Q2) sigmoid function $\sigma(a) = \dfrac{1}{1+\exp(a)} = \dfrac{1}{1+e^{-a}}$

Probit function $\Phi(a) = \displaystyle\int_{-\infty}^{a} N(\theta \mid 0,1)\, d\theta$

$N(\theta \mid 0,1) \rightarrow$ standard normal distribution (μ) mean $= 0$, variance $= 1$ (σ)

Standard distribution function

$$f(\theta \mid \mu, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(\theta-\mu)^2}{2\sigma^2}} = \frac{1}{\sqrt{2\pi(1)^2}} e^{-\frac{(\theta-0)^2}{2(1)^2}} = \frac{1}{\sqrt{2\pi}} e^{-\theta^2/2}$$

Using Probit function instead of sigmoid function from given traing data $\{(x_i, y_i) : 1 \le i \le n\}$ from distribution D

so, for Probit regression negative conditional liklihood loss is:

$$\therefore \hat{L}(\omega) = -\frac{1}{n}\sum_{y_i=1} \log \Phi(\omega^T x_i) - \frac{1}{n}\sum_{y_i=0} \log\left[1 - \Phi(\omega^T x_i)\right]$$

$$= -\frac{1}{n}\sum_{y_i=1} \log \int_{-\infty}^{\omega^T x_i} N(\theta \mid 0,1)\, d\theta - \frac{1}{n}\sum_{y_i=0} \log\left[1 - \int_{-\infty}^{\omega^T x_i} N(\theta \mid 0,1)\, d\theta\right]$$

where $N(\theta \mid 0,1) = \dfrac{1}{\sqrt{2\pi}} e^{-\theta^2/2}$

$$= -\frac{1}{n}\sum_{y_i=1} \log \int_{-\infty}^{\omega^T x_i} \frac{1}{\sqrt{2\pi}} e^{-\theta^2/2}\, d\theta - \frac{1}{n}\sum_{y_i=0} \log\left[1 - \int_{-\infty}^{\omega^T x_i} \frac{1}{\sqrt{2\pi}} e^{-\theta^2/2}\, d\theta\right]$$

so, our final equation for negative conditional liklihood loss using probit function:

$$\boxed{\hat{L}(\omega) = -\frac{1}{n}\sum_{y_i=1} \log \int_{-\infty}^{\omega^T x_i} \frac{1}{\sqrt{2\pi}} e^{-\theta^2/2}\, d\theta - \frac{1}{n}\sum_{y_i=0} \log\left[1 - \int_{-\infty}^{\omega^T x_i} \frac{1}{\sqrt{2\pi}} e^{-\theta^2/2}\, d\theta\right]}$$