IN1.6: APPLICATIONS OF INTEGRATION AREA UNDER A CURVE

Calculation of Areas

The definite integral $\int_a^b f(x)dx$ gives a measure of the *signed area* between the graph of $f(x)$ and the *x*-axis between the values of $x = a$ and $x = b$.

If the area is **above** the *x*-axis, the definite integral is **positive.**

If the area is **below** the *x*-axis, the definite integral is **negative.**

The actual area is the absolute value of the signed area.

$$A = \left| \int_a^b f(x)\,dx \right|$$

Note: If the curve crosses the *x*-axis, the areas above and below the *x*-axis must be calculated separately.

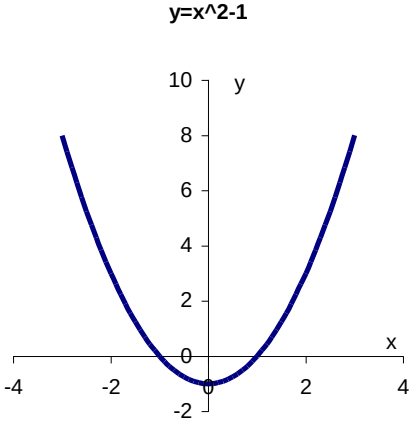
Examples

1.

Given the function $f(x) = x^2 - 1$, find the area between the curve and the *x*-axis, for the values $x = 0$ and $x = 3$.

Sketch the graph of $y = x^2 - 1$ to determine the area to be calculated

y- intercept: -1
x- intercepts: ± 1



Limits of integration: $x = 0$ to $x = 3$.

Area from $x = 0$ to $x = 1$ is **below** the *x* axis. Area from $x = 1$ to $x = 3$ is **above** the *x* axis. Integrate each area separately.

$$A = \left| \int_0^1 (x^2 - 1)\,dx \right| + \left| \int_1^3 (x^2 - 1)\,dx \right|$$

$$= \left| \left[\frac{x^3}{3} - x \right]_0^1 \right| + \left| \left[\frac{x^3}{3} - x \right]_1^3 \right|$$

$$= \left| \left[\left(\frac{1}{3} - 1 \right) - 0 \right] \right| + \left| \left[\left(\frac{27}{3} - 3 \right) - \left(\frac{1}{3} - 1 \right) \right] \right|$$

$$= \frac{2}{3} + \left(6 + \frac{2}{3} \right)$$

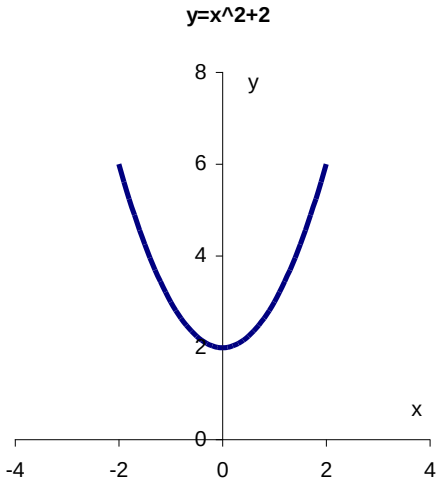
$$Area = 7\frac{1}{3}\ units$$

Given the function $f(x) = x^2 + 2$, find the area between the curve, the x-axis, and the lines $x = -1$ and $x = 1$.

Sketch the graph of and $y = x^2 + 2$ to determine the area to be calculated.

y- intercept: 2

x- intercepts: no x- intercepts



Limits of integration:

$x = -1$ to $x = 1$

$$A = \left| \int_{-1}^{1} \left(x^2 + 2 \right) dx \right| = \left| \left[\frac{x^3}{3} + 2x \right]_{-1}^{1} \right|$$

$$= \left| \left[\frac{1}{3} + 2 \right] - \left[\frac{-1}{3} - 2 \right] \right|$$

$$Area = 4\frac{2}{3} units$$

Exercises

Exercise 1

(a) Find the area enclosed by the curve $y = 6x - 2x^2$ and the x axis.

(b) Calculate the area bounded by the curve $y = x^2 - 9$ and the x axis.

(c) Find the area bounded by the curve $y = \frac{2}{x}$, the x axis and the lines $x = 2$ and $x = 4$

(d) Find the area bounded by the curve $y = \sin x$ the x – axis and the line $x = \frac{\pi}{2}$

Exercise 2

(a) Find the total area of the regions bounded by the curve $y = \cos x$ and the x – axis between x = 0 and $x = \pi$.

(b) Find the area bounded by the curve $y = x^2 - 3x + 2$ and the x – axis between $x = 0$ and $x = 2$.

Answers

1(a) 9 units, (b) 36 units, (c) $2\ln 2$, (d) 1 unit

2(a) 2 units (b) 1 unit