Assignment : 01 (RMJ916)

Q1) i) $\epsilon_{exc} = \epsilon_{app} + \epsilon_{est} + \epsilon_{opt}$

Sol^n :

$\epsilon_{app} = L(f_{opt}) - L(f^*)$

Approximation error = difference of loss of f^* and f_{opt}

$\epsilon_{est} = L(\hat{f}_{opt}) - L(f_{opt})$

estimation error = difference between loss of f_{opt} and $\hat{f}_{opt}$

$\epsilon_{opt} = L(\hat{f}) - L(\hat{f}_{opt})$

Optimization error = difference between loss of $\hat{f}_{opt}$ and $\hat{f}$

$$\text{Right side} = \epsilon_{app} + \epsilon_{est} + \epsilon_{opt}$$

$$= \cancel{L(f_{opt})} - L(f^*) + \cancel{L(\hat{f}_{opt})} - \cancel{L(f_{opt})} + L(\hat{f}) - \cancel{L(\hat{f}_{opt})} \quad (\because \text{ by defination})$$

$$= L(\hat{f}) - L(f^*)$$

$$= \epsilon_{exc} \quad (\because \text{by defination})$$

→ This is the equation of excess risk which measures the distance from the output of the algorithom to best solution possible.

L.H.S = R.H.S

$\boxed{\epsilon_{exc} = \epsilon_{exc}}$ ∴ Prove

So, $\boxed{\epsilon_{exc} = \epsilon_{app} + \epsilon_{est} + \epsilon_{opt}}$

(ii) $\boxed{\epsilon_{est} \le 2\epsilon_{con}}$

soln: $f \in H$
empirical loss concentrates around the ~~the~~ expected loss;
there $\epsilon_{con} > 0$ (sample complexity)

$$|\hat{L}(f) - L(f)| \le \epsilon_{con}$$

$$-\epsilon_{con} \le \hat{L}(f) - L(f) \le \epsilon_{con}$$

$$\therefore \epsilon_{est} = L(\hat{f}_{opt}) - L(f_{opt})$$

→ now adding value for ϵ_{est} which represent same evaluation

$$\therefore \epsilon_{est} = \hat{L}(\hat{f}_{opt}) - \underbrace{\hat{L}(\hat{f}_{opt}) + L(\hat{f}_{opt})} - L(f_{opt})$$

$$= \hat{L}(\hat{f}_{opt}) - L(f_{opt}) + L(\hat{f}_{opt}) - \hat{L}(\hat{f}_{opt})$$

$$\le \hat{L}(f_{opt}) - L(f_{opt}) + L(\hat{f}_{opt}) - \hat{L}(\hat{f}_{opt})$$

($\because$ by defination $\hat{f}_{opt}$ minimizes the empirical loss)

so, $\le |\hat{L}(f_{opt}) - L(f_{opt})| + |L(\hat{f}_{opt}) - \hat{L}(\hat{f}_{opt})|$

$$\le |\hat{L}(f_{opt}) - L(f_{opt})| + \epsilon_{con}$$

($\because \epsilon_{con}$ because $\hat{L}(f) - L(f) \le \epsilon_{con}$)

note: [$\hat{L}(f_{opt})$ showing minimize empirical loss so, difference of $\hat{L}(f_{opt}) - L(f_{opt})$ is less than or equal ϵ_{con}]

so, $\le \epsilon_{con} + \epsilon_{con}$

$$\le 2\epsilon_{con}$$

Prove: $\boxed{\epsilon_{est} \le 2\epsilon_{con}}$