

IN3 INTEGRATION OF FUNCTIONS

Integrals of the form $\frac{1}{x}$ and $\frac{1}{ax+b}$

The rule for integrating x^n cannot be used when $n = -1$. (Why?)

Differentiating $\log_e x$ gives $\frac{1}{x}$, therefore an antiderivative of $\frac{1}{x}$ is $\log_e x$

$$\int \frac{1}{x} dx = \log_e |x| + c$$

or more generally

$$\int \frac{1}{ax+b} dx = \frac{1}{a} \log_e |ax+b| + c$$

NB:

- we cannot take the log of a negative number
- this rule only applies when the denominator is a linear function

Examples

$$1. \quad \int \frac{5}{x} dx = 5 \log_e |x| + c$$

$$2. \quad \int \frac{1}{(2x-5)} dx = \frac{1}{2} \log_e |2x-5| + c \quad [a = 2 \text{ and } b = -5]$$

$$3. \quad \int \frac{1}{(1-3x)} dx = \int \frac{1}{(-3x+1)} dx \quad [a = -3 \text{ and } b = 1]$$

$$= -\frac{1}{3} \log_e |1-3x| + c$$

Exponential function

If $f(x) = e^x$ then $f'(x) = e^x$, therefore an antiderivative of e^x is e^x

$$\int e^x dx = e^x + c$$

or more generally

$$\int e^{ax+b} dx = \frac{1}{a} e^{ax+b} + c$$

Examples

1. $\int 2e^x dx = 2e^x + c$
2. $\int e^{-5x+1} dx = -\frac{1}{5}e^{-5x+1} + c$ [a = 5 and b = 1]
3. $\int e^{\frac{x}{3}+4} dx = \frac{1}{\frac{1}{3}}e^{\frac{x}{3}+4} + c = 3e^{\frac{x}{3}+4} + c$ [a = $\frac{1}{3}$ and b = 4]

Trigonometric functions

Using the derivatives of the trigonometric functions the following integrals can be deduced:

$$\begin{aligned}\int \sin(x) dx &= -\cos(x) + c \quad \text{or} \quad \int \sin(ax + b) dx = -\frac{1}{a}\cos(ax + b) + c \\ \int \cos(x) dx &= \sin(x) + c \quad \text{or} \quad \int \cos(ax + b) dx = \frac{1}{a}\sin(ax + b) + c \\ \int \sec^2(x) dx &= \tan(x) + c \quad \text{or} \quad \int \sec^2(ax + b) dx = \frac{1}{a}\tan(ax + b) + c\end{aligned}$$

Examples

1. $\int 5 \cos(x) dx = 5 \sin(x) + c$
2. $\int 3 \cos(3 - 2x) dx = -\frac{3}{2} \sin(3 - 2x) + c$ [a = -2 and b = 3]
3. $\int \sec^2\left(\frac{x}{2}\right) dx = 2 \tan\left(\frac{x}{2}\right) + c$ [a = $\frac{1}{2}$ and b = 0]

Exercise

Find the following integrals

1. (a) $\int \frac{1}{x+5} dx$ (b) $\int \frac{1}{(6-5x)} dx$ (c) $\int \frac{1}{(3x+5)} - \frac{1}{x-2} dx$
2. (a) $\int e^{3x} dx$ (b) $\int e^{2-5x} dx$ (c) $\int \frac{9e^{3x}+5}{e^{2x}} dx$ [Hint: divide through]
3. (a) $\int \sec^2(4x) dx$ (b) $\int 2 \cos(1-x) dx$ (c) $\int 2 \sin\left(\frac{5-3x}{4}\right) dx$

Answers

1. (a) $\log_e|x+5| + c$ (b) $-\frac{1}{5}\log_e|6-5x| + c$ (c) $\frac{1}{3}\log_e|3x+5| - \log_e|x-2| + c$
2. (a) $\frac{e^{3x}}{3} + c$ (b) $\frac{e^{2-5x}}{-5} + c$ (c) $9e^x - \frac{5}{2e^{2x}} + c$
3. (a) $\frac{1}{4}\tan 4x + c$ (b) $-2\sin(1-x) + c$ (c) $\frac{8}{3}\cos\left(\frac{5-3x}{4}\right) + c$