \sqrt{2x-1}$, $x \geq ½$

Answers

1) $f^{-1}(x) = x - 5$

2) $f^{-1}(x) = \frac{x}{4}$

3) $f^{-1}(x) = \frac{3x-1}{2}$

4) $f^{-1}(x) = \frac{x^2+1}{2}$, $x \geq ½$