

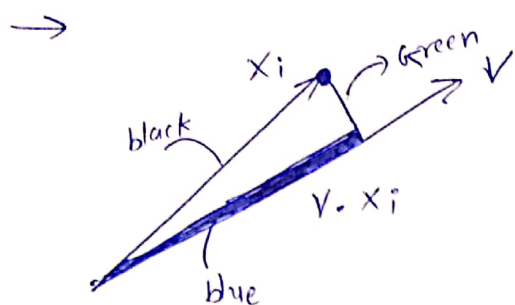
# ASSIGNMENT- 04

Q.1) (Math) Show that the optimal solution to the following two is the same:

$$\text{mean } \frac{1}{n} \sum_{i=1}^n x_i = 0$$

$$(i) \max_{V: \|V\|_2=1} \sum_{i=1}^n (V^T x_i)^2$$

$$(ii) \min_{V: \|V\|_2=1} \frac{1}{n} \sum_{i=1}^n \|x_i - V V^T x_i\|^2$$



According to the figure,  $x_i$  is the  $i$ -th data point,  $V^T x_i$  is the projection of  $x_i$  on  $V^T$  direction.

$V V^T x_i$  is a vector with direction of  $V^T$  and length of  $V^T x_i$ , ex. the vector

$$\text{blue} = V V^T x_i$$

So, vector Green = black - blue =  $x_i - V V^T x_i$

$$\text{Green} = x_i - V V^T x_i$$

$$\text{blue} = V V^T x_i$$

$$\text{black} = x_i$$

As well as,  $\text{blue}^2 + \text{Green}^2 = \text{black}^2$  where  $\text{black}^2$  is fixed.

$$\text{Maximizing } \text{blue}^2 = (V V^T x_i)^2 = x_i V V^T V V^T x_i = x_i V V^T x_i \quad (V^T V = 1) \quad \text{is } \text{---} \textcircled{1}$$

$$\text{equivalent to minimizing } \text{green}^2 = \|x_i - V V^T x_i\|^2 \quad \text{---} \textcircled{2}$$

So, the both equation  $\textcircled{1}$  &  $\textcircled{2}$  are extreme same function.