Modeling COVID-19 Spreading In A Network

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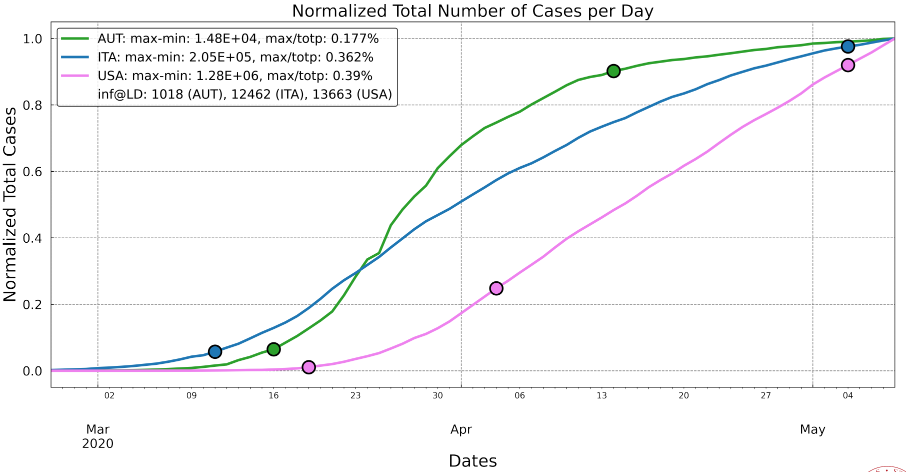
Overview

Motivations from COVID-19 Data



Goal: Study a SIR model on networks

- We will order the Social Network models (Barabási-Albert Model, Poissonian-SW Network, Regular Lattice, Caveman Model) for controlling COVID-19;
- We will estimate the time t_{max} of the peak of the infected after which the pathogen regresses;
- We will propose the *epidemic severity* to capture the final fraction of the total cases;
- We will study an *Order Parameter*:=SD(C) [2] to look for a I order phase transition-like as the average number of contacts D increases.

SIR Models

No Network Structure → Homogeneous Mean-Field (MF) Model [1]

$$\begin{cases} s + r + i = 1 \quad \text{HP: "closed + \textit{well-mixed} population"} \\ \frac{ds}{dt} = -\beta D s i \\ \frac{di}{dt} = \beta D s i - \mu i = \mu(R_0 s - 1) i, \quad R_0 := \frac{\beta D}{\mu} \equiv D\lambda \\ \frac{dr}{dt} = +\mu i \end{cases} \tag{1}$$

Network Structures → Degree-Based Mean-Field Model

$$\begin{cases} \frac{ds_k}{dt} = -\beta k s_k \Theta_k(t) \quad \text{HP: "closed population"} \\ \frac{di_k}{dt} = \beta k s_k \Theta_k(t) - \mu i_k \quad D = \langle k \rangle \text{ with } k := \text{numb. of contacts per indiv.} \end{cases}$$

Fraction of total cases at t_{max}

- Thresholds for Regression in Homogeneous SIR:

$$\begin{cases} R_0 s(t) \overset{!}{=} 1, & R_0 := \frac{\beta D}{\mu} \\ s(t) = 1 - \pi(t) \end{cases} \Leftrightarrow \pi(t_{max}) = 1 - \frac{1}{R_0} \tag{2}$$

- Decay Threshold in Heterogeneous SIR (at $t \approx 0$):

$$\begin{cases} R_0 \overset{!}{=} R_{c-net} := \frac{D^2}{\langle k^2 \rangle - D} \\ R(t) := (D - n(t))\lambda \qquad \forall\, t \\ n(t) \sim 1 + (D-1)(1-p)\pi(t) \end{cases} \Leftrightarrow \pi(t_{max}) = \frac{1}{1-p}\left[1 - \frac{R_{c-net}}{R_0 - \lambda}\right] \tag{3}$$

Epidemic Severity

Basic Reproduction Number R_0

$$R_0 := \frac{\beta D}{\mu} \tag{4}$$

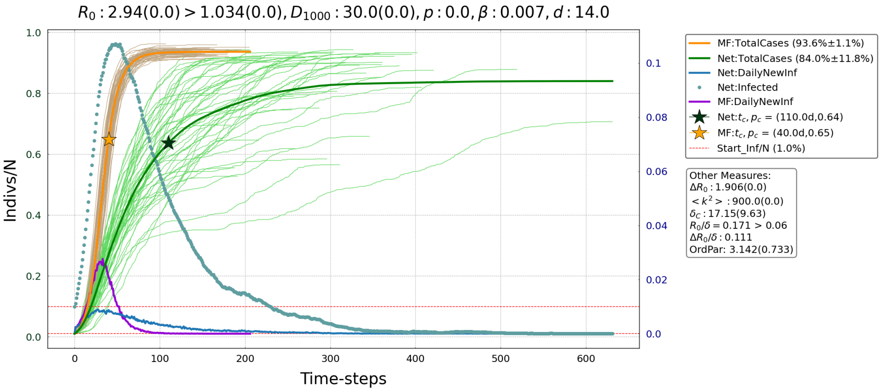
Definition of Epidemic Severity

$$\Delta R_0(\delta) := \frac{R_0 - R_{c-net}}{\delta} \tag{5}$$

where

- $R_{c-net} := \frac{D^2}{\langle k^2 \rangle - D}$. Hence, if $\langle k^2 \rangle \uparrow \implies \Delta R_0(\delta) \uparrow$;
- $\delta := \langle l \rangle$ is the average path length, e.g. from $1 \leftrightarrow 10$. δ increases as the network is less "weaved together".

Example



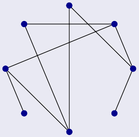
$R_0 : 2.94(0.0) > 1.034(0.0), D_{1000} : 30.0(0.0), p : 0.0, \beta : 0.007, d : 14.0$
Indivs/N
Time-steps
MF:TotalCases (93.6%±1.1%)
Net:TotalCases (84.0%±11.8%)
Net:DailyNewInf
Net:Infected
MF:DailyNewInf
Net:t_c, p_c = (110.0d,0.64)
MF:t_c, p_c = (40.0d,0.65)
Start_Inf/N (1.0%)
Other Measures:
$\Delta R_0 : 1.906(0.0)$
$< k^2 > : 900.0(0.0)$
$\delta_C : 17.15(9.63)$
$R_0/\delta = 0.171 > 0.06$
$\Delta R_0/\delta : 0.111$
OrdPar: 3.142(0.733)

First Order Phase Transition-like

Order Parameter: standard deviation (SD) of the daily new cases C

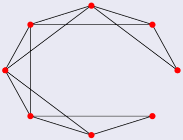
- $\mathbb{O} := SD(C)$

Critical average degree D_c for different topologies



- *Homogeneous MF*: $\frac{\beta D_c}{\mu} \stackrel{!}{=} 1 \Leftrightarrow D_{c-homog} = \frac{\mu}{\beta}$
- *Erdös-Rényi Degree-Based MF*:

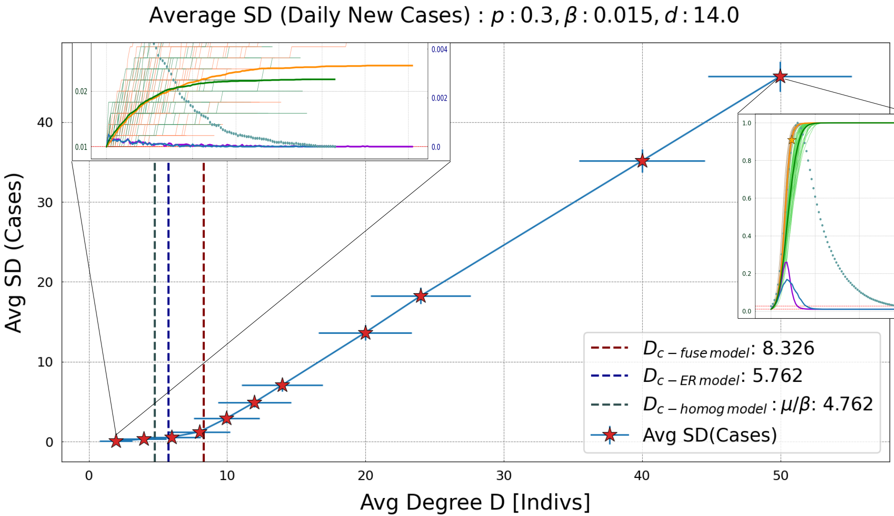
$$\begin{cases} n(t) \sim 1, \quad s(t \approx 0) \sim 1 \\ R(t) := (D - n(t))\lambda \stackrel{!}{=} R_{c-net} \sim 1 \end{cases} \Leftrightarrow D_{c-ER} = 1 + \frac{\mu}{\beta} \qquad (6)$$



- *Fuse-Model Degree-Based MF*:

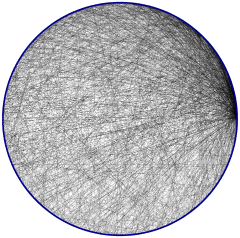
$$n(t) \sim 1 + (D-1)(1-p)/2 \quad \Leftrightarrow \quad D_{c-FM} = 1 + \frac{2\frac{\mu}{\beta}}{1+p} \qquad (7)$$

Example of Order Parameter with Long-Range interactions

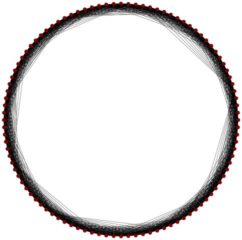


Social Networks & Results

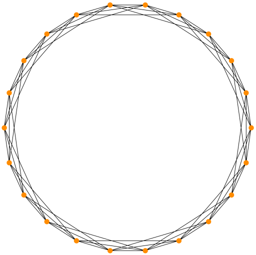
Barabási-Albert Model



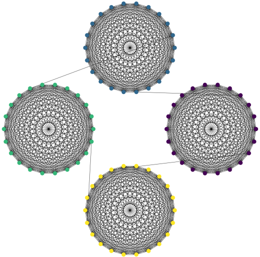
Overlapping PSW



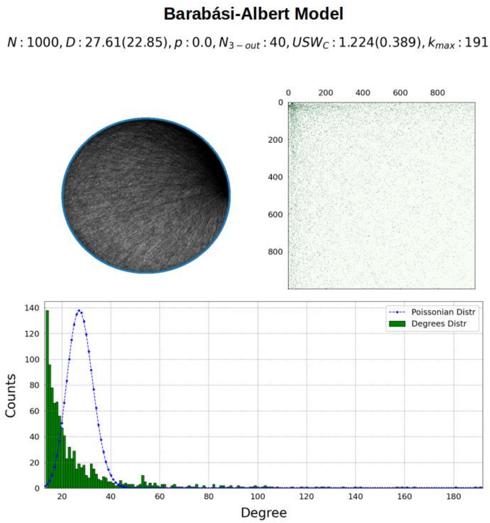
Regular Lattice



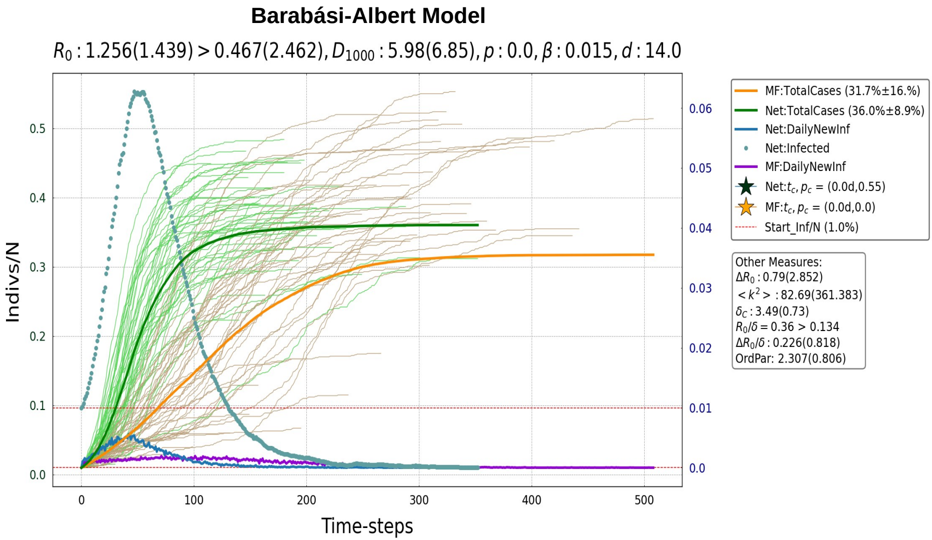
Caveman Model

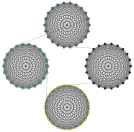
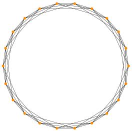
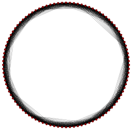
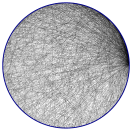


Decreasing Heterogeneity

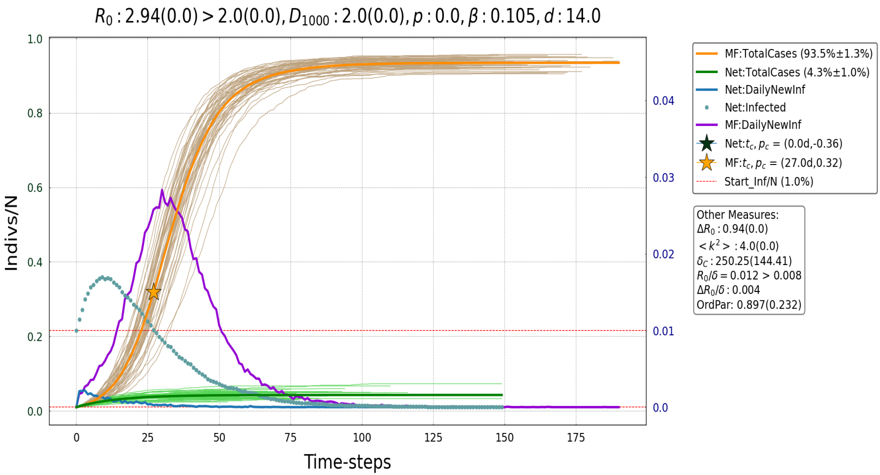


Decreasing COVID-19 Outbreaks [2]

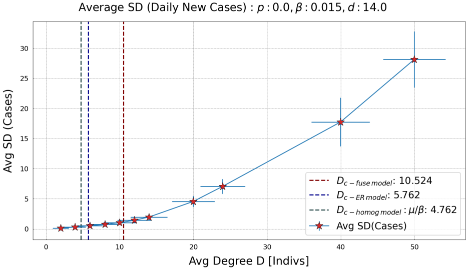


Final Outbreak Size			Degree Distribution
Networks	$D = 6$	$D = 8$	
Caveman Model 	–	$(4.3 \pm 1.4)\%$	• Low Heterogeneity • Clustered Net
Regular Lattice 	–	$(6 \pm 2.3)\%$	Peaked
Overlapping PSW 	–	$(7.4 \pm 2.5)\%$	Mid Heterogeneity (No Hubs)
Barabási-Albert Model 	$(36 \pm 8.9)\%$	–	Highly Heterogeneity (Hubs)

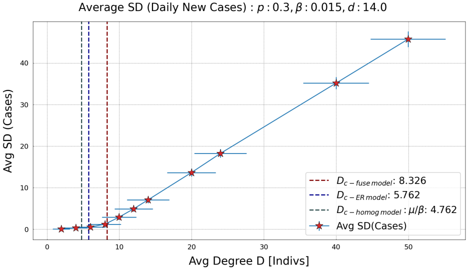
Fixed R_0 SIR + Regular Lattice



Order Parameter ($SD(C)$) of an Overlapping PSW

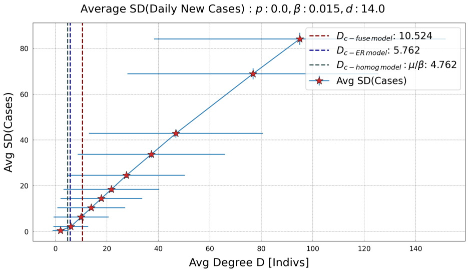


(a) Short-Range Interactions

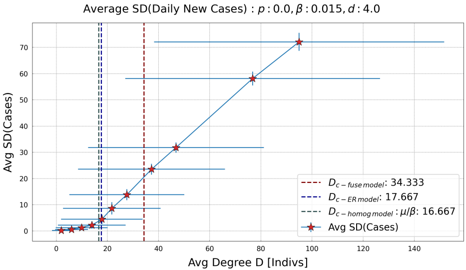


(b) Long-Range Interactions

Order Parameter ($SD(C)$) of a Barabási-Albert Model



(a) Preferential Attachment + d = 14



(b) Preferential Attachment + d = 4

Summary and Conclusions

1. Order the Social Network models for controlling COVID-19: *local* Caveman,Regular Lattice, Poissonian-SW, Barabási-Albert;
2. The peak of the infected is well estimated only for mean-field approximation (yellow star);
3. Used a Regular Lattice to test the *epidemic severity* VS R_0;
4. For Overlapping PSW, D_c is well estimated by the *Fuse model* only for long-range COVID-19 parameters;
5. For Barabási-Albert model, the *homogeneous mean-field* estimates the D_c for $d = 4$ while for $d = 14$, $SD(C) \propto D$;

Bibliography

Albert-László Barabási and Márton Pósfai.
Network science.
Cambridge University Press, 2016.

Stefan Thurner, Peter Klimek, and Rudolf Hanel.
A network-based explanation of why most covid-19 infection curves are linear.
Proceedings of the National Academy of Sciences, 117(37):22684–22689, 2020.

Thank You For Your Attention

Thresholds for Regression

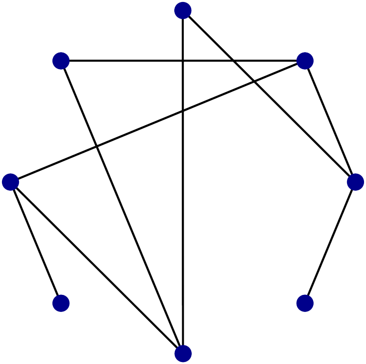
Homogeneous MF Model

$$\frac{di}{dt} = \mu(R_0 s(t) - 1)i(t) < 0 \Leftrightarrow R_0 s(t) < 1 \tag{8}$$

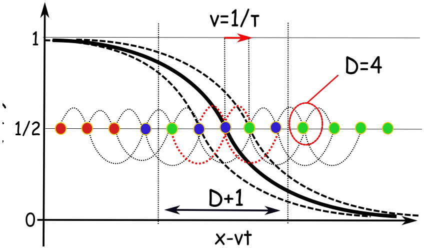
Degree-Based MF: $t \approx 0 \Rightarrow s, i \approx 1, i_0$ and $k \sim D$ for random net

$$\begin{cases} \frac{di_k}{dt} = \mu \left[\frac{\beta k}{\mu} \Theta_k(t) s_k(t) - i_k(t) \right] < 0 \\ \\ \Theta_k(t) = \frac{\sum_{k'} (k'-1) p_{k'} i_{k'}(t)}{\langle k \rangle} = i_0 \frac{D-1}{D} e^{t/\tau} = \Theta \\ \\ \tau := \left[\frac{\beta(\langle k^2 \rangle - D) - D\mu}{D} \right]^{-1} < 0 \Leftrightarrow R_0 := \frac{\beta D}{\mu} < \frac{D^2}{\langle k^2 \rangle - D} \end{cases} \tag{9}$$

Order Parameter: Erdös-Rényi and Fuse models



(a) Erdös-Rényi model ($N = 8, p = 0.4$)



(b) Fuse model with $N = 13, D = 4$

Regular Lattice model

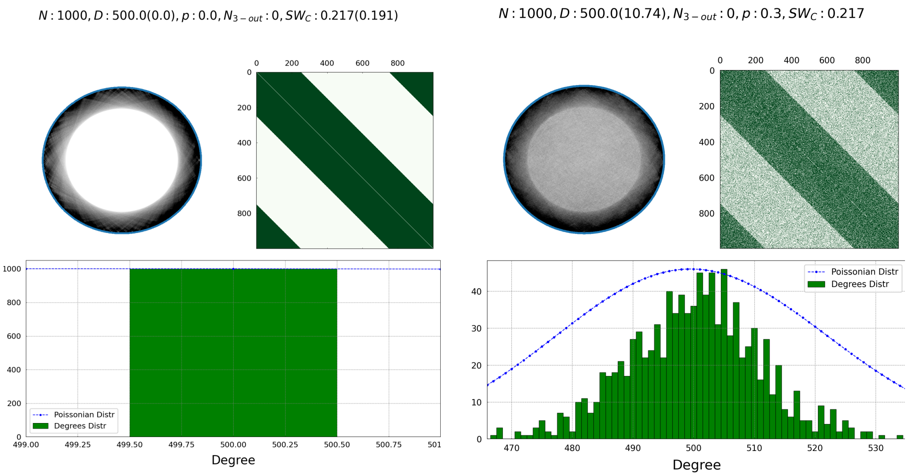
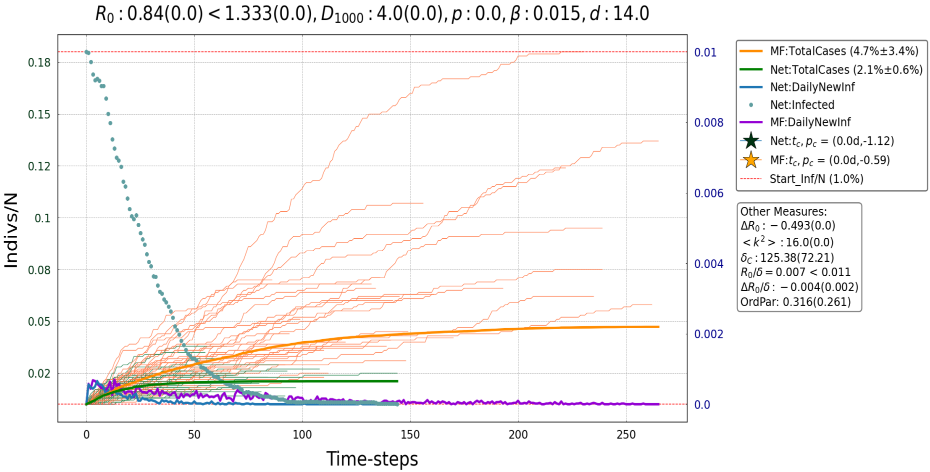
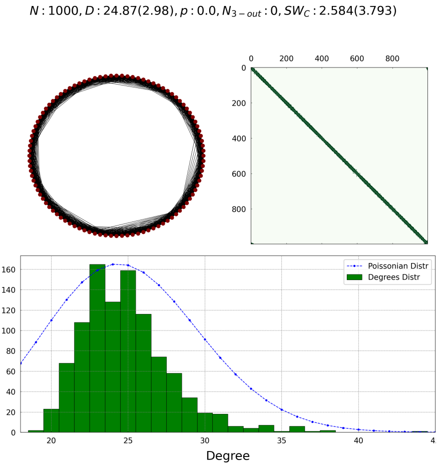


Figure: Regular lattice and extension with long-range interactions $p = 0.3$. From top left: Graph Realization, Adjacency Matrix, Degree Distribution.

COVID-19 SIR + Regular Lattice



Poissonian Small-World Networks



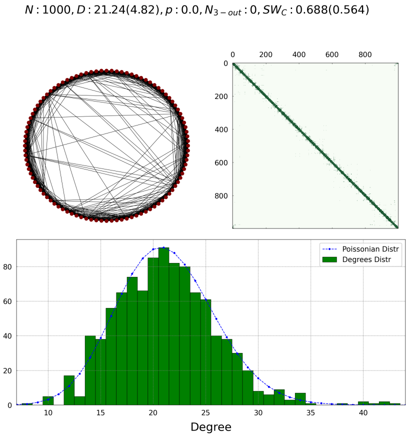
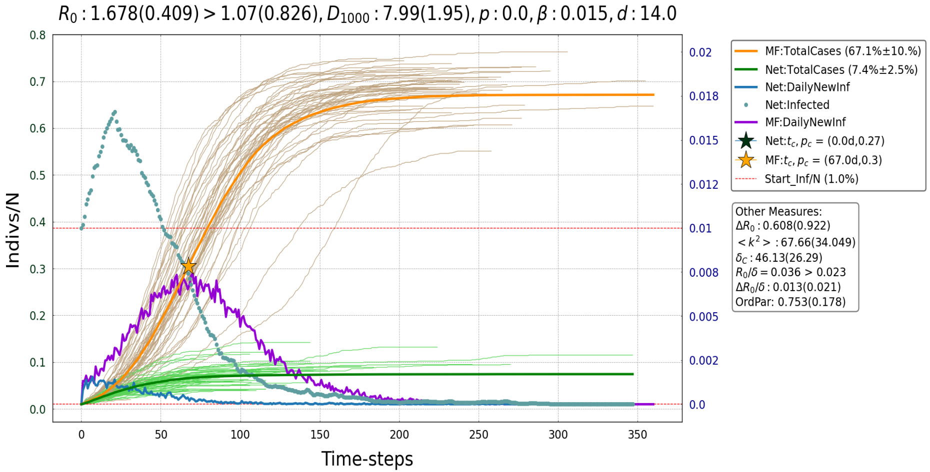


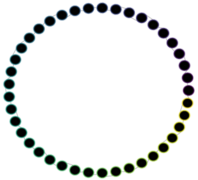
Figure: "Overlapping" PSW and "Sparse" PSW.

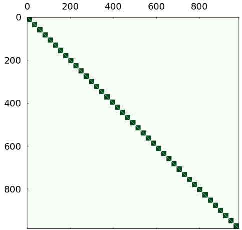
COVID-19 SIR + Overlapping PSW

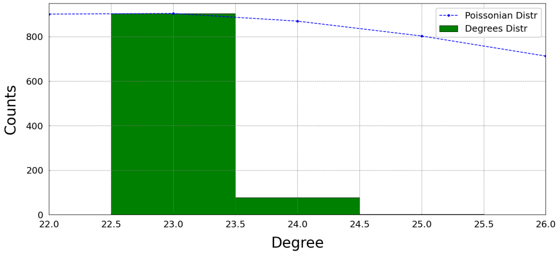


Caveman Model

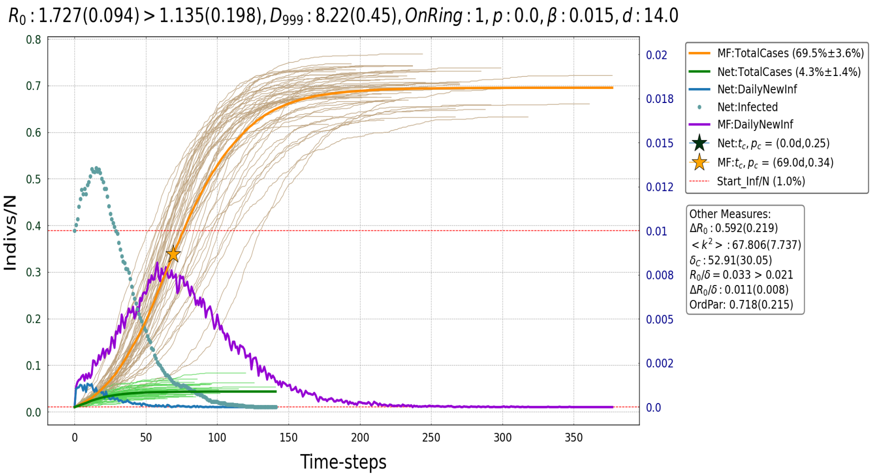
$N_{24}: 984, D: 23.08(0.28), p: 0.0, Inlinks: 1, N_{3-out}: 0, SW_C: 3.043(4.409)$





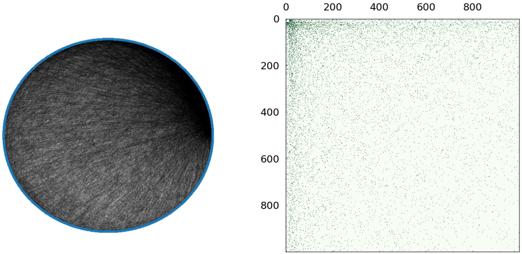


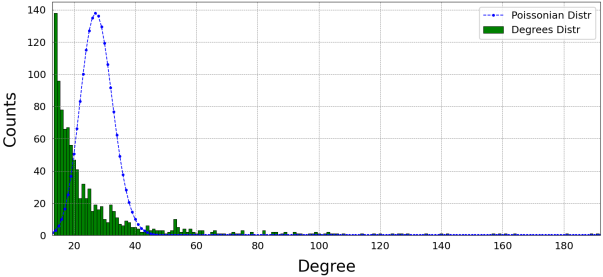
COVID-19 SIR + Caveman Model



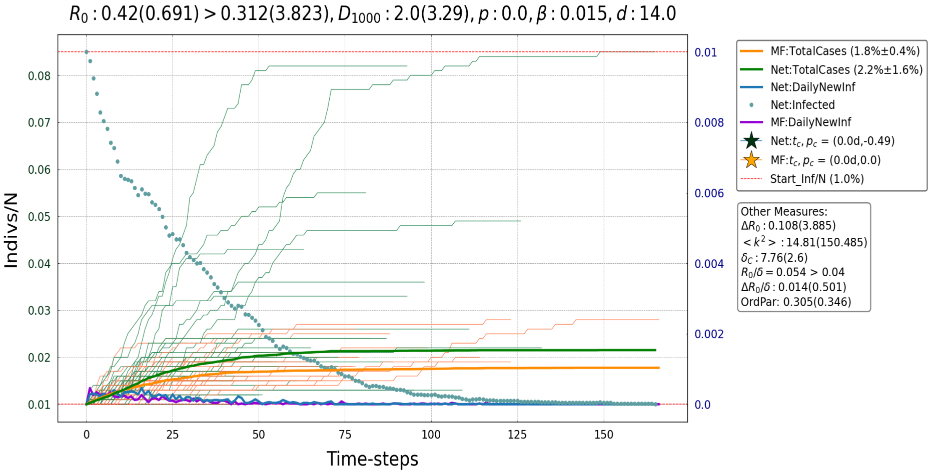
(Regular) Barabási-Albert Model

$N : 1000, D : 27.61(22.85), p : 0.0, N_{3-out} : 40, USW_C : 1.224(0.389), k_{max} : 191$

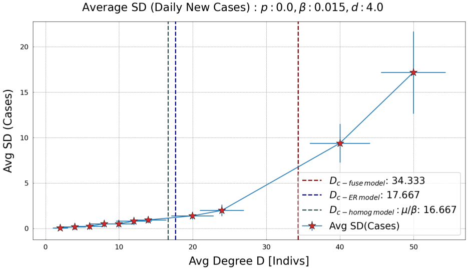




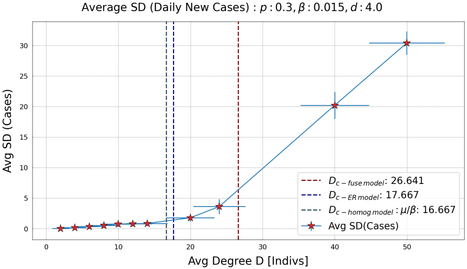
COVID-19 SIR + Barabási-Albert Model



Order Parameter ($SD(C)$) - Overl PSW - d = 4



(a) Short-Range Interactions



(b) Long-Range Interactions