# CHAPTER 4

# Linear Modulation

## 4.1 Introduction

We use the word *modulation* to mean the systematic alteration of one waveform, called the *carrier*, according to the characteristic of another waveform, the *modulating signal* or *the message.* In Continuous Wave (CW) modulation schemes, the carrier is a sinusoid. We use $c(t)$ and $m(t)$, to denote the carrier and the message waveforms respectively.

The three parameters of a sinusoidal carrier that can be varied are: amplitude, phase and frequency. A given modulation scheme can result in the variation of one or more of these parameters. Before we look into the details of various linear modulation schemes, let us understand the **need for modulation**. Three basic blocks in any communication system are: 1) transmitter 2) Channel and 3) Receiver (Fig. 4.1).


Fig. 4.1: A basic communication system

The transmitter puts the information from the source (meant for the receiver) onto the channel. The channel is the medium connecting the transmitter and the receiver and the transmitted information travels on this channel until it reaches the destination. Channels can be of two types: i) wired channels or ii) wireless channels. Examples of the first type include: twisted pair telephone channels, coaxial cables, fiber optic cable etc. Under the wireless category, we have the following examples: earth's atmosphere (enabling the propagation of ground wave and sky wave), satellite channel, sea water etc.

The main disadvantage of wired channels is that they require a man-made medium to be present between the transmitter and the receiver. Though wired channels have been put to extensive use, wireless channels are equally (if not more) important and have found a large number of applications.

In order to make use of the wireless channels, the information is to be converted into a suitable form, say electromagnetic waves. This is accomplished with the help of a transmitting antenna. The antenna at the receiver (called the receiving antenna) converts the received electromagnetic energy to an electrical signal which is processed by the receiver.

The question is: can we radiate the *baseband*[1] information bearing signal directly on to the channel?

For efficient radiation, the size of the antenna should be $\lambda/10$ or more (preferably around $\lambda/4$), where $\lambda$ is the wavelength of the signal to be radiated. Take the case of audio, which has spectral components almost from DC upto 20 kHz. Assume that we are designing the antenna for the mid frequency; that is,10 kHz. Then the length of the antenna that is required, even for the $\lambda/10$ situation is, $\dfrac{c}{10 \cdot f} = \dfrac{3 \times 10^8}{10 \times 10^4} = 3 \times 10^3$ meters, $c$ being the velocity of light.

[1] Baseband signals have significant spectral content around DC. Some of the baseband signals that are of interest to us are: a) Speech b) music and c) video (TV signals).

Approximate spectral widths of these signals are: Speech: 5 kHz, Audio : 20 kHz, Video : 5 MHz

Even an antenna of the size of 3 km, will not be able to take care of the entire spectrum of the signal because for the frequency components around 1 kHz, the length of the antenna would be $\lambda/100$. Hence, what is required from the point of view of efficient radiation is the conversion of the baseband signal into a narrowband, bandpass signal. Modulation process helps us to accomplish this; besides, modulation gives rise to some other features which can be exploited for the purpose of efficient communication. We describe below the advantages of modulation.

**1. Modulation for ease of radiation**

Consider again transmission of good quality audio. Assume we choose the carrier frequency to be 1 MHz. The linear modulation schemes that would be discussed shortly give rise to a maximum frequency spread (of the modulated signal) of 40 kHz, the spectrum of the modulated signal extending from (1000 - 20) = 980 kHz to (1000 + 20) = 1020 kHz. If the antenna is designed for 1000 kHz, it can easily take care of the entire range of frequencies involved because modulation process has rendered the signal into a NBBP signal.

**2. Modulation for efficient transmission**

Quite a few wireless channels have their own appropriate passbands. For efficient transmission, it would be necessary to shift the message spectrum into the passband of the channel intended. Ground wave propagation (from the lower atmosphere) is possible only up to about 2 MHz. Long distance ionospheric propagation is possible for frequencies in the range 2 to 30 MHz. Beyond 30 MHz, the propagation is line of sight. Preferred frequencies for satellite communication are around 3 to 6 GHz. By choosing an appropriate carrier frequency and modulation technique, it is possible for us to translate the baseband message spectrum into a suitable slot in the passband of the channel intended. That is, **modulation results in frequency translation.**

**3. Modulation for multiplexing**

Several message signals can be transmitted on a given channel, by assigning to each message signal an appropriate slot in the passband of the channel. Take the example of AM broadcast, used for voice and medium quality music broadcast. The passband of the channel used to 550 kHz to 1650 kHz. That is, the width of the passband of the channel that is being used is 1100 kHz. If the required transmission bandwidth is taken as 10 kHz, then it is possible for us to multiplex, atleast theoretically, 110 distinct message signals on the channel and still be able to separate them individually as and when we desire because the identity of each message is preserved in the frequency domain.

**4. Modulation for frequency assignment**

Continuing on the broadcast situation, let us assume that each one of the message signals is being broadcast by a different station. Each station can be assigned a suitable carrier so that the corresponding program material can be received by tuning to the station desired.

**5. Modulation to improve the signal-to-noise ratio**

Certain modulation schemes (notably frequency modulation and phase modulation) have the feature that they will permit improved signal-to-noise ratio at the receiver output, provided we are willing to pay the price in terms of increased transmission bandwidth (Note that the transmitted power need not be increased). This feature can be taken advantage of when the quality of the receiver output is very important.

Having understood the need and the potential benefits due to modulation, let us now get into the details of various linear modulation schemes. The four important types of linear modulation schemes are

1) Double SideBand, Suppressed Carrier (DSB-SC)
2) Double SideBand, Large Carrier (DSB-LC) (also called conventional AM or simply AM)

3) Single SideBand (SSB)

4) Vestigial SideBand (VSB)

We shall begin our discussion with DSB-SC.

# 4.2 DSB-SC Modulation

## 4.2.1. Modulation

The DSB-SC is the simplest of the four linear modulation schemes listed above (simplest in terms of the *mathematical description* of modulation and demodulation operations). Consider the scheme shown in Fig. 4.2

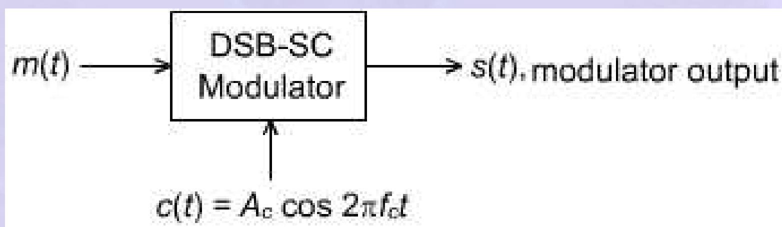



Fig. 4.2: DSB-SC modulation scheme

$m(t)$ is a baseband message signal with $M(f) = 0$ for $|f| > W$, $c(t)$ is a high frequency carrier, usually with $f_c >> W$.

DSB-SC modulator is basically a multiplier. Let $g_m$ denotes the amplitude sensitivity (or gain constant) of the modulator, with the units per volt (we assume that $m(t)$ and $A_c$ are in volts). Then the modulator output $s(t)$ is,

$$s(t) = g_m m(t)\left(A_c \cos(\omega_c t)\right) \tag{4.1a}$$

For convenience, let $g_m = 1$. Then,

$$s(t) = A_c m(t)\cos(\omega_c t) \tag{4.1b}$$

As DSB-SC modulation involves just the multiplication of the message signal and the carrier, this scheme is also known as *product modulation* and can be shown as in Fig. 4.3.

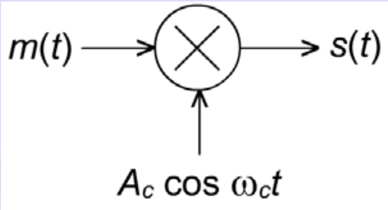



Fig. 4.3: Product Modulation scheme

The time domain behavior of the DSB-SC signal (with $A_c = 1$) is shown in Fig. 4.4(b), for the $m(t)$ shown in Fig. 4.4(a).

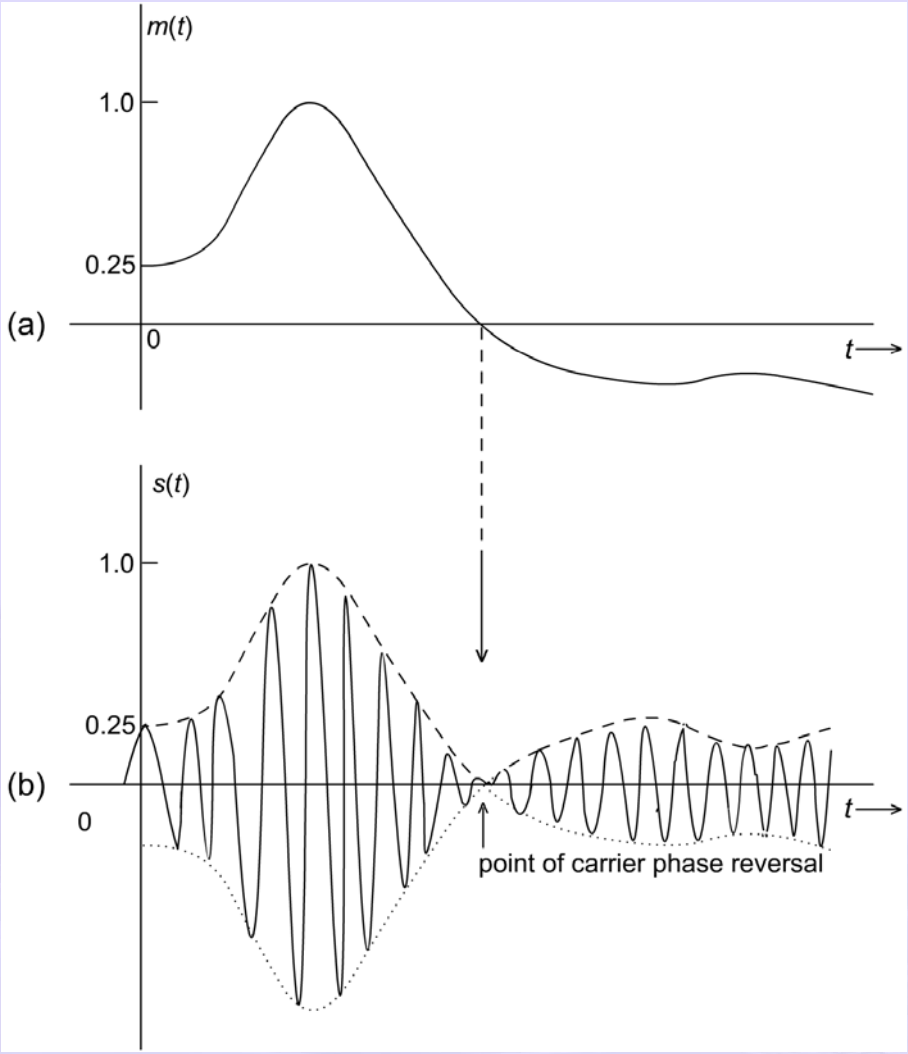



Fig. 4.4: (a) The message signal

(b) The DSB-SC signal

Note that the carrier undergoes a $180^o$ phase reversal at the zero crossings of $m(t)$. This is brought out more clearly in the oscillograms, shown in Fig. 4.5 and Fig. 4.6, where $m(t)$ is a sinusoidal signal.

With reference to Fig. 4.5, between the points 'a' and 'b', the carrier in the DSB-SC signal and the actual carrier (bottom picture) are in phase whereas between the points 'b' and 'c', they are $180^0$ out of phase. Fig. 4.6 is an expanded version of the central part of the waveforms in Fig. 4.5. Here, we can very clearly observe that to the left of ‘b’, both the carriers are in phase whereas to the right, they are $180^0$ out of phase.

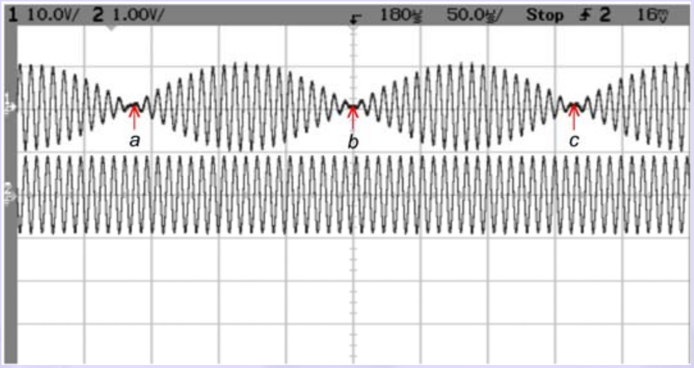



Fig. 4.5: (top) DSB-SC signal with tone modulation
(bottom) The carrier

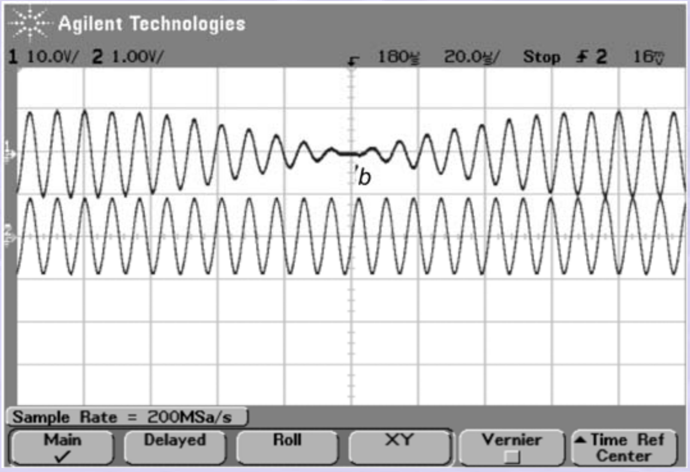



Fig. 4.6: Expanded versions of a part of the waveforms in Fig. 4.5

Consider waveforms shown in Fig. 4.7. We have on the top, modulating tone signal and at the bottom, the corresponding DSB-SC. What do we observe on the oscilloscope, if we feed the X-plates the tone signal and the Y-plates, the DSB-SC signal? The result is shown in Fig. 4.8, which can be explained as follows.

At the point ‘a’ in Fig. 4.7, the modulating tone is at its maximum and hence the DSB-SC signal has the maximum value. Point ‘a’ in Fig. 4.8 corresponds to the point ‘a’ in Fig. 4.7. Between the points ‘a’ and ‘b’ in Fig. 4.7, the tone amplitude decreases (reaching the value zero at point b); hence the maximum value reached by the DSB-SC signal during each carrier cycle keeps decreasing. As the X-plates are being fed with the same tone signal, this decrease will be linear and this corresponds to segment ‘a’ to ‘b’ in Fig. 4.8. (Note that DSB-SC signal is zero at point b). In the time interval between ‘b’ and ‘c’ of Fig. 4.7, the DSB signal increases and this increase is seen as a straight line between the points ‘b’ and ‘c’ in Fig. 4.8. Between the points ‘c’ and ‘e’ in Fig. 4.7, the tone amplitude varies from the most negative value to the most positive value. Correspondingly, the display on the oscilloscope will follow the trace $c \rightarrow d \rightarrow e$ shown in Fig. 4.8.

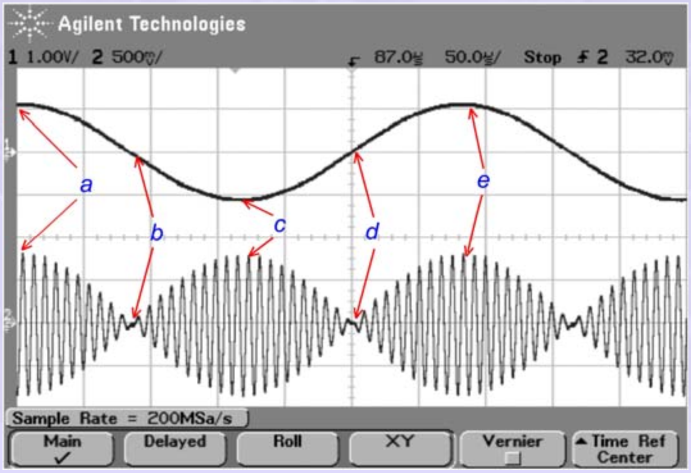



Fig. 4.7: (top) modulating signal
(bottom) DSB-SC signal

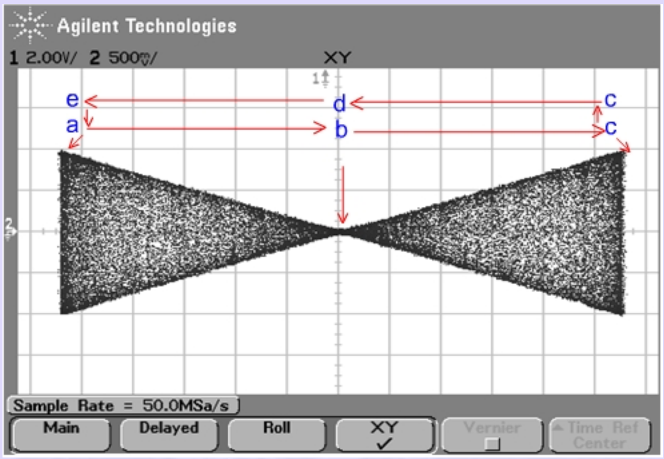



Fig. 4.8 Display on the oscilloscope with the following inputs:

X-plates: Tone signal

Y-plates: DSB-SC signal

Taking the Fourier transform of Eq. 4.1(b), we have

$$S(f) = \frac{A_c}{2}\left[M(f - f_c) + M(f + f_c)\right] \tag{4.2}$$

If we ignore the constant $\frac{A_c}{2}$ on the R.H.S of Eq. (4.2), we see that the modulation process has simply shifted the message spectrum by $\pm f_c$. As the frequency translation of a given spectrum occurs quite often in the study of modulation and demodulation operations, let us take a closer look at this.

i) Let $m(t)$ be a real signal with the spectrum $M(f)$ shown below (Fig. 4.9(a)). Let $f_c$ be 100 kHz. Assuming $\frac{A_c}{2} = 1$, we have $S(f)$ as shown in Fig. 4.9(b).

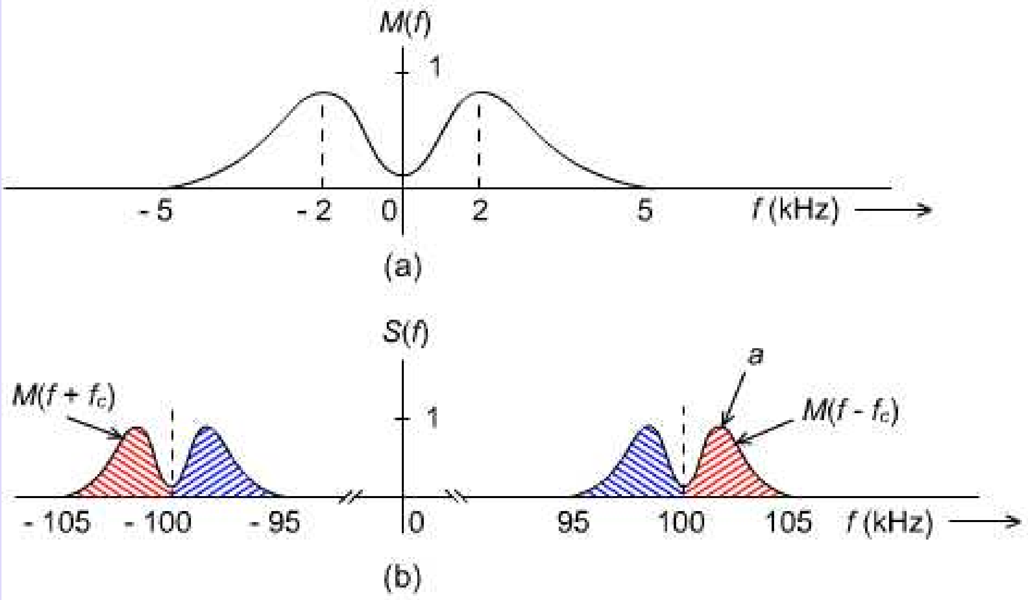



Fig. 4.9: Frequency translation (a) baseband spectrum (real signal)

(b) Shifted spectrum.

Note that $S(f)\big|_{f\,=\,102\text{ kHz}} = M(2\text{ kHz}) + M(202\text{ kHz})$

$$= 1 + 0 = 1$$

and is the point 'a' in Fig. 4.9

ii) Let $m(t)$ be a complex signal with $M(f)$ as shown in Fig. 4.10(a). The corresponding shifted spectrum (with $f_c = 100$ kHz) is shown in Fig. 4.10(b)

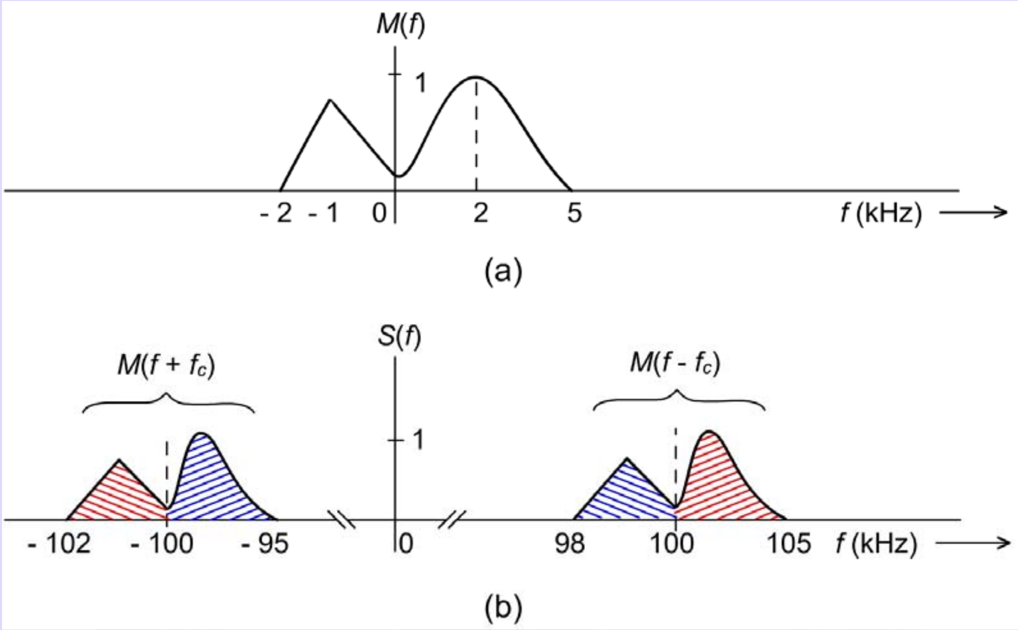



Fig. 4.10: Frequency translation (a) Baseband spectrum (complex signal)
(b) Shifted spectrum.

In figures 4.9(b) and 4.10(b), the part that is hatched in red is called the Upper Sideband (USB) and the one hatched in blue is called the Lower Sideband (LSB). Any one of these two sidebands has the complete information about the message signal. As we shall see later, SSB modulation conserves the bandwidth by transmitting only one sideband and recovering the $m(t)$ with appropriate demodulation.

**Example 4.1**

Consider the scheme shown in Fig. 4.11(a). The ideal HPF has the cutoff frequency at 10 kHz. Given that $f_1 = 10$ kHz and $f_2 = 15$ kHz, let us sketch $Y(f)$ for the $X(f)$ given at (b).

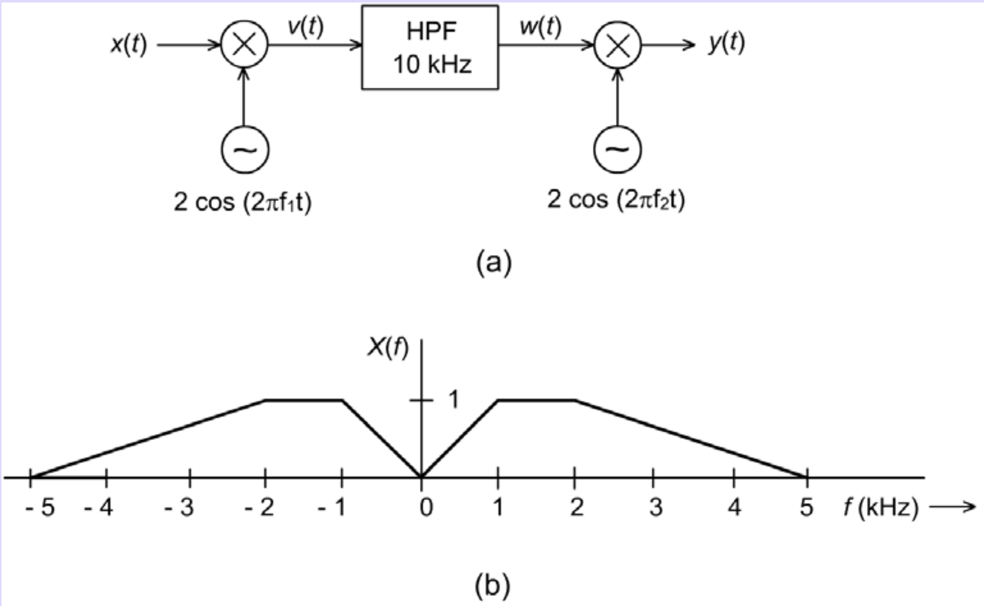



Fig. 4.11: (a) The scheme of example 4.1

(b) The input spectrum, $X(f)$

We have $V(f) = X(f - f_1) + X(f + f_1)$, which is as shown in Fig. 4.12(a).

The HPF eliminates the spectral components for $|f| \leq 10$ kHz. Hence $W(f)$ is as shown in Fig. 4.12(b).

$Y(f) = W(f - f_2) + W(f + f_2)$. This is shown in Fig. 4.12(c).

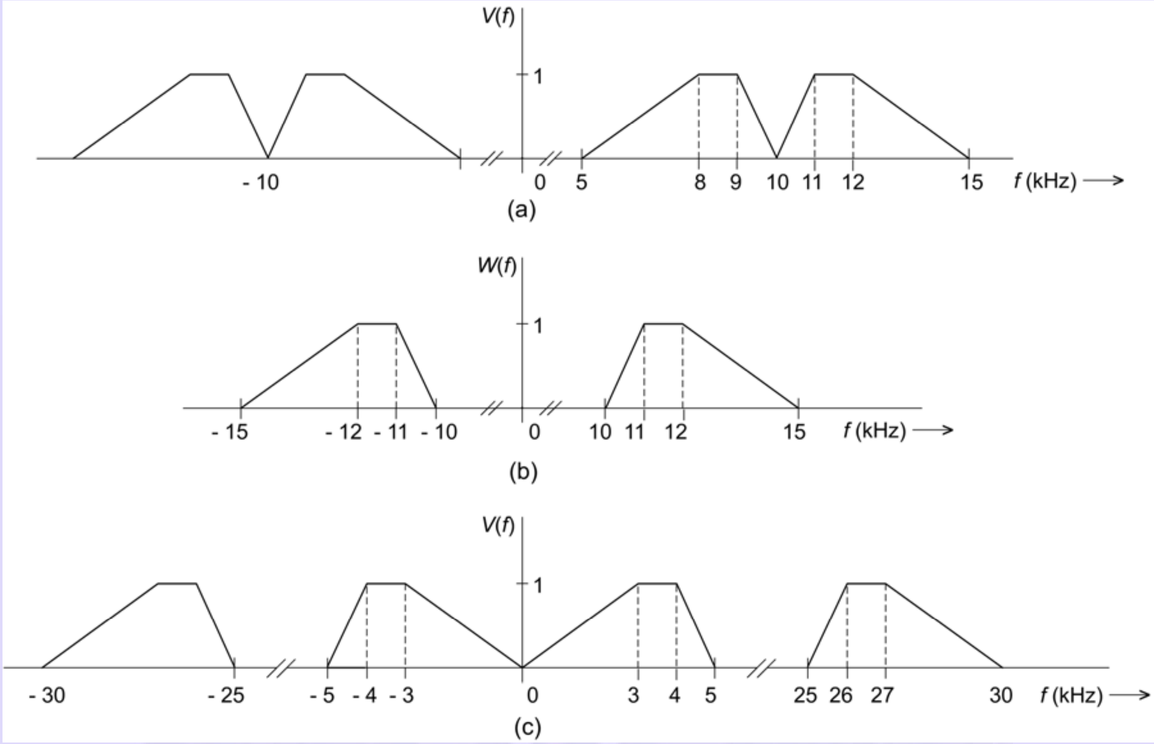



Fig. 4.12: Spectra at various points in the scheme of Fig. 4.11 ♦

### 4.2.2. Coherent demodulation

The process of demodulation of a DSB-SC signal, at least theoretically, is quite simple. Let us assume that the transmitted signal $s(t)$ has been received without any kind of distortion and is one of the inputs to the demodulator as shown in Fig. 4.13. That is, the received signal $r(t) = s(t)$. Also, let us assume that we are able to generate at the receiving end a replica of the transmitted carrier (denoted $c_r(t) = A_c^{'} \cos(\omega_c t)$ in Fig. 4.13) which is the other input to the demodulator.

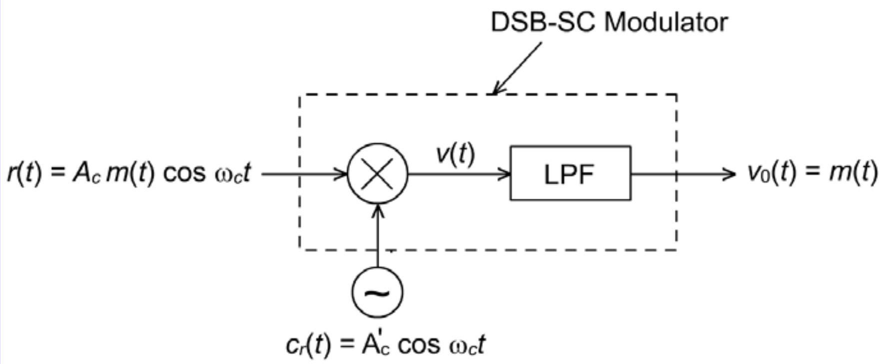



Fig. 4.13: Coherent demodulation of DSB-SC

The demodulation process consists of multiplying these two inputs and lowpass filtering the product quantity $v(t)$.

From Fig. 4.13, we have

$$v(t) = d_g \left(A_c\, m(t) \cos(\omega_c\, t)\right) \left(A_c' \cos(\omega_c t)\right)$$

where $d_g$ is the gain constant of the multiplier, called the *detector gain constant*, in the context of demodulation. For convenience, let us take $d_g = 1$

$$v(t) = A_c\, A_c'\, m(t) \cos^2(\omega_c\, t)$$

$$= A_c\, A_c'\, m(t) \frac{\left[1 + \cos(2\omega_c\, t)\right]}{2}$$

Assuming that $A_c A_c' = 2$ we have

$$v(t) = m(t) + m(t) \cos(4\pi f_c\, t) \tag{4.3}$$

The second term on the R.H.S of Eq. 4.3 has the spectrum centered at $\pm\, 2 f_c$ and would be eliminated by the lowpass filter following $v(t)$. Hence $v_0(t)$, the output of the demodulation scheme of Fig. 4.13 is the desired quantity, namely, $m(t)$.

Let us illustrate the operation of the detector in the frequency domain. Let $m(t)$ be real with the spectrum shown in Fig. 4.14(a). Let

$r(t) = s(t) = 2m(t)\cos(\omega_c t)$. Then $S(f) = M(f - f_c) + M(f + f_c)$, shown in Fig. 4.14(b).

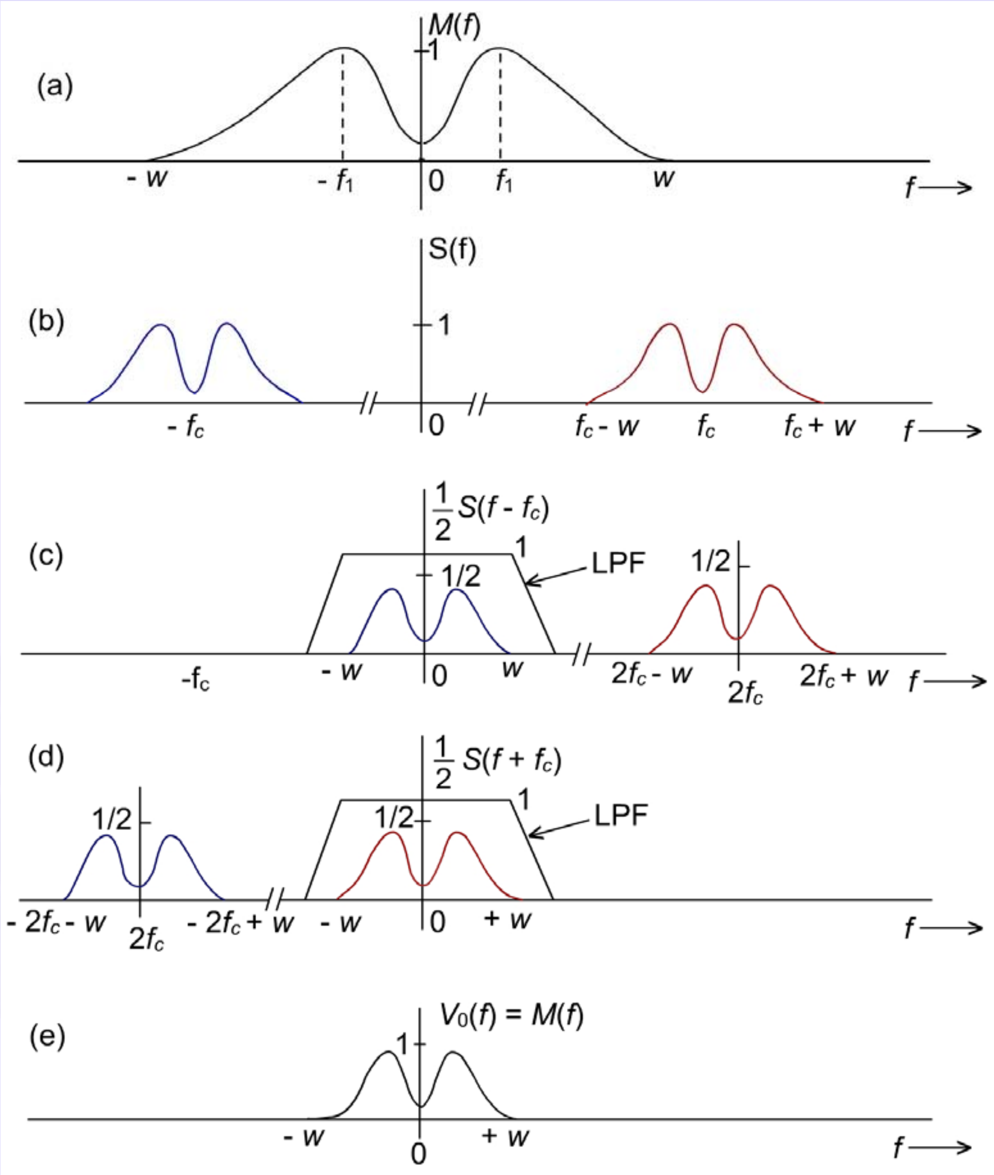



Fig. 4.14: Spectra at various points in the demodulation scheme of Fig. 4.13

(Note that the positive frequency part of $S(f)$ is shown in red and the negative frequency part in blue). Assuming $v(t) = s(t)\cos(\omega_c t)$ (Fig. 4.13 with $A_c^{'} = 1$), then $V(f) = \frac{1}{2}\left[S(f - f_c) + S(f + f_c)\right]$. $\frac{1}{2}S(f - f_c)$ and $\frac{1}{2}S(f + f_c)$ are shown in

Fig. 4.14(c) and (d) respectively. $V_0(f)$ is the sum of the outputs of the lowpass filters shown in Fig. 4.14(c) and (d) which is the desired message spectrum, $M(f)$.

From the discussion of the demodulation process so far, it appears that demodulation of DSB-SC is quite simple. In practice, it is not. In the scheme of Fig. 4.13, we have assumed that, we have available at the receiver, a carrier term that is coherent (of the same frequency and phase) with the carrier used to generate the DSB-SC signal at the transmitter. Hence this demodulation scheme is known as **coherent** (or **synchronous**) **demodulation**. As the receiver and the transmitter are, in general, not collocated, the carrier source at the receiver is different from that used at the transmitter and it is almost impossible to synchronize two independent sources. Fairly sophisticated circuitry has to be used at the receiver in order to generate the coherent carrier signal, from an $r(t)$ that has no carrier component in it. Before we discuss at the generation of the coherent carrier at the receiver, let us look at the degradation caused to the demodulated message due to a local carrier that has phase and frequency differences with the transmitted one.

**Case i):** Constant phase difference between $c(t)$ and $c_r(t)$

Let $c(t) = \cos(2\pi f_c t)$ and $c_r(t) = \cos[2\pi f_c t + \varphi]$ (the amplitude quantities, $A_c$ and $A_c^{'}$ can be treated as 1)

$$
\begin{aligned}
v(t) &= m(t)\cos(\omega_c t)\cos(\omega_c t + \varphi) \\
&= m(t)\cos(\omega_c t)\left[\cos(\omega_c t)\cos\varphi - \sin(\omega_c t)\sin\varphi\right] \\
&= m(t)\left[\cos^2(\omega_c t)\cos\varphi - \sin(\omega_c t)\cos(\omega_c t)\sin\varphi\right] \\
&= m(t)\left[\frac{1+\cos(2\omega_c t)}{2}\right]\cos\varphi - \frac{m(t)\sin(2\omega_c t)}{2}\sin\varphi
\end{aligned}
$$

At the output of the LPF, we will have only the term $\frac{m(t)\cos\varphi}{2}$. That is, the output of the demodulator, $v_0(t)$, is proportional to $m(t)\cos\varphi$. As long as $\varphi$ remains a constant, the demodulator output is a scaled version of the actual message signal. But values of $\varphi$ close to $\pi/2$ will force the output to near about zero. When $\varphi = \pi/2$ we have zero output from the demodulator. This is called the *quadrature null effect* of the coherent detector.

**Case ii):** Constant frequency difference between $c(t)$ and $c_r(t)$

Let $c(t) = \cos(2\pi f_c t)$ and $c_r(t) = \cos\left[2\pi(f_c + \Delta f)t\right]$. Then,

$$v(t) = m(t)\cos(2\pi f_c t)\cos\left[2\pi(f_c + \Delta f)t\right]$$

By carrying out the analysis similar to case (i) above, we find that

$$v_0(t) \;\alpha\; m(t)\cos\left[2\pi(\Delta f)t\right] \qquad (4.4a)$$

Let us look in some detail the implications of Eq. 4.4(a). For convenience, let

$$v_0(t) = m(t)\cos\left[2\pi(\Delta f)t\right] \qquad (4.4b)$$

Assume $\Delta f = 100$ Hz and consider the spectral component at 1 kHz in $M(f)$. After demodulation, this gives rise to two spectral components, one at 900 Hz and the other at 1100 Hz, because

$$\left[\cos\left(2\pi\times 10^3 t\right)\right]\left[\cos(2\pi\times 100t)\right] = \frac{1}{2}\left[\cos\left(2\pi\times 1100t\right) + \cos(2\pi\times 900t)\right]$$

The behavior of the sum of these two components is shown in Fig. 4.15. As can be seen from this figure, the envelope of sum signal (broken red line) attains the peak value twice in a cycle of the beat frequency $\Delta f$. Also, it goes through zero twice in a cycle of the beat frequency.

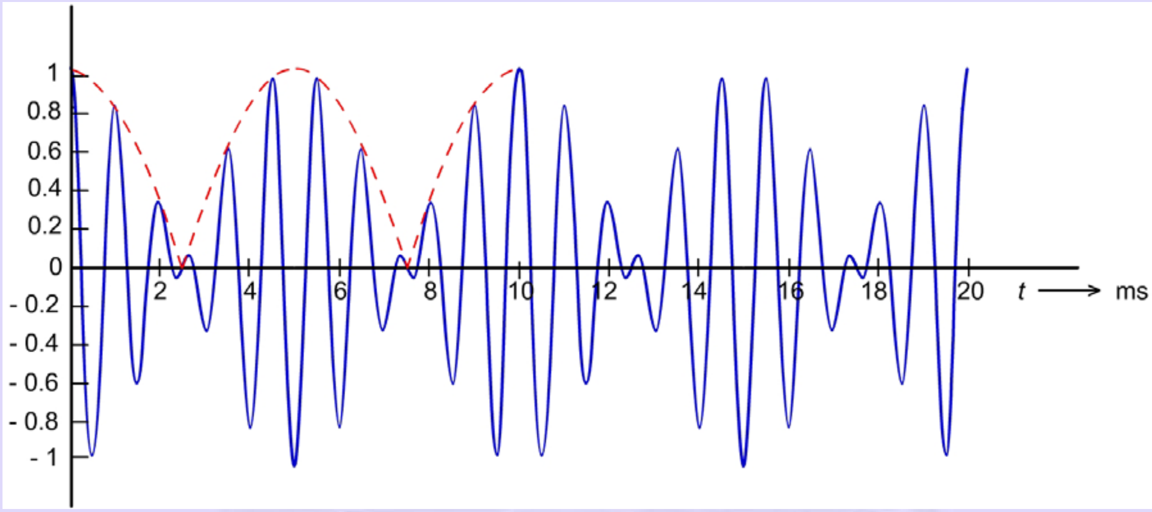



Fig. 4.15: Time-domain behavior of $\cos\left(2\pi \times 10^3 t\right) \cos\left(2\pi \times 100 t\right)$

Let us examine the effect of frequency offset in the frequency domain. Let $M(f)$ be as shown in Fig. 4.16(a). Assume $\Delta f = 300$ Hz. Then,

$V_0(f) = \frac{1}{2}\left[M(f - \Delta f) + M(f + \Delta f)\right]$ will be as shown in Fig. 4.16(d), which is one-half the sum of the spectra shown at (b) and (c). Comparing Fig. 4.16(a) and (d), we are tempted to surmise that the output of the demodulator is a fairly distorted version of the actual message signal. A qualitative feeling for this distortion can be obtained by listening to the speech files that follow.

Introduction

Output 1

Output 2

Output 3

Output 4

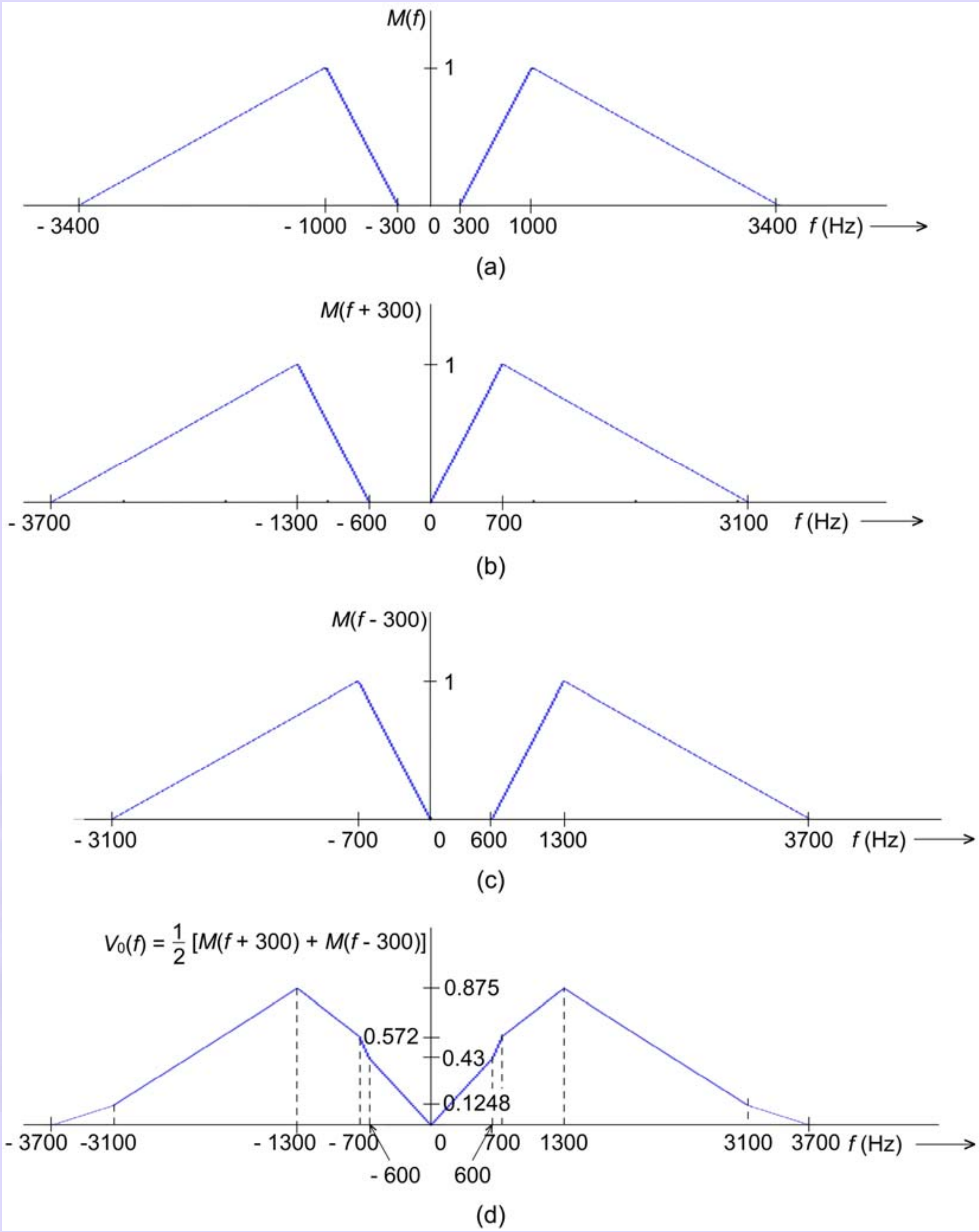



Fig. 4.16: The effect of frequency offset in the demodulation of DSB-SC:

(a) Typical message spectrum, $M(f)$

(b) $M(f+300)$

(c) $M(f-300)$

(d) $\frac{1}{2}\left[M(f+300)+M(f-300)\right]$

**Example 4.2**

In this example, we will show that the DSB-SC signal can be demodulated with the help of any periodic function $p(t)$, as long as $p(t)$ has a spectral component at $f_c$, the carrier frequency. This component could be due to the fundamental or some harmonic of the fundamental.

a) Let $s(t) = A_c\, m(t) \cos(\omega_c t)$. Consider the product $s(t) x_p(t)$ where $x_p(t)$ is any periodic signal with the period $T_0 = \dfrac{1}{f_c}$. That is,

$$x_p(t) = \sum_{n = -\infty}^{\infty} x_n\, e^{j 2 \pi n f_c t}$$

where $x_n$ is the n$^{th}$ Fourier coefficient. We will show that if $x_1 \neq 0$, then it is possible to extract $m(t)$ from the product $s(t) x_p(t)$.

b) Let $y_p(t)$ be another periodic signal with the period $T_0^{'} = N T_0$. We will show that, appropriate filtering of the product $s(t) y_p(t)$, will result in $m(t)$.

a) As $\cos(2 \pi f_c t) = \dfrac{1}{2}\left[e^{j 2 \pi f_c t} + e^{-j 2 \pi f_c t}\right]$, we have

$$s(t) x_p(t) = \frac{A_c\, m(t)}{2}\left[\sum_n x_n\, e^{j(n+1)\omega_c t} + \sum_n x_n\, e^{j(n-1)\omega_c t}\right]$$

$$= \frac{A_c\, m(t)}{2}\left[x_{-1} + \sum_{\substack{n, \\ n \neq -1}} x_n\, e^{j(n+1)\omega_c t} + x_1 + \sum_{\substack{n, \\ n \neq 1}} x_n\, e^{j(n-1)\omega_c t}\right]$$

as $\dfrac{x_{-1} + x_1}{2} = \text{Re}[x_1]$, the output, after lowpass filtering would be,

$\text{Re}[x_1]\, A_c\, m(t)$. (We assume that the LPF will reject all the other spectral components)

b) The product $s(t)\, y_p(t)$ can be written as

$$s(t)\, y_p(t) = \frac{A_c\, m(t)}{2} \left\{ \sum_n y_n\, e^{j 2\pi \left(\frac{n}{N} + 1\right) f_c\, t} + \sum_n y_n\, e^{j 2\pi \left(\frac{n}{N} - 1\right) f_c\, t} \right\}$$

$$= \frac{A_c\, m(t)}{2} \left\{ y_{-N} + \sum_{\substack{n, \\ n \neq -N}} y_n\, e^{j 2\pi \left(\frac{n}{N} + 1\right) f_c\, t} + y_N + \sum_{\substack{n, \\ n \neq N}} y_n\, e^{j 2\pi \left(\frac{n}{N} - 1\right) f_c\, t} \right\}$$

We assume that $y_N \neq 0$. Then, the output of the LPF would be $\text{Re}[y_N]\, A_c\, m(t)$. (Note that $y_p(t) s(t)$ has spectral lobes at $0, \pm \frac{f_c}{N}, \pm \frac{2 f_c}{N}$, etc. We assume that the LPF will extract the lobe at $f = 0$ and reject others). ♦

**Example 4.3**

Consider the scheme shown in Fig. 4.17. $s(t)$ is the DSB-SC signal $m(t)\cos(\omega_c\, t)$ with

$$S(f) = \begin{cases} 1\,, & 99 \text{ kHz} \leq |f| \leq 101 \text{ kHz} \\ 0\,, & outside \end{cases}$$

Let $g(t)$ be another bandpass signal with

$$G(f) = \begin{cases} 1\,, & 98 \text{ kHz} \leq |f| \leq 102 \text{ kHz} \\ 0\,, & outside \end{cases}$$

a) We will show that the output $y(t)\ \alpha\ m(t)$.

b) We will show that it would not be possible to recover $m(t)$ from $v(t)$ if

$$G(f) = \begin{cases} 1\,, & 98.5 \text{ kHz} < |f| \leq 101.5 \text{ kHz} \\ 0\,, & outside \end{cases}$$

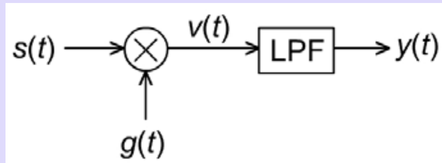



Fig. 4.17: Scheme of DSB-SC demodulation (example 4.3)

a) Let $g(t) = m_1(t)\cos(2\pi f_c t)$ where $f_c = 100$ kHz and

$$M_1(f) = \begin{cases} 2\,, & |f| \leq 2 \text{ kHz} \\ 0\,, & outside \end{cases}$$

From $S(f)$, we see that

$$M(f) = \begin{cases} 2\,, & |f| \leq 1 \text{ kHz} \\ 0\,, & outside \end{cases}$$

We have,

$$v(t) = m(t)\, m_1(t)\cos^2(\omega_c t)$$

$$= m(t)\, m_1(t)\left[\frac{1 + \cos(2\omega_c t)}{2}\right]$$

$$= \frac{m(t)\, m_1(t)}{2} + \frac{m(t)\, m_1(t)}{2}\cos(2\omega_c t)$$

We will assume that the LPF rejects the spectrum around $\pm 2 f_c$,

$$\frac{m(t)\, m_1(t)}{2} \longleftrightarrow \frac{M(f) * M_1(f)}{2}$$

$M(f) * M_1(f)$ will have a flat spectrum for $|f| \leq 1$ kHz. By using an ILPF with cutoff at 1 kHz, we can recover $m(t)$ from $v(t)$.

b) For this case $M_1(f)$ would be

$$M_1(f) = \begin{cases} 2\,, & |f| \leq 1.5 \text{ kHz} \\ 0\,, & outside \end{cases}$$

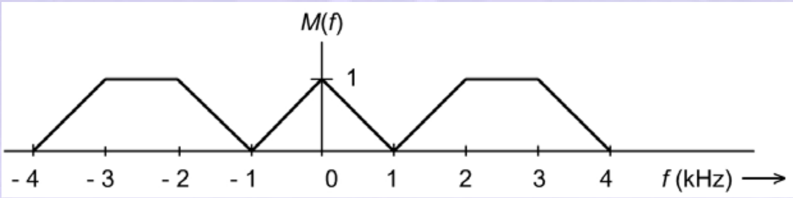
$M_1(f) * M(f)$ will be flat only for $|f| \leq 0.5$ kHz. Hence $m(t)$ cannot be recovered. ♦

**Exercise 4.1**

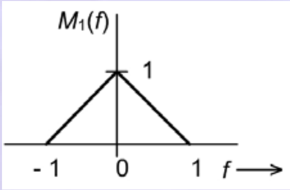
A signal $m(t)$ whose spectrum is shown in Fig. 4.18(a) is generated using the signals $m_1(t)$ and $m_2(t)$. $M_1(f)$ and $M_2(f)$ are shown at (b) and (c) respectively in Fig. 4.18. The signal $s(t) = 2\,m(t)\cos\left(10^5 \pi t\right)$ is transmitted on the channel.

a) Suggest a scheme to obtain $m(t)$ from $m_1(t)$ and $m_2(t)$.

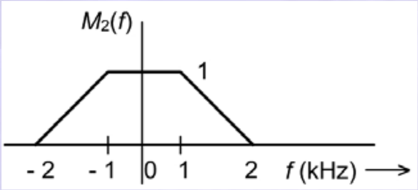
b) $m_1(t)$ and $m_2(t)$ are to be recovered from the received signal $r(t) = s(t)$. A part of this receiver is shown in Fig. 4.18(d). Complete the receiver structure by indicating the operations to be performed by the boxes with the question mark inside.



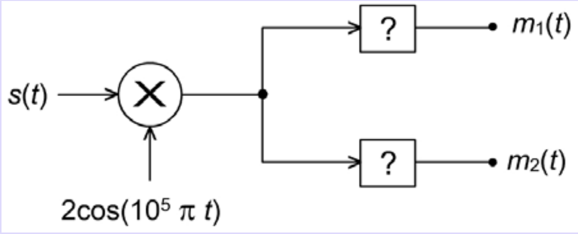

(a)




(b)




(c)




(d)

Fig. 4.18: Proposed receiver structure for the exercise 4.1

### 4.2.3 Carrier recovery for coherent demodulation

As explained in detail in sec. 4.2.2, coherent demodulation requires a carrier at the receiving end that is phase coherent with the transmitted carrier. Had there been a carrier component in the transmitted signal, it would have been possible to extract it at the receiving end and use it for demodulation. But the DSB-SC signal has no such component and other methods have to be devised to generate a coherent carrier at the receiver. Two methods are in common use for the carrier recovery (and hence demodulation) from the suppressed carrier modulation schemes, namely (a) Costas loop and (b) squaring loop.

**a) Costas loop:** This scheme is shown in Fig. 4.19.

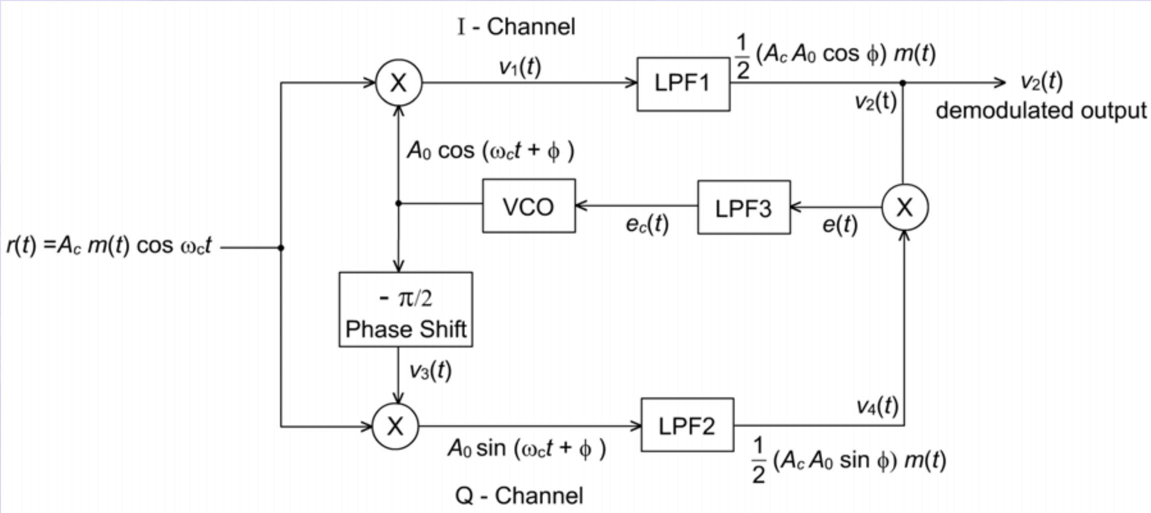



Fig. 4.19: Costas loop

The VCO (Voltage Controlled Oscillator) is a source that produces a periodic waveform[1] whose frequency is controlled by the input voltage $e_c(t)$. The output frequency $f_o$ of the VCO when $e_c(t) = 0$ is called the *free running frequency* of the VCO. The frequency put out by the VCO at any instant depends on the sign and magnitude of the control voltage, $e_c(t)$.

[1] Here, we shall assume that the VCO output is sinusoidal.

To understand the loop operation, let us assume that the frequency and phase of the VCO output are the same as that of the incoming carrier. Then,

$$v_1(t) = \left(A_c\, m(t) \cos(\omega_c t)\right)\left(A_0 \cos(\omega_c t)\right)$$

$$= A_0 A_c\, m(t) \cos^2(\omega_c t)$$

$$= A_0 A_c\, m(t) \frac{\left[1 + 2\cos(\omega_c t)\right]}{2}$$

The output of the LPF1 is

$$v_2(t) = \frac{A_0 A_c\, m(t)}{2}$$ ; that is, $v_2(t) \;\alpha\; m(t)$, the desired signal.

Similar analysis shows

$$v_4(t) = 0$$

Now suppose that VCO develops a small phase offset of $\varphi$ radians. The I-channel output will remain essentially unchanged but a small voltage will develop at the output of the Q-channel which will be proportional to $\sin\varphi$ (If the phase shift is $-\varphi$ rad, then the Q channel output is proportional to $-\sin\varphi$). Because of this, $e(t)$ is a non-zero quantity given by

$$e(t) = v_2(t)\, v_4(t) = \frac{1}{4}\left[A_c A_0\, m(t)\right]^2 \cos\varphi \sin\varphi$$

$$= \frac{1}{8}\left[A_c A_0\, m(t)\right]^2 \sin 2\varphi$$

$e(t)$ is input to LPF3, which has very narrow passband (Note that LPF1 and LPF2 should have a bandwidth of at least $W$ Hz). Hence $e_c(t) = C_0 \sin 2\varphi$ where $C_0$ is the DC value of $\frac{1}{8}\left[A_c A_0\, m(t)\right]^2$. This DC control voltage ensures that the VCO output is coherent with the carrier used for modulation.

**b) Squaring loop**

The operation of the squaring loop can be explained with the help of Fig. 4.20.

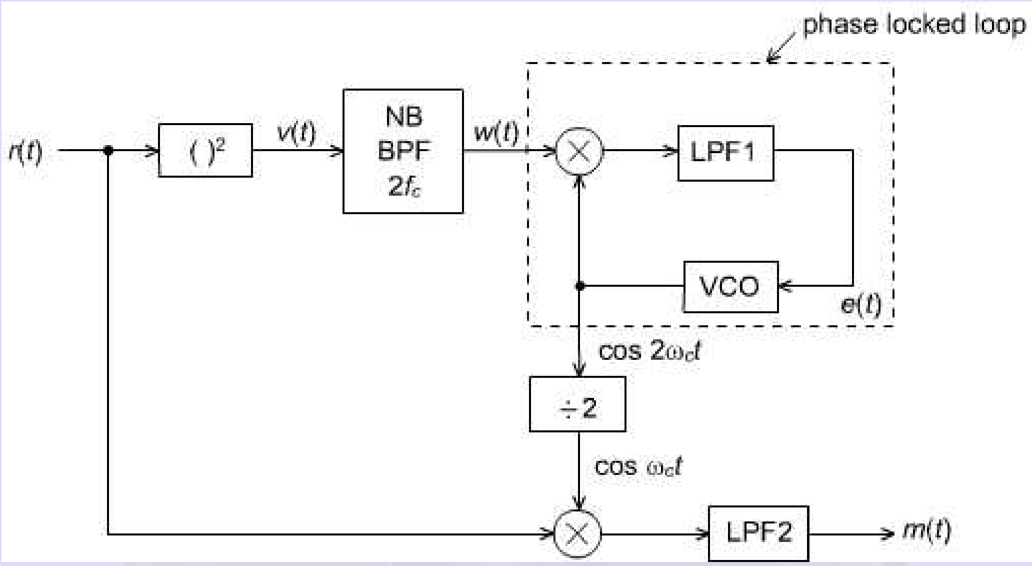



Fig. 4.20: Demodulation of DSB-SC using a squaring loop

Let $r(t) = s(t) = A_c\, m(t) \cos(\omega_c t)$. Then,

$$v(t) = r^2(t) = \frac{A_c^2}{2}\, m^2(t)\left[1 + \cos(2\omega_c t)\right]$$

$m^2(t)$ will have nonzero DC value which implies its spectrum has an impulse at $f = 0$. Because of this, $V(f)$ will have a discrete spectral component at $2f_c$. $v(t)$ is the input to a very narrowband bandpass filter, with the centre frequency $2f_c$. By making the bandwidth of LPF1 very narrow, it is possible to make the VCO to lock on to the discrete component at $2f_c$, present in $w(t)$. (The dotted box enclosing a multiplier, LPF and a VCO, connected in the feedback configuration shown is called the Phase Locked Loop (PLL)). The VCO output goes through a factor of two frequency divider, yielding a coherent carrier at its output. This carrier is used to demodulate the DSB-SC signal. Note that LPF2 must have adequate bandwidth to pass the highest frequency component present in $m(t)$.

Both the Costas loop and squaring loop have one disadvantage, namely, an $180^0$ phase ambiguity. Consider the Costas loop; if the input to the loop were to be $-A_c\,m(t)\cos(\omega_c\,t)$, output of the LPF1 would be $-\frac{1}{2}A_c\,A_0\,m(t)\cos\varphi$ and that of LPF2 would be $-\frac{1}{2}A_c\,A_0\,m(t)\sin\varphi$ with the result that, $e(t)$ would be the same as in the case discussed earlier. Similarly, for the squaring loop, $v(t)$ would be the same whether $r(t) = A_c\,m(t)\cos(\omega_c\,t)$ or $-A_c\,m(t)\cos(\omega_c\,t)$. Hence the demodulated output could be either $m(t)$ or $-m(t)$. However, this will not cause any problem for audio transmission because $m(t)$ and $-m(t)$, would sound the same to our ears.

Though DSB-SC modulation schemes place the entire transmitted power into the useful sidebands, the demodulation has to be coherent. The circuit required to generate a coherent carrier increases the cost of the receiver. If only a few receivers are to be built for a specific communication need, the cost may not be a major factor. But in a broadcast situation, there would be a large number of receivers tuned to a given station and in that scenario, it is better make the receiver fairly cheap and push the cost up of the transmitter, if required. As will seen later, the Envelope Detector(ED) is fairly cheap to implement as compared to a coherent detector. But to make use of ED, the modulated carrier should carry $m(t)$ in its envelope. Evidently, DSB-SC does not satisfy this property as explained below.

Let $\quad s(t) = A_c\,m(t)\cos(\omega_c\,t)$

Pre-envelope of $s(t) = \left[s(t)_{pe}\right] = A_c\,m(t)e^{i\,\omega_c\,t}$

Complex envelope of $s(t) = \left[s(t)_{ce}\right] = A_c\,m(t)$

Hence the envelope of DSB-SC $= \left|s(t)_{pe}\right| = \left|s(t)_{ce}\right|$

$$\alpha\ \left|m(t)\right|$$

We shall now describe a modulation scheme that has $m(t)$ as its envelope which can easily be extracted.

## 4.3 DSB-LC Modulation (or AM)

By adding a large carrier component to the DSB-SC signal, we will have DSB-LC, which, for convenience, we shall call simply as AM. By choosing the carrier component properly, it is possible for us to generate the AM signal such that it preserves $m(t)$ in its envelope. Consider the scheme shown in Fig. 4.21.

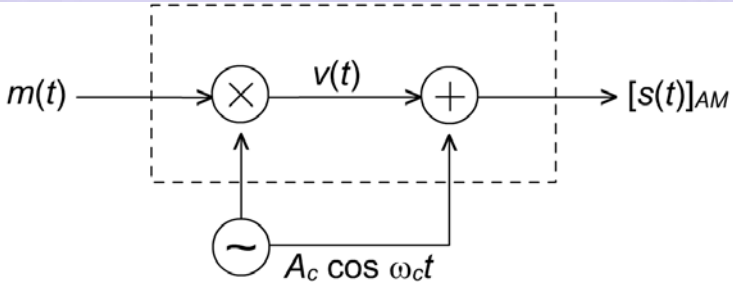



Fig. 4.21: Generation of an AM signal from a DSB-SC signal

Let $v(t) = A_c\, g_m\, m(t) \cos(\omega_c t)$. Then,

$$[s(t)]_{AM} = A_c \cos(\omega_c t) + v(t)$$

$$= A_c\left[1 + g_m\, m(t)\right]\cos(\omega_c t) \qquad (4.5)$$

In this section, unless there is confusion, we use $s(t)$ in place of $[s(t)]_{AM}$ . We shall assume that $m(t)$ has no DC component and $(m(t))_{max} = -(m(t))_{min}$. Let $g_m\, m(t)$ be such that $|g_m\, m(t)| \leq 1$ for all t. Then $\left[1 + g_m\, m(t)\right] \geq 0$ and $[s(t)]_{AM}$ preserves $m(t)$ in its envelope because

$$[s(t)]_{pe} = A_c\left[1 + g_m\, m(t)\right] e^{j \omega_c t}$$

$$[s(t)]_{ce} = A_c\left[1 + g_m\, m(t)\right]$$

As $\left[1 + g_m m(t)\right] \geq 0$, we have

Envelope of $s(t) = \left|s(t)_{ce}\right| = A_c\left[1 + g_m m(t)\right]$

The quantity after the DC block is proportional to $m(t)$.

If $\left[1 + g_m m(t)\right]$ is not nonnegative for all $t$, then the envelope would be different from $m(t)$. This would be illustrated later with a few time domain waveforms of the AM signal. Fig. 4.22(b) illustrates the AM waveform for the case $\left[1 + g_m m(t)\right] \geq 0$ for all $t$.

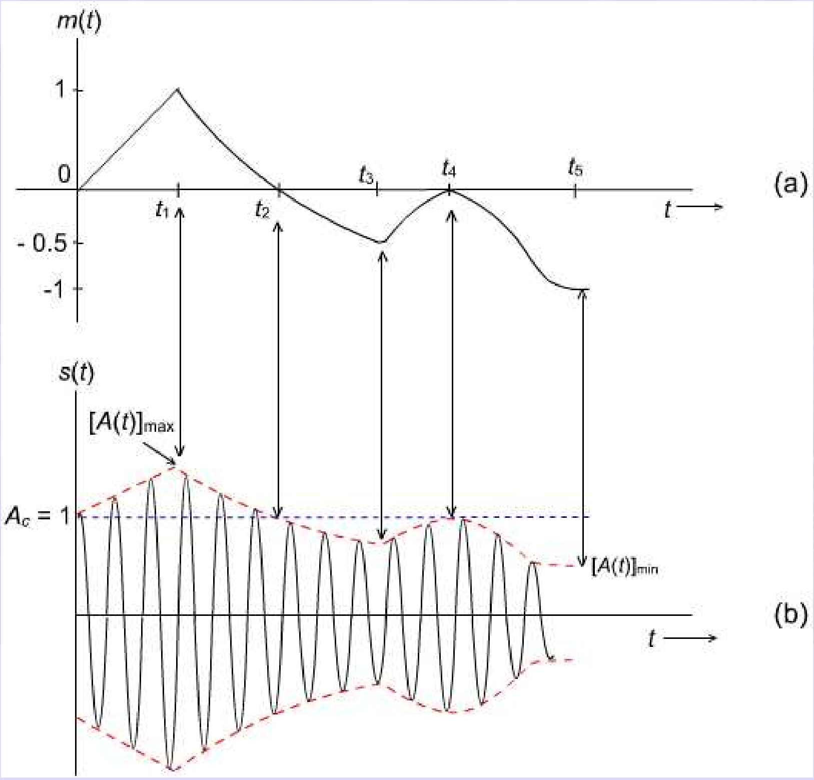



Fig. 4.22: (a) An arbitrary message waveform $m(t)$

(b) Corresponding AM waveform

A few time instants have been marked in both the figures (a) and (b). At the time instants when $m(t) = 0$, carrier level would be $A_c$ which we have assumed to be 1. Maximum value of the envelope (shown in red broken line) occurs when $m(t)$ has the maximum positive value. Similarly, the envelope will reach its minimum value when $m(t)$ is the most negative. As can be seen from the figure, the envelope of $s(t)$ follows $m(t)$ in a one-to-one fashion.

Let $\left|g_m m(t)\right|_{max} = x \leq 1$. Then $s(t)$ is said to have (100x) percentage modulation. For the case of 100% modulation, $\left[g_m m(t)\right]_{max} = -\left[g_m m(t)\right]_{min} = 1$. If $\left|g_m m(t)\right|_{max} > 1$, then we have *over modulation* which results in the envelope distortion. This will be illustrated in the context of tone modulation, discussed next.

**Exercise 4.2**

For the waveform $m(t)$ shown in Fig. 4.23, sketch the AM signal with the percentage modulation of 40. Assume $A_c = 1$ (the figure has to be shown with reference to $m(t)$)

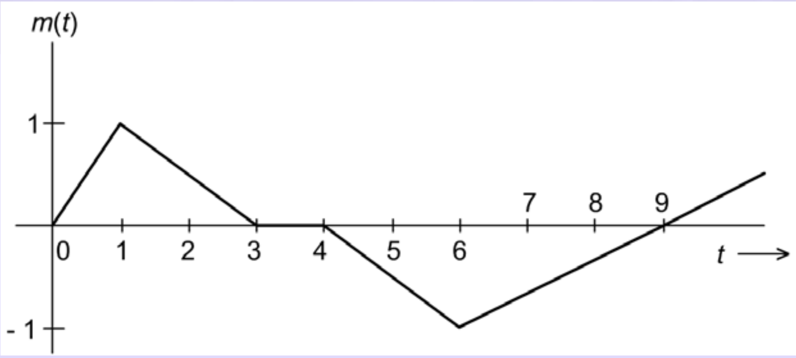



Fig. 4.23: Baseband signal for the exercise 4.2

### 4.3.1. Tone Modulation

Let $m(t)$ to be a tone signal; that is, $m(t) = A_m \cos(2\pi f_m t)$ where $f_m << f_c$. Then Eq. 4.5 becomes

$$s(t) = A_c \left[1 + g_m A_m \cos(\omega_m t)\right] \cos(\omega_c t) \quad (4.6a)$$

$$= A(t) \cos(\omega_c t) \quad (4.6b)$$

Let $g_m A_m = \mu$. Then for tone modulation,

$$s(t) = A_c \left[1 + \mu \cos(\omega_m t)\right] \cos(\omega_c t) \quad (4.7)$$

$\mu$ is called the **modulation index** or **modulation factor**. $\mu \times 100$ is the percentage modulation. To avoid envelope distortion, we require, $\mu \leq 1$.

As $A(t) = A_c \left[1 + \mu \cos(\omega_m t)\right]$, we have

$$\left[A(t)\right]_{max} = A_c \left[1 + \mu\right]$$

$$\left[A(t)\right]_{min} = A_c \left[1 - \mu\right]$$

$$\frac{\left[A(t)\right]_{max}}{\left[A(t)\right]_{min}} = \frac{1 + \mu}{1 - \mu}$$

or $$\mu = \frac{\left[A(t)\right]_{max} - \left[A(t)\right]_{min}}{\left[A(t)\right]_{max} + \left[A(t)\right]_{min}}$$

Fig. 4.24 to 4.26 illustrate the experimentally generated AM waveforms for $\mu = 0.5, 1$ and $1.5$ respectively (with $\mu > 1$, we have **overmodulation**).

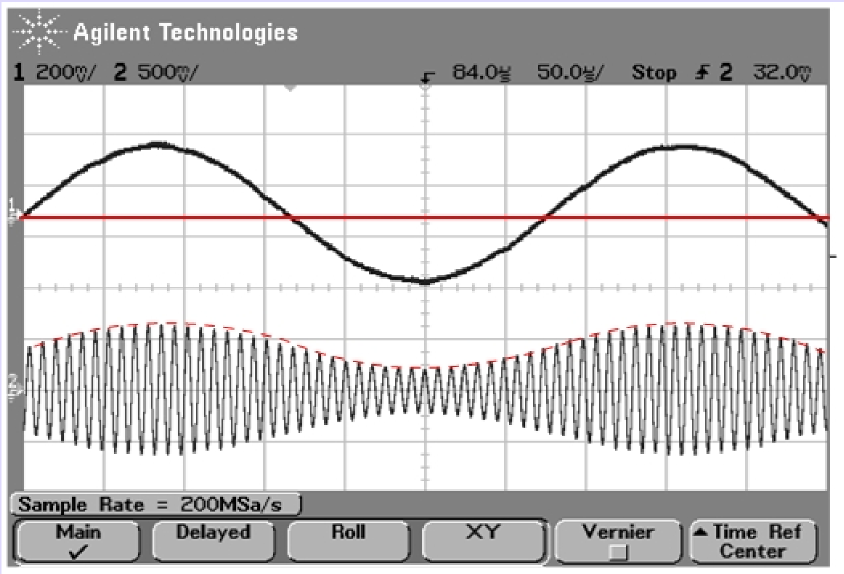



Fig. 4.24: AM with tone modulation ($\mu = 0.5$)

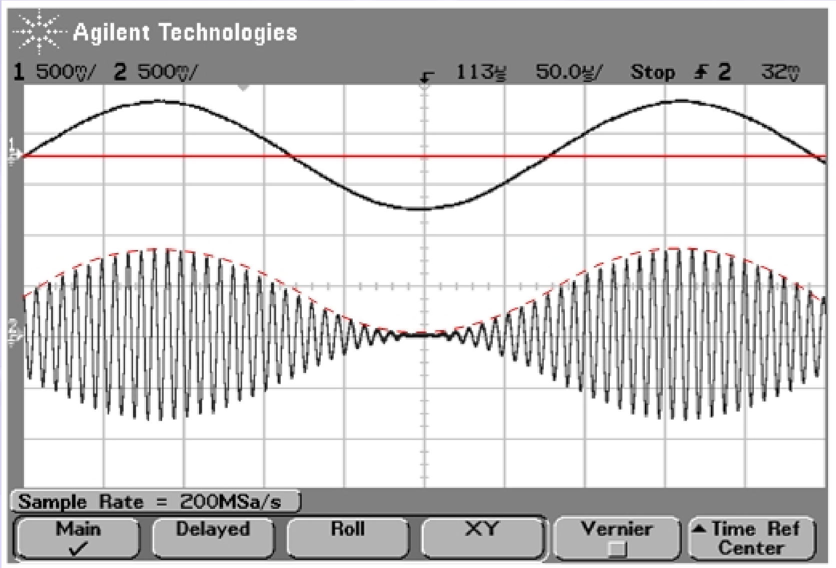



Fig. 4.25: AM with tone modulation ($\mu = 1$)

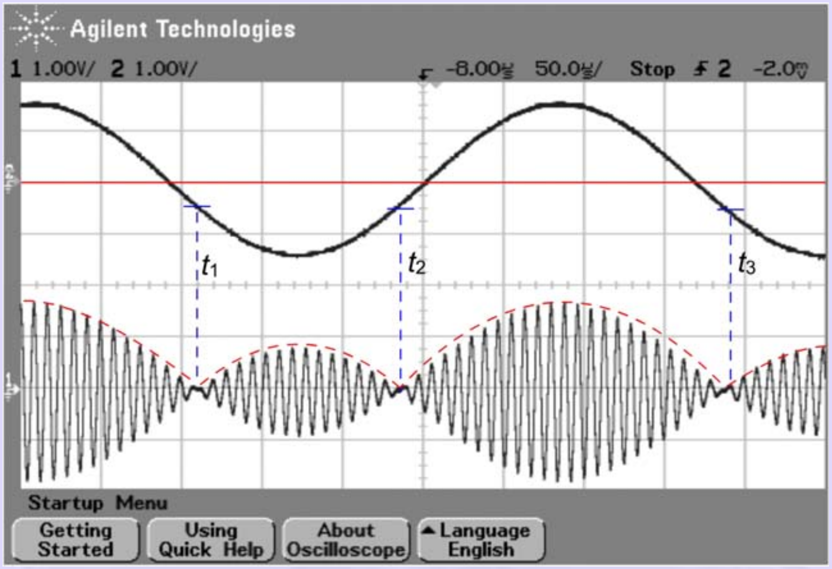



Fig. 4.26: AM with tone modulation ($\mu = 1.5$)

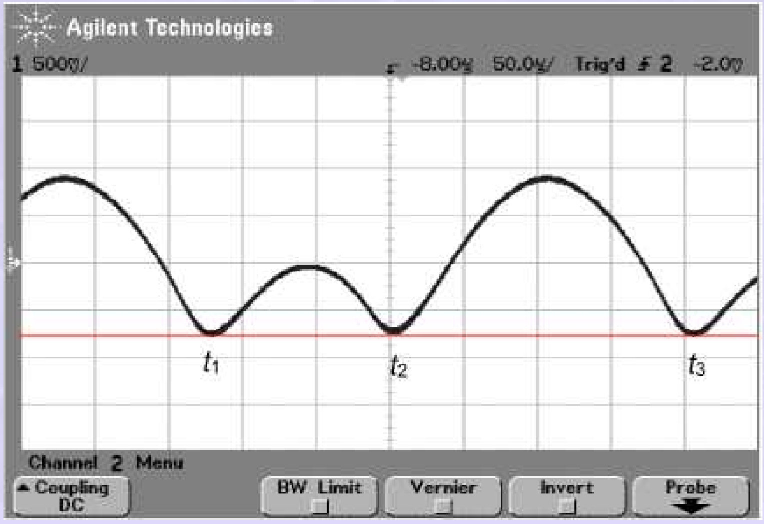



Fig. 4.27: Envelope of the AM signal of Fig. 4.26

As can be seen from 4.24 and 4.25, the envelope (shown with a red broken line) is one-to-one related to the message sinusoid. Note that, for $\mu = 1$, the carrier amplitude (and hence the envelope) goes to zero, corresponding to the time-instant when the sinusoid is going through the negative peak. However, when $\mu > 1$, the one-to-one relationship between the envelope of the modulated carrier and the modulating tone is no longer maintained. This can be more clearly seen in Fig. 4.27 which shows the output of the envelope detector when the input is the modulated carrier of Fig. 4.26. Notice that the tone signal between $(t_1, t_2)$ and to the right of $t_3$ of Fig. 4.26 is getting inverted; in other words, the output of the ED is proportional to $|(1 + \mu \cos \omega_m t)|$ which is not equal to $(1 + \mu \cos \omega_m t)$, when $\mu > 1$.

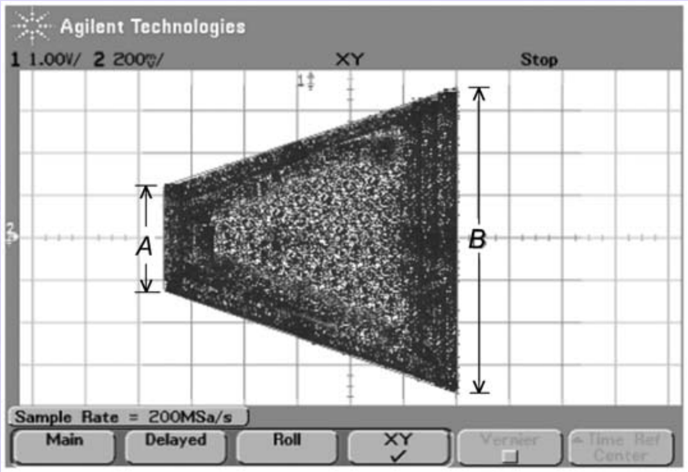



Fig. 4.28: Oscillogram when the CRO inputs are:

X-plates: tone signal

Y-plates: AM signal with $\mu = \dfrac{1}{2}$

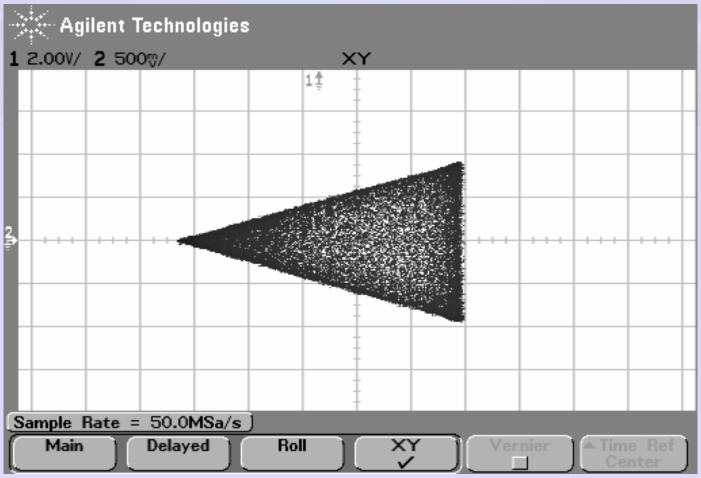



Fig. 4.29: Oscillogram when the CRO inputs are:

X-plates: tone signal

Y-plates: AM signal with $(\mu = 1)$

Fig. 4.28 and Fig. 4.29 illustrate the oscillograms when the X-plates of the CRO are fed with the modulating tone and the Y-plates with the AM signal with $\mu = 0.5$ and $\mu = 1$ respectively. In Fig. 4.28, A represents the peak-to-peak value of the carrier at its minimum (that is, $A = 2A_c[1 - \mu]$) where as B is the peak-to-peak value of the carrier at its maximum (that is, $B = 2A_c[1 + \mu]$). Hence $\mu$ can be calculated as

$$\mu = \frac{B - A}{B + A}$$

In Fig. 4.29, as A = 0 we have $\mu = 1$

**Exercise 4.3**

Picture the oscillogram when the X-plates of the CRO are fed with the modulating tone and the Y-plates with the AM signal with $\mu = 1.5$.

## 4.3.2. Spectra of AM signals

Taking the Fourier transform of Eq. 4.5,

$$\left[S(f)\right]_{AM} = \frac{A_c}{2}\left[\delta(f - f_c) + \delta(f + f_c)\right] + \frac{A_c}{2} g_m \left[M(f - f_c) + M(f + f_c)\right] \tag{4.8}$$

The plot of $[S(f)]_{AM}$ is given in Fig. 4.30.

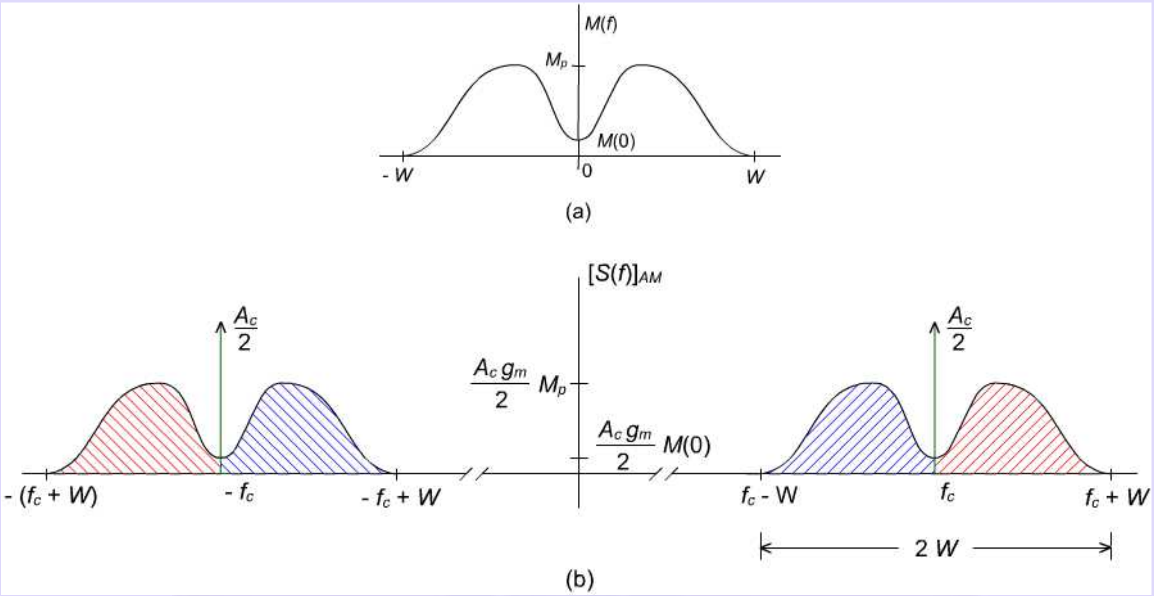



Fig. 4.30: (a) Baseband message spectrum $M(f)$

(b) Spectrum of the AM signal

Based on Fig. 4.30, we make the following observations:

1) The spectrum has two sidebands, the USB [between $f_c$ to $f_c + W$, and $(-f_c - W)$ to $-f_c$, hatched in red] and the LSB ($f_c - W$ to $f_c$ and $-f_c$ to $(-f_c + W)$, hatched in blue).

2) If the baseband signal has bandwidth $W$, then the AM signal has bandwidth $2W$. That is, the transmission bandwidth $B_T$, required for the AM signal is $2W$.

3) Spectrum has discrete components at $f = \pm f_c$, indicated by impulses of area $\frac{A_c}{2}$

4) In order to avoid the overlap between the positive part and the negative part of $S(f)$, $f_c > W$ (In practice, $f_c >> W$, so that $s(t)$ is a narrowband signal)

The discrete components at $f = \pm f_c$, do not carry any information and as such AM does not make efficient use of the transmitted power. Let us illustrate this taking the example of tone modulation.

**Example 4.4**

For AM with tone modulation, let us find $\eta = \dfrac{\text{Total sideband power}}{\text{Total power}}$, as a function of modulation index $\mu$.

For tone modulation, we have

$$s(t) = A_c \left[1 + \mu \cos(\omega_m t)\right] \cos(\omega_c t)$$

Carrier term = $A_c \cos(\omega_c t)$

Carrier Power = $\dfrac{A_c^2}{2}$

USB term = $\dfrac{A_c \mu}{2} \cos(\omega_c + \omega_m)t$

Power in USB = $\dfrac{\left(\dfrac{A_c \mu}{2}\right)^2}{2} = \dfrac{A_c^2 \mu^2}{8}$

Power in LSB = Power in USB

Total sideband Power = $2 \times A_c^2 \dfrac{\mu^2}{8} = \dfrac{A_c^2 \mu^2}{4}$

Total Power= $\dfrac{A_c^2}{2} + \dfrac{A_c^2 \mu^2}{4} = \dfrac{A_c^2}{2}\left(1 + \dfrac{\mu^2}{2}\right) = \dfrac{A_c^2 \left(2 + \mu^2\right)}{4}$

Hence, $\eta = \dfrac{\dfrac{A_c^2 \mu^2}{4}}{\dfrac{A_c^2 \left(2 + \mu^2\right)}{4}} = \dfrac{\mu^2}{2 + \mu^2}$ ♦

Calculating the value of $\eta$ for a few value of $\mu$, we have

| $\mu$ | $\eta$ |
|---|---|
| 0.25 | 0.03 |
| 0.50 | 0.11 |
| 0.75 | 0.22 |
| 1.0 | 0.33 |

As can be seen from the above tabulation, $\eta$ increases as $\mu \to 1$; however, even at $\mu = 1$, only 1/3 of the total power is in the sidebands (or side frequencies), the remaining 2/3 being in the carrier. From this example, we see that AM is not an efficient modulation scheme, in terms of the utilization of the transmitted power.

The complex envelope behavior of $s(t)$ for tone modulation is quite illustrative.

$$s(t) = A_c\left[1 + \mu \cos(\omega_m t)\right]\cos(\omega_c t)$$

$$= \text{Re}\left\{A_c e^{j\omega_c t} + \frac{A_c \mu}{2}\left[e^{j(\omega_c + \omega_m)t} + e^{j(\omega_c - \omega_m t)}\right]\right\}$$

$$\left[s(t)\right]_{ce} = A_c + \frac{A_c \mu}{2} e^{j\omega_m t} + \frac{A_c \mu}{2} e^{-j\omega_m t} \qquad (4.9)$$

Let us draw a 'phasor' diagram, using the carrier quantity as the reference. The term $\frac{A_c \mu}{2} e^{j\omega_m t}$ can be represented as a rotating vector with a magnitude of $\frac{A_c \mu}{2}$, rotating counterclockwise at the rate of $f_m$ rev/sec. Similarly, $\frac{A_c \mu}{2} e^{-j\omega_m t}$ can be shown as a vector with clockwise rotational speed of $f_m$ rev/sec.

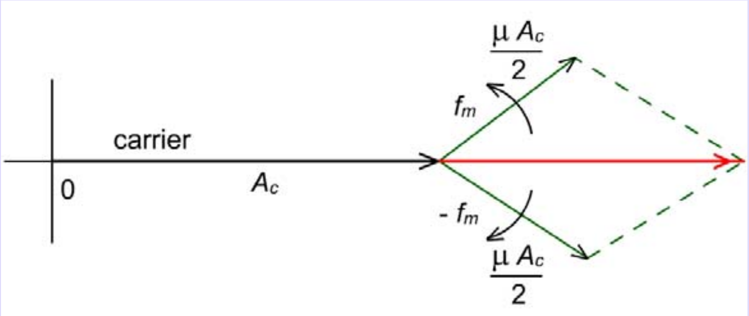



Fig. 4.31: Phasor diagram for AM with tone modulation

Fig. 4.31 depicts the behavior of all the quantities on the RHS of Eq. 4.9. From Eq.4.9, we find that the complex envelope is real and is given by $A_c\left[1 + \mu \cos(\omega_m t)\right]$. This can also be seen from the phasor diagram, because at any given time, the quadrature components of the sideband phasors cancel out where as the in-phase components add up; the resultant of the sideband components is collinear with the carrier.

The length of the in-phase component of $\left[s(t)\right]_{ce}$ depends on the sign of the resultant of the sideband phasors. As can be seen from Fig. 4.31 this varies between the limits $A_c[1 - \mu]$ to $A_c[1 + \mu]$. If the modulation index $\mu$ is less than 1, $A_c[1 - \mu] > 0$ and envelope of $s(t)$ is

$$\left|\left[s(t)\right]_{ce}\right| = \left|A_c\left[1 + \mu \cos(\omega_m t)\right]\right| = A_c\left[1 + \mu \cos(\omega_m t)\right]$$

Phasor diagrams such as the one shown in Fig. 4.31 are helpful in the study of unequal attenuation of the sideband components. We shall illustrate this with an example.

**Example 4.5**

Let $A_c = 1$, $\mu = \frac{1}{2}$ and let the upper sideband be attenuated by a factor of 2. Let us find the expression for the resulting envelope, $A(t)$.

The phasor diagram for this case is shown in Fig. 4.32.

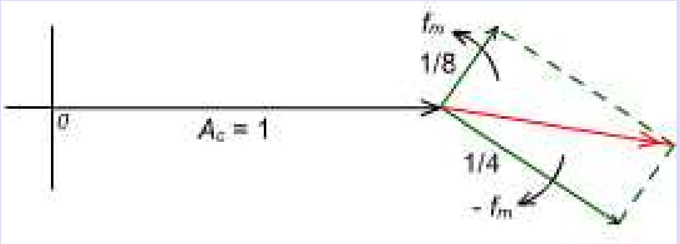



Fig. 4.32: Phasor diagram for an AM signal with unequal sidebands

As can be seen from the figure, the resultant of the sidebands is no longer collinear with the carrier.

$$\left[s(t)\right]_{ce} = 1 + \frac{1}{8}\left(\cos(\omega_m t) + j\sin(\omega_m t)\right) + \frac{1}{4}\left(\cos(\omega_m t) - j\sin(\omega_m t)\right)$$

$$= 1 + \frac{3}{8}\cos(\omega_m t) - j\,\frac{1}{8}\sin(\omega_m t)$$

$$A(t) = \left[\left(1 + \frac{3}{8}\cos(\omega_m t)\right)^2 + \left(\frac{1}{8}\sin(\omega_m t)\right)^2\right]^{\frac{1}{2}}$$

Evidently, it is not possible for us to recover the message from the above $A(t)$.

♦

## 4.4 Generation of AM and DSB-SC signals

Let $x(t)$ be the input to an LTI system with the impulse response $h(t)$ and let $y(t)$ be the output. Then,

$$y(t) = x(t) * h(t)$$

$$Y(f) = X(f)\,H(f)$$

That is, an LTI system can only alter a frequency component (either boost or attenuate), that is present in the input signal. In other words, an LTI system cannot generate at its output frequency components that are not present in $X(f)$. We have already seen that the spectrum of a DSB or AM signal is different from that of the carrier and the baseband signal. That is, to generate a DSB signal or an AM signal, we have to make use of nonlinear or time-varying systems.

### 4.4.1 Generation of AM

We shall discuss two methods of generating AM signals, one using a nonlinear element and the other using an element with time-varying characteristic.

**a) Square law modulator**

Consider the scheme shown in Fig. 4.33(a).

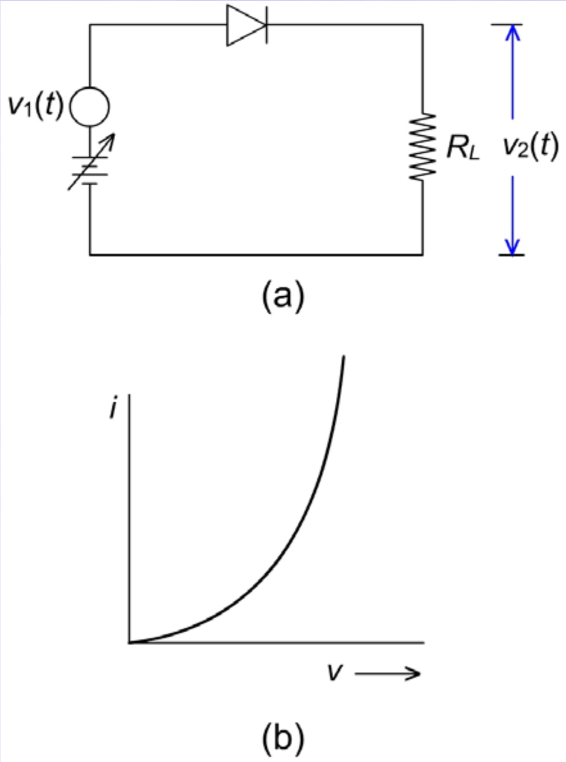



Fig. 4.33 (a): A circuit with a nonlinear element

(b): $v - i$ characteristic of the diode in Fig. 4.28(a)

A semiconductor diode, when properly biased has a $v - i$ characteristic that nonlinear, as shown in Fig. 4.33(b).

For fairly small variations of $v$ around a suitable operating point, $v_2(t)$ can be written as

$$v_2(t) = \alpha_1 v_1(t) + \alpha_2 v_1^2(t) \tag{4.10}$$

where $\alpha_1$ and $\alpha_2$ are constants.

Let $v_1(t) = A_c^{'} \cos(2\pi f_c t) + m(t)$. Then,

$$v_2(t) = \alpha_1 A_c^{'} \left[1 + \frac{2\alpha_2}{\alpha_1} m(t)\right] \cos(2\pi f_c t) + \alpha_1 m(t) + \alpha_2 m^2(t) + \alpha_2 A_c^{'2} \cos^2(2\pi f_c t) \tag{4.11}$$

The first term (on the RHS of Eq. 4.11) is $\left[s(t)\right]_{AM}$, with the carrier amplitude $A_c = \alpha_1 A_c^{'}$ and $g_m = \frac{2\alpha_2}{\alpha_1}$.

Now the question is: can we extract $\left[s(t)\right]_{AM}$ from the sum of terms on the RHS of Eq. 4.11? This can be answered by looking at the spectra of the various terms constituting $v_2(t)$. Fig. 4.34 illustrates these spectra (quantities marked A to E) for the $M(f)$ of Fig. 4.14(a). The time domain quantities corresponding to A to E are listed below.

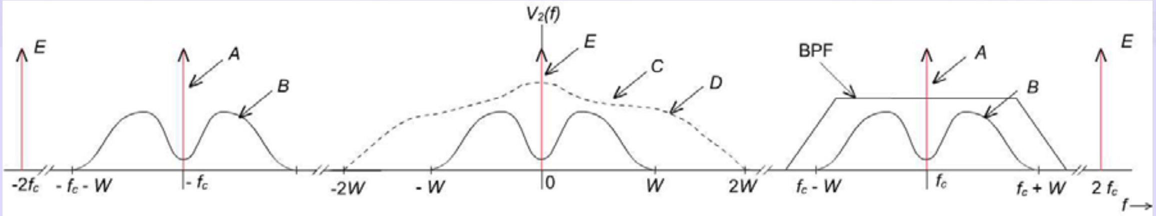



Fig. 4.34: Spectra of the components of $v_2(t)$ of Eq. 4.11

| Component | Spectrum indicated by |
|---|---|
| (i) $A_c \cos(2\pi f_c t)$ | A |
| (ii) $2\alpha_2 A_c^{'} m(t) \cos(2\pi f_c t)$ | B |
| (iii) $\alpha_1 m(t)$ | C |
| (iv) $\alpha_2 m^2(t)$ | D |
| (v) $\alpha_2 \left(A_c^{'}\right)^2 \cos^2(2\pi f_c t)$ | E |

$\left[s(t)\right]_{AM}$ consists of the components (i) and (ii) of the above list. If it is possible for us to filter out the components (iii), (iv) and (v), then the required AM signal would be available at the output of the filter. This is possible by placing a BPF with centre at $f_c$ and bandwidth 2W provided $(f_c - W) > 2W$ or $f_c > 3W$.

Usually, this is not a very stringent requirement. However, this scheme suffers from a few disadvantages.

i) The required square-law nonlinearity of a given device would be available only over a small part of the $(v - i)$ characteristic. Hence, it is possible to generate only low levels of the desired output.

ii) If $f_c$ is of the order of 3W, then we require a BPF with very sharp cut off characteristics.

**b) Switching modulator**

In the first method of generation of the AM signals, we have made use of the nonlinearity of a diode. In the second method discussed below, diode will be used as a switching element. As such, it acts as a device with time-varying characteristic, generating the desired AM signals when it is used in the circuit configuration shown in Fig. 4.35.

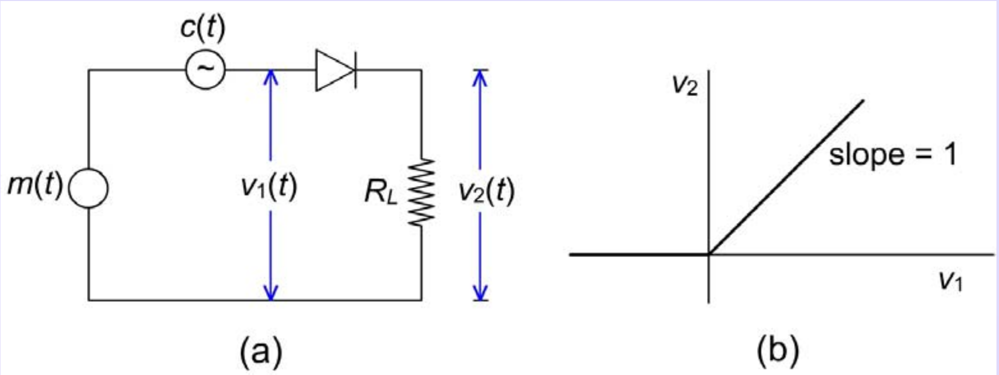



Fig. 4.35: (a) Switching modulator

(b) Switching characteristic of the diode-load combination.

The (ideal) transfer characteristics of the diode-load combination is shown at (b) in Fig. 4.35. This is explained as follows. We have,

$$v_1(t) = c(t) + m(t)$$

$$= A_c \cos(2\pi f_c t) + m(t)$$

If we assume that $|m(t)|_{max} << A_c$, then the behavior of the diode is governed by $c(t)$ and can be approximated as

$$v_2(t) \simeq \begin{cases} v_1(t), & c(t) > 0 \\ 0 \quad , & c(t) \leq 0 \end{cases}$$

(That is, the diode offers infinite impedance when reverse biased and has zero impedance, when forward biased. Hence, whether $v_1(t)$ is switched to the output or not depends on the carrier cycle)

We can express $v_2(t)$ as

$$v_2(t) = v_1(t)\, x_p(t) \tag{4.12}$$

where $x_p(t)$ is the periodic rectangular pulse train of example 1.1. That is,

$$x_p(t) = \begin{cases} 1\,, & \text{if } c(t) > 0 \ (\text{positive half cycle of } c(t)) \\ 0\,, & \text{if } c(t) < 0 \ (\text{negative half cycle of } c(t)) \end{cases}$$

But from example 1.1 ( with $f_0 = f_c$),

$$x_p(t) = \sum_{n = -\infty}^{\infty} x_n e^{j 2 \pi n f_c t}$$

where $x_n = \frac{1}{2} \operatorname{sinc}\left(\frac{n}{2}\right)$ (4.13a)

From Eq. 4.13(a), we find that $x_n = 0$, for $n = \pm 2, \pm 4$ etc. Combining the terms $x_{-n}$ and $x_n$, we obtain the trigonometric Fourier series, namely,

$$x_p(t) = \frac{1}{2} + \frac{2}{\pi} \cos(2 \pi f_c t) + \sum_{n = 2}^{\infty} \frac{(-1)^{n-1}}{2n - 1} \cos\left[2 \pi (2n - 1) f_c t\right] \qquad (4.13b)$$

From Eq. 4.12 and 4.13(b), we see that $v_2(t)$ is composed of two components, namely,

a) The desired quantity: $\frac{A_c}{2}\left[1 + \frac{4}{\pi A_c} m(t)\right] \cos(2 \pi f_c t)$

b) The undesired terms with

   i) Impulses in spectra at $f = 0, \pm 2 f_c, \pm 4 f_c$ etc.

   ii) Spectral lobes (same in shape as M(f)) of width 2W, centered at $0, \pm 3 f_c, \pm 5 f_c$ etc.

As compared to the square law modulator, switching modulator has the following advantages:

a) Generated AM signals can have larger power levels.

b) Filtering requirements are less stringent because we can separate the desired AM signal if $f_c > 2W$.

However, the disadvantage of the method is that percentage modulation has to be low in order that the switching characteristics of the diode are controlled only by the carrier.

### 4.4.2. Generation of DSB-SC

**a) Product modulator**

Generation of a DSB-SC signal involves the multiplication of $m(t)$ with $A_c \cos(\omega_c t)$. To generate this signal in the laboratory, any of the commercially available multipliers can be used. Some of them are:

| | |
|---|---|
| National: | LM 1496 |
| Motorola: | MC 1496 |
| Analog Devices: | AD 486, AD 632 etc |
| Signetics: | 5596 |

The power levels that can be generated, the carrier frequencies that can be used will depend on the IC used. The details can be obtained from the respective manuals. Generally, only low power levels are possible and that too over a limited carrier frequency range.

**b) Ring modulator**

Consider the scheme shown in Fig. 4.36. We assume that the carrier

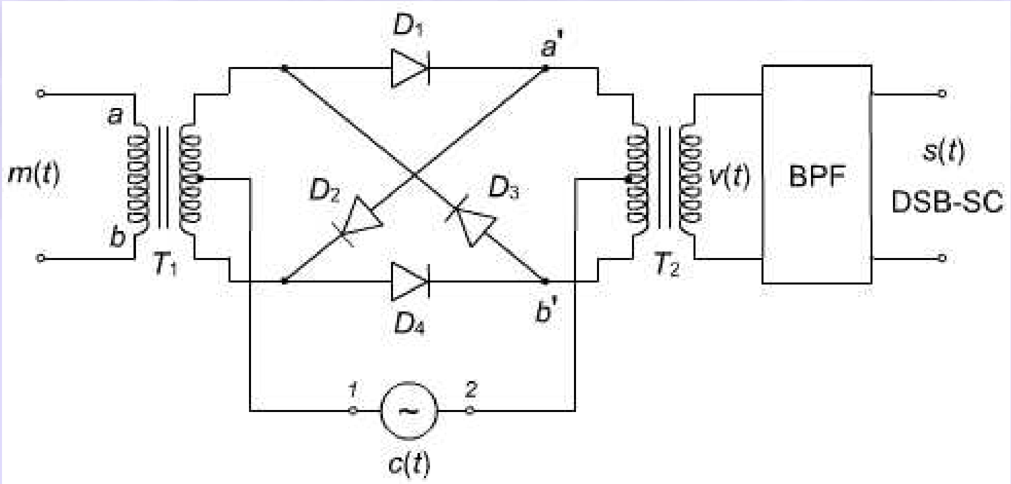



Fig. 4.36: Ring modulator

signal $c(t)$ is much larger than $m(t)$. Thus $c(t)$ controls the behavior of diodes which would be acting as ON-OFF devices. Consider the carrier cycle where the terminal 1 is positive and terminal 2 is negative. $T_1$ is an audio frequency transformer which is essentially an open circuit at the frequencies near about the carrier. With the polarities assumed for $c(t)$, $D_1$, $D_4$ are forward biased, where as $D_2$, $D_3$ are reverse biased. As a consequence, the voltage at point 'a' gets switched to $a'$ and voltage at point 'b' to $b'$. During the other half cycle of $c(t)$, $D_2$ and $D_3$ are forward biased where as $D_1$ and $D_4$ are reverse biased. As a result, the voltage at 'a' gets transferred to $b'$ and that at point 'b' to $a'$. This implies, during, say the positive half cycle of $c(t)$, $m(t)$ is switched to the output where as, during the negative half cycle, $-m(t)$ is switched. In other words, $v(t)$ can be taken as

$$v(t) = m(t)\, x_p(t) \qquad (4.14)$$

where $x_p(t)$ is square wave as shown in Fig. 4.37.

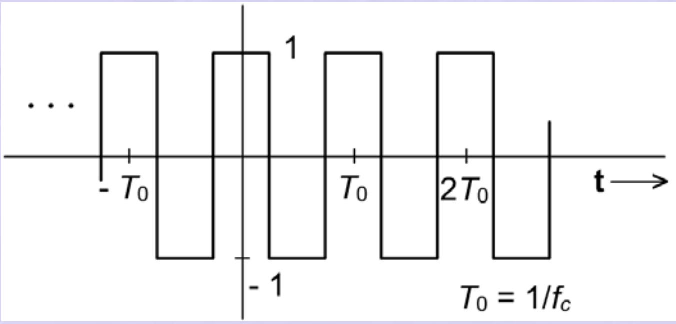



Fig. 4.37: $x_p(t)$ of Eq. 4.14

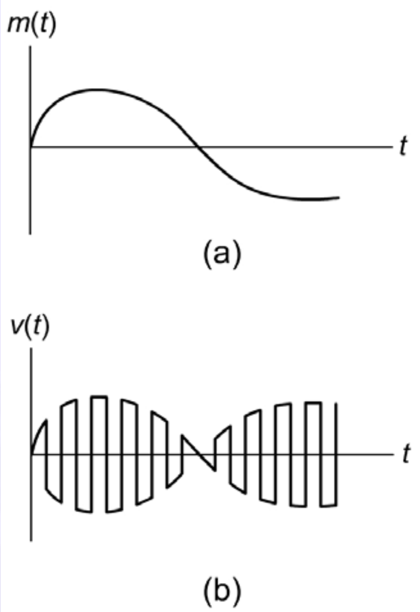



Fig. 4.38: (a) A message waveform $m(t)$

(b) $v(t)$ of the ring modulator

Fig. 4.38(b) illustrates the product quantity $m(t)\, x_p(t)$, for the $m(t)$ shown in Fig. 4.38(a). The Fourier series expansion of $x_p(t)$ can be written as

$$x_p(t) = \frac{4}{\pi} \sum_{n = 1, 3, 5, \dots} \frac{(-1)^{\frac{n-1}{2}}}{n} \cos(n \omega_c t) .$$

When $v(t)$ is passed through a BPF tuned to $f_c$, the output is the desired DSB-SC signal, namely, $s(t) = \frac{4}{\pi} m(t) \cos(\omega_c t)$.

**Example 4.6: Generation of DSB-SC**

Consider the scheme shown in Fig. 4.39. The non-linear device has the input-output characteristic given by

$$y(t) = a_0\, x(t) + a_1\, x^3(t)$$

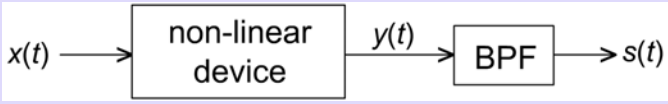



Fig. 4.39: The scheme for the example 4.6

Let $x(t) = A\cos(2\pi f_1 t) + m(t)$ where $m(t)$ is the message signal. If the required output $s(t)$ is a DSB-SC signal with a carrier frequency of 1 MHz, let us find the value of $f_1$, assuming that a suitable BPF is available.

$$y(t) = \underbrace{a_0\left[A\cos(2\pi f_1 t) + m(t)\right]}_{①} + \underbrace{a_1\left[A\cos(2\pi f_1 t) + m(t)\right]^3}_{②}$$

$$② = a_1\left[A^3\cos^3(2\pi f_1 t) + m^3(t) + 3A^2\cos^2(2\pi f_1 t)m(t) + 3A\cos(2\pi f_1 t)\, m^2(t)\right]$$

In the equation for the quantity ② above, the only term on the RHS that can give rise to the DSB-SC signal is $3a_1 A^2 m(t)\cos^2(2\pi f_1 t)$.

$$3a_1 A^2 m(t)\cos^2(2\pi f_1 t) = 3a_1 A^2 m(t)\frac{\left\{1 + \cos\left[2\pi(2f_1)t\right]\right\}}{2}$$

Assume that the BPF will pass only the components centered around $2f_1$. Then, choosing $f_1 = 500$ kHz, we will have

$$s(t) = A_c\, m(t)\cos(2\pi f_c t)$$

where $A_c = 3a_1 A^2$ and $f_c = 1$ MHz. ♦

**Exercise 4.4**

Consider the circuit configuration (called Cowan modulator) shown in Fig. 4.40. Show that the circuit can produce at its output the DSB-SC signal. $T_1$ is the audio frequency transformer where as $T_2$ and $T_3$ are designed to operate around the carrier frequency.

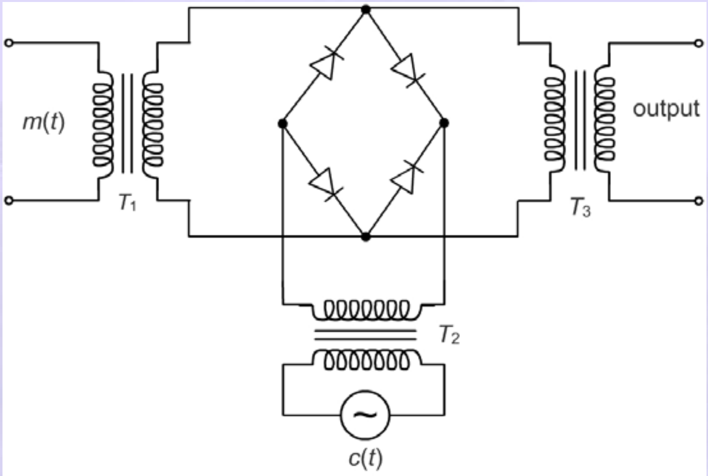



Fig. 4.40: Cowan modulator

**Exercise 4.5: Balanced Modulator (BM)**

Consider the scheme shown in Fig. 4.41. This configuration is usually called a *balanced modulator*. Show that the output $s(t)$ is a DSB-SC signal, thereby establishing that BM is essentially a multiplier.

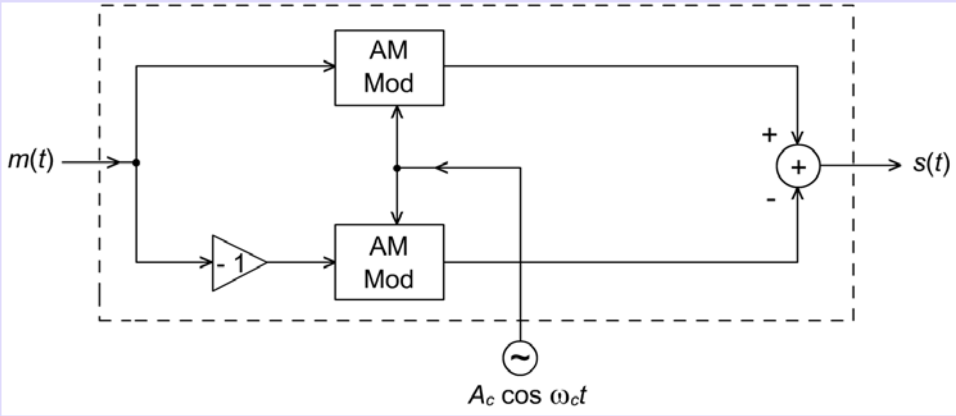



Fig. 4.41: Balanced modulator (BM)

## 4.5 Envelope Detector

As mentioned earlier, the AM signal, when not over modulated allows the recovery of $m(t)$ from its envelope. A good approximation to the ideal envelope detector can be realized with a fairly simple electronic circuit. This makes the receiver for AM somewhat simple, there by making AM suitable for broadcast applications. We shall briefly discuss the operation of the envelope detector, which is to be found in almost all the AM receivers.

Consider the circuit shown in Fig. 4.42.

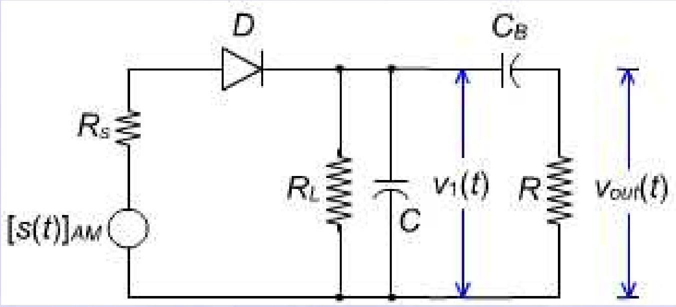



Fig. 4.42: The envelope detector circuit

We assume the diode $D$ to be ideal. When it is forward biased, it acts as a short circuit and thereby, making the capacitor $C$ charge through the source resistance $R_s$. When $D$ is reverse biased, it acts as an open circuit and $C$ discharges through the load resistance $R_L$.

As the operation of the detector circuit depends on the charge and discharge of the capacitor $C$, we shall explain this operation with the help of Fig. 4.43.

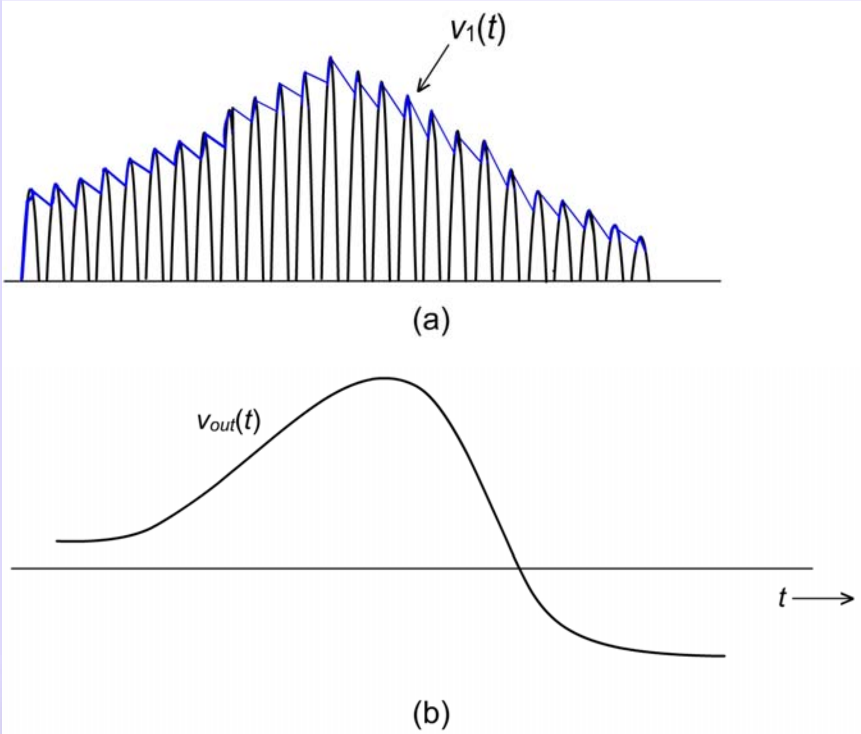



Fig. 4.43: Envelope detector waveforms

(a) $v_1(t)$ (before DC block)

(b) $v_{out}(t)$ (after DC block)

If the time constants $R_s C$ and $R_L C$ are properly chosen, $v_1(t)$ follows the envelope of $s(t)$ fairly closely. During the conduction cycle of $D$, $C$ quickly charges to the peak value of the carrier at that time instant. It will discharge a little during the next off cycle of the diode. The time constants of the circuit will control the ripple about the actual envelope. $C_B$ is a blocking capacitor and the final $v_{out}(t)$ will be proportional to $m(t)$, as shown in Fig. 4.43(b). (Note that a small high frequency ripple, at the carrier frequency could be present on $v_{out}(t)$. For audio transmission, this would not cause any problem, as $f_c$ is generally much higher than the upper limit of the audio frequency range).

How do we choose the time constants? $R_s$, though not under our control can be assumed to be fairly small. Values for $R_L$ and C can be assigned by us. During the charging cycle, we want the capacitor to charge to the peak value of the carrier in as short a time as possible. That is,

$$R_s C \ll \frac{1}{f_c} \tag{4.15a}$$

Discharge time constant should be large enough so that C does not discharge too much between the positive peaks of the carrier but small enough to be able follow the maximum rate of change of m(t). This maximum rate depends on W, the highest frequency in M(f). That is

$$\frac{1}{f_c} \ll R_L C \ll \frac{1}{W} \tag{4.15b}$$

Too small a value for $R_L C$ will make $V_1(t)$ somewhat ragged (sort of saw tooth ripple on the top) where as, with too large value for $R_L C$, ED fails to follow the envelope during the periods when $m(t)$ is decreasing. A more accurate analysis of the behavior of ED with $m(t)$ as a tone signal is given in appendix A4.1.

**Example 4.7**

Consider the scheme shown in Fig. 4.44. $x(t)$ is a tone signal given by $x(t) = \cos\left[\left(2\pi \times 10^4\right)t\right]$ and $c(t) = \cos(2\pi f_c t)$ with $f_c = 10$ MHz. $\hat{c}(t)$ is the HT of $c(t)$. $v(t)$, the output of the Balanced Modulator (BM), is applied as input to an ideal HPF, with cutoff at 10 MHz. We shall find the expression for $y(t)$, the output of an ideal envelope detector.

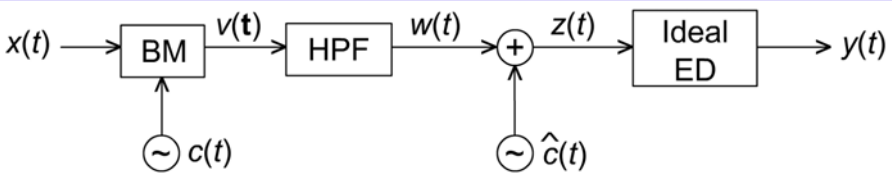



Fig. 4.44: The scheme for the example 4.7

It is not too difficult to see that

$$w(t) = \frac{1}{2}\cos\left[2\pi(10+\Delta f)10^6 t\right] \text{ where } \Delta f = 0.01 \text{ Hz. That is,}$$

$$w(t) = \frac{1}{2}\left[\cos\left[2\pi(\Delta f)t\right]\cos(\omega_c t) - \sin\left[2\pi(\Delta f)t\right]\sin(\omega_c t)\right]$$

As $z(t) = w(t) + \sin\left[(2\pi f_c)t\right]$, we have

$$z(t) = \frac{1}{2}\cos\left[2\pi(\Delta f)t\right]\cos(\omega_c t) - \left[\frac{1}{2}\sin\left[2\pi(\Delta f)t\right] - 1\right]\sin(\omega_c t)$$

$z(t)$ represents a narrowband signal with the in phase component $\frac{1}{2}\cos\left[2\pi(\Delta f)t\right]$ and the quadrature component $\left\{\frac{1}{2}\sin\left[2\pi(\Delta f)t\right] - 1\right\}$. Hence,

$$y(t) = \left\{\frac{1}{4}\cos^2\left[2\pi(\Delta f)t\right] + \left[\frac{1}{2}\sin\left[2\pi(\Delta f)t\right] - 1\right]^2\right\}^{\frac{1}{2}}$$

$$= \left\{\frac{5}{4} - \sin\left[\left(2\pi\times 10^4\right)t\right]\right\}^{\frac{1}{2}} \quad \blacklozenge$$

**Example 4.8**

Consider the scheme shown in Fig.4.45.


Fig. 4.45: The scheme for the example 4.8

Let us find the output $y(t)$ when,

a) $x(t) = \left[1 + g_m\, m(t)\right] \cos(\omega_c t)$. $\left|g_m\, m(t)\right| < 1$ and $m(t)$ is band-limited to $W$ Hz and the LPF has a bandwidth of $2W$. Assume that $f_c >> 2W$.

b) $x(t)$ is a DSB-SC signal; that is $x(t) = m(t) \cos(\omega_c t)$.

a) $v(t) = x^2(t) = \left[1 + g_m\, m(t)\right]^2 \cos^2(\omega_c t)$

$$= \left[1 + g_m\, m(t)\right]^2 \frac{\left[1 + \cos(2\omega_c t)\right]}{2}$$

$$= \frac{\left[1 + g_m\, m(t)\right]^2}{2} + \frac{\left[1 + g_m\, m(t)\right]^2}{2} \cos(2\omega_c t).$$

The second term on the RHS will be eliminated by the LPF. Hence,

$w(t) = \dfrac{\left[1 + g_m\, m(t)\right]^2}{2}$. As $\left[1 + g_m\, m(t)\right] \geq 0$, we have

$$y(t) = \frac{\left[1 + g_m\, m(t)\right]}{2}.$$

b) When $x(t) = m(t) \cos(\omega_c t)$, we have

$$v(t) = m^2(t) \cos^2(\omega_c t) = m^2(t) \frac{\left[1 + \cos(2\omega_c t)\right]}{2}$$

The output of the LPF would be

$$w(t) = \frac{m^2(t)}{2}$$

As the squaring operation removes the information about the sign of the signal, the output of $y(t)$ is

$$y(t) = \frac{\left|m(t)\right|}{\sqrt{2}}$$

♦

**Exercise 4.6**

Consider the waveform $m(t)$ shown in Fig. 4.46. A DSB-SC is generated using $m(t)$ and a suitable high frequency carrier. Sketch the output of an *ideal envelope detector* when the input to the detector is the DSB-SC signal.

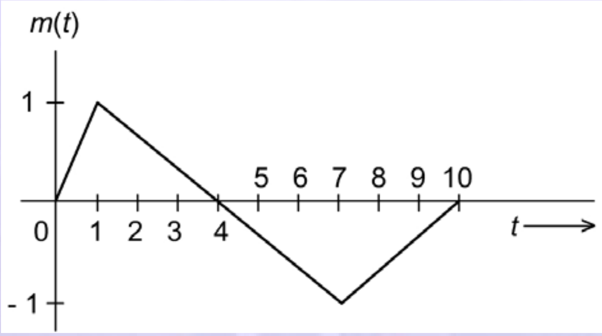



Fig. 4.46: $m(t)$ for the exercise 4.6

## 4.6 Theory of Single-Sideband

Assume that from a DSB-SC signal, we have completely suppressed one of the sidebands, say, the LSB. Let $\left[S(f)\right]_{DSB} = \frac{A_c}{2}\left[M(f - f_c) + M(f + f_c)\right]$ where $M(f)$ is as shown in Fig. 4.30(a). The resulting spectrum $\left[S(f)\right]_{USB}$ will be as shown in Fig. 4.47. Can we get back $m(t)$ from the above signal? The answer is **YES**. Let

$$v(t) = \left[s(t)\right]_{USB} \times \cos(2\pi f_c t)$$

If $S^{'}(f) = \left[S(f)\right]_{USB}$, then $V(f) = \frac{1}{2}\left[S^{'}(f - f_c) + S^{'}(f + f_c)\right]$

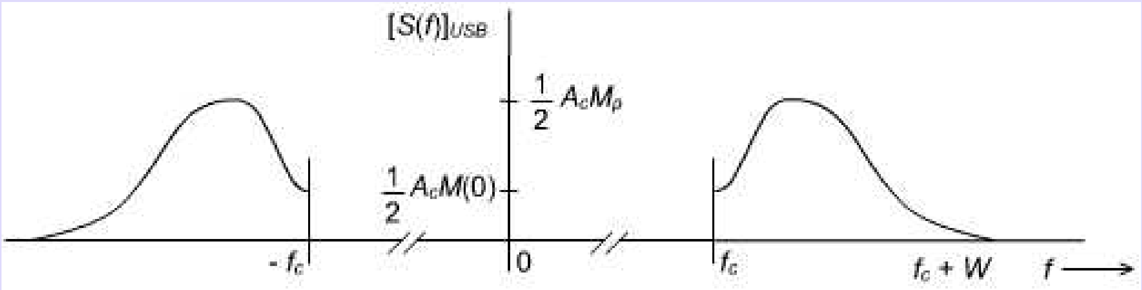



Fig 4.47: Spectrum of the upper sideband signal

By plotting the spectrum of $v(t)$ and extracting the spectrum for $|f| \leq W$, we see that it is $\frac{1}{2} A_c M(f)$. A similar analysis will show that it is possible to extract $m(t)$ from $[S(f)]_{LSB}$. In other words, with coherent demodulation, it is possible for us to recover the message signal either from USB or LSB and the transmission of both the sidebands is not a must. Hence it is possible for us to conserve transmission bandwidth, provided we are willing to go for the appropriate demodulation.

Let us now derive the time domain equation for an SSB signal. Let us start with the two-sided spectrum and then eliminate the unwanted sideband. We shall retain the upper sideband and try to eliminate the lower sideband. Consider

$$\frac{A_c}{2}\left[M(f - f_c) + \text{sgn}(f - f_c) M(f - f_c)\right]$$

But $$\text{sgn}(f - f_c) M(f - f_c) = \begin{cases} M(f - f_c), & f > f_c \\ -M(f - f_c), & f < f_c \end{cases}$$

Hence $$\frac{A_c}{2} M(f - f_c)\left[1 + \text{sgn}(f - f_c)\right] = \begin{cases} A_c M(f - f_c), & f > f_c \\ 0, & f < f_c \end{cases}$$

That is, the lower sideband has been eliminated from the positive part of the spectrum.

$$M(f - f_c) \longleftrightarrow m(t)\, e^{j 2 \pi f_c t}$$

$$\text{sgn}(f - f_c)\, M(f - f_c) \longleftrightarrow -\frac{1}{j}\, \widehat{m}(t)\, e^{j 2\pi f_c t},$$

where $\widehat{m}(t)$ is the HT of $m(t)$.

That is, $$\frac{A_c}{2} M(f - f_c)\big[1 + \text{sgn}(f - f_c)\big] \longleftrightarrow \frac{A_c}{2}\left[m(t) - \frac{1}{j}\widehat{m}(t)\right] e^{j 2\pi f_c t} \qquad (4.16a)$$

Similarly, $$\frac{1}{2} M(f + f_c)\big[1 - \text{sgn}(f + f_c)\big] = \begin{cases} M(f + f_c), & f < -f_c \\ 0 & , \ f > -f_c \end{cases}$$

$$\frac{A_c}{2} M(f + f_c)\big[1 - \text{sgn}(f + f_c)\big] \longleftrightarrow \frac{A_c}{2} m(t) e^{-j 2\pi f_c t} + \frac{A_c}{2j}\widehat{m}(t) e^{-j 2\pi f_c t} \qquad (4.16b)$$

Combining Eq. 4.16(a) and Eq. 4.16(b), we have the time domain equation for the upper single sideband signal, namely,

$$\big[s(t)\big]_{USB} = A_c\big[m(t)\cos(\omega_c t) - \widehat{m}(t)\sin(\omega_c t)\big] \qquad (4.17)$$

Assume that the USSB signal is obtained from the DSB-SC signal, $A_c\, m(t)\cos(\omega_c t)$, by filtering out the LSB part. Then,

$$\big[s(t)\big]_{USB} = \frac{A_c}{2}\big[m(t)\cos(\omega_c t) - \widehat{m}(t)\sin(\omega_c t)\big] \qquad (4.18)$$

A few authors take Eq. 4.18 as representative of the SSB signal. Eq. 4.18 has the feature that the average power of the SSB signal is one-half the average power of corresponding DSB signal. We shall make use of both Eq. 4.17 and Eq. 4.18 in our further studies.

By a procedure similar to that outlined above, we can derive a time domain expression for the LSB signal. The result would be

$$\big[s(t)\big]_{LSB} = A_c\big[m(t)\cos(\omega_c t) + \widehat{m}(t)\sin(\omega_c t)\big] \quad or \qquad (4.19a)$$

$$\big[s(t)\big]_{LSB} = \frac{A_c}{2}\big[m(t)\cos(\omega_c t) + \widehat{m}(t)\sin(\omega_c t)\big] \qquad (4.19b)$$

An SSB signal, whether upper or lower, is also a narrowband bandpass signal. Eq. 4.18 can be treated as the *canonical representation* of USB signal with $m(t)$ as the in-phase component and $\widehat{m}(t)$ as the quadrature component. Similarly

Eq. 4.19 provides the canonical representation of the LSB signal where $m(t)$ is the in-phase component and $-\widehat{m}(t)$ is the quadrature component.

We have already seen that a narrowband signal can also be expressed in the envelope and phase form. Taking the USB signal, we have for the complex envelope the quantity $\frac{A_c}{2}\left(m(t) + j\,\widehat{m}(t)\right)$. Hence the envelope $A(t)$ of the USB signal is

$$\left[A(t)\right]_{USB} = \frac{A_c}{2}\sqrt{m^2(t) + \widehat{m}^2(t)} \tag{4.20a}$$

Similarly for the phase $\varphi(t)$, we have

$$\varphi(t) = \arctan\left[\frac{\widehat{m}(t)}{m(t)}\right] \tag{4.20b}$$

Expressing the USB signal with the envelope-phase form, we have

$$\left[s(t)\right]_{USB} = A(t)\left[\cos(\omega_c t) + \varphi(t)\right] \tag{4.21}$$

where $A(t)$ and $\varphi(t)$ are given by Eqs. 4.20(a) and 4.20(b) respectively. The expression for $\left[s(t)\right]_{LSB}$ is identical to Eq. 4.21 but with $\varphi(t)$ given by

$$\varphi(t) = arc\tan\left[-\frac{\widehat{m}(t)}{m(t)}\right] \tag{4.22}$$

That is, $\left[s(t)\right]_{SSB} = A(t)\cos\left(\omega_c t + \varphi(t)\right)$ (4.23)

where $\varphi(t)$ is given either by Eq. 4.20(b) or Eq. 4.22. Eq. 4.23 indicates that an SSB signal has both **amplitude** and **phase** variations. (AM and DSB-SC signals have only the amplitude of the carrier being changed by the message signal. Note that AM or DSB-SC signals do not have quadrature components.) As such, SSB signals belong to the category of *hybrid amplitude and phase modulation.*

**Example 4.9: SSB with tone modulation**

As a simple application of the Eqs. 4.18 and 4.19, let $m(t)$ be $\cos(\omega_m t)$. Let us find the SSB signals.

$m(t) = \cos(\omega_m t) \Rightarrow \widehat{m}(t) = \sin(\omega_m t)$. Therefore,

$$[s(t)]_{USB} = \frac{A_c}{2}\left[\cos(\omega_m t)\cos(\omega_c t) - \sin(\omega_m t)\sin(\omega_c t)\right]$$

$$= \frac{A_c}{2}\left[\cos(\omega_c + \omega_m)t\right]$$

$$[s(t)]_{LSB} = \frac{A_c}{2}\left[\cos(\omega_m t)\cos(\omega_c t) + \sin(\omega_m t)\sin(\omega_c t)\right]$$

$$= \frac{A_c}{2}\left[\cos(\omega_c - \omega_m)t\right]$$

Alternatively,

$$[s(t)]_{DSB-SC} = A_c \cos(\omega_m t)\cos(\omega_c t)$$

$$= \frac{A_c}{2}\left[\cos(\omega_c + \omega_m)t + \cos(\omega_c - \omega_m)t\right]$$

Extracting the USB, we have

$$[s(t)]_{USB} = \frac{A_c}{2}\cos\left[(\omega_c + \omega_m)t\right]$$

If we eliminate the USB, then

$$[s(t)]_{LSB} = \frac{A_c}{2}\cos\left[(\omega_c - \omega_m)t\right] \quad \blacklozenge$$

**Example 4.10**

Let $m(t) = x(t)\,y(t)$ where $X(f)$ and $Y(f)$ are as shown in Fig. 4.48. An LSB signal is generated using $m(t)$ as the message signal. Let us develop the expression for the SSB signal.

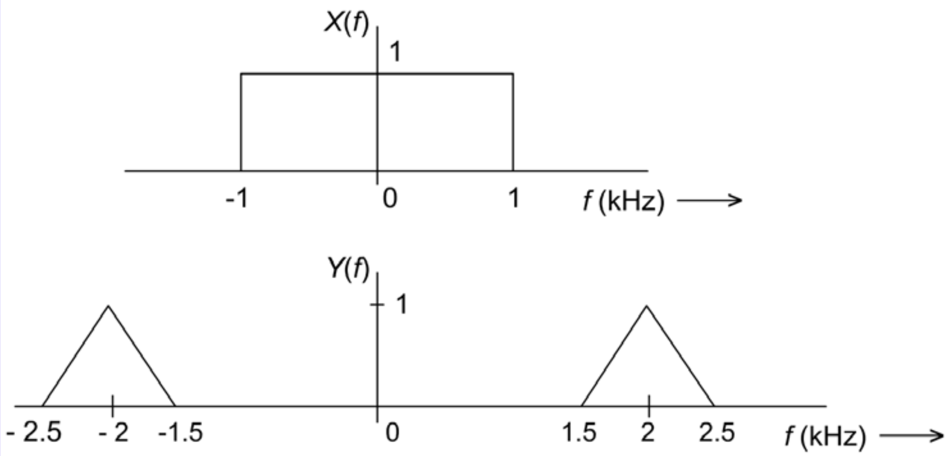



Fig. 4.48: $X(f)$ and $Y(f)$ of example 4.10

Let us take the SSB signal as

$$[s(t)]_{LSB} = m(t)\cos(\omega_c t) + \widehat{m}(t)\sin(\omega_c t) \quad (4.24)$$

We have

$$x(t) = 2\times 10^3 \operatorname{sinc}\left[\left(2\times 10^3\right)t\right]$$

$$y(t) = 10^3 \operatorname{sinc}^2(500t)\cos\left[\left(4\pi\times 10^3\right)t\right]$$

$$m(t) = x(t)\,y(t) \quad (4.25a)$$

What is required is $\widehat{m}(t)$, the HT of $m(t)$. $m(t)$ is the product of a lowpass and a bandpass signal. Hence $\widehat{m}(t) = x(t)\,\hat{y}(t)$. (See the note, after example 1.25)

But $\hat{y}(t)$, from the result of example 1.25, is

$$\hat{y}(t) = 10^3 \operatorname{sinc}^2(500t)\sin\left[\left(4\pi\times 10^3\right)t\right]$$

That is,

$$\widehat{m}(t) = 2\times 10^6 \operatorname{sinc}\left[\left(2\times 10^3\right)t\right]\operatorname{sinc}^2(500t)\sin\left[\left(4\pi\times 10^3\right)t\right] \quad (4.25b)$$

$[s(t)]_{LSB}$ is obtained by using Eq. 4.25 in Eq. 4.24. ♦

**Exercise 4.7**

Let $M(f)$ be as shown in Fig. 4.49. An upper sideband signal is generated using the signal with this $M(f)$. Compute and sketch the spectrum of the quadrature component of the SSB signal.

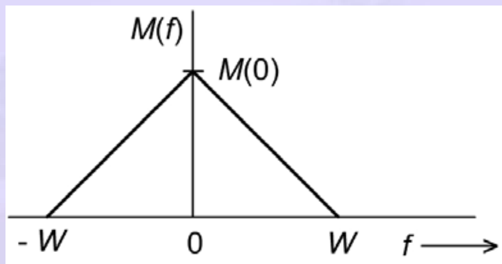



Fig. 4.49: Baseband spectrum for the Exercise 4.7

**Exercise 4.8**

Let $m(t) = \text{sinc}(t)$. Develop the expression for the following:

a) USB signal, in the canonical form.

b) USB signal, in the envelope and phase form

Ans. (b): $\left|\text{sinc}\left(\frac{t}{2}\right)\right| \cos\left[2\pi\left(f_c + \frac{1}{4}\right)t\right]$

**Exercise 4.9**

Let $s(t) = m(t)\cos(\omega_c t) - \widehat{m}(t)\sin(\omega_c t)$

where $M(f)$ is as shown in Fig. 4.49. Let $W = 5$ kHz and $M(0) = 1$.

a) Sketch the spectrum of (i) $s(t)\cos(\omega_c t)$ and (ii) $\hat{s}(t)\sin(\omega_c t)$

b) Show that sum of the spectra of part (a) is proportional to $M(f)$

c) Sketch the spectrum of $\hat{s}(t)\cos(\omega_c t) - s(t)\sin(\omega_c t)$. Is this related to $M(f)$? Explain.

## 4.7 Generation of SSB Signals

We shall consider two broad categories of SSB generation, namely, (i) frequency discrimination method and (ii) phase discrimination method. The former is based on the frequency domain description of SSB, whereas the latter in based on the time-domain description of an SSB signal.

### 4.7.1 Frequency discrimination method

Conceptually, it is a very simple scheme. First generate a DSB signal and then filter out the unwanted sideband. This method is depicted in Fig. 4.50.

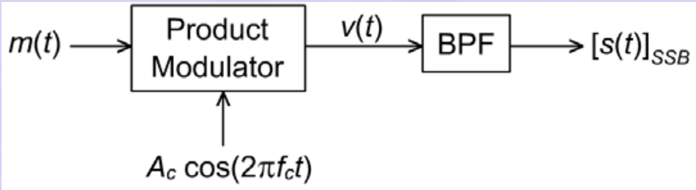



Fig. 4.50: Frequency discrimination method of SSB generation

$v(t)$ is the DSB-SC signal generated by the product modulator. The BPF is designed to suppress the unwanted sideband in $V(f)$, thereby producing the desired SSB signal.

As we have already looked at the generation of DSB-SC signals, let us now look at the filtering problems involved in SSB generation. BPFs with abrupt pass and stopbands cannot be built. Hence, a practical BPF will have the magnitude characteristic $|H(f)|$, as shown in Fig. 4.51. As can be seen from the figure, ($|H(f)|$ is shown only for positive frequencies) a practical filter, besides the PassBand (PB) and StopBand (SB), also has a TransitionBand (TB), during which the filter transits from passband to stopband. (The edges of the PB and SB depend on the attenuation levels used to define these bands. It is a common practice to define the passband as the frequency interval between the 3-dB points. Attenuation requirements for the SB depend on the application. Minimum attenuation for the SB might be in the range 30 to 50 dB.)

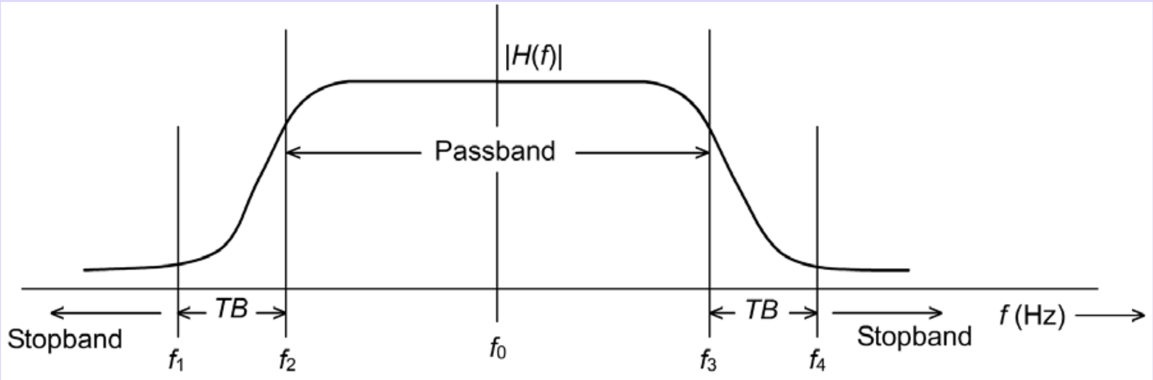



Fig. 4.51: Magnitude characteristic of a practical BPF. Centre frequency $= f_0$

Because of TB (where the attenuation is not to the desired level), a part of the undesired sideband may get through the filter. As a rule of the thumb, it is possible to design a filter if the permitted transitionband is not less than 1% of center frequency of a bandpass filter. Fortunately, quite a few signals have a spectral null around DC and if it is possible for us to fit in the transitionband into this gap, then the desired SSB signal could be generated. In order to accomplish this, it might become necessary to perform the modulation in more than one stage. We shall illustrate this with the help of an example.

**Example 4.11**

Telephone quality speech signal has a spectrum in the range 0.3 to 3.4 kHz. We will suggest a scheme to generate upper sideband signal with a carrier frequency of 5 MHz. Assume that bandpass filters are available, providing an attenuation of more than 40 dB in a TB of width $0.01\ f_0$, where $f_0$ is the centre frequency of the BPF.

Let us look at the generation of the SSB signal in one stage using a carrier of 5 MHz. When a DSB signal is generated, it will have a spectral null of 600 Hz centered at 5 MHz. That is, the transitionband is about 0.01 percent of the carrier and hence it would not be possible to design such a sideband filter. However, it would be possible to generate the desired SSB signal using two stages of modulation.

Consider the scheme shown in Fig. 4.52.

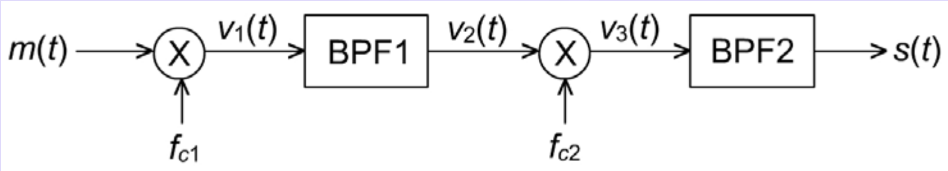



Fig. 4.52: Two stage generation of SSB signal

$v_1(t)$ is a DSB-SC signal with the USB occupying the (positive frequency) range $(f_{c1} + 300)$ Hz to $(f_{c1} + 3400)$ Hz. The frequency range of the LSB is $(f_{c1} - 3400)$ Hz to $(f_{c1} - 300)$ Hz. Let us extract the upper sideband from $v_1(t)$ with the help of BPF1. Then the centre frequency of BPF1, $(f_0)_1$, is

$$(f_0)_1 = \frac{(f_{c1} + 300) + (f_{c1} + 3400)}{2}$$

$$= (f_{c1} + 1850)$$

width of the spectral null around $f_{c1} = 600$ Hz.

Hence $\dfrac{f_{c1} + 1850}{100} \leq 600$

or $\quad f_{c1} \leq 60{,}000 - 1850$

$\qquad \leq 58.1$ kHz

Let us take $f_{c1}$ as 50 kHz. Let $M(f)$ be as shown in Fig. 4.53(a). Then $V_2(f)$ will be as shown in Fig. 4.53(b).

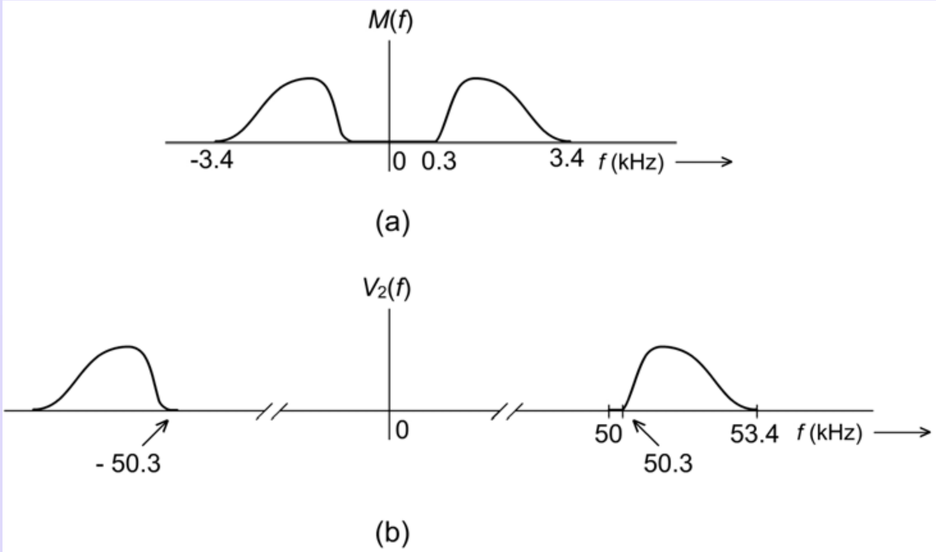



Fig. 4.53: (a) Message spectrum

(b) The spectrum of the USB signal with $f_{c1} = 50$ kHz

$v_3(t)$ is a DSB-SC signal with the carrier $f_{c2}$, $v_2(t)$ being the modulating signal. Then $V_3(f)$ will be as shown in Fig. 4.54. In this figure,

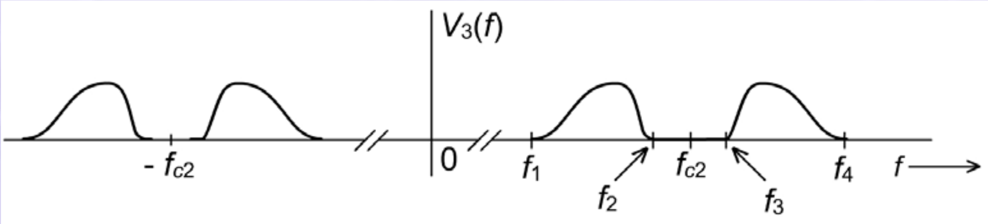



Fig. 4.54: Spectrum of $v_3(t)$ of Fig. 4.52

$$f_1 = (f_{c2} - 53{,}400) \text{ Hz}$$

$$f_2 = (f_{c2} - 50{,}300) \text{ Hz}$$

$$f_3 = (f_{c2} + 50{,}300) \text{ Hz}$$

$$f_4 = (f_{c2} + 53{,}400) \text{ Hz}$$

Hence the transition band available to design the sideband filter is $(f_3 - f_2) = 100.6$ kHz. With this TB, we can choose centre frequency of BPF2 less than or equal to 10.06 MHz. If we choose $f_{c2}$ as 4.95 MHz then we will have the upper sideband occupying frequency range (4.95 + 0.0503) = 5.0003 MHz to (4.95 + 0.0534) = 5.0034 MHz. This is exactly what would have happened if the modulation scheme was attempted in one step with 5 MHz as the carrier.

Note: As the spectral occupancy of the USB signal is from 5.0003 MHz to 5.0034 MHz, theoretical centre frequency of the BPF2 is 5.00185. With respect to this frequency, we have

$$\frac{\text{TB width}}{\text{centre freq.}} = \frac{100.6}{5001.85} \times 100 = 2.01 \text{ percent}$$

which is about twice the permitted ratio. Hence, it is possible to choose for $f_{c1}$ a value lower than 50 kHz. ♦

### 4.7.2 Phase discrimination method

This method implements Eq. 4.17 or Eq. 4.19(a), to generate the SSB signal. Consider the scheme shown in Fig. 4.55. This scheme requires two product modulators, two $\frac{\pi}{2}$ phase shifters and an adder. One of the phase shifter is actually a Hilbert transformer (HT); it should provide a $\frac{\pi}{2}$ phase shift for all the components in $M(f)$. This is not such an easy circuit to realize. Assuming it is possible to build the HT, the SSB can be generated for any $f_c$, provided the product modulators (multipliers) can work at these frequencies.

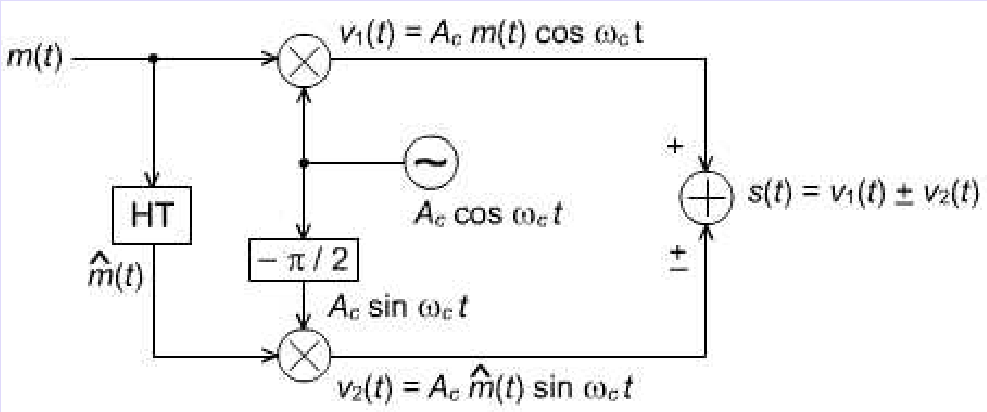



Fig. 4.55: SSB generation: Phase discrimination method

Instead of a single wide band phase shifter acting as the HT, it is possible to have an SSB generator with two Phase Shifting Networks, (PSN), one in each branch as shown in Fig. 4.56.

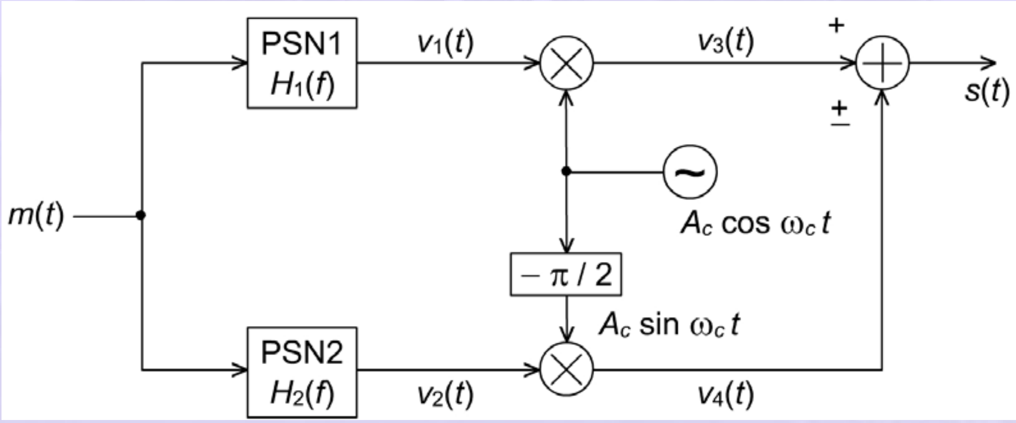



Fig. 4.56: An alternate configuration for the phase discrimination scheme

$H_1(f)$ and $H_2(f)$ are the phase shifting networks. Let $H_1(f) = e^{j\,\theta_1(f)}$ and $H_2(f) = e^{j\,\theta_2(f)}$. $\theta_1(f)$ and $\theta_2(f)$ are such that $\left[\theta_1(f) - \theta_2(f)\right] = \frac{\pi}{2}$ for the frequency range of interest. That is, PSN1 and PSN2 maintain a constant difference of $\frac{\pi}{2}$.

Let us explain the operation of the scheme shown in Fig. 4.56 by taking $m(t)$ to be a tone signal; that is, $m(t) = A_m \cos(\omega_m t)$. Let

$$\theta_1(f)\Big|_{f = f_m} = \theta_1 \text{ and } \theta_2(f)\Big|_{f = f_m} = \theta_2$$

where $\theta_2 = \theta_1 + \frac{\pi}{2}$. Then,

$$v_1(t) = A_m \cos(\omega_m t + \theta_1) \text{ and } \quad v_2(t) = A_m \cos(\omega_m t + \theta_2).$$

$$v_3(t) = A_m A_c \cos(\omega_m t + \theta_1) \cos(\omega_c t)$$

$$v_4(t) = A_m A_c \cos(\omega_m t + \theta_2) \sin(\omega_c t)$$

$$= A_m A_c \cos\left(\omega_m t + \theta_1 + \frac{\pi}{2}\right) \sin(\omega_c t)$$

$$= - A_m A_c \sin(\omega_m t + \theta_1) \sin(\omega_c t)$$

$$v_3(t) + v_4(t) = A_m A_c \cos\left[(\omega_c + \omega_m)t + \theta_1\right]$$

After coherent demodulation, we will have $\cos(\omega_m t + \theta_1)$. We shall assume that the additional phase shift $\theta_1$ which is actually frequency dependent will not cause any problem after demodulation.

As it is not too difficult to design a Hilbert transformer using digital filter design techniques, phase shift method of SSB generation is better suited for digital implementation. For a brief discussion on SSB generation using digital signal processing, the reader is referred to [1].

**Exercise 4.10**

There is a *third method* of generating the SSB signal, known as **Weaver's method**. This scheme is shown in Fig. 4.57.

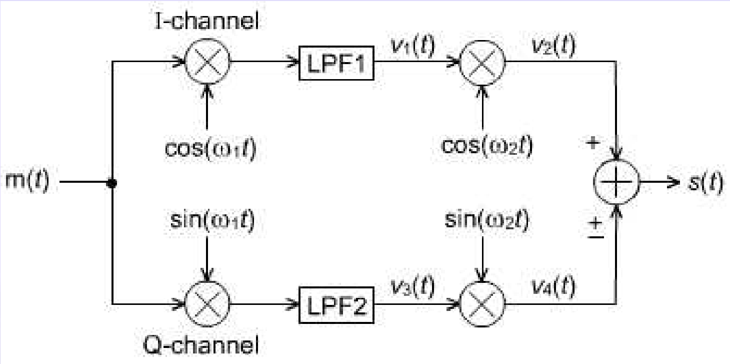



Fig. 4.57: Weaver's method of SSB generation

Let $M(f)$ be non-zero only in the interval $f_l \le |f| \le f_u$; and let $f_1 = \dfrac{f_l + f_u}{2}$. The lowpass filters in the I and Q channels are identical and have a cutoff frequency of $f_{c0} = \dfrac{f_u - f_l}{2}$. Assume that $f_2 >> f_{c0}$. By sketching the spectra at various points in the above scheme, show that $s(t)$ is an SSB signal. What is the actual carrier frequency with respect to which, $s(t)$ would be an SSB signal?

## 4.8 Demodulation of SSB

SSB signals can be demodulated using coherent demodulation as shown in Fig. 4.58.

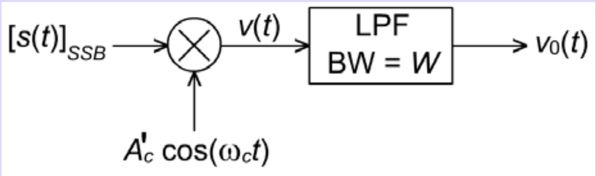



Fig. 4.58: Coherent demodulation of SSB

The received SSB signal is multiplied by the local carrier which is of the same frequency and phase as the carrier used at the transmitter. From Fig, 4.58, we have

$$v(t) = [s(t)]_{SSB} \times A_c' \cos(\omega_c t)$$

$$= \frac{A_c}{2}\left[m(t)\cos(\omega_c t) \pm \widehat{m}(t)\sin(\omega_c t)\right] \times A_c' \cos(\omega_c t)$$

$$= \frac{A_c A_c'}{2}\left[m(t)\cos^2(\omega_c t) \pm \widehat{m}(t)\cos(\omega_c t)\sin(\omega_c t)\right] \quad (4.26)$$

The second term on the RHS of Eq. 4.26 has the spectrum centered at $\pm 2f_c$ which will be eliminated by the LPF following $v(t)$. The first term of Eq. 4.26 can be written as, $\frac{A_c A_c'}{2}\left[\frac{1+\cos 2(\omega_c t)}{2}\right] m(t)$.

As $m(t)\cos(2\omega_c t)$ has the spectrum centered at $\mp 2f_c$, even this will be eliminated by the LPF. Hence $v_0(t) = \frac{A_c A_c'}{4} m(t)$. That is,

$$v_0(t) \; \alpha \; m(t)$$

The difficulty in demodulation is to have a coherent carrier at the receiver. Can we use a squaring loop or Costas loop to recover the carrier from $[s(t)]_{SSB}$?

The answer is **NO**. Let us look at the squaring loop. From Eq. 4.23,

$$[s(t)]_{SSB} = s(t) = A(t)\cos[\omega_c t + \varphi(t)]$$

After squaring, we obtain,

$$s^2(t) = A^2(t)\cos^2\left[\omega_c t + \varphi(t)\right]$$

$$= A^2(t)\left\{1 + \cos^2\left[2\left(\omega_c t + \varphi(t)\right)\right]\right\}$$

As $\varphi(t)$ is a function of time, we do not have a discrete component at $f = 2 f_c$ and hence, carrier acquisition is not possible. It is left as an exercise to show that Costas loop will not be able to put out $m(t)$ when the input to the loop is the SSB signal. Hence, when SSB is used in a communication system, highly stable crystal oscillators are used both at the transmitter and receiver. If this scheme does not work (especially, at very high frequencies) a small pilot carrier can be sent along with the SSB signal. This pilot carrier is recovered at the receiver and is used for demodulation.

Let us now look at the effects of frequency and phase offset in the carrier used for demodulation. Let the carrier term at the receiver be $A_c^{'}\cos\left[2\pi(f_c + \Delta f)t\right]$. Let the received input to the demodulator (Fig. 4.58) be

$$\frac{1}{2}A_c\left[m(t)\cos(\omega_c t) - \widehat{m}(t)\sin(\omega_c t)\right]$$

Then, $v(t) = \frac{1}{2}A_c A_c^{'}\left[m(t)\cos(\omega_c t) - \widehat{m}(t)\sin(\omega_c t)\right]\cos(\omega_c + \Delta\omega)t$

$v_0(t)$, the output of the LPF would be

$$v_0(t) \;\alpha\; \frac{1}{4}A_c A_c^{'}\left[m(t)\cos(2\pi\Delta f t) + \widehat{m}(t)\sin(2\pi\Delta f t)\right] \quad (4.27)$$

Assume that

$$v_0(t) = m(t)\cos(2\pi\Delta f t) + \widehat{m}(t)\sin(2\pi\Delta f t) \quad (4.28)$$

Consider a special case, namely, a frequency component at $f = 1$ kHz in $M(f)$ and $\Delta f = 100$ Hz. With these values, Eq. 4.28 becomes

$$v_0(t) = \cos\left(2\pi\times 10^3 t\right)\cos(2\pi\times 100t) + \sin\left(2\pi\times 10^3 t\right)\sin(2\pi\times 100t)$$

$$= \cos(2\pi\times 900t) = \frac{e^{j2\pi\times 900t} + e^{-j2\pi\times 900t}}{2} \quad (4.29)$$

As this is true of every frequency component in $M(f)$, we have the result that when $\Delta f$ is positive and the input is a USB signal, there is an inward shift of $M(f)$ by $\Delta f$. (We see from Eq. 4.28, $e^{j 2\pi \times 1000 t}$ is converted to $e^{j 2\pi \times 900 t}$ and $e^{- j 2\pi \times 1000 t}$ is converted to $e^{- j 2\pi \times 900 t}$. That is, both the spectral components have been shifted inward by 100 Hz.) By a similar analysis, we can easily see that if $\Delta f$ is negative and the input is a USB signal, then, after demodulation, the spectral components in $M(f)$ would undergo an outward shift by $\Delta f$. In all, we have four cases to be taken into account and the effects of non-zero $\Delta f$ on the resulting output after demodulation are summarized below.

Case i) $\Delta f > 0$ and the input signal is USB: Spectral components in $M(f)$ will undergo an inward shift by $\Delta f$

Case ii) $\Delta f > 0$ and the input signal is LSB: Spectral components in $M(f)$ will undergo an outward shift by $\Delta f$

Case iii) $\Delta f < 0$ and the input signal is USB: Spectral components in $M(f)$ will undergo an outward shift by $\Delta f$

Case iv) $\Delta f < 0$ and the input signal is LSB: Spectral components in $M(f)$ will undergo an inward shift by $\Delta f$

Let $M(f)$ be as shown in Fig. 4.59(a). Let $\Delta f = 300$ Hz. Then, if the input is a USB signal, the spectrum of the demodulated output, $V_0(f)$, would be as shown in Fig. 4.59(b).

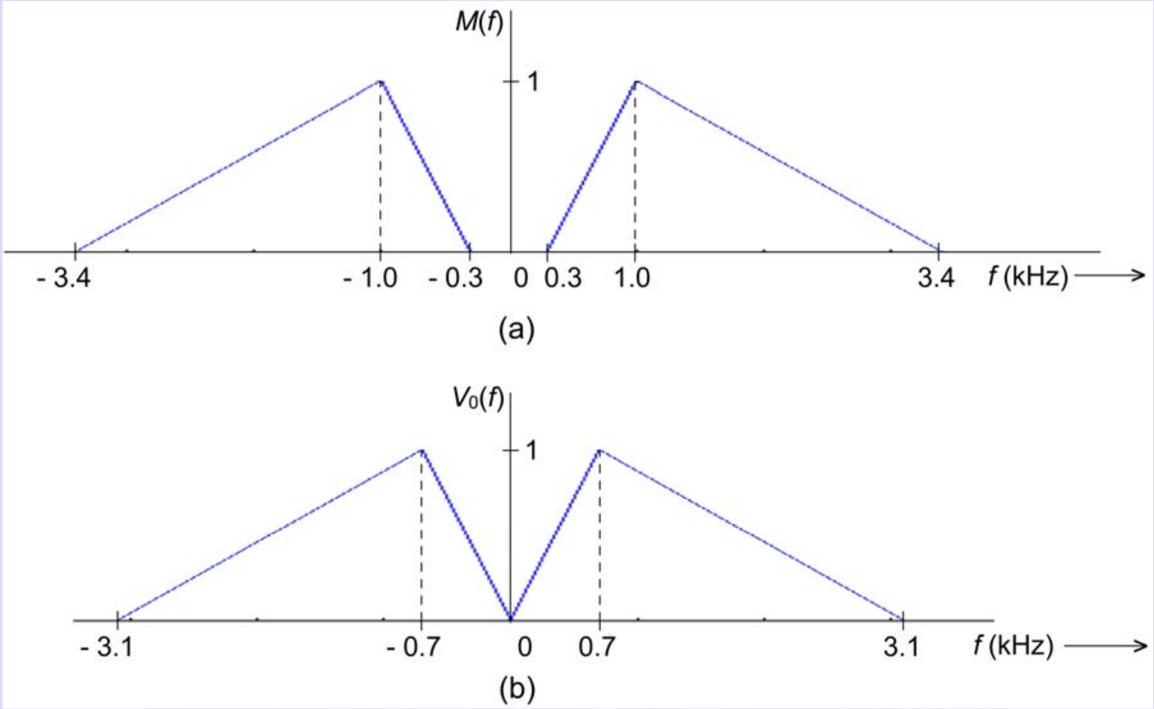



Fig. 4.59: (a) Baseband message spectrum

(b) Inward spectral shift (after demodulation) of a USB signal. Frequency offset, $\Delta f = 300$ Hz

**Exercise 4.11: Effect of phase error on SSB**

Consider the scheme shown in Fig. 4.60. Let $s(t) = \left[m(t)\cos(\omega_c t) + \widehat{m}(t)\sin(\omega_c t)\right]$. The carrier used to demodulate has a phase difference of $\theta$ with respect to the carrier used for modulation. Show that $v_0(t)$ has phase distortion (when compared to $m(t)$) by establishing

$$V_0(f) = \begin{cases} M(f)\, e^{j\theta} & , \ f > 0 \\ M(f)\, e^{-j\theta} & , \ f < 0 \end{cases}$$

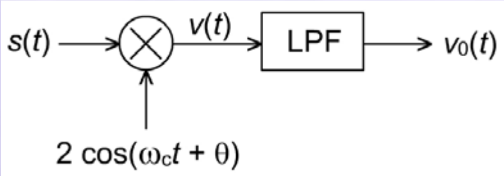



Fig. 4.60: SSB demodulation with carrier phase difference

Hint: Show that $v_0(t) \; \alpha \; \left[m(t)\cos\theta + \widehat{m}(t)\sin\theta\right]$

**Example 4.12**

Let $m(t) = A_1\cos(200\pi t) + A_2\cos(1100\pi t) + A_3\cos(4000\pi t)$. An upper sideband signal is generated using this $m(t)$. The carrier used for demodulation had a positive offset of 150 Hz. Let us find the frequencies of the spectral components after demodulation.

As the received signal is USB and $\Delta f > 0$, there would be an inward shift of the frequency components by 150 Hz. Spectral components in $M(f)$ are shown in Fig. 4.61(a).

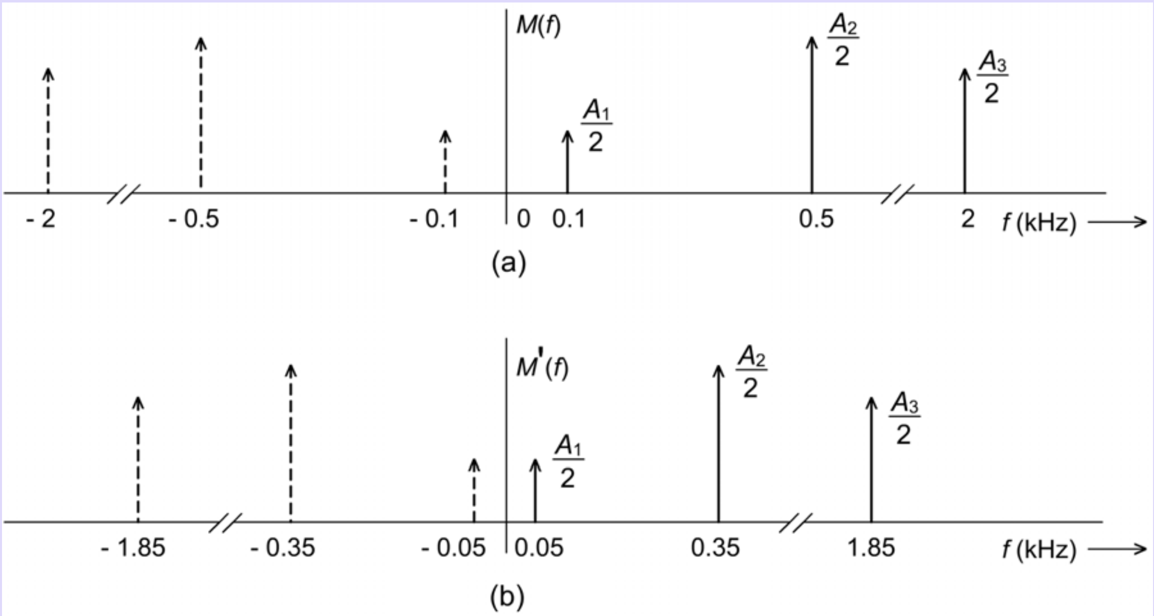



Fig. 4.61: (a) message spectrum

(b) Spectrum after demodulation with frequency error

Note that the negative frequency components have been shown with a broken line. Let $m(t)$ be the demodulated output. After demodulation, there would be an inward shift by 150 Hz and this is shown in Fig. 4.61(b). From this spectrum, we see that $m'(t)$ is consisting of components at 50 Hz, 350 Hz and 1850 Hz. ♦

The speech files that follow provide a qualitative measure of the distortion caused by the frequency offset of the local carrier in the demodulation of SSB signals. Input to the demodulator is a USB signal.

Introduction

Output 1

Output 2a

Output 2b

Output 2c

Output 3a

Output 3b

Output 3c

Output 4a

Output 4b

Output 4c

After listening to these speech outputs, one gets the feeling that, for a given frequency offset, SSB performs better than the DSB. Even in SSB, outward frequency shift of the original message spectrum ($\Delta f$ negative for USB) has better clarity than the corresponding inward shift. Of course, voice tends to become somewhat shrill, which, of course, is expected.

**Exercise 4.12: Quadrature Carrier Multiplexing (QCM)**

It is possible to transmit two DSB-SC signals with in a bandwidth of $2W$, by using a scheme called QCM, as shown in Fig. 4.62. (QCM is also referred to as Quadrature Amplitude Modulation (QAM) or simply quadrature modulation. Using QAM, we are able to achieve the BW efficiency of SSB modulation.)

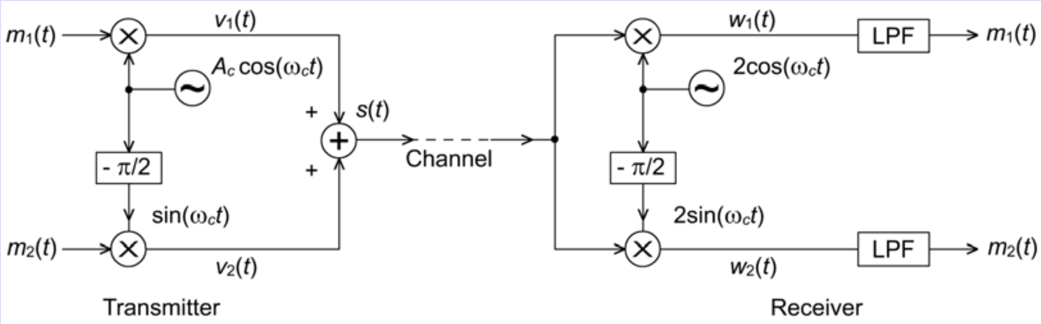



Fig. 4.62: Quadrature carrier multiplexing scheme

Two message signals $m_1(t)$ and $m_2(t)$ are used to generate two DSB-SC signals, $v_1(t)$ and $v_2(t)$ respectively. The carriers used in generating $v_1(t)$ and $v_2(t)$ are in phase quadrature. The transmitted signal $s(t) = v_1(t) + v_2(t)$. At the receiver, coherent demodulation is used to recover the two messages $m_1(t)$ and $m_2(t)$.

a) Show that $m_1(t)$ and $m_2(t)$ will be recovered by the receiver shown.

b) Let the local carrier have some frequency and phase offset; that is, instead of $2\cos(2\pi f_c t)$, let it be $2\cos\left[2\pi(f_c + \Delta f)t + \varphi\right]$. Then show that the output of the upper branch is

$$A_c\left\{m_1(t)\cos\left[2\pi(\Delta f)t + \varphi\right] - m_2(t)\sin\left[2\pi(\Delta f)t + \varphi\right]\right\}$$

where as the output of the lower branch is

$$A_c\left\{m_2(t)\cos\left[2\pi(\Delta f)t + \varphi\right] + m_1(t)\sin\left[2\pi(\Delta f)t + \varphi\right]\right\}$$

Note: We see from the above result that the carrier phase and frequency have to be fairly accurate to have proper demodulation; otherwise $m_1(t)$ will interfere with $m_2(t)$ and vice versa. This is called **cochannel interference**. QAM is used in color TV for multiplexing the chrominance signals.

## 4.9 Vestigial SideBand (VSB) Modulation

One of the widespread applications of VSB has been in the transmission of picture signals (video signals) in TV broadcast. The video signal has the characteristic that it has a fairly wide bandwidth (about 5 MHz) with almost no spectral hole around DC. DSB modulation, though somewhat easy to generate, requires too much bandwidth (about 10 MHz) where SSB, though bandwidth efficient, is extremely difficult to generate, as explained below.

With analog circuitry it is very difficult to build the $\frac{\pi}{2}$ phase shifter over a 5 MHz bandwidth; as such phase shift discrimination method is not feasible. To make use of the frequency discrimination method, we require very sharp cutoff filters. Such filters have a highly non-linear phase characteristic at the band edges and spectral components around the cut-off frequencies suffer from phase distortion (also called **group delay** distortion). The human eye (unlike the ear) being fairly sensitive to phase distortion, the quality of the picture would not be acceptable.

VSB refers to a modulation scheme where in the wanted sideband (either USB or LSB) is retained almost completely; in addition, a vestige (or a trace) of the unwanted sideband is added to the wanted sideband. This composite signal is used for transmitting the information. This vestige of the wanted sideband makes it possible to come up with a sideband filter that can be implemented in practice.

### 4.9.1 Frequency domain description of VSB

Figure 4.63 depicts the scheme of VSB generation. In this figure, $v(t)$ is a DSC-SC signal, which is applied as input to a Sideband Filter (SBF), $H_v(f)$, that shapes $V(f)$ so that $s(t)$ is a VSB signal.

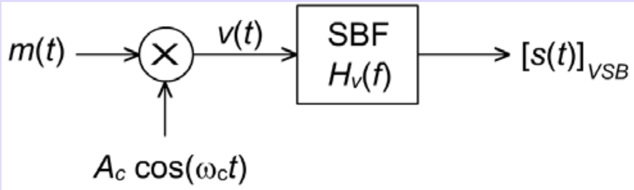



Fig. 4.63: Generation of VSB using the filtering method

Now the questions that arise are: what is the shape of the SBF and how do we demodulate such a signal. As coherent demodulation is fairly general, let us try to demodulate the VSB signal as well, using this method. In the process of demodulation, we shift the modulated carrier spectrum (bandpass spectrum) up and down by $f_c$ and then extract the relevant baseband. Because of the vestige of the unwanted sideband, we expect some overlap (in the baseband) of the shifted spectra. In such a situation, overlap should be such that, $M(f)$ is undistorted for $|f| \leq W$. In other words, $H_v(f - f_c) + H_v(f + f_c)$ should result in a filter with a rectangular passband within the frequency range $(-W \text{ to } W)$. With a little intuition, it is not too difficult to think of one such $H_v(f)$.

Assume that we are retaining the USB almost completely and permitting the vestige of the LSB. Consider the $H_v(f) = |H_v(f)| e^{j\theta_v(f)}$ shown in Fig. 4.64. We assume the phase characteristic $\theta_v(f)$ to be linear over the frequency range $f_l \leq |f| \leq f_c + W$ with $\theta_v(f_c) = -\theta_v(-f_c) = -2\pi m$, where $m$ is an integer.

If a DSB signal is given as input to the above $H_v(f)$, it will partially suppress USB (in the frequency range $f_c \leq |f| \leq f_u$) and allow the vestige of the LSB (from $f_l \leq |f| \leq f_c$). The demodulation scheme is the same as shown in Fig. 4.13, with the input to the detector being the VSB signal. $V_0(f)$, the FT of the output of the detector, is

$$V_0(f) = K_1\, M(f)\left[H_v(f - f_c) + H_v(f + f_c)\right], \text{ for } |f| \le W .$$

where $K_1$ is the constant of proportionality.

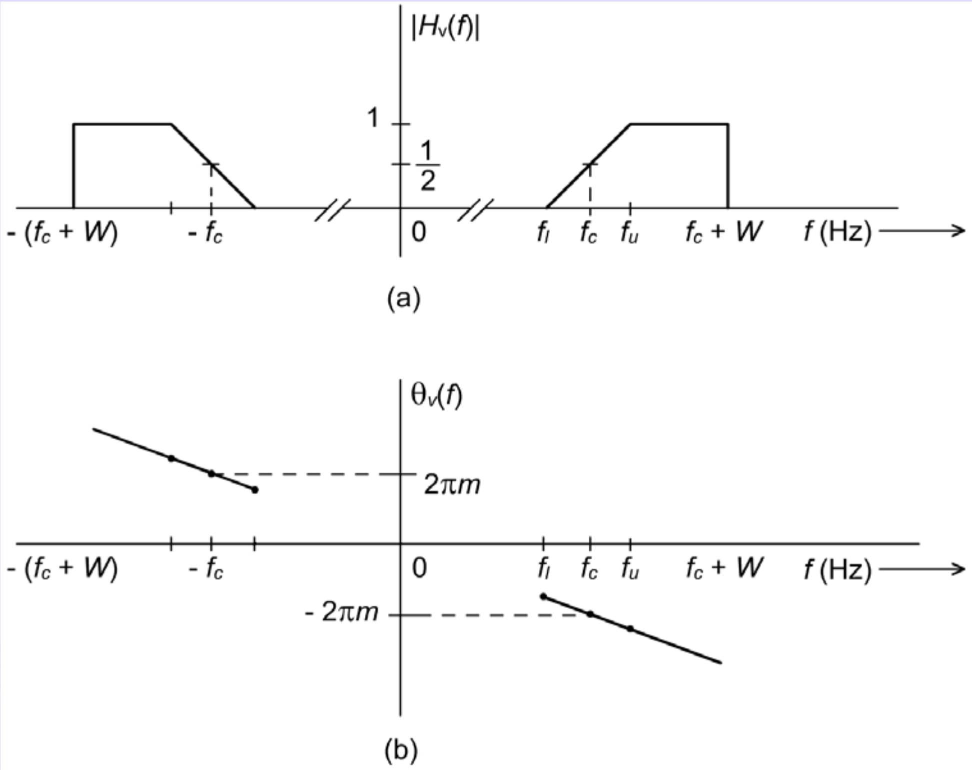



Fig. 4.64: An example of a SBF generating VSB

If $\left[H_v(f - f_c) + H_v(f + f_c)\right] = K_2\, e^{-j 2\pi f t_d},\ |f| \le W$,

where $t_d$ determines the slope of the phase characteristic and $K_2$ is a constant, then $v_0(t) = K\, m(t - t_d)$, where $K = K_1 K_2$.

Let $H_{1,v}(f) = \left|H_{1,v}(f)\right| e^{j\theta_{1,v}(f)} = H_v(f - f_c),\ -W \le f \le f_v$

where $f_v = f_c - f_l$, is the width of the vestige in LSB.

For the $H_v(f)$ shown in Fig. 4.64, $H_{1,v}(f)$ is as shown in Fig. 4.65.

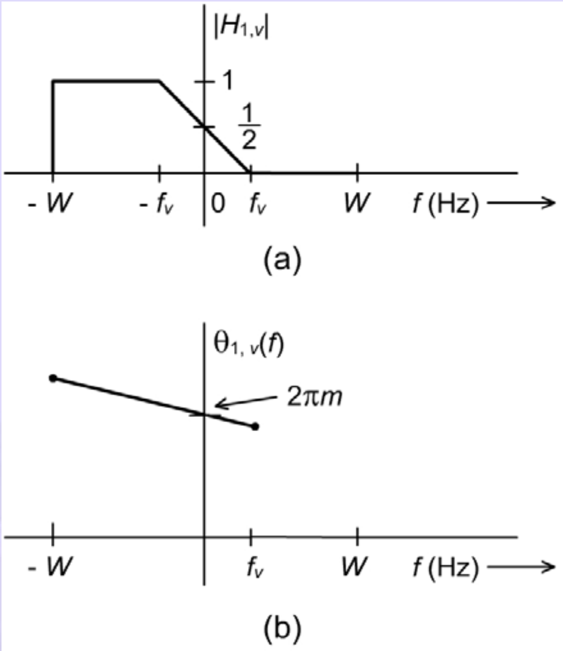



Fig. 4.65: $H_v(f - f_c)$ for $|f| \le W$

(a) magnitude characteristics

(b) phase characteristics

Let $H_{2,v}(f) = |H_{2,v}(f)| e^{j\theta_{2,v}(f)} = H_v(f + f_c)$ for $-f_v \le f \le W$. $H_{2,v}(f)$ is shown in figure 4.66.

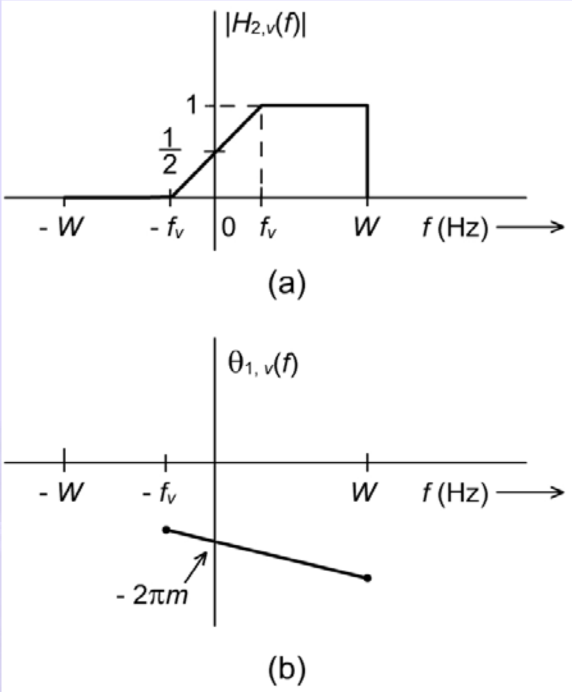



Fig. 4.66: $H_v(f + f_c)$ for $|f| \leq W$

(a) magnitude characteristics

(b) phase characteristics

As $|H_{1,v}(f)| = 0$ for $f_v \leq f \leq W$ and $e^{\pm j 2\pi m} = 1$, we can write

$$H_v(f - f_c) = |H_{1,v}(f)| \, e^{-j 2\pi f t_d}, \quad |f| \leq W \tag{4.30a}$$

By a similar reasoning,

$$H_v(f + f_c) = |H_{2,v}(f)| \, e^{-j 2\pi f t_d}, \quad |f| \leq W \tag{4.30b}$$

Therefore,

$$H_v(f - f_c) + H_v(f + f_c) = \left[|H_{1,v}(f)| + |H_{2,v}(f)|\right] e^{-j 2\pi f t_d}, \quad |f| \leq W \tag{4.30c}$$

Summing $|H_{1,v}(f)|$ and $|H_{2,v}(f)|$, we have the ideal LPF with unity gain for $|f| \leq W$.

Let us take a closer look at the sideband filter for VSB. The magnitude characteristic shown in Fig. 4.64(a) can be taken as the sum of $H_v^{(1)}(f) + H_v^{(2)}(f)$

where $H_V^{(1)}(f)$ and $H_V^{(2)}(f)$ are shown in Fig. 4.67(a) and (b) respectively.

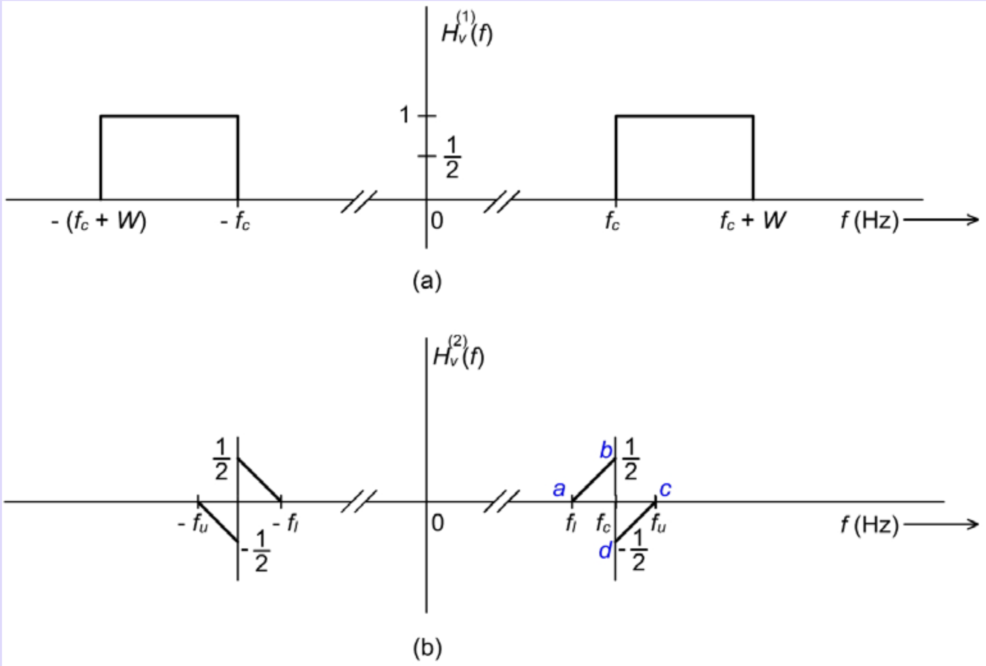



Fig. 4.67: Decomposition of the $|H_V(f)|$ of Fig. 4.64

Note that with $H_V^{(1)}(f)$ alone for the sideband filter, we would have generated the SSB signal. It is the addition of $H_V^{(2)}(f)$ to $H_V^{(1)}(f)$ that gives rise to VSB output. Consider $f > 0$; $H_V^{(2)}(f)$ consists of two straight line segments, one between the points $(a, b)$ and the other between the points $(c, d)$. Similar is the case for $f < 0$. Now the question is: should $H_V^{(2)}(f)$ consist of only straight line segments? The answer is **NO**. What is required is that $H_V^{(2)}(f)$ exhibit odd symmetry about $\pm f_c$. Two more such characteristics have been shown in fig. 4.68, for $f > 0$.

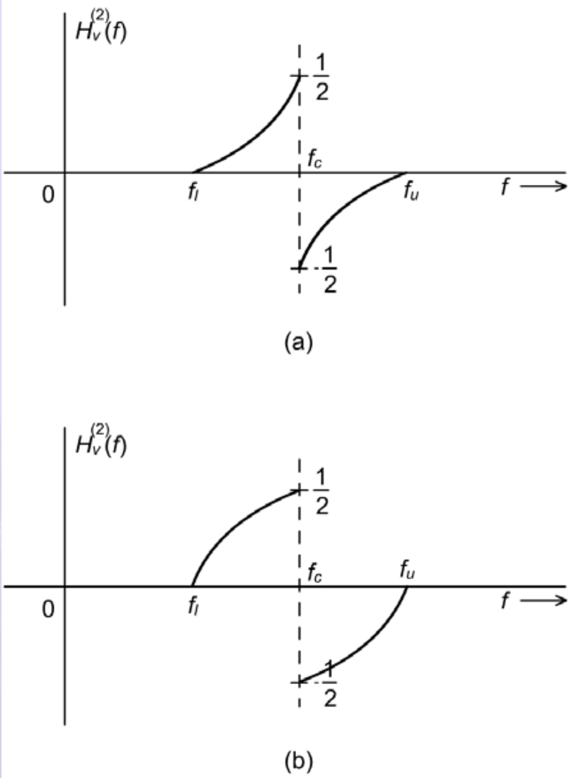



Fig. 4.68: Two more examples of $H_v^{(2)}(f)$

As such, we have a great deal of flexibility in designing $H_v(f)$. (Subject to some upper limit, there could be choice in fixing $f_v$, the width of the vestige.) These features facilitate the design and implementation of a practical sideband filter.

## 4.9.2 Time domain description of VSB

Let $h_v(t)$ denote the impulse response of the sideband filter, $H_v(f)$. From Fig. 4.63, we have

$$\left[s(t)\right]_{VSB} = s(t) = \int_{-\infty}^{\infty} h_v(\tau)\, v(t-\tau)\, d\tau$$

But $v(t) = A_c\, m(t) \cos(\omega_c t)$. For convenience, let $A_c = 1$.

Then, $s(t) = \int_{-\infty}^{\infty} h_v(\tau)\, m(t-\tau) \cos\left[\omega_c (t-\tau)\right] d\tau$

$$= \int_{-\infty}^{\infty} h_v(\tau)\, m(t-\tau) \left[\cos(\omega_c t)\cos(\omega_c \tau) + \sin(\omega_c t)\sin(\omega_c \tau)\right] d\tau$$

$$= \left[\int_{-\infty}^{\infty} h_v(\tau)\, m(t-\tau)\cos(\omega_c \tau)\, d\tau\right]\cos(\omega_c t)$$

$$-\left[-\int_{-\infty}^{\infty} h_v(\tau)\, m(t-\tau)\sin(\omega_c \tau)\, d\tau\right]\sin(\omega_c t) \qquad (4.31)$$

Eq. 4.31 is in the canonical form for the representation of narrowband signal. Let $m_c(t)$ denote the in-phase component of $s(t)$ and $m_s(t)$, the quadrature component. Then,

$$m_c(t) = \int_{-\infty}^{\infty} h_v(\tau)\, m(t-\tau)\cos(\omega_c \tau)\, d\tau \qquad (4.32a)$$

and $$m_s(t) = \left[-\int_{-\infty}^{\infty} h_v(\tau)\, m(t-\tau)\sin(\omega_c \tau)\, d\tau\right] \qquad (4.32b)$$

Then, $$\left[s(t)\right]_{VSB} = m_c(t)\cos(\omega_c t) - m_s \sin(\omega_c t) \qquad (4.33)$$

Eq. 4.33 is the canonical representation of the VSB signal.

Let $h_i(t) = h_v(t)\cos(\omega_c t)$ and $h_q(t) = -h_v(t)\sin(\omega_c t)$. Then,

$$m_c(t) = m(t) * h_i(t) \qquad (4.34a)$$

$$m_s(t) = m(t) * h_q(t) \qquad (4.34b)$$

Taking the FT of Eq. 4.34, we have

$M_c(f) = M(f)\, H_i(f)$ and $M_s(f) = M(f)\, H_q(f)$

If $M(f) = 0$ for $|f| > W$, then $M_c(f)$ and $M_s(f)$ are bandlimited to atmost $W$. This implies that $m_c(t)$ and $m_s(t)$ are lowpass signals, as required.

As $h_i(t) = h_v(t)\cos(\omega_c t)$ we have,

$$H_i(f) = \frac{H_v(f - f_c) + H_v(f + f_c)}{2} \qquad (4.35a)$$

Similarly, $H_q(f) = \dfrac{H_v(f + f_c) - H_v(f - f_c)}{2j}$ (4.35b)

But $\left[H_v(f - f_c) + H_v(f + f_c)\right] = 1$, for $|f| \leq W$ . Hence, $m_c(t) = \dfrac{m(t)}{2}$.

Let us look at $H_q(f)$.

From Eq. 4.35(b), $\dfrac{2\,H_q(f)}{j} = H_v(f - f_c) - H_v(f + f_c)$

Let $H_v(f)$ be as shown in Fig. 4.69.

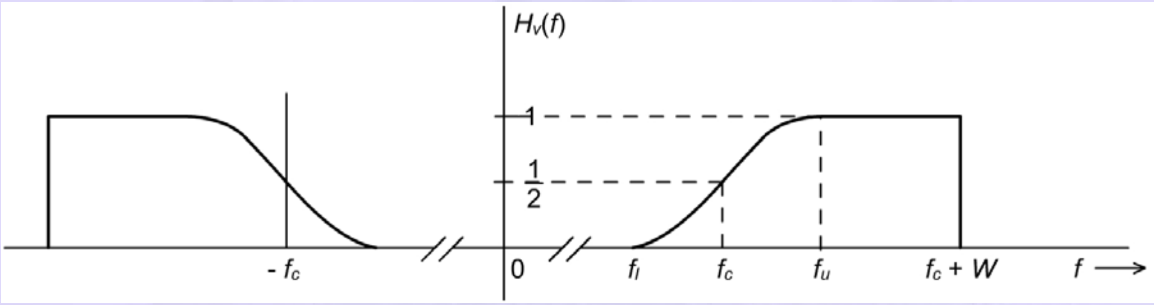



Fig. 4.69: $H_v(f)$ with vestige in LSB

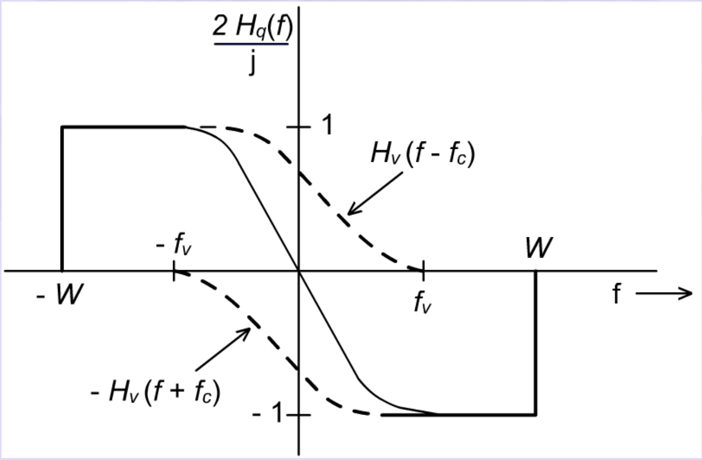



Fig. 4.70: $\dfrac{2\,H_q(f)}{j}$ (solid line) for the $H_v(f)$ of Fig. 4.69

Then, $\dfrac{2\,H_q(f)}{j}$ for $|f| \leq W$ will be as shown in Fig. 4.70.

Based on Eq. 4.33, we have another implementation for the generation of the VSB signal called the phase discrimination method of generating VSB. This scheme is shown in Fig. 4.71. Plus sign at the last summer will result in a VSB signal with a vestige in LSB whereas the minus sign will result in a VSB signal with the vestige in USB for the $H_q(f)$ shown in Fig. 4.70.

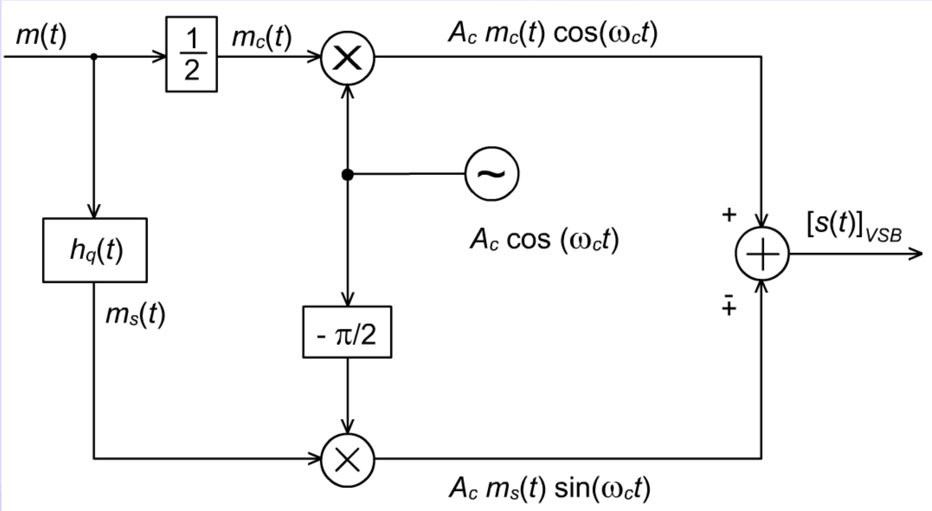



Fig. 4.71: Phase discrimination method of VSB generation

Comparing the scheme shown in Fig. 4.71 with that shown in Fig. 4.55 for the generation of SSB, we find a close resemblance between them. When $h_q(t)$ is a HT (with the magnitude response equal to 1/2), we have $[s(t)]_{VSB} = [s(t)]_{SSB}$. If $h_q(t) = 0$, we have $[s(t)]_{VSB} = [s(t)]_{DSB}$. In other terms, DSB and SSB can be treated as special cases of VSB. Transmission bandwidth of VSB is,

$$[B_T]_{VSB} = W + f_v$$

With $f_v = 0$, we have the SSB situation where as, $f_v = W$ leads to DSB.

**Example 4.13**

A VSB signal is generated from the DSB-SC signal, $2m(t)\cos(\omega_c t)$. $M(f)$ is as shown in Fig. 4.72(a). If the vestige is as shown in Fig. 4.72(b), let's find the values of all spectral components in the VSB signal for $f > f_c$, assuming that demodulation in done by multiplying the received VSB signal with $2\cos(\omega_c t)$. $M(f)$ is to be restored to its original values.

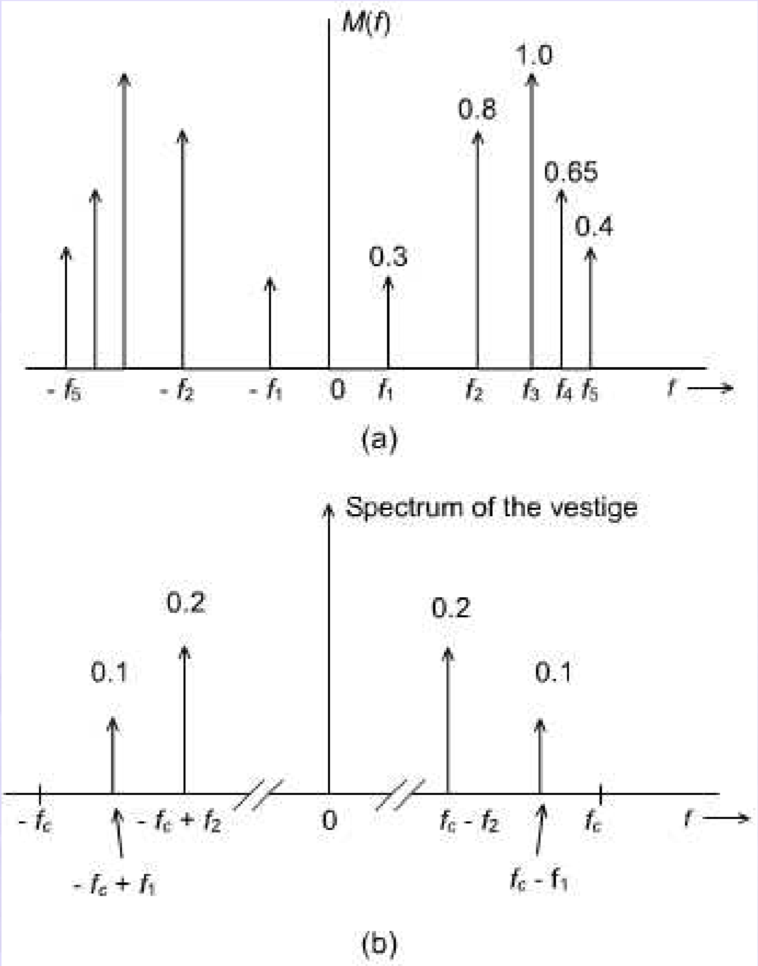



Fig. 4.72: (a) Baseband message spectrum used to generate the VSB
(b) Spectral components in the vestige in LSB

Let $\alpha_i$ be the magnitude of the spectral component at $f_c + f_i$, $i = 1, 2, ..., 5$ in the VSB spectrum. When the VSB spectrum is shifted to the left by $f_c$, we have the spectrum shown in Fig. 4.73(a). Similarly, when the VSB spectrum is shifted to the right by $f_c$, we have the spectrum shown in Fig. 4.73(b).

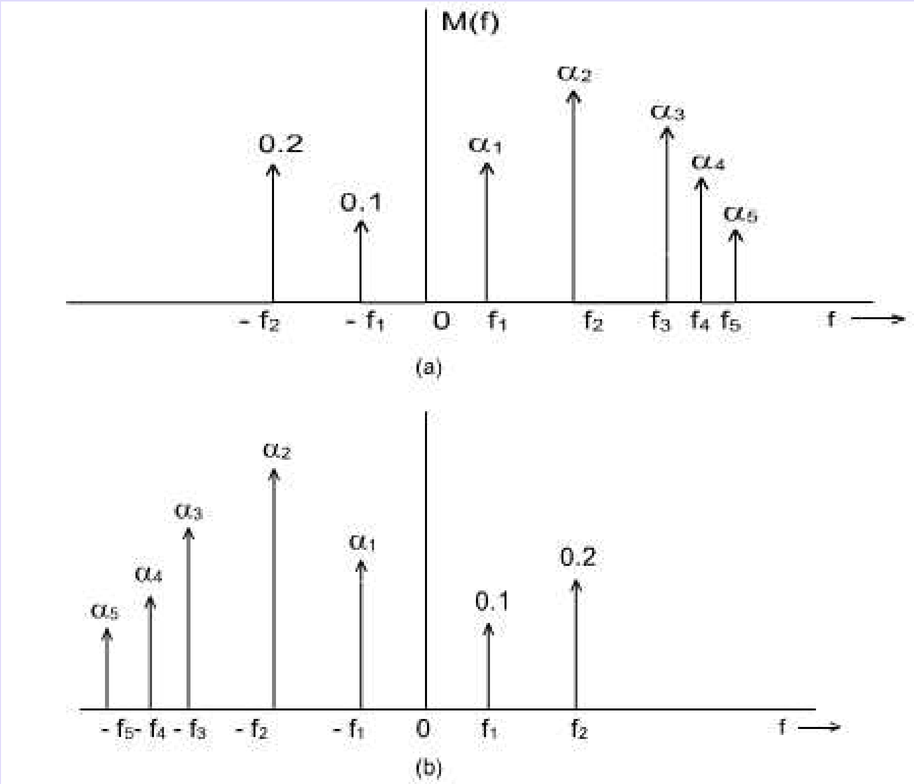



Fig. 4.73: Shifted VSB spectra: (a) left shift (b) right shift

From the spectral plots in Fig. 4.73(a) and (b), we have

$$\alpha_1 + 0.1 = 0.3 \text{; That is } \alpha_1 = 0.2$$

$$\alpha_2 + 0.2 = 0.8 \text{; That is } \alpha_2 = 0.6$$

$$\alpha_3 = 1, \alpha_4 = 0.65 \text{ and } \alpha_5 = 0.4$$

Hence the VSB spectrum for $f > 0$ is shown in Fig. 4.74.

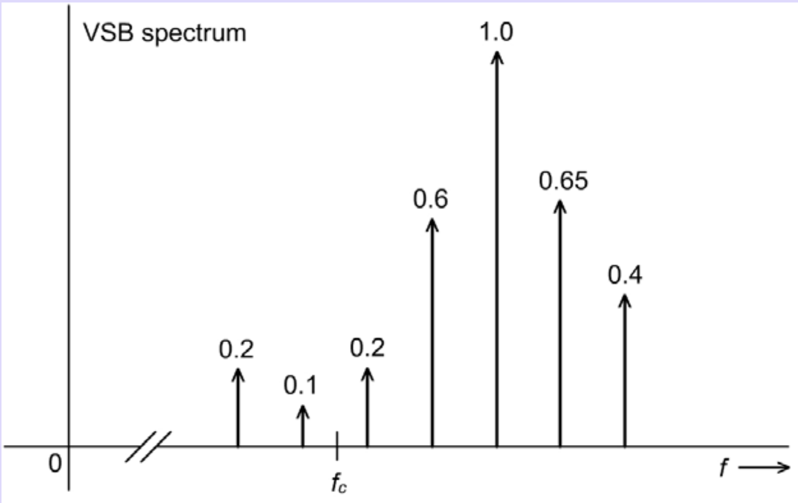



Fig. 4.74: VSB spectrum of example 4.13

♦

**Exercise 4.13**

Let $m(t) = 2 + 4\cos(200\pi t) + 6\cos(300\pi t) + 5\cos(400\pi t)$.

Specify the frequency response of a VSB filter that passes the LSB almost completely and leaves a vestige of the first frequency component in USB. It is given that the magnitude of the filter at the vestige frequency is $\frac{1}{8}$. Sketch the spectrum of the VSB signal.

## 4.10 Envelope Detection of VSB+C

As mentioned earlier, one important application of VSB has been in TV broadcast. As a large number of receivers are involved, it is preferable to make the detector circuit fairly simple and inexpensive. The envelope of a VSB signal is not one-to-one related to the message $m(t)$. Hence direct envelope detection of a VSB signal would be of no use. However, if there is a carrier component along with the VSB signal, that is, VSB+C, then ED might work. We shall now look into this.

Let the input to the ED be

$$A_c \cos(\omega_c t) + A_c\left[\beta m(t)\cos(\omega_c t) \pm \beta m_s(t)\sin(\omega_c t)\right]$$

where $\beta$ is an adjustable scale factor. Then $A(t)$, the output of the ED is

$$A(t) = A_c\left\{\left[1 + \beta m(t)\right]^2 + \left(\beta m_s(t)\right)^2\right\}^{\frac{1}{2}}$$

$$= A_c\left[1 + \beta m(t)\right]\left\{1 + \left[\frac{\beta m_s(t)}{1 + \beta m(t)}\right]^2\right\}^{\frac{1}{2}} \qquad (4.36)$$

If the distortion component $\left|\frac{\beta m_s(t)}{1 + \beta m(t)}\right| << 1$ for all $t$, then the output of the ED will be $A_c\left[1 + \beta m(t)\right]$, which, after the DC block, will provide the message output. This level of the distortion component can be reduced by

i) increasing $f_v$. Note that as $f_v \to W$, we will have the DSB signal; that is, $m_s(t) = 0$.

ii) decreasing the value of $\beta$.

In commercial TV broadcast, $f_v$ is about 75 kHz which is about 1/6 of the width of a full sideband which is about 5 MHz. It has been found that with a vestige of about 75 kHz, the distortion component is not too bothersome.

It would be instructive to compare the envelope detection of SSB+C with that of VSB+C. Let the input to the ED be SSC+C, namely,

$$A_c \cos(\omega_c t) + \left[m(t)\cos(\omega_c t) \pm \widehat{m}(t)\sin(\omega_c t)\right]$$

then, $A(t) = \left\{\left[A_c + m(t)\right]^2 + \left[\widehat{m}(t)\right]^2\right\}^{\frac{1}{2}}$

$$= A_c \left\{ \left[ 1 + \frac{m^2(t)}{A_c^2} + \frac{2\,m(t)}{A_c} \right] + \left[ \frac{\widehat{m}(t)}{A_c} \right]^2 \right\}^{\frac{1}{2}} \qquad (4.37)$$

If $\frac{m(t)}{A_c}$ and $\frac{\widehat{m}(t)}{A_c}$ are very much less than unity, then $\left[\frac{m(t)}{A_c}\right]^2$ and $\left[\frac{\widehat{m}(t)}{A_c}\right]^2$ can be dropped from Eq. 4.37, giving us

$$A(t) \simeq A_c \left[ 1 + \frac{2\,m(t)}{A_c} \right]^{\frac{1}{2}}$$

Retaining only first order term of the binomial expansion,

$$A(t) \simeq A_c \left[ 1 + \frac{m(t)}{A_c} \right] = A_c + m(t)$$

Of course, if we ensure $\left|m(t)_{\max}\right| = \left|m(t)_{\min}\right| << A_c$, then $\frac{m(t)}{A_c}$ can be neglected in comparison with unity. This may be adequate to make $\frac{\widehat{m}(t)}{A_c}$ much smaller than unity, most of the time. In any case, it is obvious that ED of SSB+C results in excessive wastage of the transmitted power. In contrast, ED of AM is reasonably efficient as the requirement to avoid envelope distortion is $\left|m(t)\right|_{\max} \leq A_c$.

**Example 4.14**

Consider the SSB+C signal,

$$s(t) = A_c \cos(\omega_c t) + m(t)\cos(\omega_c t) + \widehat{m}(t)\sin(\omega_c t)$$

where $m(t) = \frac{1}{1+t^2}$. We will show that, if $A_c >> 1$, then the output of the ED can be taken as $A_c + m(t)$.

As $m(t) = \frac{1}{1+t^2}$, we have $\widehat{m}(t) = \frac{t}{1+t^2}$. Hence $A(t)$, the envelope of $s(t)$ is,

$$A(t) = \left[\left(A_c + \frac{1}{1+t^2}\right)^2 + \left(\frac{t}{1+t^2}\right)^2\right]^{\frac{1}{2}}$$

$$= \left[A_c^2 + \frac{1}{\left(1+t^2\right)^2} + \frac{2\,A_c}{1+t^2} + \frac{t^2}{\left(1+t^2\right)^2}\right]^{\frac{1}{2}}$$

$$= \left[A_c^2 + \frac{1}{1+t^2} + \frac{2\,A_c}{1+t^2}\right]^{\frac{1}{2}}$$

$$= A_c\left\{1 + \frac{2}{A_c\left(1+t^2\right)} + \frac{1}{A_c^2\left(1+t^2\right)}\right\}^{\frac{1}{2}}$$

Neglecting the term $\frac{1}{{A_c}^2\left(1+t^2\right)}$, we have

$$A(t) \simeq A_c\left[1 + \frac{2}{A_c\left(1+t^2\right)}\right]^{\frac{1}{2}}$$

Using the binomial expansion upto the second term, we have

$$A(t) \simeq A_c\left[1 + \frac{1}{2}\,\frac{2}{A_c\left(1+t^2\right)}\right] \simeq A_c + \frac{1}{1+t^2}$$

which is the desired result. ♦

**Exercise 4.14**

Consider SSB+C with tone modulation. Let

$$s(t) = A_c \left[\cos(\omega_c t) + \beta \cos(\omega_c + \omega_m) t\right]$$

Assume that $f_c >> f_m$ and $\beta < 1$.

a) Construct the phasor diagram and develop the expression for the envelope $A(t)$.

b) Let $\beta$ be such that terms involving $\beta^m$ , $m \geq 3$ can be neglected. Show that

$$A(t) \simeq A_c \left[1 + \frac{\beta^2}{4} + \beta \cos(\omega_m t) - \frac{\beta^2}{4} \cos(2 \omega_m t)\right]$$

c) Find the value of $\beta$ so that second harmonic envelope distortion, that is, the ratio, $\dfrac{\text{Amplitude of the second harmonic in } A(t)}{\text{Amplitude of the fundamental in } A(t)}$ is less than five percent.

**A note on the linearity of AM, DSB-SC, SSB and VSB:** Having discussed these modulation schemes, let us look at the linearity aspect of these schemes. By linearity, we imply that the schemes obey the superposition property. This can be easily verified in the case of DSB-SC, SSB and VSB. Consider DSB-SC. When message signals $m_1(t)$ and $m_2(t)$ are applied separately, the resulting modulated waveforms are $A_c\, m_1(t)\cos(\omega_c t)$ and $A_c\, m_2(t)\cos(\omega_c t)$. Let

$$m(t) = \alpha_1 m_1(t) + \alpha_2 m_2(t)$$

where $\alpha_1$ and $\alpha_2$ are constants. Then the modulated carrier is

$$A_c\, m(t)\cos(\omega_c t) = A_c \left[\alpha_1 m_1(t) + \alpha_2 m_2(t)\right] \cos(\omega_c t),$$

which establishes the linearity property. Similarly, SSB and VSB can be shown to be linear. AM is not linear in its strict sense, because if $m(t)$ is applied as input to an AM modulator, the output is

$$A_c\left[1 + g_m\, m(t) \cos(\omega_c\, t)\right] = A_c\left\{1 + g_m\left[\alpha_1 m_1(t) + \alpha_2 m_2(t)\right] \cos(\omega_c\, t)\right\}$$

and this is not equal to

$$A_c\left[1 + g_m\, \alpha_1 m_1(t)\right] \cos(\omega_c\, t) + A_c\left[1 + g_m\, \alpha_2 m_2(t)\right] \cos(\omega_c\, t)$$

That is, superposition does not apply to the carrier component. As this is only a minor deviation, all the four modulation types are put under the category of linear modulation.

## 4.11 Superheterodyne Receiver

The important function of a receiver is demodulation; that is, to recover the message signal from the received modulated waveform. The other functions which become necessary for the proper reception of a signal are: *amplification and tuning or selective filtering*. Amplification becomes necessary because, most often, the received signal is quite weak and without sufficient amplification, it may not even be able to drive the receiver circuitry. Tuning becomes important especially in a broadcast situation because there is more than one station broadcasting at the same time and the receiver must pick the required station and reject the inputs from the other (unwanted) stations; tuning also ensures that out of band noise components do not affect the receiver's performance. Besides these operations, most of the receivers also incorporate certain other features such as *frequency conversion, automatic gain control* etc.

Two of the demodulation methods which we have already discussed are coherent or synchronous detection and envelope detection. Coherent detection can be used to demodulate any linear modulation scheme: DSB-SC, DSB-LC, SSB or VSB. In practice, it is used to demodulate only suppressed carrier signals. Envelope detection is mainly used in the demodulation of DSB-LC and VSB+C signals. We shall now describe the receiver used in AM broadcast.

The receiver for the broadcast AM is of the superheterodyne (or superhet) variety[1]. This is shown schematically in the Fig. 4.75.

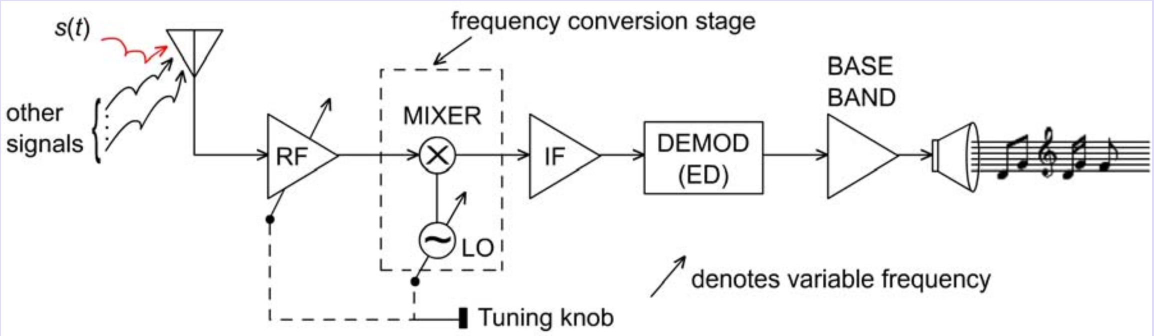



Fig. 4.75: Superheterodyne Receiver

The wanted signal $s(t)$, along with other signals and noise, is input to the Radio Frequency (RF) stage of the receiver. The RF section is tuned to $f_c$, the carrier frequency of the desired signal $s(t)$. The bandwidth of the RF stage, $B_{RF}$, is relatively broad; hence along with $s(t)$, a few adjacent signals are also passed by it. The next stage in the receiver is the frequency conversion stage consisting of a mixer and a local oscillator. The local oscillator frequency, $f_{LO}$ tracks the carrier frequency $f_c$, (with the help of a ganged capacitor) and is usually $(f_c + f_{IF})$, where $f_{IF}$ denotes the Intermediate Frequency (IF). The mixer output consists of, among others, the frequency components at $2f_c + f_{IF}$ and $f_{IF}$. The following stage, called the IF stage, is a tuned amplifier, which rejects all the other components and produces an output that is centered at $f_{IF}$. The bandwidth of the IF stage, $B_{IF}$, is approximately equal to the transmission bandwidth, $B_T$, of

[1] Some of the other applications of superhet are: reception of FM and TV broadcast signals and RADAR

the modulation scheme under consideration. For example, if the input signal is of the double sideband variety, then $B_{IF} \approx 2W$

The IF stage constitutes a very important stage in a superheterodyne receiver. It is a fixed frequency amplifier (it could consist of one or more stages of amplification) and provides most of the gain of the superhet. Also, as $B_{IF} = B_T$, it also rejects the adjacent channels (carrier frequency spacing ensures this). Next to IF, we have the *detector* or *demodulation* stage which removes the IF carrier and produces the baseband message signal at its output. Finally, the demodulator output goes through a baseband amplification stage (audio or video, depending upon the type of the signal) before being applied to the final transducer (speaker, picture-tube etc.)

The spectral drawings shown in Fig. 4.76 and 4.77 help clarify the action of a superhet receiver. We shall assume the input to the receiver is a signal with symmetric sidebands.

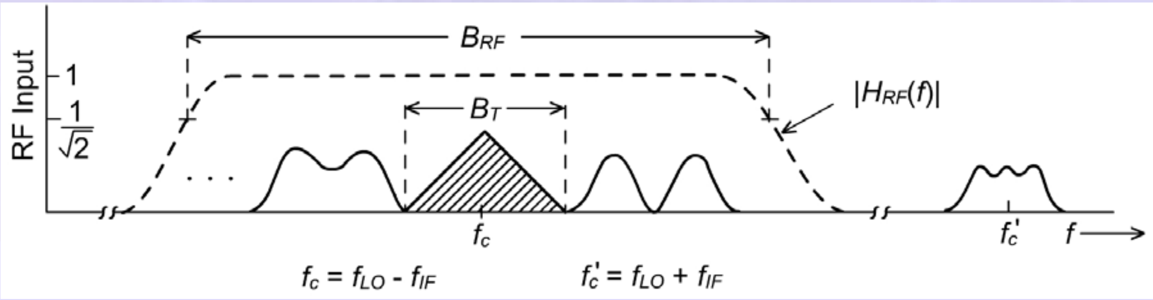



Fig. 4.76: Typical spectrum at the input to the RF stage of a superhet

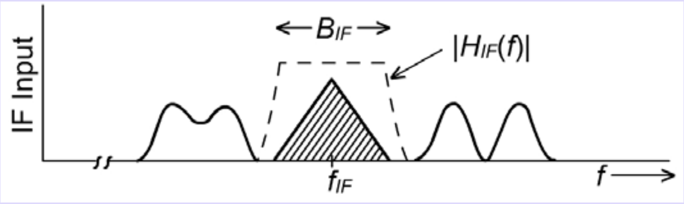



Fig. 4.77: Spectrum at the input of the IF stage of a stage of a superhet

Fig. 4.76 shows the spectrum of the input signal to the RF stage. As can be seen, it has the desired signal, plus a few adjacent channels and possibly a signal with the carrier frequency $f_c^{'} = f_c + 2 f_{IF} = f_{LO} + f_{IF} \cdot f_c^{'}$ is called the *image frequency* corresponding to $f_c$. If $f_c^{'}$ is allowed to pass through the RF stage, then it will also give rise to an output at the IF stage which would interfere with the wanted signal, namely the signal whose carrier frequency is $f_c$. Hence the main task of the RF section is to pass the frequency components in the range $f_c \pm B_{T/2}$ while rejecting the signal with the spectrum centered at $f_c^{'}$ (image frequency rejection). If the 3-dB bandwidth, $B_{RF}$ is such that

$$B_T < B_{RF} < 2 f_{1F}$$

then, $H_{RF}(f)$, should be able to provide sufficient attenuation at the image frequency.

Fig. 4.77 depicts the frequency response characteristic of the IF stage. As seen from this figure, the IF filter takes care of adjacent channel rejection.

The superheterodyne[1] structure results in several practical benefits:

i) Tuning takes place entirely in the *front end* (RF and mixer stage) so that the rest of the circuitry (IF stage, detector and the final power amplifier stage) requires no adjustments to changes in $f_c$.

ii) Separation between $f_c$ and $f_{IF}$ eliminates potential instability due to stray feedback from the amplified output to the receiver's input.

iii) Most of the gain and selectivity is concentrated in the fixed IF stage. $f_{IF}$ is so selected so that $B_{IF} / f_{IF}$ results in a reasonable fractional bandwidth (for

[1] The word superheterodyne refers to the operation of the receiver, namely, the incoming signal at the carrier frequency is heterodyned or mixed with the LO signal whose frequency is **higher** than $f_c$ $\left(f_{LO} = f_c + f_{IF}\right)$.

AM broadcast various frequency parameters are given in table 4.1). It has been possible to build superhets with about 70 dB gain at the IF stage itself.

### Table 4.1 Parameters of AM radio

| | |
|---|---|
| Carrier frequency range | 540-1600 kHz |
| Carrier spacing | 10 kHz |
| Intermediate frequency | 455 kHz |
| IF bandwidth | 6-10 kHz |
| Audio bandwidth | 3-5 kHz |

An IF of 455 kHz has been arrived at by taking the following points into consideration.

1) IF must not fall within the tuning range of the receiver. Assume that there is a station broadcasting with the carrier frequency equal to $f_{IF}$. This signal, could directly be picked off by the IF stage (every piece of wire can act as an antenna). Interference would then result between the desired station and the station broadcasting at $f_c = f_{IF}$.

2) Too high an IF would result in poor selectivity which implies poor adjacent channel rejection. Assume that IF was selected to be 2 MHz. With the required bandwidth of less than 10 kHz, we require very sharp cutoff filters, which would push up the cost of the receiver.

3) As IF is lowered, image frequency rejection would become poorer. Also, selectivity of the IF stage may increase; thereby a part of the sidebands could be lost.

We had mentioned earlier, that $f_{LO} = f_c + f_{IF}$. If we have to obtain the $f_{IF}$ component after mixing, this is possible even if $f_{LO} = f_c - f_{IF}$. But this causes the following practical difficulty. Consider the AM situation. If we select $f_{LO} = f_c - f_{IF}$, then the required range of variation of $f_{LO}$ so as to cover the entire AM band of 540-1600 kHz is (540 - 455) = 85 kHz to (1600 - 455) = 1145 kHz. Hence the tuning ratio required is $85:1145 \simeq 1:13$. If $f_{LO} = f_c + f_{IF}$, then the tuning ratio required is $995:2055 \simeq 1:2$. This is much easier to obtain than the ratio 1:13. With the exception of tuning coils, capacitors and potentiometers, all the circuitry required for proper reception of AM signals is available in IC chips (for example, BEL 700).

**Example 4.15: Double conversion superhet receiver**

In receivers operating in the VHF (30 - 300 MHz) range and meant for receiving fairly narrowband signals (say telemetry signals), to achieve good image frequency rejection and selectivity using a single mixer stage is quite difficult. Hence, receivers have been developed with more than one frequency conversion stage and more than one IF. We shall now look at the image frequency problem of a receiver with two mixer stages, usually called **double** or **dual conversion receiver**.

Consider the scheme shown in Fig. 4.78.

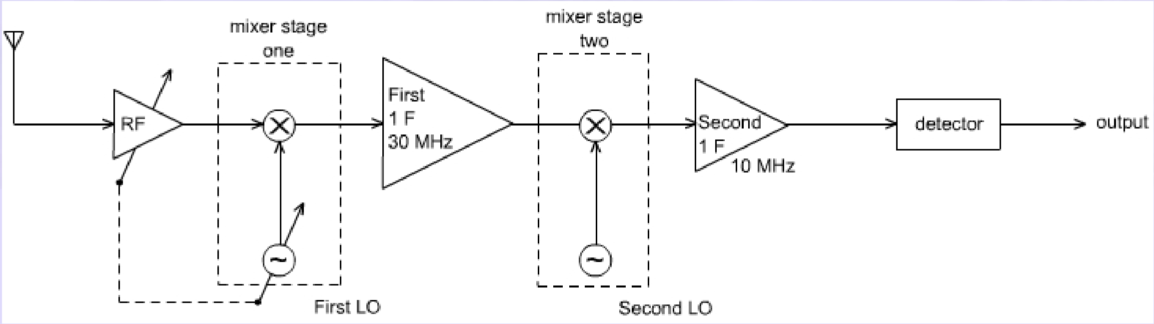



Fig. 4.78: Block diagram of a double conversion receiver

In the scheme shown, the first mixer stage has a tunable LO, its output being 30 MHz below the incoming signal frequency. The second mixer stage has an LO producing a fixed frequency output at 40 MHz. If the RF stage is tuned to 200 MHz, let us find the possible image frequencies of the incoming signal. We assume that none of the filters in the cascade are ideal.

First local oscillator frequency = (200 - 30) = 170 MHz. Hence (170 - 30) = 140 MHz will be an image frequency for the incoming signal w.r.t the first IF. As the second IF is 10 MHz, a component at 50 MHz would be an image of 30 MHz. Hence if a frequency component at $(170 \pm 50) = 220$ or 120 MHz get through the RF stage, it would interfere with the reception of the wanted carrier at 200 MHz. In other words, the image frequencies are at 220 MHz, 140 MHz and 120 MHz.

**Exercise 4.15**

a) For the tuned circuit shown in Fig. 4.79, $H(f) = \dfrac{V_0(f)}{I(f)}$ is,

$$H(f) = \frac{1}{\left(\frac{1}{R}\right) + j\left(2\pi f C - \frac{1}{2\pi f L}\right)}$$

$$= \frac{R}{1 + jR\left(\omega^2 LC - 1\right)\frac{1}{\omega L}}$$

Let $f_0$ be the resonant frequency; then $f_0 = \dfrac{1}{2\pi\sqrt{LC}}$ and Q-factor of filter is $Q = R\sqrt{\dfrac{C}{L}}$

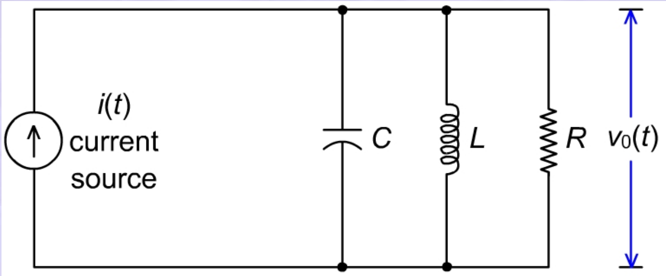



Fig. 4.79: A tuned circuit with L, C and R

Show that

$$H(f) = \frac{V_0(f)}{I(f)} = \frac{R}{1 + jQ\left[\frac{\left(f^2 - f_0^2\right)}{f f_0}\right]} = \frac{R}{1 + jQ\left(\frac{f}{f_0} - \frac{f_0}{f}\right)}$$

and hence

$$\left|H(f)\right| = \frac{R}{\sqrt{1 + Q^2 \beta^2(f)}}$$

where $\beta(f) = \left(\dfrac{f}{f_0} - \dfrac{f_0}{f}\right)$

b) Let the RF stage in a superhet consist of a simple tuned circuit (TC1) whose output is input to the mixer stage as shown in Fig. 4.80.

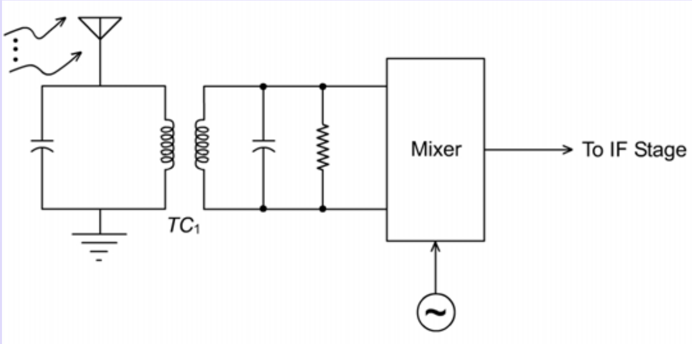



Fig. 4.80: Superhet without RF amplifier

Let $\lambda$ denote the image frequency rejection, where $\lambda = \frac{|H(f_c)|}{\left|H\left(f_c^{'}\right)\right|}$, $f_c^{'}$ being the image frequency of $f_c$.

That is, $\lambda = \sqrt{1 + Q^2 \beta^2 \left(f_c^{'}\right)}$. Calculate the value of $\lambda$ when the receiver is tuned to (i) 1.0 MHz and (ii) 20.00 MHz. Assume Q of the resonant circuit $TC_1$ to be 100.

(Ans.: (i) $\lambda = 138.6$ (ii) $\lambda = 70.6$)

c) It is required to have the value of $\lambda$ at 20 MHz the same as at $\lambda = 1.00$ MHz. For this purpose, the RF stage has been modified to include a tuned amplifier stage as shown in Fig. 4.81. Calculate the required Q of the tuned circuit, $TC_2$

Answer: Q of $TC_2 = 5.17$

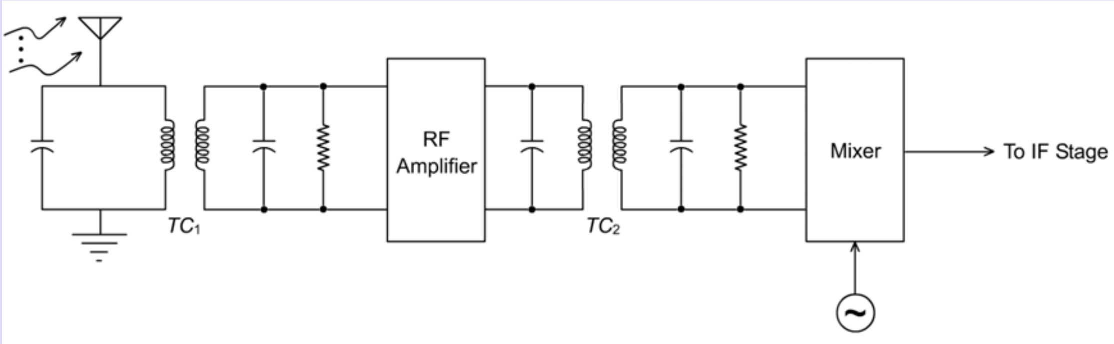



Fig. 4.81: Superhet with RF amplifier

## Appendix A4.1: Analysis of ED with Tone Modulation

Consider ED circuit shown in Fig. 4.42 which is redrawn in Fig. A4.1.

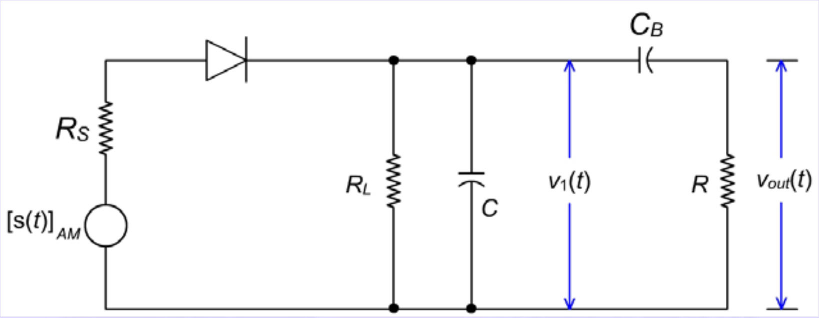



Fig. A4.1: Envelope detector

Let $\left[s(t)\right]_{AM} = \left[1 + \mu \cos(2\pi f_m t)\right] \cos(2\pi f_c t)$

We assume $\mu < 1$ so that $\left[1 + \mu \cos(2\pi f_m t)\right]$ is positive for all $t$. Then, the envelope quantity is, $A(t) = \left[1 + \mu \cos(\omega_m t)\right]$. We shall derive on upper bound on $\mu$ in terms of $R_L C$ and $\omega_m$ so that the detector is able to follow the envelope for all $t$.

Consider the waveform $v_1(t)$ shown in Fig. A4.2. The value of the envelope $A(t)$ at $t = t_0$,

$$A(t_0) = \left[1 + \mu \cos(\omega_m t_0)\right]$$

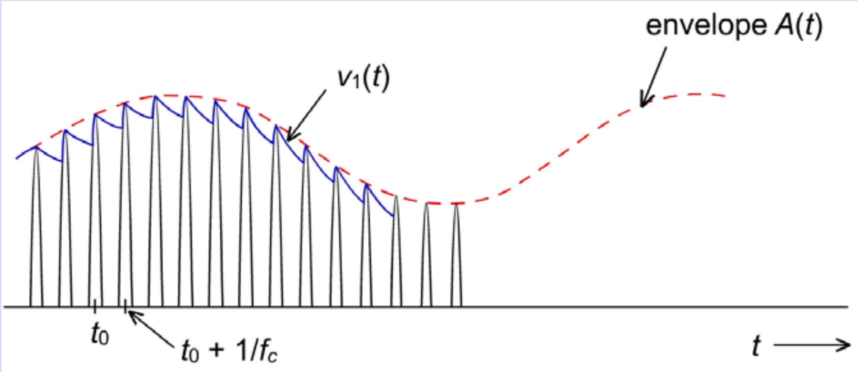



Fig. A4.2: Waveforms of ED of Fig. A4.1

Let $v_1(t_0) = A(t_0)$. Note that $v_1(t_0)$ is the voltage across the capacitor at $t = t_0$. Assuming that the capacitor discharges until the next positive peak in the carrier cycle, we have,

$$v_1\left(t_0 + \frac{1}{f_c}\right) = v_1(t_0)\, e^{-\frac{1}{R_L C}\frac{1}{f_c}}$$

From Eq. 4.15(b), we have $R_L C f_c >> 1$. Hence

$$v_1\left(t_0 + \frac{1}{f_c}\right) \simeq \left(1 + \mu\cos(\omega_m t_0)\right)\left[1 - \frac{1}{R_L C f_c}\right]$$

$$\leq A\left(t_0 + \frac{1}{f_c}\right) = \left\{1 + \mu\cos\left[\omega_m\left(t_0 + \frac{1}{f_c}\right)\right]\right\}$$

$$\leq 1 + \mu\left[\cos(\omega_m t_0)\cos\left(\frac{\omega_m}{f_c}\right) - \sin(\omega_m t_0)\sin\left(\frac{\omega_m}{f_c}\right)\right]$$

Assuming $f_m << f_c$, we approximate

$$\cos\left[2\pi\frac{f_m}{f_c}\right] \simeq 1 \quad \text{and} \quad \sin\left[2\pi\frac{f_m}{f_c}\right] \simeq \frac{2\pi f_m}{f_c}$$

Hence,

$$\left(1 + \mu\cos(\omega_m t_0)\right)\left[1 - \frac{1}{R_L C f_c}\right] \leq 1 + \mu\cos(\omega_m t_0) - \frac{\mu\,\omega_m}{f_c}\sin(\omega_m t_0)$$

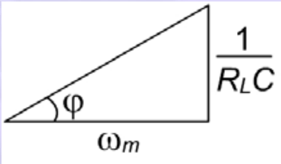
That is,

$$R_L C \leq \frac{1 + \mu \cos(\omega_m t_0)}{\mu \omega_m \sin(\omega_m t_0)} \quad \text{(A4.1a)}$$

or
$$\frac{1}{R_L C} \geq \frac{\mu \omega_m \sin(\omega_m t_0)}{1 + \mu \cos(\omega_m t_0)} \quad \text{(A4.1b)}$$

Rearranging Eq. A4.1(b), we have

$$\left(\mu \omega_m \sin(\omega_m t_0) - \frac{\mu}{R_L C} \cos(\omega_m t_0)\right) \leq \frac{1}{R_L C} \quad \text{(A4.2)}$$




Let $\varphi = \tan^{-1} \frac{1}{\omega_m R_L C}$ as shown

Then, $\cos\varphi = \dfrac{\omega_m}{\sqrt{\omega_m^2 + \left(\dfrac{1}{R_L C}\right)^2}}$ and, $\sin\varphi = \dfrac{\dfrac{1}{R_L C}}{\sqrt{\omega_m^2 + \left(\dfrac{1}{R_L C}\right)^2}}$

The inequality A4.2 can be written as

$$\mu D \cos\varphi \sin(\omega_m t_0) - \mu D \sin\varphi \cos(\omega_m t_0) \leq \frac{1}{R_L C}$$

where $D = \dfrac{1}{\sqrt{\omega_m^2 + \left(\dfrac{1}{R_L C}\right)^2}}$. That is,

$$R_L C \mu D \sin(\omega_m t_0 - \varphi) \leq 1 \quad \text{(A4.3)}$$

The inequality should hold even when $\sin(\omega_m t_0 - \varphi) = 1$. That is,

$$R_L C \mu D \leq 1$$

$$\mu \leq \frac{1}{R_L C D} = \frac{1}{\sqrt{1 + (\omega_m R_L C)^2}}$$

or
$$(R_L C) \leq \frac{1}{\omega_m} \frac{\sqrt{1 - \mu^2}}{\mu^2}$$

Evidently, we will not be able to demodulate, using the ED circuit, a tone modulated AM signal with $\mu = 1$.

Fig. A4.3 displays the experimentally generated demodulator output (top waveform) when the time constant $R_L C$ is of proper value so as to follow the envelope for all $t$ (modulating tone is shown at the bottom). Fig. A4.4 is the ED output (top waveform) when the time constant is too large. As can be seen from the figure, ED is not able to follow the negative half cycle of the tone fully, resulting in the clipping of a part of their cycle. (These waveforms are from Shreya's experimentor kit.)

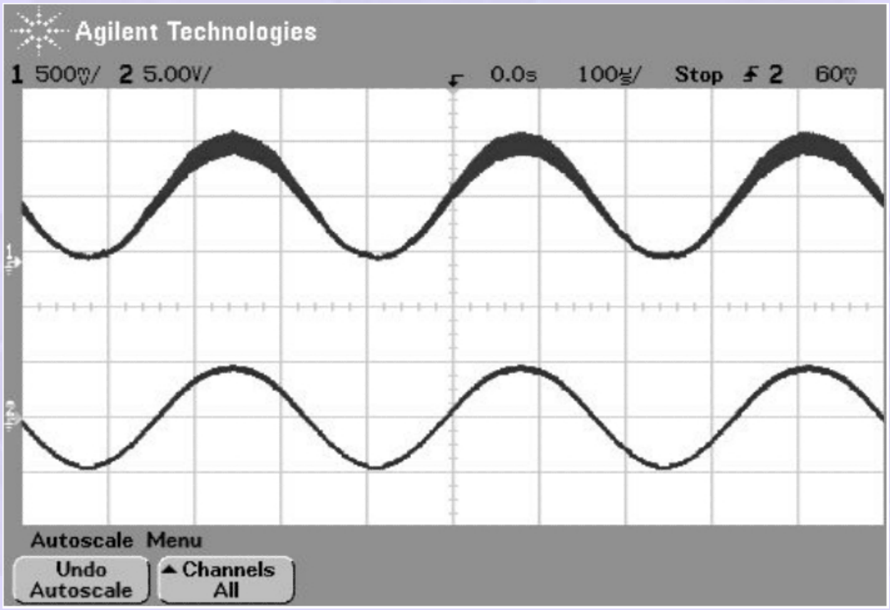



Fig. A4.3

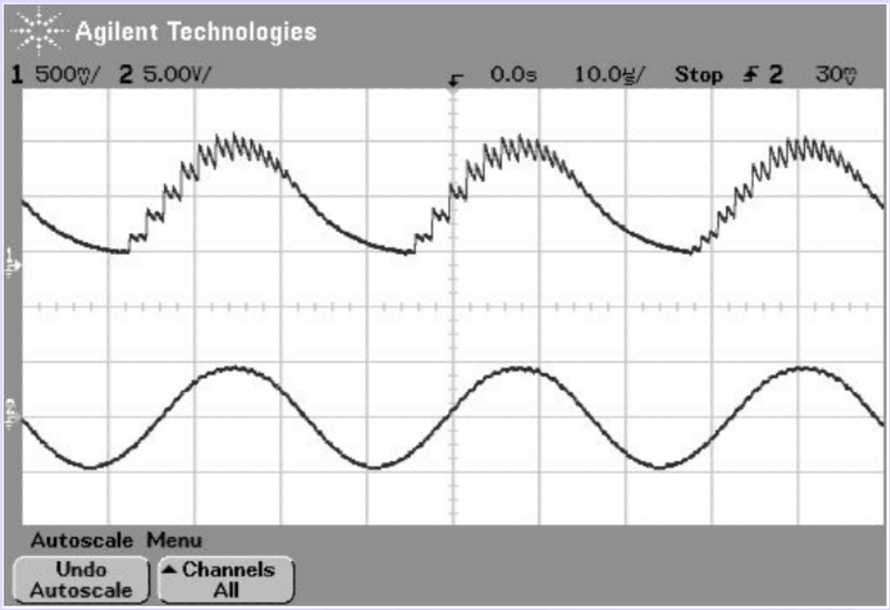



Fig. A4.4

**Exercise A4.1**

A 1 kHz square wave, switching between the levels $\pm 1$ V, amplitude modulates a carrier to a depth of 50 %. The parameters of the carrier are: $A_c = 1$ volt and $f_c = 1$ MHz.

a) Sketch the resulting AM signal.

b) Let the signal of (a) be the input of the ED of Fig. A4.1. Sketch the voltage across the capacitor C for the following cases:

i) $R_L C = 25 \, \mu\text{sec}$

ii) $R_L C = 2 \, m\text{sec}$

## References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### Some suggested references


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