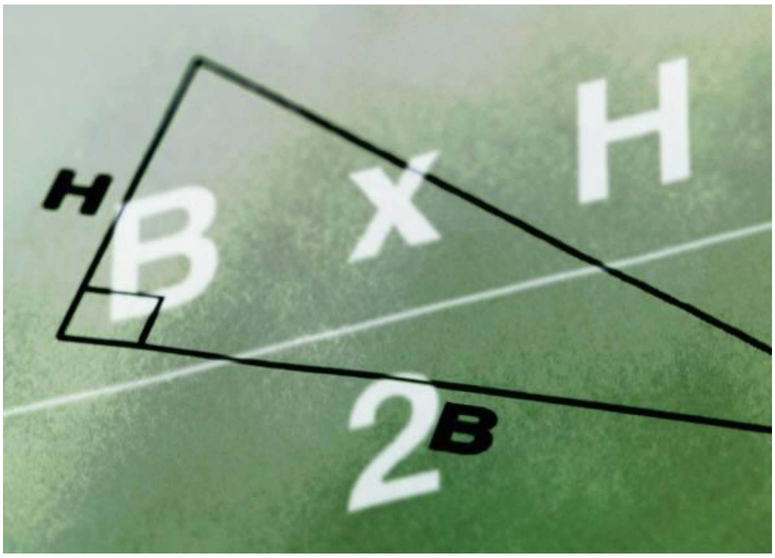
SUMMER

KNOWHOW

STUDY AND LEARNING CENTRE

RMIT UNIVERSITY

Formulae & Equations



Contents

REARRANGING FORMULAE

Some of the most important equations that we might be required to transpose occur frequently in science, engineering and economics. They are called formulae and give a general rule describing the relationship between variable quantities.

Here are some examples: $A = \Pi r^2$

$$s = u t + \tfrac{1}{2} a t^2$$

$$S = P(1 + i)^n$$

In these examples A, s and S are, respectively, the subjects of the formulae. Sometimes it is necessary to rearrange the formula to make a different variable the subject:

We know $A = \Pi r^2$ but $r = ???$

and $s = u t + \tfrac{1}{2} a t^2$ but $a = ???$

Before beginning to rearrange these more complex formulae, let us examine the processes required to make 'A' the subject in each of the following simple equations.

We may perform whichever operations we choose providing we do the same to each side of the equation.

(i)

$$A + B = C$$

$$A + B - B = C - B \qquad \text{[- B both sides]}$$

$$\therefore \quad A = C - B$$

(ii)

$$A - B = C$$

$$A - B + B = C + B \qquad \text{[+ B both sides]}$$

$$\therefore \quad A = C + B$$

(iii)

$$A B = C$$

$$\frac{AB}{B} = \frac{C}{B} \qquad [\div \text{ B both sides}]$$

$$\therefore \quad A = \frac{C}{B}$$

(iv) $$\frac{A}{B} = C$$

$$\frac{A}{B} \times B = C \times B \qquad \text{[×B both sides]}$$

$$\therefore \quad A = C \times B = CB$$

(v) $$A^2 = B$$

$$\sqrt{A^2} = \sqrt{B} \qquad [\sqrt{\ } \text{ both sides}]$$

$$\therefore \quad A = \sqrt{B}$$

(vi) $$\sqrt{A} = B$$

$$(\sqrt{A})^2 = B^2 \qquad \text{[square both sides]}$$

$$\therefore \quad A = B^2$$

Notice that - 'undoes' + and + 'undoes' -

÷ ' undoes' × and × 'undoes' ÷

$\sqrt{x}$ 'undoes' x^2 and x^2 'undoes' $\sqrt{x}$

or more generally, $\sqrt[n]{x}$ 'undoes' x^n and x^n 'undoes' $\sqrt[n]{x}$

This is because each of these pairs of operations are *inverse* operations.

Examples

1. *Transform V = A – K to make 'A' the subject*

$$V = A - K$$

$$V + K = A - K + K \qquad \text{[add k both sides]}$$

$$V + K = A \quad \text{or} \quad A = V + K$$

2. *Make 'd' the subject of* $C = \Pi \times d$

$$C = \Pi \times d$$

$$\frac{C}{\pi} = \frac{\pi \times d}{\pi} \qquad \text{[divide } \Pi \text{ both sides]}$$

$$\frac{C}{\pi} = d \quad \text{or} \quad d = \frac{C}{\pi}$$

3. *Rearrange j = 3w – 5 in terms of 'w'.*

$$j = 3w - 5$$

$$j + 5 = 3w - 5 + 5 \qquad \text{[add 5 both sides]}$$

$$j + 5 = 3w$$

$$\frac{j+5}{3} = \frac{3w}{3} \qquad \text{[divide 3 both sides]}$$

$$\frac{j+5}{3} = w \quad \text{or} \quad w = \frac{j+5}{3}$$

4. *Make 'c' the subject of $E = mc^2$*

$$E = mc^2$$

$$\frac{E}{m} = c^2 \qquad [\div \text{ m both sides}]$$

$$\sqrt{\frac{E}{m}} = c \qquad [\sqrt{\ } \text{ both sides}]$$

Exercises

1. $m = n - 2$ Find n
2. $A = 2B + C$. Find C
3. $A = 2B + C$. Find B
4. $P = \frac{k}{v}$. Find k.
5. $PV = k$. Find V.
6. $v = u + at$. Find a.
7. $v = u + at$. Find t.
8. $r = \sqrt{\frac{A}{\pi}}$. Find A.
9. $A = x^2$. Find x.
10. $A = \pi r^2$. Find r.

Answers

1. $n = m + 2$
2. $C = A - 2B$
3. $B = \frac{A-C}{2}$
4. $k = PV$
5. $V = \frac{k}{p}$
6. $a = \frac{v-u}{t}$
7. $t = \frac{v-u}{a}$
8. $A = \pi r^2$
9. $x = \pm\sqrt{A}$
10. $r = \pm\sqrt{\frac{A}{\pi}}$

TRANSPOSITION WITH BRACKETS AND FRACTIONS

Examples

1. *Transform the formula $P = 2(L - W)$ to make 'W' the subject.*

$P = 2(L - W)$

$\frac{P}{2} = L - W$ [÷ 2 both sides]

$\frac{P}{2} - L = -W$ [- L both sides]

$W = -\frac{P}{2} + L$ [× (-1) both sides and reversing equation]

2. *If $\frac{2}{k} = \frac{j+1}{3}$ find 'k'.*

$\frac{2}{k} = \frac{j+1}{3}$

$k(j + 1) = 6$ [cross multiplying is useful for removing fractions]

$k = \frac{6}{j+1}$ [÷ (j + 1)]

3. *Rearrange the formula $L = \frac{Mt-g}{b}$ to make 'M' the subject.*

$L = \frac{Mt-g}{b}$

$L \times b = Mt - g$ [×b both sides]

$bL + g = Mt$ [+ g both sides]

$\frac{bL+g}{t} = M$ [÷ t both sides]

$M = \frac{bL+g}{t}$

4. *Make 'v' the subject of $E = mgh + ½mv^2$.*

$E = mgh + ½mv^2$

$E - mgh = ½mv^2$ [- mgh both sides]

$2(E - mgh) = mv^2$ [×2 both sides]

$\frac{2}{m}(E - mgh) = v^2$ [÷ m both sides]

$V = \pm\sqrt{\frac{2}{m}(E - mgh)}$ [√ both sides]

5. Transpose $T = 2\pi\sqrt[3]{\frac{L}{G}}$ to make 'L' the subject.

$$T = 2\pi\sqrt[3]{\frac{L}{G}}$$

$$\frac{T}{2\pi} = \sqrt[3]{\frac{L}{G}} \qquad [\div\ 2\pi \text{ both sides}]$$

$$\left(\frac{T}{2\pi}\right)^3 = \frac{L}{G} \qquad [\text{raise both sides to power } 3]$$

$$L = G\left(\frac{T}{2\pi}\right)^3 \qquad [\times G \text{ both sides}]$$

Exercises

1. $S = C(A + B)$. Find A.
2. $A = \frac{h(a+b)}{2}$. Find a.
3. $I = \frac{Mr^2}{2}$. Find r .
4. $H = k(1 - bt)$. Find b
5. $v^2 = u^2 + 2as$. Find u.
6. $m = \sqrt{\frac{x+y}{z}}$. Find y.

Answers (NB: There may be equivalent forms of the correct answer)

1. $A = \frac{S}{C} - B$
2. $a = \frac{2A}{h} - b$
3. $r = {}^{\pm}\sqrt{\frac{2I}{M}}$
4. $b = \frac{k-H}{kt}$
5. $u = {}^{\pm}\sqrt{v^2 - 2as}$
6. $y = m^2z - x$

LINEAR EQUATIONS

Equations with one variable may be solved using transposition skills to make the variable the subject of the equation.
Using a sequence of inverse operations to 'undo' the equation

$$3(2x + 7) = -15....$$

.... becomes....

$$x = -6$$

Simple equations

Examples

1. $m + 4 = -2$

$m + 4 - 4 = -2 - 4$ [subtract 4 both sides]

$m = -6$

2. $p - 2 = 5$

$p - 2 + 2 = 5 + 2$ [add 2 both sides]

$p = 7$

3. $3g = 18$

$\frac{3g}{3} = \frac{18}{3}$ [divide 3 both sides]

$g = 6$

4. $\frac{y}{4} = -5$

$\frac{y}{4} \times 4 = -5 \times 4$ [multiply by 4 both sides]

$y = -20$

More complex equations

With more complicated equations more than one inverse operation may need to be applied

Examples

1. $2w - 3 = -17$

$2w = -14$ [add 3 to both sides]

$w = -7$ [divide 2 both sides]

2. $\frac{3d}{4} + 5 = 7$

$\frac{3d}{4} = 2$ [subtract 5 each side]

$3d = 8$ [multiply 4 each side]

$d = \frac{8}{3}$ [divide 3 each side]

With practice more than one operation can be done in the same line:

3. $3c + 1 = c - 5$ [move all terms with 'c' to one side of the equation]

$3c - c = -5 - 1$ [subtract c, subtract 1 each side]

$2c = -6$

$c = -3$ [divide 2 each side]

Look at the examples below for techniques to deal with equations that contain brackets and fractions

4. $3(5 - 2j) = 33$ [expand the brackets first]

$15 - 6j = 33$

$-6j = 18$ [subtract 15 each side]

$j = -3$ [divide (-6) each side]

5. $2(3k - 1) = 5(k + 7)$

$6k - 2 = 5k + 35$

$6k - 5k = 35 + 2$ [subtract 5k, add 2 each side]

$k = 37$

With just one fraction on each side of the equality use *cross multiplication:*

$$\boxed{\frac{a}{b} = \frac{c}{d} \Rightarrow ad = bc}$$

6. $\frac{h+1}{3} = \frac{h}{4}$

$4(h + 1) = 3h$ [cross multiplying]

$4h + 4 = 3h$ [expand brackets]

$h = -4$ [subtract 3h, subtract 4 each side]

7. $\frac{2z+11}{7} = \frac{z-3}{12}$

$12(2z + 11) = 7(z - 3)$ [cross multiplying]

$24z + 132 = 7z - 21$ [expand brackets]

$17z = -153$ [subtract 7z, subtract 132 each side]

$z = -9$

When cross multiplication is not appropriate fractions may be removed by multiplying both sides of the equation by a number that will cancel each of the denominators:

8. $\frac{3u}{4} - \frac{1}{3} = 7$ [NB: 12 will cancel 3 and 4]

$\frac{3u}{4} \times \frac{12}{1} - \frac{1}{3} \times \frac{12}{1} = \frac{7}{1} \times \frac{12}{1}$ [make all terms fractions and multiply through by 12]

$9u - 4 = 84$ [simplify]

$9u = 88$

$u = \frac{88}{9}$

Exercise

1. Solve the following equations

a) $x + 3 = 7$ b) $3c = 12$ c) $-r = -12$ d) $-8u = 12$

e) $4g + 4 = 16$ f) $\frac{e}{2} - 5 = -8$ g) $21 - 3t = 12$ h) $3 - \frac{u}{2} = -7$

2. Solve these equations

a) $5i + 2 = i + 10$ b) $8 - g = 5g + 14$

c) $5h - 2 = 7h - 12$ d) $6j + 13 = 4j + 13$

3. Solve

a) $3(2k - 4) = 18$ b) $5(2z + 9) = 15$

c) $3(x + 4) = 6$ d) $3(c + 3) + 2(c - 5) = 4$

e) $3(2v - 3) + 2(v - 4) = -25$ f) $3(b + 4) = 2(4b + 1)$

4 Solve

a) $\frac{9n}{5} - 4 = 5$

b) $\frac{3-2e}{11} = 1$

c) $\frac{3t}{8} + 4 = 1$

d) $\frac{5u-4}{4} = \frac{u-5}{5}$

e) $\frac{p+1}{3} + 1 = 4$

f) $\frac{d-3}{3} - 4 = \frac{d-2}{2}$

Answers

1 a) 4 b) 4 c) 12 d) $-\frac{3}{2}$ e) 3 f) -6 g) 3 h) 20

2 a) 2 b) -1 c) 5 d) 0

3 a) 5 b) -3 c) -2 d) 1 e) -1 f) 2

4 a) 5 b) -4 c) -8 d) 0 e) 8 f) -24

SIMULTANEOUS EQUATIONS

Equations with Two Variables

Two equations in two variables are said to be simultaneous if both must be considered at the same time.

An ordered pair (x, y) which satisfies both equations is said to be a solution for the simultaneous equations.

Two methods for solving a pair of simultaneous equations using algebra are
- elimination
- substitution

Elimination Method

In the elimination method we manipulate the equations so that one of the variables has coefficients that are equal and opposite in sign. Then we add the equations to eliminate that variable.

Examples

1. *Solve the simultaneous equations $x + y = 6$ and $x - y = 2$*

$x + y = 6$ [1] [Label the equations]
$x - y = 2$ [2] [NB: The 'y' coefficients are equal and opposite]

$2x = 8$ [1] + [2] [The equations are added eliminating y]
$x = 4$

Substitute $x = 4$ back into equation [1] or [2]:

$4 + y = 6$ [Using [1]]
$y = 2$

The solution is (4,2) [We can check the solution in the original equations $4 + 2 = 6$ and $4 - 2 = 2$!]

2. *Solve simultaneously $3x + 2y = 10$ and $4x + 3y = 13$*

$3x + 2y = 10$ [1] [Label the equations]
$4x + 3y = 13$ [2] [NB: The equations must be manipulated so that one variable has equal and opposite coefficients]

$12x + 8y = 40$ [1]' = [1] x 4 [Equation [1] has been multiplied by 4]
$-12x - 9y = -39$ [2]' = [2] x -3 [Equation [2] has been multiplied by -3]

$-y = 1$ [1]' + [2]' [Equations [1]' and [2]' have been added]
$y = -1$

Substituting $y = -1$ in [1]:

$3x + 2(-1) = 10$
$3x = 12$
$x = 4$

The solution is (4,-1) [Check: $3(4) + 2(-1) = 10$ and $4(4) + 3(-1) = 13$]

Substitution Method

In the substitution method the simpler equation is transposed to make one of the variables the subject, and then substitution enables the second equation to be reduced to one variable.

Examples

1. *Solve the simultaneous equations $x + y = 6$ and $x - y = 2$*

$$x + y = 6 \quad [1]$$
$$x - y = 2 \quad [2]$$ [It is convenient to label the equations]

From [1] $x + y = 6 \Rightarrow y = 6 - x$

Substitute for y in [2]: $x - (6 - x) = 2$
$$x - 6 + x = 2$$
$$2x - 6 = 2$$
$$2x = 8$$
$$x = 4$$

and from [1] $y = 6 - 4$
$$= 2$$

The solution is (4,2) which can be checked as before

2. *Solve $3x + 2y = 10$ and $4x + 3y = 13$*

$$3x + 2y = 10 \quad [1]$$
$$4x + 3y = 13 \quad [2]$$

From [1] $3x = 10 - 2y \Rightarrow x = \frac{10-2y}{3}$

Substitute in [2]: $4\left(\frac{10-2y}{3}\right) + 3y = 13$ [multiply each side by 3 to eliminate fractions]
$$4(10 - 2y) + 9y = 39$$
$$40 - 8y + 9y = 39$$
$$40 + y = 39$$
$$y = -1$$

and from [1]: $x = \frac{10-2(-1)}{3}$
$$= 4$$

The solution is (4,-1)

NB: Some pairs of simultaneous equations have no solution and some have multiple solutions. For example attempt to solve

(i) $2x - y = 6$ and $-4x + 2y = -7$

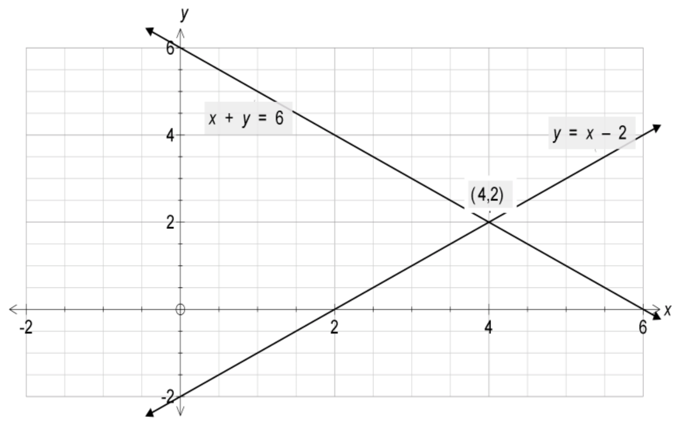
and (ii) $y = 1 - 3x$ and $6x = 2(y - 1)$

Graphical Method

In the graphical method the straight line graphs representing both equations are plotted on the same set of axes. The solution is the point of intersection of the two lines.
(the graphs must be plotted accurately for this method to give an accurate answer)

Example

Solve the simultaneous equations $x + y = 6$ and $x - y = 2$ graphically



[Graphs representing each linear equation are drawn]

[Point of intersection is read from the graph]

The solution is (4,2)

Check by substituting in each equation: $4 + 2 = 6$ and $4 - 2 = 2$

Exercises

Exercise 1

Solve simultaneously using either the elimination or substitution method:

1. $y = x - 3$ and $5x - 2y = 18$
2. $y = 3x + 4$ and $2x + 3y = 23$
3. $4x + y = 23$ and $x - y = 2$
4. $-3x + 2y = -4$ and $5x - 2y = 8$
5. $3x - 5y = 14$ and $2x + y = 5$
6. $3x + 2y = 18$ and $2x - 5y = -7$

Exercise 2

Solve the following simultaneous equations graphically.

1. $y = x + 3$ and $x + y = 5$
2. $5x - y = 8$ and $2y - 3x = -2$
3. $x + y = 1$ and $y - 2x = 7$

Answers

Exercise 1

1. (4,1) 2. (1,7) 3. (5,3) 4. (2,1) 5. (3,-1) 6. (4,3)

Exercise 2

1. (4,1) 2. (2,2) 3. (-2,3)

QUADRATIC EQUATIONS

General form

A quadratic equation can be rearranged to the form

$$ax^2 + bx + c = 0, \; a \neq 0$$

Examples:

1. $5x^2 - 3x + 9 = 0, \qquad a = 5, \; b = -3, \; c = 9$
2. $x^2 = 5x - 4 \Rightarrow x^2 - 5x + 4 = 0, \qquad a = 1, \; b = -5, \; c = 4$
3. $x = \dfrac{3}{2x} \Rightarrow 2x^2 = 3 \Rightarrow 2x^2 - 3 = 0, \; a = 1, \; b = 0, \; c = -3$

Factorisation

If the equation can be factorised then the 'null factor law' can be used to find the solutions:

Null factor law: If $m \times n = 0$, then $m = 0$ and/or $n = 0$

If the product of two or more factors is zero then any one of the individual factors may be zero and provide a solution for the equation.

Examples:

1. $y^2 = 5y$

$y^2 - 5y = 0$ [rearrange to form $ax^2 + bx + c = 0$]

$y(y - 5) = 0$ [factorise]

$y = 0$ or $y - 5 = 0$ [null factor law]

$\therefore \; y = 0$ or $y = 5$

$y = 0$: $0^2 = 5 \times 0$ ✓ and $y = 5$: $5^2 = 5 \times 5$ ✓ [check by substitution]

2. $x^2 - 5x + 4 = 0$

$(x - 4)(x - 1) = 0$ [factorise]

$x - 4 = 0$ or $x - 1 = 0$ [null factor law]

$x = 4$ or $x = 1$

$x = 4$: $42 - 5 \times 4 + 4 = 0$ ✓ [check by substitution]

$x = 1$: $12 - 5 \times 1 + 4 = 0$ ✓

3. $p^2 + 10p + 25 = 0$

$(p + 5)(p + 5) = 0$ [factorise – perfect square]

$p + 5 = 0$ [null factor law]

$p = -5$

Check: $(-5)2 + 10 \times (-5) + 25 = 0$ ✓

4. $4m^2 - 49 = 0$

$(2m + 7)(2m - 7) = 0$ [factorise - difference of squares]

$2m + 7 = 0$ or $2m - 7 = 0$ [null factor law]

$m = -\frac{7}{2}$ or $m = \frac{7}{2}$

5. $x = \frac{-6}{1-2x}$ provided $x \neq ½$

$x - 2x^2 = -6$

$2x^2 - x - 6 = 0$ [rearrange to form $ax^2 + bx + c = 0$]

$(2x + 3)(x - 2) = 0$ [factorise]

$x = -\frac{3}{2}$ or $x = 2$ [solve using null factor law]

Exercise

Solve the following quadratic equations.

(1) $x^2 - 6x + 8 = 0$ (2) $x^2 + 2x - 3 = 0$

(3) $2x^2 - 3x - 2 = 0$ (4) $6 - z - z^2 = 0$

(5) $2x^2 + 7x = 15$ (6) $11p = 3(2p^2 + 1)$

Answers

(1) $x = 4, x = 2$ (2) $x = -3, x = 1$

(3) $x = -½, x = 2$ (4) $z = -3, z = 2$

(5) $x = \frac{3}{2}$, $x = -5$ (6) $p = \frac{1}{3}, p = \frac{3}{2}$

QUADRATIC FORMULA

The solutions to any quadratic equation $ax^2+bx+c=0$ can be found by substituting the values a, b, c into the *quadratic formula:*

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

Examples

1. $x^2-5x+4=0 \rightarrow a=1, \ b=-5, \ c=4$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$x = \frac{5 \pm \sqrt{25-4(1)(4)}}{2}$$

$$x = \frac{5 \pm \sqrt{9}}{2}$$

$$x = \frac{5+\sqrt{9}}{2} \quad \text{or} \quad x = \frac{5-\sqrt{9}}{2}$$

$$x = 4 \quad \text{or} \quad x = 1$$

2. $x^2 + x + 10 = 0 \Rightarrow a = 1, \ b = 1, \ c = 10$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$x = \frac{-1 \pm \sqrt{1^2-4(1)(10)}}{2}$$

$$x = \frac{-1+\sqrt{-39}}{2}$$ which has no solution

Discriminant

NB: The part of the quadratic formula which is under the radical sign ***b² – 4ac*** is called the *discriminant.* Its value determines the number of solutions and whether they will be rational or irrational.

$b^2 - 4ac < 0 \rightarrow$ no solutions

$b^2 - 4ac = 0 \rightarrow$ one solution

$b^2 - 4ac > 0 \rightarrow$ two solutions

If $b^2 - 4ac$ is a perfect square the solutions will be rational.

Exercise

Solve the following quadratic equations (if possible)

1. $x^2 - 2x - 168 = 0$

2. $x^2 - x - 7 = 0$

3. $2x^2 + x - 2 = 0$

4. $3x^2 = x + 1$

5. $\frac{3x+1}{2} = \frac{x+1}{x}$

Answers

1. x = -12 or x = 14
2. $x = \frac{1 \pm \sqrt{29}}{2}$
3. $x = \frac{-1 \pm \sqrt{17}}{4}$
4. $x = \frac{1 \pm \sqrt{13}}{6}$
5. $x = -\frac{2}{3}$ or $x = 1$

TRANSPOSITION WITH CHALLENGES

Possible complications

- *The subject variable appears more than once in the formula!*

  For example: Make 'm' the subject of $E = m g h + \frac{1}{2} m v^2$

  Suggested procedure:

  > Move all terms containing 'm' to one side of the formula and factorise.

Examples

1. *Make 'm' the subject of $E = m g h + \frac{1}{2} m v^2$*

$E = m g h + \frac{1}{2} m v^2$ [All 'm's are already on one side of the formula]

$E = m (g h + \frac{1}{2} v^2)$ [Factorise]

$\frac{E}{gh+\frac{1}{2}v^2} = m$ [÷(gh + ½ v²)]

or $m = \frac{E}{gh+\frac{1}{2}v^2}$

2. *If $Ir = E - IR$ rearrange the equation to make I the subject*

$Ir = E - IR$

$I r + I R = E$ [move 'I's to one side of formula]

$I (r + R) = E$ [factorise]

$I = \frac{E}{r+R}$ [÷ (r + R)]

- *The formula contains lots of fractions!!*

  For example: Make 'u' the subject of $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$

  Suggested procedure:

  > - Cross multiply if possible
  >   (ie if there is only one fraction on each side of equation)
  >
  >   otherwise
  > - Eliminate fractions by multiplying both sides by the lowest common denominator.

3. *Express in terms of 'u'* $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$

$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$

$fuv \times \frac{1}{f} = fuv \times \frac{1}{v} - fuv \times \frac{1}{u}$ [multiply all terms by LCD (fuv)]

$u v = f u - f v$ [cancel and simplify]

$u v - f u = -f v$ [move all terms containing 'u' to one side]

$u (v - f) = -f v$ [factorize]

$u = \frac{-fv}{v-f}$ [÷ (v – f)]

- *the subject is an exponent!!!*

  For example: Make 't' the subject of $Q = A \times 10^{kt}$

  Suggested procedure:

Use logarithms: Remember $\log x^n = n\log x$

5. *Express in terms of 't'* $Q = A \times 10^{kt}$

$$Q = A \times 10^{kt}$$

$$\frac{Q}{A} = 10^{kt} \qquad [\div \text{ 'A' both s}]$$

$$\log\left(\frac{Q}{A}\right) = \log 10^{kt} \qquad \text{[take logs both sides]}$$

$$\log\left(\frac{Q}{A}\right) = kt\log 10 \qquad [\log x^n = n\log x]$$

$$\log\left(\frac{Q}{A}\right) = kt \qquad [\log 10 = 1]$$

$$\frac{\log\left(\frac{Q}{A}\right)}{k} = t \text{ or } t = \frac{\log\left(\frac{Q}{A}\right)}{k} \qquad [\div k \text{ both sides}]$$

Sometimes it is helpful to change a logarithmic equation into its equivalent exponential form:

$$y = \log_a x \iff a^y = x$$

6. *Transform the formula* $T = \frac{1}{c}\log_e(m - A)$ *to make 'm' the subject*

$$T = \frac{1}{c}\log_e(m - A)$$

$$cT = \log_e(m - A) \qquad [\times c \text{ both sides}]$$

$$e^{cT} = m - A \qquad \text{[change to exponential equation]}$$

$$m = e^{cT} + A \qquad [+ A \text{ both sides}]$$

Exercises

Transpose the following formulae to make the variable in brackets the subject.

1. $At = M(P + t)$ [t]
2. $I = \frac{E}{R+r}$ [r]
3. $W = \frac{2PR}{R - r}$ [R]
4. $A = \sqrt{\frac{2q(L-r)}{rL}}$ [L]
5. $E = \frac{w^2 a}{(w^2+m)b^2}$ [w]
6. $H = Ae^{-kt}$ [t]
7. $p = \frac{1}{q^2}\log_e\left(\frac{M}{2}\right)$ [M]

Answers (NB: there may be alternative answers that are algebraically equivalent)

1. $t = \frac{MP}{A-M}$
2. $r = \frac{E-IR}{I}$
3. $R = \frac{Wr}{W-2P}$
4. $L = \frac{2qr}{2q-A^2r}$
5. $w = \pm\sqrt{\frac{Eb^2m}{a-Eb^2}}$
6. $t = -\frac{1}{k}\log_e\left(\frac{H}{A}\right)$
7. $M = 2e^{pq^2}$