**RMIT UNIVERSITY**

# FU1.6: INTERVAL NOTATION

## Interval

Often the domain of a function will be restricted to a subset of $R$.
This subset is called an *interval*, and the *end points* are **a** and **b**.
An interval may represented on a real number line as follows :



In inequality notation the above number line would be written as $a \le x \le b$.

In *interval notation* the above interval would be written as [a, b].

## Closed Interval

Because the endpoints are included in the interval, this is called a *closed interval.*
Square brackets are used. eg.[2, 5].
The end points on the on the real number line are represented as solid circles (or square brackets).

## Open interval

If the endpoints are excluded, the interval is an *open interval.*
Curved brackets are used. eg (2, 5).

The end points on the real number line are represented as open circles (or curved brackets).



This is written in inequality notation as $a < x < b$.
In interval notation as (*a, b*).

## Examples

| Interval Notation | Inequality Notation | Line Graph |
|---|---|---|
| [*a, b* ] | $a \le x \le b$ |  |
| (*a, b* ) | $a < x < b$ |  |
| [*a, b* ) | $a \le x < b$ |  |

| | | |
|---|---|---|
| $(a, b]$ | $a < x \leq b$ |  |
| $[a, \infty)$ | $x \geq a$ |  |
| $(a, \infty)$ | $x > a$ |  |
| $(-\infty, b]$ | $x \leq b$ |  |
| $(-\infty, b)$ | $x < b$ |  |

In interval notation the smaller number is *always* written to the left.

i.e.    $[-3, 5) \neq (5, -3]$

Note :   the symbol $\infty$  (infinity) is **not** a numeral.

$\infty$ is the concept of continuing indefinitely to the right
$-\infty$ is the concept of continuing indefinitely to the left.

Hence we cannot write $[b, \infty]$ ,  $[-\infty, a]$   or    $b \leq x \leq \infty$ ,  $-\infty \leq x \leq a$   etc..

## Examples

**1.** Write the following in inequality notation and graph on a real number line.

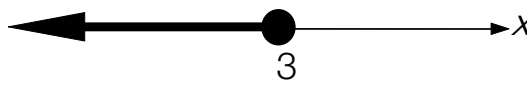(a)  $[-2, 3)$
Inequality notation   $-2 \leq x < 3$

Graph



(b)      $(-\infty, 3]$.
Inequality notation   $x \leq 3$

Graph



**2.** Write the interval notation and inequality notation for the following line graphs.
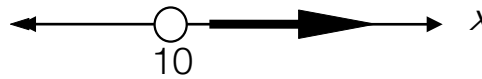
(a)



Interval notation  $(-5, 6]$.
Inequality notation $-5 < x \leq 6$

(b)



Interval notation   $(10, \infty)$
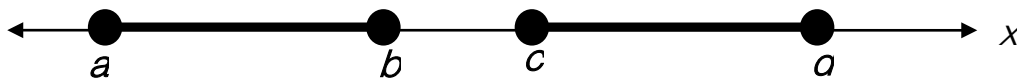Inequality notation $10 < x < \infty$

**See Exercise 1**

# Two intervals

Two (or more) subsets of R, with end points *a* and *b,* and *c* and *d,* respectively, can also be represented on a real number line.

# Examples

1.



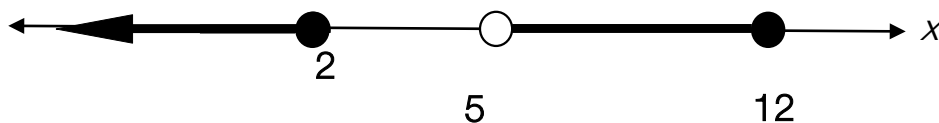This is written in interval notation as [a, b] ∪ [c, d].
The symbol as ∪ represents '*in union with*'
In inequality notation this may be written:   $a \leq x \leq b$ with $c \leq x \leq d$

2.



This is written in interval notation as (-∞, 2] ∪ (5, 12].
In inequality notation  this may be written:   $x \leq 2$  with $5 < x \leq 12$

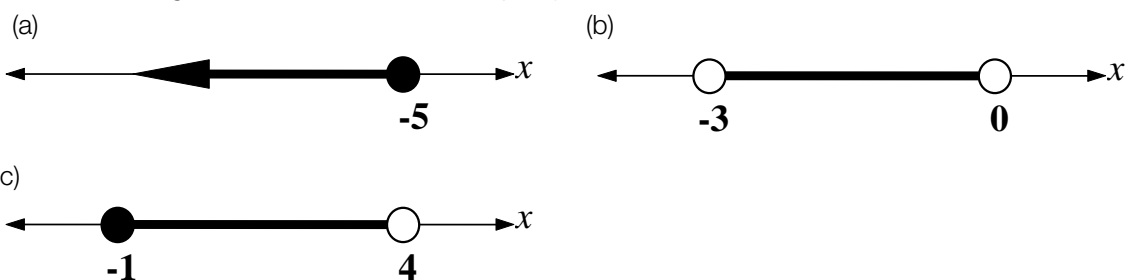See Exercise 2

# Exercises

### Exercise 1

Write the following inequalities in interval notation and graph on a real number line.

    (a)  $1 \leq x \leq 10$      (b)  $-6 \leq x < -4$      (c) $x > 5$

### Exercise 2

 Write the following in interval notation and inequality notation.

    (a)                                       (b)



    (c)



### Exercise 3

Graph the following on a real number line and write in inequality notation.

  (a)  (-∞, -3) ∪ (8, 13].     (b) [-1, 4] ∪ [6, 9].     (c) (-∞, 3] ∪ (6, ∞)
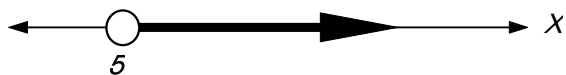
Answers

## Exercise 1

(a)  [1, 10 ]



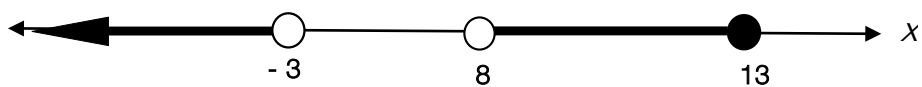(b)  [- 6, - 4 )



(c)   (5, ∞ )



## Exercise 2

(a) (-∞, -5 ], $x \leq -5$          (b) (-3, 0 ),  $-3 < x < 0$          (c) [-1, 4 ),  $-1 \leq x < 4$

## Exercise 3

(a) $-\infty < x < -3$   with   $8 < x \leq 13$



(b)  $-1 \leq x \leq 4$    with    $6 \leq x \leq 9$



(c)  $-\infty < x \leq 3$   with   $6 < x < \infty$