Department of Mathematics, Imperial College London, 2020/21
MATH97132 Introduction to Statistical Finance

Assessed Coursework (weight: 10%)

- Part A: you have to work on and submit individually. You are not allowed to discuss the question with anybody except the lecturer.
- Part B: you can submit as groups (of at most most 4 persons per group). Please designate one person from your group to submit par B, clearly stating all names of group members on the first page. Do not discuss the question with people outside of your group.
- Present your results in a written report (at most 6 pages for each part and submit these separately; links for submission via Turnitin will be on Blackboard).
- Both parts are weighted equally.
- You may use any *publicly available* free or commercial software and packages. Please indicate in your report, which software (and packages) you have used.
- Cite your sources - standard rules for academic integrity apply! **Do not use Piazza to post questions.**
- Please include your code as an appendix (which may extend beyond the page limit). Attaching Excel spreadsheets (if used in Part A) is not required.
- **Deadline:Tuesday, 23 February 2020, 4:00pm (UK time).**
- This is assessed work, you have to work on it individually (part A) or in your chosen group (B). No discussion of these questions outside of these groups is permitted.

Part A: Derivatives pricing

Consider the trinomial market model (Example P.30 in lecture notes) with parameter values:

$$\mu = 0.005, \quad h = 0.06, \quad r = 0.01, \quad S_0 = 100, \quad T = 8, \quad p_+ = 0.8, \quad p_- = 0.1. \tag{1}$$

(i) Find an equivalent martingale measure (EMM) $\mathbf{Q}$ by specifying suitable $q_+ := \mathbf{Q}[\xi_1 = 1]$ and $q_- := \mathbf{Q}[\xi_1 = -1]$.

(ii) Are there any other EMMs?

You are now asked to price a payoff $f(S_T)$ given graphically in Figure 1 below.

(iii) Work out an analytical expression for $f(x)$, $x > 0$.

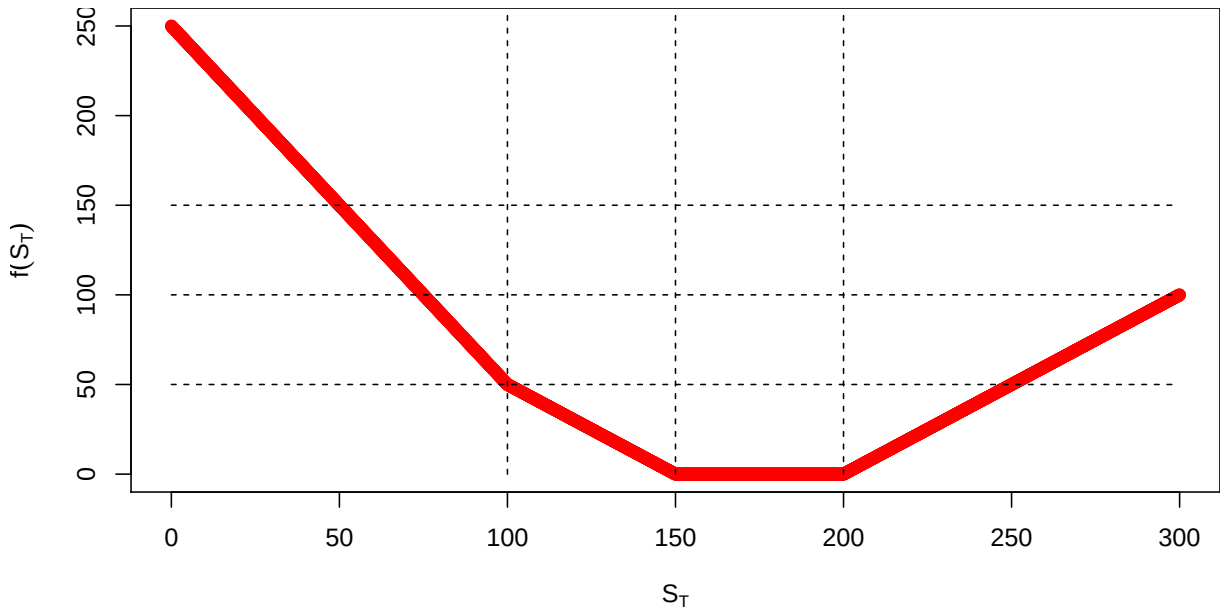

Figure 1: Payoff $f(S_T)$.

Use the values from (1) and the EMM **Q** from part (i).

(iv) Using your answer to part (iii), find an arbitrage-free price process $(\Pi_t)_{t=0}^T$ for the payoff $f(S_T)$ in the trinomial market model. (Present all values of the price process in the relevant trinomial tree.)

(v) Find a way to express $f(S_T)$ as a combination of positions (long or short) in the underlying and European put and call options. Price those instruments separately and verify that the total value of the trade agrees with your answer to part (iv).

Part B: Stylised facts - Groupwork

Choose 3 price series of different types of financial assets (e.g. an index, an exchange rate, a commodity, a stock) over a period of your choice that must include the entire year 2020, but can be longer.

(i) Compute and plot log returns for each of the three price series.

(ii) Analyse, whether each return series exhibits the stylised facts (SF_1–SF_3) introduced in Section FTS.1.

(iii) For each series, find a distribution that describes well the unconditional distribution of returns. Fit the distribution to the data and assess its goodness of fit.

This question is open-ended — there is no single correct answer. It will be assessed marked out of 10 - where 5 will be given to a basic answer with some deficiencies; 6 is a good answer; 7 and 8 is for answers that are more in-depth and answer all points and 9 or 10 is reserved for answers that also engage with the scientific literature.