Step-by-Step Guide: “Price Optimization Considering Demand” and “Price Optimization Example”

Cost and Economics in Pricing Strategy, Week 3

As you watch the videos, use the general tips as well as those tied to specific timecodes in the videos. You may wish to take careful notes throughout and save any calculations you make, as they may help when you use real-life data to perform calculations in the second video.

First, you may wish to begin by reviewing the “Calculus: Taking a Derivative” video. Once you are comfortable with the content in that video, you are ready to get started. If you need additional help, check out Khan Academy’s free calculus tutorials on taking derivatives.

“Price Optimization Considering Demand” Video

0:41
Here’s the formula for a profit function:

$$\pi = (P - MC)Q$$

π = profit
P= price
MC = marginal cost
Q = quantity sold

1:20
Next, Ron substitutes the demand function for Q:

$$\pi = (P - MC)Q$$
$$\pi = (P - MC)(10 - 2P)$$

You may remember the FOIL method from your studies of algebra. FOIL is a mnemonic device that stands for First, Outside, Inside, Last and prompts how to use the distributive property to multiply the terms of the binomials. In this problem, Ron multiplied the terms of the binomials in that order to find:

$$\pi = 10P - 2P^2 - 10MC + 2MCP$$

If you need more help understanding the FOIL method and other algebra used in these calculations, take a look at the free Khan Academy algebra tutorials.

2:07
Next, Ron notes that you’ll use the simplified profit function:

$$\pi = 10P - 2P^2 - 10MC + 2MCP$$

He explains that you’re going to maximize the function related to price by taking the derivative, setting it equal to zero, and finding the best price for this particular situation.

2:26

Ron shows you that the derivative of the profit function with respect to price is $10 - 4P + 2MC = 0$. Here are the steps you'll take to get to that point:

$$\pi = 10P - 2P^2 - 10MC + 2MCP$$

$$\frac{\partial \pi}{\partial P} = 0$$

You might remember from your studies of calculus that this is called taking a partial derivative. If you would like an overview of how to take partial derivatives when there are multiple variables, watch this [YouTube tutorial].

When taking a derivative with respect to one variable, you treat any other variables as constants. In this case, we'll treat MC as a constant.

$$10P - 2P^2 - 10\mathbf{MC} + 2\mathbf{MC}P = 0$$

When we follow the steps with this in mind, we see that 10P becomes 10 and $2P^2$ becomes 4P using the standard procedures for finding a derivative with respect to P. Treating MC as a constant results in 10(0) for 10MC and 2MC(1) for 2MCP. We'll end up with:

$$10 - 4P + 10(0) + 2MC(1) = 0$$
$$10 - 4P + 2MC = 0$$

You'll need to make similar calculations later in the video and again on the quiz when you use real-life data to perform price optimizations, so be sure to review your notes and understand these steps.

2:41

Let's look at another way to solve for the profit maximizing price. Ron explains that we know the marginal cost (MC) is equal to 1, so we can rewrite and solve this multivariable equation with respect to P:

$$10 - 4P + 2MC = 0$$

$$10 + 2MC = 4P$$

Since we know MC = 1, we can solve for P:

$$10 + 2(1) = 4P$$

$$10 + 2 = 4P$$

$$12 = 4P$$

$$P = \$3$$

Now that we know the optimal price is \$3, follow along with the video as Ron uses the demand function to determine how many units will be sold at the optimal price:

Q* = 10 - 2P

Q* = 10 - 2(3)

Q* = 10 - 6

Q* = 4 units sold

Now calculate the profit:

Profit = Quantity * (Price - Cost)

Profit = Q * (P - C)

Profit = 4 * (3 - 1)

Profit = 4 * 2

Profit = $8

Take a moment to look back over everything. Can you follow the steps to optimize the price using the demand information? If not, take some time to review Ron's video as well as the steps and videos linked here before you move on to the rest of the video.

6:09

Let's take a moment to review all of the terms in the demand model formula:

$$Q_x = a + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + e_i$$

Q = dependent variable of quantity sold
a = constant (equals quantity of X when all other variables equal 0)
X_1 = my own price
X_2 = price of a related good
X_3 = measure of disposable income
X_4 = trend variable
e_1 = error term

7:05

Let's make sure we can identify all of the numbers we'll be using in the demand model:

Q = dependent variable of quantity sold
a = constant = intercept coefficient = 779.4
X_1 = chuck price = to be determined
X_2 = price of a related good = chicken price = $1.75
X_3 = measure of disposable income = $10,000
X_4 = trend variable = the month after the data ends = 56

Let's look at how Ron fills in all of the numbers in the demand model using the beef data:

$$Q = 779.4 - 50.2 * P - 53.6 * P_C - .05 * \text{Income} + 1.67 * \text{Trend}$$

When we estimate the model, the expected value of the error term equals zero. You don't have to write the error term in your actual calculations.

Want a deeper understanding of how Ron got all of these numbers? Go back to the "Price Elasticities" video and start watching at 3:17 when Ron describes the linear model, also known as the demand model. Watch until 6:36.

Then you can watch this part of the video again and make note of how Ron uses the coefficients and other data for the variable values. A great way to practice is to write out the formula and plug the numbers in yourself.

"Price Optimization Example" Video

1:14

Take a closer look at how Ron fills in the variables from the "Building the Demand Function" slide using the real-life data in the price optimization scenario.

Ron simplified until he had only a constant and one variable. Here are the steps you'd take to get to that point:

$$Q = 779.4 - 50.2 * P - 53.6 * 1.75 - .05 * 10{,}000 + 1.67 * 56$$

$$Q = 779.4 - 50.2 * P - 93.8 - 500 + 93.52$$

$$Q = 279.12 - 50.2 * P$$

Next, Ron uses the same profit function we used before:

$$\text{Profit} = (\text{Price} - \text{Cost}) * \text{Quantity}$$

$$\text{Profit} = (P - C) * Q$$

Then he fills in the variables using the wholesale cost of $1.50 and the collapsed demand function for quantity:

$$\text{Profit} = (P - \$1.50) * (279.12 - 50.2 * P)$$

Next, he uses algebra to rewrite and collect the terms. Here's how you'd use the FOIL method to multiply:

$$279.12 * P - 50.2 * P^2 - 418.68 + 75.3 * P$$

$$- 418.68 + 354.42 * P - 50.2*P^2$$

Finally, Ron takes the partial derivative to find the optimal price:

$$- 418.68 + 354.42 * P - 50.2*P^2$$

$$\frac{\partial \pi}{\partial P} = 354.42 - 100.4*P = 0$$

$$354.42 = 100.4P$$

$$P = \$3.53$$

If you need a refresher on how to treat constants and variables when taking a partial derivative, scroll back to the notes at 2:07 for the “Price Optimization Considering Demand” video.

Congratulations--you are ready to calculate optimal prices! Be sure to review your notes and the recommended videos if any of the calculations are still unclear.