2) SVM objective can be simplified as minimizing

$$\tilde{L}(\alpha) = \sum_{i=1}^{n} \alpha_i - \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} \alpha_i \alpha_j y_i y_j x_i^T x_j \quad \text{——— eq 1}$$

$$\alpha_i \geq 0 \ \&\& \ \sum_{i=1}^{n} \alpha_i y_i = 0$$

$$\frac{1}{\gamma^2} = \sum_{i=1}^{n} \alpha_i^* \quad \text{——— have to prove}$$

So,

$$L(\omega, b, \alpha) = \frac{1}{2} ||\omega||^2 - \sum_i \alpha_i [y_i(\omega^T x_i + b) - 1] \quad \text{——— (1)}$$

as a KKT conditions $\omega = \sum_i \alpha_i y_i x_i$

$$= \frac{1}{2}(\sum_i \alpha_i y_i x_i)^2 - \sum_i \alpha_i [y_i(\omega^T x_i + b) - 1]$$

$$= \frac{1}{2}(\sum_i \alpha_i y_i x_i)^2 - \sum_i \alpha_i y_i (\sum_j \alpha_j y_j x_j)^T x_i - \sum \alpha_i y_i b + \sum \alpha_i$$

$$= \frac{1}{2}(\sum_i \alpha_i y_i x_i)^2 - \sum_{ij} \alpha_i \alpha_j \cdot y_i y_j \cdot x_i x_j^T - b \sum \alpha_i y_i + \sum \alpha_i$$

$$= \frac{1}{2}(\sum_i \alpha_i y_i x_i)^2 - (\sum_i \alpha_i y_i x_i)^2 + \sum \alpha_i$$

from (1) number $\sum_i \alpha_i [y_i(\omega^T x_i + b) - 1]$ is optimal value and it can minimized we can neglect that

So,

$$L(\omega, b, \alpha) = \frac{1}{2} ||\omega||^2 \quad \text{——— (2)}$$

and

$$\tilde{L}(\alpha) = \sum_{i=1}^{n} \alpha_i - \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} \alpha_i \alpha_j y_i y_j x_i^T x_j \quad \text{——— (3)}$$

using 2 and 3 equation

$$\frac{1}{2}||w||^2 = \sum_{i=1}^{n} \alpha_i - \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} \alpha_i \alpha_j y_i y_j x_i^T x_j$$

so, As written in KKT conditions

$$w = \Sigma_i \alpha_i y_i x_i$$

$$w = \Sigma_j \alpha_j y_j x_j$$

Therefore,

$$\frac{1}{2}||w||^2 = \sum_{i=1}^{n} \alpha_i - \frac{1}{2}||w||^2$$

$$\frac{1}{2}||w||^2 + \frac{1}{2}||w||^2 = \sum_{i=1}^{n} \alpha_i$$

$$||w||^2 = \sum_{i=1}^{n} \alpha_i$$

and margin is $\frac{1}{||w||}$ so $\gamma = \frac{1}{||w||}$ as written in chapter 5

so,
$$\frac{1}{\gamma^2} = \sum_{i=1}^{n} \alpha_i$$

so, it's proven

$$\boxed{\frac{1}{\gamma^2} = \sum_{i=1}^{n} \alpha_i^*}$$

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