Compilers and Interpreters

Bottom-Up Parsing

What is Bottom-Up Parsing?

- Idea: Apply productions **in reverse** to convert the user's program to the start symbol.
  - We can think of bottom-up parsing as the process of "reducing" a string *w* to the start symbol of the grammar. At each reduction step, a specific substring matching the body of a production is replaced by the nonterminal at the head of that production.

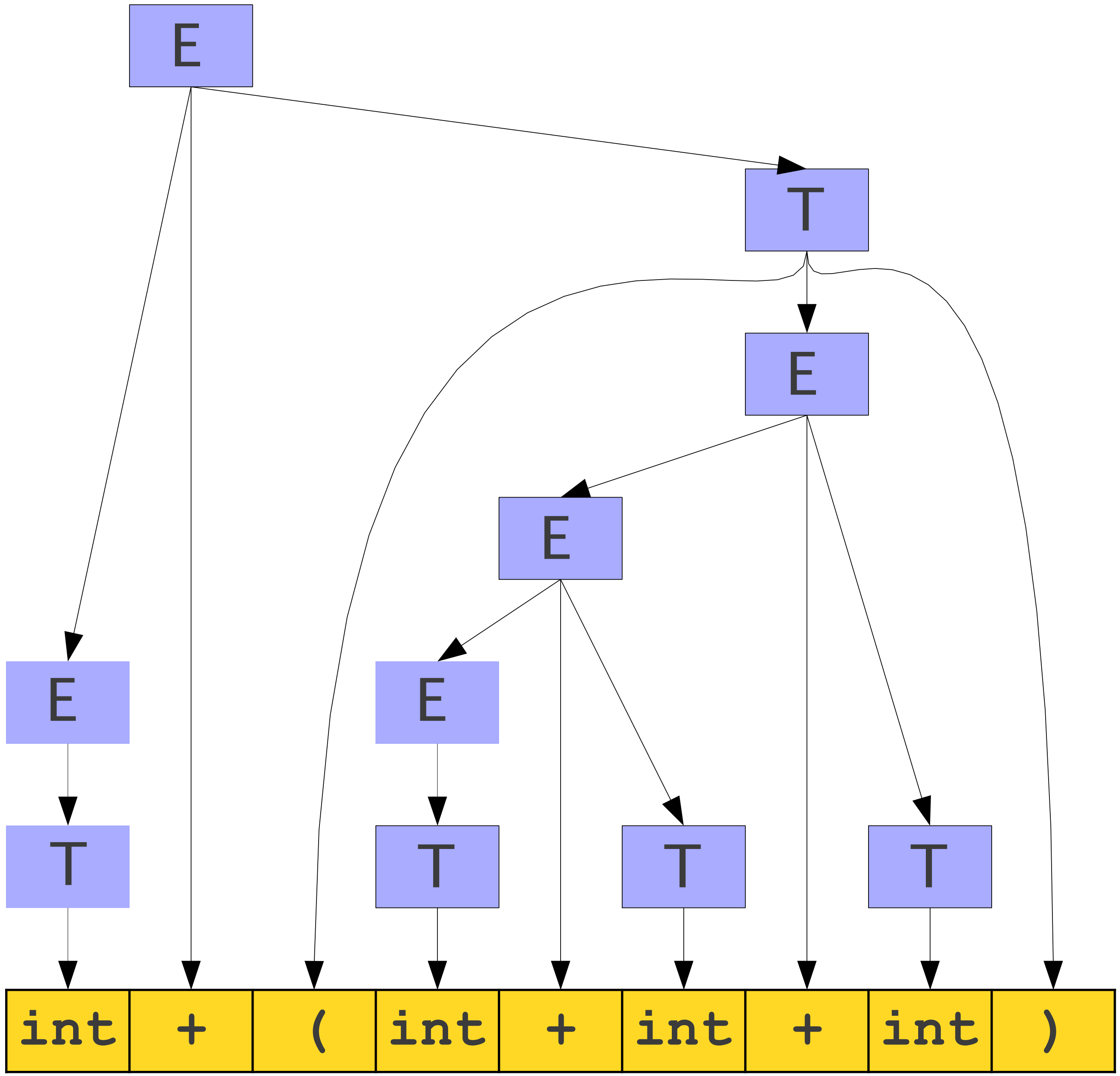
- Terms
  - Reductions, handle, shift-reduce parsing, conflicts, LR grammars

What is Bottom-Up Parsing?

- We'll be exploring four **directional**, **predictive** bottom-up parsing techniques:
  - **Directional**: Scan the input from left-to-right.
  - **Predictive**: Guess which production should be inverted.
  - The largest class of grammars for which shift-reduce parsers can be built, the LR grammars:
    LR(1), LALR(1)

One View of a Bottom-Up Parse

E → T
E → E + T
T → int
T → (E)



A Second View of a Bottom-Up Parse

$E \rightarrow T$
$E \rightarrow E + T$
$T \rightarrow \texttt{int}$
$T \rightarrow (E)$

$$
\begin{aligned}
& \texttt{int + (int + int + int)} \\
\Rightarrow\ & T \texttt{ + (int + int + int)} \\
\Rightarrow\ & E \texttt{ + (int + int + int)} \\
\Rightarrow\ & E \texttt{ + (}T\texttt{ + int + int)} \\
\Rightarrow\ & E \texttt{ + (}E\texttt{ + int + int)} \\
\Rightarrow\ & E \texttt{ + (}E\texttt{ + }T\texttt{ + int)} \\
\Rightarrow\ & E \texttt{ + (}E\texttt{ + int)} \\
\Rightarrow\ & E \texttt{ + (}E\texttt{ + }T\texttt{)} \\
\Rightarrow\ & E \texttt{ + (}E\texttt{)} \\
\Rightarrow\ & E \texttt{ + } T \\
\Rightarrow\ & E
\end{aligned}
$$

A Second View of a Bottom-Up Parse

$$
\begin{aligned}
E &\rightarrow T \\
E &\rightarrow E + T \\
T &\rightarrow \texttt{int} \\
T &\rightarrow (E)
\end{aligned}
$$

$$
\begin{aligned}
& \texttt{int + (int + int + int)} \\
\Rightarrow\ & T\ \texttt{+ (int + int + int)} \\
\Rightarrow\ & E\ \texttt{+ (int + int + int)} \\
\Rightarrow\ & E\ \texttt{+ (}T\ \texttt{+ int + int)} \\
\Rightarrow\ & E\ \texttt{+ (}E\ \texttt{+ int + int)} \\
\Rightarrow\ & E\ \texttt{+ (}E\ \texttt{+}\ T\ \texttt{+ int)} \\
\Rightarrow\ & E\ \texttt{+ (}E\ \texttt{+ int)} \\
\Rightarrow\ & E\ \texttt{+ (}E\ \texttt{+}\ T\texttt{)} \\
\Rightarrow\ & E\ \texttt{+ (}E\texttt{)} \\
\Rightarrow\ & E\ \texttt{+}\ T \\
\Rightarrow\ & E
\end{aligned}
$$

A left-to-right, bottom-up parse is a rightmost derivation traced in reverse.

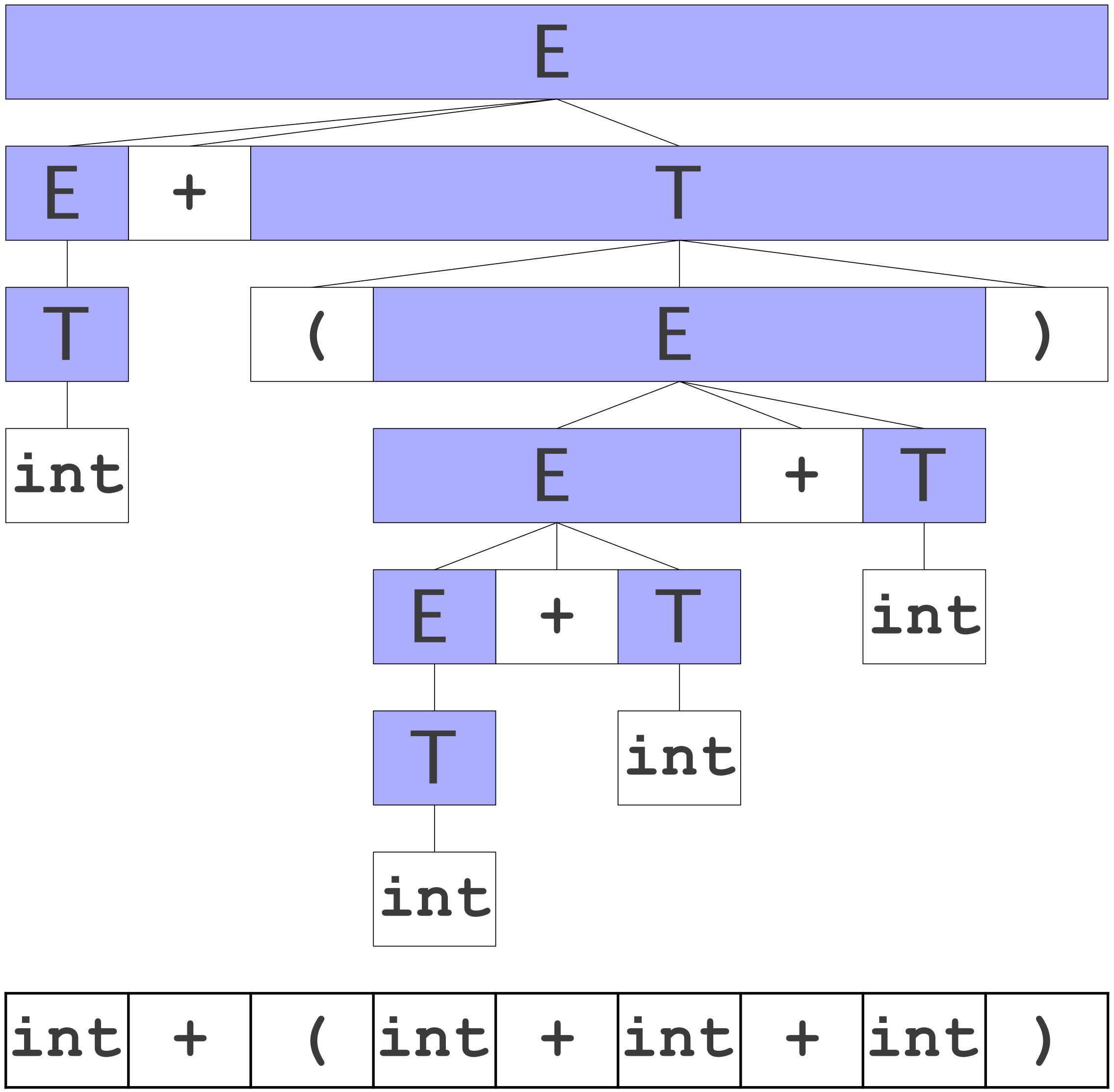
A Third View of a Bottom-Up Parse

```
  int + (int + int + int)
⇒ T + (int + int + int)
⇒ E + (int + int + int)
⇒ E + (T + int + int)
⇒ E + (E + int + int)
⇒ E + (E + T + int)
⇒ E + (E + int)
⇒ E + (E + T)
⇒ E + (E)
⇒ E + T
⇒ E
```

Each step in this bottom-up parse is called a **reduction**. We **reduce** a substring of the sentential form back to a nonterminal

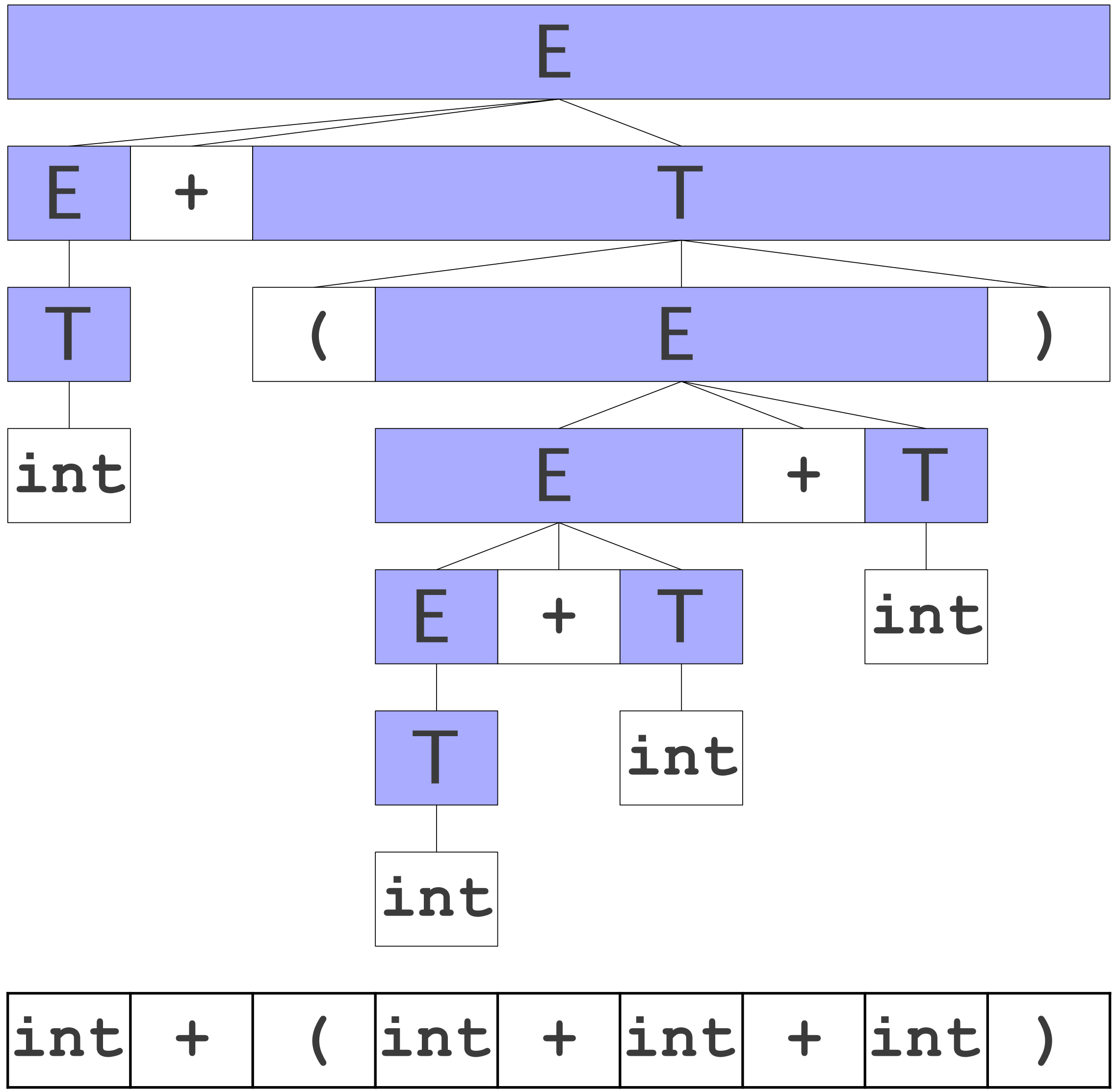
A Third View of a Bottom-Up Parse

```
  int + (int + int + int)
⇒ T + (int + int + int)
⇒ E + (int + int + int)
⇒ E + (T + int + int)
⇒ E + (E + int + int)
⇒ E + (E + T + int)
⇒ E + (E + int)
⇒ E + (E + T)
⇒ E + (E)
⇒ E + T
⇒ E
```



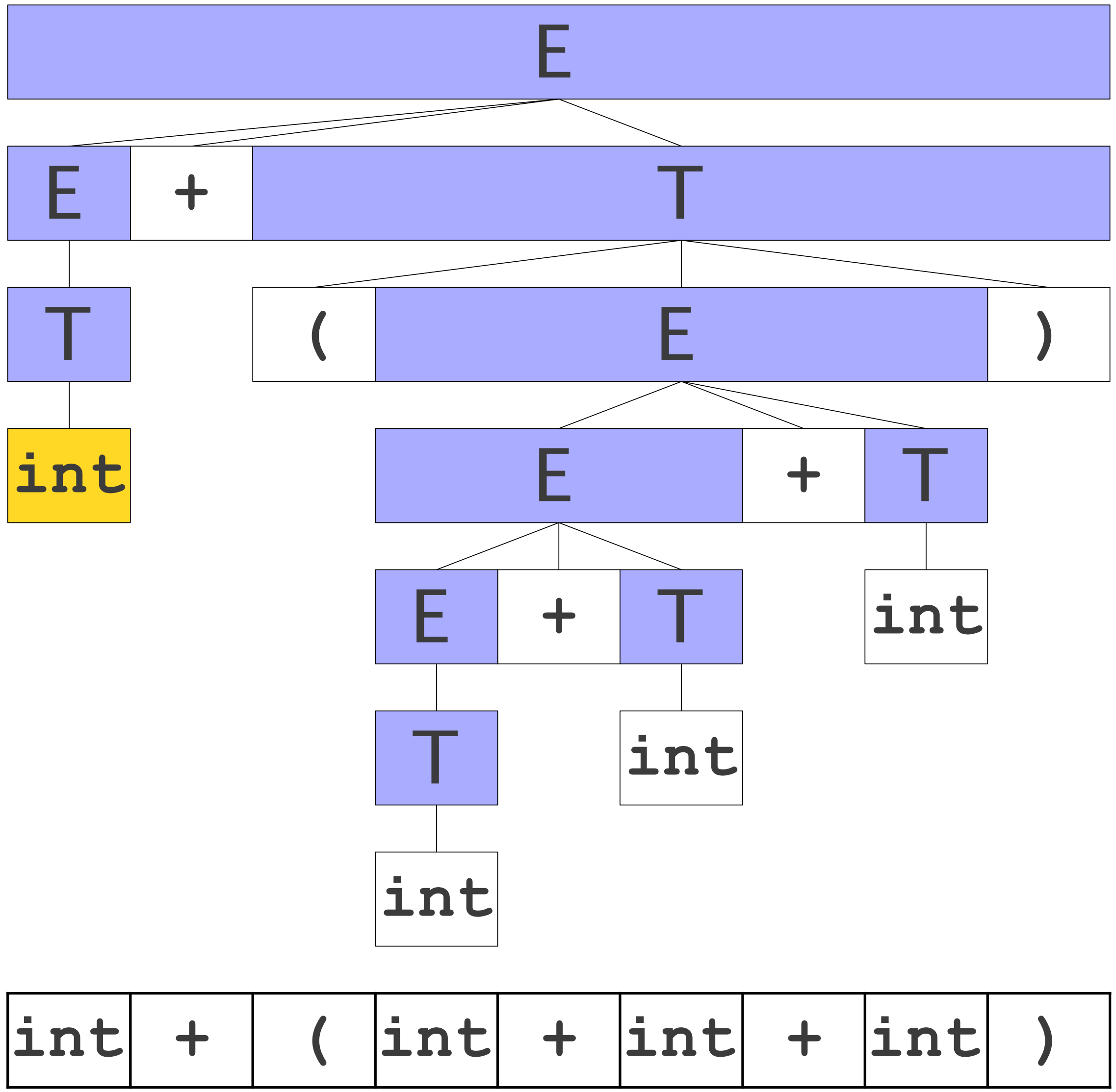
A Third View of a Bottom-Up Parse

int + (int + int + int)
⇒ T + (int + int + int)
⇒ E + (int + int + int)
⇒ E + (T + int + int)
⇒ E + (E + int + int)
⇒ E + (E + T + int)
⇒ E + (E + int)
⇒ E + (E + T)
⇒ E + (E)
⇒ E + T
⇒ E



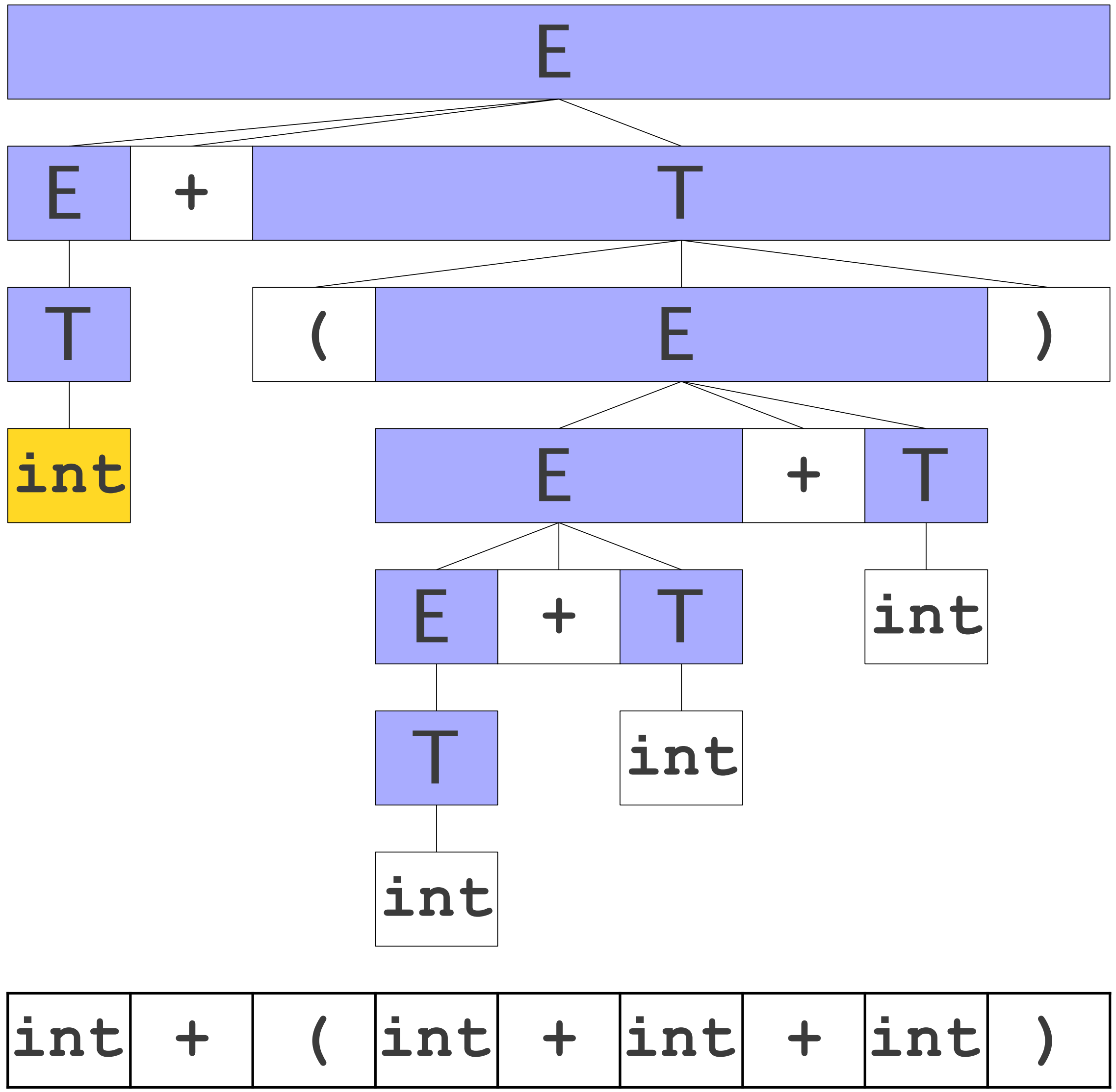
A Third View of a Bottom-Up Parse

int + (int + int + int)
⇒ T + (int + int + int)
⇒ E + (int + int + int)
⇒ E + (T + int + int)
⇒ E + (E + int + int)
⇒ E + (E + T + int)
⇒ E + (E + int)
⇒ E + (E + T)
⇒ E + (E)
⇒ E + T
⇒ E



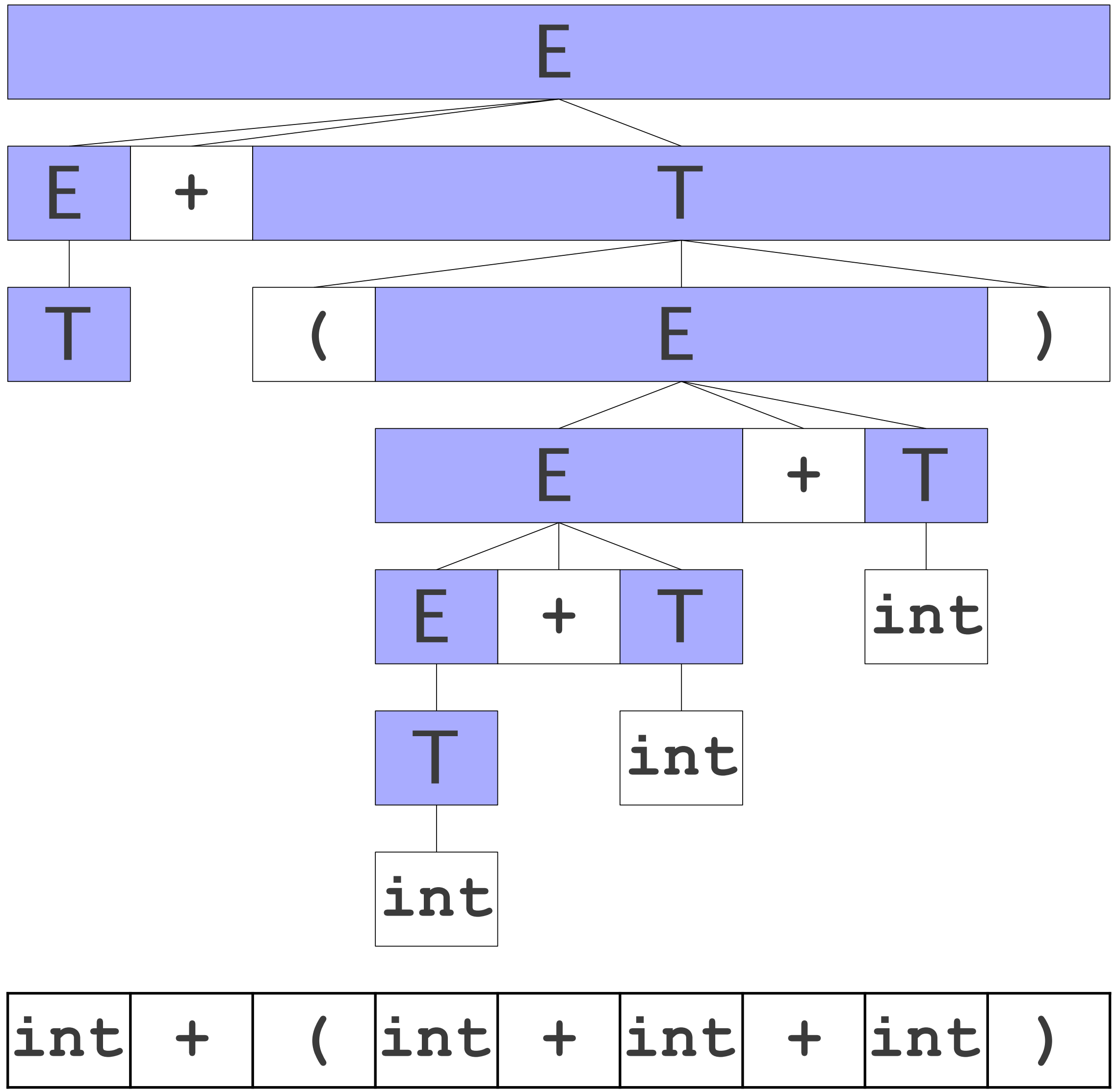
A Third View of a Bottom-Up Parse

int + (int + int + int)
⇒ T + (int + int + int)
⇒ E + (int + int + int)
⇒ E + (T + int + int)
⇒ E + (E + int + int)
⇒ E + (E + T + int)
⇒ E + (E + int)
⇒ E + (E + T)
⇒ E + (E)
⇒ E + T
⇒ E



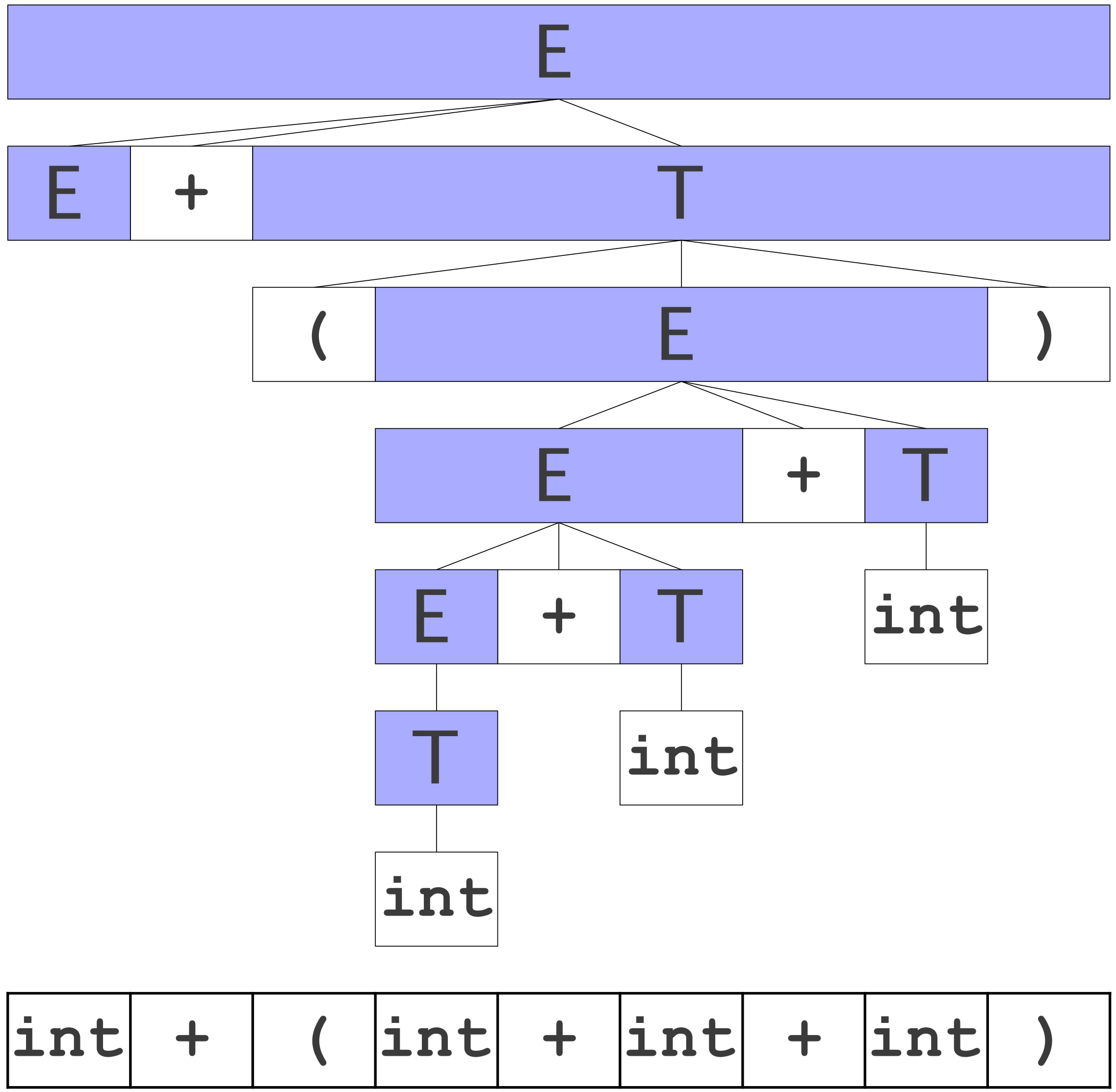
A Third View of a Bottom-Up Parse

$\Rightarrow$ **T** + (int+ int + int)
$\Rightarrow$ E + (int+ int + int)
$\Rightarrow$ E + (T + int + int)
$\Rightarrow$ E + (E + int + int)
$\Rightarrow$ E + (E + T + int)
$\Rightarrow$ E + (E + int)
$\Rightarrow$ E + (E + T)
$\Rightarrow$ E + (E)
$\Rightarrow$ E + T
$\Rightarrow$ E



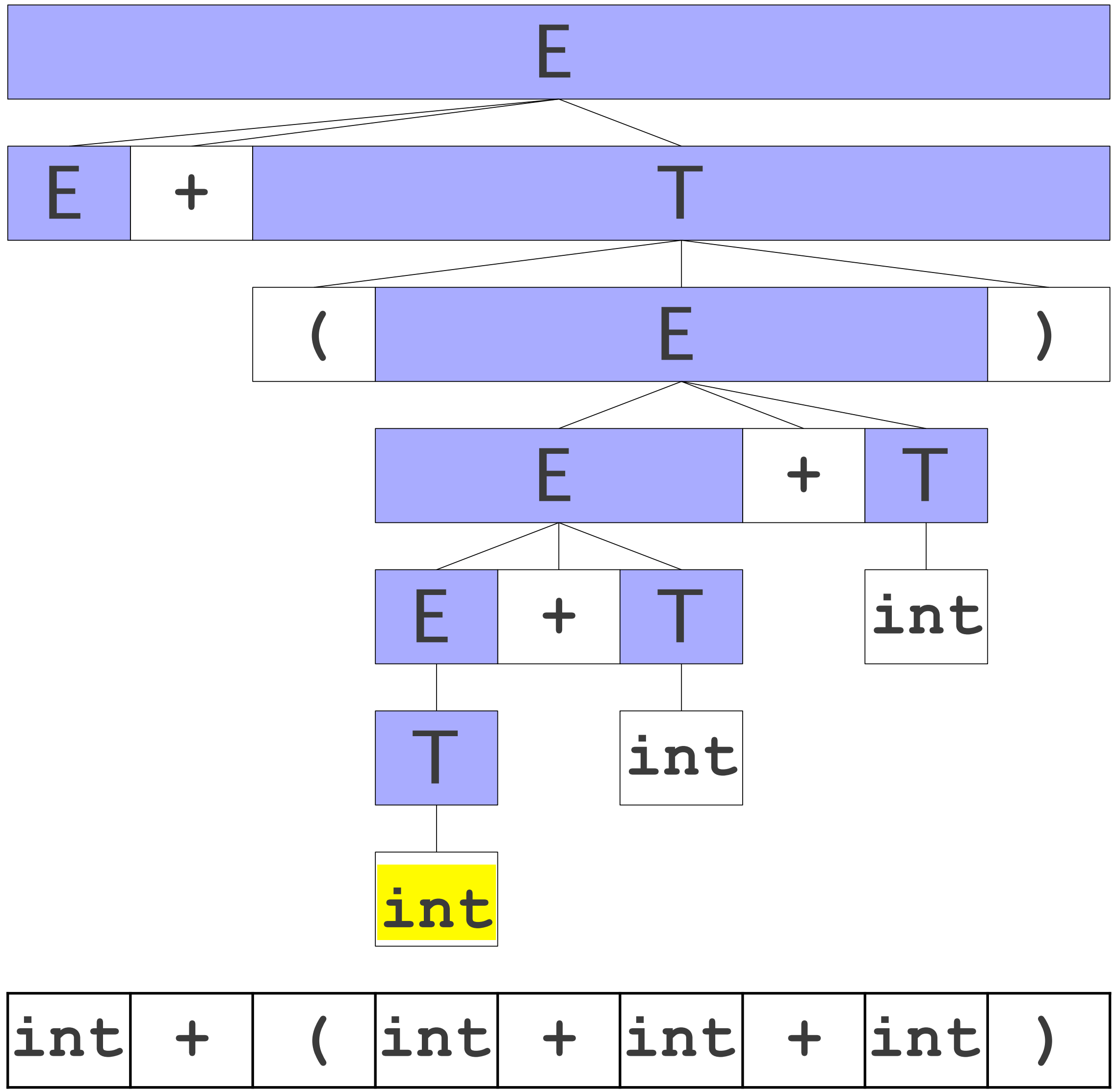
A Third View of a Bottom-Up Parse

$\Rightarrow$ `E + (int + int + int)`
$\Rightarrow$ `E + (T + int + int)`
$\Rightarrow$ `E + (E + int + int)`
$\Rightarrow$ `E + (E + T + int)`
$\Rightarrow$ `E + (E + int)`
$\Rightarrow$ `E + (E + T)`
$\Rightarrow$ `E + (E)`
$\Rightarrow$ `E + T`
$\Rightarrow$ `E`



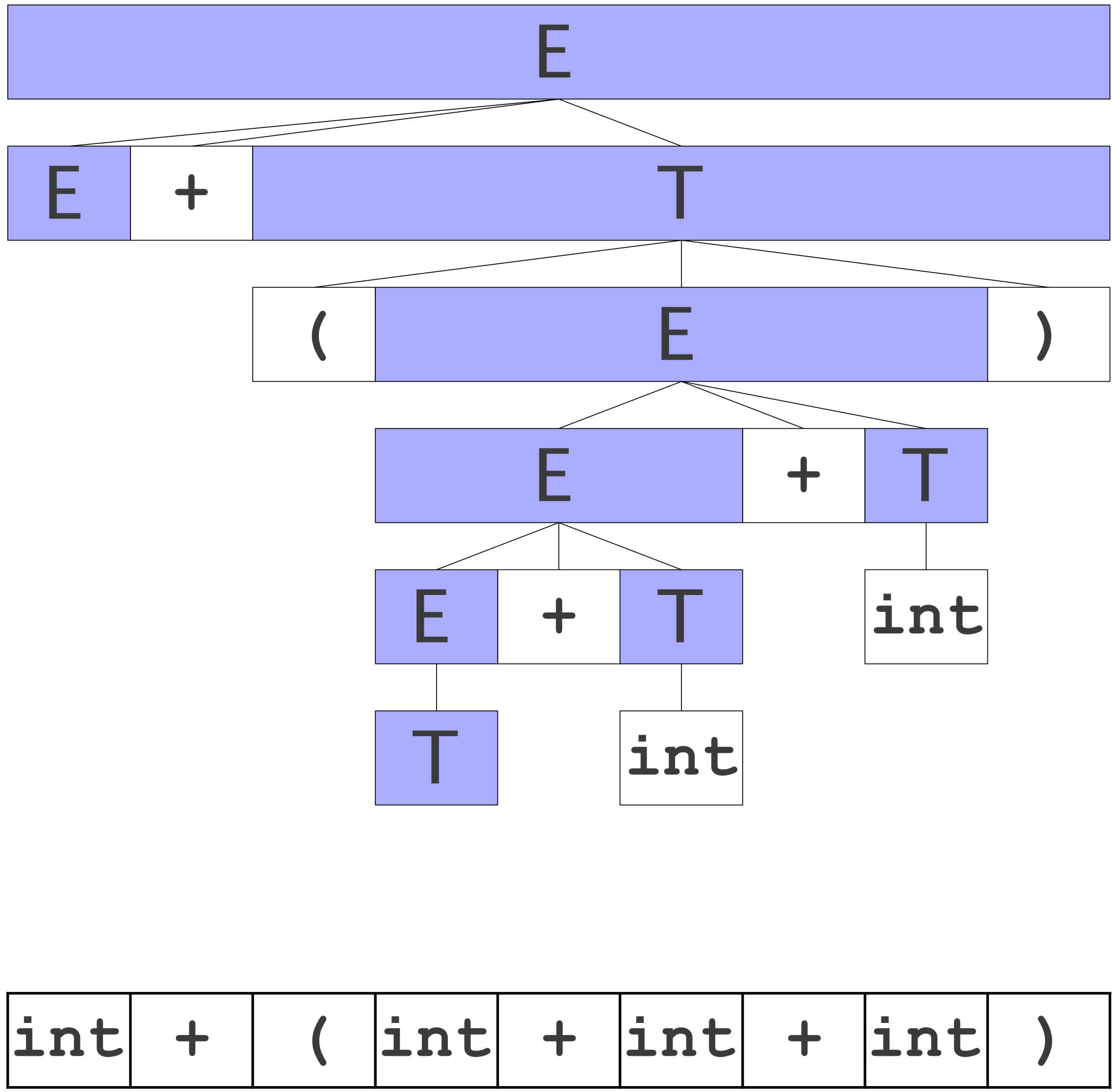
A Third View of a Bottom-Up Parse

$\Rightarrow$ `E + (int + int + int)`
$\Rightarrow$ `E + (T + int + int)`
$\Rightarrow$ `E + (E + int + int)`
$\Rightarrow$ `E + (E + T + int)`
$\Rightarrow$ `E + (E + int)`
$\Rightarrow$ `E + (E + T)`
$\Rightarrow$ `E + (E)`
$\Rightarrow$ `E + T`
$\Rightarrow$ `E`



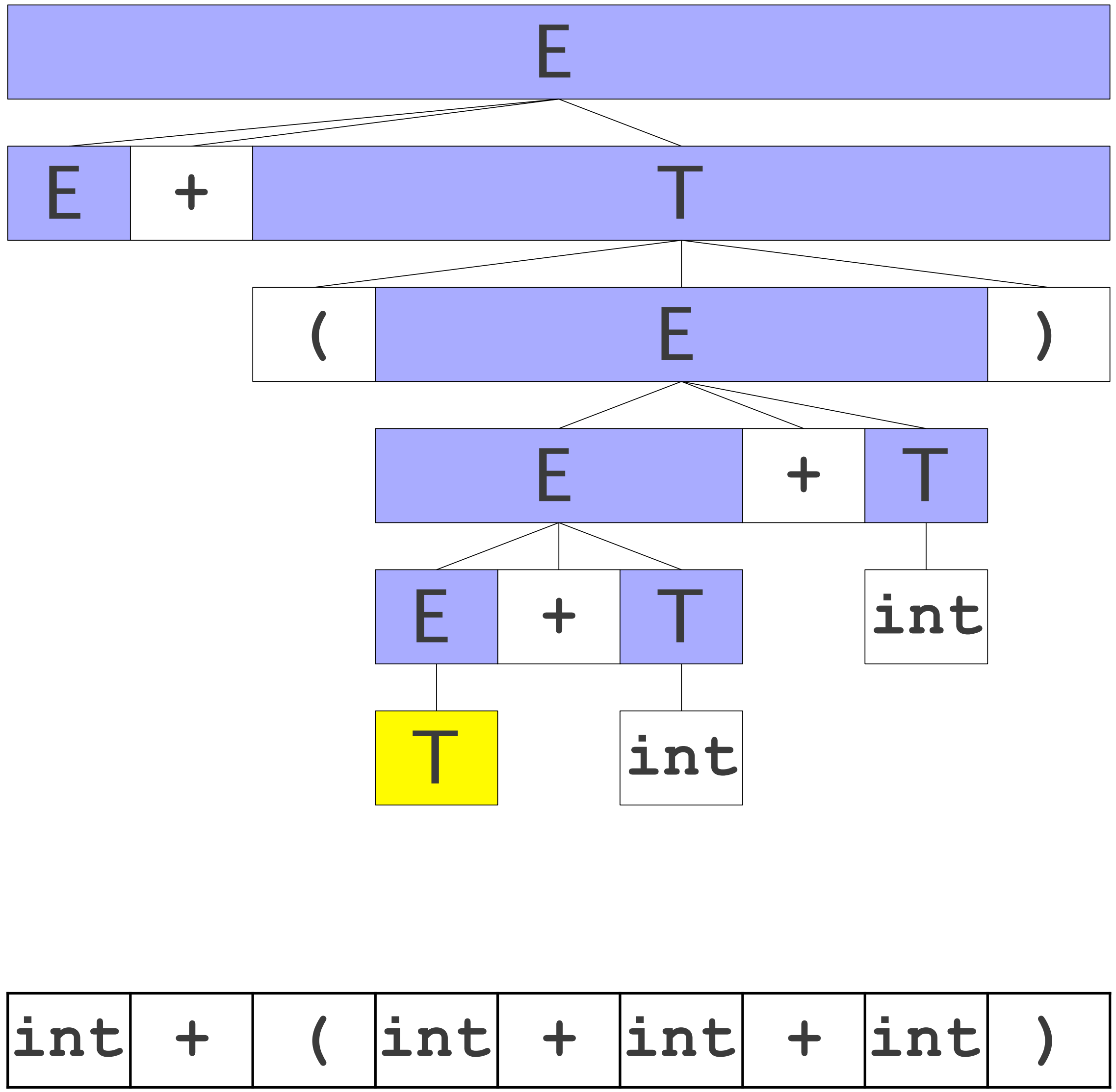
A Third View of a Bottom-Up Parse

⇒ E + (T + int + int)
⇒ E + (E + int + int)
⇒ E + (E + T + int)
⇒ E + (E + int)
⇒ E + (E + T)
⇒ E + (E)
⇒ E + T
⇒ E



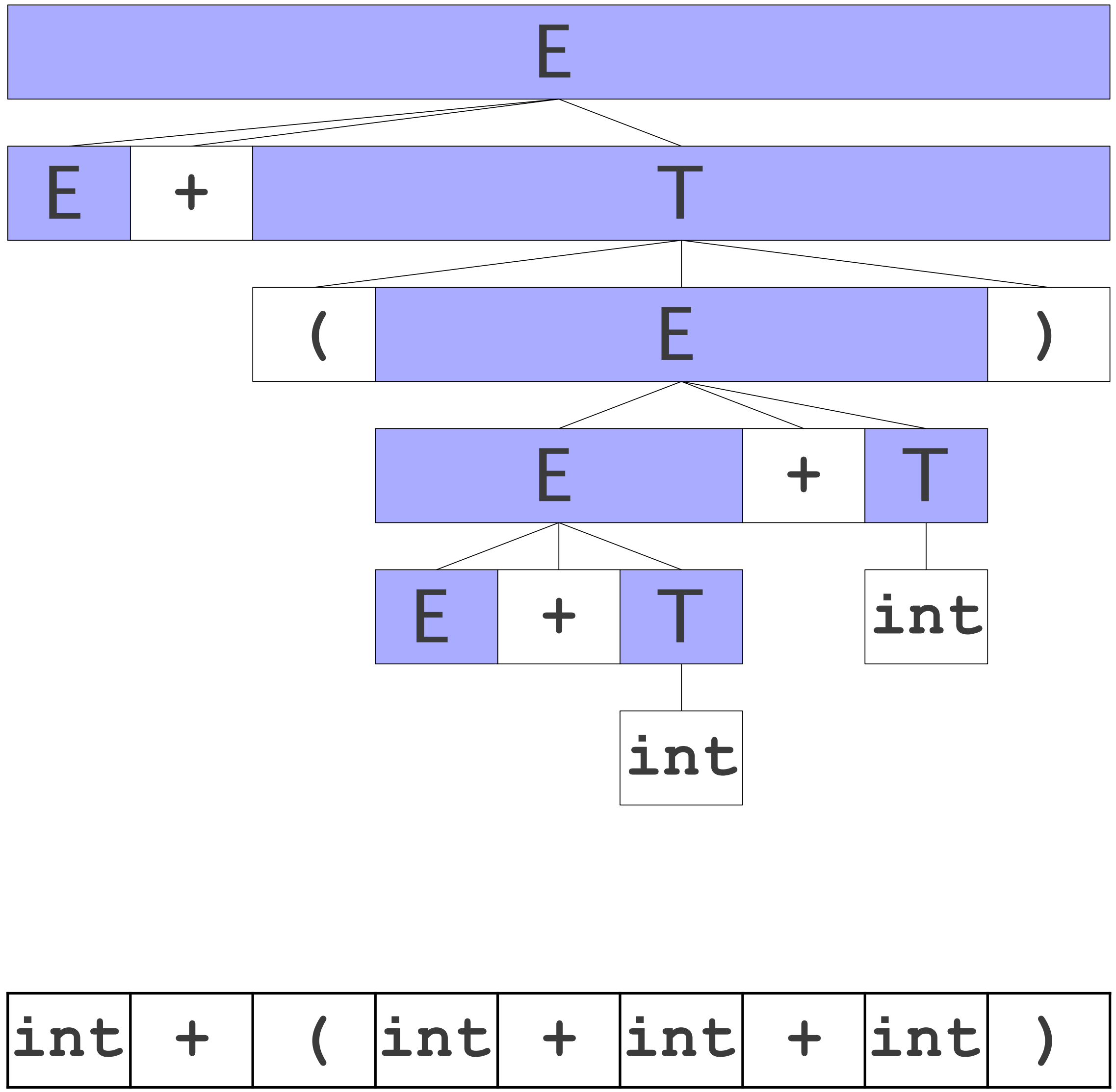
A Third View of a Bottom-Up Parse

⇒ E + (T + int + int)
⇒ E + (E + int + int)
⇒ E + (E + T + int)
⇒ E + (E + int)
⇒ E + (E + T)
⇒ E + (E)
⇒ E + T
⇒ E



A Third View of a Bottom-Up Parse

$\Rightarrow$ E + (E + int + int)
$\Rightarrow$ E + (E + T + int)
$\Rightarrow$ E + (E + int)
$\Rightarrow$ E + (E + T)
$\Rightarrow$ E + (E)
$\Rightarrow$ E + T
$\Rightarrow$ E



A Third View of a Bottom-Up Parse

$\Rightarrow$ E + (E + **int** + int)

$\Rightarrow$ E + (E + T + int)

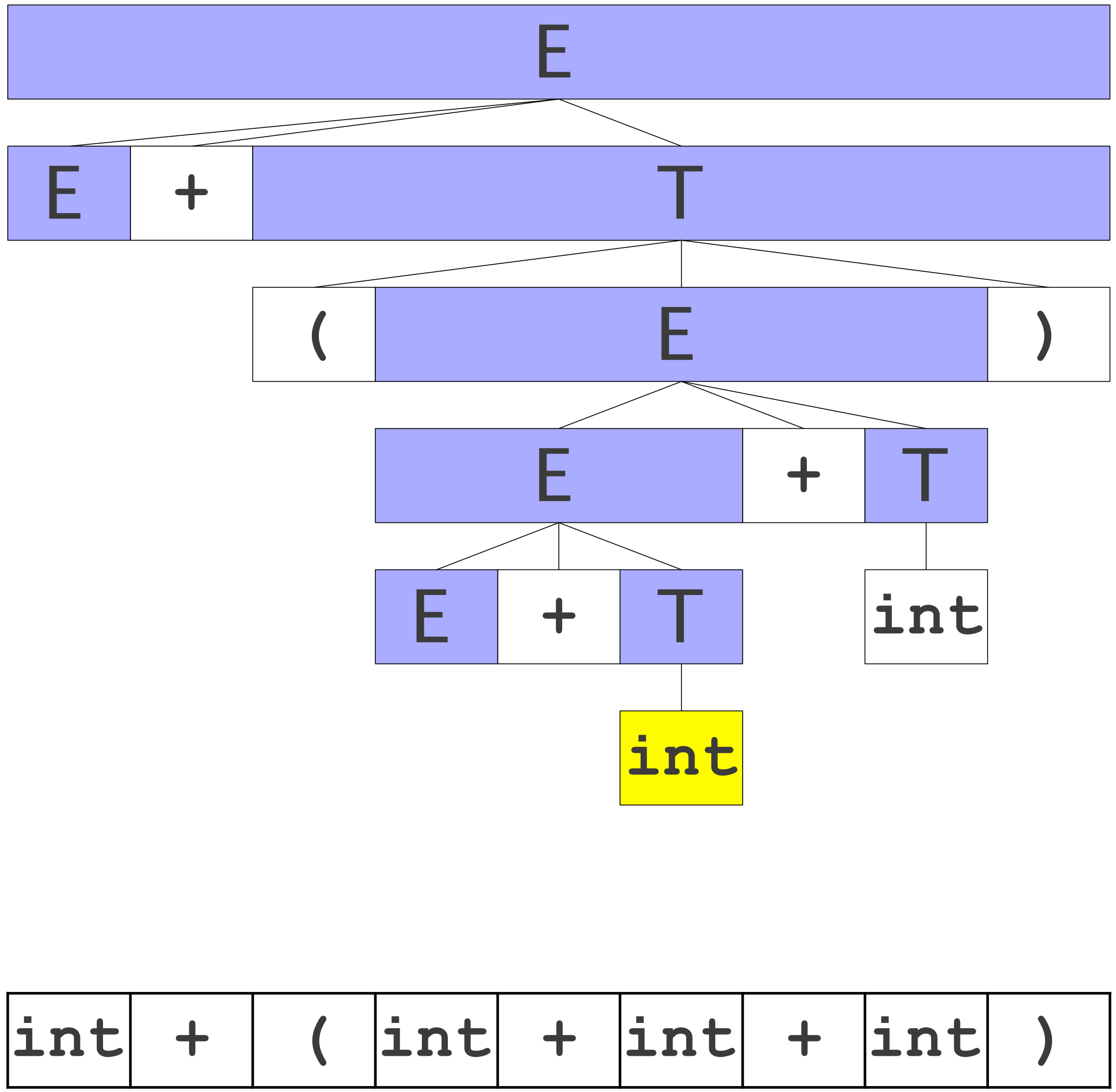
$\Rightarrow$ E + (E + int)

$\Rightarrow$ E + (E + T)

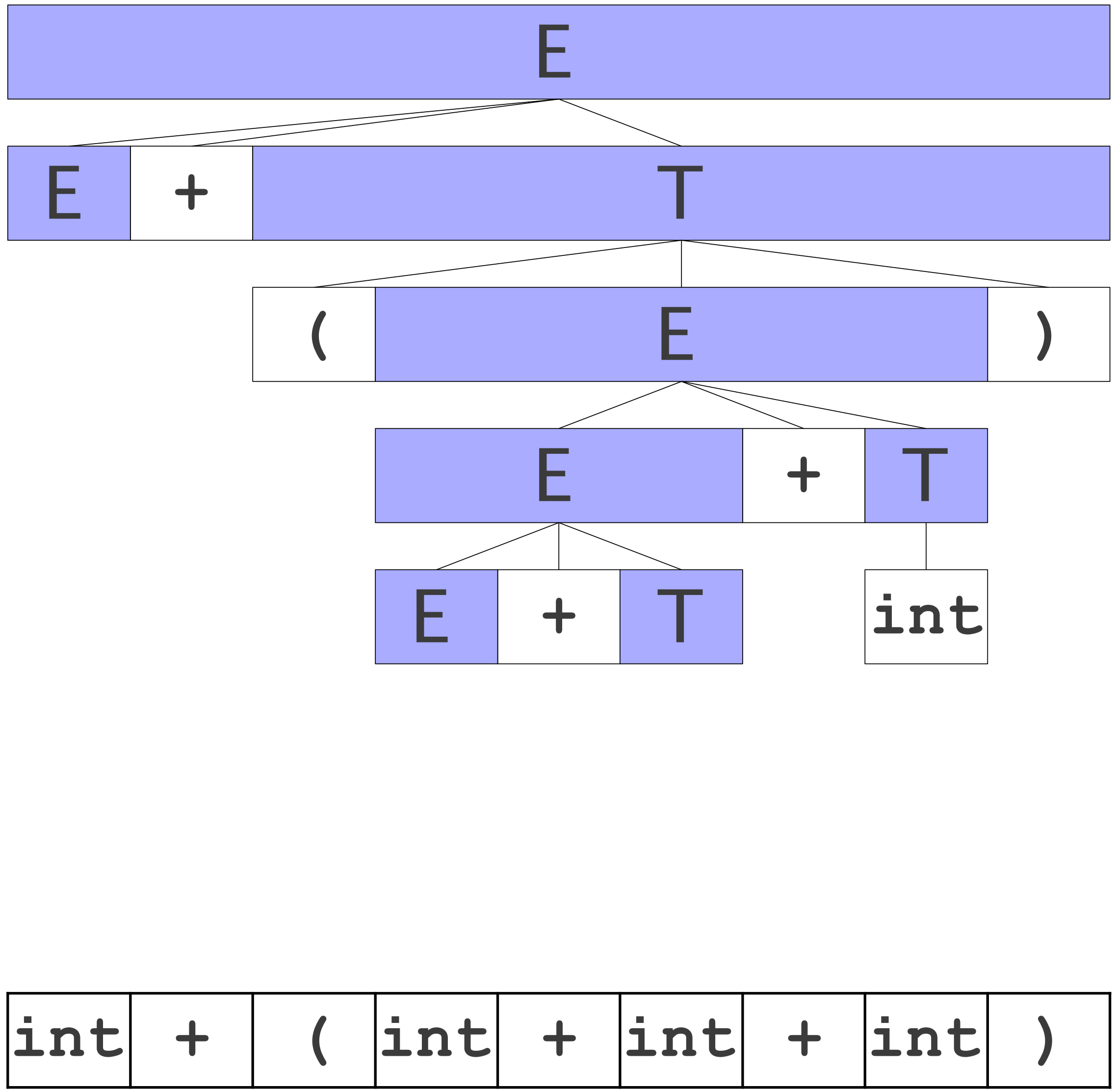
$\Rightarrow$ E + (E)

$\Rightarrow$ E + T

$\Rightarrow$ E



A Third View of a Bottom-Up Parse



$\Rightarrow$ `E + (E + T + int)`

$\Rightarrow$ `E + (E + int)`

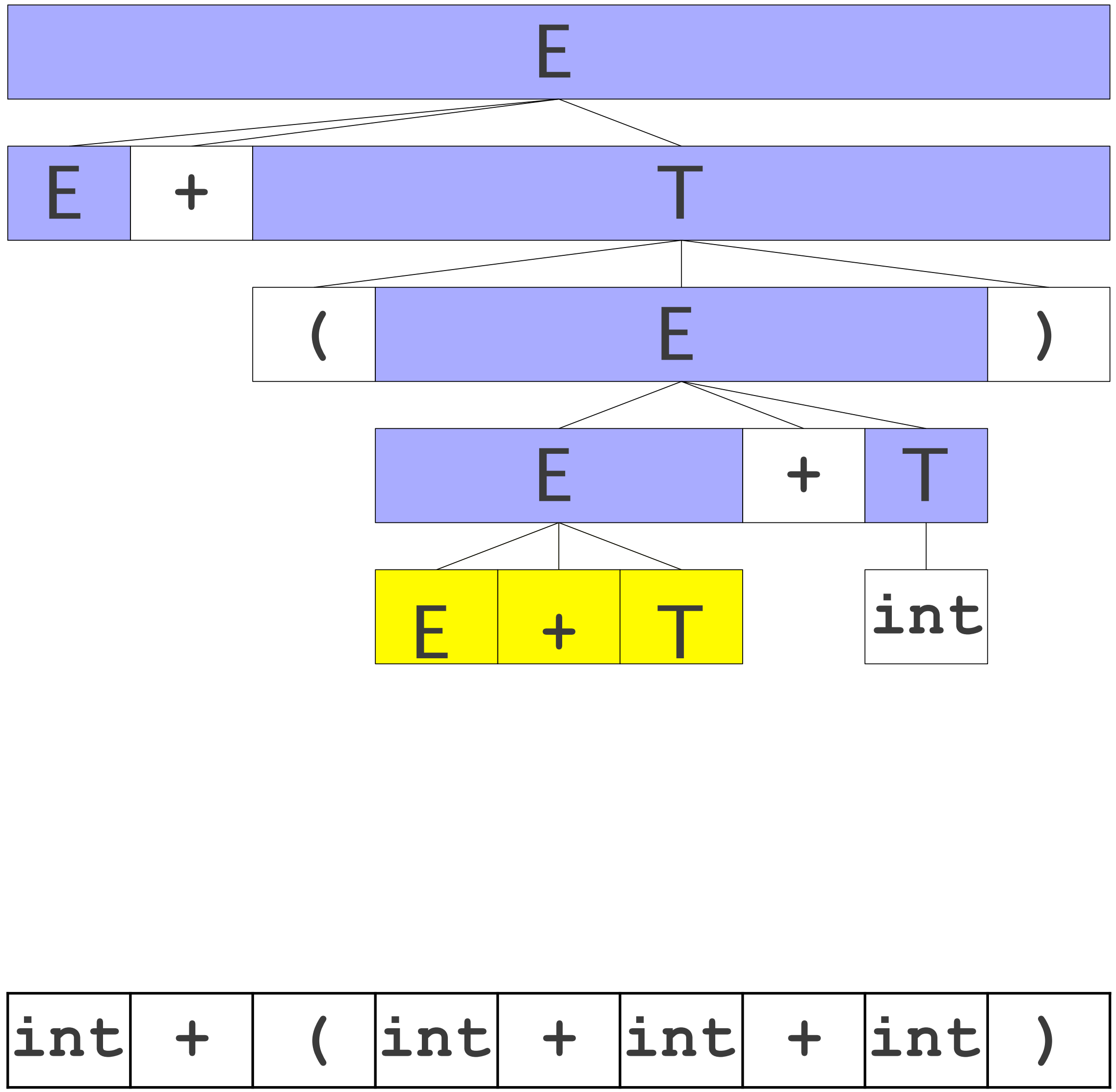
$\Rightarrow$ `E + (E + T)`

$\Rightarrow$ `E + (E)`

$\Rightarrow$ `E + T`

$\Rightarrow$ `E`

A Third View of a Bottom-Up Parse



$\Rightarrow$ E + (E + T + int)

$\Rightarrow$ E + (E + int)

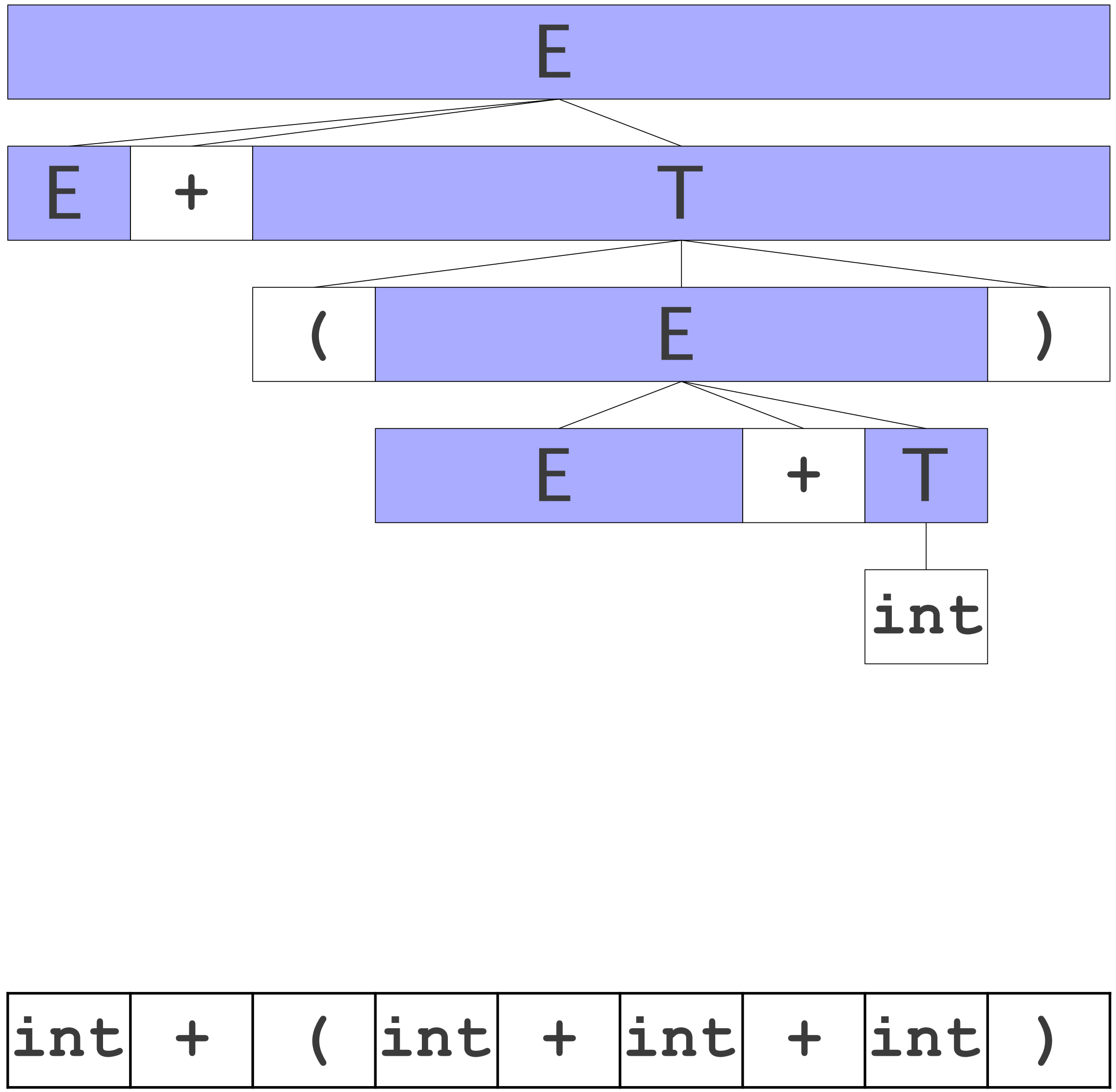
$\Rightarrow$ E + (E + T)

$\Rightarrow$ E + (E)

$\Rightarrow$ E + T

$\Rightarrow$ E

A Third View of a Bottom-Up Parse



$\Rightarrow$ `E + (E + int)`

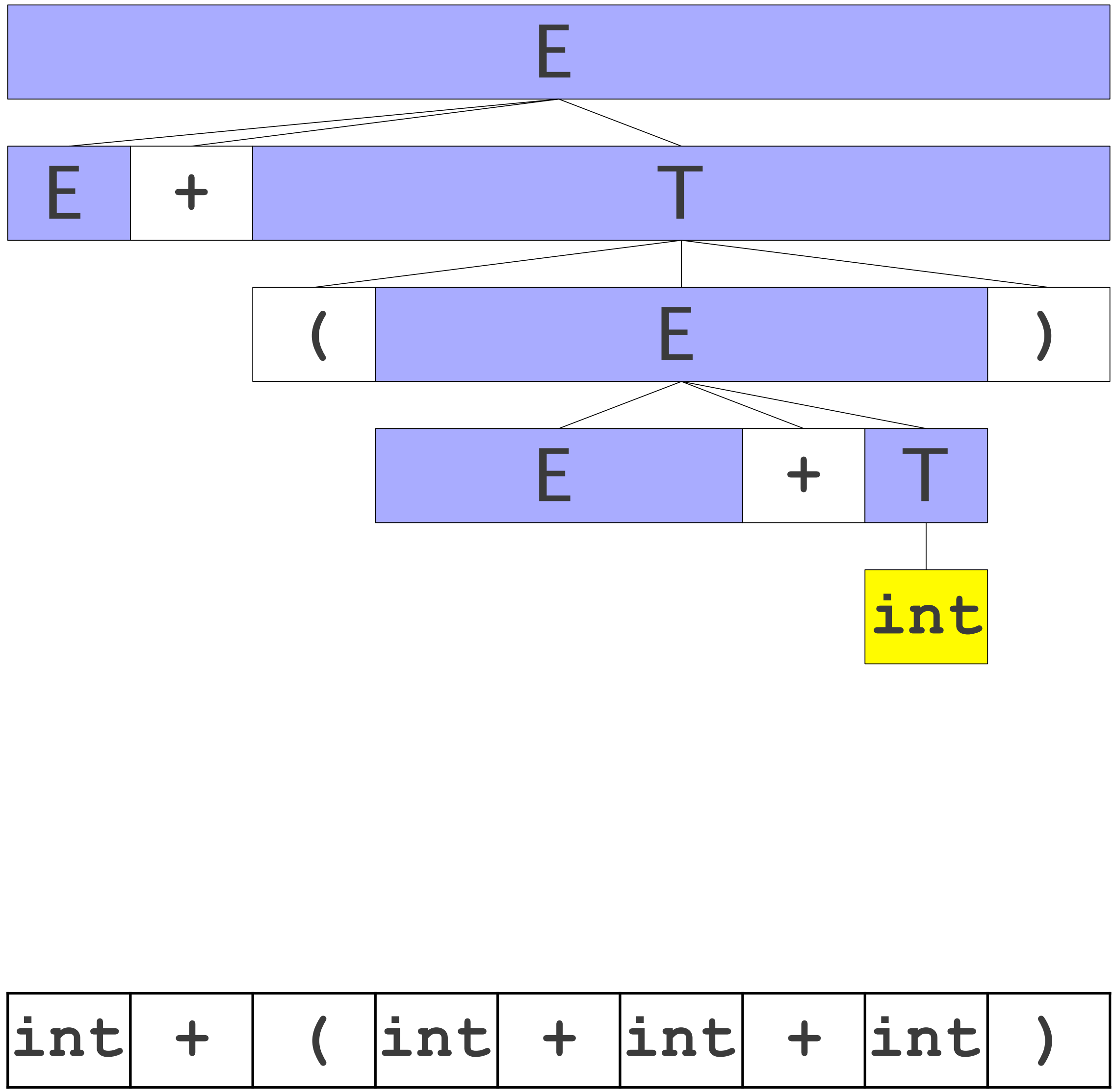
$\Rightarrow$ `E + (E + T)`

$\Rightarrow$ `E + (E)`

$\Rightarrow$ `E + T`

$\Rightarrow$ `E`

A Third View of a Bottom-Up Parse



⇒ E + (E + int)

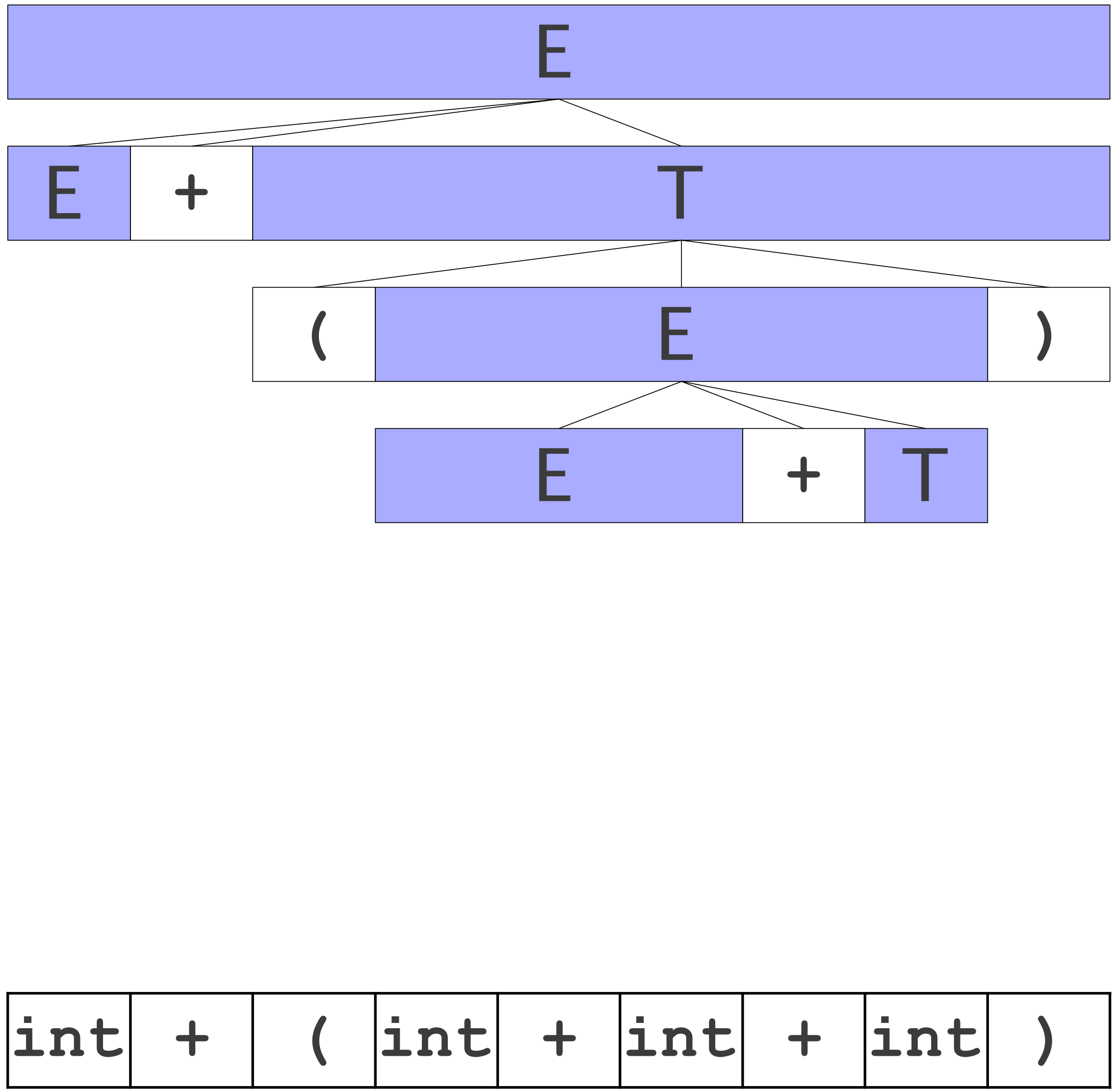
⇒ E + (E + T)

⇒ E + (E)

⇒ E + T

⇒ E

A Third View of a Bottom-Up Parse



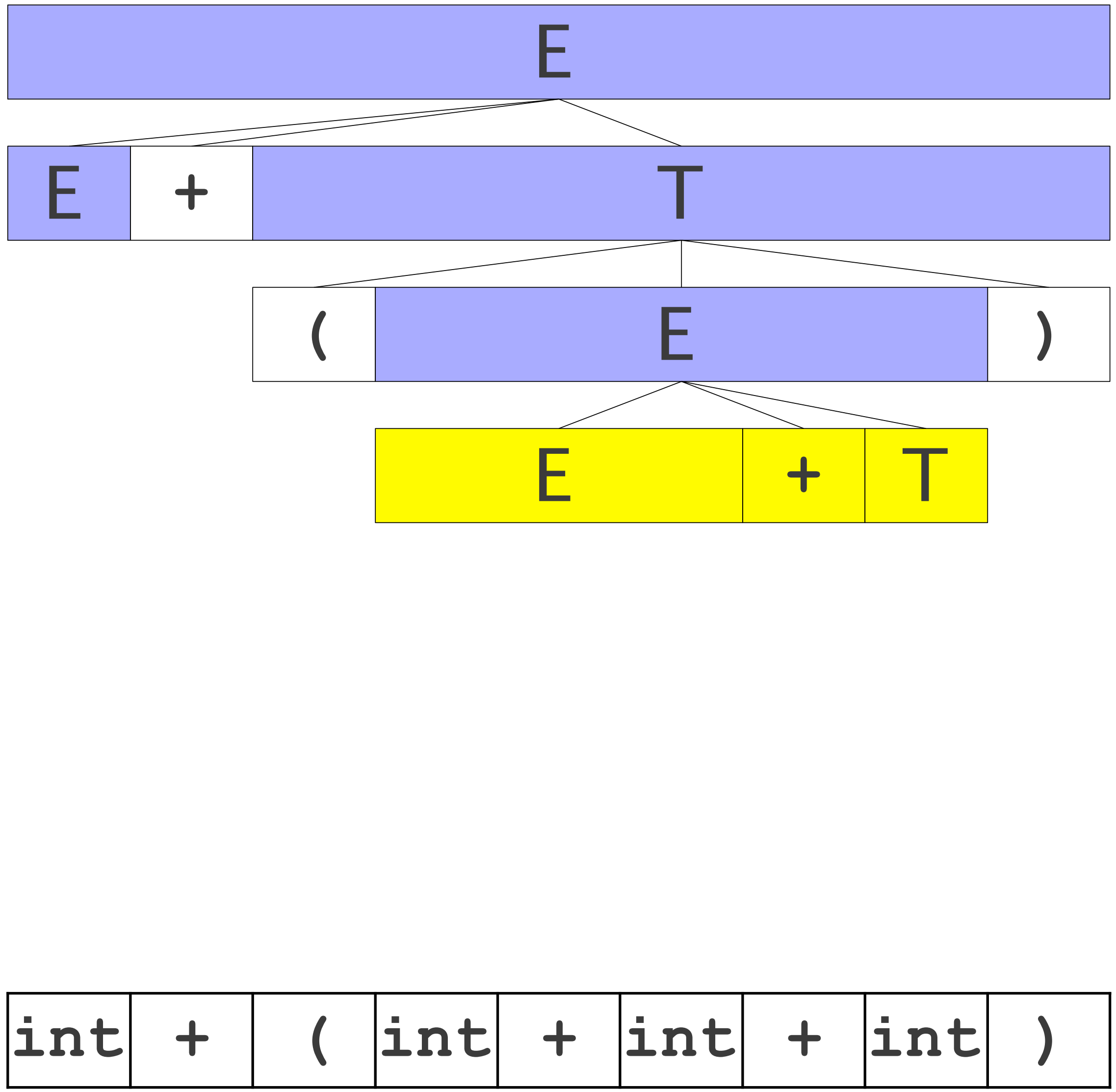
$\Rightarrow$ `E + (E + T)`

$\Rightarrow$ `E + (E)`

$\Rightarrow$ `E + T`

$\Rightarrow$ `E`

A Third View of a Bottom-Up Parse



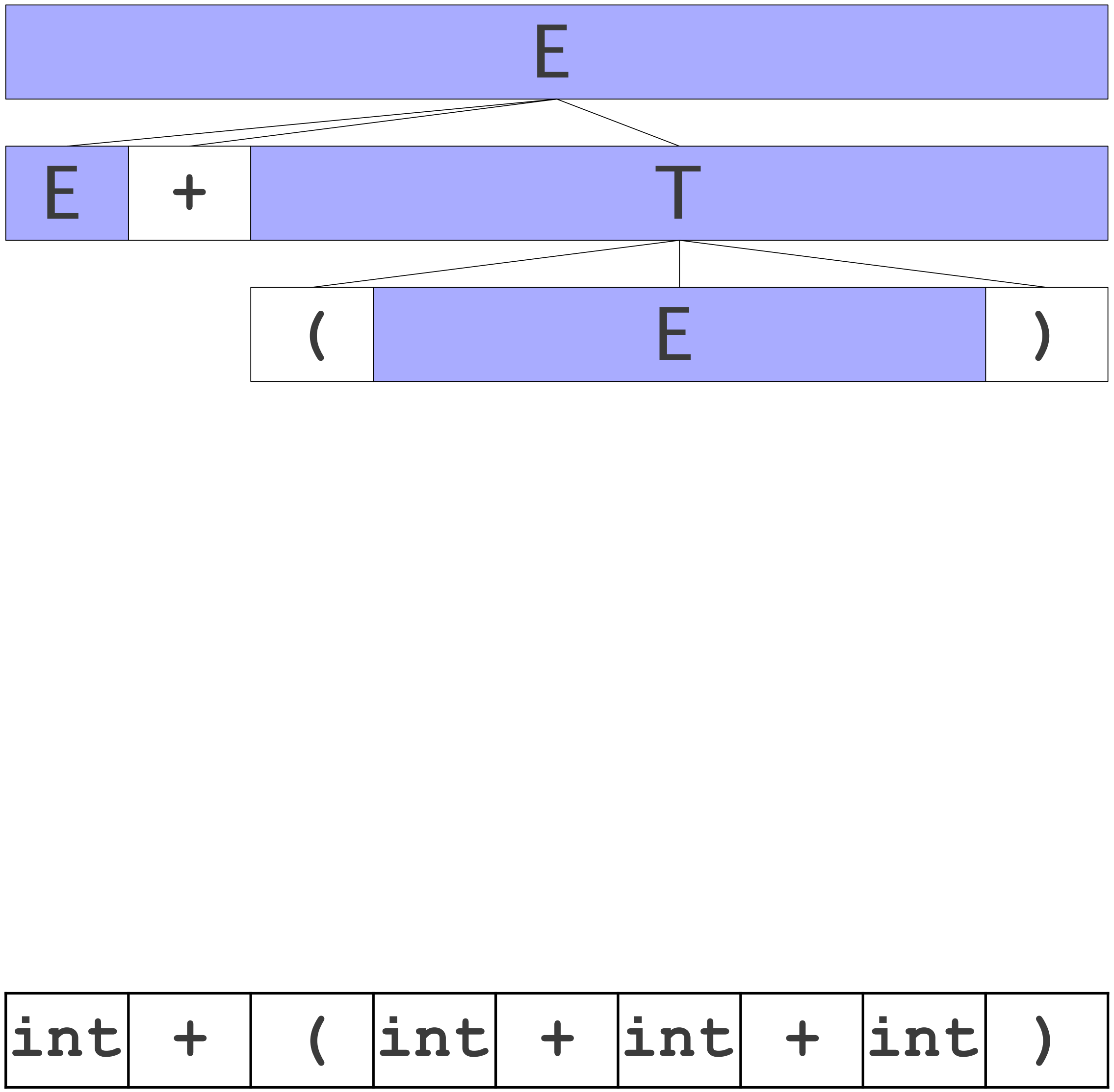
$\Rightarrow$ `E + (E + T)`

$\Rightarrow$ `E + (E)`

$\Rightarrow$ `E + T`

$\Rightarrow$ `E`

A Third View of a Bottom-Up Parse

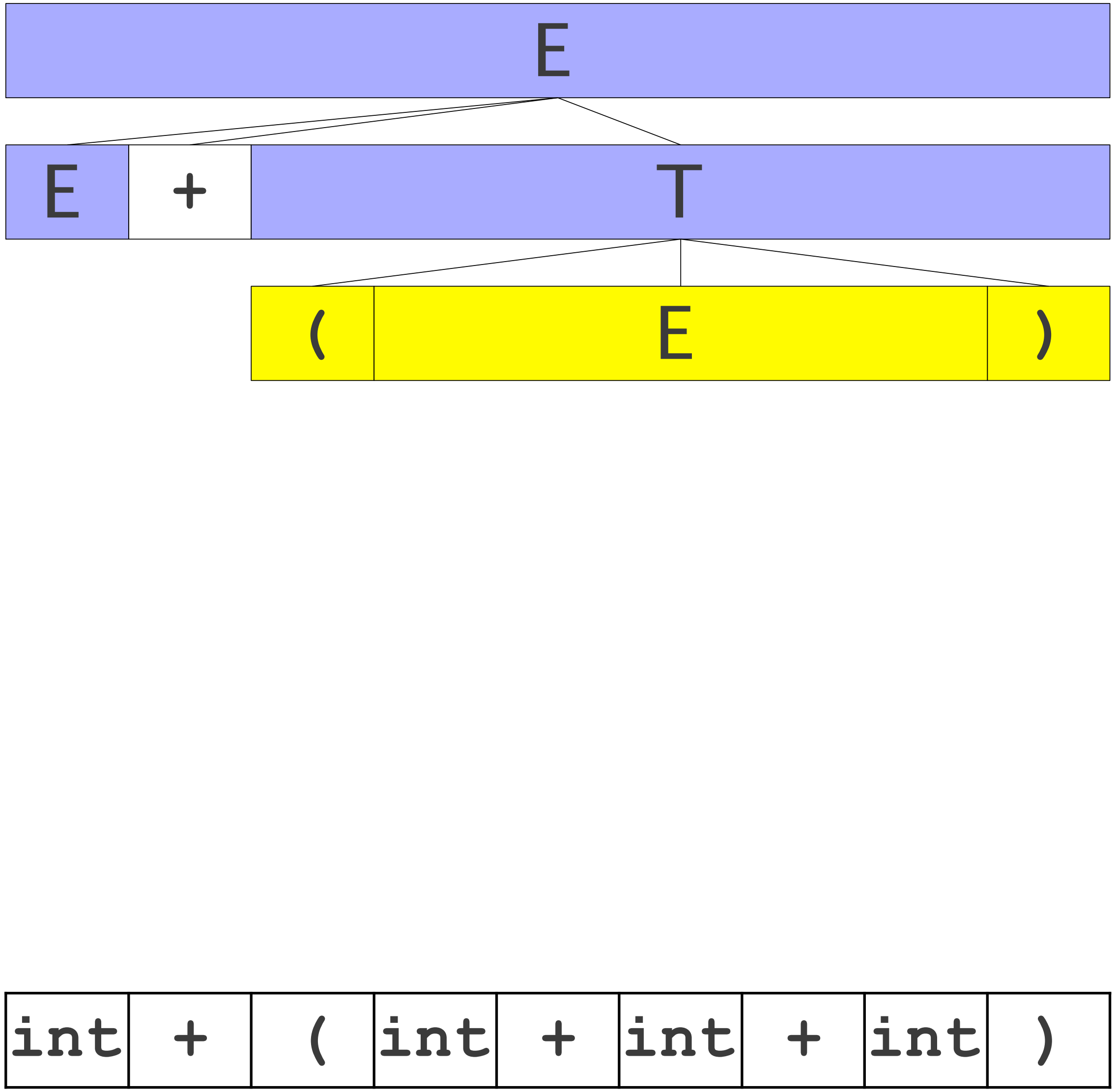


$\Rightarrow$ `E + (E)`

$\Rightarrow$ `E + T`

$\Rightarrow$ `E`

A Third View of a Bottom-Up Parse

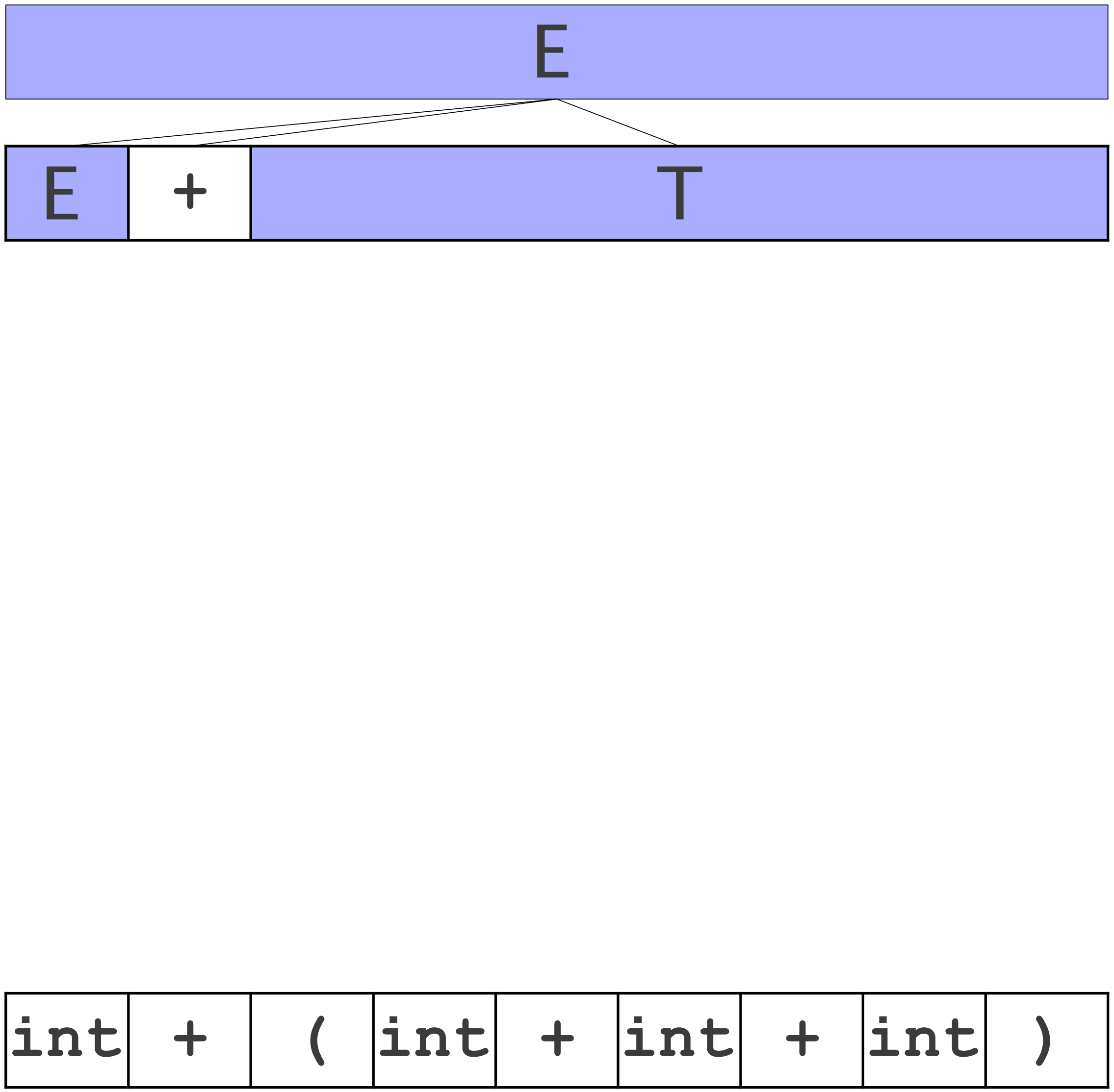


$\Rightarrow$ E + (E)

$\Rightarrow$ E + T

$\Rightarrow$ E

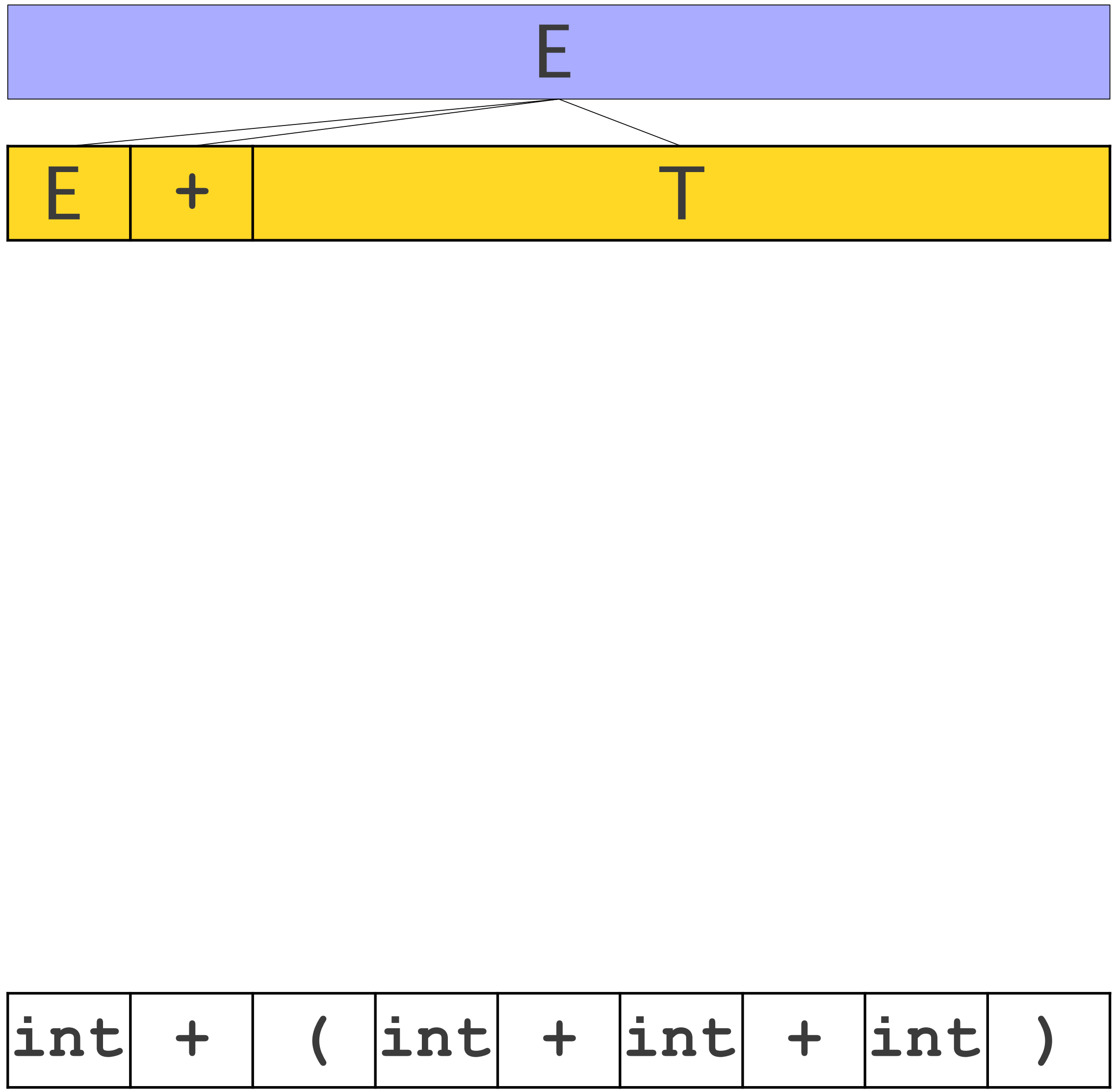
A Third View of a Bottom-Up Parse



$\Rightarrow$ **E + T**

$\Rightarrow$ **E**

A Third View of a Bottom-Up Parse



⇒ E + T
⇒ E

A Third View of a Bottom-Up Parse

E								

$\Rightarrow$ **E**

`int`	`+`	`(`	`int`	`+`	`int`	`+`	`int`	`)`

Handles

- The **handle** of a parse tree *T* is the leftmost complete cluster of leaf nodes.
- A left-to-right, bottom-up parse works by iteratively searching for a handle, then reducing the handle.

Handles

- Informally, a **handle of a string is a substring that matches the right side of a production rule**.

  But not every substring matches the right side of a production rule is handle

- A **handle** of a right sentential form γ ($\equiv \alpha\beta\omega$) is a production rule $A \rightarrow \beta$ and a position of γ where the string β may be found and replaced by A to produce the previous right-sentential form in a rightmost derivation of γ.

$$S \overset{*}{\underset{rm}{\Rightarrow}} \alpha A \omega \underset{rm}{\Rightarrow} \alpha\beta\omega$$

- If the grammar is unambiguous, then every right-sentential form of the grammar has exactly one handle.

Handle Pruning

A right-most derivation in reverse can be obtained by **handle-pruning**.

$$S=\gamma_0 \underset{rm}{\Rightarrow} \gamma_1 \underset{rm}{\Rightarrow} \gamma_2 \underset{rm}{\Rightarrow} \ldots \underset{rm}{\Rightarrow} \gamma_{n-1} \underset{rm}{\Rightarrow} \gamma_n = \omega$$

input string

Start from γ_n, find a handle $A_n \rightarrow \beta_n$ in γ_n, and replace β_n in by A_n to get γ_{n-1}.

Then find a handle $A_{n-1} \rightarrow \beta_{n-1}$ in γ_{n-1}, and replace β_{n-1} in by A_{n-1} to get γ_{n-2}.

Repeat this, until we reach S.

A Shift-Reduce Parser

$E \rightarrow E+T \mid T$

$T \rightarrow T*F \mid F$

$F \rightarrow (E) \mid id$

Right-Most Derivation of id+id*id

$E \Rightarrow E+T \Rightarrow E+T*F \Rightarrow E+T*id \Rightarrow E+F*id$

$\Rightarrow E+id*id \Rightarrow T+id*id \Rightarrow F+id*id \Rightarrow id+id*id$

Right-Most Sentential Form	Reducing Production
$\underline{id}+id*id$	$F \rightarrow id$
$\underline{F}+id*id$	$T \rightarrow F$
$\underline{T}+id*id$	$E \rightarrow T$
$E+\underline{id}*id$	$F \rightarrow id$
$E+\underline{F}*id$	$T \rightarrow F$
$E+T*\underline{id}$	$F \rightarrow id$
$E+\underline{T*F}$	$T \rightarrow T*F$
$\underline{E+T}$	$E \rightarrow E+T$
E	

Handles are red and underlined in the right-sentential forms.

A Stack Implementation of A Shift-Reduce Parser

There are four possible actions of a shift-parser action:

- **Shift** : The next input symbol is shifted onto the top of the stack.
- **Reduce**: Replace the handle on the top of the stack by the non-terminal.
- **Accept**: Successful completion of parsing.
- **Error**: Parser discovers a syntax error, and calls an error recovery routine.

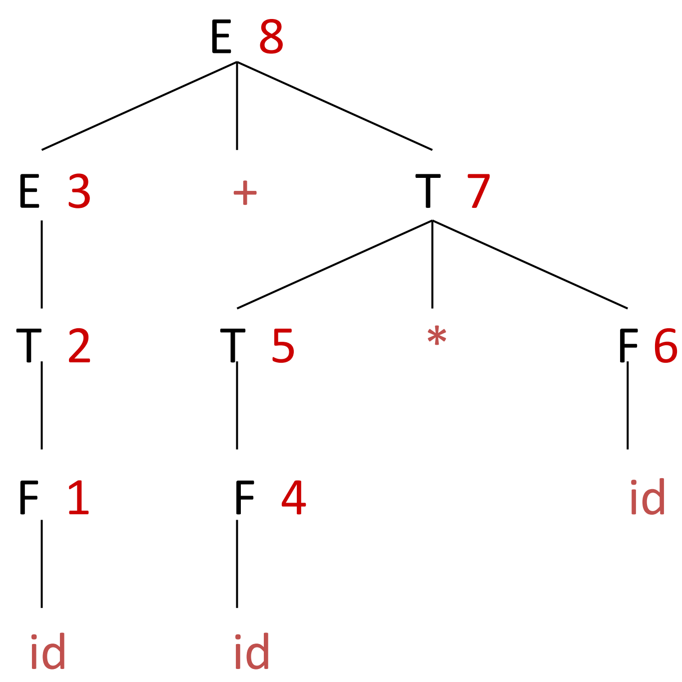
Initial stack just contains only the end-marker **$**.

The end of the input string is marked by the end-marker **$**.

A Stack Implementation of A Shift-Reduce Parser

Stack	Input	Action
$	id+id*id$	shift
$id	+id*id$	reduce by F → id
$F	+id*id$	reduce by T → F
$T	+id*id$	reduce by E → T
$E	+id*id$	shift
$E+	id*id$	shift
$E+id	*id$	reduce by F → id
$E+F	*id$	reduce by T → F
$E+T	*id$	shift
$E+T*	id$	shift
$E+T*id	$	reduce by F → id
$E+T*F	$	reduce by T → T*F
$E+T	$	reduce by E → E+T
$E	$	accept

Parse Tree



Summarizing Our Intuition

- Our first intuition (reconstructing the parse tree bottom-up) motivates how the parsing should work.
- Our second intuition (rightmost derivation in reverse) describes the order in which we should build the parse tree.
- Our third intuition (handle pruning) is the basis for the bottom-up parsing algorithms we will explore.

A Detail about Handles

$E \rightarrow F$
$E \rightarrow E + F$
$F \rightarrow F * T$
$F \rightarrow T$
$T \rightarrow \texttt{int}$
$T \rightarrow (E)$

int	+	int	*	int

A Detail about Handles

E → F

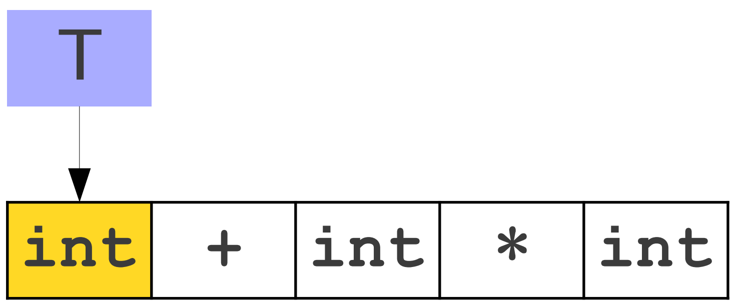
E → E + F

F → F * T

F → T

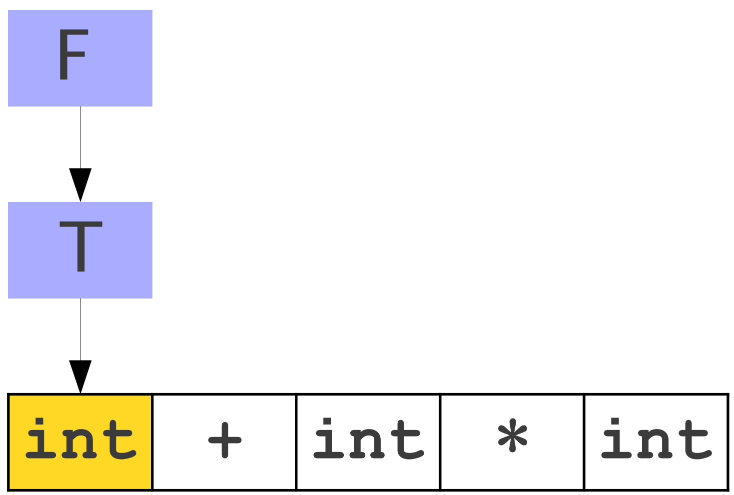
T → int

T → (E)



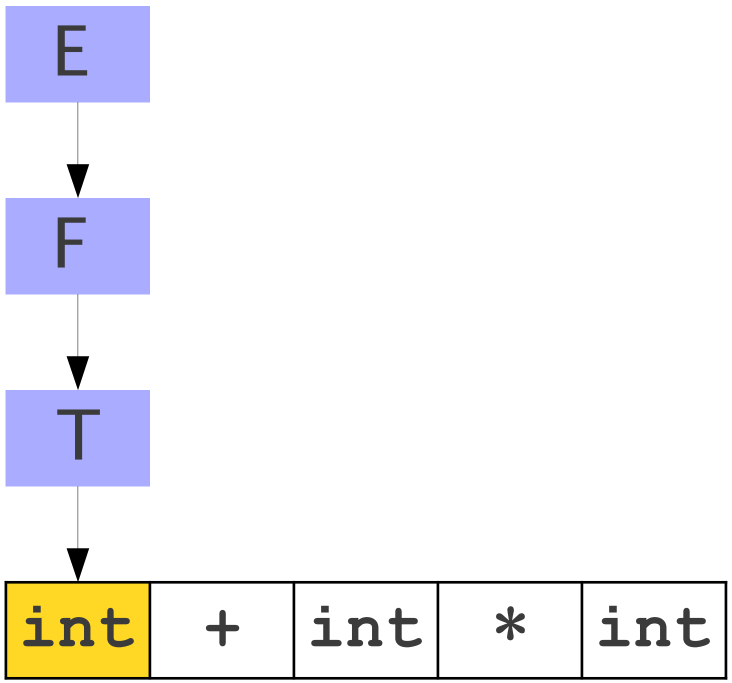
A Detail about Handles

$E \rightarrow F$
$E \rightarrow E + F$
$F \rightarrow F * T$
$F \rightarrow T$
$T \rightarrow$ `int`
$T \rightarrow$ `(E)`



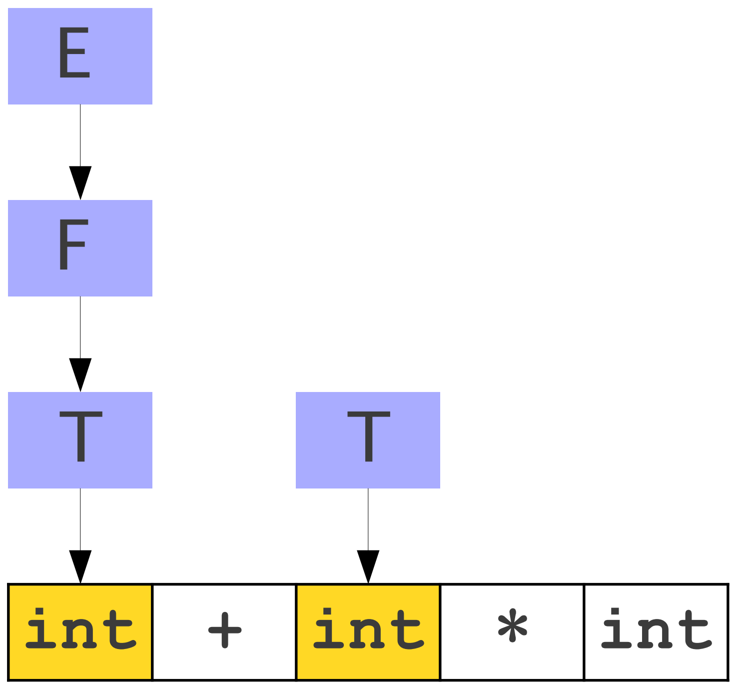
A Detail about Handles

$E \rightarrow F$
$E \rightarrow E + F$
$F \rightarrow F * T$
$F \rightarrow T$
$T \rightarrow$ `int`
$T \rightarrow (E)$



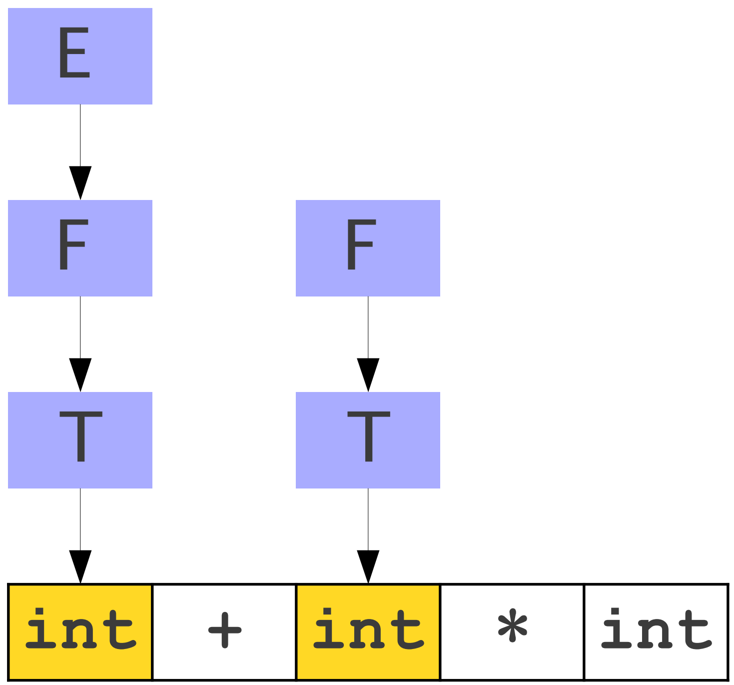
A Detail about Handles

$E \rightarrow F$
$E \rightarrow E + F$
$F \rightarrow F * T$
$F \rightarrow T$
$T \rightarrow \texttt{int}$
$T \rightarrow (E)$



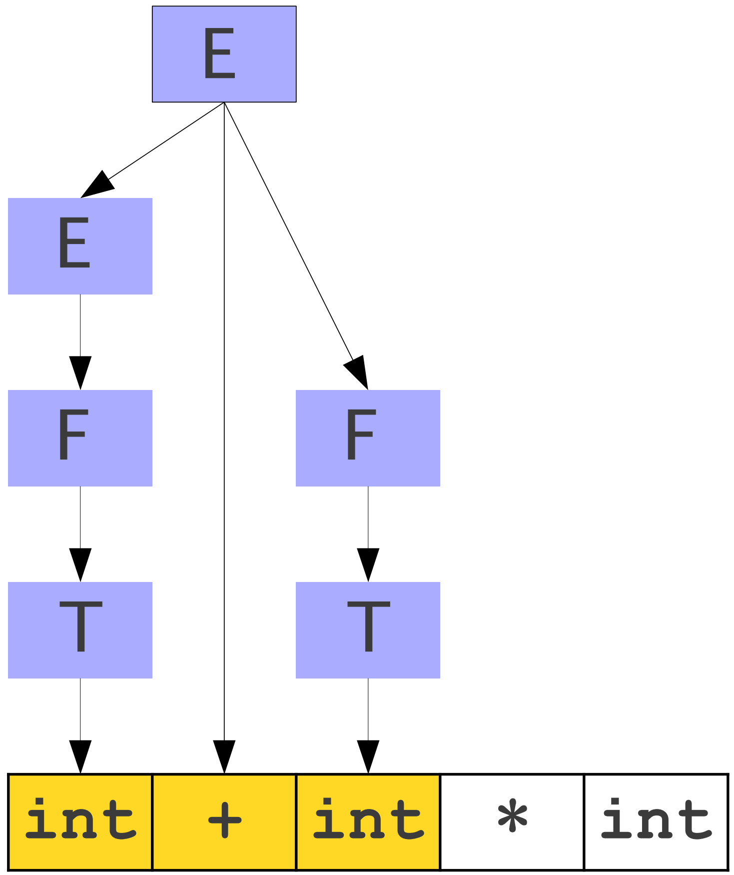
A Detail about Handles

E → F
E → E + F
F → F * T
F → T
T → int
T → (E)



A Detail about Handles

$E \rightarrow F$
$E \rightarrow E + F$
$F \rightarrow F * T$
$F \rightarrow T$
$T \rightarrow$ `int`
$T \rightarrow (E)$



A Detail about Handles

$E \rightarrow F$
$E \rightarrow E + F$
$F \rightarrow F * T$
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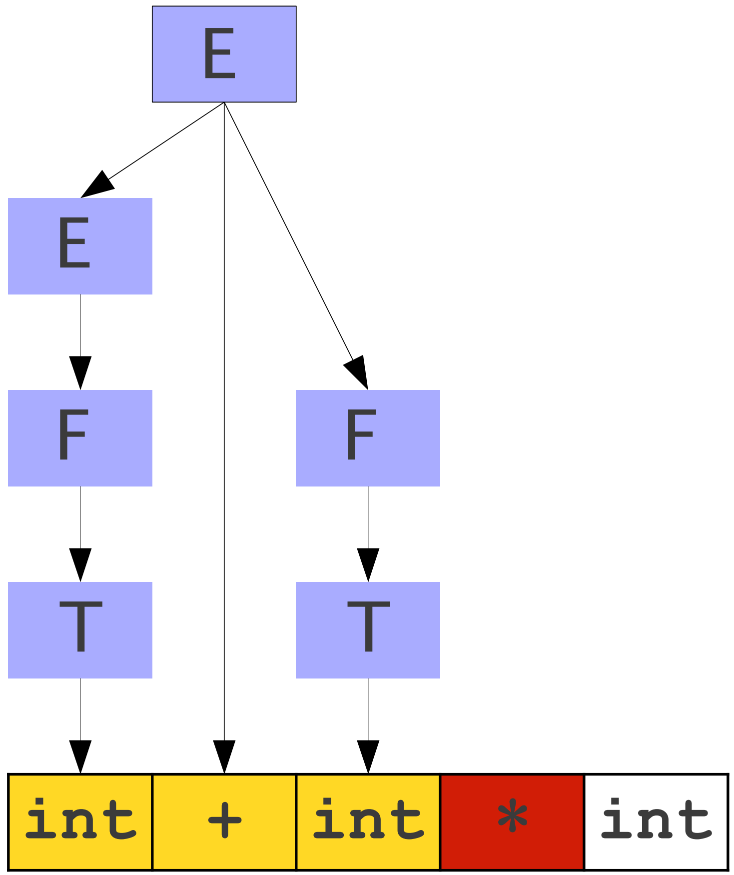
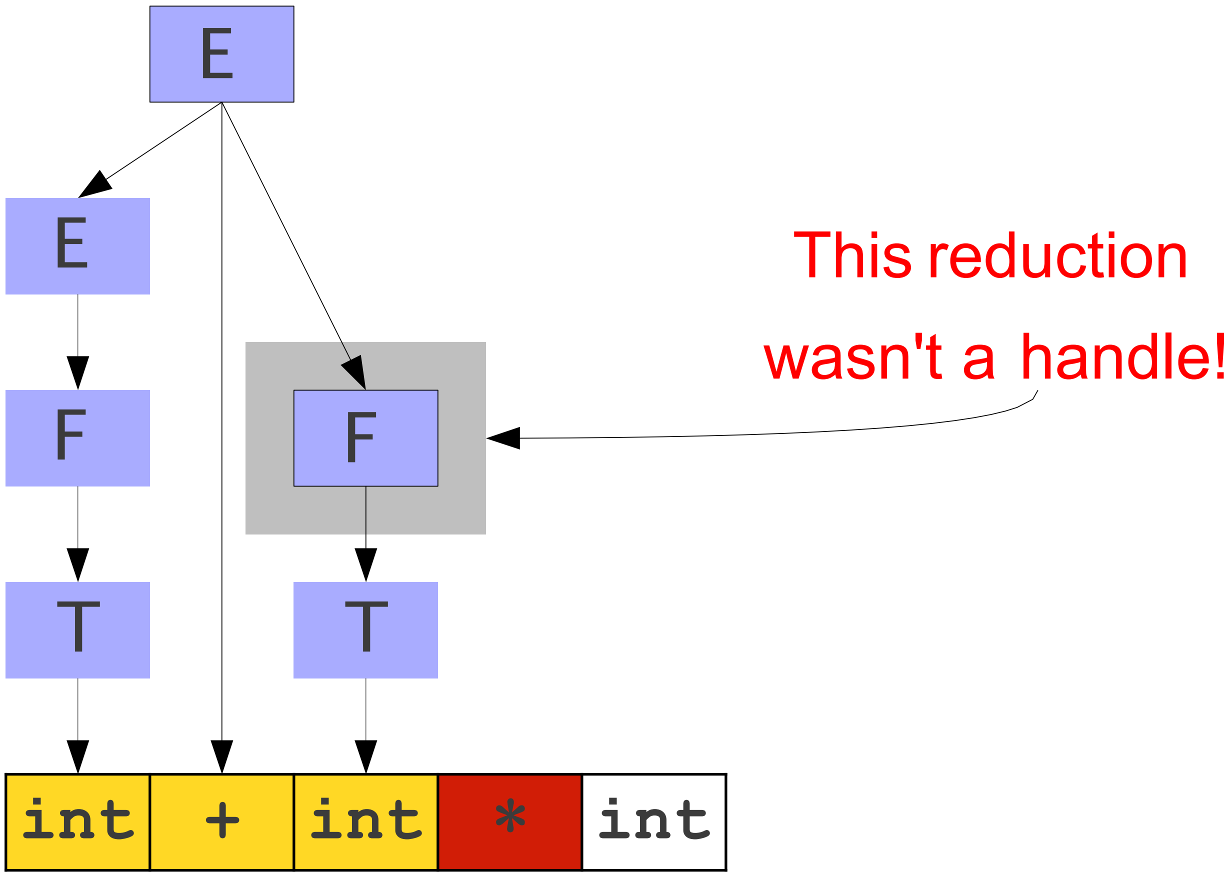
$T \rightarrow \texttt{int}$
$T \rightarrow (E)$



A Detail about Handles

E → F
E → E + F
F → F * T
F → T
T → int
T → (E)



The leftmost reduction
isn't always the handle.

Finding Handles

- Where do we look for handles?
  - Where in the string might the handle be?
- How do we search for possible handles?
  - Once we know where to search, how do we identify candidate handles?
- How do we recognize handles?
  - Once we've found a candidate handle, how do we check that it really is the handle?

Question One:

Where are handles?

Where are Handles?

- Recall: A left-to-right, bottom-up parse traces a rightmost derivation in reverse.
- Each time we do a reduction, we are reversing a production applied to the *rightmost* nonterminal symbol.
- Suppose that our current sentential form is $\alpha\gamma\omega$, where γ is the handle and $A \to \gamma$ is a production rule.
- After reducing γ back to A, we have the string $\alpha A\omega$.

  Thus ω must consist purely of terminals, since otherwise the reduction we just did was not for the rightmost terminal.

Why This Matters

- Suppose we want to parse the string γ.

  We will break γ into two parts, α and ω,
- where

  α consists of both terminals and nonterminals, and

  ω consists purely of terminals.
- Our search for handles will concentrate purely in α.
- As necessary, we will start moving terminals from ω over into α.

Shift/Reduce Parsing

- The bottom-up parsers we will consider are called **shift/reduce** parsers.

  Contrast with the LL(1) **predict/match** parser.

- Idea: Split the input into two parts:
  - Left substring is our work area; all handles must be here.
  - Right substring is input we have not yet processed; consists purely of terminals.

- At each point, decide whether to:
  - Move a terminal across the split (shift)
  - Reduce a handle (reduce)

A Sample Shift/Reduce Parse

E → **F**
E → **E + F**
F → **F * T**
F → **T**
T → **int**
T → **(E)**

int	+	int	*	int	+	int

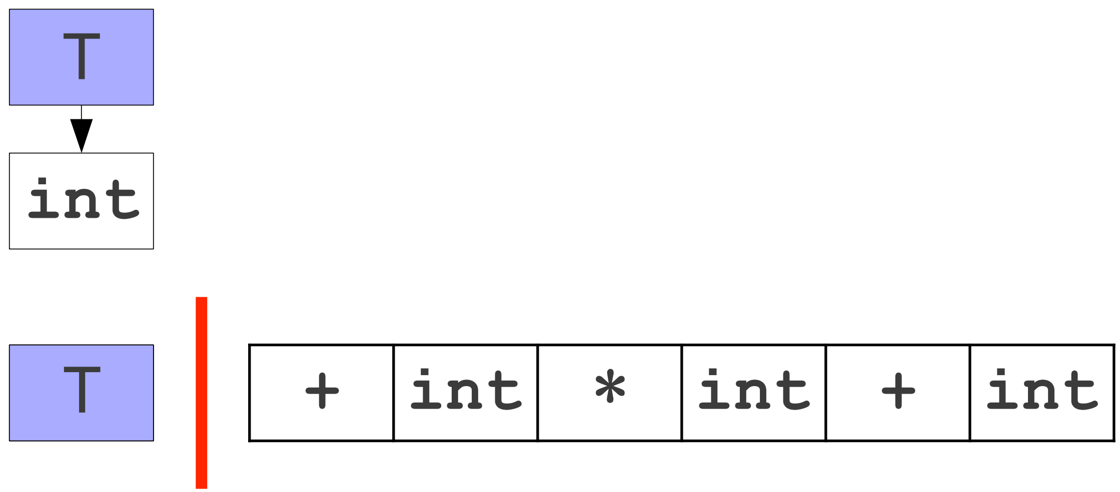
A Sample Shift/Reduce Parse

$E \to F$
$E \to E + F$
$F \to F * T$
$F \to T$
$T \to \texttt{int}$
$T \to (E)$

int	|	+	int	*	int	+	int

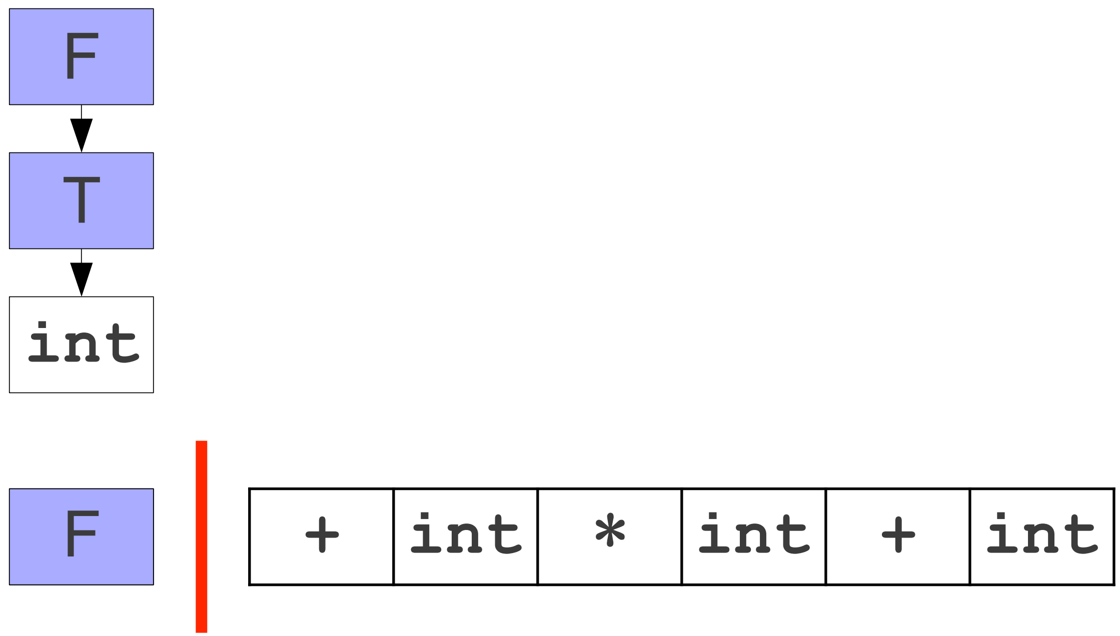
A Sample Shift/Reduce Parse

E → F
E → E + F
F → F * T
F → T
T → int
T → (E)



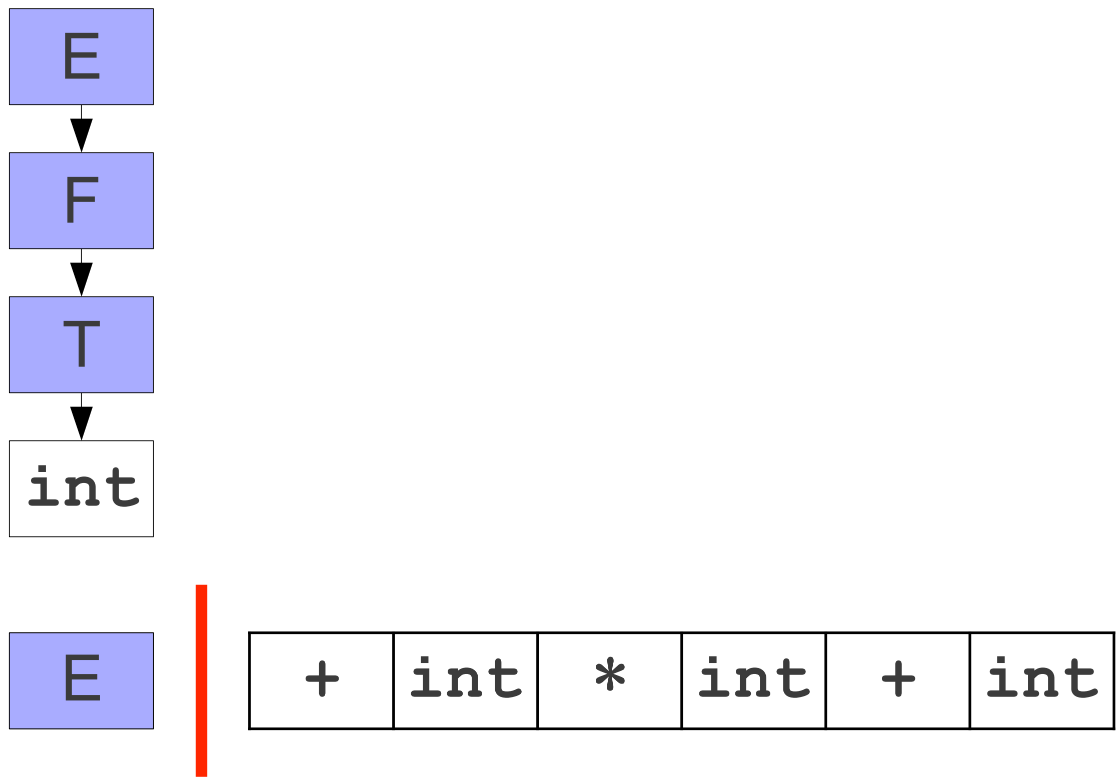
A Sample Shift/Reduce Parse

$E \rightarrow F$
$E \rightarrow E + F$
$F \rightarrow F * T$
$F \rightarrow T$
$T \rightarrow \texttt{int}$
$T \rightarrow (E)$



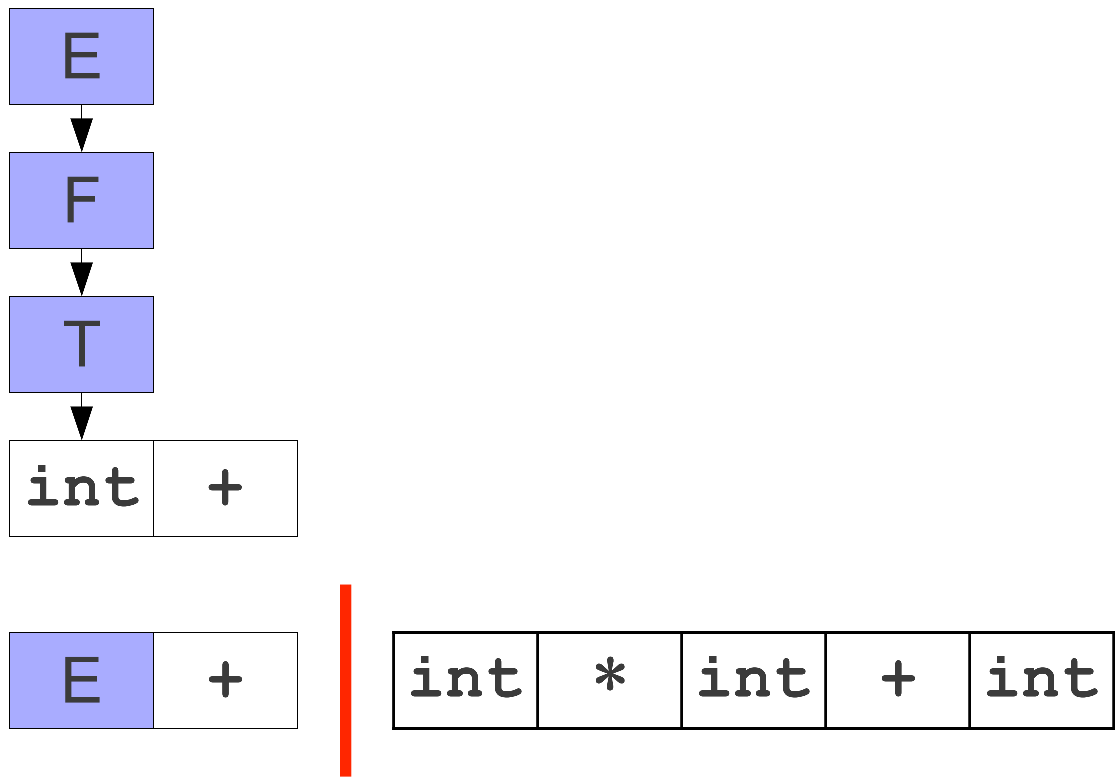
A Sample Shift/Reduce Parse

$E \rightarrow F$
$E \rightarrow E + F$
$F \rightarrow F * T$
$F \rightarrow T$
$T \rightarrow \texttt{int}$
$T \rightarrow (E)$



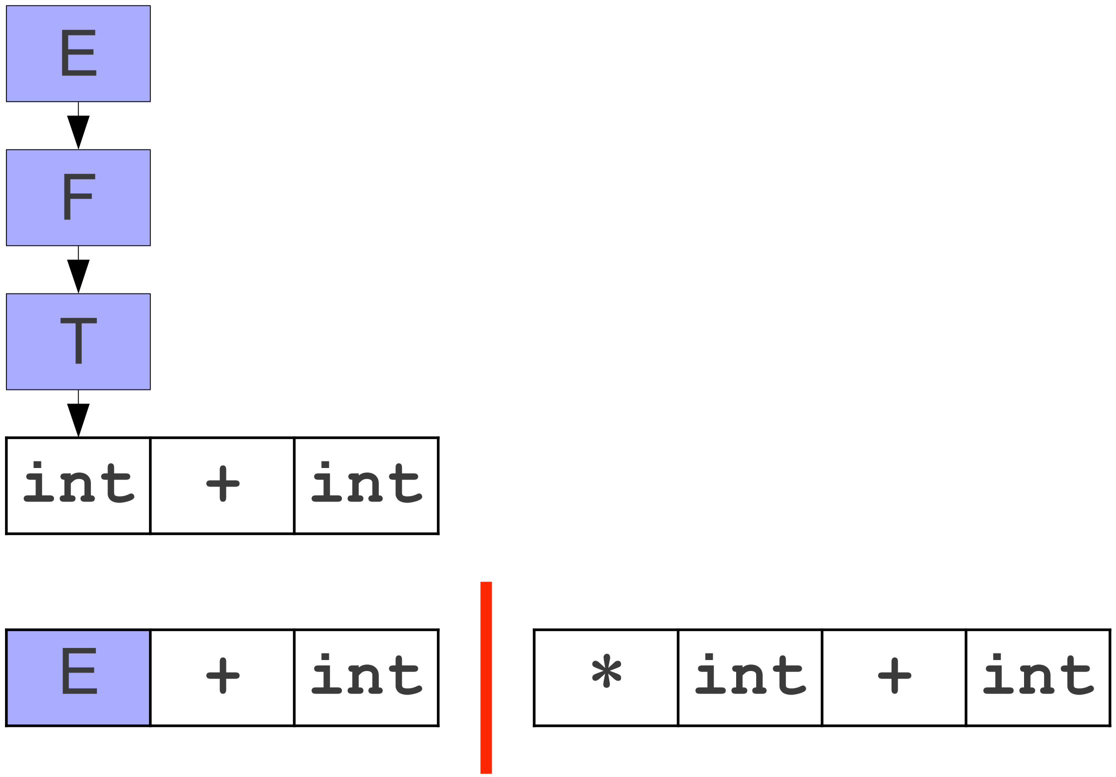
A Sample Shift/Reduce Parse

E → F
E → E + F
F → F * T
F → T
T → int
T → (E)



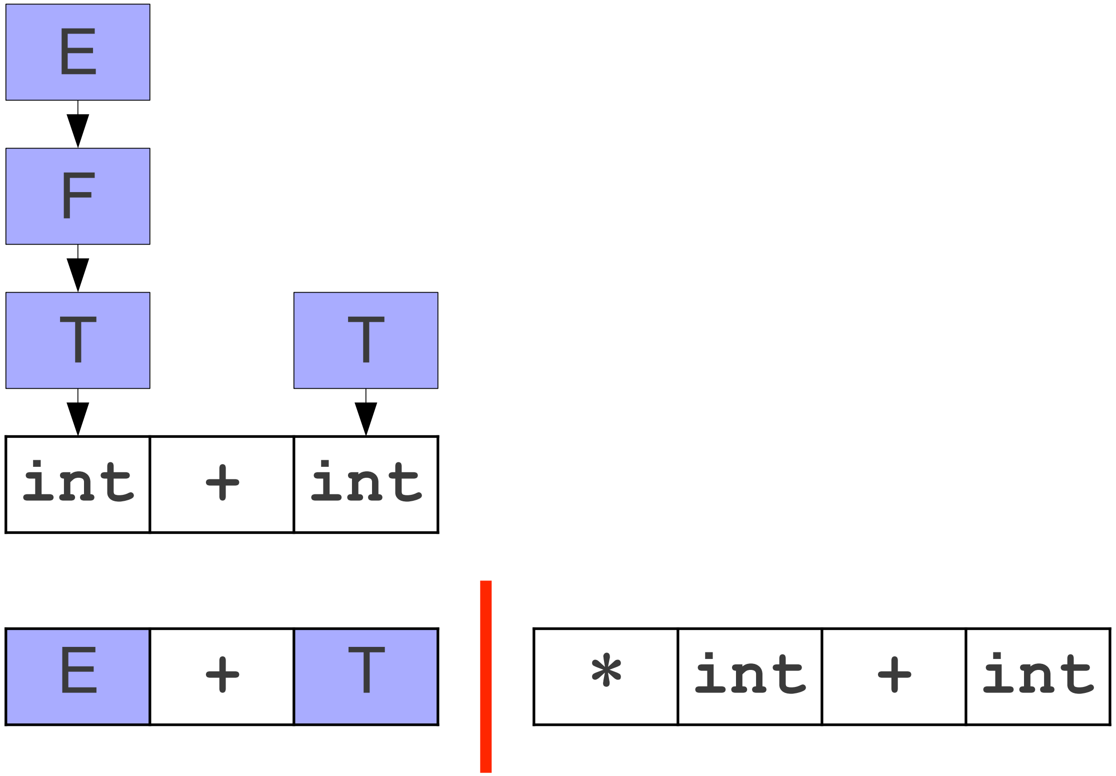
A Sample Shift/Reduce Parse

E → F
E → E + F
F → F * T
F → T
T → int
T → (E)



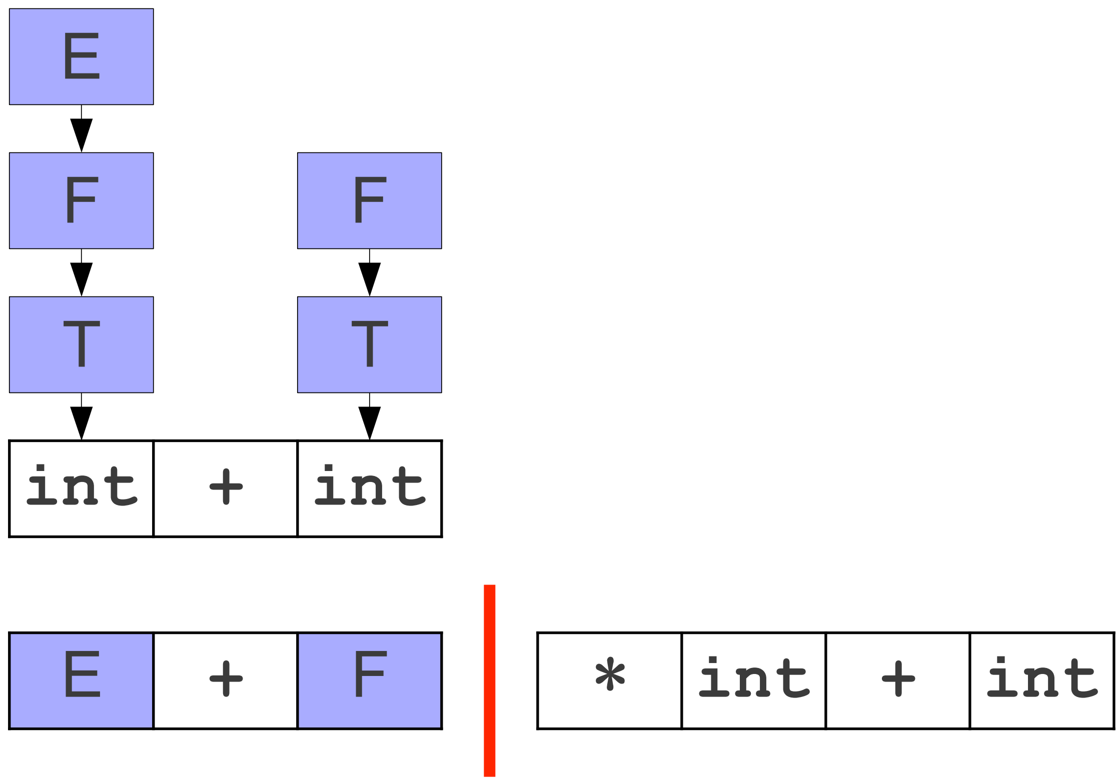
A Sample Shift/Reduce Parse

$E \rightarrow F$
$E \rightarrow E + F$
$F \rightarrow F * T$
$F \rightarrow T$
$T \rightarrow$ int
$T \rightarrow$ (E)



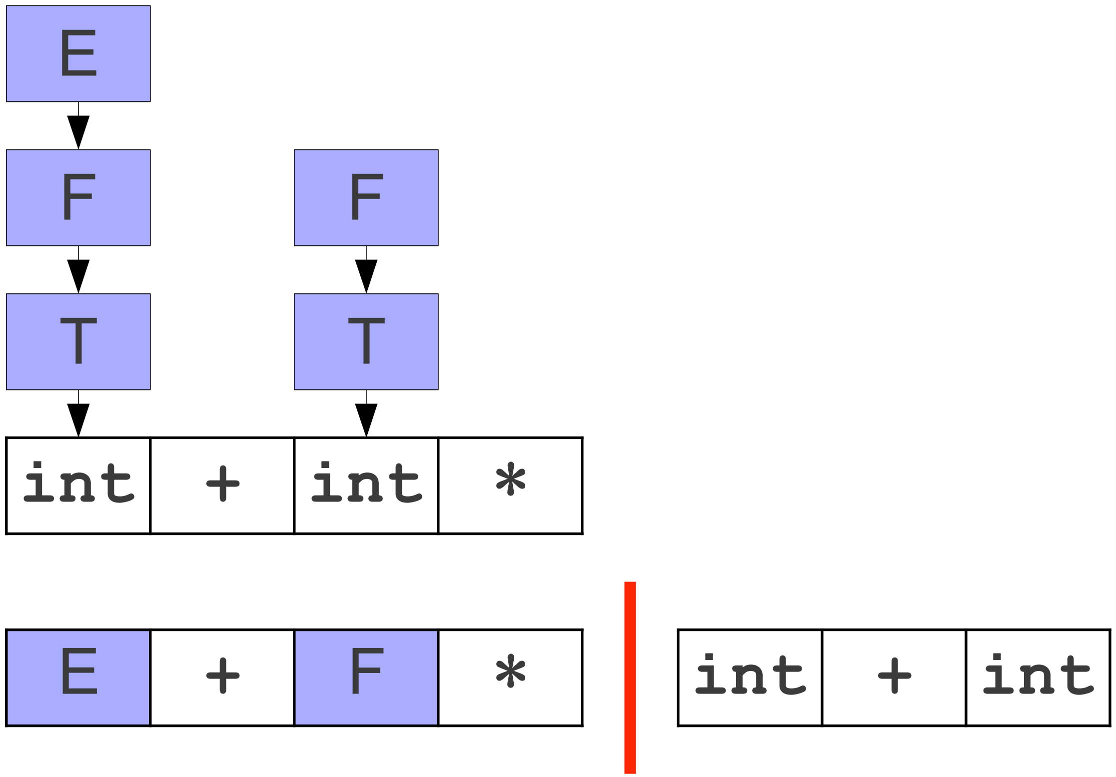
A Sample Shift/Reduce Parse

E → F
E → E + F
F → F * T
F → T
T → int
T → (E)



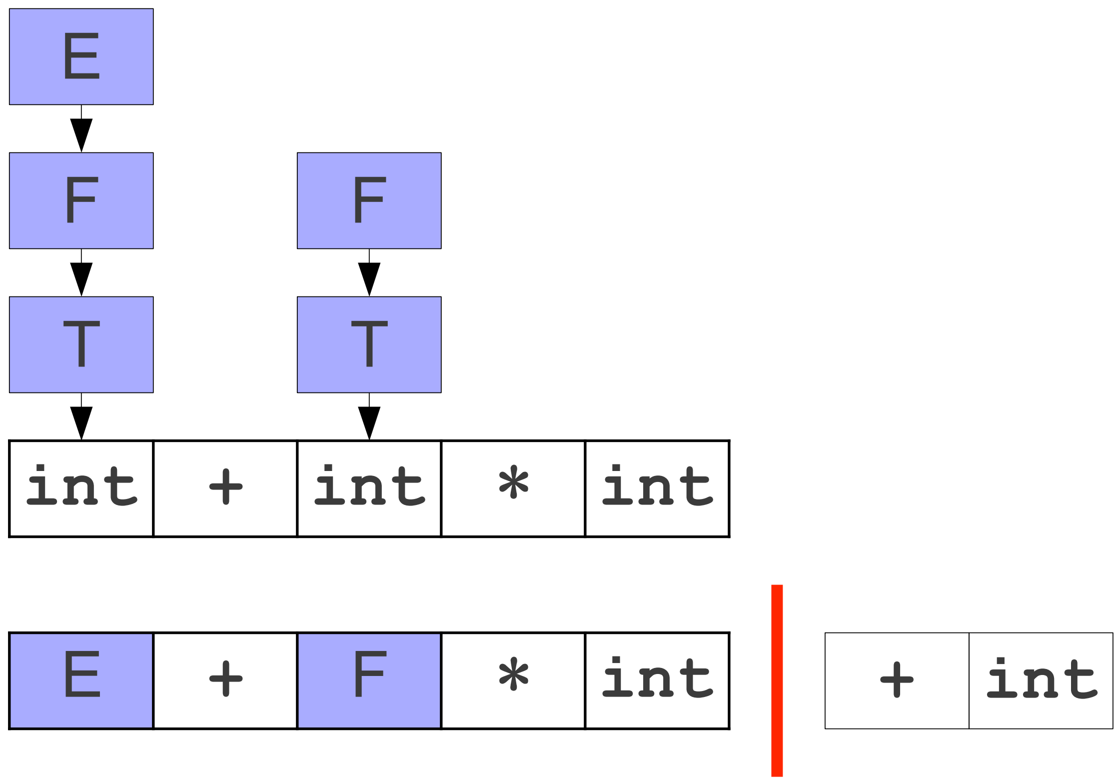
A Sample Shift/Reduce Parse

$E \rightarrow F$
$E \rightarrow E + F$
$F \rightarrow F * T$
$F \rightarrow T$
$T \rightarrow$ int
$T \rightarrow (E)$



A Sample Shift/Reduce Parse

$E \rightarrow F$
$E \rightarrow E$ **+** F
$F \rightarrow F$ ***** T
$F \rightarrow T$
$T \rightarrow$ **`int`**
$T \rightarrow$ **(**E**)**



A Sample Shift/Reduce Parse

$\mathbf{E} \to \mathbf{F}$

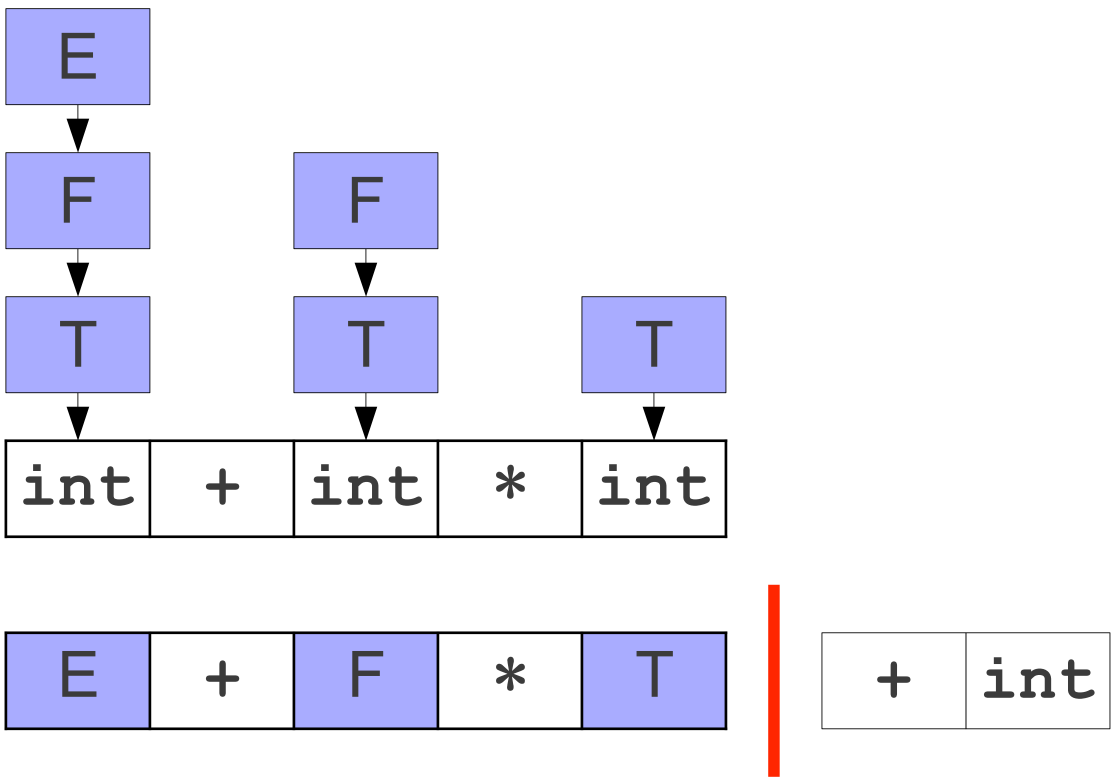
$\mathbf{E} \to \mathbf{E} + \mathbf{F}$

$\mathbf{F} \to \mathbf{F} * \mathbf{T}$

$\mathbf{F} \to \mathbf{T}$

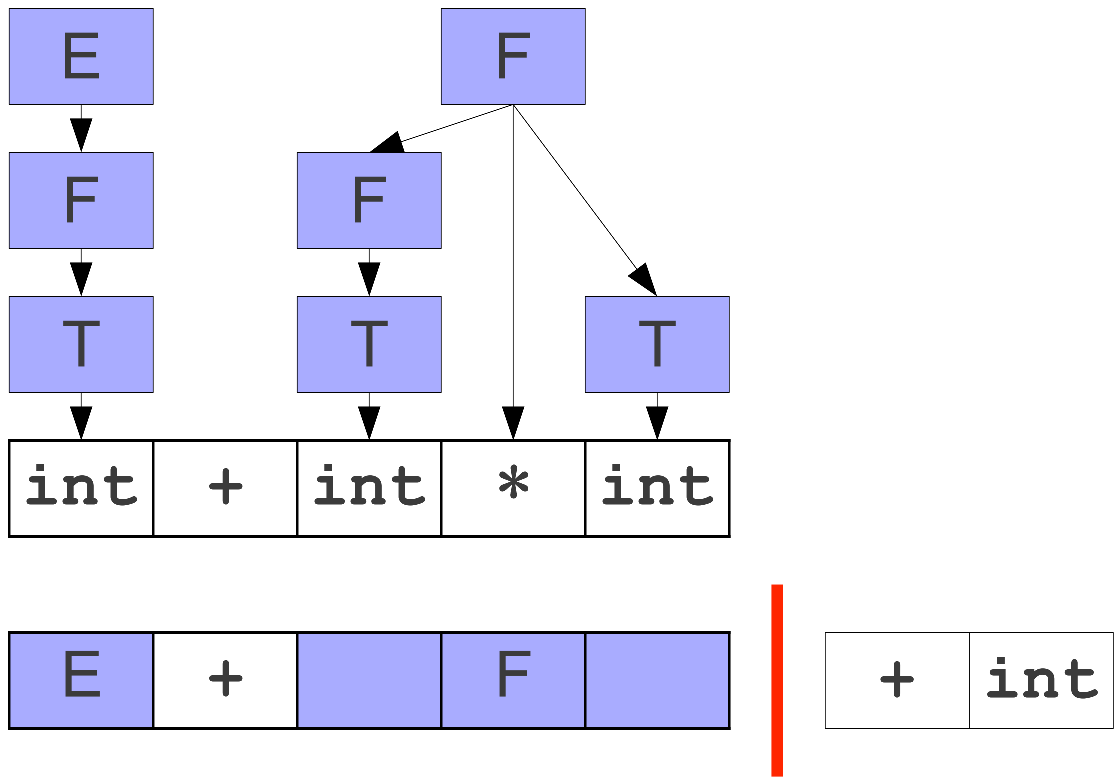
$\mathbf{T} \to$ `int`

$\mathbf{T} \to$ `(E)`



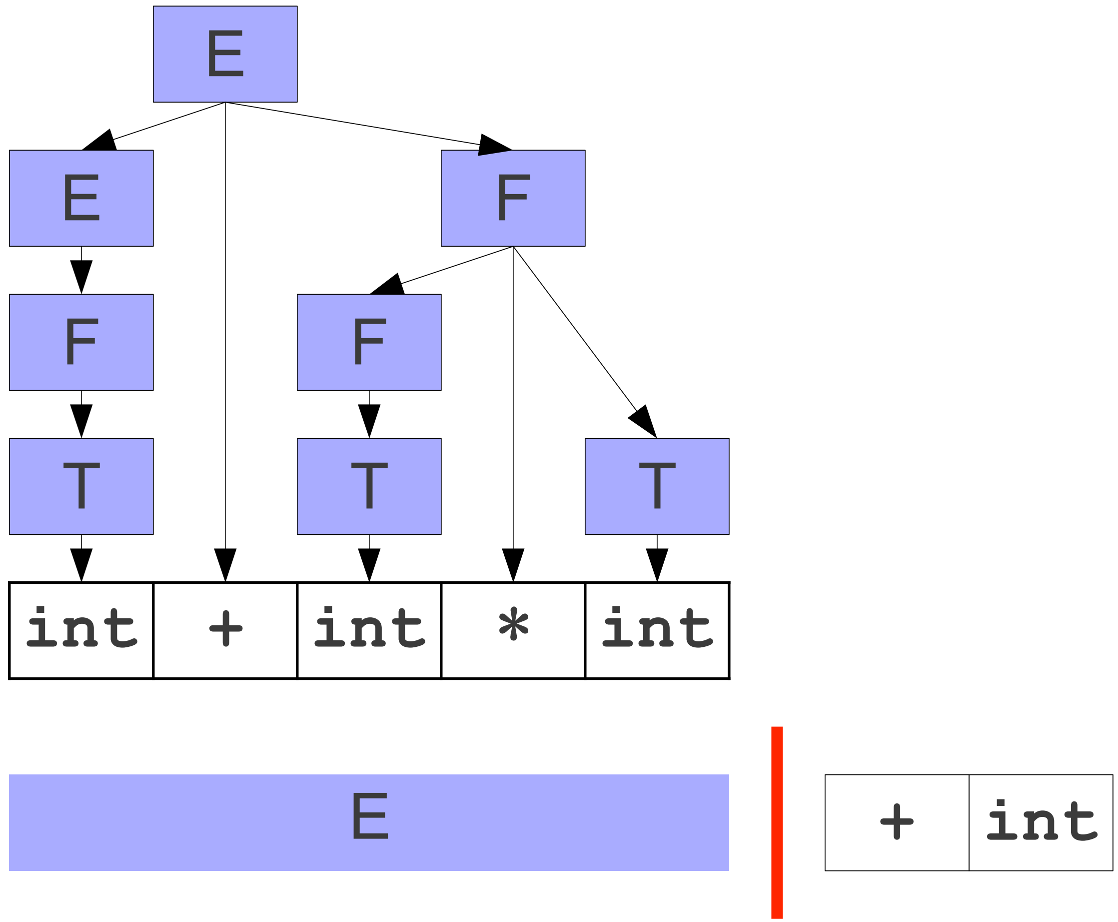
A Sample Shift/Reduce Parse

E → F
E → E + F
F → F * T
F → T
T → int
T → (E)



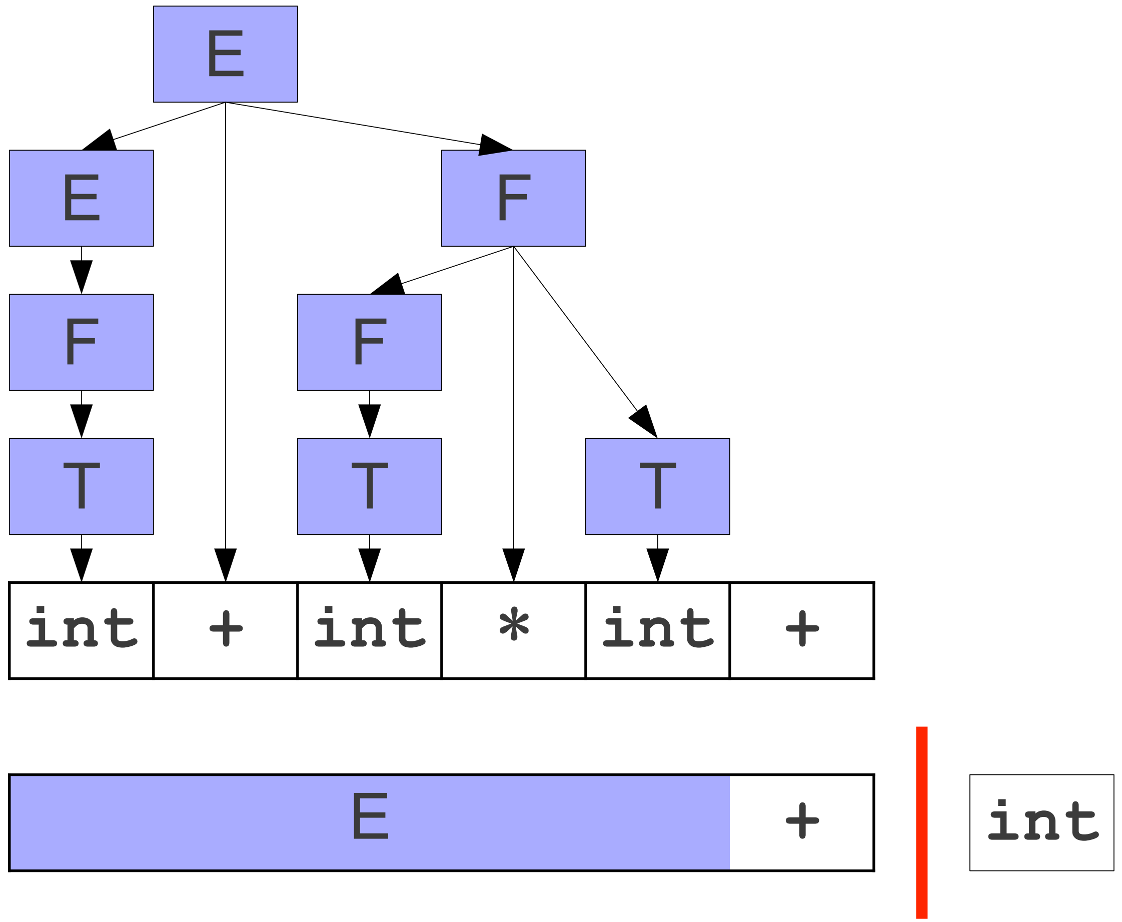
A Sample Shift/Reduce Parse

E → F
E → E + F
F → F * T
F → T
T → int
T → (E)



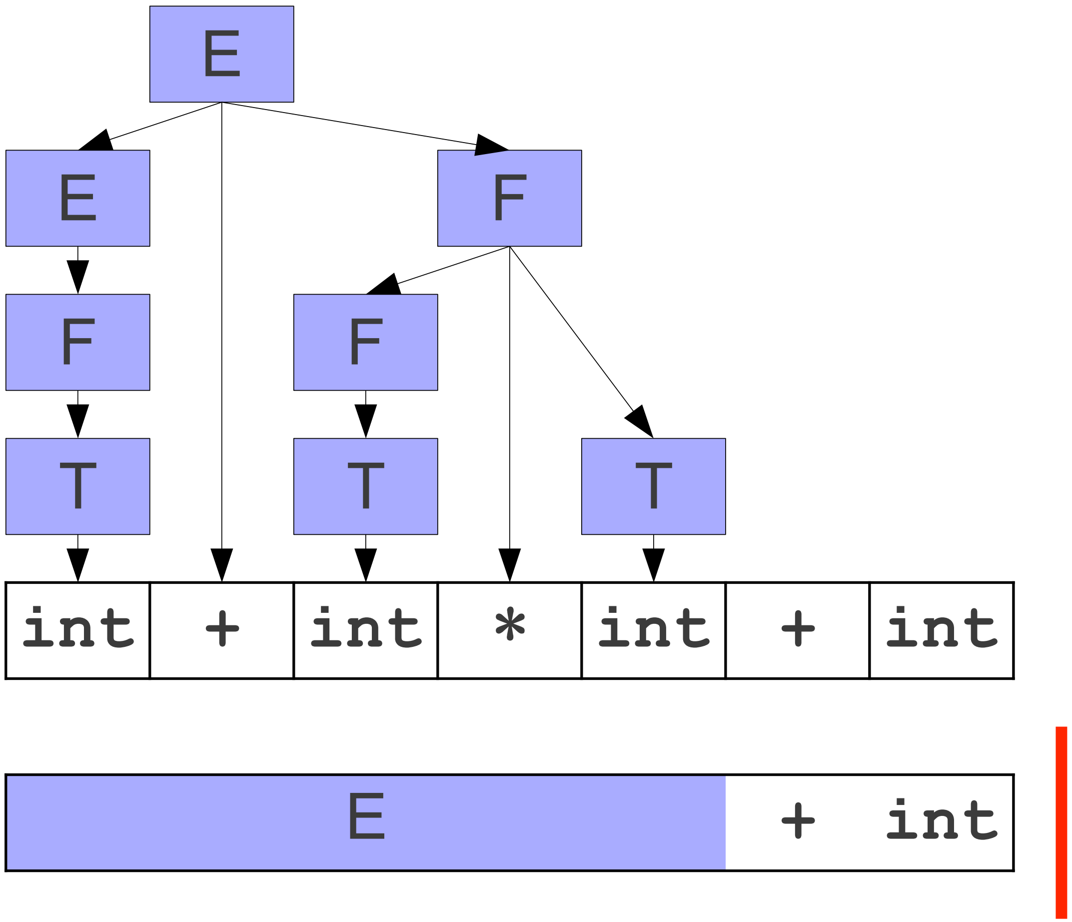
A Sample Shift/Reduce Parse

E → F
E → E + F
F → F * T
F → T
T → int
T → (E)



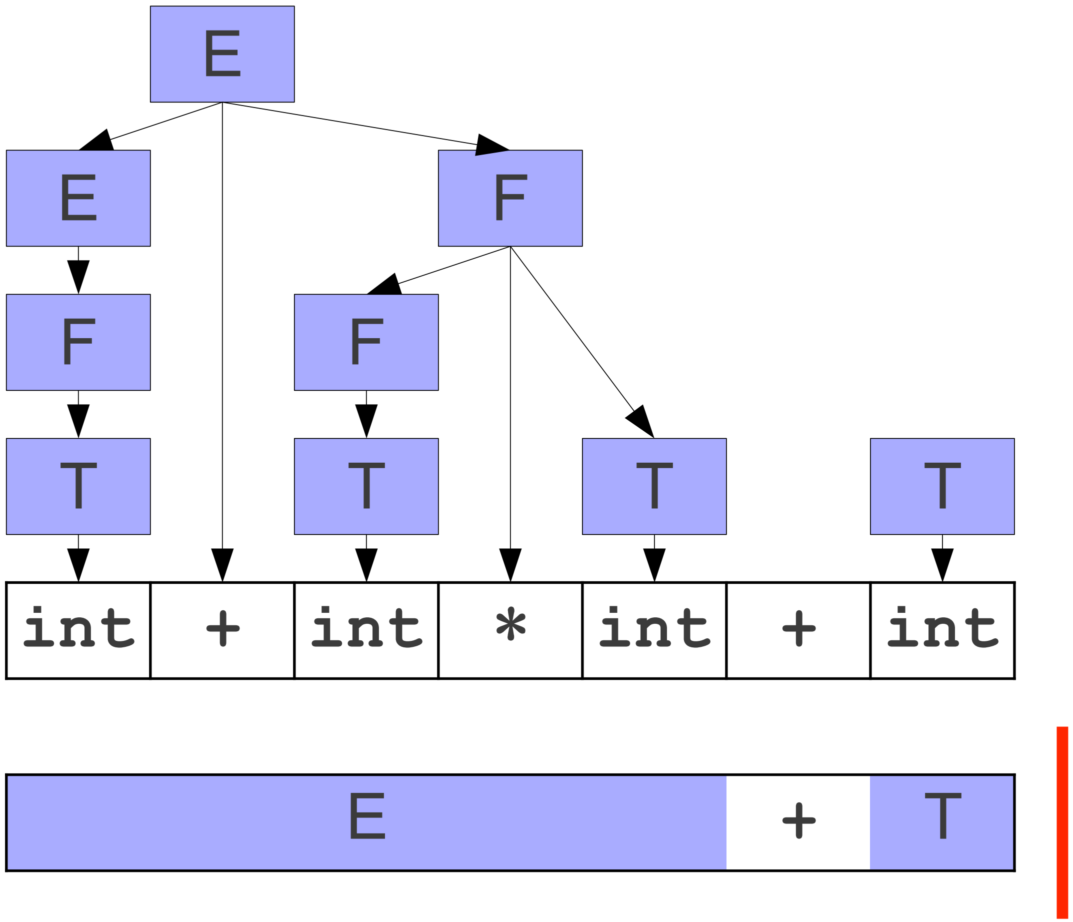
A Sample Shift/Reduce Parse

E → F
E → E + F
F → F * T
F → T
T → int
T → (E)



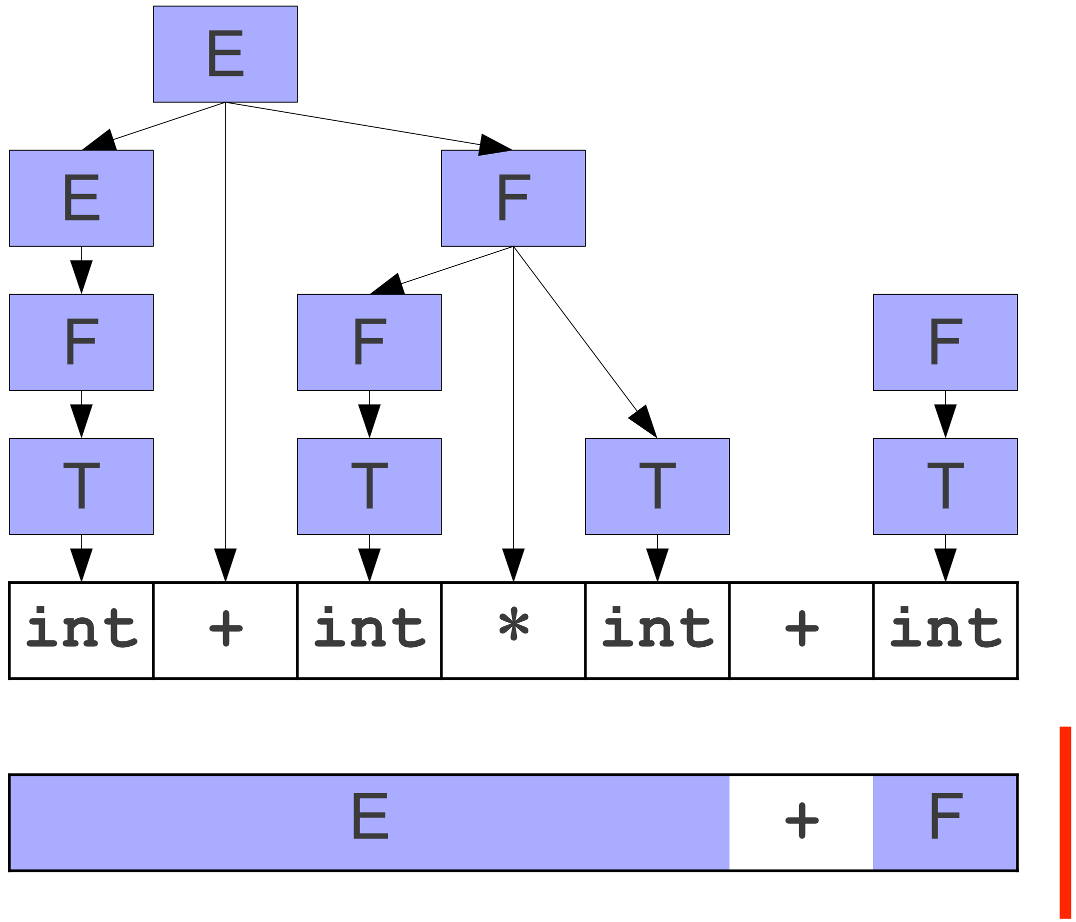
A Sample Shift/Reduce Parse

E → F
E → E + F
F → F * T
F → T
T → int
T → (E)



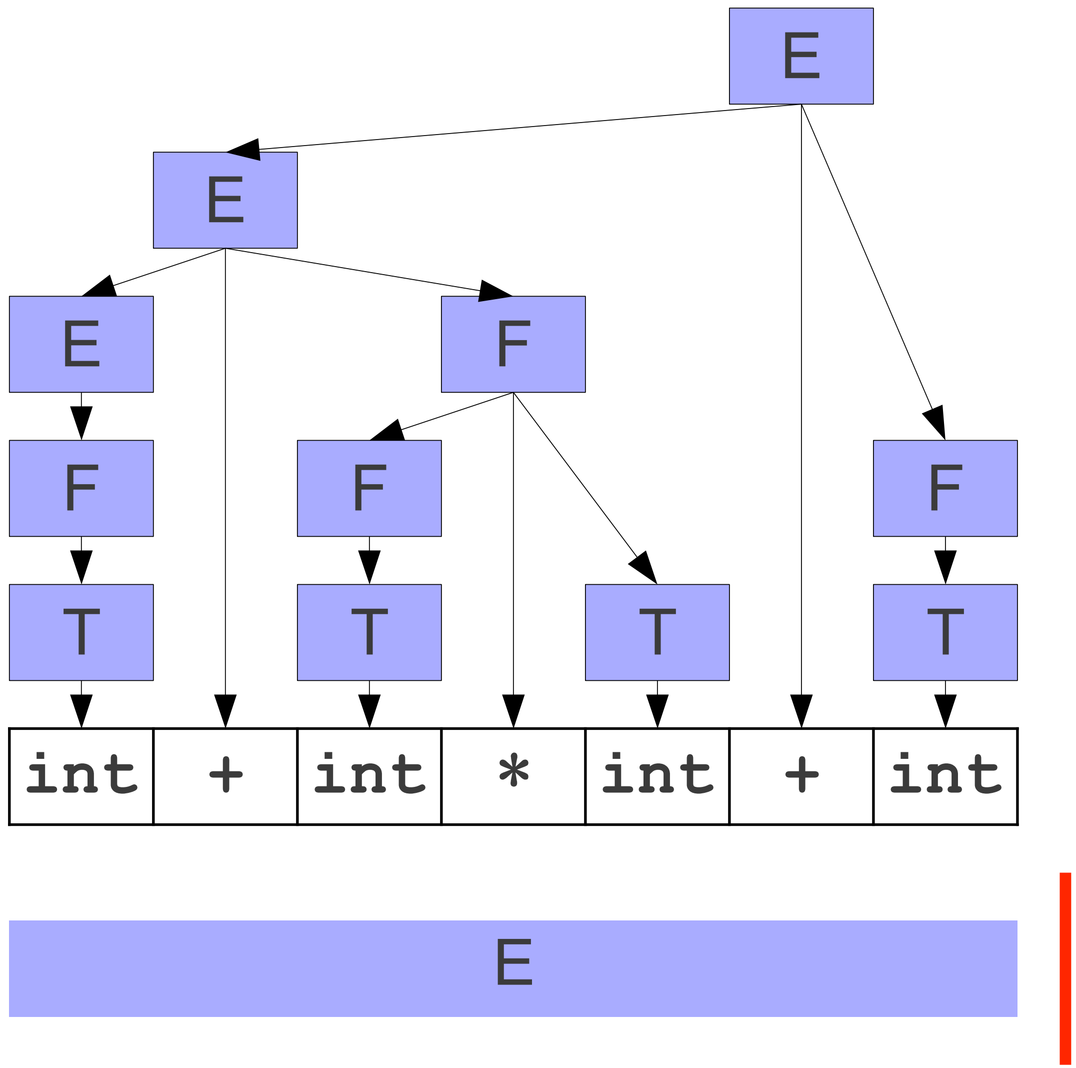
A Sample Shift/Reduce Parse

E → F
E → E + F
F → F * T
F → T
T → int
T → (E)



A Sample Shift/Reduce Parse

E → F
E → E + F
F → F * T
F → T
T → int
T → (E)



An Important Observation

- All of the reductions we applied were to the far right end of the left area.
- This is not a coincidence; all reductions are always applied all the way to the end of the left area.
- Inductive proof sketch:
  - After no reduces, the first reduction can be done at the right end of the left area.
  - After at least one reduce, the very right of the left area is a nonterminal. This nonterminal must be part of the next reduction, since we're tracing a rightmost derivation backwards.

An Important Corollary

- Since reductions are always at the right side of the left area, we never need to shift from the left to the right.
- No need to “uncover” something to do a reduction.
- Consequently, shift/reduce parsing means
  - **Shift**: Move a terminal from the right to the left area.
  - **Reduce**: Replace some number of symbols at the right side of the left area.

Simplifying our Terminology

- All activity in a shift/reduce parser is at the far right end of the left area.
- Idea: Represent the left area as a stack.
- Shift: Push the next terminal onto the stack.
- Reduce: Pop some number of symbols from the stack, then push the appropriate nonterminal.

Finding Handles

- Where do we look for handles?

  At the top of the stack.

- How do we search for handles?

  What algorithm do we use to try to discover a handle?

- How do we recognize handles?

  Once we've found a possible handle, how do we confirm that it's correct?

Question Two:

How do we search for handles?

Searching for Handles

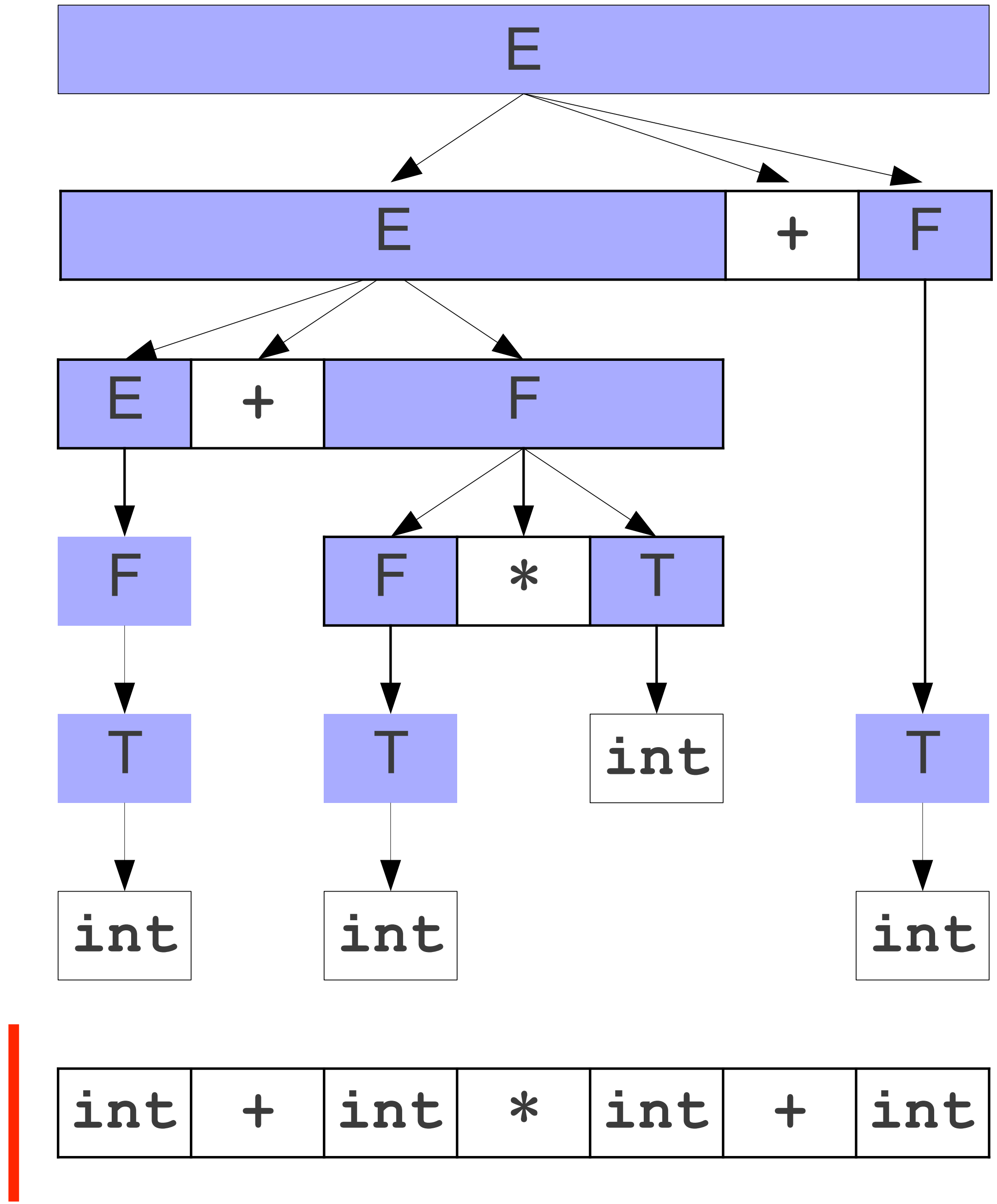
- When using a shift/reduce parser, we must decide whether to shift or reduce at each point.
- We only want to reduce when we know we have a handle.
- **Question:** How can we tell that we might be looking at a handle?

Exploring the Left Side

- The handle will always appear at the end of string in the left side of the parser.
- Can *any* string appear on the left side of the parser, or are there restrictions on what sorts of strings can appear there?
- If we can find a pattern to the strings that can appear on the left side, we might be able to exploit it to detect handles.

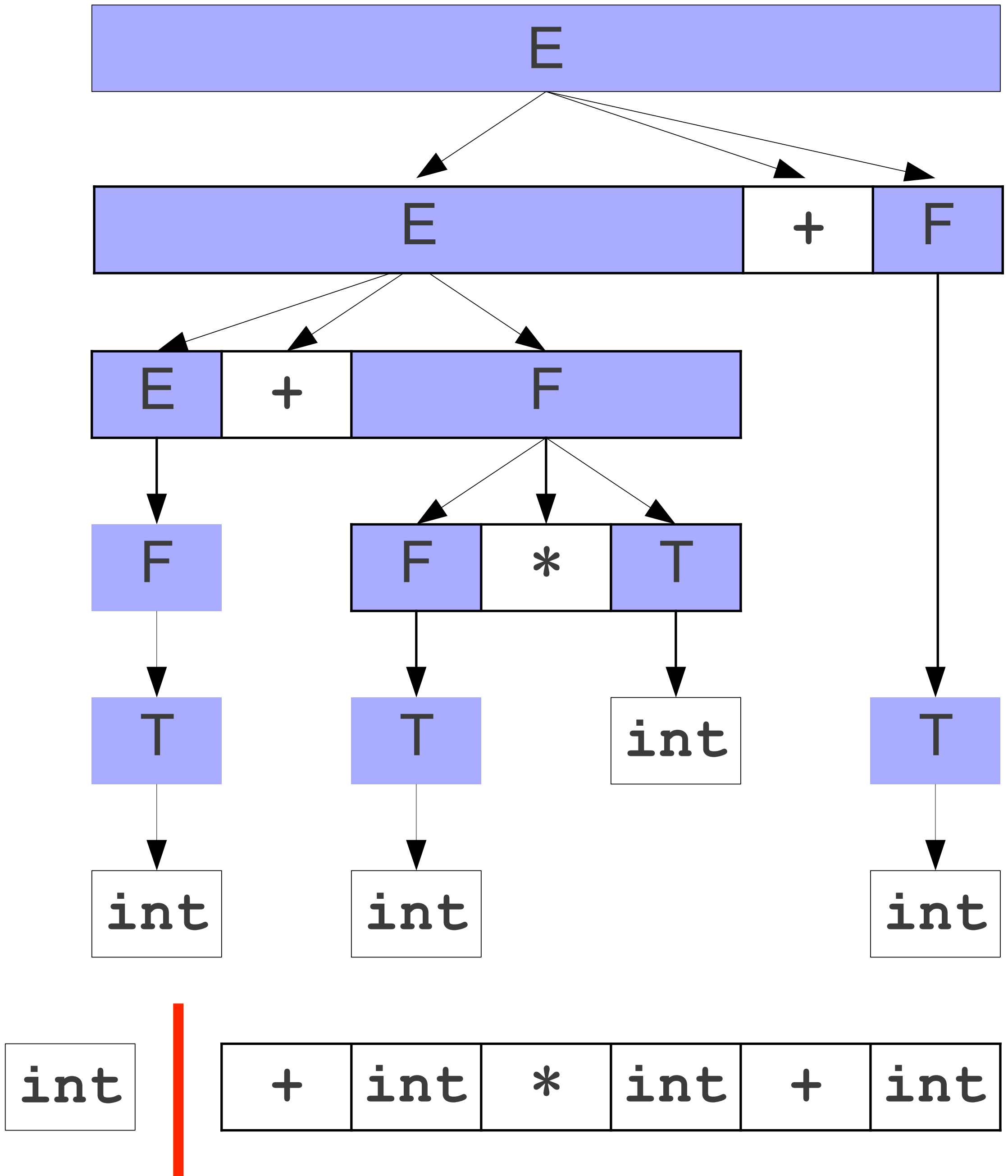
Another Look at Handles

E → F
E → E + F
F → F * T
F → T
T → int
T → (E)



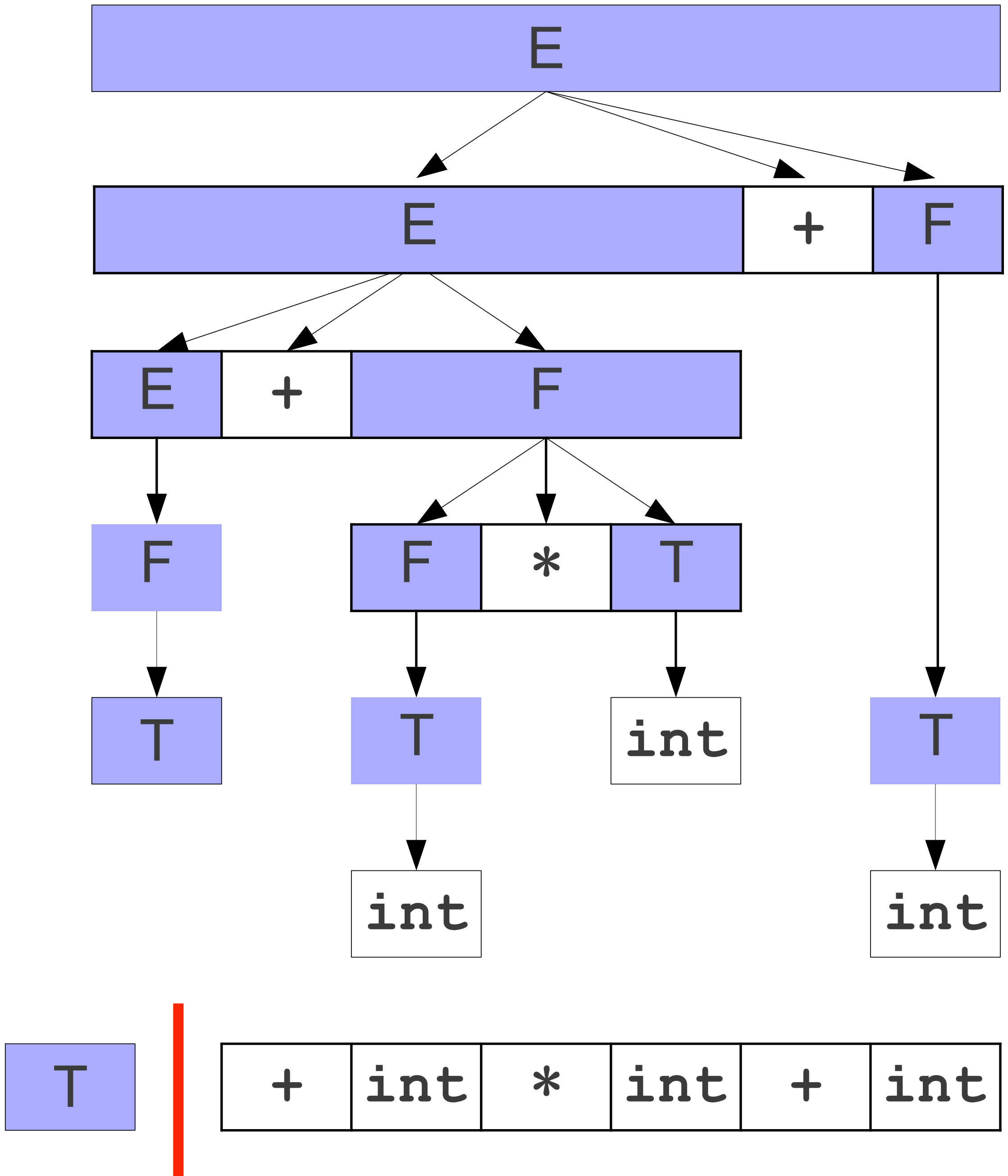
Another Look at Handles

E → F
E → E + F
F → F * T
F → T
T → int
T → (E)



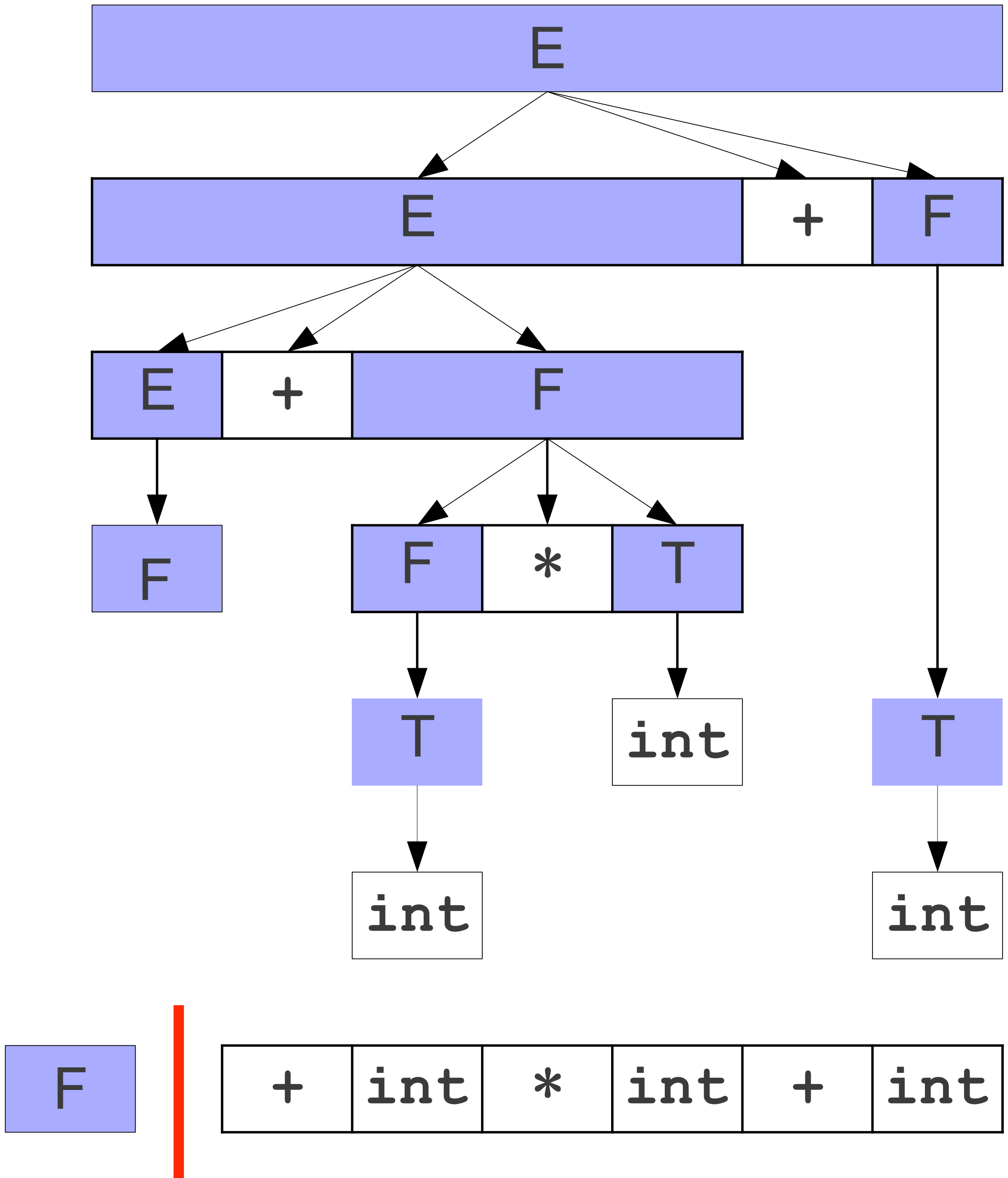
Another Look at Handles

$E \to F$
$E \to E + F$
$F \to F * T$
$F \to T$
$T \to$ `int`
$T \to$ `(E)`



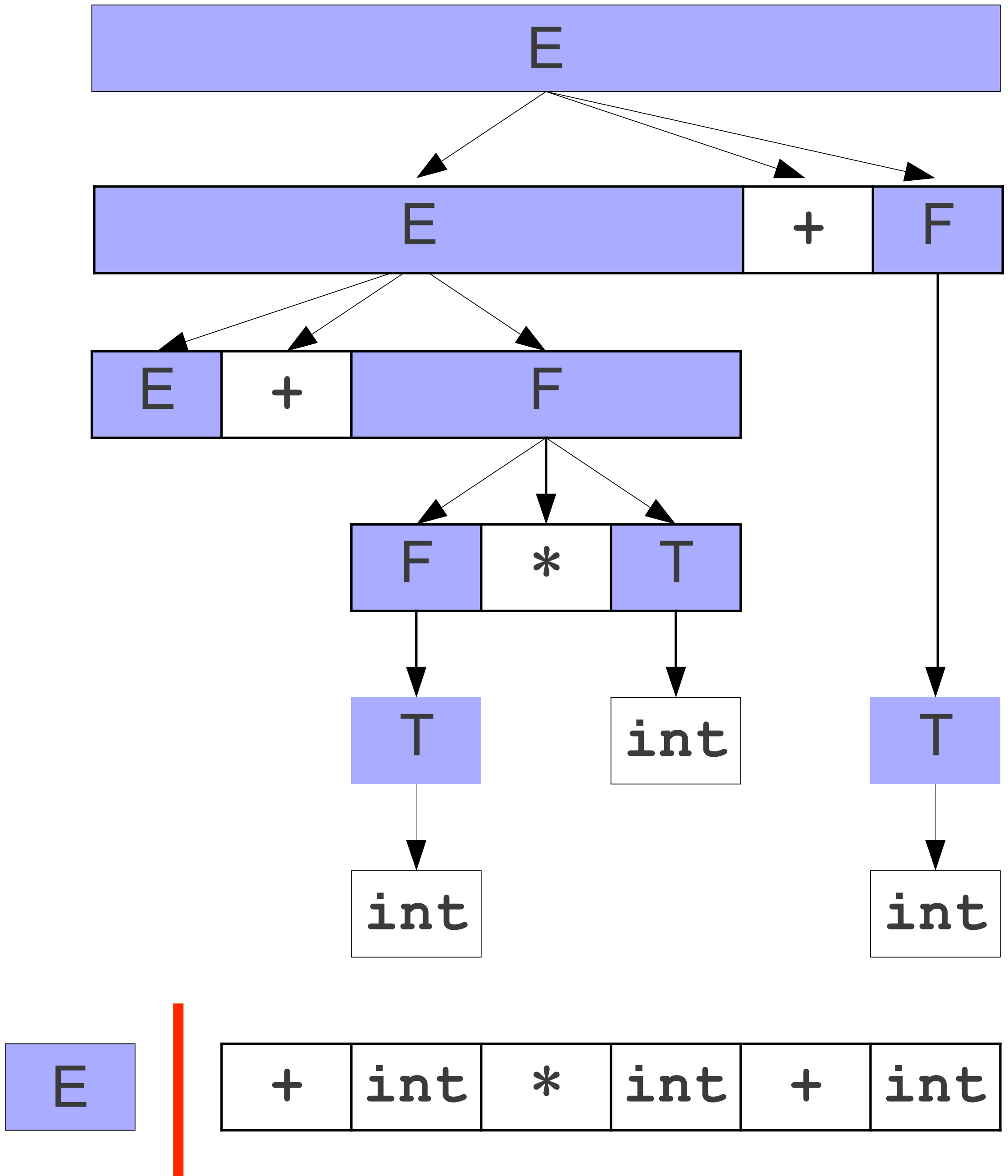
Another Look at Handles

E → F
E → E + F
F → F * T
F → T
T → int
T → (E)



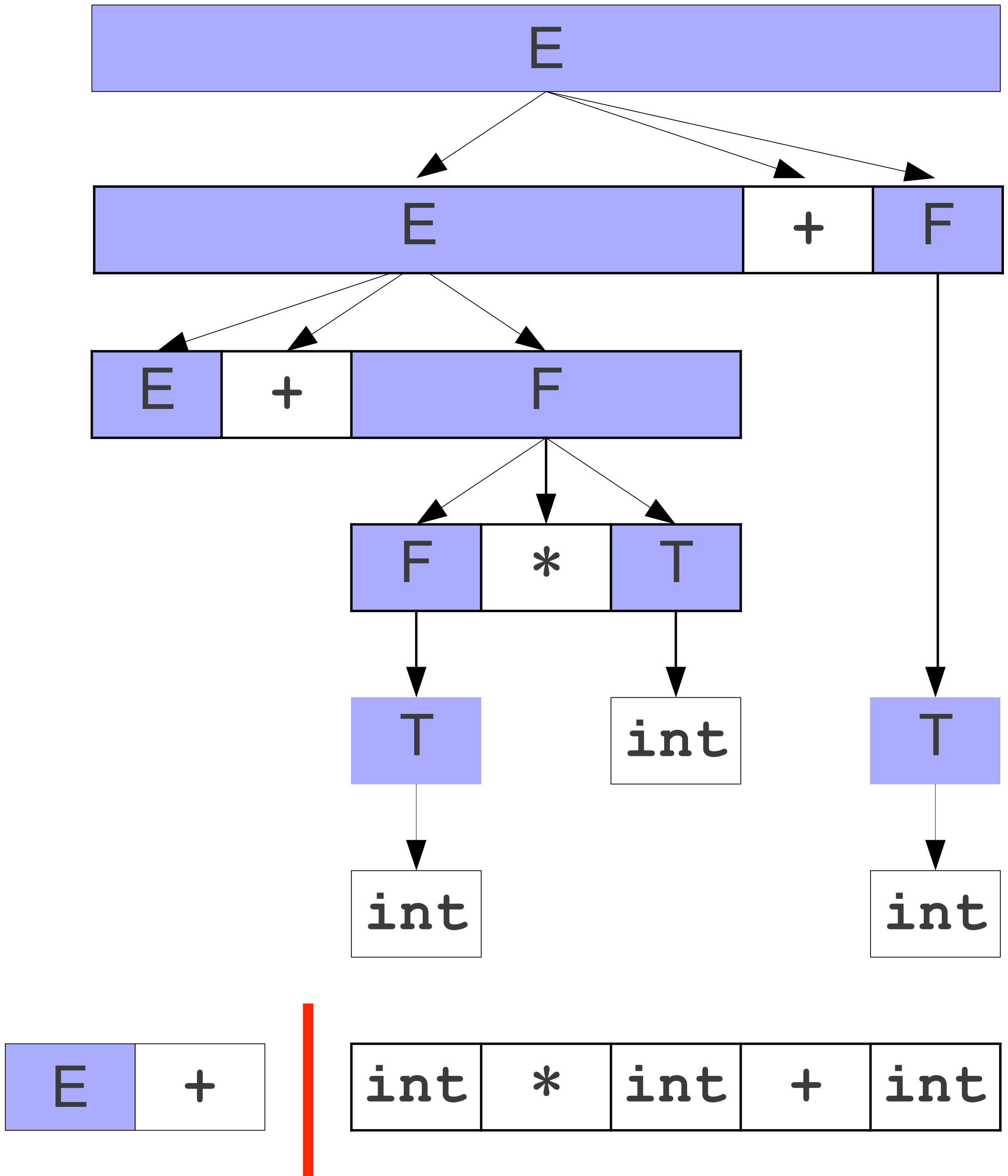
Another Look at Handles

E → F
E → E + F
F → F * T
F → T
T → int
T → (E)



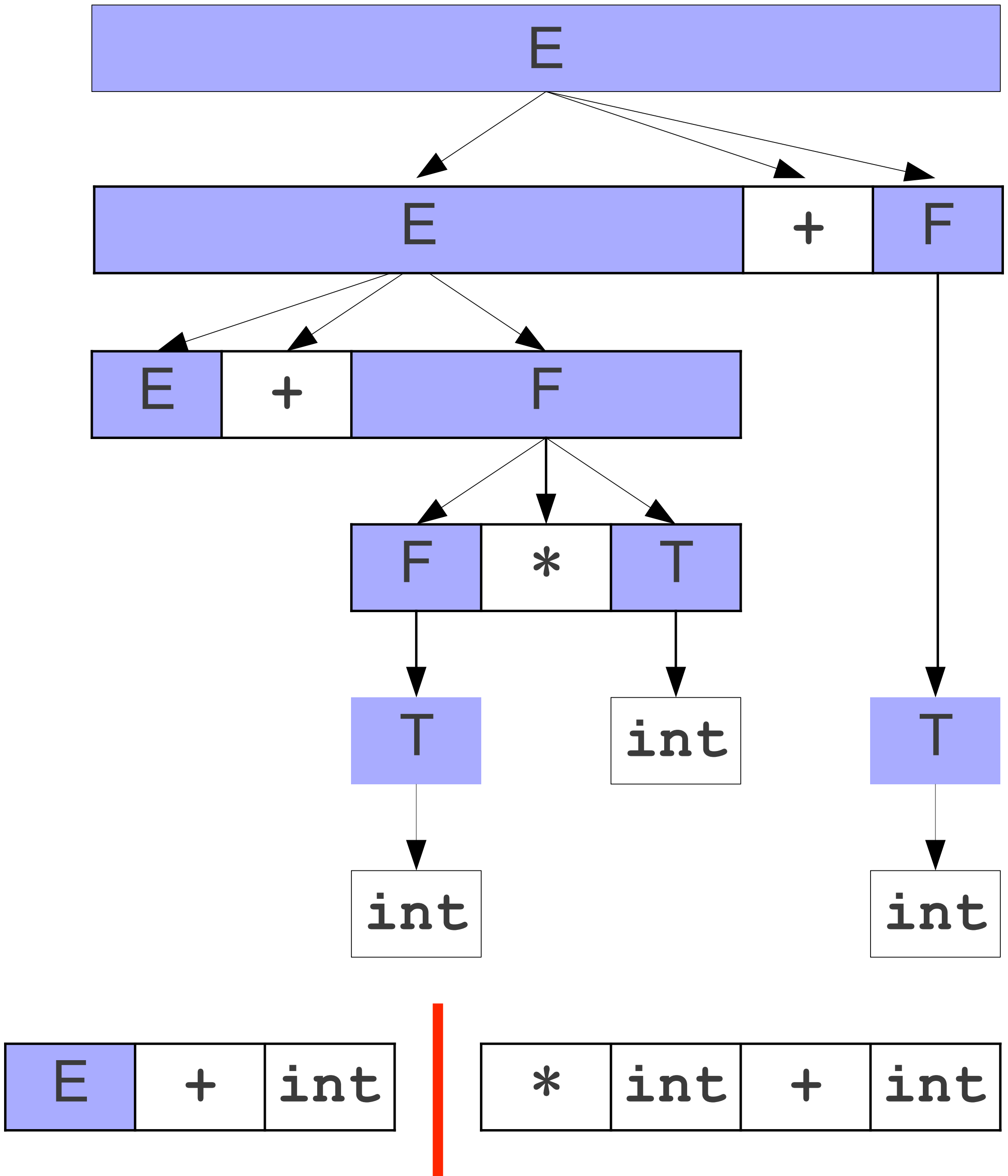
Another Look at Handles

$E \rightarrow F$
$E \rightarrow E + F$
$F \rightarrow F * T$
$F \rightarrow T$
$T \rightarrow$ int
$T \rightarrow$ (E)



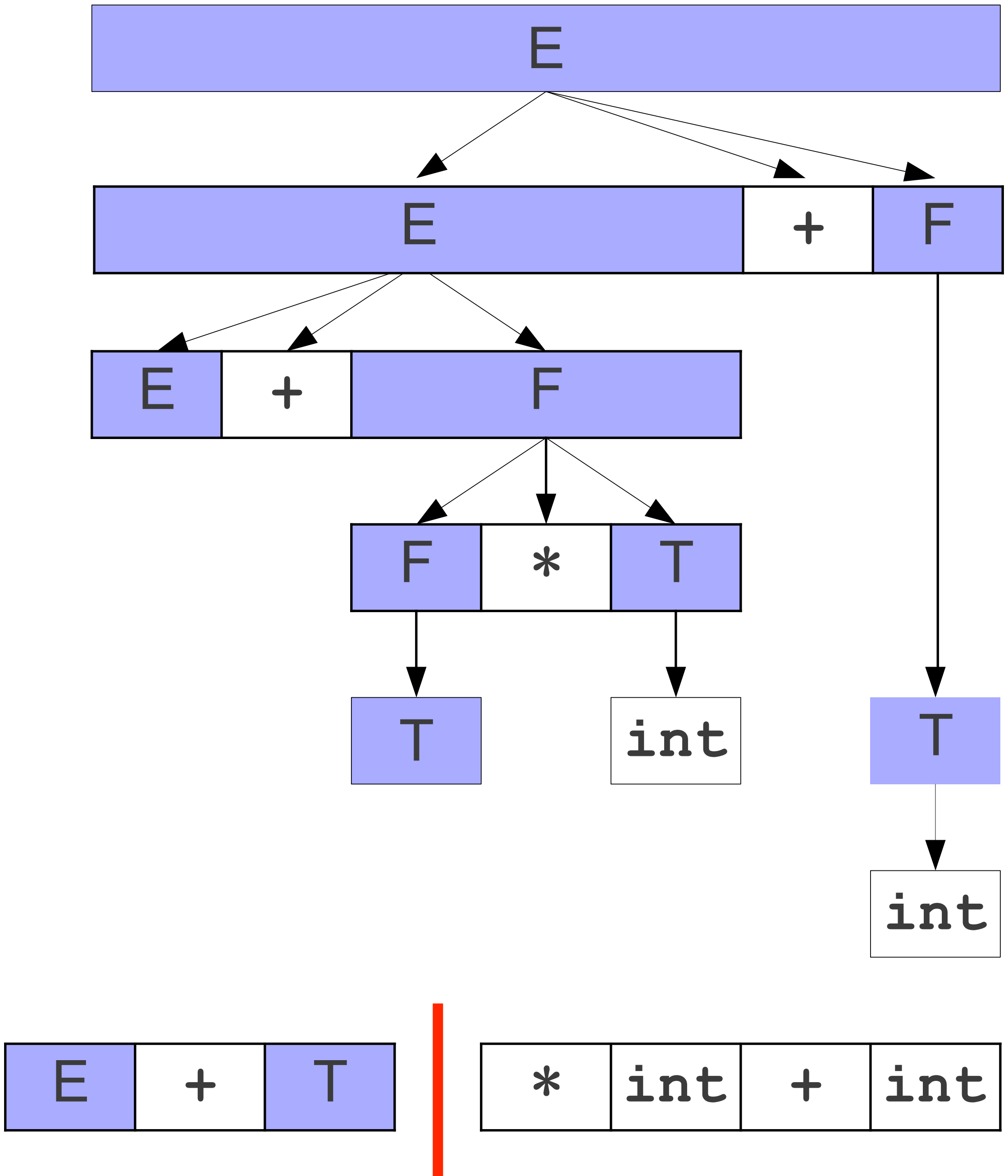
Another Look at Handles

E → F
E → E + F
F → F * T
F → T
T → int
T → (E)



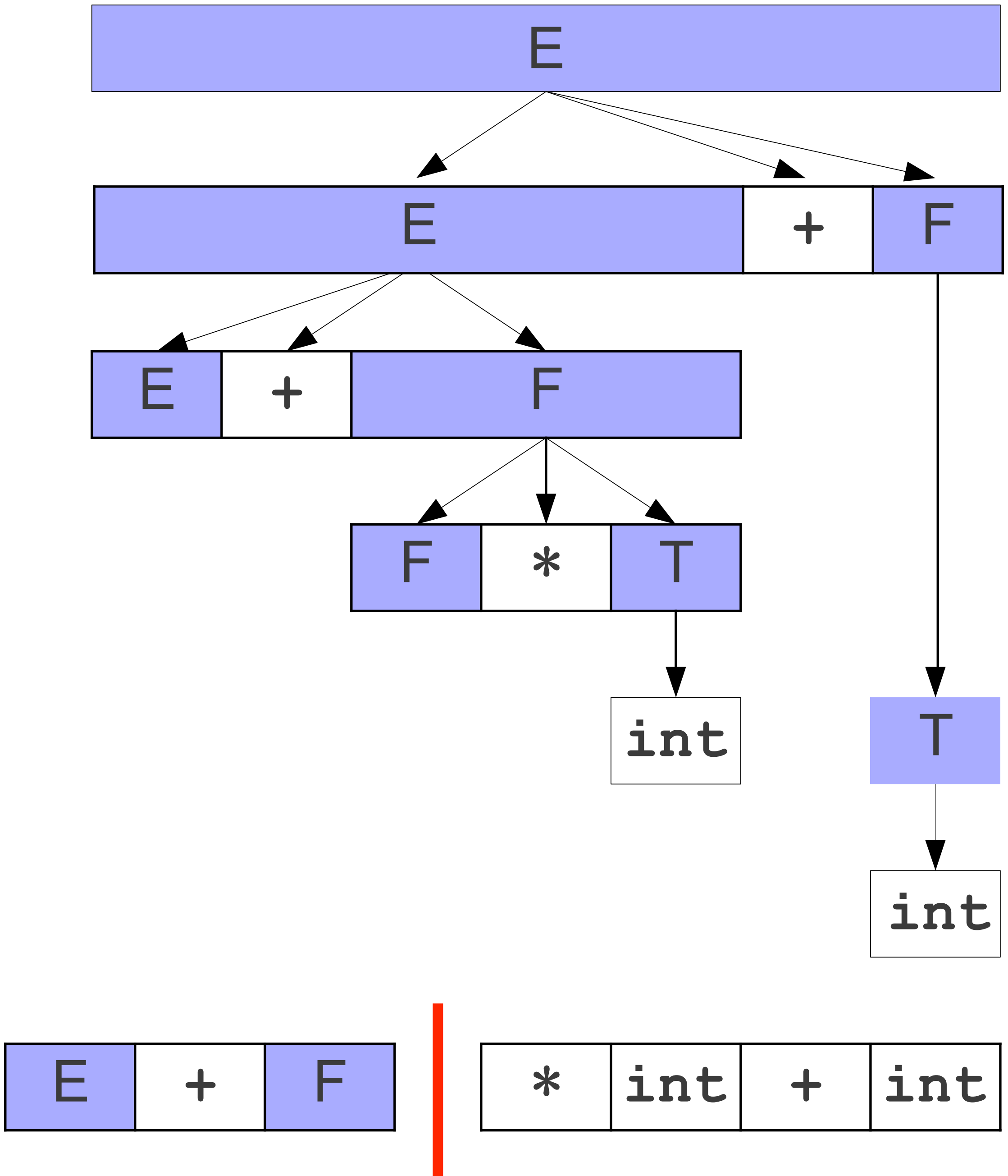
Another Look at Handles

E → F
E → E + F
F → F * T
F → T
T → int
T → (E)



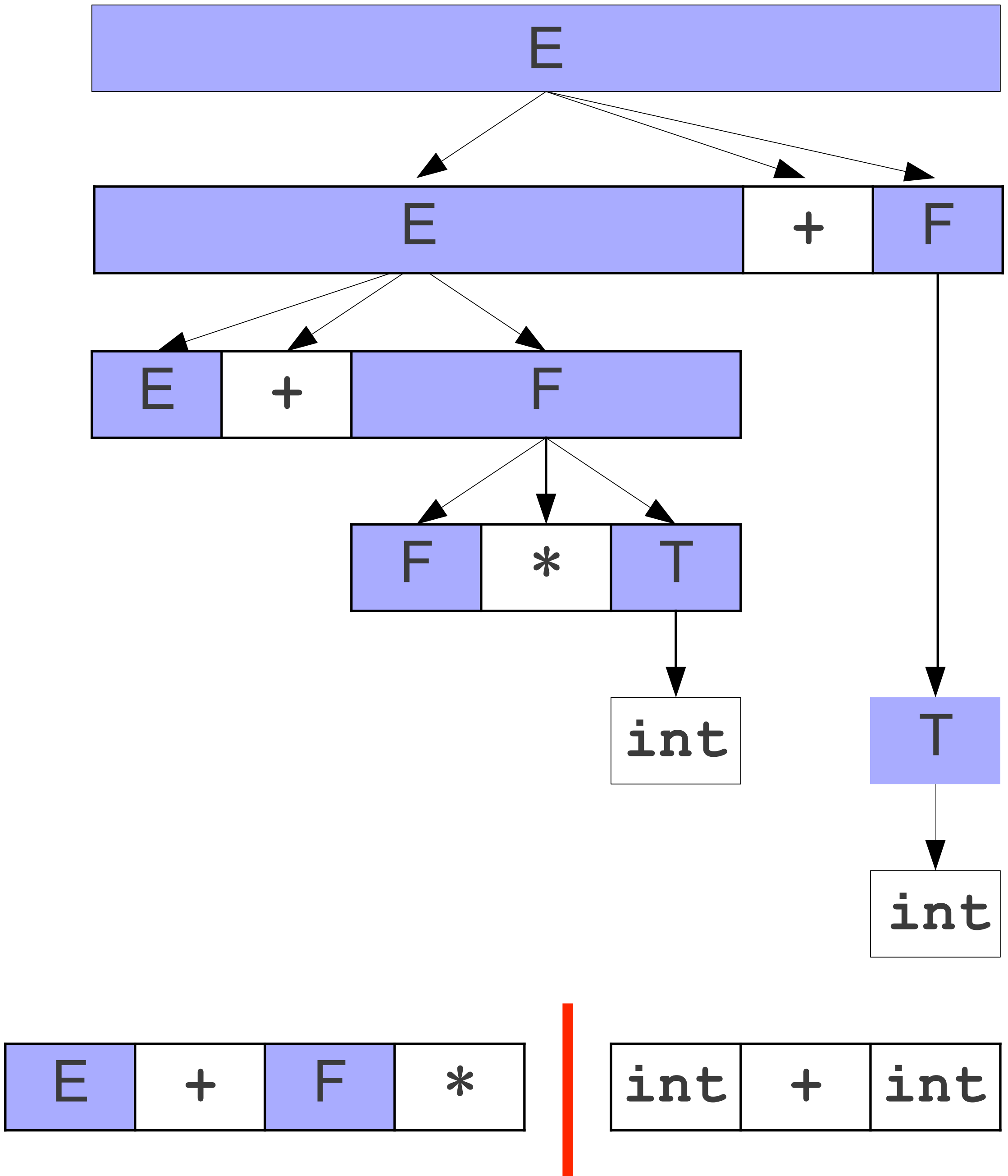
Another Look at Handles

E → F
E → E + F
F → F * T
F → T
T → int
T → (E)



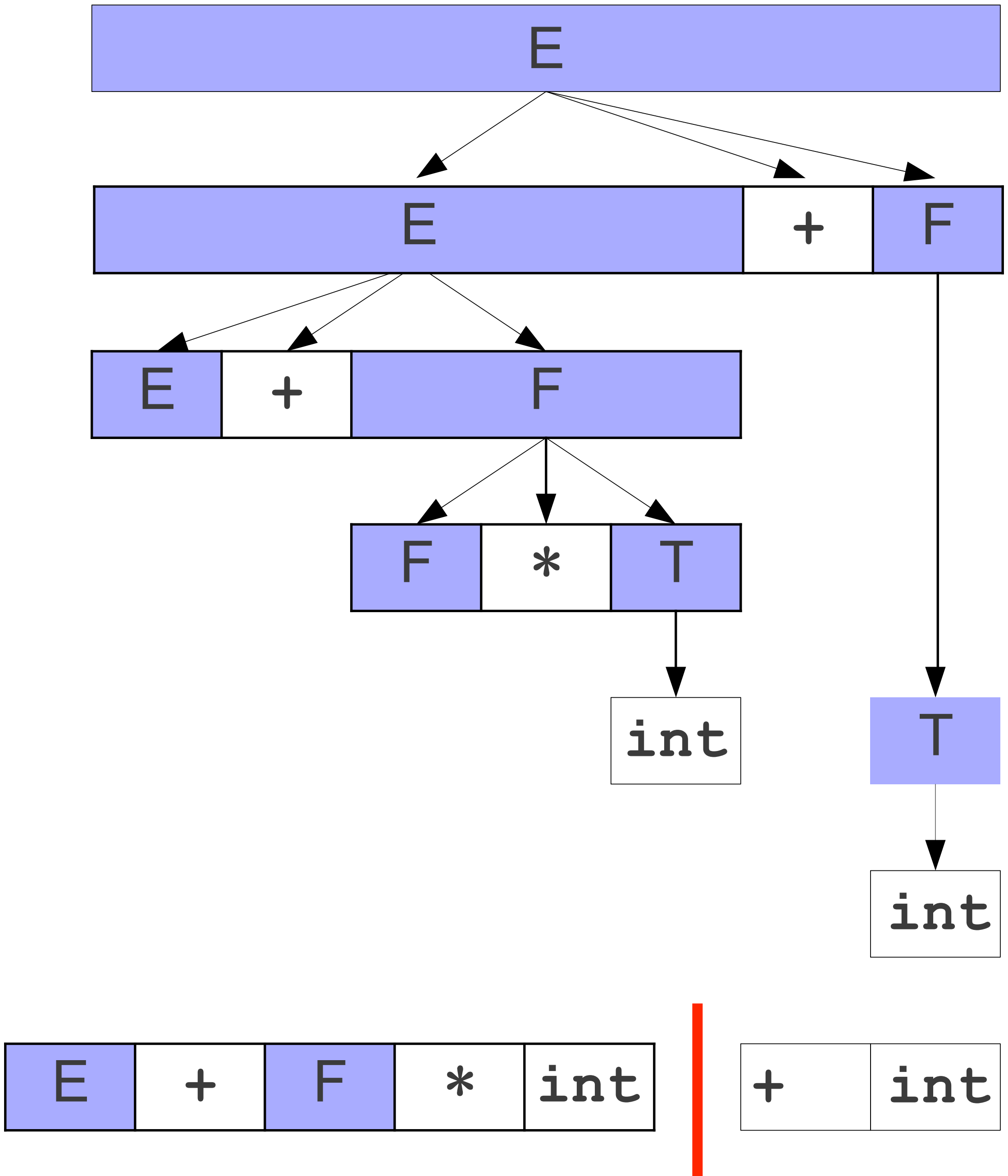
Another Look at Handles

$E \rightarrow F$
$E \rightarrow E + F$
$F \rightarrow F * T$
$F \rightarrow T$
$T \rightarrow \texttt{int}$
$T \rightarrow (E)$



Another Look at Handles

E → F
E → E + F
F → F * T
F → T
T → int
T → (E)



Another Look at Handles

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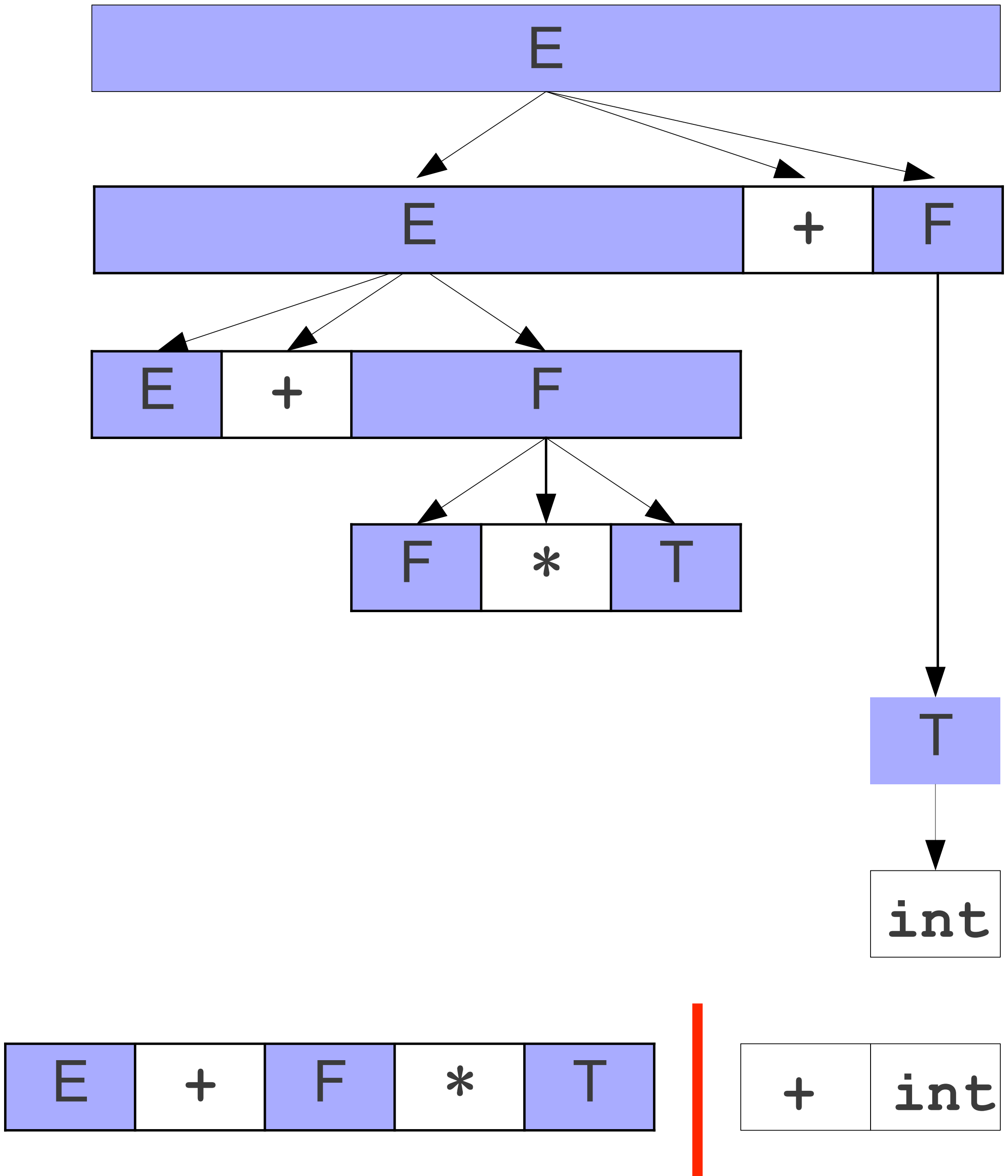
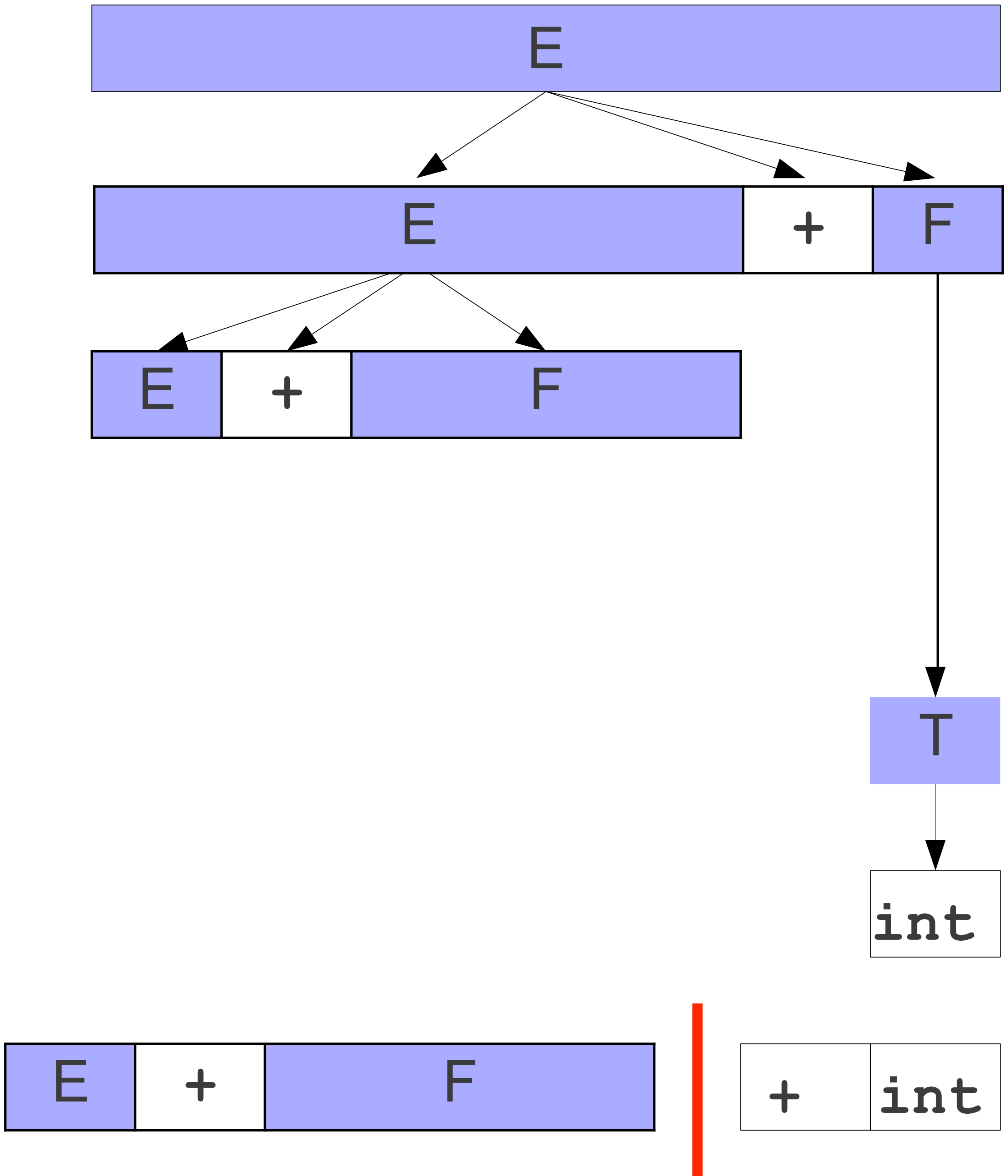
$E \rightarrow E + F$
$F \rightarrow F * T$
$F \rightarrow T$
$T \rightarrow \texttt{int}$
$T \rightarrow (E)$



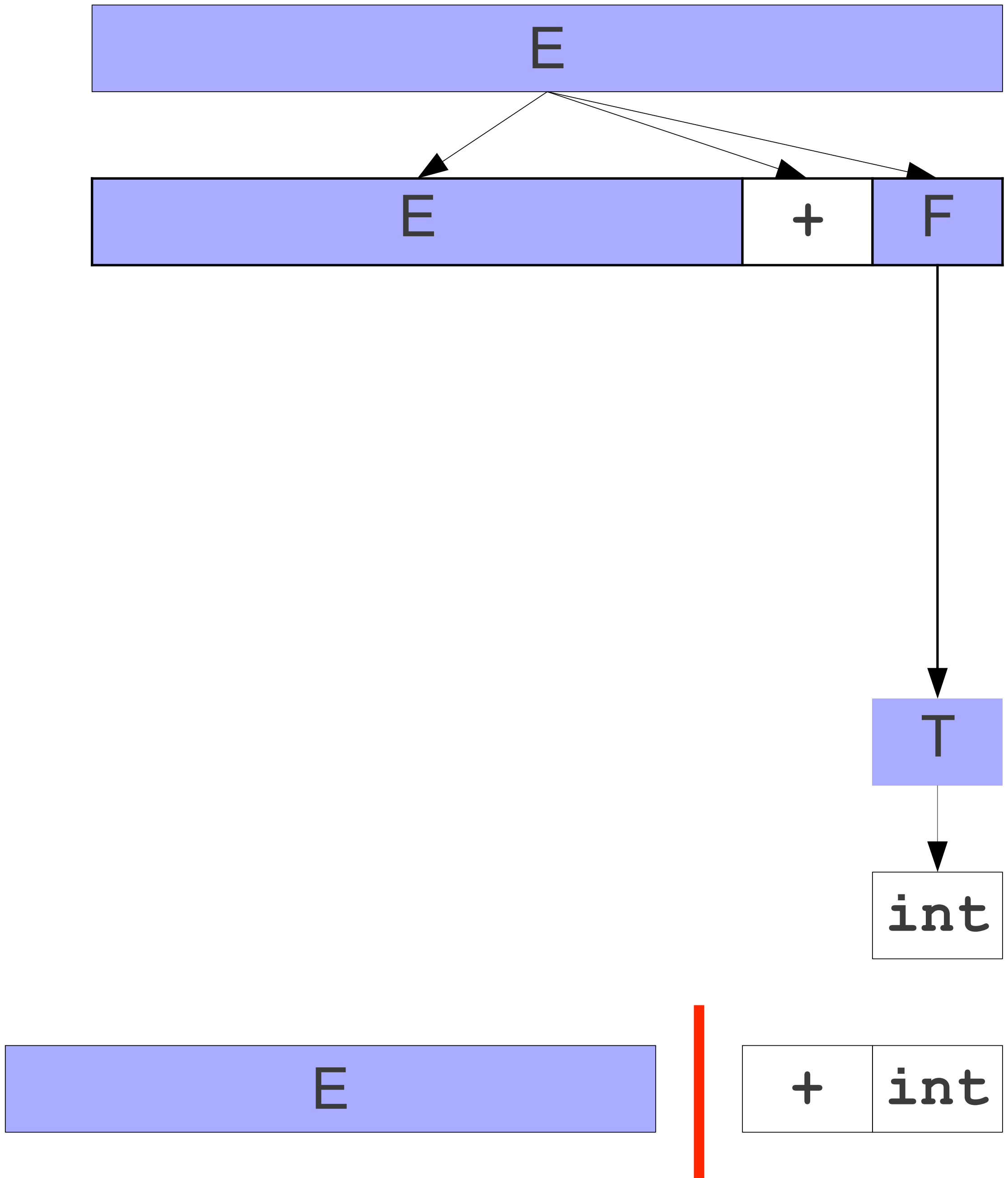
Another Look at Handles

E → F
E → E + F
F → F * T
F → T
T → int
T → (E)



Another Look at Handles

$E \to F$
$E \to E + F$
$F \to F * T$
$F \to T$
$T \to \texttt{int}$
$T \to (E)$



Another Look at Handles

$E \rightarrow F$

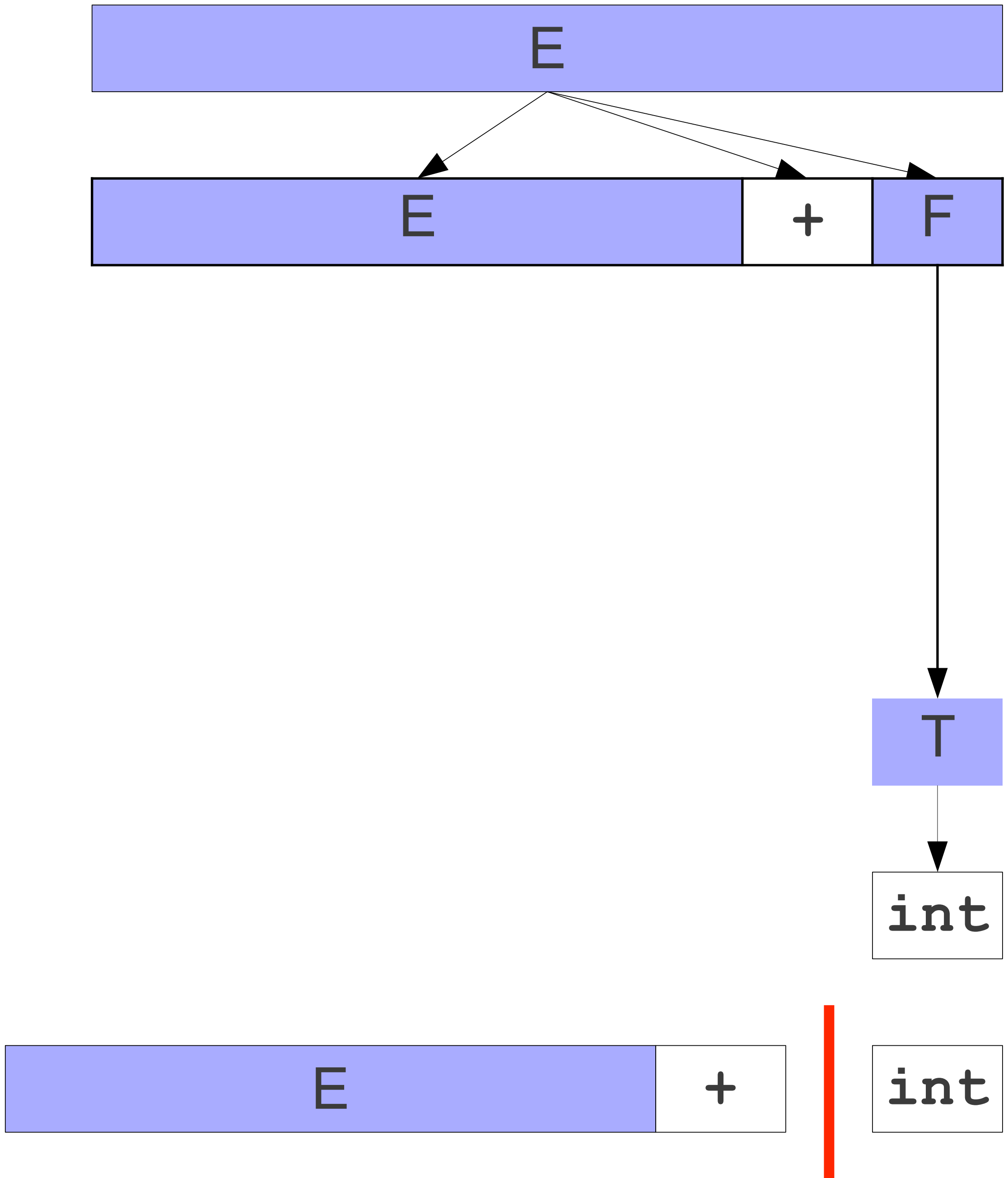
$E \rightarrow E + F$

$F \rightarrow F * T$

$F \rightarrow T$

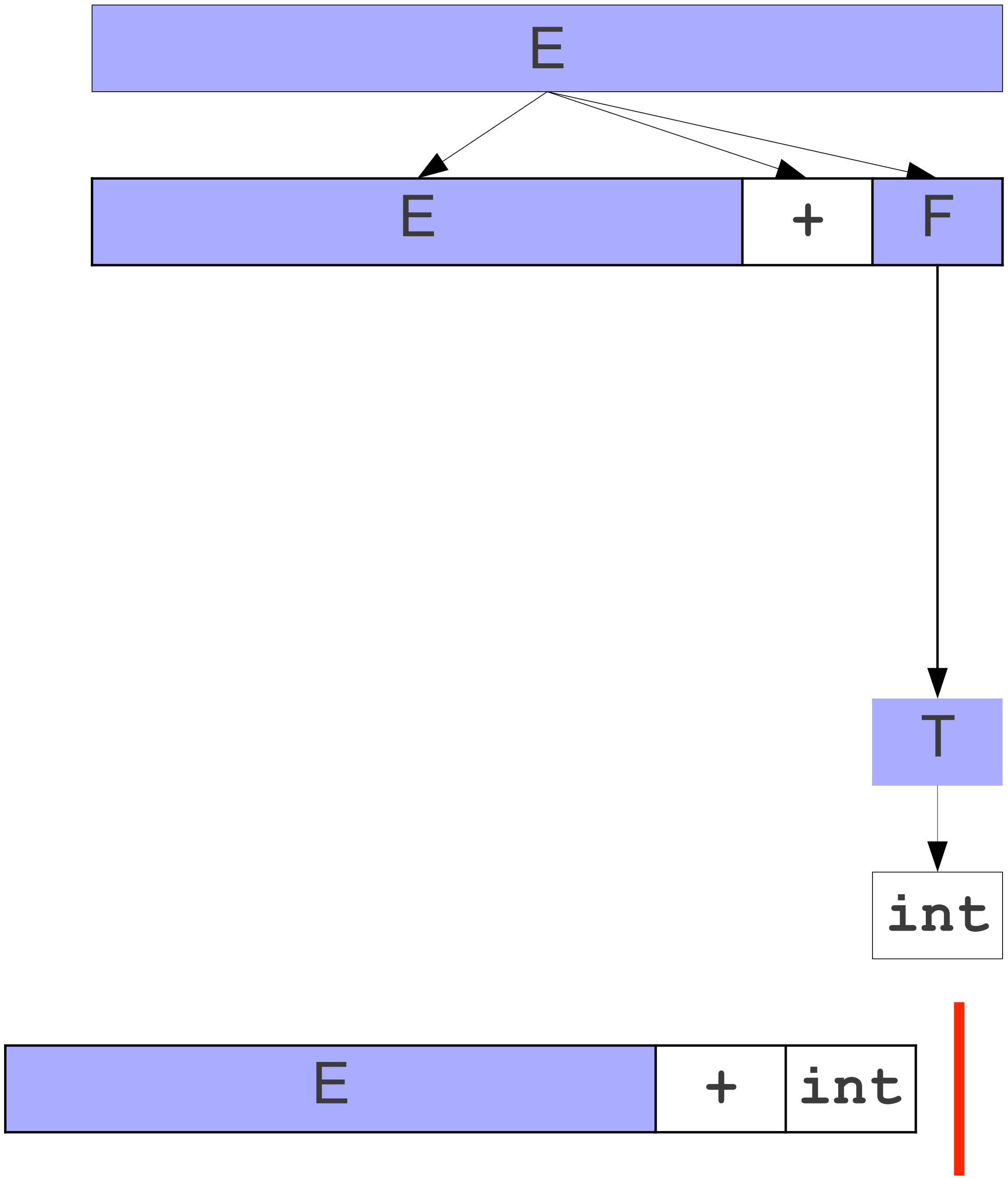
$T \rightarrow \texttt{int}$

$T \rightarrow (E)$



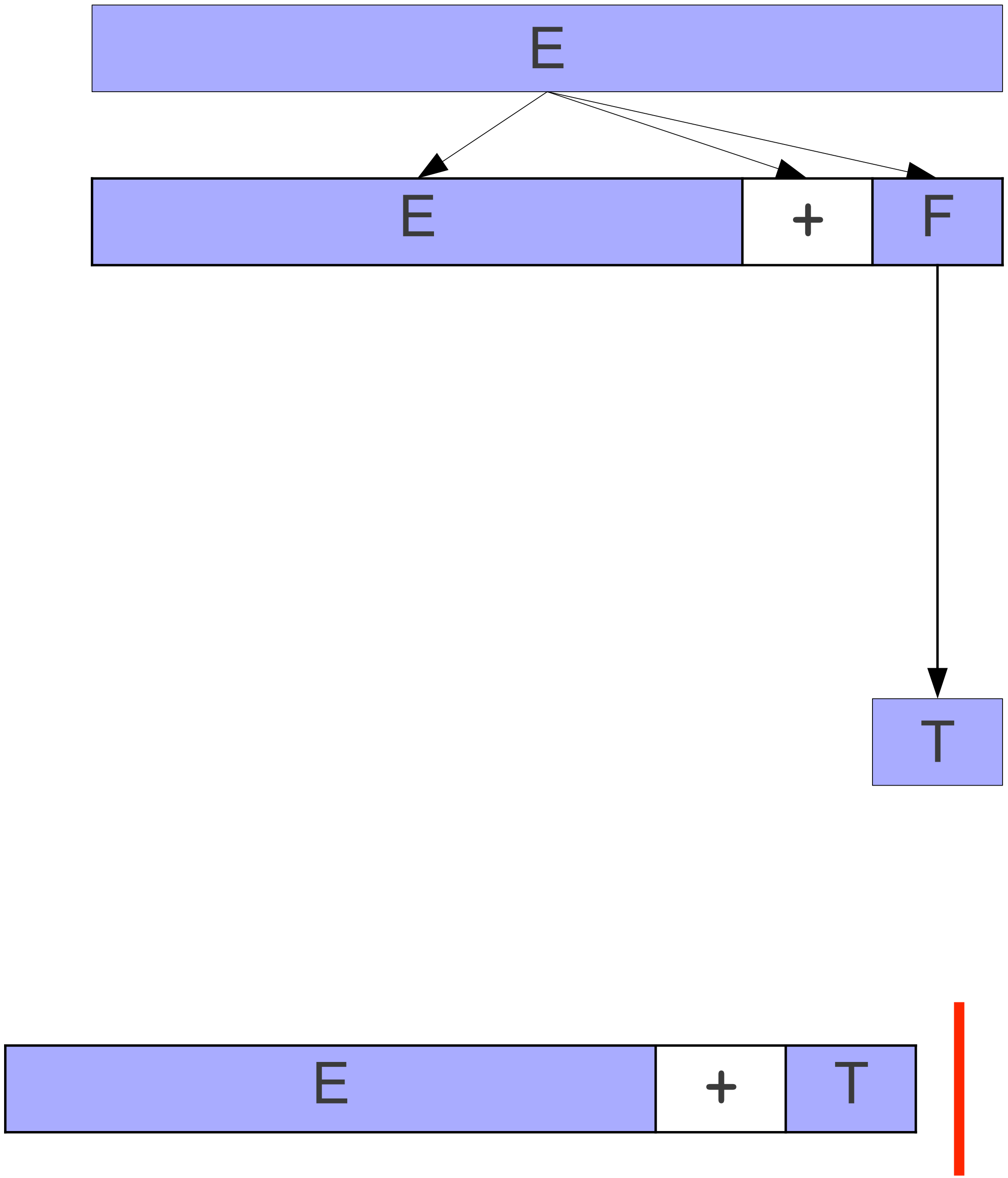
Another Look at Handles

$E \rightarrow F$
$E \rightarrow E + F$
$F \rightarrow F * T$
$F \rightarrow T$
$T \rightarrow \texttt{int}$
$T \rightarrow (E)$

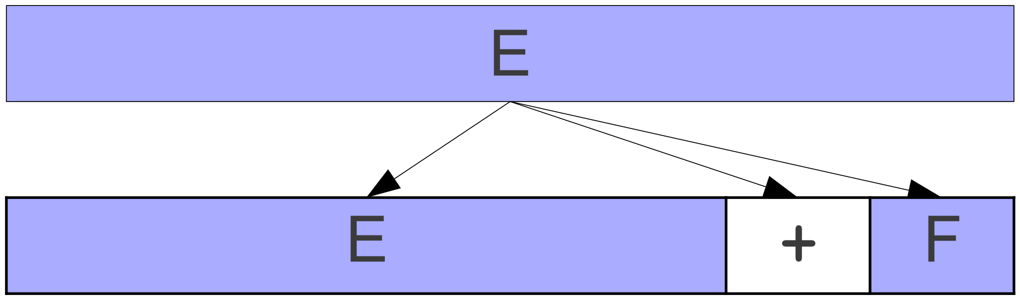


Another Look at Handles

E → F
E → E + F
F → F * T
F → T
T → int
T → (E)



Another Look at Handles



E → F

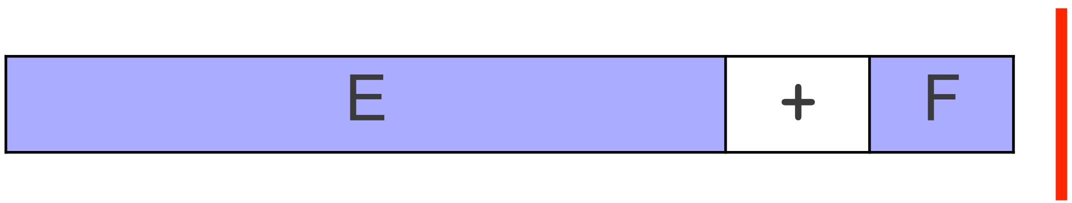
E → E + F

F → F * T

F → T

T → int

T → (E)



Another Look at Handles

E → F
E → E + F
F → F * T
F → T
T → int
T → (E)

Tracking Our Position

E → F
E → E + F
F → F * T
F → T
T → int
T → (E)

int	+	int	*	int	+	int

Tracking Our Position

$S \rightarrow E$
$E \rightarrow F$
$E \rightarrow E + F$
$F \rightarrow F * T$
$F \rightarrow T$
$T \rightarrow \texttt{int}$
$T \rightarrow (E)$

int	+	int	*	int	+	int

Tracking Our Position

$S \to .E$

$S \to E$
$E \to F$
$E \to E + F$
$F \to F * T$
$F \to T$
$T \to \texttt{int}$
$T \to (E)$

int	+	int	*	int	+	int

Tracking Our Position

S → . E
E → . E+F

S → E
E → F
E → E + F
F → F * T
F → T
T → int
T → (E)

int	+	int	*	int	+	int

Tracking Our Position

S → E
E → F
E → E + F
F → F * T
F → T
T → int
T → (E)

S → . E
E → . E+F
E → . E+F
E → . F

int	+	int	*	int	+	int

Tracking Our Position

S → E
E → F
E → E + F
F → F * T
F → T
T → int
T → (E)

S → . E
E → . E+F
E → . E+F
E → . F
F → . T

int	+	int	*	int	+	int

Tracking Our Position

S → E
E → F
E → E + F
F → F * T
F → T
T → int
T → (E)

S → . E
E → . E+F
E → . E+F
E → . F
F → . T
T → . int

int	+	int	*	int	+	int

Tracking Our Position

$S \rightarrow E$
$E \rightarrow F$
$E \rightarrow E$ **+** F
$F \rightarrow F$ ***** T
$F \rightarrow T$
$T \rightarrow$ **int**
$T \rightarrow$ **(**E**)**

S → . E
E → . E+F
E → . E+F
E → . F
F → . T
T → int.

int	|	+	int	*	int	+	int

Tracking Our Position

S → E
E → F
E → E + F
F → F * T
F → T
T → int
T → (E)

S → . E
E → . E+F
E → . E+F
E → . F
F → . T

| T |

+	int	*	int	+	int

Tracking Our Position

S → E
E → F
E → E + F
F → F * T F
→ T
T → int
T → (E)

S → . E
E → . E+F
E → . E+F
E → . F
F → T.

T	+	int	*	int	+	int

Tracking Our Position

S → . E
E → . E+F
E → . E+F
E → . F

$S \rightarrow E$
$E \rightarrow F$
$E \rightarrow E + F$
$F \rightarrow F * T$
$F \rightarrow T$
$T \rightarrow$ `int`
$T \rightarrow$ `(E)`

F	|	+	int	*	int	+	int

Tracking Our Position

$S \rightarrow E$
$E \rightarrow F$
$E \rightarrow E + F$
$F \rightarrow F * T$
$F \rightarrow T$
$T \rightarrow \texttt{int}$
$T \rightarrow (E)$

S → . E
E → . E+F
E → . E+F
E → F.

F	|	+	int	*	int	+	int

Tracking Our Position

$S \to E$
$E \to F$
$E \to E + F$
$F \to F * T$
$F \to T$
$T \to$ int
$T \to$ (E)

S → . E
E → . E+F
E → . E+F

E		+	int	*	int	+	int

Tracking Our Position

S → E
E → F
E → E + F
F → F * T
F → T
T → int
T → (E)

S → . E
E → . E+F
E → E. +F

E		+	int	*	int	+	int

Tracking Our Position

S → E
E → F
E → E + F
F → F * T
F → T
T → int
T → (E)

S → . E
E → . E+F
E → E + . F

E	+	|	int	*	int	+	int

Tracking Our Position

S → E
E → F
E → E + F
F → F * T
F → T
T → int
T → (E)

S → . E
E → . E+F
E → E + . F
F → . F *T

E	+	|	int	*	int	+	int

Tracking Our Position

$S \to E$
$E \to F$
$E \to E + F$
$F \to F * T$
$F \to T$
$T \to$ `int`
$T \to$ `(E)`

S → . E
E → . E+F
E → E + . F
F → . F *T
F → . T

E	+

int	*	int	+	int

Tracking Our Position

$S \rightarrow E$
$E \rightarrow F$
$E \rightarrow E + F$
$F \rightarrow F * T$
$F \rightarrow T$
$T \rightarrow$ int
$T \rightarrow (E)$

S → . E
E → . E+F
E → E + . F
F → . F *T
F → . T
T → . int

E	+	|	int	*	int	+	int

Tracking Our Position

S → E
E → F
E → E + F
F → F * T
F → T
T → int
T → (E)

S → . E
E → . E+F
E → E + . F
F → . F *T
F → . T
T → int.

E	+	int	|	*	int	+	int

Tracking Our Position

S → E
E → F
E → E + F
F → F * T
F → T
T → int
T → (E)

S → . E
E → . E+F
E → E + . F
F → . F *T
F → . T

E	+	T	|	*	int	+	int

Tracking Our Position

$S \to E$
$E \to F$
$E \to E$ + F
$F \to F$ * T
$F \to T$
$T \to$ int
$T \to$ (E)

S → . E
E → . E+F
E → E + . F
F → . F *T
F → T.

E	+	T	|	*	int	+	int

Tracking Our Position

S → E
E → F
E → E + F
F → F * T
F → T
T → int
T → (E)

S → . E
E → . E+F
E → E + . F
F → . F *T

E	+	F	|	*	int	+	int

Tracking Our Position

S → E
E → F
E → E + F
F → F * T
F → T
T → int
T → (E)

S → . E
E → . E+F
E → E + . F
F → F. *T

E	+	F			*	int	+	int

Tracking Our Position

S → E
E → F
E → E + F
F → F * T
F → T
T → int
T → (E)

S → . E
E → . E+F
E → E + . F
F → F* . T

E	+	F	*	|	int	+	int

Tracking Our Position

$S \to E$
$E \to F$
$E \to E + F$
$F \to F * T$
$F \to T$
$T \to \texttt{int}$
$T \to (E)$

S → . E
E → . E+F
E → E + . F
F → F* . T
T → .int

E	+	F	*	|	int	+	int

Tracking Our Position

S → E
E → F
E → E + F
F → F * T
F → T
T → int
T → (E)

S → . E
E → . E+F
E → E + . F
F → F* . T
T → int .

E	+	F	*	int	|	+	int

Tracking Our Position

$S \to E$
$E \to F$
$E \to E$ **+** F
$F \to F$ ***** T
$F \to T$
$T \to$ `int`
$T \to$ `(`E`)`

S → . E
E → . E+F
E → E + . F
F → F* . T

E	+	F	*	T	|	+	int

Tracking Our Position

$S \rightarrow E$
$E \rightarrow F$
$E \rightarrow E$ **+** F
$F \rightarrow F$ ***** T
$F \rightarrow T$
$T \rightarrow$ `int`
$T \rightarrow$ **(**E**)**

S → . E
E → . E+F
E → E + . F
F → F* T.

E	+	F	*	T	|	+	int

Tracking Our Position

S → E
E → F
E → E + F
F → F * T
F → T
T → `int`
T → (E)

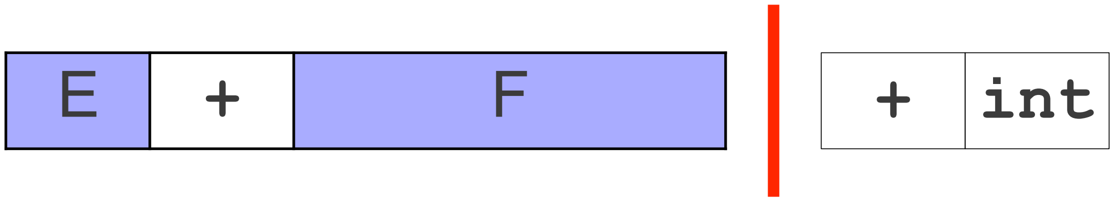
S → . E
E → . E+F
E → E + . F
F → F* T.

E	+	F	|	+	`int`

Tracking Our Position

$S \rightarrow E$
$E \rightarrow F$
$E \rightarrow E + F$
$F \rightarrow F * T$
$F \rightarrow T$
$T \rightarrow \texttt{int}$
$T \rightarrow (E)$

S → . E
E → . E+F
E → E + F.



Tracking Our Position

S → E
E → F
E → E + F
F → F * T
F → T
T → int
T → (E)

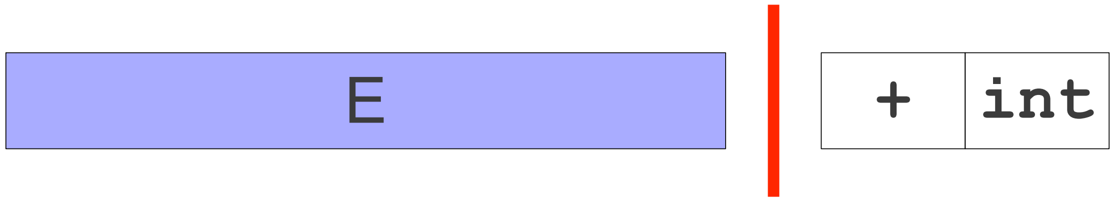
S → . E
E → . E+F

E	|	+	int

Tracking Our Position

S → E
E → F
E → E + F
F → F * T
F → T
T → int
T → (E)

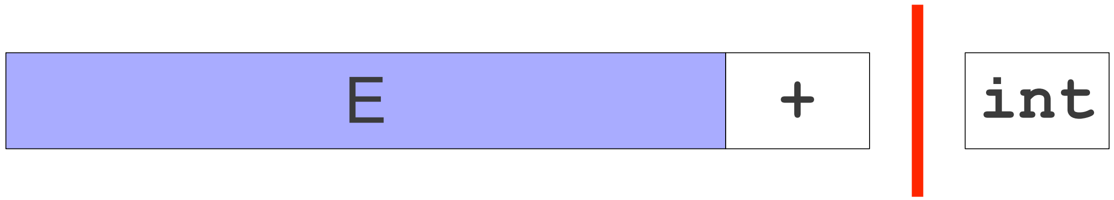
S → . E
E → E. +F



Tracking Our Position

S → . E
E → E+. F

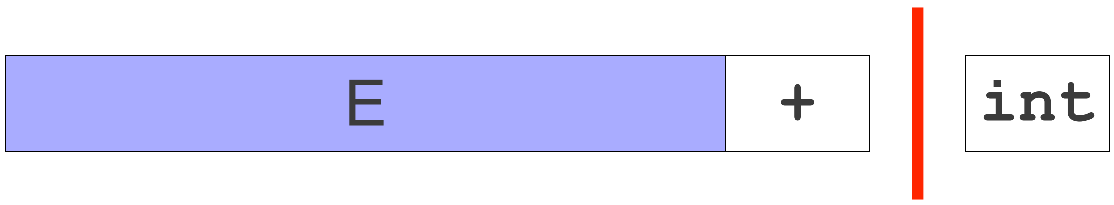
S → E
E → F
E → E + F
F → F * T
F → T
T → int
T → (E)



Tracking Our Position

$S \rightarrow E$
$E \rightarrow F$
$E \rightarrow E + F$
$F \rightarrow F * T$
$F \rightarrow T$
$T \rightarrow \texttt{int}$
$T \rightarrow (E)$

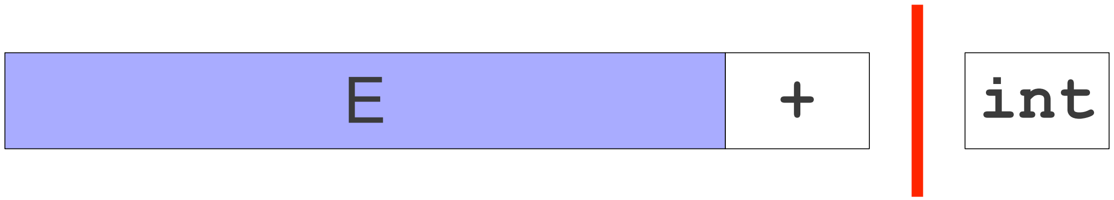
S → . E
E → E+. F
F →. T



Tracking Our Position

S → . E
E → E+. F
F → . T
T → . int

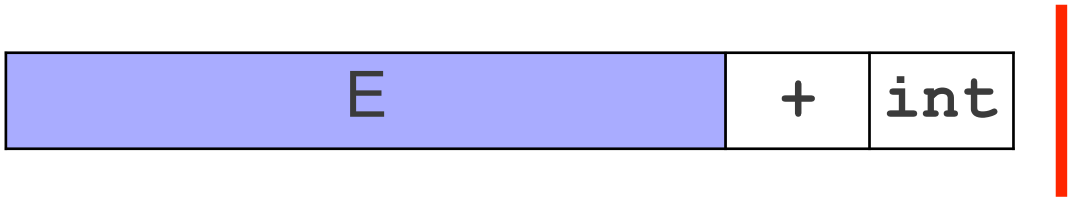
$S \rightarrow E$
$E \rightarrow F$
$E \rightarrow E$ **+** F
$F \rightarrow F$ ***** T
$F \rightarrow T$
$T \rightarrow$ **int**
$T \rightarrow$ **(**E**)**



Tracking Our Position

S → . E
E → E+. F
F → . T
T → int.

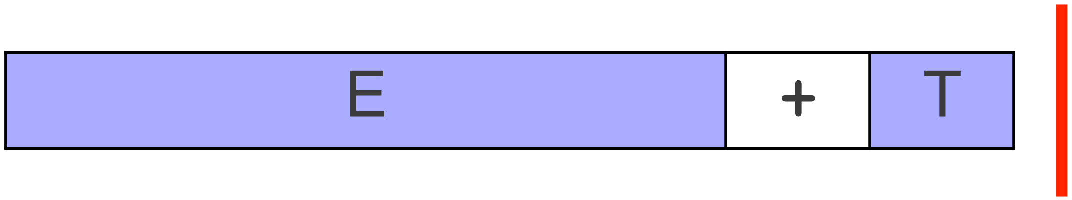
S → E
E → F
E → E + F
F → F * T F
→ T
T → int
T → (E)



Tracking Our Position

S → E
E → F
E → E + F
F → F * T
F → T
T → int
T → (E)

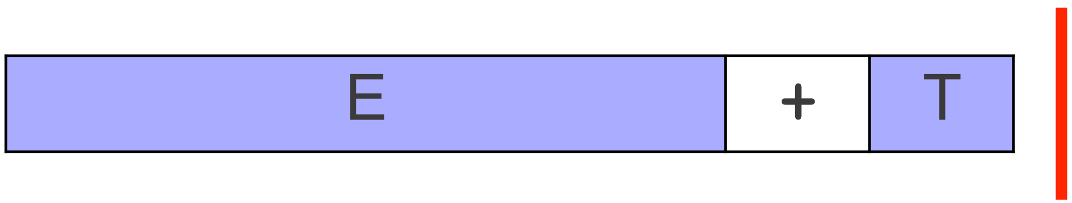
S → . E
E → E+. F
F →. T



Tracking Our Position

S → . E
E → E+. F
F → T.

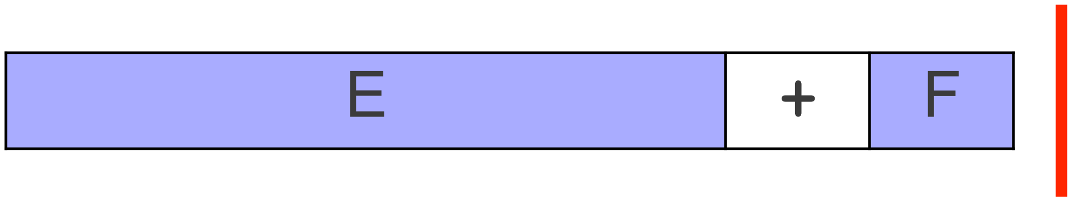
S → E
E → F
E → E + F
F → F * T
F → T
T → `int`
T → (E)



Tracking Our Position

S → . E
E → E+F.

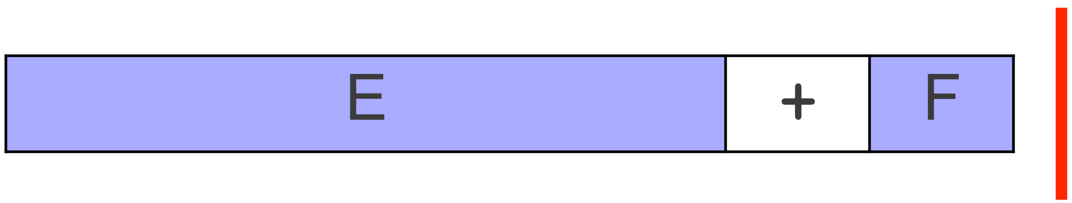
S → E
E → F
E → E + F
F → F * T
F → T
T → int
T → (E)



Tracking Our Position

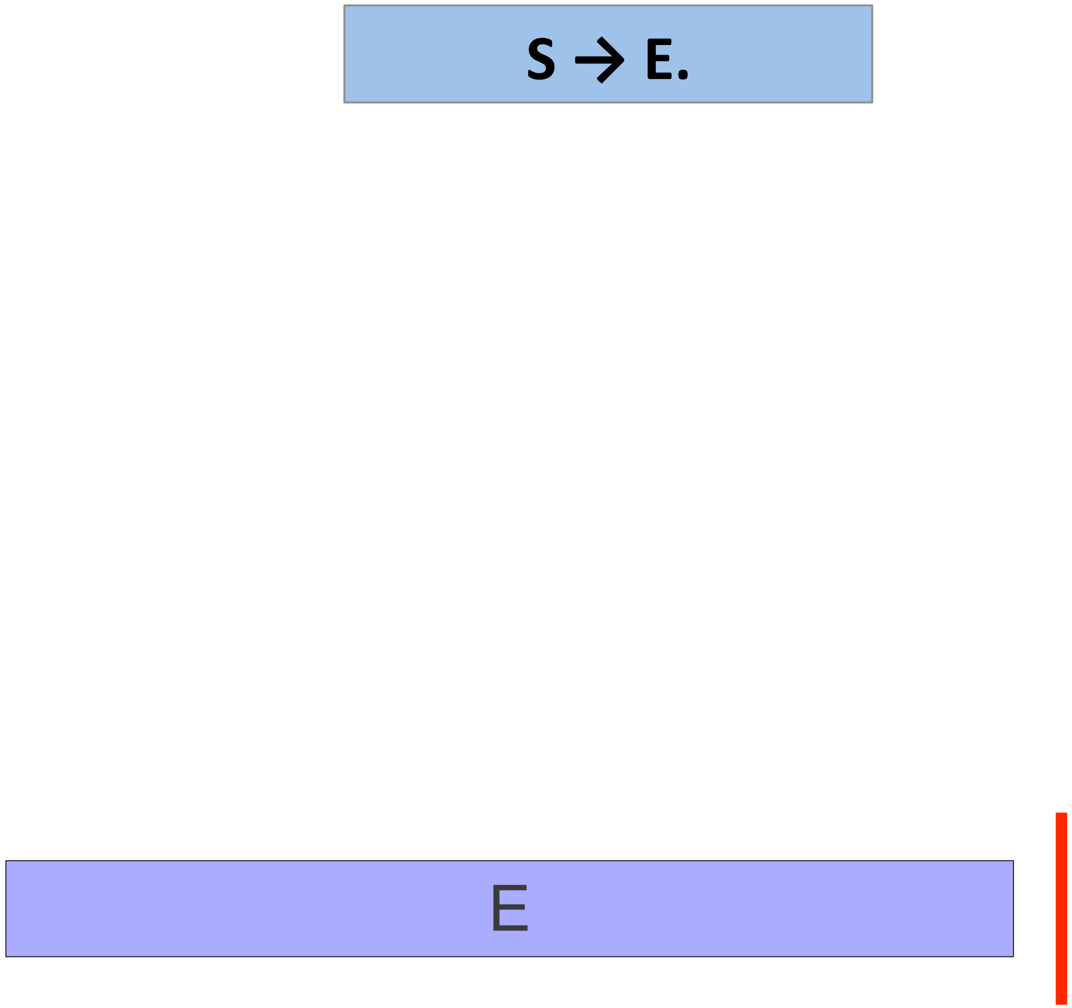
S → .E
E → E+F.

S → E
E → F
E → E + F
F → F * T
F → T
T → `int`
T → (E)



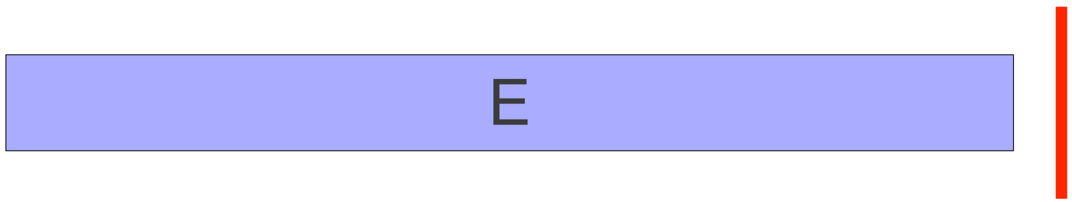
Tracking Our Position

$S \rightarrow E$
$E \rightarrow F$
$E \rightarrow E + F$
$F \rightarrow F * T$
$F \rightarrow T$
$T \rightarrow$ `int`
$T \rightarrow (E)$



Tracking Our Position

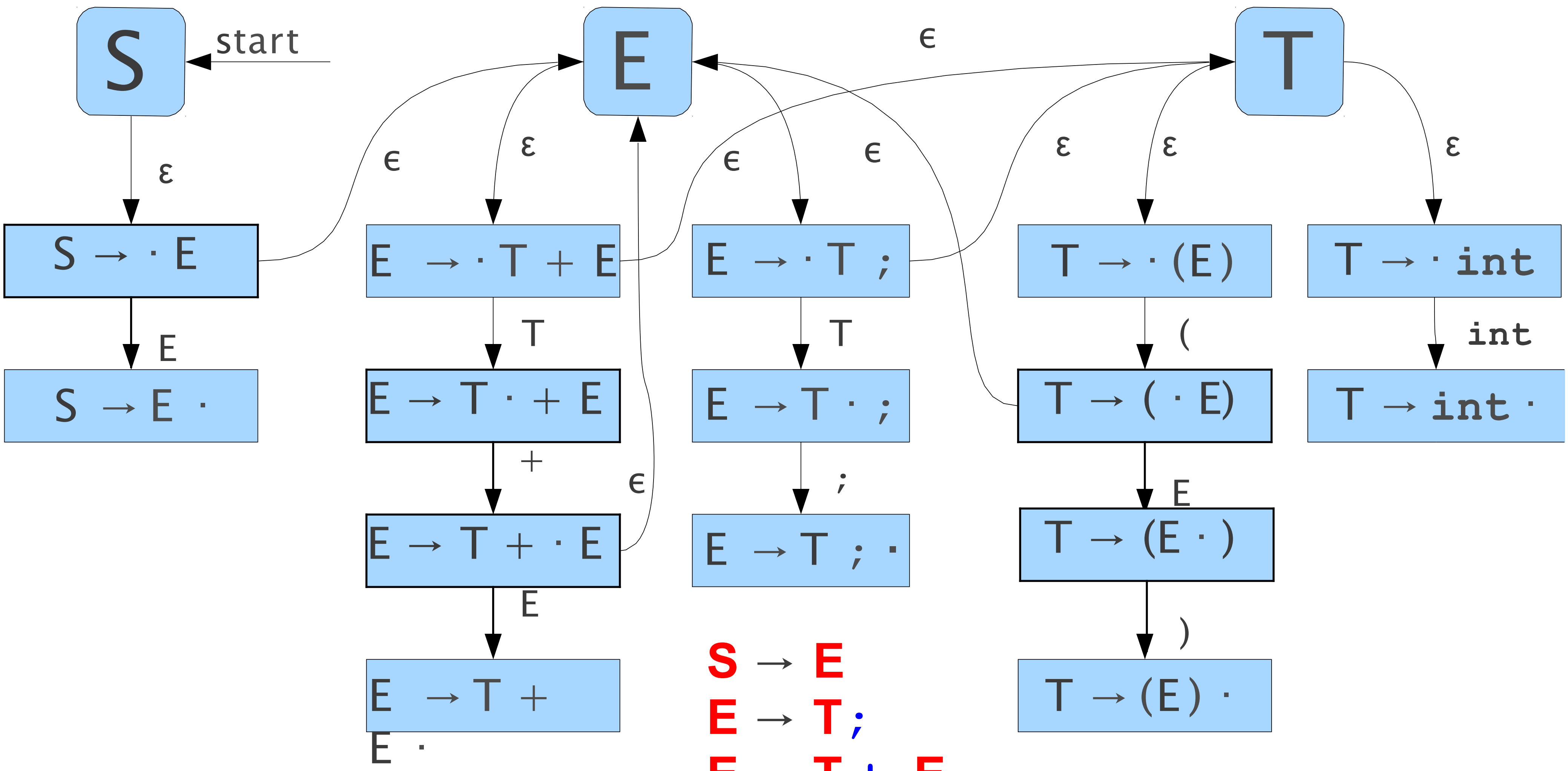
S → **E**
E → **F**
E → **E** `+` **F**
F → **F** `*` **T**
F → **T**
T → `int`
T → `(`**E**`)`



An Important Result

- There are only finitely many productions, and within those productions only finitely many positions.
- At any point in time, we only need to track where we are in one production.
- There are only finitely many options we can take at any one point.
- **We can use a finite automaton as our recognizer.**

An Automaton for Left Areas



S → E
E → T;
E → T + E
T → int
T → (E)

Constructing the Automaton

- Create a state for each nonterminal.
- For each production **A** $\rightarrow \gamma$
  - Construct states **A** $\rightarrow \alpha \cdot \omega$ for each possible way of splitting $\boldsymbol{\gamma}$ into two substrings $\boldsymbol{\alpha}$ and $\boldsymbol{\omega}$.
  - Add transitions on $\boldsymbol{x}$ between **A** $\rightarrow \alpha \cdot \boldsymbol{x}\, \omega$ and **A** $\rightarrow \alpha\, \boldsymbol{x} \cdot \omega$.
- For each state **A** $\rightarrow \alpha \cdot \mathbf{B}\, \omega$ for nonterminal **B**, add an ε–transition from **A** $\rightarrow \alpha \cdot \mathbf{B}\, \omega$ to **B**.

Why This Matters

- Our initial goal was to find handles.
- When running this automaton, if we ever end up in a state with a rule of the form

$$\mathbf{A} \rightarrow \omega \cdot$$

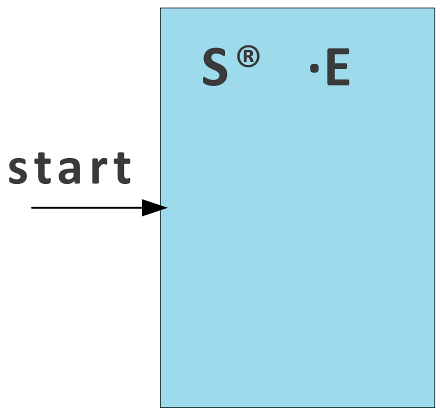
- Then we might be looking at a handle.
- This automaton can be used to discover possible handle locations!

Adding Determinism

- Typically, this handle-finding automaton is implemented deterministically.
- We could construct a deterministic parsing automaton by constructing the nondeterministic automaton and applying the subset construction, but there is a more direct approach.

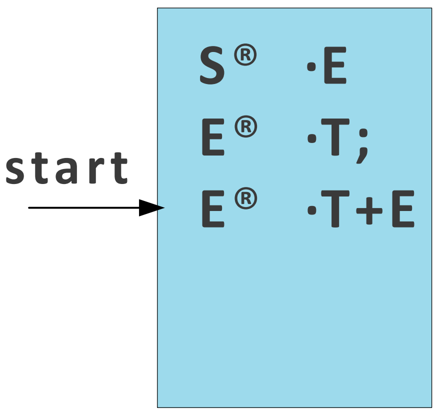
A Deterministic Automaton

S → E
E → T;
E → T + E
T → int
T → (E)



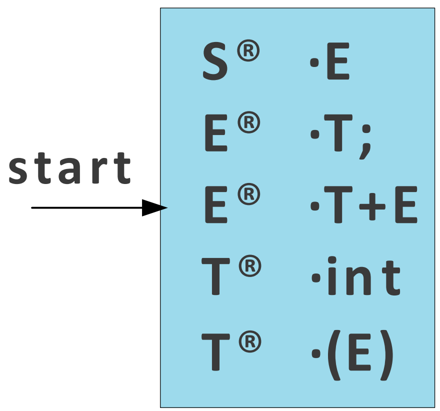
A Deterministic Automaton

$S \rightarrow E$
$E \rightarrow T;$
$E \rightarrow T + E$
$T \rightarrow \texttt{int}$
$T \rightarrow (E)$



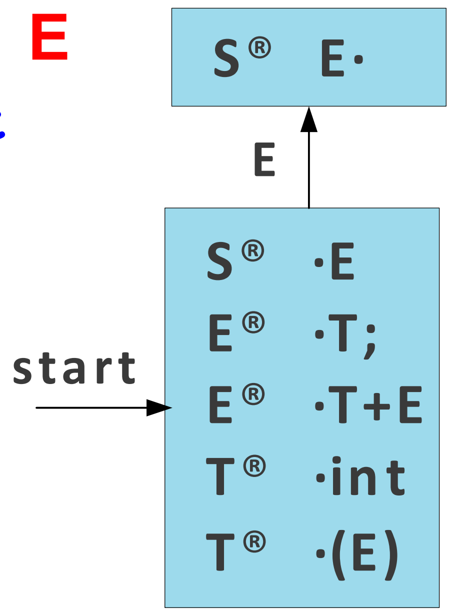
A Deterministic Automaton

S → E
E → T;
E → T+ E
T → int
T → (E)



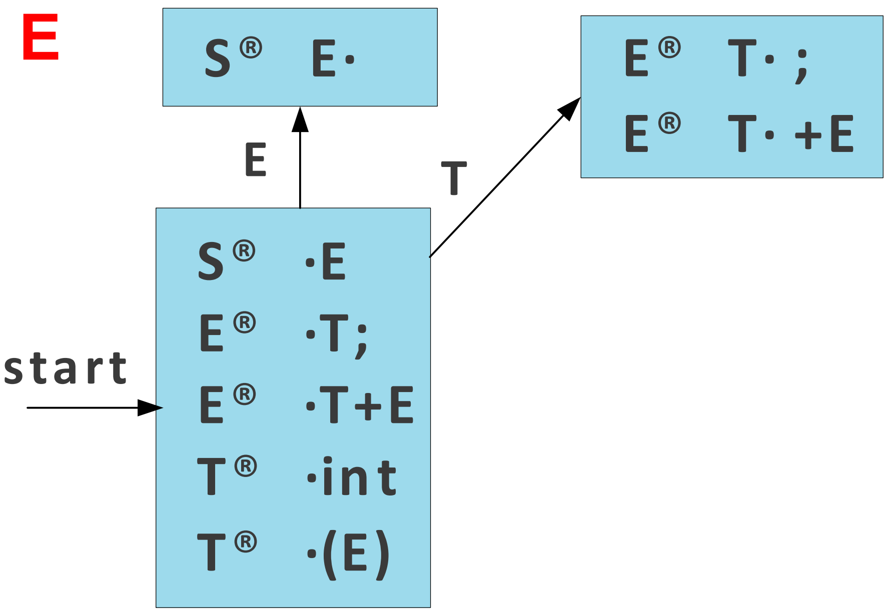
A Deterministic Automaton

$S \rightarrow E$
$E \rightarrow T;$
$E \rightarrow T + E$
$T \rightarrow \texttt{int}$
$T \rightarrow (E)$



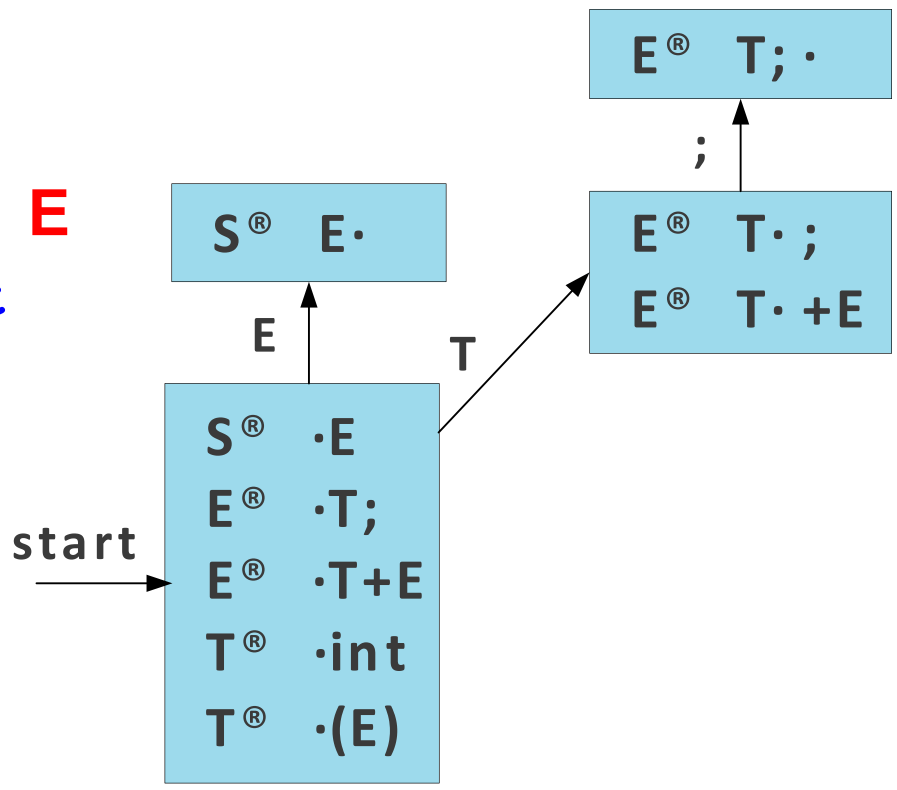
A Deterministic Automaton

S → E
E → T;
E → T + E
T → int
T → (E)



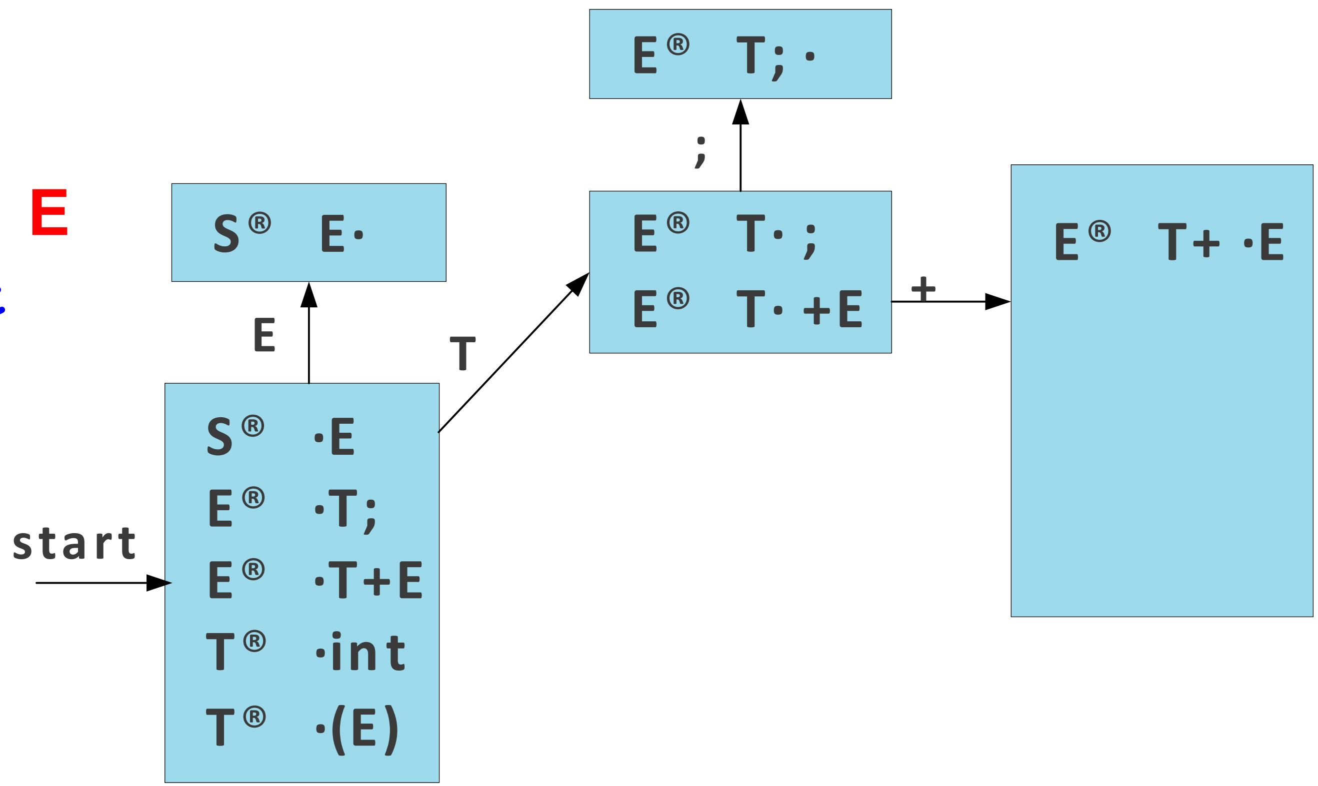
A Deterministic Automaton

S → E
E → T;
E → T + E
T → int
T → (E)



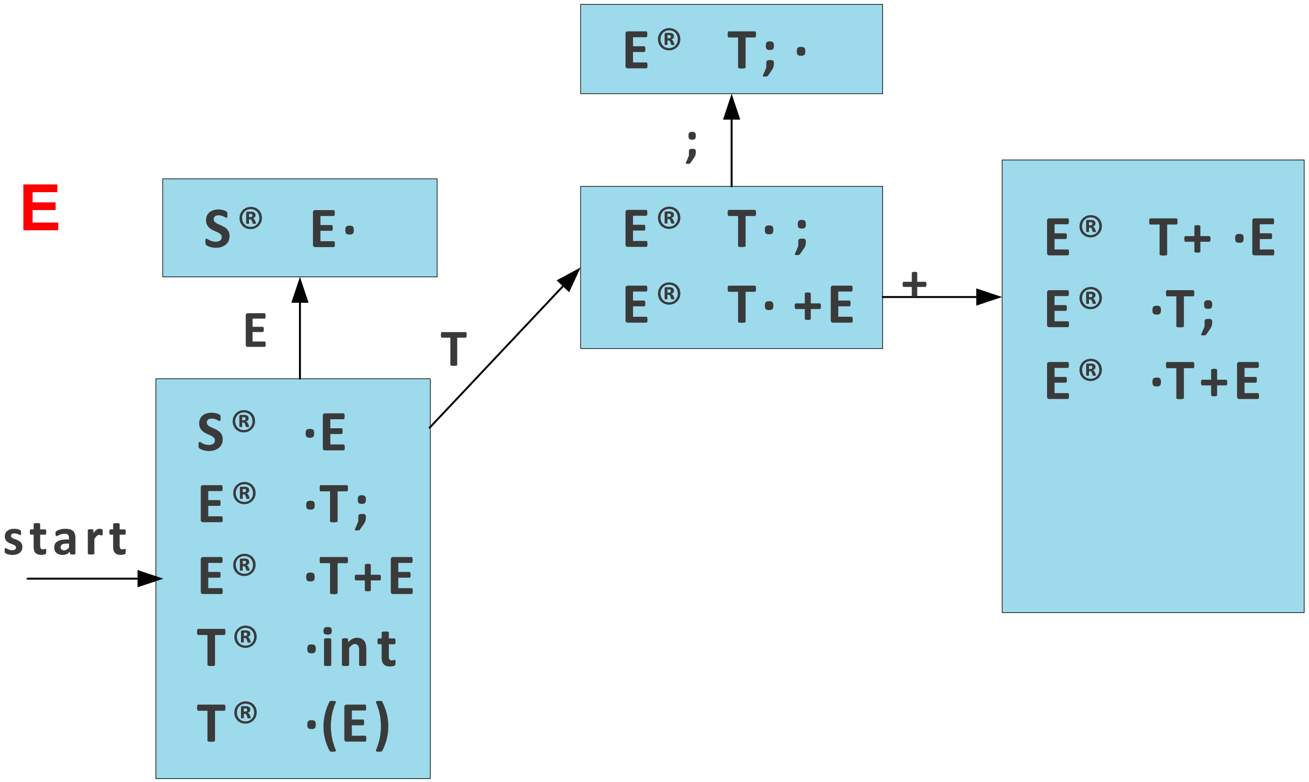
A Deterministic Automaton

S → E
E → T;
E → T + E
T → int
T → (E)



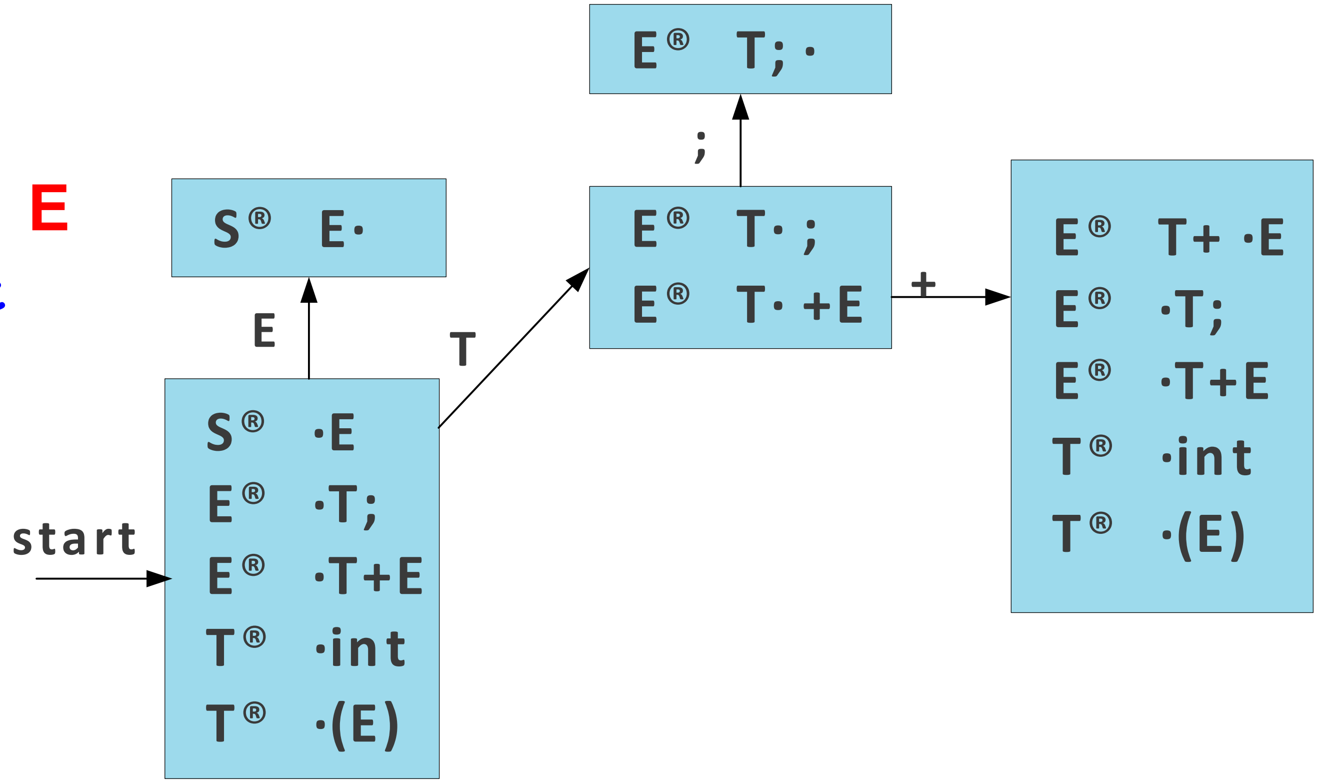
A Deterministic Automaton

S → E
E → T;
E → T + E
T → int
T → (E)



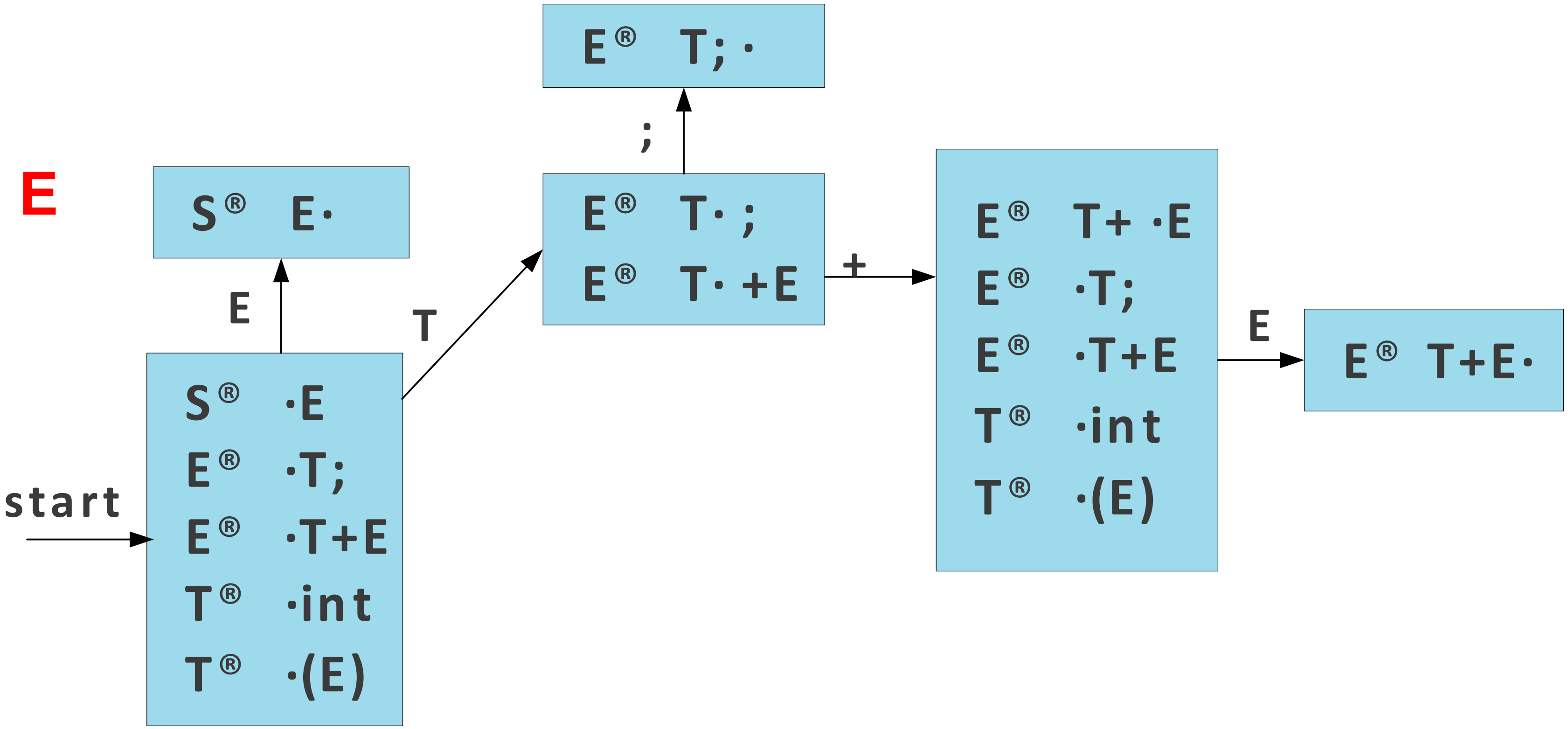
A Deterministic Automaton

S → E
E → T;
E → T + E
T → int
T → (E)



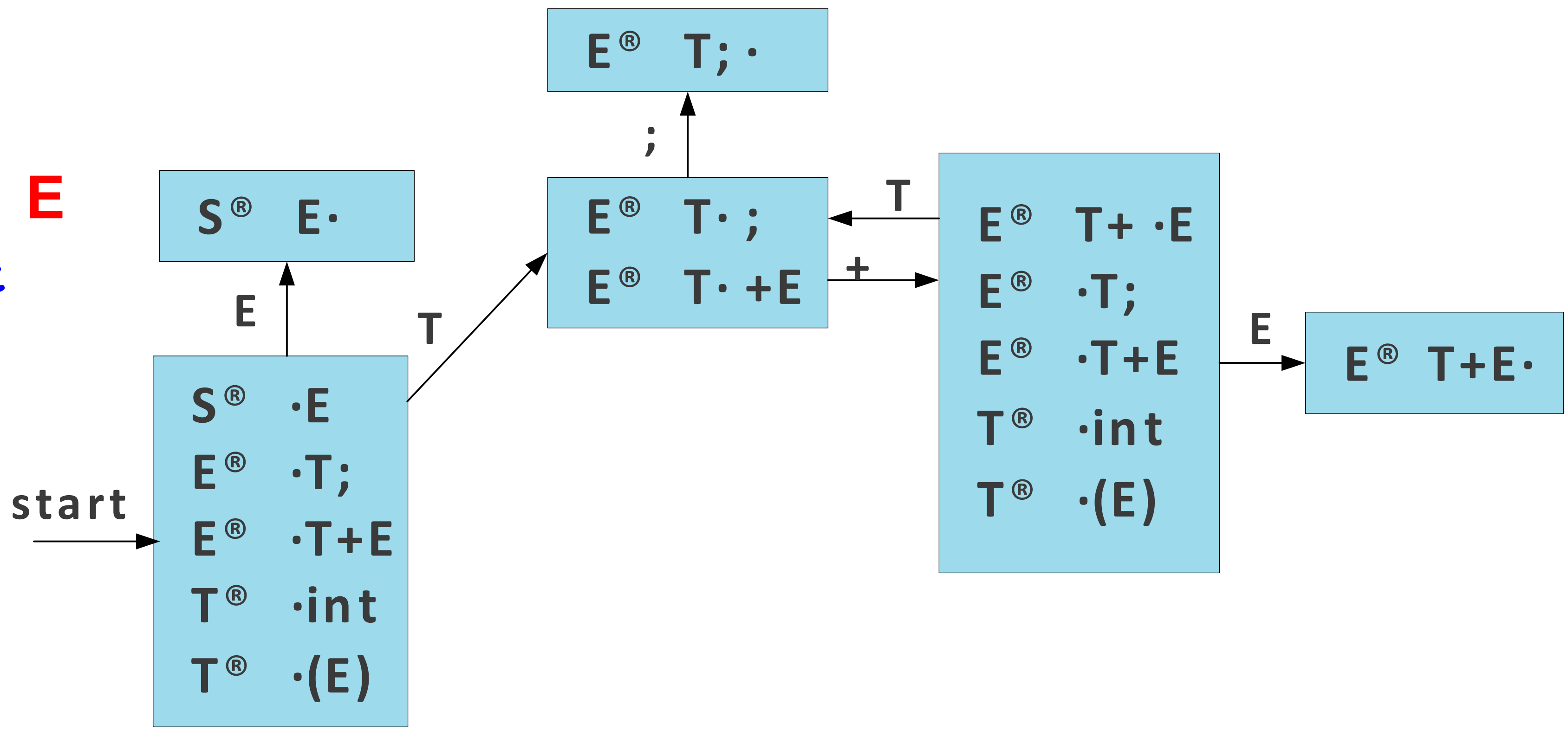
A Deterministic Automaton

S → E
E → T ;
E → T + E
T → int
T → (E)



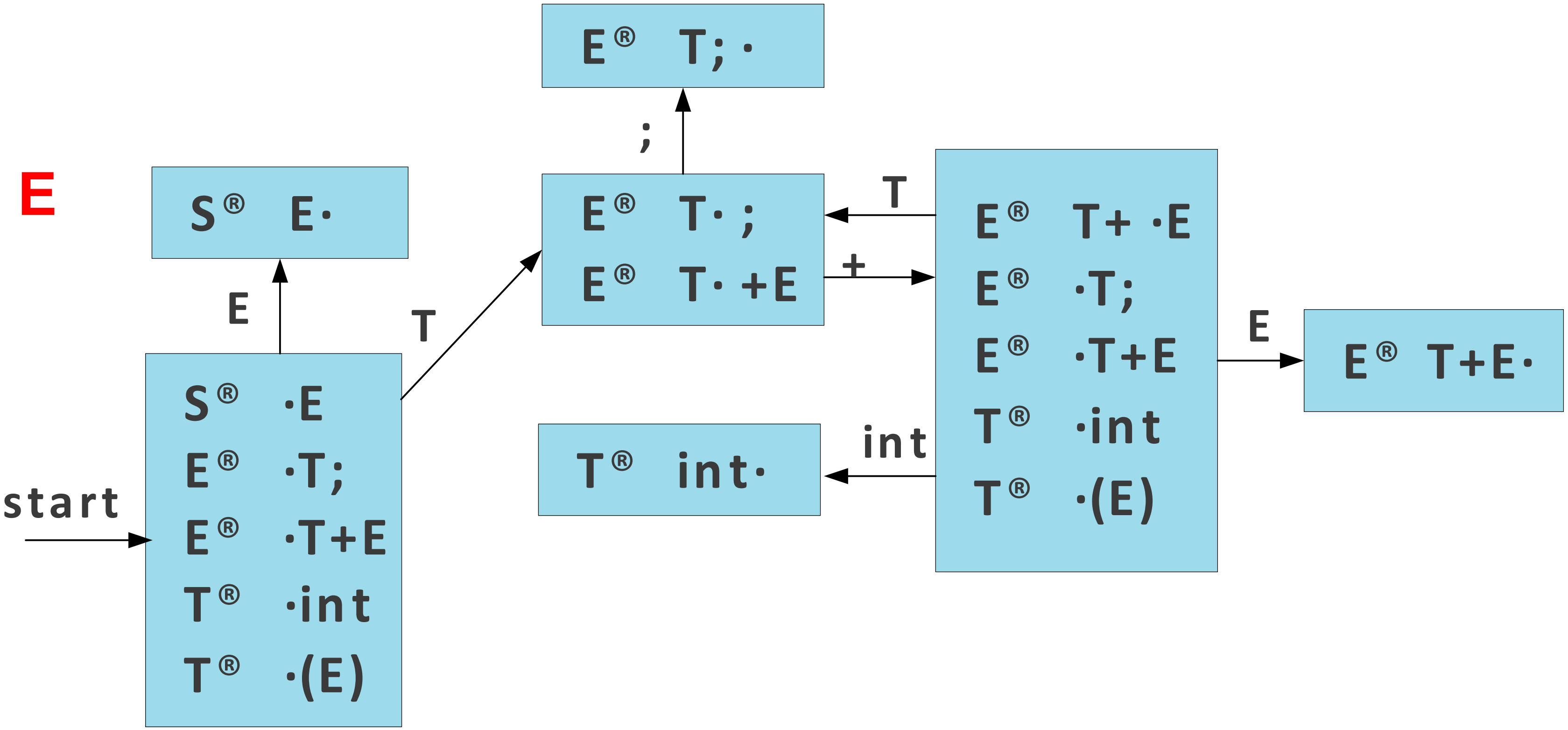
A Deterministic Automaton

S → E
E → T;
E → T + E
T → int
T → (E)



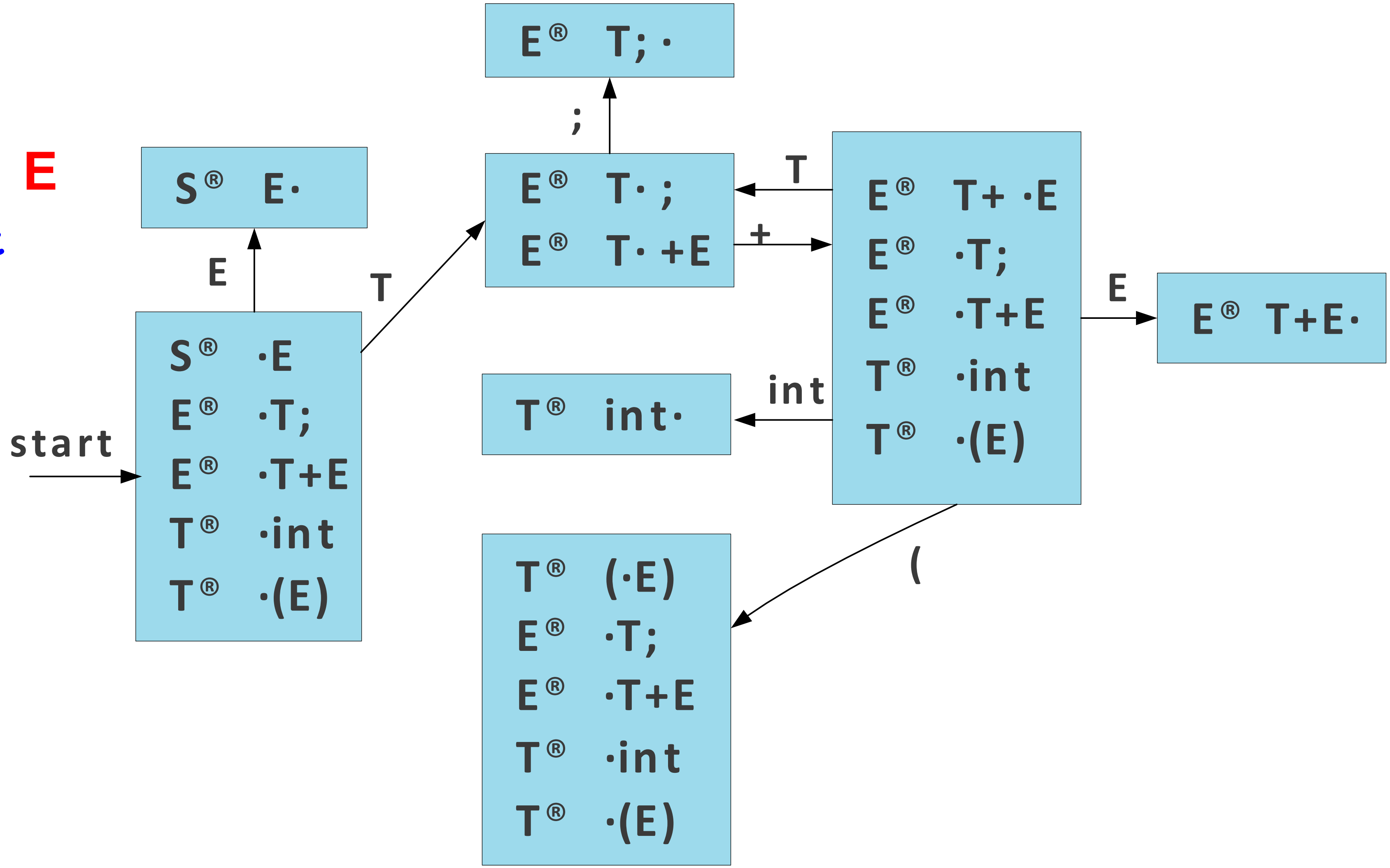
A Deterministic Automaton

S → E
E → T ;
E → T + E
T → int
T → (E)



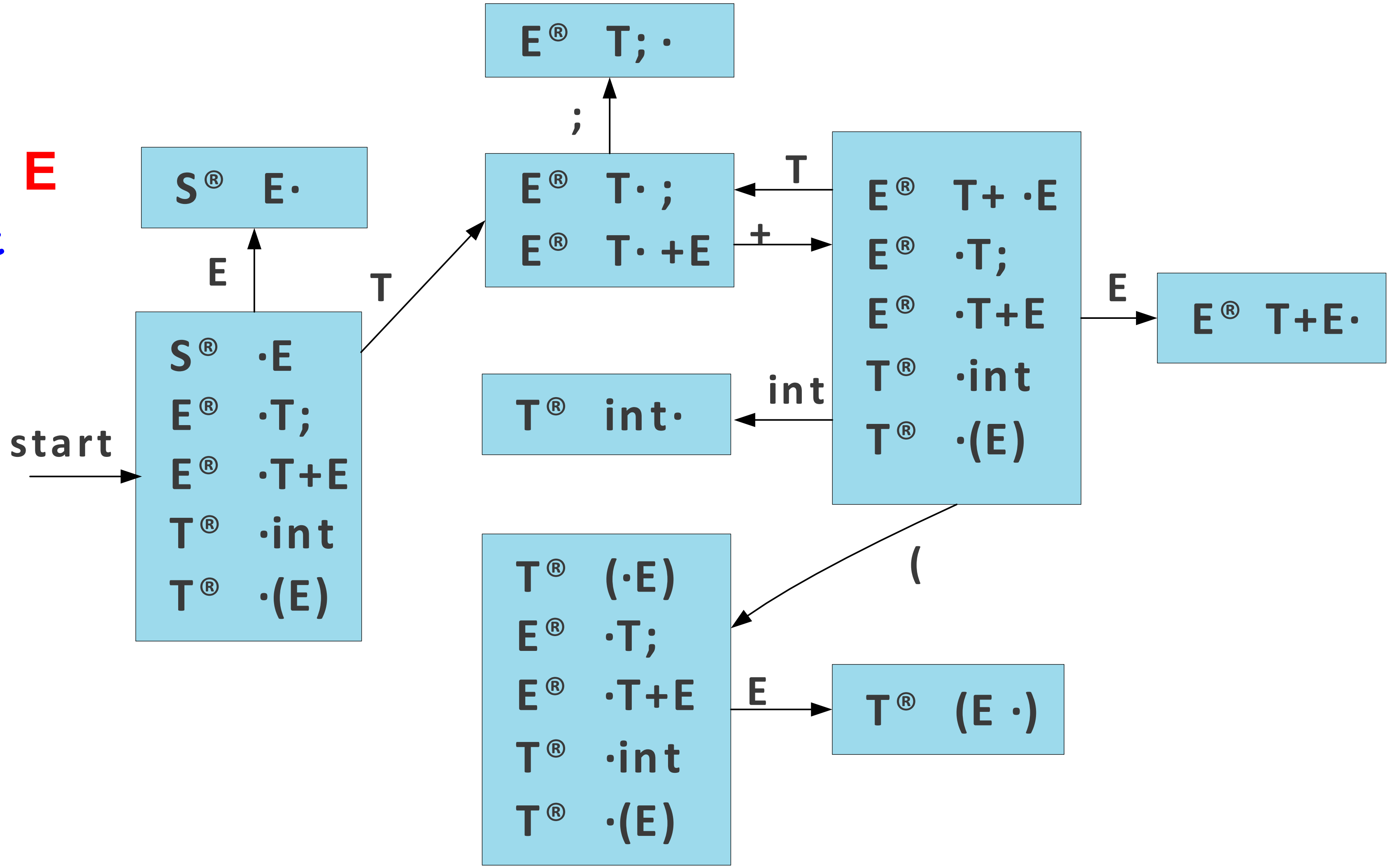
A Deterministic Automaton

$S \rightarrow E$
$E \rightarrow T\,;$
$E \rightarrow T + E$
$T \rightarrow \texttt{int}$
$T \rightarrow (E)$



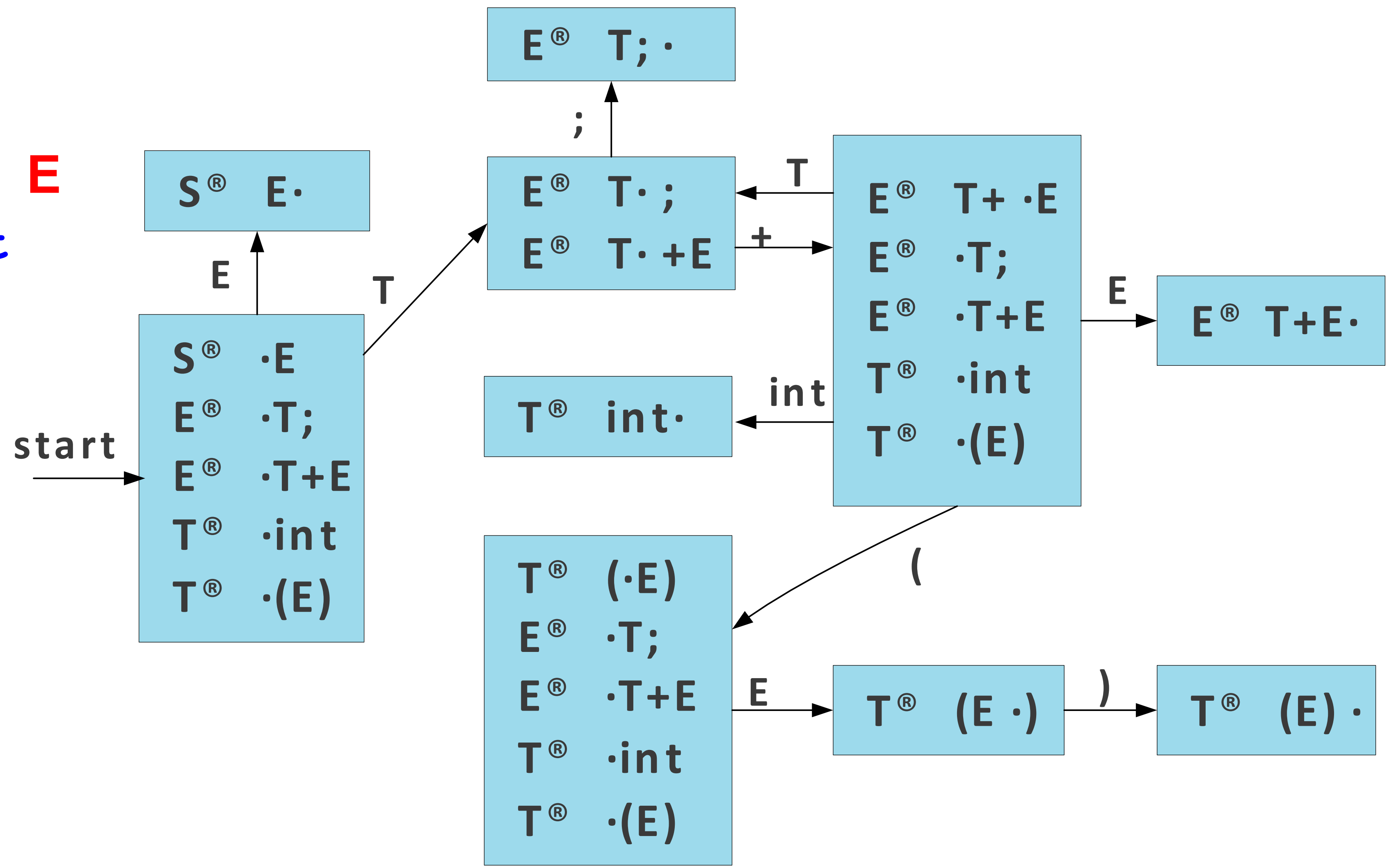
A Deterministic Automaton

$S \rightarrow E$
$E \rightarrow T;$
$E \rightarrow T + E$
$T \rightarrow \texttt{int}$
$T \rightarrow (E)$



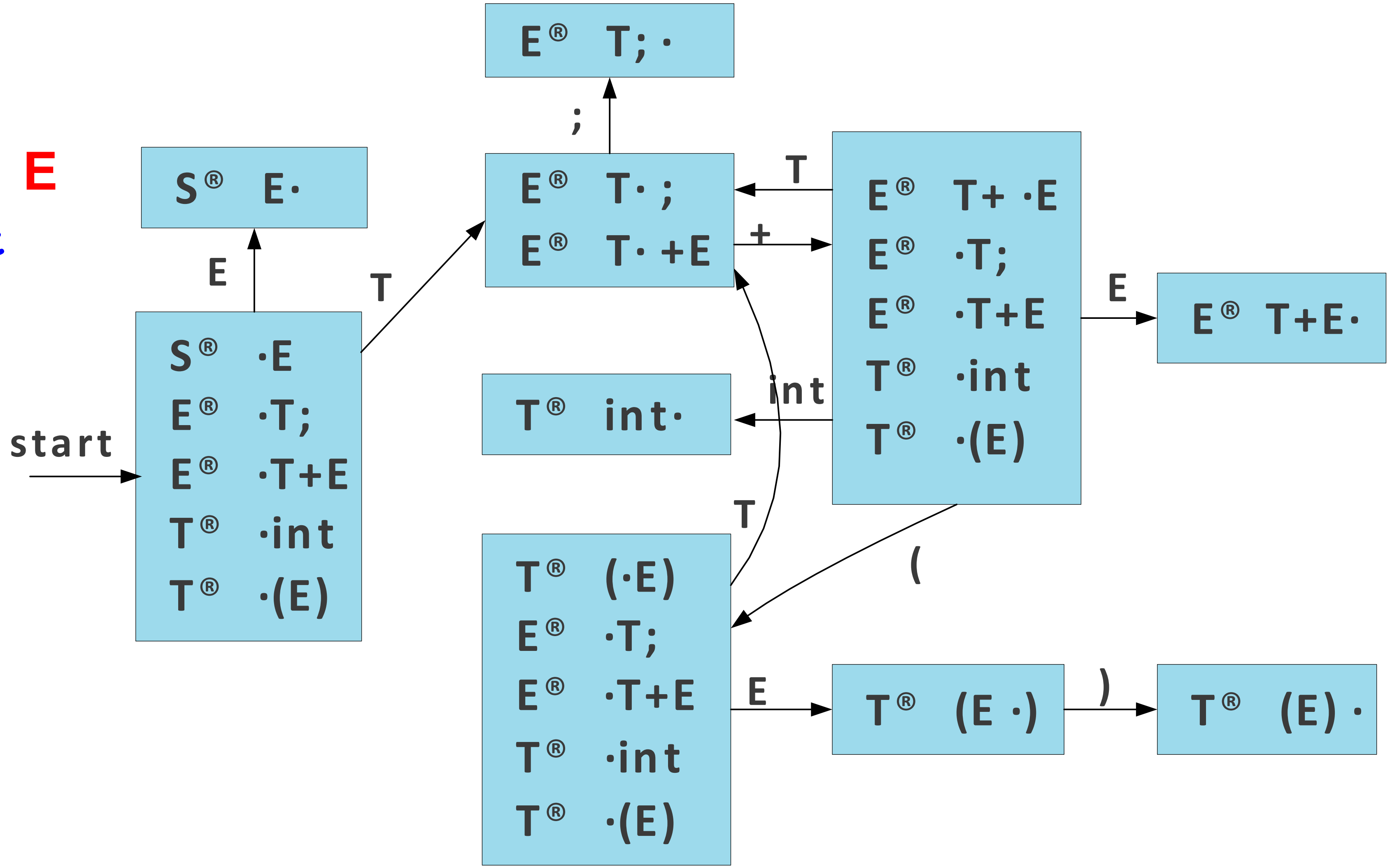
A Deterministic Automaton

S → E
E → T ;
E → T + E
T → int
T → (E)



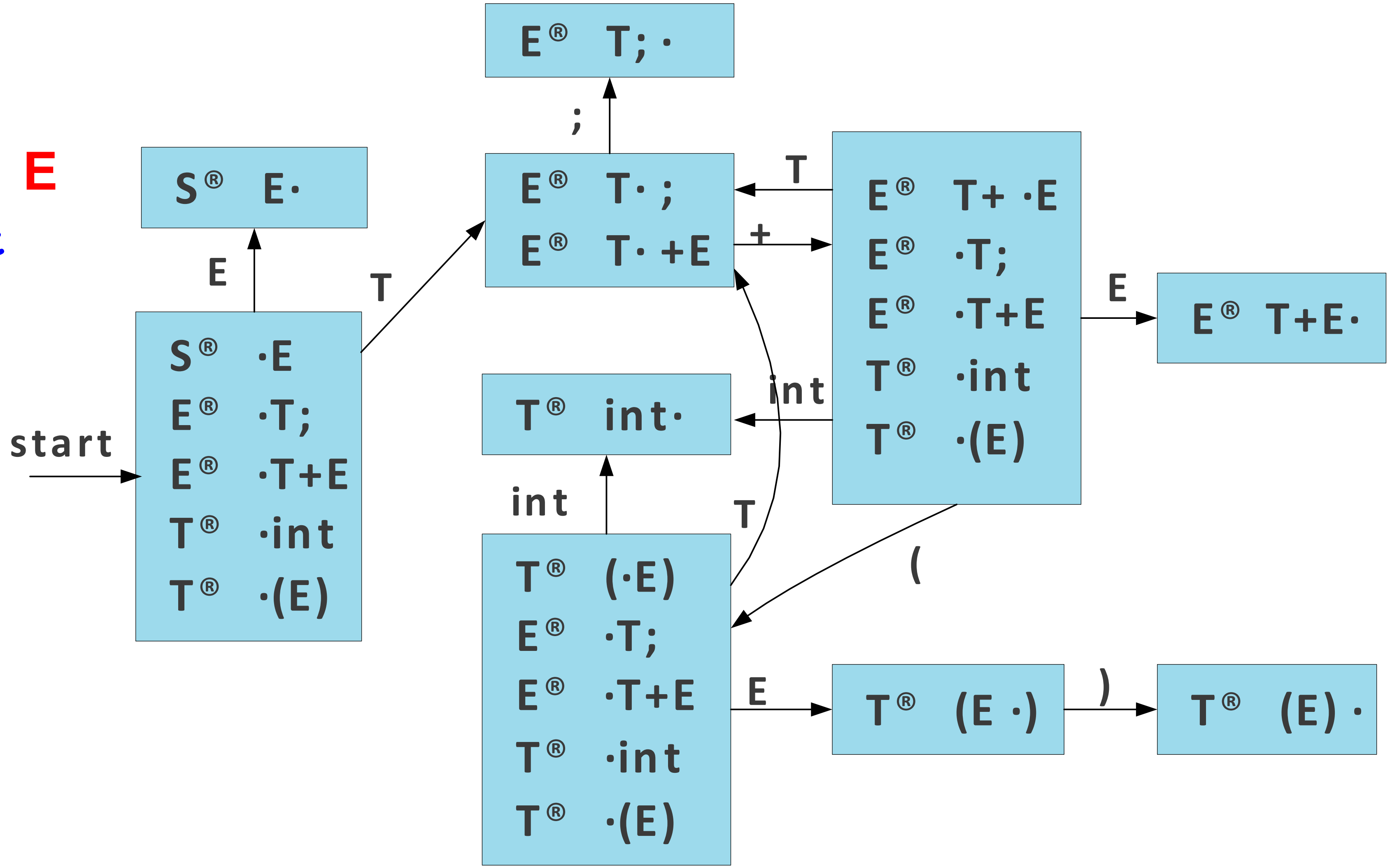
A Deterministic Automaton

S → E
E → T;
E → T + E
T → int
T → (E)



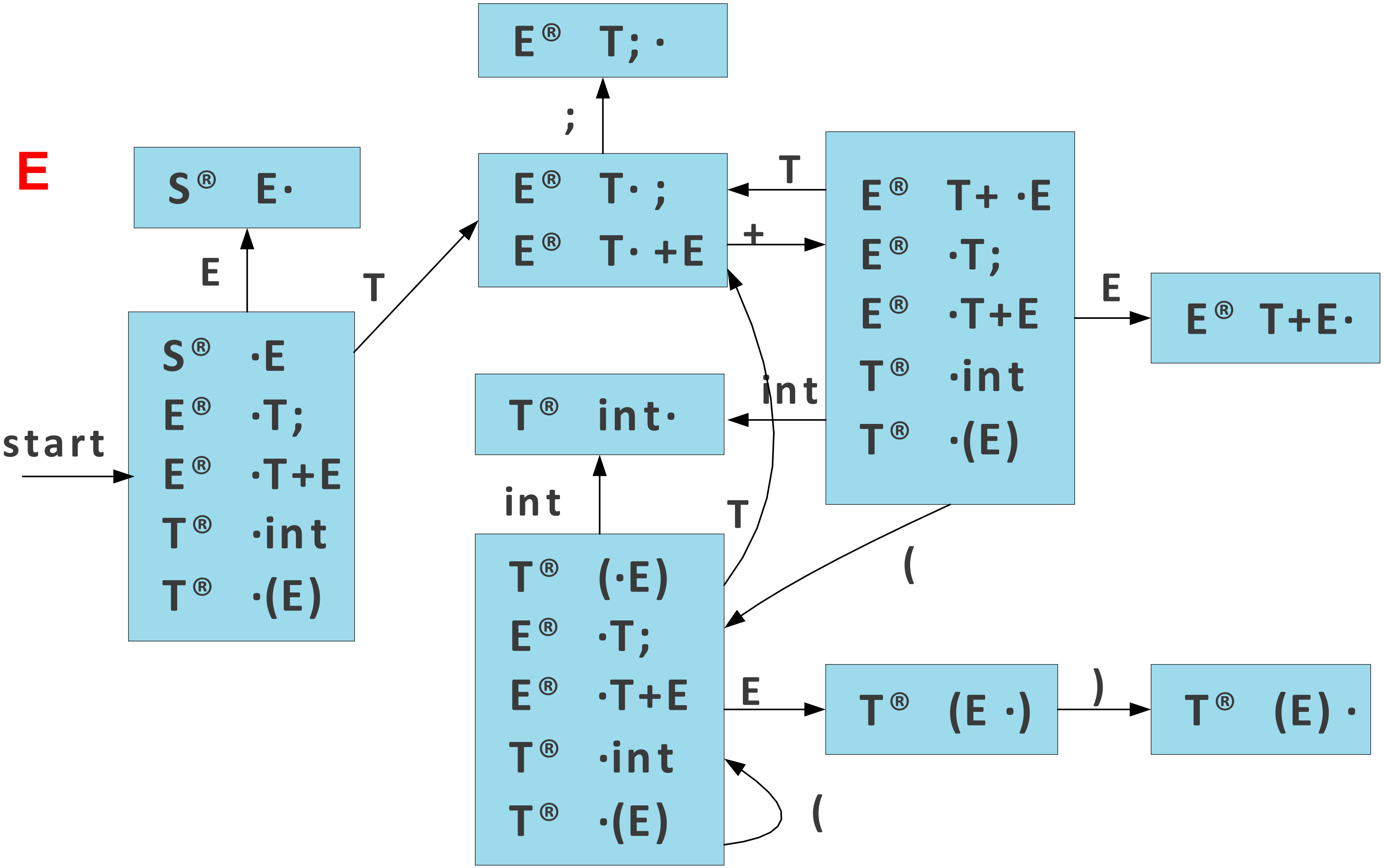
A Deterministic Automaton

S → E
E → T;
E → T + E
T → int
T → (E)



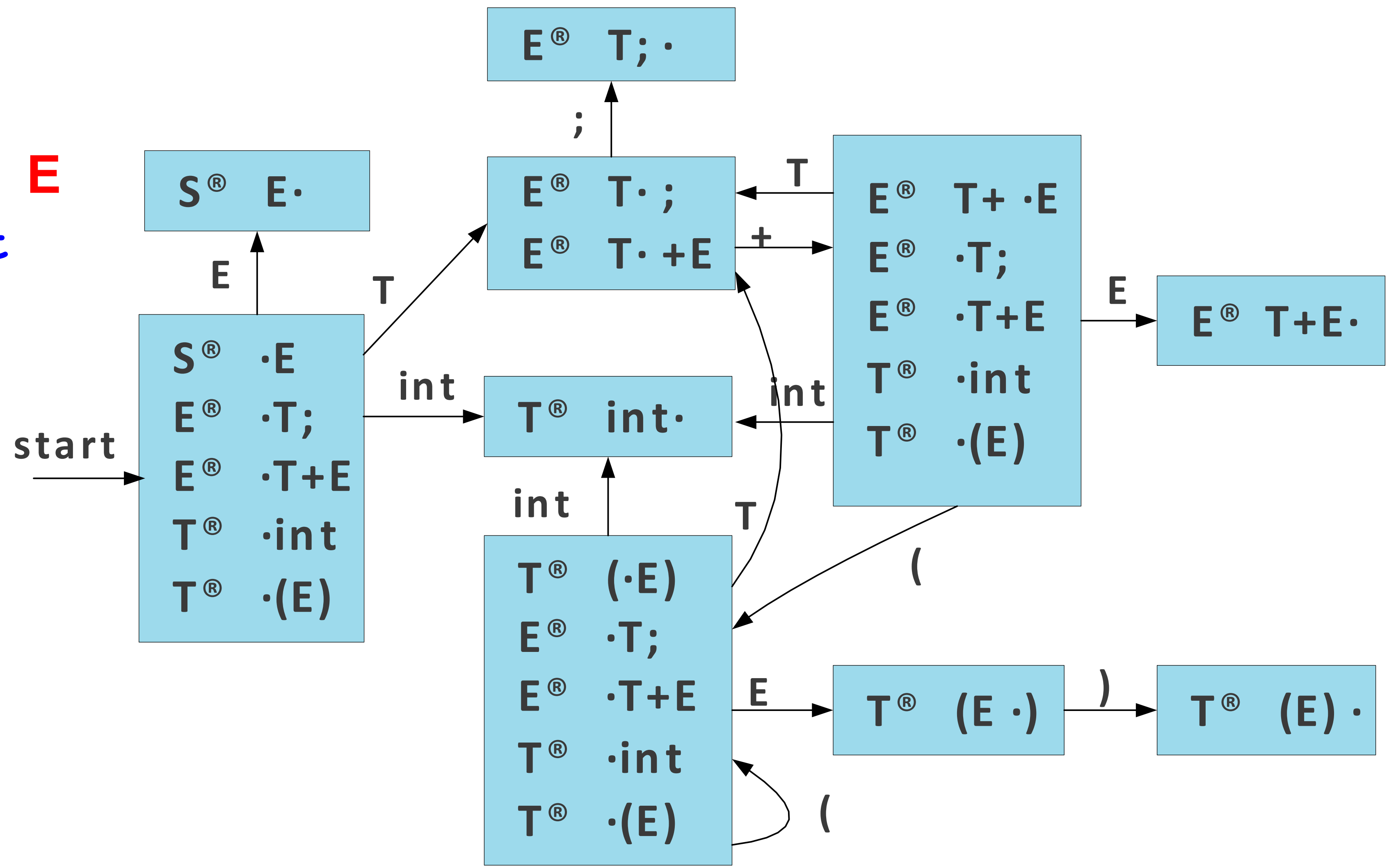
A Deterministic Automaton

S → E
E → T;
E → T + E
T → int
T → (E)



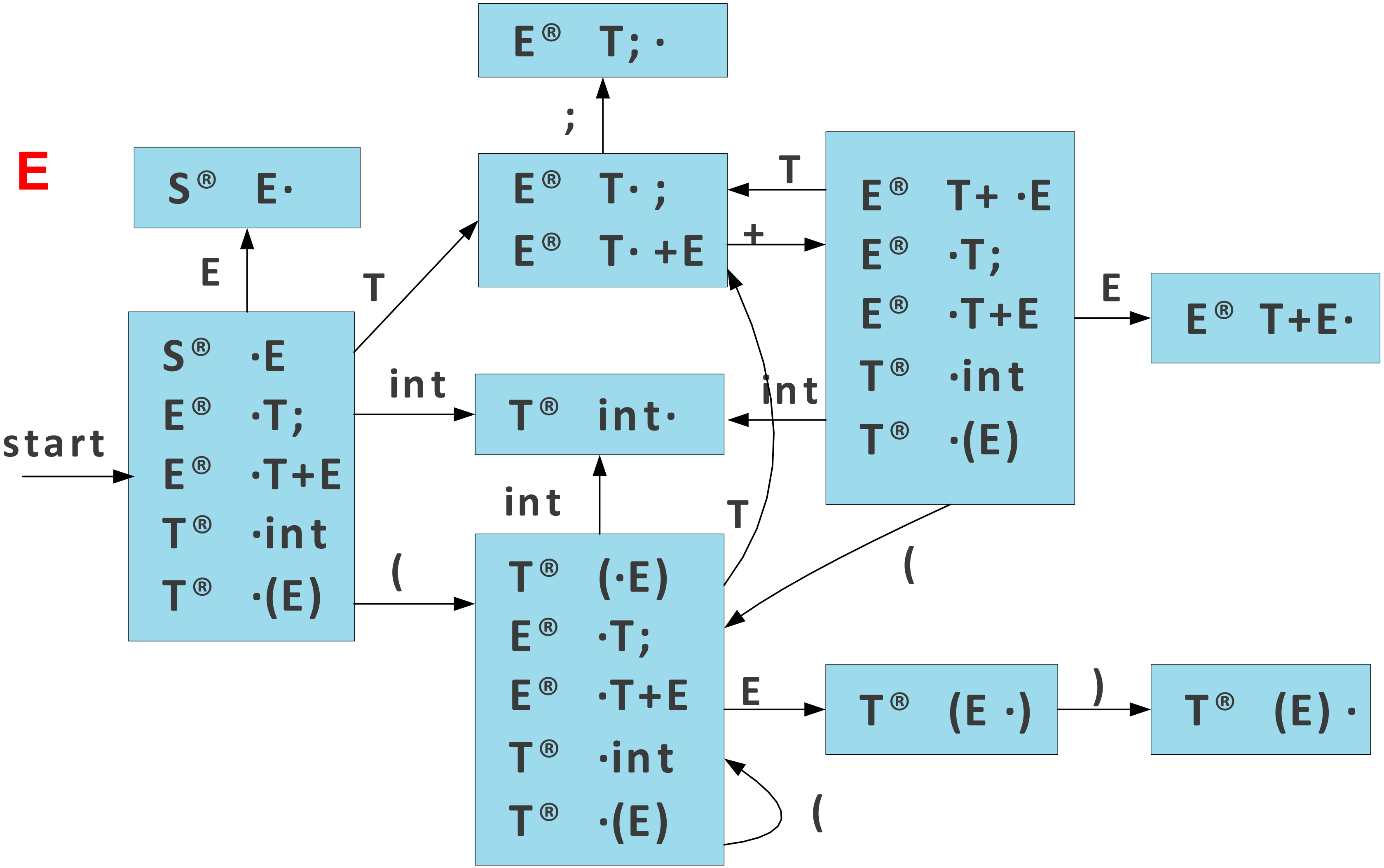
A Deterministic Automaton

S → E
E → T;
E → T + E
T → int
T → (E)



A Deterministic Automaton

S → E
E → T;
E → T + E
T → int
T → (E)



Constructing the Automaton II

- Begin in a state containing **S** $\rightarrow \cdot$ **A**, where **S** is the augmented start symbol.
- Compute the **closure** of the state:
  - If **A** $\rightarrow \alpha \cdot$ **B**ω is in the state, add **B** $\rightarrow \cdot\ \gamma$ to the state for each production **B** $\rightarrow \gamma$.
  - Yet another fixed-point iteration!
- Repeat until no new states are added:
  - If a state contains a production **A** $\rightarrow \alpha \cdot \boldsymbol{x}\omega$ for symbol $\boldsymbol{x}$, add a transition on $\boldsymbol{x}$ from that state to the state containing the closure of **A** $\rightarrow \alpha\boldsymbol{x} \cdot \omega$
- This is equivalent to a subset construction on the NFA.

Handle-Finding Automata

- Handling-finding automata can be very large.
- NFA has states proportional to the size of the grammar, so DFA can have size exponential in the size of the grammar.
  - There are grammars that can exhibit this worst-case.
  - Automata are almost always generated by tools like **bison**.

Finding Handles

- Where do we look for handles?
  - **At the top of the stack.**
- How do we search for possible handles?
  - **Build a handle-finding automaton.**
- How do we recognize handles?
  - Once we've found a candidate handle, how do we check that it really is the handle?

Question Two:

How do we recognize handles?

Handle Recognition

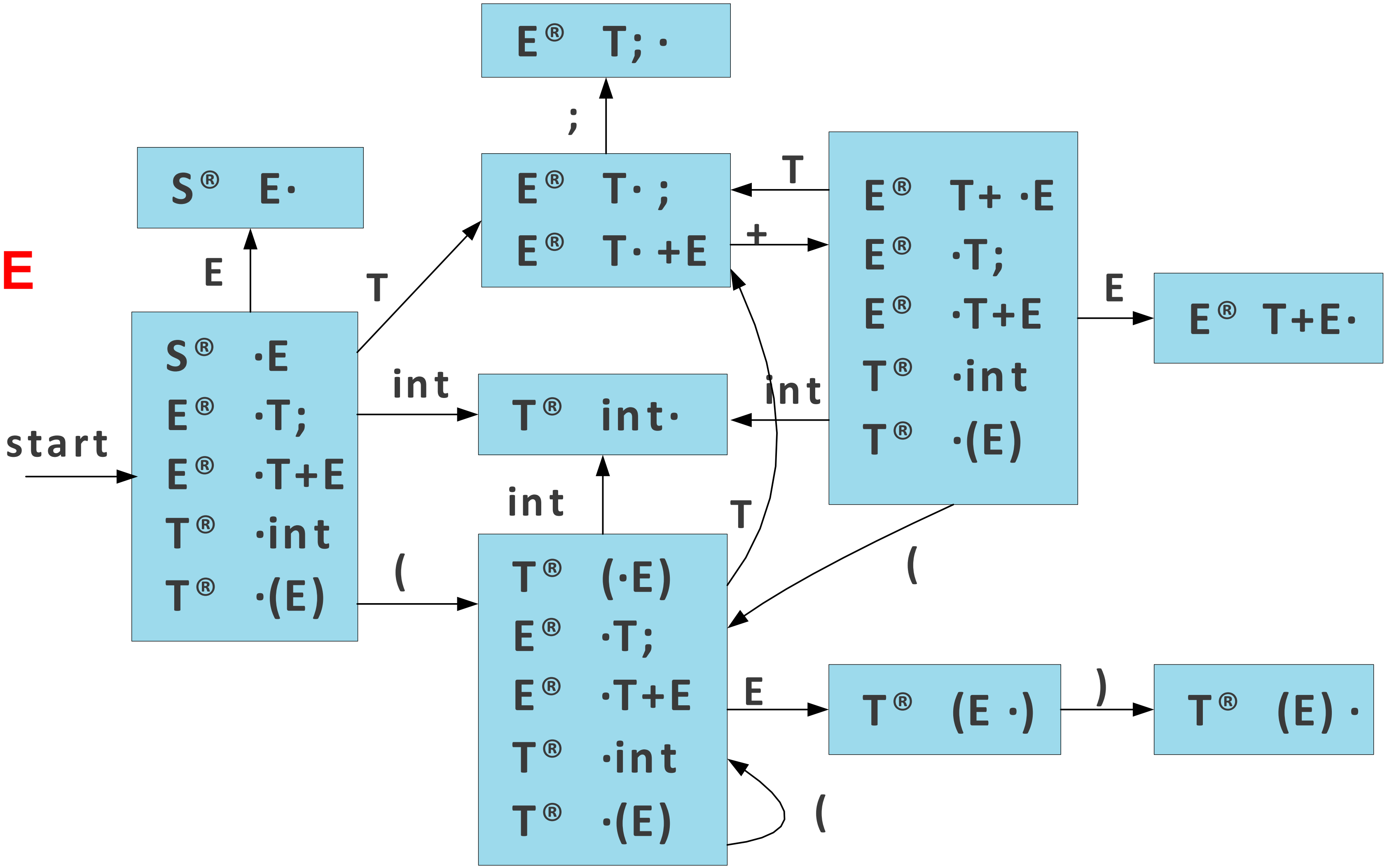
- Our automaton will tell us all places where a handle might be.
- However, if the automaton says that there might be a handle at a given point, we need a way to confirm this.
- We'll thus use **predictive bottom-up parsing**:
  - Have a deterministic procedure for guessing where handles are.
- There are many predictive algorithms, each of which recognize different grammars.

The First Algorithm: **LR(0)**

- Bottom-up predictive parsing with:
  - **L**: **L**eft-to-right scan of the input.
  - **R**: **R**ightmost derivation.
  - (**0**): Zero tokens of lookahead.
- Use the handle-finding automaton, without any lookahead, to predict where handles are.

LR(0) Parsing

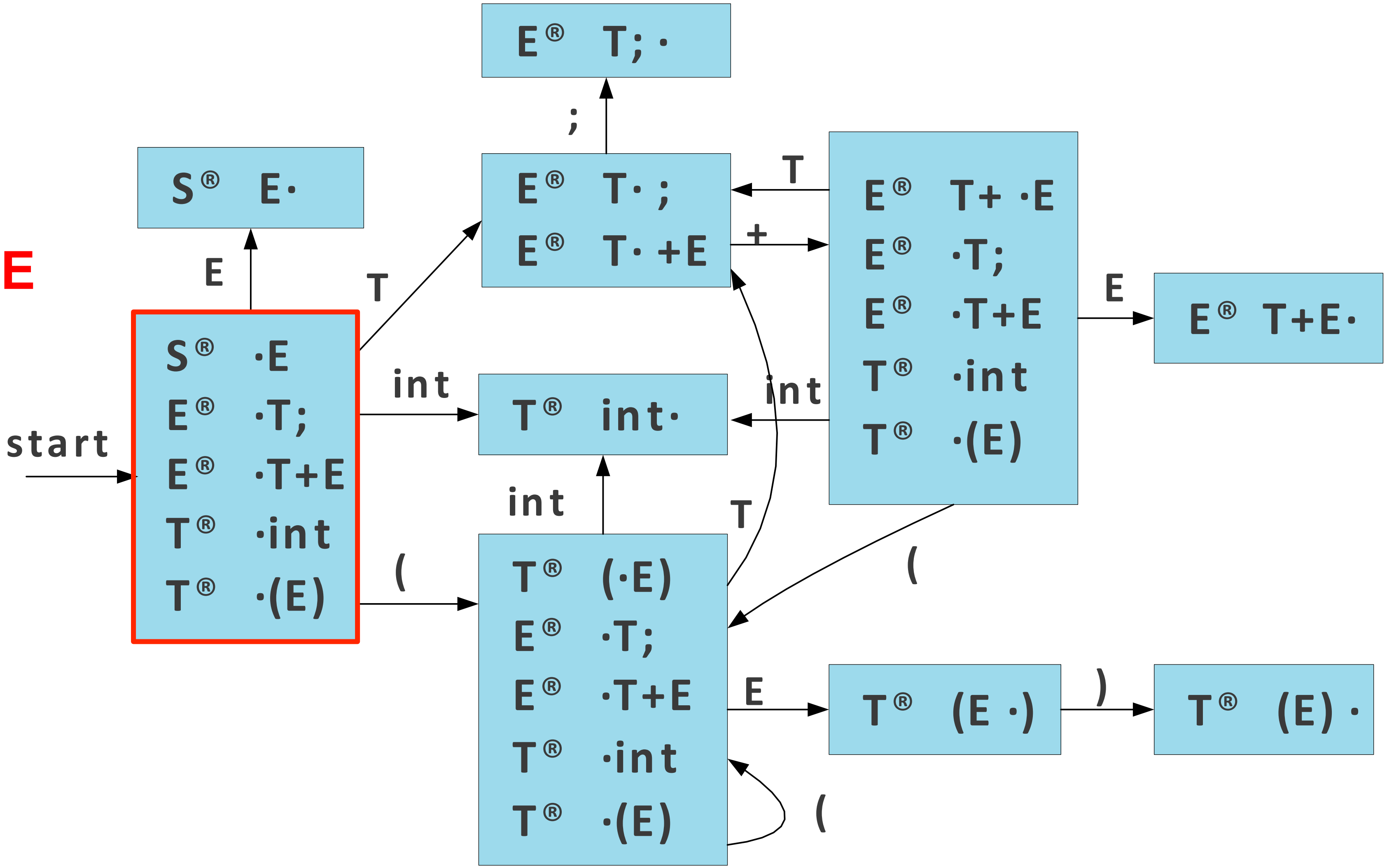
S → E
E → T;
E → T + E
T → int
T → (E)



int	+	(	int	+	int	;	)	;

LR(0) Parsing

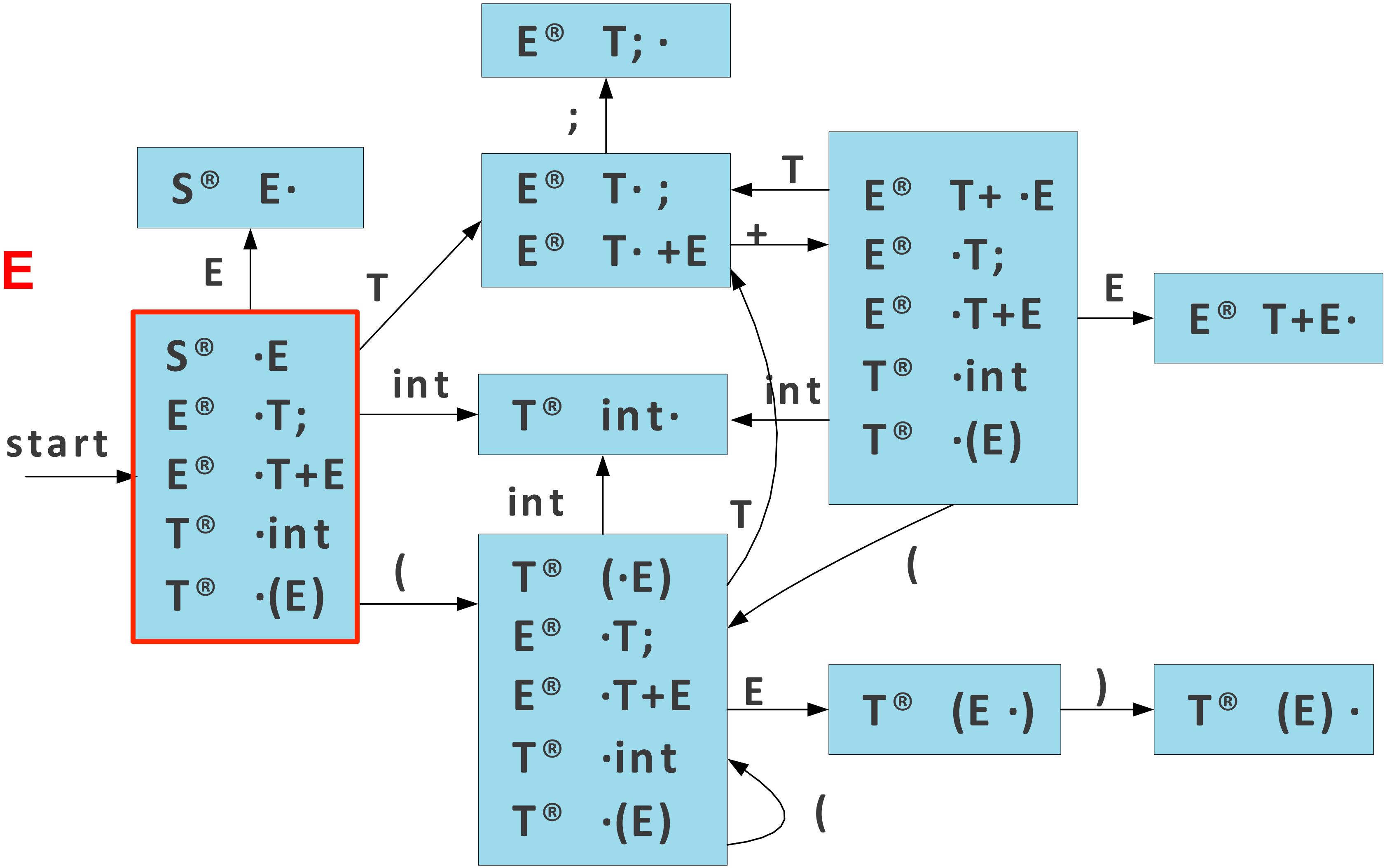
S → E
E → T;
E → T + E
T → int
T → (E)



int	+	(	int	+	int	;	)	;

LR(0) Parsing

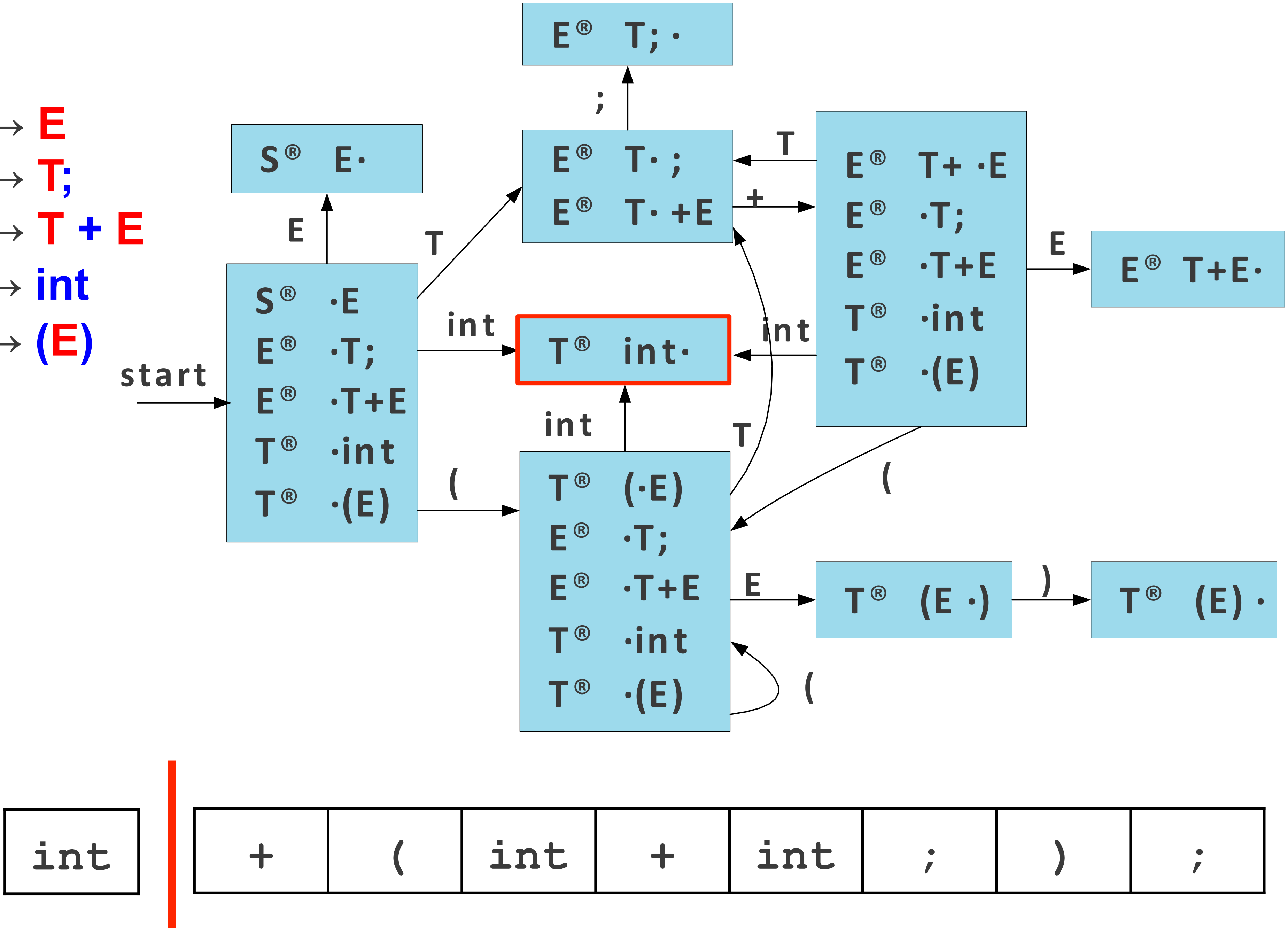
S → E
E → T;
E → T + E
T → int
T → (E)



int	+	(	int	+	int	;	)	;

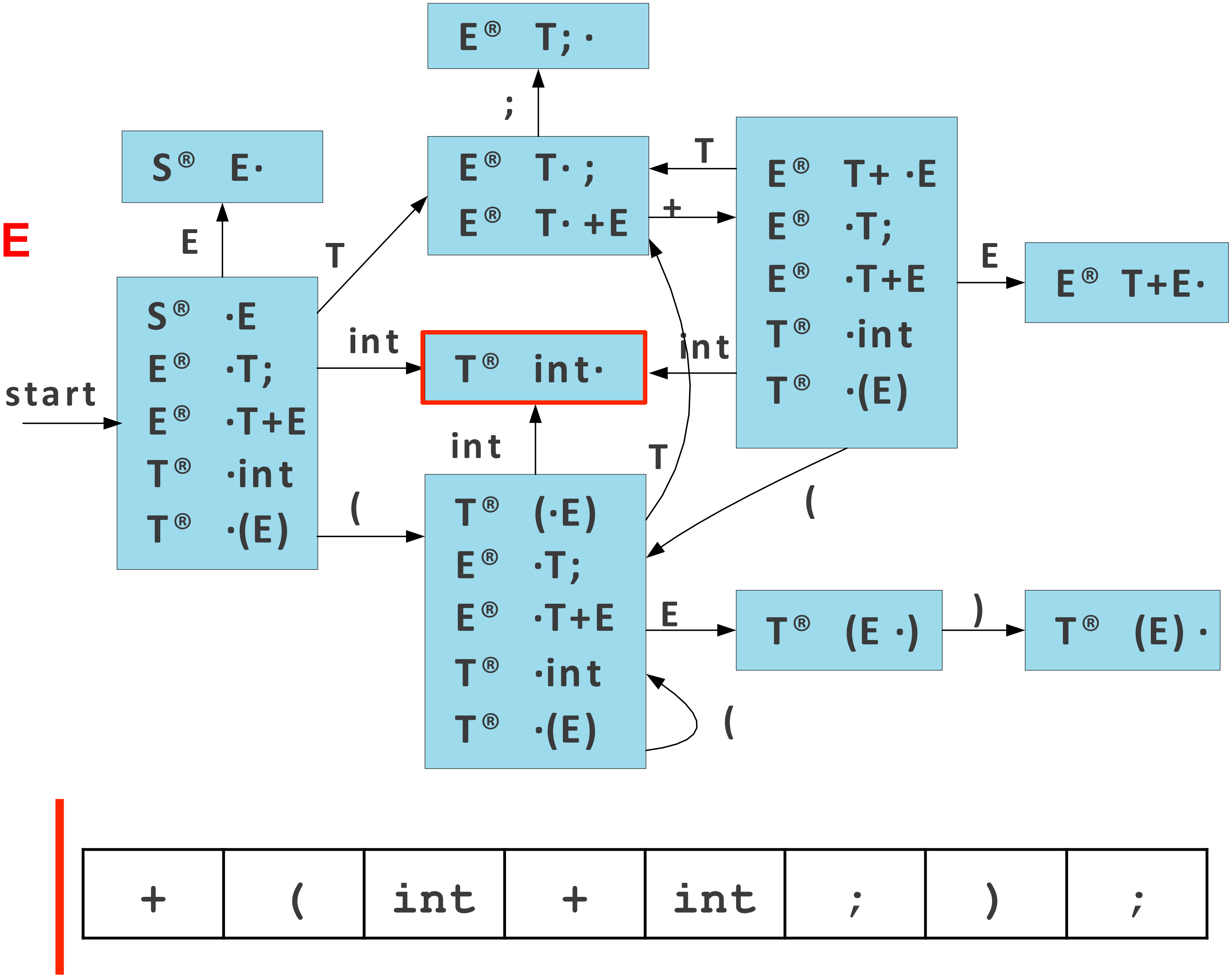
LR(0) Parsing

S → E
E → T;
E → T + E
T → int
T → (E)



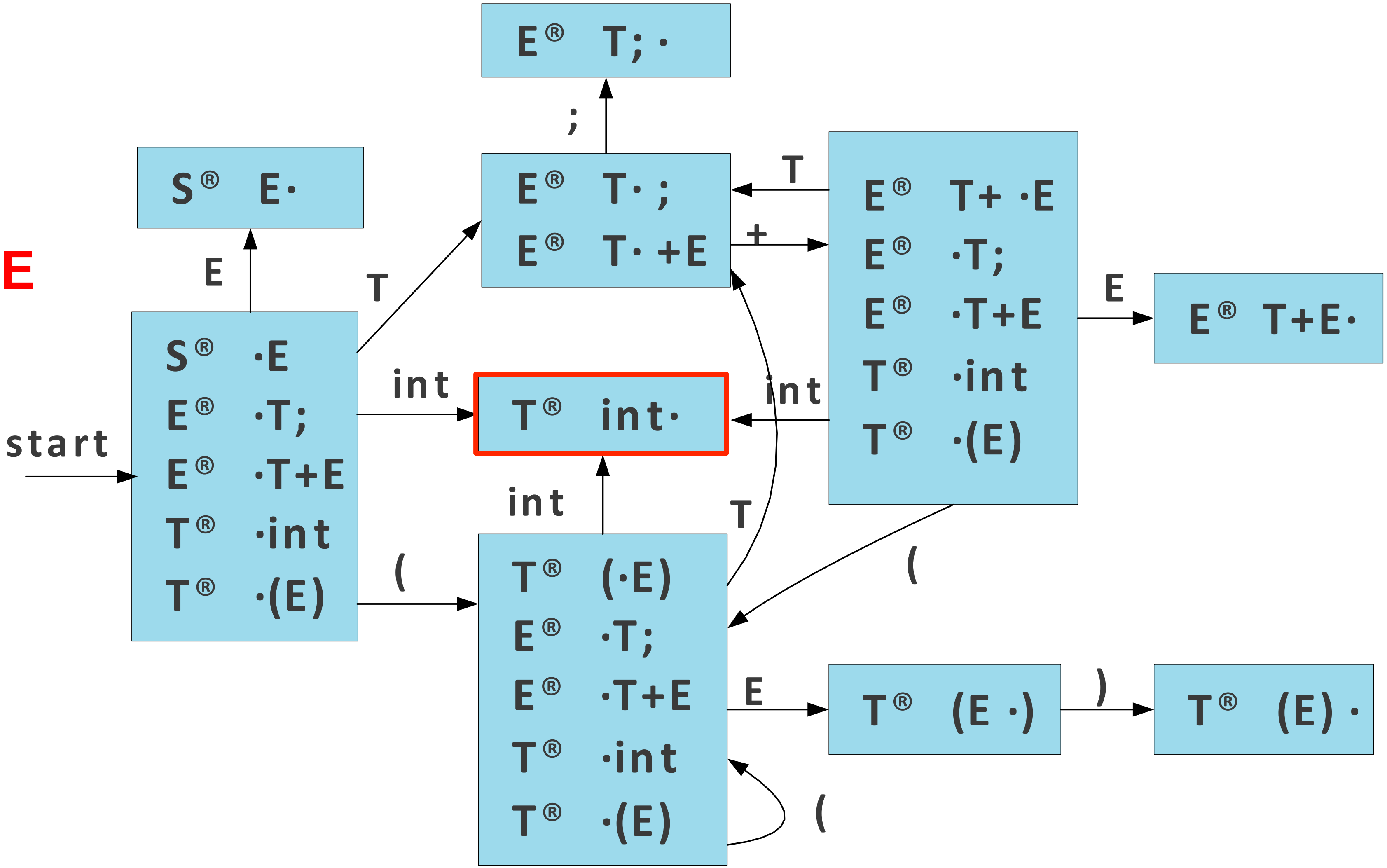
LR(0) Parsing

S → E
E → T;
E → T + E
T → int
T → (E)



LR(0) Parsing

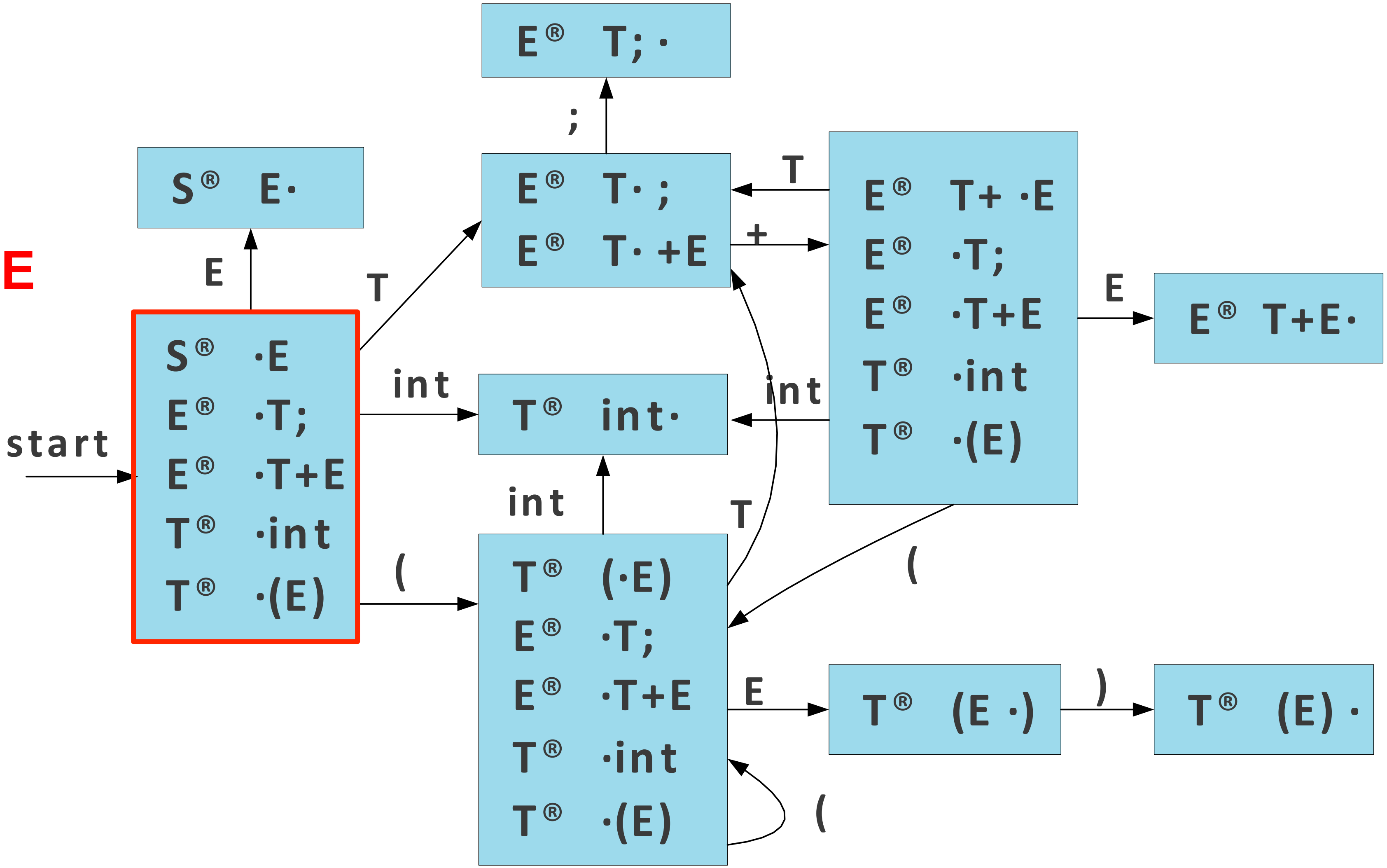
S $\rightarrow$ **E**
E $\rightarrow$ **T;**
E $\rightarrow$ **T + E**
T $\rightarrow$ **int**
T $\rightarrow$ **(E)**



T	+	(	int	+	int	;	)	;

LR(0) Parsing

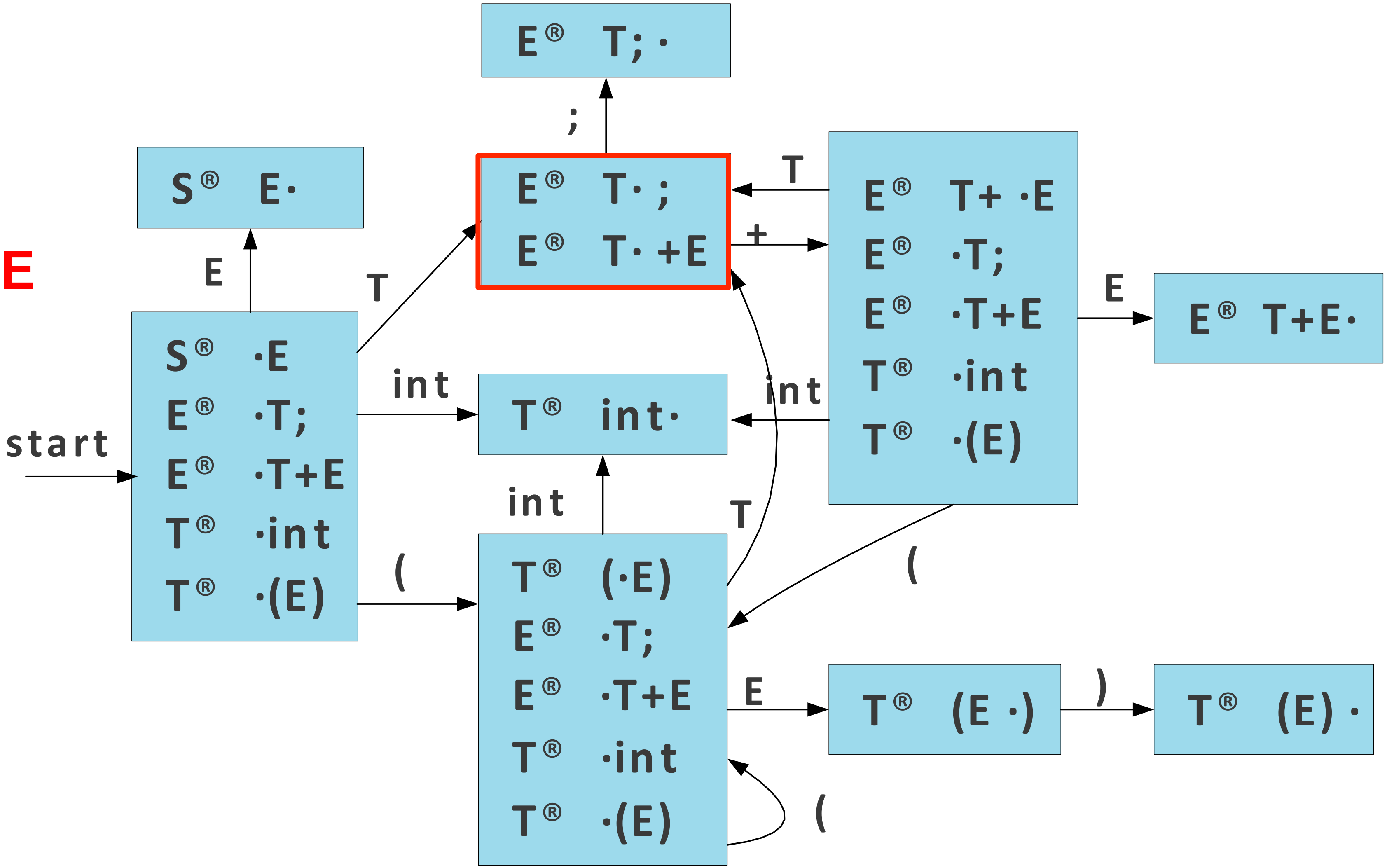
S → E
E → T;
E → T + E
T → int
T → (E)



T	+	(	int	+	int	;	)	;

LR(0) Parsing

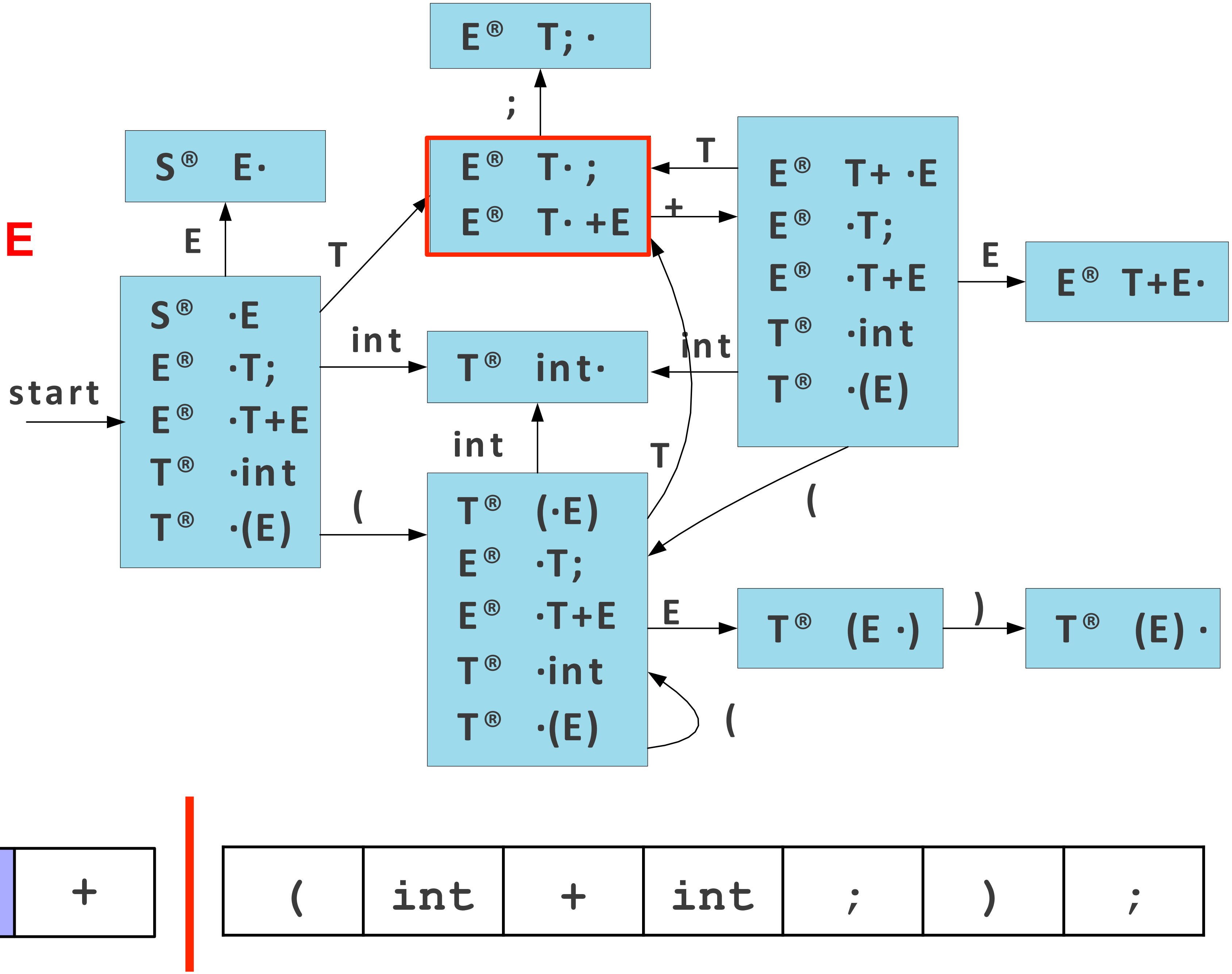
$S \rightarrow E$
$E \rightarrow T;$
$E \rightarrow T + E$
$T \rightarrow \text{int}$
$T \rightarrow (E)$



T		+	(	int	+	int	;	)	;

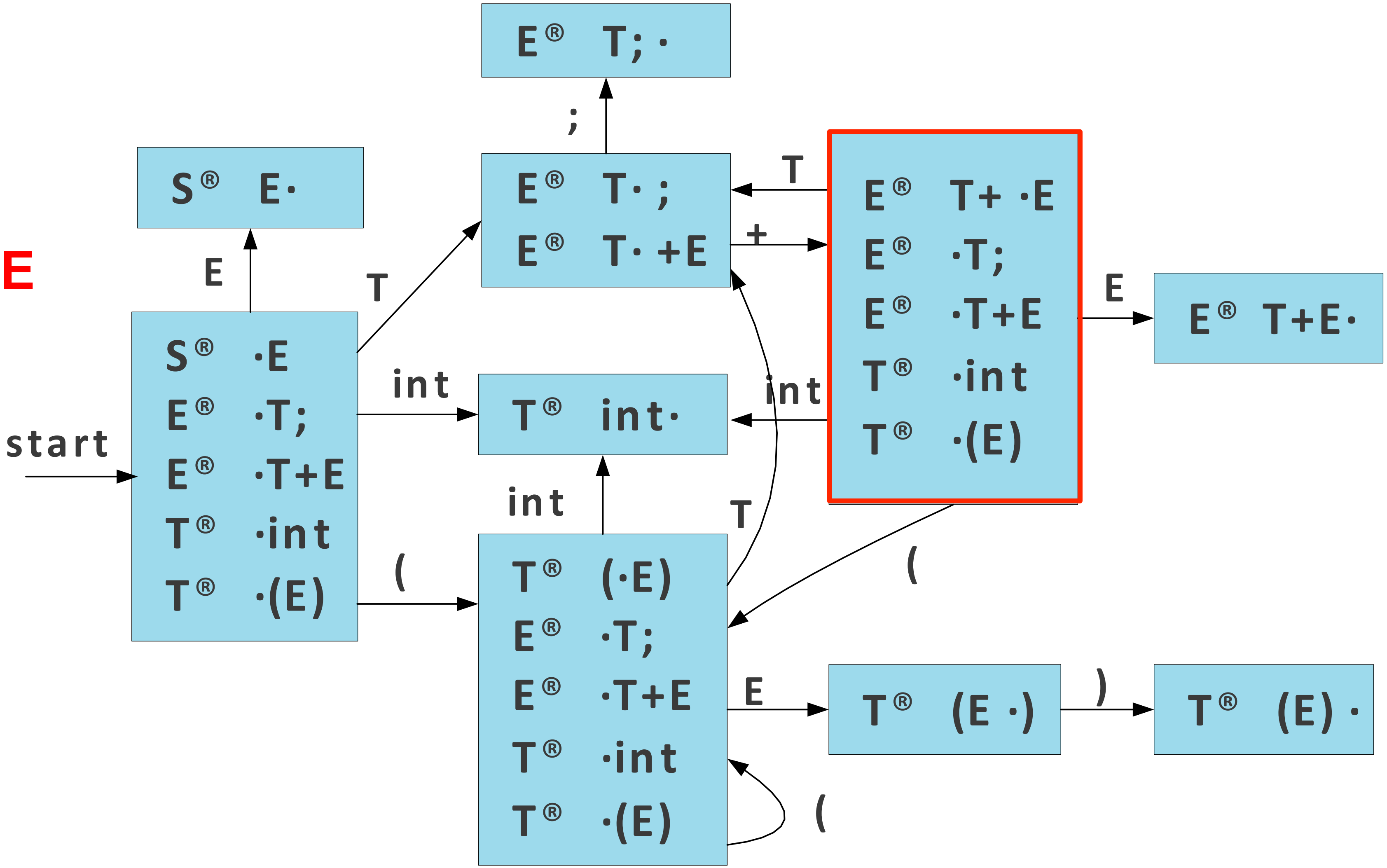
LR(0) Parsing

$S \rightarrow E$
$E \rightarrow T;$
$E \rightarrow T + E$
$T \rightarrow int$
$T \rightarrow (E)$



LR(0) Parsing

$S \rightarrow E$
$E \rightarrow T;$
$E \rightarrow T + E$
$T \rightarrow \text{int}$
$T \rightarrow (E)$

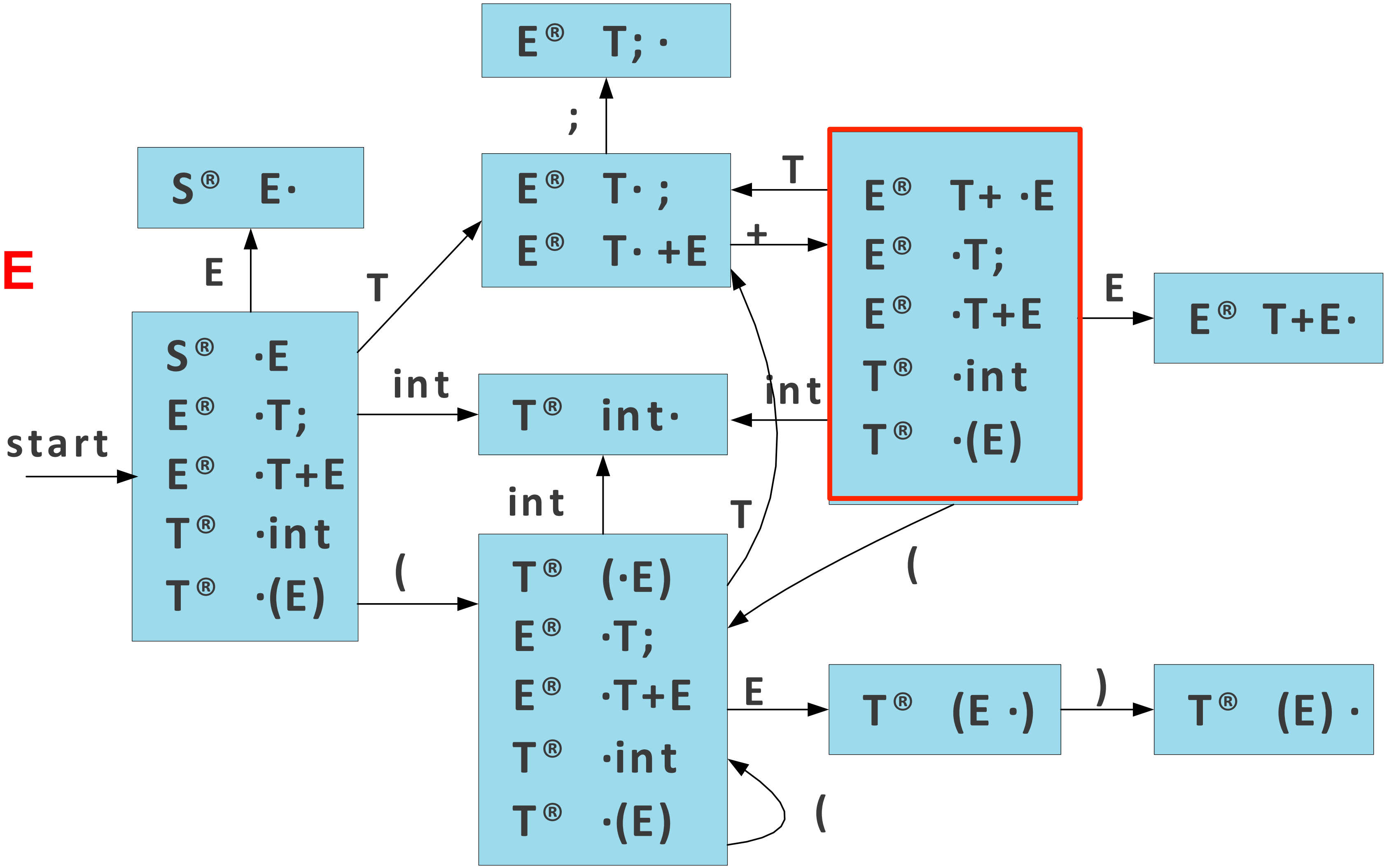


T	+

(	int	+	int	;	)	;

LR(0) Parsing

$S \rightarrow E$
$E \rightarrow T;$
$E \rightarrow T + E$
$T \rightarrow int$
$T \rightarrow (E)$

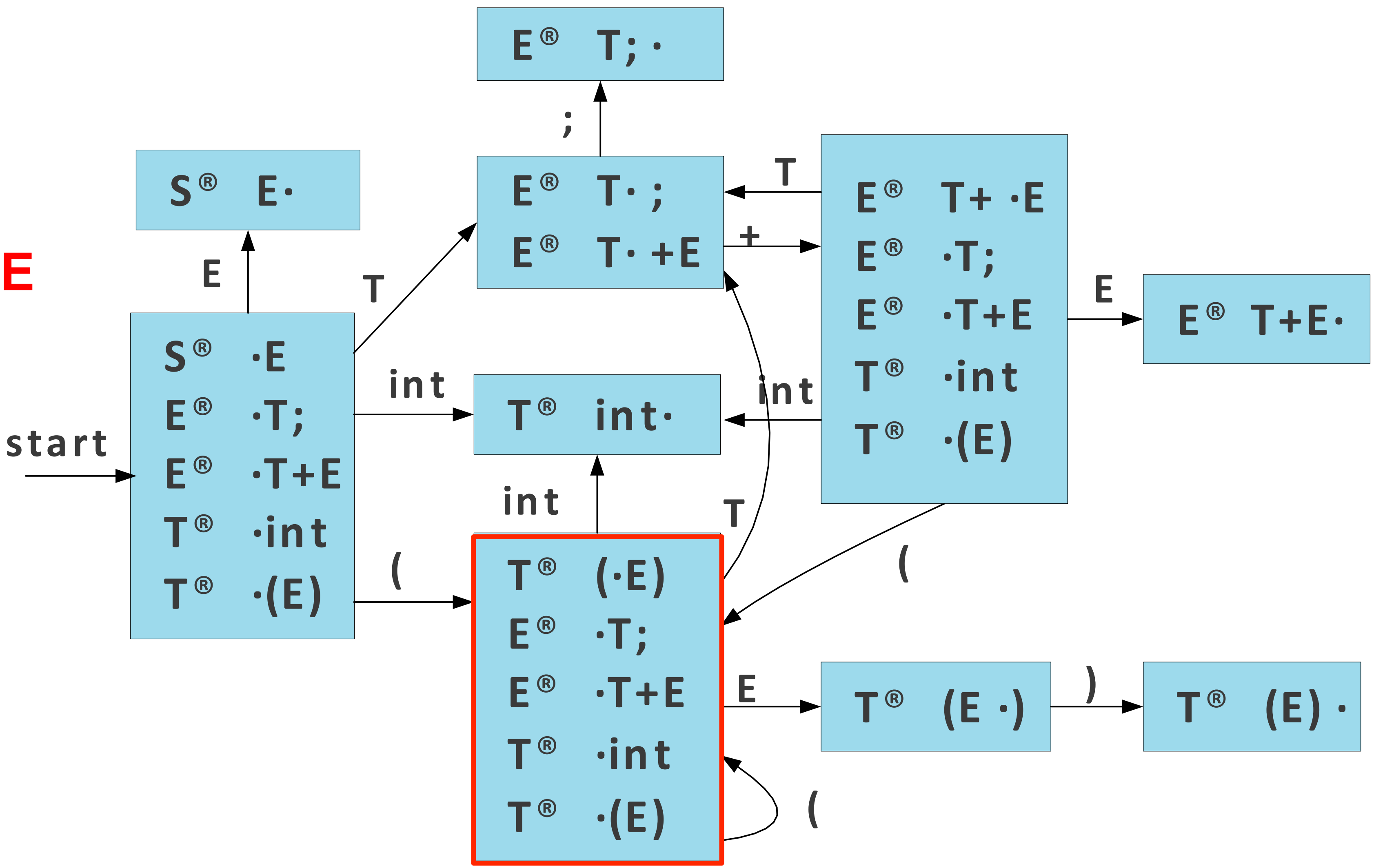


T	+	(

int	+	int	;	)	;

LR(0) Parsing

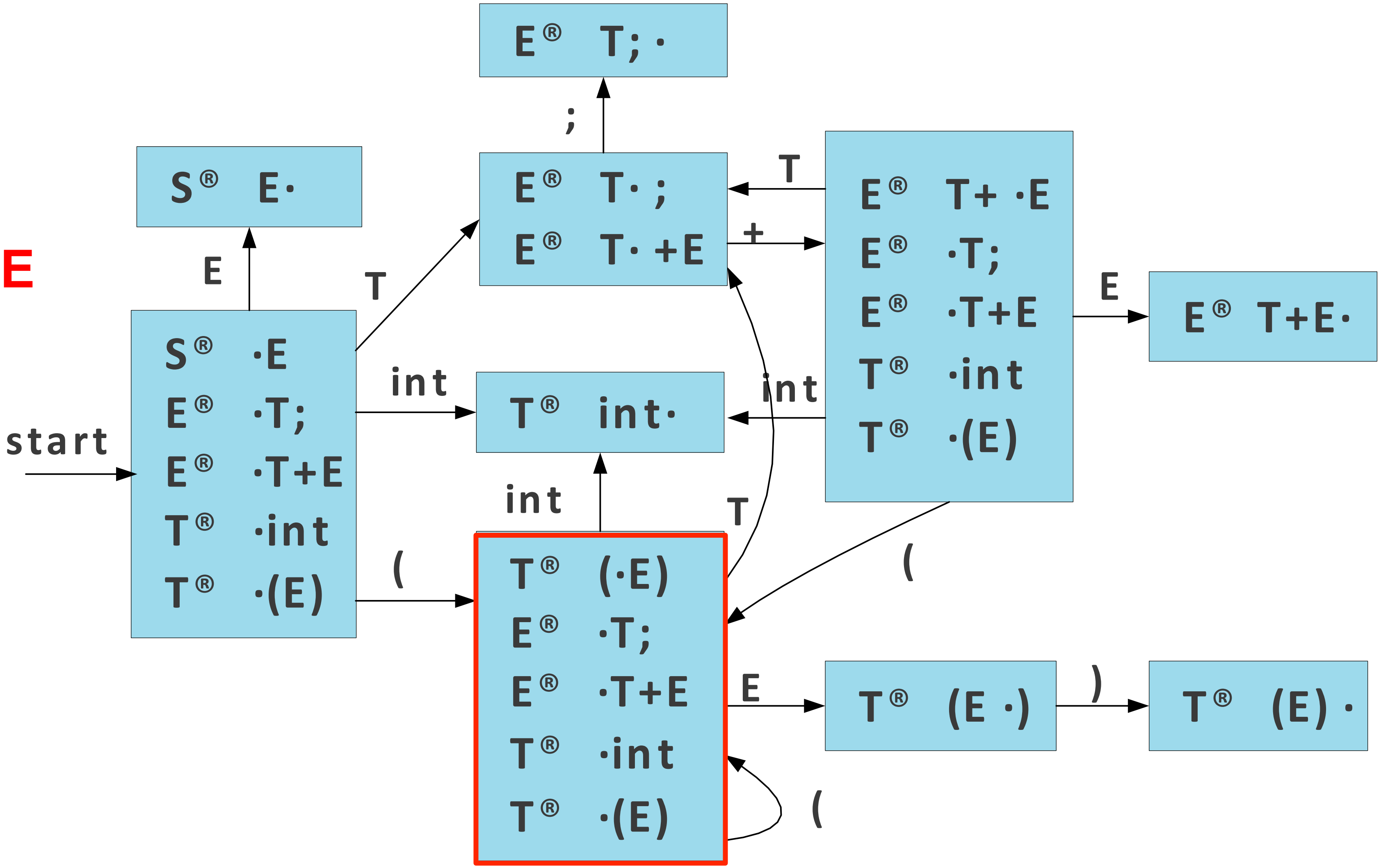
$S \rightarrow E$
$E \rightarrow T;$
$E \rightarrow T + E$
$T \rightarrow \text{int}$
$T \rightarrow (E)$



T	+	(	int	+	int	;	)	;

LR(0) Parsing

$S \rightarrow E$
$E \rightarrow T;$
$E \rightarrow T + E$
$T \rightarrow int$
$T \rightarrow (E)$



T	+	(	int

+	int	;	)	;

LR(0) Parsing

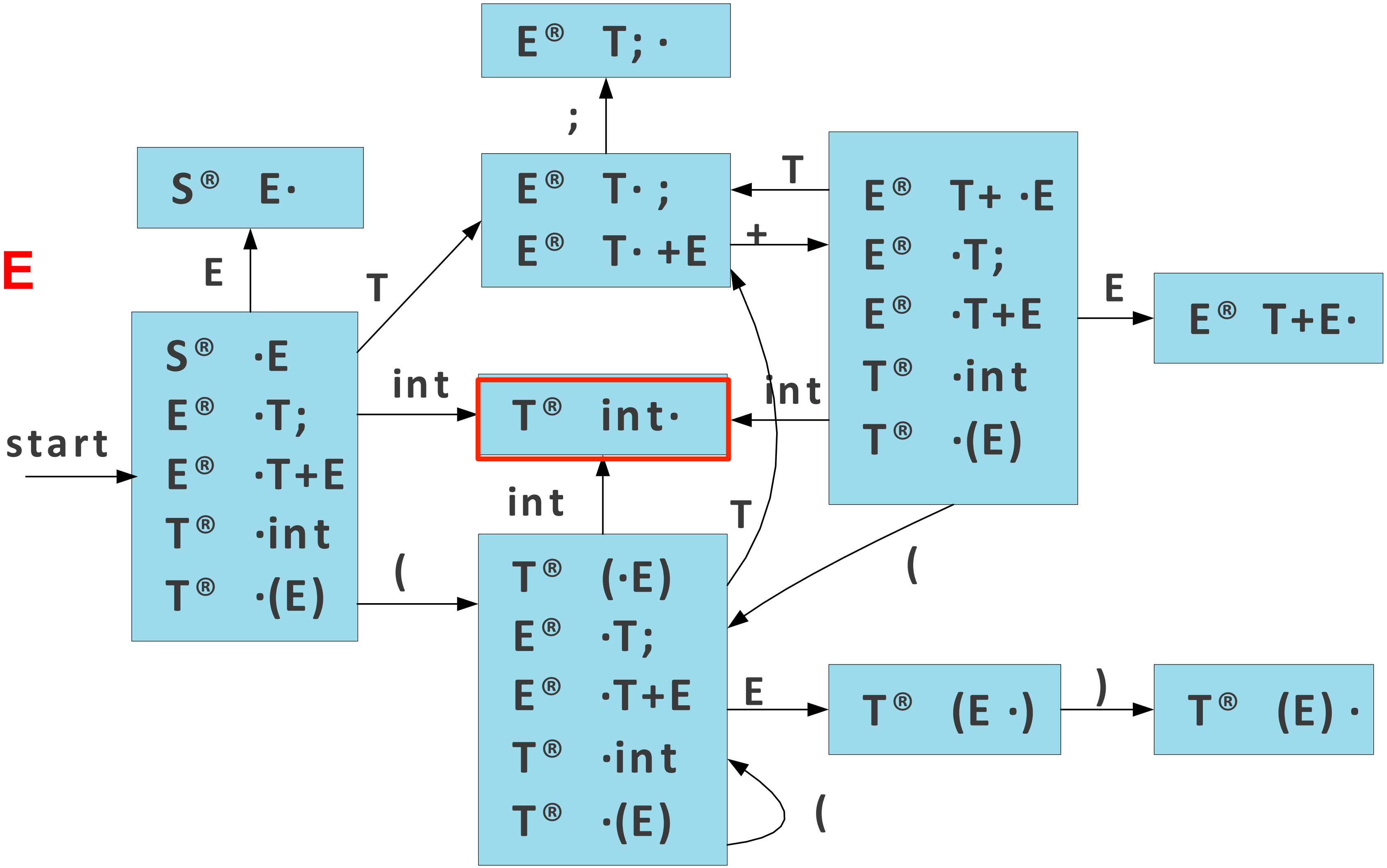
$S \rightarrow E$

$E \rightarrow T;$

$E \rightarrow T + E$

$T \rightarrow int$

$T \rightarrow (E)$



T	+	(	int

+	int	;	)	;

LR(0) Parsing

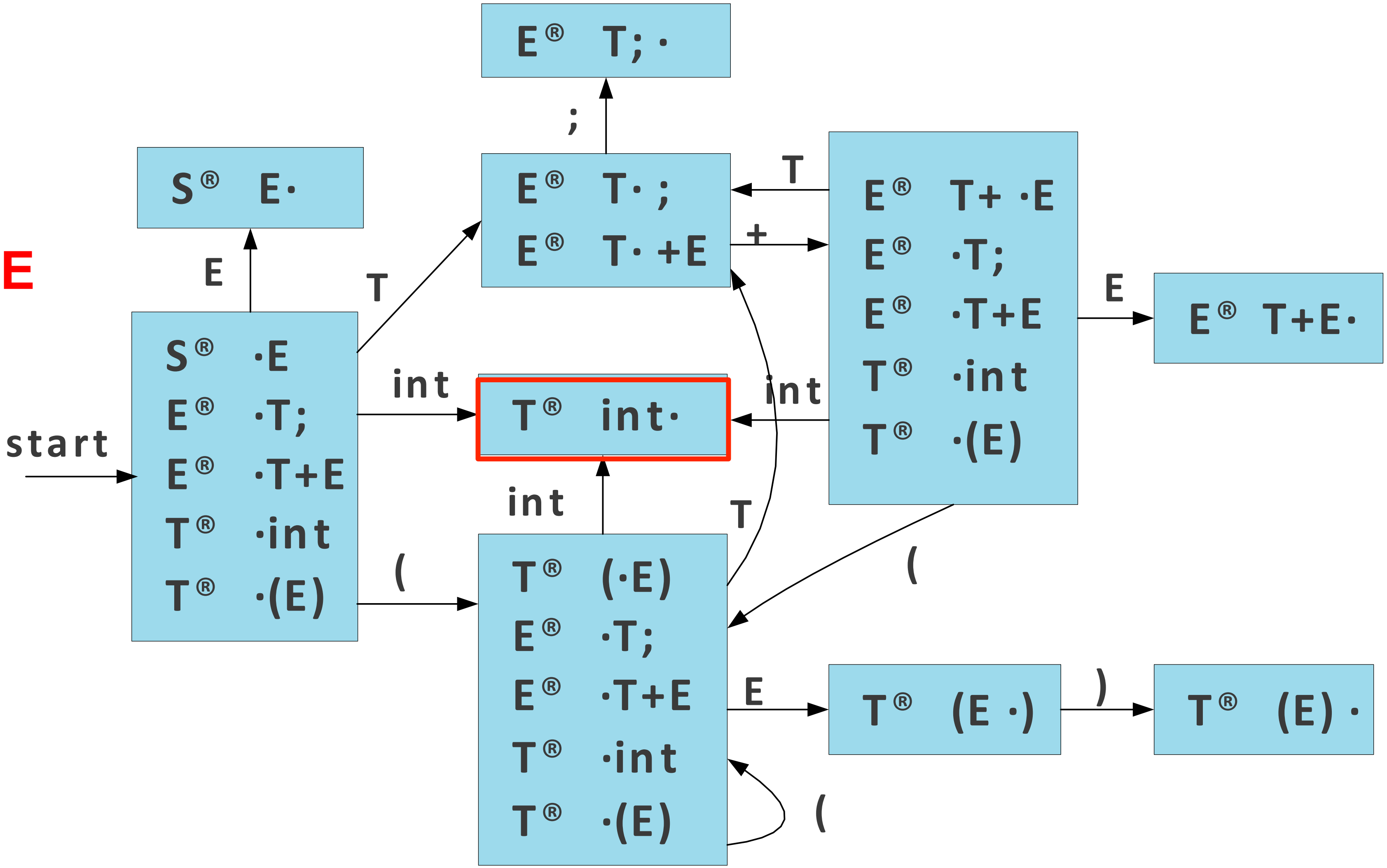
$S \rightarrow E$

$E \rightarrow T;$

$E \rightarrow T + E$

$T \rightarrow \text{int}$

$T \rightarrow (E)$

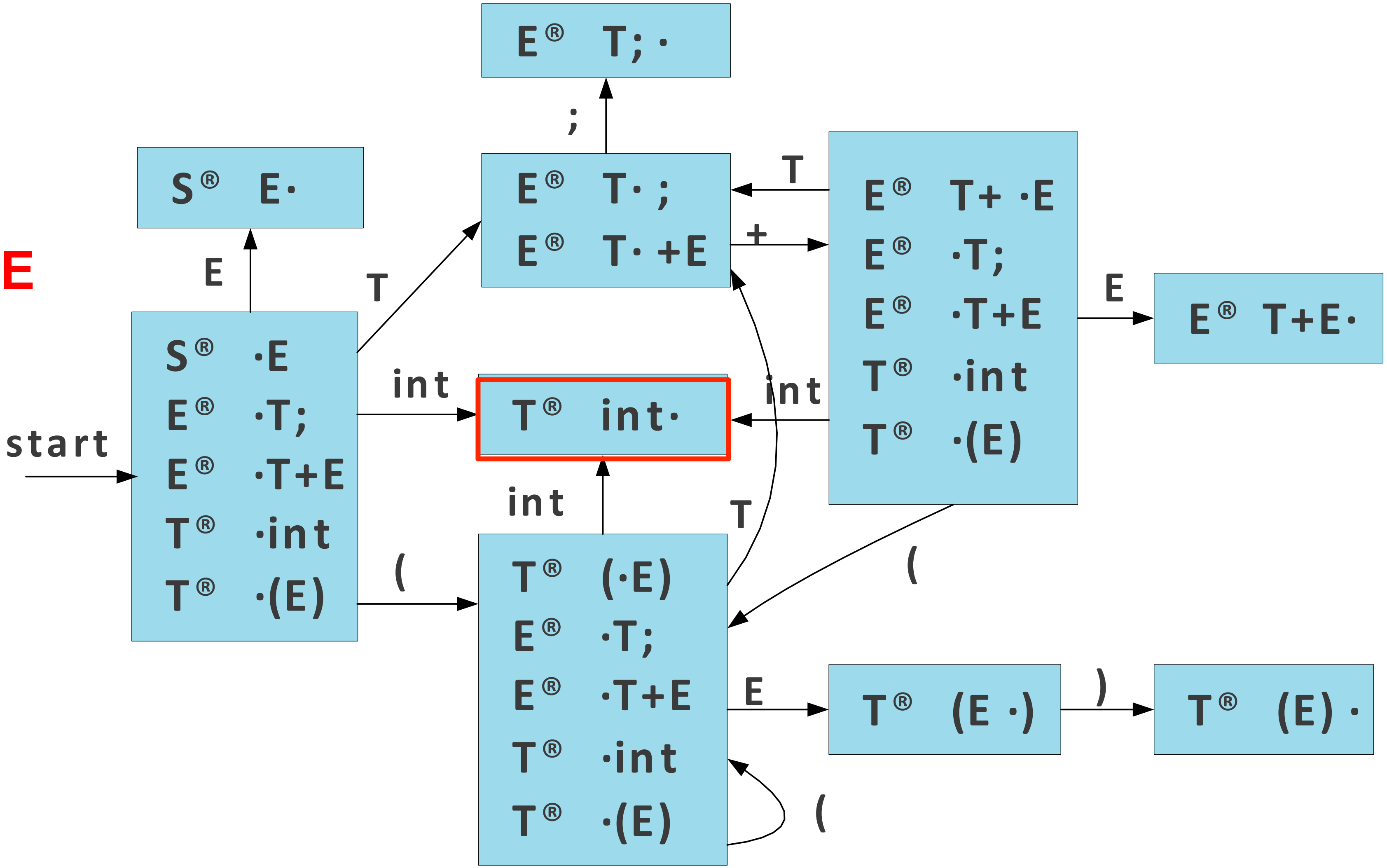


T	+	(

+	int	;	)	;

LR(0) Parsing

S → E
E → T;
E → T + E
T → int
T → (E)

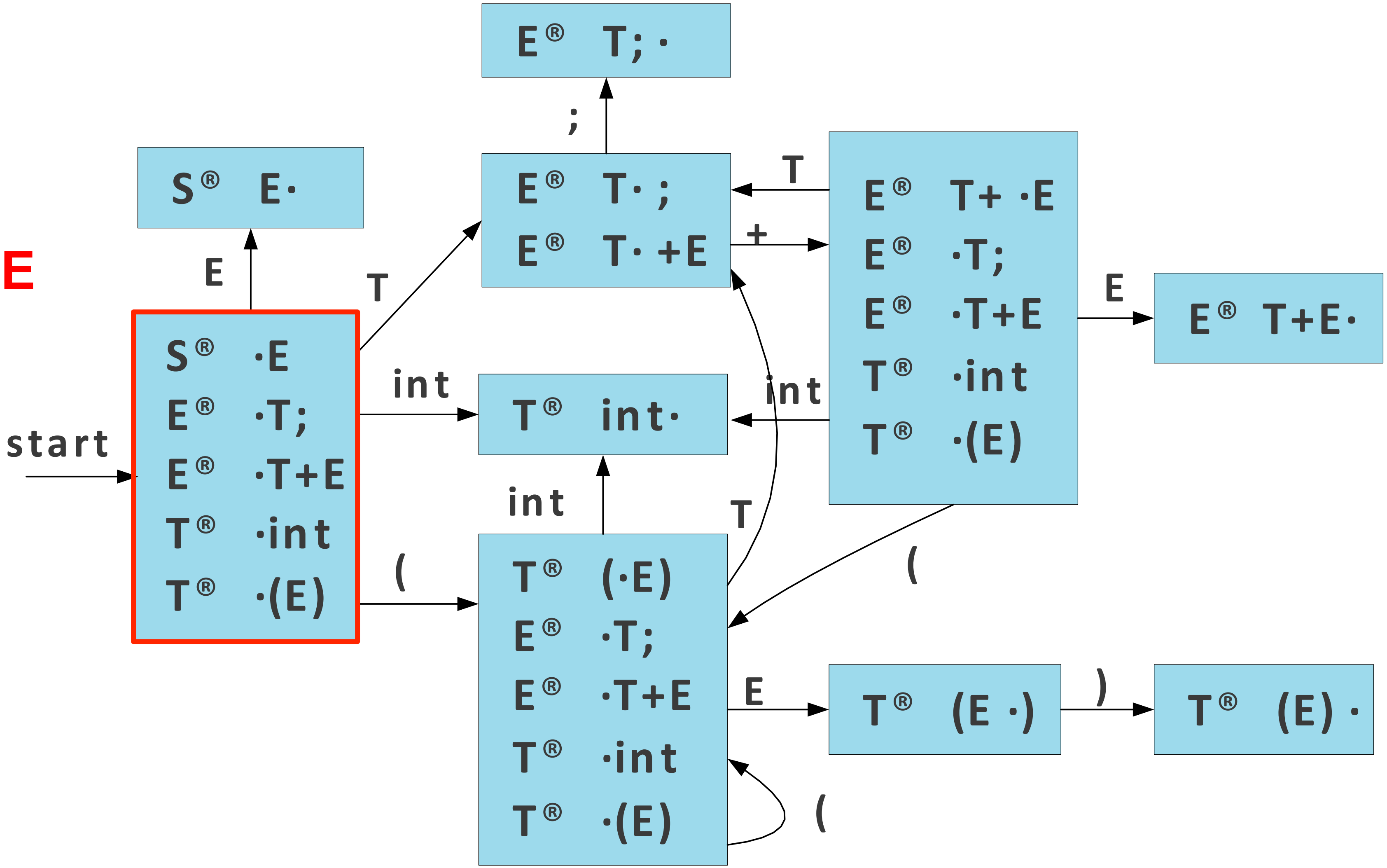


T	+	(	T

+	int	;	)	;

LR(0) Parsing

S → E
E → T;
E → T + E
T → int
T → (E)

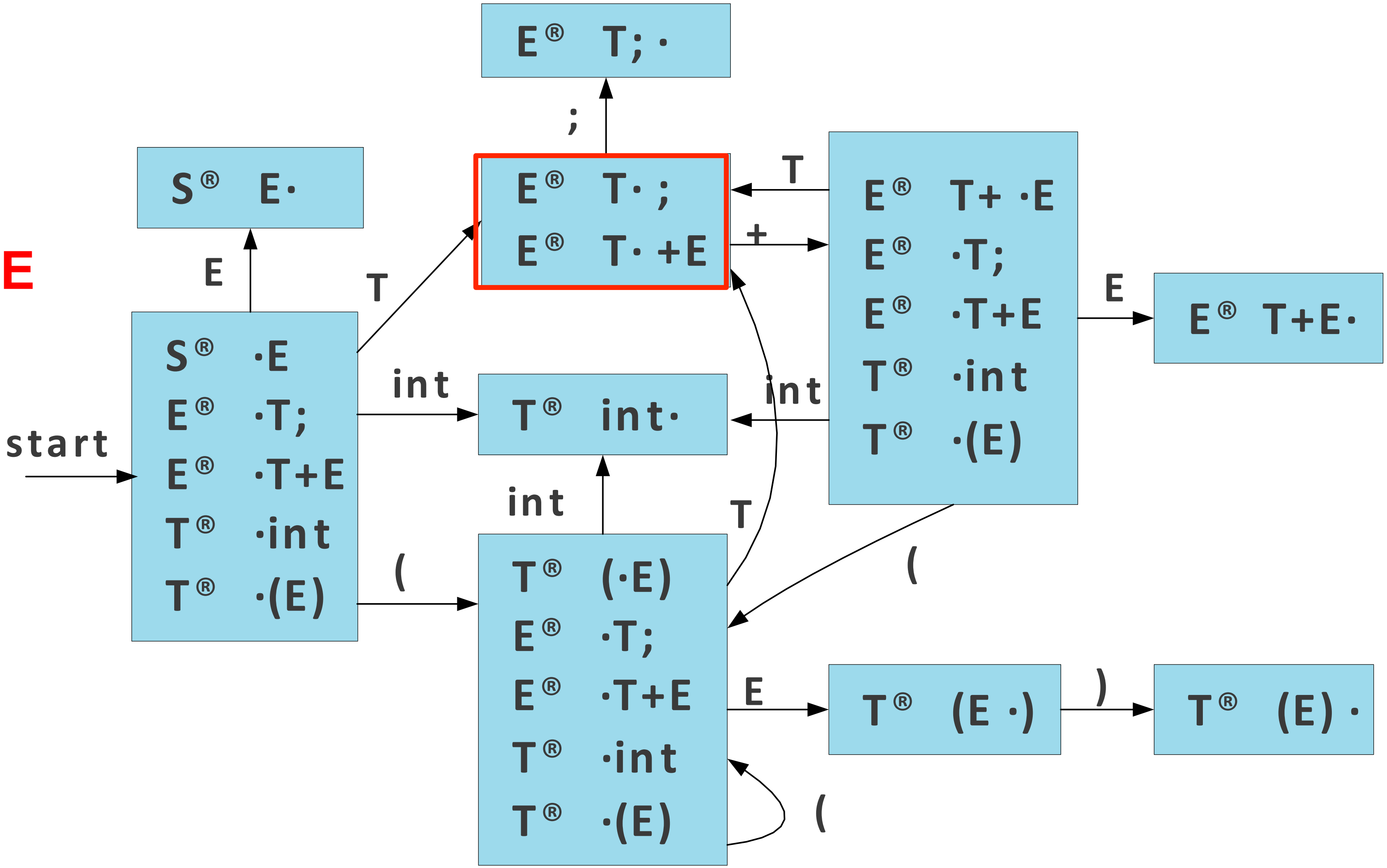


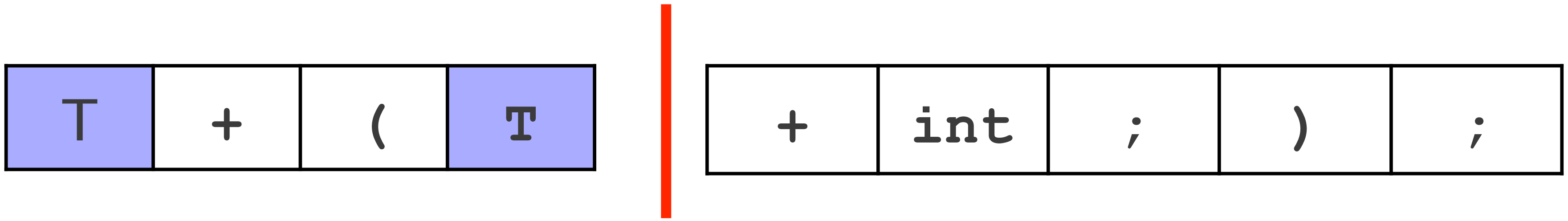
T	+	(	T

+	int	;	)	;

LR(0) Parsing

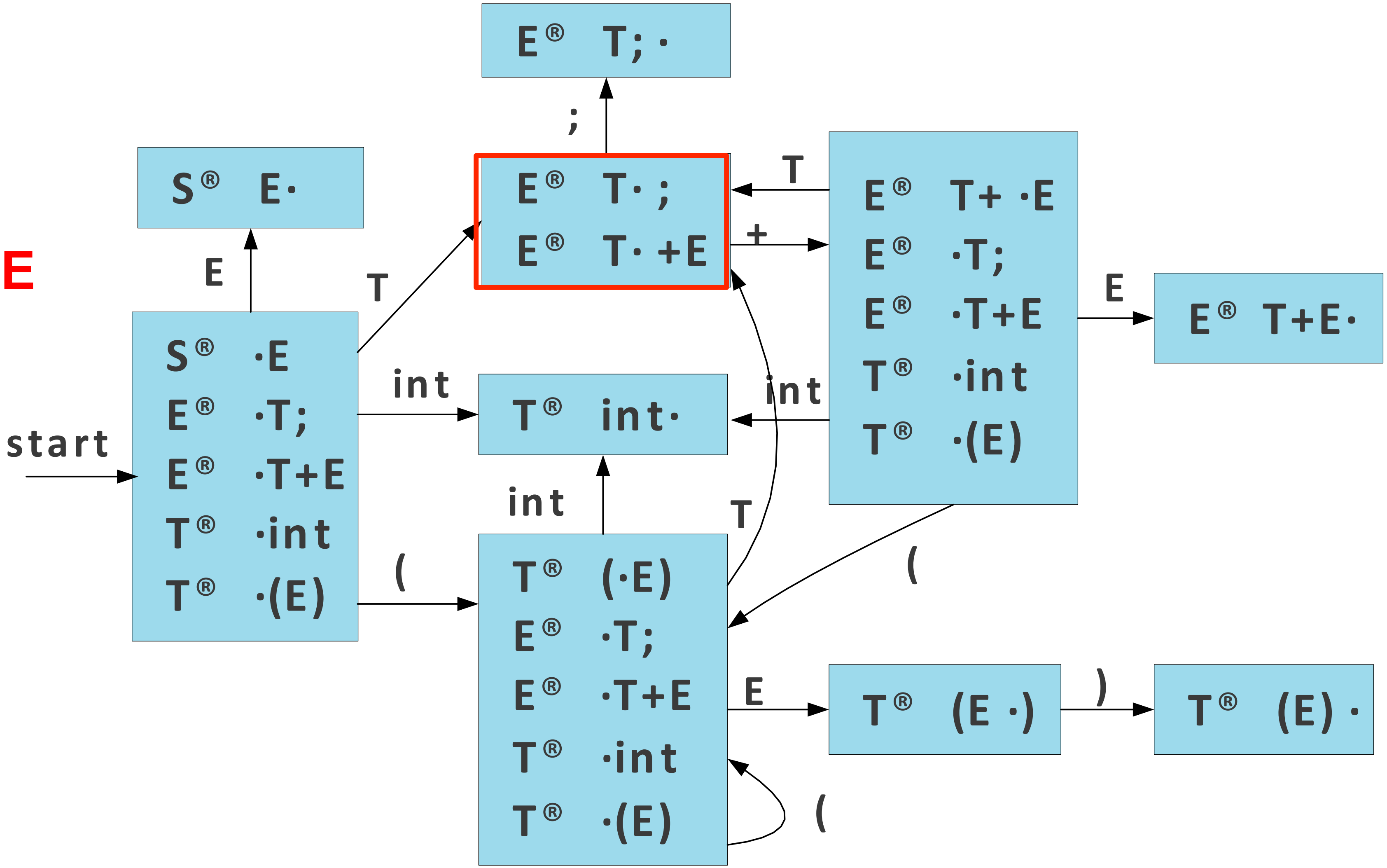
S → E
E → T;
E → T + E
T → int
T → (E)

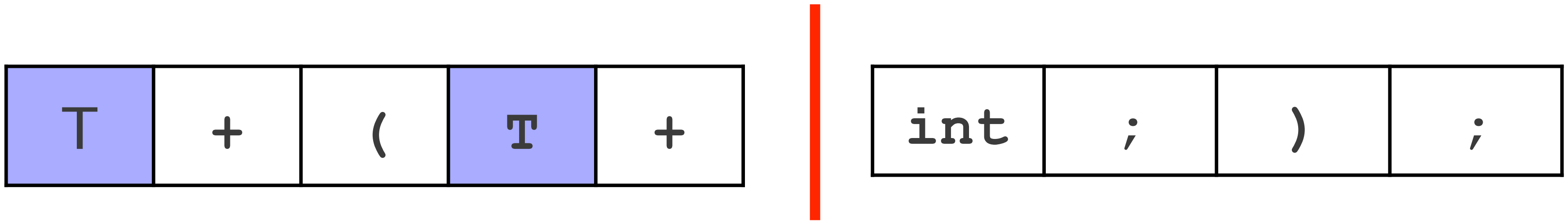




LR(0) Parsing

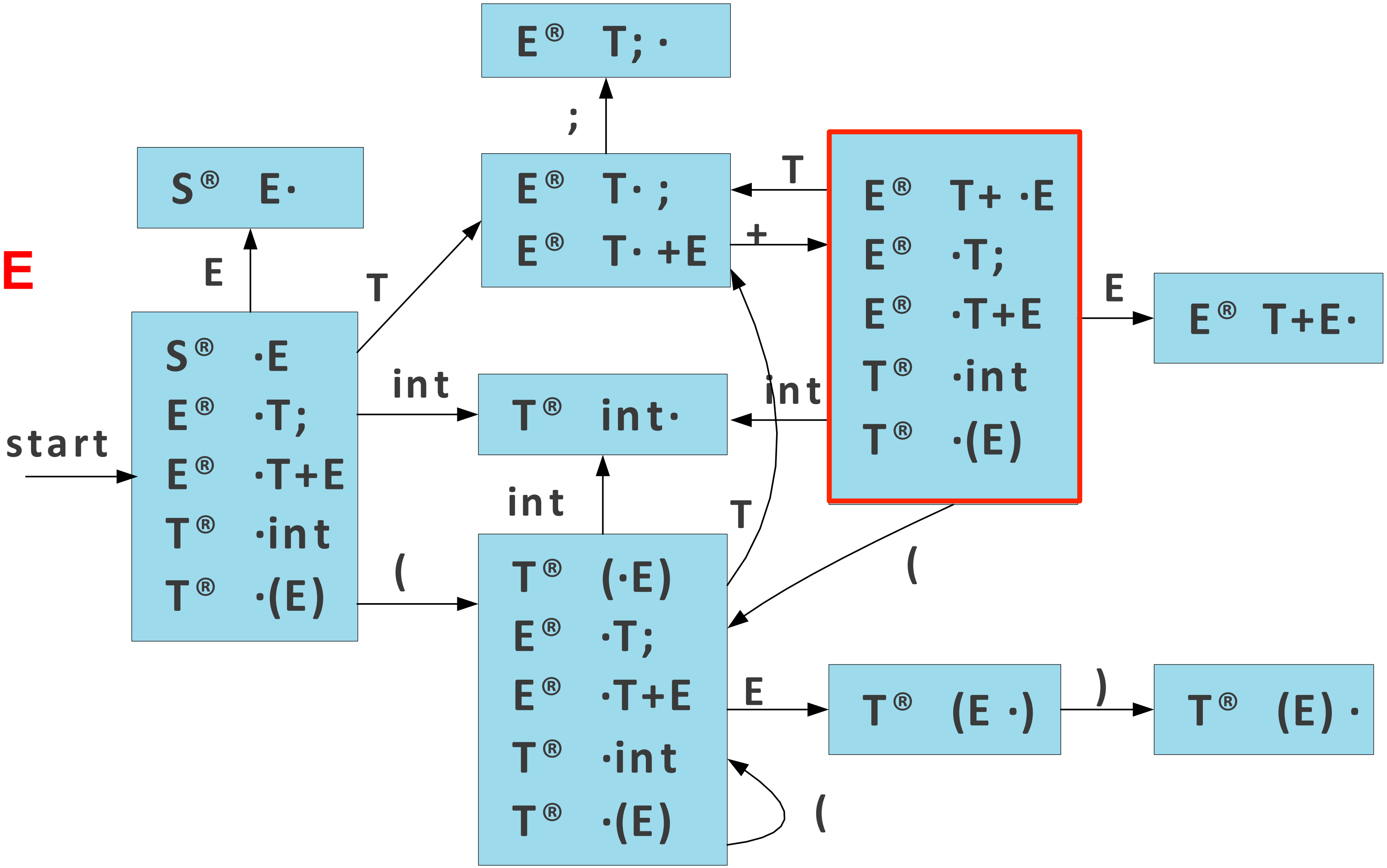
$S \rightarrow E$
$E \rightarrow T;$
$E \rightarrow T + E$
$T \rightarrow \text{int}$
$T \rightarrow (E)$





LR(0) Parsing

$S \rightarrow E$
$E \rightarrow T;$
$E \rightarrow T + E$
$T \rightarrow \text{int}$
$T \rightarrow (E)$

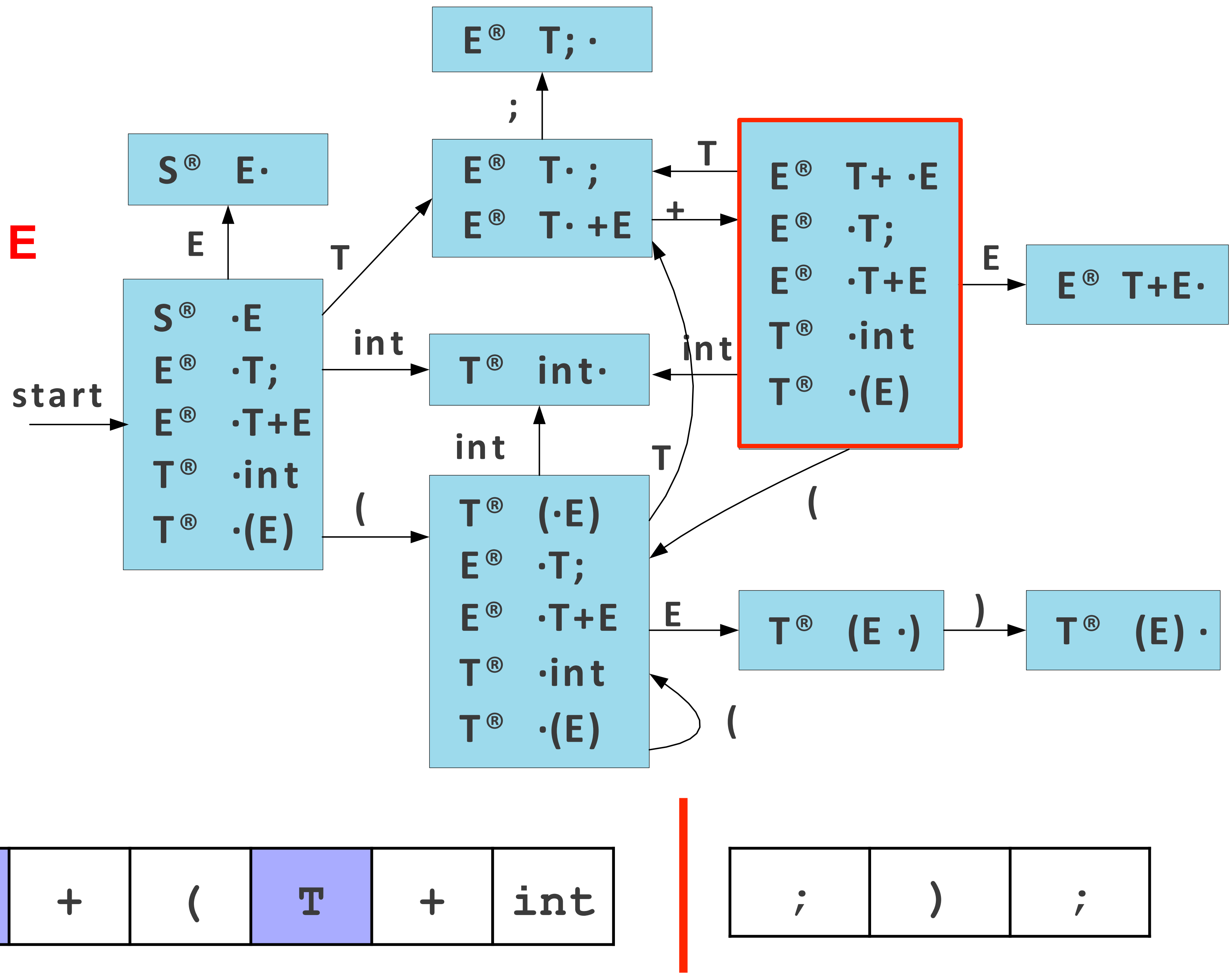


T	+	(	T	+

int	;	)	;

LR(0) Parsing

$S \rightarrow E$
$E \rightarrow T;$
$E \rightarrow T + E$
$T \rightarrow \text{int}$
$T \rightarrow (E)$



LR(0) Parsing

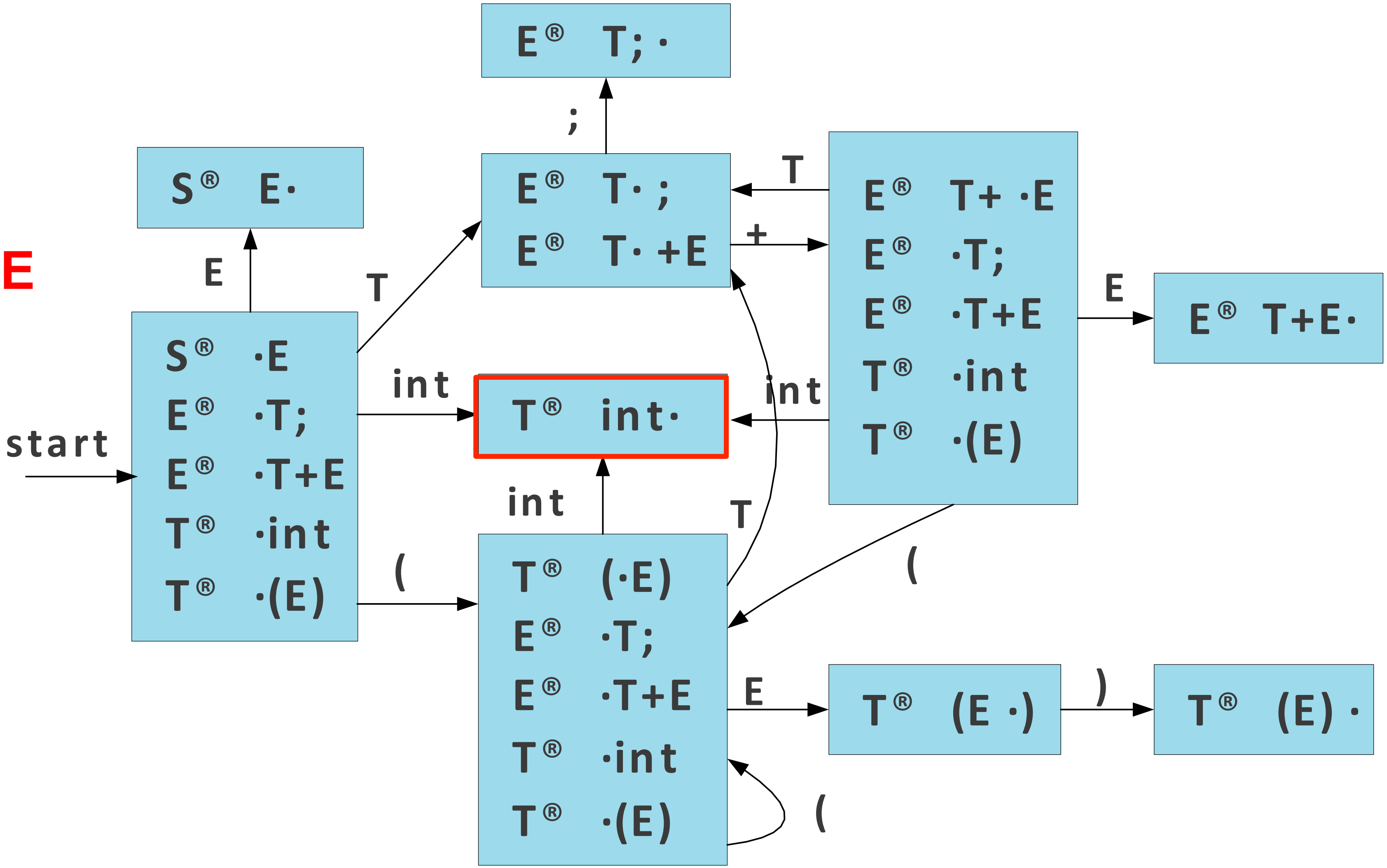
$S \rightarrow E$

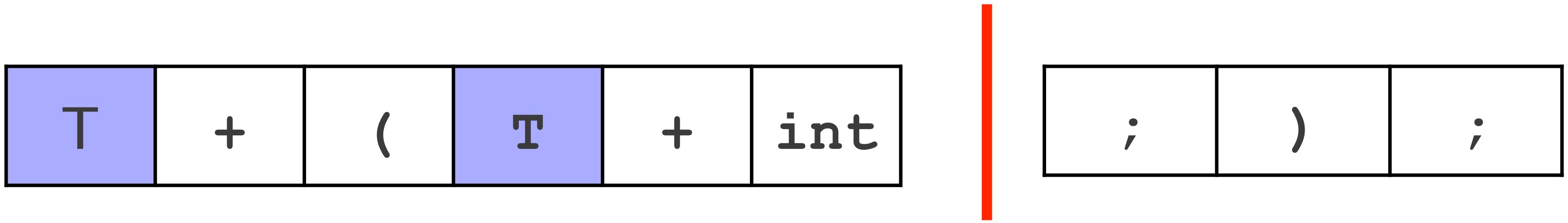
$E \rightarrow T;$

$E \rightarrow T + E$

$T \rightarrow \text{int}$

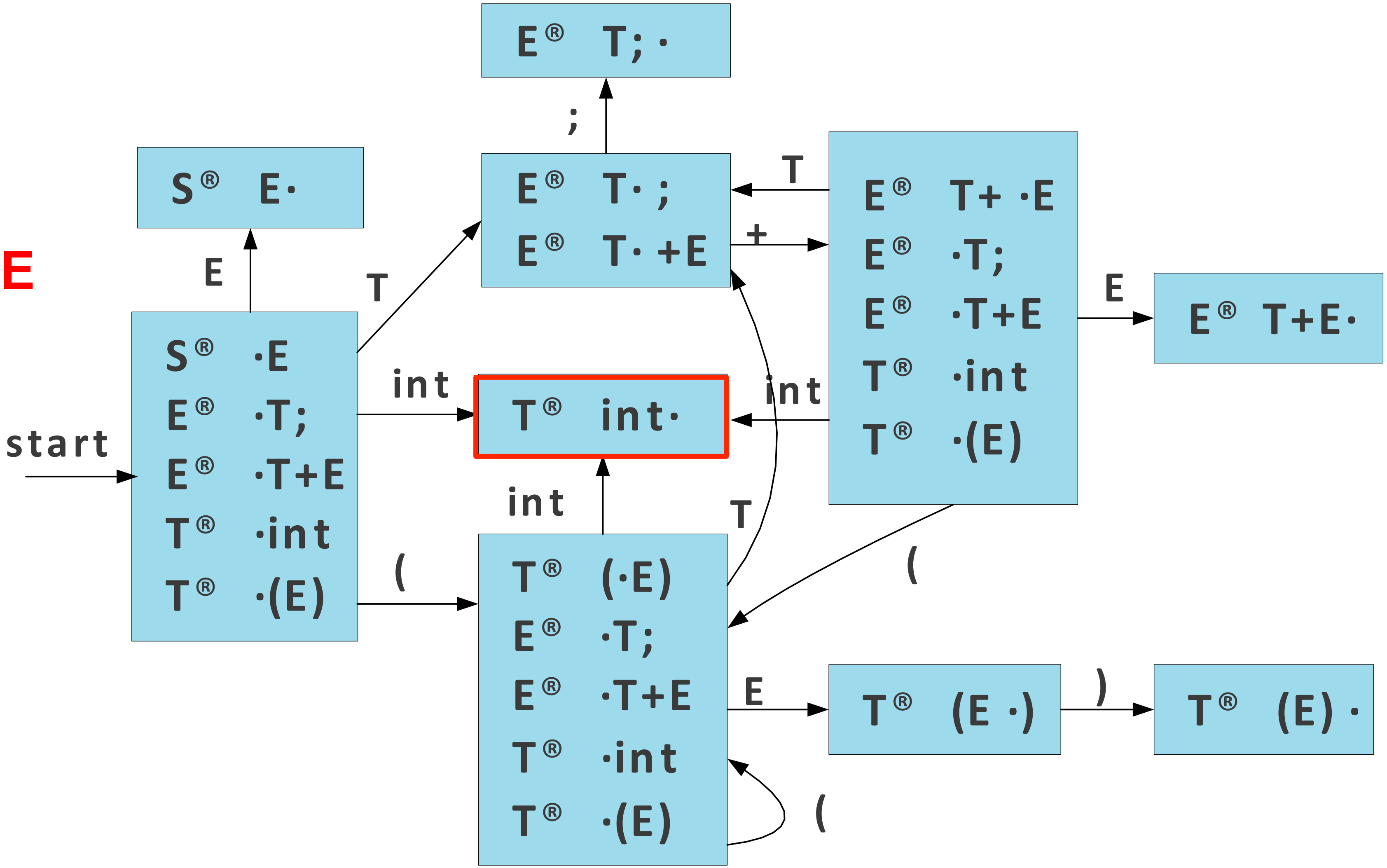
$T \rightarrow (E)$





LR(0) Parsing

S → E
E → T;
E → T + E
T → int
T → (E)

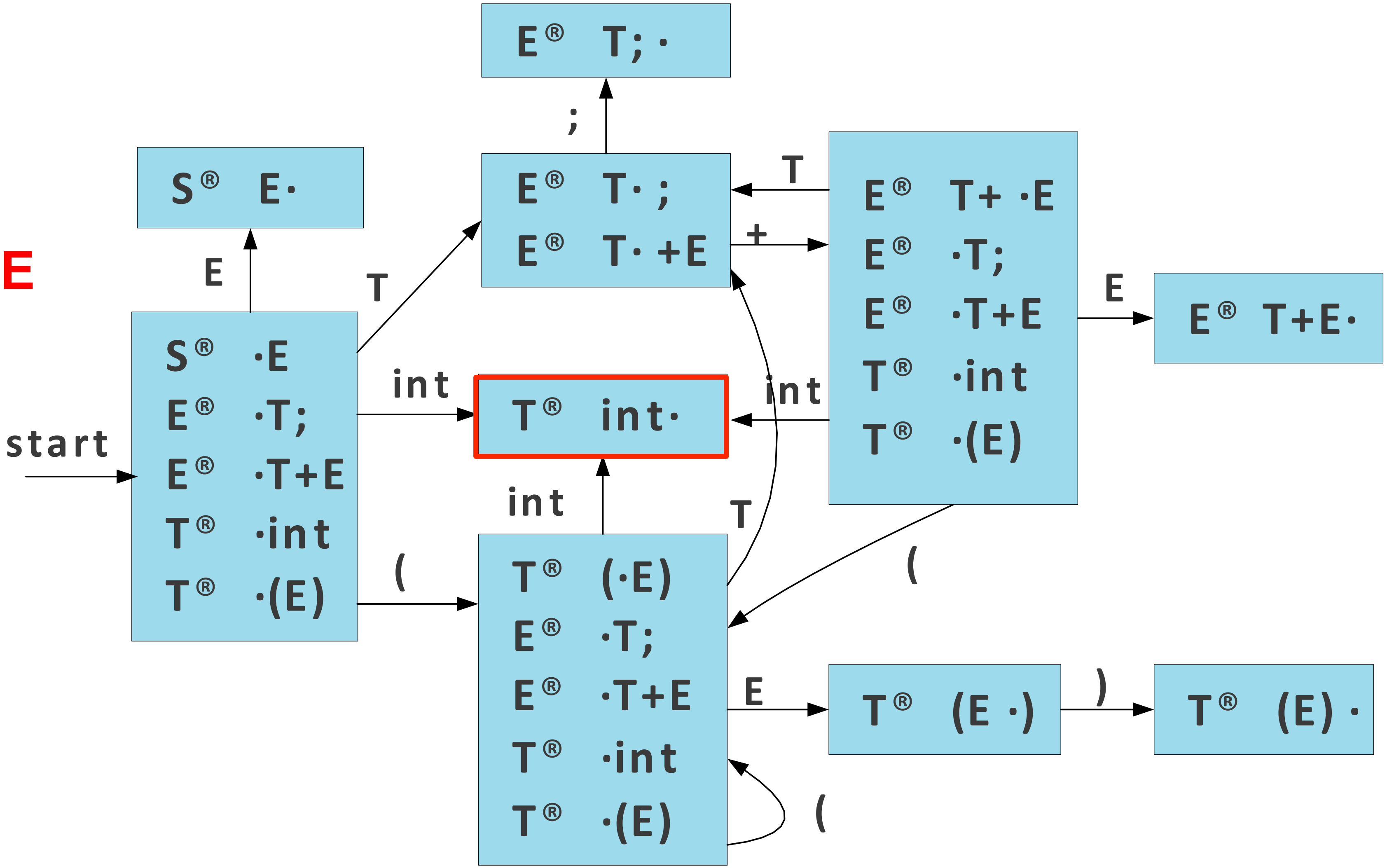


T	+	(	T	+

;	)	;

LR(0) Parsing

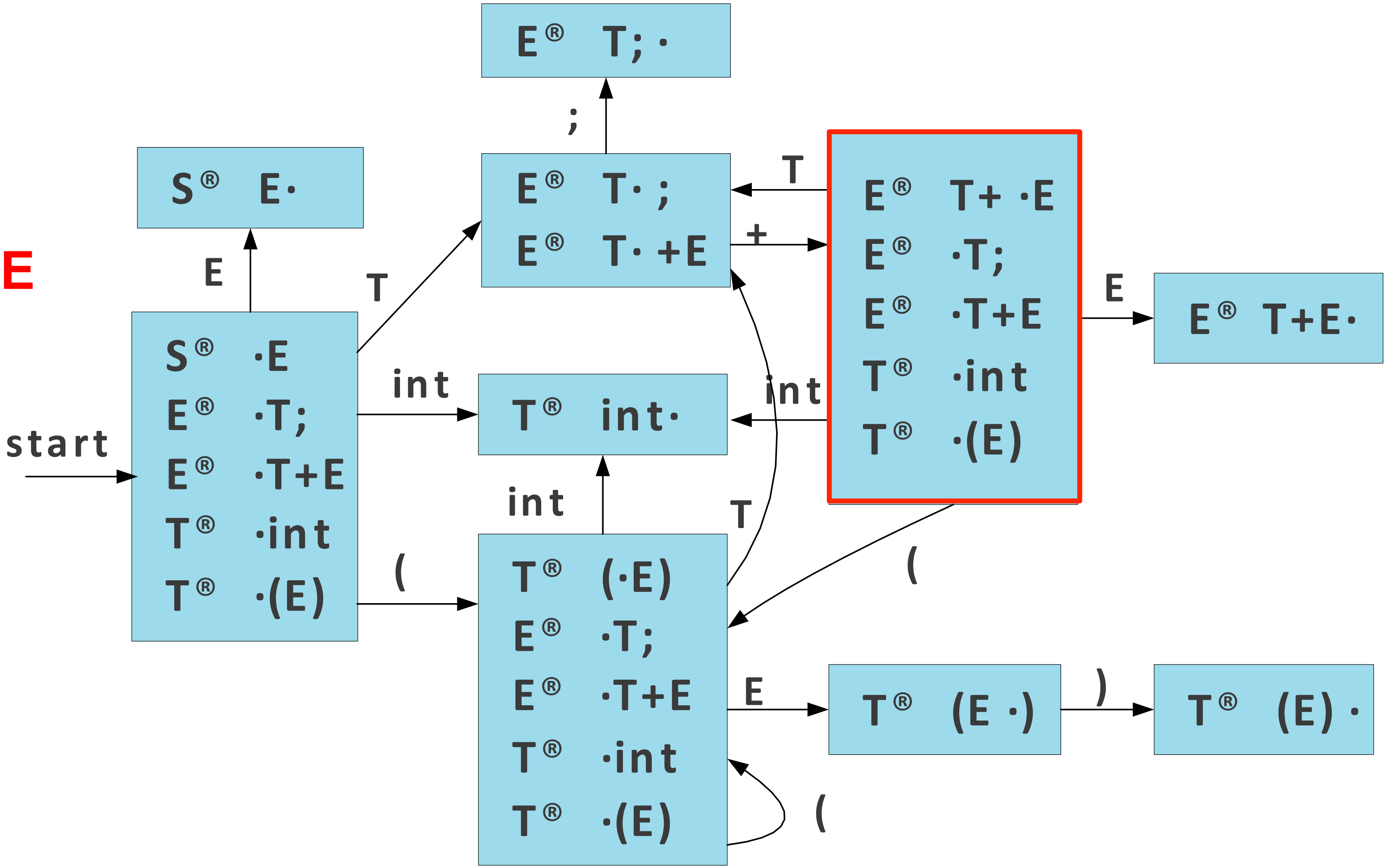
$S \rightarrow E$
$E \rightarrow T;$
$E \rightarrow T + E$
$T \rightarrow \text{int}$
$T \rightarrow (E)$



T	+	(	T	+	T		;	)	;

LR(0) Parsing

$S \rightarrow E$
$E \rightarrow T;$
$E \rightarrow T + E$
$T \rightarrow \text{int}$
$T \rightarrow (E)$

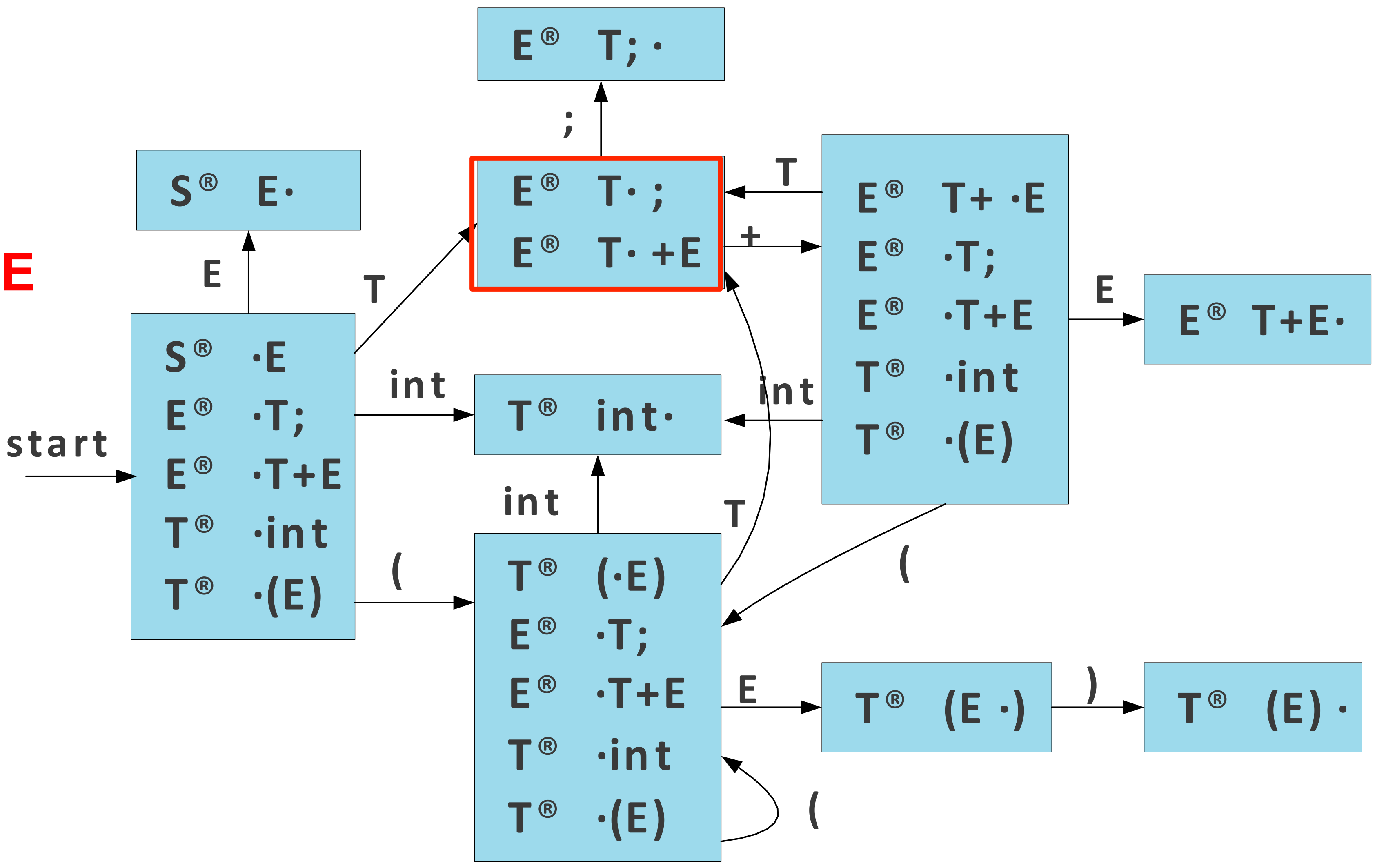


T	+	(	T	+	T

;	)	;

LR(0) Parsing

S → E
E → T;
E → T + E
T → int
T → (E)

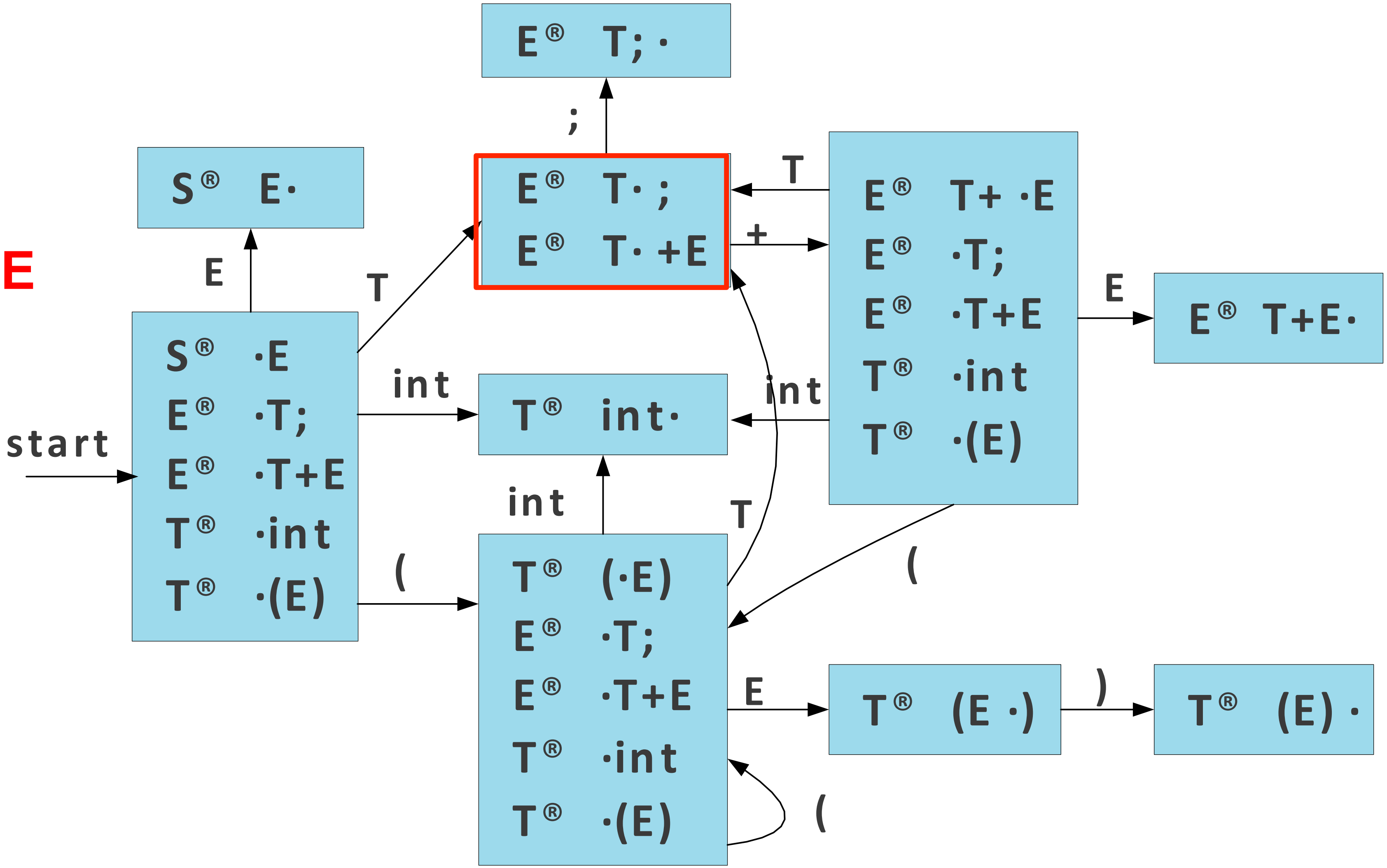


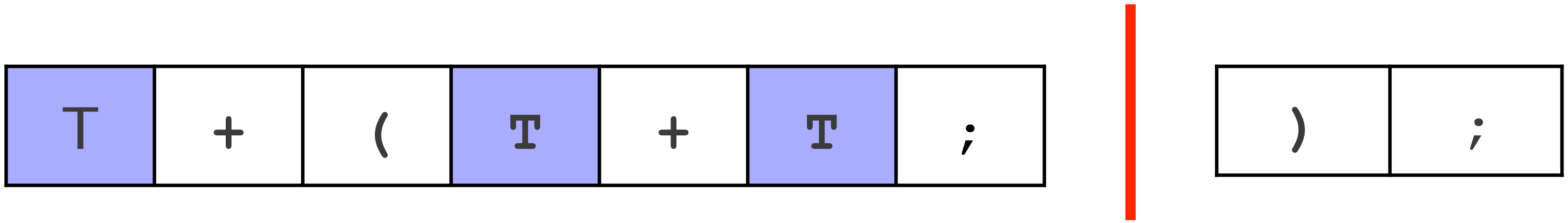
T	+	(	T	+	T

;	)	;

LR(0) Parsing

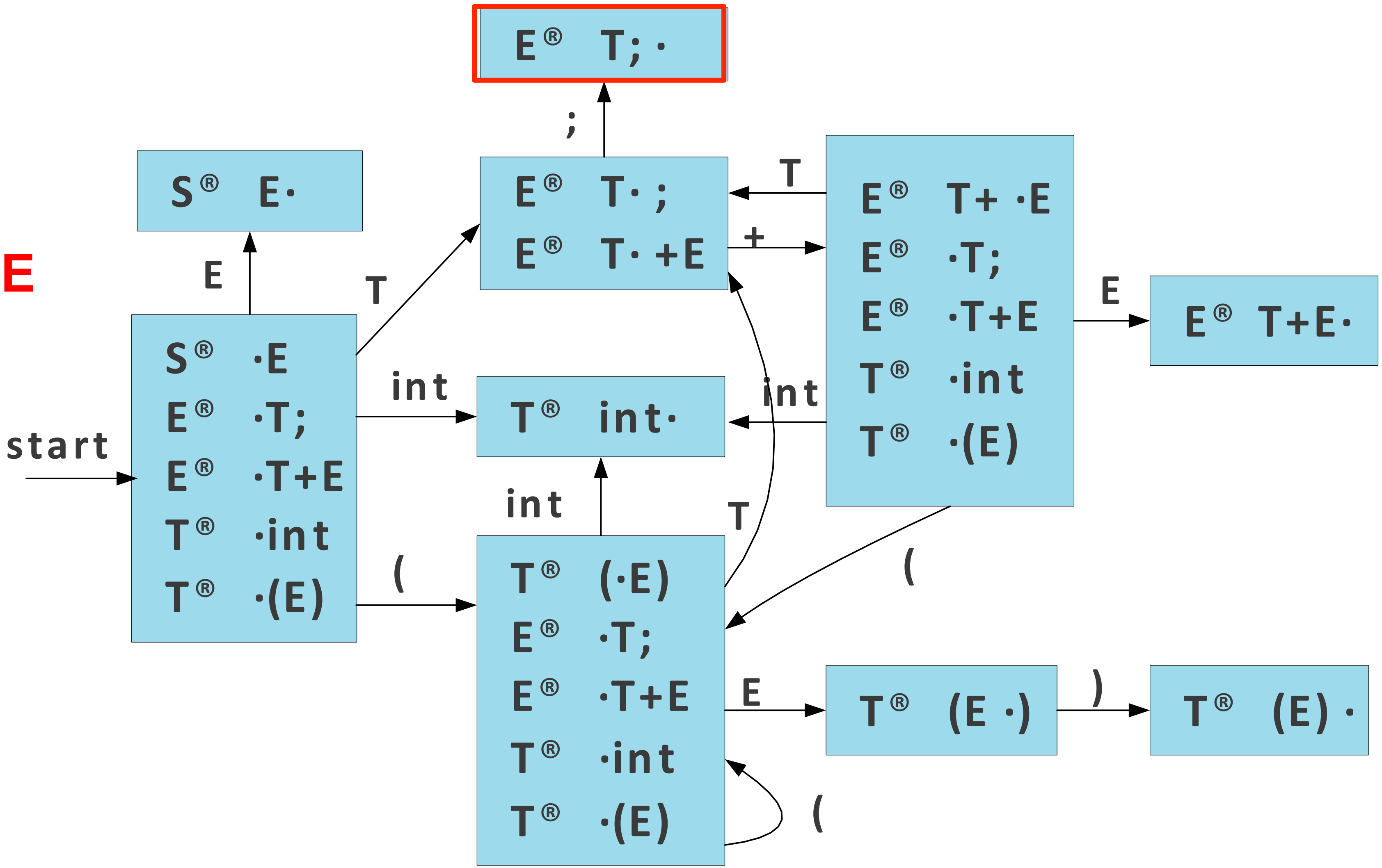
$S \rightarrow E$
$E \rightarrow T;$
$E \rightarrow T + E$
$T \rightarrow \text{int}$
$T \rightarrow (E)$





LR(0) Parsing

S → E
E → T;
E → T + E
T → int
T → (E)

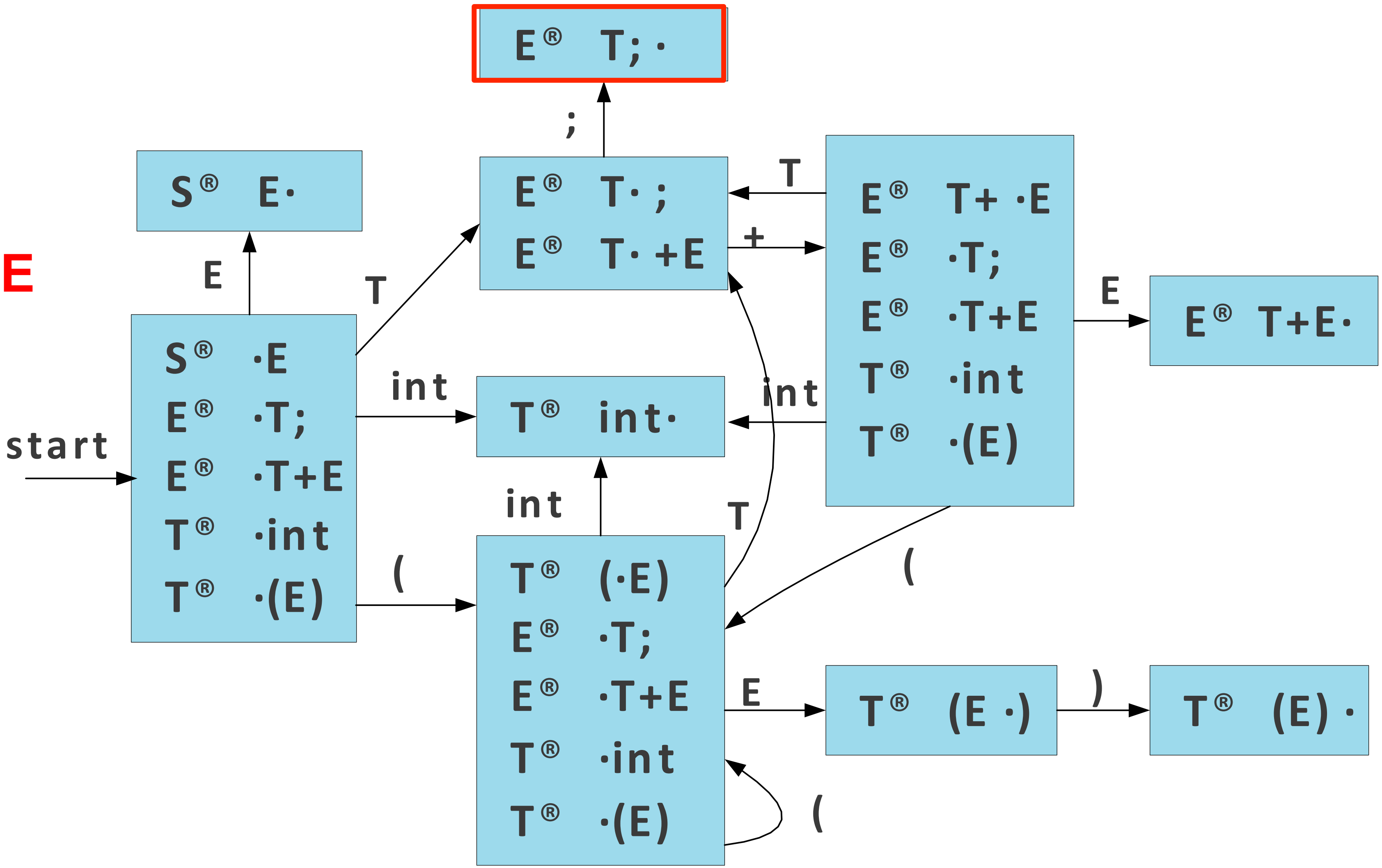


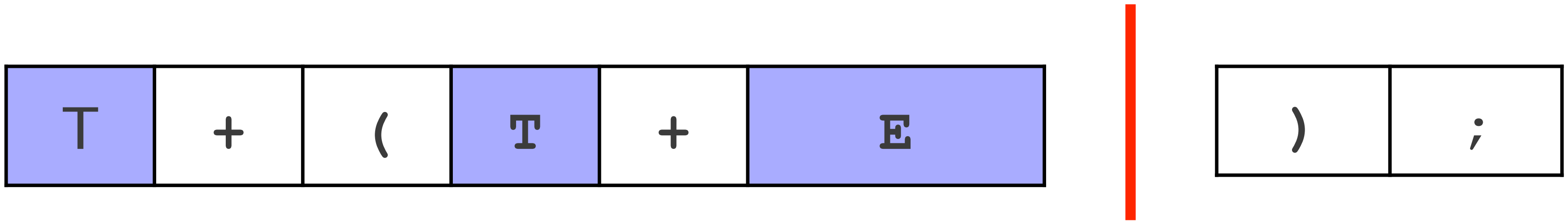
T	+	(	T	+

)	;

LR(0) Parsing

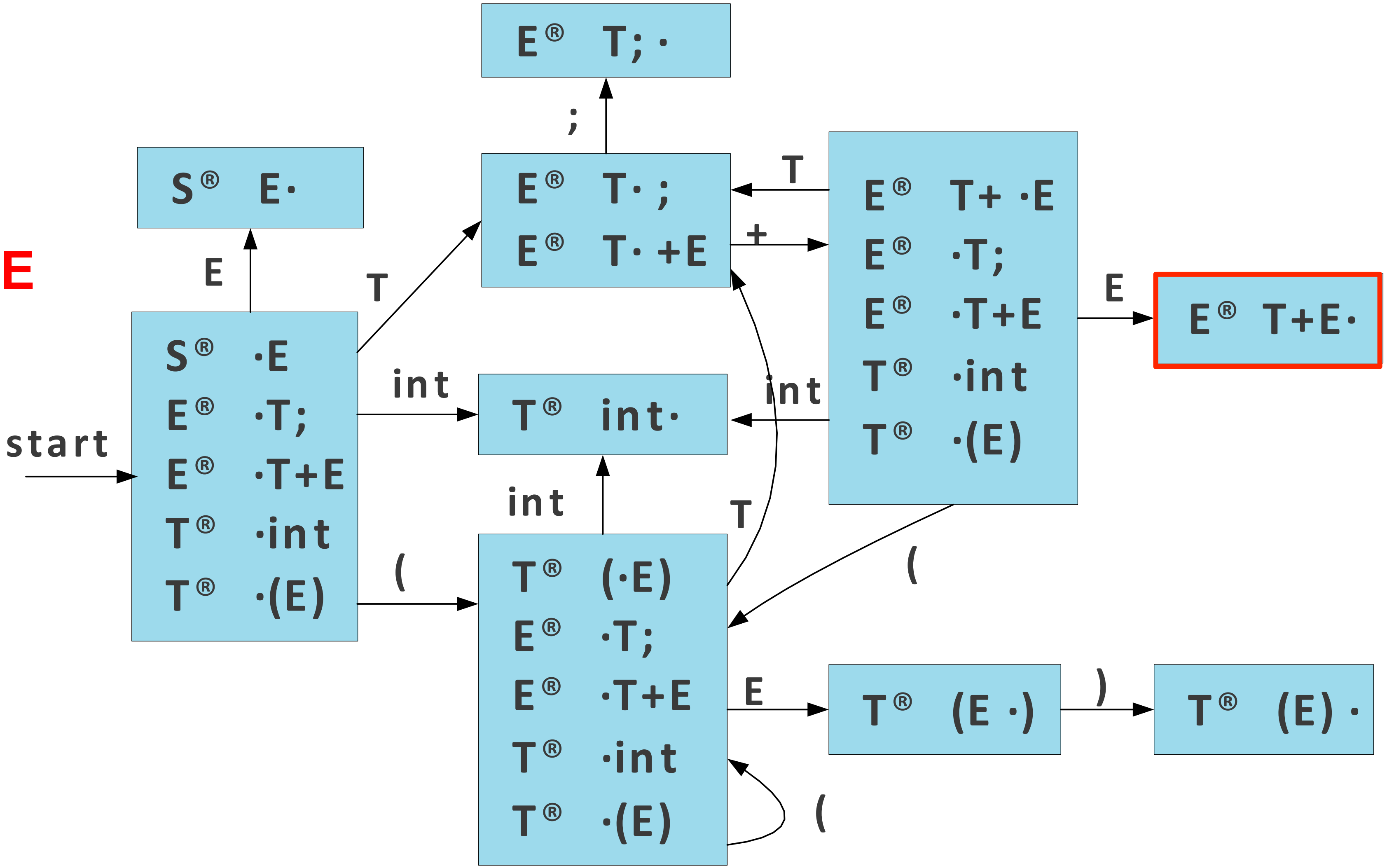
S → E
E → T;
E → T + E
T → int
T → (E)





LR(0) Parsing

S → **E**
E → **T;**
E → **T + E**
T → **int**
T → **(E)**

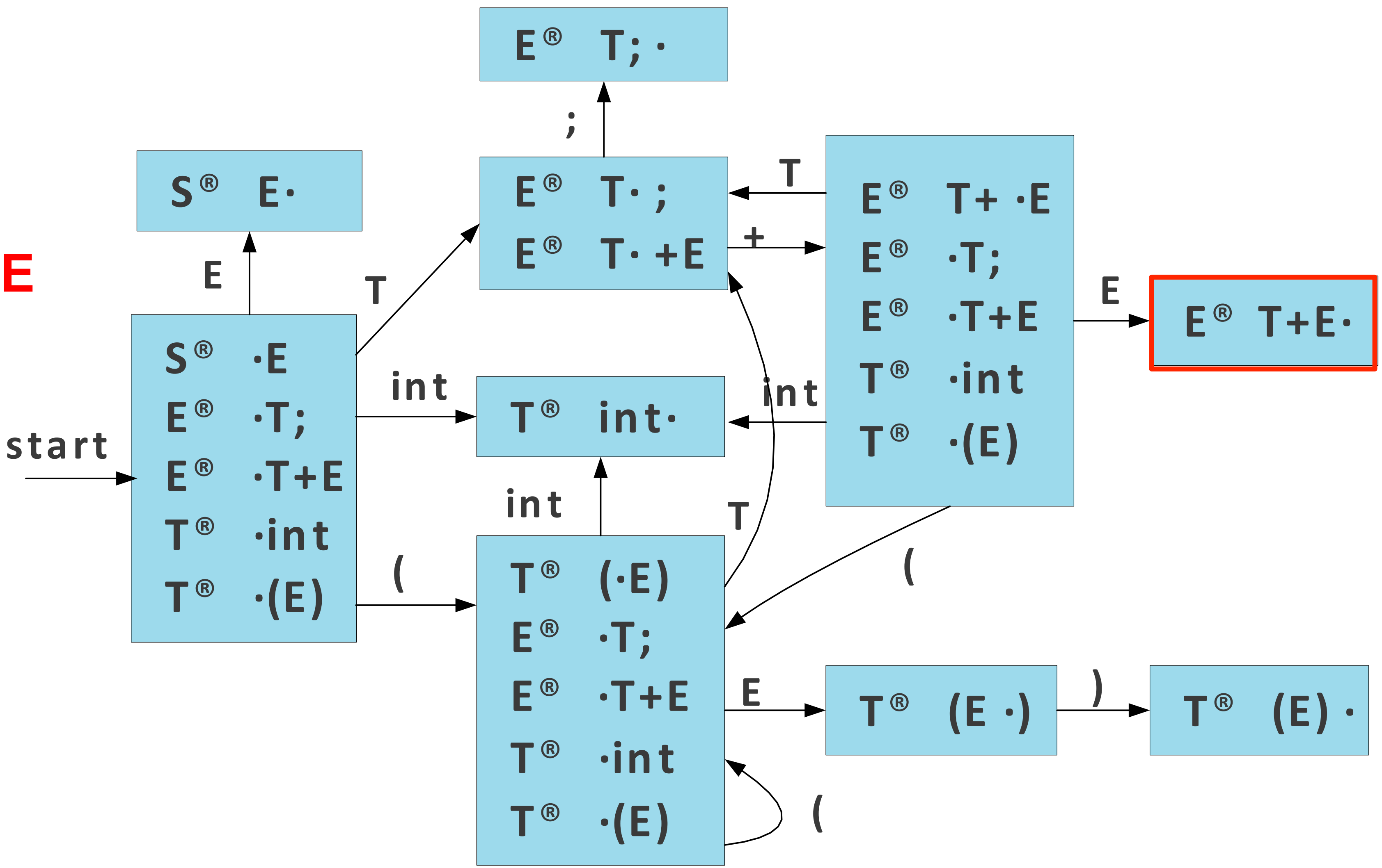


T	+	(

)	;

LR(0) Parsing

S → E
E → T;
E → T + E
T → int
T → (E)



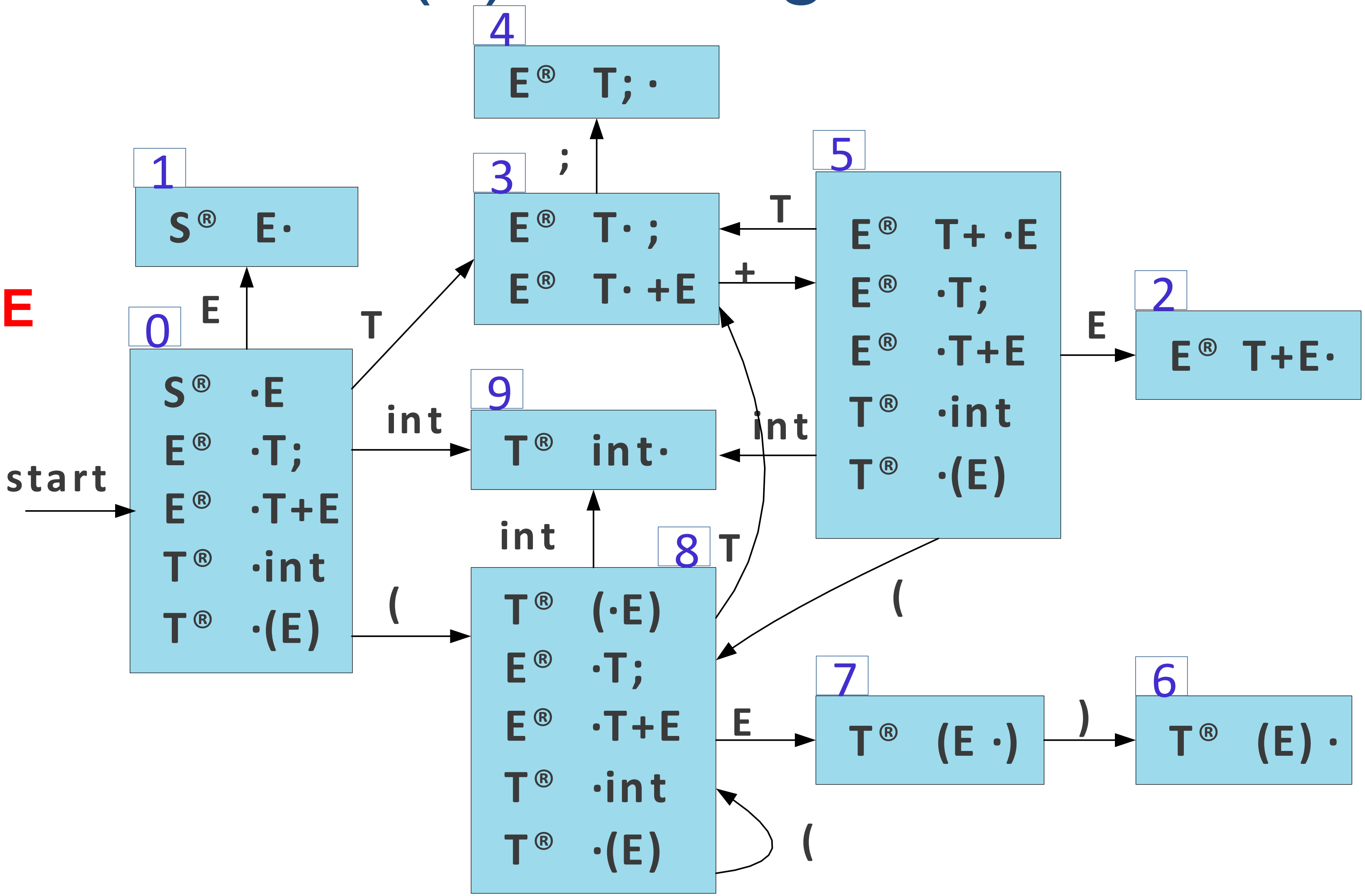
T	+	(	E	|	)	;

An optimization

- Rather than restart the automaton on each reduction, remember what state we were in for each symbol.
- When applying a reduction, restart the automaton from the last known good state.

LR(0) Parsing

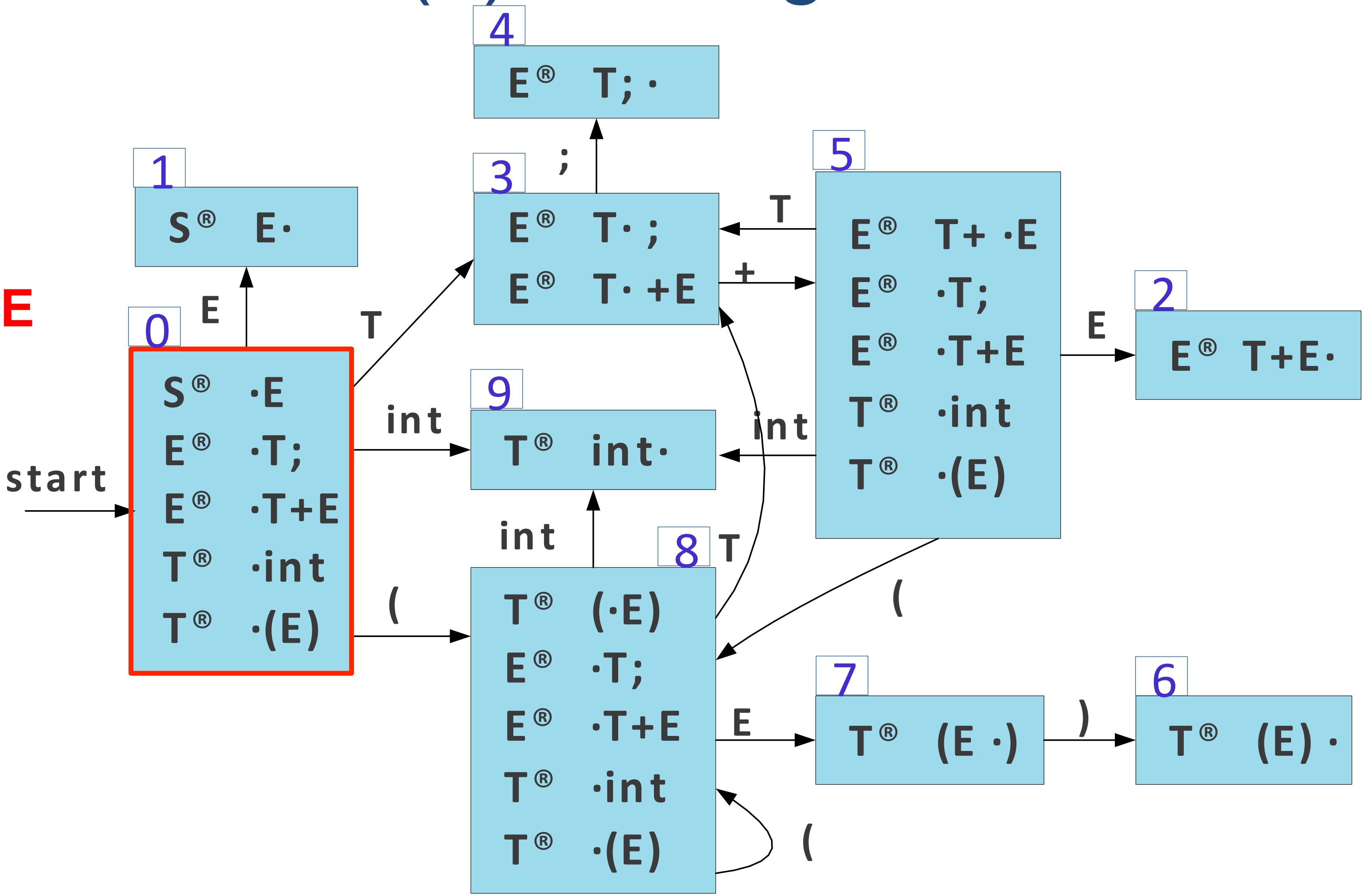
S → E
E → T;
E → T + E
T → int
T → (E)



int	+	(	int	+	int	;	)	;

LR(0) Parsing

S → E
E → T;
E → T + E
T → int
T → (E)



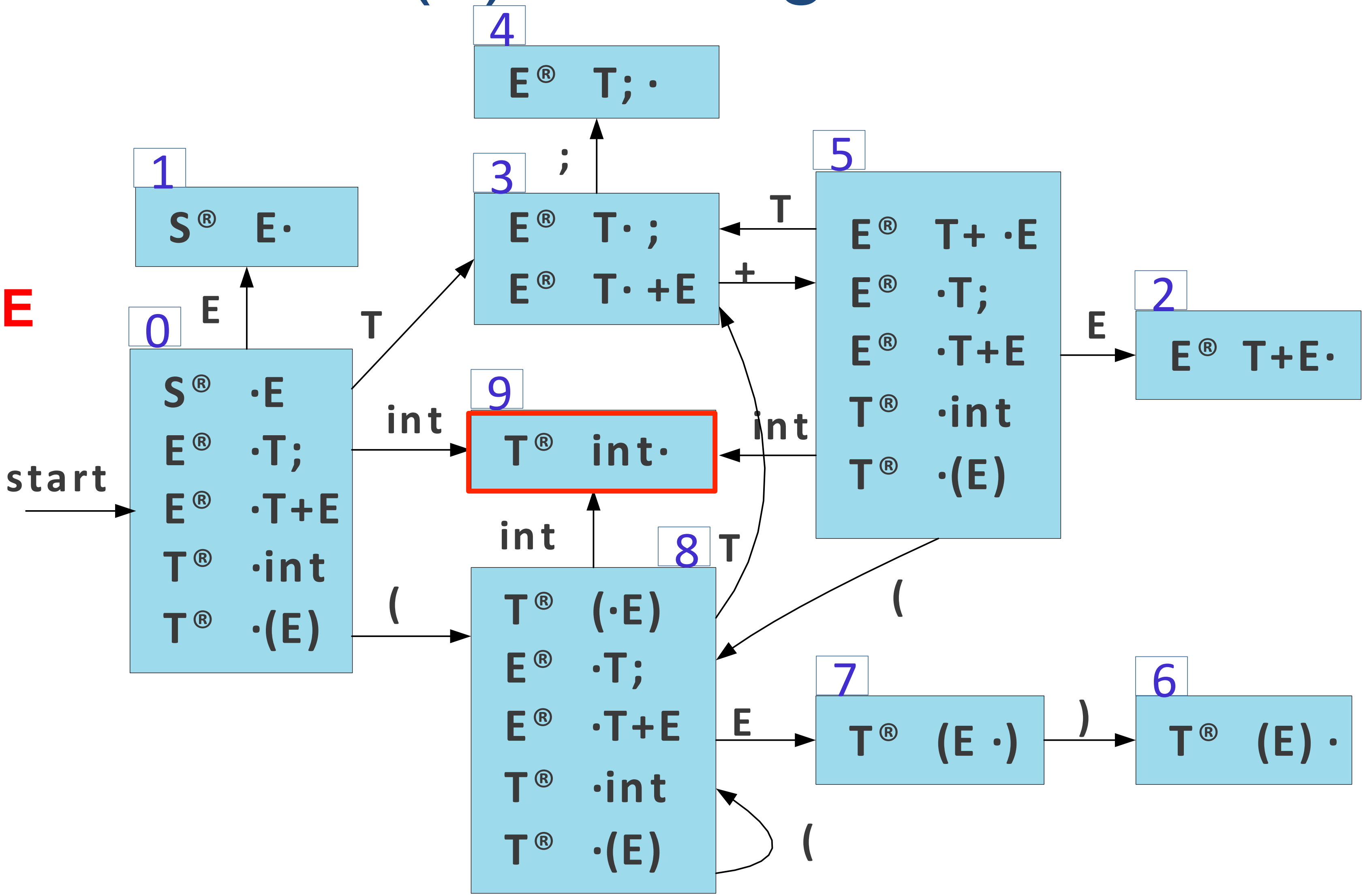
$

0

int	+	(	int	+	int	;	)	;	$

LR(0) Parsing

$S \rightarrow E$
$E \rightarrow T;$
$E \rightarrow T + E$
$T \rightarrow$ int
$T \rightarrow$ (E)

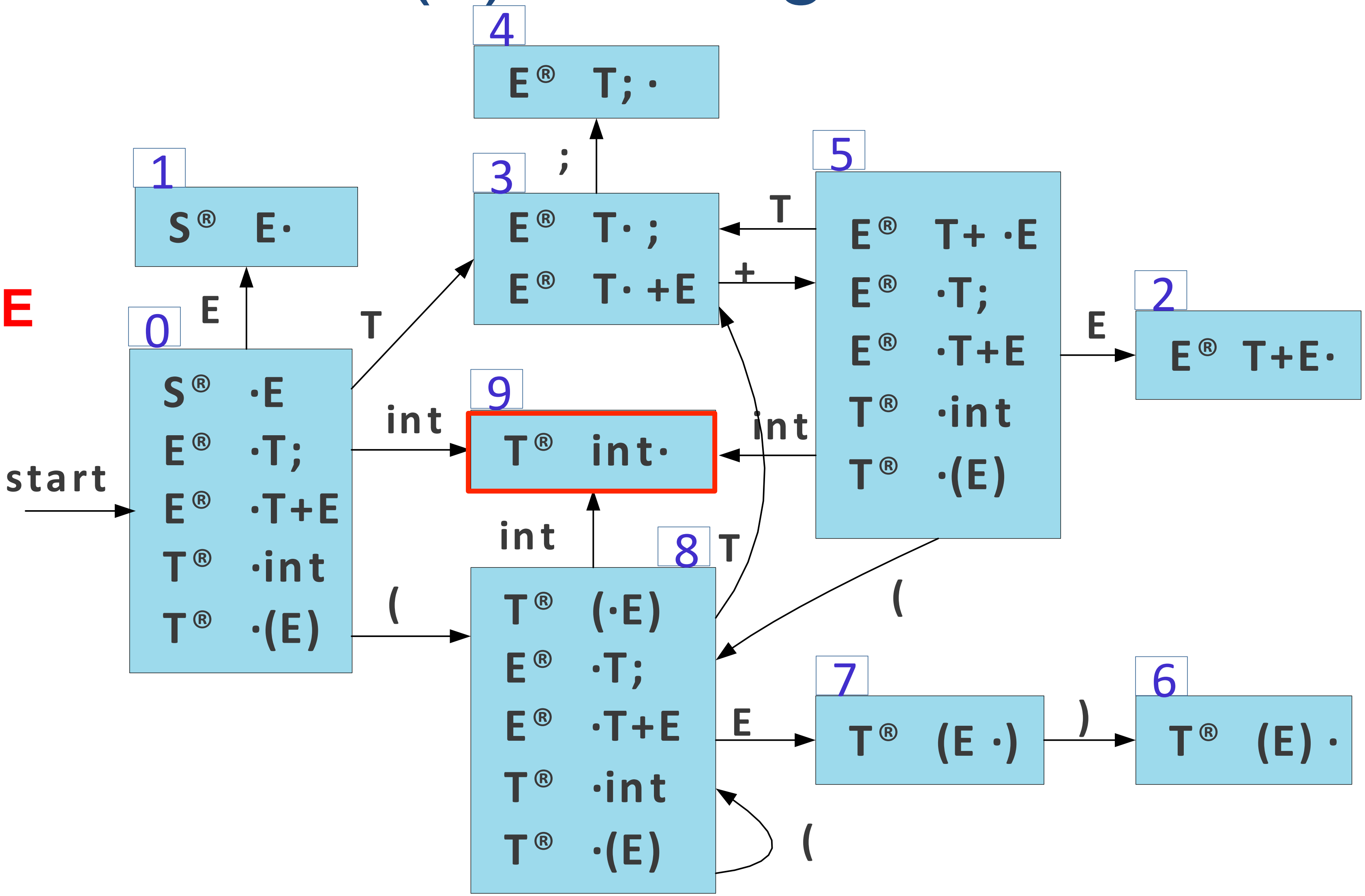


$	int
0	9

+	(	int	+	int	;	)	;	$

LR(0) Parsing

S → E
E → T;
E → T + E
T → int
T → (E)

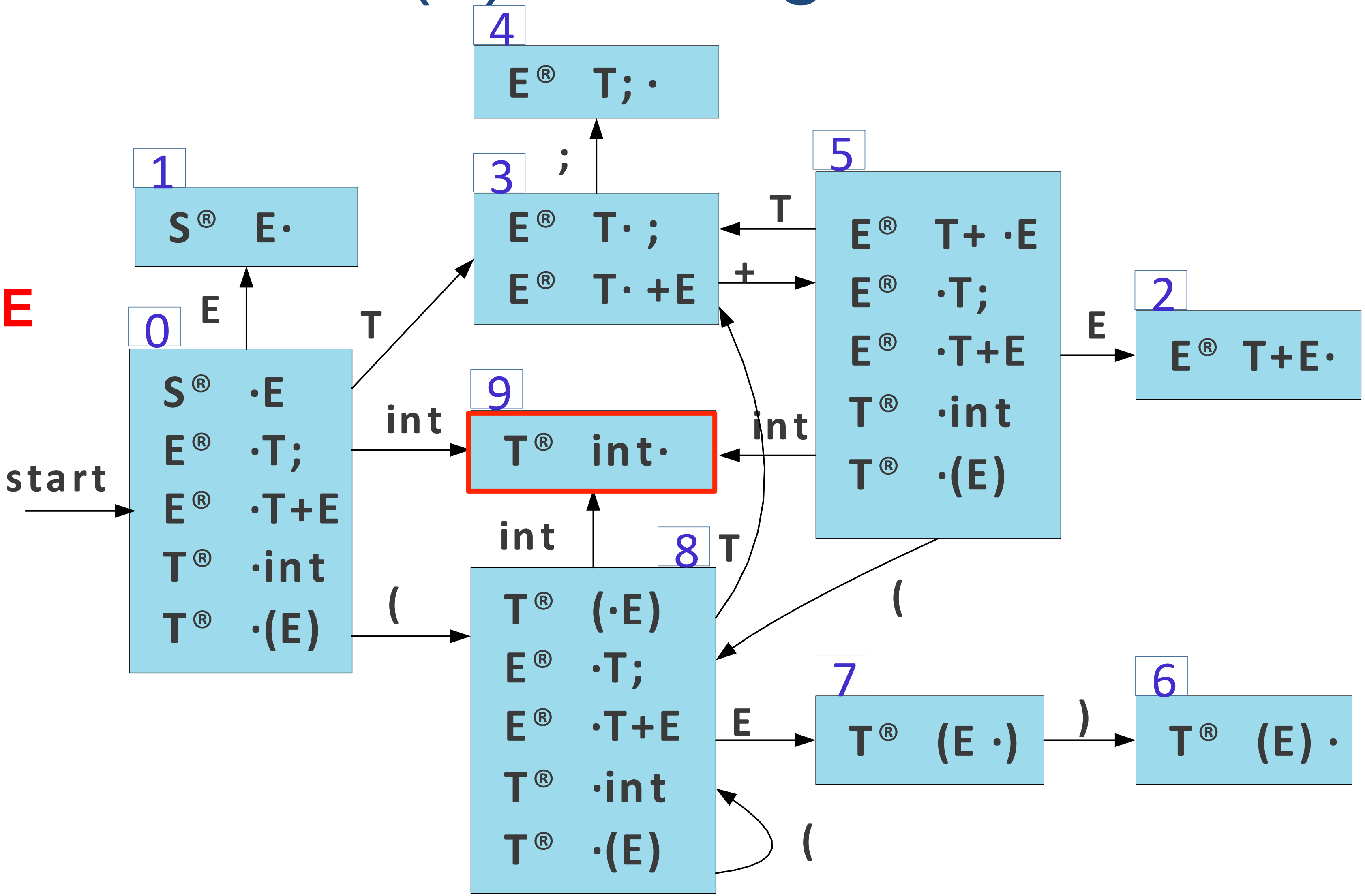


$
0

+	(	int	+	int	;	)	;	$

LR(0) Parsing

S → E
E → T;
E → T + E
T → int
T → (E)



$

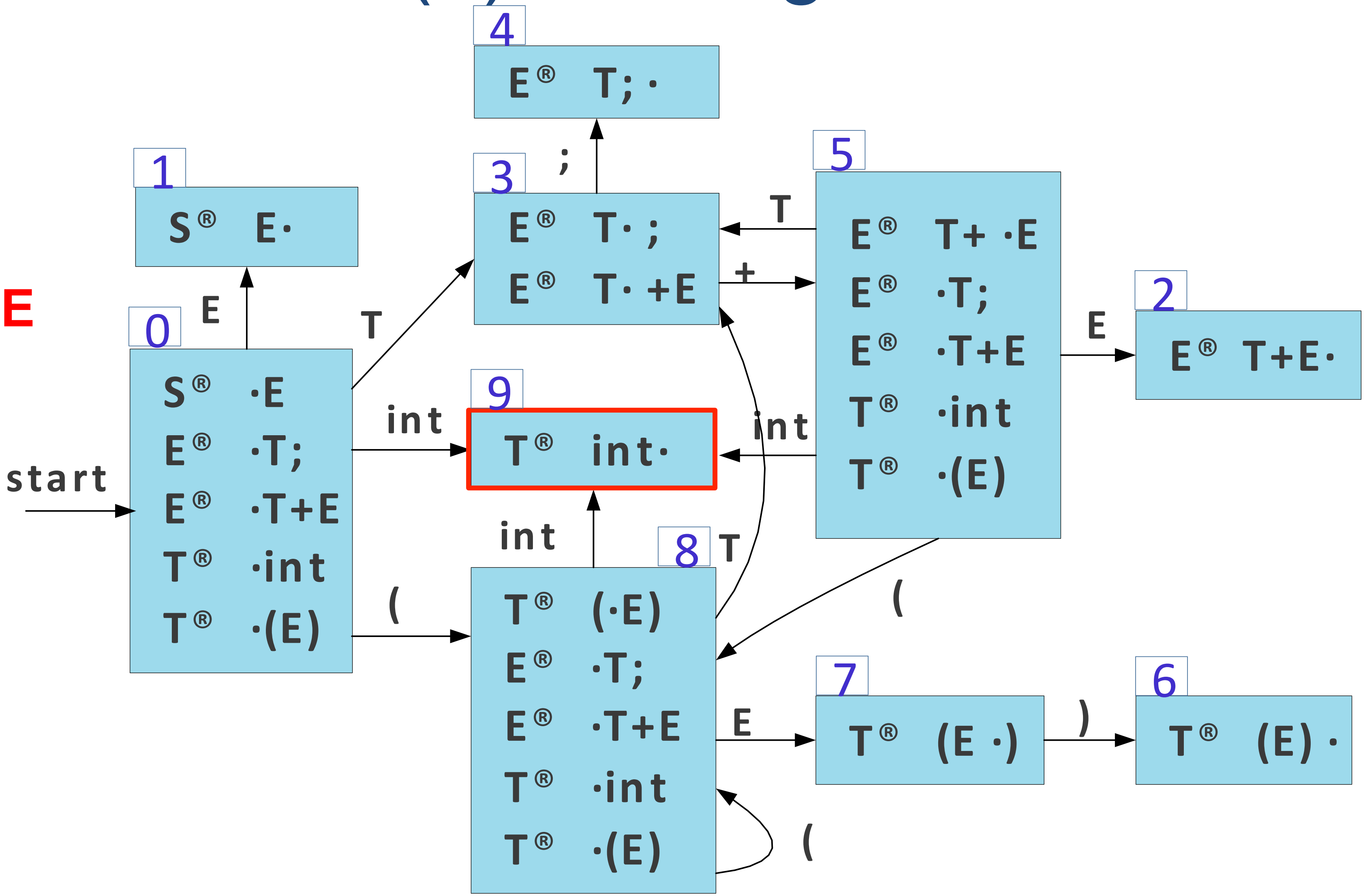
+	(	int	+	int	;	)	;

$

0

LR(0) Parsing

S → E
E → T;
E → T + E
T → int
T → (E)



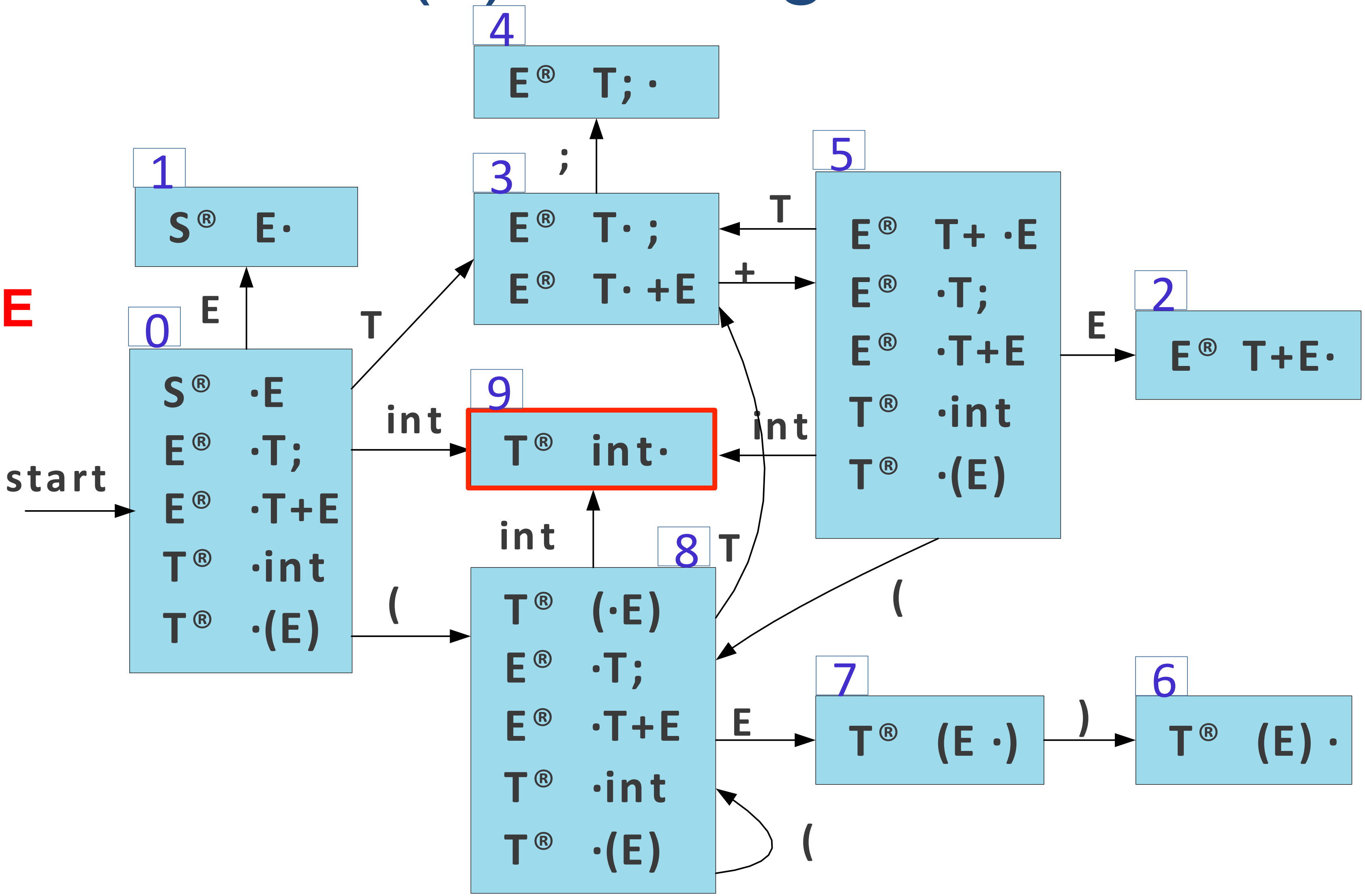
+	(	int	+	int	;	)	;

$ T

0

LR(0) Parsing

S → E
E → T;
E → T + E
T → int
T → (E)

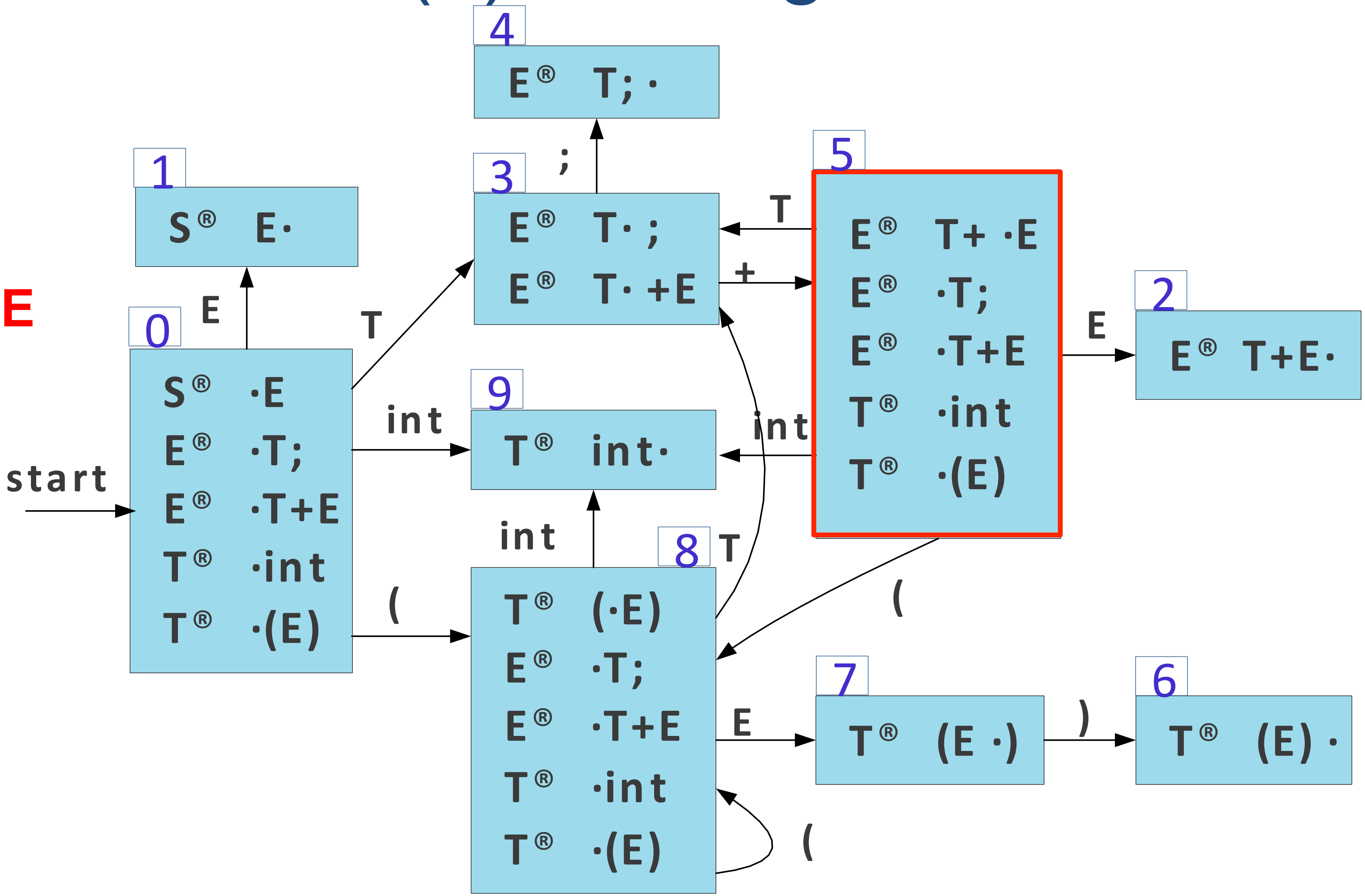


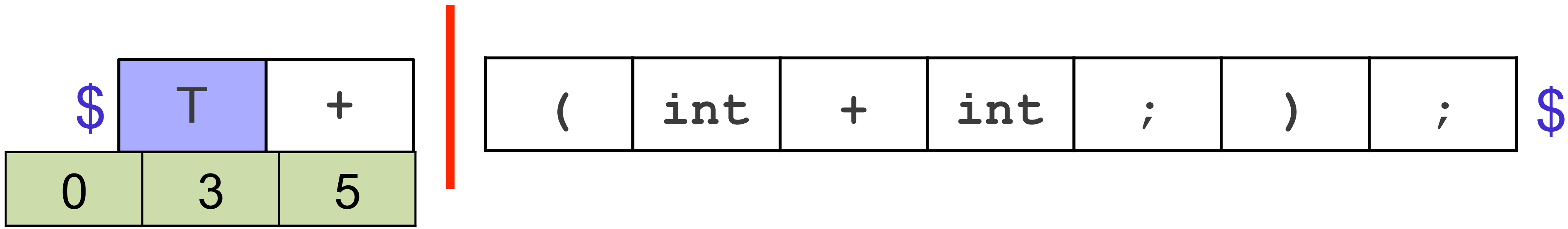
$	T
0	3

+	(	int	+	int	;	)	;	$

LR(0) Parsing

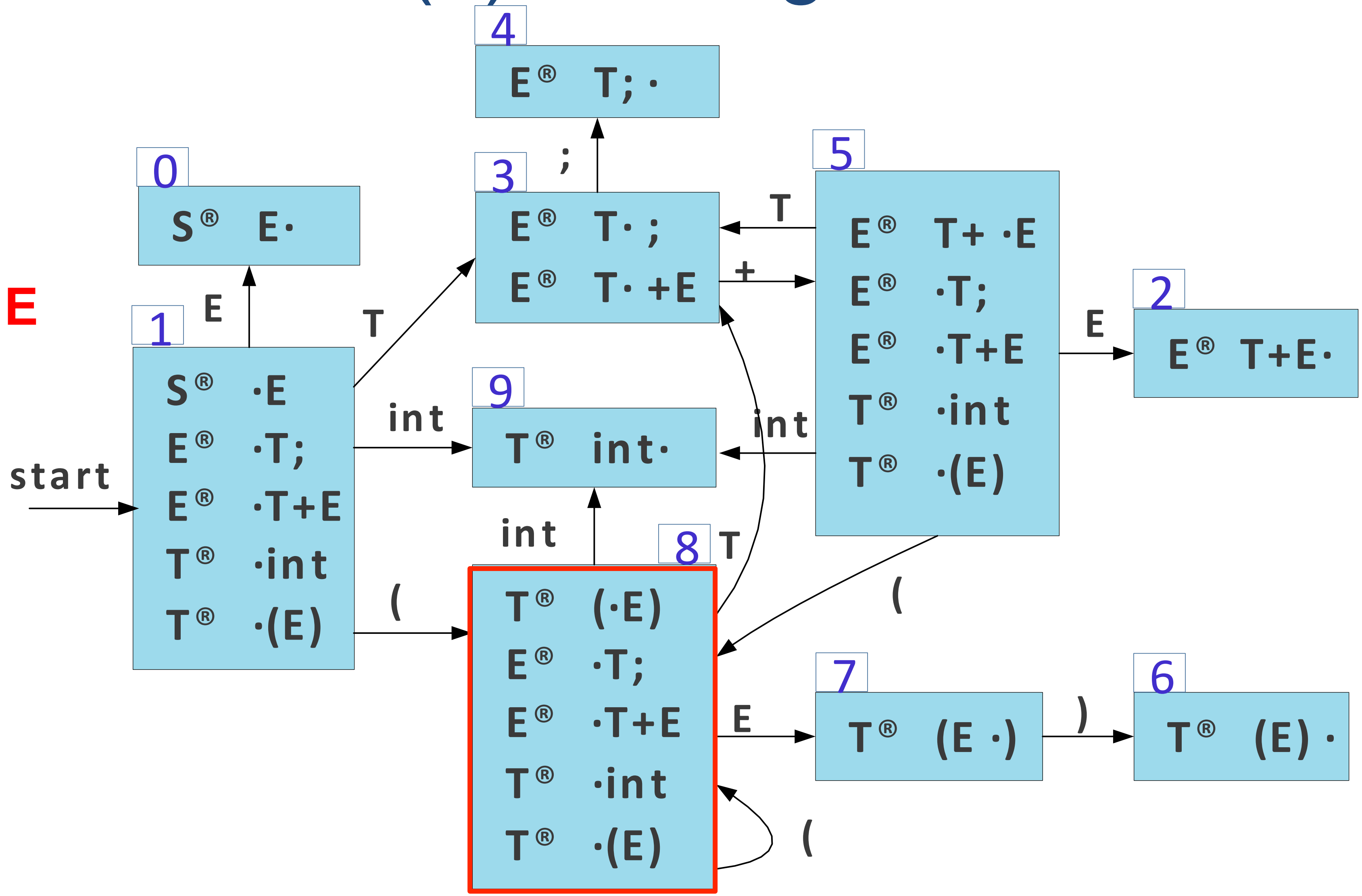
$S \rightarrow E$
$E \rightarrow T;$
$E \rightarrow T + E$
$T \rightarrow int$
$T \rightarrow (E)$





LR(0) Parsing

$S \rightarrow E$
$E \rightarrow T;$
$E \rightarrow T + E$
$T \rightarrow \text{int}$
$T \rightarrow (E)$

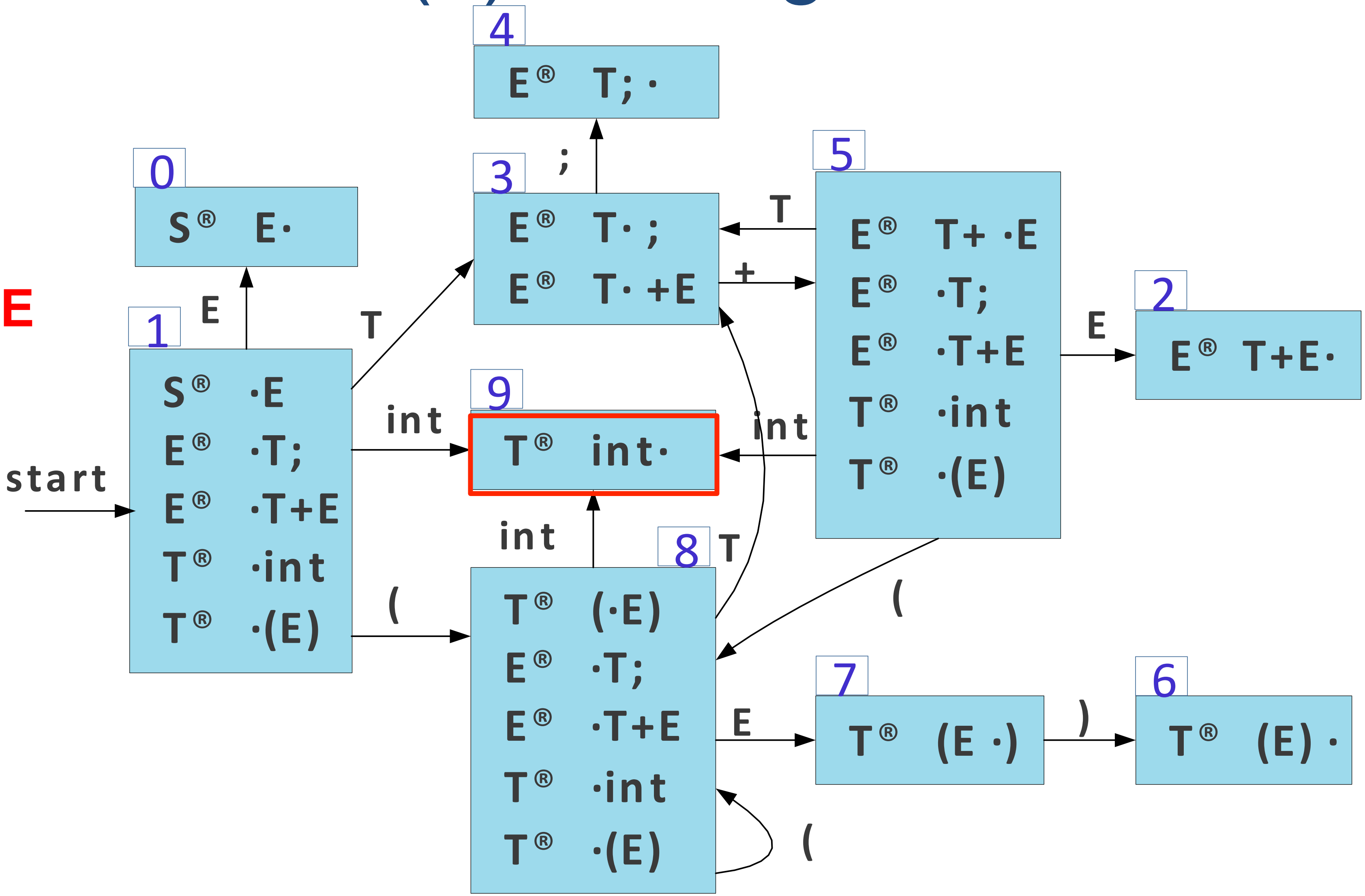


$	T	+	(	
	0	3	5	8

int	+	int	;	)	;	$

LR(0) Parsing

S → E
E → T;
E → T + E
T → int
T → (E)

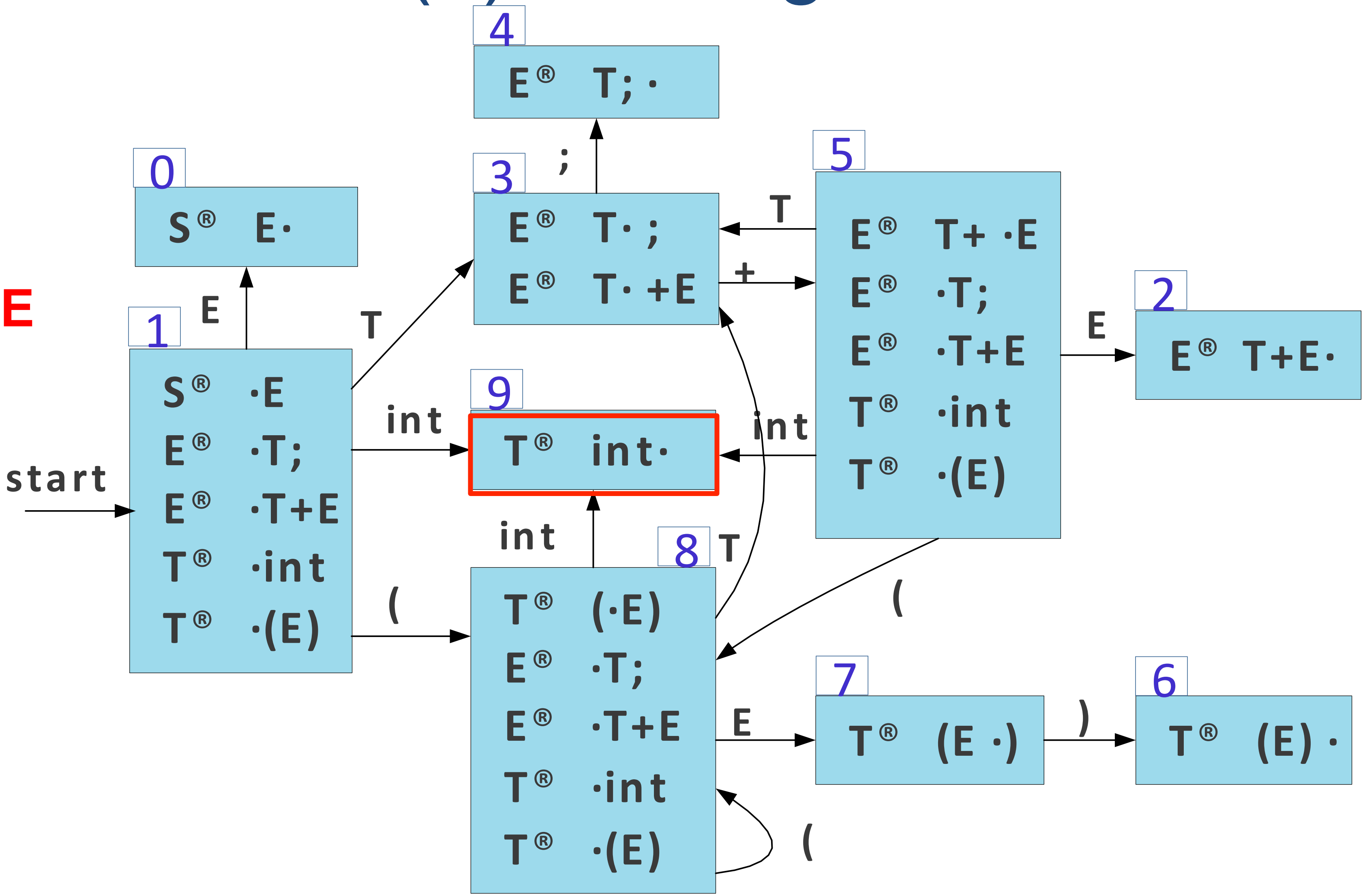


$	T	+	(	int
0	3	5	8	9

+	int	;	)	;	$

LR(0) Parsing

$S \rightarrow E$
$E \rightarrow T;$
$E \rightarrow T + E$
$T \rightarrow \text{int}$
$T \rightarrow (E)$

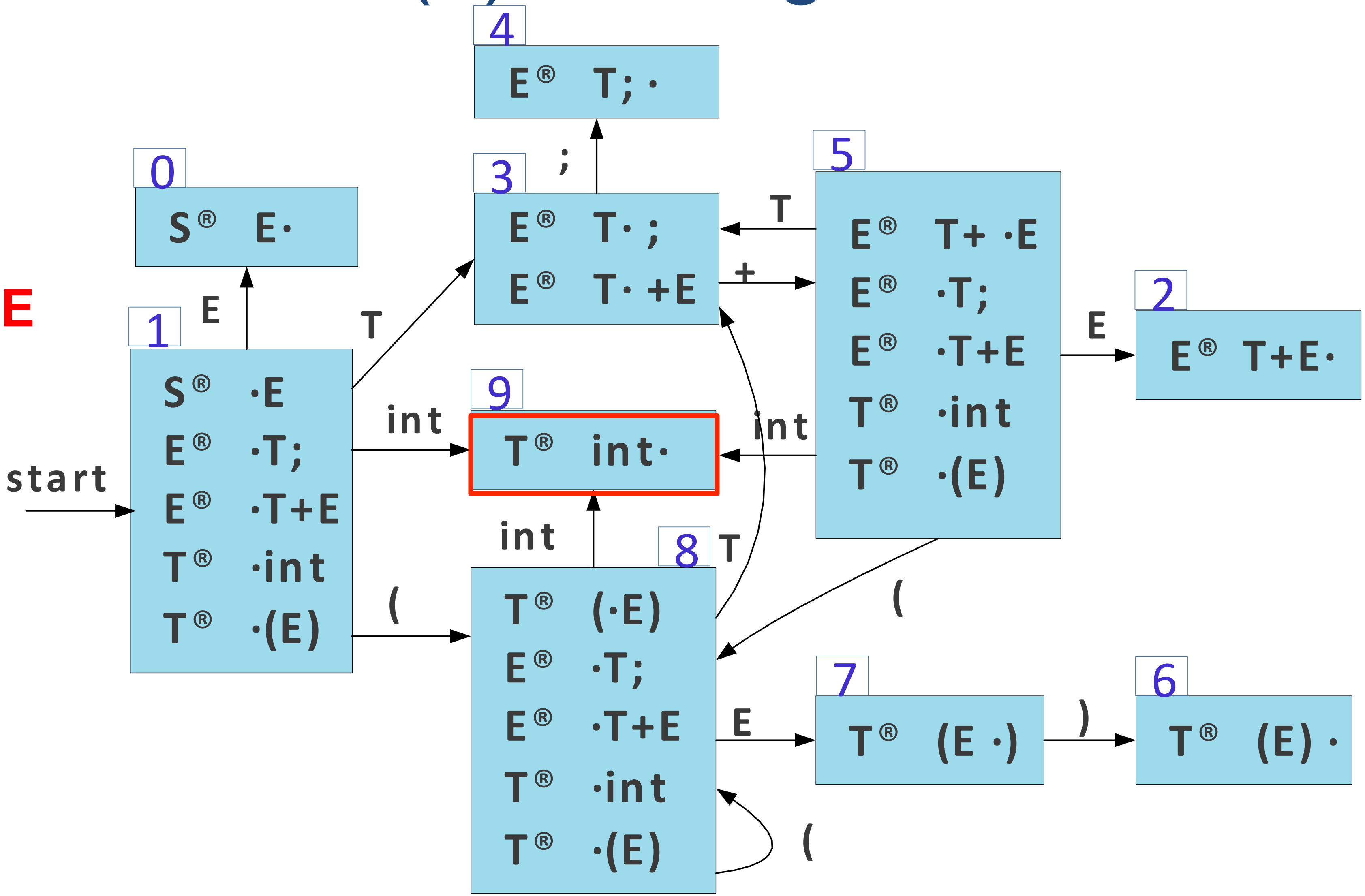


$	T	+	(	
	0	3	5	8

+	int	;	)	;	$

LR(0) Parsing

S → E
E → T;
E → T + E
T → int
T → (E)

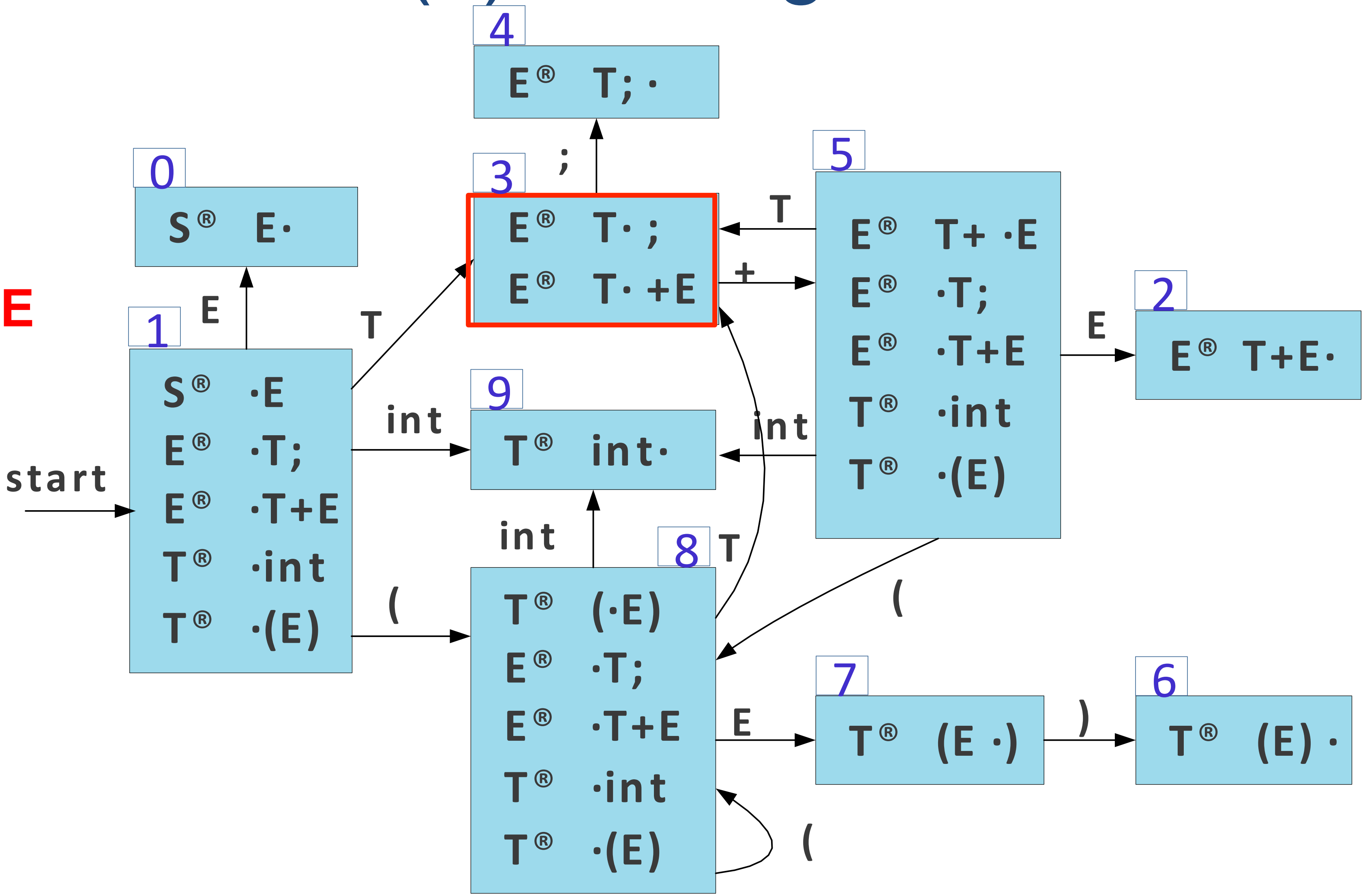


$	T	+	(	T	
	0	3	5	8	3

+	int	;	)	;	$

LR(0) Parsing

S → E
E → T;
E → T + E
T → int
T → (E)

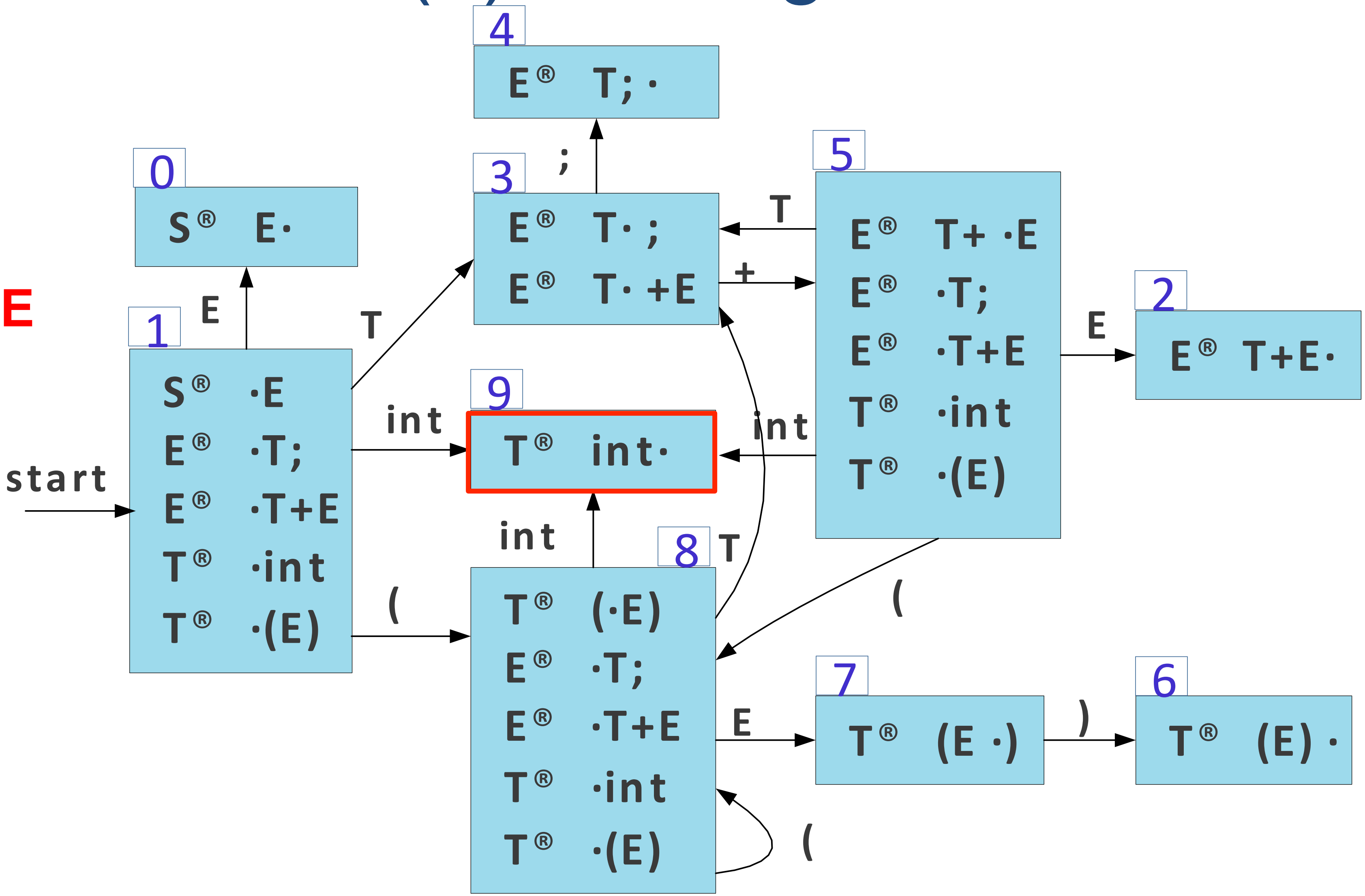


$	T	+	(	T	+
0	3	5	8	3	5

int	;	)	;	$

LR(0) Parsing

S → E
E → T;
E → T + E
T → int
T → (E)

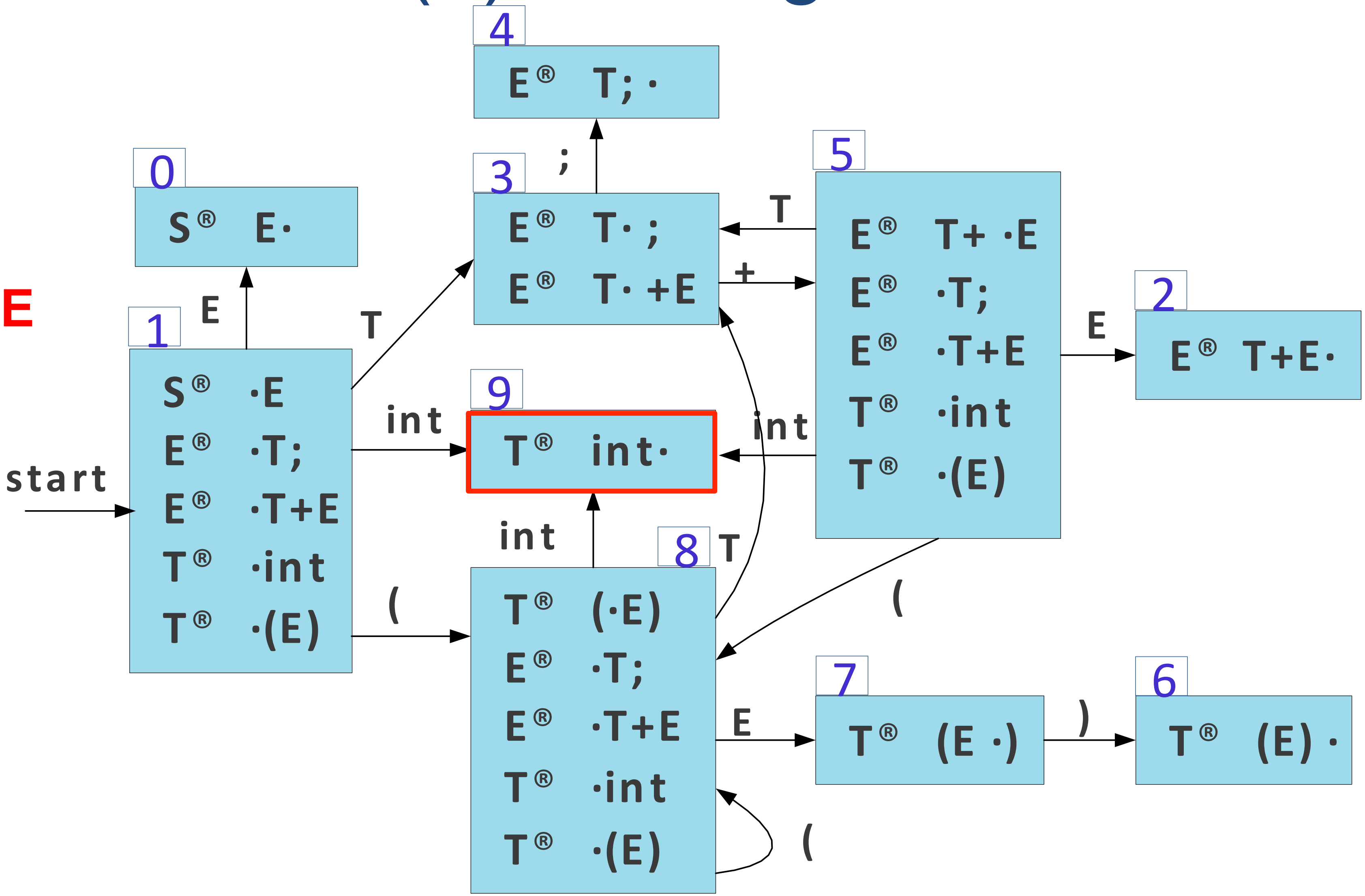


$	T	+	(	T	+	int	
0	3	5	8	3	5	9	

;	)	;	$

LR(0) Parsing

S → E
E → T;
E → T + E
T → int
T → (E)

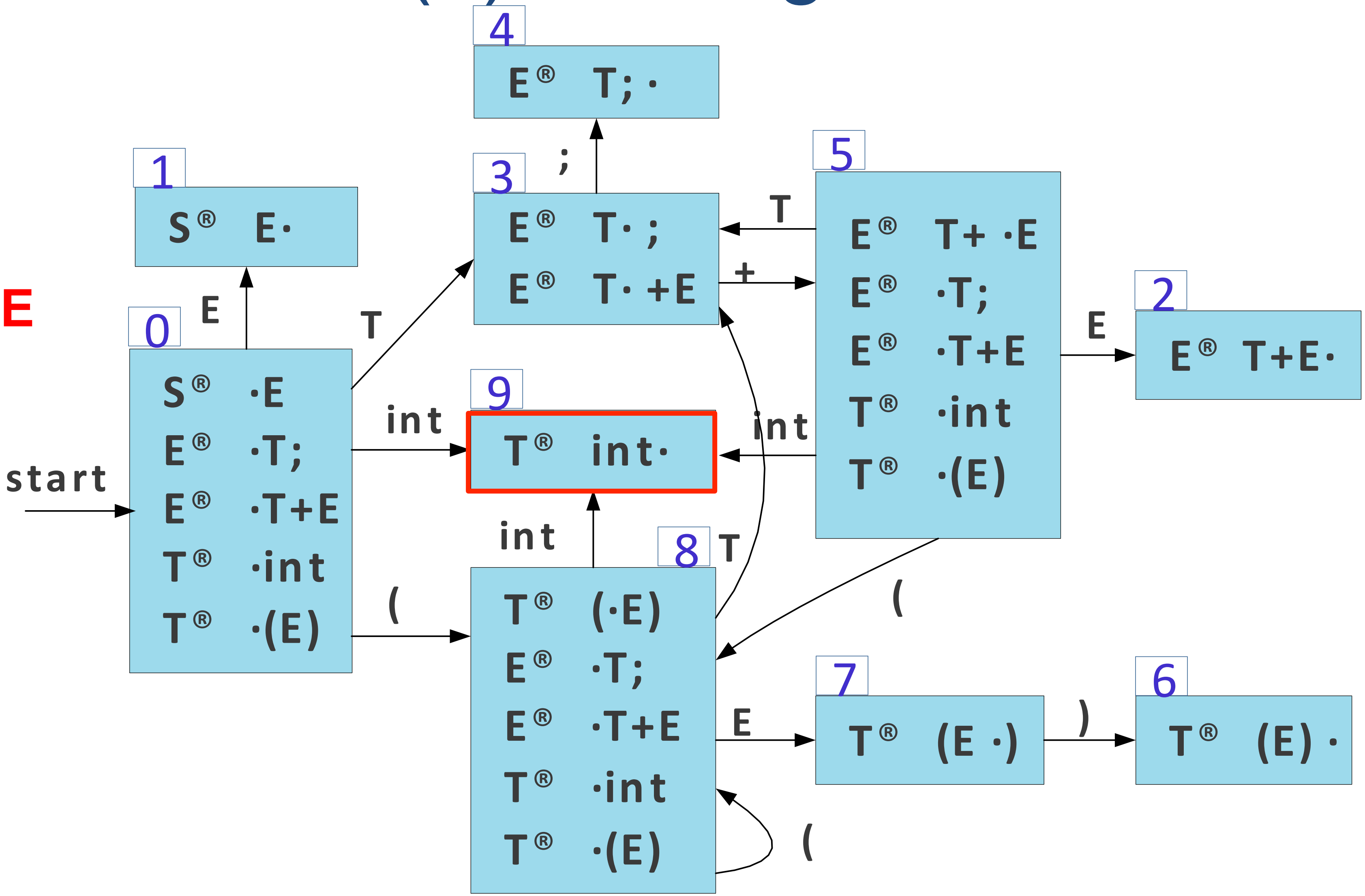


$	T	+	(	T	+	
0	3	5	8	3	5	

;	)	;	$

LR(0) Parsing

S → E
E → T;
E → T + E
T → int
T → (E)

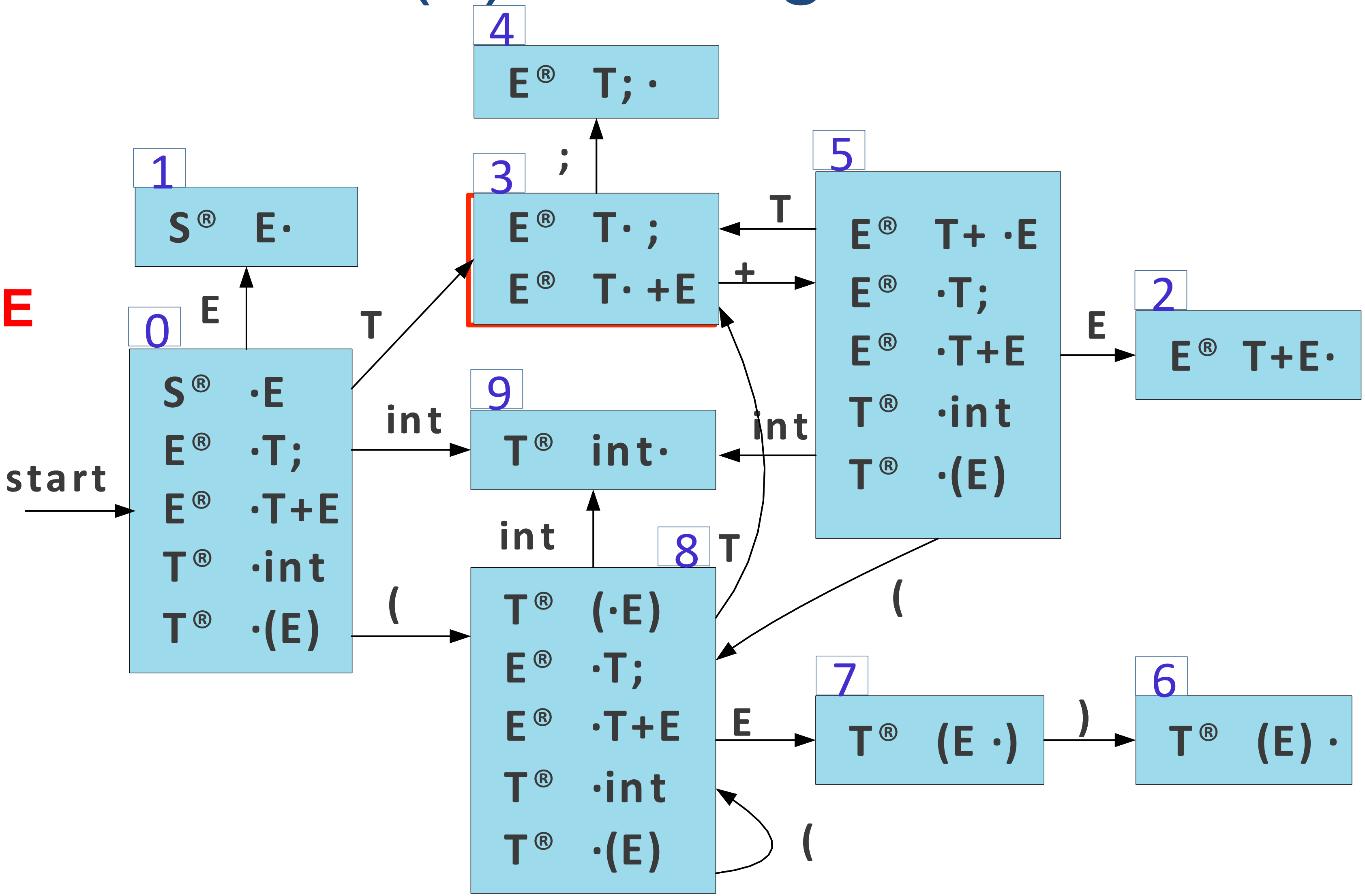


$	T	+	(	T	+	T
0	3	5	8	3	5	3

;	)	;	$

LR(0) Parsing

S → E
E → T;
E → T + E
T → int
T → (E)

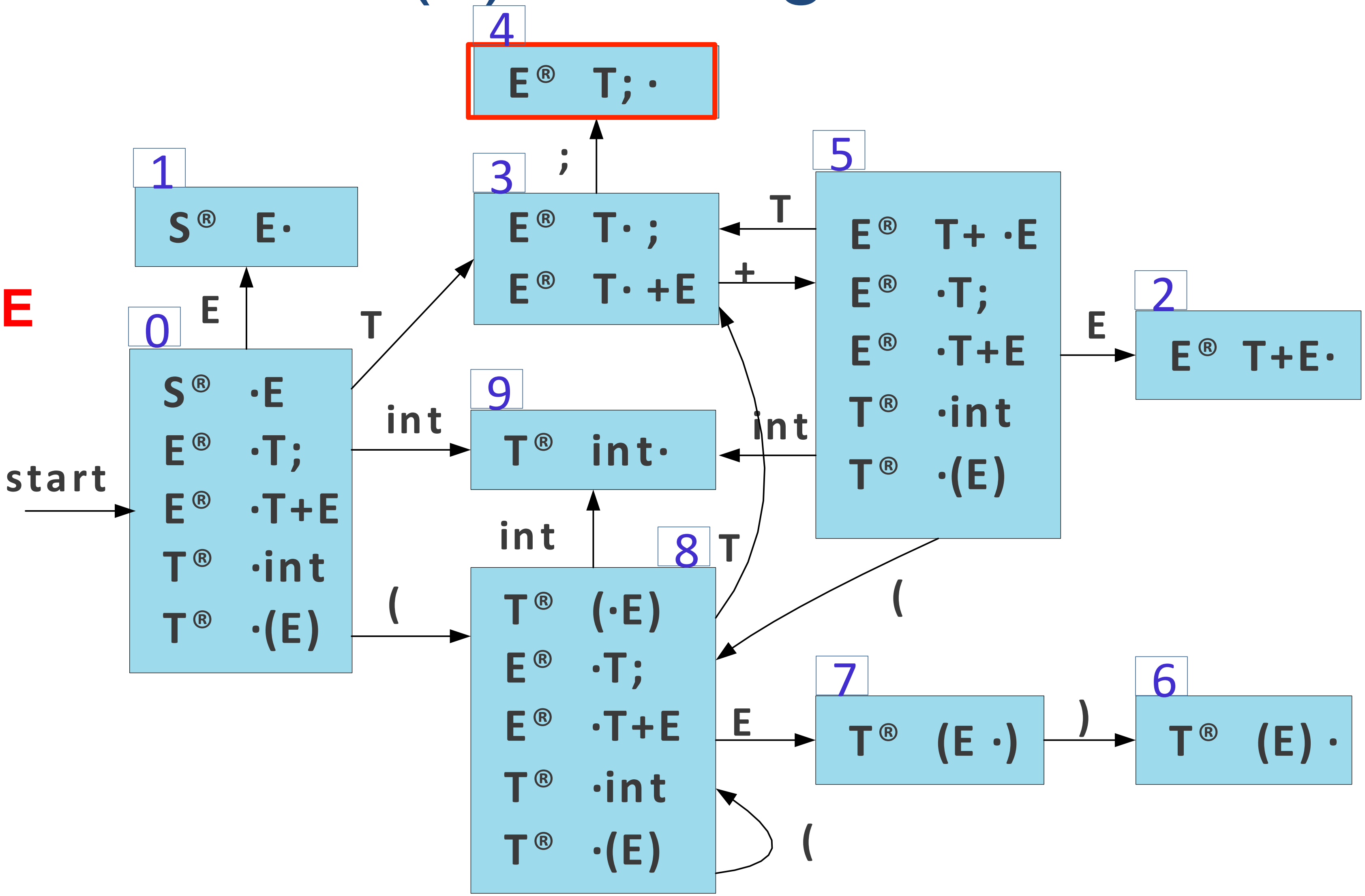


$	T	+	(	T	+	T	;
0	3	5	8	3	5	3	4

)	;	$

LR(0) Parsing

S → E
E → T;
E → T + E
T → int
T → (E)

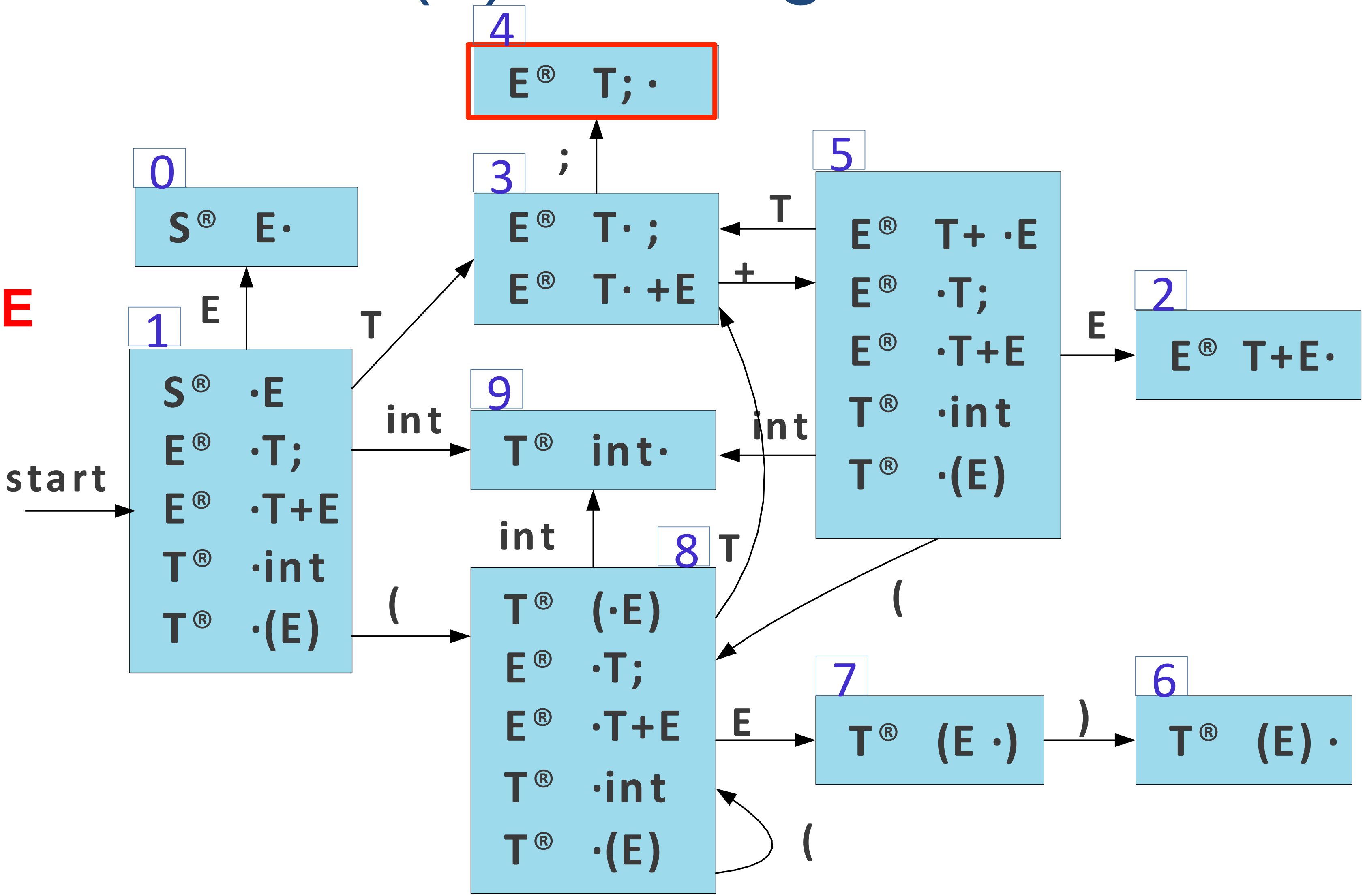


$	T	+	(	T	+
0	3	5	8	3	5

)	;	$

LR(0) Parsing

$S \rightarrow E$
$E \rightarrow T;$
$E \rightarrow T + E$
$T \rightarrow \text{int}$
$T \rightarrow (E)$

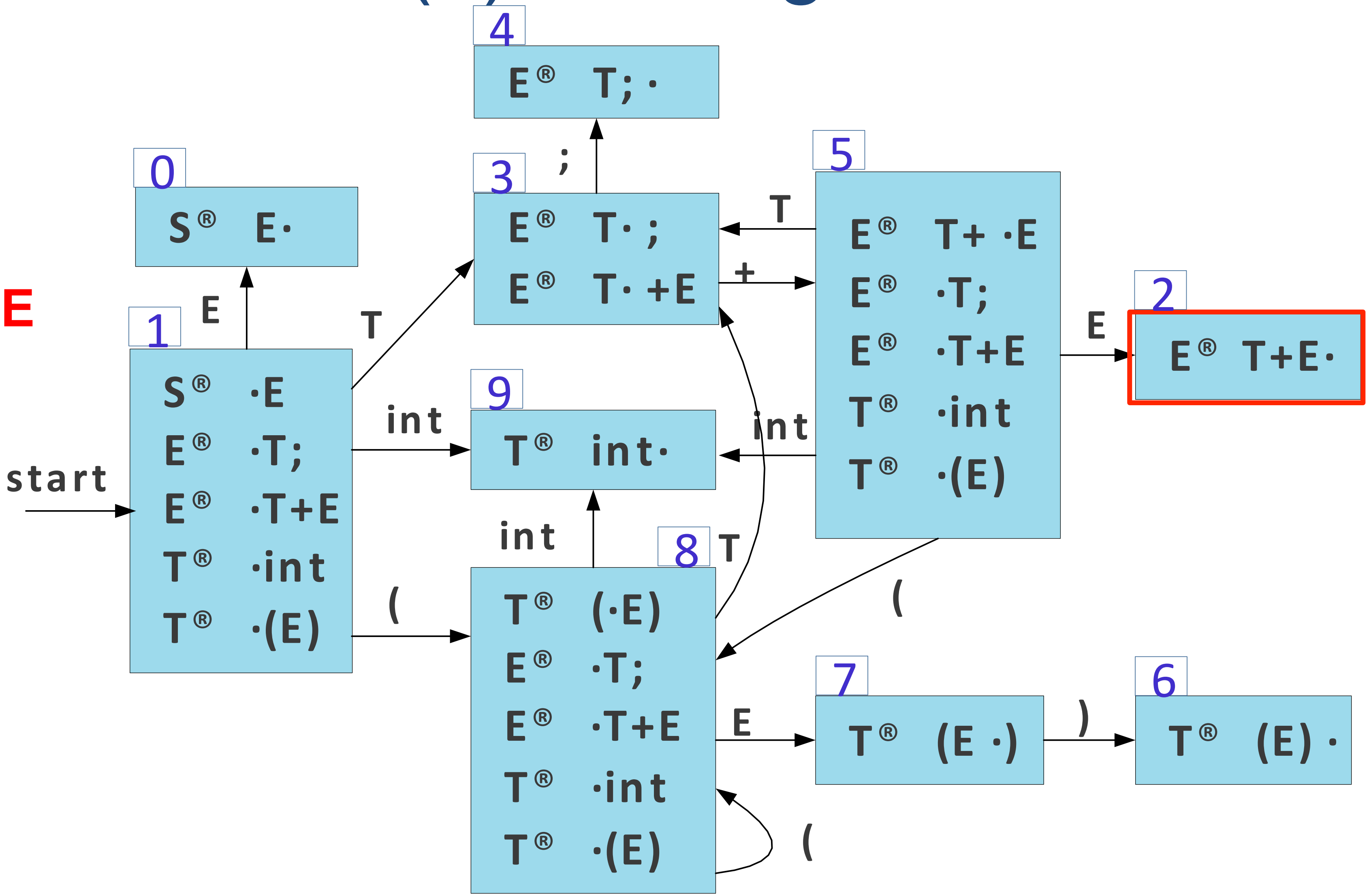


$	T	+	(	T	+	E
0	3	5	8	3	5	2

)	;	$

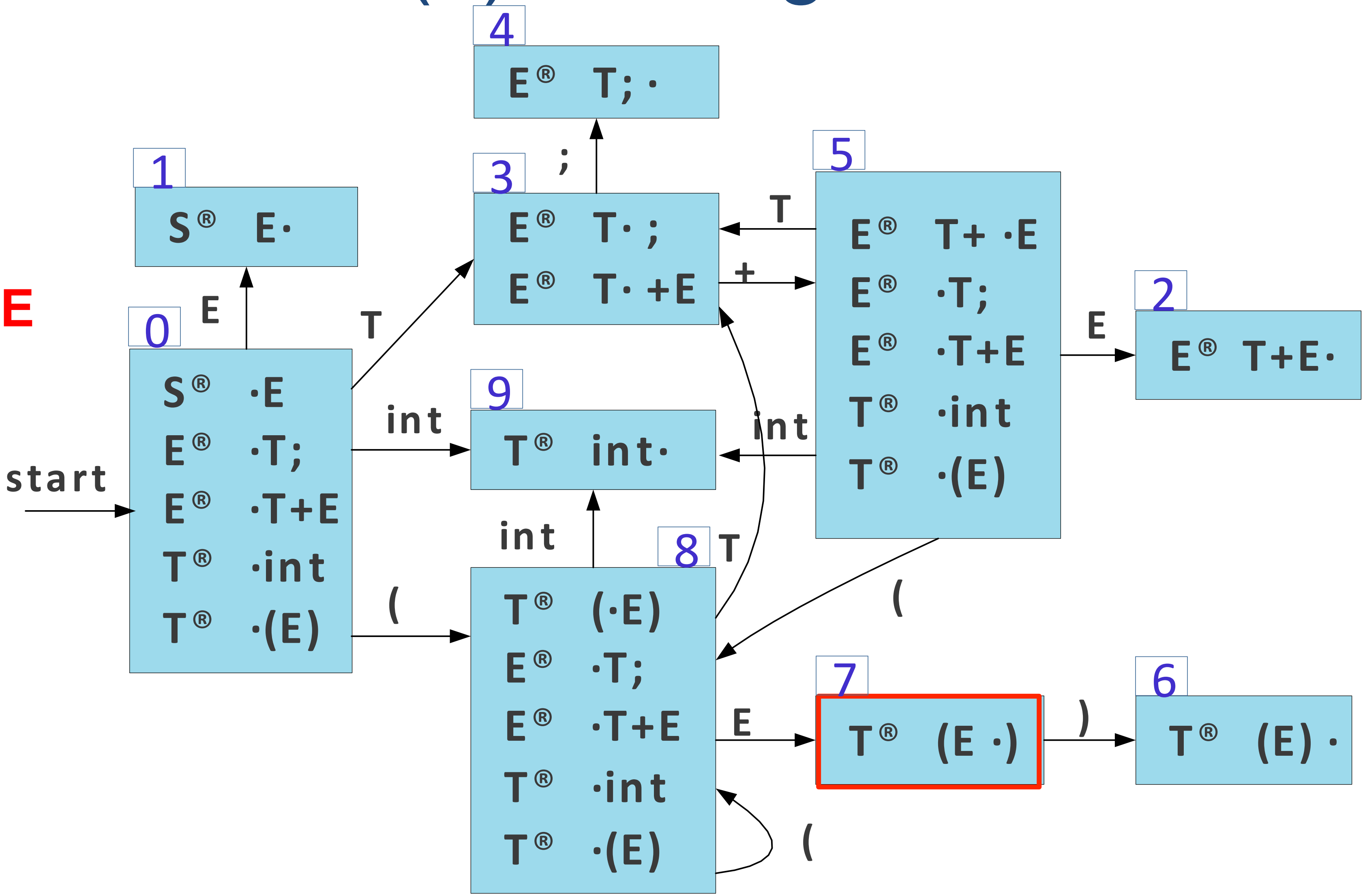
LR(0) Parsing

$S \rightarrow E$
$E \rightarrow T;$
$E \rightarrow T + E$
$T \rightarrow int$
$T \rightarrow (E)$



LR(0) Parsing

$S \rightarrow E$
$E \rightarrow T;$
$E \rightarrow T + E$
$T \rightarrow \text{int}$
$T \rightarrow (E)$

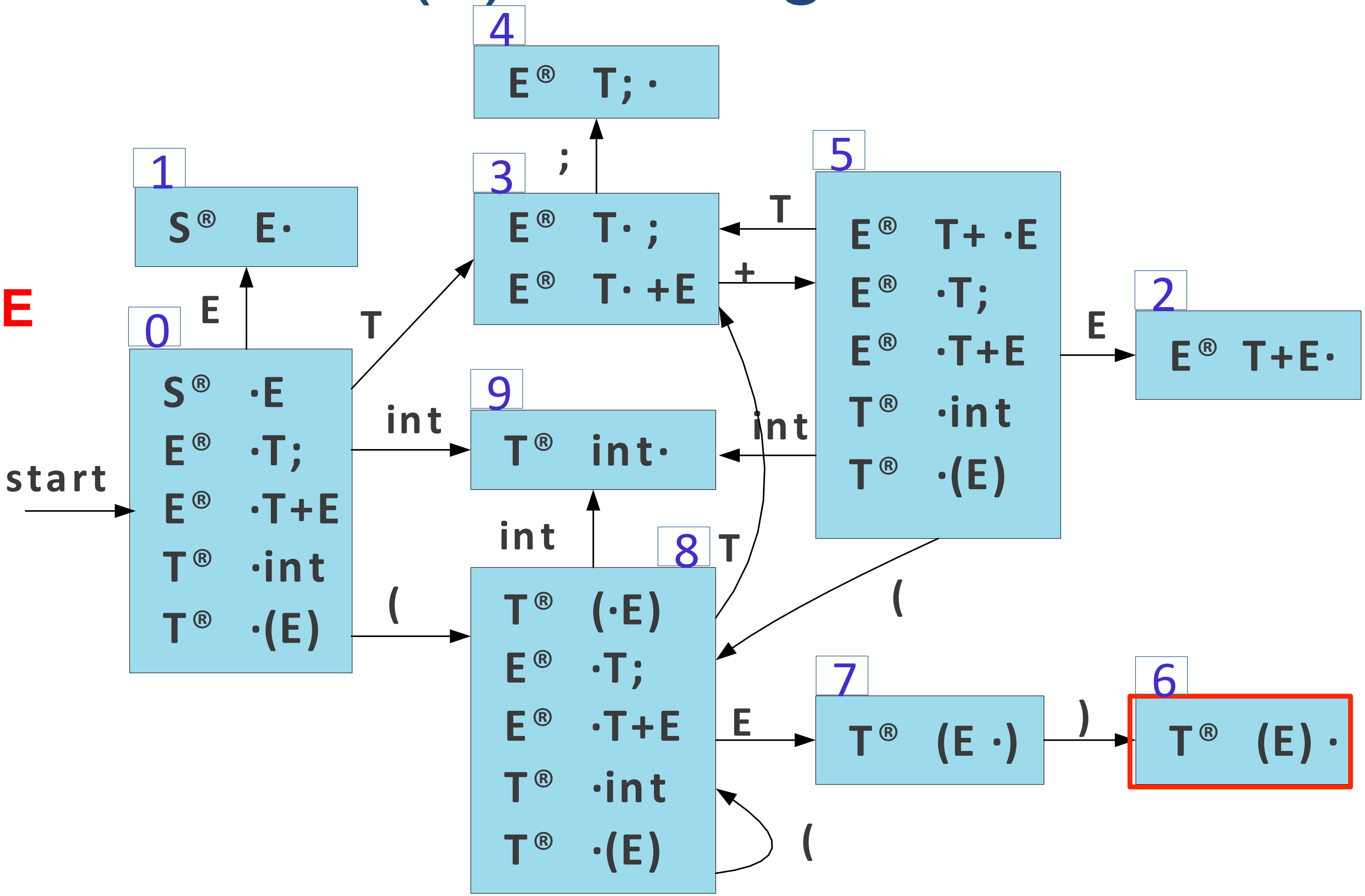


$	T	+	(	E
0	3	5	8	7

)	;	$

LR(0) Parsing

$S \rightarrow E$
$E \rightarrow T;$
$E \rightarrow T + E$
$T \rightarrow \text{int}$
$T \rightarrow (E)$

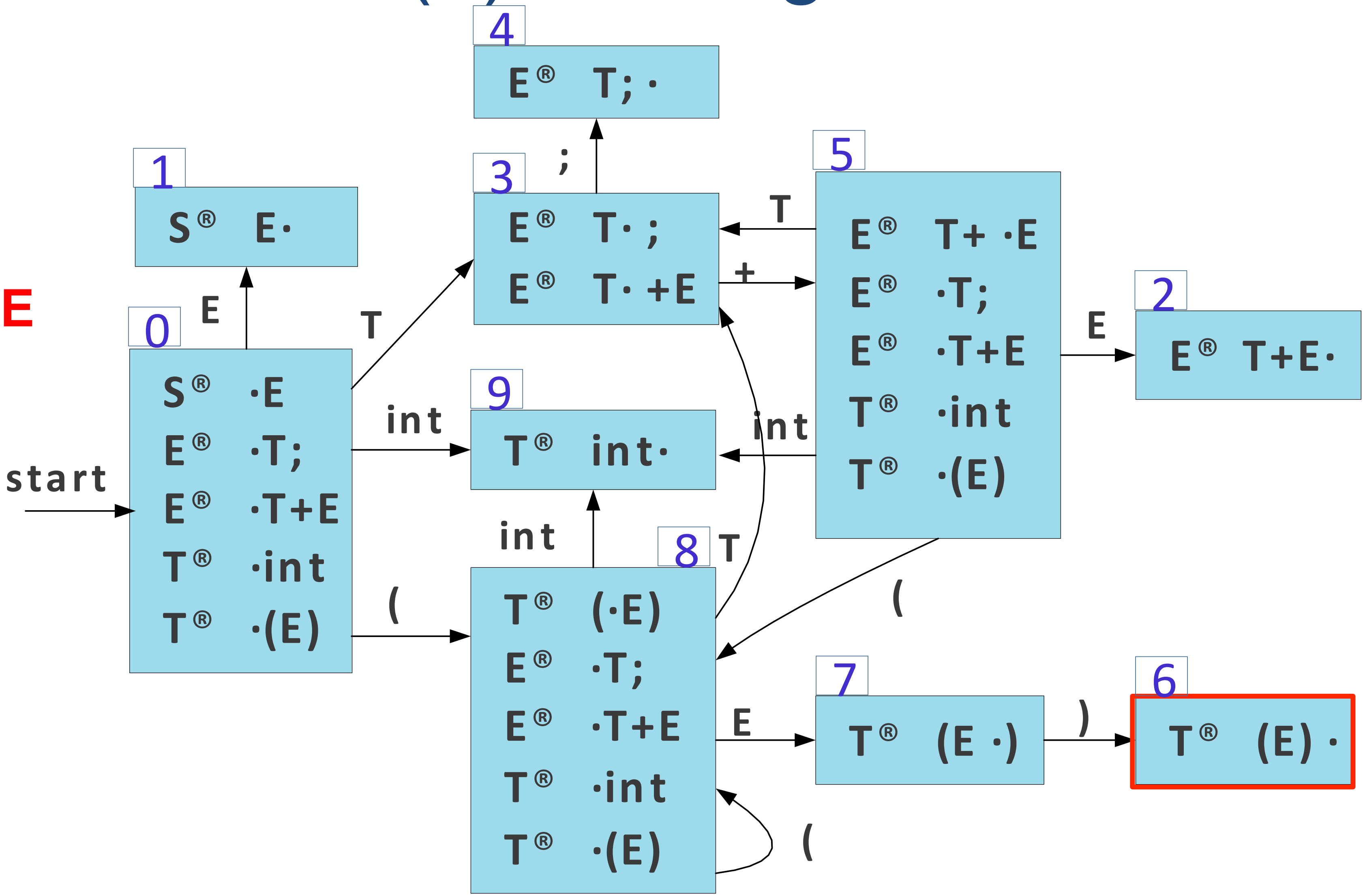


$	T	+	(	E	)
0	3	5	8	7	6

; $

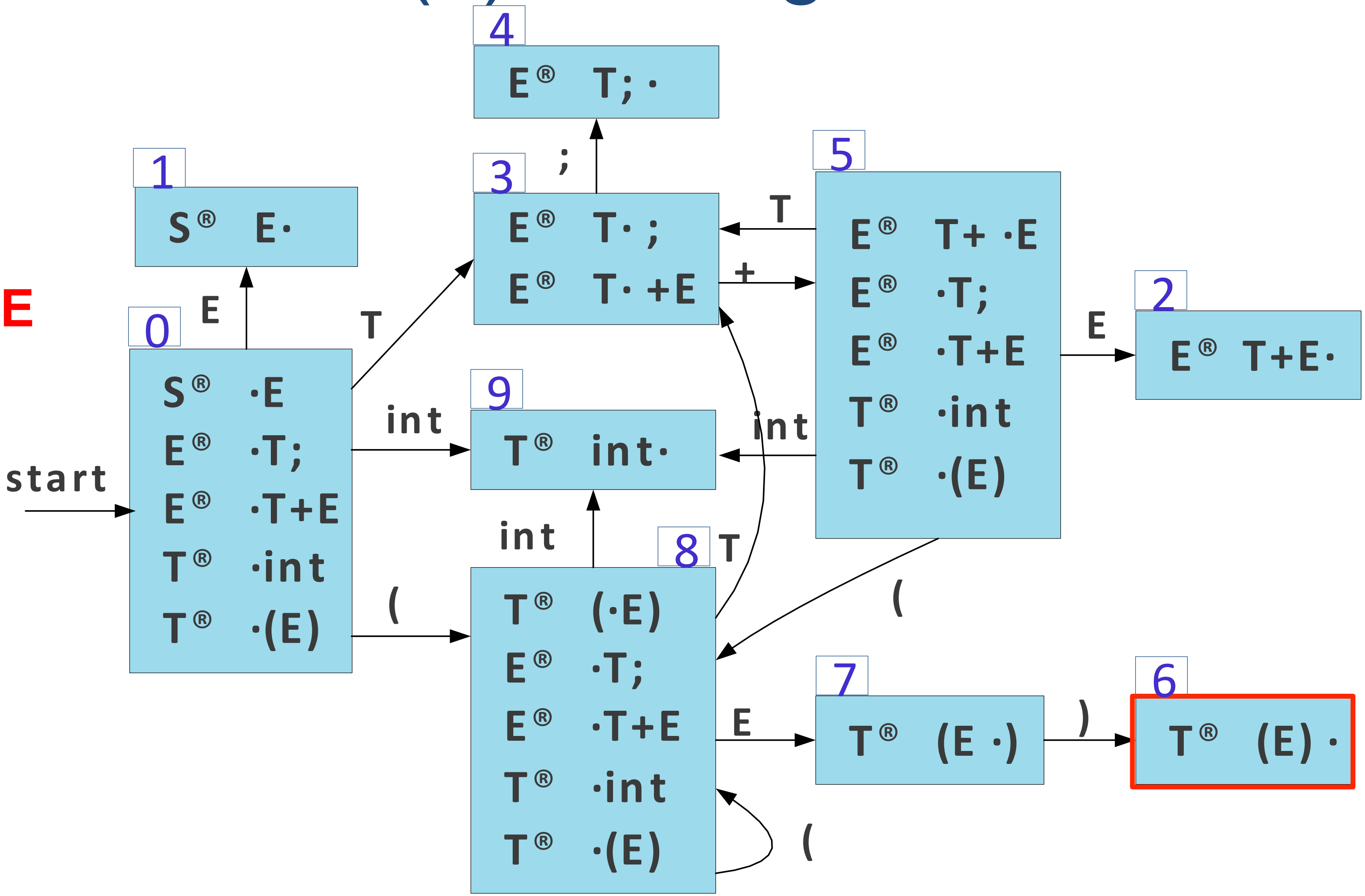
LR(0) Parsing

S → E
E → T;
E → T + E
T → int
T → (E)



LR(0) Parsing

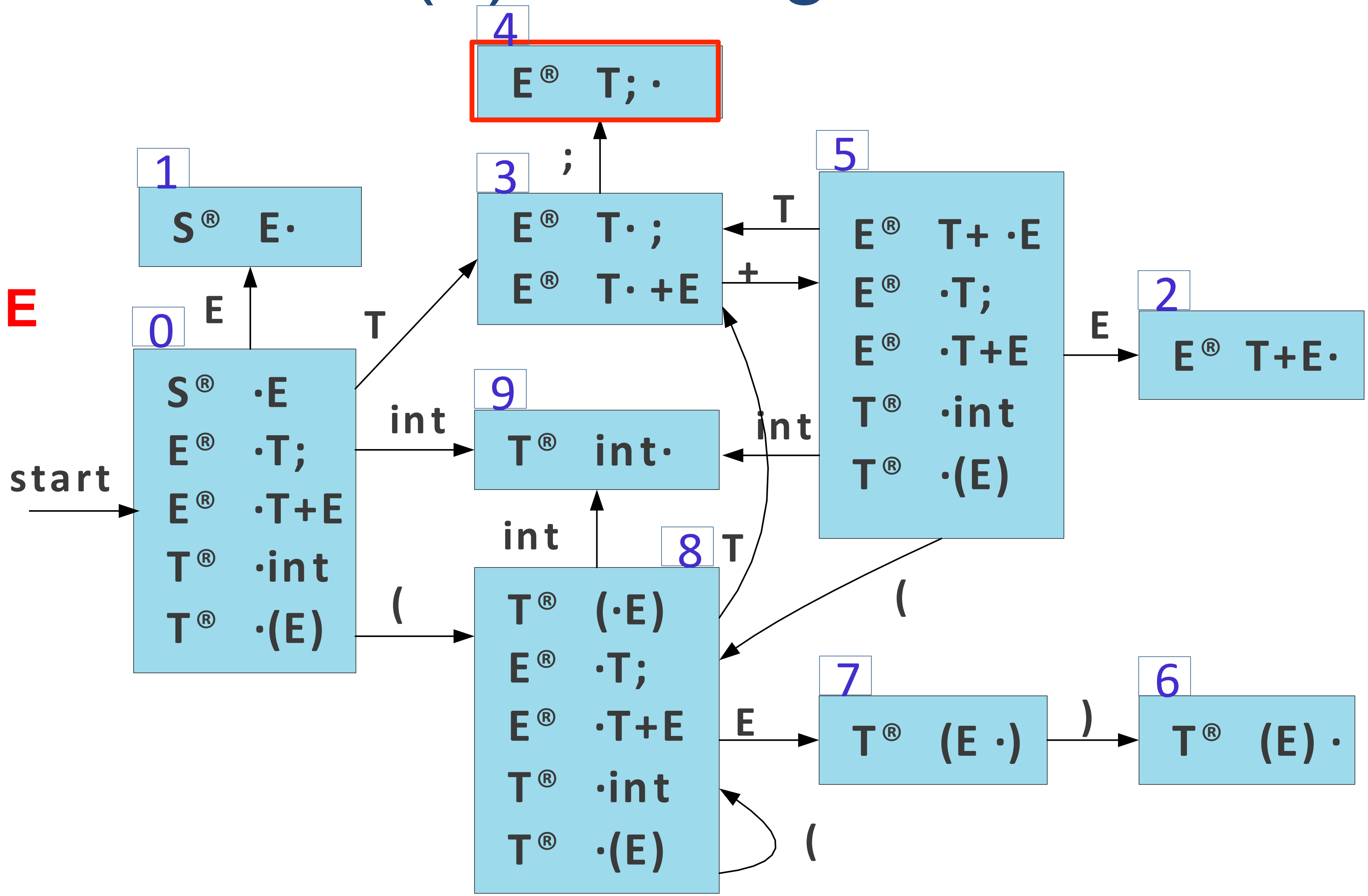
$S \rightarrow E$
$E \rightarrow T;$
$E \rightarrow T + E$
$T \rightarrow \text{int}$
$T \rightarrow (E)$



$	T	+	T	|	;	$
0	3	5	3			

LR(0) Parsing

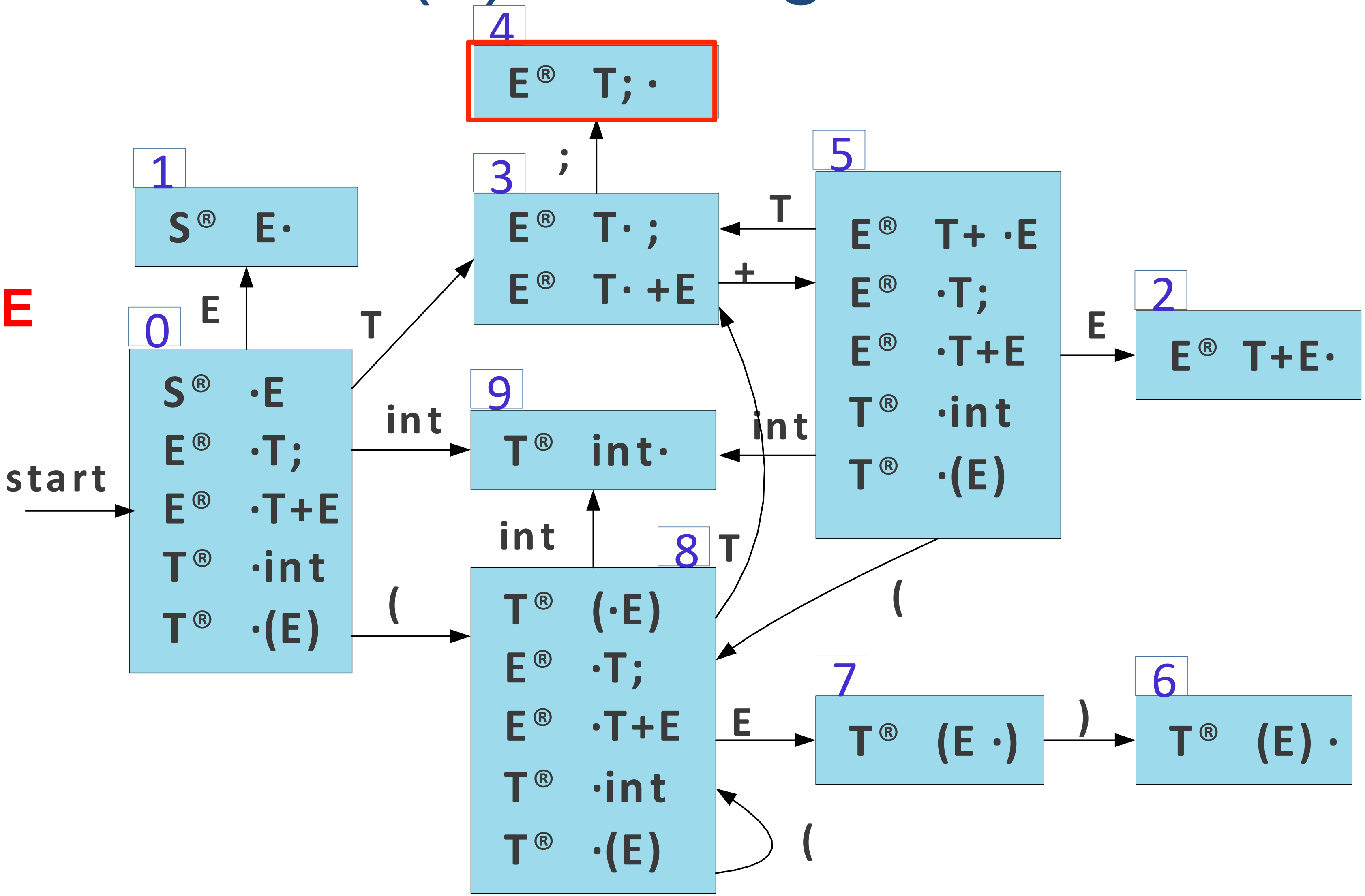
$S \rightarrow E$
$E \rightarrow T;$
$E \rightarrow T + E$
$T \rightarrow \text{int}$
$T \rightarrow (E)$



	$	T	+	T	;	$
	0	3	5	3	4	

LR(0) Parsing

S → E
E → T;
E → T + E
T → int
T → (E)

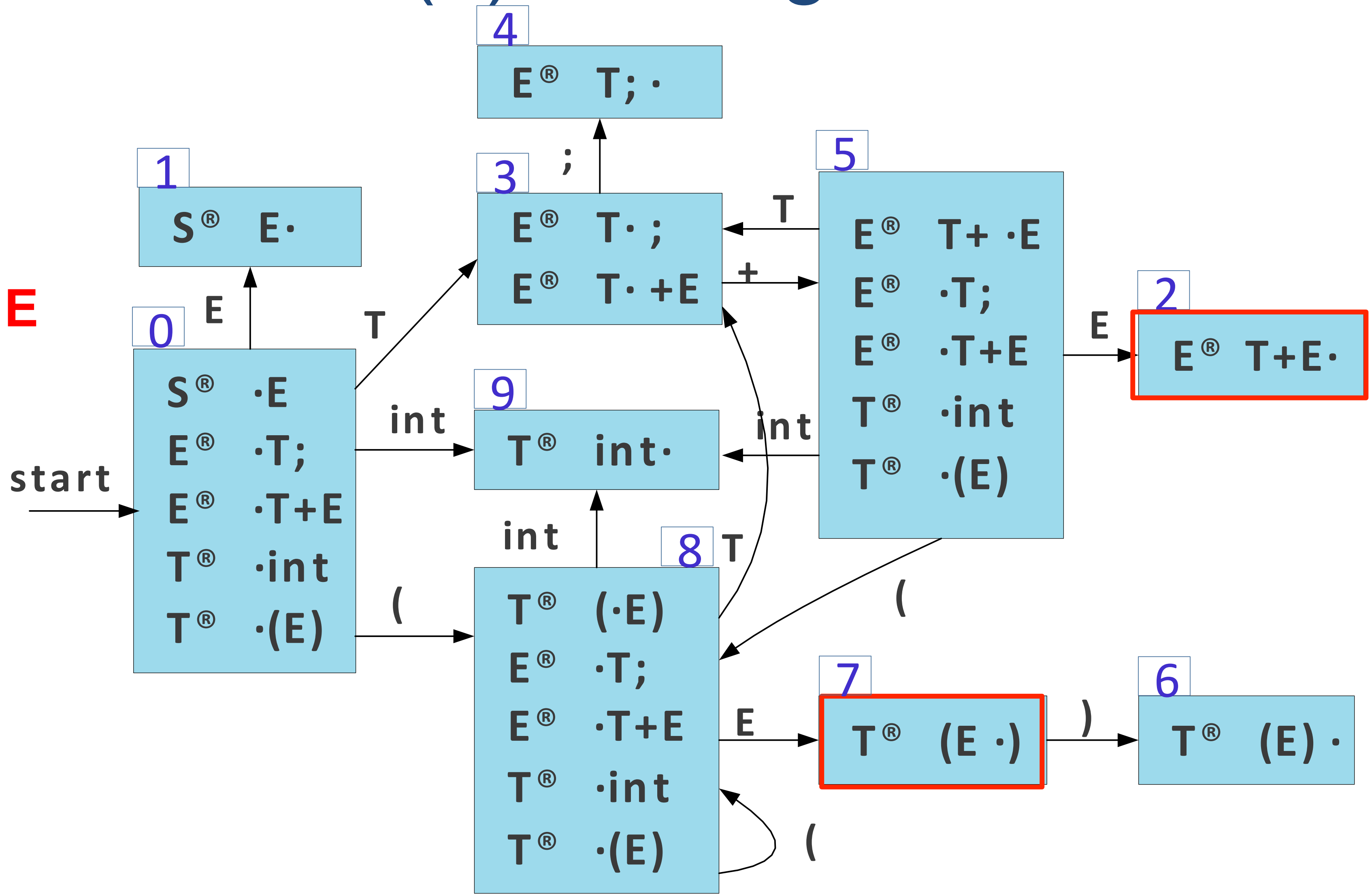


$	T	+
0	3	5

$

LR(0) Parsing

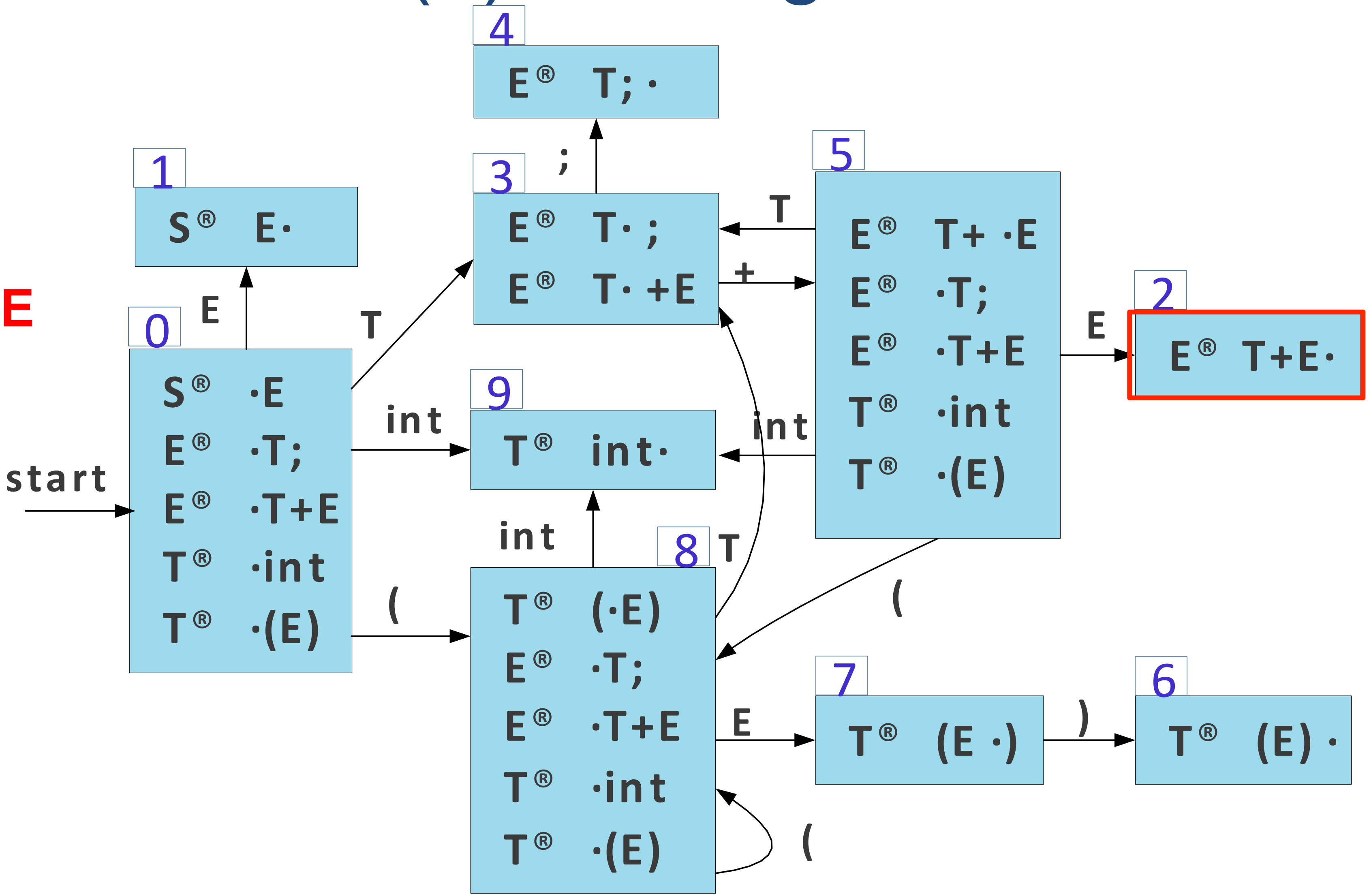
$S \rightarrow E$
$E \rightarrow T;$
$E \rightarrow T + E$
$T \rightarrow int$
$T \rightarrow (E)$



$	T	+	E	$
0	3	5	2	

LR(0) Parsing

S → E
E → T;
E → T + E
T → int
T → (E)



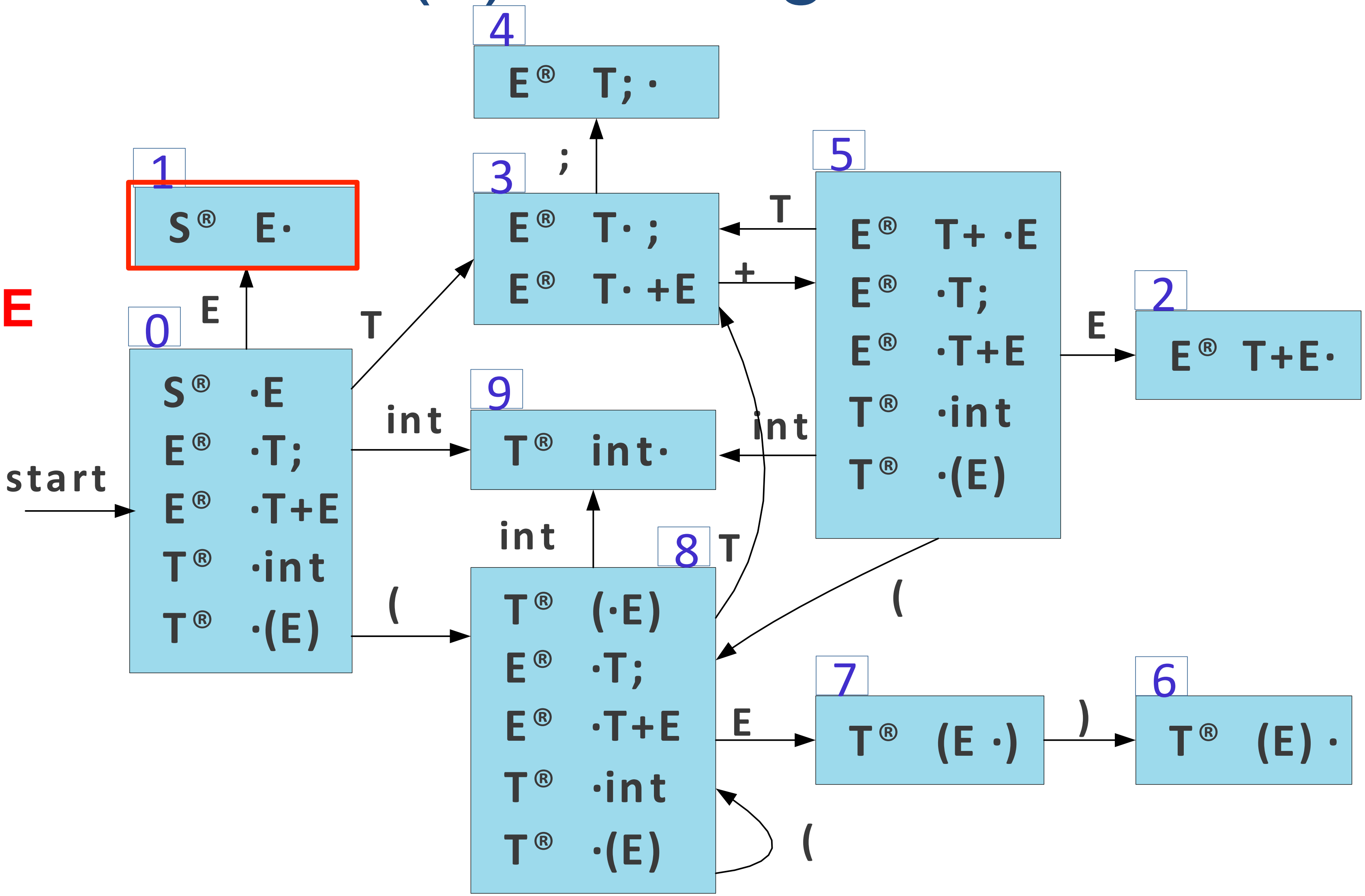
$

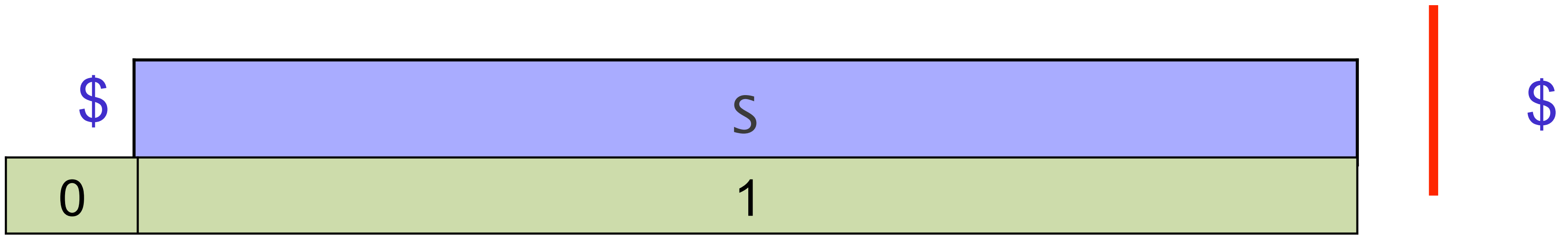
0

$

LR(0) Parsing

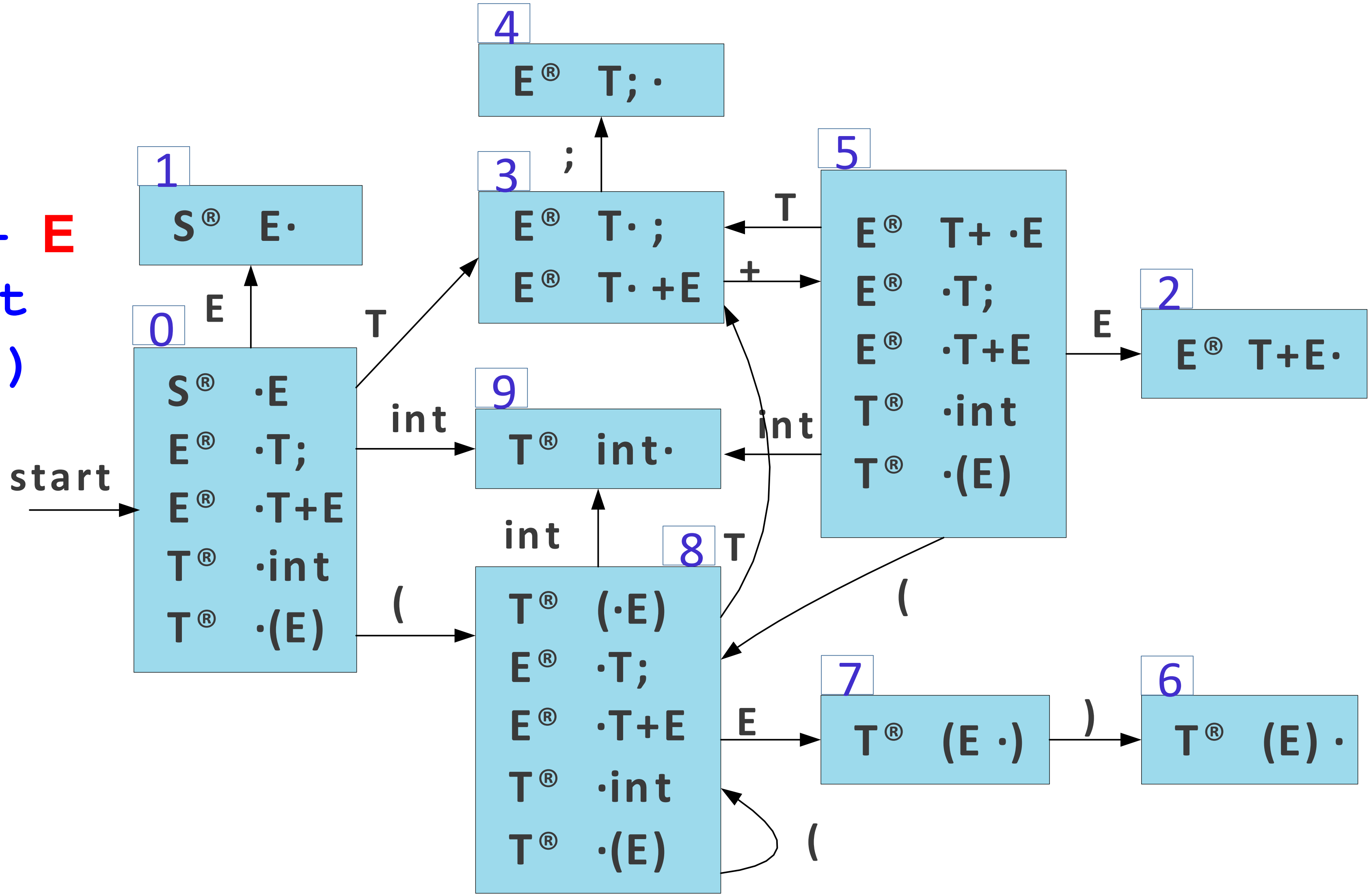
S → E
E → T;
E → T + E
T → int
T → (E)





Building LR(0) Tables

1. S → E
2. E → T;
3. E → T + E
4. T → int
5. T → (E)



LR Tables

state	int	+	;	(	)	E	T	$	Action
0	9			8		1	3		Shift
1								acc	Accept
2									Reduce **E** → **T + E**
3		5	4						Shift
4									Reduce **E** → **T ;**
5	9			8		2	3		Shift
6									Reduce **T** → **(E)**
7					6				Shift
8	9			8		7	3		Shift
9									Reduce **T** → **int**

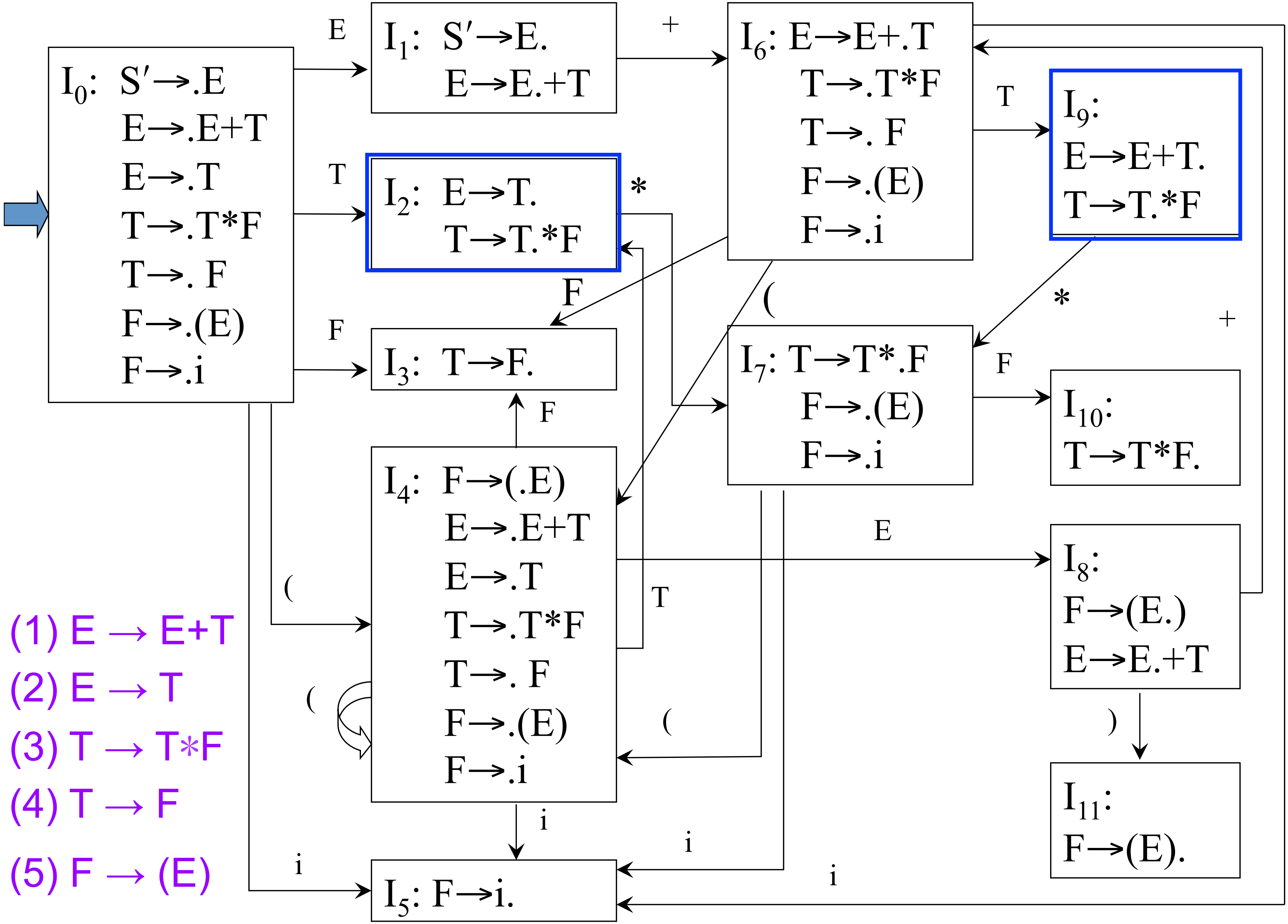
LR Tables

ACTION									GOTO	
state	int	+	;	(	)	E	T	$	E	T
0	s9			s8		s1	s3		1	3
1								acc		
2	r3	r3	r3	r3	r3	r3	r3	r3		
3		s5	s4							
4	r2	r2	r2	r2	r2	r2	r2	r2		
5	s9			s8		s2	s3		2	3
6	r5	r5	r5	r5	r5	r5	r5	r5		
7					s6					
8	s9			s8		s7	s3		7	3
9	r4	r4	r4	r4	r4	r4	r4	r4		

Representing the Automaton

- The ACTION function takes as arguments a state i al1d a terminal a (or \$, the input endmarker). The value of ACTION[i, a] can have one of four forms:
    a) Shift j , where j is a state. The action taken by the parser effectively shifts input a to the stack, but uses state j to represent a .
    b) Reduce $A \rightarrow \beta$. The action of the parser effectively reduces β on the top of the stack to head A.
    c) Accept. The parser accepts the input and finishes parsing;
    d) Error.
- We extend the GOTO function, defined on sets of items, to states: if GOTo $[I_i , A] = I_j$, then GOTO also maps a state i and a nonterminal A to state j .

The Limits of LR(0)



(1) E → E+T

(2) E → T

(3) T → T∗F

(4) T → F

(5) F → (E)

(6) F → i

LR(0) table for the expression grammar

state	ACTION					
	i	+	*	(	)	$
2	r2	r2	r2/s7	r2	r2	r2
3	r4	r4	r4	r4	r4	r4
	……					
9	r1	r1	r1/s7	r1	r1	r1
10	r3	r3	r3	r3	r3	r3
11	r5	r5	r5	r5	r5	r5

LR Conflicts

A **shift/reduce conflict** is an error where a shift/reduce parser cannot tell whether to shift a token or perform a reduction.

A **reduce/reduce conflict** is an error where a shift/reduce parser cannot tell which of many reductions to perform.

A grammar whose handle-finding automaton contains a shift/reduce conflict or a reduce/reduce conflict is not LR(0).

(1) E → E+T

(2) E → T

(3) T → T*F

(4) T → F

(5) F → (E)

(6) F → i

state	ACTION						GOTO		
	i	+	*	(	)	$	E	T	F
0	s5			s4			1	2	3
1		s6				acc			
2		r2	s7		r2	r2			
3		r4	r4		r4	r4			
4	s5			s4			8	2	3
5		r6	r6		r6	r6			
6	s5			s4				9	3
7	s5			s4					10
8		s6			s11				
9		r1	s7		r1	r1			
10		r3	r3		r3	r3			
11		r5	r5		r5	r5			

(1) E → E+T
(2) E → T
(3) T → T*F
(4) T → F
(5) F → (E)
(6) F → i

Step	state	stack	input
1)	0	$	i*i+i $
2)	05	$i	*i+i $
3)	03	$F	*i+i $
4)	02	$T	*i+i $
5)	027	$T*	i+i $

state	ACTION						GOTO		
	i	+	*	(	)	$	E	T	F
0	s5			s4			1	2	3
1		s6				acc			
2		r2	s7		r2	r2			
3		r4	r4		r4	r4			
4	s5			s4			8	2	3
5		r6	r6		r6	r6			
6	s5			s4				9	3
7	s5			s4					10
8		s6			s11				
9		r1	s7		r1	r1			
10		r3	r3		r3	r3			
11		r5	r5		r5	r5			

input： i*i+i

(1) E → E+T
(2) E → T
(3) T → T*F
(4) T → F
(5) F → (E)
(6) F → i

Step	state	stack	input
1)	0	$	i*i+i
$ 2)	05	$i	*i+i $
3)	03	$F	*i
+i $ 4)	02	$T	*i
+i $ 5)	027	$T*	i+i $
6)	0275	$T*i	+i $
7)	027 <u>10</u>	$T*F	+i $
8)	02	$T	+i $
9)	01	$E	
+i $ 10)	016	$E+	i $
11)	0165	$E+i	$
12)	0163	$E+F	$
13)	0169	$E+T	$
14)	01	$E	$

Conflicts During Shift-Reduce Parsing

- There are context-free grammars for which shift-reduce parsers cannot be used.
- Stack contents and the next input symbol may not decide action:
  - **shift/reduce conflict**: Whether make a shift operation or a reduction.
  - **reduce/reduce conflict**: The parser cannot decide which of several reductions to make.
- If a shift-reduce parser cannot be used for a grammar, that grammar is called as non-LR(k) grammar.
- left to right scanning right-most derivation k lookhead
- An ambiguous grammar can never be a LR grammar.

Shift-Reduce Parsers

- There are two main categories of shift-reduce parsers

- **Operator-Precedence Parser**
  - simple, but only a small class of grammars.

- **LR-Parsers**
  - covers wide range of grammars.
    - **SLR** – simple LR parser
    - **LR** – most general LR parser
    - **LALR** – intermediate LR parser (lookhead LR parser)
  - SLR, LR and LALR work same, only their parsing tables are different.

LR Parsers

- The most powerful shift-reduce parsing (yet efficient) is: LR(k) parsing
- LR parsing is attractive because:
  - LR parsing is most general non-backtracking shift-reduce parsing, yet it is still efficient.
  - The class of grammars that can be parsed using LR methods is a proper superset of the class of grammars that can be parsed with predictive parsers.

    LL(1)-Grammars $\subset$ **LR(1)-Grammars**
  - An LR-parser can detect a syntactic error as soon as it is possible to do so a left-to-right scan of the input.

LR Parsers

- **LR-Parsers**
  - covers wide range of grammars.
  - SLR – simple LR parser
  - LR – most general LR parser
  - LALR – intermediate LR parser (look-head LR parser)

  - SLR, LR and LALR work same (they used the same algorithm), only their parsing tables are different.

G:

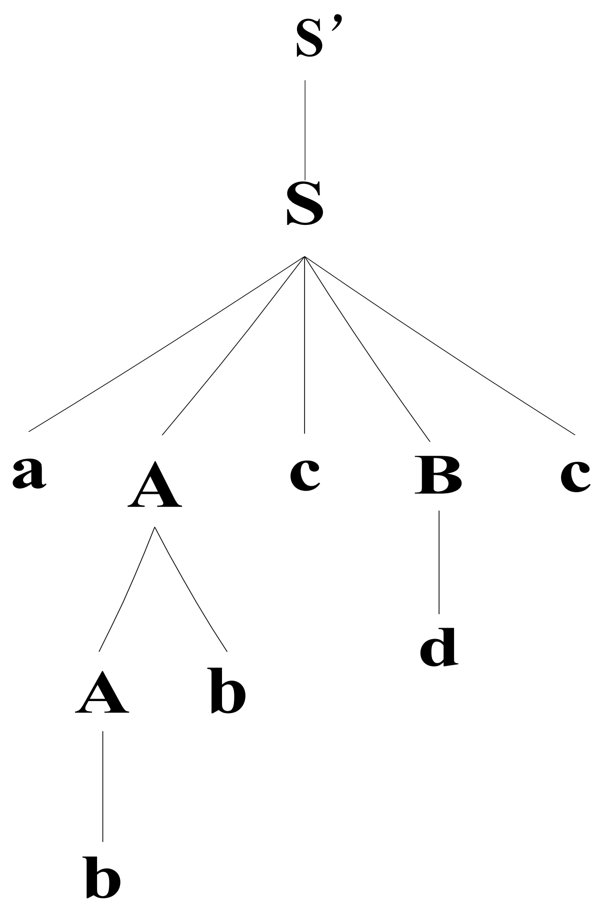
S’ → S

S → aAcBe

A → b

A → Ab

B → d

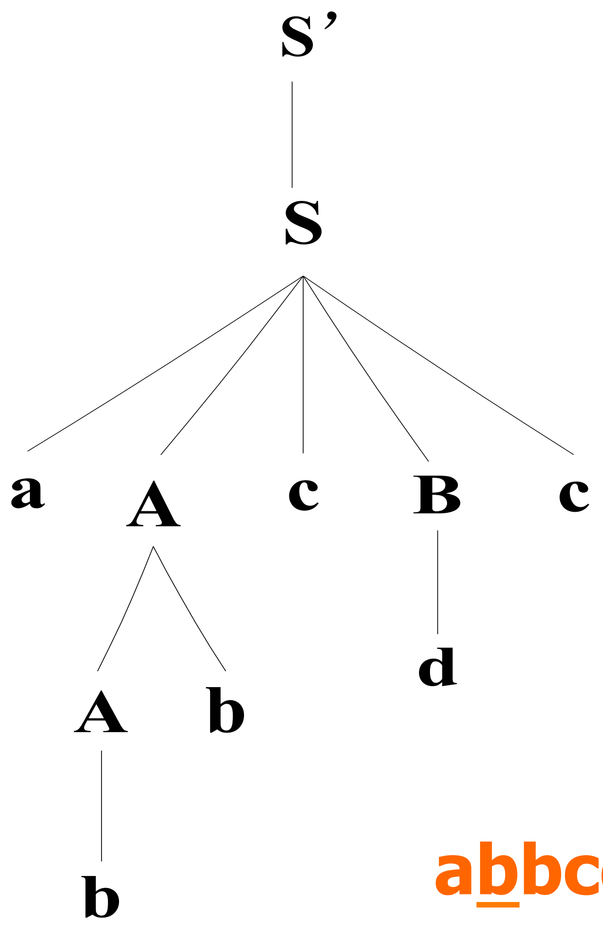


parsing:

$$a\underline{b}bcde \Leftarrow a\underline{Ab}cde \Leftarrow aAc\underline{d}e \Leftarrow \underline{aAcBe} \Leftarrow S \Leftarrow S'$$

G:

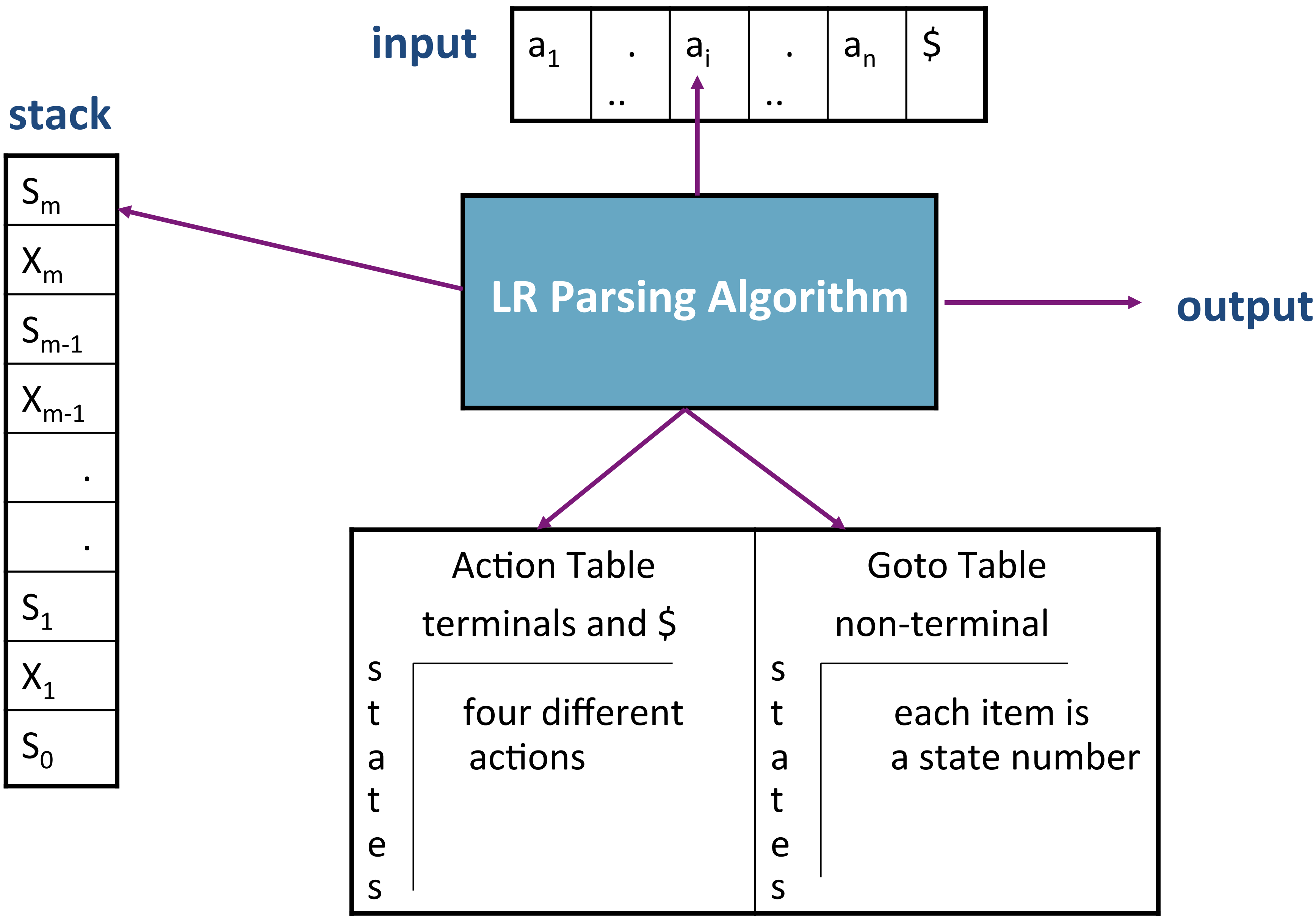
- $S' \to S$
- $S \to aAcBe$
- $A \to b$
- $A \to Ab$
- $B \to d$



steps	stack	input	output
1)	\$	abbcde\$	
2)	\$a	bbcde\$	shift
3)	\$ab	bcde\$	shift
4)	\$aA	bcde\$	reduce(A→b)
5)	\$aAb	cde\$	shift
6)	\$aA	cde\$	reduce(A→Ab)
7)	\$aAc	de\$	shift
8)	\$aAcd	e\$	shift
9)	\$aAcB	e\$	reduce(B→d)
10)	\$aAcBe	\$	shift
11)	\$S	\$	reduce(S→aAcBe)
12)	\$S'	\$	accept

abbcde ⇐ aAbcde ⇐ aAcde ⇐ aAcBe ⇐ S ⇐ S'

LR Parsing Algorithm



(SLR) Parsing Tables for Expression Grammar

1) E → E+T
2) E → T
3) T → T*F
4) T → F
5) F → (E)
6) F → id

	Action Table						Goto Table		
state	**id**	**+**	*****	**(**	**)**	**$**	**E**	**T**	**F**
0	**s5**			**s4**			**1**	**2**	**3**
1		**s6**				**acc**			
2		**r2**	**s7**		**r2**	**r2**			
3		**r4**	**r4**		**r4**	**r4**			
4	**s5**			**s4**			**8**	**2**	**3**
5		**r6**	**r6**		**r6**	**r6**			
6	**s5**			**s4**				**9**	**3**
7	**s5**			**s4**					**10**
8		**s6**			**s11**				
9		**r1**	**s7**		**r1**	**r1**			
10		**r3**	**r3**		**r3**	**r3**			
11		**r5**	**r5**		**r5**	**r5**			

Actions of A (S)LR-Parser – Example

stack	input	action	output
0	id*id+id$	shift 5	
0id5	*id+id$	reduce by F→id	F→id
0F3	*id+id$	reduce by T→F	T→F
0T2	*id+id$	shift 7	
0T2*7	id+id$	shift 5	
0T2*7id5	+id$	reduce by F→id	F→id
0T2*7F10	+id$	reduce by T→T*F	T→T*F
0T2	+id$	reduce by E→T	E→T
0E1	+id$	shift 6	
0E1+6	id$	shift 5	
0E1+6id5	$	reduce by F→id	F→id
0E1+6F3	$	reduce by T→F	T→F
0E1+6T9	$	reduce by E→E+TE→E+T	
0E1	$	accept	

Reference

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  - Chapter 4.5, 4.6
- Coursera Course – Compiler, http://www. Coursera.org
- Stanford Course CS143 by Keith Schwarz, http://cs143.stanford.edu