

FA1.4: FACTORISATION: DIFFERENCE OF TWO SQUARES

Difference of two squares (DOTS)

Consider the following expansion:

$$\begin{aligned}(x+5)(x-5) &= x^2 + 5x - 5x - 25 \\ &= x^2 - 25\end{aligned}$$

Note that:

- the terms in the brackets differ only in the sign of the second term
- the expansion is the difference of two terms, both of which are perfect squares

In general:

$$(a+b)(a-b) = a^2 - b^2$$

This can be used to factorise expressions of the form $a^2 - b^2$

Examples

- $$a^2 - 36 = (a)^2 - (6)^2$$

$$a^2 - 36 = (a+6)(a-6)$$

expression is the **difference** of two squares

'DOTS' rule (order of the terms on right hand side is not important)
- $$4x^2 - y^2 = (2x)^2 - (y)^2$$

$$4x^2 - y^2 = (2x-y)(2x+y)$$

expression is the **difference** of two squares

'DOTS' rule
- $$3x^2 - 48 = 3(x^2 - 16)$$

$$3x^2 - 48 = 3((x)^2 - (4)^2)$$

$$3x^2 - 48 = 3(x+4)(x-4)$$

not the difference of two squares but to factorise, take out a common factor of 3

factorise using 'DOTS' rule
- $$(x+2)^2 - 9 = (x+2)^2 - 3^2$$

$$(x+2)^2 - 9 = ((x+2)-3)((x+2)+3)$$

$$(x+2)^2 - 9 = (x-1)(x+5)$$

expression is the **difference** of two squares

'DOTS' rule

Simplify
- $$y^2 + 36$$

this is **not the difference** of two squares. It is the **sum** of two squares.

No real factors

Exercise

Factorise the following using the 'DOTS' rule (if possible).

1. $x^2 - 4$

4. $64x^2 - 1$

7. $5x^2 - 20$

10. $(x+2)^2 - y^2$

2. $a^2 - 100$

5. $121x^2 - 49y^2$

8. $4a^2 + 100$

11. $(x-5)^2 - 36$

3. $49 - x^2$

6. $a^2b^2 - 25$

9. $x^2y^3 - 36y$

12. $(a+1)^2 - (b-2)^2$

Answers

1. $(x-2)(x+2)$

4. $(8x-1)(8x+1)$

7. $5(x+2)(x-2)$

10. $(x-y+2)(x+y+2)$

2. $(a+10)(a-10)$

5. $(11x+7y)(11x-7y)$

8. Does not factorise

11. $(x-11)(x+1)$

3. $(7+x)(7-x)$

6. $(ab-5)(ab+5)$

9. $y(xy+6)(xy-6)$

12. $(a+b-1)(a-b+3)$