SOLUTIONS

Module Three

1.) Consider the line defined by $\mathbf{w} \cdot \mathbf{x} = b$ where $\mathbf{w} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$ and $b = 12$.

a.) Does the line go through the origin? Explain.

- ☐ Yes
- ☑ No

Because $b \neq 0$

b.) Find two possible points on the line.

There are infinitely many correct answers. The condition for $P = (p_1, p_2)$ to be on the line is that $\mathbf{w} \cdot \mathbf{p} = 3p_1 + 4p_2 = 12$.
For example, the points $P = (4, 0)$ and $Q = (0, 3)$ are on the line.

c.) Are the points $P_1 = (-1, 4)$ and $P_2 = (1, 4)$ on the same side of that line?

We have $\mathbf{w} \cdot \mathbf{p}_1 = 3 \cdot (-1) + 4 \cdot 4 = -3 + 16 = 13$ and $\mathbf{w} \cdot \mathbf{p}_2 = 3 \cdot 1 + 4 \cdot 4 = 3 + 16 = 19$.

Since both $\mathbf{w} \cdot \mathbf{p}_1 > 12$ and $\mathbf{w} \cdot \mathbf{p}_2 > 12$, the two points are on the same side of the line.

2.) Consider the line $\mathbf{w} \cdot \mathbf{x} = b$ for $\mathbf{w} = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$ and $b = -4$.

Are $P_1 = (5, -5)$ and $P_2 = (0, -1)$ on the same side of the line?

We have $\mathbf{w} \cdot \mathbf{p}_1 = (-1) \cdot 5 + 2 \cdot (-5) = -5 - 10 = -15$ and $\mathbf{w} \cdot \mathbf{p}_2 = (-1) \cdot 0 + 2 \cdot -1 = -2$.
Since $\mathbf{w} \cdot \mathbf{p}_1 < -4$ and $\mathbf{w} \cdot \mathbf{p}_2 > -4$, the two points are not on the same side of the line.

3.) Consider a vector $\mathbf{v}$ whose length is 3. Compute $\mathbf{v} \cdot \mathbf{v}$.

Remember that $\mathbf{v} \cdot \mathbf{v} = ||\mathbf{v}||^2$. Therefore, $\mathbf{v} \cdot \mathbf{v} = 3^2 = 9$.