Estimating Probability Distributions ›

Derivation of Naive Bayes Classifier

The Naive Bayes classifier decomposes a single high dimensional probability estimation problem into many 1-dimensional estimation problems by assuming the features are conditionally independent given the class label. We want to estimate $\hat{\mathrm{P}}(y|\mathbf{x})$ with the Naive Bayes classifier. To do so, we will make use of **Bayes' Rule and the Naive Bayes assumption**.

☆ **Key Points**

First you need to calculate the probabilities of the labels given the observed data, using Bayes' Rule.

Then, you choose y such that this probability is maximized, considering each dimension of the feature vector independently.

Breaking Down Naive Bayes

You can estimate $\hat{\mathrm{P}}(y|\mathbf{x})$ by calculating $\mathrm{P}(y)$ and $\mathrm{P}(\mathbf{x}|y)$, since by Bayes' Rule:

$$\mathrm{P}(y|\mathbf{x}) = \frac{\mathrm{P}(\mathbf{x}|y)\mathrm{P}(y)}{\mathrm{P}(\mathbf{x})}$$

Estimating $\mathrm{P}(y = c)$ (the probability that y takes on some value c) is simple - it is just the fraction of data points where $y = c$.

We can write this formally using an indicator variable I , which is 1 when the argument holds and 0 when it doesn't. If y takes on discrete binary values, such as 0 or 1, it simply counts how many times we observe each outcome:

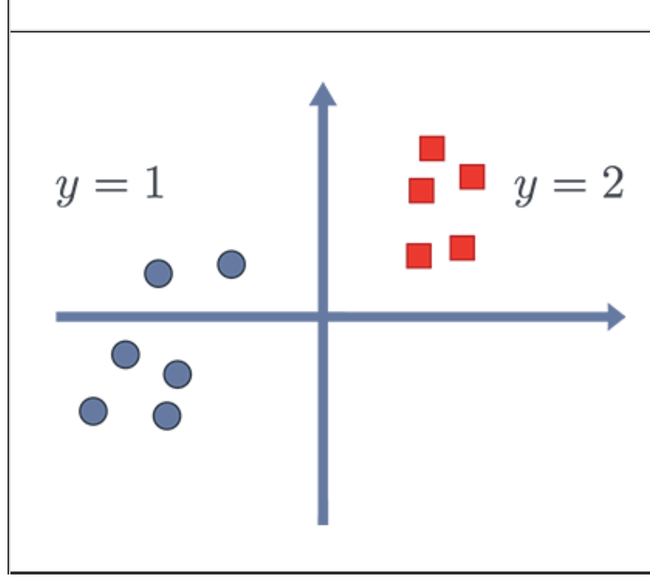
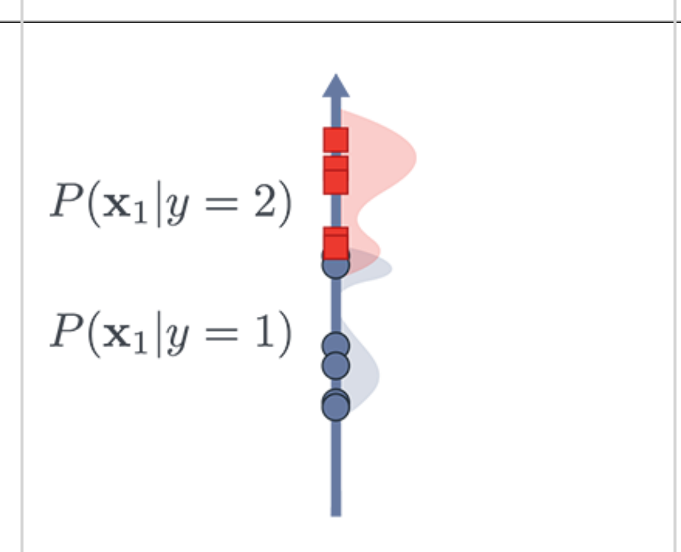
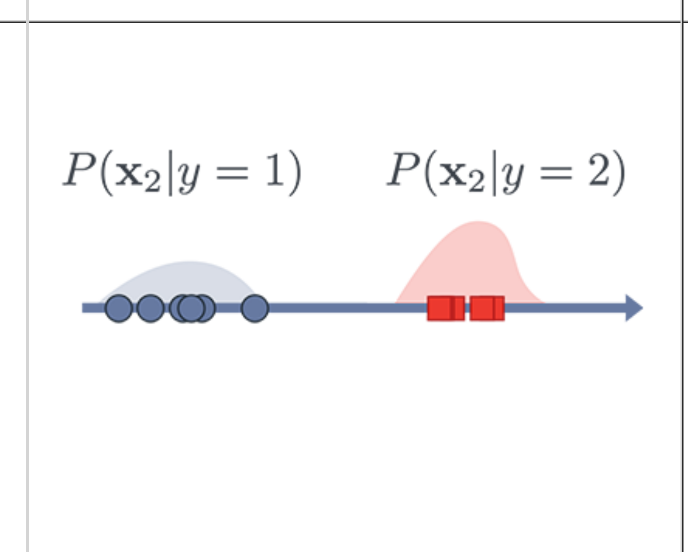
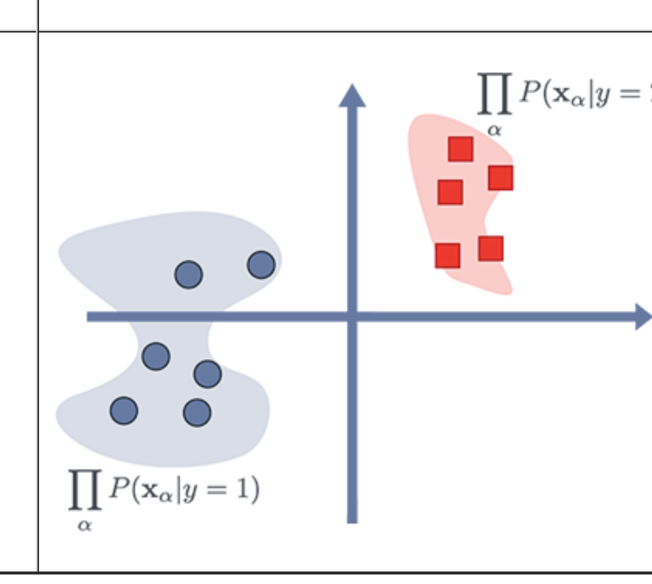
$$\mathrm{P}(y = c) = \frac{\sum_{i=1}^{n} I(y_i = c)}{n} = \hat{\pi}_c$$

Estimating $\mathrm{P}(\mathbf{x}|y)$, however, is not so simple. To do so, you will make use of the Naive Bayes assumption from earlier:

$$\mathrm{P}(\mathbf{x}|y) = \prod_{\alpha=1}^{d} \mathrm{P}(x_\alpha|y)$$

where $x_\alpha = [\mathbf{x}]_\alpha$ is the value for feature α.

Below, you'll see an illustration of how the Naive Bayes algorithm decomposes a multi-dimensional probability estimation problem (first image) into many 1-dimensional estimation problems. First, you estimate $\mathrm{P}(x_\alpha|y)$ independently in each dimension (center two images) and then obtain an estimate of the full data distribution by assuming conditional independence $\mathrm{P}(\mathbf{x}|y) = \prod_\alpha \mathrm{P}(x_\alpha|y)$ (rightmost image).

Original	Estimation of first dimension	Estimation of second dimension	Resulting data distribution
			

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Now, we can begin our derivation of the Naive Bayes classifier as follows:

$$\begin{aligned}
h(\mathbf{x}) &= \arg\max_y \mathrm{P}(y|\mathbf{x}) \\
&= \arg\max_y \frac{\mathrm{P}(\mathbf{x}|y)\mathrm{P}(y)}{\mathrm{P}(\mathbf{x})} && \text{(Bayes rule)} \\
&= \arg\max_y \mathrm{P}(\mathbf{x}|y)\mathrm{P}(y) && (\mathrm{P}(\mathbf{x}) \text{ does not depend on } y) \\
&= \arg\max_y \prod_{\alpha=1}^{d} \mathrm{P}(x_\alpha|y)\mathrm{P}(y) && \text{(by the Naive Bayes assumption)} \\
&= \arg\max_y \sum_{\alpha=1}^{d} \log(\mathrm{P}(x_\alpha|y)) + \log(\mathrm{P}(y)) && \text{(as log is a monotonic function)}
\end{aligned}$$

Estimating $\mathrm{P}(x_\alpha|y)$ is easy since we only need to consider one single dimension, so MLE should work well. Estimating $\mathrm{P}(y)$ is not affected by the assumption and can similarly done with straight-forward MLE.

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