ASSIGNMENT: 01 (RMJ916)

1) $P(x|y=i) = N(x|\mu_i, \Sigma)$

→ Standard Normal distribution

$N(x|\mu_i, \Sigma)$ → μ_i → mean, Variance $= \Sigma$

$= \frac{1}{\sqrt{2\pi\Sigma}} e^{-\frac{(x-\mu_i)^2}{2\Sigma}}$

$\therefore N(x|\mu_i, \Sigma) = \frac{1}{\sqrt{2\pi\Sigma}} e^{-\frac{(x-\mu_i)^2}{2\Sigma}}$

→ $a_i = \ln[P(x|y=i)\, P(y=1)]$

here we puting value $P(x|y=i) = N(x|\mu_i, \Sigma)$

$a_i = \ln[N(x|\mu_i, \Sigma)\, P(y=1)]$

$= \ln[(2\pi\Sigma)^{-1/2} * \exp\{-\frac{1}{2}(x-\mu_i)^2 \Sigma^{-1}\} * P(y=i)]$

$= \ln(2\pi\Sigma)^{-1/2} + \ln[\exp\{-\frac{1}{2}(x-\mu_i)^2 \Sigma^{-1}\}] + \ln[P(y=i)]$

$= \ln(2\pi)^{d/2}(\Sigma)^{1/2} + \{-\frac{1}{2}(x-\mu_i)^2 \Sigma^{-1}\} + \ln[P(y=i)]$

$= \ln(2\pi)^{d/2} \cdot (\Sigma)^{1/2} - \frac{1}{2\Sigma}[x^T x - 2x \cdot \mu_i + \mu_i^T \mu_i] + \ln[P(y=i)]$ $(\because (x-y)^2 = x^2 - 2xy + y^2)$

$= \ln(2\pi)^{d/2} \cdot (\Sigma)^{1/2} - \frac{1}{2\Sigma} x^T x + \frac{2}{2\Sigma} x \mu_i - \frac{1}{2\Sigma} \mu_i^T \mu_i + \ln[P(y=i)]$

$= -\frac{1}{2\Sigma} x^T \cdot x + \frac{(\mu_i x)}{\Sigma} + \ln((2\pi)^d \Sigma)^{1/2} - \frac{1}{2\Sigma} \mu_i^T \mu_i + \ln[P(y=i)]$

$\therefore \omega^T = \frac{\mu_i}{\Sigma}$

so, $a_i = -\frac{1}{2\Sigma} x^T \cdot x + (\omega^i)^T x + b_i$

$\therefore b_i = -\frac{1}{2\Sigma} \mu_i^T \mu_i + \ln[P(y=i)] + \ln((2\pi)^d \cdot \Sigma)^{1/2}$

so, now we have as a chapter 8

$$p(y=i/x) = \frac{\exp(a_i)}{\Sigma_j \exp(a_j)}$$

$$= \frac{\exp\{-\frac{1}{2\Sigma}x^T \cdot x + (w^i)^T x + b_i\}}{\Sigma_j \exp(-\frac{1}{2\Sigma}x^T \cdot x + (w^j)^T x + b_j)}$$

$$= \frac{\exp(-\frac{1}{2\Sigma}x^T \cdot x) \cdot \exp((w^i)^T x + b_i)}{\Sigma_j \exp(\frac{1}{2\Sigma}x^T \cdot x) \cdot \exp((w^j)^T x + b_j)}$$

$$= \frac{\cancel{\exp(-\frac{1}{2\Sigma}x^T x)} \cdot \exp((w^i)^T x + b_i)}{\cancel{\exp(-\frac{1}{2\Sigma}x^T x)} \cdot \Sigma_j \exp((w^j)^T x + b_j)}$$

($\because$ cancel out $-\frac{1}{2\Sigma}x^T x$)

$$= \frac{\exp((w^i)^T x + b_i)}{\Sigma_j \exp((w^j)^T x + b_j)}$$

$$\therefore a_i = (w^i)^T x + b_i$$

is linear. so,

→ here, in this equation x $\boxed{p(x|y=i) = N(x|\mu_i, \Sigma)}$

compute $a_i = \ln[p(x|y=i)p(y=i)]$

is $\boxed{\text{linear.}}$

→ But, if $p(x|y=i)$ has different covariance as, $\boxed{p(x|y=i) = N(x|\mu_i, \Sigma_i)}$

it is $\boxed{\text{not linear}}$

That time Σ_i value is differ we can't cancel and we can not say that it is constant and linear so, it is not linear.

see explanation in below,

$\rightarrow N(x|\mu_i, \Sigma_i) = \frac{1}{\sqrt{2\pi\Sigma_i}} e^{-\frac{(x-\mu_i)^2}{2\Sigma_i}}$

$\rightarrow \omega^T = \frac{\mu_i}{\Sigma_i}$ && $b_i = -\frac{1}{2\Sigma_i} \mu_i^T \mu_i + \ln(p(y=i)) + \ln \frac{1}{(2\pi)^{d/2} (\Sigma_i)^{1/2}}$

so, now we have put value

$$p(y=i/x) = \frac{\exp(a_i)}{\Sigma_j \exp(a_j)}$$

$$= \frac{\exp(-\frac{1}{2\Sigma_i} x^T x + (\omega^i)^T x + b^i)}{\Sigma_j \exp(-\frac{1}{2\Sigma_j} x^T x + (\omega^i)^T x + b^i)}$$

we cannot cancel out $-\frac{1}{2\Sigma_i} x^T x$

so, α_i is not linear.