Desc. Experience 1

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Sub-Experience One: The Mad Mail Carrier
Suppose Sue is a Mail Carrier who is crazy. He likes to ensure that non of the n houses on his delivery route get the mail they are supposed to. Your goal, should you choose to accept it, for this sub-experience is to determine the number of ways Sue can deliver mail so that no one gets their mail in two ways. One method is to use the Principle of Inclusion/Exclusion (PIE). The other mothod is to use an exponential generating function to solve a recurrence which you'll develop. Put D_n equal to the number of ways Sue can distribute mail to n houses so that none of them gets the correct mail.

Sub-Experience One, Part One: PIE Approach Use the PIE to determine D_n.

Solution:

Sub-Experience Two, Part One: PIE Approach The formula you obtain above should involve a truncated power series for e^{-1}. Show that $D_n = \lfloor \frac{n!}{e} + \frac{1}{2} \rfloor, forn > 0$. (For $n = 0$, the formula doesn't work: $D_0 = 1$, but the formula gives 0.)
Solution:

Sub-Experience Three, Part One: PIE Approach SE 1.3.1. Prove the recurrence $D_n = (n-1)D_{n-1} + (n-1)D_{n-2},\ for\ n \geq 2,\ and\ D_0 = 1, D_1 = 0.$
SE 1.3.2. Deduce, from the above recurrence $D_n = nD_{n-1} + (-1)^n,\ for\ n \geq 1,\ and\ D_0 = 1.$
SE 1.3.3. Use an exponential generating function to solve the recurrence from part 1.3.2
Solution:

Sub-Experience Two: Stirling Numbers of the Second Kind
Recall that a partition of a set X into k blocks is a set $\prod = \{B_1, ..., B_k\}$ where the B_is are disjoint nonempty subsets of X whose union is X. Define $\binom{n}{k}$ to be the number of partitions of a set with n elements into k blocks.
2.1 Use the Principle of Inclusion/Exclusion to create a formula for $\binom{n}{k}$. Please verify that the formula you find actually works.
2.2 Use exponential generating functions to create a formula for $\binom{n}{k}$. Please verify that the formula you find actually works. It may appear different than the found in part 2.1, so be careful.
Solution:

Sub-Experience Three: Catalan Numbers
Roll up your sleeves and prepare to get down and dirty with generating functions. The goal of the following sequence of problems is to prove that the sequence $(b_n)_n \geq 0$ generated by the recurrence relation

$$b_n = b_0 b_{n-1} + b_1 b_{n-2} + ... + b_{n-1} b_0 \ with b_0 = 1$$

gives the answer to the following counting questions, and that $\binom{b_n = \frac{1}{n+1} 2n}{n}$. These numbers are called the *Catalan Numbers* and are nearly as ubiquitous as the Fibonacci Numbers.

How many binary trees on n vertices are there? Follow the steps below to produce a closed formula for b_n using a generating function.

3.0 Denote by b_n the number of binary trees on vertices. Please prove that $b_n = b_0 b_{n-1} + b_1 b_{n-2} + ... + b_{n-1} b_0$ with $b_0 = 1$.
3.1 Let $g(x) = \sum_{n=0}^{\infty} b_n x^n$ be the generating function for the sequence $b_n n \geq 0$ of the number of binary trees on n vertices. Show that $g(x) = 1 + x(g(x))^2$.
3.2 Use the quadratic formula to show that $g(x) = 1 - \frac{\sqrt{1-4x}}{2x}$. Note that you will have two options to use for $g(x)$, one of which you must rule out.
3.3 Recall $\binom{The\ Binomial\ Theorem:\ For\ any\ real\ number\ r,\ (1+z)^r = \sum_{c=0}^{\infty} r}{kz^k,\ where}$

$$\left(\begin{matrix} r \\ k = \begin{cases} \frac{r(r-1)(r-2)\ldots(r-k+1)}{k!}, r\in\mathbb{R}, 0\le k\in\mathbb{Z}; \\ 0, k<0. \end{cases} \end{matrix}\right)$$

Use the Binomial Theorem to obtain $g(x) = \frac{1}{2x}$
Solution:

Sub-Experience Four: Non-Standard Dice
Solution:

Sub-Experience Five: Eul-ing the GF Machine
Solution:

Sub-Experience Six: The Twelve-Fold Way
Solution:

[Bonus] Sub-Experience Seven:
Solution: