

Modeling COVID-19 Spreading In A Network

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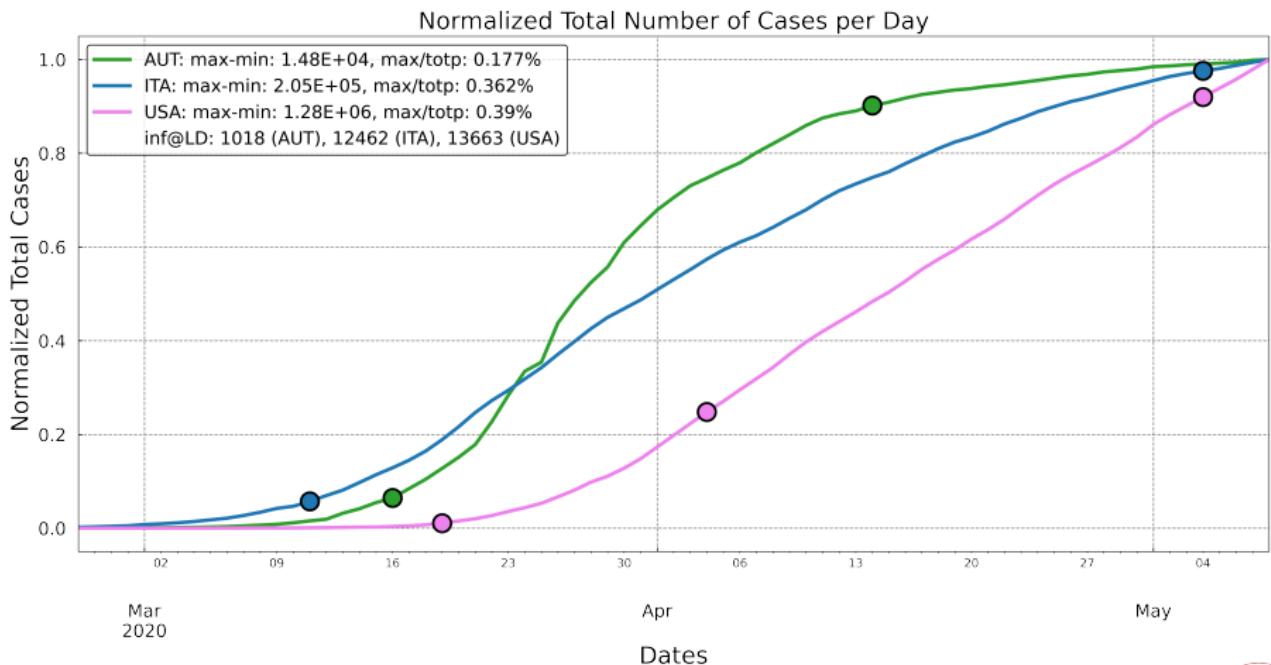


Overview

- 1 Motivations
- 2 Goals of the research
- 3 Epidemiology Background: SIR Model
- 4 Social Networks & Results
- 5 Summary and Conclusions



Motivations from COVID-19 Data



Goal: Study a SIR model on networks

- Order the Social Network models (Watts-Strogatz, Caveman, Poissonian-SW, Barabási-Albert) for controlling COVID-19;
- Estimate the time t_c of the peak of the infected after which the pathogen regresses;
- Propose the epidemic severity to capture the final fraction of the total cases;
- Introduce an Order Parameter:= $SD(C)$ to look for a phase-like transition among regimes;
- Estimate the critical average of social contacts (D_c):
 - for $D < D_c$, linear growth at early times;
 - for $D > D_c$, exponential growth at early times.



SIR Models

No Network Structure → Homogeneous Mean-Field (MF) Model

$$\left\{ \begin{array}{l} s + r + i = 1 \quad \text{HP: "closed + well-mixed population"} \\ \frac{ds}{dt} = -\beta D s i, \quad D := \langle k \rangle \\ \frac{di}{dt} = \beta D s i - \mu i = \mu (R_0 s - 1) i, \quad R_0 := \frac{\beta D}{\mu} \equiv D \lambda \\ \frac{dr}{dt} = +\mu i \end{array} \right. \quad (1)$$

Network Structures → Degree-Based Mean-Field Model

$$\left\{ \begin{array}{l} \frac{ds_k}{dt} = -\beta k s_k \Theta_k \\ \frac{di_k}{dt} = \beta k s_k \Theta_k - \mu i_k = \beta k (1 - i_k - r_k) \Theta_k - \mu i_k \end{array} \right. \quad (2)$$

Thresholds for Regression

Homogeneous MF Model

$$\frac{di}{dt} = \mu(R_0 s(t) - 1)i(t) < 0 \Leftrightarrow R_0 s(t) < 1 \quad (3)$$

Degree-Based MF: $t \approx 0 \Rightarrow s, i \approx 1, i_0$ and $k \sim D$ for random net

$$\begin{cases} \frac{di_k}{dt} = \mu \left[\frac{\beta k}{\mu} \Theta_k(t) s_k(t) - i_k(t) \right] < 0 \\ \Theta_k(t) = \frac{\sum_{k'} (k'-1) p_{k'} i_{k'}(t)}{\langle k \rangle} = i_0 \frac{D-1}{D} e^{t/\tau} = \Theta \quad \text{at } t \approx 0 \\ \tau := \left[\frac{\beta(\langle k^2 \rangle - D) - D\mu}{D} \right]^{-1} < 0 \Leftrightarrow R_0 := D \frac{\beta}{\mu} < \frac{D^2}{\langle k^2 \rangle - D} \quad \text{at } t \approx 0 \end{cases} \quad (4)$$

Decay

Critical fraction of total cases $\pi(t_c)$

- Pathogen regression in Homogeneous SIR:

$$R(t) := Ds(t)\lambda < 1 \Leftrightarrow \pi(t_c) = 1 - \frac{1}{R_0}, \quad R_0 := \frac{\beta D}{\mu} \quad (5)$$

- Pathogen decaying in Heterogenous SIR (at $t \approx 0$):

$$\begin{cases} R_0 < R_{c-net} := \frac{D^2}{\langle k^2 \rangle - D} \\ R(t) := (D - n(t))\lambda \quad \forall t \quad \Leftrightarrow \pi(t_c) = \frac{1}{1-p} \left[1 - \frac{R_{c-net}}{R_0 - \lambda} \right] \\ n(t) \sim 1 + (D - 1)(1 - p)\pi(t) \end{cases} \quad (6)$$

Epidemic Severity (ES)

Basic Reproduction Number R_0

$$R_0 := \frac{\beta D}{\mu} \quad (7)$$

Definition of ES

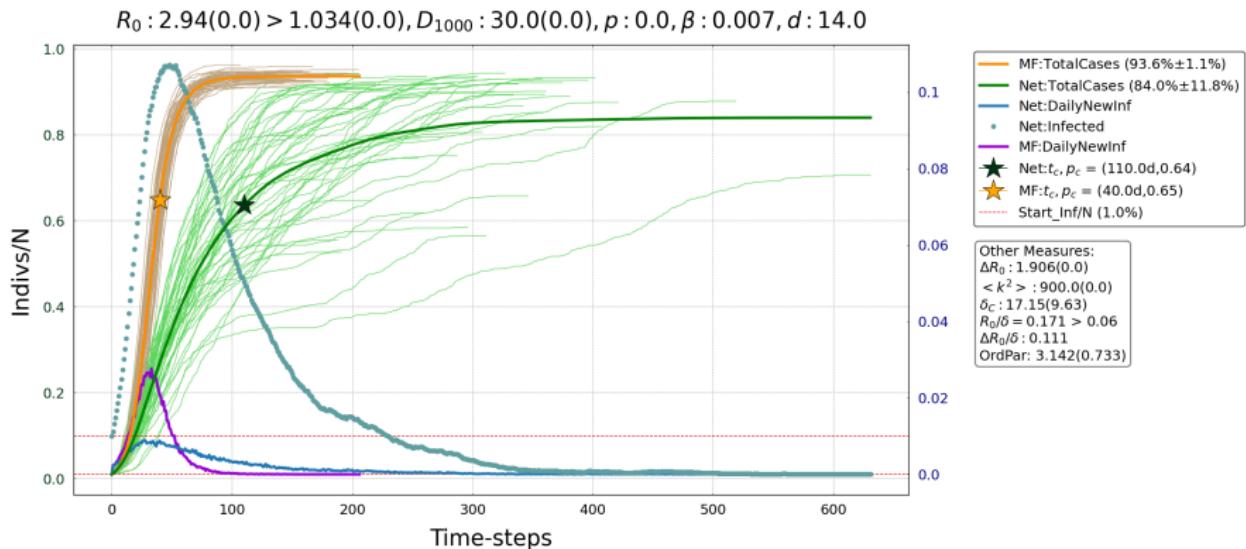
$$\Delta R_0(\delta) := \frac{R_0 - R_{c-net}}{\delta} \quad (8)$$

where $\delta := \langle l \rangle$ is the average path length, e.g. from $1 \leftrightarrow 10$.

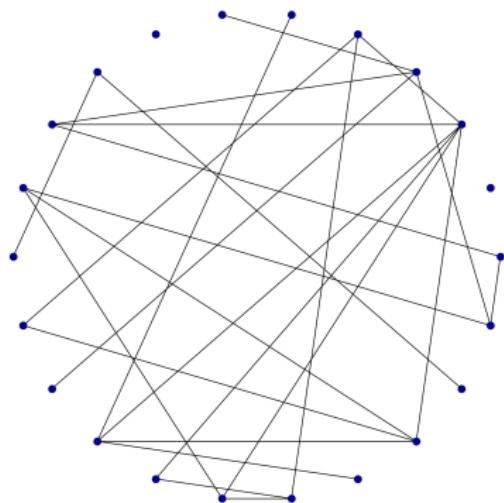
δ increases as the network is less “weaved together”.



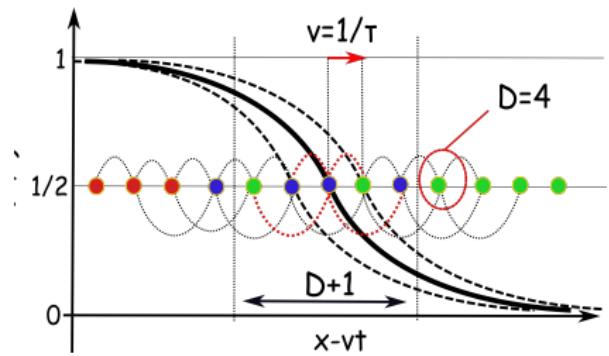
Example



Erdös-Rényi and Fuse models



(a) Erdös-Rényi model ($N = 22, p = 0.1$)



(b) Fuse model with $N = 13, D = 4$

First Order-like Phase Transition

Order Parameter: standard deviation (SD) of the daily new cases C

- $\mathbb{O} := SD(C)$ where $C(t)$ are the new daily infected;

Critical average degree D_c for different topologies

- Homogenous MF: $\frac{\beta D_c}{\mu} \stackrel{!}{=} 1 \Leftrightarrow D_{c-homog} = \frac{\mu}{\beta}$
- Erdös-Rényi Degree-Based MF:

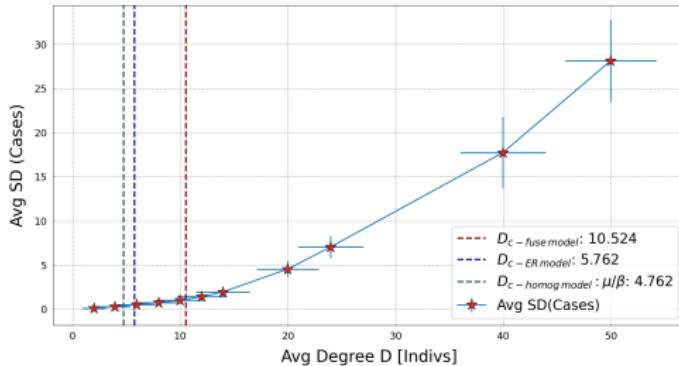
$$\begin{cases} n(t) \sim 1, \quad s(t \approx 0) \sim 1 \\ R(t) := (D - n(t))\lambda \stackrel{!}{=} R_{c-net} \sim 1 \end{cases} \Leftrightarrow D_{c-ER} = 1 + \frac{\mu}{\beta} \quad (9)$$

- Fuse-Model Degree-Based MF:

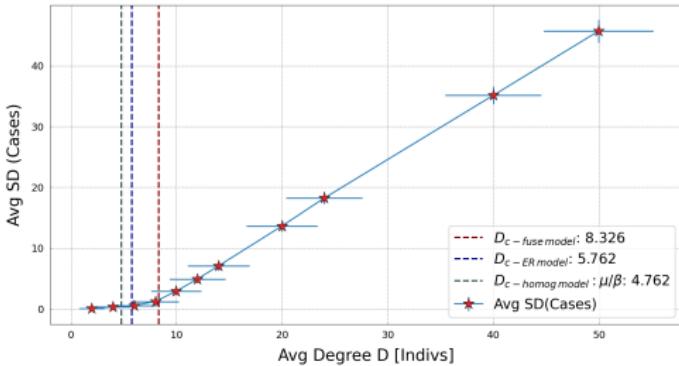
$$n(t) \sim 1 + (D - 1)(1 - p)/2 \quad \Leftrightarrow \quad D_{c-FM} = 1 + \frac{2\frac{\mu}{\beta}}{1 + p} \quad (10)$$

Examples of Order Parameters

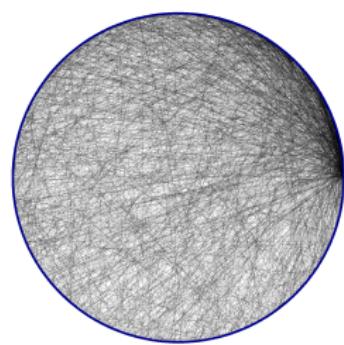
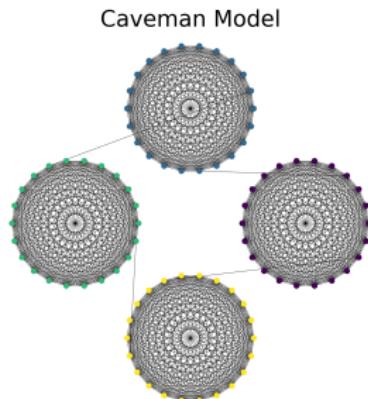
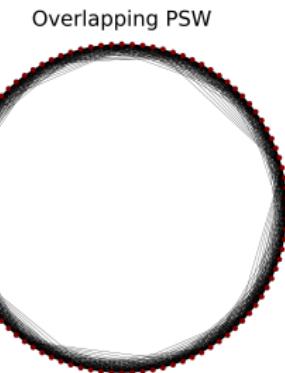
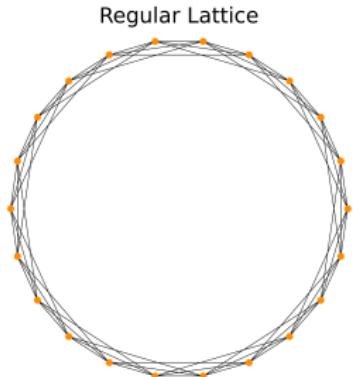
Average SD (Daily New Cases) : $p : 0.0, \beta : 0.015, d : 14.0$



Average SD (Daily New Cases) : $p : 0.3, \beta : 0.015, d : 14.0$

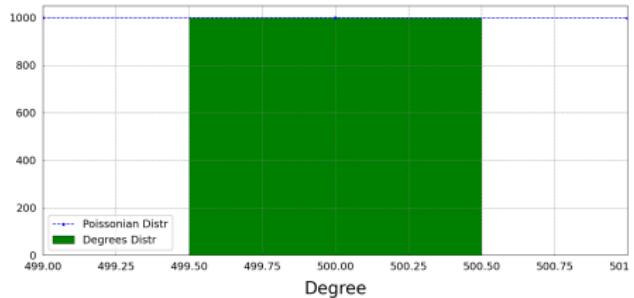
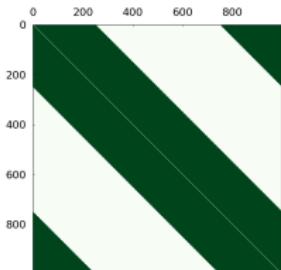
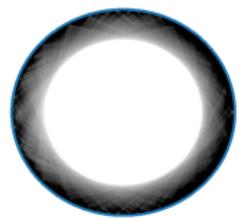


Social Networks & Results



Watts-Strogatz model

$N: 1000, D: 500.0(0.0), p: 0.0, N_{3-out}: 0, SW_C: 0.217(0.191)$



$N: 1000, D: 500.0(10.74), N_{3-out}: 0, p: 0.3, SW_C: 0.217$

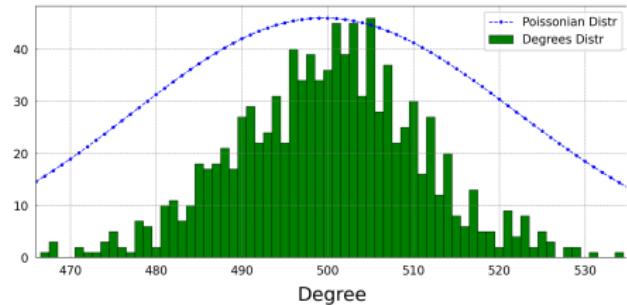
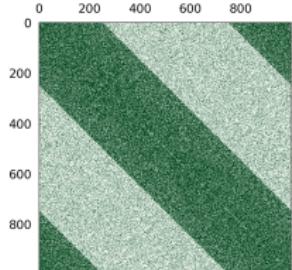
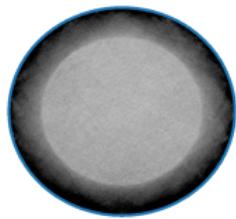


Figure: Regular lattice and extension with long-range interactions $p = 0.3$.
From top left: Graph Realization, Adjacency Matrix, Degree Distribution.



Fixed R_0 SIR + Regular Lattice

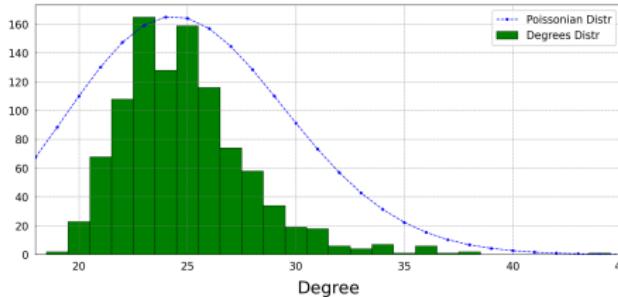
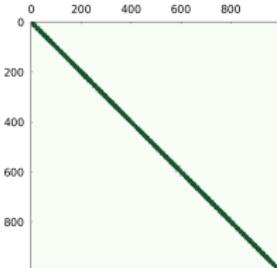


COVID-19 SIR + Regular Watts-Strogatz model



Poissonian Small-World Networks

$N : 1000, D : 24.87(2.98), p : 0.0, N_{3-out} : 0, SW_C : 2.584(3.793)$



$N : 1000, D : 21.24(4.82), p : 0.0, N_{3-out} : 0, SW_C : 0.688(0.564)$

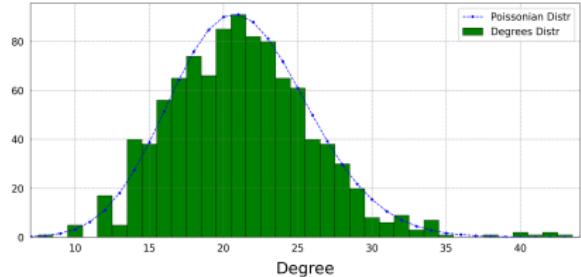
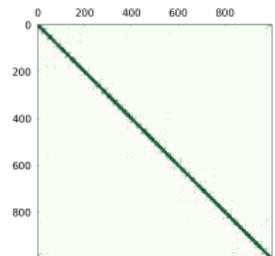
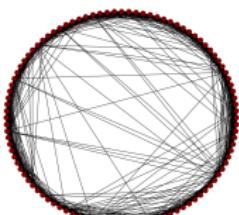


Figure: “Overlapping” PSW and “Sparse” PSW.

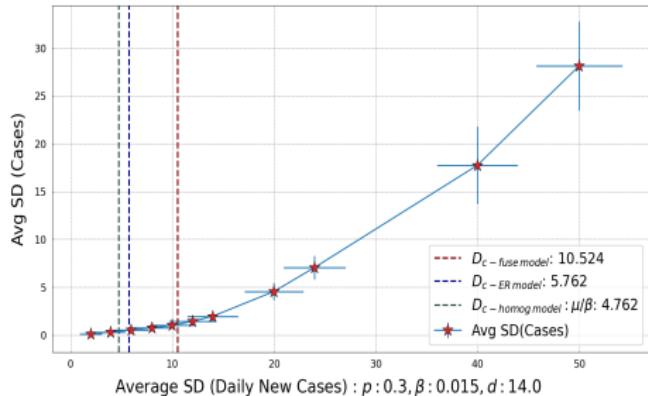


COVID-19 SIR + Overlapping PSW

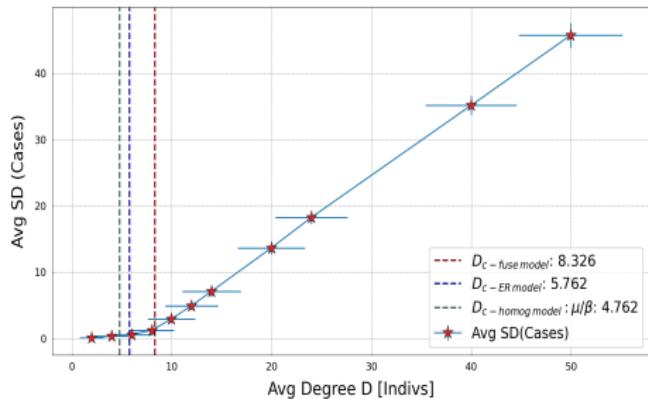


Order Parameter := $SD(C)$

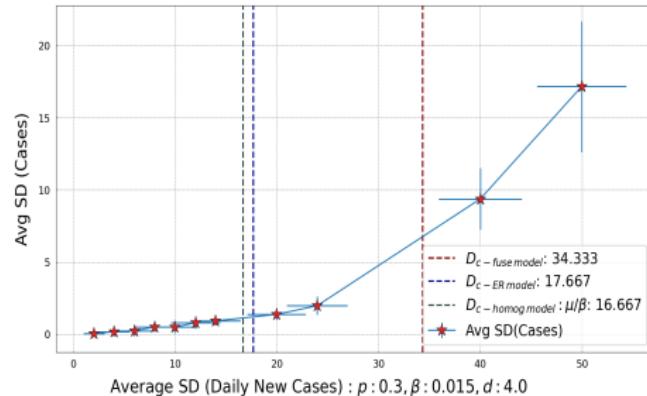
Average SD (Daily New Cases) : $p : 0.0, \beta : 0.015, d : 14.0$



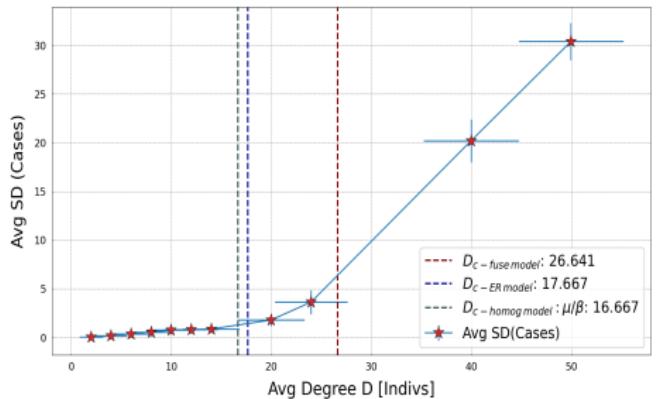
Average SD (Daily New Cases) : $p : 0.3, \beta : 0.015, d : 14.0$



Average SD (Daily New Cases) : $p : 0.0, \beta : 0.015, d : 4.0$



Average SD (Daily New Cases) : $p : 0.3, \beta : 0.015, d : 4.0$



Caveman Model

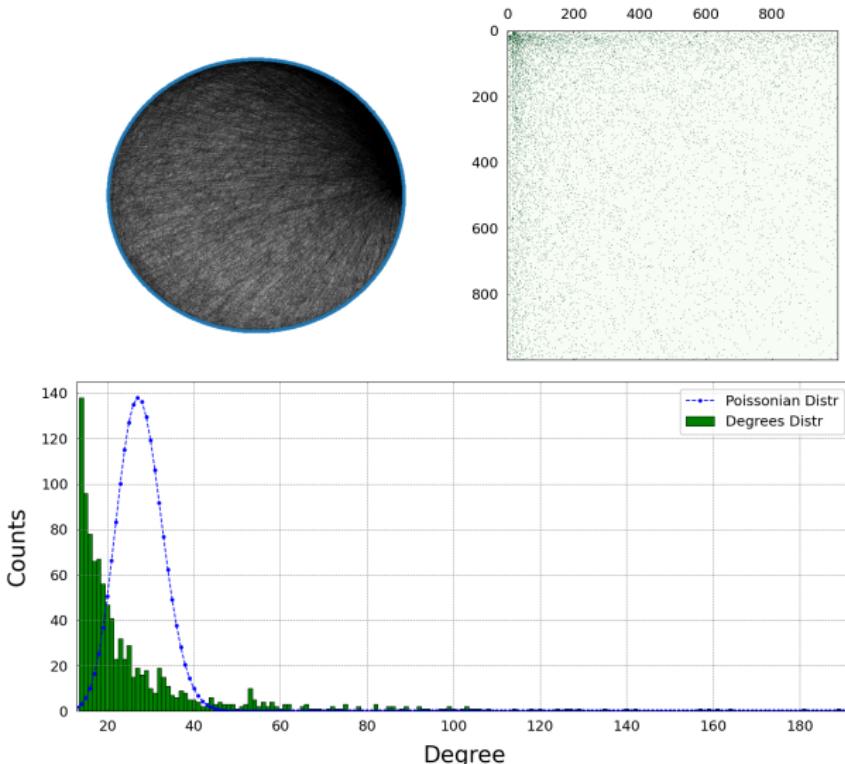


COVID-19 SIR + Caveman Model



(Regular) Barabási-Albert Model

$N : 1000, D : 27.61(22.85), p : 0.0, N_{3-out} : 40, USW_C : 1.224(0.389), k_{max} : 191$

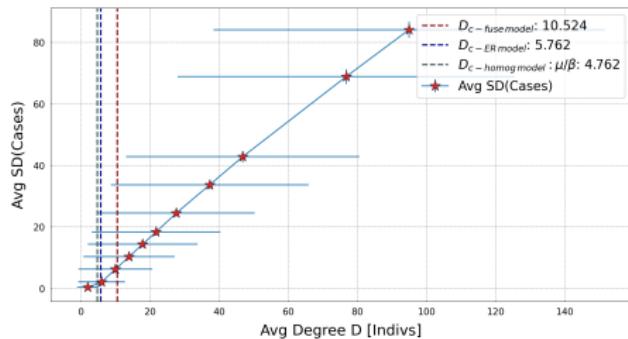


COVID-19 SIR + Barabási-Albert Model

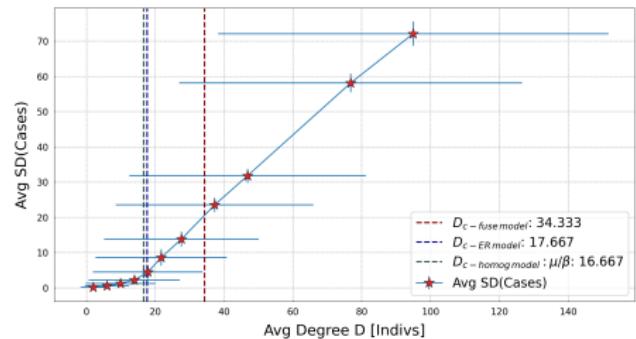


Order Parameter := $SD(C)$

Average SD(Daily New Cases) : $p : 0.0, \beta : 0.015, d : 14.0$



Average SD(Daily New Cases) : $p : 0.0, \beta : 0.015, d : 4.0$



Final Outbreak Size		Degree Distribution	
Networks	$D = 6$	$D = 8$	
Caveman Model 	–	$(4.3 \pm 1.4)\%$	<ul style="list-style-type: none"> • Low Heterogeneity • Clustered Net
Regular Lattice 	–	$(6 \pm 2.3)\%$	Peaked
Overlapping PSW 	–	$(7.4 \pm 2.5)\%$	Mid Heterogeneity (No Hubs)
Barabási-Albert Model 	$(36 \pm 8.9)\%$	–	Highly Heterogeneity (Hubs)

Summary and Conclusions

- ① Used a Watts-Strogatz model to test the *epidemic severity* VS R_0 ;
- ② The peak of the infected is well estimated only for mean-field approximation (yellow star);
- ③ Order the Social Network models for controlling COVID-19: *local* Caveman, Watts-Strogatz, Poissonian-SW, Barabási-Albert;
- ④ For Overlapping PSW, D_c is well estimated by the *Fuse model* only for $\beta = 0.015$ and $d = 14$;
- ⑤ For Barabási-Albert model, estimates of D_c comes from *homogeneous mean-field*;



Thank You For Your Attention

