INTRODUCTORY PROBABILITY

A probability is written as a number between zero and one: $0 \le \Pr(A) \le 1$

$\Pr(A) = 0$ means that event A is impossible.

$\Pr(A) = 1$ means that event A is certain.

When considering a set of all possible outcomes an ***event*** is a particular outcome of interest.
For example,

- In tossing a coin the particular event of interest might be 'obtaining a head'
- In considering the weather for Saturday the event of interest might be 'it doesn't rain'
- In planning a two child family he particular event of interest might be 'a boy and a girl'.

The probability of an event E can be found with the formula:

$$\Pr(E) = \frac{\text{number of ways E can occur}}{\text{total number of possible outcomes}}$$

[assuming all outcomes are equally likely]

Examples:

1. If two coins are tossed find the probability of obtaining two heads.

   Let E be the event 'two heads'

   The possible outcomes are HH HT TH TT

   $$\therefore \quad \Pr(E) = \frac{\text{number of ways E can occur}}{\text{total number of possible outcomes}}$$

   $$= \frac{1}{4}$$

2. If a die is thrown find the probability of obtaining an odd number

   Let E be the event 'an odd number'

   The possible outcomes are 1 2 3 4 5 6

   $$\therefore \quad \Pr(E) = \frac{\text{number of ways E can occur}}{\text{total number of possible outcomes}}$$

   $$= \frac{3}{6}$$

   $$= \frac{1}{2}$$

The multiplication principle

Two events, A and B, are independent if the fact that A occurs does not affect the probability of B occurring. Because successive tosses of a coin are independent events, an alternate way of calculating the probability in example one would be to use the multiplication principle.

If A and B are independent events then
$\Pr(A \text{ and } B) = \Pr(A \cap B) = \Pr(A) \times \Pr(B)$

The probability of a head on the first toss (H_1) and a head on the second toss (H_2)

$$= \Pr(H_1 \cap H_2)$$

$$= \frac{1}{2} \times \frac{1}{2}$$

$$= \frac{1}{4}$$

The addition principle

$$\Pr(A \text{ or } B) = \Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$$
If A and B are mutually exclusive (cannot happen together) then
$$\Pr(A \text{ or } B) = \Pr(A \cup B) = \Pr(A) + \Pr(B)$$

If we are tossing a single die twice and want to calculate the probability that a 6 occurs, then the 6 could occur on the first toss (S_1) or on the second toss (S_2):

$\Pr(S_1 \text{ or } S_2) = \Pr(S_1 \cup S_2) = \Pr(S_1) + \Pr(S_2) - \Pr(S_1 \cap S_2)$ [because the events are not mutually exclusive]

$$= \frac{1}{6} + \frac{1}{6} - \frac{1}{36}$$
$$= \frac{11}{36}$$

Complementary events

If E is an event in then (not E) or $\overline{E}$ or E' is called the complement of E.

Examples of complementary events:

- 'winning the grand final' and 'not winning the grand final'
- 'passing a test' and 'failing a test'
- 'being left handed' and 'being right handed'

Because $P(E) + P(E') = 1$ it follows that
$$P(E') = 1 - P(E)$$

In the previous example where a die was tossed twice the probability of not getting a 6 on either the first or second toss $= 1 - \frac{11}{36}$

$$= \frac{25}{36}$$

Exercise

1. If 1000 tickets are sold in a raffle and one winning ticket is chosen at random, what is my probability of winning the raffle if I buy 5 tickets?

2. If I roll a die, what is the probability that the number uppermost is greater than 4?

3. A bag contains 6 white marbles and 4 black marbles. A marble is chosen, the colour recorded and then replaced three times. What is the probability that all three marbles are white?

4. The probability that person A is alive in 30 years time is 0.7. The probability that person B is alive in 30 years time is 0.4 .

Find the probability that:

(a) both are alive in 30 years. (b) neither are alive in 30 years

(c) only one is alive in 30 years time (d) at least one is alive in 30 years time.

Answers

1. $\frac{1}{200}$ 2. $\frac{1}{3}$ 3. 0.216 4. (a) 0.28 (b) 0.18 (c) 0.54 (d) 0.82