

## Desc. Experience 1

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### Sub-Experience One: The Mad Mail Carrier

Suppose Sue is a Mail Carrier who is crazy. He likes to ensure that none of the  $n$  houses on his delivery route get the mail they are supposed to. Your goal, should you choose to accept it, for this sub-experience is to determine the number of ways Sue can deliver mail so that no one gets their mail in two ways. One method is to use the Principle of Inclusion/Exclusion (PIE). The other method is to use an exponential generating function to solve a recurrence which you'll develop. Put  $D_n$  equal to the number of ways Sue can distribute mail to  $n$  houses so that none of them gets the correct mail.

**Sub-Experience One, Part One: PIE Approach** Use the PIE to determine  $D_n$ .

**Solution:**

**Sub-Experience Two, Part One: PIE Approach** The formula you obtain above should involve a truncated power series for  $e^{-1}$ . Show that  $D_n = \lfloor \frac{n!}{e} + \frac{1}{2} \rfloor$ , for  $n > 0$ . (For  $n = 0$ , the formula doesn't work:  $D_0 = 1$ , but the formula gives 0.)

**Solution:**

**Sub-Experience Three, Part One: PIE Approach SE 1.3.1.** Prove the recurrence  $D_n = (n - 1)D_{n-1} + (n - 1)D_{n-2}$ , for  $n \geq 2$ , and  $D_0 = 1, D_1 = 0$ .

**SE 1.3.2.** Deduce, from the above recurrence  $D_n = nD_{n-1} + (-1)^n$ , for  $n \geq 1$ , and  $D_0 = 1$ .

**SE 1.3.3.** Use an exponential generating function to solve the recurrence from part 1.3.2

**Solution:**

### Sub-Experience Two: Stirling Numbers of the Second Kind

Recall that a partition of a set  $X$  into  $k$  blocks is a set  $\Pi = \{B_1, \dots, B_k\}$  where the  $B_i$ s are disjoint nonempty subsets of  $X$  whose union is  $X$ . Define  $\binom{n}{k}$  to be the number of partitions of a set with  $n$  elements into  $k$  blocks.

**2.1** Use the Principle of Inclusion/Exclusion to create a formula for  $\binom{n}{k}$ . Please verify that the formula you find actually works.

**2.2** Use exponential generating functions to create a formula for  $\binom{n}{k}$ . Please verify that the formula you find actually works. It may appear different than the found in part 2.1, so be careful.

$\{\binom{n}{k}\}$   
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**Solution:**

### Sub-Experience Three: Catalan Numbers

**Solution:**

### Sub-Experience Four: Non-Standard Dice

**Solution:**

### Sub-Experience Five: Eul-ing the GF Machine

**Solution:**

### Sub-Experience Six: The Twelve-Fold Way

**Solution:**

### [Bonus] Sub-Experience Seven:

**Solution:**