CIS532v2:
Estimating Probability Distributions

Live Session 1

Ernest Green

Today's Live Session

CIS532v2 - Course Info

Bayes Optimal Classifier, MLE, Naive Bayes

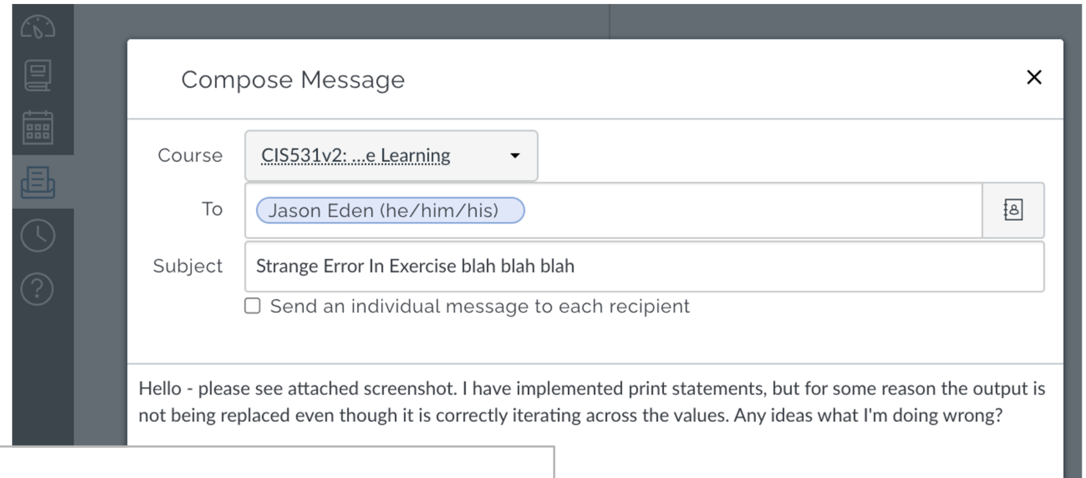
Pre-project coding exercises

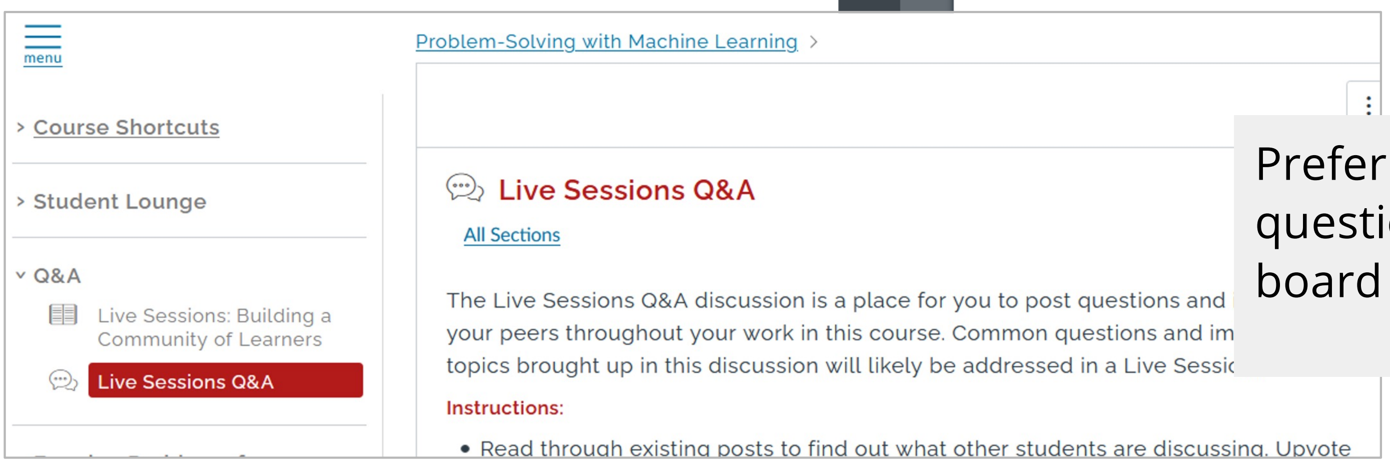
Reminder about avoiding loops whenever possible

Ernest Green, Course Facilitator

How to get assistance:

- Private message me through Canvas





Preferred: You can also post questions to the Q&A discussion board

Additional Resources

Lecture 7 "Estimating Probabilities from Data: Maximum Likelihood Estimation" -Cornell CS4780 SP17

https://www.youtube.com/watch?v=RIawrYLVdIw

Machine Learning Lecture 8 "Estimating Probabilities from Data: Naive Bayes" -Cornell CS4780 SP17

https://www.youtube.com/watch?v=pDHEX2usCS0

Machine Learning Lecture 9 "Naive Bayes continued" -Cornell CS4780 SP17

https://www.youtube.com/watch?v=VDK0nkjFh5U

Machine Learning Lecture 10 "Naive Bayes continued" -Cornell CS4780 SP17

https://www.youtube.com/watch?v=rqB0XWoMreU

Machine Learning Lecture 11 "Logistic Regression" -Cornell CS4780 SP17

https://www.youtube.com/watch?v=GnkDzIOxfzI

Avoid the Appearance of Plagiarism

Key points:

- **Show your work -** comment out, do not delete, mistakes, print statements, test cells, etc. Messy notebook that shows original thought is better than pristine code that happens to look like someone else's work
- **Reference helpful websites** - put links to discussion boards, Q&A forums (Stack Overflow, etc.) that contained information you found useful in developing your code
- **When in doubt, ask your facilitator!**

Statistics Refreshers

- Bayes’ Rule

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

- Chain Rule

$$P(A, B, C, D) = P(A)P(B|A)P(C|A, B)P(D|A, B, C)$$

Bayes Basics

The quick proof of Bayes' theorem

https://www.youtube.com/watch?v=U_85TaXbeIo

Bayes theorem

https://www.youtube.com/watch?v=HZGCoVF3YvM

Naive Bayes, Clearly Explained!!!

https://www.youtube.com/watch?v=O2L2Uv9pdDA

Distributions

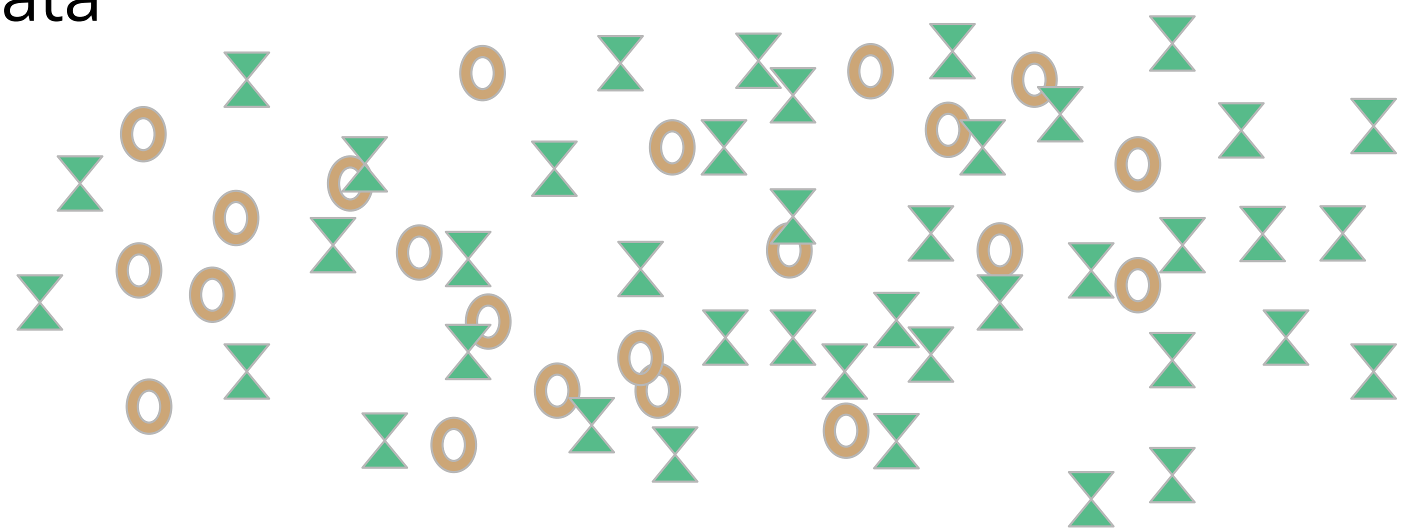
- Gaussian (a.k.a. Normal)
  - Continuous outcome variables
  - Bell curve, symmetric at center
  - Values for mean = mode = median

- Binomial
  - Two discrete outcome variables (ex: coin flips)

- Other distribution types
  https://www.unf.edu/~cwinton/html/cop4300/s09/class.notes/DiscreteDist.pdf

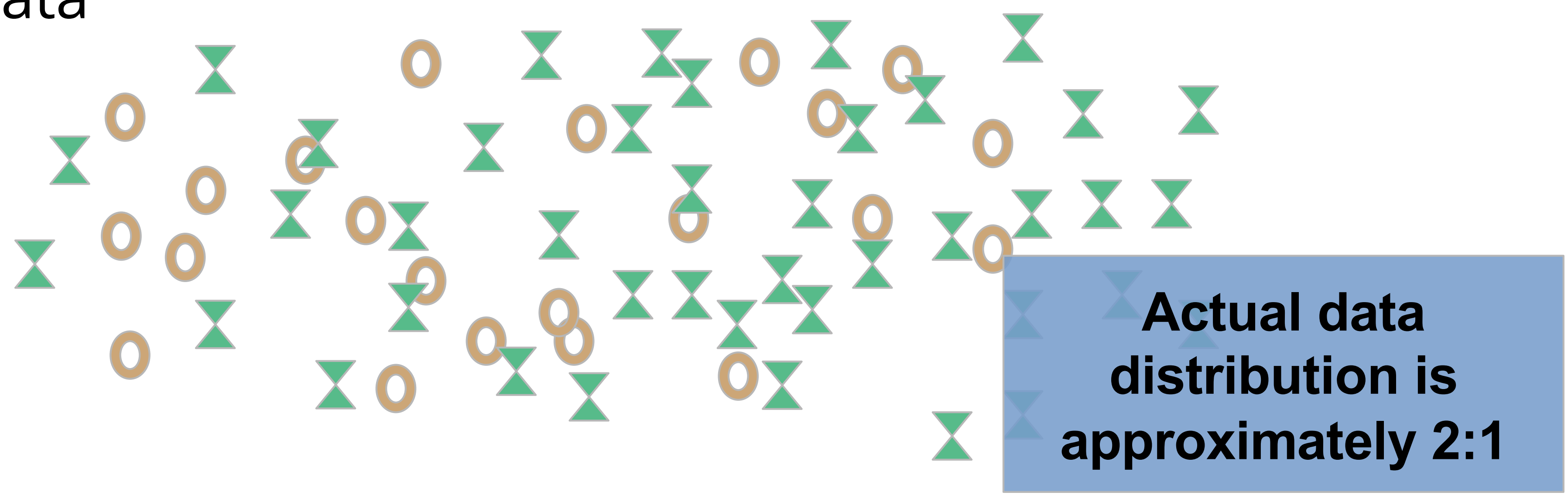
Estimating Distributions from Data

- **Bayes Optimal Classifier** - Bayes Theorem applied, but only works if you know the actual distribution of all data



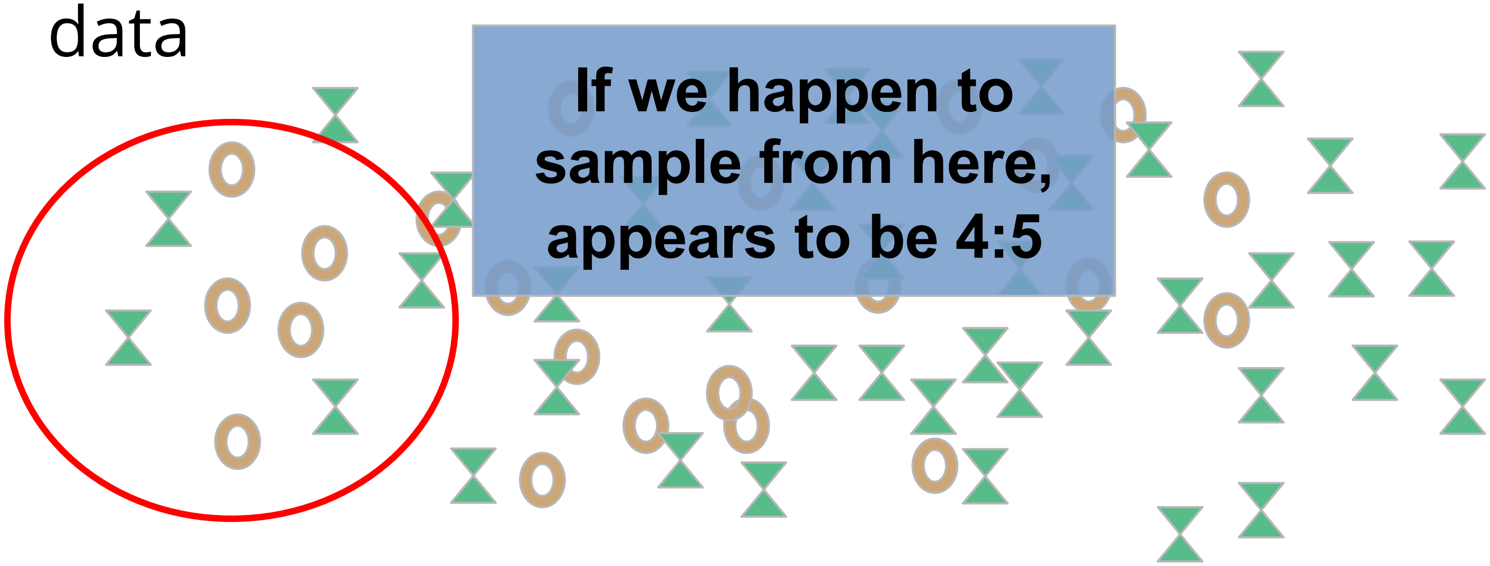
Estimating Distributions from Data

- **Bayes Optimal Classifier** - Bayes Theorem applied, but only works if you know the actual distribution of all data



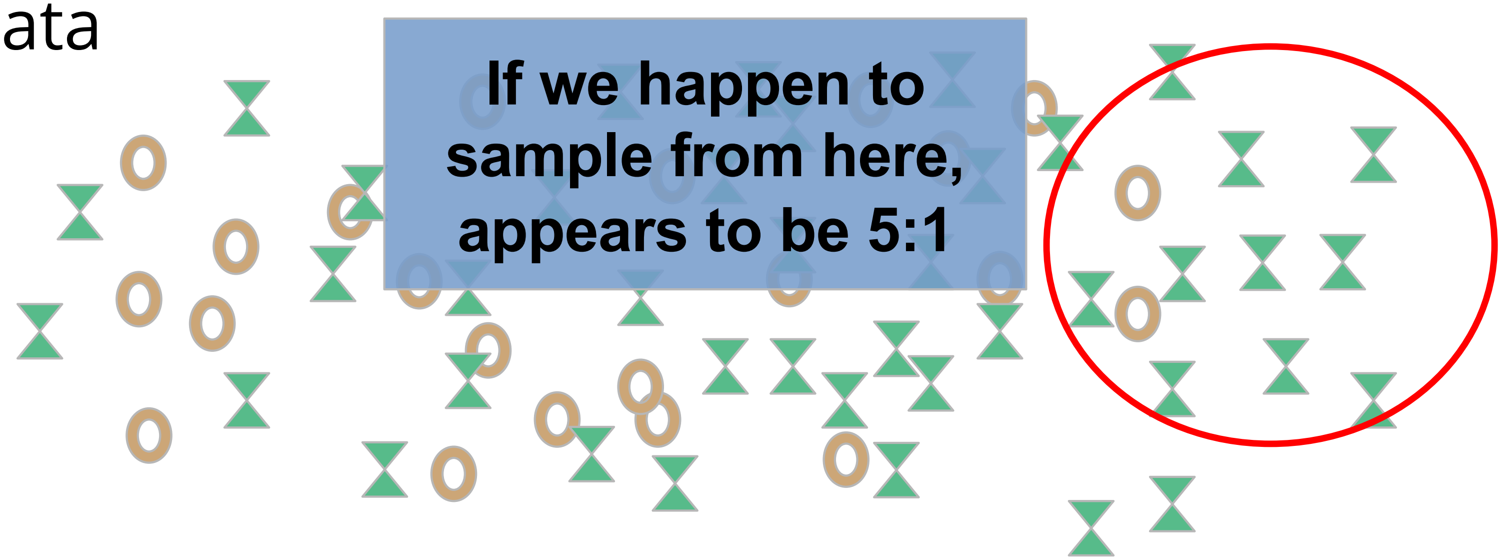
Estimating Distributions from Data

- **Bayes Optimal Classifier** - Bayes Theorem applied, but only works if you know the actual distribution of all data



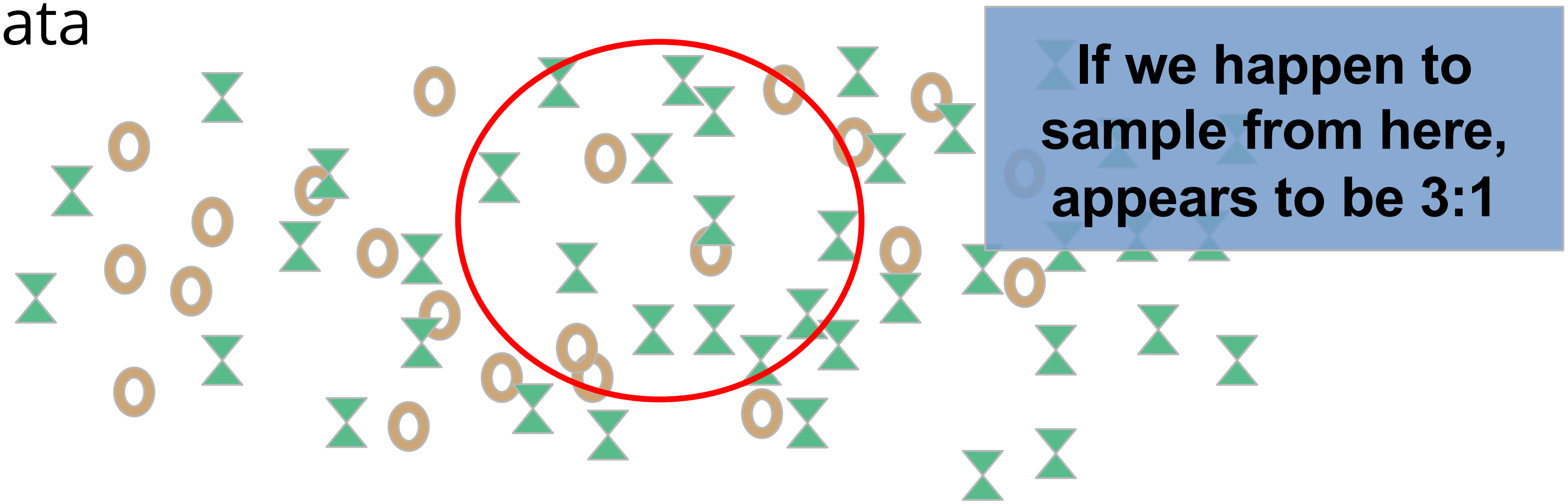
Estimating Distributions from Data

- **Bayes Optimal Classifier** - Bayes Theorem applied, but only works if you know the actual distribution of all data



Estimating Distributions from Data

- **Bayes Optimal Classifier** - Bayes Theorem applied, but only works if you know the actual distribution of all data



Estimating Distributions from Data

- **Bayes Optimal Classifier** - Bayes Theorem applied, but only works if you know the actual distribution of all data
    - ...and you never actually know the underlying distribution of your data

Maximum Likelihood Estimation

Step 1: Estimate the distribution of your data based on known parameters

Coin flip: it's either heads or tails, so binomial distribution

Take the data you observe, tweak those parameters until what you observed becomes as likely as possible (becomes maximized)

Estimating Theta

In a binomial distribution, theta is the likelihood of a target outcome

The probability of theta for given data approximately equals theta to the power of an observed outcome times 1-theta to the power of the other outcome

$$P_\theta(Data) \approx \theta^h(1-\theta)^t$$

Simplifying Theta

But first… a note about log values

The Case for Log Values - Simplification

Log (natural log) is the inverse of Exponential

Has some interesting properties that can be used to simplify calculations:

log(a * b) = log(a) + log(b)

log(a / b) = log(a) - log(b)

$$\theta^h(1-\theta)^t \quad == \quad h\log(\theta) + t\log(1-\theta)$$

Simplifying Theta - First Derivative

To maximize a function, take the first derivative and set it equal to zero

$$h\log(\theta) + t\log(1-\theta) \rightarrow \frac{h}{\theta} + \frac{t}{1-\theta}(-1) = 0$$

If you're rusty on derivatives of logs (note - log and natural log are used interchangeably in portions of the course material, and numpy.log() refers to natural log): https://www.youtube.com/watch?v=Dp9sgIvaKPk

Simplifying Theta - Solve for Theta

$$\frac{h}{\theta} + \frac{t}{1-\theta}(-1) = 0$$ -> $$\theta = \frac{h}{t+h}$$

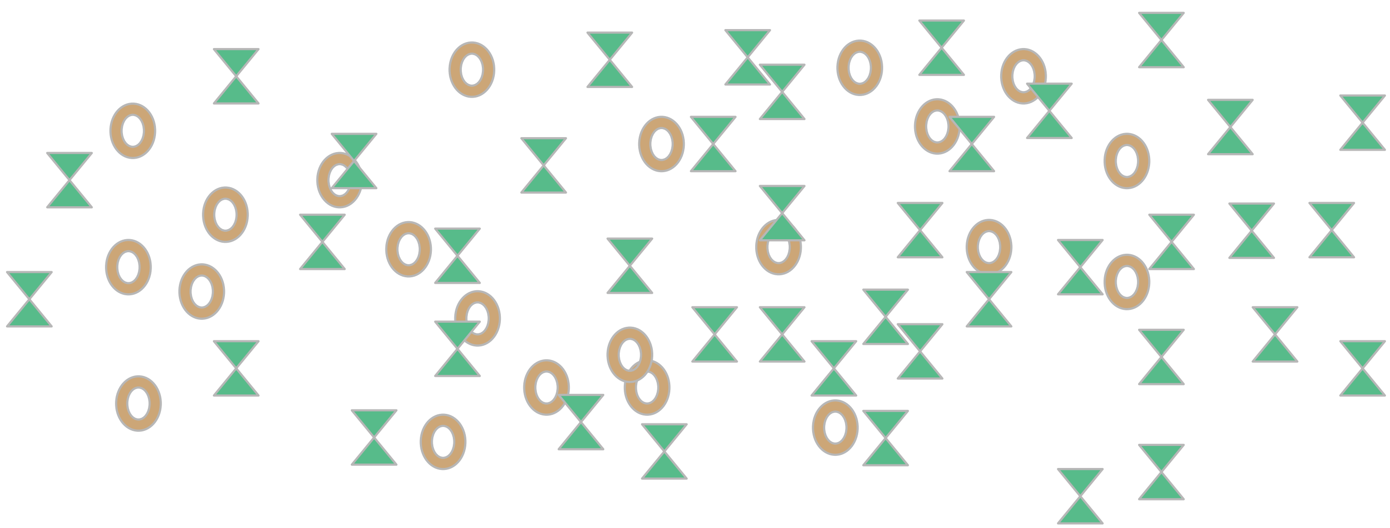
Simplifying Theta - Solve for Theta

$$\frac{h}{\theta} + \frac{t}{1-\theta}(-1) = 0 \quad \textbf{->} \quad \theta = \frac{h}{t+h}$$

...which makes intuitive sense, but now we have the mathematical underpinning.

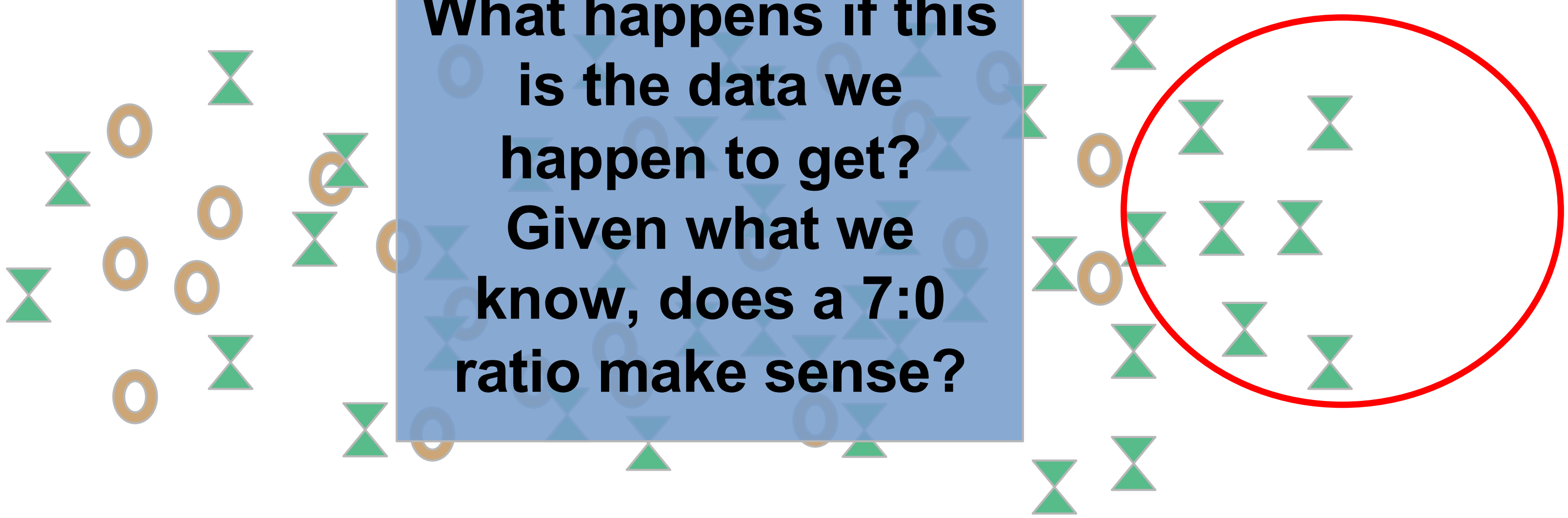
Smoothing - Why?

- Avoids “divide by 0” or “multiply by 0” errors
- Corrects for non-representative samples



Smoothing - Accounting for Unlikely Samples

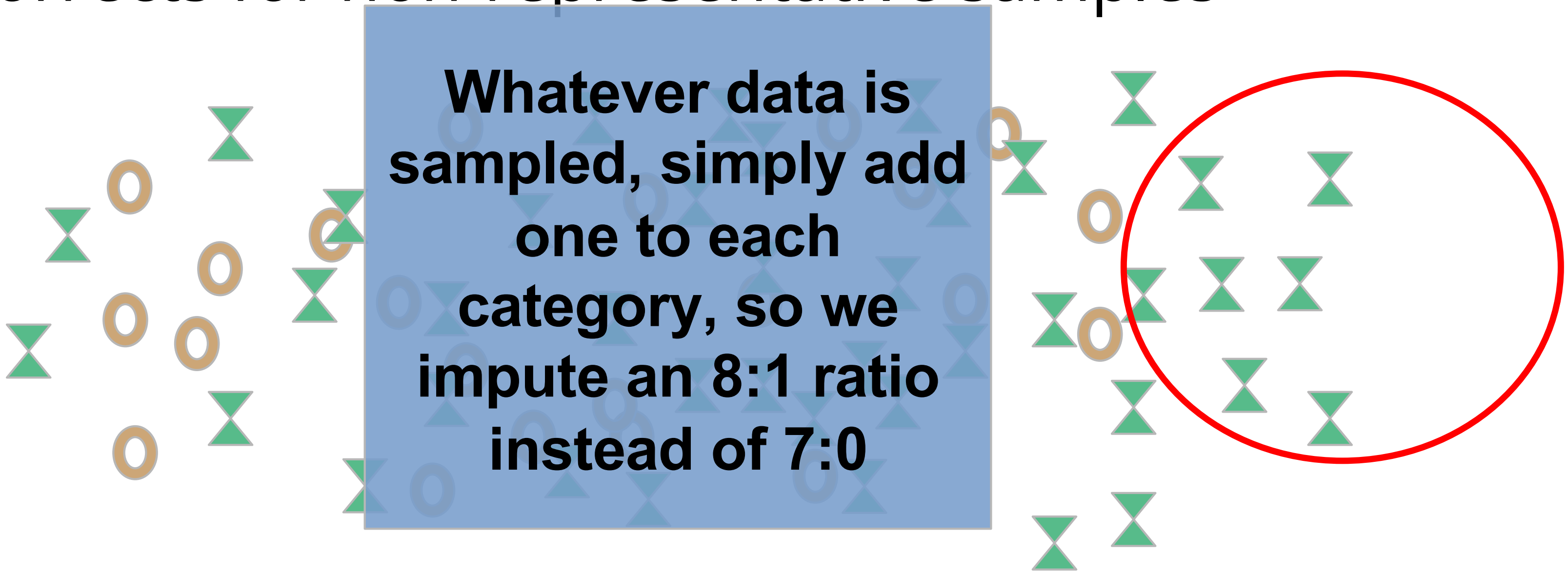
- Avoids “divide by 0” or “multiply by 0” errors
- Corrects for non-representative samples



What happens if this is the data we happen to get? Given what we know, does a 7:0 ratio make sense?

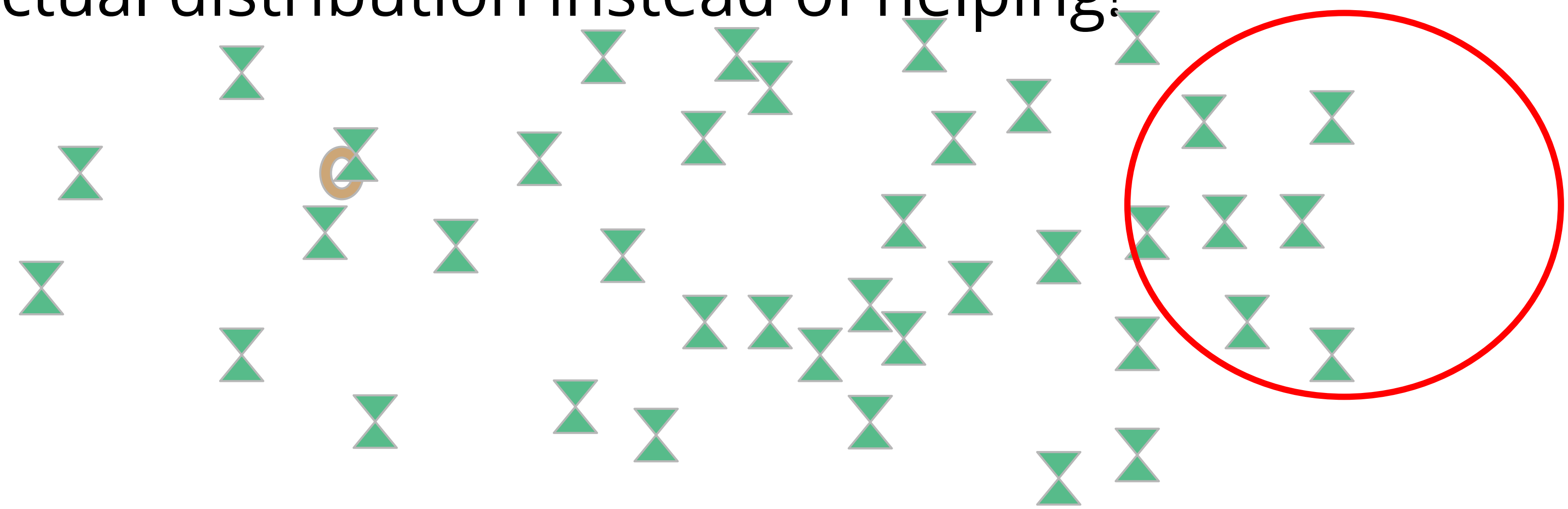
Smoothing - Laplace (+1)

- Avoids “divide by 0” or “multiply by 0” errors
- Corrects for non-representative samples



Smoothing - Beware of Assumptions

- If our data ratio really was 40:1 or similar, our +1 smoothing would hinder our progress in getting to the actual distribution instead of helping!



Additional Reading

- Maximum A Posterior (MAP)
    - https://wiseodd.github.io/techblog/2017/01/01/mle-vs-map/
- Basically: MLE makes distribution assumptions, whereas MAP assumes prior knowledge is used to determine theta value

Classification with the Naive Bayes Algorithm

- MLE Curse of Dimensionality
  - The higher d, lower chance of there being an exact match
  - Thus, lots of cases where no probability exists
- Naive Bayes Assumption
  - “Naively” assume that all data points conditionally independent, even if you know they’re not
  - Allows us to use chain rule to determine probabilities - exact match not required for labeling

Naive Bayes in Action

Binary features:

- **C** = “Guy wears a cape.”
- **M** = “Guy wears a mask.”
- **U** = “Guy wears his underwear outside his pants”

Labels:

- **G** = “Guy is good”
- **B** = "Guy is bad"

Class Prior Probabilites

P(G)

P(B)

Probabilities of Features for "Good" Class

P(C|G)

P(M|G)

P(U|G)

Probabilities of Features for "Bad" Class

P(C|B)

P(M|B)

P(U|B)

Naive Bayes in Action

Question 1: Estimate the probability that Suzie's date wears a mask and cape, given that he is good, using the Naive Bayes assumption.

P (M,C|G)

Multiply P (M|G) by P (C|G)

Question 2: Before Suzie meets her date, what's the probability he is wearing a mask and cape (she does not know if he is good or bad)? (Use NB Assumption)

P (M,C) = P(M,C|G)P(G) + P(M,C|B)P(B)

Naive Bayes in Action - Question 3

Using Naive Bayes assumption and Bayes' Rule, predict the probability that he is good if he wears a mask and a cape.   Hint: **P (G|M,C)**

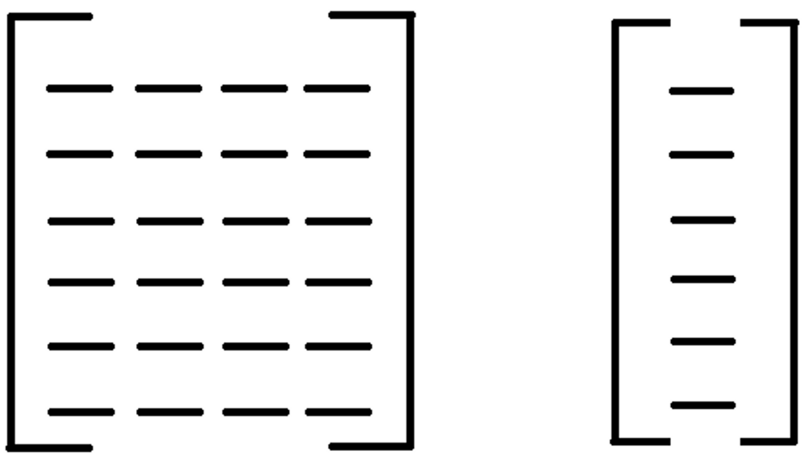
$$P(G|M,C) = \frac{\left(P(M,C|G)\right)\left(P(G)\right)}{P(M,C)}$$

[Machine Learning Lecture 9 "Naive Bayes continued" -Cornell CS4780 SP17](https://www.youtube.com/watch?v=VDK0nkjFh5U)

https://www.youtube.com/watch?v=VDK0nkjFh5U

Reminder: Avoid Loops, Utilize Indexing and Slicing

Matrix X Matrix Y



```
ind = X[condition]
subset_Y = Y[ind]

I, D = findknn(xTr, xTe, k)

yTe = yTr[I]


ind = Y[condition]
subset_X = X[ind]
```

Questions?

Thank You For Attending!

End of Live Session 1