Social Polarization in Response to Contagions

September 25, 2020

Abstract

I construct a very simple utility-based model of social connections which leads to variation in the number of social connections each person has. When the threat of a contagion is introduced, highly connected individuals have less incentive to break off marginal social connections than do less connected individuals. This increases the variance of the distribution of connections, which has important implications in understanding the spread of contagions such as disease.

1 Introduction

Why does it seem that some highly secluded people with very low exposure risk are highly concerned about social distancing, while other more gregarious individuals at very high risk of exposure don't seem to care at all?

If the personal risk from a contagion is the same for two individuals, then an individual with a very high number of social connections expects to be exposed to the contagion regardless of any marginal changes to their behavior.

2 A simple model of endogenous social connections

I make assumptions about the utility of maintaining social connections, and about the disutility of being exposed to contagion, such that individuals with lots of neighbors are precisely those who won't decide to socially distance.

Social Connections

There is a unit mass of individuals, indexed by x from 0 to 1. Each individual person can be thought of as one of the vertices in a social graph, where the edges of the graph represent person-to-person connections along which contagions, such as disease or information, can potentially spread.

Each person $x \in (0, 1)$ has some randomly determined exogenous level of social skill or social capital κ_x. If the process for generating κ has distribution function F_κ, then without loss of generality, we can order the indices so that $F_\kappa(\kappa_x) = Pr(\kappa \leq \kappa_x) = x$

Each person chooses how many $n \geq 0$ social connections (edges in the social graph) they wish to maintain, and uses these connections to improve their own welfare. These connections could be thought of employment which allows the person to buy sandwiches, agreements with neighbors to cooperate on household-production, or intrinsically rewarding social interactions like friendships and dance partners. Higher levels of social capital enable a person to get more utility out of each connection, but there are diminishing returns to seeking out new connections. The utility that person x with social capital κ_x gains from maintaining n connections is $\frac{1}{1-\alpha}\kappa_x n^{1-\alpha}$ where $\alpha \in (0, 1)$ is a parameter common across people. Here, the connections are formed randomly, with each person not knowing the index or social capital of their neighbors. And non-integer numbers of connections are allowed.

If the maintenance of each social connection requires effort which incurs exogenous disutility ϖ, then person x's utility as a function of their choice of social connections is

$$U_x(n) = \frac{1}{1-\alpha}\kappa_x n^{1-\alpha} - \varpi n$$

If $n(x)$ denotes person x's optimal number of social connections, then

$$n(x) = \left(\frac{\kappa_x}{\varpi}\right)^{1/\alpha}$$

The above equation on also gives us the distribution over degrees in the social graph. $F_N\left(n(x)\right) = Pr\left(n(x) \leq \kappa_x\right) = x$

Transmission of the contagion

There is some contagion which spreads through the network in an SIR manner. Like in (Newman 2002) [9], which provides a general description of evaluating the spread of disease through a network, I'm considering the spread of the epidemic by looking at it's final outcomes rather than by looking at the time-path of infectious frequency among the population.

If a person is infected with the contagion, then they experience additional disutility δ. Multiple infectious exposures don't incur any additional disutility. Let $I_{\delta x} = 1$ iff person x is exposed to the contagion at some point during the epidemic, and 0 otherwise.

When a person first hears about the epidemic, they choose how many social connections to maintain for it's duration. Their expected utility is

$$\begin{aligned}
\mathbb{E}\left[U_x(n,\delta)\right] &= \mathbb{E}\left[\frac{1}{1-\alpha}\kappa_x n^{1-\alpha} - \varpi n - I_{\delta x}\delta\right] \\
&= \frac{1}{1-\alpha}\kappa_x n^{1-\alpha} - \varpi n - Pr(I_{\delta x} = 1)\cdot\delta
\end{aligned}$$

T is the transmissibility of the contagion. It is the chance that an infected person will transmit the disease to an uninfected person given that they share a social connection.[1] T here is the same for all connections.

Let R_∞ denote the portion of the population that is exposed to the disease throughout the entire course of the epidemic. Because people are assumed to not know the connectivity of their neighbors, each person expects that each of their neighbors will become infected with probability R_∞. R_∞ is determined in equilibrium through the choices of n by the individuals.

The chance that the contagion is transmitted along any particular social connection is TR_∞. This is the probability that that neighbor gets sick times the probability that they transmit along that particular social connection.

But infection occurs in person x iff *at least* one of their neighbors transmits the infection to them. If you are a person in this social network with n neighbors, then the chance that none of your neighbors transmits to you is $[1 - TR_\infty]^n$. And the the chance that at least one of your neighbors transmits to you is $1 - [1 - TR_\infty]^n$.

So

$$\mathbb{E}\left[U_x(n,\delta)\right] = \frac{1}{1-\alpha}\kappa_x n^{1-\alpha} - \varpi n - \delta + \delta\left[1 - TR_\infty\right]^n$$

If $\delta = 0$, then this is the same as the baseline model without contagion. If $\delta > 0$, then the contagion is something bad like a disease. And if $\delta < 0$, then the contagion is something beneficial like technological diffusion.

If people choose their social connections for the duration of the epidemic in an expected utility maximizing way, then the optimal number of connections for person x, $n(x,\delta)$ satisfies

$$0 = \kappa_x n(x,\delta)^{-\alpha} - \varpi + \delta n(x,\delta)\left[1 - TR_\infty\right]^{n(x,\delta)-1}$$

[1] Contrast T with R_0, the basic reproduction number, which is the average number of new infections generated by each infection when the entire population is initial susceptible. R_0 is determined by a combination of the transmission rate and structure of social interactions.[8]

Main Consequence

Because any individual can only become infected with a contagion once, and because the chances of avoiding infection exponentially decay with number of connections, highly connected individuals will change their behavior less in response to the threat of contagion. An introvert with very low social capital and hence very few connections can drastically increase their safety by cutting off a marginal social connection. But an extrovert with a very large number of connections would experience almost no benefit from doing so. The same highly connected individuals who drive the spread of contagion, are also the individuals least likely to self-interestedly take steps to mitigate the spread.

conjecture 1:

Let $g(x, \delta)$ be person x's proportional decrease in connections when news of an impending pandemic arrives:

$$g(x, \delta) = \frac{n(x, \delta) - n(x)}{n(x)}$$

For $\delta > 0$, $|g(x, \delta)|$ is decreasing in x.

For $\delta < 0$, $|g(x, \delta)|$ is increasing in x.

conjecture 2:

Given that $\delta > 0$ and [insert sufficient assumptions about distribution of social capital] ,the variance of the distribution of $n(x, \delta)$ over x is greater than that for $n(x)$.

3 Implications for Disease Outbreaks

References

[1] Lloyd-Smith, J., Schreiber, S., Kopp, P. et al. Superspreading and the effect of individual variation on disease emergence. Nature 438, 355–359 (2005).

https://doi.org/10.1038/nature04153 (Highly cited paper about the variation of these connections and their effects)

[2] Fogli, Alessandra, and Laura Veldkamp. Germs, social networks and growth. No. w18470. National Bureau of Economic Research, 2012. (Discussion of how diffusion through a network can have positive effects when talking about technology.)

[3] Kremer, Michael. "Integrating behavioral choice into epidemiological models of AIDS." The Quarterly Journal of Economics 111.2 (1996): 549-573. (Pretty much already does the differential response based on connectivity thing. Well, that one uses random partners and die-off and only looks at the steady state. So maybe my BS is close enough)

[4] Kremer, Michael, and Charles Morcom. "The effect of changing sexual activity on HIV prevalence." Mathematical biosciences 151.1 (1998): 99-122. (Similar to the above)

[5] Britton, Tom, Frank Ball, and Pieter Trapman. "The disease-induced herd immunity level for Covid-19 is substantially lower than the classical herd immunity level." arXiv preprint arXiv:2005.03085 (2020). (Just plots a few toy examples of how variations in activity can alter the herd immunity threshold)

[6] Gomes, M. Gabriela M., et al. "Individual variation in susceptibility or exposure to SARS-CoV-2 lowers the herd immunity threshold." medRxiv (2020).("Models that curtail individual variation in susceptibility or exposure to infection overestimate epidemic sizes and herd immunity thresholds." Don't quite understand the setup of the model.)

[7] Mossong, Joël, et al. "Social contacts and mixing patterns relevant to the spread of infectious diseases." PLoS Med 5.3 (2008): e74. (Empirical analysis of social contact structure)

[8] Pourbohloul, Babak, et al. "Modeling control strategies of respiratory pathogens." Emerging infectious diseases 11.8 (2005): 1249. (Talks about the importance of trans-

misability instead of just basic reproductive number. Also talks about the importance in the variability of the connections)

[9] Newman, Mark EJ. "Spread of epidemic disease on networks." Physical review E 66.1 (2002): 016128.(Has the rigourous probabilistic treatment but no endogeneity. peanut butter and chocolate. Power law stuff)

[10] Amaral, Luıs A. Nunes, et al. "Classes of small-world networks." Proceedings of the national academy of sciences 97.21 (2000): 11149-11152. (Power law and exponential distributions)

[11] Barabási, Albert-László, and Réka Albert. "Emergence of scaling in random networks." science 286.5439 (1999): 509-512. (emergence of scaling using a sort of superstar effect explanation)