Supplementary Material to:

Proprioceptive Control for Accurate Foothold Placement Aiding Fast-Legged Locomotion

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I. Preliminaries and Notation

This material complements the paper in []. It is a detailed description of the singularly perturbed model used in []. The equations presented in the accompanying paper are highlighted in red.

II. Kinetic and Potential Energies

III. Equations of Motion

The equation of motion during swing phase in dependent generalized coordinates is given by:

$$M(q_d)\ddot{q}_d + C(q_d, \dot{q}_d)\dot{q}_d + g(q_d) = Bu \qquad (1)$$

Here

$M_{1,1}(q_d) = m + m_1 + m_2 + m_3$

$M_{1,2}(q_d) = 0$

$M_{1,3}(q_d) = l_h m_1 \cos(\phi) + l_h m_2 \cos(\phi)$
$\quad + l_{cm,2} m_2 \cos(\phi + q_1 + q_2)$
$\quad + l_1 m_2 \cos(\phi + q_1) + l_{cm,3} m_3 \cos(\phi + q_3)$

$M_{1,4}(q_d) = l_{cm,2} m_2 \cos(\phi + q_1 + q_2) + l_1 m_2 \cos(\phi + q_1)$

$M_{1,5}(q_d) = l_{cm,2} m_2 \cos(\phi + q_1 + q_2)$

$M_{1,6}(q_d) = l_{cm,3} m_3 \cos(\phi + q_3)$

$M_{2,1}(q_d) = 0$

$M_{2,2}(q_d) = m + m_1 + m_2 + m_3$

$M_{2,3}(q_d) = l_h m_1 \sin(\phi) + l_h m_2 \sin(\phi)$
$\quad + l_{cm,2} m_2 \sin(\phi + q_1 + q_2)$
$\quad + l_1 m_2 \sin(\phi + q_1) + l_{cm,3} m_3 \sin(\phi + q_3)$

$M_{2,4}(q_d) = l_{cm,2} m_2 \sin(\phi + q_1 + q_2) + l_1 m_2 \sin(\phi + q_1)$

$M_{2,5}(q_d) = l_{cm,2} m_2 \sin(\phi + q_1 + q_2)$

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$M_{2,6}(q_d) = l_{cm,3} m_3 \sin(\phi + q_3)$

$M_{3,1}(q_d) = l_h m_1 \cos(\phi) + l_h m_2 \cos(\phi)$
$\quad + l_{cm,2} m_2 \cos(\phi + q_1 + q_2)$
$\quad + l_1 m_2 \cos(\phi + q_1) + l_{cm,3} m_3 \cos(\phi + q_3)$

$M_{3,2}(q_d) = l_h m_1 \sin(\phi) + l_h m_2 \sin(\phi)$
$\quad + l_{cm,2} m_2 \sin(\phi + q_1 + q_2)$
$\quad + l_1 m_2 \sin(\phi + q_1) + l_{cm,3} m_3 \sin(\phi + q_3)$

$M_{3,3}(q_d) = I_1 + I_2 + I_3 + I + l_1^2 m_2 + l_{cm,2}^2 m_2 + l_{cm,3}^2 m_3$
$\quad + l_h^2 m_1 + l_h^2 m_2 + 2 l_{cm.2} l_h m_2 \cos(q_1 + q_2)$
$\quad + 2\, l_1 \mathrm{l}_{\mathrm{cm},2} \mathrm{m}_2 \cos(q_2) + 2\, l_1 \mathrm{l}_{\mathrm{h}} \mathrm{m}_2 \cos(q_1)$

$M_{3,4}(q_d) = m_2 l_1^2 + 2\, m_2 \cos(q_2)\, l_1 l_{cm,2}$
$\quad + l_h m_2 \cos(q_1)\, l_1 + m_2 l_{cm,2}^2$
$\quad + l_h m_2 \cos(q_1 + q_2)\, l_{cm,2} + I_1 + I_2$

$M_{3,5}(q_d) = I_2 + l_{cm,2}^2 m_2 + l_{cm,2} l_h m_2 \cos(q_1 + q_2)$
$\quad + l_1 l_{cm,2} m_2 \cos(q_2)$

$M_{3,6}(q_d) = m_3 l_{cm,3}^2 + I_3$

$M_{4,1}(q_d) = l_{cm,2} m_2 \cos(\phi + q_1 + q_2)$
$\quad + l_1 m_2 \cos(\phi + q_1)$

$M_{4,2}(q_d) = l_{cm,2} m_2 \sin(\phi + q_1 + q_2)$
$\quad + l_1 m_2 \sin(\phi + q_1)$

$M_{4,3}(q_d) = m_2 l_1^2 + 2\, m_2 \cos(q_2)\, l_1 l_{cm,2} + l_h m_2 \cos(q_1)\, l_1$
$\quad + m_2 \mathrm{l}_{\mathrm{cm},2}^2 + \mathrm{l}_{\mathrm{h}} m_2 \cos(q_1 + q_2)\, l_{cm,2} + I_1 + I_2$

$M_{4,4}(q_d) = m_2 l_1^2 + 2\, m_2 \cos(q_2)\, l_1 l_{cm,2} + m_2 l_{cm,2}^2$
$\quad + I_1 + I_2$

$M_{4,5}(q_d) = m_2 l_{cm,2}^2 + l_1 m_2 \cos(q_2)\, l_{cm,2} + I_2$

$M_{4,6}(q_d) = 0$

$M_{5,1}(q_d) = l_{cm,2} m_2 \cos(\phi + q_1 + q_2)$

$M_{5,2}(q_d) = l_{cm,2} m_2 \sin(\phi + q_1 + q_2)$

$M_{5,3}(q_d) = I_2 + l_{cm,2}^2 m_2 + l_{cm,2} l_h m_2 \cos(q_1 + q_2)$

$$+ l_1 l_{cm,2} m_2 \cos(q_2)$$
$$M_{5,4}(q_d) = m_2 l_{cm,2}^2 + l_1 m_2 \cos(q_2)\, l_{cm,2} + I_2$$
$$M_{5,5}(q_d) = m_2 l_{cm,2}^2 + I_2$$
$$M_{5,6}(q_d) = 0$$
$$M_{6,1}(q_d) = l_{cm,3} m_3 \cos(\phi + q_3)$$
$$M_{6,2}(q_d) = l_{cm,3} m_3 \sin(\phi + q_3)$$
$$M_{6,3}(q_d) = m_3 l_{cm,3}^2 + I_3$$
$$M_{6,4}(q_d) = 0$$
$$M_{6,5}(q_d) = 0$$
$$M_{6,6}(q_d) = m_3 l_{cm,3}^2 + I_3$$

$$C_{1,1}(q_d) = 0$$
$$C_{1,2}(q_d) = 0$$
$$\begin{aligned} C_{1,3}(q_d) = &-\dot{q}(l_{cm,2} m_2 \sin(\phi + q_1 + q_2) \\ &+ l_1 m_2 \sin(\phi + q_1)) \\ &-\dot{\phi}(l_h m_1 \sin(\phi) + l_h m_2 \sin(\phi) \\ &+ l_{cm,2} m_2 \sin(\phi + q_1 + q_2) \\ &+ l_1 m_2 \sin(\phi + q_1) + l_{cm,3} m_3 \sin(\phi + q_3)) \\ &-\dot{q}_3 l_{cm,3} m_3 \sin(\phi + q_3) \\ &-\dot{q}_2 l_{cm,2} m_2 \sin(\phi + q_1 + q_2) \end{aligned}$$
$$\begin{aligned} C_{1,4}(q_d) = &-\dot{\phi}(l_{cm,2} m_2 \sin(\phi + q_1 + q_2) \\ &+ l_1 m_2 \sin(\phi + q_1)) \\ &-\dot{q}_1(l_{cm,2} m_2 \sin(\phi + q_1 + q_2) \\ &+ l_1 m_2 \sin(\phi + q_1)) \\ &-\dot{q}_2 l_{cm,2} m_2 \sin(\phi + q_1 + q_2) \end{aligned}$$
$$C_{1,5}(q_d) = -l_{cm,2} m_2 \sin(\phi + q_1 + q_2)(\dot{\phi} + \dot{q}_1 + \dot{q}_2)$$
$$C_{1,6}(q_d) = -l_{cm,3} m_3 \sin(\phi + q_3)(\dot{\phi} + \dot{q}_3)$$
$$C_{2,1}(q_d) = 0$$
$$C_{2,2}(q_d) = 0$$
$$\begin{aligned} C_{2,3}(q_d) = &\dot{\phi}(l_h m_1 \cos(\phi) + l_h m_2 \cos(\phi) \\ &+ l_{cm,2} m_2 \cos(\phi + q_1 + q_2) \\ &+ l_1 m_2 \cos(\phi + q_1) + l_{cm,3} m_3 \cos(\phi + q_3)) \\ &+ \dot{q}_1(l_{cm,2} m_2 \cos(\phi + q_1 + q_2) \\ &+ l_1 m_2 \cos(\phi + q_1)) + \dot{q}_3\, l_{cm,3} m_3 \cos(\phi + q_3) \\ &+ \dot{q}_2 l_{cm,2} m_2 \cos(\phi + q_1 + q_2) \end{aligned}$$
$$\begin{aligned} C_{2,4}(q_d) = &-\dot{\phi}(l_{cm,2} m_2 \cos(\phi + q_1 + q_2) \\ &+ l_1 m_2 \cos(\phi + q_1)) \\ &-\dot{q}_1(l_{cm,2} m_2 \cos(\phi + q_1 + q_2) \\ &+ l_1 m_2 \cos(\phi + q_1)) \\ &-\dot{q}_2 l_{cm,2} m_2 \cos(\phi + q_1 + q_2) \end{aligned}$$
$$C_{2,5}(q_d) = l_{cm,2} m_2 \cos(\phi + q_1 + q_2)\,(\dot{\phi} + \dot{q}_1 + \dot{q}_2)$$
$$C_{2,6}(q_d) = l_{cm,3} m_3 \cos(\phi + q_3)\,(\dot{\phi} + \dot{q}_3)$$
$$C_{3,1}(q_d) = 0$$
$$C_{3,2}(q_d) = 0$$
$$\begin{aligned} C_{3,3}(q_d) = &-\dot{q}_2\big(l_{cm,2} l_h m_2 \sin(q_1 + q_2)\big) \\ &+ l_1 l_{cm,2} m_2 \sin(q_2)) \\ &-\dot{q}_1(l_{cm,2} l_h m_2 \sin(q_1 + q_2) \\ &+ l_1 l_h m_2 \sin(q_1)) \end{aligned}$$
$$\begin{aligned} C_{3,4}(q_d) = &-\dot{q}_2\big(l_{cm,2} l_h m_2 \sin(q_1 + q_2)\big) \\ &+ l_1 l_{cm,2} m_2 \sin(q_2)) \\ &-\dot{\phi}(l_{cm,2} l_h m_2 \sin(q_1 + q_2) \\ &+ l_1 l_h m_2 \sin(q_1)) \\ &-\dot{q}_1(l_{cm,2} l_h m_2 \sin(q_1 + q_2) + l_1 l_h m_2 \sin(q_1)) \end{aligned}$$
$$\begin{aligned} C_{3,5}(q_d) = &-l_{cm,2} m_2(l_h \sin(q_1 + q_2) + l_1 \sin(q_2)) \\ &* (\dot{\phi} + \dot{q}_1 + \dot{q}_2) \end{aligned}$$
$$C_{3,6}(q_d) = 0$$
$$C_{4,1}(q_d) = 0$$
$$C_{4,2}(q_d) = 0$$
$$\begin{aligned} C_{4,3}(q_d) = &\dot{\phi}\big(l_{cm,2} l_h m_2 \sin(q_1 + q_2) + l_1 l_h m_2 \sin(q_1)\big) \\ &-\dot{q}_2 l_1 l_{cm,2} m_2 \sin(q_2) \end{aligned}$$
$$C_{4,4}(q_d) = -\dot{q}_2 l_1 l_{cm,2} m_2 \sin(q_2)$$
$$C_{4,5}(q_d) = -l_1 l_{cm,2} m_2 \sin(q_2)(\dot{\phi} + \dot{q}_1 + \dot{q}_2)$$
$$C_{4,6}(q_d) = 0$$
$$C_{5,1}(q_d) = 0$$
$$C_{5,2}(q_d) = 0$$
$$\begin{aligned} C_{5,3}(q_d) = &\dot{\phi}\big(l_{cm,2} l_h m_2 \sin(q_1 + q_2) + l_1 l_{cm,2} m_2 \sin(q_2)\big) \\ &+ \dot{q}_1 l_1 l_{cm,2} m_2 \sin(q_2) \end{aligned}$$
$$C_{5,4}(q_d) = l_1 l_{cm,2} m_2 \sin(q_2)(\dot{\phi} + \dot{q}_1)$$
$$C_{5,5}(q_d) = 0$$
$$C_{5,6}(q_d) = 0$$
$$C_{6,1}(q_d) = 0$$
$$C_{6,2}(q_d) = 0$$
$$C_{6,3}(q_d) = 0$$
$$C_{6,4}(q_d) = 0$$

$C_{6,5}(q_d) = 0$

$C_{6,6}(q_d) = 0$

$G_1(q_d) = 0$

$G_2(q_d) = g(m + m_1 + m_2 + m_3)$

$$G_3(q_d) = g(l_h m_1 \sin(\phi) + l_h m_2 \sin(\phi) \\ + l_{cm,2} m_2 \sin(\phi + q_1 + q_2) + l_1 m_2 \sin(\phi + q_1) \\ + l_{cm,3} m_3 \sin(\phi + q_3))$$

$G_4(q_d) = g m_2 (l_1 \sin(\phi + q_1) + l_{cm,2} \sin(\phi + q_1 + q_2))$

$G_5(q_d) = g l_{cm,2} m_2 \sin(\phi + q_1 + q_2)$

$G_6(q_d) = g l_{cm,3} \mathrm{m}_3 \sin(\phi + q_3)$

IV. SINGULARLY PERTURBED EQUATIONS OF MOTION