SOLUTIONS

Module Two

1.) Consider the vectors: $\mathbf{u} = \begin{pmatrix} 10 \\ 4 \end{pmatrix}$ $\mathbf{v} = \begin{pmatrix} -3 \\ -5 \end{pmatrix}$ $\mathbf{w} = \begin{pmatrix} -2 \\ 5 \end{pmatrix}$ $\mathbf{x} = \begin{pmatrix} 1 \\ -3 \end{pmatrix}$

Compute the following:

a.) $\mathbf{u} \cdot \mathbf{w}$

$$\boxed{\mathbf{u} \cdot \mathbf{w} = 10 \cdot (-2) + 4 \cdot 5 = -20 + 20 = 0}$$

b.) $\mathbf{u} \cdot \mathbf{x}$

$$\boxed{\mathbf{u} \cdot \mathbf{x} = 10 \cdot 1 + 4 \cdot (-3) = 10 - 12 = -2}$$

c.) $\mathbf{v} \cdot \mathbf{w}$

$$\boxed{\mathbf{v} \cdot \mathbf{w} = (-3) \cdot (-2) + (-5) \cdot 5 = 6 - 25 = -19}$$

d.) $\mathbf{v} \cdot \mathbf{x}$

$$\boxed{\mathbf{v} \cdot \mathbf{x} = (-3) \cdot 1 + (-5) \cdot (-3) = -3 + 15 = 12}$$

e.) $\mathbf{u} \cdot \mathbf{u}$

$$\boxed{\mathbf{u} \cdot \mathbf{u} = 10 \cdot 10 + 4 \cdot 4 = 100 + 16 = 116}$$

f.) $\mathbf{u} \cdot \mathbf{v}$

$$\boxed{\mathbf{u} \cdot \mathbf{v} = 10 \cdot (-3) + 4(-5) = -30 - 20 = -50}$$

2.) Consider the vectors: $\mathbf{u} = \begin{pmatrix} 10 \\ 4 \end{pmatrix}$ $\mathbf{v} = \begin{pmatrix} -3 \\ -5 \end{pmatrix}$ $\mathbf{w} = \begin{pmatrix} -2 \\ 5 \end{pmatrix}$ $\mathbf{x} = \begin{pmatrix} 1 \\ -3 \end{pmatrix}$

a.) Which vectors are orthogonal to $\mathbf{u}$? Check the correct answer.

- ☐ $\mathbf{u}$
- ☐ $\mathbf{v}$
- ☐ $\mathbf{x}$
- ☑ $\mathbf{w}$

To answer this question, refer to the previous exercise to see which vectors have a dot product with $\mathbf{u}$ that equals 0. This is the case only for $\mathbf{w}$. Therefore, $\mathbf{w}$ is orthogonal to $\mathbf{u}$.

b.) Which vectors are orthogonal to $\mathbf{v}$? Check the correct answer.

- ☐ $\mathbf{u}$
- ☐ $\mathbf{v}$
- ☐ $\mathbf{x}$
- ☐ $\mathbf{w}$
- ☑ None of the above

None of the vectors have a dot product with $\mathbf{v}$ that gives 0. Hence, no vector is orthogonal to $\mathbf{v}$.

3.) Consider the vectors: $\mathbf{x} = \begin{pmatrix} 1 \\ 4 \end{pmatrix}$ and $\mathbf{y} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$

Compute the length of the orthogonal projection of $\mathbf{y}$ onto $\mathbf{x}$; in other words, compute the value of d .

We know that $\mathbf{x} \cdot \mathbf{y} = \frac{||\mathbf{x}|| \cdot ||\mathbf{y}|| \cdot d}{||\mathbf{y}||} = d \cdot ||\mathbf{x}||$. Thus, $d = \frac{\mathbf{x} \cdot \mathbf{y}}{||\mathbf{x}||}$.

Here, $\mathbf{x} \cdot \mathbf{y} = (1 \cdot 3) + (4 \cdot 4) = 3 + 16 = 19$ and $||\mathbf{x}|| = \sqrt{1^2 + 4^2} = \sqrt{17}$.

Therefore $d = \frac{19}{\sqrt{17}}$.

4.) Consider the vectors: $\mathbf{x} = \begin{pmatrix} 1 \\ 4 \end{pmatrix}$ and $\mathbf{y} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$

Compute the projection of $\mathbf{x}$ onto $\mathbf{y}$.

It is given by $\mathbf{x} \cdot \mathbf{y} = d \cdot ||\mathbf{y}||$ or $d = \frac{\mathbf{x} \cdot \mathbf{y}}{||\mathbf{y}||}$.

Here again $\mathbf{x} \cdot \mathbf{y} = (1 \cdot 3) + (4 \cdot 4) = 3 + 16 = 19$.

The norm of $\mathbf{y}$ is given by $||\mathbf{y}|| = \sqrt{3^2 + 4^2} = \sqrt{25} = 5$. Therefore, $d = \frac{19}{5}$.

5.) Consider the zero vector: $\mathbf{0} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$. Notice that it has length 0.

a.) Is $\mathbf{0}$ orthogonal to the vector $\mathbf{v} = \begin{pmatrix} -2 \\ 3 \end{pmatrix}$?

- ☑ Yes
- ☐ No

Why?

Yes, because $\mathbf{0} \cdot \mathbf{v} = (0 \cdot (-2)) + (0 \cdot 3) = 0$.

b.) Consider any vector: $\mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$. Is $\mathbf{0}$ orthogonal to this vector?

- ☑ Yes
- ☐ No

Why?

Yes, because $\mathbf{0} \cdot \mathbf{x} = (0 \cdot x_1) + (0 \cdot x_2) = 0$.