ASSIGNEMENT: 5

1) For Restricted Boltzman machine is defined over binary vectors $v, h \in \{0,1\}^d$ as

$$\therefore P(V,h) := \frac{\exp(-E(V,h))}{Z}$$

where E is the energy function

$$\therefore E(V,h) = -V^T W h - b^T V - C^T h$$

with parameters $W \in R^{d \times d}$, $b, C \in R^d$ and Z is the partition function

$$\therefore Z = \sum_V \sum_h \exp(-E(V,h))$$

For simplicity assume that the dimension $d = 1$

show that

$$\therefore P(h = 1/V) = \sigma(C + WV)$$

where $\sigma(x) := \frac{1}{1+\exp(-x)}$ is the logistic function

directly calculate $P(h = 1/V) = \frac{P(V,1)}{\sum_h P(V,h)}$

$\rightarrow$ $E(V,h) = -V^T W h - b^T V - C^T h$

$$\therefore Z = \sum_V \sum_h \exp(-E(V,h)) \qquad \text{equ } 1$$

where $d = 1$

$$P(V,h) = \frac{\exp(-E(V,h))}{Z} \qquad \text{eq } 2$$

Now,

Left side $= p(h=1/v)$

$$= \frac{p(v,1)}{\sum_h p(v,h)} \quad \text{———} \left(\because p(h/v) = \frac{p(v,h)}{\sum_h p(v,h)} \right)$$

$$= \frac{\frac{\exp(-E(v,1))}{Z}}{\sum_h p(v,h)} \quad \text{———— using equ (2)}$$

$$= \frac{\exp(-E(v,1))}{Z, \sum_h p(v,h)}$$

$$= \frac{\exp(-E(v,1))}{Z \cdot \sum_h \frac{\exp(-E(v,h))}{Z}}$$

$$= \frac{\exp(-E(v,1))}{\sum_h \exp(-E(v,h))}$$

$$= \frac{\exp(v^T w + b^T v + c^T)}{\sum_h \exp(v^T w h + b^T v + c^T h)}$$

$$= \frac{\exp(v^T w + c^T)}{\sum_h \exp(v^T w h + c^T h)} \quad \text{———— equ (3)}$$

So, we have $v, h \in \{0,1\}^d$ and $d=1$;

$v, h = \{0,1\}$

$$\therefore p(h=1/v) = \frac{\exp(v^T w + c^T)}{\exp(v^T w \cdot 0 + c^T \cdot 0) + \exp(v^T w \cdot 1 + c^T \cdot 1)}$$

$$= \frac{\exp(V^T\omega + C^T)}{1 + \exp(V^T\omega + C^T)}$$

$$= \frac{1}{\frac{1}{\exp(V^T\omega + C^T)} + 1}$$

$$= \frac{1}{1 + \exp(-(V^T\omega + C^T))}$$

$$= \sigma(V^T\omega + C^T)$$

$$= \text{Right side}$$

So, $P(h=1/V) = \sigma(V^T\omega + C^T)$

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