EXERCISES

Module Four

1.) Consider the vectors $\mathbf{x} = \begin{pmatrix} 5 \\ 1 \end{pmatrix}$ and $\mathbf{y} = \begin{pmatrix} 1 \\ 6 \end{pmatrix}$.

Compute the length d of the projection of $\mathbf{x}$ onto $\mathbf{y}$ as well as the vector $\mathbf{proj}_{\mathbf{y}}(\mathbf{x})$.
Round your answers to two decimal places.

$$d = \frac{11}{\sqrt{37}} = 1.81 \quad \mathbf{proj}_{\mathbf{y}}(\mathbf{x}) = \begin{pmatrix} \frac{11}{37} \\ \frac{66}{37} \end{pmatrix} = \begin{pmatrix} 0.30 \\ 1.78 \end{pmatrix}$$

2.) Consider the vectors $\mathbf{x} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$ and $\mathbf{y} = \begin{pmatrix} -12 \\ 5 \end{pmatrix}$.

Compute the length d of the projection of $\mathbf{x}$ onto $\mathbf{y}$ as well as the vector $\mathbf{proj}_{\mathbf{y}}(\mathbf{x})$.
Round your answers to two decimal places.

$$d = \frac{-33}{13} = -2.54 \quad \mathbf{proj}_{\mathbf{y}}(\mathbf{x}) \begin{pmatrix} \frac{396}{169} \\ -\frac{165}{169} \end{pmatrix} = \begin{pmatrix} 2.34 \\ -0.98 \end{pmatrix}$$

3.) Consider the vectors $\mathbf{x} = \begin{pmatrix} 4 \\ -3 \end{pmatrix}$ and $\mathbf{y} = \begin{pmatrix} 6 \\ 8 \end{pmatrix}$.

Compute the length d of the projection of $\mathbf{x}$ onto $\mathbf{y}$ as well as the vector $\mathbf{proj}_{\mathbf{y}}(\mathbf{x})$.
Round your answers to two decimal places.

It turns out that these two vectors are orthogonal (the dot product is zero).
Therefore, $d = 0$ and $\mathbf{proj}_{\mathbf{y}}(\mathbf{x}) = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$.

4.) Consider the line $\mathbf{w} \cdot \mathbf{x} = b$ given by $\mathbf{w} = \begin{pmatrix} -2 \\ 5 \end{pmatrix}$ and $b = 0$.

a.) Does this line pass through the origin? Explain.

Yes, because $b = 0$.

b.) Consider the points $P_1 = (3, -1)$ and $P_2 = (-5, 1)$.
Compute the distance between the two points and the line.

For P_1: $\mathbf{w} \cdot \mathbf{p}_1 = -11$ and $||\mathbf{w}|| = \sqrt{29}$. Thus, the distance is $\frac{-11}{\sqrt{29}} = -2.04$.

For P_2: $\mathbf{w} \cdot \mathbf{p}_2 = 15$ and $||\mathbf{w}|| = \sqrt{29}$. Thus, the distance is $\frac{15}{\sqrt{29}} = 2.79$.

5.) Consider the line $\mathbf{w} \cdot \mathbf{x} = b$ given by $\mathbf{w} = \begin{pmatrix} 15 \\ 8 \end{pmatrix}$ and $b = -16$.

a.) Does this line pass through the origin? Explain.

No, because $b \neq 0$.

b.) Consider the points $P_1 = (-5, 0)$, $P_2 = (1, 1)$, and $P_3 = (0, -2)$.
Compute the distance between the three points and the line.

For P_1: $\mathbf{w} \cdot \mathbf{p}_1 = -75$, $b = -16$, and $||\mathbf{w}|| = 17$. Thus, $d_1 = \dfrac{-75 - (-16)}{17} = \dfrac{-59}{17} = -3.47$.

For P_2: $\mathbf{w} \cdot \mathbf{p}_2 = 23$, $b = -16$, and $||\mathbf{w}|| = 17$. Thus, $d_2 = \dfrac{23 - (-16)}{17} = \dfrac{39}{17} = 2.29$.

For P_3: $\mathbf{w} \cdot \mathbf{p}_3 = -16$, $b = -16$, and $||\mathbf{w}|| = 17$. Thus, $d_3 = \dfrac{-16 - (-16)}{17} = \dfrac{0}{17} = 0$.