Generating Confidence Intervals

eCornell

7/28/2021

This R Markdown file demonstrates how to use bootstrap methods with regression analysis to find the confidence interval (CI) around a regression line. The example here finds the 95% CI of the best-fit line that passes through the scatterplot of prestige vs. education. These data are from the Prestige data set.

Step 1: Load the data and define colors.

```
knitr::opts_chunk$set(echo = TRUE)
# The Prestige data set is available in the carData library
library(carData)
# Load the Prestige data set
data(Prestige)
# Exclude any observations that do not have an entry in the type column
Prestige = Prestige[!is.na(Prestige$type),]

#eCornell Hex Codes:
crimson = '#b31b1b' #Crimson
lightGray = '#cecece' #lightGray
darkGray = '#606366' #darkGray
skyBlue = '#92b2c4' #skyblue
gold = '#fbb040' #gold
ecBlack = '#393f47' #ecBlack
```

Step 2: Generate many bootstrapped data sets.

In the code below, you'll generate a bootstrapped data set 10,000 times, and store the values of correlations, intercepts, and slopes for each data set.

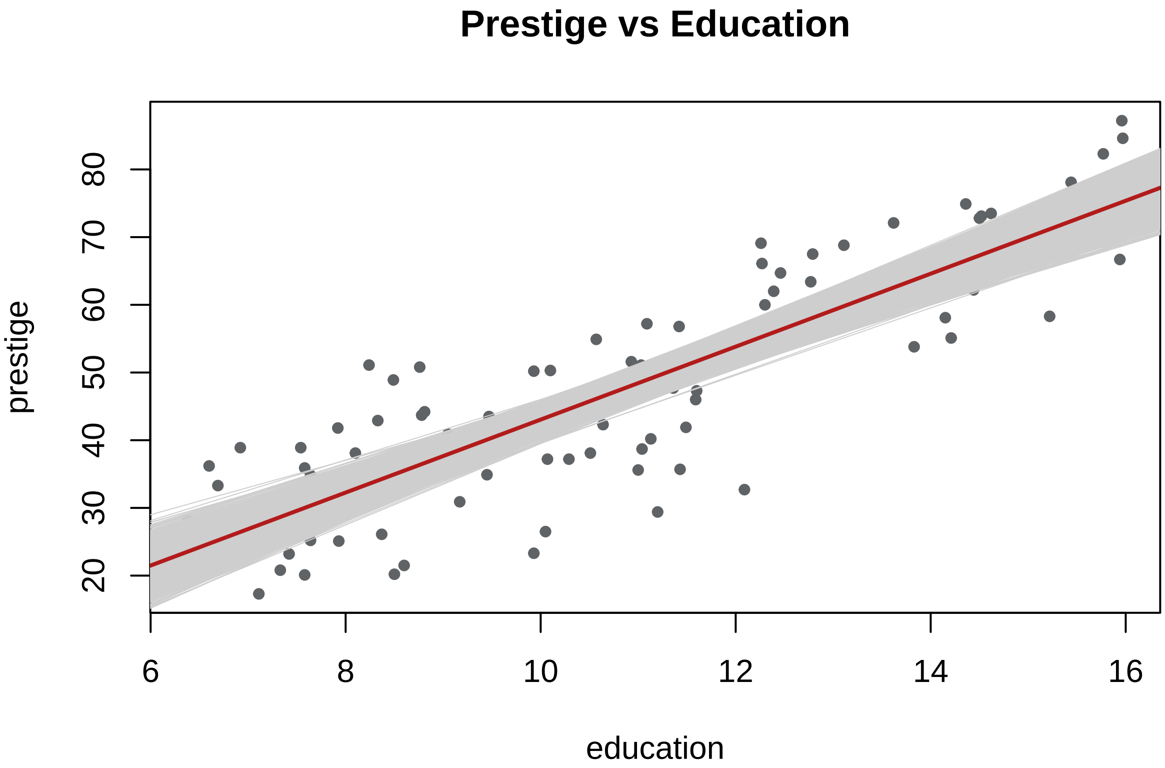
```
set.seed(1) # Set the seed for reproducibility
B = 10000   # Number of bootstrapped data sets
corr.boot = rep(0, B) # vector to store correlation coefficients
a.boot = rep(0,B)     # vector to store intercept values
b.boot = rep(0, B)    # vector to store slope values

# Create plot of observed data:
plot(Prestige$education, Prestige$prestige,
     pch = 20, col = darkGray,
     xlab = 'education', ylab = 'prestige',
     main = 'Prestige vs Education')

# Use a for loop to generate B bootstrapped data sets:
for (b in 1:B){
  boot.id = sample(98, replace = TRUE)
  Prestige.boot = Prestige[boot.id,]

# Store coefficients on each bootstrapped sample:
  corr.boot[b] = cor(Prestige.boot$education, Prestige.boot$prestige)
  fit.boot <- lm(prestige ~ education, data = Prestige.boot)
  a.boot[b] = fit.boot$coefficients[1]
  b.boot[b] = fit.boot$coefficients[2]

# Visualize each bootstrapped regression line on the plot in gray:
  abline(fit.boot, lwd = 0.5, col = lightGray)
}
# Add the regression line for the observed data in red
fit <- lm(prestige ~ education, data = Prestige)
abline(fit, col = crimson, lwd = 2)
```



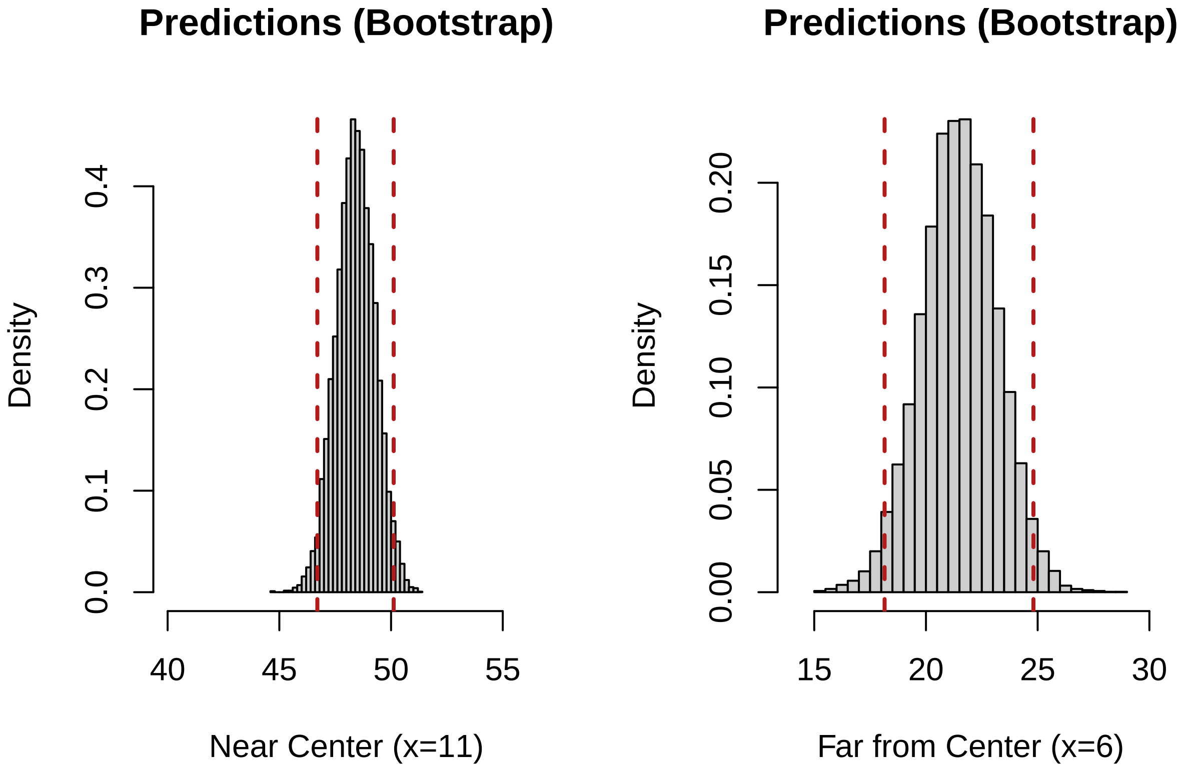
Step 3: Examine bootstrap values.

You can use the vectors of correlation and regression coefficients you created above to examine how uncertainty varies as X changes. Use the following code to compare the variation when X = 6 with the variation when X = 11:

```
par(mfrow = c(1,2))

# fitted values near the center of data set
prestige.pred.11 = a.boot + 11*b.boot
hist(prestige.pred.11, breaks = 30, freq = FALSE, col = lightGray,
     xlim = c(40,56),
     main = 'Predictions (Bootstrap)',
     xlab = 'Near Center (x=11)')
abline(v = quantile(prestige.pred.11, probs = c(0.025, 0.975)),
       lty = 2, col = crimson, lwd = 2)

# fitted values near the left end of data set
prestige.pred.6 = a.boot + 6*b.boot
hist(prestige.pred.6, breaks = 30, freq = FALSE, col = lightGray,
     xlim = c(14, 30),
     main = 'Predictions (Bootstrap)',
     xlab = 'Far from Center (x=6)')
abline(v = quantile(prestige.pred.6, probs = c(0.025, 0.975)),
       lty = 2, col = crimson, lwd = 2)
```



```
par(mfrow = c(1,1))
```

Step 4: Generate the 95% confidence band.

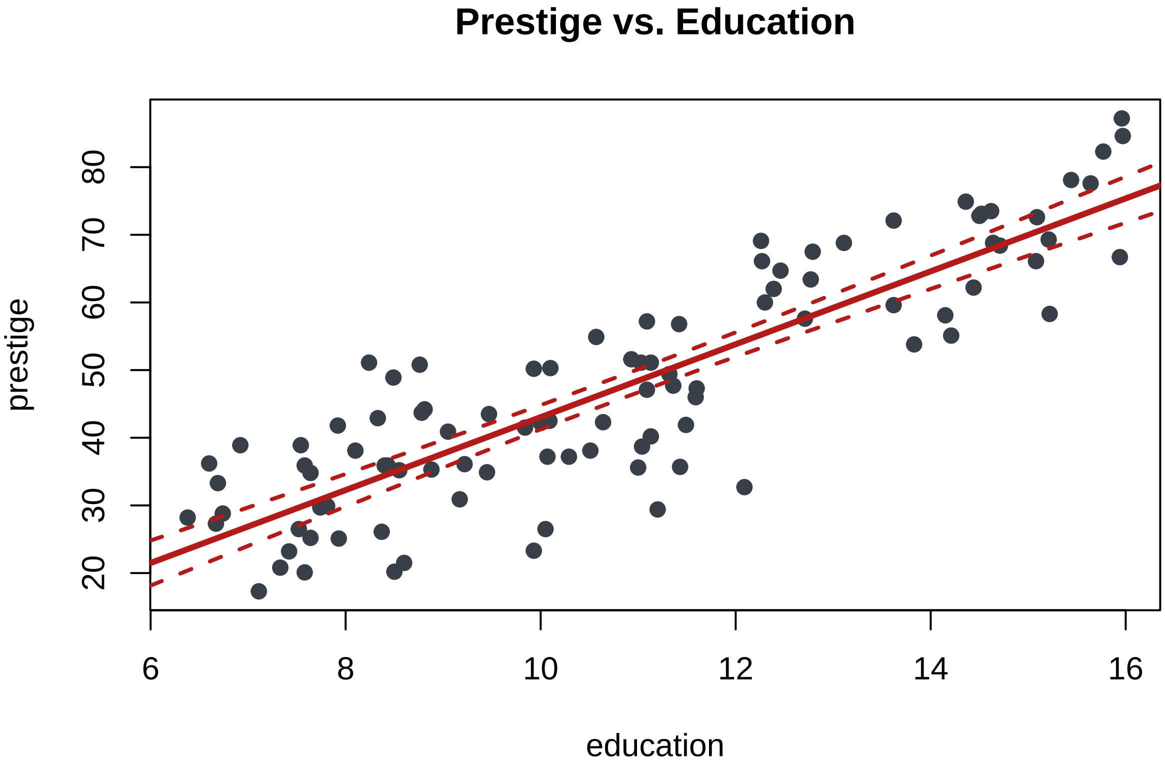
Repeatedly calculating such 95% intervals for every value of X, we get a 95% *confidence band* around the regression line.

```
# Create a vector of points across the range of X values
x.all = seq(6, 18, by = 0.01)
# Create a vector to store the upper confidence intervals
ci.upper = rep(0, length(x.all))
# Create a vector to store the lower confidence intervals
ci.lower = rep(0, length(x.all))


# Use a for loop to calculate the 95% CI at each point in x.all:
for (i in 1:length(x.all)){
  x = x.all[i]
  prestige.pred.x = a.boot + x*b.boot
  ci.upper[i] = quantile(prestige.pred.x, probs = 0.975)
  ci.lower[i] = quantile(prestige.pred.x, probs = 0.025)
}

# Plot prestige vs. education and add the regression line:
plot(prestige ~ education, data = Prestige, pch = 19, col = ecBlack,
     main = "Prestige vs. Education")
fit.prestige <- lm(prestige ~ education, data = Prestige)
abline(fit.prestige, col = crimson, lwd = 3)

# Add the 95% CIs to the plot:
lines(x.all, ci.upper, lty = 2, col = crimson, lwd = 2)
lines(x.all, ci.lower, lty = 2, col = crimson, lwd = 2)
```


Prestige vs. Education
prestige
education